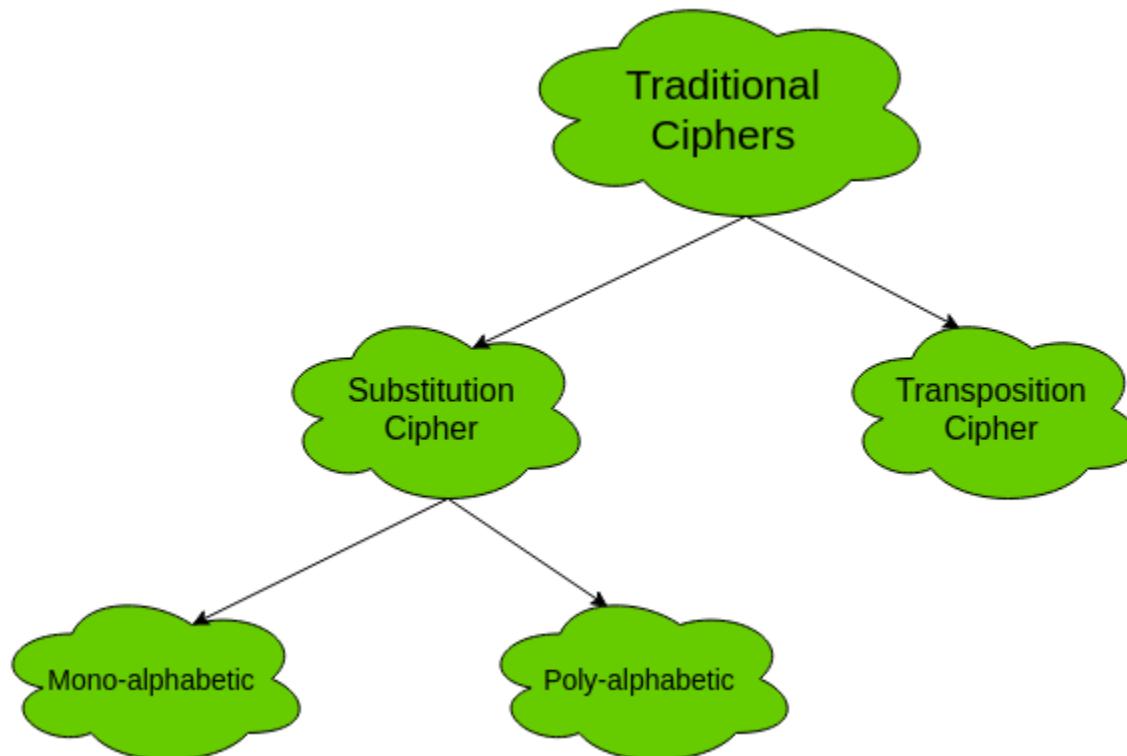


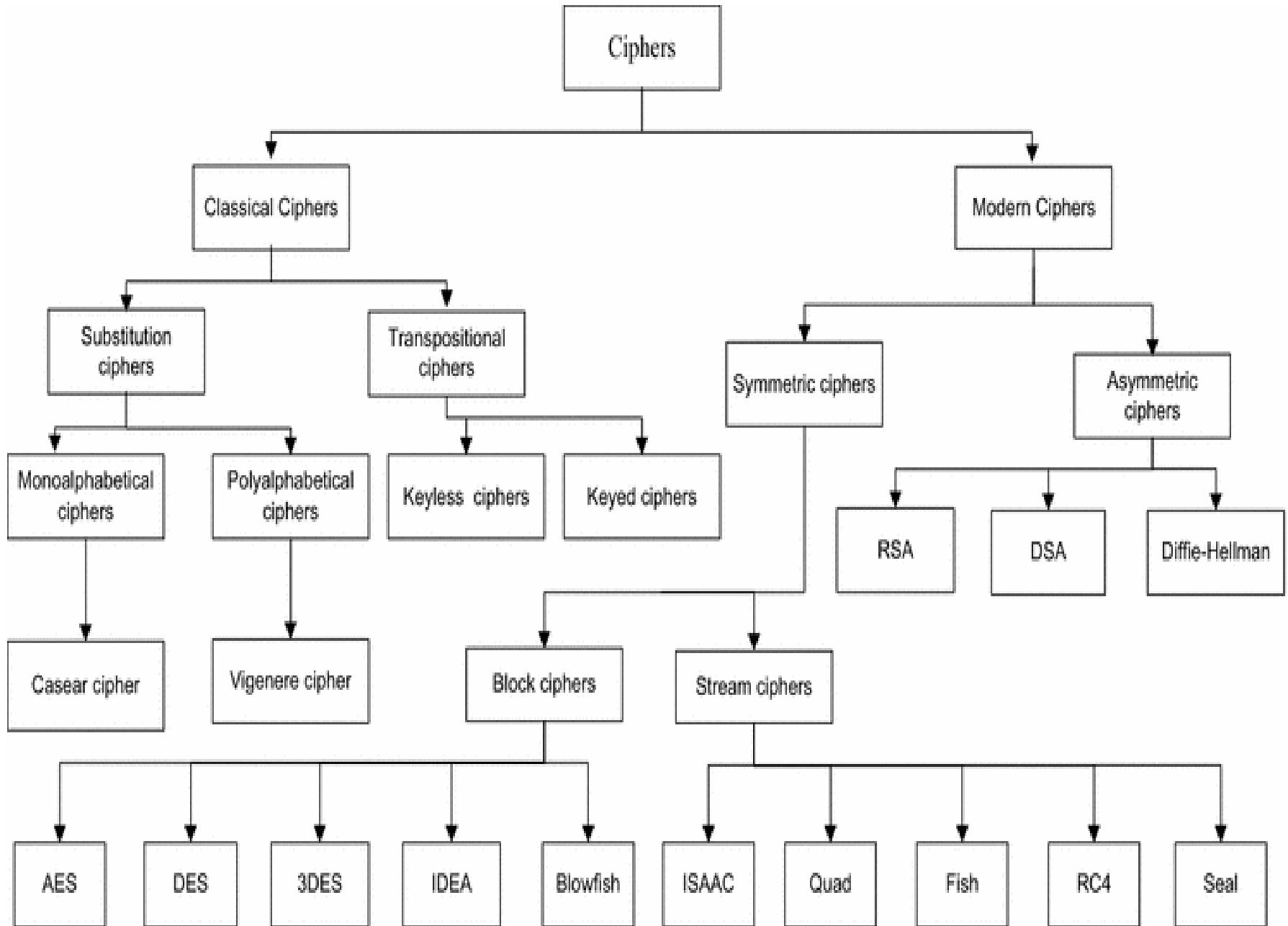
Cryptography

Classical Ciphers

Traditional Symmetric Ciphers

The two types of traditional symmetric ciphers are **Substitution Cipher** and **Transposition Cipher**.
The following flowchart categories the traditional ciphers:





Classical vs Modern

