

Final Exam Part A, Math 3012 QHS, Fall 2020

Instructor: Dr. Su

Please administer on 12/04/2020 at 8:00 am.

Students should have 12 hours to type and submit the solutions.

PLEASE DO NOT PHOTOCOPY THIS EXAM

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Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise. Solutions to the exam will give an idea of how much to writing is needed.
- You will have 12 hours to take the exam, type your solutions and submit.
- This is take-home exam. Meaning that this exam will be open book: you can use any resources (including online calculators and Mathematica) available to them to answer the questions that are given. cannot communicate with anyone during these tests including using Reddit or online message boards or using solutions provided from another student or third party.
- You can ask the instructor questions during the exam via email or through Canvas messaging. Piazza will be temporarily inactive during the exam.
- A small amount points may be allocated for organization and following instructions during the upload process. Please indicate where questions are located and rotate pages to the proper orientation.

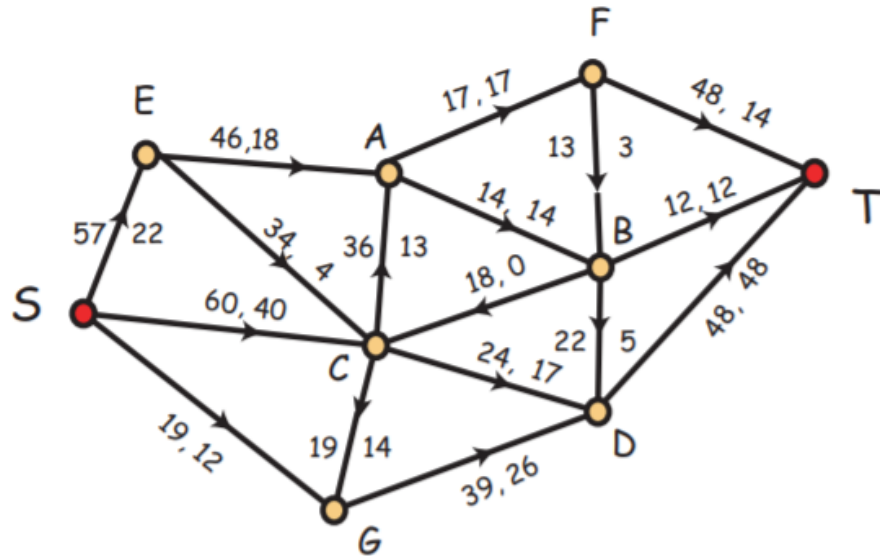
1. (20 points) (a) (10 points) Use the Euclidean algorithm to find $d = \gcd(115, 161)$.

(b) (10 points) Use your work in the preceding problem to find integers a and b so that $d = 115a + 161b$.

2. (20 points) (a) (10 points) For a positive integer n , let q_n count the number of quaternary strings (alphabet = $\{0, 1, 2, 3\}$) that do not contain either 11 or 002 as substrings of consecutive characters. Find initial conditions by specifying q_1 , q_2 and q_3 .

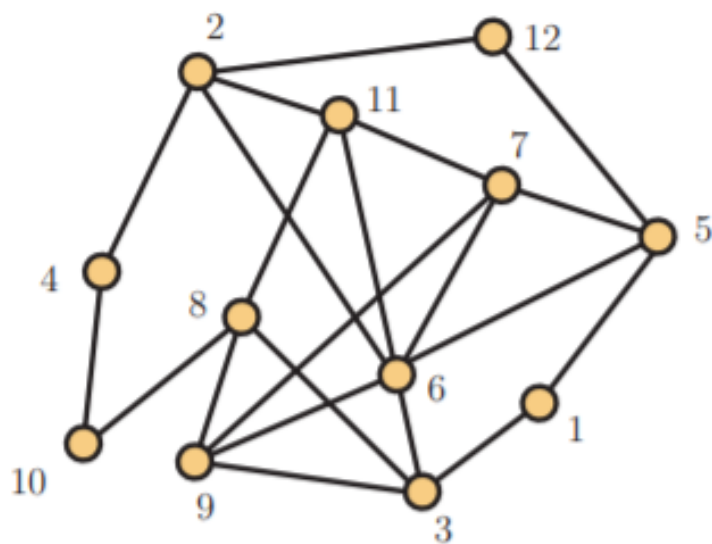
- (b) (10 points) Develop a recurrence for q_n when $n \geq 4$ and use it to compute q_4 and q_5 .

3. (20 points) Consider the following network flow:



- (a) (7 points) The value of the current flow is:
- (b) (7 points) The capacity of the cut $\{S, B, D, E, G\} \cup \{A, C, F, T\}$ is:
- (c) (6 points) Carry out the labeling algorithm, using the pseudo-alphabetic order on the vertices and list below the labels which will be given to the vertices.

4. (20 points) Show that the graph below is Hamiltonian. You may give your answer by darkening appropriate edges on the figure, or by giving an appropriate permutation of the vertex set.



5. (20 points) Is the binary relation

$$P = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 3), (2, 4), (3, 4)\}$$

a partial order on the set $X = \{1, 2, 3, 4\}$? If so, discuss what properties you verified and how. If not, list the ordered pairs that must be added to P to make it a partial order, in case it is possible.