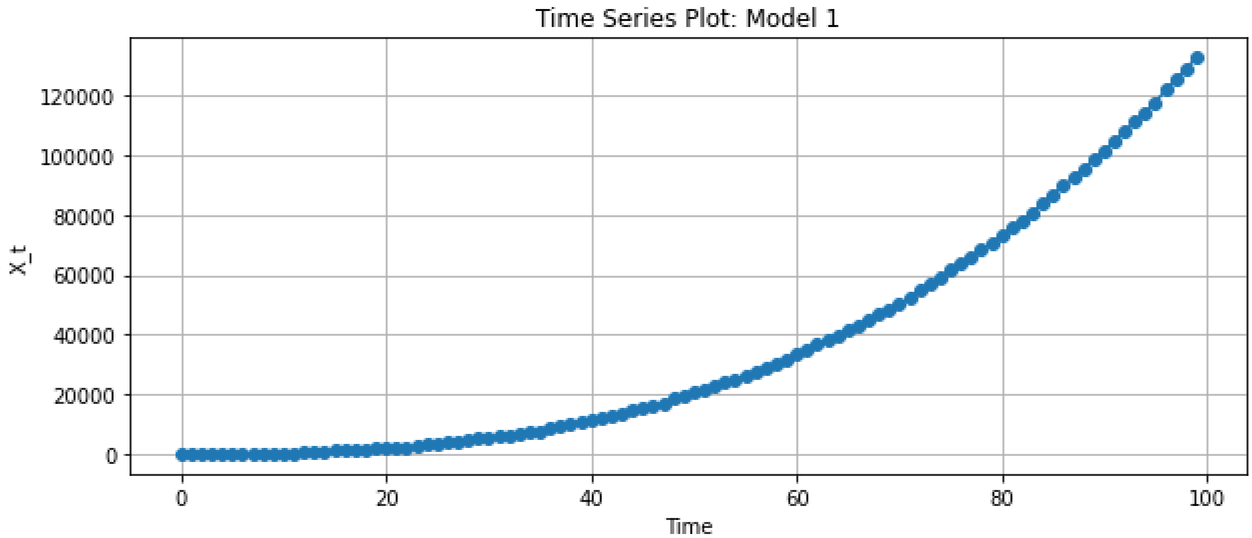
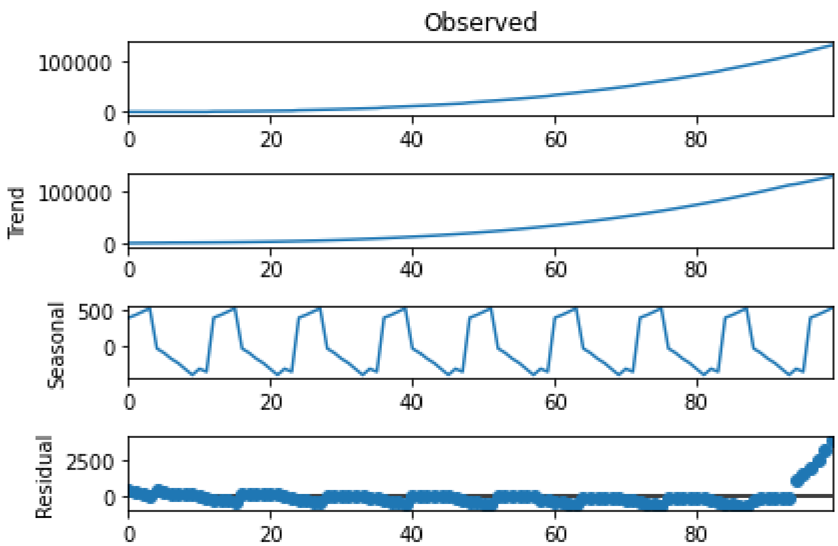
Q.1 Aim: To generate 100 observations from the given models.

