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Program Structures & Algorithms
Spring 2023 (Sec -8)
Assignment No. 5

Task

- (Part 1) *A cutoff (defaults to, say, 1000) which you will update according to the first argument in the command line when running. It's your job to experiment and come up with a good value for this cutoff. If there are fewer elements to sort than the cutoff, then you should use the system sort instead.*

- (Part 2) *Recursion depth or the number of available threads. Using this determination, you might decide on an ideal number (t) of separate threads (stick to powers of 2) and arrange for that number of partitions to be parallelized (by preventing recursion after the depth of lg t is reached)*

- (Part 3) *An appropriate combination of these.*

Relationship Conclusion:

After running the algorithm for different array sizes and cutoff values, and using varying thread counts for each array size, the following results are observed:

$$Optimal \left(\frac{cutoff\ value}{Array\ Size} \right) = 0.2$$

$$Optimal\ Thread\ Count = 8$$

The cutoff values are always maintained below array size, and $\frac{cutoff\ value}{Array\ Size}$ is varied from 0.1 to 1.0 in steps of 0.1; this helps accurately measure run time of parallelized sort and prevents system sort.

Maximum depth is given by:

$$Max\ Depth = \log_2 \left(\frac{Array\ Size}{cutoff} \right)$$

Evidence to support the conclusion:

Cutoff/Array Size	Average Run Time (ms) for Array Size = 100000					
	2 Threads	4 Threads	8 Threads	16 Threads	32 Threads	64 Threads
0.1	19	16	11.4	7.6	6.4	8.6
0.2	7.3	11.6	8.2	6.5	5.9	7.2
0.3	12.2	9.4	8.8	7.3	6.9	7.9
0.4	9.4	9.3	8.1	7.4	7.2	8.8
0.5	7.1	9.5	8.4	7.3	7.3	8.7
0.6	7.3	11.3	9.2	8.3	7.9	11.2
0.7	8.1	11.2	9	8.3	8	10.8
0.8	9.7	11.6	8.9	8.4	9.1	10.6
0.9	9.6	10.9	8.8	7.9	8.6	9.8
1	15.3	11.1	8.3	7.9	9.1	9.3
Optimal (cutoff/Array Size) Ratio:	0.5	0.4	0.2	0.2	0.2	0.2
Optimal Thread Count:	32					

There are a few outliers for optimal **(cutoff/Array Size) Ratio**, but the majority **optimal ratio** which gives the **lowest run-time** is found to be 0.2. The optimal thread count is found to be 32, but larger array sizes need to be considered as this could be an outlier.

Cutoff/Array Size	Average Run Time (ms) for Array Size = 200000					
	2 Threads	4 Threads	8 Threads	16 Threads	32 Threads	64 Threads
0.1	21.3	18.6	15.9	15.3	16.5	13.2
0.2	18.8	16	11.4	11.7	16	12.7
0.3	23.6	14.8	14.5	14.7	16.5	14.3
0.4	24.3	14.2	14.3	15.2	16.2	15
0.5	26	14.6	14.2	15.3	16	15.1
0.6	21.4	17	16	18	18.7	17.6
0.7	19.7	16.4	15.7	17	18	17.4
0.8	18.8	16.2	15.8	16.7	17.5	17.4
0.9	17.2	15.7	15.6	16.4	16.6	16.3
1	17.1	15.6	15.5	19.1	16.4	16.4
Optimal (cutoff/Array Size) Ratio :	1	0.4	0.2	0.2	0.2	0.2
Optimal Thread Count :	8					

There are a few outliers for optimal **(cutoff/Array Size) Ratio**, but the majority **optimal ratio** which gives the **lowest run-time** is found to be 0.2. The **optimal thread count** for which performance time is least is found to be 8,
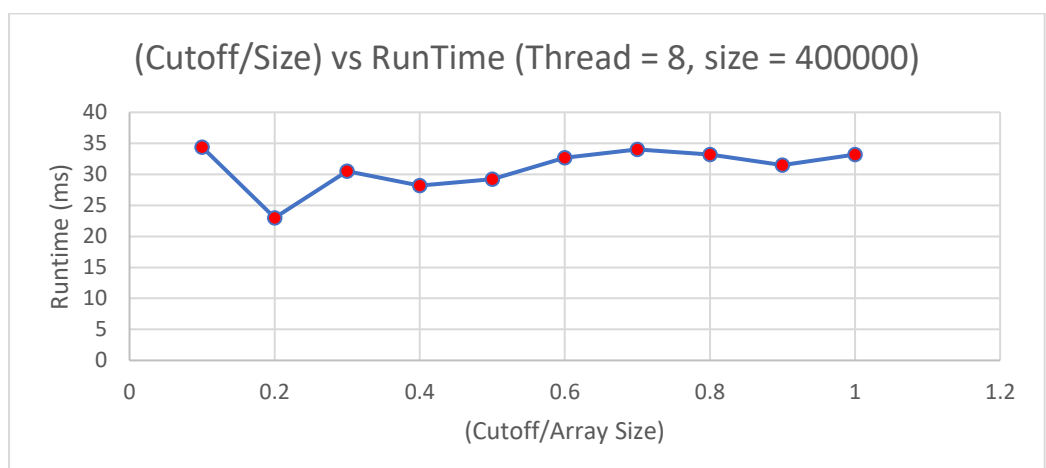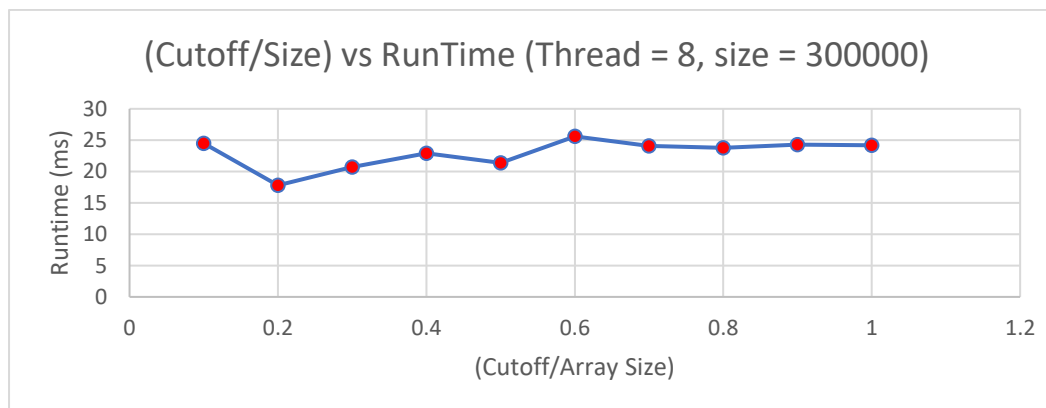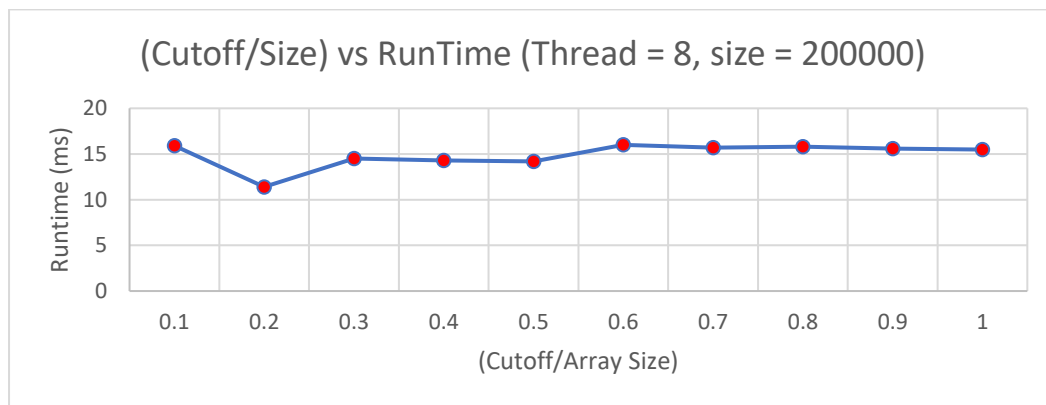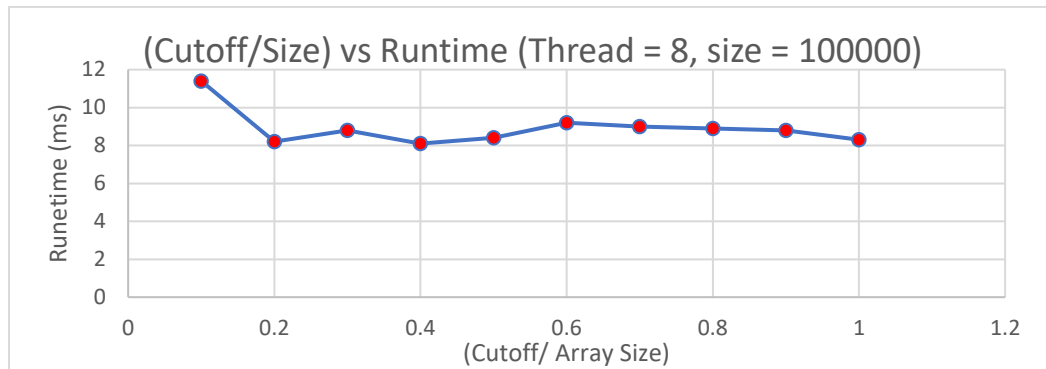
Cutoff/Array Size	Average Run Time (ms) for Array Size = 300000					
	2 Threads	4 Threads	8 Threads	16 Threads	32 Threads	64 Threads
0.1	28.6	28.9	24.5	22.9	17.6	18.6
0.2	24.4	23.8	17.8	17.8	17.8	18.1
0.3	26.7	22	20.7	21.9	21.5	22.8
0.4	25.7	21.1	22.9	21.3	22.4	20.7
0.5	27.1	21.6	21.4	21.8	22.1	22.6
0.6	24.1	27.2	25.6	24.6	25	25.7
0.7	24.9	25	24.1	23.7	24	23.9
0.8	24	24.8	23.8	23.9	24	24.3
0.9	25.4	24.2	24.3	24.7	24.1	24.2
1	25.8	24.1	24.2	23	23.8	24
Optimal (cutoff/Array Size) Ratio :	0.8	0.4	0.2	0.2	0.1	0.2
Optimal Thread Count :	8					

