

# Math 2552 Written HW Set 7

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**Trench 3.1.7.** Use Euler's method with step sizes  $h = 0.1$ ,  $h = 0.05$ , and  $h = 0.025$  to find approximate values of the solution of the initial value problem

$$y' + \frac{2}{x}y = \frac{3}{x^3} + 1, \quad y(1) = 1$$

at  $x = 1.0, 1.1, 1.2, 1.3, \dots, 2.0$ . Compare these approximate values with the values of the exact solution

$$y = \frac{1}{3x^2}(9 \ln x + x^3 + 2),$$

which can be obtained by the method of Section 2.1. Present your results in a table like Table 3.1.1.

| $x$ | $h = 0.1$   | $h = 0.05$  | $h = 0.025$ | Exact       |
|-----|-------------|-------------|-------------|-------------|
| 1.0 | 1.000000000 | 1.000000000 | 1.000000000 | 1.000000000 |
| 1.1 | 1.200000000 | 1.174813735 | 1.163906015 | 1.153937085 |
| 1.2 | 1.307212622 | 1.272318906 | 1.256964040 | 1.242799540 |
| 1.3 | 1.362954963 | 1.325650055 | 1.309013781 | 1.293546032 |
| 1.4 | 1.389819425 | 1.353514366 | 1.337139286 | 1.321811247 |
| 1.5 | 1.400603239 | 1.366805485 | 1.351411241 | 1.336916440 |
| 1.6 | 1.402745029 | 1.372014729 | 1.357897143 | 1.344535503 |
| 1.7 | 1.400644088 | 1.373071526 | 1.360308069 | 1.348172348 |
| 1.8 | 1.396924918 | 1.372374545 | 1.360932547 | 1.350008229 |
| 1.9 | 1.393151367 | 1.371388136 | 1.361182553 | 1.351402121 |
| 2.0 | 1.390242009 | 1.370996758 | 1.361921132 | 1.353193719 |

**Trench 3.2.7.** Use the improved Euler method with step sizes  $h = 0.1, h = 0.05$ , and  $h = 0.025$  to find approximate values of the solution of the initial value problem

$$y' + \frac{2}{x}y = \frac{3}{x^3} + 1, \quad y(1) = 1$$

at  $x = 1.0, 1.1, 1.2, 1.3, \dots, 2.0$ . Compare these approximate values with the values of the exact solution

$$y = \frac{1}{3x^2}(9 \ln x + x^3 + 2),$$

which can be obtained by the method of Section 2.1. Present your results in a table like Table 3.2.2.

| $x$ | $h = 0.1$   | $h = 0.05$  | $h = 0.025$ | $h = 0.1$      | $h = 0.05$  | $h = 0.025$ | Exact       |
|-----|-------------|-------------|-------------|----------------|-------------|-------------|-------------|
| 1.0 | 1.000000000 | 1.000000000 | 1.000000000 | 1.000000000    | 1.000000000 | 1.000000000 | 1.000000000 |
| 1.1 | 1.200000000 | 1.174813735 | 1.163906015 | 1.153606311    | 1.153879293 | 1.153925558 | 1.153937085 |
| 1.2 | 1.307212622 | 1.272318906 | 1.256964040 | 1.242464607    | 1.242748907 | 1.242790763 | 1.242799540 |
| 1.3 | 1.362954963 | 1.325650055 | 1.309013781 | 1.293314190    | 1.293522398 | 1.293544147 | 1.293546032 |
| 1.4 | 1.389819425 | 1.353514366 | 1.337139286 | 1.321704368    | 1.321817246 | 1.321816580 | 1.321811247 |
| 1.5 | 1.400603239 | 1.366805485 | 1.351411241 | 1.336924820    | 1.336948622 | 1.336928009 | 1.336916440 |
| 1.6 | 1.402745029 | 1.372014729 | 1.357897143 | 1.344639720    | 1.344588782 | 1.344552009 | 1.344535503 |
| 1.7 | 1.400644088 | 1.373071526 | 1.360308069 | 1.348351961    | 1.348241722 | 1.348192555 | 1.348172348 |
| 1.8 | 1.396924918 | 1.372374545 | 1.360932547 | 1.350244948    | 1.350089364 | 1.350031080 | 1.350008229 |
| 1.9 | 1.393151367 | 1.371388136 | 1.361182553 | 1.351680683    | 1.351491477 | 1.351426765 | 1.351402121 |
| 2.0 | 1.390242009 | 1.370996758 | 1.361921132 | 1.353501839    | 1.353288493 | 1.353219485 | 1.353193719 |
|     | Euler       |             |             | Improved Euler |             |             | Exact       |

**Trench 3.3.7.** Use the Runge-Kutta method with step sizes  $h = 0.1, h = 0.05$ , and  $h = 0.025$  to find approximate values of the solution of the initial value problem

$$y' + \frac{2}{x}y = \frac{3}{x^3} + 1, \quad y(1) = 1$$

at  $x = 1.0, 1.1, 1.2, 1.3, \dots, 2.0$ . Compare these approximate values with the values of the exact solution

$$y = \frac{1}{3x^2}(9 \ln x + x^3 + 2),$$

which can be obtained by the method of Section 2.1. Present your results in a table like Table 3.3.1.

| $x$ | $h = 0.1$      | $h = 0.05$  | $h = 0.025$ | $h = 0.1$   | $h = 0.05$  | $h = 0.025$ | Exact       |
|-----|----------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1.0 | 1.000000000    | 1.000000000 | 1.000000000 | 1.000000000 | 1.000000000 | 1.000000000 | 1.000000000 |
| 1.1 | 1.153606311    | 1.153879293 | 1.153925558 | 1.153933943 | 1.153936905 | 1.153937074 | 1.153937085 |
| 1.2 | 1.242464607    | 1.242748907 | 1.242790763 | 1.242795524 | 1.242799309 | 1.242799526 | 1.242799540 |
| 1.3 | 1.293314190    | 1.293522398 | 1.293544147 | 1.293541986 | 1.293545800 | 1.293546018 | 1.293546032 |
| 1.4 | 1.321704368    | 1.321817246 | 1.321816580 | 1.321807471 | 1.321811030 | 1.321811234 | 1.321811247 |
| 1.5 | 1.336924820    | 1.336948622 | 1.336928009 | 1.336913020 | 1.336916244 | 1.336916429 | 1.336916440 |
| 1.6 | 1.344639720    | 1.344588782 | 1.344552009 | 1.344532439 | 1.344535327 | 1.344535493 | 1.344535503 |
| 1.7 | 1.348351961    | 1.348241722 | 1.348192555 | 1.348169612 | 1.348172191 | 1.348172339 | 1.348172348 |
| 1.8 | 1.350244948    | 1.350089364 | 1.350031080 | 1.350005783 | 1.350008089 | 1.350008220 | 1.350008229 |
| 1.9 | 1.351680683    | 1.351491477 | 1.351426765 | 1.351399928 | 1.351401996 | 1.351402114 | 1.351402121 |
| 2.0 | 1.353501839    | 1.353288493 | 1.353219485 | 1.353191745 | 1.353193606 | 1.353193712 | 1.353193719 |
|     | Improved Euler |             |             | Runge-Kutta |             |             | Exact       |