Home Work of Week 6

Deadline: 9:00am, December 9(Tuesday), 2014

1. Given an n-vertex undirected graph $G = (V, E)$, consider the following method of generating an independent set. Given a permutation σ of the vertices, define a subset $S(\sigma)$ of the vertices as follows: for each vertex i, $i \in S(\sigma)$ if and only if no neighbor j of i precedes i in the permutation σ.

   - Show that each $S(\sigma)$ is an independent set in G.
   - Suggest a natural randomized algorithm to produce σ for which you can show that the expected cardinality of $S(\sigma)$ is $\sum_{i=1}^{n} \frac{1}{d_i+1}$, where d_i denotes the degree of vertex i.
   - De-randomizing the above algorithm.

2. Prove that, for every integer n, there exists a way to 2-color the edges of K_x so that there is no monochromatic clique of size k when $x = n - \binom{n}{k} 2^{1-\binom{k}{2}}$. Note that K_x stands for the x-vertex complete graph. (Hint, start by 2-coloring the edges of K_n and fix things up.)

3. Prove the following claims.

   - For every integer n, there exists a coloring of the edges of the complete graph K_n by two colors so that the total number of monochromatic copies of K_4 is at most $\binom{n}{4} 2^{-5}$.
   - Give a randomized algorithm for finding a coloring with at most $\binom{n}{4} 2^{-5}$ monochromatic copies of K_4 that runs in expected time polynomial in n.
   - Show how to construct such a coloring deterministically in polynomial time using the method of conditional expectations.

4. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string $s_1 s_2 ... s_i ... s_{20}$, where s_i is 1 if the i^{th} trial gets Head, and otherwise is 0.