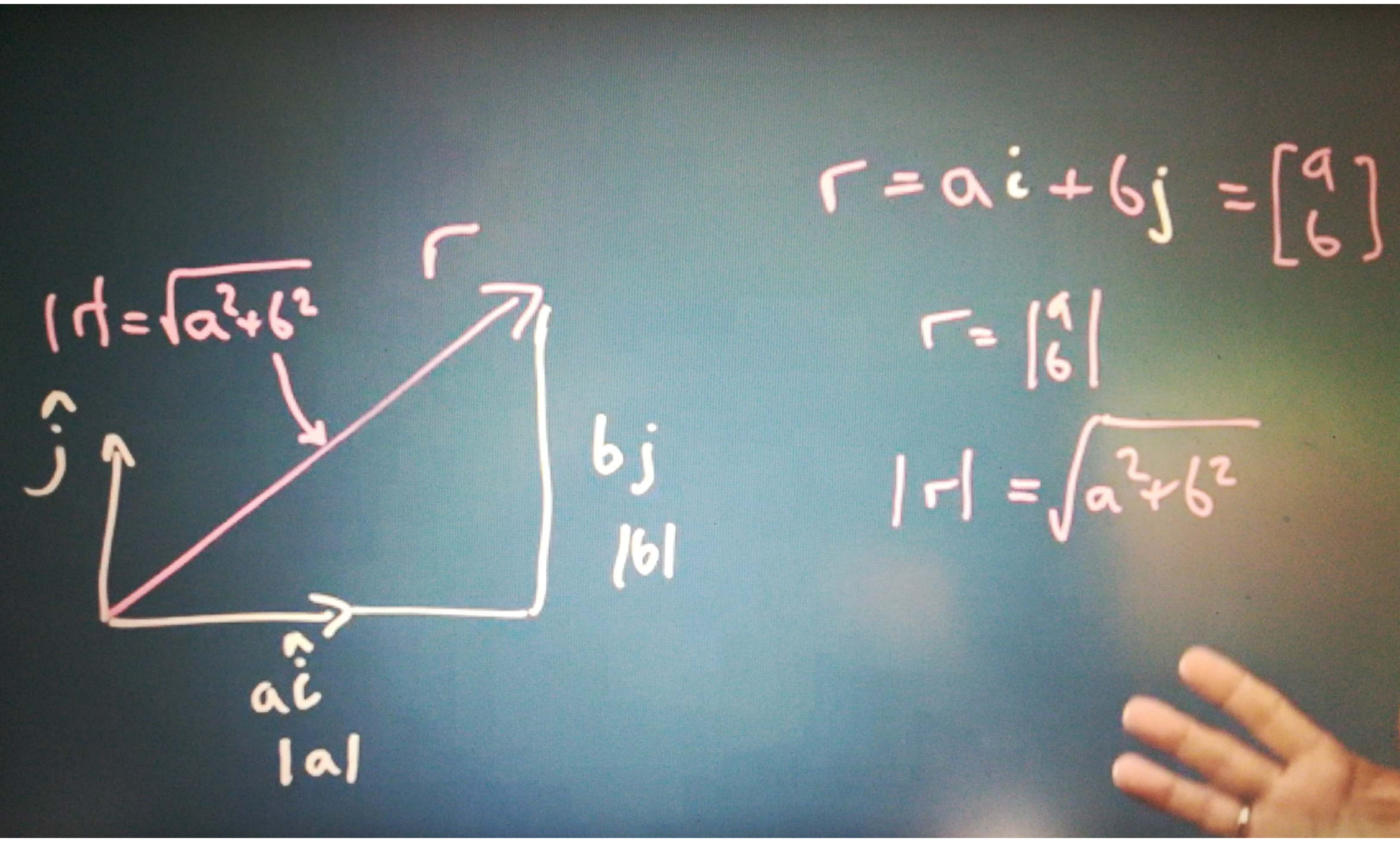


|r| = √(a² + b²)
r
ĵ
bj
|b|
aî
|a|
r = ai + bj = [a b]
r = |a b|
|r| = √(a² + b²)

✓ Congratulations! You passed!

