## Multivariate chain rule exercise

## **TOTAL POINTS 5**

1. In this quiz, you will practice calculating the multivariate chain rule for various functions.

For the following functions, calculate the expression  $rac{df}{dt}=rac{\partial f}{\partial \mathbf{x}}rac{d\mathbf{x}}{dt}$  in matrix form, where  $\mathbf{x}=(x_1,x_2)$ .

$$f(\mathbf{x}) = f(x_1, x_2) = x_1^2 x_2^2 + x_1 x_2$$

$$x_1(t) = 1 - t^2$$

$$x_2(t) = 1 + t^2$$

$$egin{aligned} \bigcirc & rac{df}{dt} = rac{\partial f}{\partial \mathbf{x}} rac{d\mathbf{x}}{dt} = \left[2x_1^2x_2 + x_1, 2x_1x_2^2 + x_2
ight] egin{bmatrix} -2t \ 2t \end{bmatrix} \end{aligned}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = \left[2x_1^2x_2 + x_1, 2x_1x_2^2 + x_2\right] \begin{bmatrix} 2t \\ -2t \end{bmatrix}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = \left[2x_1x_2^2 + x_2, 2x_1^2x_2 + x_1\right] \begin{bmatrix} 2t \\ -2t \end{bmatrix}$$

✓ Correct

Well done!

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			0.4			
2. For the following functions	s, calculate the expr	ression 4	= 27 9	in matrix form.	where $\mathbf{x} =$	$(x_1, x_2, x_3)$
		dt	OX.	at		1, 2, 3,

$$f(\mathbf{x}) = f(x_1, x_2, x_3) = x_1^3 cos(x_2)e^{x_3}$$

$$x_1(t) = 2t$$

$$x_2(t) = 1 - t^2$$

$$x_3(t) = e^t$$

$$egin{aligned} igcolum_{rac{df}{dt} = rac{\partial f}{\partial \mathbf{x}} rac{d\mathbf{x}}{dt} = [3x_1^2 cos(x_2)e^{x_3}, x_1^3 cos(x_2)e^{x_3}, x_1^3 sin(x_2)e^{x_3}] egin{bmatrix} 2 \ 2t \ -e^t \end{bmatrix} \end{aligned}$$

$$\bigcirc_{\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} = \left[3x_1^2 cos(x_2)e^{x_3}, -x_1^3 sin(x_2)e^{x_3}, x_1^3 sin(x_2)e^{x_3}\right] \begin{bmatrix} 2\\2t\\e^t \end{bmatrix}$$

$$\stackrel{\textbf{(b)}}{=} \frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} = \left[3x_1^2 cos(x_2)e^{x_3}, -x_1^3 sin(x_2)e^{x_3}, x_1^3 cos(x_2)e^{x_3}\right] \begin{bmatrix} 2\\ -2t\\ e^t \end{bmatrix}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} = [3x_1^2 cos(x_2)e^{x_3}, -x_1^3 cos(x_2)e^{x_3}, x_1^3 cos(x_2)e^{x_3}] \begin{bmatrix} 2\\2t\\e^t \end{bmatrix}$$

Correct

3. For the following functions, calculate the expression  $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{u}} \frac{d\mathbf{u}}{dt}$  in matrix form, where  $\mathbf{x} = (x_1, x_2)$  and  $\mathbf{u} = (u_1, u_2)$ .

$$f(\mathbf{x}) = f(x_1, x_2) = x_1^2 - x_2^2$$

$$x_1(u_1,u_2)=2u_1+3u_2$$

$$x_2(u_1,u_2)=2u_1-3u_2$$

$$u_1(t) = \cos(t/2)$$

$$u_2(t) = sin(2t)$$

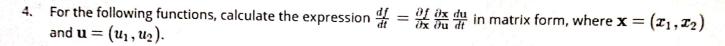
$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{u}} \frac{d\mathbf{u}}{dt} = \begin{bmatrix} -2x_1, -2x_2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -\sin(t/2)/2 \\ 2\cos(t) \end{bmatrix}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{u}} \frac{d\mathbf{u}}{dt} = [2x_1, 2x_2] \begin{bmatrix} 2 & -3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} \sin(t/2) \\ 2\cos(2t) \end{bmatrix}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{u}} \frac{d\mathbf{u}}{dt} = [2x_1, 2x_2] \begin{bmatrix} 2 & -3 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -\cos(t/2)/2 \\ 2\sin(2t) \end{bmatrix}$$

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## ✓ Correct



$$f(\mathbf{x}) = f(x_1, x_2) = cos(x_1)sin(x_2)$$

$$x_1(u_1,u_2) = 2u_1^2 + 3u_2^2 - u_2$$

$$x_2(u_1,u_2)=2u_1-5u_2^3$$

$$u_1(t) = e^{t/2}$$

$$u_2(t) = e^{-2t}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{u}} \frac{d\mathbf{u}}{dt} = \left[ -sin(x_1)cos(x_2), cos(x_1)cos(x_2) \right] \begin{bmatrix} u_1 & 6u_2 - 1 \\ 2 & -u_2^2 \end{bmatrix} \begin{bmatrix} e^{t^2/2}/2 \\ -2e^{-2t} \end{bmatrix}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{u}} \frac{d\mathbf{u}}{dt} = \left[ -\cos(x_1) \sin(x_2), \cos(x_1) \cos(x_2) \right] \begin{bmatrix} u_1 & 6u_2 - 1 \\ 2 & -15u_2^2 \end{bmatrix} \begin{bmatrix} e^t \\ e^t \end{bmatrix}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt} = \left[ -\sin(x_1)\cos(x_2), \cos(x_1)\cos(x_2) \right] \begin{bmatrix} 41u_1 & 6u_2 - 1 \\ 2 & -15u_2 \end{bmatrix} \begin{bmatrix} e^{\frac{1}{1}\sqrt{8}} / 8 \\ -2e^{\frac{2t}{2}} \end{bmatrix}$$

/ Correct

5. For the following functions, calculate the expression  $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{u}} \frac{d\mathbf{u}}{dt}$  in matrix form, where  $\mathbf{x} = (x_1, x_2)$  and  $\mathbf{u} = (u_1, u_2)$ .

$$f(\mathbf{x}) = f(x_1, x_2, x_3) = sin(x_1)cos(x_2)e^{x_3}$$

$$x_1(u_1, u_2) = sin(u_1) + cos(u_2)$$

$$x_2(u_1, u_2) = cos(u_1) - sin(u_2)$$

$$x_3(u_1,u_2)=e^{u_1+u_2}$$

$$u_1(t) = 1 + t/2$$

$$u_2(t) = 1 - t/2$$

$$[cos(x_1)cos(x_2)e^{x_3}, -sin(x_1)sin(x_2)e^{x_3}, sin(x_1^i)cos(x_2)e^{x_3}] egin{bmatrix} cos(u_1) & -sin(u_2) \ -sin(u_1) & -cos(u_2) \ e^{u_1-u_2} & e^{u_1-u_2} \end{bmatrix} egin{bmatrix} 1/2 \ -1/2 \end{bmatrix}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{u}} \frac{d\mathbf{u}}{dt} =$$

$$[cos(x_1)cos(x_2)e^{x_3}, -sin(x_1)^2sin(x_2)e^{x_3}, sin(x_1)cos(x_2)e^{x_3}] egin{bmatrix} sin(u_1) & -sin(u_2) \ -sin(u_1) & -cos(u_2) \ 3e^{u_1+u_2} & e^{u_1+u_2} \end{bmatrix} egin{bmatrix} -1/2 \ -1/2 \end{bmatrix}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} \frac{du}{dt} =$$

$$\left[cos(x_1)cos(x_2)e^{x_3}, -sin(x_1)cos(x_2)e^{x_3}, sin(x_1)cos(x_2)e^{x_3}\right] \begin{bmatrix} cos(u_1) & sin(u_2) \\ -sin(u_1) & -cos(u_2) \\ e^{u_1+u_2} & -e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$

$$\bigcirc \frac{df}{dt} = \frac{\partial f}{\partial t} \frac{\partial x}{\partial t} \frac{dy}{dt} =$$

$$\left[ \cos(x_1)\cos(x_2)e^{x_1}, \sin(x_1)\sin(x_2)e^{x_1}, \sin(x_1)\cos(x_2)e^{x_1} \right] \begin{bmatrix} -\cos(u_1) & -\sin(u_2) \\ -\sin(u_1) & -\cos(u_2) \\ e^{u_1+u_2} & 2e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

/ Correct