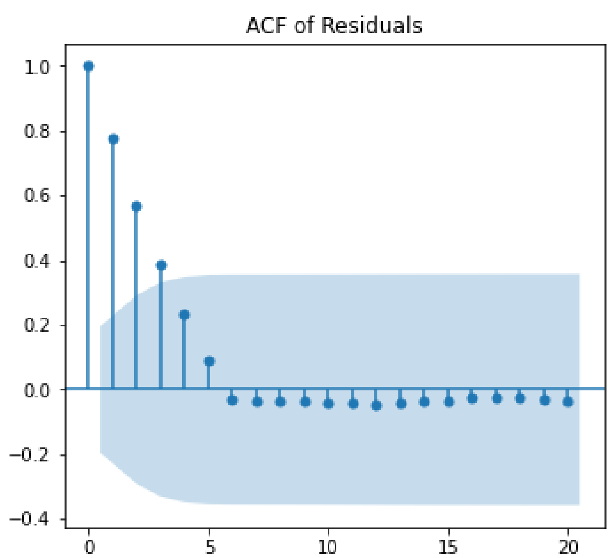
Model 1

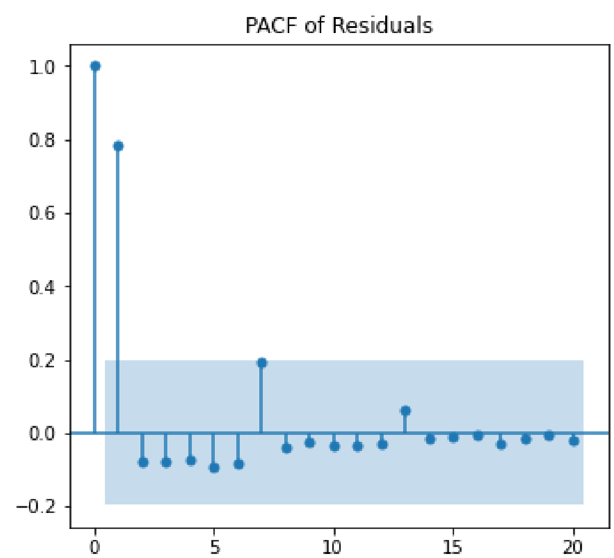
In [17]:

```
import numpy as np
import matplotlib.pyplot as plt
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

# 1. Generate data
np.random.seed(0)
n = 100
t = np.arange(1, n+1)

Z = np.random.normal(0, np.sqrt(2), n)
X = np.zeros(n)

for i in range(12, n):
    X[i] = 5 + 2*t[i] + 4*t[i]**2 + X[i-12] + Z[i]
# 2. Plot the series
plt.figure(figsize=(10,4))
plt.plot(X, marker='o')
plt.title("Time Series Plot: Model 1")
plt.xlabel('Time')
plt.ylabel('X_t')
plt.grid(True)
plt.show()

# 3. Decompose series
# As no real seasonality, just trying to remove trend
result = seasonal_decompose(X, model='additive', period=12, extrapolate_trend='freq')
result.plot()
plt.show()

# 4. Extract and plot residuals
residuals = result.resid
residuals = residuals[~np.isnan(residuals)]  # Remove NaNs

# 5. Plot ACF and PACF
plt.figure(figsize=(12,5))
plt.subplot(121)
plot_acf(residuals, ax=plt.gca(), title="ACF of Residuals")
plt.subplot(122)
plot_pacf(residuals, ax=plt.gca(), title="PACF of Residuals")
plt.show()
```









