Homework of Week 10

Deadline: 9:00am, January 12 (Monday), 2015

1. Consider a Markov chain on the states $\{0, 1, ...n\}$, where for $i < n$ we have $P_{i,i+1} = 1/2$ and $P_{i,0} = 1/2$. Also, $P_{n,n} = 1/2$ and $P_{n,0} = 1/2$. This process can be viewed as a random walk on a directed graph with vertices $\{0, 1, ...n\}$, where each vertex has two directed edges: one that returns to 0 and one that moves to the vertex with the next higher number (with a self loop at vertex n). Find the stationary distribution of this chain. (This example shows that random walks on directed graphs are different than random walks on undirected graphs.)

2. Let n equidistant points be marked on a circle. Without loss of generality, we think of the points as being labeled clockwise from 0 to $n-1$. Initially, a wolf begins at 0 and there is a sheep at each of the remaining $n-1$ points. The wolf takes a random walks on the circle. For each step, it moves with probability 1/2 to one neighboring point and with probability 1/2 to the other neighboring point. At the first visit to a point, the wolf eats the sheep at the point. Which sheep is most likely to be the last eaten?

3. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string $s_1s_2...s_i...s_{20}$, where s_i is 1 if the i^{th} trial gets Head, and otherwise is 0.