Theory assignment 0

A theorem in Complex Analysis says that every polynomial has at least one root in the complex plane. So,

$$p(x) = q(x)(x - x_1)$$

where $q(x)$ must be a polynomial of one lower order. But $q(x)$ must also have a zero, etc. A proof by induction yields that an n^{th}order polynomial must take the form

$$p(x) = \prod_{i=1}^{n} (x - x_i)\, q(x)$$

where $q(x)$ is just a constant (ie a zero order polynomial). Thus any n^{th} order polynomial has exactly n roots. The only exception of course, is the zero polynomial that is zero everywhere and has an infinite number of zeros.

1. Show that the only n^{th} order polynomial that is equal to zero at $n+1$ points must be the zero polynomial.
2. Show that there is a unique polynomial of order n that passes through $n+1$ points. (Prove by contradiction - assume two distinct solutions exist and consider the difference of those two polynomials)
3. Consider Lagrange interpolation. If each operation (addition, multiplication or division) creates a relative error δ, estimate the overall error in the answer. Assume that errors are Gaussian and white, and have zero mean.
4. How will the computer precision used (float, double, etc) affect the final accuracy in Q4?