TO PASS 80% or higher

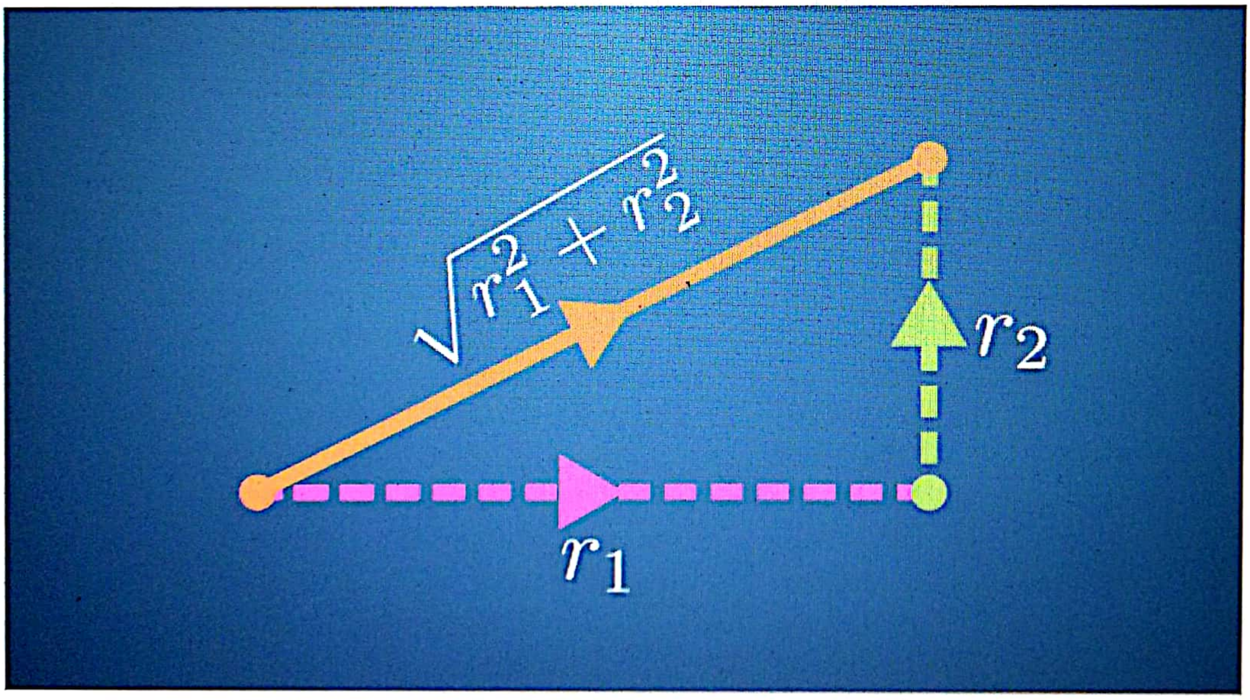
Keep Lear

Dot product of vectors

TOTAL POINTS 6

1. As we have seen in the lecture videos, the dot product of vectors has a lot of applications. Here, you will complete some exercises involving the dot product.

   We have seen that the size of a vector with two components is calculated using Pythagoras' theorem, for example the following diagram shows how we calculate the size of the orange vector $\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$:



   In fact, this definition can be extended to any number of dimensions; the size of a vector is the square root of the sum of the squares of its components. Using this information, what is the size of the vector $\mathbf{s} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$?

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In fact, this definition can be extended to any number of dimensions; the size of a vector is the square root of the sum of the squares of its components. Using this information, what is the size of the vector

$$\mathbf{s} = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 2 \end{bmatrix}?$$

- ○ $|\mathbf{s}| = 10$
- ◉ $|\mathbf{s}| = \sqrt{30}$
- ○ $|\mathbf{s}| = \sqrt{10}$
- ○ $|\mathbf{s}| = 30$

✓ **Correct**

The size of the vector is the square root of the sum of the squares of the components.

2. Remember the definition of the dot product from the videos. For two n component vectors, $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$.

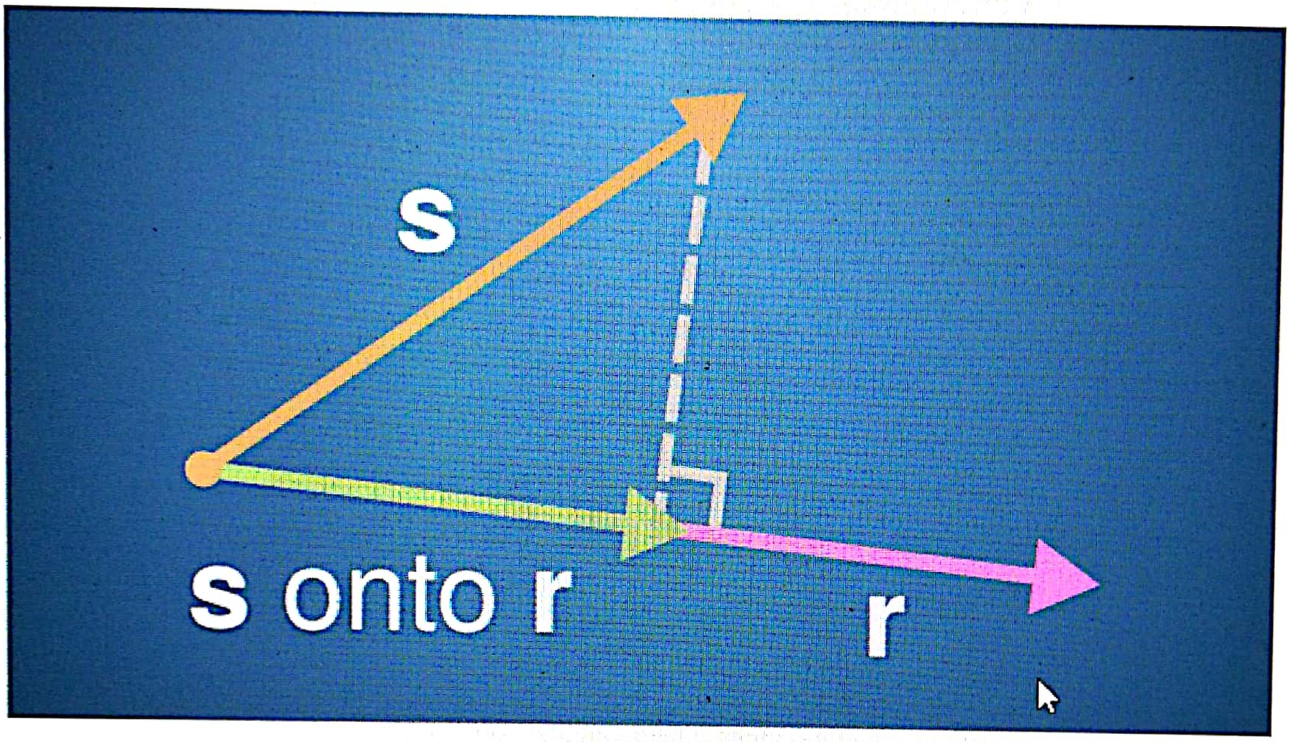
What is the dot product of the vectors $\mathbf{r} = \begin{bmatrix} -5 \\ 3 \\ 2 \\ 8 \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}$?

- ◉ $\mathbf{r} \cdot \mathbf{s} = -1$
- ○ $\mathbf{r} \cdot \mathbf{s} = 1$
- ○ $\mathbf{r} \cdot \mathbf{s} = \begin{bmatrix} -5 \\ 6 \\ -2 \\ 0 \end{bmatrix}$
- ○ $\mathbf{r} \cdot \mathbf{s} = \begin{bmatrix} -4 \\ 5 \\ 1 \\ 9 \end{bmatrix}$

✓ **Correct**

The dot product of two vectors is the total of the component-wise products.

3. The lectures introduced the idea of projecting one vector onto another. The following diagram shows the projection of s onto $\mathbf{r}$ when the vectors are in two dimensions:



Remember that the scalar projection is the *size* of the green vector. If the angle between $\mathbf{s}$ and $\mathbf{r}$ is greater than $\pi/2$, the projection will also have a minus sign.

We can do projection in any number of dimensions. Consider two vectors with three components, $\mathbf{r} = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} 10 \\ 5 \\ -6 \end{bmatrix}$.

What is the scalar projection of $\mathbf{s}$ onto $\mathbf{r}$?

- ○ $-\frac{1}{2}$
- ○ $\frac{1}{2}$
- ○ -2

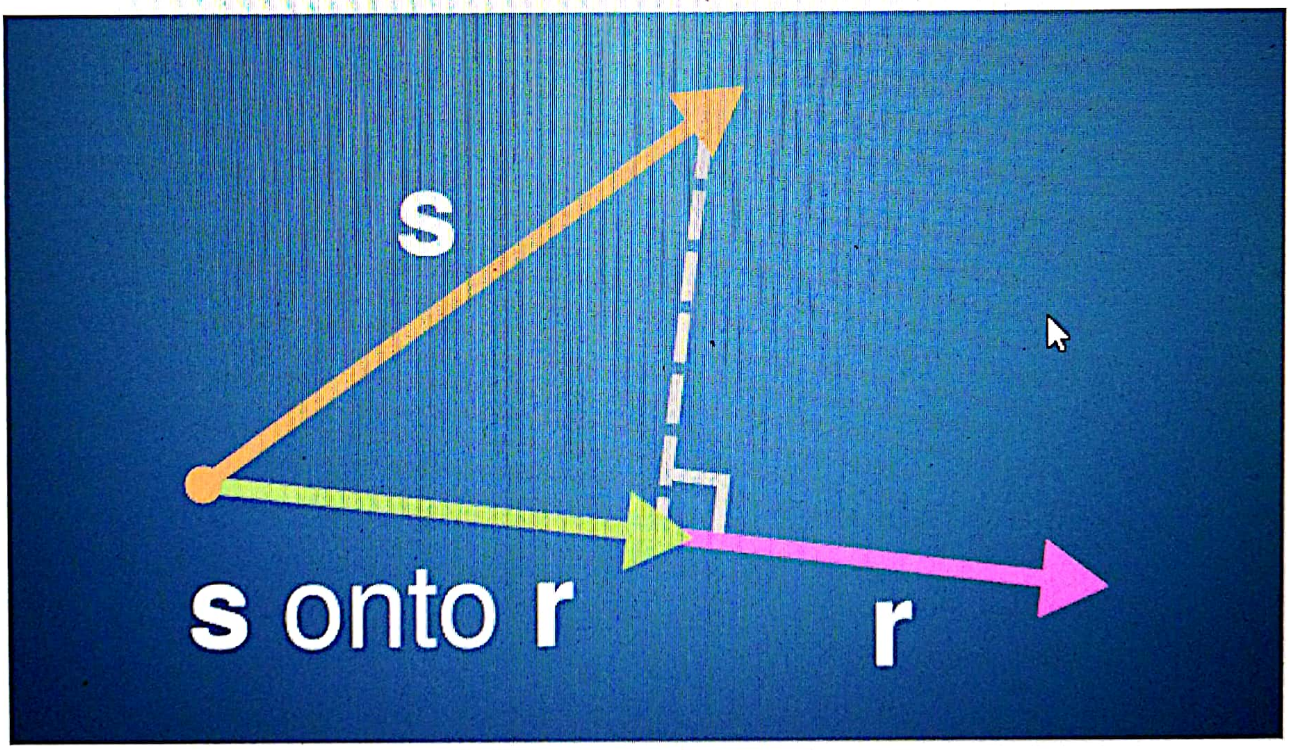
What is the scalar projection of s onto $\mathbf{r}$?

