Math 2552 Written HW Set 7

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Trench 3.1.7. Use Euler's method with step sizes $h = 0.1, h = 0.05$, and $h = 0.025$ to find approximate values of the solution of the initial value problem

$$y' + \frac{2}{x}y = \frac{3}{x^3} + 1, \quad y(1) = 1$$

at $x = 1.0, 1.1, 1.2, 1.3, \ldots, 2.0$. Compare these approximate values with the values of the exact solution

$$y = \frac{1}{3x^2}(9\ln x + x^3 + 2),$$

which can be obtained by the method of Section 2.1. Present your results in a table like Table 3.1.1.

x	$h = 0.1$	$h = 0.05$	$h = 0.025$	Exact
1.0	1.000000000	1.000000000	1.000000000	1.000000000
1.1	1.200000000	1.174813735	1.163906015	1.153937085
1.2	1.307212622	1.272318906	1.256964040	1.242799540
1.3	1.362954963	1.325650055	1.309013781	1.293546032
1.4	1.389819425	1.353514366	1.337139286	1.321811247
1.5	1.400603239	1.366805485	1.351411241	1.336916440
1.6	1.402745029	1.372014729	1.357897143	1.344535503
1.7	1.400644088	1.373071526	1.360308069	1.348172348
1.8	1.396924918	1.372374545	1.360932547	1.350008229
1.9	1.393151367	1.371388136	1.361182553	1.351402121
2.0	1.390242009	1.370996758	1.361921132	1.353193719

Trench 3.2.7. Use the improved Euler method with step sizes $h = 0.1, h = 0.05$, and $h = 0.025$ to find approximate values of the solution of the initial value problem

$$y' + \frac{2}{x}y = \frac{3}{x^3} + 1, \quad y(1) = 1$$

at $x = 1.0, 1.1, 1.2, 1.3, \ldots, 2.0$. Compare these approximate values with the values of the exact solution

$$y = \frac{1}{3x^2}(9\ln x + x^3 + 2),$$

which can be obtained by the method of Section 2.1. Present your results in a table like Table 3.2.2.

x	$h = 0.1$	$h = 0.05$	$h = 0.025$	$h = 0.1$	$h = 0.05$	$h = 0.025$	Exact
1.0	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000
1.1	1.200000000	1.174813735	1.163906015	1.153606311	1.153879293	1.153925558	1.153937085
1.2	1.307212622	1.272318906	1.256964040	1.242464607	1.242748907	1.242790763	1.242799540
1.3	1.362954963	1.325650055	1.309013781	1.293314190	1.293522398	1.293544147	1.293546032
1.4	1.389819425	1.353514366	1.337139286	1.321704368	1.321817246	1.321816580	1.321811247
1.5	1.400603239	1.366805485	1.351411241	1.336924820	1.336948622	1.336928009	1.336916440
1.6	1.402745029	1.372014729	1.357897143	1.344639720	1.344588782	1.344552009	1.344535503
1.7	1.400644088	1.373071526	1.360308069	1.348351961	1.348241722	1.348192555	1.348172348
1.8	1.396924918	1.372374545	1.360932547	1.350244948	1.350089364	1.350031080	1.350008229
1.9	1.393151367	1.371388136	1.361182553	1.351680683	1.351491477	1.351426765	1.351402121
2.0	1.390242009	1.370996758	1.361921132	1.353501839	1.353288493	1.353219485	1.353193719
	Euler			Improved Euler			Exact

Trench 3.3.7. Use the Runge-Kutta method with step sizes $h = 0.1, h = 0.05$, and $h = 0.025$ to find approximate values of the solution of the initial value problem

$$y' + \frac{2}{x}y = \frac{3}{x^3} + 1, \quad y(1) = 1$$

at $x = 1.0, 1.1, 1.2, 1.3, \dots, 2.0$. Compare these approximate values with the values of the exact solution

$$y = \frac{1}{3x^2}(9\ln x + x^3 + 2),$$

which can be obtained by the method of Section 2.1. Present your results in a table like Table 3.3.1.

x	$h = 0.1$	$h = 0.05$	$h = 0.025$	$h = 0.1$	$h = 0.05$	$h = 0.025$	Exact
1.0	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000
1.1	1.153606311	1.153879293	1.153925558	1.153933943	1.153936905	1.153937074	1.153937085
1.2	1.242464607	1.242748907	1.242790763	1.242795524	1.242799309	1.242799526	1.242799540
1.3	1.293314190	1.293522398	1.293544147	1.293541986	1.293545800	1.293546018	1.293546032
1.4	1.321704368	1.321817246	1.321816580	1.321807471	1.321811030	1.321811234	1.321811247
1.5	1.336924820	1.336948622	1.336928009	1.336913020	1.336916244	1.336916429	1.336916440
1.6	1.344639720	1.344588782	1.344552009	1.344532439	1.344535327	1.344535493	1.344535503
1.7	1.348351961	1.348241722	1.348192555	1.348169612	1.348172191	1.348172339	1.348172348
1.8	1.350244948	1.350089364	1.350031080	1.350005783	1.350008089	1.350008220	1.350008229
1.9	1.351680683	1.351491477	1.351426765	1.351399928	1.351401996	1.351402114	1.351402121
2.0	1.353501839	1.353288493	1.353219485	1.353191745	1.353193606	1.353193712	1.353193719
	Improved Euler			Runge-Kutta			Exact