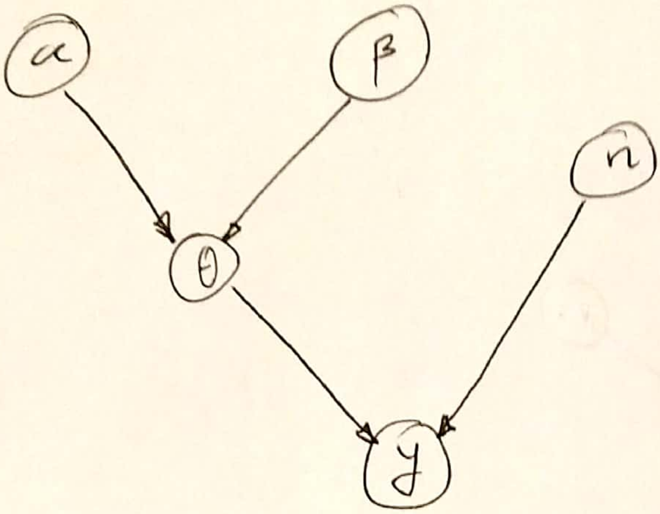
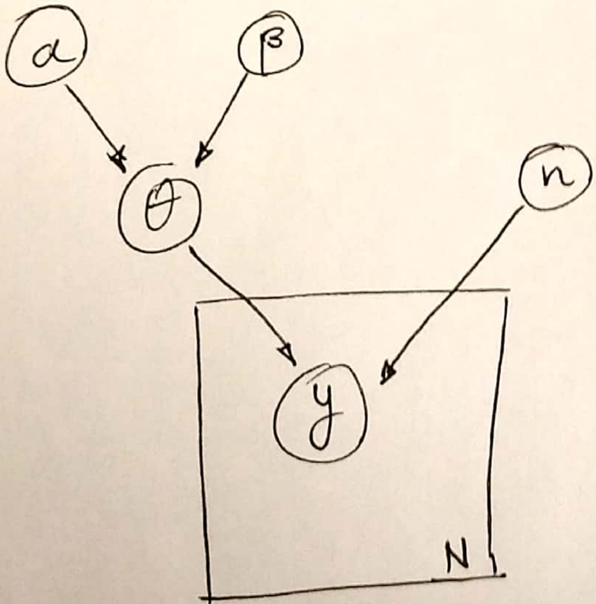
Q. 1

(a)



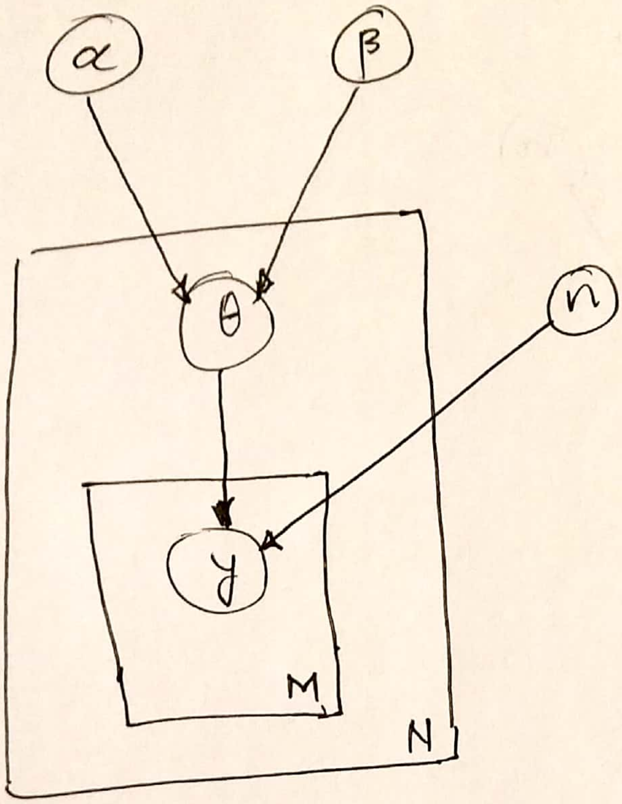
Q. 1

(b)



Scanned by CamScanner

Q.1 (c)



Q.2

Sequence of observation

$$y = \{y_1, y_2 \cdots y_k \cdots y_n\}$$

$$y_i \sim \begin{cases} \text{Poisson}(y_i;\lambda_1) & 1 \le i \le k-1 \\ \text{Poisson}(y_i;\lambda_2) & k \le i \le n \end{cases}$$

Q. 2(a)

Likelihood

$$p(y|\lambda_1, \lambda_2, k) = p(y|\lambda_1, \lambda_2, k)$$

$$= \prod_{i=1}^{k-1} \text{Poisson}(y_i;\lambda_1) \prod_{i=k}^{n} \text{Poisson}(y_i;\lambda_2)$$

Q. 2(b)

Gamma prior over λ_1 and λ_2

$$\lambda_i \sim \text{Gamma}(\lambda_i; a, b)$$

$$\therefore p(\lambda_1) = \text{Gamma}(\lambda_1; a, b)$$

$$p(\lambda_2) = \text{Gamma}(\lambda_2; a, b)$$

uniform prior over k.

$$k \equiv \{1, 2, \cdots n\}$$

$$p(k) = \frac{1}{n-1}$$

Posterior

$$p(\lambda_1, \lambda_2, k \mid y) \propto p(y \mid \lambda_1, \lambda_2, k)\, p(\lambda_1, \lambda_2, k)$$

$$\propto p(y \mid \lambda_1, \lambda_2, k)\, p(\lambda_1)\, p(\lambda_2)\, p(k)$$

Q.2(C)

$$p(\lambda_1 \mid \lambda_2, k, y) \propto p(y \mid \lambda_1, \lambda_2, k)\, p(\lambda_1)$$

$$\propto \prod_{i=1}^{k-1} \text{Poisson}(y_i, \lambda_1) \times \prod_{i=k-1}^{n} \text{Poisson}(y_i, \lambda_2) \times \text{Gamma}(\lambda_1; a, b)$$

$$\propto \left(\prod_{i=1}^{k-1} \text{Poisson}(y_i, \lambda_1) \right) \times \text{Gamma}(\lambda_1; a, b)$$

$$\propto \left(\prod_{i=1}^{k-1} \frac{\lambda_1^{y_i} e^{-\lambda_1}}{y_i!} \right) \times \frac{1}{\Gamma a} b^a \lambda_1^{a-1} e^{-b\lambda_1}$$

$$\propto \lambda_1^{\left(\sum_{i=1}^{k-1} y_i\right)} e^{-(k-1)\lambda_1} \lambda_1^{a-1} e^{-b\lambda_1}$$

$$\propto \lambda_1^{\left(a + \sum_{i=1}^{k-1} y_i - 1\right)} e^{-(k+b-1)\lambda_1}$$

$$\therefore p(\lambda_1 \mid \lambda_2, k, y) \approx \text{Gamma}\left(\lambda_1; a + \sum_{i=1}^{k-1} y_i, k+b-1\right)$$

Q.2(d) Similarly,

$p(\lambda_2 | \lambda_1, k, y)$

$$\propto p(y | \lambda_1, \lambda_2, k)\, p(\lambda_2)$$

$$\propto \prod_{i=1}^{k-1} Poisson(y_i, \lambda_1) \times \prod_{i=k}^{n} Poisson(y_i, \lambda_2) \times Gamma(\lambda_2; a, b)$$

$$\propto \left(\prod_{i=k}^{n} \frac{\lambda_2^{y_i} e^{-\lambda_2}}{y_i!} \right) \frac{1}{\Gamma a} b^a \lambda_2^{a-1} e^{-b\lambda_2}$$

$$\propto \lambda_2^{\sum_{i=k}^{n} y_i} e^{-\lambda_2 (n-k+1)} \lambda_2^{a-1} e^{-b\lambda_2}$$

$$\propto \lambda_2^{\left(a + \sum_{i=k}^{n} y_i - 1\right)} e^{-\lambda_2 (n+b-k+1)}$$

$$\therefore p(\lambda_2 | \lambda_1, k, y) \propto Gamma\left(\lambda_2; a + \sum_{i=k}^{n} y_i, (n+b-k+1)\right)$$

Q 2 (e)

$$p(k \mid \lambda_1, \lambda_2, y) \propto p(y \mid \lambda_1, \lambda_2\, k)\; p(k)$$

$$\propto \frac{\prod_{i=1}^{k-1} \text{Poisson}(y_i, \lambda_1) \prod_{i=k}^{n} \text{Poisson}(y_i, \lambda_2)}{n-1}$$

$$\therefore p(k \mid \lambda_1, \lambda_2, y) \propto \prod_{i=1}^{k-1} \text{Poisson}(y_i, \lambda_1) \prod_{i=k}^{n} \text{Poisson}(y_i, \lambda_2)$$

In [43]:
```
using Distributions, Gadfly, Cairo, Fontconfig, StatsBase;
```

In [44]:
```
white_panel = Theme(
    panel_fill = colorant"white",
    default_color = colorant"purple", bar_spacing=10mm,
    major_label_font_size = 18pt,
    minor_label_font_size = 14pt,
    key_title_font_size = 18pt,
    key_label_font_size = 14pt,
    major_label_color = colorant"black",
    minor_label_color = colorant"black"
);
```

Q2(f). Write Julia code to generate data from this above generative model and plot the resultant sequence of counts for 100 time steps.

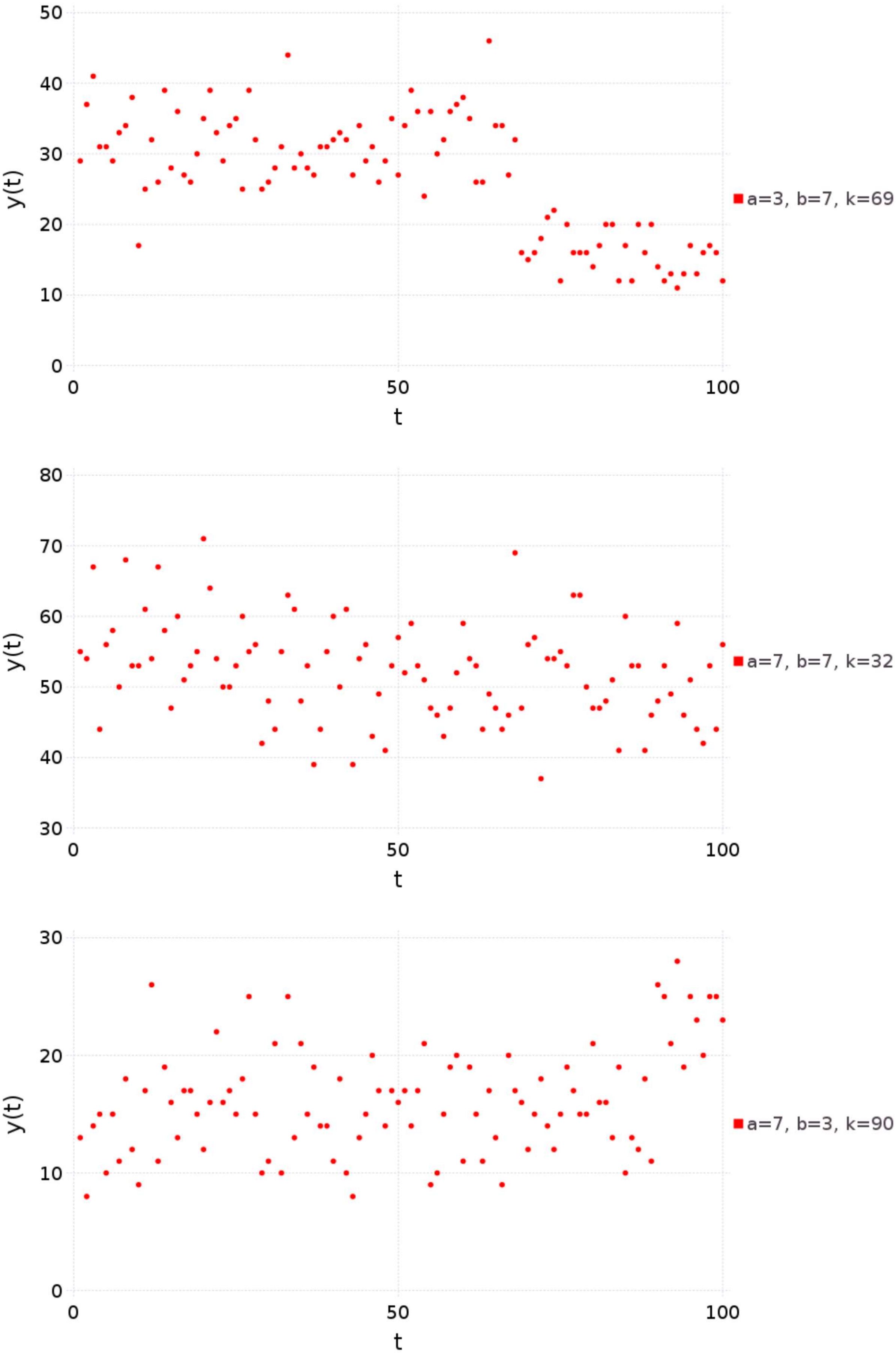
In [45]:
```
function generativeModel(n, a, b)

    pdf_k = DiscreteUniform(1, n);
    pdf_lambda1 = Gamma(a, b);
    pdf_lambda2 = Gamma(a, b);

    lambda1 = rand(pdf_lambda1);
    lambda2 = rand(pdf_lambda2);
    k = rand(pdf_k);


    pdf_y1 = Poisson(lambda1);
    pdf_y2 = Poisson(lambda2);

    y = zeros(n);

    for i = 1:k-1
