Midterm Akashdeep Vasisht

1) Base Case: $n = 1$ $1 = 2^0$

Induction step: assume theorm to be true for all positive integers up to n.

Now for $n+1$

k = largest power of 2

$k \leq n+1$

$k = 2^{a_1} + 2^{a_2} + 2^{a_3} + 2^{a_4} + 2^{a_x}$ where

$a1 > a2 > a3 > a4 > ax$

$k = n+1$ ✓

IF $k < n+1$ we add 2^b to k

$b < a1$

$b = \max(ax-1 : ax-1 > a1-1) = b < a1$

$n+1 = k + 2^b + (n+1k - 2^b)$

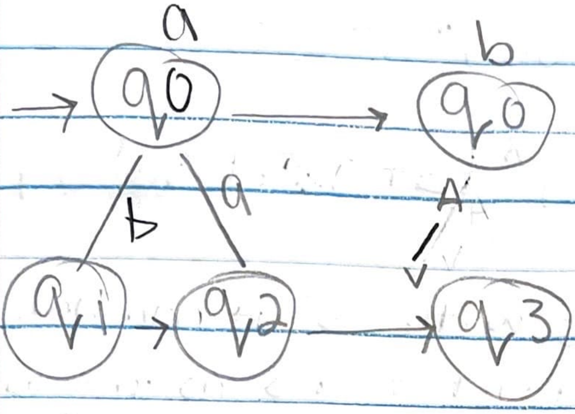
hence $b < a1$, power of 2, 2^b is distinct from it being used in repsentation of k

$n+1-k-2^b$

2) $S \to A \mid B \mid AB$ S = starting symbol
$A \to Sa \mid aa$ A = "aa"
$B \to Sb \mid bb$ B = "bb"

S can generate "aa" for substring (A)
or "bb" for substring (B)
or both AB.
A can generate "a" followed by A (Sa)
or produce "aa" (aa).
B can generate "b" followed by B (Sb)
or produce "bb" (bb).

3)



intial state q_0

input a $q_0 \rightarrow q_1$ input b $q_0 \rightarrow q_0$

input a $q_1 \rightarrow q_2$ input b $q_1 \rightarrow q_0$

input a $q_2 \rightarrow q_1$ input b $q_2 \rightarrow q_3$

input a $q_3 \rightarrow q_0$ input b $q_3 \rightarrow q_0$

4) $\cancel{(a^+ bb + b^+ a^* bb)^* a^* b^*}$

$(b + aa^* bb)^* (\varepsilon + aa^* + aa^* b)$

if string starts with "b" it can have any amount of a's as long as it ends with "b".

if string starts with "a" it can have any number of a's but not end with "a"

ε makes the string b empty which also does not have aba in it.