Lec. 16 Ex

1] Since the data given follows Poisson Distribution:-

$$p(y|\lambda) = \frac{\lambda^y}{y!} e^{-\lambda}$$

a] For Fisher Information:-

$$I_y(\theta) = -E_y\left[\frac{\partial^2}{\partial \lambda^2}\left(\log p(y|\lambda)\right)\right] \cdot p(y|\lambda)$$

$$= -E_y\left[\frac{\partial^2}{\partial \lambda^2} \log\left(\frac{\lambda^y e^{-\lambda}}{y!}\right)\right] \cdot p(y|\lambda)$$

$$= -E_y\left[\frac{\partial^2}{\partial \lambda^2}\left[\underbrace{y\log\lambda}_{①} - \underbrace{\lambda\log e}_{②} - \underbrace{\log(y!)}_{②}\right]\right] \cdot p(y|\lambda)$$

$$\Rightarrow \frac{\partial}{\partial\lambda}\left[y\log\lambda - \lambda\log e - \log(y!)\right]$$

$$= y \cdot \frac{1}{\lambda} - 1 = y(\lambda)^{-1} - 1$$

$$\Rightarrow \frac{\partial^2}{\partial\lambda^2} = \frac{y}{\lambda} - 1 = \frac{y}{\lambda^2}$$

Plugging back in Equation

$$= -E_y\left[\frac{y}{\lambda^2}\right] \cdot p(y|\lambda)$$

$$= -E_y\left[\frac{y}{\lambda^2}\right] \cdot \frac{\cancel{\lambda^y}\, \cancel{e}}{\cancel{y!}} \cdot p(y|\lambda)$$

$$= \frac{1}{\lambda^2}\left[-E_y\,(y) \cdot p(y|\lambda)\right]$$

$$= \frac{1}{\lambda^2}(\lambda)$$

$$\boxed{I_y(\theta) = \frac{1}{\lambda}}$$

For n data points: $\boxed{I_y(\theta) = \frac{n}{\lambda}}$

b] Jeffreys' Prior:

$$p(\theta) \propto \sqrt{I_y(\theta)}$$

$$\Rightarrow p(\lambda) \propto \sqrt{\frac{n}{\lambda}}$$

c] <u>Posterior:-</u>

$$p(\lambda|y) = p(\lambda|y_1 y_2 \dots y_n)$$

$$= \frac{p(y_1 y_2 \dots y_n | \lambda)\ p(\lambda)}{p(y_1 \dots y_n)}$$

$$\propto \frac{\lambda^{(\sum_1^n y_i)}\ e^{(-n\lambda)}}{\prod_{i=1}^{n} (y_i!)} \sqrt{\frac{n}{\lambda}}$$

$$\boxed{p(\lambda|y) \propto \frac{\sqrt{n}}{\prod_{i=1}^{n}(y_i!)}\ e^{(-n\lambda)} \cdot \lambda^{[(\sum_{i=1}^{n} y_i) - \frac{1}{2}]}}$$

e] <u>Jeffreys' Prior for ϕ</u>, where $\lambda = \phi^2$.

<u>We have</u>, $p(\lambda) \propto \sqrt{\frac{n}{\lambda}}$

Substituting $\lambda = \phi^2$

$$p(\lambda) \propto \sqrt{\frac{n}{\phi^2}} \Rightarrow p(\lambda) \propto \frac{\sqrt{n}}{\phi}$$

1(d)

In [4]:

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using Distributions, Gadfly;
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In [21]:

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#plotting Jeffreys prior

lambda_y=0:0.0015:10;

y = zeros(length(lambda_y));

