Program Structures and Algorithms
Spring 2023(SEC – 8)

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Task:
Random Walk - After m steps, how far (*d*), generally speaking, is the man from the lamp post? (Considering step length is 1)

Relationship Conclusion*:*
Distance (d) from the lamp post to current position is directly proportional to the square root of the number of steps (m) taken starting from the lamp post (considering step length of 1).
$\boldsymbol{d \propto \sqrt{m}}$
The coefficient of proportionality is approximately found to be 0.875.
$\boldsymbol{d = 0.875\sqrt{m}}$

Evidence to support that conclusion:

Varying 'm' over the values [1,2,3,4,5,6,7,8,9,10], the mean distance ('d') and mean squared distance ('d^2') are observed over 100,000 trials for each value of 'm'. The observed values strongly suggest that:
mean squared distance: $\boldsymbol{d^2 = m}$
mean distance: $\boldsymbol{d = 0.875\sqrt{m}}$

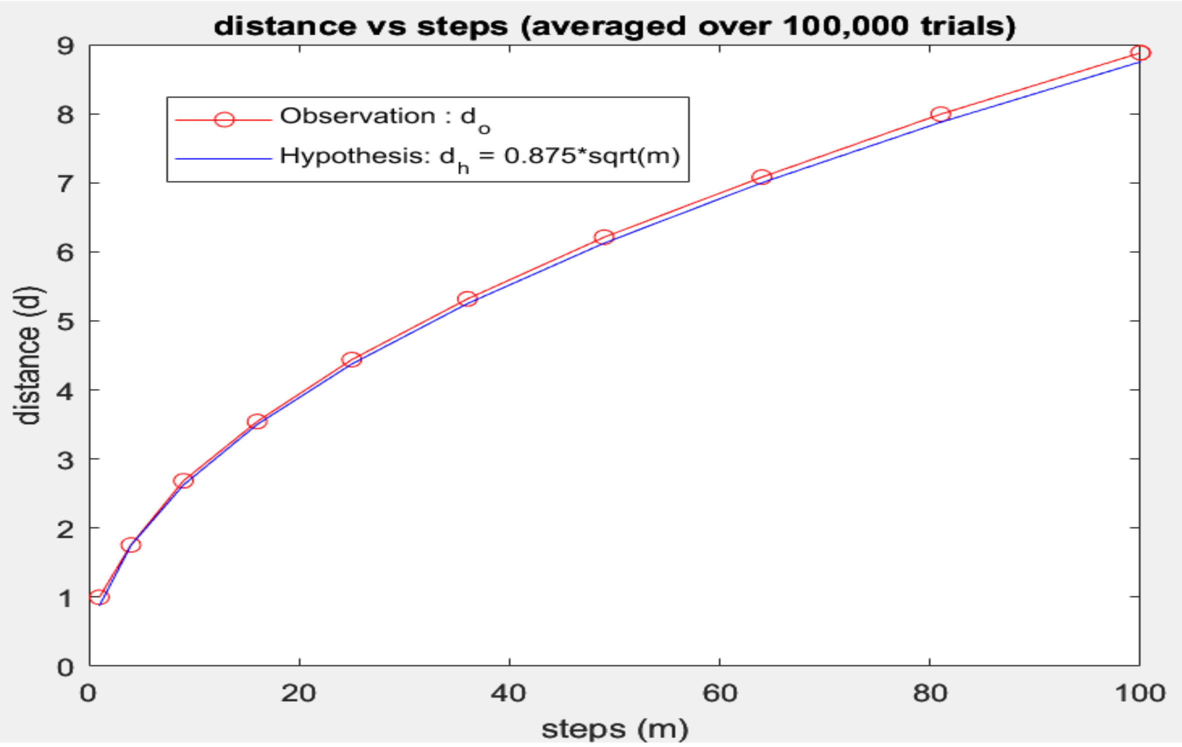
Screenshot of results in IDE:

