This is a assignment of Statistics Advance Part 1

1. What is a random variable in probability theory?

A **random variable** is a variable that takes **numerical values** based on the outcome of a **random experiment**.

- It assigns a number to each possible outcome.
- It helps us **quantify uncertainty** in probability.

Example:

Tossing a coin:

- If Head → assign 1
- If Tail → assign 0
  Then the random variable `X` = outcome of coin toss:
- Possible values: 0 or 1

So, **`X` is a random variable** which can take different values depending on chance.

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2. What are the types of random variables?

There are **two main types**:

A. Discrete Random Variable

- Takes **countable** values (finite or infinite but countable)
- Often results from **counting** something

Examples:

- Number of students in a class
- Number of heads in 5 coin tosses: {0, 1, 2, 3, 4, 5}
- Rolling a die: {1, 2, 3, 4, 5, 6}

Graph:

Bar graph with spikes at each value

B. Continuous Random Variable

- Takes **infinite values in a range**
- Often results from **measuring** something

Examples:

- Height of a person (like 160.5 cm, 161.2 cm, etc.)
- Time taken to complete a task
- Temperature readings

Graph:

Smooth **curve** (like the bell curve for normal distribution)

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3. What is the difference between discrete and continuous distributions?

Discrete Distribution:

- Represents probabilities of **discrete random variables** (countable outcomes).
- Each individual value has a **non-zero probability**.
- The **sum** of all probabilities = **1**

Example:

Rolling a die → outcomes: 1, 2, 3, 4, 5, 6
Each has probability = 1/6

Graph:

Bar graph with **spikes** at each value.

Continuous Distribution:

- Represents probabilities of **continuous random variables** (infinite outcomes in a range).
- The probability of any exact value is **0**
- Probability is calculated over an **interval** (like between 2 and 4)
- The **area under the curve** = **1**

Example:

Height of people → measured as 160.2 cm, 160.25 cm, etc. (infinite possibilities)

Graph:

A **smooth curve** (e.g., normal distribution bell curve)

Key Differences Table:

Feature	Discrete Distribution	Continuous Distribution
Random Variable Type	Discrete (countable)	Continuous (infinite values)
Probability of single value	> 0 (e.g., P(X=2) = 0.2)	0 (e.g., P(X=2) = 0)
Total Probability	Sum of all points = 1	Area under curve = 1
Example	Coin toss, dice roll	Height, weight, time
Visualization	Bar chart	Smooth curve (PDF)

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4. What are probability distribution functions (PDF)?

A **Probability Distribution Function (PDF)** is a function that **describes the likelihood** of a random variable taking on a particular value.

There are **two types** based on the type of random variable:

A. PDF for Discrete Random Variables

It gives **P(X = x)** for every possible outcome x.

This is also called a **Probability Mass Function (PMF)**.

Example:

For a fair die:
P(X = 3) = 1/6

B. PDF for Continuous Random Variables

- It's a **curve** that represents **probability density**, not exact probabilities.
- The actual probability is calculated over an **interval** using **area under the curve**.

Example:

If X is a normal random variable,
P(2 ≤ X ≤ 3) = Area under PDF curve from x = 2 to x = 3

For continuous variables, P(X = a) = 0
(because one point has zero width ⇒ no area under curve)

Properties of a PDF:

1. PDF(x) ≥ 0 for all x
2. Total area under the curve = 1
3. For continuous case:
   $P(a \le X \le b) = \int_a^b f(x)dx$

5. How do cumulative distribution functions (CDF) differ from probability distribution functions (PDF)?

Probability Distribution Function (PDF):

- **PDF** shows **how likely** a random variable is to take a specific value (or range of values).
- For **discrete variables**, it's called **PMF** (Probability Mass Function).
- For **continuous variables**, the PDF is a **curve** — the probability over an interval is the **area under the curve**.

Key Point:

- PDF gives **instantaneous probability density**.
- In continuous case: probability at a single point is **0**.
- Mathematically,for continuous random variable
- XP(a≤X≤b)=Area under PDF from a to b

Cumulative Distribution Function (CDF):

- **CDF** gives the **cumulative probability** that the random variable is **less than or equal to** a value.
- It **adds up** the probability from the lowest value up to x.

