



Today's agenda

- ↳ understanding sorting
- ↳ Problems on sorting
- ↳ Sorting techniques



AlgoPrep



Sorting: Arranging data in increasing/decreasing.

↳ on what parameter.

Ex1: 2 4 10 15 27 → true

Ex2: 20 7 3 -5 -8 → true

Ex3: 1 2 3 7 4 9 6

→ not sorted on the basis of value.

#/factor: 1 2 2 2 3 3 4 → sorted on the basis of factor count.

- bubble sort
- selection sort
- insertion sort
- merge sort
- quick sort
- bucket sort

→ merge sort

arr[4]: 3 1 9 8

↓

Arrays.sort(arr);

→ inc. order.

↓

↳ T.C: $O(N \log N)$

arr[4]: 1 3 8 9



Q) Order of Removal

↳ Given n elements at every step remove an array element. Cost to remove element = Sum of array elements present. Find min cost to remove all elements.

Note: Add cost first and then remove.

$$\text{Ex1: } \text{arr}[3] = \{ \overset{\times}{\underset{0}{3}} \overset{\times}{\underset{1}{2}} \overset{\times}{\underset{1}{5}} \}$$

$$\text{remove } 3: 10$$

$$\text{remove } 2: 7$$

$$\text{remove } 5: \underline{5}$$

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$$\text{arr}[3] = \{ \overset{\times}{\underset{0}{3}} \overset{\times}{\underset{1}{2}} \overset{\times}{\underset{1}{5}} \}$$

$$\text{remove } 2: 10$$

$$\text{remove } 5: 8$$

$$\text{remove } 3: \underline{3}$$

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$$\text{Ex2: } \text{arr}[4] = \{ \overset{\times}{\underset{0}{4}} \overset{\times}{\underset{1}{6}} \overset{\times}{\underset{2}{2}} \overset{\times}{\underset{2}{7}} \}$$

$$\text{remove } 7: 4 + 6 + 2 + 7$$

$$\text{remove } 2: 4 + 6 + 2$$

$$\text{remove } 6: 4 + 6$$

$$\text{remove } 4: \underline{4}$$

minimize overall cost

1st min Contribution = 0th index = min ele

2nd min Contribution = 1st index = 2nd min ele.

3rd min ele.

→ Array should be sorted in inc. order.



$arr[4] = \{ 4^0, 6^1, 2^2, 7^3 \}$

→ removal cost: $N-i$

\downarrow
 $\{ 2^0, 4^1, 6^2, 7^3 \}$
costs ↓ gains ↓ time ↓ time

// pseudo code

```
int removalOrder (int arr[N]) {
```

```
    Arrays.sort(arr);
```

```
    int ans = 0;
```

```
    for (int i = N-1; i >= 0; i--) {  
        ans = ans + (arr[i] * (N-i));  
    }
```

```
    return ans;
```

T.C: $O(N \log N + N)$

$= O(N \log N)$

S.C: $O(1)$



arr[4]: 4 6 2 7

```
int removeOrder (int arr[N]) {  
    Arrays.sort (arr);  
    int ans = 0;
```

```
    for (int i = N-1; i >= 0; i--) {  
        ans = ans + (arr[i] * (N-i));  
    }
```

```
    return ans;
```

arr[4]: { 2 4 6 7 }

ans = 0 + 7 + 2*6 + 2*4 + 2*2 = 39

i = 3 : 2 + 4 + 6 + 7

i = 2 : 2 + 4 + 6

i = 1 : 2 + 4

i = 0 : 2

3



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An element is said to be good if

{No. of element $< \text{ele} :: \text{ele itself}$ }

↓
a00 β)

Ex 1: $\{ \overset{0}{-1} \overset{1}{-4} \overset{2}{3} \overset{3}{5} \overset{4}{-15} \overset{5}{4} \} \Rightarrow \text{ans} = 3$

Index: $\begin{matrix} 2 & 1 & 3 & 5 & 0 & 4 \end{matrix}$

↳ for every number, count smaller than that number and check if they are same. {nested loop}.

