Assignment 1

1) DeMorgan's Law given

Base Case:

$n=1$ satisfy condition for base case

$\overline{\bigcup_{p=1,2,3...} S_p} = \overline{S_1 \cup (\bigcup_{p=2,3...} S_p)}$

$\overline{S_1 \cup (\bigcup_{p=2,3,...} S_p)} = \overline{S_1} \cap \overline{(\bigcup_{p=2,3...} S_p)}$ (∵ induction hypothesis)

$\overline{(\bigcup_{p=2,3...} S_p)} = \bigcap_{p=2,3...} \overline{S_p}$

$\overline{\bigcup_{p=1,2,3..} S_p} = \overline{S'} \cap (\bigcap_{p=2,3...} \overline{S_p})$

$= \bigcap_{p=1,2,3...} \overline{S_p}$

2) $(S_1 \cup S_2) - (S_1 \cap S_2') = S_2$

$S - T = S \cap T'$

$(S_1 \cup S_2) \cap (S_1 \cap S_2')'$

$(S_1 \cup S_2) \cap (S_1' \cup S_2)$

$(S_1 \cap (S_1' \cup S_2))$ ← using distributive law

$\cup (S_2 \cap (S_1' \cup S_2))$

$((S_1 \cap S_1') \cup (S_1 \cap S_2)) \cup ((S_2 \cap S_1') \cup (S_2 \cap S_2))$

$(\varnothing \cup (S_1 \cap S_2)) \cup ((S_2 \cap S_1') \cup S_2)$

$(S_1 \cap S_2) \cup ((S_2 \cap S_1') \cup S_2)$

$(S_1 \cap S_2) \cup S_2$

$= S_2$

3) $S \to aaA$ This gives strings $\{aab, aabaab, aabaabaab, \ldots\}$
$A \to bS$
$S \to \lambda$ This grammer gives combinations of "aab"

$$(a\,ab)^n \text{ or } (a^2 b)^n$$
$$L(G) = (a^2 b)^n : n \geq 0 \}$$

4) ~~$L = \{a^{n^2} \mid n >= 0\}$~~

Gives string $Aa \to Ba \to Aaa \to Baa \to Aaaa \to aaaa$

$$L(G) = \{a^\infty\}$$

This grammer gives infinite amount of "a".

5) a) $S \to aA \mid bS$

$A \to aB \mid bA$

$B \to aB \mid bB \mid \lambda$

The only way for a string to end is if it gets to B. It can only do so if it has 2 "a"'s.

b) $S \to aA \mid bS \mid \lambda$

$A \to aB \mid bA \mid \lambda$

$B \to aC \mid bB \mid \lambda$

$C \to bC \mid \lambda$

Once a string gets to 3 "a"'s it can only have b's. It can end anytime before that happens.