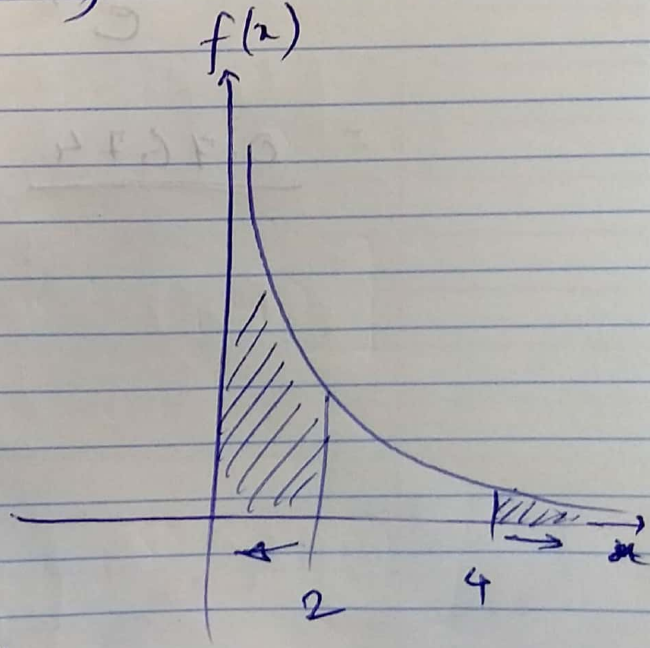
Q1

$$f(x) = \begin{cases} \frac{1}{2} e^{-x/2} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a)

$$P\left(x \leq \left(\frac{200}{100}\right) \cup x \geq \left(\frac{400}{100}\right)\right)$$

$$= P(x \leq 2 \cup x \geq 4)$$



$$= P(x \leq 2) + P(x \geq 4)$$

$$= \int_{-\infty}^{2} f(x)\,dx + \int_{4}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{2} f(x)\,dx + \int_{4}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{2} f(x)\,dx + \int_{4}^{\infty} f(x)\,dx$$

$$= \int_0^2 \frac{1}{2} e^{-x/2}\, dx + \int_4^\infty \frac{1}{2} e^{-x/2}\, dx$$

$$= \frac{1}{2}\left(\frac{e^{-x/2}}{-1/2}\right)_0^2 + \frac{1}{2}\left(\frac{e^{-x/2}}{(-1/2)}\right)_4^\infty$$

$$= \left(\frac{1}{2}\right)\frac{1}{(-1/2)}\left[e^{-1} - 1\right] + \left(e^{-4/2} - 0\right)$$

$$= 1 - \frac{1}{e} + e^{-2}$$

$$= \underline{0.7674}$$

Q. 1

(b) $P\left(x > \left(\frac{300}{100}\right) \middle| x > \left(\frac{200}{100}\right)\right)$

$$= \frac{P(x \geq 3, x > 2)}{P(x > 2)} \quad \text{— (Baye's Rule)}$$

$$= \frac{P(x \geq 3)}{P(x > 2)} \quad \ldots (x \geq 2) \subset (x > 3)$$

$$= \frac{\int_3^{\infty} \left(\frac{1}{2} e^{-x/2}\right)}{\int_2^{\infty} \left(\frac{1}{2} e^{-x/2}\right)}$$

$$= \frac{e^{-3/2}}{e^{-2/2}}$$

$$= e^{\left[\left(\frac{3}{2}\right)+\left(\frac{2}{2}\right)\right]} = e^{-0.5} = \underline{\underline{0.6065}}$$

Q. 2

$$E(x) = \int_{-\infty}^{\infty} x f(x) \, dx$$

$$= \int_{-\infty}^{0} 0 \, x \, dx$$

$$+ \int_{0}^{2} \frac{x}{4} \cdot x \, dx$$

$$+ \int_{2}^{3} x \left(\frac{1}{2}\right) dx$$

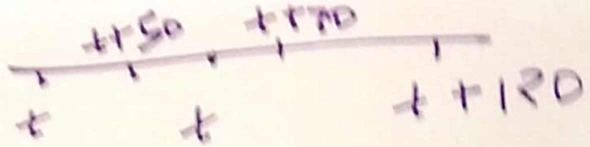
$$+ \int_{3}^{\infty} x \cdot 0 \, dx$$

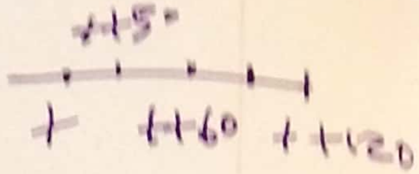
$$= \left[\frac{x^3}{12}\right]_0^2 + \left(\frac{x^2}{4}\right)_2^3$$

$$= \frac{(2)^3}{12} + \frac{3^2}{4} - \frac{2^2}{4}$$

$$= \frac{2}{3} + \frac{5}{4} =$$

$\therefore E(x) \approx 1.92$





Assignment 4

$$\frac{1}{120+t-t}$$

③ $f(x) = \begin{cases} 1/120 & \cancel{t} \le x \le 120+t \\ 0 & \text{elsewhere} \end{cases}$

10 min from center = $x+10$, $x-10$.

$$p(t) = \int_{-\infty}^{\infty} \frac{1}{120}\,dt$$

$$= \int_{t+50}^{t+60} \frac{1}{120}\,dt + \int_{t+60}^{t+70} \frac{1}{120}\,dt$$

$$= \frac{1}{2}[t]_{t+50}^{t+60} + \frac{1}{2}[t]_{t+60}^{t+70}$$

$$= \frac{1}{2}[t+60-t-50] + \frac{1}{2}[t+70-t-60]$$

$$= \frac{1}{120}[10] + \frac{1}{120}[10]$$

$$= \underline{0.1667}$$

Scanned by CamScanner

4) a) $\theta = 2.4$

$$P(X>2.5) = \int_{2.5}^{\infty} \frac{1}{\theta} e^{-\frac{x}{\theta}} d(x) = \int_{2.5}^{\infty} \frac{1}{2.4} e^{-\frac{x}{2.4}} dx =$$

$$-e^{-\frac{x}{2.4}} \Big|_{2.5}^{\infty} = e^{-\frac{2.5}{2.4}} = e^{-1.042} = 0.35$$

$$1 - P(X>2.5) = 1 - 0.35 = 0.65$$

b) $$\int_{2}^{3} \frac{1}{\theta} e^{-\frac{x}{\theta}} d(x) = \int_{2}^{3} \frac{1}{2.4} e^{-\frac{x}{2.4}} dx =$$

$$= -e^{-\frac{x}{2.4}} \Big|_{2}^{3} = -e^{-\frac{3}{2.4}} + e^{-\frac{2}{2.4}} = -0.29 + 0.43 = 0.14$$