Bigger Jacobians!

TOTAL POINTS 5

1. In this quiz, you will calculate the Jacobian matrix for some vector valued functions.

For the function $u(x, y) = x^2 - y^2$ and $v(x, y) = 2xy$, calculate the Jacobian matrix $J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$.

○ $J = \begin{bmatrix} 2x & 2y \\ -2y & 2x \end{bmatrix}$

◉ $J = \begin{bmatrix} 2x & -2y \\ 2y & 2x \end{bmatrix}$

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○ $J = \begin{bmatrix} 2x & -2y \\ -2y & 2x \end{bmatrix}$

✓ **Correct**

Well done!

2. For the function $u(x, y, z) = 2x + 3y$, $v(x, y, z) = \cos(x)\sin(z)$ and $w(x, y, z) = e^x e^y e^z$, calculate the

Jacobian matrix $J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$.

○ $J = \begin{bmatrix} 2 & 3 & 0 \\ -\cos(x)\sin(z) & 0 & -\sin(x)\cos(z) \\ e^x e^y e^z & e^x e^y e^z & e^x e^y e^z \end{bmatrix}$

○ $J = \begin{bmatrix} 2 & 3 & 0 \\ \sin(x)\sin(z) & 0 & -\cos(x)\cos(z) \\ e^x e^y e^z & e^x e^y e^z & e^x e^y e^z \end{bmatrix}$

◉ $J = \begin{bmatrix} 2 & 3 & 0 \\ -\sin(x)\sin(z) & 0 & \cos(x)\cos(z) \\ e^x e^y e^z & e^x e^y e^z & e^x e^y e^z \end{bmatrix}$

○ $J = \begin{bmatrix} 2 & 3 & 0 \\ \cos(x)\sin(z) & 0 & -\sin(x)\cos(z) \\ e^x e^y e^z & e^x e^y e^z & e^x e^y e^z \end{bmatrix}$

✓ **Correct**
Well done!

3. Consider the pair of linear equations $u(x, y) = ax + by$ and $v(x, y) = cx + dy$, where a, b, c and d are all constants. Calculate the Jacobian, and notice something kind of interesting!

◉ $J = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

○ $J = \begin{bmatrix} b & c \\ d & a \end{bmatrix}$

○ $J = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

○ $J = \begin{bmatrix} b & c \\ a & d \end{bmatrix}$

✓ **Correct**

Well done!

A succinct way of writing this down is the following:

$$\begin{bmatrix} u \\ v \end{bmatrix} = J \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

This is a generalisation of the fact that a simple linear function $f(x) = a \cdot x$ can be re-written as $f(x) = f'(x) \cdot x$, as the Jacobian matrix can be viewed as the multi-dimensional derivative. Neat!

4. For the function $u(x, y, z) = 9x^2y^2 + ze^x$, $v(x, y, z) = xy + x^2y^3 + 2z$ and $w(x, y, z) = \cos(x)\sin(z)e^y$, calculate the Jacobian matrix and evaluate at the point $(0, 0, 0)$.

○ $J = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

○ $J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

⦿ $J = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

○ $J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

✓ **Correct**

Well done!

5. In the lecture, we calculated the Jacobian of the transformation from Polar co-ordinates to Cartesian co-ordinates in 2D. In this question, we will do the same, but with Spherical co-ordinates to 3D.

For the functions $x(r, \theta, \phi) = r\cos(\theta)\sin(\phi)$, $y(r, \theta, \phi) = r\sin(\theta)\sin(\phi)$ and $z(r, \theta, \phi) = r\cos(\phi)$, calculate the Jacobian matrix.

○ $J = \begin{bmatrix} r^2\cos(\theta)\sin(\phi) & -\sin(\theta)\sin(\phi) & \cos(\theta)\cos(\phi) \\ r\sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) \\ \cos(\phi) & 1 & r\sin(\phi) \end{bmatrix}$

○ $J = \begin{bmatrix} r\cos(\theta)\sin(\phi) & -r\sin(\theta)\sin(\phi) & r\cos(\theta)\cos(\phi) \\ r\sin(\theta)\sin(\phi) & r^2\cos(\theta)\sin(\phi) & \sin(\theta)\cos(\phi) \\ \cos(\phi) & -1 & -r\sin(\phi) \end{bmatrix}$

⦿ $J = \begin{bmatrix} \cos(\theta)\sin(\phi) & -r\sin(\theta)\sin(\phi) & r\cos(\theta)\cos(\phi) \\ \sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) \\ \cos(\phi) & 0 & -r\sin(\phi) \end{bmatrix}$

○ $J = \begin{bmatrix} r\cos(\theta)\sin(\phi) & -\sin(\theta)\sin(\phi) & \cos(\theta)\cos(\phi) \\ r\sin(\theta)\sin(\phi) & \cos(\theta)\sin(\phi) & \sin(\theta)\cos(\phi) \\ r\cos(\phi) & 0 & -\sin(\phi) \end{bmatrix}$

✓ **Correct**

Well done! The determinant of this matrix is $-r^2\sin(\phi)$, which does not vary only with θ.