Grammar:

alphabet set (terminals)

$G = \{ V, \Sigma, P, S$ → start symbol

V → set of non-terminals

P → production

$\alpha \in \{V^+ \cup \Sigma\}^*$

~~* mean~~

* → 0 or more repitition

\+ → 1 or more repitition

$\Sigma = \{$ symbol's $\}$

$\{0, 1, 2, \ldots, 9\}$

$V =$ set of NT

A - Z

$P(\alpha \rightarrow \beta)$ — production ~~expansion~~

should have a non-terminal

Eg: $w = 0110$, $y = 0aa$, $x = adbcaa$

$z = 111$

special string : ε or λ

concatenation : wz 0110111

Length : $|w| = 4$, $|\varepsilon| = 0$, $|x| = 6$

Reversal: $y^R = aa0$

Special strings:

Σ^* All strings of symbols from Σ

$$\Sigma^{\cancel{*}+} = \Sigma^* - \{\varepsilon\}$$

ε - empty strings

if $\Sigma = \{a, b\}$

$$\Sigma^* = \{\underbrace{\epsilon}_{\text{0 reps}}, \overbrace{a, b}^{\text{1 rep}}, \overbrace{aa, ab, ba, bb, \ldots}^{\text{2 reps}}\}$$

$$\Sigma^+ = \{\overbrace{a, b}^{\text{1 rep}}, \overbrace{aa, ab, ba, bb, \ldots}^{\text{2 rep}}\}$$

$$\boxed{\begin{aligned} \Sigma^* &= \Sigma^+ \cup \{\epsilon\} \\ \Sigma^+ &= \Sigma^* - \{\epsilon\} \end{aligned}}$$

ϕ ~~= empt~~

ϕ - null languag - has no strings $\{\}$

ϵ - is an empty string

$$\boxed{\phi \neq \{\epsilon\}}$$

$\Sigma = \{0, 1\}$

Language rules

1) $L\{\epsilon\} = \{\epsilon\}L = L$

2) $(L_1 L_2) L_3 = L_1 (L_2 L_3)$

3) $L^+ = LL^*$

4) $L^+ = L^* - \{\epsilon\}$

5) $L = \{\} = \phi$

6) $L\phi = \phi L = \phi$

7) $L_1 \cup L_2 = \{$

no. of string in

$|a \cdot b| = |a| \times |b|$

$|a \cdot \phi| = |a| \times \overset{=0}{|\phi|}$

$a\phi \overset{=0}{=} \phi$

Production:

$P : \alpha \to \beta$

$\alpha \in \{V \cup \Sigma\}^*$

eg: $L = \{a^n b^n \mid n \geq 0\}$ — set builder form

$S \to \epsilon \mid aSb$

production

eg. to derive $aabb$

$S \to \epsilon \mid aSb$

$S \to aSb$

$aaSbb$

$aa\epsilon bb$

$aabb$

Regex

- $\cdot$ concatenation
- $|$ or
- $*$ zero/more rep
- $+$ one/more rep

Precedence:

$(\;) \quad \underset{+}{*} \quad \cdot \quad |$

$L = \{ a^n b^{n+1} \mid n \geq 0 \}$

$S \rightarrow b \mid aSb$

To derive aabbb

$S \rightarrow aSb$
aaSbb
aabbb

$L = \{ a^n b^m \mid$ m may not be equal to n $\}$

$S \rightarrow \epsilon \mid aS \mid Sb$

derive abbb

$S \rightarrow aS$
aSb
aSbb
aSbbb
$a\epsilon bbb$
abbb

aS
abS
abbS
abbbS
$abbb\epsilon$
abbb

pallindrome: $\{a, b\}$

$P: S \to \epsilon \mid aSa \mid bSa \mid a \mid b$

$P: \alpha \to \beta$

RE $\beta \in \{V \cup \Sigma\}^*$

recursively enumerable

$\alpha \in \{V^+ \cup \Sigma\}^*$

Context sensitive $|\alpha| \leq |\beta|$

context free

$|\alpha| = 1$

Regular Language

$A \to a$ ie β can be

$A \to aB$ (i) a terminal

(i) a term & a non termin

$S \to ~~Ba~~ \mid Sba$

$a^n b^n c^n$

$(ab)^n$

$S \to \epsilon \mid abS$

$A \to \epsilon \mid aA$

$B \to \epsilon \mid bB$

$C \to \epsilon \mid cB$

$S \to aAbBcC$

Regular expressions

regular grammar: $S \rightarrow aS \mid bS \mid \epsilon$

① ( )

