Home Work of Week 5

Deadline: 9:00am, December 2(Tuesday), 2014

1. Bloom filters can be used to estimate set differences. Suppose Alice has a set X and Bob has a set Y, both with n elements. For example, the sets might represent their 100 favorite songs. Alice and Bob create Bloom filters of their sets respectively, using the same number of bits m and the same k hash functions. Determine the expected number of bits where our Bloom filters differ as a function of m, n, k and $|X \bigcap Y|$. Explain how this could be used as a tool to find people with the same taste in music more easily than comparing lists of songs directly.

2. We have shown that any event that occurs with small probability in the balls-and-bins setting where the number of balls in each bin is an independent Poisson random variable also occurs with small probability in the standard balls-and-bins model. Prove a similar statement for random graphs: every event that happens with small probability in the $G_{n,p}$ model also happens with small probability in the $G_{n,N}$ model for $N = \binom{n}{2}p$.

3. We have an algorithm (Algorithm 5.2 in the textbook) which finds Hamiltonian cycles. We have shown that the algorithm can be applied to find a Hamiltonian cycle with high probability in a graph chosen randomly from $G_{n,p}$ when p is known and sufficiently large, by initially placing edges in the edge lists appropriately. Argue that the algorithm can similarly be applied to find a Hamiltonian cycle with high probability on a graph chosen randomly from $G_{n,N}$ when $N = c_1 n \ln n$ for a suitably large constant c_1.

4. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string $s_1 s_2 ... s_i ... s_{20}$, where s_i is 1 if the i^{th} trial gets Head, and otherwise is 0.