Lec - 14 Ex

1]

a] Data Likelihood :-

Poisson :- $p(x) = \frac{\lambda^x}{x!} e^{-\lambda}$

$$P(y|\theta) = \prod_{i=1}^{n} P(y_i|\theta)$$

$$= \prod_{i=1}^{n} P(y_1|\theta)\, P(y_2|\theta) \, \ldots\ldots \, P(y_n|\theta)$$

$$\boxed{P(y|\theta) = \prod_{i=1}^{10} \frac{\lambda^{y_i}}{y_i!} e^{-\lambda}} = \frac{(e)^{-10\lambda} (\lambda)^{22}}{(2!)^4 (6!)^2 (1!)^2}$$

(b) $P(\lambda) = k$

$$\int_0^\infty P(\lambda)\, d\lambda = k(\lambda)_0^\infty$$

$$= \infty$$

Since this prior is not integrating to 1, this is not a proper prior.

c) Using prior $p(\lambda) = k$

$$p(\theta|y) \propto p(y|\theta) \cdot p(\theta)$$

i.e. $$p(\lambda|y) \propto P(y|\lambda) \cdot P(\lambda)$$

$$\propto \prod_{i=1}^{10} \frac{\lambda^{y_i}}{y_i!} e^{-\lambda} (k)$$

$$\propto \frac{\lambda^{\sum_i^{10} y_i}}{\prod_i^{10} y_i!} e^{-10\lambda}$$

$$p(\lambda|y) \propto \frac{\lambda^{\sum y_i}}{\prod y_i!} e^{-10\lambda}$$

(i) $\beta = 0.1$ $\quad \alpha - 1 = \sum_i^{10} y_i$

(ii) $\alpha = \sum_i^{10} y_i + 1 = 22+1 = 23$

$\Gamma(\alpha) =$

(iv) $\Gamma(\alpha) = \text{Gamma}(\sum_i^{10} y_i + 1)$

(v) Of the form $\text{Gamma}(\sum_i^{10} y_i + 1, 0.1)$

(c) Gamma$(\underline{23, 0.1})$

e) Here Prior;

$$p(\lambda) = \text{Gamma}(\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \lambda^{\alpha-1} e^{-\lambda/\beta}$$

$$p(\theta|y) \propto p(y|\theta) \cdot p(\theta)$$

i.e. $p(\lambda|y) \propto p(y|\lambda) \cdot p(\lambda)$

$$\propto \left(\prod_{i=1}^{10} \frac{\lambda^{y_i}}{y_i!} e^{-\lambda}\right)\left(\frac{1}{\Gamma(\alpha)\beta^{\alpha}} \lambda^{\alpha-1} e^{-\lambda/\beta}\right)$$

$$\propto \left(\frac{\lambda^{\sum_i^{10} y_i} e^{-10\lambda}}{\prod_i^{10} y_i!}\right)\left(\frac{1}{\Gamma(\alpha)\beta^{\alpha}} \lambda^{\alpha-1} e^{-\lambda/\beta}\right)$$

$$\boxed{P(\lambda|y) \propto \frac{\lambda^{\left(\sum_i^{10} y_i + \alpha - 1\right)} e^{\left(-10\lambda - \lambda/\beta\right)}}{\left(\Gamma(\alpha)\beta^{\alpha}\right)\left(\prod_i^{10} y_i!\right)}}$$

$$\sum_i^{10} y_i + \alpha - 1 \Rightarrow 22 + 20 - 1 \Rightarrow \underline{\underline{41}}$$

For the above ~~Equation~~ Gamma Function:

$$\alpha - \cancel{1} = \sum_{i}^{10} y_i + \alpha - \cancel{1}$$

$$\alpha - 1 = 41 \quad \therefore \quad \alpha = 42$$

$$\frac{-\cancel{\lambda}}{\beta} = -\lambda\left(\frac{10\beta + 1}{\beta}\right) = -\cancel{\lambda}(20)$$

$$\beta = \frac{1}{20} = 0.05$$

$$p(\lambda \mid y) \propto \text{Gamma}(42, 1/20)$$

(d) Prior $p(\lambda) = \text{Gamma}(20, 0.1)$

Posterior of (c) i.e. $\text{Gamma}(23, 0.1)$

From (1), (c) $\Rightarrow$ Non-informative prior

$$\alpha_1 = \left(\sum_{i=1}^{n} y_i + 1\right) \quad \beta_1 = \frac{1}{n}$$

From (1) e) with informative prior.

$$\alpha_1 = \sum_{i=1}^{n} y_i + \alpha \qquad \beta_1 = \left(\frac{1}{n + 1/\beta} \right)$$

When we observe the values α & β provided in (1).e as the posterior of non-informative prior, it can be said that for $\alpha = 20$, Total number of crimes which were counted were $\sum_{i=1}^{n} y_i = \alpha - 1 = \underline{\underline{19}}$

and for $\beta = 0.1$, total no. of samples considered for the data were $n = \frac{1}{\beta} = \underline{\underline{10}}$

Thus, the informative prior incorporates the data about previous information while evaluating the parameter for posterior.

In [1]:
```
using Distributions, Gadfly;
```

In [2]:
```
d = Gamma(23,0.1)
```

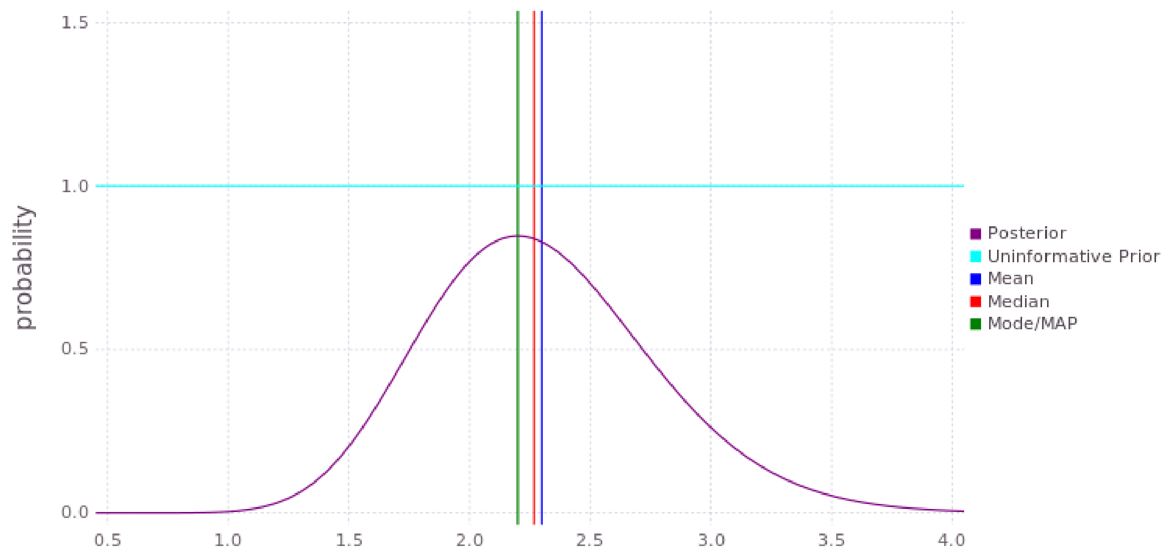
Out[2]:
```
Distributions.Gamma{Float64}(α=23.0, θ=0.1)
```

In [4]:
```
[Base.mean(d) Base.median(d) Distributions.modes(d)]
```

Out[4]:
```
1×3 Array{Float64,2}:
 2.3  2.26675  2.2
```

1.d

In [17]:
```
x = collect(0:0.001:10);
prior = ones(length(x));
median_val = Base.median(d);
mean_val = Base.mean(d);
mode_val = Distributions.modes(d);
posterior1 = pdf.(d,x);
myplot = Gadfly.plot(
layer(x=x,y=posterior1,Geom.line,Theme(default_color=colorant"purple")),
layer(x=x,y=prior,Geom.line,Theme(default_color=colorant"cyan")),
layer(xintercept=[mean_val],Geom.vline(color=colorant"blue")),
layer(xintercept=[median_val],Geom.vline(color=colorant"red")),
layer(xintercept=[mode_val[1]],Geom.vline(color=colorant"green")),
Coord.Cartesian(xmin=0.5, xmax=4,ymax=1.5), Guide.ylabel("probability"),
Guide.xlabel(""),
Guide.manual_color_key("", ["Posterior", "Uninformative Prior",
"Mean", "Median", "Mode/MAP"], ["purple","cyan","blue","red","green"]));
draw(PNG(8inch, 4inch), myplot)
```



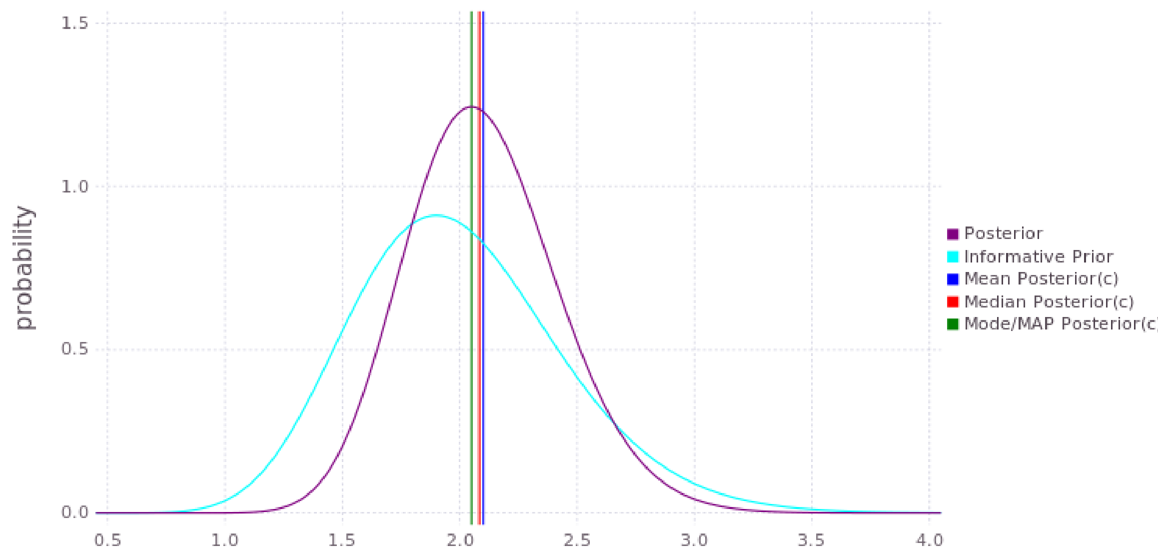
Out[17]: false

```
WARNING: both Gadfly and Base export "cross"; uses of it in module Main must
be qualified
```

1.f

In [19]:
```
posterior_2nd = Gamma(42,0.05)
posterior2 = pdf.(posterior_2nd,x);
prior_2nd = Gamma(20,0.1)
prior2 = pdf.(prior_2nd,x);
```

In [20]:
```
x = collect(0:0.001:10);
median_val = Base.median(posterior_2nd);
mean_val = Base.mean(posterior_2nd);
mode_val = Distributions.modes(posterior_2nd);
myplot_2 = Gadfly.plot(
layer(x=x,y=posterior2,Geom.line,Theme(default_color=colorant"purple")),
layer(x=x,y=prior2,Geom.line,Theme(default_color=colorant"cyan")),
layer(xintercept=[mean_val],Geom.vline(color=colorant"blue")),
layer(xintercept=[median_val],Geom.vline(color=colorant"red")),
layer(xintercept=[mode_val[1]],Geom.vline(color=colorant"green")),
Coord.Cartesian(xmin=0.5, xmax=4,ymax=1.5), Guide.ylabel("probability"),
Guide.xlabel(""),
Guide.manual_color_key("", ["Posterior", "Informative Prior",
"Mean Posterior(c)", "Median Posterior(c)", "Mode/MAP Posterior(c)"], ["purpl
e","cyan","blue","red","green"]));
draw(PNG(8inch, 4inch), myplot_2)
```



Out[20]: false

The informative prior tells us that 19 crimes were previously observed over 10 days. The new posterior with informative prior is plotted above and shows that 21 crimes are observed with higher probability as compared to posterior with non informative prior.

In [21]:
```
[Base.mean(posterior_2nd) Base.median(posterior_2nd) Distributions.modes(poste
rior_2nd)]
```

Out[21]:
```
1×3 Array{Float64,2}:
 2.1  2.08336  2.05
```

In [22]:
```
crime = Poisson(2.05)
```

Out[22]:
```
Distributions.Poisson{Float64}(λ=2.05)
```

In [24]:
```
prob_4orMore_crimes = 1 - cdf(crime,4)
```

Out[24]:
```
0.05727694947895323
```

The Probability of 4 or more crimes today is 0.05727

In [ ]: