# Math 2552 Written HW Set 9

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**Trench 8.2.1a.** Use the table of Laplace transforms to find the inverse Laplace transform.

$$\frac{3}{(s-7)^4}$$

*Solution.* Using the linearity of the inverse Laplace transform and the Shifting Theorem, we may write

$$L^{-1}\left(\frac{3}{(s-7)^4}\right) = 3L^{-1}\left(\frac{1}{(s-7)^4}\right) = \frac{3}{6}e^{7t}L^{-1}\left(\frac{6}{s^4}\right)$$

We evaluate the rightmost side with the table of Laplace transforms, noting that $L(s)[t^n] = n!/s^{n+1}$, and find

$$L^{-1}\left(\frac{3}{(s-7)^4}\right) = \frac{e^{7t}t^3}{2}$$

$\square$

**Trench 8.2.1b.** Use the table of Laplace transforms to find the inverse Laplace transform.

$$\frac{2s-4}{s^2-4s+13}$$

*Solution.* We start by completing the square in the denominator, yielding

$$\frac{2s-4}{s^2-4s+13} = \frac{2s-4}{(s-2)^2+9}$$

Using the shifting theorem and the linearity of the inverse Laplace transform, we obtain

$$L^{-1}\left(\frac{2s-4}{(s-2)^2+9}\right) = 2e^{2t}L^{-1}\left(\frac{s}{s^2+9}\right)$$

Using the table of Laplace transforms, we note that $L(s)[\cos\omega t] = s/(s^2+\omega^2)$. Letting $\omega = 3$, we obtain

$$L^{-1}\left(\frac{2s-4}{s^2-4s+13}\right) = 2e^{2t}\cos(3t)$$

□

**Trench 8.2.3a.** Use Heaviside's method to find the inverse Laplace transform.

$$\frac{3-(s+1)(s-2)}{(s+1)(s+2)(s-2)}$$

*Solution.* We write

$$\frac{3-(s+1)(s-2)}{(s+1)(s+2)(s-2)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-2}$$

Setting $s=-1$ and ignoring the $(s+1)$ factor in the denominator yields

$$A = \frac{3-(0)(-3)}{(1)(-3)} = -1$$

Doing the same for the other coefficients gives

$$B = \frac{3-(-1)(-4)}{(-1)(-4)} = -\frac{1}{4}$$

and

$$C = \frac{3-(3)(0)}{(3)(4)} = \frac{1}{4}$$

Thus, we find

$$\begin{aligned} L^{-1}\left(\frac{3-(s+1)(s-2)}{(s+1)(s+2)(s-2)}\right) &= -L^{-1}\left(\frac{1}{s+1}\right) - \frac{1}{4}L^{-1}\left(\frac{1}{s+2}\right) + \frac{1}{4}L^{-1}\left(\frac{1}{s-2}\right) \\ &= -e^{-t} - \frac{1}{4}e^{-2t} + \frac{1}{4}e^{2t} \end{aligned}$$

□