- $-\frac{1}{2}$
- $\frac{1}{2}$
- -2
- 2

✓ **Correct**

The scalar projection of of s onto $\mathbf{r}$ can be calculated with the formula $\frac{\mathbf{s} \cdot \mathbf{r}}{|\mathbf{r}|}$

4. Remember that in the projection diagram, the vector projection *is* the green vector:



Let $\mathbf{r} = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ and let $\mathbf{s} = \begin{bmatrix} 10 \\ 5 \\ -6 \end{bmatrix}$.

What is the vector projection of $\mathbf{s}$ onto $\mathbf{r}$?

Let $\mathbf{r} = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ and let $\mathbf{s} = \begin{bmatrix} 10 \\ 5 \\ -6 \end{bmatrix}$.

What is the vector projection of $\mathbf{s}$ onto $\mathbf{r}$?

- ○ $\begin{bmatrix} 6 \\ -8 \\ 0 \end{bmatrix}$
- ○ $\begin{bmatrix} 30 \\ -20 \\ 0 \end{bmatrix}$
- ○ $\begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$
- ◉ $\begin{bmatrix} 6/5 \\ -8/5 \\ 0 \end{bmatrix}$

✓ **Correct**

The vector projection of $\mathbf{s}$ onto $\mathbf{r}$ can be calculated with the formula $\frac{\mathbf{s}\cdot\mathbf{r}}{\mathbf{r}\cdot\mathbf{r}}\mathbf{r}$.

5. Let $\mathbf{a} = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 5 \\ 12 \end{bmatrix}$.

Which is larger, $|\mathbf{a}+\mathbf{b}|$ or $|\mathbf{a}| + |\mathbf{b}|$?

- ○ $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$
- ○ $|\mathbf{a}+\mathbf{b}| > |\mathbf{a}| + |\mathbf{b}|$
- ◉ $|\mathbf{a}+\mathbf{b}| < |\mathbf{a}| + |\mathbf{b}|$

✓ **Correct**

In fact, it has been shown that $|\mathbf{a}+\mathbf{b}| < |\mathbf{a}| + |\mathbf{b}|$ for every pair of vectors $\mathbf{a}$ and $\mathbf{b}$. This is

- ○ $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$
- ○ $|\mathbf{a}+\mathbf{b}| > |\mathbf{a}| + |\mathbf{b}|$
- ◉ $|\mathbf{a}+\mathbf{b}| < |\mathbf{a}| + |\mathbf{b}|$

✓ **Correct**

In fact, it has been shown that $|\mathbf{a}+\mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$ for every pair of vectors $\mathbf{a}$ and $\mathbf{b}$. This is called the triangle inequality; try to think about it in the 2d case and see if you can understand why.

6. Which of the following statements about dot products are correct?

1/1

- [x] The size of a vector is equal to the square root of the dot product of the vector with itself.

✓ **Correct**

We saw in the video lectures that $|\mathbf{r}| = \sqrt{\mathbf{r}\cdot\mathbf{r}}$.

- [x] We can find the angle between two vectors using the dot product.

✓ **Correct**

We saw in the lectures that $\mathbf{r}\cdot\mathbf{s} = |\mathbf{r}||\mathbf{s}|\cos\theta$, where θ is the angle between the vectors. This can then be used to find θ.

- [ ] The scalar projection of $\mathbf{s}$ onto $\mathbf{r}$ is always the same as the scalar projection of $\mathbf{r}$ onto $\mathbf{s}$.

- [x] The vector projection of $\mathbf{s}$ onto $\mathbf{r}$ is equal to the scalar projection of $\mathbf{s}$ onto $\mathbf{r}$ multiplied by a vector of unit length that points in the same direction as $\mathbf{r}$.

✓ **Correct**

The vector projection is equal to the scalar projection multiplied by $\frac{\mathbf{r}}{|\mathbf{r}|}$.

- [ ] The order of vectors in the dot product is important, so that $\mathbf{s}\cdot\mathbf{r} \neq \mathbf{r}\cdot\mathbf{s}$.