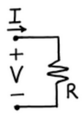
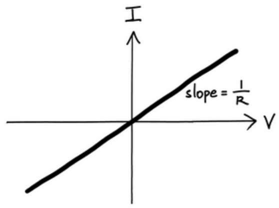
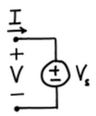
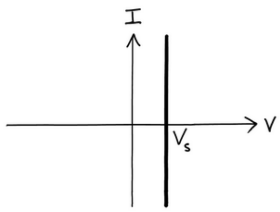
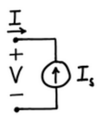
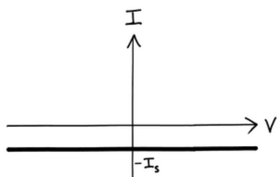
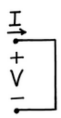
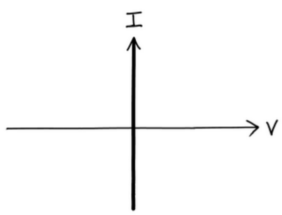
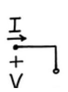
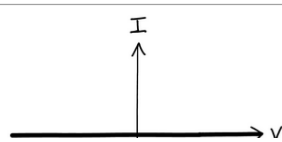
Element I-V Characteristics

A circuit element has a specific current-voltage relationship.

If you vary V_x in the test circuit in Fig. 1, then the measured data point (V, I) varies as well. In this way, you can trace a plot of the current-voltage relationship called the I-V characteristic of the element. When the element is in the load subcircuit, we plot the I-V characteristic with V and I in standard labeling.

Element	Schematic	I-V Characteristic	Explanation
Resistor			A resistor R satisfies Ohm's law $I = V/R$, so its I-V characteristic goes through the origin and has slope $1/R$.
Voltage Source			A voltage source V_s maintains a fixed voltage drop and can allow any current, so its I-V characteristic is a vertical line at $V = V_s$.
Current Source			A current source I_s maintains a fixed current and can allow any voltage drop, so its I-V characteristic is a horizontal line at $I = -I_s$. Note that there is a negative sign because the current arrow labels on I and I_s are in opposite directions.
Short Circuit			A short circuit is a direct connection between two terminals. The short circuit maintains zero voltage drop and can allow any current, so its I-V characteristic is a vertical line at $V = 0$. Notice that a short circuit behaves identically to a zero voltage source with $V_s = 0$ V.
Open Circuit			An open circuit is the absence of a connection between two terminals. The open circuit maintains zero current and can allow any voltage drop, so its I-V characteristic is a horizontal line at $I = 0$.

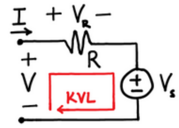
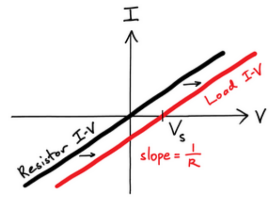
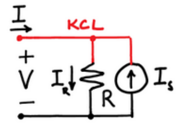
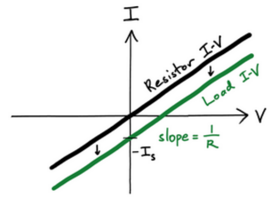
Subcircuit	Schematic	I-V Characteristic	Explanation
Voltage Source and Resistor in Series (Load)			KVL implies that $V = V_R + V_s$, so the load I-V characteristic is the resistor I-V characteristic shifted to the right by V_s (i.e. along the V-axis pointing right by an amount $+V_s$).
Current Source and Resistor in Parallel (Load)			KCL implies that $I = I_R - I_s$, so the load I-V characteristic is the resistor I-V characteristic shifted down by I_s (i.e. along the I-axis pointing up by an amount $-I_s$).

Table 2: I-V characteristics of combinations of circuit elements in load subcircuit.

Source I-V Characteristics

A source I-V characteristic has an equal but opposite slope compared to the equivalent load I-V characteristic.

If a combination of elements is in the source subcircuit instead of the load subcircuit, then V and I are in nonstandard labeling. This difference occurs because the current arrow label of I points out of the top terminal of the source subcircuit but into the top terminal of the load subcircuit. Therefore, the source I-V characteristic is a vertically flipped copy of the equivalent load I-V characteristic.

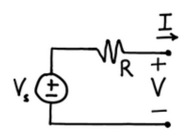
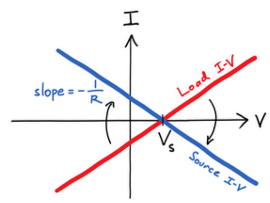
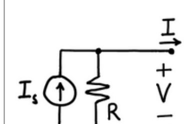
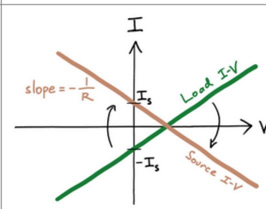
Subcircuit	Schematic	I-V Characteristic	Explanation
Voltage Source and Resistor in Series (Source)			The source I-V characteristic is a line with slope $-1/R$ and V-intercept of V_s. It is flipped vertically compared to the equivalent load I-V characteristic.
Current Source and Resistor in Parallel (Source)			The source I-V characteristic is a line with slope $-1/R$ and I-intercept of I_s. It is flipped vertically compared to the equivalent load I-V characteristic.

Table 3: I-V characteristics of combinations of circuit elements in source subcircuit

Load Line Method

Analyze a circuit graphically by plotting the I-V characteristics of both subcircuits together.

The I-V characteristic of the load subcircuit represents all the (V, I) values at which it can operate. Likewise, the source subcircuit's I-V characteristic represents all of its possible (V, I) values. Therefore, when you plot both I-V characteristics together on the same I-V plane, their point of intersection is the (V, I) value at which both subcircuits operate together. For this reason, the intersection is called the operating point.

We can use this insight to analyze a circuit graphically. Finding the operating point in this way is called the load line method. (Confusingly, the source I-V characteristic is known as the load line because it intersects the load I-V characteristic.) Table 5 below shows two examples for the test circuit in Fig. 1 with different load subcircuits, a resistor and current source, respectively.

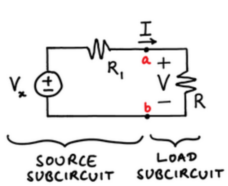
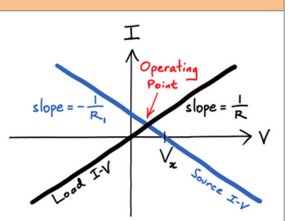
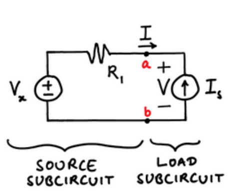
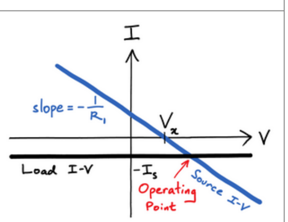
Circuit	Schematic	Load Line Method Plot
Fig. 1 with Resistor as Load Subcircuit		
Fig. 1 with Current Source as Load Subcircuit		

Table 5: Load line method examples.

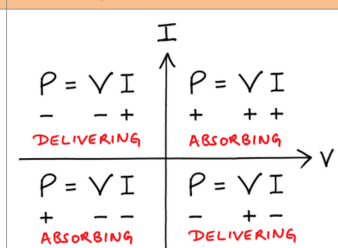
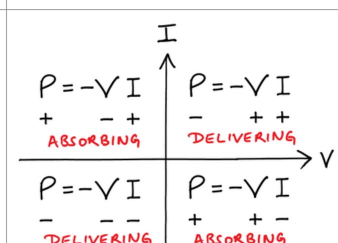
Power On The I-V Plane

Power is absorbed in two quadrants of the I-V plane and delivered in the other two.

We extend the formula for power to the case of nonstandard labeling by reversing its sign to account for switching the relative orientation of the voltage polarity and current arrow labels.

$$P = \begin{cases} VI, & \text{under standard labeling,} \\ -VI, & \text{under nonstandard labeling.} \end{cases} \tag{1}$$

The I-V plane spans the axes on which an I-V characteristic is plotted. In Table 4 below, we use equation (1) to identify the sign of P in each quadrant under both standard and nonstandard labelings. We then determine whether a subcircuit operating in a given quadrant is absorbing or delivering power, depending on whether it is a load or source subcircuit.