        y[i] = Int(rand(pdf_y1));
    end

    for i = k:n
        y[i] = Int(rand(pdf_y2));
    end

    return y, k;
end
```

Out[45]:
```
generativeModel (generic function with 2 methods)
```

In [ ]:

```
n =100;

a = 3;
b = 7;
y, k = generativeModel(n, a, b);

legend_label = string("a=", a, ", b=", b, ", k=", k);
myPlot1 = plot(layer(x=1:100, y=y, Geom.point, Theme(default_color=colorant"r
        Guide.manual_color_key("", [legend_label], ["red","SteelBlue"]),
        Guide.ylabel("y(t)"), Guide.xlabel("t"), white_panel);


a = 7;
b = 7;
y, k = generativeModel(n, a, b);

legend_label = string("a=", a, ", b=", b, ", k=", k);
myPlot2 = plot(layer(x=1:100, y=y, Geom.point, Theme(default_color=colorant"r
        Guide.manual_color_key("", [legend_label], ["red","SteelBlue"]),
        Guide.ylabel("y(t)"), Guide.xlabel("t"), white_panel);


a = 7;
b = 3;
y, k = generativeModel(n, a, b);

legend_label = string("a=", a, ", b=", b, ", k=", k);
myPlot3 = plot(layer(x=1:100, y=y, Geom.point, Theme(default_color=colorant"r
        Guide.manual_color_key("", [legend_label], ["red","SteelBlue"]),
        Guide.ylabel("y(t)"), Guide.xlabel("t"), white_panel);

myplot = vstack(myPlot1,myPlot2,myPlot3);
```

In [51]:
```
1 draw(PNG(10inch, 15inch), myplot);
```



In [ ]:

```
1
```