

Mathematics

Textbook for Class VII



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Foreword

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in science and mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Dr H.K. Dewan for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time and contribution. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

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20 November 2006

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We also know that most children in upper primary and secondary classes develop a fear of mathematics and it is one of the reasons for students not being able to continue in schools. NCF-2005 has also mentioned this problem and has therefore emphasised the need to develop a programme which is relevant and meaningful. The need for conceptualising mathematics teaching allows children to explore concepts as well as develop their own ways of solving problems. This also forms corner-stone of the principles highlighted in the NCF-2005.

In Class VI we have begun the process of developing a programme which would help children understand the abstract nature of mathematics while developing in them the ability to construct their own concepts. As suggested by NCF-2005, an attempt has been made to allow multiple ways of solving problems and encouraging children to develop strategies different from each other. There is an emphasis on working with basic principles rather than on memorisation of algorithms and short-cuts.

The Class VII textbook has continued that spirit and has attempted to use language which the children can read and understand themselves. This reading can be in groups or individual and at some places require help and support by the teacher. We also tried to include a variety of examples and opportunities for children to set problems. The appearance of the book has sought to be made pleasant by including many illustrations. The book attempts to engage the mind of the child actively and provides opportunities to use concepts and develop her/his own structures rather than struggling with unnecessarily complicated terms and numbers.

We hope that this book would help all children in their attempt to learn mathematics and would build in them the ability to appreciate its power and beauty. We also hope that this would enable to revisit and consolidate concepts and skills that they have learnt in the primary school. We hope to strengthen the foundation of mathematics, on which further engagement with studies as well as her daily life would become possible in an enriched manner.

The team in developing the textbook consists of many teachers who are experienced and brought to the team the view point of the child and the school. We also had people who have done research in learning of mathematics and those who have been writing textbooks for mathematics for many years. The team has tried to make an effort to remove fear of mathematics from the minds of children and make it a part of their daily routine even outside the school. We had many discussions and a review process with some other teachers of schools across the country. The effort by the team has been to accommodate all the comments.

In the end, I would like to place on record our gratefulness to Prof Krishna Kumar, Director, NCERT, Prof G. Ravindra, Joint Director, NCERT and Prof Hukum Singh, Head, DESM, for giving opportunity to me and the team to work on this challenging task. I am also grateful to

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A Note for the Teachers

This book is a continuation of the process and builds on what was initiated in Class VI. We had shared with you the main points reflected in NCF-2005. These include relating mathematics to a wider development of abilities in children, moving away from complex calculations and algorithms following, to understanding and constructing a framework of understanding. The mathematical ideas in the mind of the child grow neither by telling nor by merely giving explanations. For children to learn mathematics, to be confident in it and understand the foundational ideas, they need to develop their own framework of concepts. This would require a classroom where children discuss ideas, look for solutions of problems, set new problems and find not only their own ways of solving problems but also their own definitions with the language they can use and understand. These definitions need not be as general and complete as the standard ones.

In the mathematics class it is important to help children read with understanding the textbook and other references. The reading of materials is not normally considered to be related to learning of mathematics but learning mathematics any further would require the child to comprehend the text. The text in mathematics uses a language that has brevity. It requires the ability to deal with terseness and with symbols, to follow logical arguments and appreciate the need for keeping certain factors and constraints. Children need practice in translating mathematical statements into normal statements expressing ideas in words and vice-a-versa. We would require children to become confident of using language in words and also being able to communicate through mathematical statements.

Mathematics at the upper primary stage is a major challenge and has to perform the dual role of being both close to the experience and environment of the child and being abstract. Children often are not able to work in terms of ideas alone. They need the comfort of context and/or models linked to their experience to find meaning. This stage presents before us the challenge of engaging the children while using the contexts but gradually moving them away from such dependence. So while children should be able to identify the principles to be used in a contextual situation, they should not be dependent or be limited to contexts. As we progress further in the middle school there would be greater requirement from the child to be able to do this.

