Logistic Regression

AC

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Logistic Regression - Introduction

Logistic regression is a predictive analysis approach conducted when the dependent variable is dichotomous (binary). It is used to describe data and to explain the relationship between one dependent binary variable and one or more metric independent variables.

Logistic Regression - Basic Equation

The logistic function can be understood simply as finding the β parameters that best fit:

y = 1 if $\beta_0 + \beta_1 x + \epsilon > 0$

y = 0 where ϵ is an error distributed by the standard logistic distribution. (If standard normal distribution is used instead, it is a probit regression.)

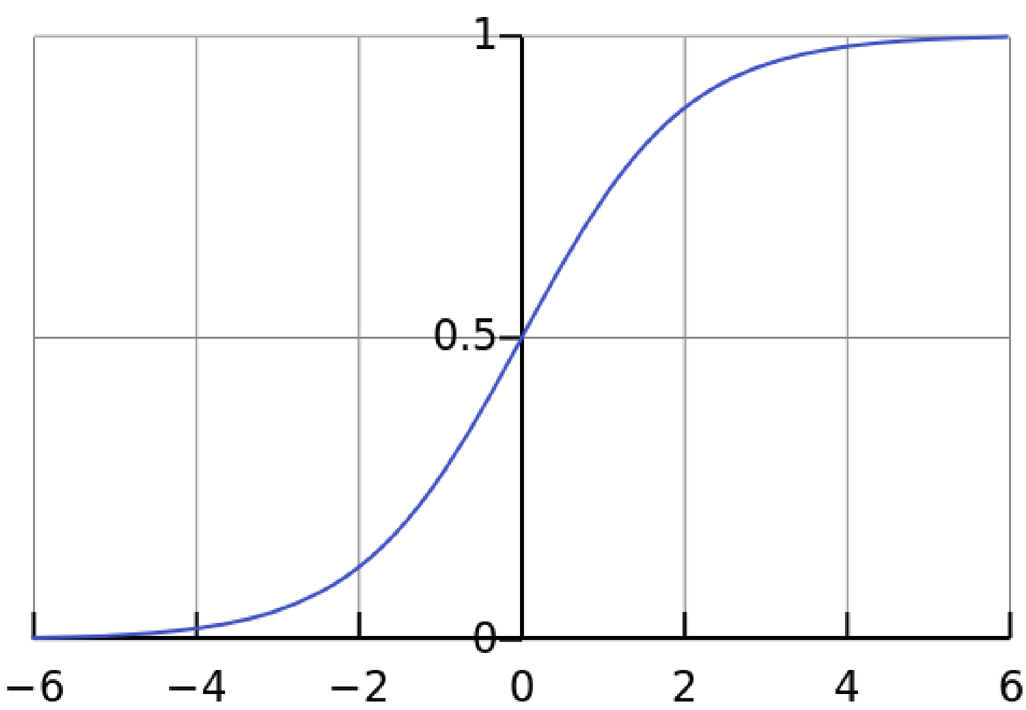


Figure 1. The standard logistic function $\sigma(t)$ (where $\sigma(t) \in (0,1)$)

The logistic function $\sigma(t)$ is defined as follows:

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$

In our case, $t = \beta_0 + \beta_1 x$.

So the logistic function can now be written as:

$$F(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

Logistic Regression - Assumptions

The major assumptions are:

1. The outcomes must be discrete i.e.the dependent variable should be binary in nature (eg. presence vs absence).
2. There should be no outliers in the data, this can be done by converting the continuous predictors to standardized z scores and remove values below -3.29 or greater than 3.29.
3. There should be no high intercorrelations (multicollinearity) among the predictors (Correlation coefficients should be less than 0.9).

