1) $P(w_h) = 0.85$
$P(g_A) = 0.9$
$P(g_A|w_h) = 0.99$

(a) $P(w_h/g_A) = \dfrac{P(g_A|w_h) \times P(w_h)}{P(g_A)}$

$\therefore \boxed{P(w_h/g_A) = \dfrac{0.99 \times 0.85}{0.9} = 0.935}$

Here, we apply Bayes Rule, we know the probability of a student who got an A given that he worked hard. We also know the probabilities of a student working hard and student getting an A.

So, By formula $P(A/B) = \dfrac{P(B/A) \times P(A)}{P(B)}$

we calculated $P(w_h/g_A)$.

(b) $P(g_A/\neg w_h) = \dfrac{P(\neg w_h/g_A) \times P(g_A)}{P(\neg w_h)}$.

as $P(w_h) + P(\neg w_h) = 1$
$P(\neg w_h) = 1 - P(w_h) = 1 - 0.85 = 0.15$

as $P(w_h/g_A) + P(\neg w_h/g_A) = 1$ .. as $P(A/B) + P(\neg A/B) = 1$.

$\therefore P(\neg w_h/g_A) = 1 - P(w_h/g_A)$
$= 1 - 0.935$
$= 0.065$

$\boxed{P(g_A/\neg w_h) = \dfrac{0.065 \times \cancel{0.9}6}{\cancel{0.15}} = 0.39}$

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As it is inevitable that if event B has occured, it means that the prob of A occured given B occured and A did not occur given B occured must be equal to 1.

Ans 3]

$P(Happy = No)$
$= \frac{5}{8} = 0.625$

$P(Happy = Yes)$
$= \frac{3}{8} = 0.375$

(a) $P(W = Good, S = Pass, N = Out) = ?$

Ans] So here we calculate
$P(W = Good, S = Pass, N = Out / Happy = Yes)$

and $P(W = Good, S = Pass, N = Out / Happy = No)$

and whichever is the highest value, that class value is the answer.

Now $P(W = Good, S = Pass, N = Out | Happy = Yes)$

$= P(W = Good / Happy = Yes) \times P(S = Pass | H = Yes) \times P(N = Out / H = Yes) \times P(H = Yes)$

--- as all attributes are independent of each other in Naive Bayes → ①

$P(W = Good | H = Yes) = 1/3$
$P(S = Pass / H = Yes) = 3/3 = 1$
$P(N = Out / H = Yes) = 1/3$.

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$\therefore P(W = Good, S = Pass, N = Out / Happy = Yes)$

$= 1/3 \times 1/3 \times 1 \times 3/8$

$= 1/9 = 0.11 \times 0.375 = 0.04125$

Now, $P(W = Good, S = Pass, N = Out / H = No)$

$= P(W = Good / H = No) \times P(S = Pass / H = No) \times P(H = No) \times P(N = Out / H = No)$ — same as 1

$P(W = Good / H = No) = \frac{3}{5}$

$P(S = Pass / H = No) = 1/5$

$P(N = Out / H = No) = 3/5$

$\therefore P(W = Good, S = Pass, N = Out / H = No)$

$= \frac{3}{5} \times \frac{1}{5} \times \frac{3}{5} \times \frac{5}{8} = \frac{9 \times 5}{125 \times 8} \cancel{0.072}$

$= 0.045$

$\therefore P(W = G, S = P, N = O / H = Yes) < P(W = G, S = P, N = O / H = No)$

$\therefore$ We predict that Jim is not happy for this ~~distance~~ instance.

(b) We calculate
$P(W = Good, S = Pass, N = Out \mid Happy = Yes)$

and $P(W = Good, S = Pass, N = Out \mid Happy = No)$

By Bayes classifier:

$P(W = Good, S = Pass, N = Out \mid Happy = Yes)$

$\times P(Happy = Yes)$

$= P(Happy = Yes \mid W = Good, S = Pass, N = Out)$

Here $W = Good, S = Pass, N = Out$
is only in 1 instance

$\therefore P(W = Good, S = Pass, N = Out \mid Happy = Yes) \times P(H = Yes)$

$= \frac{1}{3} \times P(Happy = Yes) = \frac{3}{8} \times \frac{1}{3} = \frac{1}{8}$

$P(W = Good, S = Pass, N = Out \mid Happy = No) \times P(H = No)$

$= 0 \times \frac{5}{8} = 0$

$\therefore P(W = Good, S = Pass, N = Out \mid H = Yes)$

$> P(W = Good, S = Pass, N = Out \mid H = No)$

$\therefore$ Tim is happy is our prediction in Bayes Classifier.

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ANS 4]

(a) $P(\text{Prog in C++} / \text{Programmer}) = 0.5 = \dfrac{P(\text{Programmer}/\text{Prog in C++}) \times P(\text{Prog in C++})}{P(\text{Programmer})}$

$P(\text{Prog in Java} / \text{Programmer}) = 0.4$

$P(\text{Work in Microsoft} \mid \text{Programmer}) = 0.01$

$P(\text{Prog in C++} \mid \cancel{\text{Pro}}\ \text{Work in Microsoft}) = 0.99$

$P(\text{Prog in Java} \mid \text{Work in Microsoft}) = 0.98$

$P(\text{Works in Microsoft} / \text{Prog in C++}, \text{Prog in Java})$

$= P(C / A, B) = P(C/\cancel{A}) \times P(\cancel{C}/B) \times P(A) \times P(B)$

$= \dfrac{P(A, B / C) \times P(C)}{P(A, B)}$

$= \dfrac{P(A/C) \times P(B/C) \times P(C)}{P(A) \times P(B)}$

$= \dfrac{(P(\text{Prog in C++} / \text{Work in Microsoft}) \times P(\text{Prog in Java} / \text{Microsoft}) \times P(\text{Work in Microsoft}))}{P(\text{Prog in C++}) \times P(\text{Prog in Java})}$

$= \cancel{\dfrac{0.5 \times 0.4 \times 0.1 \times 0.1 \times 0.1}{0.5 \times 0.4}}$

$= \dfrac{0.99 \times 0.98 \times 0.d}{0.5 \times 0.4}$

$= 0.4851 \times 0.1 = 0.0485$

ANS] 48.51 %

So, this means that Probability that a programmer who knows both C++ and Java is a Microsoft Employee is 0.485 ie (4.85%) of all the Programmers

Ans (a) P(taxi = blue) = 0.5
P(taxi = green) = 0.5

P(taxi appears what it is) = 0.75

∴ P(taxi appears blue and is actually blue) = P(taxi = blue) × P(taxi appears what it is)
= 0.5 × 0.75 = 0.375 — ①

∴ P(taxi appears green and is actually green) = 0.5 × 0.75
= 0.375 — ②

∴ P(taxi appears blue) × P(taxi = green)
= 0.25 × 0.5
= 0.125 — ③

∴ P(taxi appears green and it is actually blue
= 0.25 × 0.5 — ④
= 0.125.

So, here we cannot predict the most likely color. (As the probabilities of ① and ② are same, and that of ③ and ④ are same.

(b) $P(\text{taxi} = \text{green}) = 0.9$
$P(\text{taxi} = \text{blue}) = 0.1$

P(taxi appears green when it is green)
$= 0.9 \times 0.75$
$= 0.675$ — ①

P(taxi appears blue when it is blue)
$= 0.1 \times 0.75$
$= 0.075$ — ②

P(taxi appears blue when it is green)
$= 0.25 \times 0.9 = 0.225$ — ③

P(taxi appears green when it is blue)
$= 0.1 \times 0.25 = 0.025$ — ④

Here we can predict that most likely color of the taxi is green. (from ①, ②, ③, ④).

Ans 5] In General, Bayes Rule is

$$P(Y=1 \mid X_i) = \frac{P(Y=1) \cdot P(X_i \mid Y=1)}{P(Y=1) \cdot P(X_i \mid Y=1) + P(Y=0) \cdot P(X_i \mid Y=0)}$$

$$= \frac{P(Y=1) \cdot P(X_i=0 \mid Y=1)}{P(Y=1) \cdot P(X_i=0 \mid Y=1) + P(Y=0) \cdot P(X_i=0 \mid Y=0)}$$

$$+ \frac{P(Y=1) \cdot P(X_i=1 \mid Y=1)}{P(Y=1) \cdot P(X_i=1 \mid Y=1) + P(Y=0) \cdot P(X_i=1 / Y=0)}$$

