Final Exam MakeUp Part A, Math 2552

Instructions (PLEASE READ)

Formatting and Timing

- **Show your work** and justify your answers for all questions unless stated otherwise.
- Please write neatly, and use dark and clear writing so that the scan is easy to read.
- Please write your name or initials at the top of every page
- Please solve the questions in the exam in the order they are given.
- You do not need to print the exam. As long as you solve problems in the order they are given (just like the written homework sets), you can write your answers on your own paper. But students can print the exam and write their answers on the printed copy if they prefer.

Submission

- Students should scan their work and submit it through Gradescope. There should be an **assignment** in Gradescope for this exam. The process for submitting your work will be similar to what you have used for homework.
- Work must be submitted by May 7, 8:00 pm, ET.
- Please upload your work as a single PDF file. If this is not possible you can email your work to your instructor.
- During the upload process in Gradescope, please indicate which page of your work corresponds to each question in the exam.

Questions

- If there are questions during the exam, students can email their instructor or message them through Canvas.
- Our course Piazza forum will be temporarily inactive during the exam.
- If you run into any technical issues or any unanticipated emergencies, please email your instructor as soon as you can.

Integrity

- Students can use any resources while taking these tests including online calculators and Mathematica
- Students cannot communicate with anyone during these tests.
- Students cannot use solutions provided from another student or third party.
- In other words: do your own work but you can use technology to solve problems.

1. (4 points) Solve the IVP.

$$y' = 9t^2 y^2, \quad y(0) = \frac{1}{2}$$

2. (10 points) Solve the differential equation using the method of undetermined coefficients.

$$y'' - 5y' + 4y = 68\cos t - 12e^t$$

3. (10 points) Consider the homogeneous linear system of differential equations.

$$\vec{x}' = A\vec{x} = \begin{pmatrix} 5 & \frac{1}{2} \\ -\frac{1}{2} & 4 \end{pmatrix} \vec{x}, \quad \vec{x} = \vec{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

(a) (2 points) Determine the eigenvalues of A. Show your work.

(b) (5 points) Express the general solution of the system in terms of real valued functions.

(c) (3 points) Sketch the phase portrait of the system. Please include the eigenspaces in your sketch, indicate the direction of motion of your solution curves and eigenspaces, and do not forget to label your axes.

4. (10 points) Consider the differential equation $\dfrac{dy}{dt} = (y+2)(y-3)$, where y is a real function of t, and $t \geq 0$, and $k > 0$.

   (a) State the critical points of the differential equation.

   (b) Draw the phase line, and determine whether the critical points (if any) are stable, semi-stable, or unstable. Show your work.

   (c) Determine where y is concave up and concave down for $y \in \mathbb{R}$. Show your work.

   (d) Using results from parts (a), (b), and (c) to sketch several solution curves in the ty-plane for $t \geq 0$.

5. (5 points) The position of a moving object, $y(t)$, for time $t \geq 0$ satisfies the IVP

$$y'' + y' - 2y = 0, \quad y = y(t)$$

(a) Express the differential equation in the IVP as a first-order system in the form $\vec{x}' = A\vec{x}$.

(b) Solve the DE using any method you like. Show your work.

(c) Sketch the trajectory of the object for $t \geq 0$ in the phase plane. Please include the eigenspaces of A in your sketch, indicate the direction of motion on your solution curves and eigenspaces, and do not forget to label your axes.

6. (1 point) One point will be allocated for presentation, neatness, and organization. Please ensure that

   1. your work is legible in the scan

   2. your name or initials are at the top of every page

   3. questions are answered in the order in which they were given

   4. during the upload process you have indicated which pages correspond to which question, and made sure that none of your pages are upside down or sideways (you can also change the orientation of the pages when you upload in Gradescope)

   Ensuring that these criteria are met helps ensure that your exam is graded efficiently and accurately.

Please sign and date the following GT Honor Code statement.

Georgia Tech Honor Code
Having read the Georgia Institute of Technology Academic Honor Code, I understand and accept my responsibility as a member of the Georgia Tech community to uphold the Honor Code at all times.

_____   _____

signature   date