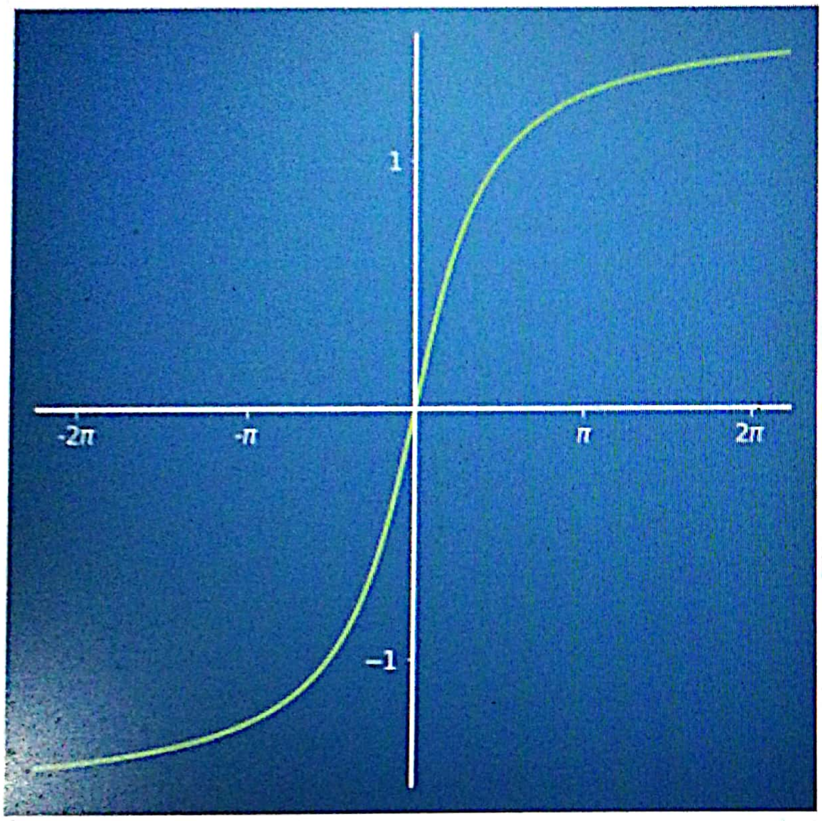
Taylor series - Special cases

TOTAL POINTS 5

1. The graph below shows the function $f(x) = \tan^{-1}(x)$



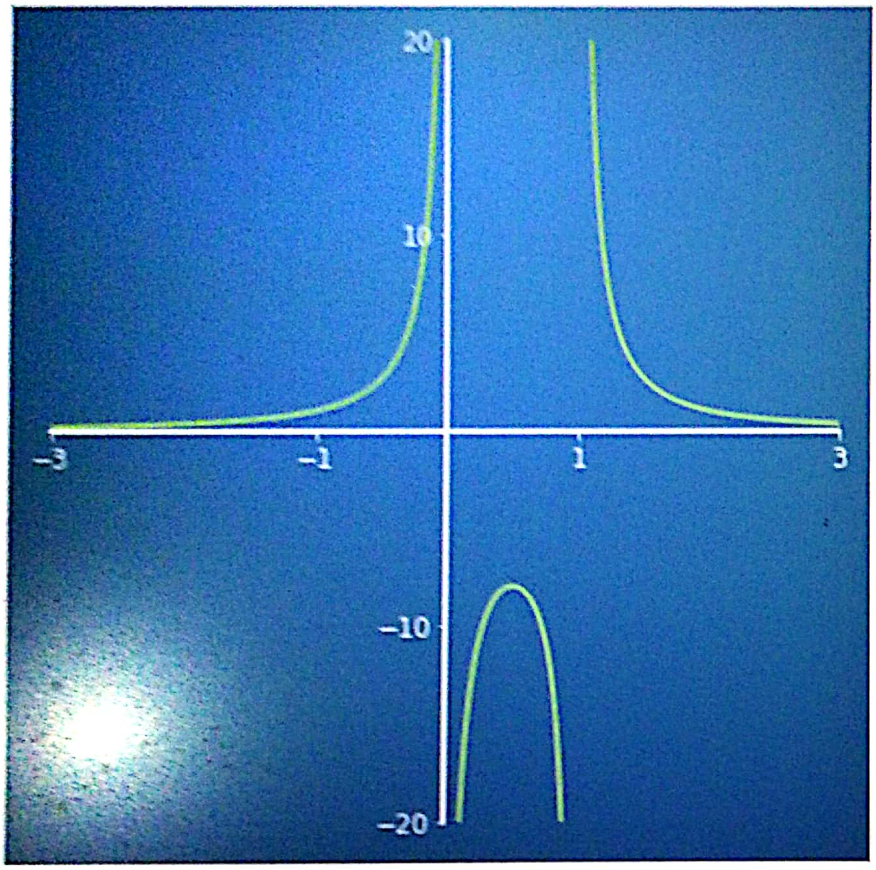
By using the Maclaurin series or otherwise, determine whether the function shown above is even, odd or neither.

