✓ **Congratulations! You passed!**

TO PASS 75% or higher

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Non-square matrix multiplication

TOTAL POINTS 8

1. In the previous lecture we saw the Einstein summation convention, in which we sum over any indices which are repeated. In traditional notation we might write, for example, $\sum_{j=1}^{3} A_{ij} v_j = A_{i1} v_1 + A_{i2} v_2 + A_{i3} v_3$. With the Einstein summation convention we can avoid the big sigma and write this as $A_{ij} v_j$. We know that we sum over j because it appears twice.

   We saw that thinking about this type of notation helps us to multiply non-square matrices together. For example, consider the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix},$$

   and remember that in the A_{ij} notation the first index i represents the row number and the second index j represents the column number. For example, $A_{12} = 2$.

   Let's define the matrix $C = AB$. Then in Einstein summation convention notation $C_{mn} = A_{mj} B_{jn}$.

   Using the Einstein summation convention, calculate $C_{21} = A_{2j} B_{j1}$.

   - ○ $C_{21} = 3$
   - ○ $C_{21} = 4$
   - ◉ $C_{21} = 5$
   - ○ $C_{21} = 6$

   ✓ **Correct**

   Writing out the summation we see that $C_{21} = A_{2j} B_{j1} = A_{21} B_{11} + A_{22} B_{21} + A_{23} B_{31}$. We

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✓ **Correct**

Writing out the summation we see that $C_{21} = A_{2j}B_{j1} = A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}$. We can get the same answer as acting the 2nd row of A on the 1st column of B, which is exactly how we would multiply square matrices together.

2. We can use the same method to calculate every element of $C = AB$. Doing so we see that we are multiplying A's rows with B's columns in exactly the same way as we would for square matrices.

In fact, we can multiply any matrices together as long as the terms which we sum over have the same number of elements. For example, there are the same number of values for j in $C_{mn} = A_{mj}B_{jn}$. The resulting matrix C will have as many rows as A and as many columns as B.

Using the same matrices as before, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, what is $C = AB$?

○ $C = \begin{bmatrix} 4 & 5 \\ 3 & 4 \\ 5 & 1 \end{bmatrix}$

○ $C = \begin{bmatrix} 4 & 5 & 3 \\ 5 & 2 & 5 \end{bmatrix}$

○ $C = \begin{bmatrix} 7 & 1 & 4 \\ 2 & 5 & 3 \end{bmatrix}$

◉ $C = \begin{bmatrix} 4 & 3 & 5 \\ 5 & 4 & 1 \end{bmatrix}$

✓ **Correct**

Did you calculate this by considering each element $C_{mn} = A_{mj}B_{jn}$ or by multiplying row by column? Make sure you understand why they give the same answer!

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3. Let's practice multiplying together a few more matrices which are not square.

Calculate the product:

$$\begin{bmatrix} 2 & 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

- ○ 29
- ◉ 30
- ○ 31
- ○ 32

✓ **Correct**

This is another way to define the dot product of two vectors!

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4. Calculate the product:

$$\begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 5 & 6 \end{bmatrix}$$

- ○ 32
- ○ 30
- ○ $\begin{bmatrix} 2 & 4 & 5 & 6 \\ 6 & 15 & 12 & 18 \\ 8 & 4 & 12 & 10 \\ 2 & 4 & 6 & 5 \end{bmatrix}$
- ◉ $\begin{bmatrix} 2 & 4 & 5 & 6 \\ 6 & 12 & 15 & 18 \\ 4 & 8 & 10 & 12 \\ 2 & 4 & 5 & 6 \end{bmatrix}$

✓ **Correct**

Well done.

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5. Calculate the product:

$$\begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 4 & -1 \\ -2 & 0 & 0 & 2 \end{bmatrix}$$

- (selected) $\begin{bmatrix} 2 & 2 & 8 & -4 \\ -6 & 0 & 0 & 6 \\ 0 & 1 & 4 & -1 \end{bmatrix}$
- $\begin{bmatrix} 2 & 2 & -8 & 4 \\ -6 & 0 & 0 & -6 \\ 0 & -1 & -1 & -4 \end{bmatrix}$
- $\begin{bmatrix} -2 & 2 & 8 & 4 \\ -6 & 0 & 6 & 6 \\ 0 & 4 & 4 & 1 \end{bmatrix}$
- $\begin{bmatrix} 2 & -2 & 8 & -4 \\ 6 & 0 & 0 & 0 \\ 6 & -1 & 4 & 1 \end{bmatrix}$

✓ **Correct**

Well done!

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6. We have seen that we can multiply an $m \times n$ matrix with an $n \times k$ matrix, and the resultant matrix will be an $m \times k$ matrix. You can check this is consistent with your previous answers.

1/1

Let $D = ABC$ where A is a 5×3 matrix, B is a 3×7 matrix and C is a 7×4 matrix.

What are the dimensions of the matrix D?

- ○ D is a 3×7 matrix
- ○ A, B and C cannot be multiplied together because they have the wrong dimensions.
- ○ D is a 5×7 matrix
- ◉ D is a 5×4 matrix
- ○ D is a 4×5 matrix

✓ **Correct**

We can use the rule about multiplying non square matrices to more than two matrices.

7. Calculate the product:

1/1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- ○ $\begin{bmatrix} 5 & 1 & 6 \\ 4 & 3 & 2 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$
- ◉ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$
- ○ $\begin{bmatrix} 6 & 1 & 3 \\ 4 & 5 & 2 \end{bmatrix}$

✓ **Correct**

... [1 0] ... [1 2 3] ...

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- ○ $\begin{bmatrix} 6 & 1 & 3 \\ 4 & 5 & 2 \end{bmatrix}$

✓ **Correct**

We can see that the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ doesn't change $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ even though the second matrix is not square.

8. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors with n elements. Which of the following are equal to the dot product of these two vectors?

- ☐ $u_i v_j$
- ☑ $\mathbf{u} \cdot \mathbf{v}$

✓ **Correct**

This is the typically how we write the dot product.

- ☐ $\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix}$
- ☑ $u_i v_i$

✓ **Correct**

This is the dot product in Einstein summation convention notation. It doesn't matter which letter we use for our indices as long as they are the same.

- ☑ $\begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

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✓ **Correct**

This is the typically how we write the dot product.

☐ $\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix}$

☑ $u_i v_i$

✓ **Correct**

This is the dot product in Einstein summation convention notation. It doesn't matter which letter we use for our indices as long as they are the same.

☑ $\begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

✓ **Correct**

This gives the same result as the dot product, like in question 3.

☑ $\sum_{i=1}^{n} u_i v_i$

✓ **Correct**

This is the dot product with the full sum written out.

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