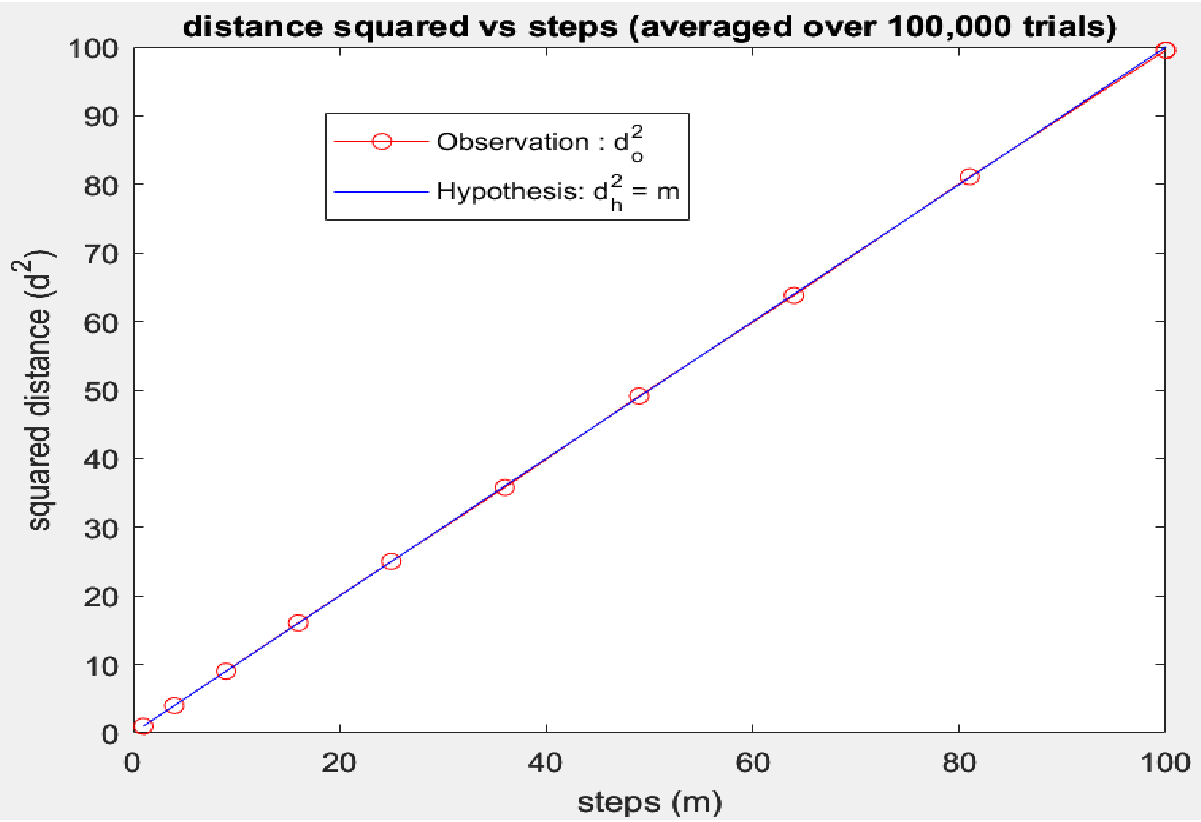
```
Please enter different values of 'm' separated by ',' : 1,4,9,16,25,36,49,64,81,100
____Mean distance vs steps____
1 steps: 1.0 over 100000 experiments
4 steps: 1.7599454368328271 over 100000 experiments
9 steps: 2.6935952020698384 over 100000 experiments
16 steps: 3.5336671048557826 over 100000 experiments
25 steps: 4.438363075092173 over 100000 experiments
36 steps: 5.315860280234762 over 100000 experiments
49 steps: 6.217083238145446 over 100000 experiments
64 steps: 7.111106673299705 over 100000 experiments
81 steps: 7.991829068104304 over 100000 experiments
100 steps: 8.877272344807551 over 100000 experiments

____Mean squared distance vs steps____
1 steps: 1.0(mean squared distance) over 100000 experiments
4 steps: 3.9978(mean squared distance) over 100000 experiments
9 steps: 8.98828(mean squared distance) over 100000 experiments
16 steps: 15.94718(mean squared distance) over 100000 experiments
25 steps: 25.15628(mean squared distance) over 100000 experiments
36 steps: 36.01996(mean squared distance) over 100000 experiments
49 steps: 48.94224(mean squared distance) over 100000 experiments
64 steps: 64.32644(mean squared distance) over 100000 experiments
81 steps: 80.60808(mean squared distance) over 100000 experiments
100 steps: 99.72374(mean squared distance) over 100000 experiments
```

The evidence for the expressions from the observed data in IDE is further substantiated through graphical representations below.

