# Numerical Method for ODE's

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# Why use Numerical Methods

- So far in our course we have been solving Ordinary Differential Equations **analytically** i.e. we have been finding **exact** solutions.
- But a lot of the problems we solve may not have analytical solutions or might be too difficult to solve analytically.
- And a lot of the time, the problems we are trying to solve will be too complicated and computers will be employed to help us solve the problem.
- In both of these an many other cases, Numerical Methods are employed
- **Note:** Numerical methods do not give exact **solutions** but **approximations**

# Euler's Method

- The method we will be discussing today is Euler's Method

**Euler's Method Algorithm:**

Let:

$$\frac{dy}{dt} = f(y,t)$$

be an ODE with solution $y(t)$ on the interval $[a,b]$ with initial value $y(a) = y(t_0) = y_0$

Let $t_k = t_{k-1} + h \Rightarrow t_k = t_0 + kh$ where

$$h = \frac{b-a}{n}$$

and n is the number of data points. Then

$$\boldsymbol{y_k = y_{k-1} + f(t_{k-1}, y_{k-1})h}$$

where $y_k \approx y(t_k)$ for $t_k$ where $k \in \{1, \ldots, n\}$.

- The key **mathematical idea** is:

> At each step the approximation of the graph of the unknown solution $y(t)$ is done by the **tangent line**.

- In this presentation I will show you the algorithm in play by solving the following initial value problem (IVP):

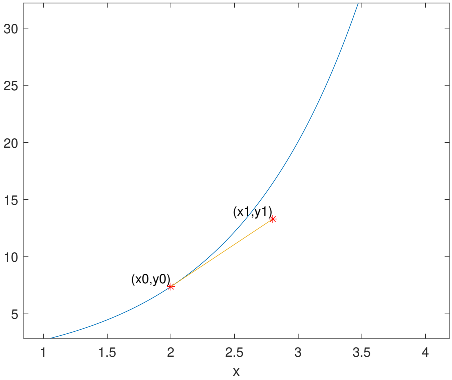
$$\frac{dy}{dx} = e^x \quad \text{on the interval} \quad [2, 3]$$

- **Note:** This example was only chosen for simplicity. I could have chosen many other examples.
- The exact solution to this IVP (not surprisingly) is:

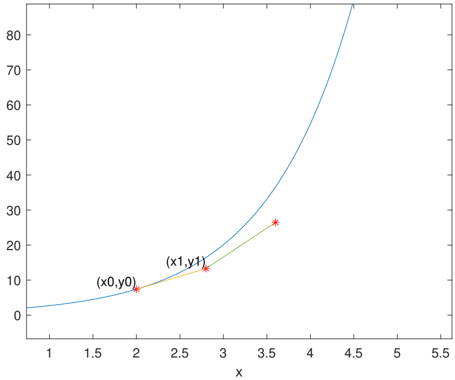
$$y = e^x$$

# Euler's Method in action!!!!!!!

- First I will show you how it works for just two **2 data points** 1 iteration at a time:
    1. Iteration 1:



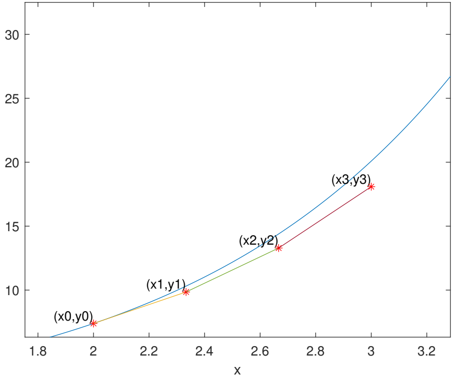
- The second iteration:
  2. Iteration 2:



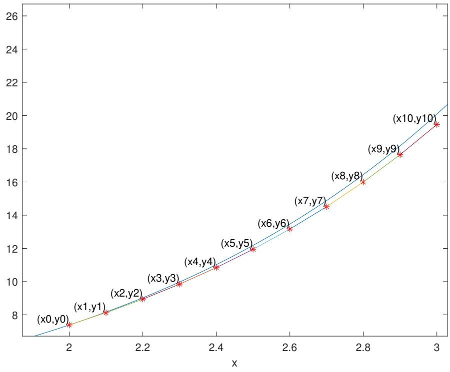
# Wait a minute!, that is not a good approximation...

Well as you can see the approximation does not look very good. But lets see what happens when we increase the number of data points

- When n = 3, i.e. We use 3 data points:



- When n = 10, i.e. We use 3 data points:



- When n = 100, i.e. We use 3 data points: