

Theorem

For any $x, y \in \mathbb{R}$,

$$\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}$$

Proof: Without loss of generality, let x20, y20

$$\frac{|x+y|}{|+|x+y|} = \frac{x+y}{|+x+y|} = \frac{y}{|+x+y|}$$

$$\leq x + y$$

$$1 + x + y$$

$$= \frac{|\alpha|}{1+|\alpha|} + \frac{|y|}{1+|y|}$$

Theorem, Hölder's Inequality

Let xi≥0 and yi≥0 \ i=1,2,...,n and suppose that p>1 and q>1 such that

$$\frac{1}{p} + \frac{1}{q} = 1$$
.

Then

$$\sum_{i=1}^{n} x_i y_i \leq \left(\sum_{i=1}^{n} x_i^{p}\right)^{1/p} \left(\sum_{i=1}^{n} y_i^{q}\right)^{1/q}$$

When, p=q=2, we get Cauchy-Schwartz inequality

$$\sum_{i=1}^{n} x_{i} y_{i} \leq \left(\sum_{i=1}^{n} x_{i}^{2}\right)^{1/2} \left(\sum_{i=1}^{n} y_{i}^{2}\right)^{1/2}$$

Proof:

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	$\frac{1}{p} + \frac{1}{q}$	=1.				
	1 1					
Then			1/0 / 10	. 1/.		
	n	(N P)	$\int_{i=1}^{n} y^{i}$	9/9		
	\geq $\alpha_{i}y_{i}$			• /		
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When p=q=2, we	Jet Cauc	My - OCHWAY	LZ INEQUAL			
		-	1/0 / n	\1/2		
	71	/N 2	$\int_{1}^{1/2} \left(\sum_{i=1}^{n} y_{i} \right)^{1/2} $	2/2		
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Theorem: Minkowski	They wall to					
Let x;≥0 and	y; ≥0 -	for $i=1.2$	2. n ar	od suppose.	that p>1	. Then
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