Key Point:

- CDF is **always increasing** from 0 to 1.
- Mathematically:
  $F(x)=P(X\leq x)$

Difference Summary Table:

Feature	PDF	CDF
Meaning	Probability density at a value	Probability up to a value
Range of Values	Can be > 1 (for densities)	Always between 0 and 1
Shape	Bell curve, spikes, etc.	Always non-decreasing curve
Usage	Find density at point	Find cumulative probability
Relation	CDF is the integral of PDF	Derivative of CDF gives PDF

Simple Example (Dice Roll):

- PDF/PMF:
    - $P(X = 3) = 1/6$
- CDF:
    - $P(X \leq 3) = P(X=1) + P(X=2) + P(X=3) = 1/6 + 1/6 + 1/6 = 0.5$

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6. What is a discrete uniform distribution?

Discrete Uniform Distribution:

- A **discrete uniform distribution** is when **all possible outcomes** have the **same probability**.
- Each outcome is **equally likely**.

Examples:

- Rolling a **fair six-sided die**:
  Each side {1, 2, 3, 4, 5, 6} has probability = 1/6
- Picking a random day of the week:
  {Monday, Tuesday, ..., Sunday} → Probability = 1/7 each

Key Properties:

- **Equal probability** for all values
- If there are n possible outcomes:
  P(X=xi)=n/1
- Mean (Expected Value):
  E(X)=a+b/2
- Variance:
  Var(X)=(b−a+1)square−1/12
  where a and b are the minimum and maximum values.

Summary:

Feature	Discrete Uniform Distribution
Probability of each outcome	Equal (1/n)
Examples	Dice roll, random lottery numbers
Shape of PMF	Flat horizontal line (equal spikes)

7. What are the key properties of a Bernoulli distribution?

Bernoulli Distribution:

A **Bernoulli distribution** models a random experiment with only **two possible outcomes**:

- **Success** (usually coded as 1)
- **Failure** (usually coded as 0)

It is the **simplest discrete probability distribution**.

Examples:

- Tossing a coin → Head (1), Tail (0)
- Passing or failing a test
- Yes or No survey responses

Probability Function:

Let p = probability of success (1), then:

$$P(X=x)=\{p1-p \quad \text{if } x=1 \quad \text{if } x=0$$

Key Properties:

Property	Value / Formula
Possible values	0 or 1
Mean (Expected value)	E(X)=p
Variance	Var(X)=p(1−p)
Skewness	1−2p/under root p(1-p)
Distribution type	Discrete

Graph:

A bar graph with spikes at:

- 0 → height = 1 - p
- 1 → height = p

💡 Use:

Bernoulli is used as the **building block** for:

- **Binomial distribution**
- **Logistic regression**
- **Binary classification in ML**

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8. What is the Binomial Distribution, and how is it used in probability?

Binomial Distribution:

The **Binomial Distribution** models the **number of successes** in a **fixed number of independent Bernoulli trials**, each with the same probability of success.

Real-Life Examples:

- Number of heads in 10 coin tosses
- Number of correct answers in a multiple-choice test
- Number of defective items in a batch

Key Parameters:

- n = number of trials
- p = probability of success in a single trial

Probability Mass Function (PMF):

$P(X=k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$

Where:

$\binom{n}{k}$ = combinations of k successes out of n trials

- k = number of successes ($0 \leq k \leq n$)

Key Properties:

Property	Formula / Value
Mean (Expected Value)	$E(X) = np$
Variance	$Var(X) = np(1-p)$
Distribution Type	Discrete
Shape	Bell-shaped (when n is large)

Conditions to Use Binomial:

1. Fixed number of trials n
2. Each trial is **independent**
3. Each trial has only **two outcomes**: success/failure
4. Constant probability p across trials

Use in Probability:

- To find probability of **exact number of successes** (e.g., exactly 3 heads)
- To **model real-world situations** like quality control, marketing, medicine

