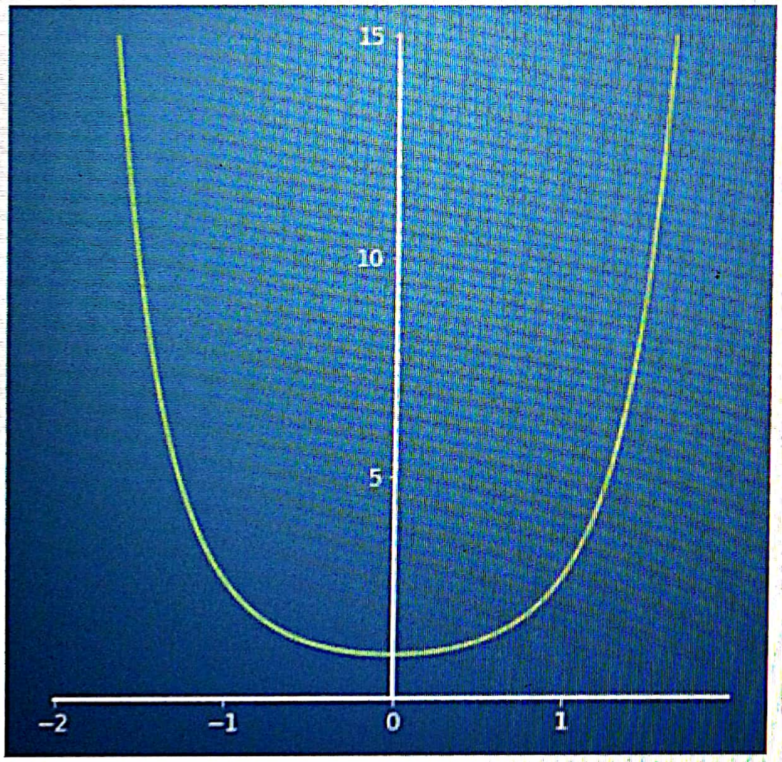
Applying the Taylor series

TOTAL POINTS 5

1. In the two previous videos, we have shown the short mathematical proofs for the Taylor series, and for special cases when the starting point is $x = 0$, the Maclaurin series. In these set of questions, we will begin to work on applying the Taylor and Maclaurin series formula to obtain approximations of functions.



For the function $f(x) = e^{x^2}$ about $x = 0$, using the the Maclaurin series formula, obtain an approximation up to the first three non zero terms.

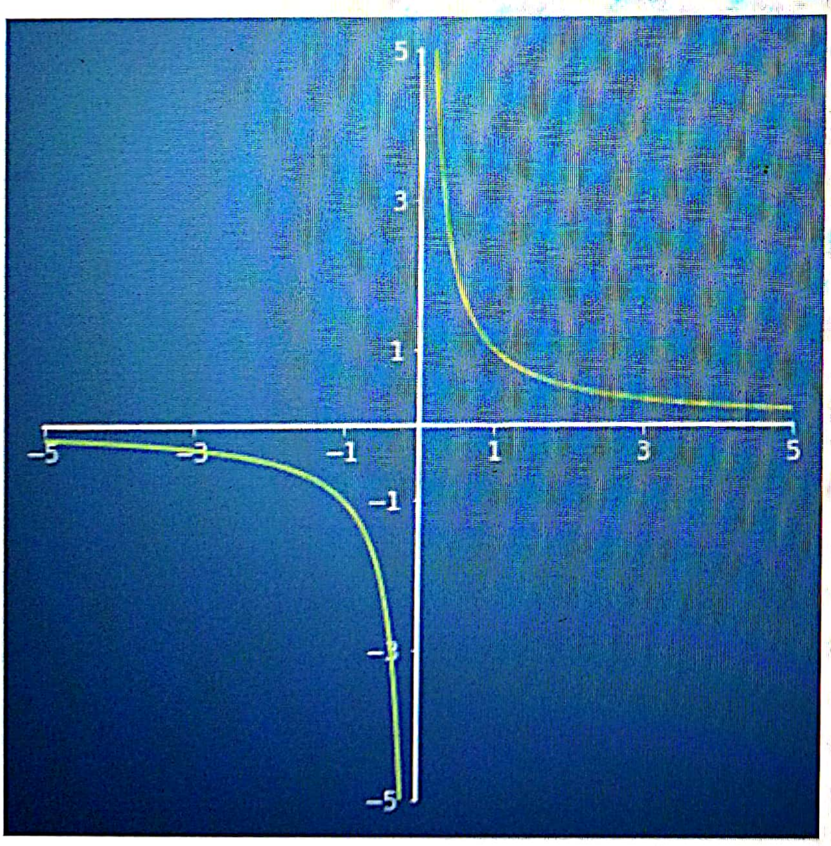
- ○ $f(x) = 1 - x^2 - \frac{x^4}{2} \ldots$
- ○ $f(x) = x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \ldots$
- ◉ $f(x) = 1 + x^2 + \frac{x^4}{2} + \ldots$

