EE6505: Assignment on Minimization
Finding Minima of functions in 1-D and n-D

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December 11, 2021

This is a long assignment, as I have merged the one and N dimensional assignments to create it. See what you can manage with it.

1 Reading Assignment

Minimization chapter in Numerical Recipes.

2 Finding Roots in 1-D

Find all the positive roots of

$$\tan x = \sqrt{\pi\alpha - x}$$

for any positive α. This type of equation typically arises in the solution of modes in a dielectric waveguide.

1. Plot the left and right hand sides. Determine the existence of roots for $\alpha = 10$.
2. Bracket the roots using the plots to determine the bracketing regions. Draw vertical lines through your bracketing points. This kind of visualization is extremely important to ensure that your bracketing happened correctly. Do this for $\alpha = 10$ and then generalize for any α. **Note:** you may not use $\tan\pi/2, \tan 3\pi/2$, etc as bracketing points, as the tangent goes to infinity at those points.
3. **Create a function** that will take α and return the set of roots of the equation. Note that α can be any real number.
4. Use bisection, Brent and Newton Ralphston to see how quickly the roots are obtained.

Note: The bracketing algorithm should be independent of α. You should be able to defend the logic in the viva.

3 Minimization in 1-D

We wish to find the first ten minima of

$$\sin x + \frac{1}{1+x^2}$$

1. Graph the function
2. Bracket the minima by assuming that they lie between the zeros of $\sin x$. Draw vertical lines through the zeros and see if this assumption is correct.
3. Use the golden section search to zero in on the minimum.
4. Compare with Brent method.
5. Determine the "dead zone", the range of x values around a root in which the function does not change to computer resolution.
6. Convert the method to a "find the zero" method by differentiating the above function. Then find the zeros with a root finding algorithm. **Compare the accuracy of the minimum.**

4 Minimization of "nice" functions in N Dimensions

The function to be minimized is

$$f(x,y,z) = 1.5 - J_0\left(5\,(x-0.5)^2 + (0.5x+y-0.5)^2 + 0.2\,(0.25x+0.5y+z-0.5)^2\right)$$

Clearly a global minimum is at $(0.5, 0.25, 0.25)$ since $J_0(x)$ is maximum when its (real) argument is zero. The initial point is the origin. Use

- Minimizing along Coordinate directions
- Powell,
- Minimization along the gradient
- Conjugate Gradient

to determine how the minimum is reached, and how quickly.

Obtain the Hessian for this problem and determine its eigenvalues and eigen directions. Compare them to the conjugate directions obtained by Powell and the Conjugate Gradient methods. Are they the same?

5 Minimization of difficult functions

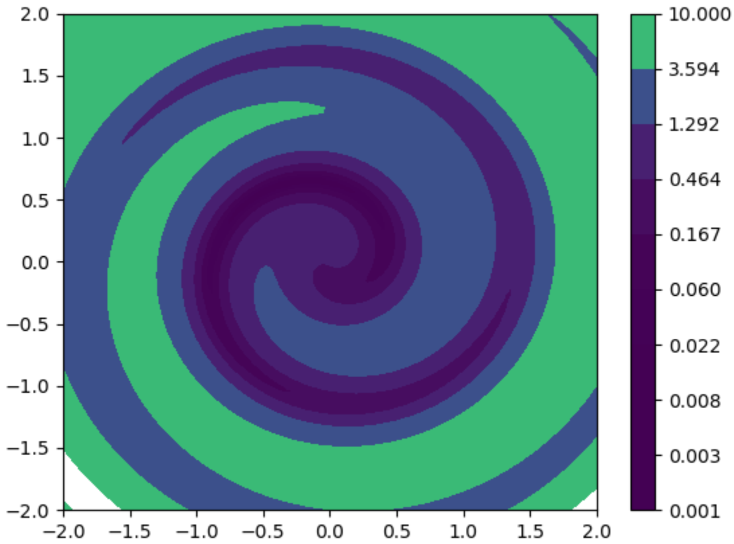
The function to be minimized is the following:

$$f(x,y) = u^2 + v^2 \tag{1}$$

where

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 5 \\ 5 \end{pmatrix} \tag{2}$$

where $\alpha = 5\left(1.5\sqrt{x^2+y^2} - 1.5\right)$.



- Apply Gradient Descent, Powell and Conjugate Gradient methods and find the approach to the minimum.
- In the approach region, see how much effect there is due to the $(g_{i+1} - g_i) \cdot g_{i+1}$ modification in the Conjugate Gradient method.
- In the final region where the function is nearly quadratic, study the way the g_i and h_i are generated and whether they satisfy the orthogonality conditions.
- Animate the solution process to show how the algorithms progress.

Note: The desired graphs should be overlaid on top of the contour plot obtained in part 1.