- **Hypothesis testing** (binomial test)

Example:

Q: What's the probability of getting **2 heads** in **3 coin tosses**?
Here:

- `n = 3, p = 0.5, k = 2`
- `P(X=2)=(23)·(0.5)2·(0.5)1=3·0.25·0.5=0.375`

9. What is the Poisson Distribution and Where is it Applied?

Poisson Distribution:

The **Poisson distribution** models the **number of times an event occurs** in a **fixed interval of time or space**, **if**:

- Events occur **independently**,
- The **rate of occurrence is constant**, and
- Two events cannot happen at **exactly the same instant**.

Formula (PMF):

If λ is the **average rate** (mean number of occurrences), and k is the **actual number of occurrences**, then:

$P(X=k)=e^{-\lambda} \cdot \lambda^k / k!$

Where:

- e≈2.718e \approx 2.718e≈2.718
- λ>0λ > 0λ>0
- k=0,1,2,…k = 0, 1, 2, \dotsk=0,1,2,…

Key Properties:

Property	Value / Formula
Mean (Expected value)	λ
Variance	λ
Skewness	1/under root λ
Distribution Type	Discrete
Support	k ∈ {0, 1, 2, ...}

Applications:

- Number of calls at a call center per minute
- Number of emails received per hour
- Number of accidents at an intersection per day
- Number of typing errors per page
- Number of customers entering a shop in an hour

When to Use Poisson:

Use when you're counting **how many times** something happens in a **fixed interval** of:

- Time
- Space
- Area
- Volume

10. What is a Continuous Uniform Distribution?

Continuous Uniform Distribution:

A **continuous uniform distribution** is where **every value within a given interval** is **equally likely** to occur.

Example:

- Random number between 0 and 1
- Choosing a time uniformly between 2 PM and 4 PM

PDF Formula:

If a variable X is uniformly distributed between a and b:

f(x)={b−a10for a≤x≤ botherwise

Key Properties:

Property	Formula

Range	[a, b]
Mean	E(X)=a+b/2
Variance	ar(X)=(b−a)2 square / 12
Probability	All values in [a, b] equally likely
Distribution Type	Continuous

Applications:

- Simulating random wait times
- Random sampling in simulations
- Choosing a point at random on a straight line
- Any situation where every outcome in a range is equally likely

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Comparison with Discrete Uniform:

Feature	Discrete Uniform	Continuous Uniform
Values	Countable (e.g. dice)	Infinite (e.g. time)

Probability	**Specific values**	**Intervals (area under PDF)**
Graph	**Bar chart**	**Flat horizontal line**

11. What are the Characteristics of a Normal Distribution?

Normal Distribution:

The **normal distribution** is a **bell-shaped** and **symmetric** probability distribution used to describe many **natural phenomena**.

Key Characteristics:

Feature	Description
Shape	Bell-shaped curve (Gaussian)
Symmetry	Perfectly symmetric about the mean
Mean = Median = Mode	All three are equal and lie at the center
Tails	Extend infinitely in both directions but never touch the x-axis
Spread	Controlled by standard deviation (σ)

68-95-99.7 Rule	Describes how data falls within standard deviations from the mean

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68-95-99.7 Rule:

- 68% of data lies within **±1σ**
- 95% of data lies within **±2σ**
- 99.7% of data lies within **±3σ**

Mathematical Formula (PDF):

f(x)=σ 2πunder root /⋅e−(x−μ)/2σ 2square

Where:

- μ = mean
- σ = standard deviation

Real-World Examples:

- Heights, weights, IQ scores, exam scores, etc.

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12. What is the Standard Normal Distribution, and Why is It Important?

Standard Normal Distribution:

A **standard normal distribution** is a **special case** of the normal distribution with:

- **Mean (μ) = 0**
- **Standard deviation (σ) = 1**

Importance:

1. **Used for Z-scores**:
   Transforms any normal variable into a standard form:

   $$Z=X-\mu/\sigma$$

2. This allows comparisons between **different normal distributions**.

3. **Simplifies calculations**:
   Most statistical tables (like probability charts) are based on the standard normal distribution.

4. **Foundation for inferential statistics**:
   It's the core of **hypothesis testing, confidence intervals**, and **p-values**.

Visual:

The graph is bell-shaped and centered at **0**, with spread controlled by standard deviation = 1.

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13. What is the Central Limit Theorem (CLT), and Why is It Critical in Statistics?

Central Limit Theorem (CLT):

The **CLT states** that:

> When you take **many random samples** from any population (with any shape) and calculate the **mean** of each sample, the **distribution of those sample means** will tend to be **normal**, **if the sample size is large enough**.

Key Points:

- The **original data** can be skewed or non-normal.
- But the **sampling distribution of the mean** becomes **normal** as `n` increases.
- Usually, `n ≥ 30` is considered "large enough."

Why is it important?

It **justifies using normal-based methods** (like Z-tests, t-tests) even if the data isn't perfectly normal.

Forms the **foundation of inferential statistics**: estimates, confidence intervals, and hypothesis tests.

Formula (Sampling Distribution):

Mean of sample means=μ
Standard deviation (standard error)=σ/under root n

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14. How Does the Central Limit Theorem Relate to the Normal Distribution?

Connection:

- The **CLT explains why** the **normal distribution appears so often** in statistics.
- It shows that the **sampling distribution of the mean** will be **normal**, even if the original data is not.
- It gives the **normal distribution** a **central role** in:
    - Hypothesis testing
    - Confidence intervals
    - Statistical modeling

- **Visual Summary:**
- **Original data sample size sampling distribution**
- **Skewed Small (n=5) not normal**
- **Skewed Large (n=30+) Approaches Normal**
- **Normal Any n Always Normal**

15. What is the Application of Z-Statistics in Hypothesis Testing?

Z-statistic (Z-test):

The Z-statistic is used in hypothesis testing when:

- **The population standard deviation (σ) is known.**
- **The sample size is large (n ≥ 30).**

Applications of Z-Statistic:

1. **One-sample Z-test:**
    - **Compare the sample mean to a known population mean.**
2. **Two-sample Z-test:**
    - **Compare the means of two independent large samples.**
3. **Z-test for proportions:**
    - **Compare sample proportion to a known population proportion.**

Z-Test Process:

1. **State null (H_0) and alternative (H_1) hypotheses.**
2. **Calculate the Z-score.**
3. **Find the critical value or p-value.**
4. **Make a decision:**
    - **If $|Z|$ > critical value → reject H_0.**
    - **If p-value < α (e.g. 0.05) → reject H_0.**

Example Use Cases:

- **Quality control**
- **Medical trials**
- **Comparing exam performance to national average**

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16. How Do You Calculate a Z-Score, and What Does It Represent?

Z-score:

The Z-score tells you how many standard deviations a value is from the mean.

Formula: $Z=X-\mu/\sigma$

Where:

- **X = individual data point**
- **μ\muμ = mean**
- **σ\sigmaσ = standard deviation**

Interpretation:

Z-Score	Meaning
0	Exactly at the mean
+1	1 standard deviation above the mean
-1	1 standard deviation below the mean
±2 or more	Far from the mean (outliers)

Use in Statistics:

- **Comparing scores across different distributions**
- **Outlier detection**
- **Hypothesis testing (Z-tests)**
- **Normalization in machine learning**

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17. What Are Point Estimates and Interval Estimates in Statistics?

Point Estimate:

A single value used to estimate a population parameter.

Example:
Sample mean $\bar{x}$ is a point estimate of population mean μ

Interval Estimate:

A range of values used to estimate the population parameter, along with a confidence level.

Example:
"We are 95% confident the population mean lies between 68 and 72."

Comparison:

Point estimate **single best guest** **$\bar{x}=70$**

Interval estimate **range with confidence** **$68 \leq \mu \leq 72$ with 95% confidence**

18. What is the Significance of Confidence Intervals in Statistical Analysis?

Confidence Interval (CI):

A range around a point estimate that likely contains the true population parameter with a specified confidence level (e.g., 95%).

