Lec 20

Q.1.

$$\left.\begin{array}{l} g(x) = \theta \\ p(x) = p(\theta|y) \end{array}\right\}$$ Since $p(x)$ is already a following the requirement $p(x) \geq 0$ and $\int p(x)dx = 1$.

Q.2.

(a). ~~g(x)~~

$p(x) \sim U(1,7)$

$\frac{1}{\delta} = \frac{1}{7-1} = \frac{1}{6}$ $\delta = 6$

$g(x) = \delta f(x)$

$g(x) = 6\left[x^{2.7} e^{-x}\right]$

(b). ① $p(x) \geq 0$ and $\int p(x)dx = 1$

② $g(x) = \delta f(x)$

(c).

```
① Initialize $x_1 \dots x_n$ to $0_s$
② for $i = 1 \dots n$ times
③     Draw $x_i \sim U(1, 7)$
④ end
⑤ Compute $S_n = \frac{1}{n} \sum_{i=1}^{n} \delta f(x_i)$
⑥ Return $S_n$
```

(d). Julia Code

(e). Julia Code

Q.3.

(a). Julia Code

(b). $q(x) = \frac{(7-x)}{18}$

$f(x) = x^{2.7} e^{-x}$

$$I(f) = \int f(x)\,dx = \int \frac{f(x)}{q(x)} q(x)\,dx$$

Algorithm:

```
① Initialize $x_1 \ldots x_n$ to 0s
② for i = 1... n times
③     Draw $x_i \sim \left(\frac{7-x}{18}\right)$
④ end
⑤ Compute $S_n = \frac{1}{n} \sum_{i=1}^{n} \frac{x^{2.7} e^{-x}}{\left(\frac{7-x}{18}\right)}$
⑥ Return $S_n$
```

(c). Julia code

(d). Julia code

Q.4.

The mean and variance of both ordinary Monte Carlo method and importance Sampling method had a somewhat similar values and hence we can infer that the choice of q(x) was not appropriate for p(x). Since both of the methods resulted in similar values for variance, we cannot pick one method in particular that resulted in lower variance.

In [1]:

```
using Distributions, Gadfly, FontConfig, Cairo;
```

```
INFO: Recompiling stale cache file /mnt/juliabox/.julia/lib/v0.6/Fontconfig.j
i for module Fontconfig.
INFO: Recompiling stale cache file /mnt/juliabox/.julia/lib/v0.6/Compose.ji f
or module Compose.
INFO: Recompiling stale cache file /mnt/juliabox/.julia/lib/v0.6/Gadfly.ji fo
r module Gadfly.
INFO: Recompiling stale cache file /mnt/juliabox/.julia/lib/v0.6/FontConfig.j
i for module FontConfig.
```

Q. 2d

In [2]:

```
n = 10000
delta = 6
f(x) = x^2.7.*e^-x
x = rand(Uniform(1,7),n);
S = sum(delta.*f.(x))/n
```

Out[2]:

```
3.760910630394306
```

```
WARNING: replacing module Fontconfig.
WARNING: requiring "FontConfig" in module "Main" did not define a correspondi
ng module.
```

Q. 2e

In [3]:

```
n = 10000;
delta = 6;
f(x) = x^2.7.*e^-x
S = zeros(100);
for i=1:100
    x = rand(Uniform(1,7),n);
    S[i] = sum(delta.*f.(x))/n;
end
mean(S)
```