Learning mathematics is not about remembering solutions or methods but knowing how to solve problems. Problem-solving strategies give learners opportunities to think rationally, enabling them to understand and create methods as well as processes; they become active participants in the construction of new knowledge rather than being passive receivers. Learners need to identify and define a problem, select or design possible solutions and revise or redesign the steps, if required. The role of a teacher gets modified to that of a guide and facilitator. Students need to be provided with activities and challenging problems, along with sets of many problem-solving experiences.

On being presented a problem, children first need to decode it. They need to identify the knowledge required for attempting it and build a model for it. This model could be in the form of an illustration or a situation construct. We must remember that for generating proofs in geometry the figures constructed are also models of the ideal dimensionless figure. These diagrams are, however, more abstract than the concrete models required for attempting problems in arithmetic and algebra. Helping children to develop the ability to construct appropriate models by breaking up the problems and evolving their own strategies and analysis of problems is extremely important. This should replace prescriptive algorithms to solve problems.

Teachers are expected to encourage cooperative learning. Children learn a lot in purposeful conversation with each other. Our classrooms should develop in the students the desire and capacity to learn from each other rather than compete. Conversation is not noise and consultation is not cheating. It is a challenge to make possible classroom groups that benefit the most from being with each other

and in which each child contributes to the learning of the group. Teachers must recognise that different children and different groups will use distinct strategies. Some of these strategies would appear to be more efficient and some not as efficient. They would reflect the modelling done by each group and would indicate the process of thinking used. It is inappropriate to identify the best strategy or pull down incorrect strategies. We need to record all strategies adopted and analyse them. During this, it is crucial to discuss why some of the strategies are unsuccessful. The class as a group can improve upon the ineffective and unsuccessful strategies and correct them. This implies that we need to complete each strategy rather than discard some as incorrect or inappropriate. Exposures to a variety of strategies would deepen mathematical understanding and ability to learn from others. This would also help them to understand the importance of being aware of what one is doing.

Enquiry to understand is one of the natural ways by which students acquire and construct knowledge. The process can even begin with casual observations and end in generation and acquisition of knowledge. This can be aided by providing examples for different forms of questioning-explorative, open-ended, contextual, error detection etc. Students need to get exposed to challenging investigations. For example in geometry there could be things like, experimenting with suitable nets for solids, visualising solids through shadow play, slicing and elevations etc. In arithmetic we can make them explore relationships among members, generalise the relationships, discover patterns and rules and then form algebraic relations etc.

Children need the opportunity to follow logical arguments and find loopholes in the arguments presented. This will lead them to understand the requirement of a proof.

At this stage topics like Geometry are poised to enter a formal stage. Provide activities that encourage students to exercise creativity and imagination while discovering geometric vocabulary and relationships using simple geometric tools.

Mathematics has to emerge as a subject of exploration and creation rather than an exercise of finding answers to old and complicated problems. There is a need to encourage children to find many different ways to solve problems. They also need to appreciate the use of many alternative algorithms and strategies that may be adopted to solve a problem.

Topics like Integers, Fractions and Decimals, Symmetry have been presented here by linking them with their introductory parts studied in earlier classes. An attempt has been made to link chapters with each other and the ideas introduced in the initial chapters have been used to evolve concepts in the subsequent chapters. Please devote enough time to the ideas of negative integers, rational numbers, exploring statements in Geometry and visualising solids shapes.

We hope that the book will help children learn to enjoy mathematics and be confident in the concepts introduced. We want to recommend the creation of opportunity for thinking individually and collectively. Group discussions need to become a regular feature of mathematics classroom thereby making learners confident about mathematics and make the fear of mathematics a thing of past.

We look forward to your comments and suggestions regarding the book and hope that you will send interesting exercises, activities and tasks that you develop during the course of teaching, to be included in the future editions.

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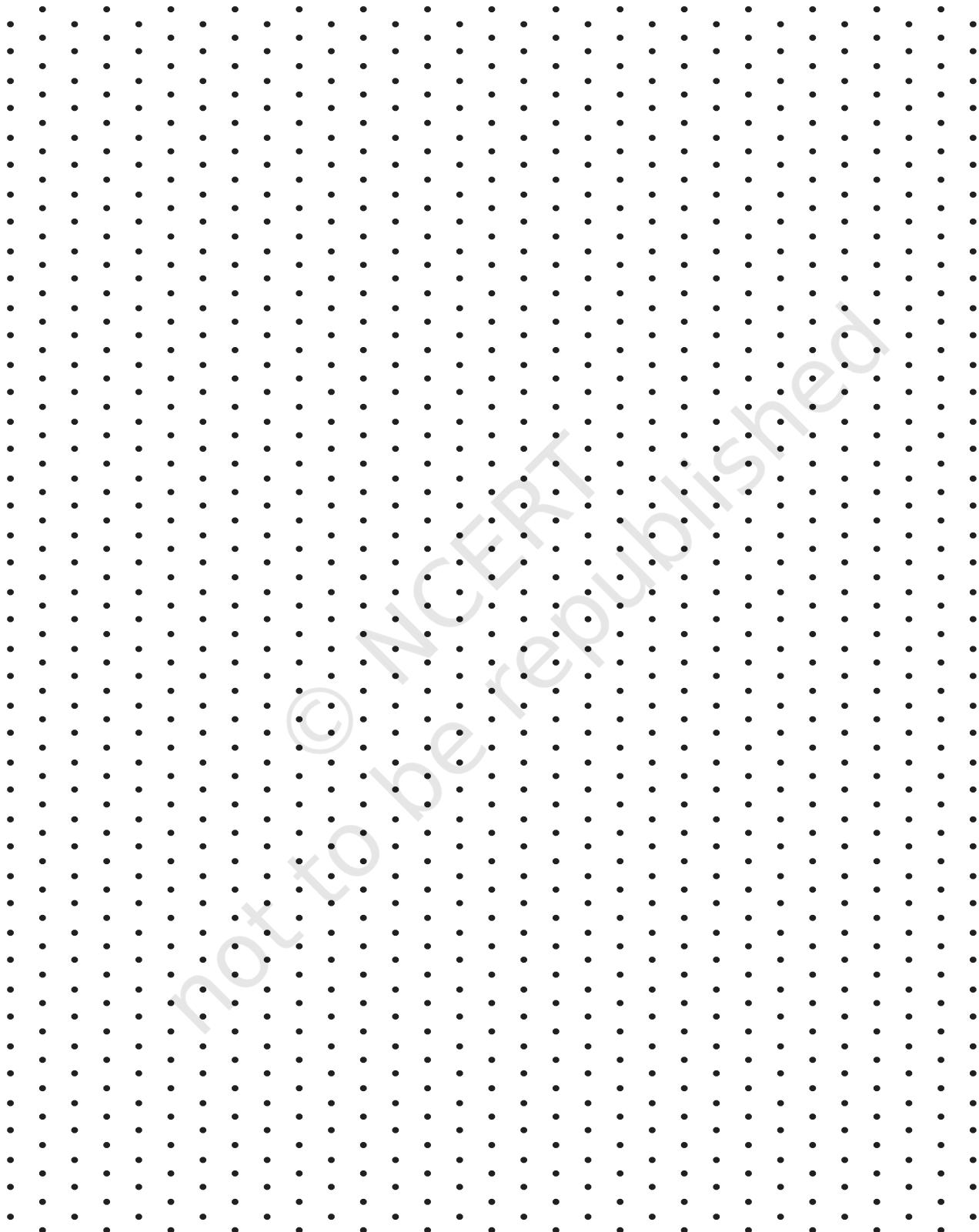
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Isometric Dot Sheet



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We look forward to your comments and suggestions regarding the book and hope that you will send interesting exercises, activities and tasks that you develop during the course of teaching, to be included in the future editions.

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Isometric Dot Sheet

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