1. If we generate random samples from an exponential distribution, yes we expect to pass the three tests. Because rand(Exponential(1), 10000) would generate a random Exponential distribution with 10000 samples. In the first histogram (first test), we get a histogram which is more inclined towards left as expected from Exponential data. In the second test of testing successive points we get a non-linear scatter plot and in the third test we get a autocorrelation of about 0, since exponential values are not correlated. Hence we expect the rand(Exponential(1), 10000) to pass 3 tests

2. Poisson distribution with $\lambda = 2 \rightarrow \dfrac{\lambda^x e^{-\lambda}}{x!}$

$$f(x) = 2^x e^{-2}/x!$$

① step 1: $F(x) = e^{-2} \int_0^x \frac{2^x}{x!} dx$

② step 2: $u = F(x) = e^{-2} \int_0^x \frac{2^x}{x!} dx$

③ step 3: solving for x, $x = F^{-1}(u)$.

Step ③ cannot be solved, as we cannot perform Integration for the factor.

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b. Since we cannot find the Inverse of F, we cannot draw $u \sim U(0,1)$, Hence Poisson samples cannot be drawn from this method without knowing $F^{-1}(x)$.

3. Gaussian Distribution with $\mu=0, \sigma^2=1$

$$\rightarrow N(0,1) \quad \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-(x)^2/2}$$

① Step 1 : $F(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-x^2/2} dx$

② Step 2 : $u_1 = F(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-x_1^2/2} dx$ $\quad u_2 = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-x_2^2/2} dx$

③ Step 3 : $x = F^{-1}(u)$

Given $x_1, x_2 \sim N(0,1)$

$x_1 = \sqrt{-2\ln u_1} \cos(2\pi u_2)$ and

$x_2 = \sqrt{2\ln u_1} \sin(2\pi u_2)$

④ Draw $u_1, u_2 \sim U(0,1)$, u_1, u_2 are uniform samples

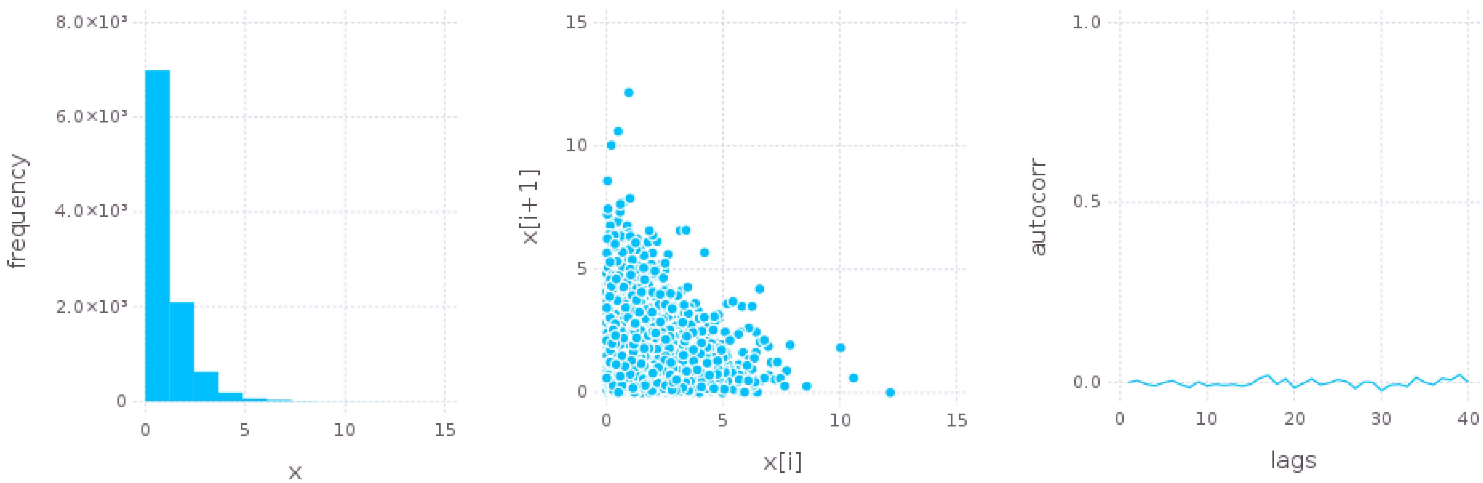
$x_1 = \sqrt{-2\ln u_1} \cos(2\pi u_2) \sim N(0,1)$