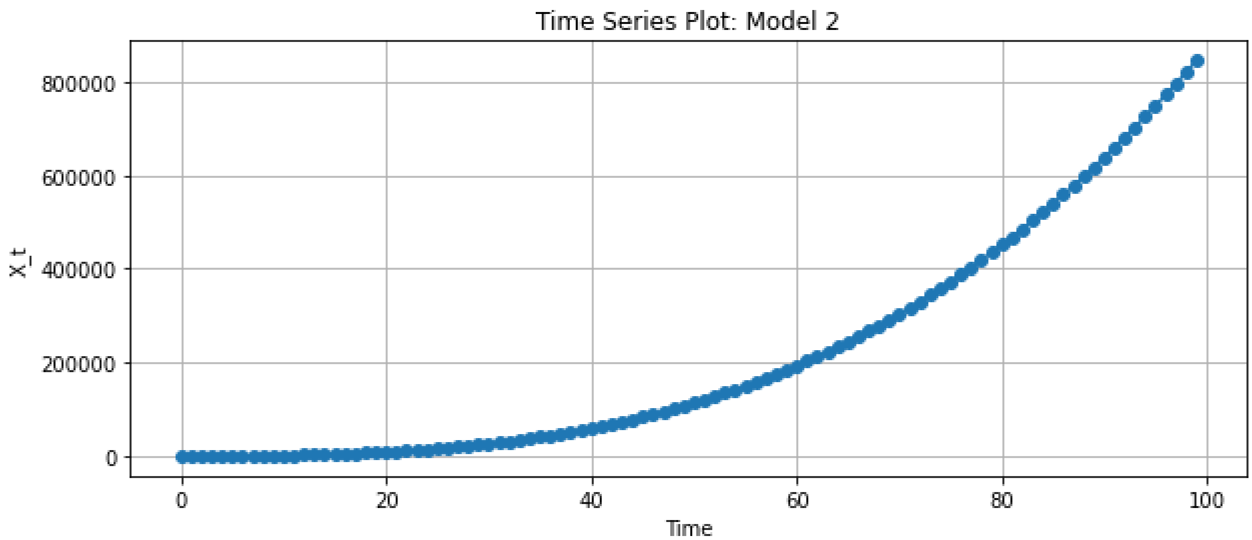
Interpretation:

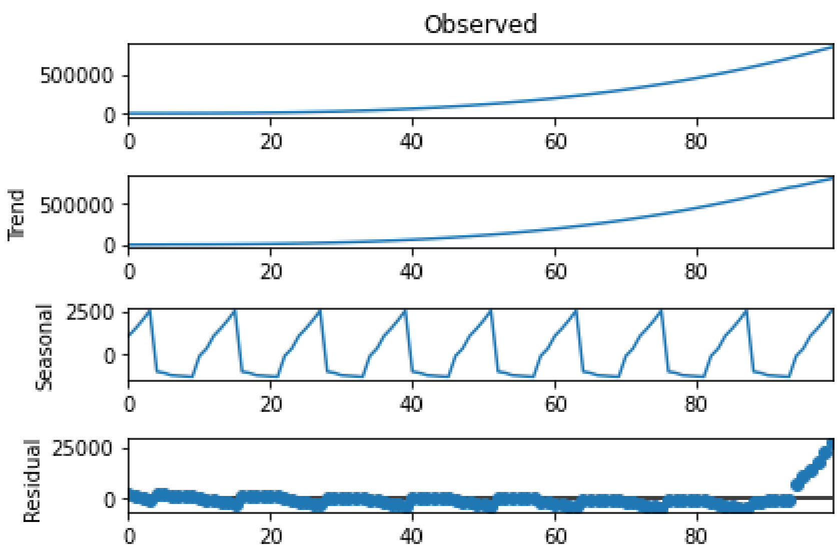
The time series plot shows a strong upward quadratic trend with clear seasonality repeating every 12 periods. Decomposition confirms the steep trend and regular seasonal pattern, while the ACF and PACF of residuals indicate slight autocorrelation at lag 1 and lag 12, suggesting minor AR(1) and seasonal effects remain after detrending.

