STATISTICAL RETHINKING 2023
WEEK 3 SOLUTIONS

1. Because there are no back-door paths from `area` to `avgfood`, we only need to include `area` to the causal effect of area. No other variables are needed. In fact, adding other variables could cause bias. Here is a model using standardized versions of the variables and those standardized priors from the book:

```
library(rethinking)
data(foxes)
d <- foxes
d$W <- standardize(d$weight)
d$A <- standardize(d$area)
d$F <- standardize(d$avgfood)
d$G <- standardize(d$groupsize)

m1 <- quap(
    alist(
        F ~ dnorm( mu , sigma ),
        mu <- a + bA*A,
        a ~ dnorm(0,0.2),
        bA ~ dnorm(0,0.5),
        sigma ~ dexp(1)
    ), data=d )

precis(m1)
```

```
      mean   sd  5.5% 94.5%
a     0.00 0.04 -0.07  0.07
bA    0.88 0.04  0.81  0.95
sigma 0.47 0.03  0.42  0.52
```

Territory size seems to have a substantial effect on food availability. These are standardized variables, so `bA` above means that each standard deviation change in area results on average in about 0.9 standard deviations of change in food availability.

2. To infer the causal influence of `avgfood` on `weight`, we need to close any back-door paths. There are no back-door paths in the DAG. So again, just use a model with a single predictor.

```
m2 <- quap(
    alist(
        W ~ dnorm( mu , sigma ),
        mu <- a + bF*F,
        a ~ dnorm(0,0.2),
        bF ~ dnorm(0,0.5),
        sigma ~ dexp(1)
    ), data=d )
precis(m2)
```

```
       mean   sd  5.5% 94.5%
a      0.00 0.08 -0.13  0.13
bF    -0.02 0.09 -0.17  0.12
sigma  0.99 0.06  0.89  1.09
```

There seems to be only a small total effect of food on weight, if there is any effect at all. It's about equally plausible that it's negative as positive, and it's small either way.

3. Now for the direct effect. We need to block the mediated path through group size *G*. That means stratify by group size.

```
m3a <- quap(
    alist(
        W ~ dnorm( mu , sigma ),
        mu <- a + bF*F + bG*G,
        a ~ dnorm(0,0.2),
        c(bF,bG) ~ dnorm(0,0.5),
        sigma ~ dexp(1)
    ), data=d )
precis(m3a)
```

```
       mean   sd  5.5% 94.5%
a      0.00 0.08 -0.13  0.13
bF     0.48 0.18  0.19  0.76
bG    -0.57 0.18 -0.86 -0.29
sigma  0.94 0.06  0.84  1.04
```

The direct effect of food on weight is positive (0.19–0.76), it seems. That makes sense. This model also gives us the direct effect (also the total effect) of group size on weight. And it is the opposite and of the same magnitude as the direct effect of food. These two effects seem to cancel one another. That may be why the total effect of food is about zero: the direct effect is positive but the mediated effect through groups size is negative.

What is going on here? Larger territories increase available food (problem 1). But increases in food (and territory) do not influence fox weight. The reason seems to be because adding more food directly increases weight, but the path through group size cancels that increase. To check this idea, we can estimate the causal effect of food on groups size:

```
m3b <- quap(
    alist(
        G ~ dnorm( mu , sigma ),
        mu <- a + bF*F,
        a ~ dnorm(0,0.2),
        bF ~ dnorm(0,0.5),
        sigma ~ dexp(1)
    ), data=d )
precis(m3b)
```

```
       mean   sd  5.5% 94.5%
a      0.00 0.04 -0.06  0.06
bF     0.90 0.04  0.83  0.96
sigma  0.43 0.03  0.39  0.48
```

Food appears to have a reliably large (0.83–0.96) effect on group size. That is, more food means more foxes. This is consistent with the idea that the mediating influence of group size cancels the direct influence of more food on individual fox body weight. In simple terms, the benefits of more food are canceled by more foxes being attracted to the food, so each fox gets the same amount.

The ecologists will recognize this situation as an *ideal free distribution*.

4-OPTIONAL CHALLENGE. This is a trick question. I am sorry. The total causal effect cannot be estimated, because the introduction of U adds a backdoor path. That path can be closed by conditioning on G, but them you don't have the total causal effect. You can however get the direct effect of F on W, but conditioning of G as before.