

the input voltage  $V_{in}$ ; the circuit constants  $V_{CC}$ ,  $R_1$  and  $R_2$ ; and the BJT parameters  $\beta$ ,  $V_{BE,on}$  and  $V_{CE,sat}$ . We now show how equations (1) to (7) define three operating modes for the BJT, called OFF, ACTIVE and SATURATED, and derive expressions for  $V_{BE}$ ,  $I_B$ ,  $I_C$  and  $V_{CE}$  in each mode.

Conditions	Mode	Behavior: Derivation and Reasons
$V_{in} < V_{BE,on}$	OFF	$V_{BE} = V_{in}$ from BJT input model (3) $I_B = 0$ from BJT input model (3) $I_C = 0$ from BJT output model (5) $V_{CE} = V_{CC}$ from output loop KVL (2)
$V_{in} > V_{BE,on}$ $V_{CE} > V_{CE,sat}$	ACTIVE	$V_{BE} = V_{BE,on}$ from BJT input model (4) $I_B = \frac{V_{in} - V_{BE,on}}{R_1}$ from input loop KVL (1) $I_C = \beta I_B$ from BJT output model (6) $V_{CE} = V_{CC} - I_C R_2$ from output loop KVL (2)
$V_{in} > V_{BE,on}$ $V_{CE} \not> V_{CE,sat}$	SATURATED	$V_{CE} = V_{CE,sat}$ from BJT output model (7) $I_C = \frac{V_{CC} - V_{CE,sat}}{R_2} \equiv I_{C,sat}$ from output loop KVL (2) $V_{BE} = V_{BE,on}$ from BJT input model (4) $I_B = \frac{V_{in} - V_{BE,on}}{R_1}$ from input loop KVL (1)

Table 1: BJT operating modes.

Note that  $I_{C,sat}$ , which is called the saturation current, is the maximum value of  $I_C$  among all the modes. In contrast,  $V_{CE,sat}$  is the minimum value of  $V_{CE}$ . After some practice, you are expected to recall and evaluate the derivation for each mode by looking at a circuit diagram like Fig. 1 or Fig. 3.

### LOAD LINE METHOD

Analyze a circuit graphically by plotting the I-V characteristics of both subcircuits together.

The I-V characteristic of the load subcircuit represents all the  $(V, I)$  values at which it can operate. Likewise, the source subcircuit's I-V characteristic represents all of its possible  $(V, I)$  values. Therefore, when you plot both I-V characteristics together on the same I-V plane, their point of intersection is the  $(V, I)$  value at which both subcircuits operate together. For this reason, the intersection is called the operating point.

We can use this insight to analyze a circuit graphically. Finding the operating point in this way is called the load line method. (Confusingly, the source I-V characteristic is known as the load line because it intersects the load I-V characteristic.) Table 5 below shows two examples for the test circuit in Fig. 1 with different load subcircuits, a resistor and current source, respectively.

Circuit	Schematic	Load Line Method Plot
Fig. 1 with Resistor as Load Subcircuit		
Fig. 1 with Current Source as Load Subcircuit		

Table 5: Load line method examples.

Notice that, when the load subcircuit is a resistor, the operating point is in the upper-right quadrant, so the source subcircuit is delivering power ( $P < 0$ ) and the load subcircuit is absorbing power ( $P > 0$ ). But when the load subcircuit is a current source, the operating point is in the lower-right quadrant, so the source subcircuit is absorbing power ( $P > 0$ ) and the load subcircuit is delivering power ( $P < 0$ ).

Figure 7

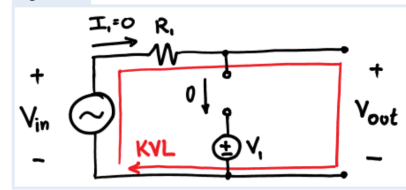


Figure 7: Diode circuit. When  $V_{in} < V_1 + V_{on}$  in Fig. 1, the diode is OFF and is replaced by an open circuit in this figure. Now, we apply KVL to obtain:  $V_{in} = I_1 R_1 + V_{out}$ . Note that  $I_1 = 0$  since  $R_1$  is not part of a complete circuit. Therefore,  $V_{out} = V_{in}$ .

Figure 8

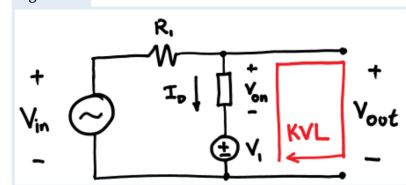


Figure 8: Diode clipper with the diode ON. When  $V_{in} > V_1 + V_{on}$  in Fig. 1, the diode is ON and is replaced by an element with voltage drop  $V_{on}$  in this figure. Now, we apply KVL to obtain:  $V_1 + V_{on} = V_{out}$ . Therefore,  $V_{out} = V_1 + V_{on}$ .

Combining the results from Fig. 7 and Fig. 8 gives

$$V_{out} = \begin{cases} V_{in}, & \text{if } V_{in} < V_1 + V_{on}, \\ V_1 + V_{on}, & \text{if } V_{in} > V_1 + V_{on}. \end{cases} \quad (2)$$

$$G = \frac{V_{CE,sat} - V_{CC}}{V_{in} - V_{BE,on}} \quad (1)$$

The constant output voltages  $V_{CC}$  in OFF mode and  $V_{CE,sat}$  in SATURATED mode are explained by the behavior of  $V_{CE}$  in those modes. The transition input voltage  $V_{BE,on}$  between OFF and ACTIVE is the boundary between the conditions on  $V_{in}$  in those modes.

At the transition input voltage between ACTIVE and SATURATED, the BJT obeys the behavior of both of these modes. In particular, we follow these steps to derive  $V_{in}^*$ :

$$I_C = I_{C,sat} = \frac{V_{CC} - V_{CE,sat}}{R_2} \quad \text{SATURATED mode } I_C \text{ formula}$$

$$I_B = \frac{I_C}{\beta} \quad \text{from ACTIVE mode } I_C \text{ formula}$$

$$V_{in}^* = V_{in} = V_{BE,on} + I_B R_1 \quad \text{from ACTIVE mode } I_B \text{ formula}$$

The overall expression is:

$$V_{in}^* = V_{BE,on} + \frac{R_1}{\beta R_2} (V_{CC} - V_{CE,sat}), \quad (2)$$

where  $V_{CC}$ ,  $R_1$  and  $R_2$  are known circuit constants and  $\beta$ ,  $V_{BE,on}$  and  $V_{CE,sat}$  are known BJT parameters. Substituting equation (2) into equation (1), gives another formula for the gain of the common emitter amplifier:

$$G = -\frac{\beta R_2}{R_1} \quad (3)$$

Figure 1

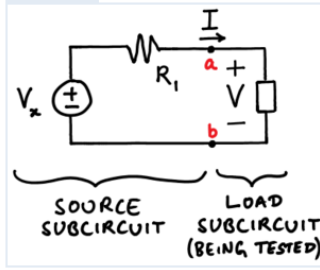


Fig. 1: Test circuit. The voltage source  $V_x$  and the resistor  $R_1$  form the fixed part of the test circuit and we say they comprise the source subcircuit. The element being tested is interchangeable and we say it forms the load subcircuit. Typically, the source subcircuit delivers electrical energy to the load subcircuit, but this is not always the case.  $V$  is the voltage drop from terminal  $a$  to terminal  $b$  and  $I$  is the current from  $a$  to  $b$  through the load subcircuit.

### SOURCE I-V CHARACTERISTICS

A source I-V characteristic has an equal but opposite slope compared to the equivalent load I-V characteristic.

If a combination of elements is in the source subcircuit instead of the load subcircuit, then  $V$  and  $I$  are in nonstandard labeling. This difference occurs because the current arrow label of  $I$  points out of the top terminal of the source subcircuit but into the top terminal of the load subcircuit. Therefore, the source I-V characteristic is a vertically flipped copy of the equivalent load I-V characteristic.

Subcircuit	Schematic	I-V Characteristic	Explanation
Voltage Source and Resistor in Series (Source)			The source I-V characteristic is a line with slope $-1/R$ and $V$ -intercept of $V_s$ . It is flipped vertically compared to the equivalent load I-V characteristic.
Current Source and Resistor in Parallel (Source)			The source I-V characteristic is a line with slope $-1/R$ and $I$ -intercept of $I_s$ . It is flipped vertically compared to the equivalent load I-V characteristic.

Labeling	Subcircuit Type	Sign of P by I-V Plane Quadrant
Standard Labeling	Load Subcircuit	$P = \begin{cases} V I > 0 & \text{DELIVERING} \\ V I < 0 & \text{ABSORBING} \end{cases}$
Nonstandard Labeling	Source Subcircuit	$P = \begin{cases} V I < 0 & \text{DELIVERING} \\ V I > 0 & \text{ABSORBING} \end{cases}$

its in source subcircuit.

Table 4: Signs of P on the I-V plane.

### POWER ON THE I-V PLANE

Power is absorbed in two quadrants of the I-V plane and delivered in the other two.

$$P = \begin{cases} V I > 0 & \text{under standard labeling} \\ V I < 0 & \text{under nonstandard labeling} \end{cases} \quad (1)$$

We extend the formula for power to the case of nonstandard labeling by reversing its sign to account for switching the relative direction of the voltage polarity and current arrow labels.

The I-V plane spans the axes on which an I-V characteristic is plotted. In Table 4 below, we use standard labeling. We then determine whether a subcircuit operating in a given quadrant is absorbing or delivering power, depending on whether it is a load or source subcircuit.

Figure 7

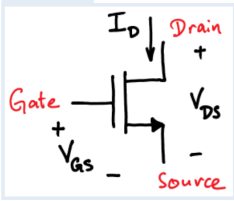


Fig. 7: The circuit symbol for an NMOS transistor. The input voltage  $V_{GS}$  drops from gate to source. (Note that  $V_{GS} = V_G$  in Fig. 3 and Fig. 4 since the body and source terminals are grounded in those diagrams.) On the output side,  $I_D$  flows into the drain and  $V_{DS}$  drops from drain to source. Notice that the NMOS transistor symbol incorporates the symbol for a capacitor to signify the gate's capacitive properties.

If the input  $V_{GS}$  is fixed, an output I-V characteristic can be traced relating  $I_D$  to  $V_{DS}$ . Varying  $V_{GS}$  therefore produces a family of curves. We model the operation of an NMOS transistor by dividing the I-V space into 3 modes, OFF, ACTIVE and OHMIC, and fitting linear equations to each I-V curve in each mode.

Figure 9

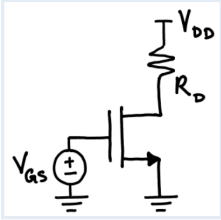


Fig. 9: NMOS transistor connected a Thévenin subcircuit. You can apply the load line method to analyze an NMOS transistor connected to a Thévenin subcircuit, consisting of  $V_{DD}$  and  $R_D$ . Intersect the NMOS output I-V characteristic in Fig. 8 with the Thévenin subcircuit's I-V characteristic to find the operating point ( $V_{DS}$ ,  $I_D$ ) and the mode which it represents.

Figure 6

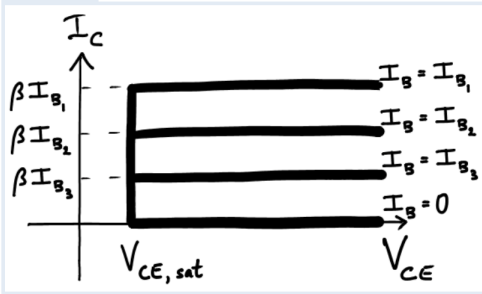


Fig. 6: Model for output I-V characteristic. In this model, all the I-V curves start from  $I_C = 0$  and  $V_{CE} = V_{CE,sat}$ , called the saturation voltage. If  $I_B = 0$ , the curve is  $I_C = 0$  for all  $V_{CE} \geq V_{CE,sat}$ . If  $I_B > 0$ , then  $I_C = \beta I_B$  for all  $V_{CE} > V_{CE,sat}$ . The typical value of  $V_{CE,sat}$  for a silicon BJT is 0.2 V.

From this model, we draw the following conclusions about the output loop of a common emitter circuit:

If  $I_B = 0$ , then  $I_C = 0$ .

If  $I_B > 0$  and  $V_{CE} > V_{CE,sat}$ , then  $I_C = \beta I_B$ .

Otherwise,  $V_{CE} = V_{CE,sat}$ .

(5)

(6)

(7)

Element	Schematic	I-V Characteristic	Explanation
Resistor			A resistor $R$ satisfies Ohm's law $I = V/R$ , so its I-V characteristic goes through the origin and has slope $1/R$ .
Voltage Source			A voltage source maintains a fix drop and can't allow current, so its characteristic is a vertical line at $V = V_s$ .
Current Source			A current source maintains a fix and can allow voltage drop, its characteristic is a horizontal line. Note that the negative sign in current arrow and $I_s$ are in c directions.
Short Circuit			A short circuit connection be terminals. The circuit maintain voltage drop a allow any current, so its characteristic is a vertical line at $V = 0$ . Notice that a short circuit behaves identically to a current source with $I_s = 0$ A.
Open Circuit			An open circuit absence of a c between two t. The open circuit maintains zero and can allow voltage drop, its characteristic is a horizontal line. Notice that an circuit behave identically to a current source with $I_s = 0$ A.

### LOAD I-V CHARACTERISTICS

A load subcircuit consisting of sources and resistors has a linear I-V characteristic.

The load subcircuit may consist of more than one element in series and/or parallel combinations. The load I-V characteristic can be obtained from the elements' I-V characteristics using KVL and KCL. In fact, if the load subcircuit consists of sources and resistors only, the load I-V characteristic must be linear, as illustrated by the examples below.

Subcircuit	Schematic	I-V Characteristic	Explanation
Voltage Source and Resistor in Series (Load)			KVL implies that $V = VR + V_s$ , so the load I-V characteristic is the resistor I-V characteristic shifted to the right by $V_s$ (i.e. along the V-axis pointing right by an amount $+V_s$ ).
Current Source and Resistor in Parallel (Load)			KCL implies that $I = I_s - IR$ , so the load I-V characteristic is the resistor I-V characteristic shifted down by $I_s$ (i.e. along the I-axis pointing up by an amount $-I_s$ ).

Table 2: I-V characteristics of combinations of circuit elements in load subcircuit.

Subcircuit	Schematic	I-V Characteristic	Explanation
Unknown subcircuit			Any subcircuit containing exclusively sources and resistors has a linear I-V characteristic. This line has V-intercept equal to $V_{oc}$ since $I_{sc} = 0$ , and it has I-intercept equal to $I_{sc}$ since $V_{sc} = 0$ . Therefore, the slope is $-I_{sc}/V_{oc}$ .
Thévenin subcircuit			This line is the I-V characteristic of a voltage source and resistor in series.
Norton subcircuit			This line is the I-V characteristic of a current source and resistor in parallel.

Figure 5

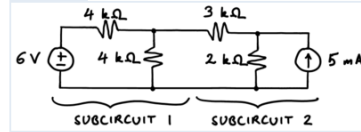


Fig. 5: Circuit consisting of two subcircuits. Either of the known subcircuits can be replaced by a simpler Thévenin or Norton equivalent without changing the behavior of the other known subcircuit.

The table below gives example procedures for determining the Thévenin and Norton equivalents of the two subcircuits in Fig. 5.

Step	Subcircuit 1	Subcircuit 2
Draw known subcircuit		
Find one of the values $V_T$ , $I_N$ or $R_{eff}$		
Find another value out of $V_T$ , $I_N$ or $R_{eff}$		
Find the remaining value $V_T$ , $I_N$ or $R_{eff}$	$I_N = \frac{V_T}{R_{eff}} = \frac{3V}{2k\Omega} = \frac{3}{2} \text{ mA}$	$V_T = I_N R_{eff} = (2 \text{ mA})(5 \text{ k}\Omega) = 10 \text{ V}$
Draw Thévenin equivalent		
Draw Norton equivalent		

Table 3: Example procedures for finding Thévenin and Norton equivalents for known subcircuits.