UNIT-V

Detection of Radar Signals in noise:

Introduction: The two basic operations performed by radar are:

1. detection of the presence of reflecting objects, and
2. extraction of information from the received waveform to obtain such target data as position, velocity and perhaps size.

→ The operation of detection and extraction may be performe separately and in either order, although a radar that is good detection device is usually a good radar for extracting information and vice-versa.

→ The detection of radar signals in noise, clutter requires special circuitary.

→ Methods for detection of desired signals along with rejection of undesired noise, clutter and interference in radar is called signal processing.

→ Important radar signal processor includes. Matched filter Correlation receiver, matched filter with non-white noise logarithemic detector I, Q detector. Choherent detector etc.

Matched-filter Receiver:

Def: A linear network which maximizes the output peak signa-to-noise power ratio of a radar receiver maximizes the detectability of a target is called a matched filter.

(or)

A network whose frequency response function maximizes the output Peak-Signal-to-mean noise (power) ratio is called a matched filter.

→ Thus a matched filter, or a close approximation to $|H(f)|$ basis for the design of almost all radar receivers.

Matched filter frequency response function:

Matched filter maximizes the output peak SNR when the input noise spectral density is uniform (white noise). The frequency response function of matched filter is given by

$$H(f) = G_a S^*(f) e^{-j2\pi f t_m}$$

where

G_a is a constant

t_m is time at which output of matched filter is maximum.

$S^*(f)$ is complex conjugate of spectrum of input signal $s(t)$.

→ The fourier transform of received signal $s(t)$ is

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt$$

→ The received signal spetrum is now

$$S(f) = |S(f)| e^{\{j\phi_s(f)\}}$$

where $|S(f)|$ is amplitude spectrum

$|\phi_s(f)|$ is phase spectrum

→ The matched filter frequency response function in terms of amplitude and phase is expressed as

$$H(f) = |H(f)| e^{\{j\phi_m(f)\}}$$

Let $G_a = 1$ then.

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eived

$$|H(f)|\, e^{-j\phi_m(f)} = |G_a\, S^*(f)\, e^{-j2\pi f t_m}|$$

$$= |G_a|\, |S^*(f)|\, |e^{-j2\pi f t_m}|$$

$$= |S(f)|^*\, e^{-j2\pi f t_m}$$

$$= |S(f)|\, e^{+j\phi_s(f)}\, e^{-j2\pi f t_m}$$

$$|H(f)|\, e^{-j\phi_m(f)} = |S(f)|\, e^{j(\phi_s(f) - 2\pi f t_m)}$$

Equating amplitude and phases in above equation.

$$|H(f)| = |S(f)|$$

$$-\phi_m(f) = \phi_s(f) - 2\pi f t_m$$

$$\phi_m(f) = -\phi_s(f) + 2\pi f t_m$$

Above expression indicates that the magnitude of matched filter frequency response function is the Same as the amplitude Spectrum of input Signal and the phase of the matched filter frequency response is negitive ~~sign~~ before of phase spectrum of signal plus phase shift proportional to frequency

→ The negitive Sign before $\phi_s(f)$ Cancels the phase Components of received Signal so that all frequency Components at output of filter are of Same phase and add Coherently to maximize the signal.

Matched filter Impulse response:

Matched filter is also described by its impulse response h(t), which is the inverse fourier transform of the frequency response function H(f) and is given by

$$h(t) = \int_{-\infty}^{\infty} H(f)\, e^{j2\pi f t}\, df$$

But $H(f) = G_a S^*(f)\, e^{-j2\pi f t_m}$

$$= \int_{-\infty}^{\infty} G_a S^*(f)\, e^{-j2\pi f t_m} e^{j2\pi f t}\, df$$

$$= G_a \int_{-\infty}^{\infty} S^*(f)\, e^{-j2\pi f (t_m - t)}\, df$$

Since $S^*(f) = S(-f)$

$$h(t) = G_a \int_{-\infty}^{\infty} S(f)\, e^{-j2\pi (-f)(t_m - t)}\, df$$

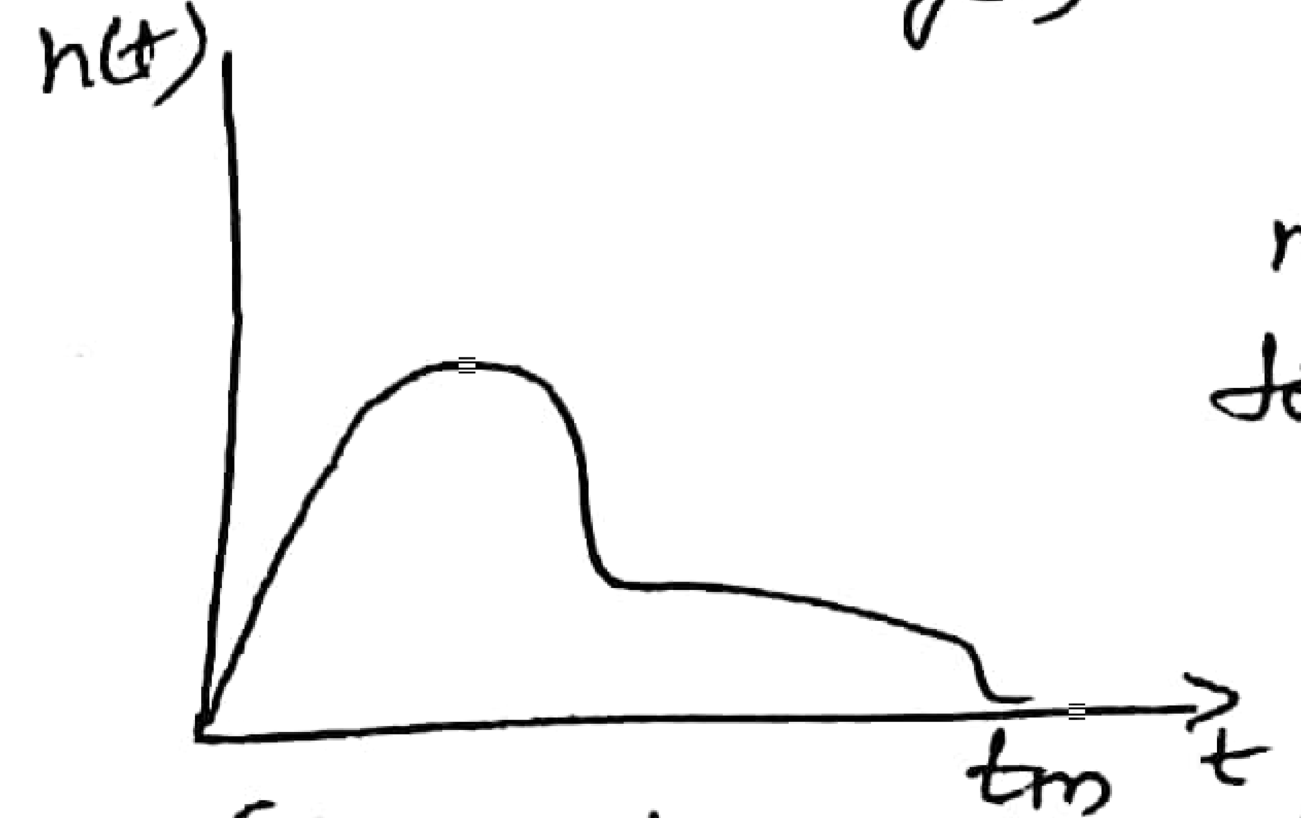
$$h(t) = G_a \int_{-\infty}^{\infty} S(f)\, e^{j2\pi f (t_m - t)}\, df$$

$$h(t) = G_a\, s(t_m - t).$$

→ This equation shows impulse response of a matched filter is the time inverse of the received signal. The received signal is reversed in time i.e starting from fixed time t_m.

→ The received signal s(t) and its impulse response h(t) of matched filter is shown in below fig(1).

(a) a received wave form s(t)



(b) impulse response h(t) of matched filter for the i/p slg

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Received signal $s(t)$ and impulse response $h(t)$ of matched filter.

· The impulse response of a filter must not have any output if the input signal is not applied. Therefore, impulse response of filter to be realizable should have $(t_m - t) > t < t_m$. This Condition is equivalent to the frequency response function with phase $(e^{-j2\pi f t_m})$, which means a time delay of t_m.

→ For Convenience, the impulse response is often written simply as $s(t)$ and the frequency response function as $S^*(f)$, with realizability Conditions understood.

Derivation of the Matched-filter frequency Response:

→ The frequency response function of matched filter is derived by using Schwartz's inequality. The frequency response function of the linear, time invarient filter which maximizes the output SNR is given by

$$H(f) = G_a S^*(f)\, e^{-j2\pi f t_m}$$

→ when the input noise is stationary and has uniform spectral density (white). The ratio to be maximized is

$$R_f = \frac{|S_o(t)|^2_{max}}{N}$$

Where

$|S_o(t)|_{max}$ = is maximum output signal

N is mean square noise power at receiver output

→ The ratio R_f is twice the average SNR when the input signal $s(t)$ is a rectangular sine pulse. The magnitude of the output voltage of a filter with frequency response function $H(f)$ is

$$|S_o(t)| = \left| \int_{-\infty}^{\infty} S(f) \cdot H(f)\, e^{j2\pi f t}\, df \right|$$

Where, $S(f)$ is the fourier transform of the input signal.

→ The mean output noise power is given as

$$N = \frac{N_0}{2} \int_{-\alpha}^{\alpha} |H(f)|^2 \, df$$

Where N_0 is the input noise power per unit bandwidth

→ The $\frac{1}{2}$ factor before integral sign is because the limits are taken from $-\alpha$ to $+\alpha$. But N_0 is defined as noise power per unit BW over positive values of f.

→ Subsitituting expression of $|S_0(t)|$ and N is the expression of R_f

$$R_f = \frac{\left| \int_{-\alpha}^{\alpha} S(f) \cdot H(f) \, e^{j2\pi f t_m} \, df \right|^2}{\frac{N_0}{2} \int_{-\alpha}^{\alpha} |H(f)|^2 \, df}$$

Assuming t_m is the time t at which output $|S_0(t)|^2$ is maximum.

→ According to Schwartz's inequality, if P and Q are two complex functions.

$$\int P^* P \, dx \int Q^* Q \, dx \geq \left| \int P^* Q \, dx \right|^2$$

→ The equality sign applies when $P = kQ$ where k is a constant

Let $P^* = S(f) \, e^{j2\pi f t_m}$ and

$Q = H(f)$

also $\int P^* P \, dx = \int |P|^2 \, dx$

→ Applying Schwartz's inequality to numerator of R_f

$$R_f = \frac{\int_{-\alpha}^{\alpha} |H(f)|^2 \, df \int_{-\alpha} |S(f)|^2 \, df}{\frac{N_0}{2} \int_{-\alpha}^{\alpha} |H(f)|^2 \, df}$$

$$= \frac{\int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_0}{2}}$$

-> Parseval's Theorem states energy in frequency and time domain states Signal energy is

$$E = \int_{-\infty}^{\infty} |S(f)|^2 df = \int_{-\infty}^{\infty} |s(t)|^2 dt = \text{Signal energy}$$

Therefore, $R_f \leq \frac{2E}{N_0}$

-> The above expression indicates that the output Peak to mean noise ratio from a matched filter depends total energy of received Signal and noise power per unit bandwidth only. It does not depend on shape of the Signal duration and bandwidth. Therefore these characteristics of signals Can be used to achieve radar Capabilities.

-> when Constant k is Set equal to $1/G_a$ then the frequency response function which maximizes peak Signal to mean noise ratio (R_f) is given by

$$H(f) = G_a S^*(f) e^{-j2\pi f t_m}$$

-> An important property of matched filter is that ~~time~~ irrespective of shape, time duration or bandwidth of input Signal waveform, the maximum ratio of output Peak Signal-to-mean noise power is twice the energy (E) Contained in the received Signal divided by noise power Per unit BW (N_0)

-> The noise power per hertz of bandwidth $n = kT_0F_n$

where

k = Boltzman's Constant

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T_0 = Standard Temperature (290°K)

F_n = Receiver noise figure.

