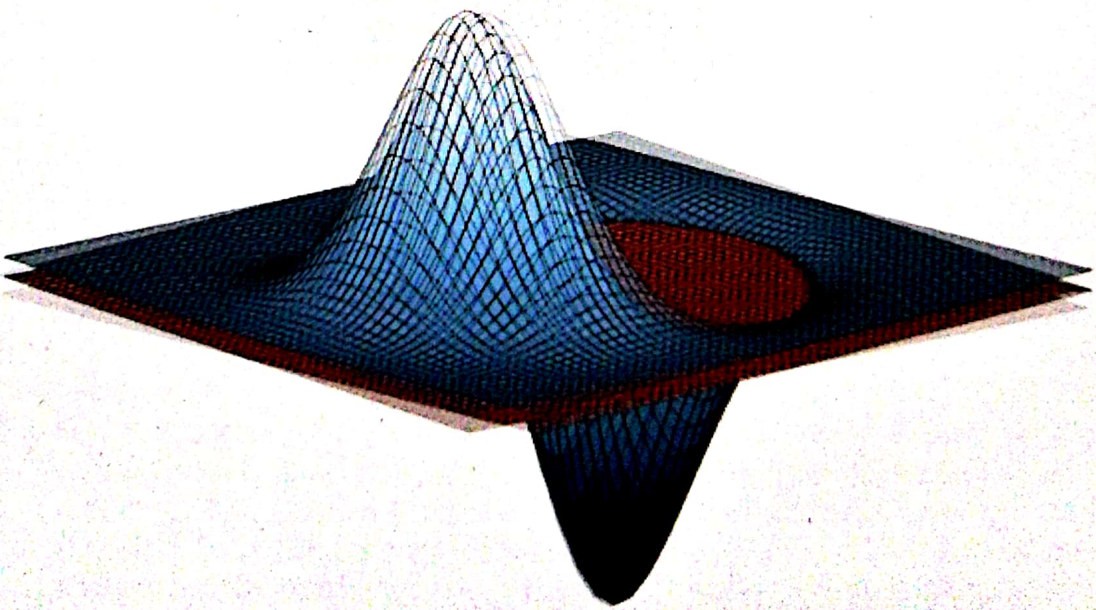
2D Taylor series

TOTAL POINTS 5

1. Now you have seen how to calculate the multivariate Taylor series, and what zeroth, first and second order approximations look like for a function of 2 variables. In this course we won't be considering anything higher than second order for functions of more than one variable.

   In the following questions you will practise recognising these approximations by thinking about how they behave with different x and y, then you will calculate some terms in the multivariate Taylor series yourself.

   The following plot features a surface and its Taylor series approximation in red, around a point given by a red circle. What order is the Taylor series approximation?



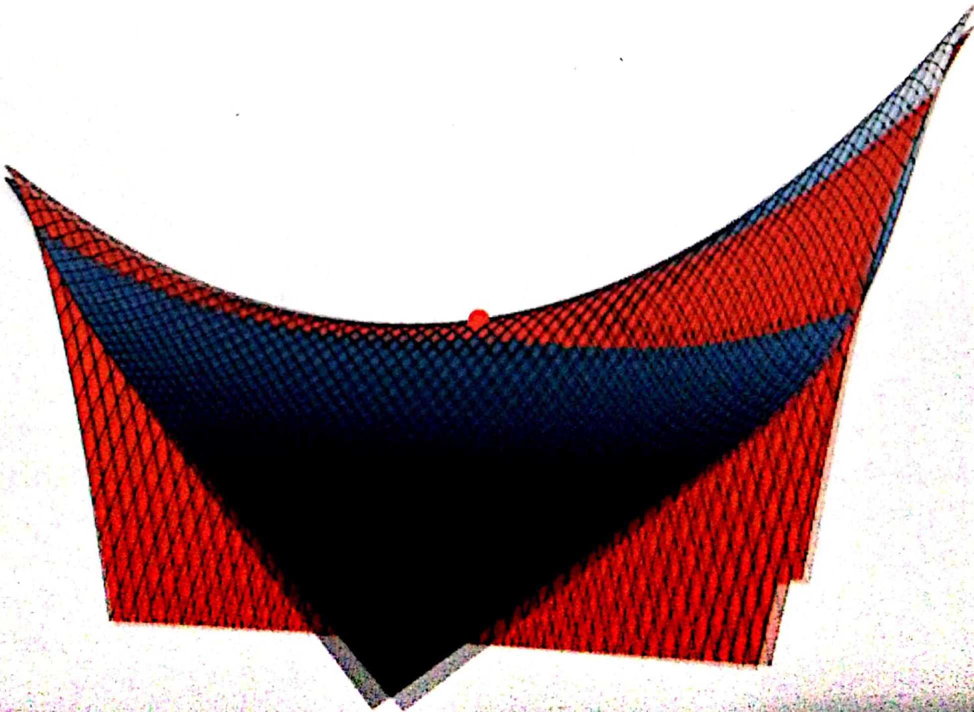
- [x] Zeroth order
- [ ] First order
- [ ] Second order
- [ ] None of the above