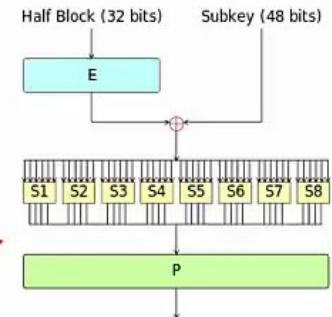
Classical

- Confidentiality
- Plain text
- Military
- Secrecy of protocol/algorithm



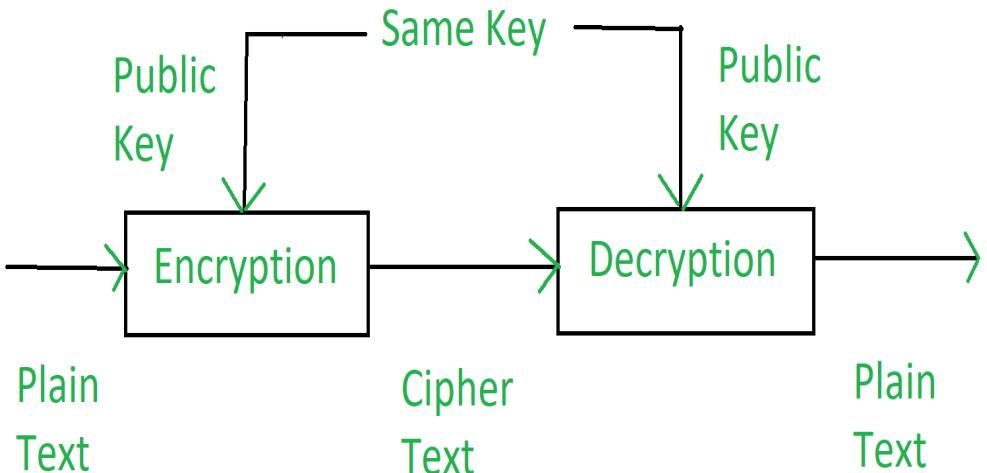
Modern

- Confidentiality, Integrity
 - Further digital cash, secure voting etc
- Deals with bits
- Every one
- Provable security based on mathematics (protocol /algorithm often open)