There are a few outliers for optimal **(cutoff/Array Size) Ratio**, but the majority **optimal ratio** which gives the **lowest run-time** is found to be 0.2. The **optimal thread count** for which performance time is least is found to be 8,

Cutoff/Array Size	Average Run Time (ms) for Array Size = 400000					
	2 Threads	4 Threads	8 Threads	16 Threads	32 Threads	64 Threads
0.1	33.7	33.8	34.4	27	24	24.2
0.2	41.5	30.5	23	23.5	24	23.7
0.3	42.7	26.1	30.5	28.4	29.4	30.1
0.4	37.8	29	28.2	28.7	31.2	32.1
0.5	37.6	27.3	29.2	29.3	31	30
0.6	32.6	33	32.7	34	34.1	36.6
0.7	32.1	32.8	34	32.4	32.3	35.4
0.8	33.2	32.4	33.2	32.2	32.6	34.1
0.9	33.2	32.7	31.5	32.6	31.6	32.9
1	32.3	30.8	33.2	32.1	30.9	32.5
Optimal (cutoff/Array Size) Ratio :	0.7	0.3	0.2	0.2	0.2	0.2
Optimal Thread Count :	8					

There are a few outliers for optimal **(cutoff/Array Size) Ratio**, but the majority **optimal ratio** which gives the **lowest run-time** is found to be 0.2. The **optimal thread count** for which performance time is least is found to be 8,

Graphs – (Cutoff/Size) vs Runtime for majority optimal Thread Count of 8:

(Cutoff/Size) vs Runtime (Thread = 8, size = 100000)

(Cutoff/Size) vs RunTime (Thread = 8, size = 200000)

(Cutoff/Size) vs RunTime (Thread = 8, size = 300000)

(Cutoff/Size) vs RunTime (Thread = 8, size = 400000)

Console Output (raw cutoff values and 10-run timings):

Array Size: 100,000:

```
Size of Array: 100000
Degree of parallelism: 2
cutoff: 10000          10times Time:190ms
cutoff: 20000          10times Time:73ms
cutoff: 30000          10times Time:122ms
cutoff: 40000          10times Time:94ms
cutoff: 50000          10times Time:71ms
cutoff: 60000          10times Time:73ms
cutoff: 70000          10times Time:81ms
cutoff: 80000          10times Time:97ms
cutoff: 90000          10times Time:96ms
cutoff: 100000         10times Time:153ms
Degree of parallelism: 4
cutoff: 10000          10times Time:160ms
cutoff: 20000          10times Time:116ms
cutoff: 30000          10times Time:94ms
cutoff: 40000          10times Time:93ms
cutoff: 50000          10times Time:95ms
cutoff: 60000          10times Time:113ms
cutoff: 70000          10times Time:112ms
cutoff: 80000          10times Time:116ms
cutoff: 90000          10times Time:109ms
cutoff: 100000         10times Time:111ms
```

```
Degree of parallelism: 8
cutoff: 10000          10times Time:114ms
cutoff: 20000          10times Time:82ms
cutoff: 30000          10times Time:88ms
cutoff: 40000          10times Time:81ms
cutoff: 50000          10times Time:84ms
cutoff: 60000          10times Time:92ms
cutoff: 70000          10times Time:90ms
cutoff: 80000          10times Time:89ms
cutoff: 90000          10times Time:88ms
cutoff: 100000         10times Time:83ms
Degree of parallelism: 16
cutoff: 10000          10times Time:76ms
cutoff: 20000          10times Time:65ms
cutoff: 30000          10times Time:73ms
cutoff: 40000          10times Time:74ms
cutoff: 50000          10times Time:73ms
cutoff: 60000          10times Time:83ms
cutoff: 70000          10times Time:83ms
cutoff: 80000          10times Time:84ms
cutoff: 90000          10times Time:79ms
cutoff: 100000         10times Time:79ms
```

```
Degree of parallelism: 32
cutoff: 10000          10times Time:64ms
cutoff: 20000          10times Time:59ms
cutoff: 30000          10times Time:69ms
cutoff: 40000          10times Time:72ms
cutoff: 50000          10times Time:73ms
cutoff: 60000          10times Time:79ms
cutoff: 70000          10times Time:80ms
cutoff: 80000          10times Time:91ms
cutoff: 90000          10times Time:86ms
cutoff: 100000         10times Time:91ms
Degree of parallelism: 64
cutoff: 10000          10times Time:86ms
cutoff: 20000          10times Time:72ms
cutoff: 30000          10times Time:79ms
cutoff: 40000          10times Time:88ms
cutoff: 50000          10times Time:87ms
cutoff: 60000          10times Time:112ms
cutoff: 70000          10times Time:108ms
cutoff: 80000          10times Time:106ms
cutoff: 90000          10times Time:98ms
cutoff: 100000         10times Time:93ms
```

