Midterm2, Math 3012 QHS, Fall 2020

Instructor: Dr. Su
Please administer on 10/22/2020 at 8:00 am.
Students should have 12 hours to type and submit the solutions.

PLEASE DO NOT PHOTOCOPY THIS EXAM

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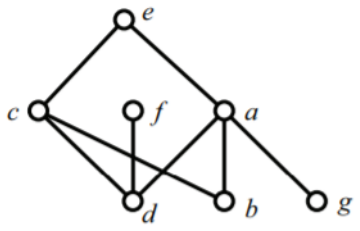
Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise. Solutions to the exam will give an idea of how much to writing is needed.
- You will have 12 hours to take the exam, type your solutions and submit.
- This is take-home exam. Meaning that this exam will be open book: you can use any resources (including online calculators and Mathematica) available to them to answer the questions that are given. cannot communicate with anyone during these tests including using Reddit or online message boards or using solutions provided from another student or third party.
- You can ask the instructor questions during the exam via email or through Canvas messaging. Piazza will be temporarily inactive during the exam.
- A small amount points may be allocated for organization and following instructions during the upload process. Please indicate where questions are located and rotate pages to the proper orientation.

1. (20 points) Shown below is the diagram of an interval order. Use the algorithm taught in class to find an interval representation by computing the down-sets and up-sets in the space provided. Then use the First Fit coloring algorithm to find the width ω and a partition of the poset into ω chains.



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$D(b) =$ $U(b) =$
$D(c) =$ $U(c) =$
$D(d) =$ $U(d) =$
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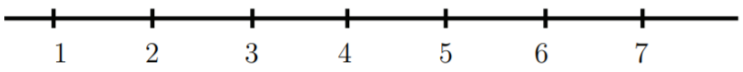
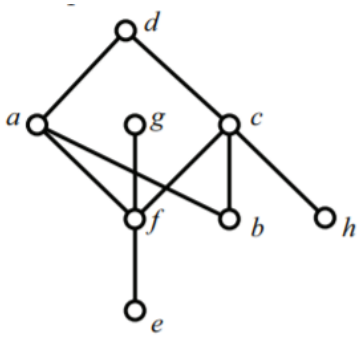


2. (20 points) Consider the poset shown below. The ground set is

$$X = \{a, b, c, d, e, f, g, h\}.$$

Write the reflexive, antisymmetric and transitive relation on X which defines this poset.



3. (20 points) Let 2^{15} be the poset consisting of all subsets of $\{1, 2, 3, ..., 15\}$, ordered by inclusion.
   (a) (5 points) What is the height of this poset?
   (b) (5 points) What is the width of this poset?
   (c) (5 points) How many maximal chains does the poset have?
   (d) (5 points) How many maximal chains in this poset pass through the set $\{2, 3, 8, 13\}$?

4. (20 points) (a) (10 points) Let $X = \{a, b, c, d, e\}$ and let

$$P = \{(a, a), (b, b), (c, c), (d, d), (e, e), (d, b), (d, c), (a, e), (a, b), (e, b)\}.$$

Draw a diagram for the poset (X, P).

(b) (10 points) How many symmetric binary relations are there on $\{1, 2, ..., n\}$? Of these how many are reflexive?

5. (20 points) Verify Euler's formula for this planar graph.