Characteristic polynomials, eigenvalues and eigenvectors

TOTAL POINTS 10

1. Given a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, recall that one can calculate its eigenvalues by solving the characteristic polynomial $\lambda^2 - (a+d)\lambda + (ad - bc) = 0$. In this quiz, you will practice calculating and solving the characteristic polynomial to find the eigenvalues of simple matrices.

1/1 point

For the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

- ○ $\lambda^2 + 3\lambda - 2 = 0$

  $\lambda_1 = -1, \lambda_2 = 2$

- ○ $\lambda^2 - 3\lambda - 2 = 0$

  $\lambda_1 = 1, \lambda_2 = -2$

- ◉ $\lambda^2 - 3\lambda + 2 = 0$

  $\lambda_1 = 1, \lambda_2 = 2$

- ○ $\lambda^2 + 3\lambda + 2 = 0$

  $\lambda_1 = -1, \lambda_2 = -2$

✓ **Correct**

Well done! This matrix has two distinct eigenvalues.

2. Recall that for a matrix A, the eigenvectors of the matrix are vectors for which applying the matrix transformation is the same as scaling by some constant.

1/1 point

For $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

2. Recall that for a matrix A, the eigenvectors of the matrix are vectors for which applying the matrix transformation is the same as scaling by some constant.

1/1 point

For $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

- [x] $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

  ✓ **Correct**

  Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

- [x] $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

  ✓ **Correct**

  Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

- [x] $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

  ✓ **Correct**

  Well done! One way to check that a vector is an eigenvector is to simply apply the matrix transformation and see if this is the same as multiplying by a scalar. Another way is to calculate the eigenvector by hand, as in the lecture videos.

- [ ] $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

3. For the matrix $A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1/1 point

- (●) $\lambda^2 - 8\lambda + 15 = 0$

  $\lambda_1 = 3, \lambda_2 = 5$

- ( ) $\lambda^2 + 8\lambda - 15 = 0$

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3. For the matrix $A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

- ○ $\lambda^2 - 8\lambda - 15 = 0$

  $\lambda_1 = -3, \lambda_2 = 5$

- ○ $\lambda^2 + 8\lambda - 15 = 0$

  $\lambda_1 = 3, \lambda_2 = -5$

- ○ $\lambda^2 + 8\lambda + 15 = 0$

  $\lambda_1 = -3, \lambda_2 = -5$

- ◉ $\lambda^2 - 8\lambda + 15 = 0$

  $\lambda_1 = 3, \lambda_2 = 5$

✓ **Correct**

Well done! This matrix has two distinct eigenvalues.

4. For the matrix $A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

0 / 1 point

- ☑ $\begin{bmatrix} -1 \\ -1/2 \end{bmatrix}$

✓ **Correct**

Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

- ☑ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

4. For the matrix $A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

0 / 1 point

- [x] $\begin{bmatrix} -1 \\ -1/2 \end{bmatrix}$

✓ **Correct**

Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

- [x] $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

! **This should not be selected**

Recall that we never consider the zero-vector as an eigenvector of a matrix.

- [ ] $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$

- [x] $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

✓ **Correct**

Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

5. For the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

◯ $\lambda^2 + 5\lambda - 4 = 0$

$\lambda_1 = 1, \lambda_2 = -4$

◉ $\lambda^2 - 5\lambda + 4 = 0$

$\lambda_1 = 1, \lambda_2 = 4$

◯ $\lambda^2 + 5\lambda + 4 = 0$

$\lambda_1 = -1, \lambda_2 = -4$

5. For the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

- ○ $\lambda^2 + 5\lambda - 4 = 0$

  $\lambda_1 = 1, \lambda_2 = -4$

- ◉ $\lambda^2 - 5\lambda + 4 = 0$

  $\lambda_1 = 1, \lambda_2 = 4$

- ○ $\lambda^2 + 5\lambda + 4 = 0$

  $\lambda_1 = -1, \lambda_2 = -4$

- ○ $\lambda^2 - 5\lambda - 4 = 0$

  $\lambda_1 = -1, \lambda_2 = 4$

✓ **Correct**

Well done! This matrix has two distinct eigenvalues.

6. For the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

1 / 1 point

- [x] $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

✓ **Correct**

Well done! One way to check that a vector is an eigenvector is to simply apply the matrix transformation and see if this is the same as multiplying by a scalar. Another way is to calculate the eigenvector by hand, as in the lecture videos.

- [x] $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

✓ **Correct**

Well done! One way to check that a vector is an eigenvector is to simply apply the matrix transformation and see if this is the same as multiplying by a scalar. Another way is to calculate the eigenvector by hand, as in the lecture videos.

✓ **Correct**

Well done! One way to check that a vector is an eigenvector is to simply apply the matrix transformation and see if this is the same as multiplying by a scalar. Another way is to calculate the eigenvector by hand, as in the lecture videos.

- [ ] $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

- [ ] $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

7. For the matrix $A = \begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

○ $\lambda^2 + 25 = 0$

$\lambda_1 = -5, \lambda_2 = 5$

○ $\lambda^2 + 25 = 0$

$\lambda_1 = \lambda_2 = -5$

○ $\lambda^2 - 25 = 0$

$\lambda_1 = \lambda_2 = 5$

◉ $\lambda^2 - 25 = 0$

$\lambda_1 = -5, \lambda_2 = 5$

✓ **Correct**

Well done! This matrix has two distinct eigenvalues.

8. For the matrix $A = \begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

0 / 1 point

- [x] $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

✓ Correct

8. For the matrix $A = \begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

0 / 1 point

- [x] $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

✓ **Correct**

Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

- [x] $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

✓ **Correct**

Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

- [ ] $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$
- [ ] $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

You didn't select all the correct answers

9. For the matrix $A = \begin{bmatrix} 5 & 4 \\ -4 & -3 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

- ( ) $\lambda^2 - 2\lambda + 1 = 0$

  $\lambda_1 = \lambda_2 = -1$
- ( ) $\lambda^2 - 2\lambda + 1 = 0$

  No real solutions.
- (•) $\lambda^2 - 2\lambda + 1 = 0$

  $\lambda_1 = \lambda_2 = 1$
- ( ) $\lambda^2 \quad 2\lambda + 1 = 0$

9. For the matrix $A = \begin{bmatrix} 5 & 4 \\ -4 & -3 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1/1 point

- $\lambda^2 - 2\lambda + 1 = 0$

  $\lambda_1 = \lambda_2 = -1$
- $\lambda^2 - 2\lambda + 1 = 0$

  No real solutions.
- $\lambda^2 - 2\lambda + 1 = 0$

  $\lambda_1 = \lambda_2 = 1$
- $\lambda^2 - 2\lambda + 1 = 0$

  $\lambda_1 = -1, \lambda_2 = 1$

✓ **Correct**

Well done! This matrix has one repeated eigenvalue - which means it may have one or two distinct eigenvectors (which are not scalar multiples of each other).

10. For the matrix $A = \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1/1 point

- $\lambda^2 - \lambda - 1 = 0$

  $\lambda_1 = \frac{1-\sqrt{5}}{2}, \lambda_2 = \frac{1+\sqrt{5}}{2}$
- $\lambda^2 - \lambda + 1 = 0$

  No real solutions.
- $\lambda^2 + \lambda - 1 = 0$

  $\lambda_1 = \frac{-\sqrt{5}-1}{2}, \lambda_2 = \frac{\sqrt{5}-1}{2}$
- $\lambda^2 + \lambda + 1 = 0$

  No real solutions.

✓ **Correct**