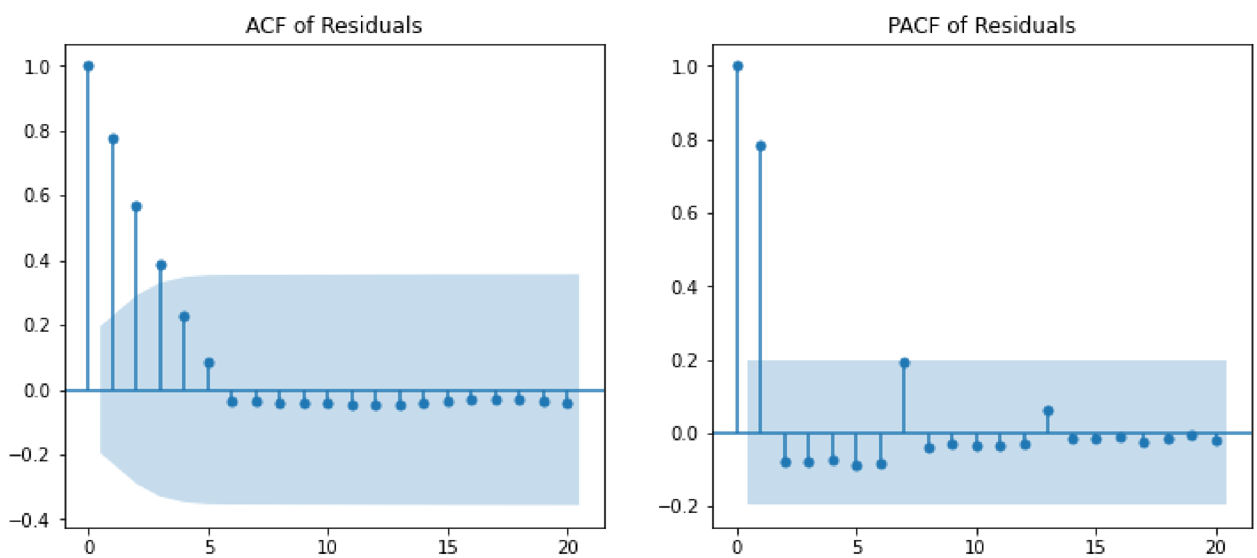
Model 2

In [12]:

```
import numpy as np
import matplotlib.pyplot as plt
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

# 1. Generate data
np.random.seed(0)
n = 100
t = np.arange(1, n+1)

Z = np.random.normal(0, np.sqrt(0.2), n)
X = np.zeros(n)

for i in range(1, n):
    X[i] = 0.5 + 2.5*t[i]**2 + X[i-1] + Z[i]

# 2. Plot the series
plt.figure(figsize=(10,4))
plt.plot(X, marker='o')
plt.title("Time Series Plot: Model 2")
plt.xlabel('Time')
plt.ylabel('X_t')
plt.grid(True)
plt.show()

# 3. Decompose series
result = seasonal_decompose(X, model='additive', period=12, extrapolate_trend='freq')
result.plot()
plt.show()

# 4. Extract and plot residuals
residuals = result.resid
residuals = residuals[~np.isnan(residuals)]

# 5. Plot ACF and PACF
plt.figure(figsize=(12,5))
plt.subplot(121)
plot_acf(residuals, ax=plt.gca(), title="ACF of Residuals")
plt.subplot(122)
plot_pacf(residuals, ax=plt.gca(), title="PACF of Residuals")
plt.show()
```







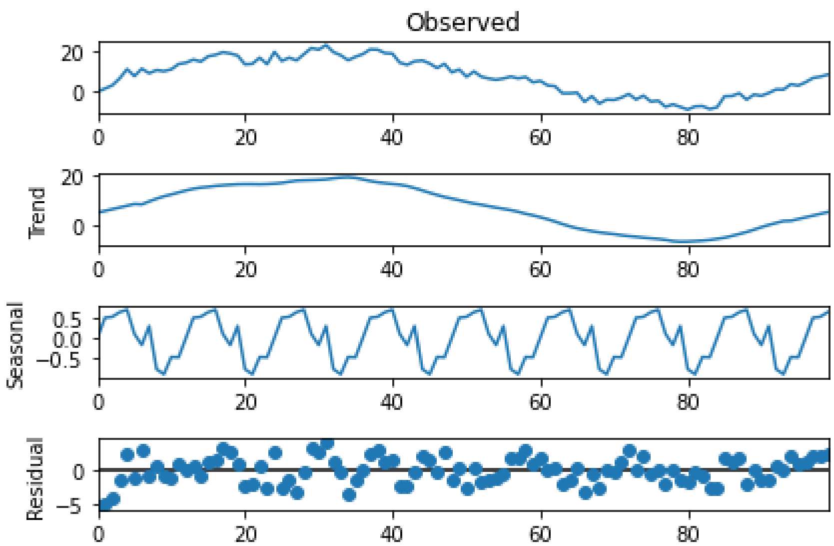
Interpretation:

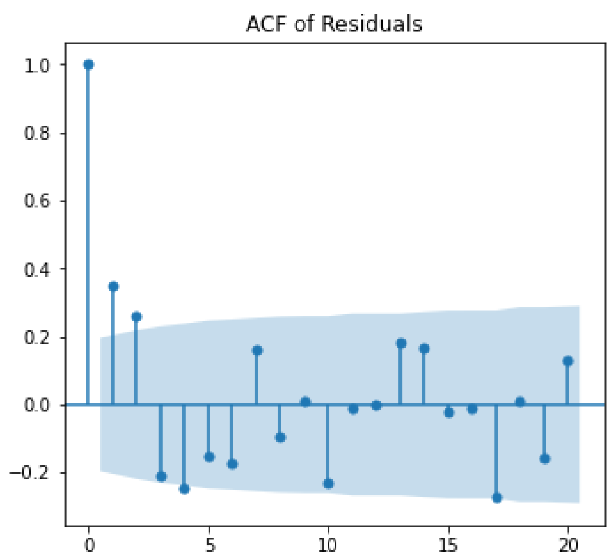
The time series plot shows a strong upward nonlinear trend, but slightly less steep compared to Model 1. Decomposition confirms an increasing trend and seasonal pattern with smaller fluctuations. The ACF and PACF of residuals show autocorrelations at initial lags, especially lag 1, suggesting minor AR(1) structure with some leftover seasonality.

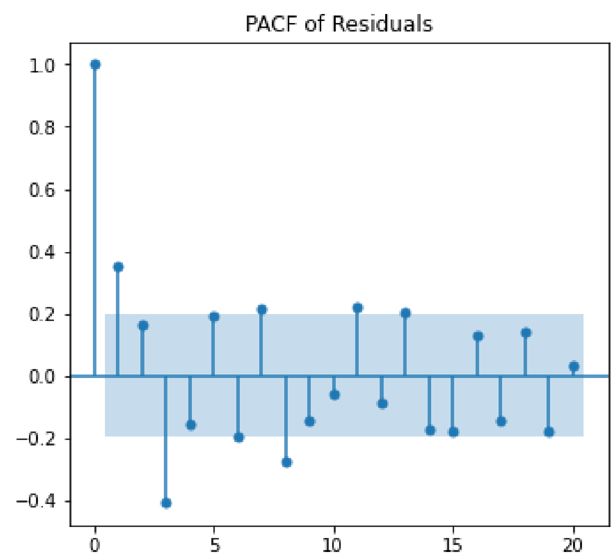
Model 3

In [13]:

```
import numpy as np
import matplotlib.pyplot as plt
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

# 1. Generate data
np.random.seed(0)
n = 100
t = np.arange(1, n+1)

Z = np.random.normal(0, np.sqrt(3.2), n)
X = np.zeros(n)

for i in range(1, n):
    X[i] = 2 * np.sin(2 + 3.5 * t[i]) + X[i-1] + Z[i]

# 2. Plot the series
plt.figure(figsize=(10,4))
plt.plot(X, marker='o')
plt.title("Time Series Plot: Model 3")
plt.xlabel('Time')
plt.ylabel('X_t')
plt.grid(True)
plt.show()

# 3. Decompose series
result = seasonal_decompose(X, model='additive', period=12, extrapolate_trend='freq')
result.plot()
plt.show()

# 4. Extract and plot residuals
residuals = result.resid
residuals = residuals[~np.isnan(residuals)]

# 5. Plot ACF and PACF
plt.figure(figsize=(12,5))
plt.subplot(121)
plot_acf(residuals, ax=plt.gca(), title="ACF of Residuals")
plt.subplot(122)
plot_pacf(residuals, ax=plt.gca(), title="PACF of Residuals")
plt.show()
```