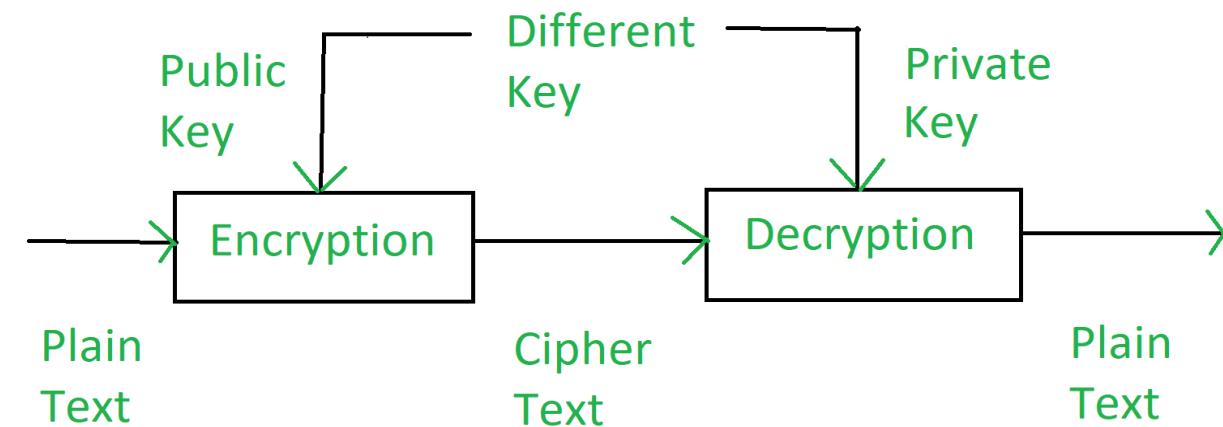
Role of Key

In **cryptography**, a **key** is a piece of information (a parameter) that determines the functional output of a **cryptographic** algorithm.



Symmetric Cryptography

For **encryption** algorithms, a **key** specifies the transformation of plaintext into ciphertext, and vice versa for decryption algorithms.



Asymmetric Cryptography

Various Ciphers

1. Caesar Cipher
 2. Vigenere Cipher
 3. Affine Cipher
 4. Playfair Cipher
 5. Pigpen Cipher
 6. ADFGVX Cipher
 7. Vernam Cipher
 8. Hill Cipher
 9. Digraph Cipher
 10. Rotor Cipher
 11. Book Cipher
 12. Decimation
-
1. Rail fence Cipher
 2. Keyless transposition Cipher
 3. Keyed transposition Cipher
 4. Double Transposition Cipher