Array Size: 200,000:

```
Size of Array: 200000
Degree of parallelism: 2
cutoff: 20000        10times Time:213ms
cutoff: 40000        10times Time:188ms
cutoff: 60000        10times Time:236ms
cutoff: 80000        10times Time:243ms
cutoff: 100000       10times Time:260ms
cutoff: 120000       10times Time:214ms
cutoff: 140000       10times Time:197ms
cutoff: 160000       10times Time:188ms
cutoff: 180000       10times Time:172ms
cutoff: 200000       10times Time:171ms
Degree of parallelism: 4
cutoff: 20000        10times Time:186ms
cutoff: 40000        10times Time:160ms
cutoff: 60000        10times Time:148ms
cutoff: 80000        10times Time:142ms
cutoff: 100000       10times Time:146ms
cutoff: 120000       10times Time:170ms
cutoff: 140000       10times Time:164ms
cutoff: 160000       10times Time:162ms
cutoff: 180000       10times Time:157ms
cutoff: 200000       10times Time:156ms
```

```
Degree of parallelism: 8
cutoff: 20000        10times Time:159ms
cutoff: 40000        10times Time:114ms
cutoff: 60000        10times Time:145ms
cutoff: 80000        10times Time:143ms
cutoff: 100000       10times Time:142ms
cutoff: 120000       10times Time:160ms
cutoff: 140000       10times Time:157ms
cutoff: 160000       10times Time:158ms
cutoff: 180000       10times Time:156ms
cutoff: 200000       10times Time:155ms
Degree of parallelism: 16
cutoff: 20000        10times Time:153ms
cutoff: 40000        10times Time:117ms
cutoff: 60000        10times Time:147ms
cutoff: 80000        10times Time:152ms
cutoff: 100000       10times Time:153ms
cutoff: 120000       10times Time:180ms
cutoff: 140000       10times Time:170ms
cutoff: 160000       10times Time:167ms
cutoff: 180000       10times Time:164ms
cutoff: 200000       10times Time:191ms
```

```
Degree of parallelism: 32
cutoff: 20000        10times Time:165ms
cutoff: 40000        10times Time:160ms
cutoff: 60000        10times Time:165ms
cutoff: 80000        10times Time:162ms
cutoff: 100000       10times Time:160ms
cutoff: 120000       10times Time:187ms
cutoff: 140000       10times Time:180ms
cutoff: 160000       10times Time:175ms
cutoff: 180000       10times Time:166ms
cutoff: 200000       10times Time:164ms
Degree of parallelism: 64
cutoff: 20000        10times Time:132ms
cutoff: 40000        10times Time:127ms
cutoff: 60000        10times Time:143ms
cutoff: 80000        10times Time:150ms
cutoff: 100000       10times Time:151ms
cutoff: 120000       10times Time:176ms
cutoff: 140000       10times Time:174ms
cutoff: 160000       10times Time:174ms
cutoff: 180000       10times Time:163ms
cutoff: 200000       10times Time:164ms
```

Array Size: 300,000:

```
Size of Array: 300000
Degree of parallelism: 2
cutoff: 30000        10times Time:286ms
cutoff: 60000        10times Time:244ms
cutoff: 90000        10times Time:267ms
cutoff: 120000       10times Time:257ms
cutoff: 150000       10times Time:271ms
cutoff: 180000       10times Time:241ms
cutoff: 210000       10times Time:249ms
cutoff: 240000       10times Time:240ms
cutoff: 270000       10times Time:254ms
cutoff: 300000       10times Time:258ms
Degree of parallelism: 4
cutoff: 30000        10times Time:289ms
cutoff: 60000        10times Time:238ms
cutoff: 90000        10times Time:220ms
cutoff: 120000       10times Time:211ms
cutoff: 150000       10times Time:216ms
cutoff: 180000       10times Time:272ms
cutoff: 210000       10times Time:250ms
cutoff: 240000       10times Time:248ms
cutoff: 270000       10times Time:242ms
cutoff: 300000       10times Time:241ms
```

```
Degree of parallelism: 8
cutoff: 30000        10times Time:245ms
cutoff: 60000        10times Time:178ms
cutoff: 90000        10times Time:207ms
cutoff: 120000       10times Time:229ms
cutoff: 150000       10times Time:214ms
cutoff: 180000       10times Time:256ms
cutoff: 210000       10times Time:241ms
cutoff: 240000       10times Time:238ms
cutoff: 270000       10times Time:243ms
cutoff: 300000       10times Time:242ms
Degree of parallelism: 16
cutoff: 30000        10times Time:229ms
cutoff: 60000        10times Time:178ms
cutoff: 90000        10times Time:219ms
cutoff: 120000       10times Time:213ms
cutoff: 150000       10times Time:218ms
cutoff: 180000       10times Time:246ms
cutoff: 210000       10times Time:237ms
cutoff: 240000       10times Time:239ms
cutoff: 270000       10times Time:247ms
cutoff: 300000       10times Time:230ms
```

```
Degree of parallelism: 32
cutoff: 30000        10times Time:176ms
cutoff: 60000        10times Time:178ms
cutoff: 90000        10times Time:215ms
cutoff: 120000       10times Time:224ms
cutoff: 150000       10times Time:221ms
cutoff: 180000       10times Time:250ms
cutoff: 210000       10times Time:240ms
cutoff: 240000       10times Time:240ms
cutoff: 270000       10times Time:241ms
cutoff: 300000       10times Time:238ms
Degree of parallelism: 64
cutoff: 30000        10times Time:186ms
cutoff: 60000        10times Time:181ms
cutoff: 90000        10times Time:228ms
cutoff: 120000       10times Time:207ms
cutoff: 150000       10times Time:226ms
cutoff: 180000       10times Time:257ms
cutoff: 210000       10times Time:239ms
cutoff: 240000       10times Time:243ms
cutoff: 270000       10times Time:242ms
cutoff: 300000       10times Time:240ms
```

