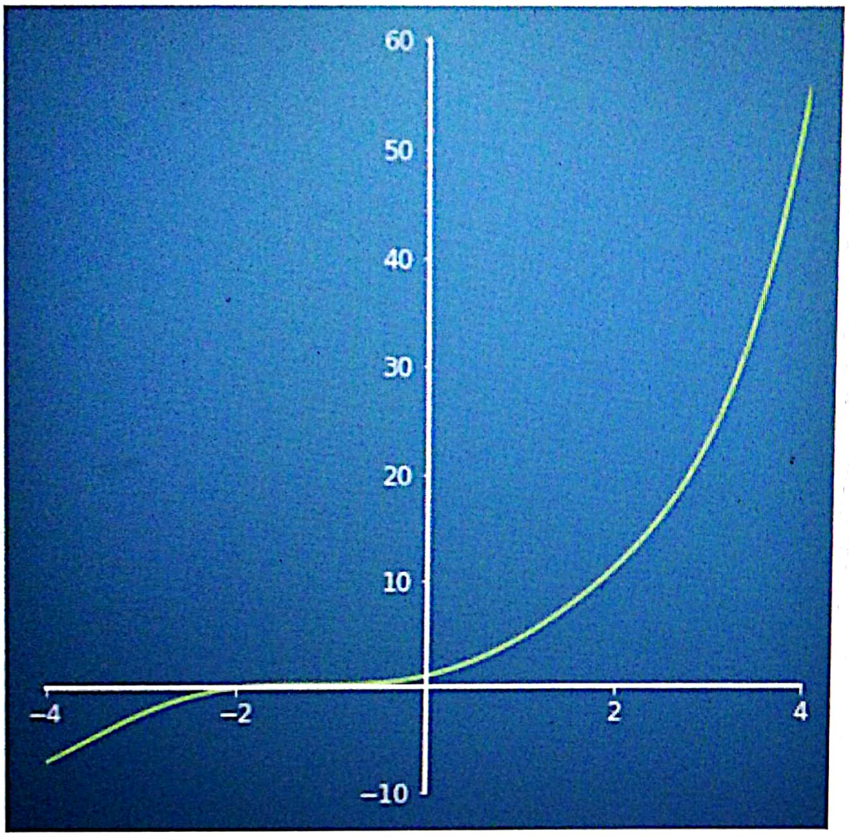
2. Find the first four non zero terms of the Taylor expansion for the function $f(x) = e^x + x + \sin(x)$ about $x = 0$. The function is shown below:



- [ ] $$f(x) = 1 + 3x - \frac{x^2}{2} + \frac{x^4}{24} + \ldots$$

- [ ] $$f(x) = 3x + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{720} + \ldots$$

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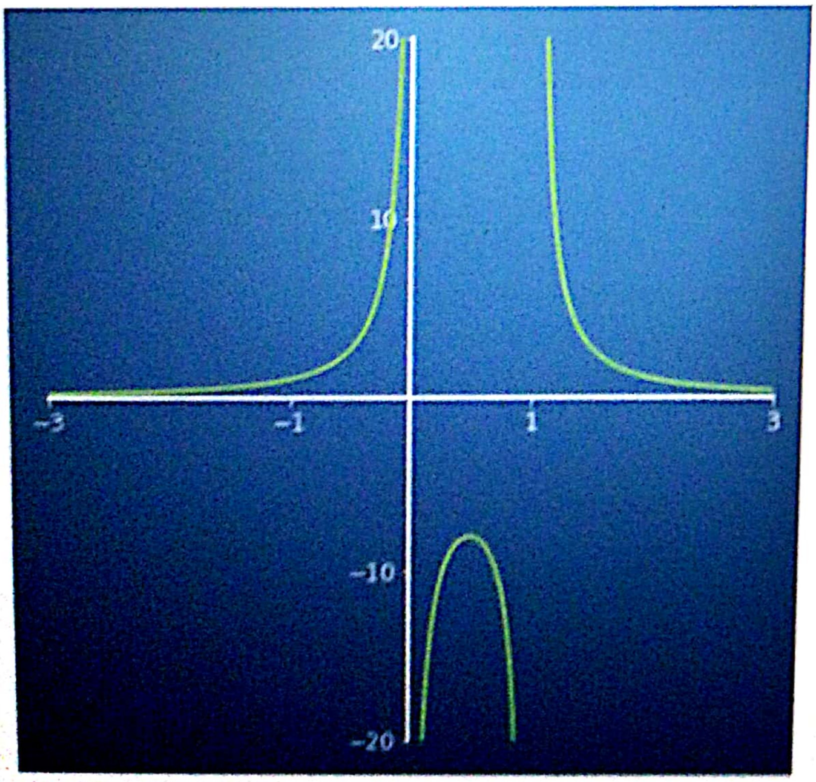
$$f(x) = 1 + 3x + \frac{x^2}{2} + \frac{x^4}{24} + \ldots$$