$x_2 = \sqrt{-2\ln u_2} \sin(2\pi u_2) \sim N(0,1)$

In [3]:
```
1ST QUESTION PART B
```

In [2]:
```
using StatsBase
```

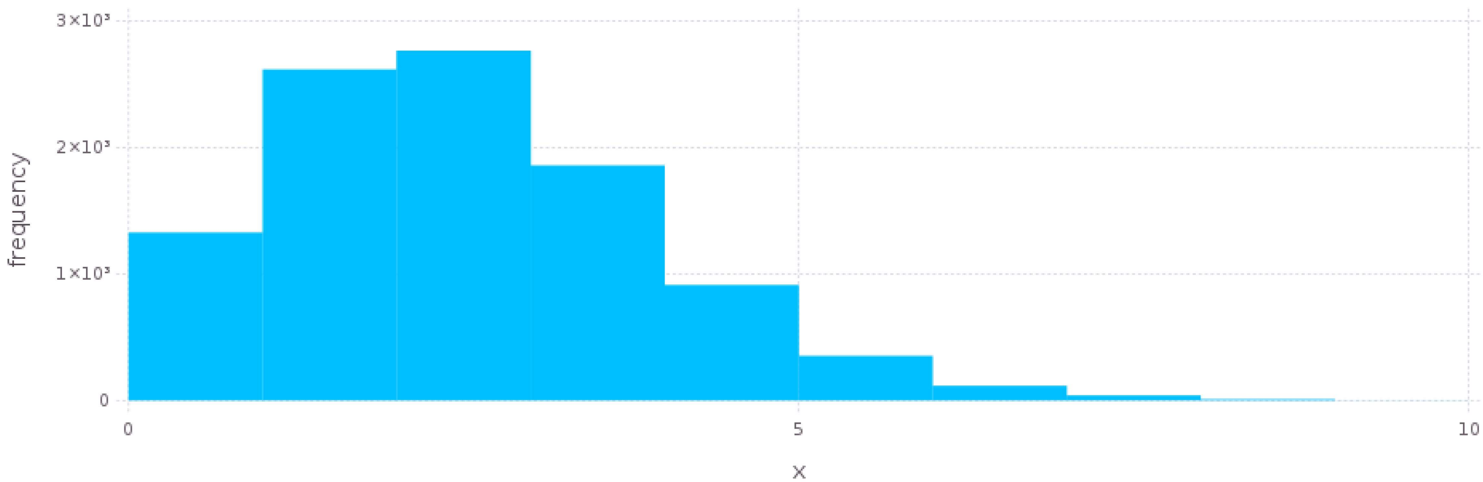
In [7]:
```
using Gadfly;
using Distributions;
x = rand(Exponential(1),10000);
myplot1 = plot(x=x, Geom.histogram(bincount=10),
Guide.xlabel("x"),Guide.ylabel("frequency"));
myplot2 = plot(x=x[1:end-1],y=x[2:end], Geom.point,
Guide.xlabel("x[i]"),Guide.ylabel("x[i+1]"));
myplot3 = plot(x=1:40,y=autocor(x,1:40), Geom.line,
Coord.Cartesian(ymax=1),Guide.xlabel("lags"),
Guide.ylabel("autocorr"));
myplot = hstack(myplot1,myplot2,myplot3)
draw(PNG(10inch, 3.5inch), myplot);
```