- (•) Odd
- ( ) Even
- ( ) Neither odd nor even

✓ **Correct**

For an odd function, $-f(x) = f(-x)$. We can also determine if a function is odd by looking at

2. The graph below shows the discontinuous function $f(x) = \frac{2}{(x^2 - x)}$. For this function, select the starting points that will allow a Taylor approximation to be made.



- [x] $x = 2$

✓ **Correct**

A Taylor approximation centered at $x = 2$ will allow us to approximate $f(x)$ for $x > 1$only.

- [ ] $x = 1$

☑ $x = 2$

✓ **Correct**

A Taylor approximation centered at $x = 2$ will allow us to approximate $f(x)$ for $x > 1$only.

☐ $x = 1$

☑ $x = -3$

✓ **Correct**

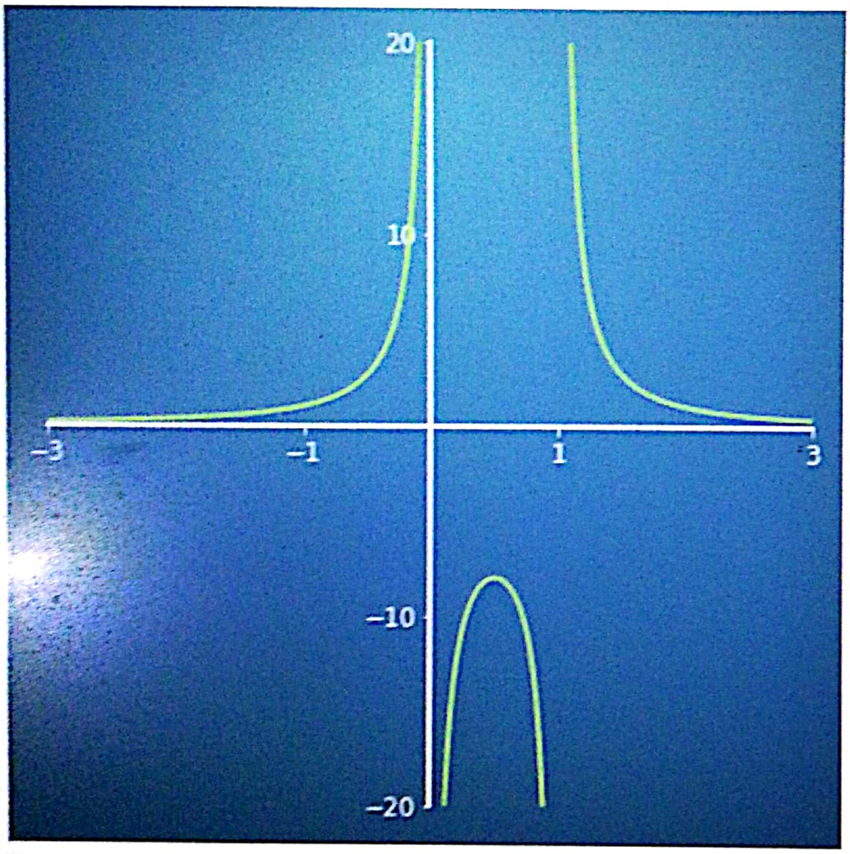
A Taylor approximation centered at $x = -3$ will allow us to approximate $f(x)$ for $x < 0$ only.

☑ $x = 0.5$

✓ **Correct**

A Taylor approximation centered at $x = 0.5$ will allow us to approximate $f(x)$ for $0 < x < 1$ only.

3. For the same function as previously discussed, $f(x) = \frac{2}{(x^2 - x)}$, select all of the statements that are true about the resulting Taylor approximation.



- [ ] This is a well behaved function
- [ ] The approximation converges quickly
- [x] Approximation ignores segments of the function

☑ Approximation ignores segments of the function

✓ **Correct**

Due to the discontinues function and the range of x values in which it remains well behaved, the starting point of the Taylor series dictates the domain of the function we are trying to approximate.

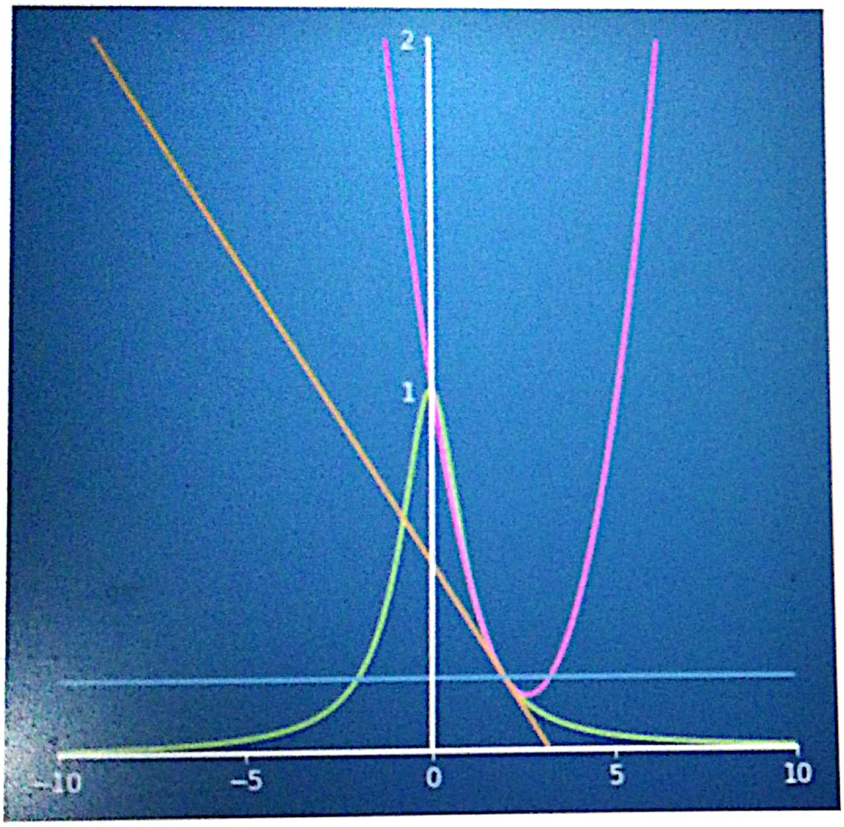
☐ Approximation accurately captures the asymptotes

☑ Approximation ignores the asymptotes

✓ **Correct**

Taylor series approximations often find it difficult to capture asymptotes correctly. For example, the zeroth and first order terms cut directly through an asymptote in most cases.

4. The graph below highlights the function $f(x) = \frac{1}{(1+x^2)}$ (green line), with the Taylor expansions for the first 3 terms also shown about the point $x = 2$. The Taylor expansion is $f(x) = \frac{1}{5} - \frac{4(x-2)}{25} + \frac{11(x-2)^2}{125} + \ldots$. Although the function looks rather normal, we find that the Taylor series does a bad approximation further from its starting point, not capturing the turning point. What could be the reason why this approximation is poor for the function described.



- [ ] Function does not differentiate well
- [x] Asymptotes are in the complex plane

✓ **Correct**

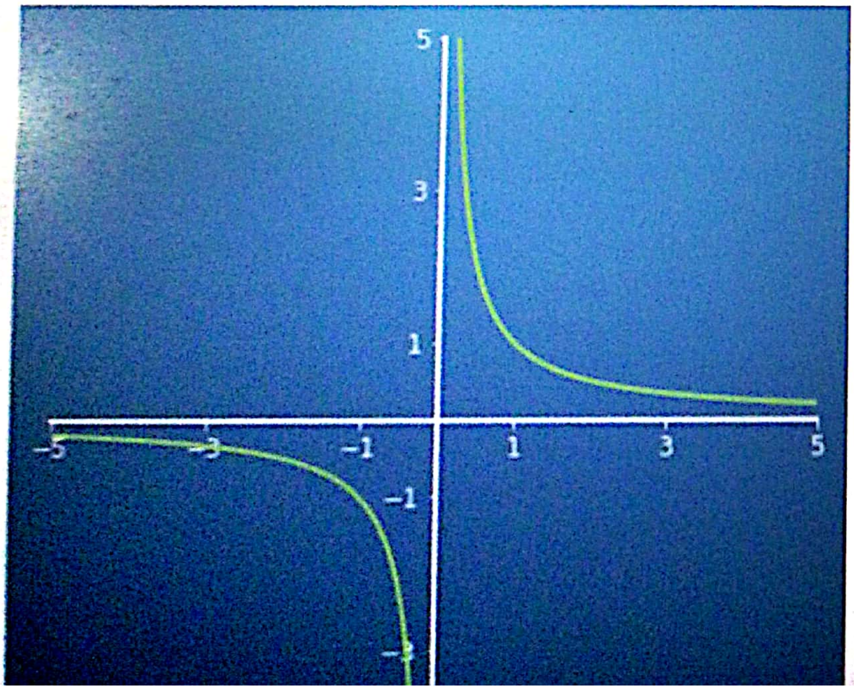
Although this function is well behaved in the real plane, in the imaginary plane, the asymptotes limit its convergence and the behaviour of the Taylor expansion, which is shown to behave badly for functions that are discontinuous.

- [ ] The function has no real roots
- [ ] None of these options
- [x] It is a discontinuous function in the complex plane

✓ **Correct**

Although this function is well behaved in the real plane, in the imaginary plane, the asymptotes limit its convergence and the behaviour of the Taylor expansion, which is shown to behave badly for functions that are discontinuous.

5. For the function $f(x) = \frac{1}{x}$, provide the linear approximation about the point $x = 4$, ensuring it is second order accurate.



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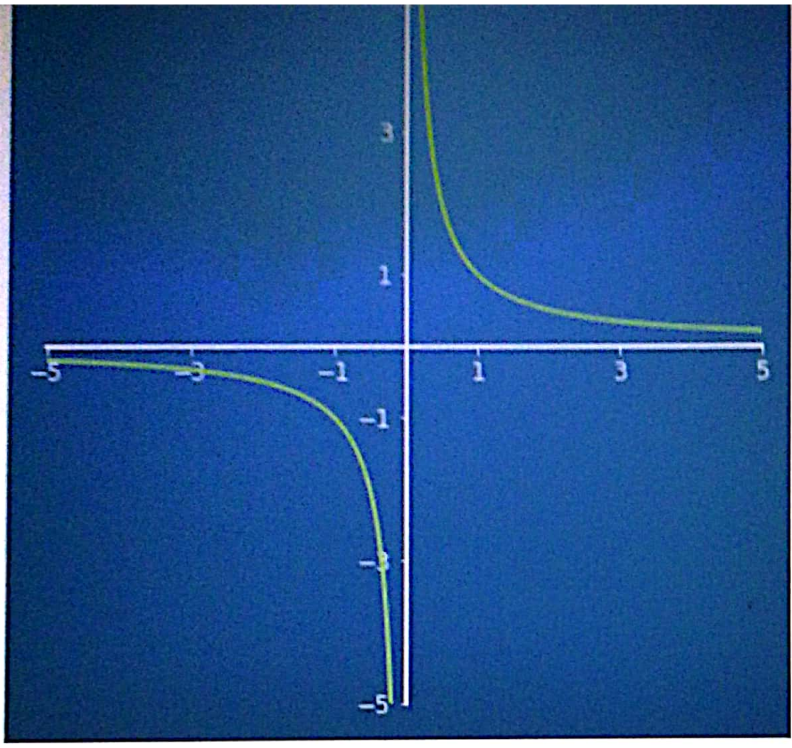

- [x] $f(x) = 1/4 - (x-4)/16 + O(\Delta x^2)$
- [ ] $f(x) = 1/4 + x/16 - O(\Delta x^2)$
- [ ] $f(x) = 1/4 - (x-4)/16 + O(\Delta x)$
- [ ] $f(x) = 1/4 - x/16 + O(\Delta x^2)$

✓ **Correct**

Second order accurate means we have a first order Taylor series. All the terms above are sufficiently small, assuming Δx is small.