concatenation

③ . ④ | ② *

union

kleene closure

RE definition

Basis 1: if a is any symbol, then a is an RE and $L(a) = \{a\}$

Basis 2: ϵ is a RE, $L(\epsilon) = \{\epsilon\}$

Basis 3: ϕ is a RE, $L(\phi) = \phi$ or $\{\}$

Induction 1: $L(E_1 \mid E_2) = L(E_1) \cup L(E_2)$

ie $L(a \mid b) = \{a, b\} = \{a\} \cup \{b\}$

or $L(A+B)$

Induction 2: $L(E_1 E_2) = L(E_1)\, L(E_2)$

$L(ab) = \{ab\} = \{a\} \cdot \{b\}$

Induction 3: $L(E^*) = (L(E))^*$

$L(a^*) = \{\epsilon, a, aa, aaa\} = \{a\}^*$

$L(01) = \{0, 1\}$ RE : 01

L

all strings of 0's and 1's having 101 as a substring

$$L = (0+1)^* \cdot 101 \cdot (0+1)^*$$

All strings of 0s and 1s having a no. of 0s that is a multiple of 3

$$1^*(1^*0 1^*0 1^*0 1^*)^* 1^*$$

$$(r+s)^* = (s+r)^*$$

$$(r^*)^* = r^*$$

$$(r^* s^*)^* = \text{all } \text{~~strings with~~ any num of r's ~~The set~~}$$

$$= (r+s)^*$$

$\bar{0}$

0.

$L(M+N) = LM + LN$ distributes $+$ over $\cdot$

but not

but not

$\cdot$ over $+$

~~$(L+M)\cdot N$~~

as ~~$(L+M)N = LN + MN$~~

$(L\cdot M) + N = LM + N$

Exercise

- even

~~$(\Sigma^+\Sigma^+)^*$~~ ~~$(\Sigma\Sigma)^*$~~ $((0+1)(0+1))^*$

- odd

~~$(\Sigma^+ \ldots)^*$~~ $(0+1)^*1$

$((0+1)(0+1))^*1$

- ~~ending with 1 and not containing 00~~

$$0^+((0^*10^+10^*)1)^*0^*$$

$$L = \{1^n 0^m \mid n+m \text{ is even}\}$$

$$((11)^*(00)^*) + ((11)^*1(00)^*0)$$

- Even no of 0's foll by odd no of 1's

$$((00)^*)((11)^*1)$$

- Begin with 110

$$110(0+1)^*$$

- Begin with 1 foll by any no of 0's or a single 1

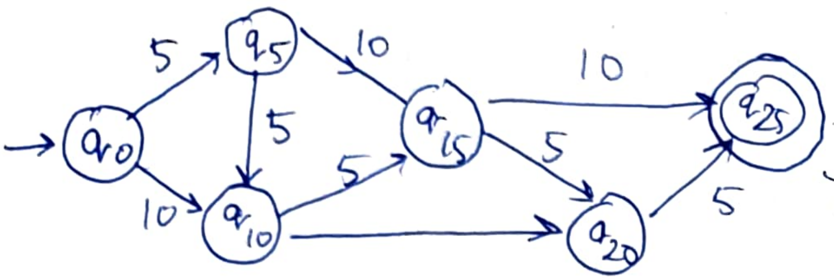
$$1(0^*+1)$$

- All strings with no more than 3 ones

$$0^*(\cancel{0}1\,0^*\,1\,0^*\,1)\,0^*$$

-

Automaton



accept state
(final state)

