MATHEMATICAL DERIVATION OF LEAST SQUARES

EQUATION : $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

$$S = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left(y_i - (\beta_0 + \beta_1 x_i)\right)^2$$

Partial Derivative w.r.t. β_0

$$\frac{\partial S}{\partial \beta_0} = \frac{\partial}{\partial \beta_0}\left[\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2\right]$$

$$\frac{\partial S}{\partial \beta_0} = \sum_{i=1}^{n}\left[\frac{\partial}{\partial \beta_0} (y_i - \beta_0 - \beta_1 x_i)^2\right]$$

Differentiating the squared quantity in parantheses. Since the quantity is a composite use chain rule - to obtain partial derivative of S. (y_i, β_1, x_i - constants)

$$\frac{\partial S}{\partial \beta_0} = \sum_{i=1}^{n}\left[2\,(y_i - \beta_0 - \beta_1 x_i)\right](-1)$$

$$\frac{\partial S}{\partial \beta_0} = 2 \sum_{i=1}^{N} (y_i - \beta_0 - \beta_1 x_i)(-1)$$

$$\left[\frac{\partial S}{\partial \beta_0} \Rightarrow 0\right]$$

$$-2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} (\beta_0 + \beta_1 x_i)$$

$$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \beta_0 + \beta_1 \sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{n} y_i = n\beta_0 + \beta_1 \sum_{i=1}^{n} x_i$$

($\div$ by n) both sides

$$\sum_{i=1}^{n} \frac{y_i}{n} - \beta_1 \sum_{i=1}^{n} \frac{x_i}{n} = \beta_0$$

$$\boxed{\beta_0 = \bar{y} - \beta_1 \bar{x}}$$