Final Exam Part B, Math 3012 QHS, Fall 2020

Instructor: Dr. Su
Please administer on 12/04/2020 at 8:00 am.
Students should have 12 hours to type and submit the solutions.

PLEASE DO NOT PHOTOCOPY THIS EXAM

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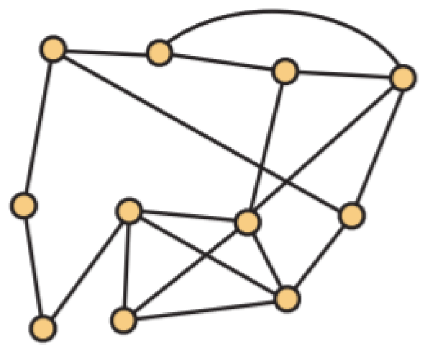
Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise. Solutions to the exam will give an idea of how much to writing is needed.
- You will have 12 hours to take the exam, type your solutions and submit.
- This is take-home exam. Meaning that this exam will be open book: you can use any resources (including online calculators and Mathematica) available to them to answer the questions that are given. cannot communicate with anyone during these tests including using Reddit or online message boards or using solutions provided from another student or third party.
- You can ask the instructor questions during the exam via email or through Canvas messaging. Piazza will be temporarily inactive during the exam.
- A small amount points may be allocated for organization and following instructions during the upload process. Please indicate where questions are located and rotate pages to the proper orientation.

1. (20 points) Consider the graph G shown below:



(a) (10 points) Find $\omega(G)$ for this graph, Show that $\chi(G) = \omega(G)$ by providing a proper coloring of G. You may indicate your coloring by writing directly on the figure.

(b) (10 points) Explain why the graph G is not perfect.

2. (20 points) Using induction, prove that for every positive integer n,

$$1 + 2 + ... + n = \frac{n(n+1)}{2}$$

3. (20 points) (a) (10 points) Find the general solution to the advancement operator equation:

$$A^4(A-5+4i)^3(A-1)^2(A+8)(A-9)f=0$$

(b) (10 points) Find the solution to the advancement operator equation:

$$(A^2-11A+28)f(n)=0, f(0)=-2 \text{ and } f(1)=1.$$

4. (20 points) In a Bernoulli trial set up, there are three outcomes ξ_1, ξ_2 and ξ_3 with probabilities p_1, p_2 and p_3, respectively. The trial is repeated until either ξ_1 occurs (this is a win) or ξ_2 occurs (this is a loss). As long as ξ_3 occurs, the trial is repeated. What is the probability of a win?

5. (20 points) (a) (10 points) Write the inclusion/exclusion formula for the number of onto functions from $\{1, 2, ..., n\}$ to $\{1, 2, ..., m\}$.

   (b) (10 points) Evaluate your formula when $n = 6$ and $m = 3$.