EE5471: Assignment on Random Number Generation

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1 Reading Assignment

Chapter on Random Numbers

2 The Assignment

1. Implement the function to generate normally distributed random numbers in Python. Plot a histogram of the random numbers generated and compare to the theoretical pdf.

2. Use the Chi-squared test and determine if 100 random numbers generated above belong to a gaussian (zero mean, unit variance) distribution. Vary the bin sizes and the number of random numbers and determine the optimal test. Apply the test to random numbers generated from the above function and offset by 0.1. Vary the offset and determine when the chi-squared test can distinguish between the correct random numbers and the offset ones.

3. Repeat Q2 with the KS test.

4. Repeat Q2 and Q3 where the mean remains zero, while the variance changes from 1.0 to 1.1. At what change in variance do the two tests detect a problem? What does this say about the two tests?

5. Consider now that the function in Q1 was the sum of two random distributions:

$$x = \begin{cases} \text{randn()} & 99\% \text{ of the time} \\ \text{randn()*100} & 1\% \text{ of the time} \end{cases}$$

   How do the two tests handle the distribution? Do they recognise that this is no longer a gaussian. Does the answer confirm what you learned in Q2, Q3 and Q4?

6. Consider the following function:

$$f(x,y) = u^2 + v^2 \tag{1}$$

   where

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \tag{2}$$

where $\alpha = \pi \sin\left(10\left(\sqrt{x^2+y^2}-0.5\right)\right)$. We wish to compute

$$I = \int_{|f|<1} dxdy$$

(a) Write a Python function to implement this function of two variables, x and y.

(b) Plot a contour plot (use `contourf` to get filled contours) of the function with contour values `[0.0,1.0]` and consider the region corresponding to $|f| < 1$.

(c) Find a covering function that contains all the places where $|f|$<1, and use it to estimate the integral above. Can you think of another way of computing this integral?