✓ **Correct**

The red surface is constant everywhere and so has no terms in $\Delta \mathbf{x}$ or $\Delta \mathbf{x}^2$

2. What order Taylor series approximation, expanded around the red circle, is the red surface in the following plot?

1/1



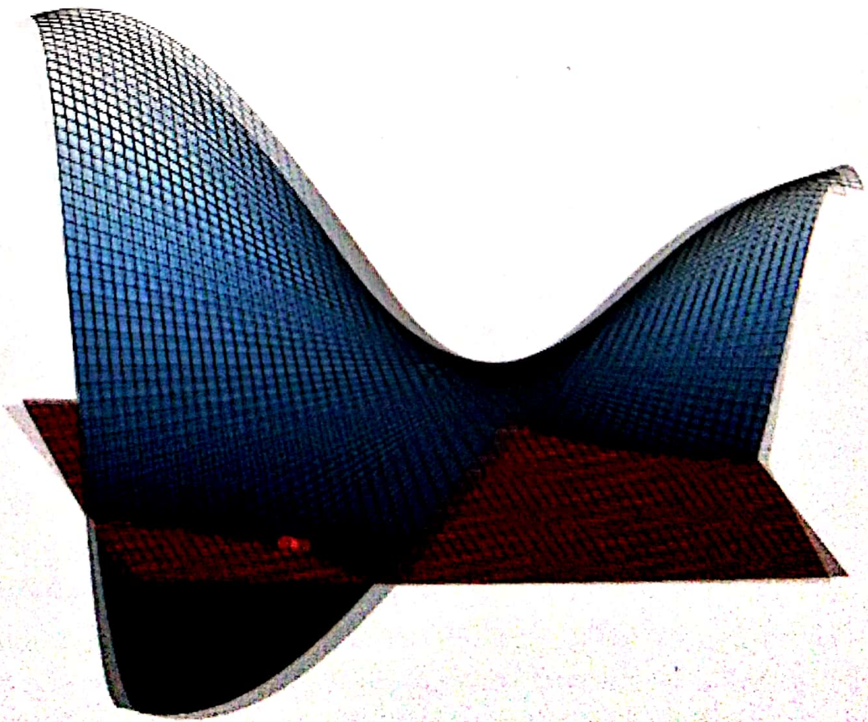
- [ ] Zeroth order
- [ ] First order
- [x] Second order
- [ ] None of the above

