Final Exam MakeUp Part B, Math 2552

Instructions (PLEASE READ)

Formatting and Timing

- **Show your work** and justify your answers for all questions unless stated otherwise.
- Please write neatly, and use dark and clear writing so that the scan is easy to read.
- Please write your name or initials at the top of every page
- Please solve the questions in the exam in the order they are given.
- You do not need to print the exam. As long as you solve problems in the order they are given (just like the written homework sets), you can write your answers on your own paper. But students can print the exam and write their answers on the printed copy if they prefer.

Submission

- Students should scan their work and submit it through Gradescope. There should be an **assignment** in Gradescope for this exam. The process for submitting your work will be similar to what you have used for homework.
- Work must be submitted by May 7, 8:00 pm, ET.
- Please upload your work as a single PDF file. If this is not possible you can email your work to your instructor.
- During the upload process in Gradescope, please indicate which page of your work corresponds to each question in the exam.

Questions

- If there are questions during the exam, students can email their instructor or message them through Canvas.
- Our course Piazza forum will be temporarily inactive during the exam.
- If you run into any technical issues or any unanticipated emergencies, please email your instructor as soon as you can.

Integrity

- Students can use any resources while taking these tests including online calculators and Mathematica
- Students cannot communicate with anyone during these tests.
- Students cannot use solutions provided from another student or third party.
- In other words: do your own work but you can use technology to solve problems.

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1. (10 points) Use the variation of parameters method to identify the general solution to

$$\vec{x}\,' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \vec{x} + \begin{pmatrix} 2e^{3t} \\ -4e^{3t} \end{pmatrix}.$$

Please show your work.

2. (10 points) Consider the non-linear system below.

$$\frac{dx}{dt} = y^2 - 1, \qquad \frac{dy}{dt} = x + y - 1$$

(a) Plot and label the nullclines of the system. Please label your axes.

(b) Identify all critical points of the system. Show your work.

(c) (Question 7 continued) Compute the Jacobian matrix. For each critical point you identified in part (b), use eigenvalues to classify the critical points according to stability (stable, unstable, asymptotically stable) and type (saddle, proper node, etc).

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3. (5 points) A tank holds 200 litres of salt water. Initially, there are 2 kg of salt in the tank. Salt water containing 0.2 kg of salt per litre is pumped into the tank at a rate of 4 litres per minute. The well-mixed solution is pumped out at a rate of 0.5 litres per minute. Construct an initial value problem that models the amount of salt in the tank for $t \in [0, T]$, where T is some positive constant. Do not solve your initial value problem.

4. (4 points) Construct an initial value problem for the following situation.

   A 0.1 Newtons (N) force stretches a spring 0.01 m. A mass weighing 4 kg is attached to the spring, and the spring is also attached to a viscous damper that applies a force of 0.6 N when the velocity of the mass is 0.1 m/s. The mass is pulled up 0.1 m above its equilibrium position and given an initial upward velocity of 0.12 m/s.

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5. (10 points) Use the Laplace transform to solve the IVP.

$$y'' - 3y' + 2y = 20e^{3t}, \quad y(0) = 5, \quad y'(0) = 4$$

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6. (1 point) One point will be allocated for presentation, neatness, and organization. Please ensure that

    1. your work is legible in the scan
    2. your name or initials are at the top of every page
    3. questions are answered in the order in which they were given
    4. during the upload process you have indicated which pages correspond to which question, and made sure that none of your pages are upside down or sideways (you can also change the orientation of the pages when you upload in Gradescope)

    Ensuring that these criteria are met helps ensure that your exam is graded efficiently and accurately.

Please sign and date the following GT Honor Code statement.

Georgia Tech Honor Code
Having read the Georgia Institute of Technology Academic Honor Code, I understand and accept my responsibility as a member of the Georgia Tech community to uphold the Honor Code at all times.

_____ _____

signature date