$$f(x) = 1 + 3x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$

Correct

As there is a variety of functions here i.e. $\sin(x)$ and an exponential, we are not likely to get expansions that are often only for odd or even powers of x.

3. The graph below shows the discontinuous function $f(x) = \frac{2}{(x^2 - x)}$. Approximate the section of this function that covers the domain $0 < x < 1$. Use the Taylor series forumla and $x = 0.5$ as your starting point, find the first two non zero terms.



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- [ ] $f(x) = -8 - 32x^2 \ldots$
- [ ] $f(x) = -8 + 32(x-0.5)^2 \ldots$
- [x] $f(x) = -8 - 32(x-0.5)^2 \ldots$
- [ ] $f(x) = -4 - 16(x-0.5)^2 \ldots$

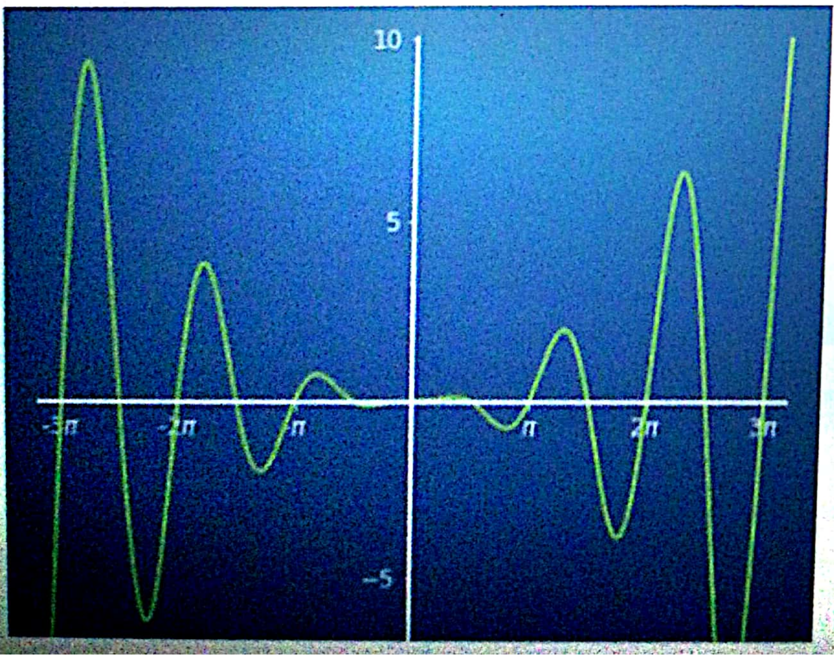
✓ **Correct**

This second order approximation is only valid within the domain $0 < x < 1$, and is, therefore, a poor approximation for the entire function, but behaves well within the defined domain.

4. Determine if the function:

$$f(x) = \left(\frac{x}{2}\right)^2 \frac{\sin(2x)}{2}$$

shown below is odd, even or neither.



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Odd

Even

Neither odd nor even

✓ **Correct**

For an odd function, $-f(x) = f(-x)$. We can also determine if a function is odd by looking at its symmetry. If it has rotational symmetry with respect to the origin, it is an odd function.

5. Take the Taylor expansion of the function

$$f(x) = e^{-2x}$$

about the point $x = 2$ and subsequently linearise the function.

$$f(x) = \left(\frac{1}{e^2}\right)[2(x-2)] + O(\Delta x^2)$$

$$f(x) = \left(\frac{1}{e^4}\right)[1-2(x-2)] + O(\Delta x^2)$$

$$f(x) = \left(\frac{1}{e^4}\right)[1+2(x-2)]$$

$$f(x) = \left(\frac{1}{e^4}\right)[1-2(x-2)] + 4(x-2)^2 + O(\Delta x^3)$$

✓ **Correct**

Here we are taking a complicated function and simplifying it into its linear components, making sure to still note down its level of accuracy.

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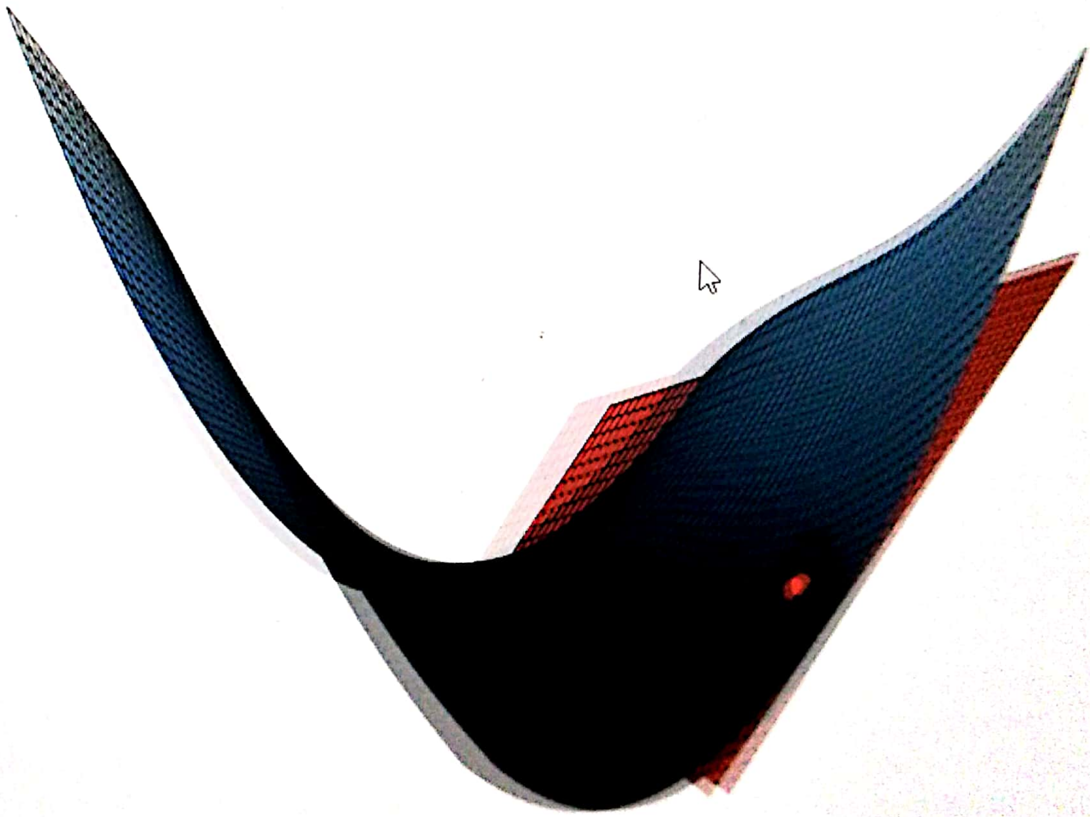
6. The figures below feature functions of two variables with proposed Taylor series approximations in red, expanded around the red circle. Which of the following features a valid second order approximation?

- ○ $f(x, y) = x^2 - xy + \sin(y)$



- ◉ $f(x, y) = xe^{-x^2 - y/4}$

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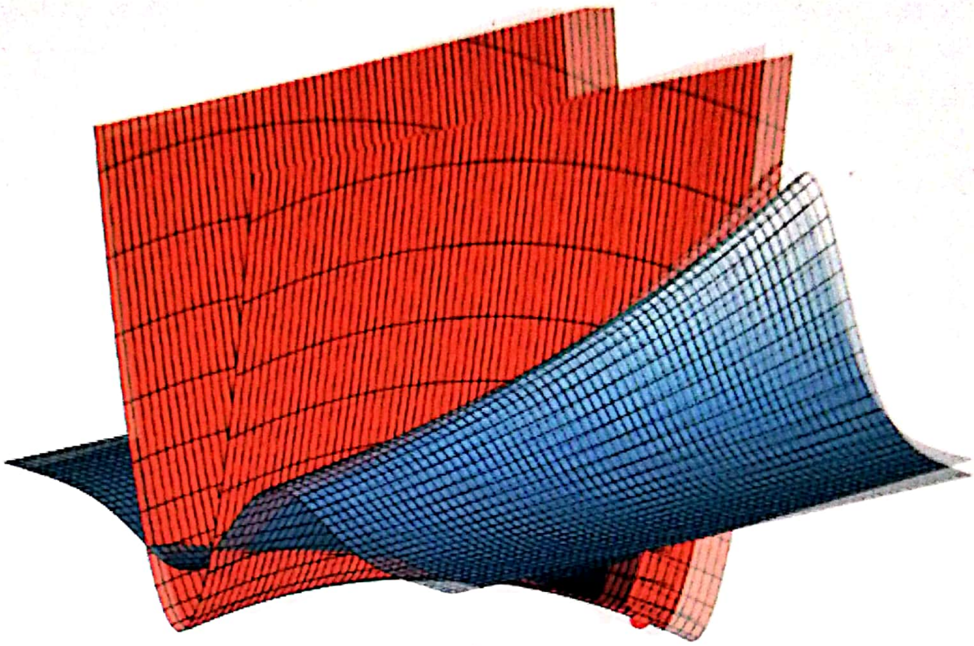
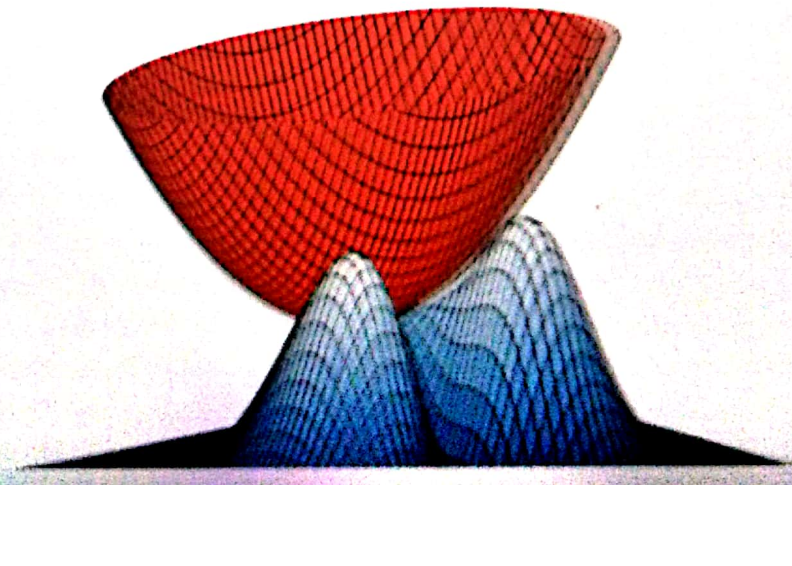
$f(x, y) = xe^{-x^2-y/4}$



$f(x, y) = (x^3 + 2y^2)e^{-x^2-y^2}$



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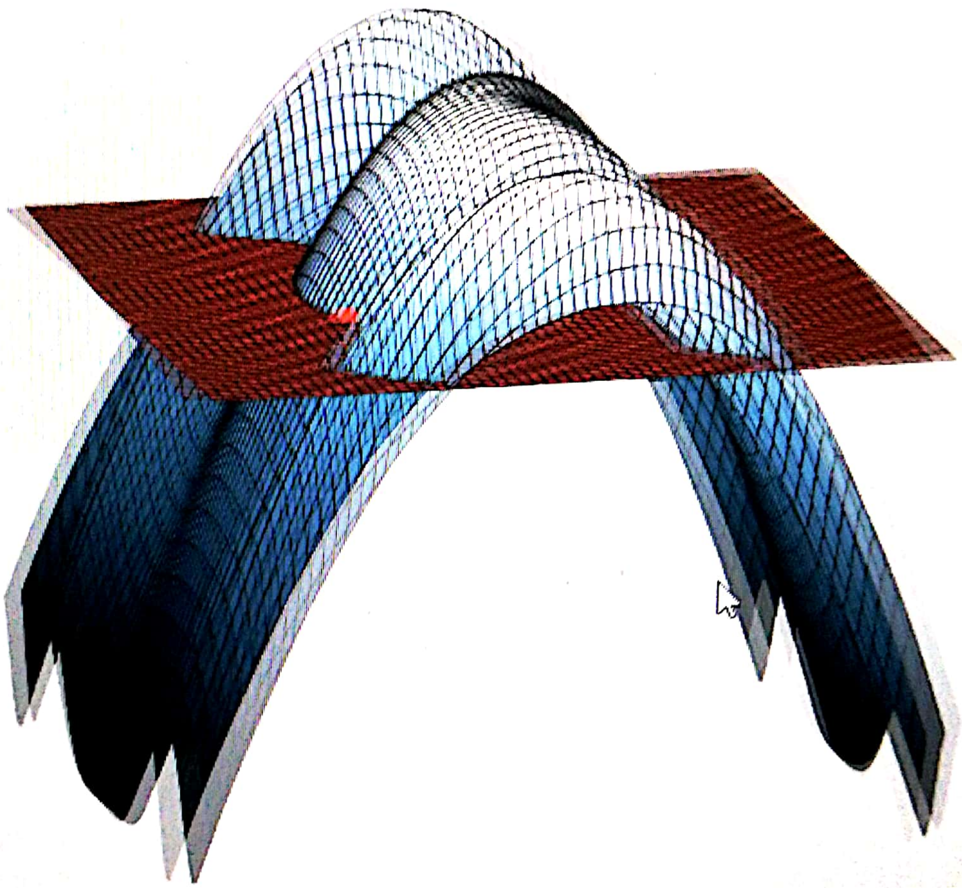
○ $f(x, y) = \cos(x - y^2) - x^2$



✓ **Correct**

The red surface has a gradient which changes with x and y, and both the function and its approximation have the same behaviour near the point of expansion, unlike the other second order red surface in this question. It is therefore the second order Taylor series approximation.

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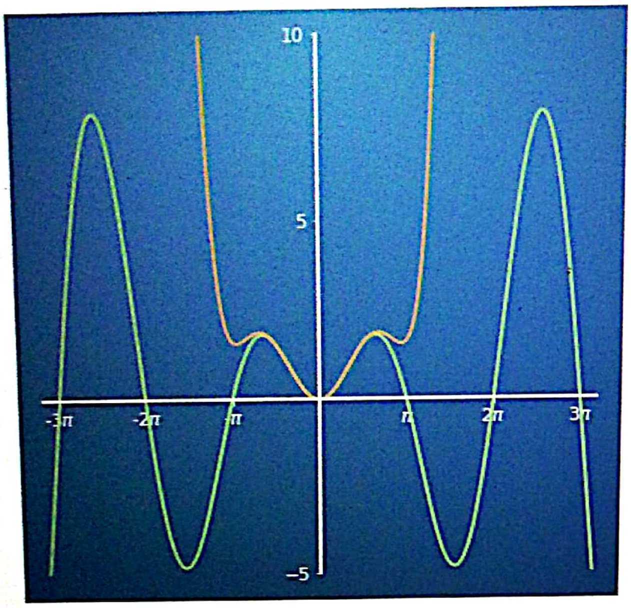
0 / 1 point

Now that we have completed the set of Taylor series lectures and answered all the quiz questions, we now need to test our understanding of Taylor series. We have looked at the derivation of Taylor series, broken it down into a power series approximation, explored special cases and developed the idea of multivariant Taylor series, that is required in order for us to develop a good grounding for the next chapters in this course.

For the function $f(x) = x\sin(x)$ shown below, determine what order approximation is shown by the orange curve, where the Taylor series approximation was centered about $x = 0$.



- [ ] Second Order
- [ ] Third Order
- [ ] Fourth Order
- [ ] Sixth Order
- [x] None of the above

3rd order

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