Q 1

(a) Given distribution: exponential.

pdf of exponential:

$$f(x) = f(x|\theta) = \frac{1}{\theta} e^{-x/\theta}$$

$$\theta = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{\theta}.$$

$$\boxed{\therefore \; f(x|\lambda) = \lambda e^{-x\lambda}}$$

Likelihood function: $L(\lambda)$

$$L(\lambda) = p(x_1, x_2 \dots x_n | \lambda)$$

Assuming each sample is iid in "n" samples

$$\therefore L(\lambda) = p(x_1|\lambda) \dots p(x_n|\lambda)$$

$$\therefore L(\lambda) = \prod_{i=1}^{n} p(x_i|\lambda) = \prod_{i=1}^{n} f(x_i|\lambda)$$

$$= \prod_{i=1}^{n} \left(\lambda e^{-x_i\lambda}\right)$$

$$L(\lambda) = \lambda^n \prod_{i=1}^{n} \left(e^{-x_i \lambda}\right)$$

(1 (a))

$$\boxed{\therefore L(\lambda) = \lambda^n e^{-\lambda \sum_{i=1}^{n}(x_i)}}$$

... ~~log~~ (Likelihood function)

[1(b)] Log liklihood function

$$\ln[L(\lambda)] = \ln\left[\lambda^n e^{-\lambda \sum_{i=1}^{n}(x_i)}\right]$$

$$= \boxed{n \ln(\lambda) - \lambda \sum_{i=1}^{n} x_i}$$

[1(c)]

from [1(b)]

when $\ln[L(\lambda)]$ is maximum,

$$\frac{d}{d(\lambda)}\left(\ln[L(\lambda)]\right) = 0$$

$$\therefore \frac{d}{d(\lambda)}\left[n \ln(\lambda) - \lambda \sum_{i=1}^{n} x_i\right] = 0$$

$$\therefore \frac{n}{\lambda} - \sum_{i=1}^{n} x_i = 0$$

$$\therefore \boxed{\lambda \sum_{i=1}^{n} (x_i) - n = 0} \Rightarrow \boxed{\hat{\lambda}_{MLE} = \frac{n}{\sum_{i=1}^{n} (x_i)}}$$

1(d) $\Rightarrow$ $n = 5$ $\quad x_1 = 2,\ x_2 = 3,\ x_3 = 1,\ x_4 = 3$

$x_5 = 4$

$$\therefore \hat{\lambda}_{MLE} = \frac{5}{\sum_{i=1}^{5} x_i} = \frac{5}{13}$$

$$\therefore \boxed{\hat{\lambda}_{MLE} = \frac{5}{13}}$$

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(2) $P(A) = \theta \qquad P(a) = 1-\theta$

genotype	AA	Aa	aa
Probability	θ^2	$2\theta(1-\theta)$	$(1-\theta)^2$

(a). $L(\theta) = P(AA)^{20} P(Aa)^{10} P(aa)^{70}$

$= (\theta^2)^{20} [2\theta(1-\theta)]^{10} [(1-\theta)^2]^{70}$

$= \theta^{40} \, 2^{10} \, \theta^{10} (1-\theta)^{10} (1-\theta)^{140}$

$\boxed{L(\theta) = 2^{10} \theta^{50} (1-\theta)^{150}} \rightarrow$ (Likelihood fn.)

(b). $l(\theta) = \log[L(\theta)]$

$= \log[2^{10} \cdot \theta^{50} \cdot (1-\theta)^{150}]$

$\boxed{l(\theta) = 10\log 2 + 50\log\theta + 150\log(1-\theta)}$ (log likelihood fn)

(c). $\frac{d}{d\theta} l(\theta) = 0 + \frac{50}{\theta} + \left(-\frac{150}{1-\theta}\right)$.

$\boxed{\text{Equation} = \frac{50}{\theta} - \frac{150}{1-\theta}}$

(d).

~~$\frac{50}{\theta} - \frac{150}{1-\theta} = 0$~~

~~$50(1-\theta) - 15\theta = 0$~~

~~$50 - 50\theta - 15\theta = 0$~~

~~$50 = 155\theta$~~

~~$\theta = \frac{50}{155}$~~

~~$\hat{\theta}_{MLE} = 0.0322$~~

$\frac{50}{\theta} - \frac{150}{1-\theta} = 0$

$50(1-\theta) - 150(\theta) = 0$

$50 - 50\theta - 150\theta = 0$

$50 = 200\theta$

$\theta = \frac{50}{200}$

$\boxed{\hat{\theta}_{MLE} = 0.25}$