for i=1:length(y)
      y[i] = 1/sqrt(lambda_y[i]);
end

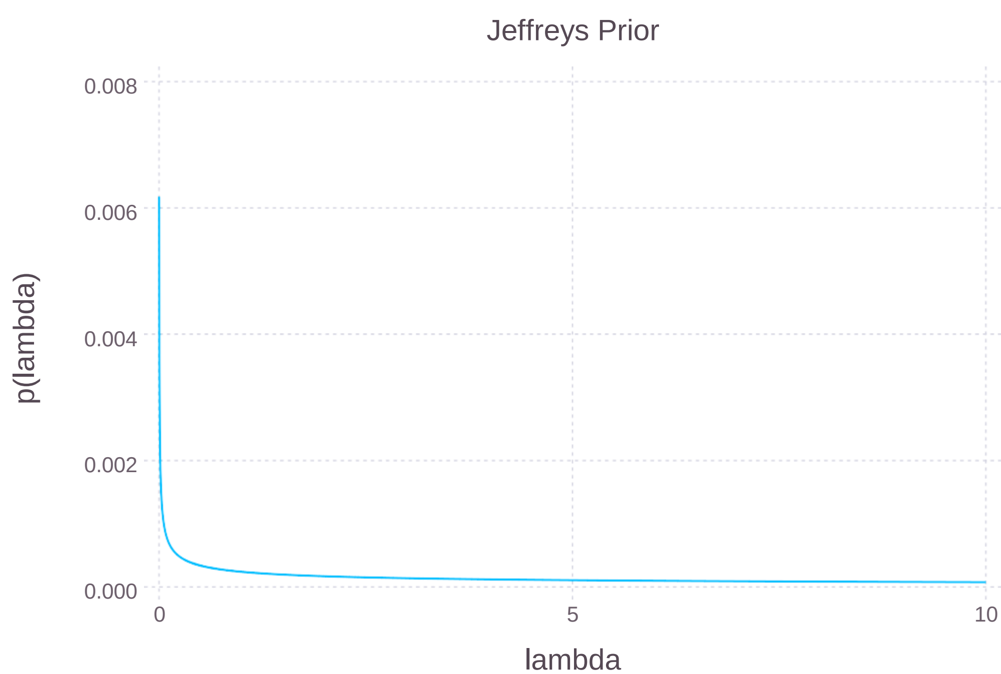
y = y./sum(y[y.!=Inf]);

p_lambda = y;

myplot1 = plot(x=lambda_y, y = y, Geom.line,Guide.ylabel("p(lambda)"
      Guide.xlabel("lambda"), Guide.title("Jeffreys Prior"))

```

Out[21]:



In [22]:

```
#plotting posterior
lambda_y=collect(0:0.01:10);

y_sum = 22;
n = 10;

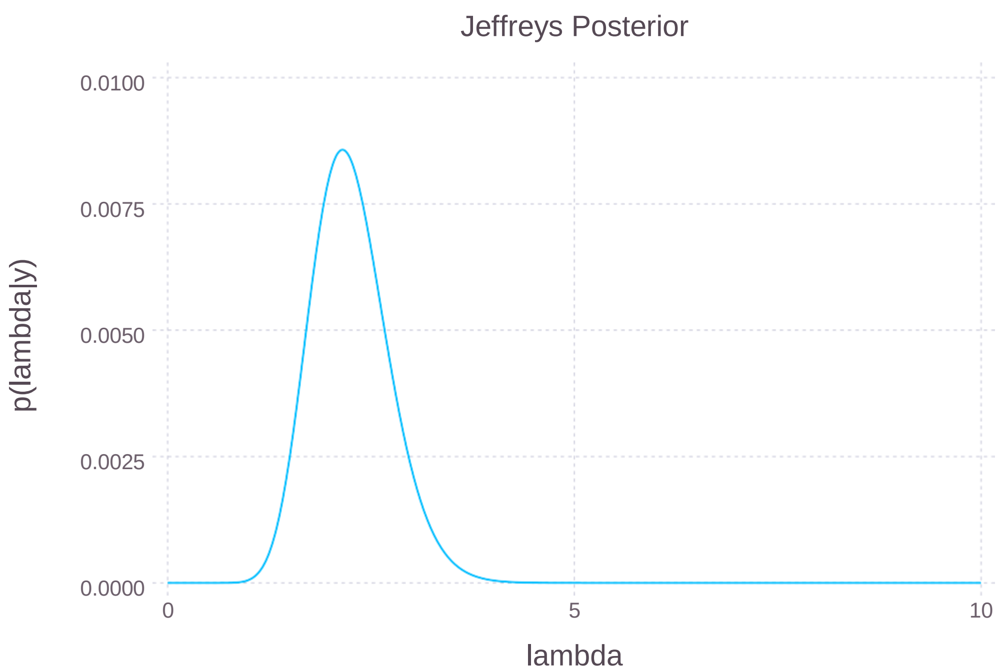
posterior = zeros(length(lambda_y));

for i=1:length(posterior)
      posterior[i] = exp(-lambda_y[i]*n) * (lambda_y[i])^(y_sum-0.5);
#       posterior[i] = (lambda_y[i]^2)*((1-lambda_y[i])^8)*p_lambda[i]
end

posterior = posterior./sum(posterior[.!isnan.(posterior)]);

myplot2 = plot(x=lambda_y, y = posterior, Geom.line,Guide.ylabel("p(
      Guide.xlabel("lambda"),Guide.title("Jeffreys Posterior "))
```

Out[22]:



In [ ]:

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Earlier when we plotted posterior that were mapped to a standard distribution, we derived a equation that resembled a particular distribution and used that distribution to plot the posterior. Here in this case, we are discretizing the values of the parameter and using the proportionality we are just estimating the shape of the posterior and can choose the argmax using the mode of the distribution.