Graphical Representation:



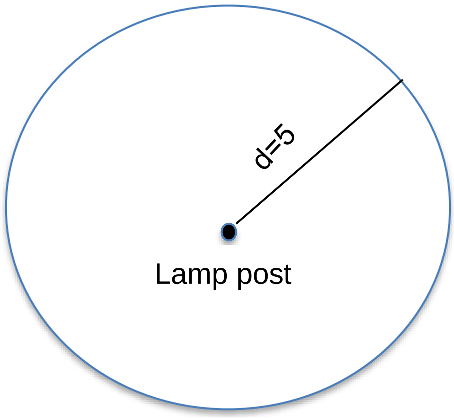


Explanation for Observations:

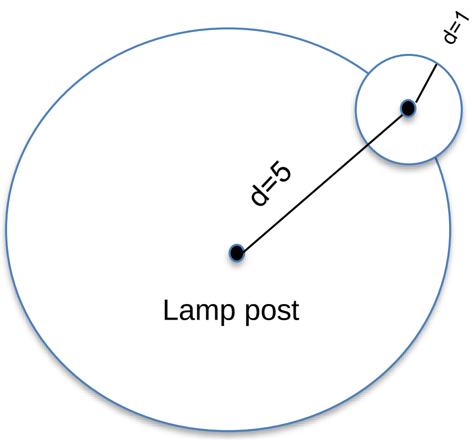
Reasoning for the relationship**:** $\boldsymbol{d \propto \sqrt{m}}$

Assuming the relationship is true, i.e, $d \propto \sqrt{m}$, or for simplicity that $d = \sqrt{m}$, it helps to imagine a circle to denote the locus of all points a distance ‘d’ away from the lamp post.

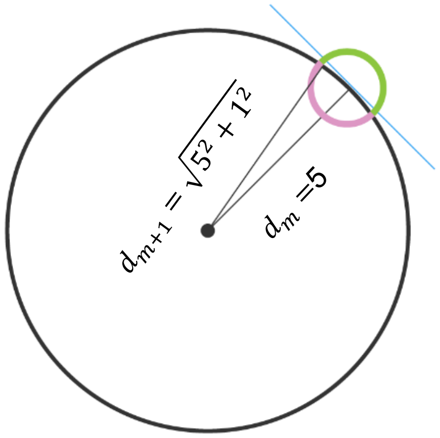
Suppose 25 steps have been committed, and assuming the relationship is true, the current position away from the lamp post = $\sqrt{25} = 5$.



From the new position, considering a step length of 1, the new locus of possible future positions can be depicted with a new circle of radius 1 as follows:



The circumference of the new circle denotes all possible future displacements from that point for a step length of 1.



By colour coding the new circle and drawing a tangent, it becomes apparent that the distance from the lamp post generally increases with additional steps: We notice that the green region going farther away from the lamp post in the circumference is larger than the pink region closing in towards the lamp post.

Also, the average distance contributed by this step can be calculated by considering all points along this circumference (green and pink), but the most "neutral" or "average" step from this point would be to take the step perpendicular to the radius of the big circle along the tangent. By Pythagorean theorem, the overall mean distance from the lamp post is then :
$\sqrt{25 + 1^2} = \sqrt{26}$.
Since this would be step 26, and $d = \sqrt{26}$, we have approximately shown that:
$d_{m+1} = \sqrt{m + 1}$ assuming that $d_m = \sqrt{m}$.

Unit Test Screenshots: