

Graduate Assistants:
Clinton Curry
Brent Shivers

UAB/EdGrid
John C. Mayer
UAB/Mathematics

Mathematical Modeling

- Creates a mathematical representation of some phenomenon to better understand it.
- Matches observation with symbolic representation.
- Informs theory and explanation.

The success of a mathematical model depends on how easily it can be used, and how accurately it predicts and how well it explains the phenomenon being studied.



Mathematical Modeling

- A mathematical model is central to most computational scientific research.
- Other terms often used in connection with mathematical modeling are
 - Computer modeling
 - Computer simulation
 - Computational mathematics
 - Scientific Computation





Mathematical Modeling and the Scientific Method

 How do we incorporate mathematical modeling/computational science in the scientific method?





Mathematical Modeling Problem-Solving Steps

- Identify problem area
- Conduct background research
- State project goal
- Define relationships
- Develop mathematical model

- Develop computational algorithm
- Perform test calculations
- Interpret results
- Communicate results







Syllabus: MA 261/419/519

Introduction to Mathematical Modeling

- Prerequisite: Calculus 1
- Goals
 - Learn to build models
 - Understand mathematics behind models
 - Improve understanding of mathematical concepts
 - Communicate mathematics
- Models may be mathematical equations, spreadsheets, or computer simulations.





Contact Information

- Instructor: Dr. John Mayer CH 490A
 - mayer@math.uab.edu 934-2154
 - **-** ??
- Assistant: Mr. Robert Cusimano CH 478B
 - rob5236@uab.edu 934-2154
 - ??MW 4:00 5:30 PM (computer lab CB 112)
- Assistant: Ms. Jeanine Sedjro ??
 - **-** ??
 - ??MW 6:45 8:00 PM (computer lab CB 112)





Class Meetings

• Lectures.

- Monday andWednesday
- -5:30 6:45 PM
- -CB15 112.

Computer lab.

- Monday and Wednesday
- -4:00-5:30 PM
- -6:45 8:00 PM
- -CB15 112.





Software Tools

- Spreadsheet:
 - Microsoft EXCEL
- Compartmental Analysis and System Dynamics programming:
 - Isee Systems STELLA





Computer Lab

- The only computer labs that have STELLA installed are CB15 112 and EDUC 149A.
- Always bring a formatted 3.5" diskette to the lab.
- Label disk: "Math Modeling, Spring, 2006, Your Name,

Math Phone Number: 934-2154"





Assignments

- One or two assignments every week.
- First few assignments have two due dates.
 - Returned next class.
 - Re-Graded for improved grade.
- Written work at most two authors.
 - You may work together with a partner on the computer.





Midterm Tests

- Two tests, one about every 5 weeks.
- Given in the computer lab.
- Basic building blocks relevant to the kinds of models we are constructing.
- Mathematics and logic behind the computer models.





System Dynamics Stories and Projects

- System Dynamics Stories.
 - Scenarios describing realistic situations to be modeled.
 - Entirely independent work no partners.
 - Construct model and write 5-10 page technical paper (template provided).
- Project due date to be announced.
 - Begin work about middle of term.
 - Preliminary evaluation three weeks prior to due date.



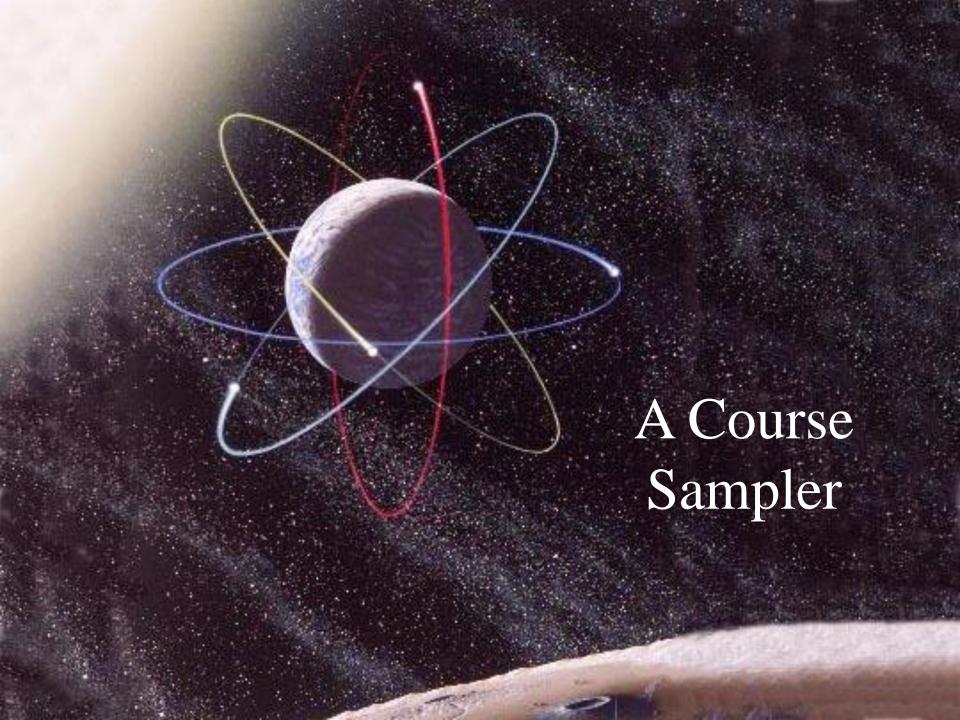


Grading

	MA 261	MA 419	MA 519
Assignments	40%	30%	30%
Tests (2)	30%	30%	25%
Model 1	10%	10%	10%
Model 2	na	na	10%
Paper	20%	20%	15%
Lesson Plan	na	10%	10%







Topics for Tools

Spreadsheet Models

- Data analysis: curve fitting.
- Recurrence Relations
 and difference
 equations.
- Cellular Automata:
 nearest-neighbor
 averaging.

Compartmental Models

- Introduction toSystem Dynamics.
- Applications of System Dynamics.
- System DynamicsStories (guided projects).





Spreadsheet Models: Excel

- Curve fitting introduction to (linear) regression
- Difference Equations: modeling growth
- Nearest-neighbor averaging





Morteville

by Doug Childers

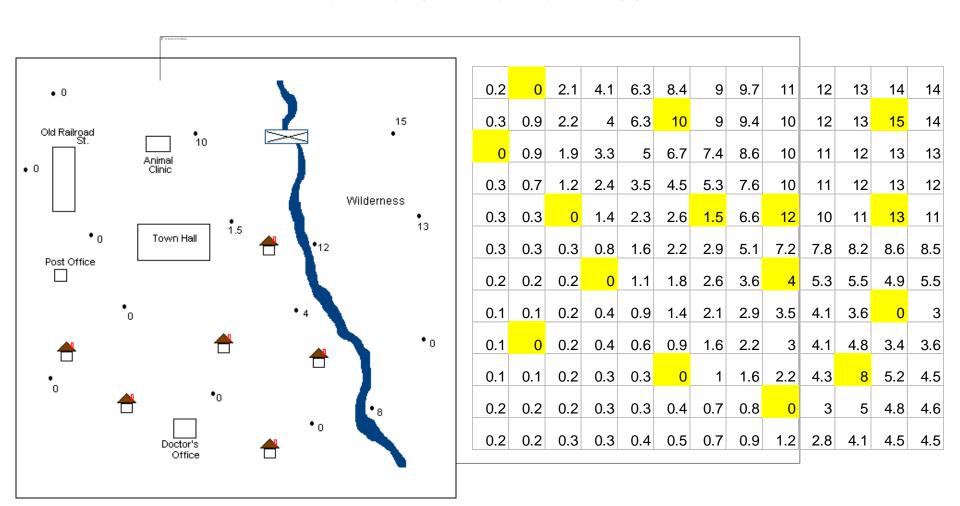
- Anthrax detected in Morteville
- Is terrorism the source?
- Infer geographic distribution from measures at several sample sites

- Build nearest neighbor averaging automaton in Excel
- Form hypothesis
- Get more data and compare
- Revise hypothesis





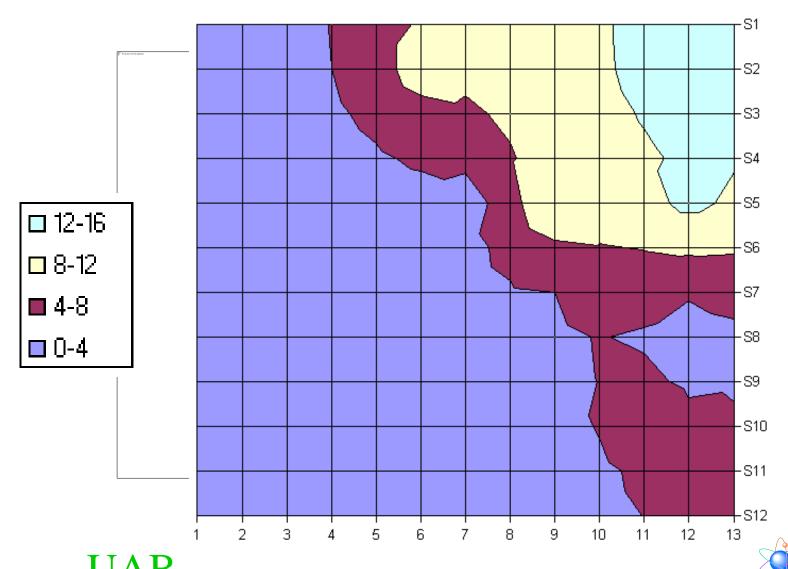
Morteville View 1



UAB



Anthrax Distribution 1



Compartmental Modeling

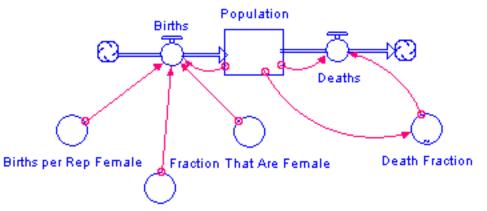
- How to Build a Stella Model
- Simple Population Models
- Generic Processes

- Advanced Population Models
- Drug Assimilation
- Epidemiology
- System Stories

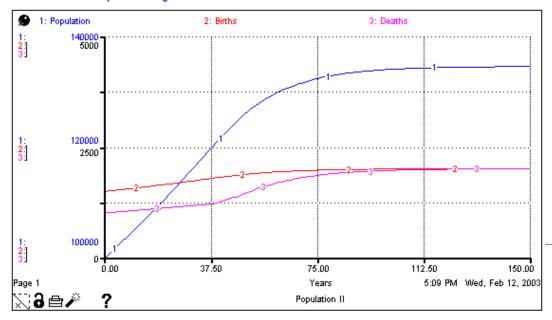




Population Model



Reproducing Females



•Stella Model

Equations

•Graphical Output

Population(t) = Population(t - dt) + (Births - Deaths) * dt INIT Population = 100000 {people}

INFLOWS:

Births =

Population*Fraction_That_Are_Female*Reproducing_Females*Births_per_Rep_Female

{people/year}
OUTFLOWS:

Deaths = Population*Death_Fraction

{people/year}

Births_per_Rep_Female = 66/1000 {people/1000

rep females/year}

Fraction_That_Are_Female = 0.5

{females/people}

Reproducing_Females = .45 {rep females/female}

Death_Fraction = GRAPH(Population)

 $(120000,\,0.01),\,(125000,\,0.011),\,(130000,\,0.013),$

(135000, 0.015), (140000, 0.017), (145000,

0.019), (150000, 0.02)



Generic Processes

• Linear model with external resource

Exponential growth or decay model

Convergence model

weight gained weight lost

ght gain weight loss rate

Typewriters made per employee

Temperature

weight gain per month

Ed Grid

cooling rate

Typewriters in warehouse

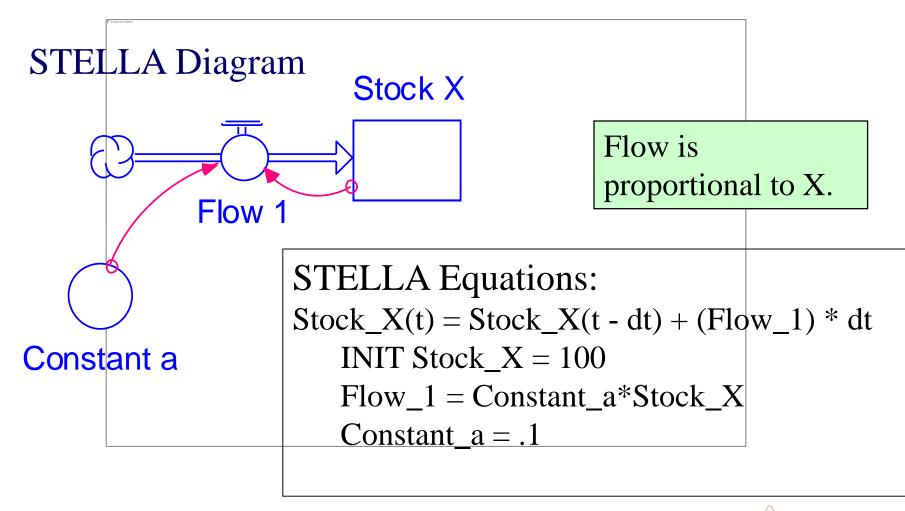
Employees

Typewriters made per week

Cooling

24

Calculus Example





Connecting the Discrete to the

Continuous

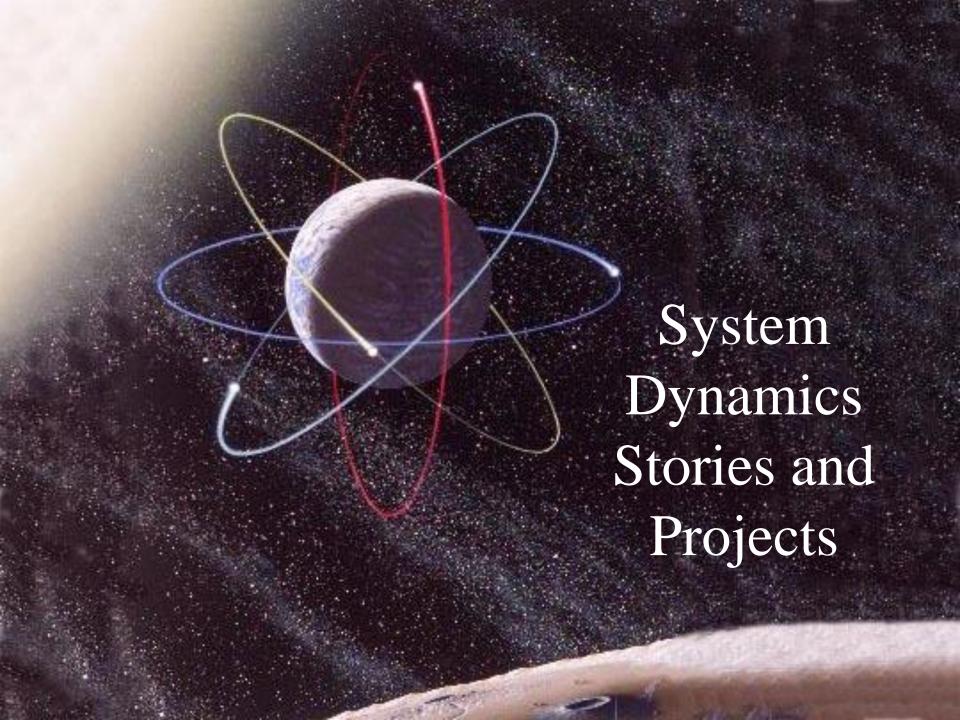
- $Stock_X(t) = Stock_X(t dt) + (Flow_1) * dt$
- $(Stock_X(t) Stock_X(t dt))/dt = Flow_1$
 - Flow_1 = Constant_a*Stock_X(t dt)
- (X(t) X(t dt))/dt = a X(t dt)
- Let dt go to 0
- dX/dt = a X(t) (a Differential Equation)



Solution to DE

- dX/dt = a X(t)
- dX/X(t) = a dt
- Integrate
- $\log |(\mathbf{X}(\mathbf{t}))| = \mathbf{at} + \mathbf{C}$
- $X(t) = \exp(C) \exp(at)$
- $X(t) = X(0) \exp(at)$





System Dynamics Stories and Projects

- Scenarios describing realistic situations to be modeled.
- Fully independent work.
- Construct model.
- Write 5-10 page technical paper (template provided).