Sample Example

```
rm(list = ls())

library(ggplot2)
library(aod)

## Warning: package 'aod' was built under R version 3.3.1

library(Rcpp)

dat0 <- data.frame(read.csv("http://www.ats.ucla.edu/stat/data/binary.csv"))
head(dat0)

##   admit gre  gpa rank
## 1     0 380 3.61    3
## 2     1 660 3.67    3
## 3     1 800 4.00    1
## 4     1 640 3.19    4
## 5     0 520 2.93    4
## 6     1 760 3.00    2

summary(dat0)

##      admit             gre             gpa             rank      
##  Min.   :0.0000   Min.   :220.0   Min.   :2.260   Min.   :1.000  
##  1st Qu.:0.0000   1st Qu.:520.0   1st Qu.:3.130   1st Qu.:2.000  
##  Median :0.0000   Median :580.0   Median :3.395   Median :2.000  
```

```
##  Mean   :0.3175   Mean   :587.7   Mean   :3.390   Mean   :2.485
##  3rd Qu.:1.0000   3rd Qu.:660.0   3rd Qu.:3.670   3rd Qu.:3.000
##  Max.   :1.0000   Max.   :800.0   Max.   :4.000   Max.   :4.000

sapply(dat0, sd)

##       admit         gre         gpa        rank
##   0.4660867 115.5165364   0.3805668   0.9444602

#table(dat0$admit,dat0$rank)
#For categorical data, looking at contingency table
xtabs(~ admit + rank, data = dat0)

##      rank
## admit  1  2  3  4
##     0 28 97 93 55
##     1 33 54 28 12

#Using the logit model
dat0$rank <- as.factor(dat0$rank)

logit <- glm(admit ~ gre + gpa + rank, data = dat0, family = "binomial")
summary(logit)

##
## Call:
## glm(formula = admit ~ gre + gpa + rank, family = "binomial",
##     data = dat0)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.6268  -0.8662  -0.6388   1.1490   2.0790
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -3.989979   1.139951  -3.500 0.000465 ***
## gre          0.002264   0.001094   2.070 0.038465 *
## gpa          0.804038   0.331819   2.423 0.015388 *
## rank2       -0.675443   0.316490  -2.134 0.032829 *
## rank3       -1.340204   0.345306  -3.881 0.000104 ***
## rank4       -1.551464   0.417832  -3.713 0.000205 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 499.98  on 399  degrees of freedom
## Residual deviance: 458.52  on 394  degrees of freedom
## AIC: 470.52
##
## Number of Fisher Scoring iterations: 4
```

```r
#Obtaining confidence intervals using profiled log-liklihood
confint(logit)

## Waiting for profiling to be done...

##                     2.5 %       97.5 %
## (Intercept) -6.2716202334 -1.792547080
## gre          0.0001375921  0.004435874
## gpa          0.1602959439  1.464142727
## rank2       -1.3008888002 -0.056745722
## rank3       -2.0276713127 -0.670372346
## rank4       -2.4000265384 -0.753542605

#Obtaining confidence intervals using standard errors
confint.default(logit)

##                     2.5 %       97.5 %
## (Intercept) -6.2242418514 -1.755716295
## gre          0.0001202298  0.004408622
## gpa          0.1536836760  1.454391423
## rank2       -1.2957512650 -0.055134591
## rank3       -2.0169920597 -0.663415773
## rank4       -2.3703986294 -0.732528724

#Can test the overall effect of rank
wald.test(b = coef(logit), Sigma = vcov(logit), Terms = 4:6)

## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 20.9, df = 3, P(> X2) = 0.00011

#Interpreting the odds ratios
exp(coef(logit))

## (Intercept)         gre         gpa       rank2       rank3       rank4
##   0.0185001   1.0022670   2.2345448   0.5089310   0.2617923   0.2119375

#Interpreting the odds ratio and 95% CI
exp(cbind(OR = coef(logit), confint(logit)))

## Waiting for profiling to be done...

##                    OR       2.5 %    97.5 %
## (Intercept) 0.0185001 0.001889165 0.1665354
## gre         1.0022670 1.000137602 1.0044457
## gpa         2.2345448 1.173858216 4.3238349
## rank2       0.5089310 0.272289674 0.9448343
## rank3       0.2617923 0.131641717 0.5115181
## rank4       0.2119375 0.090715546 0.4706961
```

```r
#Calculating the predicted probability of admission at each
#value of rank, holding gre and gpa at their means.

dat1 <- with(dat0, data.frame(gre = mean(gre), gpa = mean(gpa),
             rank = factor(1:4)))
dat1
```

```
##     gre    gpa rank
## 1 587.7 3.3899    1
## 2 587.7 3.3899    2
## 3 587.7 3.3899    3
## 4 587.7 3.3899    4
```

```r
#Prediction

dat1$rankprob <- predict(logit, newdata = dat1, type = "response")
dat1
```

```
##     gre    gpa rank  rankprob
## 1 587.7 3.3899    1 0.5166016
## 2 587.7 3.3899    2 0.3522846
## 3 587.7 3.3899    3 0.2186120
## 4 587.7 3.3899    4 0.1846684
```

```r
#Preparing data to plot

dat2 <- with(dat0,
             data.frame(gre = rep(seq(from = 200, to = 800, length.out =
100), 4),
             gpa = mean(gpa), rank = factor(rep(1:4, each = 100))))


dat3 <- cbind(dat2, predict(logit, newdata = dat2, type = "link", se = TRUE))
dat3 <- within(dat3, {
               PredictedProb <- plogis(fit)
               LL <- plogis(fit - (1.96*se.fit))
               UL <- plogis(fit + (1.96*se.fit))
})

head(dat3)
```

