

# Practicing the product rule

TOTAL POINTS 6

1. In this quiz you'll have some practice using the product rule alongside the rules you've already learned.

In the previous video we considered the product of two functions,  $A(x) = f(x)g(x)$ , and saw that its derivative is given by  $A'(x) = f'(x)g(x) + f(x)g'(x)$ .

Which of the following is the product rule in  $\frac{d}{dx}$  notation?

- ☒  $\frac{dA(x)}{dx} = \frac{df(x)}{dx}g(x) + f(x)\frac{dg(x)}{dx}$
- ☐  $\frac{dA(x)}{dx} = \frac{df(x)}{dx}g(x) - f(x)\frac{dg(x)}{dx}$
- ☐  $\frac{dA(x)}{dx} = \frac{df(x)}{dx}\frac{dg(x)}{dx}$
- ☐  $\frac{dA(x)}{dx} = \frac{df(x)}{dx}\frac{dg(x)}{dx} + f(x)g(x)$

✓ Correct

It's useful to be able to translate between these different notations as you will see both in the real world.

2. When using the product rule it may help to consider how the function can be broken up into two parts, which can then be labelled  $f(x)$  and  $g(x)$ . Use this method to differentiate the function  $A(x) = (x + 2)(3x - 3)$  with respect to  $x$ .

- ☐  $A'(x) = 3$
- ☐  $A'(x) = 3x + 3$

2. When using the product rule it may help to consider how the function can be broken up into two parts, which can then be labelled  $f(x)$  and  $g(x)$ . Use this method to differentiate the function  $A(x) = (x + 2)(3x - 3)$  with respect to  $x$ .

- ☐  $A'(x) = 3$
- ☐  $A'(x) = 3x + 3$
- ☒  $A'(x) = 6x + 3$
- ☐  $A'(x) = 3x + 6$

✓ **Correct**

You can see that this gives the same result as expanding the brackets for  $(x + 2)(3x - 3)$  and then differentiating the quadratic.

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3. Remember that how we choose to label the function,  $A(x)$  or  $u(x)$  or  $f(x)$ , is not important. The key is to see if the function can be written as a product of two functions, and if so, use the product rule.

Differentiate the function  $f(x) = x^3 \sin(x)$  with respect to  $x$ .

- ☐  $f'(x) = x^3 \sin(x) - 3x^2 \cos(x)$
- ☐  $f'(x) = 3x^2 \sin(x) - x^3 \cos(x)$
- ☒  $f'(x) = 3x^2 \sin(x) + x^3 \cos(x)$
- ☐  $f'(x) = x^3 \sin(x) + 3x^2 \cos(x)$

✓ **Correct**

You identified the two functions as  $x^3$  and  $\sin(x)$  and applied the product rule correctly.

4. Using the same approach, differentiate the function  $f(x) = \frac{e^x}{x}$  with respect to  $x$ . (HINT:  $f(x) = \frac{e^x}{x} = f(x) = e^x \frac{1}{x}$ ).

☒  $f'(x) = e^x \left( \frac{1}{x} - \frac{1}{x^2} \right)$

☐  $f'(x) = -\frac{e^x}{x^2}$

☐  $f'(x) = e^x \left( \frac{1}{x} + \frac{1}{x^2} \right)$

☐  $f'(x) = \frac{e^x}{x}$

✓ **Correct**

You identified the two functions,  $e^x$  and  $\frac{1}{x}$ , and applied the product rule correctly.

5. We can extend the product rule to products of more than two functions. I

Consider the function  $u(x) = f(x)g(x)h(x)$ . Substitute  $A(x) = f(x)g(x)$  and then use the product rule *twice* to find the expression for  $u'(x)$ . This is the product rule for a product of three functions!

☒  $u'(x) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

☐  $u'(x) = f'(x)g'(x)h'(x)$

☐  $u'(x) = f(x)g(x)h'(x) + f'(x)g'(x)h(x)$

☐  $u'(x) = [f'(x)g(x) + f(x)g'(x)] h'(x)$

✓ **Correct**

You might be able to see from this how the product rule can be extended to as many functions as necessary.



6. Using your answer to the previous question, differentiate the function  $f(x) = xe^x \cos(x)$  with respect to  $x$ .

- ☐  $f'(x) = -(1+x)e^x \sin(x)$
- ☒  $f'(x) = e^x [(x+1) \cos(x) - x \sin(x)]$
- ☐  $f'(x) = -e^x \sin(x)$
- ☐  $f'(x) = e^x (x \cos(x) - \sin(x))$

✓ **Correct**

You spotted that the functions are  $x$ ,  $e^x$  and  $\cos(x)$ , then applied the product rule correctly.