Homework of Week 9

Deadline: 9:00am, January 2 (Friday), 2015

1. Assume that the transition probability matrix of a Markov chain satisfies that the sum of the entries in each column is 1. Prove that the uniform distribution is a stationary distribution of this Markov chain.

2. Let X_n be the sum of n independent rolls of a fair die. Show that, for any $k \geq 2$,

$$\lim_{n\to\infty} Pr(X_n \text{ is divisible by } k) = \frac{1}{k} \tag{1}$$

3. Given a finite Markov chain, prove that a state i is recurrent if and only if $Pr[N_{ii} = \infty] = 1$, where N_{ii} is the times of returning to i if the chain starts at state i.

4. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string $s_1 s_2 ... s_i ... s_{20}$, where s_i is 1 if the i^{th} trial gets Head, and otherwise is 0.