Out[3]:

```
3.786332561773042
```

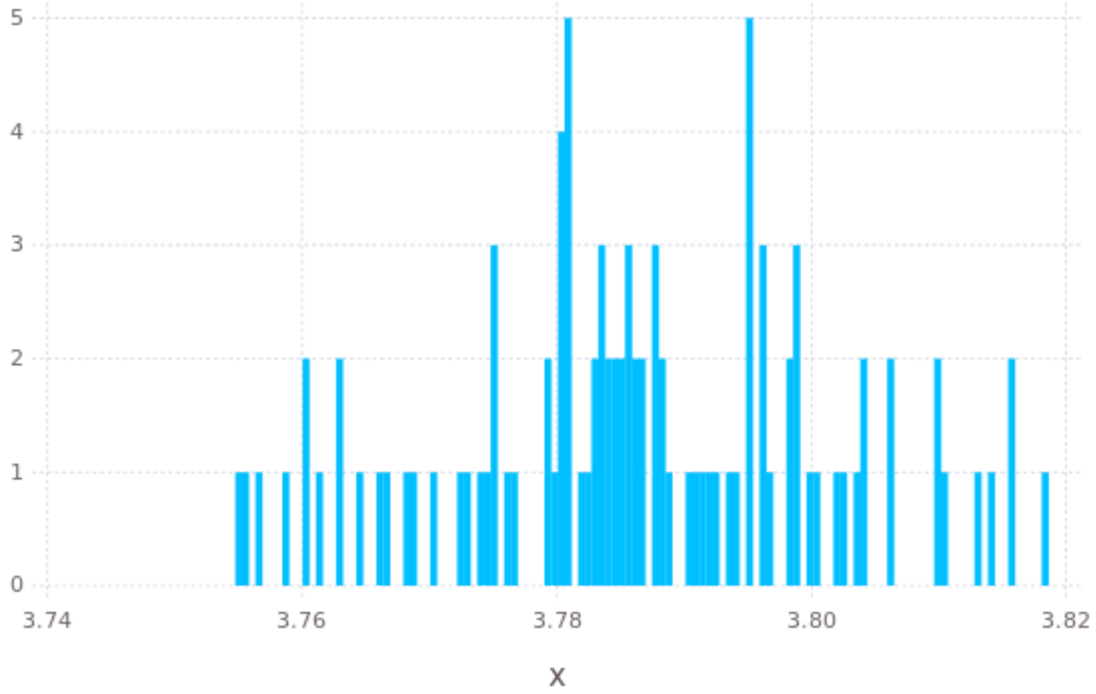
In [4]:

```
var(S)
```

Out[4]:

```
0.00021200196200736715
```

In [5]:
```
myplot = Gadfly.plot(x=S, Geom.histogram);
draw(PNG(6inch, 4inch),myplot)
```



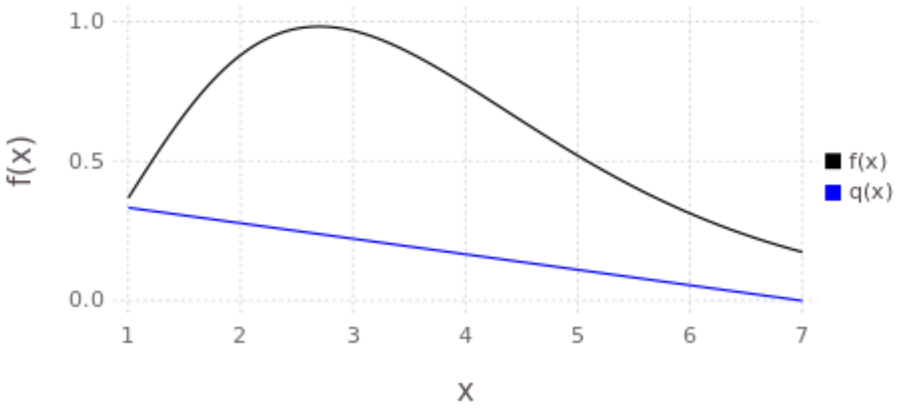
Out[5]:
```
false
```

Q. 3a

In [6]:
```
x = collect(1:0.01:7);
f(x) = x^2.7.*e^-x
g(x) = (7-x)/18
#ordinary_px = rand(Uniform(1,7));
```

In [7]:

```
myplot = plot(
layer(x=x,y=f.(x),Geom.line,Theme(default_color=colorant"black")),
layer(x=x,y=g.(x),Geom.line,Theme(default_color=colorant"blue")),
#layer(x=x,y=ordinary_px, Geom.line,Theme(default_color=colorant"red")),
Guide.ylabel("f(x)"),Guide.xlabel("x"),Coord.Cartesian(xmin=1, xmax=7),
Guide.manual_color_key("", ["f(x)", "q(x)"], ["black","blue"]));
draw(PNG(5inch, 2.5inch), myplot);
```



In [8]:

```
function accept_reject_method(n)
x = 1:0.01:7;
f(x) = x^2.7.*e^-x
g(x) = (7-x)/18

M = maximum(f.(x)./g.(x));
count = 0;
samples = [];
while(count<n)
y = rand(Uniform(1,7));
u = rand(Uniform(0,1));
if(u<f(y)/(M*g(y)))
samples = [samples; y];
count +=1;
end
end
return samples;
end
```

Out[8]:

```
accept_reject_method (generic function with 1 method)
```

In [ ]:

```
f(x) = x^2.7.*e^-x
g(x) = (7-x)/18

n= 10000;
x = accept_reject_method(n);
S = sum(f.(x)./(q.(x)))/length(x)
```

In [ ]:

```
n = 10000;
f(x) = 4/(1+x^2);
p(x) = (10-2x)/25;
S = zeros(100);
for i= 1:100
x = accept_reject_method(n);
S[i] = sum(f.(x)./(p.(x)))/n;
end
mean(S)
var(S)
```

In [ ]: