UNIT-3
IMAGE RESTORATION AND RECONSTRUCTION

Introduction

Image Restoration is the process to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon. These techniques are oriented toward modeling the degradation and applying the inverse process to recover the original image. Restoration improves image in some predefined sense. Image enhancement techniques are subjective process, where as image restoration techniques are objective process.

3.1 A Model of Image Degradation/Restoration Process

Image Degradation process operates on a degradation function that operates on an input image with an additive noise term to produce degraded image. The image degradation model is shown below.

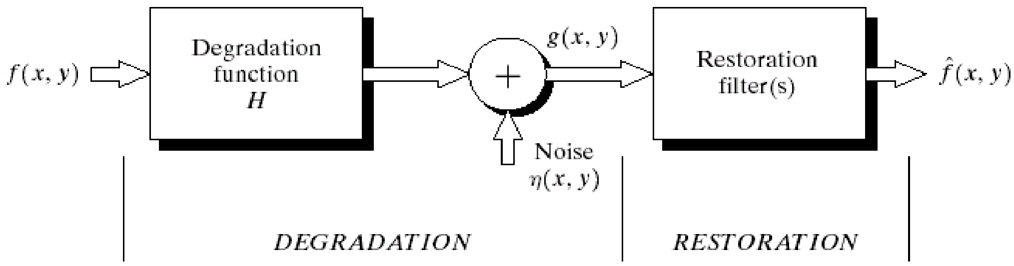


Fig: A Model of Image Degradation/Restoration Process

Let $f(x, y)$ is an input image and $g(x, y)$ is the degraded image with some knowledge about the degradation function H and some knowledge about the additive noise term $\eta(x, y)$. The objective of the restoration is to obtain an estimate $\hat{f}(x, y)$ of the original image. If H is a linear, position-invariant process, then the degraded image is given in the spatial domain by

$$g(x,y)=f(x,y)*h(x,y)+\eta(x,y)$$

Where $h(x, y)$ is the spatial representation of the degraded function. The degrade image in frequency domain is represented as

$$G(u,v)=F(u,v)H(u,v)+N(u,v)$$

The terms in the capital letters are the Fourier Transform of the corresponding terms in the spatial domain.

3.2 Noise Models

The principle sources of noise in digital image are due to image acquisition and transmission.

- During image acquisition, the performance of image sensors gets affected by a variety of factors such as environmental conditions and the quality of sensing elements.
- During image transmission, the images are corrupted due to the interference introduced in the channel used for transmission.

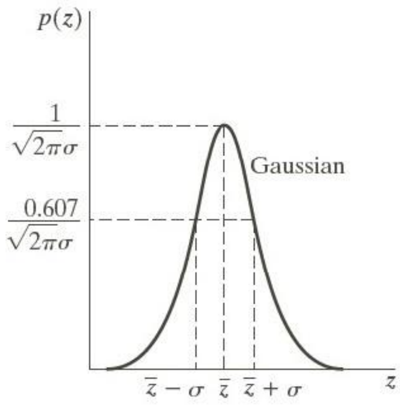
The Noise components are considered as random variables, characterized by a probability density function. The most common PDFs found in digital image processing applications are given below.

Gaussian Noise

Gaussian noise is also known as 'normal' noise. The Probability density function of a Gaussian random variable z is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$

Where z represents intensity, $\bar{z}$ is the mean and σ is its standard deviation and its square (σ^2) is called the variance of z. The values of Gaussian noise is approximately 70% will be in the range $[(\bar{z} - \sigma), (\bar{z} + \sigma)]$ and 95% will be in the range $[(\bar{z} - 2\sigma), (\bar{z} + 2\sigma)]$.

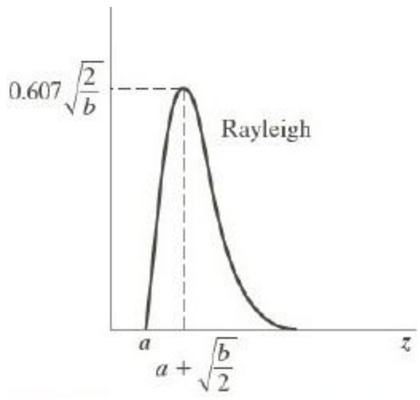


Rayleigh Noise

The PDF of Rayleigh Noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

$$\text{Mean:} \quad \bar{z} = a + \sqrt{\pi b/4} \qquad \text{Variance:} \quad \sigma^2 = \frac{b(4-\pi)}{4}$$



Applications:

- It is used for characterizing noise phenomenon in range imaging.
- It describes the error in the measurement instrument.
- It describes the noise affected in radar.
- It determines the noise occurred when the signal is passed through the band pass filter.

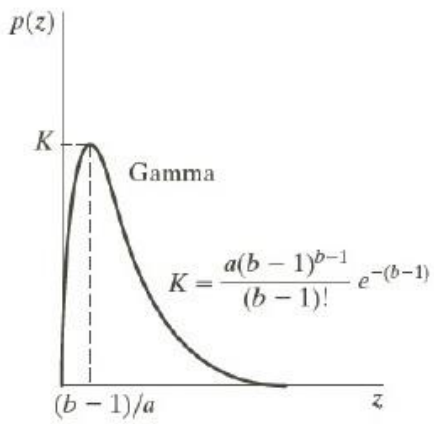
Erlang (gamma) Noise

The probability density function of Erlang noise is given by

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \qquad a > 0$$

b positive integer

Mean: $\bar{z} = \frac{b}{a}$ Variance: $\sigma^2 = \frac{b}{a^2}$



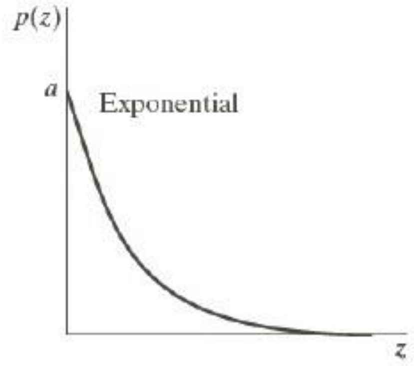
Exponential Noise

The probability density function of Exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

p(z) is maximum at z=0

Mean: $z = \frac{1}{a}$ Variance: $\sigma^2 = \frac{1}{a^2}$



Applications:

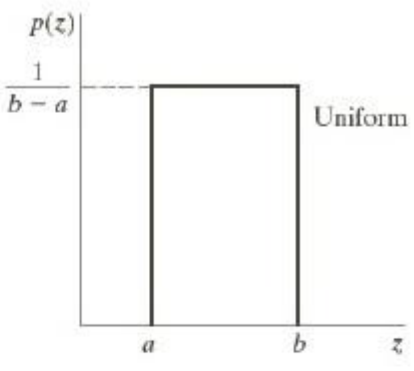
- It is used to describe the size of the raindrop.
- It is used to describe the fluctuations in received power reflected from certain targets
- It finds application in Laser imaging.

Uniform Noise

The probability density function of Uniform noise is given by

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

Mean: $\bar{z} = \frac{a+b}{2}$ Variance: $\sigma^2 = \frac{(b-a)^2}{12}$

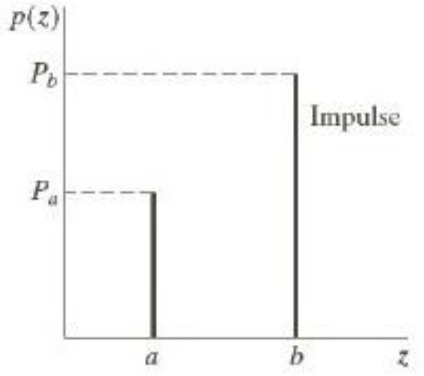


Salt and Pepper Noise (Impulse Noise)

The probability density function of Salt and Impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

$$P_a = P_b \Rightarrow \textit{unipolar} \text{ noise}$$



If b>a, gray level b will appear as a light dot in image. Level a will appear like a dark dot. The salt and pepper noise is also called as bi-polar impulse noise or Data-drop-out and spike noise.

Periodic Noise

Periodic noise in an image occurred from electrical or elecrtomechnaical interference during image acquisition. This is the only type of spatially dependent noise and the parameters are estimated by the Fourier spectrum of the image. Periodic noise tends to produce frequency spikes that often can be detected even by visual analysis. The mean and variance are defined as

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_S(z_i)$$

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_S(z_i)$$

3.3 Restoration in the Presence of Noise only-Spatial Filtering

When the only degradation present in an image is noise,

$$g(x, y) = f(x,y) + \eta(x,y)$$

and

$$G (u, v) = F (u, v) + N (u, v)$$

The noise terms are unknown so subtracting them from g(x, y) or G (u, v) is not a realistic approach. In the case of periodic noise it is possible to estimate N (u, v) from the spectrum G (u, v). So N (u, v) can be subtracted from G (u, v) to obtain an estimate of original image. Spatial filtering can be done when only additive noise is present.

Mean Filters

Arithmetic Mean Filter:

It is the simplest mean filter. Let S_{xy} represents the set of coordinates in the sub image of size m*n centered at point (x, y). The arithmetic mean filter computes the average value of the corrupted image g(x, y) in the area defined by S_{xy}. The value of the restored image f at any point (x, y) is the arithmetic mean computed using the pixels in the region defined by S_{xy}.

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

This operation can be using a convolution mask in which all coefficients have value 1/mn. A mean filter smoothes local variations in image Noise is reduced as a result of blurring.

Geometric Mean Filter:

An image restored using a geometric mean filter is given by the expression

$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}.$$

Here, each restored pixel is given by the product of the pixel in the sub-image window, raised to the power 1/mn. A Geometric means filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose image details in the process.

Harmonic Mean Filter:

The harmonic mean filtering operation is given by the expression

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}.$$

The harmonic mean filter works well for salt noise but fails for pepper noise. It does well also with other types of noise.

Contra harmonic Mean Filter:

The contra harmonic mean filter yields a restored image based on the expression

$$\hat{f}(x, y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Where Q is called the order of the filter and this filter is well suited for reducing the effects of salt and pepper noise. For positive values of Q the filter eliminates pepper noise. For negative values of Q it eliminates salt noise. It cannot do both simultaneously. The contra harmonic filter reduces to arithmetic mean filter if Q=0 and to the harmonic filter if Q= -1.

<u>Order-Static Filters</u>

Order statistics filters are spatial filters whose response is based on ordering the pixel contained in the image area encompassed by the filter. The response of the filter at any point is determined by the ranking result.

Median Filter:

It is the best known order statistic filter. It replaces the value of a pixel by the median of gray levels in the Neighborhood of the pixel.

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s,t)\}$$

The value of the pixel at *(x, y)* is included in the computation of the median. Median filters are quite popular because for certain types of random noise, they provide excellent noise reduction capabilities with considerably less blurring than smoothing filters of similar size. These are effective for bipolar and unipolar impulse noise.

Max and Min Filters:

The median filter represents the 50th percentile of a ranked set of numbers. If using the 100th percentile results is called “Max Filter”. It can be defined as

$$\hat{f}(x, y) = \max_{(s,t)\in S_{xy}} \{g(s, t)\}$$

This filter is used for finding the brightest point in an image. Pepper noise in the image has very low values, it is reduced by max filter using the max selection process in the sublimated area S_{XY}.

The 0th percentile filter is min filter

$$\hat{f}(x, y) = \min_{(s,t)\in S_{xy}} \{g(s, t)\}$$

This filter is useful for flinging the darkest point in image. Also, it reduces salt noise as a result of the min operation.

Midpoint Filter:

The midpoint filter simply computes the midpoint between the maximum and minimum values in the area encompassed by the filter.

$$\hat{f}(x, y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s, t)\} + \min_{(s,t)\in S_{xy}} \{g(s, t)\}\right].$$

It combines the order statistics and averaging .This filter works best for randomly distributed noise like Gaussian or uniform noise.

Alpha-trimmed mean Filter:

If we delete the d/2 lowest and the d/2 highest intensity values of *g(s, t)* in the neighborhood S_{XY}. Let $g_r(s, t)$ represents the remaining *mn-d* pixels. A filter formed by averaging the reaming pixels is called alpha-trimmed mean filter.

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s,t)\in S_{xy}} g_r(s, t)$$

The value of d can range from 0 to *mn*-1. If d=0 this filter reduces to arithmetic mean filter. If d=*mn*-1, the filter becomes a median filter. For the other values of d the alpha-trimmed median filter is useful for multiple types of noise, such as combination of salt-and-pepper and Gaussian noise.

3.4 Adaptive Filters

Adaptive filter whose behavior changes based on the statistical characteristics of the image inside the filter region S_{xy}.

Adaptive, local noise reduction filter

The simplest statistical measures of a random variable are its mean and variance. The mean gives a measure of average intensity in the region over which the mean is computed and the variance gives a measure of contrast in that region.

Let the filter is operate on a local region S_{XY}. The response of the filter at any point *(x, y)* is based on four quantities: (a) *g(x, y),* the value of noisy image at *(x, y)*; (b) σ_η^2 the variance of the noise corrupting *f(x, y)* to form *g(x, y)*; (c) m_L, the local mean of the pixels in S_{XY}; and (d) σ_L^2, the local variance of the pixels in S_{XY}. Hence the behavior of the filter is,

- If σ_η^2 is zero, the filter should return simply the value of *g(x, y).*
- If the local variance is high relative to σ_η^2 that means ($\sigma_L^2 > \sigma_\eta^2$), the filter should return a value close to *g(x, y).*
- If the two variances are equal, the filter returns the arithmetic mean value of the pixel in S_{XY}.

An adaptive filter for obtaining the restored image is

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

The only quantity that needs to be known or estimated is the variance of the overall noise is σ_η^2. The other parameters are computed from the pixels in S_{XY}.

Adaptive median filter

Adaptive median filters are used to preserve the details while smoothing non impulse. However it changes the size of S_{XY} during the filtering operation, depending on certain conditions. The output of the filter is a single value used to replace the value of pixel at (*x, y*). Let us consider the following parameters,

z_{min} = minimum intensity value in S_{XY}

z_{max} = maximum intensity value in S_{XY}

z_{med} = median of intensity values in S_{XY}

z_{xy} = intensity value at co-ordinates (*x, y*)

S_{max} = maximum allowed size of S_{XY}

The adaptive median filtering algorithm works in two stages, denoted as stage A and stage B as follows:

Stage A: A1 = z_{med} - z_{min}

A2 = z_{med} - z_{max}

If A1>0 AND A2<0, go to stage B

Else increase the window size

If window size ≤ S_{max} repeat stage A

Else output z_{med}

Stage B: B1 = z_{xy} - z_{min}

B2 = z_{xy} - z_{max}

If B1>0 AND B2<0, output z_{xy}.

Else output z_{med}

3.5 Periodic Noise Reduction by Frequency Domain Filtering

Periodic noise in images are appears as concentrated bursts of energy in the Fourier transform at locations corresponding to the frequencies of the periodic interference. This can be removed by using selective filters.

Band Reject Filter:

The Band Reject Filter transfer function is defined as

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

Where ‘W’ is the width of the band, D is the distance *D (u, v)* from the centre of the filter, D_o is the cutoff frequency and n is the order of the Butterworth filter. The band reject filters are very effective in removing periodic noise and the ringing effect normally small. The perspective plots of these filters are

Fig: Perspective plots of (a) Ideal (b) Butterworth and (c) Gaussian Band Reject Filters

Band Pass Filter:

A *band pass* filter performs the opposite operation of a band reject filter. A Band pass filter is obtained from the band reject filter as

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

Notch Filters:

A Notch filter Reject (or pass) frequencies in a predefined neighborhood about the centre of the frequency rectangle. It is constructed as products of high pass filters whose centers have been translated to the centers of the notches. The general form is defined as

$$H_{NR}(u,v) = \prod_{k=1}^{Q} \underbrace{H_k(u,v)}_{\substack{\text{centre at} \\ (u_k,\, v_k)}} \underbrace{H_{-k}(u,v)}_{\substack{\text{centre at} \\ (-u_k,\, -v_k)}}$$

Where $H_k(u,v)$ and $H_{-k}(u,v)$ are high pass filters whose centers are at (u_k, v_k) and ($-u_k$, $-v_k$) respectively. These centers are specified with respect to the center of the frequency rectangle (M/2, N/2).

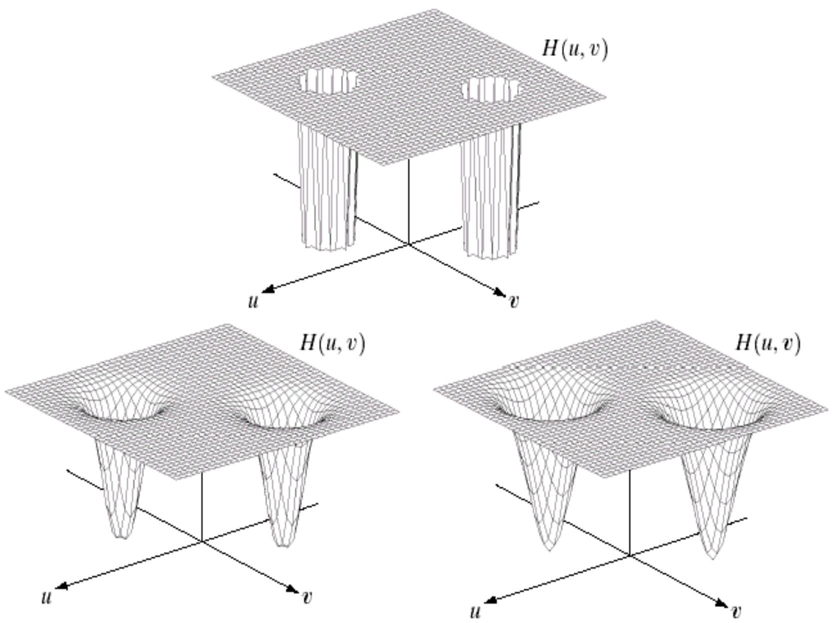


Fig: Perspective plots of (a) Ideal (b) Butterworth and (c) Gaussian Notch Reject Filters

A *Notch Pass filter* (NP) is obtained from a *Notch Reject filter* (NR) using:

$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

3.6 Linear, Position-Invariant Degradations

The input –output relation relationship before the restoration stage is expressed as

$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

Let us assume that $\eta(x, y) = 0$ then

$$g(x, y) = H[f(x,y)]$$

If H is linear

$$H[af_1(x,y)+bf_2(x,y)] = aH[f_1(x,y)]+bH[f_2(x,y)]$$

Where a and b are scalars. $f_1(x,y)$ and $f_2(x,y)$ are any two input images. If a=b=1

$$H[f_1(x,y)+f_2(x,y)] = H[f_1(x,y)]+H[f_2(x,y)]$$

It is called the property of additivity. This property says that, if H is a linear operator, the response to a sum of two inputs is equal to the sum of the two responses.

An operator having the input –output relation relationship $g(x, y) = H[f(x,y)]$ is said to be position invariant if

$$H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)$$

It indicates that the response at any point in the image depends only on the value of the input at that point not on its position. If the impulse signal can be considered

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\boxed{\delta(x-\alpha, y-\beta)}\,d\alpha\, d\beta$$

impulse

$$g(x,y) = H[f(x,y)] = H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha, y-\beta)\,d\alpha\, d\beta\right]$$

linear

$$g(x,y) = H[f(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H\left[f(\alpha,\beta)\delta(x-\alpha, y-\beta)\right]d\alpha\, d\beta$$

$$g(x,y) = H[f(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\boxed{H\left[\delta(x-\alpha, y-\beta)\right]}d\alpha\, d\beta$$

$$h(x,\alpha,y,\beta) = H\left[\delta(x-\alpha, y-\beta)\right]$$

$$h(x,y) = H\left[\delta(x,y)\right]$$

Impulse response (point spread function)

If position-invariant

$$H\left[\delta(x-\alpha, y-\beta)\right] = h(x-\alpha, y-\beta)$$

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha, y-\beta)\,d\alpha\, d\beta$$

$\eta(x,y) \neq 0$

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha, y-\beta)\,d\alpha\, d\beta + \eta(x,y)$$

3.7 Estimating the Degradation Function

There are three principle ways to estimate the degradation function for use in image restoration: (1) Observation, (2) Experimentation and (3) mathematical modeling.

Estimation by Image observation:

In this process the degradation function is estimated by observing the Image. Select the sub image whose signal content is strong. Let the observed sub image be denoted by $g_s(x, y)$ and the processed sub image is $\hat{f}\,(x, y)$. The estimated degradation function can be expressed as

$$H_s(u, v) = \frac{G_s(u, v)}{\hat{F}_s(u, v)}$$

From the characteristics of this equation, we then deduce the complete degradation function *H (u, v)* based on the consideration of position invariance.

Estimation by Experimentation:

The degrade image can be estimated accurately when the equipment is identical to the one used to obtain the degraded image. Images similar to the degraded image can be acquired with various system settings until they are degraded as closely as possible to the image we wish to restore. Now obtain the impulse response of the degradation by imaging an impulse using the same system settings.

An impulse is simulated by a bright dot of light, as bright as possible to reduce the effect of noise. Then the degradation image can be expressed as

$$H(u, v) = \frac{G(u, v)}{A}$$

Where *G(u, v)* is the Fourier transform of observed image and A is a constant describing the strength of the impulse.

Estimation by Modeling:

Image degradation function can be estimated by modeling includes the environmental conditions that cause degradations. It can be expressed as

$$H(u, v) = e^{-k(u^2+v^2)^{5/6}}$$

Where k is a constant that depends on the nature of the turbulence. Let *f(x, y)* is an image that undergoes planar motion and that x_o (t) and y_o (t) are the time varying components in the direction of x and y. The total blurring image $g(x, y)$ is expressed as

$$g(x, y) = \int_0^T f[x - x_0(t), y - y_0(t)]\, dt$$

The Fourier Transform of *g*(*x, y*) is *G(u, v)*

$$G(u, v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x, y) e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_0^T f[x - x_0(t), y - y_0(t)]\, dt\right] e^{-j2\pi(ux+vy)}\, dx\, dy$$

Reversing the order of the integration then

$$G(u, v) = \int_0^T \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f[x - x_0(t), y - y_0(t)] e^{-j2\pi(ux+vy)}\, dx\, dy\right] dt$$

$$G(u, v) = \int_0^T F(u, v) e^{-j2\pi[ux_0(t)+vy_0(t)]}\, dt$$

$$= F(u, v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}\, dt$$

Let us define *H (u, v)*

$$H(u, v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}\, dt$$

Then the above expression can be expressed as

$$G(u, v) = H(u, v)F(u, v)$$

If the motion variables $x_o(t)$ and $y_o(t)$ are known then the degradation function *H (u, v)* can becomes

$$H(u, v) = \frac{T}{\pi(ua + vb)}\sin[\pi(ua + vb)]e^{-j\pi(ua+vb)}$$

3.8 Inverse Filtering

The simplest approach to restoration is direct inverse filtering, where we can estimate $\hat{F}$(u, v) of the transform of the original image simply by dividing the transform of the degraded image *G* (u,v) by the degraded function

$$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)}$$

We know that $G(u,v) = H(u, v)F(u, v) + N(u, v)$

Then

$$\hat{F}(u, v) = F(u, v) + \frac{N(u, v)}{H(u, v)}$$

From the above expression we can observe that if we know the degrade function we cannot recover the un-degraded image exactly because *N* (u, v) is not known. If the degrade function has zero or very small values, then the ratio *N* (u, v)/ *H*(u, v) could easily dominate the $\hat{F}$(u, v). To avoid this disadvantage we limit the filter frequency values near the origin because *H*(u, v) values are maximum at the origin.

3.9 Minimum Mean Square Error Filtering (Wiener Filtering)

In this filtering process, it incorporates both the degrade function and statistical characteristics of noise .The images and noise in this method are considered as random variables. The objective is to find an estimate $\hat{f}$ of the uncorrupted image *f* such that the mean square error between them is minimized. This error is measured by

$$e^2 = E\{(f - \hat{f})^2\}$$

Where E { } is the expected value of the argument. It is assumed that noise and image are uncorrelated; one or the other has zero mean; the intensity levels in the estimate are a linear function of the levels in the degraded image. Based on these conditions the minimum of the error function in frequency domain is given by

$$\begin{aligned}\hat{F}(u, v) &= \left[\frac{H^*(u, v)S_f(u, v)}{S_f(u, v)|H(u, v)|^2 + S_\eta(u, v)}\right]G(u, v)\\ &= \left[\frac{H^*(u, v)}{|H(u, v)|^2 + S_\eta(u, v)/S_f(u, v)}\right]G(u, v)\\ &= \left[\frac{1}{H(u, v)}\frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v)/S_f(u, v)}\right]G(u, v)\end{aligned}$$

This result is known as “*Wiener filter*”. The terms inside the bracket is commonly referred as the minimum mean square error filter or the least square error filter. It does not have the same problem as the inverse filter with zeros in the degraded function, unless the entire denominator is zero for the same values of *u* and *v*.

H(u, v) = degraded function

*H**(u, v) = complex conjugate of H(u, v)

$$S_\eta(u, v) = |N(u, v)|^2 = \text{power spectrum of the noise}$$

$$S_f(u, v) = |F(u, v)|^2 = \text{power spectrum of the undegraded image}$$

The signal to noise ratio in frequency domain

$$\text{SNR} = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u, v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u, v)|^2}$$

The signal to noise ratio in spatial domain

$$\text{SNR} = \frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\hat{f}(x, y)^2}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x, y) - \hat{f}(x, y)]^2}$$

The mean square error is obtained by using the expression

$$\text{MSE} = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x, y) - \hat{f}(x, y)]^2$$

The modified expression to estimate $\hat{f}$ by using minimum mean square error filtering

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)}\frac{|H(u, v)|^2}{|H(u, v)|^2 + K}\right]G(u, v)$$

3.10 Constrained Least Square Filtering

The problem with the wiener filter is that it is necessary to know the power spectrum of noise and image. The degraded image is given in the spatial domain by

$$g(x,y)=f(x,y)*h(x,y)+\eta(x,y)$$

In vector-matrix form

$$\mathbf{g} = \mathbf{Hf} + \boldsymbol{\eta}$$

Where $\boldsymbol{g}$, $\boldsymbol{f}$, $\boldsymbol{\eta}$ vectors of dimension MNx1.$\boldsymbol{H}$ matrix of dimension MN x MN is very large and is highly sensitive to noise. Optimality of restoration based on a measure of smoothness: using Laplacian operator. The restoration must be constrained by the parameters is to find the minimum criterion function c is defined as

$$C = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\nabla^2 f(x, y)]^2$$

subject to the constraint

$$\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\|^2 = \|\boldsymbol{\eta}\|^2$$

The frequency domain solution to this optimization problem is given by the expression

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right]G(u,v)$$

Where γ is a parameter that must be adjusted so that constraint is satisfied and *P* (*u, v*) is the Fourier transform of the function

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

It is possible to adjust the parameter γ interactively until acceptable results are achieved. A procedure for computing γ by iteration is as follows. Define a residual vector r as

$$\mathbf{r} = \mathbf{g} - \mathbf{H}\hat{\mathbf{f}}$$

Hence $\hat{F}$(u, v) is a function of γ, then r also a function of this parameter is a monotonically increasing function of γ .

$$\begin{aligned} \phi(\gamma) &= \mathbf{r}^T\mathbf{r} \\ &= \|\mathbf{r}\|^2 \end{aligned}$$

We want to adjust γ so that

$$\|\mathbf{r}\|^2 = \|\boldsymbol{\eta}\|^2 \pm a$$

Where a is an accuracy factor. If $\|\mathbf{r}\|^2 = \|\boldsymbol{\eta}\|^2$ the constarint is satisfied. Because φ(γ) is monotonic , finding the desired value of γ is not difficult. The appraoch

1. Specify an initial value of γ.
2. Compute $\|\mathbf{r}\|^2$.
3. Stop if Eq. (5.9-8) is satisfied; otherwise return to Step 2 after increasing γ if $\|\mathbf{r}\|^2 < \|\boldsymbol{\eta}\|^2 - a$ or decreasing γ if $\|\mathbf{r}\|^2 > \|\boldsymbol{\eta}\|^2 + a$. Use the new value of γ in Eq. (5.9-4) to recompute the optimum estimate $\hat{F}(u,v)$.

The variance and the mean of the entire image is

$$\sigma_\eta^2 = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\eta(x,y) - m_\eta\right]^2$$

$$m_\eta = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \eta(x, y)$$

Hence the noise is

$$\|\boldsymbol{\eta}\|^2 = MN\left[\sigma_\eta^2 + m_\eta^2\right]$$

3.11 Geometric Mean Filter

Geometric mean filter is slightly generalized wiener filter in the form

$$\hat{F}(u, v) = \left[\frac{H^*(u, v)}{|H(u, v)|^2}\right]^\alpha \left[\frac{H^*(u, v)}{|H(u, v)|^2 + \beta\left[\frac{S_\eta(u, v)}{S_f(u, v)}\right]}\right]^{1-\alpha} G(u, v)$$

α and β being positive real constants. Based on the values of α and β, geometric mean filter performs the different actions

α = 1 => inverse filter

α = 0 => *parametric Wiener filter* (standard Wiener filter when β = 1)

α = 1/2 => actual geometric mean

α = 1/2 and β = 1 => *spectrum equalization filter*

PREVIOUS QUESTIONS

1. What is meant by image restoration? Explain the image degradation model
2. Discuss about the noise models
3. Explain the concept of algebraic image restoration
4. Discuss the advantages and disadvantages of wiener filter with regard to image restoration.
5. Explain about noise modeling based on distribution function
6. Explain about wiener filter in noise removal
7. What is geometric mean filter? Explain
8. Explain the following. a) Minimum Mean square error filtering. b) Inverse filtering.
9. Discuss about Constrained Least Square restoration of a digital image in detail.
10. Explain in detail about different types of order statistics filters for Restoration.
11. Name different types of estimating the degradation function for use in image restoration and explain in detail estimation by modeling.
12. Explain periodic noise reduction by frequency domain filtering
13. Explain adaptive filter and also what the two levels of adaptive median filtering algorithms are