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ON THE OSCILLATORY INTEGRATION OF SOME ORDINARY DIFFERENTIAL EQUATIONS

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Abstract. Conditions are given for a class of nonlinear ordinary differential equations $x'' + a(t)w(x) = 0$, $t \geq t_0 \geq 1$, which includes the linear equation to possess solutions $x(t)$ with prescribed oblique asymptote that have an oscillatory pseudo-wronskian $x'(t) - \frac{x(t)}{t}$.

1. Introduction

A certain interest has been shown recently in studying the existence of bounded and positive solutions to a large class of elliptic partial differential equations which can be displayed as

$$\Delta u + f(x,u) + g(|x|)x \cdot \nabla u = 0\,, \qquad x \in G_R\,, \tag{1}$$

where $G_R = \{x \in \mathbb{R}^n : |x| > R\}$ for any $R \geq 0$ and $n \geq 2$. We would like to mention the contributions [3], [1], [8] – [11], [13, 14], [18] and their references in this respect.

It has been established, see [8, 9], that it is sufficient for the functions f, g to be Hölder continuous, respectively continuously differentiable in order to analyze the asymptotic behavior of the solutions to (1) by the comparison method [15]. In fact, given $\zeta > 0$, let us assume that there exist a continuous function $A\colon [R,+\infty) \to [0,+\infty)$ and a nondecreasing, continuously differentiable function $W\colon [0,\zeta] \to [0,+\infty)$ such that

$$0 \leq f(x,u) \leq A\big(|x|\big)W(u) \quad \text{for all} \quad x \in G_R,\, u \in [0,\zeta]$$

and $W(u) > 0$ when $u > 0$. Then we are interested in the positive solutions $U = U\big(|x|\big)$ of the elliptic partial differential equation

$$\Delta U + A\big(|x|\big)W(U) = 0\,, \qquad x \in G_R\,,$$

for the role of super-solutions to (1).

M. Ehrnström [13] noticed that, by imposing the restriction

$$x \cdot \nabla U(x) \leq 0\,, \qquad x \in G_R\,,$$

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