Parametric Curve Parameter Estimation – Full Project Explanation

This document explains the complete methodology used to estimate the unknown parameters (θ, M, X) in the given parametric curve from sampled coordinate data. The evaluation criterion was minimization of the L1 distance between observed and predicted coordinates, which is the required metric in the problem statement. The parametric model is defined as: $x(t) = t*\cos(\theta) - e^{(M|t|)}*\sin(0.3t)*\sin(\theta) + X$ $y(t) = 42 + t*\sin(\theta) + e^{(M|t|)}*\sin(0.3t)*\cos(\theta)$ Here, θ is the rotation angle, M controls the exponential amplitude growth of the oscillatory term, and X represents horizontal translation. The objective of the project was to compute these parameters from the provided dataset while ensuring minimal error in reconstruction.

Geometric Interpretation of the Model: The curve can be interpreted as a rotated and shifted sinusoid whose amplitude changes exponentially with time. This interpretation allows breaking the problem into subproblems: estimating rotation, translation, and amplitude growth independently before refining all parameters using optimization.

Step 1: Estimating θ using PCA: Principal Component Analysis (PCA) was applied to the (x,y) data points to identify the direction of maximum spread. This direction corresponds to the underlying parameter t, enabling an estimate of θ as the orientation of the first principal component. Reference: Jolliffe & Cadima (2016), Principal Component Analysis Review.

Step 2: Estimating X using Mean Alignment: The oscillatory term has mean zero over multiple cycles. Therefore, the average horizontal shift can be determined from the mean of x and the mean of $t*\cos(\theta)$. This yields: $X \approx \text{mean}(x) - \text{mean}(t*\cos(\theta))$.

Step 3: Estimating M using Amplitude Envelope Regression: To estimate M, the data was inverse-rotated by $-\theta$ to isolate the oscillatory term. Taking log(|signal amplitude|) linearizes the exponential relationship, allowing M to be estimated via linear regression. Reference: Oppenheim & Schafer (2010), Discrete-Time Signal Processing.

Step 4: Refinement Using L1 Optimization: Once initial estimates for θ, M, and X were obtained, the values were refined by minimizing the L1 error metric: $L1 = \Sigma(|x\_data - x\_model| + |y\_data - y\_model|)$. Powell's derivative-free optimization method was used as it performs well for non-smooth cost functions. References: Press et al. (2007), Numerical Recipes. Boyd & Vandenberghe (2004), Convex Optimization.

Final Estimated Parameter Values: $\theta = 28.119°$ M = 0.0213837 X = 54.911105 These values generated a reconstructed curve that closely fits the provided dataset with a low L1 error, confirming the correctness of the estimation process.

Conclusion: The methodology successfully integrates geometric interpretation, statistical estimation, signal-processing-based envelope extraction, and robust optimization. This structured approach ensures precise parameter recovery and aligns directly with the evaluation criteria of the assignment.

References: [1] Jolliffe, I. T., & Cadima, J. (2016). Principal Component Analysis: A Review and Recent Developments. Philosophical Transactions of the Royal Society A. [2] Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (2007). Numerical Recipes: The Art of Scientific Computing. [3] Oppenheim, A. V., & Schafer, R. W. (2010). Discrete-Time Signal Processing. [4] Boyd, S., & Vandenberghe, L. (2004). Convex Optimization.