What does it mean for a vector $\mathbf{b_3}$ to be linearly independent to the vectors $\mathbf{b_1}$ and $\mathbf{b_2}$? Select all correct answers.

- [ ] $\mathbf{b_3} = a_1\mathbf{b_1} + a_2\mathbf{b_2}$, for some a_1 or a_2.

Un-selected is correct

- [x] $\mathbf{b_3} \neq a_1\mathbf{b_1} + a_2\mathbf{b_2}$, for any a_1 or a_2.

Correct

This is an algebraic way of understanding linear independence.

- [ ] $\mathbf{b_3} = \mathbf{b_1} + \mathbf{b_2}$.

Un-selected is correct

- [x] $\mathbf{b_3}$ does not lie in the plane spanned by $\mathbf{b_1}$ and $\mathbf{b_2}$.

Correct

This is a geometric way of understanding linear independence.

Congratulations! You passed!

TO PASS 80% or higher

Keep Learning

Linear dependency of a set of vectors

TOTAL POINTS 6

1. In the lecture videos you saw that vectors are linearly dependent if it is possible to write one vector as a linear combination of the others. For example, the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are linearly dependent if $\mathbf{a} = q_1 \mathbf{b} + q_2 \mathbf{c}$ where q_1 and q_2 are scalars.

   1/1

   Are the following vectors linearly dependent?

   $\mathbf{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.

   - (•) Yes
   - ( ) No

   ✓ **Correct**

   When there are two vectors we only need to check if one can be written as a scalar multiple of the other. We can see that the vectors are linearly dependent because $\mathbf{a} = \frac{1}{2}\mathbf{b}$.

2. We say that two vectors are linearly independent if they are *not* linearly dependent, that is, we cannot write one of the vectors as a linear combination of the others. Be careful not to mix the two definitions up!

Are the following vectors linearly independent?

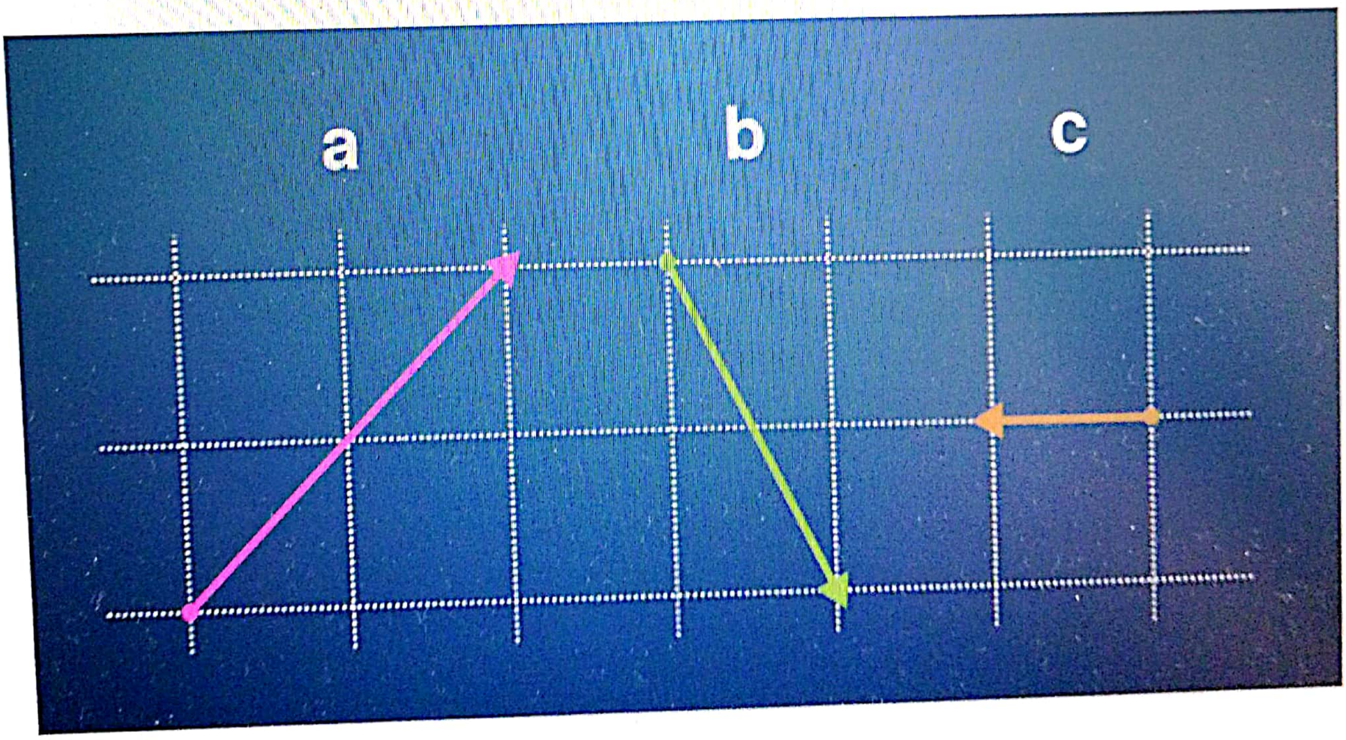
$\mathbf{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

- (●) Yes
- ( ) No

✓ **Correct**

These vectors are linearly independent as one is not a scalar multiple of the other.

3. We also saw in the lectures that three vectors that lie in the same two dimensional plane must be linearly dependent. This tells us that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are linearly dependent in the following diagram:



? Put your answer in the following

What are the values of q_1 and q_2 that allow us to write $\mathbf{a} = q_1\mathbf{b} + q_2\mathbf{c}$? Put your answer in the following codeblock:

```
# Assign the correct values for q1 and q2 to write a as a linear combination of b and c
q1 = -1
q2 = -3
```

Run

Reset

```
-3
```

✓ **Correct**

Good job!

4. In fact, an n-dimensional space can have as many as n linearly independent vectors. The following three vectors are three dimensional, which means that we must check if they are linearly dependent or independent.

Are the following vectors linearly independent?

$\mathbf{a} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

- (●) Yes
- ( ) No

✓ **Correct**

These vectors are linearly independent as one can not be written as a linear sum of the other two.

6. The following set of vectors cannot be used as a basis for a three dimensional space. Why?

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} \text{ and } \mathbf{c} = \begin{bmatrix} 4 \\ 3 \\ -3 \end{bmatrix}.$$

- [x] The vectors do not span three dimensional space

✓ **Correct**

There are three vectors but they are linearly dependent. If we remove one of the vectors the remaining two are linearly independent, which means that the vectors only span two dimensions.

- [ ] There are too many vectors for a three dimensional basis

- [x] The vectors are not linearly independent

✓ **Correct**

We can see that $\mathbf{c} = 2\mathbf{a} - \mathbf{b}$, so the vectors are linearly dependent. The definition of a basis requires that the vectors are linearly independent.

- [ ] The vectors are linearly independent