Finite automaton

~~FA~~ =

$$F = \{Q, \Sigma, q_0, A, \delta\}$$

Q - finite set of state

Σ - final input alphabet

$q_0 \in Q$ is the initial state

$A \subseteq Q$ is the set of accepting states

$\delta : Q \times \Sigma \rightarrow Q$ is the transition function

FA

DFA
deterministic FA
takes input
and gives a unique next state

NFA
non-deterministic FA
can take ϵ too
but has two or more next states

DFA

will not acc ϵ

for one state on acc i/p transfer another state

$\delta : Q \times \cancel{\Sigma} \to Q$

representation example

$A \xrightarrow{1} B$

$\boxed{\delta(A, 1) = B}$

NFA

will acc ϵ

" multiple states

$\delta : Q \times \{\Sigma \cup \epsilon\} \to 2^Q$

power set of states

RE to NFA

using Thompson's rules : Basis

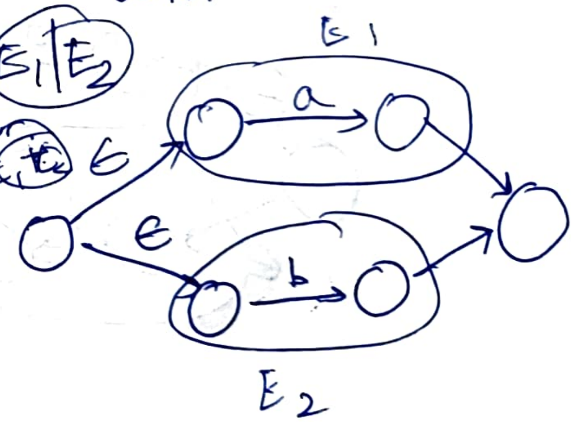
RE	
a	$\bigcirc \xrightarrow{a} \bigcirc$
ϵ	$\bigcirc \xrightarrow{\epsilon} \bigcirc$
ϕ	$\bigcirc \quad \bigcirc$

