SUPPORT VECTOR MACHINE - LINEAR EXAMPLE

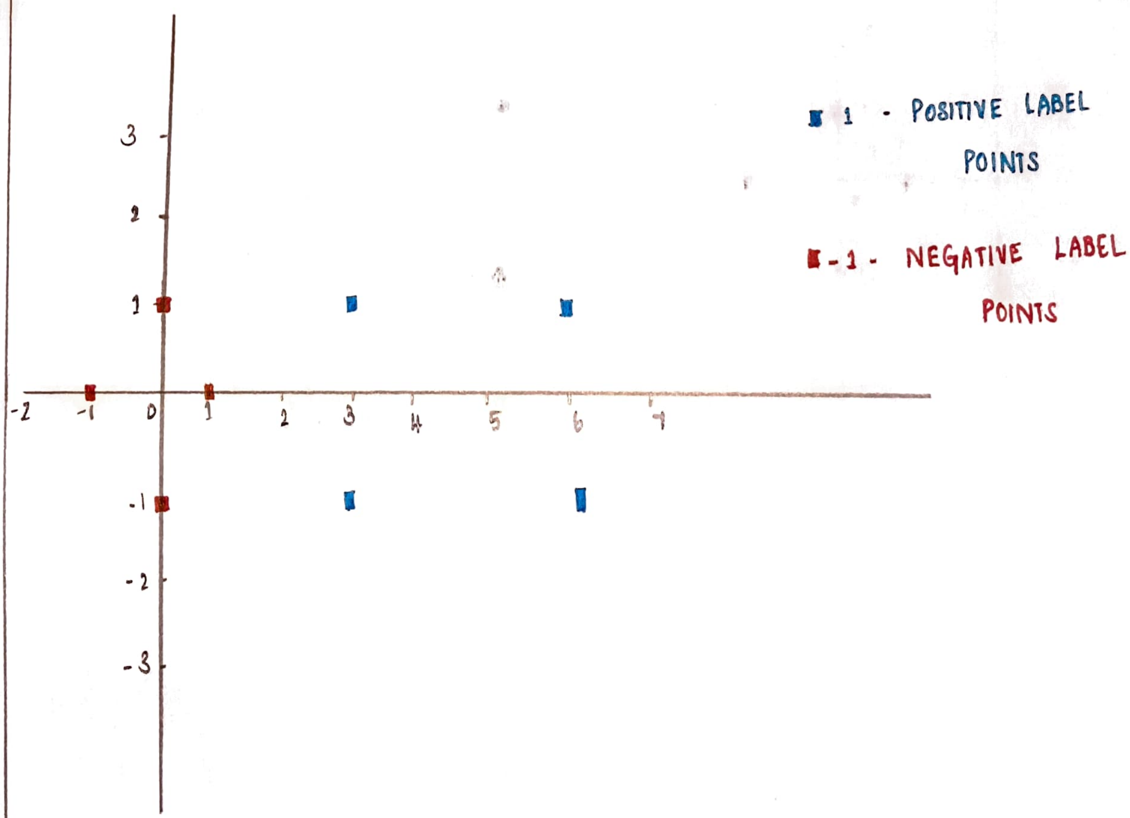
Suppose we are given the following positively labeled data points,

$$\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \end{pmatrix} \right\}$$

and the following negatively labeled data points,

$$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}$$

STEP 1 : PLOTTING THE POINTS



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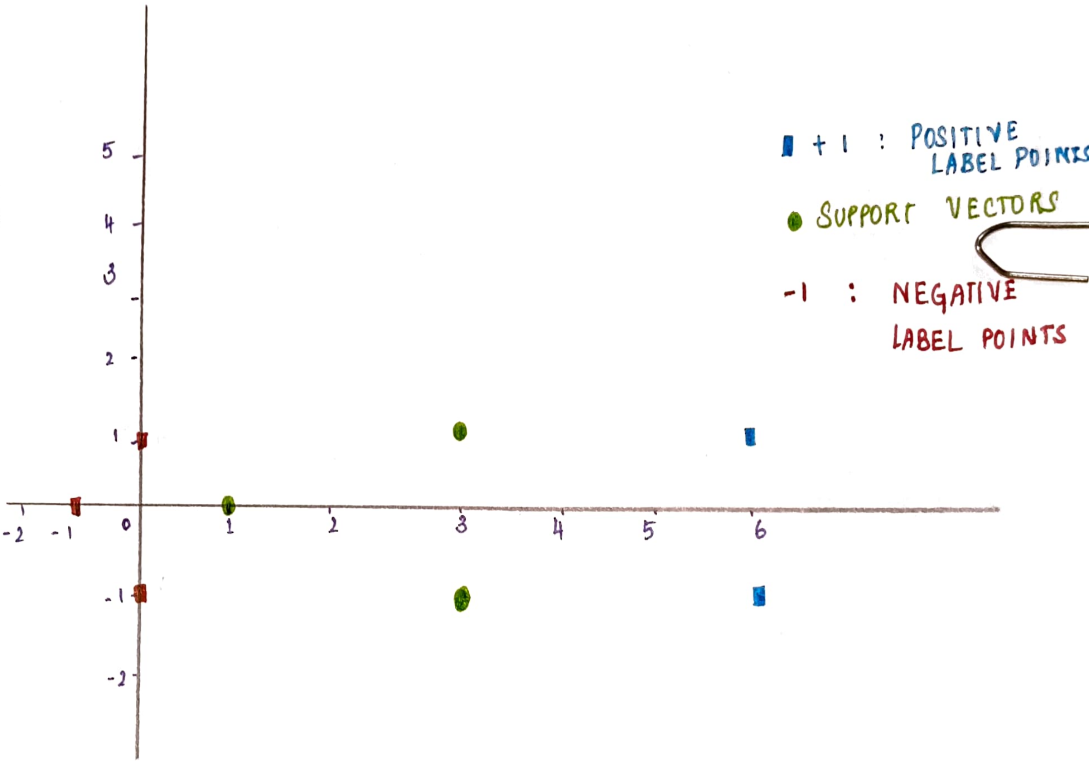
STEP 2 : IDENTIFICATION OF THREE SUPPORT VECTORS

