UNIT-3
IMAGE RESTORATION

IMAGE RESTORATION:

Restoration improves image in some predefined sense. It is an objective process. Restoration attempts to reconstruct an image that has been degraded by using a priori knowledge of the degradation phenomenon. These techniques are oriented toward modeling the degradation and then applying the inverse process in order to recover the original image. Restoration techniques are based on mathematical or probabilistic models of image processing. Enhancement, on the other hand is based on human subjective preferences regarding what constitutes a "good" enhancement result. Image Restoration refers to a class of methods that aim to remove or reduce the degradations that have occurred while the digital image was being obtained. All natural images when displayed have gone through some sort of degradation:

- During display mode

- Acquisition mode, or

- Processing mode

    - Sensor noise

    - Blur due to camera mis focus

    - Relative object-camera motion

    - Random atmospheric turbulence

- Others

Degradation Model:

Degradation process operates on a degradation function that operates on an input image with an additive noise term. Input image is represented by using the notation f(x,y), noise term can be represented as η(x,y).These two terms when combined gives the result as g(x,y). If we are given g(x,y), some knowledge about the degradation function H or J and some knowledge about the additive noise teem η(x,y), the objective of restoration is to obtain an estimate *f'(x,y)* of the original image. We want the estimate to be as close as possible to the original image. The more we know about h and η , the closer f(x,y) will be to f'(x,y). If it is a linear position invariant process, then degraded image is given in the spatial domain by

$$g(x,y)=f(x,y)*h(x,y)+\eta(x,y)$$

h(x,y) is spatial representation of degradation function and symbol * represents convolution. In frequency domain we may write this equation as

$$G(u,v)=F(u,v)H(u,v)+N(u,v)$$

The terms in the capital letters are the Fourier Transform of the corresponding terms in the spatial domain.
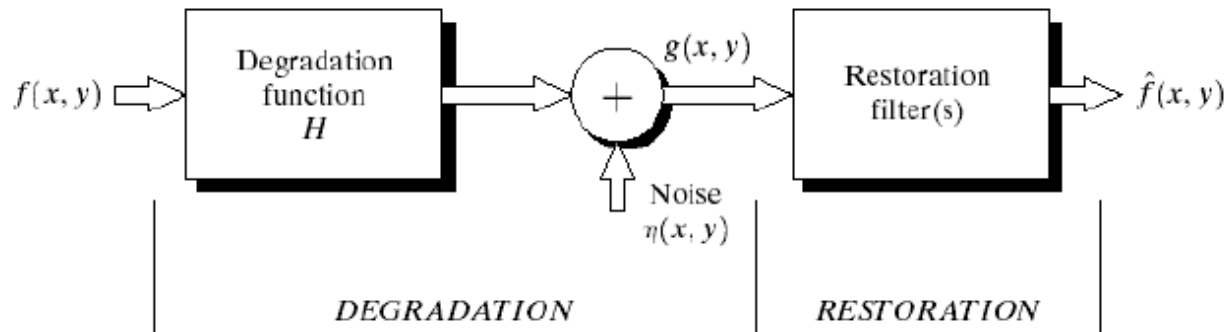

Fig: A model of the image Degradation / Restoration process

Noise Models:

The principal source of noise in digital images arises during image acquisition and /or transmission. The performance of imaging sensors is affected by a variety of factors, such as environmental conditions during image acquisition and by the quality of the sensing elements themselves. Images are corrupted during transmission principally due to interference in the channels used for transmission. Since main sources of noise presented in digital images are resulted from atmospheric disturbance and image sensor circuitry, following assumptions can be made i.e. the noise model is spatial invariant (independent of spatial location). The noise model is uncorrelated with the object function.