In [ ]:
```
From the plots seen above, the first histogram gives a random histogram but since we have passed an exponential distribution,
therefore the graph is more inclined towards left but it is a random sample though.
The second distribution shows the correlation between successive samples and it shows that there is no clear correlation
and the samples are random
The third distribution shows the correlation after a lag and clearly it is around 0 and there is no corrlation
in between the samples.
Hence the given sample is a random sample.
```

In [ ]:
```
2ND QUESTION PART C
```

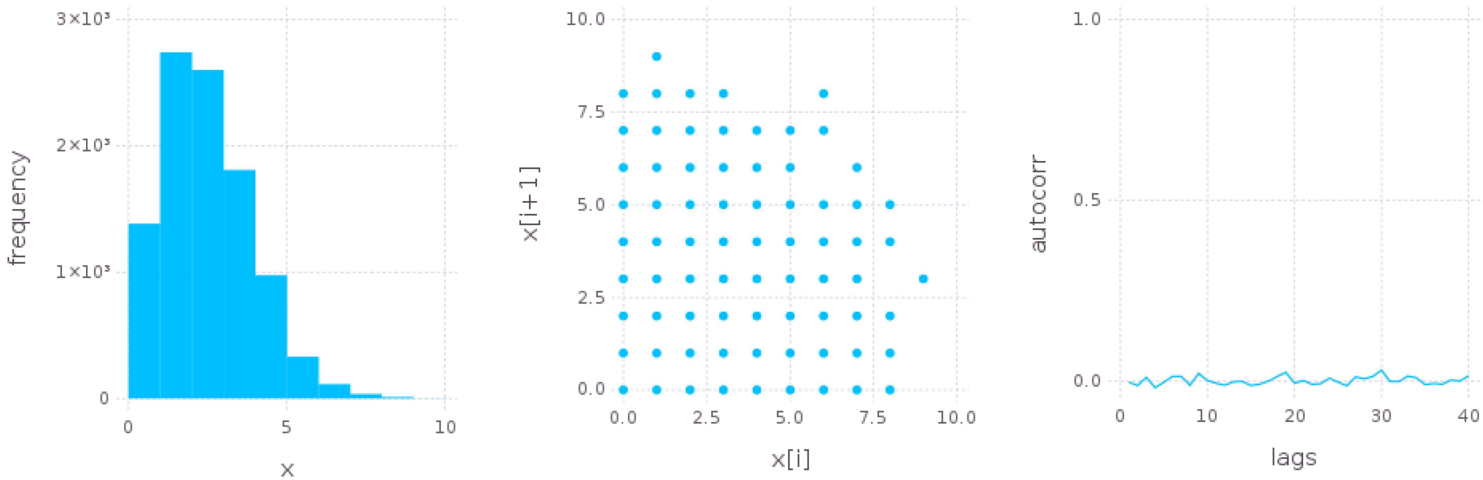
In [8]:
```
x = rand(Poisson(2),10000);
myplot = plot(x=x, Geom.histogram(bincount=10),
Guide.xlabel("x"),Guide.ylabel("frequency"));
Coord.Cartesian(ymax=1),Guide.xlabel("x"),
Guide.ylabel("Poisson(2)");
draw(PNG(10inch, 3.5inch), myplot);
```



In [ ]:
```
The plot shows a random Poisson distribution,Since we were unable to draw the samples from F Inverse,we cannot compare the
plots.
```

In [ ]:
```
2ND QUESTION PART D
```

In [9]:
```
x = rand(Poisson(2),10000);
myplot1 = plot(x=x, Geom.histogram(bincount=10),
Guide.xlabel("x"),Guide.ylabel("frequency"));
myplot2 = plot(x=x[1:end-1],y=x[2:end], Geom.point,
Guide.xlabel("x[i]"),Guide.ylabel("x[i+1]"));
myplot3 = plot(x=1:40,y=autocor(x,1:40), Geom.line,
Coord.Cartesian(ymax=1),Guide.xlabel("lags"),
Guide.ylabel("autocorr"));
myplot = hstack(myplot1,myplot2,myplot3);
draw(PNG(10inch, 3.5inch), myplot);
```



In [ ]:
```
From the 3 plots shown above, the Poisson sample is an uncorrelated sample and hence it is a random sample.
From the 1st plot the values are random but follow a poisson distribution,From the second plot the successive values
are uncorrelated,From the third plot the values are uncorrelated after a lag x.
Hence this distribution from a random sample.
```

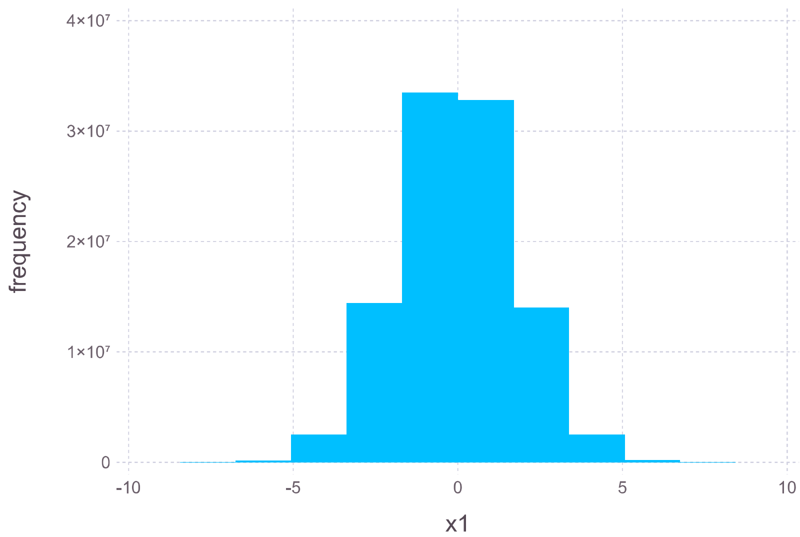
In [ ]:
```
3RD QUESTION PART B
```

In [24]:
```
using Gadfly;
using Distributions;
u1 = rand(10000,3);
u2 = rand(3,10000);
x1 = sqrt(-2*log(u1))*cos(2*pi*u2);
x2 = sqrt(-2*log(u1))*sin(2*pi*u2);
```

```
WARNING: log(x::AbstractArray{T}) where T <: Number is deprecated, use log.(x) instead.
Stacktrace:
 [1] depwarn(::String, ::Symbol) at ./deprecated.jl:70
 [2] log(::Array{Float64,2}) at ./deprecated.jl:57
 [3] include_string(::String, ::String) at ./loading.jl:522
 [4] include_string(::Module, ::String, ::String) at /users/PES0767/ucn3125/.julia/v0.6/Compat/src/Compat.jl:88
 [5] execute_request(::ZMQ.Socket, ::IJulia.Msg) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/execute_request.jl:180
 [6] (::Compat.#inner#14{Array{Any,1},IJulia.#execute_request,Tuple{ZMQ.Socket,IJulia.Msg}})() at /users/PES0767/ucn3125/.julia/v0.6/Compat/
src/Compat.jl:332
 [7] eventloop(::ZMQ.Socket) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/eventloop.jl:8
 [8] (::IJulia.##15#18)() at ./task.jl:335
while loading In[24], in expression starting on line 5
WARNING: log(x::AbstractArray{T}) where T <: Number is deprecated, use log.(x) instead.
Stacktrace:
 [1] depwarn(::String, ::Symbol) at ./deprecated.jl:70
 [2] log(::Array{Float64,2}) at ./deprecated.jl:57
 [3] include_string(::String, ::String) at ./loading.jl:522
 [4] include_string(::Module, ::String, ::String) at /users/PES0767/ucn3125/.julia/v0.6/Compat/src/Compat.jl:88
 [5] execute_request(::ZMQ.Socket, ::IJulia.Msg) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/execute_request.jl:180
 [6] (::Compat.#inner#14{Array{Any,1},IJulia.#execute_request,Tuple{ZMQ.Socket,IJulia.Msg}})() at /users/PES0767/ucn3125/.julia/v0.6/Compat/
src/Compat.jl:332
 [7] eventloop(::ZMQ.Socket) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/eventloop.jl:8
 [8] (::IJulia.##15#18)() at ./task.jl:335
while loading In[24], in expression starting on line 6
```

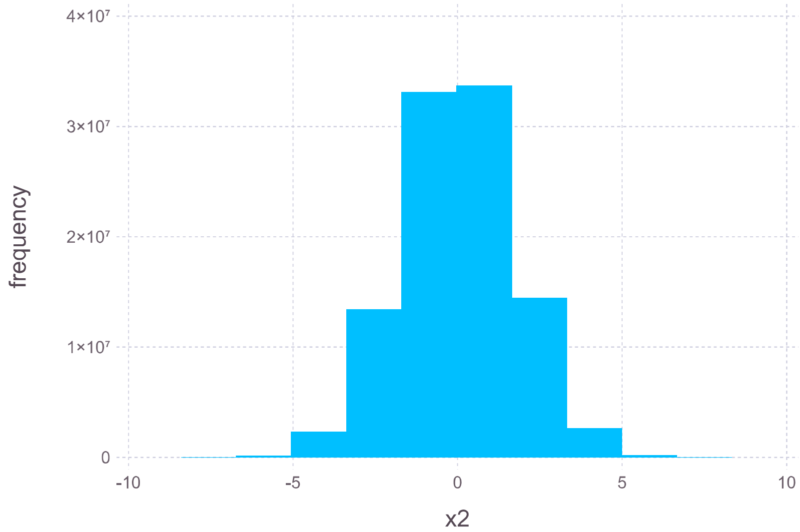
In [29]:
```
using Gadfly;
myplot1 = plot(x=x1, Geom.histogram(bincount=10),
Guide.xlabel("x1"),Guide.ylabel("frequency"))
```

Out[29]:



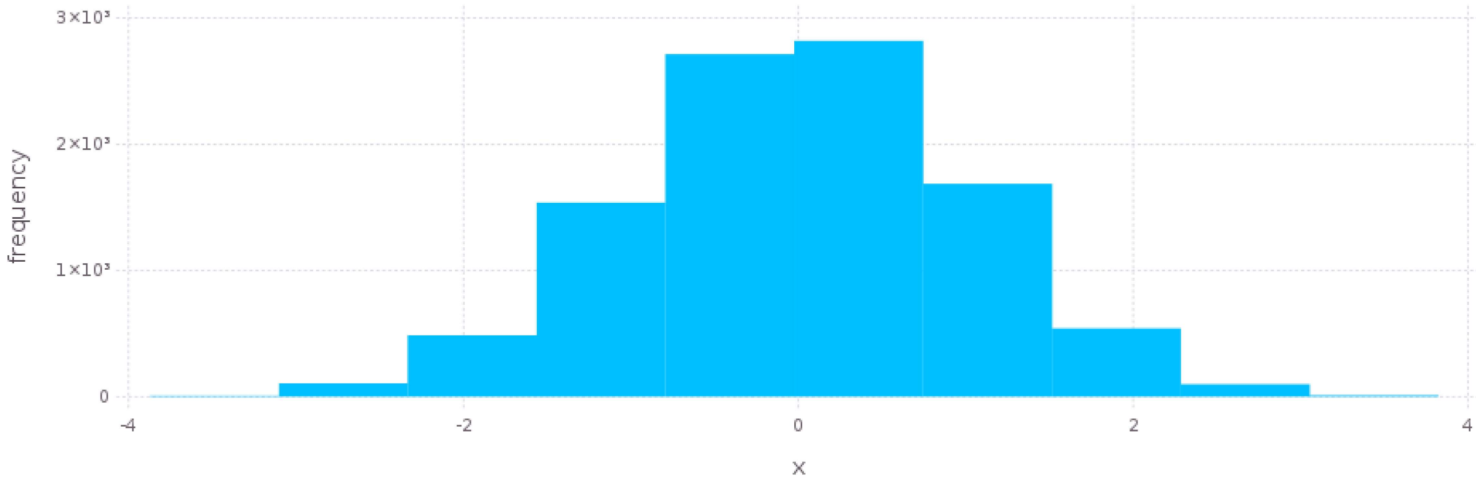
In [30]:
```
using Gadfly;
myplot1 = plot(x=x2, Geom.histogram(bincount=10),
Guide.xlabel("x2"),Guide.ylabel("frequency"))
```

Out[30]:



In [ ]:
```
3RD QUESTION PART C
```

In [16]:
```
x = rand(Normal(0,1),10000);
myplot = plot(x=x, Geom.histogram(bincount=10),
Guide.xlabel("x"),Guide.ylabel("frequency"));
Coord.Cartesian(ymax=1),Guide.xlabel("x"),
Guide.ylabel("Normal(0,1)");
draw(PNG( 10inch, 3.5inch), myplot)
```

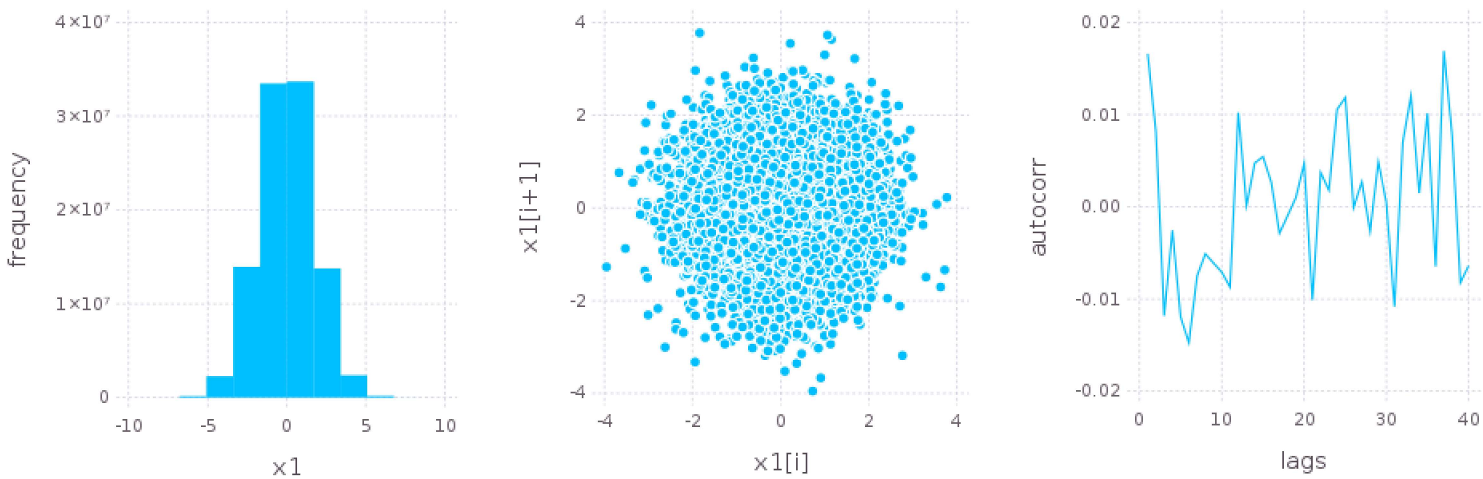


Out[16]: false

In [ ]:
```
By comparing the above plot with the plot drawn from the samples of poisson distribution, we observe both the plots to be same.
Both are same because of the Inverse Transform Method by which we draw samples of the distributions.
```

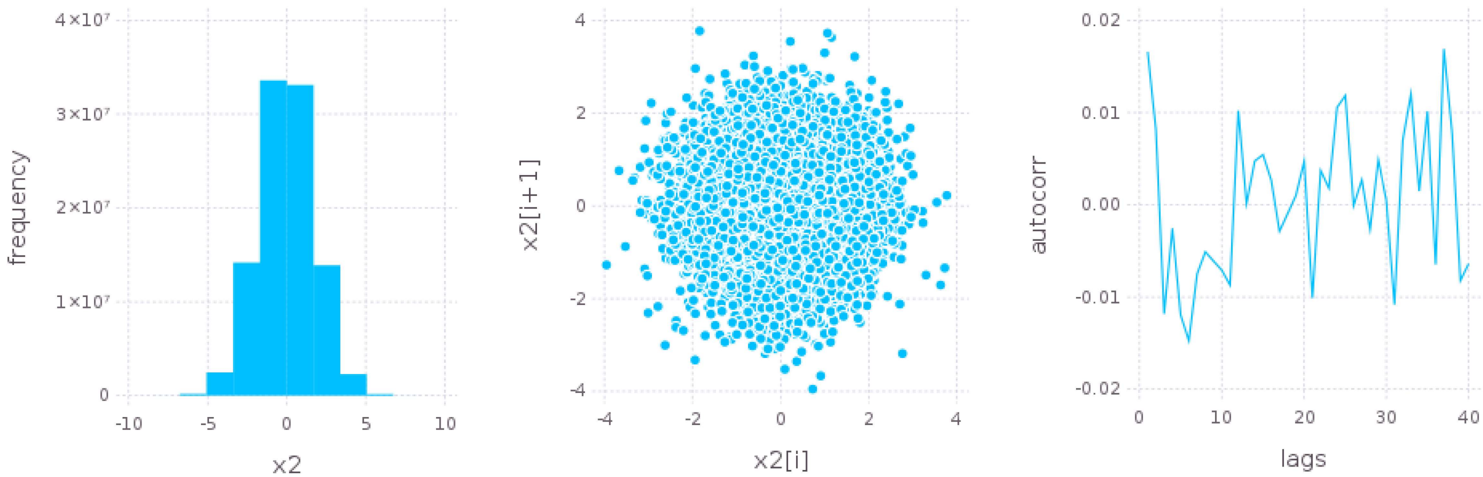
In [ ]:
```
3RD QUESTION D PART
```

In [18]:
```
myplot1 = plot(x=x1, Geom.histogram(bincount=10),
Guide.xlabel("x1"),Guide.ylabel("frequency"));
myplot2 = plot(x=x[1:end-1],y=x[2:end], Geom.point,
Guide.xlabel("x1[i]"),Guide.ylabel("x1[i+1]"));
myplot3 = plot(x=1:40,y=autocor(x,1:40), Geom.line,
Guide.xlabel("lags"),
Guide.ylabel("autocorr"));
myplot = hstack(myplot1,myplot2,myplot3);
draw(PNG( 10inch, 3.5inch), myplot)
```



Out[18]: false

In [19]:
```
myplot1 = plot(x=x2, Geom.histogram(bincount=10),
Guide.xlabel("x2"),Guide.ylabel("frequency"));
myplot2 = plot(x=x[1:end-1],y=x[2:end], Geom.point,
Guide.xlabel("x2[i]"),Guide.ylabel("x2[i+1]"));
myplot3 = plot(x=1:40,y=autocor(x,1:40), Geom.line,
Guide.xlabel("lags"),
Guide.ylabel("autocorr"));
myplot = hstack(myplot1,myplot2,myplot3);
draw(PNG( 10inch, 3.5inch), myplot)
```



Out[19]: false

In [ ]:
```
From the above plots,the three distributions pass the Tests of Random functions and the samples are uncorrelated ,
Hence the Gaussian samples are random numbers.
```