Efficiency of nonmatched filters. In practice the matched filter cannot always be obtained exactly. It is appropriate, therefore, to examine the efficiency of nonmatched filters compared with the ideal matched filter. The measure of efficiency is taken as the peak signal-to-noise ratio from the nonmatched filter divided by the peak signal-to-noise ratio ($2E/N_0$) from the matched filter. Figure 10.2 plots the efficiency for a single-tuned (RLC) resonant filter and a rectangular-shaped filter of half-power bandwidth B, when the input is a rectangular pulse of width τ. The maximum efficiency of the single-tuned filter occurs for $B\tau \approx 0.4$. The corresponding loss in signal-to-noise ratio is 0.88 dB as compared with a matched filter. Table 10.1 lists

Table 10.1 Efficiency of nonmatched filters compared with the matched filter

Input signal	Filter	Optimum $B\tau$	Loss in SNR compared with matched filter, dB
Rectangular pulse	Rectangular	1.37	0.85
Rectangular pulse	Gaussian	0.72	0.49
Gaussian pulse	Rectangular	0.72	0.49
Gaussian pulse	Gaussian	0.44	0 (matched)
Rectangular pulse	One-stage, single-tuned circuit	0.4	0.88
Rectangular pulse	2 cascaded single-tuned stages	0.613	0.56
Rectangular pulse	5 cascaded single-tuned stages	0.672	0.5

the values of $B\tau$ which maximize the signal-to-noise ratio (SNR) for various combinations of filters and pulse shapes. It can be seen that the loss in SNR incurred by use of these non-matched filters is small.

Matched filter with nonwhite noise. In the derivation of the matched-filter characteristic [Eq. (10.15)], the spectrum of the noise accompanying the signal was assumed to be white; that is, it was independent of frequency. If this assumption were not true, the filter which maximizes the output signal-to-noise ratio would not be the same as the matched filter of Eq. (10.15). It has been shown[11-13] that if the input power spectrum of the interfering noise is given by $[N_i(f)]^2$, the frequency-response function of the filter which maximizes the output signal-to-noise ratio is

$$H(f) = \frac{G_a S^*(f) \exp(-j2\pi f t_1)}{[N_i(f)]^2} \tag{10.19}$$

When the noise is nonwhite, the filter which maximizes the output signal-to-noise ratio is called the NWN (nonwhite noise) matched filter. For white noise $[N_i(f)]^2 =$ constant and the NWN matched-filter frequency-response function of Eq. (10.19) reduces to that of Eq. (10.15). Equation (10.19) can be written as

$$H(f) = \frac{1}{N_i(f)} \times G_a\left(\frac{S(f)}{N_i(f)}\right)^* \exp(-j2\pi f t_1) \tag{10.20}$$

This indicates that the NWN matched filter can be considered as the cascade of two filters. The first filter, with frequency-response function $1/N_i(f)$, acts to make the noise spectrum uniform, or white. It is sometimes called the *whitening filter*. The second is the matched filter described by Eq. (10.15) when the input is white noise and a signal whose spectrum is $S(f)/N_i(f)$.