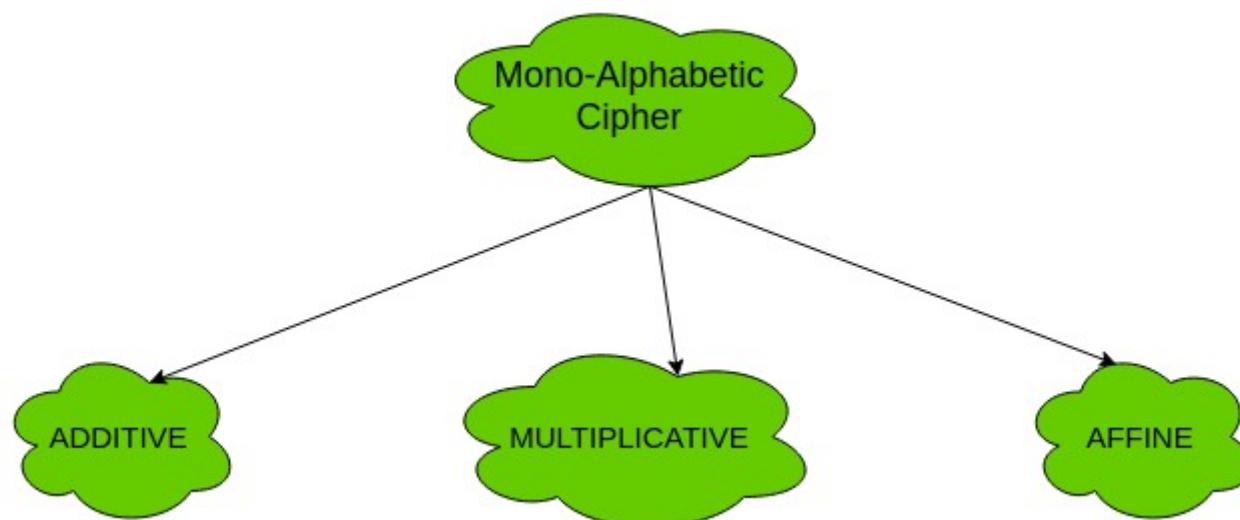
Digraph Cipher

- Playfair Cipher
- Hill cipher

Transposition Cipher

1. Keyless transposition Cipher
2. Keyed transposition Cipher
3. Double Transposition Cipher

Types of mono-alphabetic ciphers are:



Caesar Cipher

- earliest known substitution cipher
- by Julius Caesar
- first attested use in military affairs
- replaces each letter by 3rd letter on
- example:

meet me after the toga party

PHHW PH DIWHU WKH WRJD SDUWB

Caesar Cipher

- can define transformation as:

a b c d e f g h i j k l m n o p q r s t u v w x y z
D E F G H I J K L M N O P Q R S T U V W X Y Z A B C

- mathematically give each letter a number

a b c d e f g h i j k l m n o p q r s t u v w x y z
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

- then have Caesar cipher as:

$$c = E(p) = (p + k) \bmod 26$$

$$p = D(c) = (c - k) \bmod 26$$

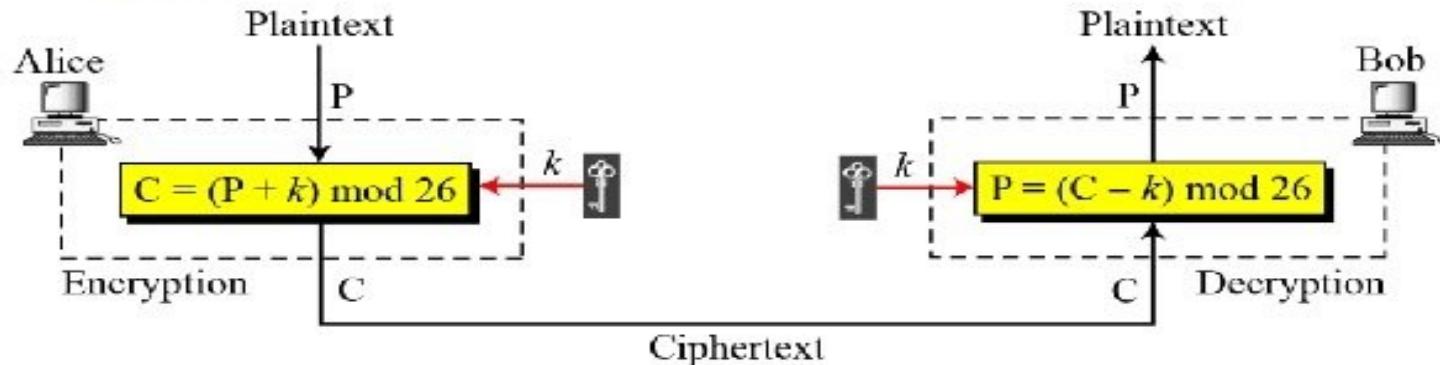
Cryptanalysis of Caesar Cipher

- only have 26 possible ciphers
 - A maps to A,B,..Z
- could simply try each in turn
- a **brute force search**
- given ciphertext, just try all shifts of letters
- do need to recognize when have plaintext

Additive Cipher

3.2.1 Continued

Figure 3.9 Additive cipher



Note

When the cipher is additive, the plaintext, ciphertext, and key are integers in \mathbb{Z}_{26} .

3.2.1 Continued

Example 3.3

Use the additive cipher with **key = 15** to encrypt the message “hello”.