Interpretation:

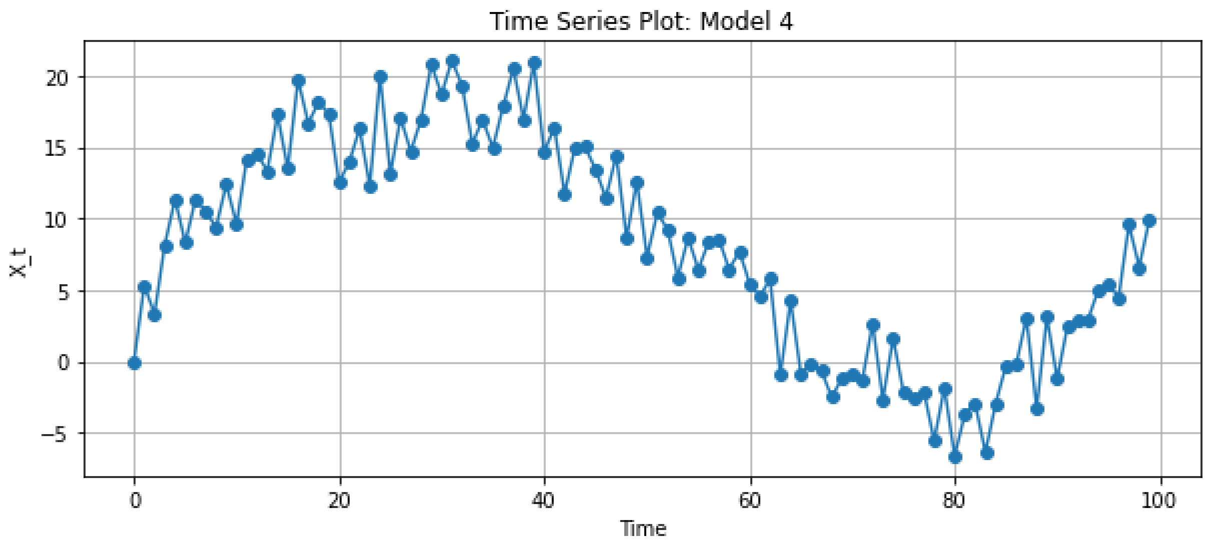
The time series plot of Model 3 shows a wave-like oscillating pattern around zero without a clear trend, indicating a strong seasonal behavior. Decomposition confirms periodic seasonality, while

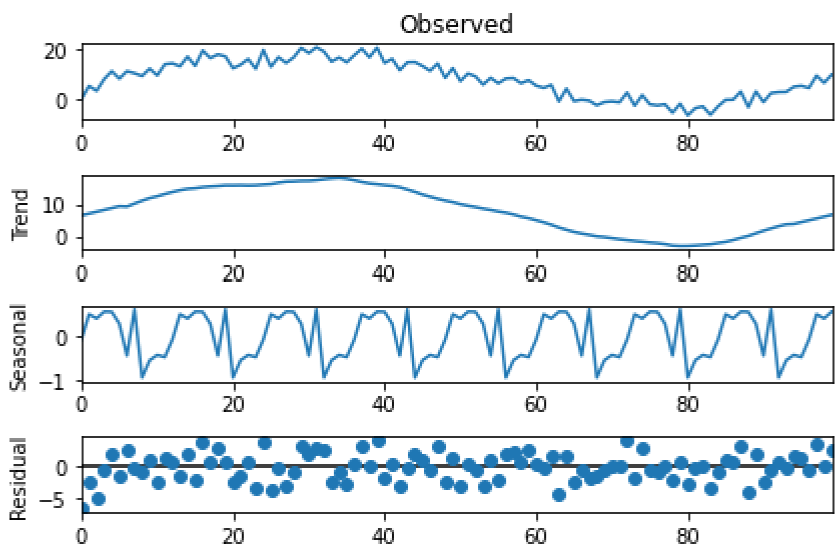
ACF and PACF plots show significant spikes at seasonal lags, suggesting strong dependence on past seasonal values.

