Assignment - 4

1). Trivial Functional Dependency

→ A functional dependency is trivial if

$X \longrightarrow Y$ and $X \cap Y \neq \phi$ and $Y \subseteq X$

Eg: Fid, Fname → Fname

→ Non trivial Functional dependency

* A functional dependency is non trivial if

$X \longrightarrow Y$ and $X \cap Y = \phi$

i.e X and Y have no attribute in common.

Eg: Area code → Area Name

→ Transitive functional dependency

* A functional dependency is transitive

if $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$

Eg. if Stud_Id → Ins_Id

and

Ins_Id → Course_Id

Then Stud_Id → Course_Id.

→ 1NF

* If a table does not have multivalued attributes, then it is said to be in 1NF.

Eg. Table R can be decomposed into R_1 and R_2 to change into 1NF.

R

Roll	Name	Mob
1	Satyam	94706/1687
2	Sanjay	76784/34687
3	Lakshya	12345

Roll	Name
1	Satyam
2	Sanjay
3	Lakshya

1NF

Roll	Mob
1	94706
1	1687
2	76784
2	36887
3	12345

1NF

→ 2NF

* If all the non prime attribute are fully functional dependent on the Candidate Key, then the table is in 2NF (given the table follows 1NF as well).

R

cusId	state	City
1	A	B
2	C	D
3	E	B
4	G	H

R_1

cusId	State
1	A
2	B
3	C
4	D

R_2

State	City
A	B
C	D
G	H

→ 3NF

If a table is in 2NF and also none of the prime attributes refer to other non prime attributes, then the table is in 3NF.

Eg. $R \rightarrow R_1$, $R \rightarrow R_2$

R	CusId	State	City
	1	A	B
	2	C	D
	3	A	B
	4	E	F

R_1	CustId	State
	1	A
	2	C
	3	A
	4	E

R_2	State	City
	A	B
	C	D
	E	F

→ BCNF

If a table is in 3NF and it is in the functional dependency, all the entries on the LHS are superkey, then the table is said to be in BCNF.

Eg: $CK = \{SSN, aadhar\}$

$FD = \{SSN \rightarrow Emp\ Name, aadhar \rightarrow age,$
$SSN \rightarrow aadhaar, aadhar \rightarrow SSN\}$

Q2) $R = \{P, Q, R, S, T, U, V, W, X, Y, Z\}$

$FD = \{ \{P,R\} \rightarrow \{Q\}$

$\{P\} \rightarrow \{S,T\}$

$\{R\} \rightarrow \{U\}$

$\{U\} \rightarrow \{V,W\}$

$\{S\} \rightarrow \{X,Y\}$

$\{U\} \rightarrow \{Z\} \}$

$P \not\rightarrow R \not\rightarrow T \not\rightarrow U \not\rightarrow W \not\rightarrow Y \not\rightarrow Z^{+} = PQRSTUVWXYZ$

Properties used ① $P,R \rightarrow Q$

$P \rightarrow ST$

$R \rightarrow U$

$R \rightarrow U \rightarrow V,W \Rightarrow R \rightarrow V,W$

$P \rightarrow ST \Rightarrow P \rightarrow S$ and $R \rightarrow T$

$P \rightarrow S \rightarrow X,Y \Rightarrow P \rightarrow X,Y$

$R \rightarrow U \rightarrow Z \Rightarrow R \rightarrow Z$

Super key :- That can infer all the attributes of the relation.

Super key Found : $PR^{+} = PQRSTUVWXYZ$

Check wether this super key can be a candidate key.

⇒ Cond^{n} = No proper subset of PR i.e {P}, {R} should be a super key.

$P^+ = \{P\ S\ T\ X\ Y\}$

$R^+ = \{R\ U\ V\ W\ Z\}$

So neither of P and R are i.e proper subset of PR are Super key so PR is a CK:

$CK = \{PR\}$

⇒ Prime Attr. = $\{PR\}$

⇒ Non-Prime Attr. = $\{Q\ S\ T\ U\ V\ W\ X\ Y\ Z\}$

→ Since none of the Prime Attributes are on RHS of FD we have PR as the only CK as of now.

*Checking 2NF condition : i.e Partial dependency

If proper subset of CK i.e $\{P\}$ and $\{S\}$ derives Non Prime attribute then it is called Partial dependency (P.D.).

$P.D. = \{P \rightarrow ST,\ R \rightarrow U\}$

$P^+ = PSTXY \qquad R^+ = RUVWZ$

$R_1 = \{PSTXY\} \qquad R_2 = \{RUVWZ\}$

Left attributes $R_3 = \{Q\}$ —Adding common attributes→ $\{Q, P, R\}$ Preferably PK.

So now we have relation which is in 2NF.

$R_1 = \{PSTXY\}$ $R_2 = \{RUVWZ\}$

$R_3 = \{QPR\}$

Now we need to understand the normalization state of the table so we first write down Functional dependencies of the tables above. To find FD, we will use prev. table FD.

$FD_{R_1} = \{ P \rightarrow STXY, S \rightarrow XY \}$

Since P derives all the attributes, no need to check for PS, PT, PX, PY — other combination as it will be a repetition. Similarly for PS, ST, SX, SY etc as it will be repetition and T, X, Y are not deriving anything.

Similarly for other tables.

$R_2 = \{RUVWZ\}$

$FD = \{ R \rightarrow UVWZ, U \rightarrow VWZ \}$
(composition)

For same reason as above we won't find any other dependency.