Array Size: 400,000:

```
Size of Array: 400000
Degree of parallelism: 2
cutoff: 40000        10times Time:337ms
cutoff: 80000        10times Time:415ms
cutoff: 120000       10times Time:427ms
cutoff: 160000       10times Time:378ms
cutoff: 200000       10times Time:376ms
cutoff: 240000       10times Time:326ms
cutoff: 280000       10times Time:321ms
cutoff: 320000       10times Time:332ms
cutoff: 360000       10times Time:332ms
cutoff: 400000       10times Time:323ms
Degree of parallelism: 4
cutoff: 40000        10times Time:338ms
cutoff: 80000        10times Time:305ms
cutoff: 120000       10times Time:261ms
cutoff: 160000       10times Time:290ms
cutoff: 200000       10times Time:273ms
cutoff: 240000       10times Time:330ms
cutoff: 280000       10times Time:328ms
cutoff: 320000       10times Time:324ms
cutoff: 360000       10times Time:327ms
cutoff: 400000       10times Time:308ms
```

```
Degree of parallelism: 8
cutoff: 40000        10times Time:344ms
cutoff: 80000        10times Time:230ms
cutoff: 120000       10times Time:305ms
cutoff: 160000       10times Time:282ms
cutoff: 200000       10times Time:292ms
cutoff: 240000       10times Time:327ms
cutoff: 280000       10times Time:340ms
cutoff: 320000       10times Time:332ms
cutoff: 360000       10times Time:315ms
cutoff: 400000       10times Time:332ms
Degree of parallelism: 16
cutoff: 40000        10times Time:270ms
cutoff: 80000        10times Time:235ms
cutoff: 120000       10times Time:284ms
cutoff: 160000       10times Time:287ms
cutoff: 200000       10times Time:293ms
cutoff: 240000       10times Time:340ms
cutoff: 280000       10times Time:324ms
cutoff: 320000       10times Time:322ms
cutoff: 360000       10times Time:326ms
cutoff: 400000       10times Time:321ms
```

```
Degree of parallelism: 32
cutoff: 40000        10times Time:240ms
cutoff: 80000        10times Time:240ms
cutoff: 120000       10times Time:294ms
cutoff: 160000       10times Time:312ms
cutoff: 200000       10times Time:310ms
cutoff: 240000       10times Time:341ms
cutoff: 280000       10times Time:323ms
cutoff: 320000       10times Time:326ms
cutoff: 360000       10times Time:316ms
cutoff: 400000       10times Time:309ms
Degree of parallelism: 64
cutoff: 40000        10times Time:242ms
cutoff: 80000        10times Time:237ms
cutoff: 120000       10times Time:301ms
cutoff: 160000       10times Time:321ms
cutoff: 200000       10times Time:300ms
cutoff: 240000       10times Time:366ms
cutoff: 280000       10times Time:354ms
cutoff: 320000       10times Time:341ms
cutoff: 360000       10times Time:329ms
cutoff: 400000       10times Time:325ms
```