Union

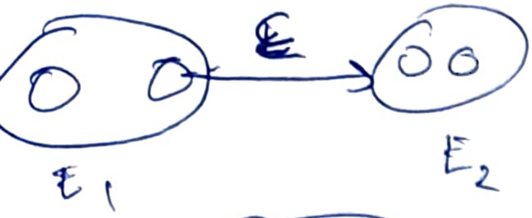
$E_1 | E_2$

or

$E_1 + E_2$

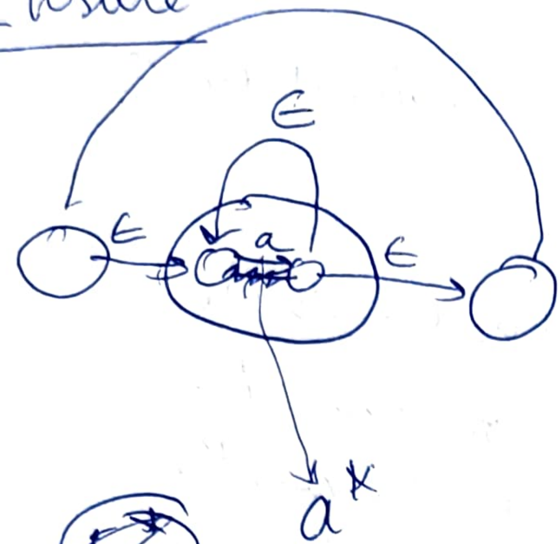


Concatenation



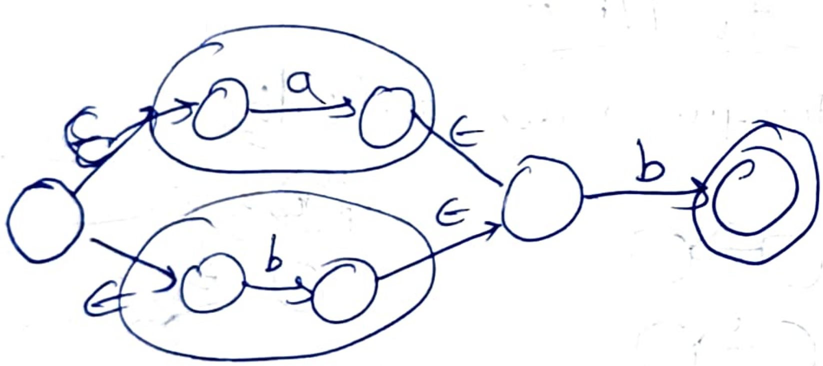
$E_1 E_2$

Kleene Closure

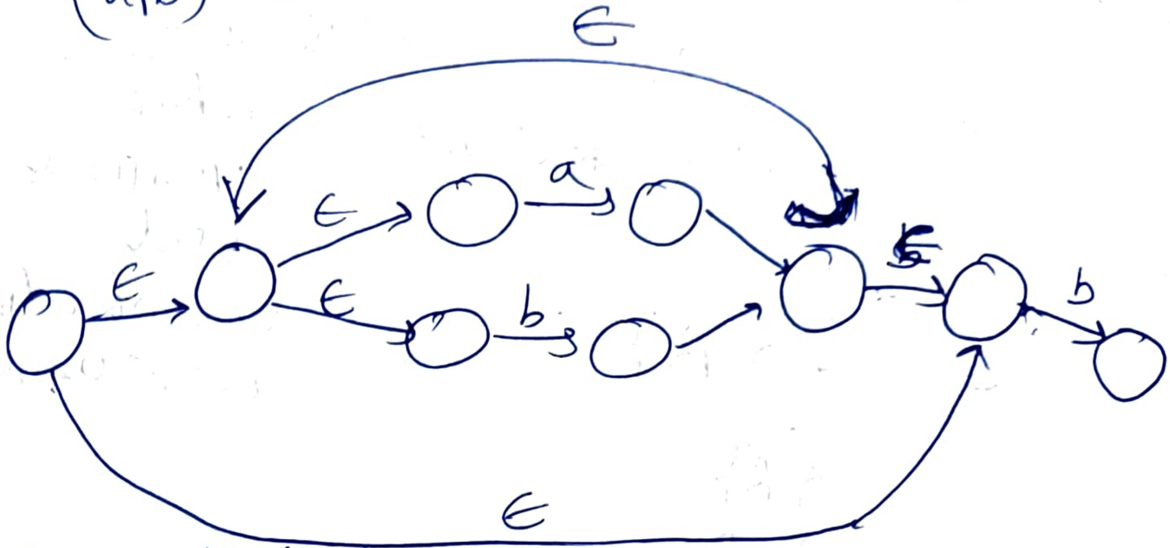


Eg: abb

(a|b) b

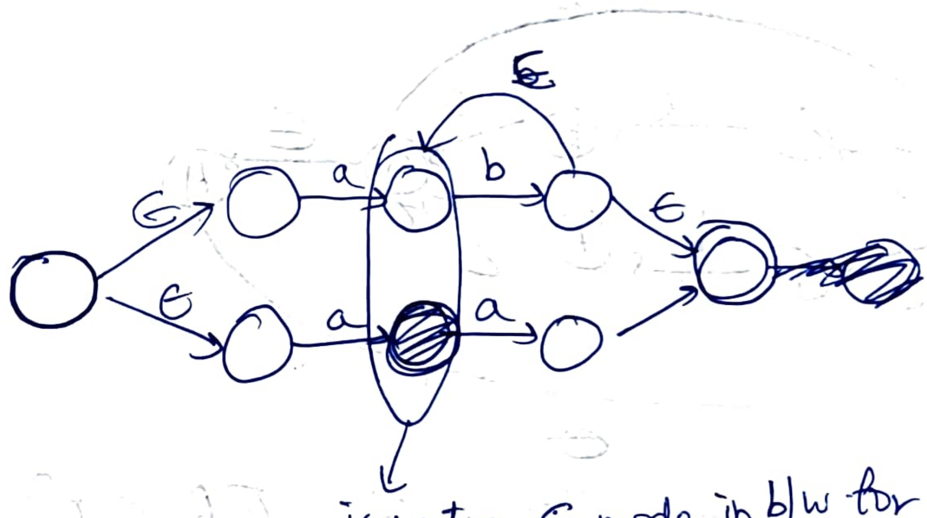


$(a|b)^* b$



Construct NFA for

$ab^+ | aa$ $\qquad$ $a(b^+|a)$



insert a ∈ node in b/w for concatenation

m

NFA to DFA

1) ε closure – set of states to which a particular state transition to on ε transition

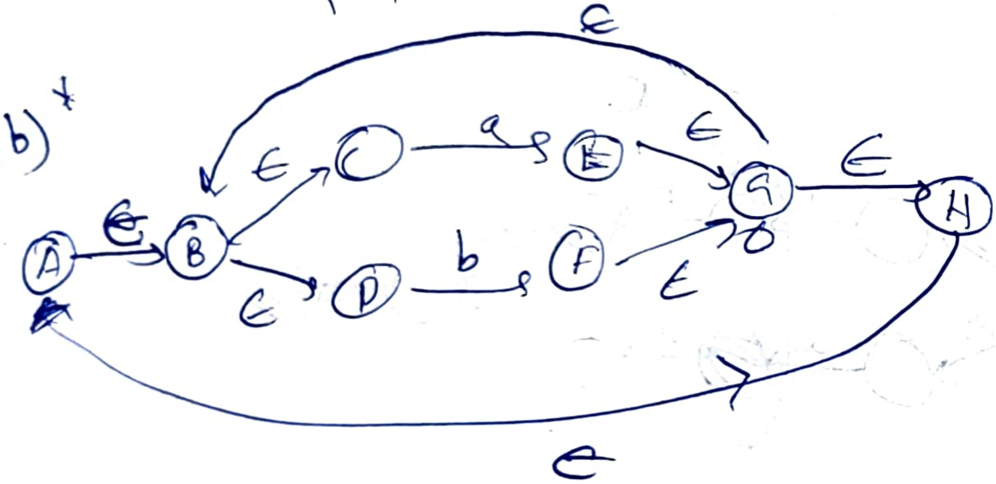
2) Subset construction

(1)

$\epsilon\text{-cl}\{B\} = \{B\}$

$\epsilon\text{-cl}\{A\} = \{A, C, B\}$

$(a|b)^*$



$\epsilon\text{-cl}\{F\} = \{F, G, H, \cancel{E}\ B, C, D\}$..

$\epsilon\text{-cl}\{B\} = \{B, C, D\}$

$\epsilon\text{-cl}\{A\} = \{A, B, H, C, D\}$

2) Subset constructions

Steps:

1) ϵ - cl { start state of NFA }

consider the same ~~ex~~ eg

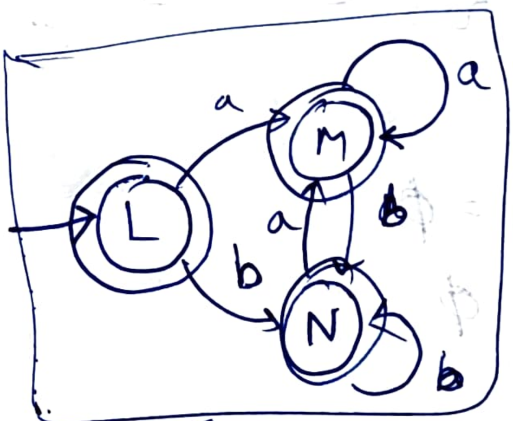
ϵ-cl(A) = { A, B, C, D, H } = ~~B~~ L

2) $L \xrightarrow{a} E$ $\quad \epsilon$-cl{E} = {E, G, H, B, C, D}

$\neq L$ $\therefore$ new state = M

$L \xrightarrow{b} F$ $\quad \epsilon$-cl{F} = {F, G, B, H, C, D}

$\neq L$ $\therefore$ new state N



~~Size~~

$\delta(M, a) = E$

$M \xrightarrow{a} \{E\}$ $\quad \epsilon$-cl{~~M~~} = M

$\therefore$ loop

$M \xrightarrow{b} \{F\}$ $\quad \epsilon$-cl{F} = N

Same for N...