$$= \frac{1}{1 + \frac{P(Y=0) \cdot P(X_i=0 \mid Y=0)}{P(Y=1) \cdot P(X_i=0 \mid Y=1)}} + \frac{1}{1 + \frac{P(Y=0) \cdot P(X_i=1 / Y=0)}{P(Y=1) \cdot P(X_i=1 / Y=1)}}$$

$$= \frac{1}{1 + \exp\left(\ln\left(\frac{P(Y=0) \cdot P(X_i=0 \mid Y=0)}{P(Y=1) P(X_i=0 \mid Y=1)}\right)\right)} + \frac{1}{1 + \exp\left(\ln\left(\frac{P(Y=0) \cdot P(X_i=1 \mid Y=0)}{P(Y=1) P(X_i=1 \mid Y=1)}\right)\right)}$$

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$$= \frac{1}{1 + \exp\left(\ln \frac{P(Y=0)}{P(Y=1)} + \sum_i \ln \frac{P(X_i=0 \mid Y=0)}{P(X_i=0 \mid Y=1)}\right)}$$

$$+ \frac{1}{1 + \exp\left(\ln \left(\frac{P(Y=0)}{P(Y=1)}\right) + \sum_i \ln \frac{P(X_i=1 \mid Y=0)}{P(X_i=1 \mid Y=1)}\right)}$$

$$= \frac{1}{1 + \exp\left(\ln \left(\frac{1-\pi}{\pi}\right) + \sum_i \ln \left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right)}$$

$$+ \frac{1}{1 + \exp\left(\ln \left(\frac{1-\pi}{\pi}\right) + \sum_i \ln \left(\frac{\theta_{i0}}{\theta_{i1}}\right)\right)}$$

Now, the final step expresses $P(Y=0)$ and $P(Y=1)$ in terms of the binomial parameter $\pi$.

$$= \frac{1}{1 + \exp\left(\ln\left(\frac{1-\pi}{\pi}\right) + \sum_i \ln \frac{\theta_{i0}^{x_i}(1-\theta_{i0})^{(1-x_i)}}{\theta_{i1}^{x_i}(1-\theta_{i1})^{(1-x_i)}}\right)}$$

$$\cancel{= \frac{1}{1 + \exp\left[\ln\left(\frac{1-\pi}{\pi}\right) + \sum_i \ln\left(\frac{(1-\theta_{i0})}{(1-\theta_{i1})}\right)\right]} + \frac{1}{1 + \exp\left[\ln\left(\frac{1-\pi}{\pi}\right) + \sum_i \ln\left(\frac{\theta_{i0}}{\theta_{i1}}\right)\right]}}$$

$$\cancel{= 2 + \exp\left[\ln(1-\right.}$$

$$= \frac{1}{1+\exp\left(\ln\left(\frac{1-\pi}{\pi}\right) + \sum_{i=1} \ln \frac{\theta_{i0}^{x_i}}{\theta_{i1}^{x_i}} + \sum \ln \frac{(1-\theta_{i0})^{(1-x_i)}}{(1-\theta_{i1})^{(1-x_i)}}\right)}$$

$$= \frac{1}{1+\exp\left(\ln\left(\frac{1-\pi}{\pi}\right) + \sum_{i=1}^{n}\left(x_i \ln \frac{\theta_{i0}}{\theta_{i1}} + (1-x_i)\ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right)\right)}$$

$$= \frac{1}{1+\exp\left(\ln\left(\frac{1-\pi}{\pi}\right) + \sum_{i=1}^{n} x_i\left[\ln \frac{\theta_{i0}}{\theta_{i1}} + \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right] + \sum_{i=1}^{n} \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right)}$$

$$= \therefore \quad w_0 = \ln\left(\frac{1-\pi}{\pi}\right) + \sum_{i=1}^{n} \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)$$

$$w_i = \ln \frac{\theta_{i0}}{\theta_{i1}} + \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)$$

$$= \frac{1}{1+\exp\left(w_0 + \sum_{i=1}^{n} w_i x_i\right)}$$

Also $P(Y=0|X) = 1 - P(Y=1|X)$

$$= 1 - \frac{1}{1+\exp\left(w_0 + \sum_{i=1}^{n} w_i x_i\right)}$$

$$= \frac{\exp\left(w_0 + \sum_{i=1}^{n} w_i x_i\right)}{1+\exp\left(w_0 + \sum_{i=1}^{n} w_i x_i\right)}$$

— X —

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