Labeling	Subcircuit Type	Sign of P by I-V Plane Quadrant
Standard Labeling	Load Subcircuit	
Nonstandard Labeling	Source Subcircuit	

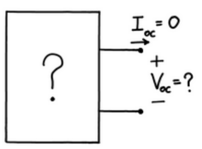
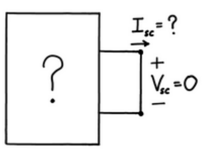
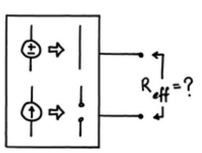
Value	Diagram/Equation	Method
V_T		• Keep the terminals of the unknown subcircuit open (i.e. disconnected) so that the open-circuit current $I_{oc} = 0$. • Find the open-circuit voltage V_{oc} across the terminals. Note that V_{oc} can be measured in lab using a voltmeter across the terminals since an ideal voltmeter's infinite resistance keeps the circuit open. • Set $V_T = V_{oc}$.
I_N		• Short the terminals of the unknown subcircuit so that the short-circuit voltage $V_{sc} = 0$. • Find the short-circuit current I_{sc} between the terminals. Note that I_{sc} can be measured in lab using an ammeter across the terminals since an ideal ammeter's zero resistance shorts the circuit. • Set $I_N = I_{sc}$.
R_{eff}		• Turn off (i.e. zero) all the sources in the unknown subcircuit. Specifically, zero voltage sources behave as short circuits and zero current sources behave as open circuits. Remember these facts by noticing that there is always zero voltage drop across a short circuit and always zero current through an open circuit. • Set R_{eff} equal to the equivalent resistance between the terminals.
V_T, I_N or R_{eff}	$R_{eff} = \frac{V_T}{I_N}$	Given any two of V_T, I_N and R_{eff}, solve $R_{eff} = V_T/I_N$ to find the other one.

Table 1: Methods for finding Thévenin and Norton equivalent subcircuit values.

Known Subcircuits

Use Thévenin and Norton equivalents to simplify known subcircuits.

We have seen that Thévenin and Norton equivalents can represent unknown subcircuits, but they can also be used to simplify known subcircuits. Consider two subcircuits connected together as shown below.

Figure 5

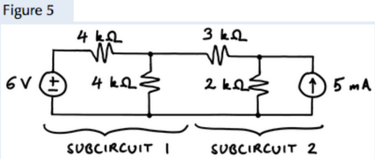


Fig. 5: Circuit consisting of two subcircuits. Either of the known subcircuits can be replaced by a simpler Thévenin or Norton equivalent without changing the behavior of the other known subcircuit.

The table below gives example procedures for determining the Thévenin and Norton equivalents of the two subcircuits in Fig. 5.

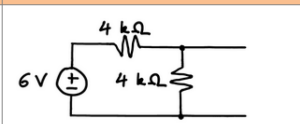
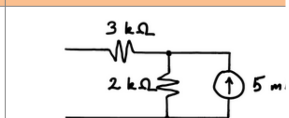
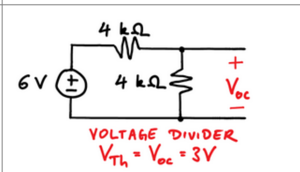
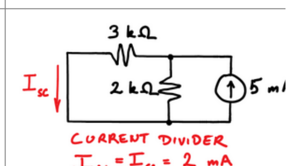
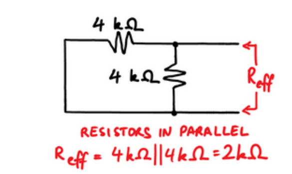
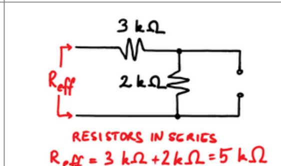
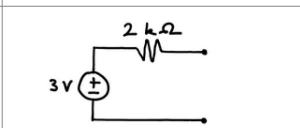
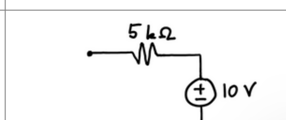
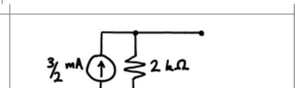
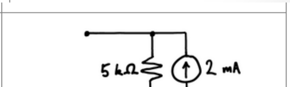
Step	Subcircuit 1	Subcircuit 2
Draw known subcircuit		
Find one of the values V_T, I_N or R_{eff}		
Find another value out of V_T, I_N or R_{eff}		
Find the remaining value V_T, I_N or R_{eff}	$I_N = \frac{V_T}{R_{eff}} = \frac{3\text{ V}}{2\text{ k}\Omega} = \frac{3}{2}$ mA	$V_T = I_N R_{eff} = (2\text{ mA})(5\text{ k}\Omega) = 10$ V
Draw Thévenin equivalent		
Draw Norton equivalent		

Table 3: Example procedures for finding Thévenin and Norton equivalents for known subcircuits.