- ◉ $f(x) = 1 + x^2 + \frac{x^4}{2} + \ldots$
- ○ $f(x) = 1 + 2x + \frac{x^2}{2} + \ldots$

✓ **Correct**

We find that only even powers of x appear in the Taylor approximation for this function.

2.



Use the Taylor series formula to approximate the first three terms of the function $f(x) = 1/x$, expanded around the point $p = 4$.

- ○ $f(x) = \frac{(x-4)}{16} + \frac{(x-4)^2}{64} - \frac{(x-4)^3}{256} \ldots$

○ $f(x) = -\frac{1}{4} - \frac{(x+4)}{16} - \frac{(x+4)^2}{64} \ldots$

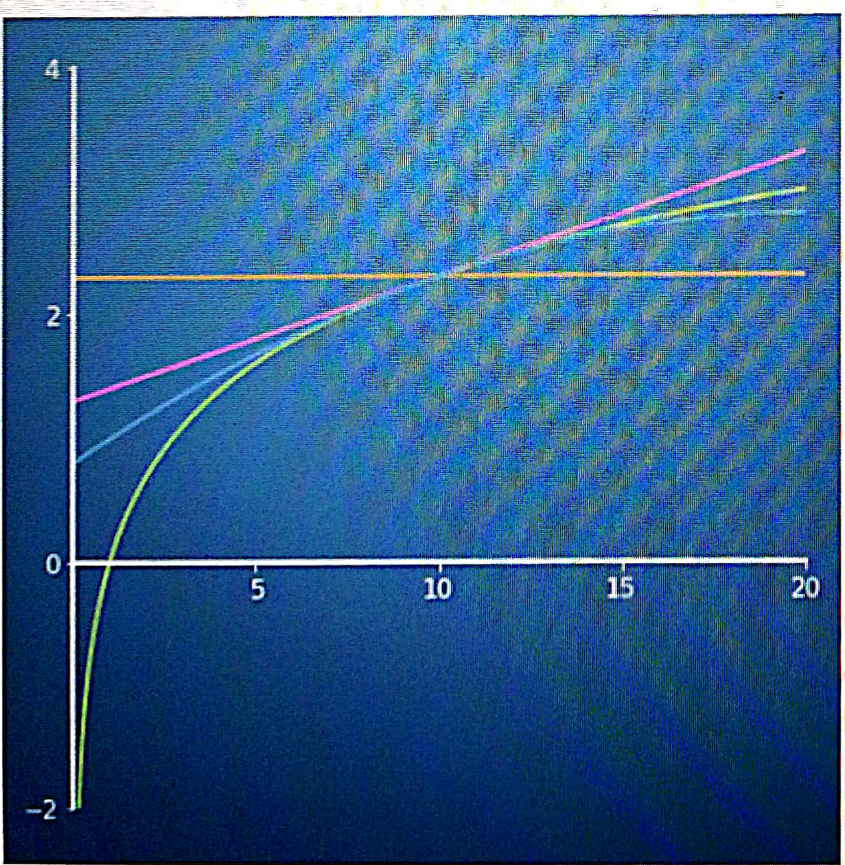
○ $f(x) = \frac{1}{4} - \frac{(x+4)}{16} + \frac{(x+4)^2}{64} + \ldots$