→ The matched filter assumes that the input sign $s(t)$ is same as transmitted signal except the amplitu It requires that the shape of transmitted signal does not change due to reflection by target or by propa- -gation through at-mosphere

<u>Correlation Detection</u> :

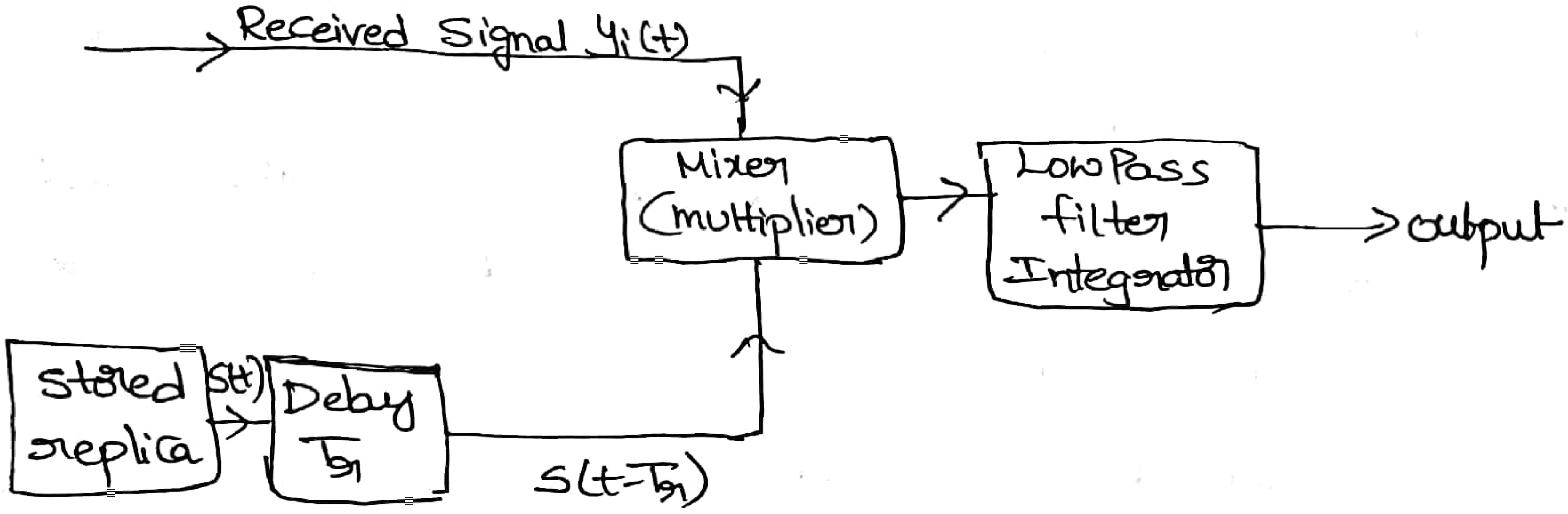

fig: Block diagram of a cross-correlation receiver

→ The output of the matched filter is the cross correlation function of the received signal and the transmitted signal

→ In a correlation receiver, the input signal $y_i(t)$ is multiplied by a delayed replica of transmitted signal $s(t-T_R)$, where T_R is an estimate of the time delay of the target echo signal

→ The product is passed through a low pass filter to perform the integration.

→ If the output of the integrator (filter) exceeds a pre-determined threshold at a time T_R, a target is said.

be at a range $R = \frac{CT_R}{2}$, where c is the velocity of propagation.

Sig ⇒ The cross correlation receiver tests for the presence of a target at only a single time delay T_R. Targets at time delay other than T_R are found by varying T_R on successive transmissions, Searching possible values of T_R complicates the correlation receiver.

→ Mathematically the cross-correlation receiver and matched filter receiver are equivalent. Hence Selection as which to use in a particular radar application is determined by ease of implementation. The matched filter receiver is preferred over correlation filter in most radar applications

→ The cross correlation receiver correlates the received signal $y_i(t)$ with stored delayed replica of known signal $s(t)$.

→ The above fig. shows the block diagram of cross-correlation receiver.

→ The correlation receiver performs cross-correlation between signal $y_i(t)$ corrupted by noise and replica of transmitted signal $S(t)$. The correlation receiver is a linear, time-invarient receiver and linear, time invariant filter which maximizes output peak signal to mean noise power ratio for a fixed input signal to noise ratio.

→ The signal energy is given by

$$E = \int_{-\infty}^{\infty} |S(f)|^2 \, df$$

→ The maximum ratio of peak signal power to the noise power is proportional to energy spectral density of the input signal, irrespective of the shape of input wave.

Detection Criteria:

→ Detection of signals is equivalent to deciding whether the receiver output is due to noise alone or to signal plus noise. This is the type of decision made by a human operator from the information presented on a radar display

→ When the detection process is carried out automatically by electronic means without the aid of an operator, the detect criterion must be carefully specified and built into the decision making device.

→ The radar detection process was described in terms of threshold detection. If the envelope of the receiver output exceeds a pre-established threshold, a signal is said to be present.

→ The threshold level divides the output into a region of no detection and a region of detection.

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...ncy of non-matched filters: The efficiency of non-matched filters compared with the ideal matched filter. The measure of efficiency is taken as the peak signal to-noise ratio from the non-matched filter divided by the peak signal-to-noise ratio from the matched filter.

-> The efficiency for a single-tuned (RLC) resonant filter and rectangular-shaped filter of half-power bandwidth B, when the input is a rectangular pulse of width τ. The maximum efficiency of the single-tuned filter occurs for $B\tau \approx 0.4$

-> Efficiency of non-matched filters compared with the matched filter

Input Signal	Filter	optimum $B\tau$	Loss in SNR compared with matched filter, dB
Rectangular pulse	Rectangular	1.37	0.85
Rectangular Pulse	Gaussian	0.72	0.49
Gaussian Pulse	Rectangular	0.72	0.49
Rectangular pulse	one-stage Gaussian	0.44	0.88
Rectangular pulse	2 Cascaded Single-tuned Stages	0.613	0.56
Rectangular pulse	5 Cascaded Single-tuned Stages	0.672	0.5

-> The values of $B\tau$ which maximize the signal-to-noise-ratio (SNR) for various combinations of filters and pulse shapes. It can be seen that the loss in SNR incurred by use of these non-matched filters is small.

Matched filter with non white noise:

→ The signal was assumed to be white. that is it was in-dent of frequency. If this assumption were not true, the filter which maximizes the output signal-to-noise ratio would not be the same as the matched filter.

The frequency-response function of the filter which maximizes the output signal-to noise ratio is

$$H(f) = \frac{G_a S^*(f)\, e^{-j2\pi f t_1}}{[N_i(f)]^2}$$

When the noise in non-white, the filter which maximizes the output signal-to-noise ratio is, called the NWN (non-white noise) matched filter. for white noise $[N_i(f)]^2$ = constant and the NWN matched-filter frequency-response function and the equation can be written as.

$$H(f) = \frac{1}{N_i(f)} \times G_a \left(\frac{S(f)}{N_i(f)} \right) e^{-j2\pi f t_1}$$

→ This indicates that the NWN matched filter can be considered as the cascade of two filters. The first filter with frequency-response function $1/N_i(f)$, acts to make the noise spectrum uniform or white. It is sometimes called the whitening filter. The second is the matched filter described. when the input is white noise and a signal whose spectrum is $S(f)/N_i(f)$.

Noise figure of a receiver:

The noise figure of a receiver can be described as a measure of the noise produced by a practical receiver compared to the noise of an ideal receiver.

-> The noise figure, F_n of a linear network may be defined as

$$\text{Noise figure } F_n = \frac{\text{Input Signal-to-noise ratio}}{\text{output Signal-to-noise ratio}}$$

$$F_n = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} = \frac{N_{out}}{kT_0 B_n G}$$

where

S_{in} = Available input signal power

N_{in} = Available input noise power $= kT_0 B_n$

S_{out} = Available output Signal power

N_{out} = Available output Noise power

$G = \frac{S_{out}}{S_{in}}$ = Available gain

k = Boltzman's Constant $= 1.38 \times 10^{-23}$ J/deg

T_0 = Standard Temparature of 290K

B_n = Noise Bandwidth

-> The above equation allows two different but equivalent interpretations of noise figure.

-> It may be interpreted as the degradation of the signal to noise ratio caused by the receiver or may be considered a the ratio of the actual available output noise power to the noise power which would be available if the network amplified the thermal noise.

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→ The noise figure can alternately be expressed.

$$F_n = \frac{kT_0 B_n G_1 + \Delta N}{kT_0 B_n G_1} = 1 + \frac{\Delta N}{kT_0 B_n G_1}$$

Where ΔN is the additional noise introduced by the practical network.

→ The noise figure is commonly expressed in decibels
i.e

$$F_n(dB) = 10 \log_{10} F_n$$

→ Sometimes instead of the term noise figure the term noise factor is also used, when F_n is expressed as a ratio.

Noise figure of networks in Cascade:

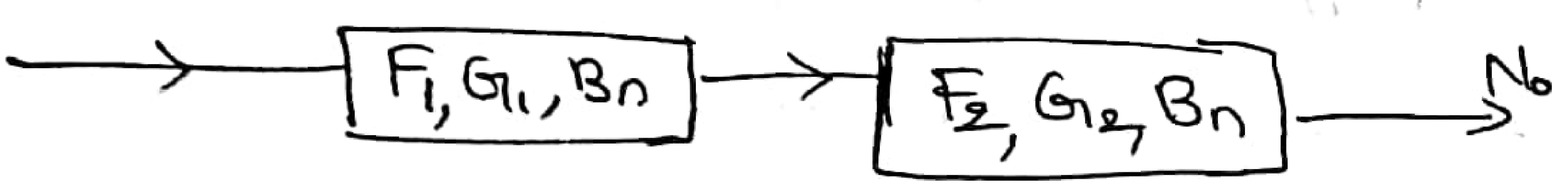

fig: Two networks in Cascade.

→ Let us consider two networks in cascade, each of the same noise bandwidth B_n but of different noise figures F_1 and F_2 and available gains G_1 and G_2 respectively. This is shown in above fig.

→ To find the overall noise figure F_0 of the two circuits in cascade, we may write the output noise N_0 of two circuits as.

$N_0 = F_0 G_1 G_2 kT_0 B_n$ = Noise from network 1 at output of network 2 + noise ΔN_2 introduced by network 2

$$= kT_0 B_n F_1 G_1 G_2 + \Delta N_2$$

$$= kT_0 B_n F_1 G_1 G_2 + (F_2 - 1) kT_0 B_n G_2$$

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$$F_0 = \frac{kT_0 B_n F_1 G_1 G_2 + (F_2-1) kT_0 B_n G_2}{kT_0 B_n G_1 G_2}$$

$$F_0 = F_1 + \frac{F_2-1}{G_1}$$

the If the gain of the first network is large than one may neglect the contribution of the second network. For the design of multistage receivers, this concept is important

-> If N number of networks are cascaded then the noise figure can be shown, in the above manner to be.

$$F_0 = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1 G_2} + \cdots + \frac{F_n-1}{G_1 G_2 \cdots G_{N-1}}$$

Noise Temperature: (Effective Noise temperature T_e)

-> The noise introduced by a network can be expressed as an effective noise temperature of the receiver system, including the effects of antenna temperature T_a.

-> If T_e represents the effective noise temperature, then

$$T_S = T_a + T_e = T_0 T_S$$

where F_S = System noise figure.

-> The effective noise temperature of receiver consisting of a number of networks in cascade is

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots$$

where T_1 and G_1 are the effective noise temperature and gain of the 1st network, T_2 and G_2 are those for the 2nd network and so on.

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Noise figure Measurement:

-> The receiver noise figure is measured with a broad-band noise source of known intensity. The noise figure F_n can't shown to be.

$$F_n = \frac{T_2/T_0 - 1}{Y - 1}$$

Where $Y = \frac{N_2}{N_1}$

The noise figure is found by measuring

N_1 = the noise power output N_1 of the receiver when an Impedance at $T_0 = 290^\circ k$ is Connected to the receiver input

N_2 = the noise power output N_2 when a matched noise generator at T_2 is Connected to the receiver input