Midterm1, Math 3012 QHS, Fall 2020

Instructor: Dr. Su
Please administer on 09/24/2020 at 8:00 am.
Students should have 12 hours to type and submit the solutions.

PLEASE DO NOT PHOTOCOPY THIS EXAM

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Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise. Solutions to the exam will give an idea of how much to writing is needed.
- You will have 12 hours to take the exam, type your solutions and submit.
- This is take-home exam. Meaning that this exam will be open book: you can use any resources (including online calculators and Mathematica) available to them to answer the questions that are given. cannot communicate with anyone during these tests including using Reddit or online message boards or using solutions provided from another student or third party.
- You can ask the instructor questions during the exam via email or through Canvas messaging. Piazza will be temporarily inactive during the exam.
- A small amount points may be allocated for organization and following instructions during the upload process. Please indicate where questions are located and rotate pages to the proper orientation.

1. (20 points) (a) (10 points) Use the Euclidean algorithm to find $d = \gcd(420, 245)$.

(b) (10 points) Use your work in the preceding problem to find integers a and b so that d = 420a + 245b.

2. (20 points) How many integer valued solutions to the following equations and inequalities:

   (a) (10 points) $x_1 + x_2 + x_3 = 42, x_1, x_2, x_3 > 0$.

   (b) (10 points) $x_1 + x_2 + x_3 = 42, x_1, x_3 > 0, 0 < x_2 \leq 6$.

3. (20 points) (a) (10 points) Find the coefficient of $x^4y^7z^{24}$ in $(6x - 5y + 8z^2)^{23}$,
(b) (10 points) Find the coefficient of $x^4y^7z^{21}$ in $(6x - 5y + 8z^2)^{23}$.

4. (20 points) How many lattice paths from $(0, 0)$ to $(24, 31)$ do not pass through $(15, 19)$

5. (20 points) For a positive integer n, let t_n count the number of ternary strings of length n that do not contain 102 as a substring. Note that $t_1 = 3, t_2 = 9$ and $t_3 = 26$. Develop a recurrence relation for tn and use it to compute t_4, t_5 and t_6.