Solution

We apply the encryption algorithm to the plaintext, character by character:

| | | |
|-------------------|---|--------------------|
| Plaintext: h → 07 | Encryption: $(07 + 15) \text{ mod } 26$ | Ciphertext: 22 → W |
| Plaintext: e → 04 | Encryption: $(04 + 15) \text{ mod } 26$ | Ciphertext: 19 → T |
| Plaintext: l → 11 | Encryption: $(11 + 15) \text{ mod } 26$ | Ciphertext: 00 → A |
| Plaintext: l → 11 | Encryption: $(11 + 15) \text{ mod } 26$ | Ciphertext: 00 → A |
| Plaintext: o → 14 | Encryption: $(14 + 15) \text{ mod } 26$ | Ciphertext: 03 → D |

$$\text{Encryption } EK(x) = x + K \text{ mod } 26$$

3.2.1 Continued

Example 3.4

Use the additive cipher with **key = 15** to decrypt the message “WTAAD”.

Solution

We apply the decryption algorithm to the plaintext character by character:

Ciphertext: W → 22

Decryption: $(22 - 15) \bmod 26$

Plaintext: 07 → h

Ciphertext: T → 19

Decryption: $(19 - 15) \bmod 26$

Plaintext: 04 → e

Ciphertext: A → 00

Decryption: $(00 - 15) \bmod 26$

Plaintext: 11 → l

Ciphertext: A → 00

Decryption: $(00 - 15) \bmod 26$

Plaintext: 11 → l

Ciphertext: D → 03

Decryption: $(03 - 15) \bmod 26$

Plaintext: 14 → o

$$\text{Decryption } DK(x) = x - K \bmod 26$$

3.2.1 Continued

Example 3.5

Eve has intercepted the ciphertext “UVACLYFZLJBYL”. Show how she can use a brute-force attack to break the cipher.

Solution

Eve tries keys from 1 to 7. With a key of 7, the plaintext is “not very secure”, which makes sense.

Ciphertext: UVACLYFZLJBYL

| | | |
|---------|---------------|--------------------------|
| $K = 1$ | \rightarrow | Plaintext: tuzbkxeykiaxk |
| $K = 2$ | \rightarrow | Plaintext: styajwdxjhzwj |
| $K = 3$ | \rightarrow | Plaintext: rsxzivcwigyvi |
| $K = 4$ | \rightarrow | Plaintext: qrwyhubvhfxuh |
| $K = 5$ | \rightarrow | Plaintext: pqvxgtaugewtg |
| $K = 6$ | \rightarrow | Plaintext: opuwfsztfdvsf |
| $K = 7$ | \rightarrow | Plaintext: notverysecure |

Multiplicative Cipher

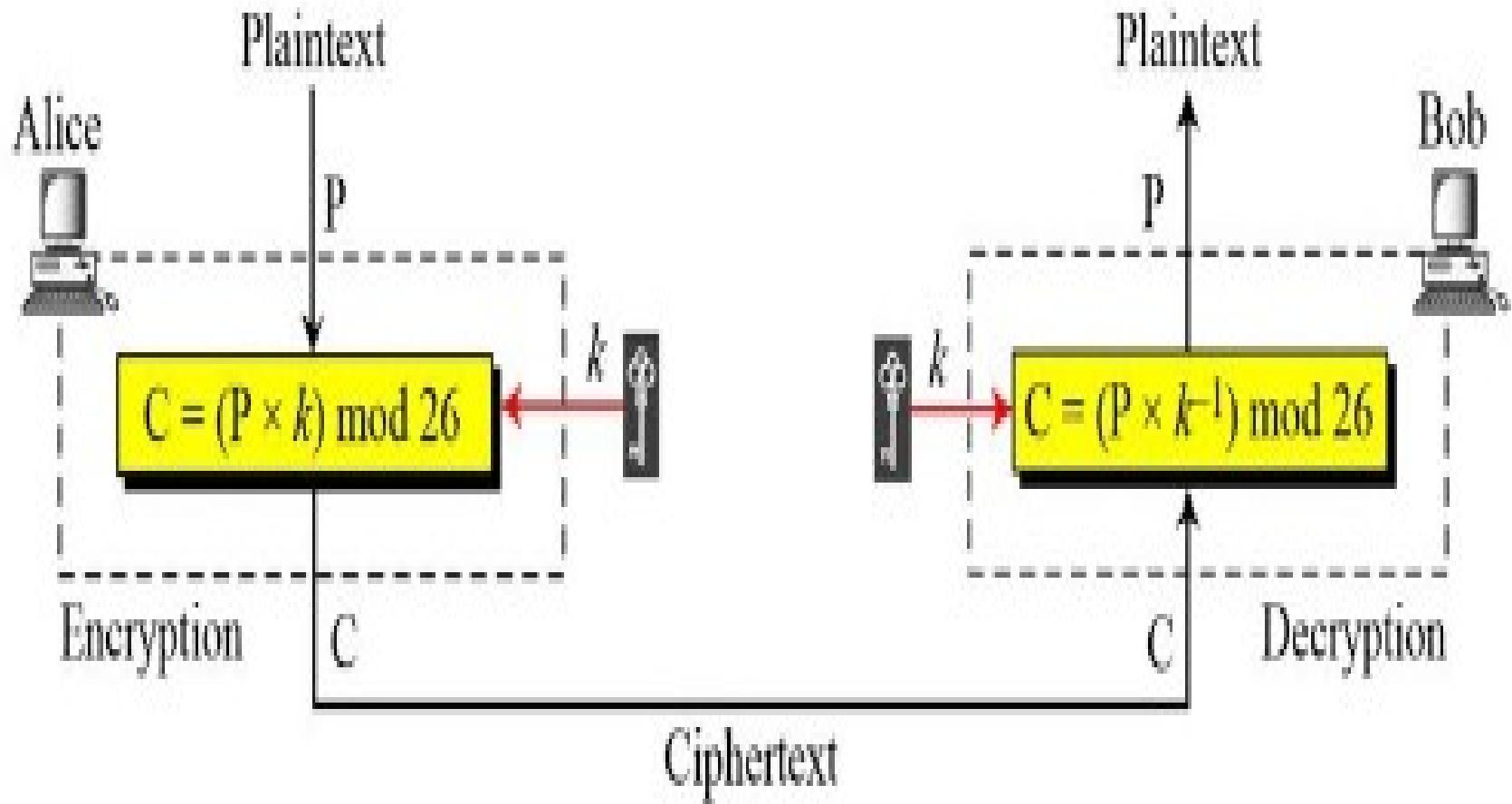
- The multiplicative cipher is similar to additive cipher except the fact that the key bit is multiplied to the plain-text symbol during encryption. Likewise, the cipher-text is multiplied by the multiplicative inverse of key for decryption to obtain back the plain-text.

$$C = (M * k) \bmod n$$

$$M = (C * k^{-1}) \bmod n$$

- where,
 k^{-1} -> multiplicative inverse of k (key)
- The key space of multiplicative cipher is 12. Thus, it is also not very secure.

Multiplicative Cipher



Encryption

$$C = E(K, P) = (P * K) \bmod 26$$

Decryption

- Decryption algorithm :

$$P=D(K,C)=(C \cdot K^{-1}) \bmod 26$$

Example

encrypt the message "HELLO" using multiplicative cipher with key = 7

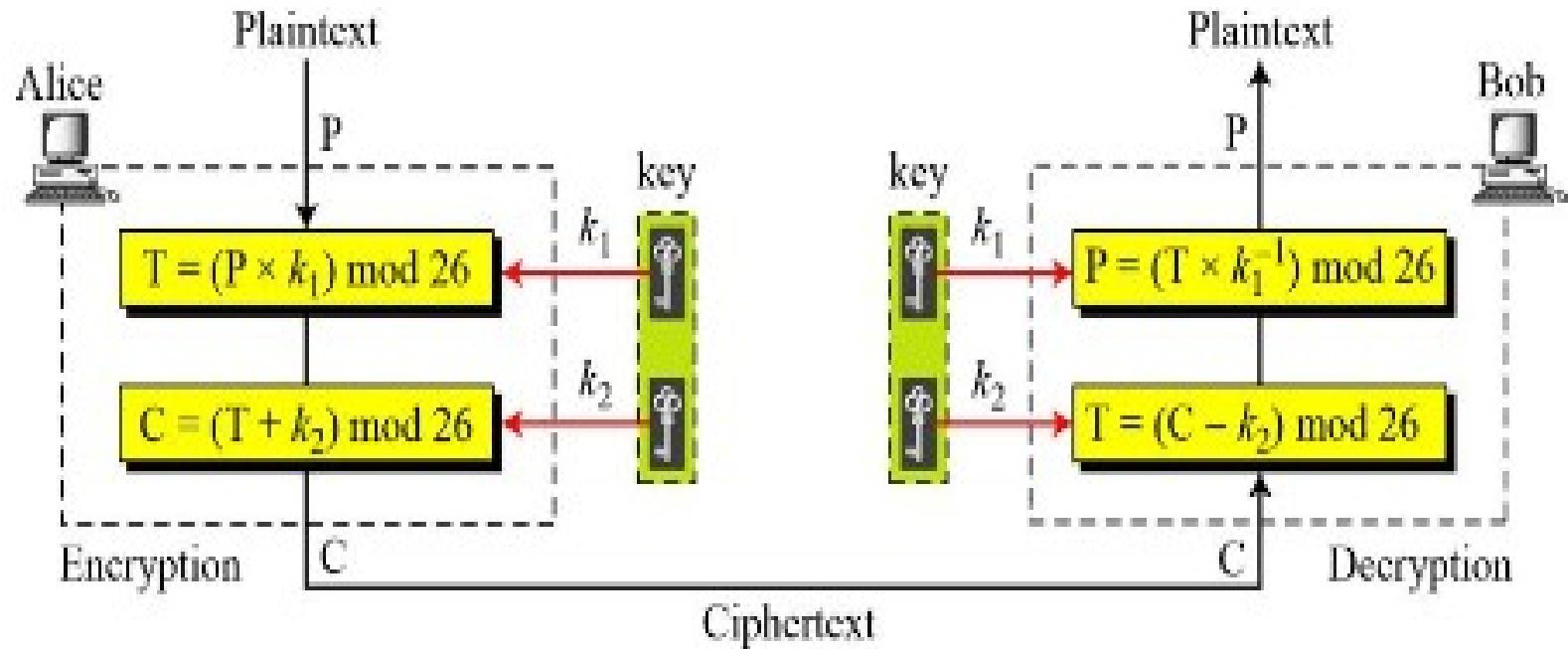
| Plaintext: Letters → Numeric Value | Encryption: $(P * K) \text{ mod } 26$ | Ciphertext: Numeric Value → letters |
|------------------------------------|---|-------------------------------------|
| Plaintext: H = 07 | Encryption: $(07 * 07) \text{ mod } 26$ | Ciphertext: 23 = X |
| Plaintext: E = 04 | Encryption: $(04 * 07) \text{ mod } 26$ | Ciphertext: 02 = C |
| Plaintext: L = <u>11</u> | Encryption: $(11 * 07) \text{ mod } 26$ | Ciphertext: 25 = Z |
| Plaintext: L = <u>11</u> | Encryption: $(11 * 07) \text{ mod } 26$ | Ciphertext: 25 = Z |
| Plaintext: O = 14 | Encryption: $(14 * 07) \text{ mod } 26$ | Ciphertext: 20 = U |

Table of Multiplicative Inverse

we must use a multiplier which is co-prime (the values do not share any factors when dividing in relation to the size of the alphabet (26), so you should use either 1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23 or 25.

| key a | key a^{-1} |
|---------|--------------|
| 1 | 1 |
| 3 | 9 |
| 5 | 21 |
| 7 | 15 |
| 9 | 3 |
| 11 | 19 |
| 15 | 7 |
| 17 | 23 |
| 19 | 11 |
| 21 | 5 |
| 23 | 17 |
| 25 | 25 |

Affine Cipher



$$C = (P \times k_1 + k_2) \bmod 26$$

$$P = ((C - k_2) \times k_1^{-1}) \bmod 26$$

where k_1^{-1} is the multiplicative inverse of k_1 and $-k_2$ is the additive inverse of k_2

Example

Encrypt the message "HELLO" using Affine Cipher with key pair (7,2)

| Plaintext: Letters → Numeric Value | Encryption: $((P * K1) + K2) \text{mod } 26$ | Ciphertext: Numeric Value → letters |
|------------------------------------|--|-------------------------------------|
| Plaintext: H = 07 | Encryption: $(07 * 07 + 2) \text{mod } 26$ | Ciphertext: 25 = Z |
| Plaintext: E = 04 | Encryption: $(04 * 07 + 2) \text{mod } 26$ | Ciphertext: 04 = E |
| Plaintext: L = 11 | Encryption: $(11 * 07 + 2) \text{mod } 26$ | Ciphertext: 01 = B |
| Plaintext: L = 11 | Encryption: $(11 * 07 + 2) \text{mod } 26$ | Ciphertext: 01 = B |
| Plaintext: O = 14 | Encryption: $(14 * 07 + 2) \text{mod } 26$ | Ciphertext: 22 = W |

Affine Ciphers...