Model 4

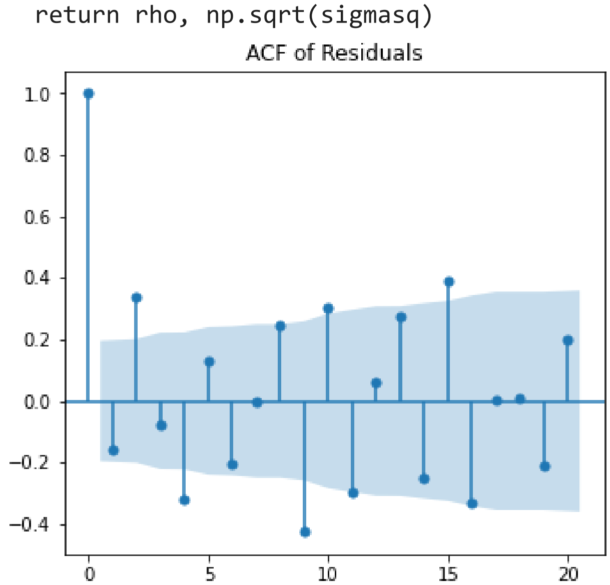
In [14]:

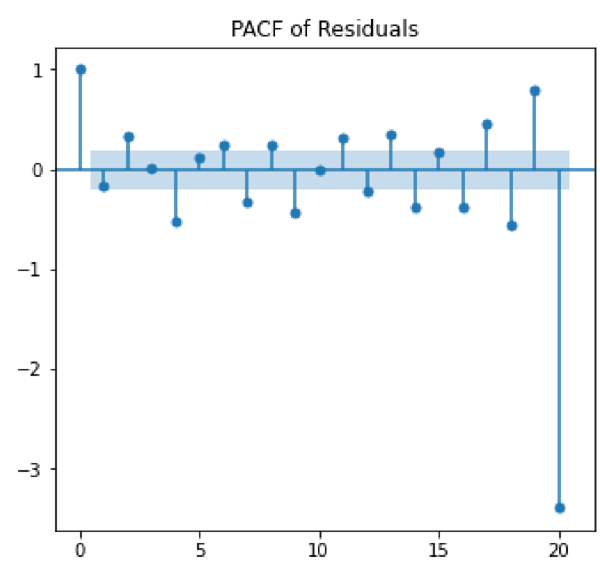
```
import numpy as np
import matplotlib.pyplot as plt
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

# 1. Generate data
np.random.seed(0)
n = 100
t = np.arange(1, n+1)

Z = np.random.normal(0, np.sqrt(2.2), n)
X = np.zeros(n)

for i in range(1, n):
    X[i] = 2 * np.cos(3 * t[i]) + 3 * np.sin(2.5 + 2.5 * t[i]) + X[i-1] + Z[i]

# 2. Plot the series
plt.figure(figsize=(10,4))
plt.plot(X, marker='o')
plt.title("Time Series Plot: Model 4")
plt.xlabel('Time')
plt.ylabel('X_t')
plt.grid(True)
plt.show()

# 3. Decompose series
result = seasonal_decompose(X, model='additive', period=12, extrapolate_trend='freq')
result.plot()
plt.show()

# 4. Extract and plot residuals
residuals = result.resid
residuals = residuals[~np.isnan(residuals)]

# 5. Plot ACF and PACF
plt.figure(figsize=(12,5))
plt.subplot(121)
plot_acf(residuals, ax=plt.gca(), title="ACF of Residuals")
plt.subplot(122)
plot_pacf(residuals, ax=plt.gca(), title="PACF of Residuals")
plt.show()
```






```
C:\Users\sgmck\anaconda3\lib\site-packages\statsmodels\regression\linear_model.py:1434:
RuntimeWarning: invalid value encountered in sqrt
  return rho, np.sqrt(sigmasq)
```





Interpretation:

The time series plot of Model 4 shows a clear periodic sinusoidal pattern with no strong trend, indicating strong seasonality. The ACF and PACF plots also show significant cyclical spikes, confirming the presence of seasonal correlations.