2.3 RECEIVER NOISE

Since noise is the chief factor limiting receiver sensitivity, it is necessary to obtain some means of describing it quantitatively. Noise is unwanted electromagnetic energy which interferes with the ability of the receiver to detect the wanted signal. It may originate within the receiver itself, or it may enter via the receiving antenna along with the desired signal. If the radar were to operate in a perfectly noise-free environment so that no external sources of noise accompanied the desired signal, and if the receiver itself were so perfect that it did not generate any excess noise, there would still exist an unavoidable component of noise generated by the thermal motion of the conduction electrons in the ohmic portions of the receiver input stages. This is called *thermal noise, or Johnson noise*, and is directly proportional to the temperature of the ohmic portions of the circuit and the receiver bandwidth.[60] The available thermal-noise power generated by a receiver of bandwidth B_n (in hertz) at a temperature T (degrees Kelvin) is equal to

$$\text{Available thermal-noise power} = kTB_n \tag{2.2}$$

where k = Boltzmann's constant = 1.38×10^{-23} J/deg. If the temperature T is taken to be 290 K, which corresponds approximately to room temperature (62°F), the factor kT is 4×10^{-21} W/Hz of bandwidth. If the receiver circuitry were at some other temperature, the thermal-noise power would be correspondingly different.

A receiver with a reactance input such as a parametric amplifier need not have any significant ohmic loss. The limitation in this case is the thermal noise seen by the antenna and the ohmic losses in the transmission line.

For radar receivers of the superheterodyne type (the type of receiver used for most radar applications), the receiver bandwidth is approximately that of the intermediate-frequency stages. It should be cautioned that the bandwidth B_n of Eq. (2.2) is not the 3-dB, or half-power, bandwidth commonly employed by electronic engineers. It is an integrated bandwidth and is given by

$$B_n = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \, df}{|H(f_0)|^2} \tag{2.3}$$

where $H(f)$ = frequency-response characteristic of IF amplifier (filter) and f_0 = frequency of maximum response (usually occurs at midband).

When $H(f)$ is normalized to unity at midband (maximum-response frequency), $H(f_0) = 1$. The bandwidth B_n is called the *noise bandwidth* and is the bandwidth of an equivalent rectangular filter whose noise-power output is the same as the filter with characteristic

$H(f)$. The 3-dB bandwidth is defined as the separation in hertz between the points on the frequency-response characteristic where the response is reduced to 0.707 (3 dB) from its maximum value. The 3-dB bandwidth is widely used, since it is easy to measure. The measurement of noise bandwidth, however, involves a complete knowledge of the response characteristic $H(f)$. The frequency-response characteristics of many practical radar receivers are such that the 3-dB and the noise bandwidths do not differ appreciably. Therefore the 3-dB bandwidth may be used in many cases as an approximation to the noise bandwidth.[7]

The noise power in practical receivers is often greater than can be accounted for by thermal noise alone. The additional noise components are due to mechanisms other than the thermal agitation of the conduction electrons. For purposes of the present discussion, however, the exact origin of the extra noise components is not important except to know that it exists. No matter whether the noise is generated by a thermal mechanism or by some other mechanism, the total noise at the output of the receiver may be considered to be equal to the thermal-noise power obtained from an "ideal" receiver multiplied by a factor called the *noise figure*. The noise figure F_n of a receiver is defined by the equation

$$F_n = \frac{N_o}{kT_0 B_n G_a} = \frac{\text{noise out of practical receiver}}{\text{noise out of ideal receiver at std temp } T_0} \tag{2.4a}$$

where N_o = noise output from receiver, and G_a = available gain. The standard temperature T_0 is taken to be 290 K, according to the Institute of Electrical and Electronics Engineers definition. The noise N_o is measured over the linear portion of the receiver input-output characteristic, usually at the output of the IF amplifier before the nonlinear second detector. The receiver bandwidth B_n is that of the IF amplifier in most receivers. The available gain G_a is the ratio of the signal out S_o to the signal in S_i, and $kT_0 B_n$ is the input noise N_i in an ideal receiver. Equation (2.4*a*) may be rewritten as

$$F_n = \frac{S_i/N_i}{S_o/N_o} \tag{2.4b}$$

The noise figure may be interpreted, therefore, as a measure of the degradation of signal-to-noise-ratio as the signal passes through the receiver.

Rearranging Eq. (2.4*b*), the input signal may be expressed as

$$S_i = \frac{kT_0 B_n F_n S_o}{N_o} \tag{2.5}$$

If the minimum detectable signal S_{min} is that value of S_i corresponding to the minimum ratio of output (IF) signal-to-noise ratio $(S_o/N_o)_{min}$ necessary for detection, then

$$S_{min} = kT_0 B_n F_n \left(\frac{S_o}{N_o}\right)_{min} \tag{2.6}$$

Substituting Eq. (2.6) into Eq. (2.1) results in the following form of the radar equation:

$$R_{max}^4 = \frac{P_t G A_e \sigma}{(4\pi)^2 kT_0 B_n F_n (S_o/N_o)_{min}} \tag{2.7}$$

Before continuing the discussion of the factors involved in the radar equation, it is necessary to digress and review briefly some topics in probability theory in order to describe the signal-to-noise ratio in statistical terms.