I-V Characteristics

The Thévenin and Norton theorems can be proved by equating I-V characteristics.

Most of the results in Table 1 can be derived by comparing the I-V characteristic of an unknown subcircuit consisting of sources and resistors with those of the Thévenin and Norton subcircuits.

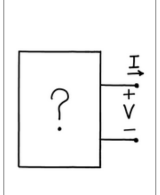
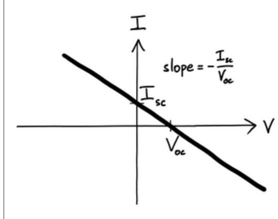
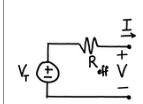
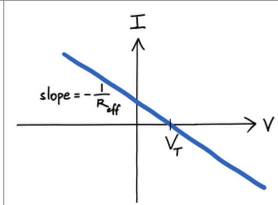
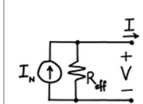
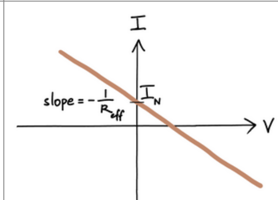
Subcircuit	Schematic	I-V Characteristic	Explanation
Unknown subcircuit			Any subcircuit containing exclusively sources and resistors has a linear I-V characteristic. This line has V-intercept equal to V_{oc} since $I_{oc} = 0$, and it has I-intercept equal to I_{sc} since $V_{sc} = 0$. Therefore, the slope is $-I_{sc}/V_{oc}$.
Thévenin subcircuit			This line is the I-V characteristic of a voltage source and resistor in series.
Norton subcircuit			This line is the I-V characteristic of a current source and resistor in parallel.

Table 2: Comparison of I-V characteristics of unknown, Thévenin and Norton subcircuits.

For the three subcircuits in Table 2 to be equivalent, their I-V characteristics must be identical. Therefore, we equate the intercepts and slopes to obtain three of the results in Table 1.

$$V_T = V_{oc} \quad \text{by equating } V\text{-intercepts} \tag{1}$$

$$I_N = I_{sc} \quad \text{by equating } I\text{-intercepts} \tag{2}$$

$$R_{eff} = \frac{V_{oc}}{I_{sc}} = \frac{V_T}{I_N} \quad \text{by equating slopes} \tag{3}$$

Subcircuits

A circuit element can be tested by connecting it to a subcircuit.

Suppose you want to investigate the devices that you find in your lab kit. You build a simple test circuit by connecting a voltage source V_x, a resistor R_1 and the circuit element you want to test, all in series. (The resistor is there to prevent a short circuit in case the element is another voltage source.) If you measure the voltage drop V across the element and the current I through it, you have a data point (V, I) about the element's behavior.

Figure 1

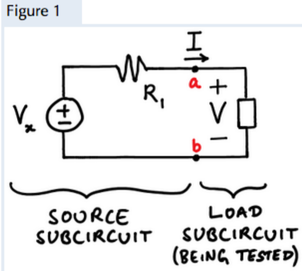


Fig. 1: Test circuit. The voltage source V_x and the resistor R_1 form the fixed part of the test circuit and we say they comprise the source subcircuit. The element being tested is interchangeable and we say it forms the load subcircuit. Typically, the source subcircuit delivers electrical energy to the load subcircuit, but this is not always the case. V is the voltage drop from terminal a to terminal b and I is the current from a to b through the load subcircuit.

Notice that V and I are in standard labeling with respect to the load subcircuit. But they are in nonstandard labeling in relation to the source subcircuit because I flows from terminal b to terminal a through that part of the circuit. The source subcircuit in Fig. 1 is called a Thévenin equivalent subcircuit.