Why Important?

1. **Provides uncertainty range:**
   Shows how much variability is in your estimate.

2. **More informative than point estimate:**
   Tells you *how precise* your estimate is.

3. **Used in decision-making:**
   Helps determine statistical significance in hypothesis testing.

4. **Linked to hypothesis testing:**
   If the hypothesized value (like μ_0) lies outside the confidence interval, we reject the null hypothesis.

Common Confidence Levels:

- **90%**
- **95% (most common)**
- **99%**

Formula (for mean, known σ):

CI=x¯±Zα/2⋅σ/under root n

19. What is the Relationship Between a Z-Score and a Confidence Interval?

Z-Score and Confidence Interval (CI) are closely connected.

- A **Z-score** determines **how far** a sample mean is from the population mean in terms of **standard deviations**.
- A **Confidence Interval** uses the **Z-score** as a **critical value** to define the range where the true parameter is likely to lie.

Formula (for known σ):

CI=x¯±Zα/2⋅σ/under root n

x¯\bar{x}x¯ = sample mean
Zα/2 = Z-score corresponding to the desired confidence level (e.g. 1.96 for 95%)

= standard error σ/under root n
The Z-score sets the width of the confidence interval. Higher confidence level → higher Z-score → wider interval.

20. How Are Z-Scores Used to Compare Different Distributions?

Z-score standardizes values from different distributions to a common scale.

$Z = X - \mu / \sigma$

- This lets you compare values **even if** the means and standard deviations of the distributions are **different**.

Example:

- Alice scores 85 on a test with mean 80 and SD 5 → Z = 1
- Bob scores 90 on a test with mean 88 and SD 4 → Z = 0.5

Alice performed **better relative to her group** (Z=1 vs Z=0.5).

Use Cases:

- Comparing test scores across different subjects
- Outlier detection
- Normalization in machine learning

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21. What Are the Assumptions for Applying the Central Limit Theorem (CLT)?

The **Central Limit Theorem (CLT)** allows us to assume that the **sampling distribution of the sample mean** is approximately **normal**, **regardless of population shape**, if certain assumptions are met.

- **Key Assumptions**:
- Assumption Description
- Random Sampling Data must be selected randomly and independently.
- Sample Size (n ≥ 30) A large enough sample size is required (n ≥ 30 is a rule of thumb). Smaller n is okay if the population is already normal.
- Finite Variance The population should have a finite standard deviation (σ).
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Conclusion:

If these assumptions are satisfied, we can use **normal distribution methods** (like Z-tests), even for **non-normal data**.

22. What Is the Concept of Expected Value in a Probability Distribution?

Expected Value (E[X]):

The **expected value** of a random variable is the **long-run average** outcome if the experiment is repeated many times.

Formula:

- For **discrete** random variable:

$E[X]=\sum(x_i \cdot P(x_i))$
For **continuous** random variable:
$E[X]=\int_{-\infty}^{\infty} x \cdot f(x)dx$

Example:

For a fair 6-sided die:

$E[X]=1(1/6)+2(1/6)+...+6(1/6)=3.5$

It doesn't mean you'll roll a 3.5, but on **average**, over many rolls, you'll get that value.

Use Cases:

- Decision making
- Insurance risk calculations
- Expected profit/loss

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23. How Does a Probability Distribution Relate to the Expected Outcome of a Random Variable?

Relationship:

A **probability distribution** assigns **probabilities to each possible outcome** of a random variable.

The **expected value** is the **weighted average** of all outcomes based on that distribution.

How They Connect:

- The **distribution** tells us *how likely* each value of the random variable is.
- The **expected value** summarizes that information into a **single number** representing the **average outcome over time**.

Example:

In a game:

- Win ₹100 with 0.3 probability
- Lose ₹50 with 0.7 probability

$E[X] = (100 \cdot 0.3) + (-50 \cdot 0.7) = 30 - 35 = -₹5$

So, on average, you lose ₹5 per game.

######### the End ##########

I done this assignment on google doc

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