✓ **Correct**

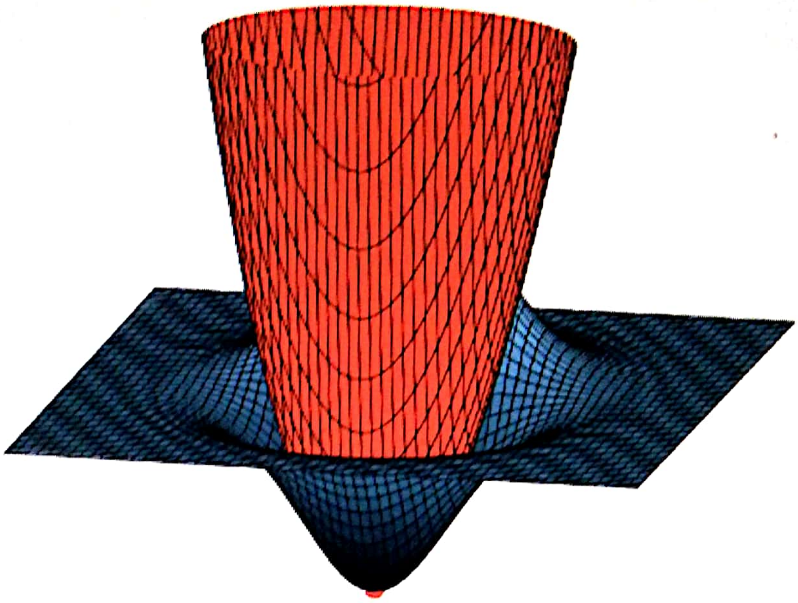
The gradient of the surface is not constant, so we must have a term of higher order than $\Delta\mathbf{x}$.

3. Which red surface in the following images is a first order Taylor series approximation of the blue surface? The original functions are given, but you don't need to do any calculations.

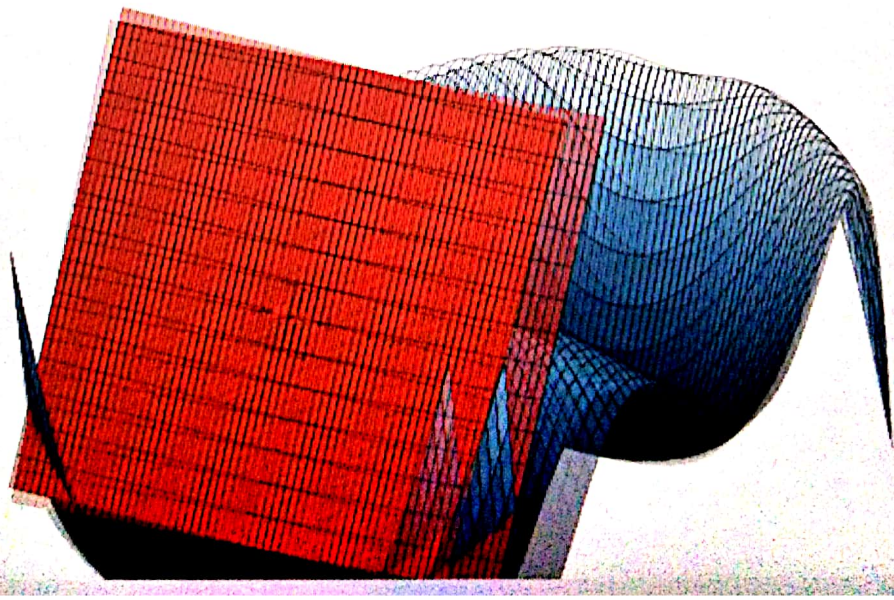
- [ ] $f(x, y) = \sin(xy/5)$



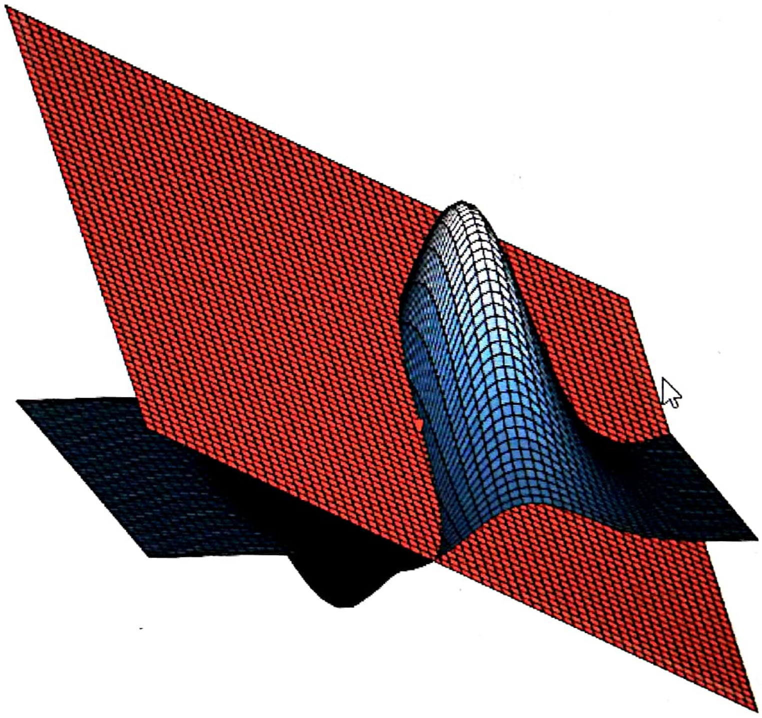
○ $f(x, y) = xe^{-x^2-y^2}$



◉ $f(x, y) = x\sin(x^2/2 + y^2/4)$



○ $f(x, y) = (x^2 + 2x)e^{-x^2 - y^2/5}$



✓ **Correct**

The gradient of the red surface is non-zero and constant, so the $\mathbf{\Delta x}$ terms are the highest order.

4. Recall that up to second order the multivariate Taylor series is given by $f(\mathbf{x} + \boldsymbol{\Delta}\mathbf{x}) = f(\mathbf{x}) + J_f \boldsymbol{\Delta}\mathbf{x} + \frac{1}{2}\boldsymbol{\Delta}\mathbf{x}^T H_f \boldsymbol{\Delta}\mathbf{x} + \ldots$

Consider the function of 2 variables, $f(x, y) = xy^2 e^{-x^4 - y^2/2}$. Which of the following is the first order Taylor series expansion of f around the point $(-1, 2)$?

- ○ $f_1(-1+\Delta x, 2+\Delta y) = -4e^{-3} - 4e^{-3}\Delta x + 4e^{-3}\Delta y$
- ○ $f_1(-1+\Delta x, 2+\Delta y) = -4e^{-3} + 16e^{-3}\Delta x - 8e^{-3}\Delta y$
- ○ $f_1(-1+\Delta x, 2+\Delta y) = 2e^{-33/2} - 63e^{-33/2}\Delta x - 2e^{-33/2}\Delta y$
- ◉ $f_1(-1+\Delta x, 2+\Delta y) = -4e^{-3} - 12e^{-3}\Delta x + 4e^{-3}\Delta y$

✓ **Correct**

5. Now consider the function $f(x, y) = \sin(\pi x - x^2 y)$. What is the Hessian matrix H_f that is associated with the second order term in the Taylor expansion of f around $(1, \pi)$?

- ○ $H_f = \begin{pmatrix} -2 & -2\pi \\ 2\pi & 1 \end{pmatrix}$
- ○ $H_f = \begin{pmatrix} -\pi^2 & \pi \\ \pi & -1 \end{pmatrix}$
- ◉ $H_f = \begin{pmatrix} -2\pi & -2 \\ -2 & 0 \end{pmatrix}$
- ○ $H_f = \begin{pmatrix} -2\pi & -2 \\ -2 & 1 \end{pmatrix}$

✓ **Correct**

Good, you can check your second order derivatives here:

$\partial_{xx} f(x, y) = -2y\cos(\pi x - x^2 y) - (\pi - 2xy)^2 \sin(\pi x - x^2 y)$

$\partial_{xy} f(x, y) = -2x\cos(\pi x - x^2 y) - x^2(\pi - 2xy)\sin(\pi x - x^2 y)$

$\partial_{yx} f(x, y) = -2x\cos(\pi x - x^2 y) - x^2(\pi - 2xy)\sin(\pi x - x^2 y)$

$\partial_{yy} f(x, y) = -x^4 \sin(\pi x - x^2 y)$