In $R_3 = \{QPR\}$

$FD = \{PR \rightarrow Q\}$, $CK = \{PR\}$

In this we have $P^+ = PSTXY$ (STXY not part of R_3)

$R^+ = RUVWZ$ (UVWZ " " ")

$\therefore PR^+ = PQRSTUVWXYZ$

(except PQR others are not part of R_3)

$QP^+ = QPSTXY$ (only Reflexive FD, so not useful)

$QR^+ = QRUVWZ$ (only Reflexive FD, others are not part of R_3 so not usefull)

Let's find out CK for R_1.

$R_1 = \{PSTXY\}$ $FD = \{P \rightarrow PSTXY, S \rightarrow XY\}$

$\Rightarrow P^+ = PSTXY$

$\Rightarrow P^+ = PSTXY \rightarrow P.A = \{P\}$, $NPA = \{STXY\}$

Since none of PA is on RHS of FD we can say we have only P as CK. $CK = \{P\}$

In 3NF we can have

{ L.H.S of FD can be C.K. or S.K.
OR
~~LHS or~~ RHS is a Prime Attribute }

But in R_1 we have NPA → NPA which violates the condⁿ of 3NF. ($S \rightarrow XY$)

So Now Since $S \rightarrow XY$ is causing rule violation we will ~~use~~ find S^+.

$S^+ = SXY$ (we will use F.D of Relation R_1 and not the original relation)

$R_{11} = \{SXY\}$ $\quad R_{12} = \{PT\} \xrightarrow[\text{Common}]{\text{Adding}} \{PTS\}$

Now left find out FD for R_{11} and R_{12}.

R_{11}:

$S^+ = SXY$

$X^+ = X$ {Reflexive}

$Y^+ = Y$ {Reflexive}

SX^+ = Repetitive

SY^+ = Repetitive

$XY^+ = XY$ {Reflexive}

$FD = \{S \rightarrow XY\}$

$CK = \{S\}$

R_{12}:

$P^+ = PSTXY$ ✓

$T^+ = T$ (Only Reflexive)

$S^+ = S$ (only Reflexive)

$PT^+ = PTSXY$ (Repetitive)

$TS^+ = TSXY$ (X, Y not in R_{12} so only Reflexive)

$PS^+ = PSTXY$ (Repetitive)

$FD = \{P \rightarrow TS\}$

$CK = \{P\}$

R_{11} and R_{12} both have CK on LHS of FD, so we can say both follow BCNF criteria.

Lets check R_2 for CK.

$R_2 = \{R\ U\ V\ W\ Z\}$

$F.D. = \{R \to UVWZ,\ U \to VWZ\}$

$RUVWZ^+ = RUVWZ$

$R^+ = RUVWZ$

Since none of PA is on RHS of FD we can say that we have on R as CK. $CK = \{R\}$

P.A = {R}

NPA = {U V W Z}

Since $U \to VWZ$ we have NPA → NPA which is voilating the relation of 3NF.

So we find U^+. ~~$RUVWZ$~~ $VUWZ$

$R_{21} = UVWZ$ $\quad R_{22} = \{R\}$ Adding common $\{RU\}$

Lets find FD for R_{21} and R_{22}.

R_{21}

$U^+ = UVWZ$

$V^+ = V$
$W^+ = W$
$Z^+ = Z$
} Reflexive

Others are just repetitive.

$F.D = \{U \rightarrow VWZ\}$

$C.K = \{U\}$

R_{22} :

$R^+ = RUVWZ$ (V, W, Z N.P.)

$U^+ = UVWZ$ (Reflexive and not present in R_{22})

$F.D = \{R \rightarrow U\}$

$CK = \{R\}$

Both R_{21} and R_{22} have C.K on LHS of FD so we can say both follow BCNF criteria.

Finally, if we check for R_3 we can say R_3 already follow BCNF rules.

Q2) $R = \{P, Q, R, S, T, U, V, W, X, Y, Z\}$

$FD = \{ \{P, R\} \rightarrow \{Q\}, \{P\} \rightarrow \{S, T\}, \{R\} \rightarrow \{U\}, \{U\} \rightarrow \{V, W\}, \{S\} \rightarrow \{X, Y\}, \{U\} \rightarrow \{Z\} \}$

Q3) Employees (Aadhar NO., Union ID)

Technician (Name, Salary, address, Phone NO., Aadhar NO.)

Test (Test NO., Test Name, Max. Score)

Bus (Reg. NO.)

Union (Union Name, Union ID)

Model (Model Name, Model NO., Capacity)

Testing Event (Reg NO, Test NO, Date, hours, Score)

ExperIn (Aadhar NO, Model NO.)

The complete information can be broken down into seven relations

① Employees : FD - { Aadhar NO. → Union ID }
CK - { Aadhar NO }, PA - { Aadhar NO. }
NPA - { Union ID }

② Technician : FD - { Aadhar NO. → Name, Salary, address, Phone NO. }
CK - { Aadhar NO. } PA - { Aadhar NO. }
NPA - { Name, Salary, address, phone NO. }

③ Test : FD - { Test NO. → Test Name, Max. Score }
CK - { Test NO. } PA - { Test NO. }
NPA - { Test Name, Max. Score }

④ Union : FD - { Union ID → Union Name }
CK - { Union ID } PA - { Union ID }
NPA - { Union Name }

⑤ Model : FD - { Model NO. → Model Name, Capacity }
CK - { Model NO. } PA - { Model NO. }
NPA - { Capacity }

⑥ Testing Event : FD - { Reg. NO., Test NO., Date → hours, Score }
CK - { Reg NO, Test NO, Date }
PA - { Reg NO, Test NO, Date } NPA - { hours, Score }

⑦ ExpertIn : FD - { Aadhar NO. → Model NO. }
CK - { Aadhar NO, Model NO }
PA - { Aadhar NO, Model NO }