There is a connection, known as the ***Fundamental Theorem of Calculus***, between indefinite integral and definite integral which makes the definite integral as a practical tool for science and engineering. The definite integral is also used to solve many interesting problems from various disciplines like economics, finance and probability.

In this Chapter, we shall confine ourselves to the study of indefinite and definite integrals and their elementary properties including some techniques of integration.

## 7.2 Integration as an Inverse Process of Differentiation

Integration is the inverse process of differentiation. Instead of differentiating a function, we are given the derivative of a function and asked to find its primitive, i.e., the original function. Such a process is called *integration* or *anti differentiation*.

Let us consider the following examples:

We know that $$\frac{d}{dx}(\sin x) = \cos x \qquad \text{... (1)}$$

$$\frac{d}{dx}\left(\frac{x^3}{3}\right) = x^2 \qquad \text{... (2)}$$

and $$\frac{d}{dx}(e^x) = e^x \qquad \text{... (3)}$$

We observe that in (1), the function cos $x$ is the derived function of sin $x$. We say that sin $x$ is an anti derivative (or an integral) of cos $x$. Similarly, in (2) and (3), $\frac{x^3}{3}$ and $e^x$ are the anti derivatives (or integrals) of $x^2$ and $e^x$, respectively. Again, we note that for any real number C, treated as constant function, its derivative is zero and hence, we can write (1), (2) and (3) as follows :

$$\frac{d}{dx}(\sin x + \text{C}) = \cos x\,,\ \frac{d}{dx}\left(\frac{x^3}{3} + \text{C}\right) = x^2 \text{ and } \frac{d}{dx}(e^x + \text{C}) = e^x$$

Thus, anti derivatives (or integrals) of the above cited functions are not unique. Actually, there exist infinitely many anti derivatives of each of these functions which can be obtained by choosing C arbitrarily from the set of real numbers. For this reason C is customarily referred to as ***arbitrary constant***. In fact, C is the ***parameter*** by varying which one gets different anti derivatives (or integrals) of the given function.

More generally, if there is a function F such that $\frac{d}{dx}\text{F}(x) = f(x)$, $\forall\, x \in$ I (interval), then for any arbitrary real number C, (also called *constant of integration*)

$$\frac{d}{dx}\left[\text{F}(x) + \text{C}\right] = f(x),\ x \in \text{I}$$