Use the affine cipher to decrypt the message “ZEBBW” with the key pair **(7, 2)** in modulus **26**.

| | | |
|-----------|---|-----------|
| C: Z → 25 | Decryption: $((25 - 2) \times 7^{-1}) \bmod 26$ | P: 07 → h |
| C: E → 04 | Decryption: $((04 - 2) \times 7^{-1}) \bmod 26$ | P: 04 → e |
| C: B → 01 | Decryption: $((01 - 2) \times 7^{-1}) \bmod 26$ | P: 11 → l |
| C: B → 01 | Decryption: $((01 - 2) \times 7^{-1}) \bmod 26$ | P: 11 → l |
| C: W → 22 | Decryption: $((22 - 2) \times 7^{-1}) \bmod 26$ | P: 14 → o |

The additive cipher is a special case of an affine cipher in which $k_1 = 1$. The multiplicative cipher is a special case of affine cipher in which $k_2 = 0$.

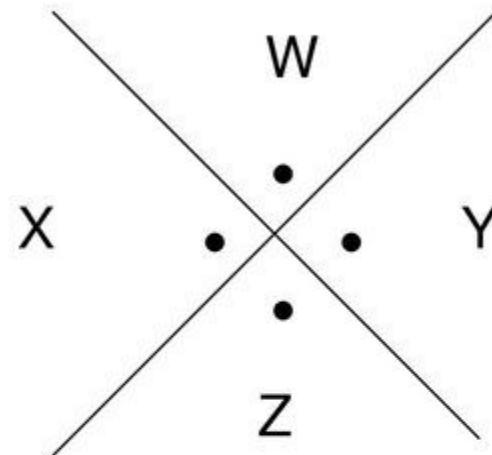
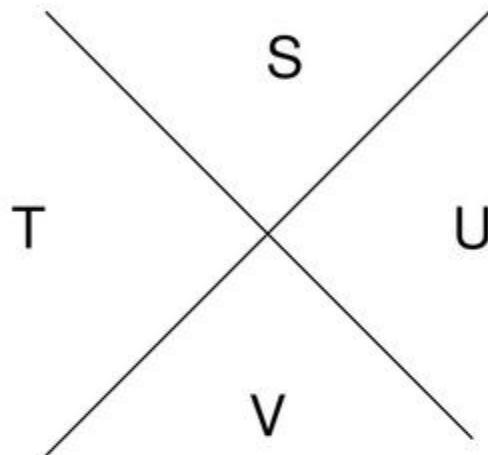
Pigpen Cipher

- Pigpen cipher is a variation on letter substitution
- Alphabets are arranged as follows:

| | | |
|-------|---|---|
| A | B | C |
| <hr/> | | |
| D | E | F |
| <hr/> | | |
| G | H | I |

| | | | | |
|---|---|---|---|---|
| J | • | K | • | L |
| M | • | N | • | O |
| P | • | Q | • | R |
| | | | | |

Pigpen Cipher diagram (cont'd)



A =

C =

G =

W =

Pigpen Cipher

- Alphabets will be represented by the corresponding diagram
- E.g., WAG would be A diagram illustrating the Pigpen cipher mapping for the letters W, A, and G. The letter 'W' is represented by a diamond shape with a dot at the top vertex. The letter 'A' is represented by a vertical line segment with a dot at its top end. The letter 'G' is represented by a vertical line segment with a horizontal line segment extending from its right side.
- This is a weak cipher

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | b | c | d | e | f | g | h | i | j |
| ˩ | ˨ | ˧ | ˥ | ߱ | ߳ | ߷ | ߸ | ߹ | ߻ |
| k | l | m | n | o | p | q | r | s | t |
| ܊ | ܉ | ܌ | ܍ | ܎ | ܏ | ܐ | ܑ | ܒ | ܔ |
| u | v | w | x | y | z | | | | |
| ܕ | ܈ | ܌ | ܔ | ܏ | ܐ | | | | |

Decode the following pigpen ciphertext:

L F A O L E B N J D C T F E V T O F

Encode the following message using the pigpen cipher:

the truth is out there