Gaussian Noise:

These noise models are used frequently in practices because of its tractability in both spatial and frequency domain. The PDF of Gaussian random variable is

$$p_z(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

Where z represents the gray level, μ= mean of average value of z, σ= standard deviation.
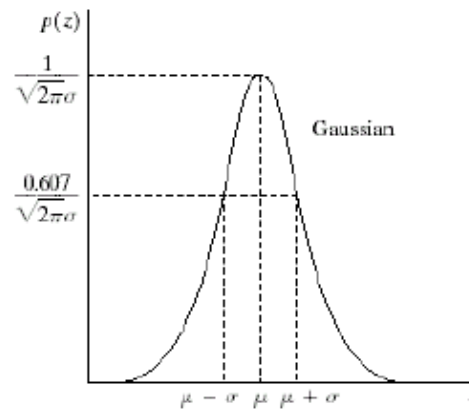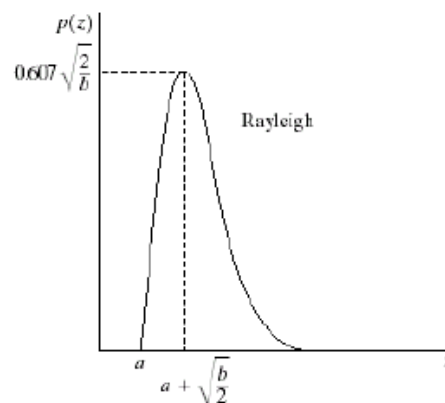
Rayleigh Noise:

Unlike Gaussian distribution, the Rayleigh distribution is no symmetric. It is given by the formula.

$$p_z(z) = \begin{cases} \dfrac{2}{b}(z - a)e^{-(z-a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$$

The mean and variance of this density is

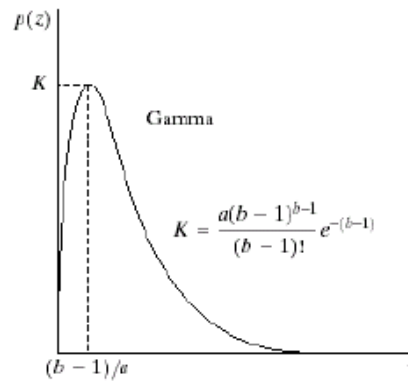$$m = a + \sqrt{\pi b/4}, \sigma^2 = \frac{b(4 - \pi)}{4}$$

(iii)Gamma Noise:

The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!}e^{-az}, & \text{for } z \geq 0 \\ 0, & \text{for } z < 0 \end{cases}$$

The mean and variance of this density are given by

$$\text{mean}: \mu = \frac{b}{a} \quad \text{variance}: \sigma^2 = \frac{b}{a^2}$$

$$K = \frac{a(b-1)^{b-1}}{(b-1)!} e^{-(b-1)}$$

Its shape is similar to Rayleigh disruption. This equation is referred to as gamma density
it is correct only when the denominator is the gamma function.
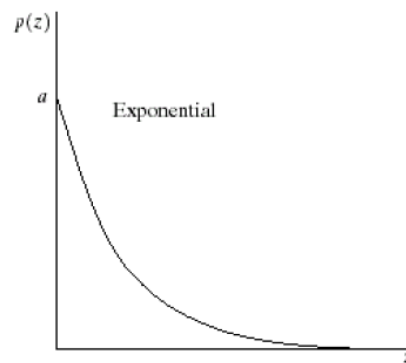
(iv)Exponential Noise:

Exponential distribution has an exponential shape. The PDF of exponential noise is given as

$$p_z(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

Where a>0. The mean and variance of this density are given by

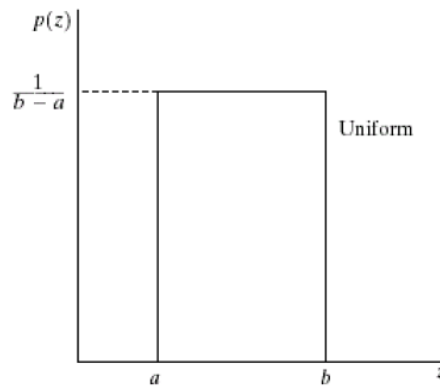$$m = \frac{1}{a}, \quad \sigma^2 = \frac{1}{a^2}$$

(v)Uniform Noise:

The PDF of uniform noise is given by

$$p_z(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance of this noise is

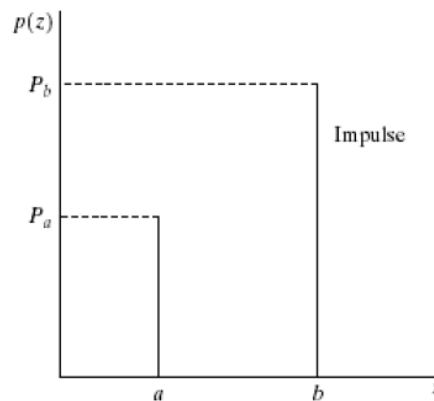$$m = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}$$

(vi)Impulse (salt & pepper) Noise:

In this case, the noise is signal dependent, and is multiplied to the image.

The PDF of bipolar (impulse) noise is given by

$$p_z(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

$$b > a$$

If b>a, gray level b will appear as a light dot in image. Level a will appear like a dark dot.

Restoration in the presence of Noise only- Spatial filtering:

When the only degradation present in an image is noise, i.e.

$$g(x,y)=f(x,y)+\eta(x,y)$$

or

$$G(u,v)= F(u,v)+ N(u,v)$$

The noise terms are unknown so subtracting them from g(x,y) or G(u,v) is not a realistic approach. In the case of periodic noise it is possible to estimate N(u,v) from the spectrum G(u,v).

So N(u,v) can be subtracted from G(u,v) to obtain an estimate of original image. Spatial filtering can be done when only additive noise is present. The following techniques can be used to reduce the noise effect:

i) Mean Filter:

ii) (a)Arithmetic Mean filter:

It is the simplest mean filter. Let Sxy represents the set of coordinates in the sub image of size m*n centered at point (x,y). The arithmetic mean filter computes the average value of the corrupted image g(x,y) in the area defined by Sxy. The value of the restored image f at any point (x,y) is the arithmetic mean computed using the pixels in the region defined by Sxy.

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in Sxy} g(s,t)$$

This operation can be using a convolution mask in which all coefficients have value 1/mn A mean filter smoothes local variations in image Noise is reduced as a result of blurring. For every pixel in the image, the pixel value is replaced by the mean value of its neighboring pixels with a weight .This will resulted in a smoothing effect in the image.

(b)Geometric Mean filter:

An image restored using a geometric mean filter is given by the expression

$$\hat{f}(x,y) = \left(\prod_{(s,t) \in Sxy} g(s,t) \right)^{1/mn}$$

Here, each restored pixel is given by the product of the pixel in the sub image window, raised to the power 1/mn. A geometric mean filters but it to loose image details in the process.

(c)Harmonic Mean filter:

The harmonic mean filtering operation is given by the expression

$$\hat{f}(x,y) = \left. \sum_{(s,t) \in Sxy} g(s,t)^{Q+1} \middle/ \sum_{(s,t) \in Sxy} g(s,t)^{Q} \right.$$

The harmonic mean filter works well for salt noise but fails for pepper noise. It does well with Gaussian noise also.

(d)Order statistics filter:

Order statistics filters are spatial filters whose response is based on ordering the pixel contained in the image area encompassed by the filter. The response of the filter at any point is determined by the ranking result.

(e)Median filter:

It is the best order statistic filter; it replaces the value of a pixel by the median of gray levels in the Neighborhood of the pixel.

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s,t)\}$$

The original of the pixel is included in the computation of the median of the filter are quite possible because for certain types of random noise, the provide excellent noise reduction capabilities with considerably less blurring then smoothing filters of similar size. These are effective for bipolar and unipolor impulse noise.

(e)Max and Min filter:

Using the l00th percentile of ranked set of numbers is called the max filter and is given by the equation

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{\max}\{g(s,t)\}$$

It is used for finding the brightest point in an image. Pepper noise in the image has very low values, it is reduced by max filter using the max selection process in the sublimated area sky. The 0th percentile filter is min filter.

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{\min}\{g(s,t)\}$$

This filter is useful for flinging the darkest point in image. Also, it reduces salt noise of the min operation.

(f)Midpoint filter:

The midpoint filter simply computes the midpoint between the maximum and minimum values in the area encompassed by

$$\hat{f}(x, y) = \left(\underset{(s,t)\in S_{xy}}{\max}\{g(s,t)\} + \underset{(s,t)\in S_{xy}}{\min}\{g(s,t)\} \right) \Big/ 2$$

It comeliness the order statistics and averaging .This filter works best for randomly distributed noise like Gaussian or uniform noise.

Periodic Noise by Frequency domain filtering:

These types of filters are used for this purpose-

Band Reject Filters:

It removes a band of frequencies about the origin of the Fourier transformer.

 Ideal Band reject Filter:

An ideal band reject filter is given by the expression

$$H(u,v) = \begin{cases} 1 & \text{if} & D(u,v) < D_0 - W/2 \\ 0 & \text{if} & D_0 - W/2 \leq D(u,v) \leq D_0 + W/2 \\ 1 & \text{if} & D(u,v) > D_0 + W/2 \end{cases}$$

$D(u,v)$- the distance from the origin of the centered frequency rectangle.

W- the width of the band

Do- the radial center of the frequency rectangle.

Butterworth Band reject Filter:

$$H(u,v) = 1 \bigg/ \left[1 + \left(\frac{D(u,v)W}{D^2(u,v) - D_0^2} \right)^{2n} \right]$$

Gaussian Band reject Filter:

$$H(u,v) = 1 - \exp \left[-\frac{1}{2} \left(\frac{D^2(u,v) - D_0^2}{D(u,v)W} \right)^2 \right]$$

These filters are mostly used when the location of noise component in the frequency domain is known. Sinusoidal noise can be easily removed by using these kinds of filters because it shows two impulses that are mirror images of each other about the origin. Of the frequency transform.

a b c

FIGURE From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Band pass Filter:

The function of a band pass filter is opposite to that of a band reject filter It allows a specific frequency band of the image to be passed and blocks the rest of frequencies. The transfer function of a band pass filter can be obtained from a corresponding band reject filter with transfer function Hbr(u,v) by using the equation

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

These filters cannot be applied directly on an image because it may remove too much details of an image but these are effective in isolating the effect of an image of selected frequency bands.

Notch Filters:

A notch filter rejects (or passes) frequencies in predefined neighborhoods about a center frequency.

Due to the symmetry of the Fourier transform notch filters must appear in symmetric pairs about the origin.

The transfer function of an ideal notch reject filter of radius D_0 with centers a (u_0, v_0) and by symmetry at $(-u_0, v_0)$ is

$$D_1(u,v) = \sqrt{(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2}$$

$$D_2(u,v) = \sqrt{(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2}$$

Ideal, butterworth, Gaussian notch filters

$$H(u,v) = \begin{cases} 0 & \text{if} \quad D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 1 & \text{otherwise} \end{cases}$$

$$H(u,v) = 1 \Big/ \left[1 + \left(\frac{D_0^2}{D_1(u,v)D_2(u,v)} \right)^n \right]$$

$$H(u,v) = 1 - \exp\left[-\frac{1}{2}\left(\frac{D_1(u,v)D_2(u,v)}{D_0^2} \right) \right]$$
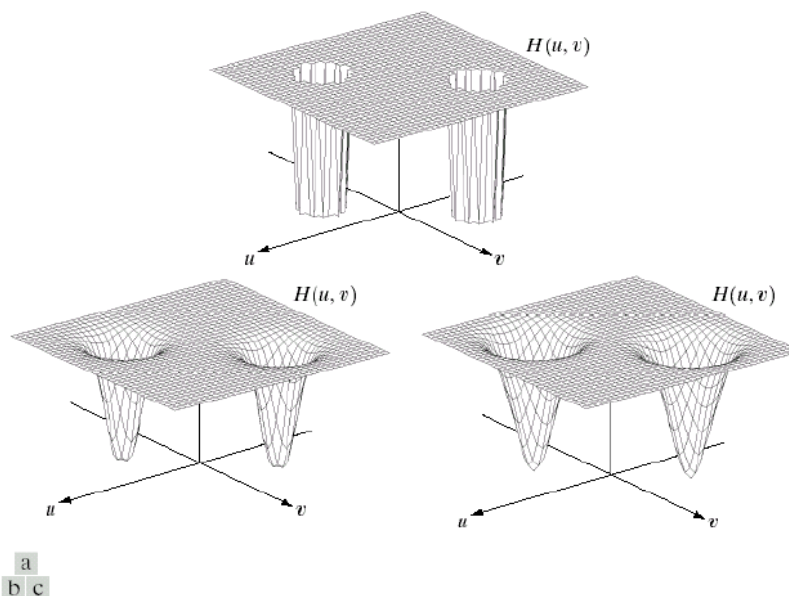

a
b c

FIGURE Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

Inverse Filtering:

The simplest approach to restoration is direct inverse filtering where we complete an estimate $\hat{F}(u, v)$ of the transform of the original image simply by dividing the transform of the degraded image G(u,v) by degradation function H(u,v)

$$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)}$$

We know that

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Therefore

$$\hat{F}(u, v) = F(u, v) + \frac{N(u, v)}{H(u, v)}$$

From the above equation we observe that we cannot recover the undegraded image exactly because N(u,v) is a random function whose Fourier transform is not known.

One approach to get around the zero or small-value problem is to limit the filter frequencies to values near the origin.

We know that H(0,0) is equal to the average values of h(x,y).

By Limiting the analysis to frequencies near the origin we reduse the probability of encountering zero values.

Minimum mean Square Error (Wiener) filtering:

The inverse filtering approach has poor performance. The wiener filtering approach uses the degradation function and statistical characteristics of noise into the restoration process.

The objective is to find an estimate $\hat{f}$ of the uncorrupted image f such that the mean square error between them is minimized.

The error measure is given by

$$e^2 = E\{[f(x) - \hat{f}(x)]^2$$

Where E{.} is the expected value of the argument.

We assume that the noise and the image are uncorrelated one or the other has zero mean.

The gray levels in the estimate are a linear function of the levels in the degraded image.

$$\hat{F}(u, v) = \left[\frac{H^*(u, v)S_f(u, v)}{S_f(u, v)|H(u, v)|^2 + S_\eta(u, v)}\right]G(u, v)$$

$$= \left[\frac{H^*(u, v)}{|H(u, v)|^2 + S_\eta(u, v)/S_f(u, v)}\right]G(u, v)$$

$$= \left[\frac{1}{H(u, v)}\frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v)/S_f(u, v)}\right]G(u, v)$$

Where H(u,v)= degradation function

H*(u,v)=complex conjugate of H(u,v)

$| H(u,v)|^2$=H* (u,v) H(u,v)

S_n(u,v)=$|N(u,v)|^2$= power spectrum of the noise

S_f(u,v)=$|F(u,v)|^2$= power spectrum of the underrated image

The power spectrum of the undegraded image is rarely known. An approach used frequently when these quantities are not known or cannot be estimated then the expression used is

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)}\frac{|H(u, v)|^2}{|H(u, v)|^2 + K}\right]G(u, v)$$

Where K is a specified constant.

Constrained least squares filtering:

The wiener filter has a disadvantage that we need to know the power spectra of the undegraded image and noise. The constrained least square filtering requires only the knowledge of only the mean and variance of the noise. These parameters usually can be calculated from a given degraded image this is the advantage with this method. This method produces a optimal result. This method require the optimal criteria which is important we express the

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

in vector-matrix form

$$\mathbf{g} = \mathbf{Hf} + \boldsymbol{\eta}$$

The optimality criteria for restoration is based on a measure of smoothness, such as the second derivative of an image (Laplacian).

The minimum of a criterion function C defined as

$$C = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\nabla^2 f(x, y)]^2$$

Subject to the constraint

$$\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\|^2 = \|\boldsymbol{\eta}\|^2$$

Where $\|\mathbf{w}\|^2 \triangleq \mathbf{w}^T\mathbf{w}$ is a euclidean vector norm $\hat{\mathbf{f}}$ is estimate of the undegraded image. ∇^2 is laplacian operator.

The frequency domain solution to this optimization problem is given by

$$\hat{F}(u, v) = \left[\frac{H^*(u, v)}{|H(u, v)|^2 + \gamma |P(u, v)|^2} \right] G(u, v)$$

Where γ is a parameter that must be adjusted so that the constraint is satisfied.

P(u,v) is the Fourier transform of the laplacian operator

$$p(x, y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$