◉ $f(x) = \frac{1}{4} - \frac{(x-4)}{16} + \frac{(x-4)^2}{64} + \ldots$

✓ **Correct**

We find that only even powers of x appear in the Taylor approximation for this function.

3.



By finding the first three terms of the Taylor series shown above for the function $f(x) = \ln(x)$ (green line) about $x = 10$, determine the magnitude of the difference of using the second order taylor expansion against the first order Taylor expansion when approximating to find the value of $f(2)$.

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- ○ $\Delta f(2) = 1.0$
- ○ $\Delta f(2) = 0$
- ◉ $\Delta f(2) = 0.32$
- ○ $\Delta f(2) = 0.5$

✓ **Correct**

The second order Taylor approximation about the point $x = 10$ is $f(x) = \ln(10) + \frac{(x-10)}{10} - \frac{(x-10)^2}{200} \ldots$

So the first order approximation is

$$g_1 = \ln(10) + \frac{(x-10)}{10}$$

and the second order approximation is

$$g_2 = \ln(10) + \frac{(x-10)}{10} - \frac{(x-10)^2}{200}.$$

So, the magnitude of the difference is

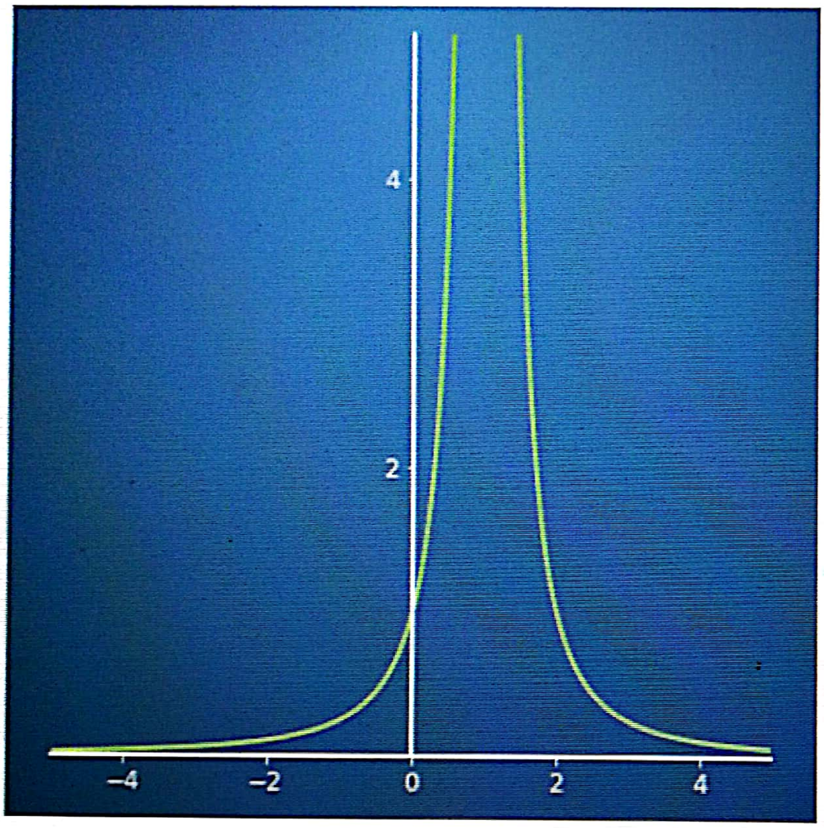
$$|g_2(2) - g_1(2)| = \left| -\frac{(x-10)^2}{200} \right|$$

and substituting in $x = 2$ gives us

$$|g_2(2) - g_1(2)| = \left| -\frac{(2-10)^2}{200} \right| = 0.32$$

4. In some cases, a Taylor series can be expressed in a general equation that allows us to find a particular n^{th} term of our series. For example the function $f(x) = e^x$ has the general equation $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. Therefore if we want to find the 3^{rd} term in our Taylor series, subsitituting $n = 2$ into the general equation gives us the term $\frac{x^2}{2}$. We know the Taylor series of the function e^x is $f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \ldots$. Now let us try a further working example of using general equations with Taylor series.

let us try a further working example of using general equations with Taylor series.



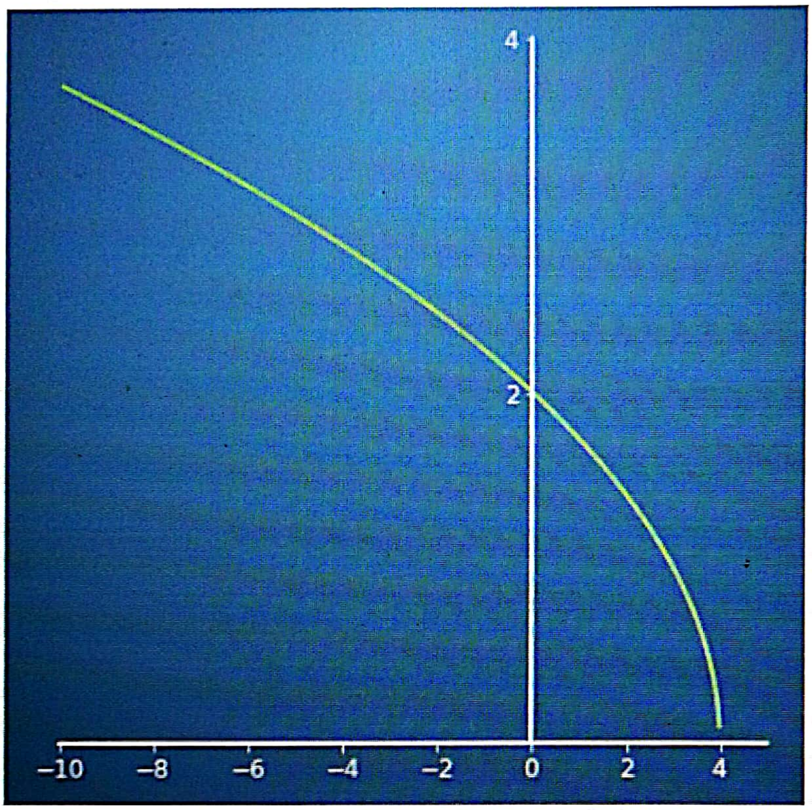
By evaluating the function $f(x) = \frac{1}{(1-x)^2}$ about the origin $x = 0$, determine which general equation for the n^{th} order term correctly represents $f(x)$.

- ○ $f(x) = \sum_{n=0}^{\infty} (2+n)(x)^n$
- ○ $f(x) = \sum_{n=0}^{\infty} (1+2n)(x)^n$
- ◉ $f(x) = \sum_{n=0}^{\infty} (1+n)x^n$
- ○ $f(x) = \sum_{n=0}^{\infty} (1+n)(-x)^n$

✓ **Correct**

By doing a Maclaurin series approximation, we obtain $f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \ldots$, which satisfies the general equation shown.

5.



By evaluating the function $f(x) = \sqrt{4 - x}$ at $x = 0$, find the quadratic equation that approximates this function.

- ○ $f(x) = 2 - x - \frac{x^3}{64} \ldots$
- ○ $f(x) = 2 + x + x^2 \ldots$
- ○ $f(x) = \frac{x}{4} - \frac{x^2}{64} \ldots$
- ◉ $f(x) = 2 - \frac{x}{4} - \frac{x^2}{64} \ldots$

✓ **Correct**

The quadratic equation shown is the second order approximation.