CS 5135/6035 Learning Probabilistic Models

Exercise Questions for Lecture 3 (Introduction to Probability)

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Questions

1. Seven friends decide to order pizzas by telephone from Pizza4U based on a flyer pushed through their letterbox. Pizza4U has only 4 kinds of pizza, and each person chooses a pizza independently. Bob phones Pizza4U and places the combined pizza order. How many different combined orders are possible? **[2 points]**

2. For each of the probability tables (i) list the random variable, (ii) domain of the random variable, and (iii) determine if it is a probability distribution: **[3 points]**

    (a)
    $$\begin{aligned} p(a = 1) & & 0.6 \\ p(a = 2) &= & 0.04 \\ p(a = 3) & & 0.34 \end{aligned}$$

    (b)
    $$\begin{aligned} p(x = a) & & 0.46 \\ p(x = b) &= & 0.16 \\ p(x = c) & & 0.38 \end{aligned}$$

    (c)
    $$\begin{aligned} p(z = aa) & & 0.4 \\ p(z = bc) &= & 0.1 \\ p(z = de) & & 0.5 \end{aligned}$$

3. There are two boxes. *Box 1* contains three red and five white balls and *box 2* contains two red and five white balls. A box is chosen at random $p(box = 1) = p(box = 2) = 0.5$ and a ball chosen at random from this box turns out to be red. What is the posterior probability that the red ball came from box 1? **[3 points]**

4. Answer the following questions: **[5+3+3 points]**

    (i) Prove that
    $$p(x|z) = \sum_y p(x|y, z)p(y|z) = \sum_{y,w} p(x|w, y, z)p(w|y, z)p(y|z)$$

    (ii) When y is unconditionally idependent of z, prove that
    $$p(x|z) = \sum_y p(x|y, z)p(y)$$

    (iii) When x is conditionally idependent of y, given z, prove that
    $$p(x|z) = \sum_y p(x|z)p(y|z)$$

5. The joint distribution $p(A, B)$ is as follows: (rows: A, columns: B) **[4 points]**

	b1	b2	b3
a1	0.42	0.05	0.02
a2	0.02	0.02	0.01
a3	0.02	0.02	0.42

Compute the marginal probabilities $p(A)$ and $p(B)$.

6. Based on the above joint distribution, determine if A and B are independent. **[2 points]**

7. [**Bonus question**] 1. Seven friends decide to order pizzas by telephone from Pizza4U based on a flyer pushed through their letterbox. Pizza4U has only 4 kinds of pizza, and each person chooses a pizza independently. Bob phones Pizza4U and places the combined pizza order, simply stating how many pizzas of each kind are required. Unfortunately, the precise order is lost, so the chef makes seven randomly chosen pizzas and then passes them to the delivery boy. What is the probability that the delivery boy has the right order?