S.C: $O(1)$

↳ $\{ \overset{0}{-1} \overset{1}{-4} \overset{2}{3} \overset{3}{9} \overset{4}{-15} \overset{5}{7} \}$

$\{ -15 \quad -4 \quad -1 \quad 3 \quad 7 \quad 9 \}$
 $\downarrow \quad \downarrow \quad \downarrow$
 $2 \quad 4 \quad 5$

After sorting, Count of elements smaller than $arr[i] = i$



// Pseudo Code

```
int goodIntegers (int arr[N]) {  
    Arrays.sort (arr);  
    int count = 0;  
  
    for (int i = 0; i < N; i++) {  
        if (arr[i] == i) { count++; }  
    }  
    return count;  
}
```

T.C: $O(N \log N)$
S.C: $O(1)$



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{No. of element < ele == ele itself}



Good integers : {Data can repeat}

Ex1: { 0 2 2 3 3 8 } \rightarrow ans = 3
#less : 0 1 1 3 3 5

Ex2: { -4 -4 2 2 5 5 5 5 8 8 8 8 10 17 }
#less : 0 0 2 2 4 4 4 4 8 8 8 11 12

After sorting

ans = 5

obs1: All the occurrences of same number will be either good or bad.

obs2: if ele is 1st occ \rightarrow arr[i] != arr[i-1]

\Downarrow

No. of ele < ele == i

i == arr[i]

obs3: if ele is not the 1st occ.



// Pseudo code

```
int goodintegerduplicate (int arr[N]) {  
    Arrays.sort (arr);  
    int count = 0;  
  
    int lesscount = 0;  
    for (int i = 1; i < N; i++) {  
        if (arr[i] != arr[i-1]) { // first occ.  
            lesscount = i;  
        }  
        else { // next occ.  
        }  
        if (arr[i] == lesscount) { count++; }  
    }  
}
```

T.C: $O(N \log N)$
S.C: $O(1)$



Ex2: { -4 -4 2 2 5 5 5 5 8 8 8 10 12 }

if less: 0 0 2 2 4 4 4 4 8 8 8

```
int goodintegerduplicate (int arr[n]) {
```

```
    Arrays.sort(arr);
```

```
    int count = 0;
```

```
    if (arr[0] == 0) { count++; }
```

```
    int lesscount = 0;
```

```
    for (int i = 1; i < n; i++) {
```

```
        if (arr[i] != arr[i-1]) { // first occ.
```

```
            lesscount = i;
```

```
        }
```

```
        else { // next occ.
```

```
            // nothing
```

```
        if (arr[i] == lesscount) { count++; }
```

```
    }
```

3

Count = 5 lesscount = 8



// Sorting techniques

① Bubble Sort

↳ Sort the array in asc. order but we can swap adjacent elements only.

arr[8] = { 5 7 5 4 10 -2 6 3 }

iter 0: { 5 7 5 4 10 -2 6 3 } → { 0, N-2 }
5 7 5 4 10 -2 6 3
{ 5 5 4 7 -2 6 3 10 }

iter 1: { 5 7 5 4 10 -2 6 3 } → { 0, N-3 }
5 7 5 4 10 -2 6 3
{ 5 4 5 -2 6 3 7 10 }

iter 2: { 0, N-4 }

iter 6

iter 7 ??

↳ N length → N-1 iteration
{ 0, N-2 }



// Pseudo code

```
void bubbleSort (int arr[N]) {  
     $i \leq N-1$   
    for (i=0; i <= N-2; i++) {  
        for (int j=0; j <= N-2-i; j++) {  
            if (arr[j] > arr[j+1]) {  
                int temp = arr[j];  
                arr[j] = arr[j+1];  
                arr[j+1] = temp;  
            }  
        }  
    }  
}
```

T.C: $O(N^2)$
S.C: $O(1)$

3
3
3



arr[8] = { 5 7 5 4 10 -2 6 3 }

void bubbleSort (int arr[]) {

for (i=0; i<N-1; i++) {

for (int j=0; j<N-2-i; j++) {
if (arr[j] > arr[j+1]) {
int temp = arr[j];
arr[j] = arr[j+1];
arr[j+1] = temp;
}

}

}

i=0

arr[8] = { 5 7 5 4 10 -2 6 3 }

{ 5 5 4 7 -2 6 3 10 }

i=1

j → 10, 5

arr[8] = { 5 5 4 7 -2 6 3 10 }

{ 5 4 5 -2 6 3 7 10 }