Chapter I

Complex Numbers

1 The Algebra of Complex Numbers

1.1 Arithmetic Operations

Problem 1.1.1. Find the values of

$$(1+2i)^3, \quad \frac{5}{-3+4i}, \quad \left(\frac{2+i}{3-2i}\right)^2, \quad (1+i)^n + (1-i)^n.$$

Problem 1.1.2. If $z = x + iy$ (x and y real), find the real and imaginary parts of

$$z^4, \quad \frac{1}{z}, \quad \frac{z-1}{z+1}, \quad \frac{1}{z^2}$$

Problem 1.1.3. Show that

$$\left(\frac{-1 \pm i\sqrt{3}}{2}\right)^3 = 1 \quad \text{and} \quad \left(\frac{\pm 1 \pm i\sqrt{3}}{2}\right)^6 = 1$$

for all combinations of signs.