Midterm3, Math 3012 QHS, Fall 2020

Instructor: Dr. Su
Please administer on 11/12/2020 at 8:00 am.
Students should have 12 hours to type and submit the solutions.

PLEASE DO NOT PHOTOCOPY THIS EXAM

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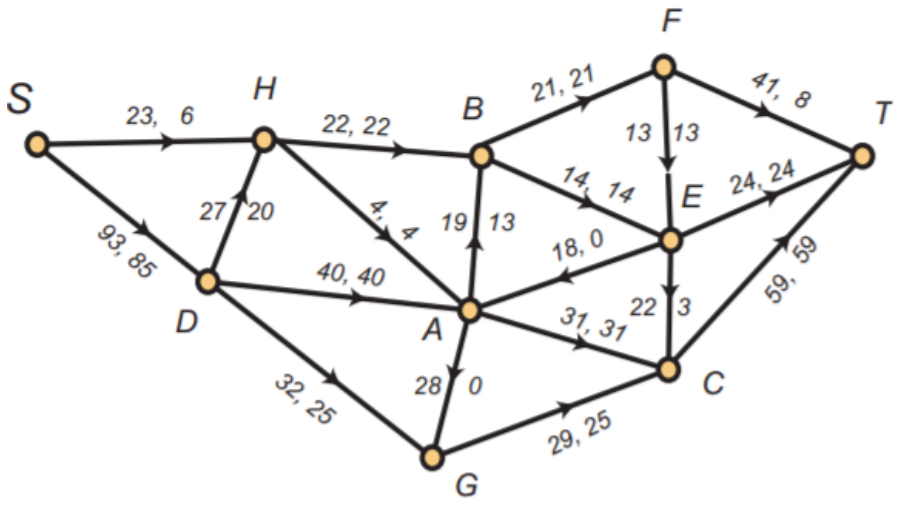
Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise. Solutions to the exam will give an idea of how much to writing is needed.
- You will have 12 hours to take the exam, type your solutions and submit.
- This is take-home exam. Meaning that this exam will be open book: you can use any resources (including online calculators and Mathematica) available to them to answer the questions that are given. cannot communicate with anyone during these tests including using Reddit or online message boards or using solutions provided from another student or third party.
- You can ask the instructor questions during the exam via email or through Canvas messaging. Piazza will be temporarily inactive during the exam.
- A small amount points may be allocated for organization and following instructions during the upload process. Please indicate where questions are located and rotate pages to the proper orientation.

1. (20 points) Write the inclusion formula for the number d_n of derangements of $\{1, 2, ..., n\}$. Then use this formula to calculate d_6.

2. (20 points) Write the inclusion-exclusion formula for $S(n, m)$, the number of surjections from $\{1, 2, ..., n\}$ to $\{1, 2, ..., m\}$. Then use this formula to calculate $S(6, 4)$.

3. (20 points) Consider the following network flow:



(a) (7 points) The value of the current flow is:

(b) (7 points) The capacity of the cut $\{S, A, D, G, H\} \cup \{B, C, E, F, T\}$ is:

(c) (6 points) Write below the labels that are applied by carrying out the Ford-Fulkerson labeling algorithm.

4. (20 points) (a) (10 points) Find a particular solution to the advancement operator equation:

$$(A^2 - 3A + 5)f = 4 \cdot 3^n.$$

(b) (10 points) Write the general solution to the homogeneous advancement operator equation:

$$[A - (7 - 2i)]^3(A - 1)^4 f = 0.$$

5. (20 points) Note that $1800 = 25 \cdot 9 \cdot 8$. Use this information and the inclusion-exclusion formula to determine $\phi(1800)$, where ϕ is the Euler ϕ-function studied in class.