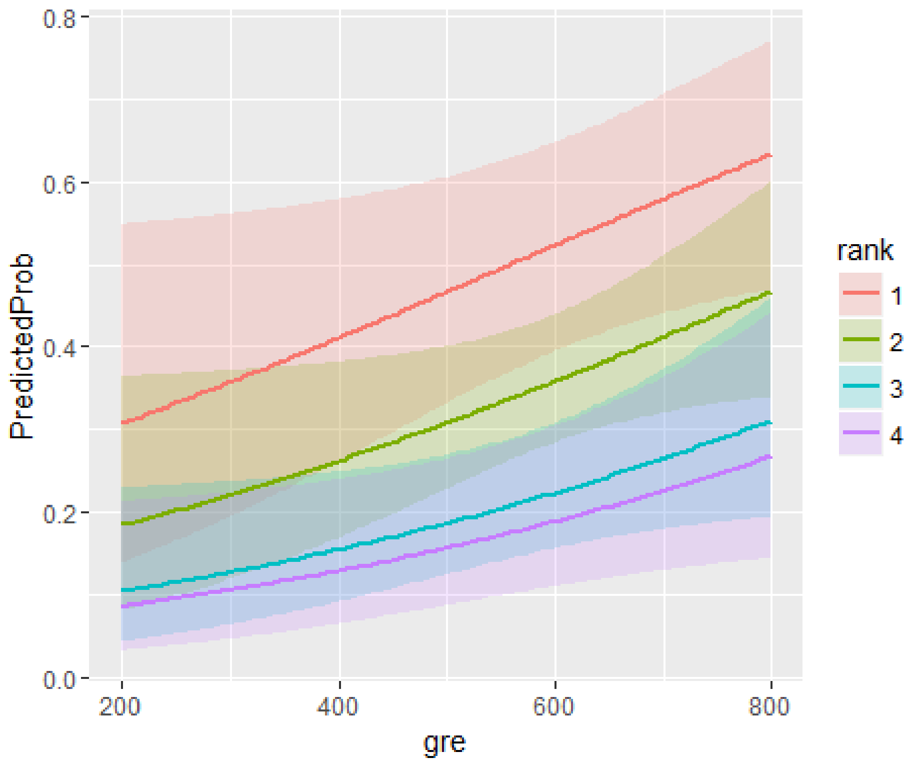
```
##         gre    gpa rank        fit    se.fit residual.scale        UL
## 1 200.0000 3.3899    1 -0.8114870 0.5147714              1 0.5492064
## 2 206.0606 3.3899    1 -0.7977632 0.5090986              1 0.5498513
## 3 212.1212 3.3899    1 -0.7840394 0.5034491              1 0.5505074
## 4 218.1818 3.3899    1 -0.7703156 0.4978239              1 0.5511750
## 5 224.2424 3.3899    1 -0.7565919 0.4922237              1 0.5518545
## 6 230.3030 3.3899    1 -0.7428681 0.4866494              1 0.5525464
##          LL PredictedProb
## 1 0.1393812     0.3075737
```

```
## 2 0.1423880     0.3105042
## 3 0.1454429     0.3134499
## 4 0.1485460     0.3164108
## 5 0.1516973     0.3193867
## 6 0.1548966     0.3223773

#Testing Model Fit

with(logit, null.deviance - deviance)

## [1] 41.45903

with(logit, df.null - df.residual)

## [1] 5

#Evaluating p-value
with(logit, pchisq(null.deviance - deviance, df.null - df.residual,
lower.tail = FALSE))

## [1] 7.578194e-08

#Log Liklihood
logLik(logit)

## 'log Lik.' -229.2587 (df=6)
```

Plots

The above result can be graphically shown as follows:

```
ggplot(dat3, aes(x = gre, y = PredictedProb)) +
  geom_ribbon(aes(ymin = LL, ymax = UL, fill = rank), alpha = 0.2) +
  geom_line(aes(colour = rank), size = 1)
```



References

1. R Data Analysis Examples: Logit Regression from http://www.ats.ucla.edu/stat/r/dae/logit.htm
2. Statistics Solutions http://www.statisticssolutions.com/what-is-logistic-regression/
3. Wikipedia https://en.wikipedia.org/wiki/Logistic_regression