A Kash
Vasisht

Quiz

1) a) $P(\bar{X} > 1.520)$ $= P(Z > 2.828)$
$= P(Z > \dfrac{1.520-1.5}{\frac{0.1}{\sqrt{200}}}$
$= 1 - 0.99766 = 0.0023$

b) $P(\bar{X} < 1.490) = 0.0495$ $\dfrac{1.490-1.5}{\frac{0.1}{\sqrt{n}}} = -1.65$
$n = 272.25 \approx 272$

2) a) $P(X > 15) = \dfrac{1}{12}\int_{15}^{\infty} e^{-x/12} dx = \dfrac{1}{12}[-12e^{-x/12}]\Big|_{15}^{\infty} = e^{-15/12}$
$= 0.2865$

b) $P(X > 30 | X > 10) = P(X > (20+10) | X > 10)$
$= P(X > 20) = \dfrac{1}{12}\int_{20}^{\infty} e^{-x/12} dx = e^{-20/12} = 0.1889$

3) $P(R_2 - R_1 > 15) = P(Z > \dfrac{15-20}{5\sqrt{5}}) = P(Z > 1.118) = 0.868$

4) a) $P = 100/1000 = 0.1$ $n = 25$ $p = 0.01$

b) $P(X=8) = \dfrac{25!}{(8!)17!} = 1,771,030 (0.1)^8 (0.9)^{17} = 0.175$

5) a) $^{n-1}C_{k-1} \, P^k (1-P)^{n-k}$

b) $P(X=5) = {}^{(5-1)}C_{(3-1)} (0.40)^3 (1-0.40)^{5-3}$
$= 12 \times 0.064 \times 0.36$
$= 0.27648$

c) $= \sqrt{np(1-p)} = \sqrt{n(0.4)(1-0.4)} = \sqrt{n(0.4)(.06)} = \sqrt{n(0.24)}$
$= n(0.4)$

6) a) X_1, X_2, X_3 $P_1 = 0.5$ $P_2 = 0.1$ $P_3 = 0.40$

b) $\dfrac{25!}{10!\,6!\,9!}(0.5)^{10}(0.1)^6(0.40)^9$

c) $P(X_3 = 4) = 25_{C_4}(0.40)^4(1-0.40)^{25-4}$

$$= 12650 \times 0.0256 \times 2.19 \times 10^{-5}$$
$$= 0.007104$$

7) a) $X_1 \sim Pos(\lambda = 4)$
$$P(X_1 = 6) = \frac{e^{-4}\,4^6}{6!} = 0.1042$$

b) $P(T_1 < 1) = 1 - e^{-4 \times 1} = 0.2212$

c) $X_2 \sim Pos(\lambda = 4 \cdot 3 = 12)$
$$P(X_2 = 10) = e^{-12}\frac{12^{10}}{10!} = 0.1048$$

d) $= T_2 \sim exp(4/3$ $P(T_2 < 2)\displaystyle\int_0^2 (4/3)e^{-4t/3}\,dt$

e) $= \displaystyle\int_0^2 e^{-\lambda t}\frac{\lambda^3 t^{3-1}}{(3-1)!}\,dt$

8) a) $P(X > 1) = 1 - P(X = 0) + P(X = 1)$
$$= 25(0.2)^0(0.8)^{25} + 25(0.2)^1(0.8)^{24}$$
$$= 0.9726$$
$$E(X) = np = 25 \times 0.2 = 5$$

b) 1) $X \sim Geometric$
$$P = 0.2 \quad q = 1 - p = 0.8$$
2) $\sigma_X^2 = q/p^2 = 0.8/(0.2)^2 = 20$
3) $P(X = 4) = q^{x-1}P$
$$= (0.8)^{4-1}(0.2) = 0.1024$$

9) a) $P(13.5 \le x \le 15) = P\left(\dfrac{13.5-5}{1.25} \le \dfrac{x-\mu}{\sigma} \le \dfrac{15-15}{1.25}\right)$

$P(-1.2 \le z \le 0) = P(z \le 0) - P(z \le -1.2)$
$= 0.5 - 0.1151 = 0.3849$

b) $\dfrac{x-15}{1.25} = 1.91 \quad x = 12.6125$

10) a) $\mu = 18 \times \tfrac{1}{9} = 2$

b) $P(x \ge 1) = 1 - P(x<1)$
$= 1 - 18 C_0 \left(\tfrac{1}{9}\right)^0 \left(\tfrac{8}{9}\right)^{18} = 0.88$

11) a) $P(x>3) = \displaystyle\int_3^\infty f(x)\,dx \quad \int_3^\infty \tfrac{1}{2} e^{-x/2}\,dx$

$= -e^{-x/2}\Big|_3^\infty = e^{-3/2} = 0.2231$

b) $P(x>4+3 \mid x>4) = P(x>3)$
$= .2231$

12) a) $X \sim G(0.01)$

b) $E(x) = \dfrac{1}{p} \quad \dfrac{1}{0.01} = 100$

13) a) $P(x=2)$ $\dfrac{\binom{20}{2}\binom{80}{2}}{\binom{100}{4}} = \dfrac{\dfrac{20!}{2!\,2!} \times \dfrac{80!}{2!\,78!}}{\dfrac{100!}{4!\,96!}}$

b) $P(x=2) = \dbinom{4}{2}(0.2)^2 \times (0.8)^2$

$= 6 \times 0.0256 = 0.1536$

14) a) $P(97 \leq x \leq 100) = \displaystyle\int_{97}^{100} f(x)\,dx = \int_{97}^{100} \dfrac{1}{6}\,dx = \dfrac{100-97}{6}$

$= 0.5$ $= 3/6$

b) $X \sim Bin(n,p)$

$P(x=2) = \dbinom{7}{2}(0.5)^2 (1-0.5)^5$

$= 7! / 5!\,2!\,(0.5)^7$

$= 0.1641$