Minimum mean Square Error (Wiener) filtering:

The inverse filtering approach has poor performance. The wiener filtering approach uses the degradation function and statistical characteristics of noise into the restoration process.

The objective is to find an estimate $\hat{f}$ of the uncorrupted image f such that the mean square error between them is minimized.

The error measure is given by

$$e^2 = E\{[f(x) - \hat{f}(x)]^2$$

Where E{.} is the expected value of the argument.

We assume that the noise and the image are uncorrelated one or the other has zero mean.

The gray levels in the estimate are a linear function of the levels in the degraded image.

$$\hat{F}(u, v) = \left[\frac{H^*(u, v)S_f(u, v)}{S_f(u, v)|H(u, v)|^2 + S_\eta(u, v)} \right] G(u, v)$$

$$= \left[\frac{H^*(u, v)}{|H(u, v)|^2 + S_\eta(u, v)/S_f(u, v)} \right] G(u, v)$$

$$= \left[\frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v)/S_f(u, v)} \right] G(u, v)$$

Where $H(u,v)$= degradation function

$H^*(u,v)$=complex conjugate of $H(u,v)$

$|H(u,v)|^2 = H^*(u,v)\,H(u,v)$

$S_n(u,v) = |N(u,v)|^2$= power spectrum of the noise

$S_f(u,v) = |F(u,v)|^2$= power spectrum of the underrated image

The power spectrum of the undegraded image is rarely known. An approach used frequently when these quantities are not known or cannot be estimated then the expression used is

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + K} \right] G(u, v)$$

Where K is a specified constant.