Modeling a Dam 2



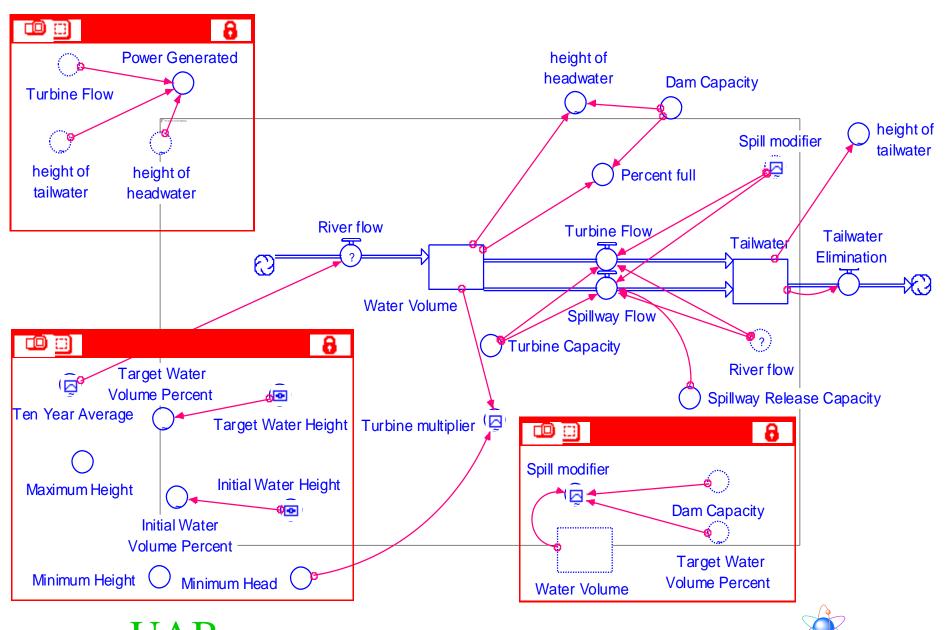
Boysen Dam

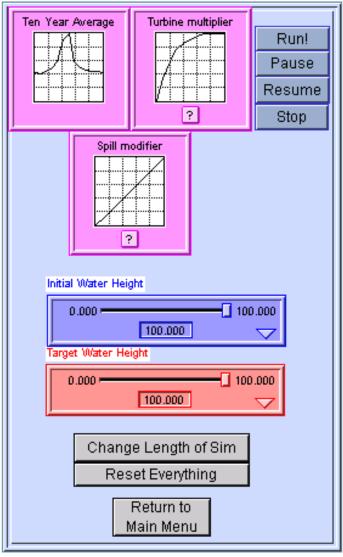
Boysen Dam has several purposes: It "provides regulation of the streamflow for power generation, irrigation, flood control, sediment retention, fish propagation, and recreation development." The United States Bureau of Reclamation, the government agency that runs the dam, would like to have some way of predicting how much power will be generated by this dam under certain conditions.

Clinton Curry

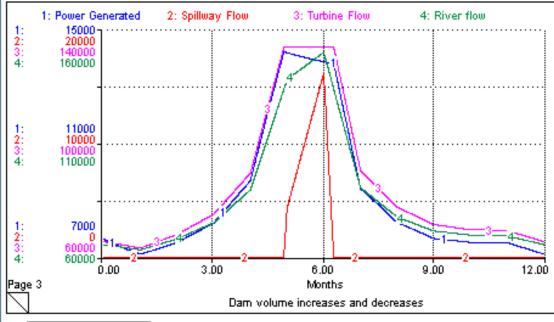


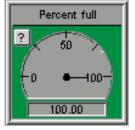






Boysen Dam





Simulation Time	12.0	
Water Volume	892,226.0	
Target Water Height	100.0	
Power Generated	7,094.0	





Modeling a Smallpox Epidemic One infected terrorist comes to town How does the system handle the epidemic under different assumptions? – Alicia Wilson