HW #7 2.4 #15, 19, 20, 23

15) A) $\int_{-\infty}^{0} 0t\,dt + \int_{0}^{\infty} 0.1te^{-0.1t}\,dt = -\int_{0}^{\infty} t\,d(e^{-0.1t})$

$= -te^{-0.1t}\Big|_0^{\infty} + \int_0^{\infty} e^{-0.1t}\,dt$

$= 0 - \frac{1}{0.1} e^{-0.1t}\Big|_0^{\infty}$

$= 10$

B) $0 - \int_0^{\infty} t^2\,d(e^{-0.1t}) - 100$

$= -t^2e^{-0.1}\Big|_0^{\infty} + \int_0^{\infty} 2te^{-0.1t}\,dt - 100$

$= -20te^{-0.1t}\Big|_0^{\infty} + 20\int_0^{\infty} e^{-0.1t}\,dt - 100$

$= 0 - 200e^{-0.1t}\Big|_0^{\infty} - 100$

$= 200 - 100 = 100$

C) $\int_{-\infty}^{t} 0\,dy = 0 \quad \int_{-\infty}^{0} 0\,dt + \int_0^t 0.1e^{-0.1y}\,dy$

$= 0 - e^{-0.1y}\Big|_0^t = 1 - e^{-0.1t}$

$F(t) = \begin{cases} 0 & t \le 0 \\ 1 - e^{-0.1t} & t > 0 \end{cases}$

D) $P(T < 12) = F(12)$

$= 1 - e^{-(0.1)12} = 1 - e^{-1.2} = 0.6988$

19) A) $\int_3^4 3/64\, x^2(4-x)\,dx + \int_4^\infty 0\,dx$

$= 3/64\left(\frac{4x^3}{3} - \frac{x^4}{4}\right|_3^4 + 0$

$= 3/64\left(\frac{256}{3} - \frac{256}{4}\right) - \left(\frac{108}{3} - \frac{81}{4}\right)$

$= 3/64(256/12 - 189/12)$

$= 3/64(67/12) = 67/256 = 0.26$

B) $\int_2^3 \frac{3}{64}x^2(4-x)\,dx$

$= 3/64\left(\frac{4x^3}{3} - \frac{x^4}{4}\right|_2^3$

$= 3/64\left[\frac{108}{3} - \frac{81}{4}\right)\left(\frac{32}{3} - \frac{16}{4}\right)$

$= \frac{109}{256} = 0.4258$

C) $\int_0^4 x\left[\frac{3}{64}x^3(4-x)\right]dx$

$3/64\int_0^4 4x^3 - x^4\,dx$

$= 3/64\left(256 - \frac{1024}{5}\right) = 3/64\left(\frac{256}{5}\right)$

$= 12/5 = 2.4$

D) $\int_0^4 x^2(3/64\, x^2(4-x)\,dx) - \mu^2 x$

$$= 3/64 \left(\frac{4x^5}{5} - \frac{x^6}{6}\right)\Big|_0^4 - (2.4)^2$$

$$= 3/64 \left(\frac{4096}{5} - \frac{4096}{6}\right) - (2.4)^2$$

$$= 3/64 \left(\frac{4096}{30}\right) - (2.4)^2$$

$$= 6.4 - 5.76 = 0.64$$

E) $\int_{-\infty}^{0} 0\,dt + \int_0^x \frac{3}{64} t^2 (4-t)\,dt$

$$= 0 + \int_0^x \frac{3}{64} (4t^2 - t^3)\,dt$$

$$= 3/64 \left[\frac{4x^3}{3} - \frac{x^4}{4}\right]$$

$$= 3/64 \left(\frac{16x^3 - 3x^4}{12}\right) = \frac{x^3}{16} - \frac{3x^4}{256}$$

$$= \int_{-\infty}^{0} 0\,dt + \int_0^4 \frac{3}{64} t^2 (4-t)\,dt + \int_4^x 0\,dt = 1$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{16} - \frac{3x^4}{256} & 0 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$

20) A) $\int_{-\infty}^{0} 0\,dx + \int_{0}^{0.02} 625x\,dx$

$= 0 + \frac{625x^2}{2}\Big|_0^{0.02} = 0.125$

B) $\int_{-\infty}^{0} 0x\,dx + \int_{0}^{0.04} 625x^2\,dx + \int_{.04}^{.08} 50x - 625x^2\,dx$

$+ \int_{.08}^{\infty} 0x\,dx$

$0 + \frac{625x^3}{3}\Big|_0^{0.04} + \left(25x^2 - \frac{625x^3}{3}\right)\Big|_{.04}^{.08} + 0$

$= 0.039 = 0.04$

C) $\int_{-\infty}^{0} 0x^2\,dx + \int_{0}^{.04} 625x^3\,dx + \int_{.04}^{.08} (50x^2 - 625x^3)\,dx$

$+ \int_{.08}^{\infty} 0x^2\,dx - (0.04)^2$

$= 0 + \frac{625x^4}{4}\Big|_0^{.04} + \left(\frac{50x^3}{3} - \frac{625x^4}{4}\right)\Big|_{.04}^{.08}$

$+ 0 - 0.0016$

$= \sqrt{0.000267} = 0.016$

D) $\int_{-\infty}^{x} 0\,dt = 0$ $\quad \int_{-\infty}^{0} 0\,dt + \int_{0}^{x} 625t\,dt = \frac{625x^2}{2}$

$$\int_{-\infty}^{0} 0\,dt + \int_{0}^{.04} 625t\,dt + \int_{0.04}^{x} (50 - 625t)\,dt$$

$$= 50x - \frac{625x^2}{2} - 1$$

$$\int_{-\infty}^{0} 0\,dt + \int_{0}^{.04} 625t\,dt + \int_{.04}^{.08} 50 - 625t\,dt$$
$$+ \int_{.08}^{\infty} 0\,dt$$
$$= 1$$

$$F(x) \begin{cases} 0 & x \le 0 \\ 625x^2/2 & 0 < x \le .04 \\ 50x - 625x^2/2 - 1 & 0.04 < x < 0.08 \\ 1 & x > 0.08 \end{cases}$$

E) $625x_m^2/2 = 0.5$ $\quad 50x_m - 625x_m^2/2 - 1$
$x = 0.04$ $\quad = 0.5$
$x_{m1} = 0.04,\ x_{m2} = 0.12$
$x_m = 0.04$

F) $\int_{.015}^{.04} 625x\,dx + \int_{.04}^{.063} (50 - 625x)\,dx$

$$= \frac{625x^2}{2}\Big|_{.015}^{.04} + \left(50x - \frac{625x^2}{2}\right)\Big|_{.04}^{.063}$$

$= 0.84$

23) A) $\int_{-\infty}^{2} 0\,dx + \int_{2}^{2.5} \frac{3}{52} x(6-x)\,dx$

$0 + \frac{3}{52}\left(2x^3 - \frac{x^4}{4}\right|_{2}^{2.5}$

$= 0.2428$

B) $\int_{2.5}^{3.5} \frac{3}{52} x(6-x)\,dx$

$= \frac{3}{52}\left(3x^2 - \frac{x^3}{3}\right)\Big|_{2.5}^{3.5}$

$= 0.5144$

c) $\int_{-\infty}^{2} 0x\,dx + \int_{2}^{4} \frac{3}{52} x^2(6-x)\,dx + \int_{4}^{\infty} 0x\,dx$

$= 0 + \frac{3}{52}\left(2x^3 - \frac{x^4}{4}\right|_{2}^{4} + 0$

$= 3$

D) $\int_{-\infty}^{2} 0x^2\,dx + \int_{2}^{4} \frac{3}{52} x^3(6-x)\,dx$

$+ \int_{4}^{\infty} 0x^2\,dx - 3^2$

$= 0 + \frac{3}{52}\left(\frac{3x^4}{2} - \frac{x^5}{5}\right|_{2}^{4} + 0 - 9$

$= 0.3231$

e) $\int_{2.43}^{3.57} 3/52\, x(6-x)\,dx$

$= 3/52\left(3x^2 - \frac{x^3}{3}\right)\Big|_{2.43}^{3.57}$

$= 0.5848$

F) $\int_{-\infty}^{x} 0\,dt = 0$

$\int_{-\infty}^{2} 0\,dt + \int_{2}^{x} 3/52\, t(6-t)\,dt$

$= \left(\frac{9x^2 - x^3 - 28}{52}\right)$

$\int_{-\infty}^{2} 0\,dt + \int_{2}^{4} 3/52\, t(6-t)\,dt + \int 0\,dt$

$= 0 + 3/52\left(3t^2 - \frac{t^3}{3}\right)\Big|_{2}^{4} + 0$

$= 1$

$0 \quad x < 2$

$\frac{9x^2 - x^2 - 28}{52} \quad 2 \leq x < 4$

$1 \quad x \geq 4$