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Exercise for lecture 19

Q.1. $f(x) \propto x^{\alpha-1} \left(\frac{1-x}{1+x}\right)^{\beta-1}$ $(\alpha = 2.7 \ \& \ \beta = 6.3)$
(Target)

Beta distribution $(\alpha = 2 \ \& \ \beta = 2)$ → (candidate)

$g(x) \propto x^{\alpha-1}(1-x)^{\beta-1}$

(a).

① The candidate density fn. should be a standard distribution of which we know how to draw samples from. In our case we have Beta dist. as our candidate dist & hence we do know how to draw samples from Beta dist.

② f and g have compatible supports $(g(x) > 0 \ \& \ f(x) > 0)$

③ There is a constant M such that $\frac{f(x)}{g(x)} \le M$ such that $Mg(x)$ envelopes $f(x)$ as tightly as possi

(b). Pseudo-code for Accept-Reject Method:

① We generate a sample x from the candidate dist. Beta $(\alpha=2, \beta=2)$.
② We generate a sample u from $u \sim U(0,1)$.
③ If $u \le \frac{1}{M} \frac{f(x)}{g(x)}$, we accept the sample x
④ otherwise we reject x as a sample from target.

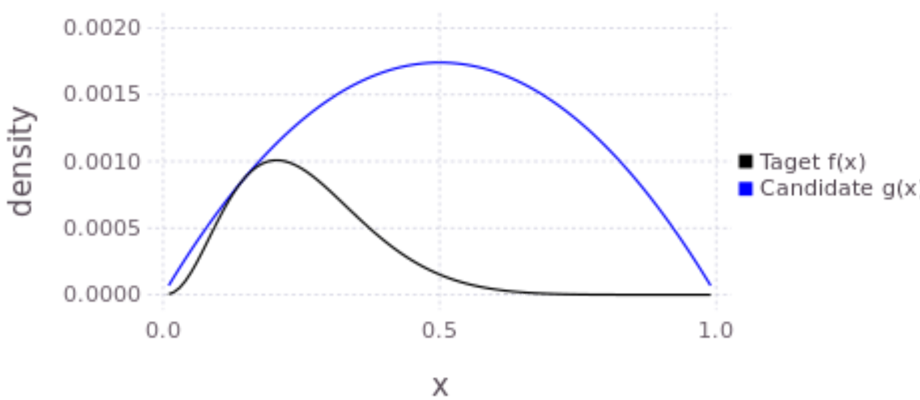
In [49]:
```
using Gadfly, Distributions, Cairo, StatsBase;
```

In [50]:
```
x = collect(0.01:0.01:0.99);
d = Beta(2,2);
```

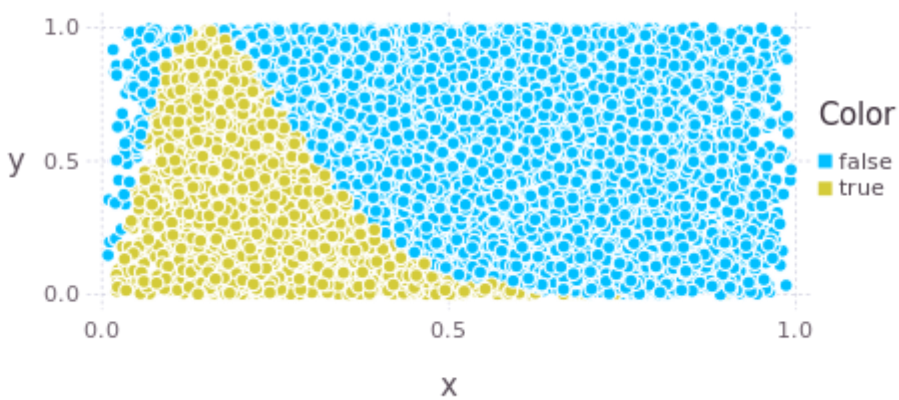
In [51]:
```
f(x) = x.^2.7 .* (((1.-x)./(1.+x)).^6.3);
g_x = pdf.(d,x);
M = maximum(f.(x)./g_x)
```

Out[51]:
```
0.0011607208936631457
```

In [52]:
```
myplot = Gadfly.plot(
layer(x=x,y=f.(x),Geom.line,Theme(default_color=colorant"black")),
layer(x=x,y=M.*g_x,Geom.line,Theme(default_color=colorant"blue")),
Guide.ylabel("density"),Guide.xlabel("x"),
Guide.manual_color_key("", ["Taget f(x) ", "Candidate g(x)"], ["black","blu
draw(PNG(5inch, 2.5inch), myplot);
```



In [53]:
```
n = 10000;
y = rand(d,n);
u = rand(Uniform(0,1),n);
g_x_y = pdf.(d,y);
x_samples = y[u.<f.(y)./(M.*g_x_y)];
myplot = plot(x=y,y=u,color = u.<f.(y)./(g_x_y.*M),Geom.point)
draw(PNG(5inch, 2.5inch), myplot);
```

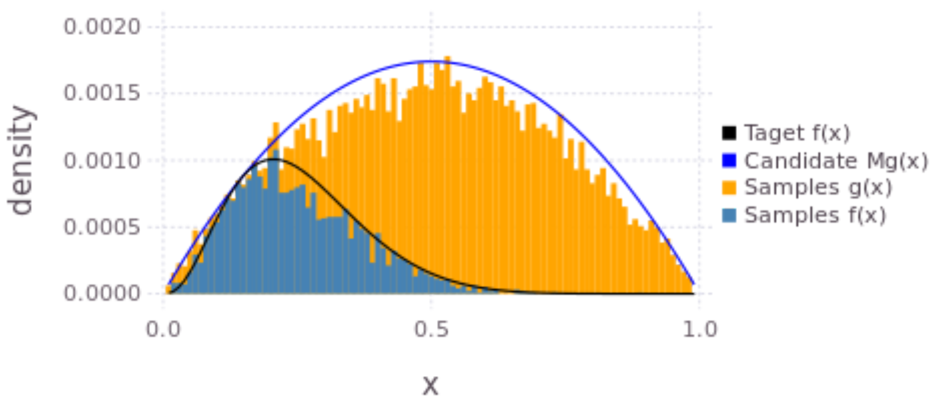


In [54]:
```
length(x_samples)
```

Out[54]: 2574

In [55]:

```
samples1 = y;
hist1 = [fit(Histogram,samples1,x).weights; 0]./96500;
samples2 = x_samples;
hist2 = [fit(Histogram,samples2,x).weights; 0]./99000;
myplot = plot(
layer(x=x,y=f.(x),Geom.line,Theme(default_color=colorant"black")),
layer(x=x,y=M.*g_x,Geom.line,Theme(default_color=colorant"blue")),
layer(x=x,y=hist2, Geom.bar,
Theme(default_color=colorant"SteelBlue")),
layer(x=x,y=hist1, Geom.bar,
Theme(default_color=colorant"orange")),
Coord.Cartesian(xmin=0, xmax=1),
Guide.ylabel("density"),Guide.xlabel("x"),
Guide.manual_color_key("", ["Taget f(x) ", "Candidate Mg(x)", "Samples g(x)
"Samples f(x)"], ["black","blue","orange","SteelBlue"]));
draw(PNG(5inch, 2.5inch), myplot);
```



In [56]:

```
x = collect(0.01:0.01:0.99);
d = Beta(2,5);
f(x) = x.^2.7 .* (((1.-x)./(1.+x)).^6.3);
g_x = pdf.(d,x);
M = maximum(f.(x)./g_x)
```

Out[56]: 0.00041081474534106004

(c). Julia code

(d). Julia code

(e). Fraction of total samples accepted : $\frac{\text{x\_samples}}{\text{total samples}}$

$= \cancel{\frac{1736}{10000}}$ $\frac{2574}{10000}$

$= \cancel{0.1736}$ $= \underline{0.2574}$

Q.2.

(a). ① The candidate density fn. should be a standard dist. of which we know how to draw samples from. In our case, we have Beta dist. as our candidate dist & hence we know how to draw samples from it.

② f & g must have compatible supports. [$g(x)>0$ & $f(x)>0$]

③ There is constant M such that $f(x)/g(x) \leq M$ Such that $Mg(x)$ envelopes $f(x)$ as tightly as possible.

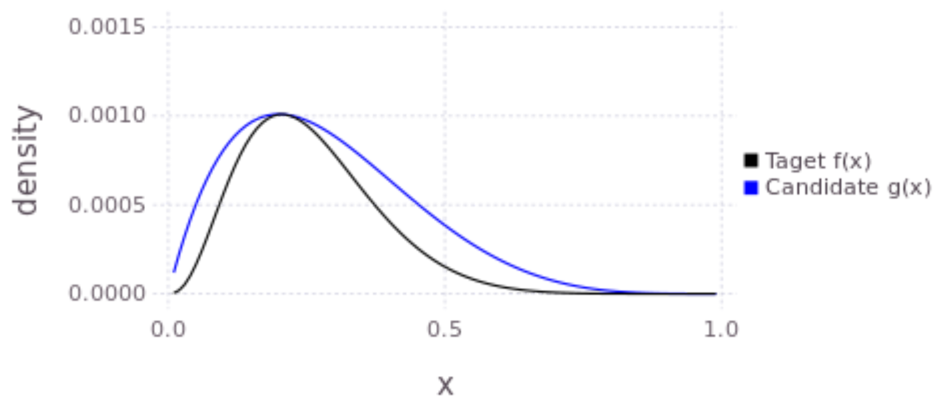
(b). Julia Code

(c). Fraction of total samples accepted : $\frac{\text{x\_samples}}{\text{Total samples}}$

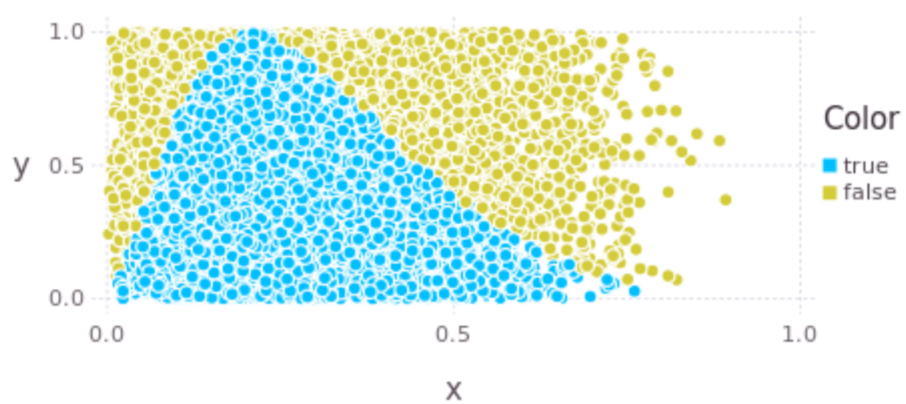
$= \frac{7234}{10000}$

$= \underline{\underline{0.7234}}$

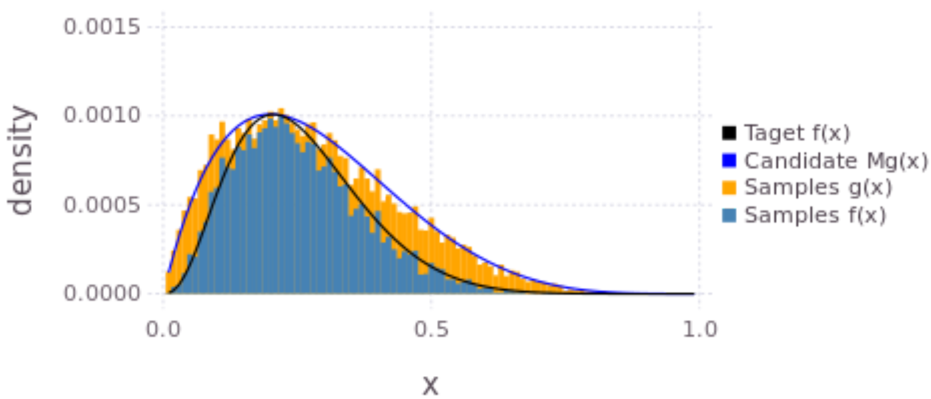
In [57]:
```
myplot = Gadfly.plot(
layer(x=x,y=f.(x),Geom.line,Theme(default_color=colorant"black")),
layer(x=x,y=M.*g_x,Geom.line,Theme(default_color=colorant"blue")),
Guide.ylabel("density"),Guide.xlabel("x"),
Guide.manual_color_key("", ["Taget f(x) ", "Candidate g(x)"], ["black","blu
draw(PNG(5inch, 2.5inch), myplot);
```



In [58]:
```
n = 10000;
y = rand(d,n);
u = rand(Uniform(0,1),n);
g_x_y = pdf.(d,y);
x_samples = y[u.<f.(y)./(M.*g_x_y)];
myplot = plot(x=y,y=u,color = u.<f.(y)./(g_x_y.*M),Geom.point)
draw(PNG(5inch, 2.5inch), myplot);
```



In [65]:
```
samples1 = y;
hist1 = [fit(Histogram,samples1,x).weights; 0]./250000;
samples2 = x_samples;
hist2 = [fit(Histogram,samples2,x).weights; 0]./260000;
myplot = plot(
layer(x=x,y=f.(x),Geom.line,Theme(default_color=colorant"black")),
layer(x=x,y=M.*g_x,Geom.line,Theme(default_color=colorant"blue")),
layer(x=x,y=hist2, Geom.bar,
Theme(default_color=colorant"SteelBlue")),
layer(x=x,y=hist1, Geom.bar,
Theme(default_color=colorant"orange")),
Coord.Cartesian(xmin=0, xmax=1),
Guide.ylabel("density"),Guide.xlabel("x"),
Guide.manual_color_key("", ["Taget f(x) ", "Candidate Mg(x)", "Samples g(x)
"Samples f(x)"], ["black","blue","orange","SteelBlue"]));
draw(PNG(5inch, 2.5inch), myplot);
```



In [60]:
```
length(x_samples)
```

Out[60]: 7234

In [ ]:

In [ ]:

Q.3. Among candidate dist. used in 1 & 2, the dist. of 2 i.e. Beta(2,5) results in a tighter envelope.

(a). ~~This~~ This can be determined without visualizing by just looking at the value of M. The smaller the value of M, the tighter is the envelope.

Case 1 : M = ~~[illegible]~~ ... 0.00116...

Case 2 : M = ~~[illegible]~~ ... 0.000414...

Hence since 2 has smaller M, it results in tighter envelope.

(b). This can be determine by fraction of accepted samples. Higher the fraction of accepted samples, more the target dist. agrees with the candidate dist. and hence resulting in a tighter envelope.

~~(c). Advantage of tighter envelope is that it gives us the ability to draw samples from a candidate dist. that agrees most to the target dist. & has the maximum support. Drawing from a candidate dist. that tightly envelopes target dist. results in more no. of samples samples being accepted.~~

(c). Advantage of tighter envelope is that the fraction of samples accepted when we have a tight envelope is very high and hence it is computationally efficient.