Notation

$A \xrightarrow{a} B$

$\delta(A, a) = B$

Since ~~all~~ L, M, N have H

they're all acception

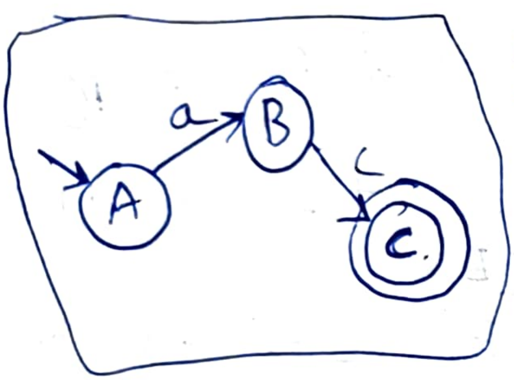
ab

DFA:

$\epsilon\text{-}cl\{0\} = \{0\} = A$

$\delta(A, a) = \{1\}$ $\quad \epsilon\text{-}cl\{1\} = \{1, 2\} = B$

$\delta(A, b) = \cancel{\{\}}$ ~~nothing~~ ϕ



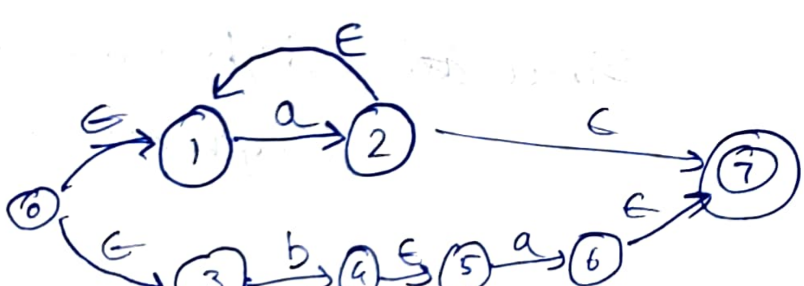
$\delta(B, a) = \cancel{\{\}}\ \phi$

$\delta(B, b) = \{3\}$ $\quad \epsilon\text{-}cl\{3\} = \{3\}$
$= C$

$\delta(C, a) = \phi$

$\delta(C, b)\ \phi$

$a^+ \mid ba$



$\epsilon\text{-}cl\{0\} = \{0, 1, 3\} = A$

$\delta(A, a) = \{2\}$ $\quad \epsilon\text{-cl}\{2\} = \{2, 1, 7\}$

$= B$

$\delta(A, b) = \{4\}$ $\quad \epsilon\text{-cl}\{4\} = \{4, 5\}$

$= C$

$\delta(B, a) = \{2\} = B$

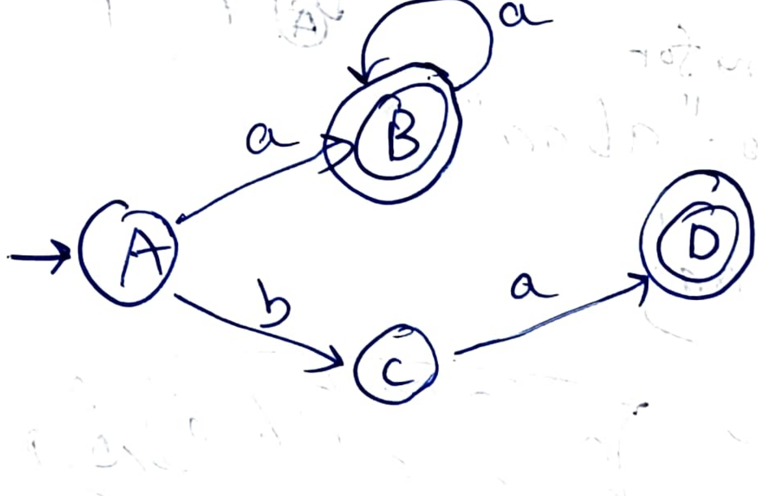
$\delta(B, b) = \phi$

$\delta(C, a) = \{6\}$ $\quad \epsilon\text{-cl}\{6\} = \{6, 7\}$

$= D$

$\delta(C, b) = \phi$

$\delta(D, a) = \phi$ $\quad \delta(D, b) = \phi$



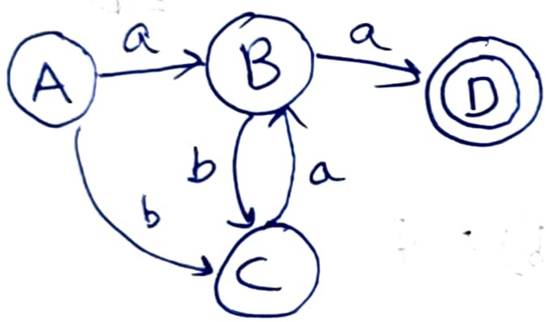
Minimal DFA-

1) get the set of accepting and non-accepting state

$\{A, B, C, D\}$

$\{A, C\}$ non accepting

$\{B, D\}$ accepting



	a	b
A	B	C
B	D A	C
C	B	ϕ
D A	ϕ	ϕ

Show transitions for i/p string "abaa"

