

# Assign 6 Akash Vasisht

1a)  $L = \{0^n 1^h 2^h \mid n \geq 0\}$

- 1) assume  $L$  regular
- 2)  $N = 2 \quad n \geq 0$

$0^2 1^2 2^2$

$x = 00$	$ x  = 2$	Case 1	$n = 6$	Case 3 $L = x^i z^j$
$y = 11$	$ y  = 2$		$ x^i y^j  \leq n$	$L = 00(11)^2 22$
$z = 22$	$ z  = 2$		$4 \leq 6$	$L = 00111122$

Assumption incorrect

(Case 2  $|y| \geq 1$ ) Failed

Since lang. 0's 1's & 2's  
must be equal.

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1b)  $L = \{www \mid w \in \{a, b\}^*\}$

- 1) assume regular

$$w = ab \quad L = \{ababab\}$$

$$x = ab \quad y = ab \quad z = ab$$

$$|w| = 2 \quad |y| = 2 \quad |z| = 2$$

Case 1  $n = 6$

Case 2  $|y| \geq 1$

$$|xy| \leq n$$

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$$4 \leq 6$$

Case 3  $L = x^i y^j z^k \mid i > 0$

$$i = 2$$

$$L = ab(ab)^2 ab = abababab$$

Not regular since string must be divisible by 3

1)  $L = \{a^{2^n} \mid n \geq 0\}$  ( $a^{2^n}$  string of  $2^n$  0's)

String  $w = a^{2^2}$       aaaa  
 $a^4$                   xyz      1)  $|y| > 0$   
                          xyz      2)  $|xy| \leq p$   
                          xyz      3)  $xy^iz \in L$   
                           $a(a^2)aaa$   
                           $a^5 \in L$

Not regular

2) When it says "but if we consider ..."

$x^2z$  will have more 0's than 1's

But its wrong.  $0^*1^*$  means 0000  
or 1111

Meaning the number of 0's & 1's don't have  
to be equal

3)  $w \in L$   $w = a^4b^2$   $4 \leq 2^2$

$w = aaaaabbb$

1)  $V/x$  contains only one type of alphabet

$w = aaaaabb$  Pumping V we get

$w' = aaaaaabb$

$w' = a^7 b^2 \neq 2^2$

$w'$  doesn't belong to L  $a_1 n \leq 2^2$

2)  $v \mid x$ : contains more than one alphabet

$$w = aa \ aab \ b \\ u \quad v \quad w$$

$$w' = aa \ adbaab \ b \\ u \quad v \quad w$$

$w'$  do not belong to  $L$

3b)  $w \in L \quad w = a^2 b^3 c^4 \quad i > 2, j > 3$

1)  $v \mid x$  contain one type alphabet

$$w = a \ a b b b \ c c c \\ u \ v \ w \ y$$

$$w' = a^3 b^5 c^4 \quad n=3 \ j=5 \ k=4$$

$w'$  does not belong to  $L$

2)  $v \mid x$  contain more than one type

$$w = a \ ab \ b \ bc \ ccc \\ u \ v \ w \ x \ y$$

$$w' = a \ abab \ b \ bcbc \ ccc$$

$$w' = a^2 b a b^3 c^5 d$$

$w'$  does not belong to  $L$  as it is

not form  $a^n b^j c^k$

4) No, suppose  $L_2 = \{a^n b^n \mid n \geq 0\}$  which

is context free lang. &  $L_1$  is regular

so  $L_1 = (a+b)^*$  the union of  $L_1 \cup L_2$

$$= L_1 \cup L_2 = (a+b)^* \cup \{a^n b^n \mid n \geq 0\} = (a+b)^*$$

which is regular, if  $L_1 \cup L_1 \cup L_2$  are regular  
then  $L_2$  need not to be regular.

5) Let  $P$  be pumping length  $w = a^P z b^P$   
 $|w| > P \Rightarrow |w| = 2P + 2$   
 since  $n = P - 2 \Rightarrow k = P$

1)  $|xy| \leq P$

2)  $|y| > 0$

3)  $xz^0 = x(y)^k z \in L$

$x = a^{P-3}, y = a, z = b^P$

1)  $|xy| = (P-3) + 1 = P + 2 > P$

2)  $|y| = 1 > 0$

3)  $P = 2, x(y)^k z = a^{P-3} a a b^k = a^{P-1} b^P$

$a^{P-1} b^P \notin L$

3 is not satisfied

b)  $L_1 = \{a^n b^n : n \geq 0\}$  which is context free

$L_2 = \{a^n b^n : n \text{ is a multiple of } 5\}$  lang.

$S \rightarrow AaAaAaAa$

$A \rightarrow aaaa$

$B \rightarrow bbbb$

Since, context free lang. are closed under complement  
 and intersection with regular langs.

We can use these properties to say -  
 $L$  is context free.

7)  $S \rightarrow aSb | b$

$$\{a^n b^{2n} | n \geq 1\}$$

For every one a, two b's must generate

It is not regular because we want to count  
the number we need a's, we need  
external memory - DFA has no memory.