↓

Data points closer to the Hyperplane.

$$\left\{ S_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad S_3 = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right\}$$



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STEP 3 : AUGMENTING SUPPORT VECTOR WITH BIAS 1

(i) Each vector is augmented with a 1 as a bias input

(ii) $S_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\tilde{S}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$S_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\tilde{S}_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

$S_3 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $\tilde{S}_3 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$

STEP 4 : COMPUTATION OF $\alpha 1$, $\alpha 2$, $\alpha 3$

$$\alpha 1 \, \tilde{S}_1 \cdot \tilde{S}_1 + \alpha_2 \, \tilde{S}_2 \cdot \tilde{S}_1 + \alpha_3 \, \tilde{S}_3 \cdot \tilde{S}_1 = -1$$

$$\alpha 1 \, \tilde{S}_1 \cdot \tilde{S}_2 + \alpha_2 \, \tilde{S}_2 \cdot \tilde{S}_2 + \alpha_3 \, \tilde{S}_3 \cdot \tilde{S}_2 = +1$$

$$\alpha_1 \, \tilde{S}_1 \cdot \tilde{S}_3 + \alpha_2 \, \tilde{S}_2 \cdot \tilde{S}_3 + \alpha_3 \, \tilde{S}_3 \cdot \tilde{S}_3 = +1$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} = +1$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = +1$$

$$\alpha_1(1+0+1) + \alpha_2(3+0+1) + \alpha_3(3+0+1) = -1$$

$$\alpha_1(3+0+1) + \alpha_2(9+1+1) + \alpha_3(9-1+1) = 1$$

$$\alpha_1(3+0+1) + \alpha_2(9-1+1) + \alpha_3(9+1+1) = 1$$

$$2\alpha_1 + 4\alpha_2 + 4\alpha_3 = -1$$

$$4\alpha_1 + 11\alpha_2 + 9\alpha_3 = 1$$

$$4\alpha_1 + 9\alpha_2 + 11\alpha_3 = 1$$

$$\alpha_1 = -3.5$$

$$\alpha_2 = 0.75$$

$$\alpha_3 = 0.75$$

STEP 5 : COMPUTATION OF WEIGHT VECTOR

$$\vec{w} = \sum_i \alpha_i \tilde{s}_i$$

$$= -3.5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 0.75 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + 0.75 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{w} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$

(i) Vectors are augmented with a bias.

(ii) ∴ Equate the last entry in $\tilde{w}$ as the hyperplane offset b.

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(iii) Hyperplane equation

$$\boxed{y = wx + b}$$

$w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$b = -2$

Hyperplane Equation

$w^T \cdot x = -b$

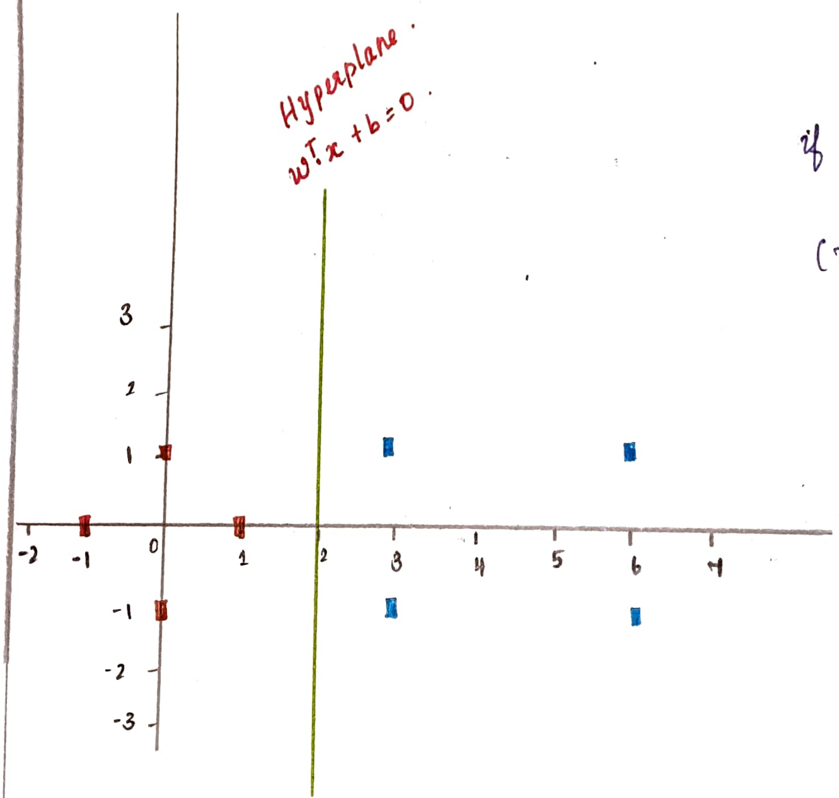
$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot x = -(-2)$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot x = 2$

if $x = 1$
$y = 0$
(The Line parallel to y - axis)

if $x = 0$
$y = 1$
(The Line parallel to x - axis)

if $x = 1$
$y = 1$
(The Line is at 45° w.r.t x and y axis)



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