$\delta(A, abaa)$

$\delta(B, baa)$

$\delta(C, aa)$

$\delta(B, a)$

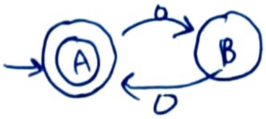
$= D$

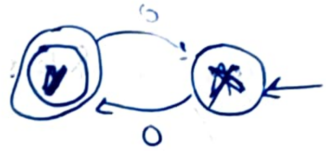
$\boxed{\delta(\delta(\delta(\delta(A, a), b), a), a)}$

$\Sigma = \{0\}$

$L = (00)^* 0$

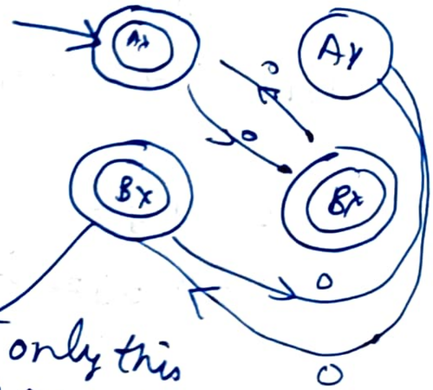
$M / (00)^*$





combined automaton

$L \cup M$	0	1
AX^*	BY	
BX^*	AY	
AY^*	BX	
BY^*	AX	



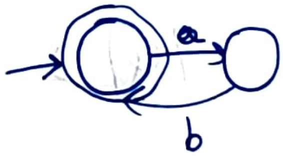
$L \cap M$ only this is acceptor

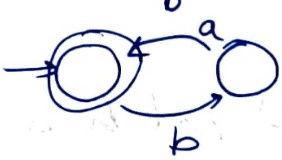
To get the reverse ~~thp~~ of the automaton

$L = (ab)^*$

L: 01

L^R: 10





L^R: $(ba)^*$

Pumping Lemma

FA → RE

$$\begin{tikzcd}
& RE \arrow[dl, bend right, "O(n)"'] & \\
\epsilon NFA \arrow[dr, bend right, "O(n^3)"'] & & DFA \arrow[ul, bend right, "O(n^2)"'] \\
& NFA \arrow[ur, bend right, "O(2^n \times n^3)"'] &
\end{tikzcd}$$

To prove that a language is not regular

$w = xyz$

if $xyyz$ is not in L, then it is not a regular language

$w = \{0^k \mid k \text{ is prime}\}$

$w = 0^k$

$0^{k-i-j} \; 0^i \; 0^j$

$x \quad y \quad \cancel{z}$

$|xy| \le k$

$|y| > 1$

$0^{k-i-j} \; 0^{i+1} \; 0^j$

$= 0^{k+1}$ ((k+1) need not be a prime ~~then~~ for every k which is a prime no.)

$\notin L$

$\therefore$ L is not a regular language.

3) ww^r is not regular. $w \in \{0,1\}^*$

4) $0^i 1^j$ $i > j$ is not regular

$$\underset{x}{w}\ \underset{y}{w^r} = w\, w^r w^r$$

ww^rw^r need not be a palindrome

$\therefore$ $ww^rw^r \notin L$

$\therefore$ L is not regular

$w = 0^i 1^j$ $\quad i > j$

$\cancel{0^i 0^j 0^j}$

$\cancel{0^i 0^{j+1} 0^j}$

$0^{i-j}\, 0^j\, \phi^j$

$0^{i-j}\, 0^{j+1}\, \phi^j$

$= 0^{i+1}\, 0^j$ $\quad i+1$ need not be $> j$

$\therefore \notin L$