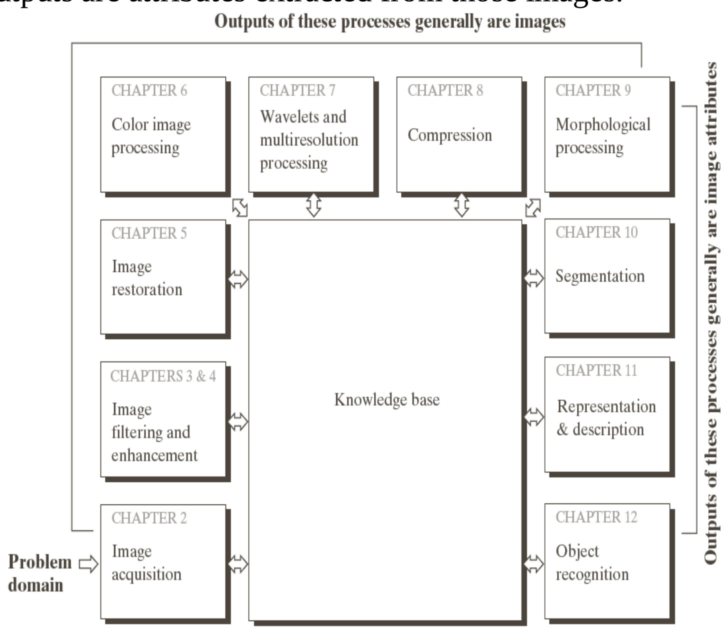
Introduction

What Is Digital Image Processing?

An image may be defined as a two-dimensional function, f(x, y), where x and y are *spatial* (plane) coordinates, and the amplitude of f at any pair of coordinates (x, y) is called the *intensity* or *gray level* of the image at that point. When x, y, and the amplitude values of f are all finite, discrete quantities, we call the image a *digital image*. The field of *digital image processing* refers to processing digital images by means of a digital computer. Note that a digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are referred to as *picture elements*, *image elements*, *pels*, and *pixels*. *Pixel* is the term most widely used to denote the elements of a digital image.

Fundamental Steps in Digital Image Processing

There are two categories of the steps involved in the image processing
1. Methods whose outputs are input are images.
2. Methods whose outputs are attributes extracted from those images.



i) Image acquisition

It could be as simple as being given an image that is already in digital form. Generally the image acquisition stage involves processing such as scaling.

ii) Image Enhancement

It is among the simplest and most appealing areas of digital image processing. The idea behind this is to bring out details that are obscured or simply to highlight certain features of interest in image. Image enhancement is a very subjective area of image processing.

iii) Image Restoration
It deals with improving the appearance of an image. It is an objective approach, in the sense that restoration techniques tend to be based on mathematical or probabilistic models of image processing. Enhancement, on the other hand is based on human subjective preferences regarding what constitutes a "good" enhancement result.

iv) Color image processing
It is an area that is been gaining importance because of the use of digital images over the internet. Color image processing deals with basically color models and their implementation in image processing applications.

v) Wavelets and Multiresolution Processing
These are the foundation for representing image in various degrees of resolution

vi) Compression
It deals with techniques reducing the storage required to save an image, or the bandwidth required to transmit it over the network. It has to major approaches:
a) Lossless Compression
b) Lossy Compression

vii) Morphological processing
It deals with tools for extracting image components that are useful in the representation and description of shape and boundary of objects. It is majorly used in automated inspection Applications.

viii) Representation and Description
It always follows the output of segmentation step that is, raw pixel data, constituting either the boundary of an image or points in the region itself. In either case converting the data to a form suitable for computer processing is necessary.

ix) Recognition
It is the process that assigns label to an object based on its descriptors. It is the last step of image processing which use artificial intelligence software.

x) Knowledge base
Knowledge about a problem domain is coded into an image processing system in the form of a knowledge base. This knowledge may be as simple as detailing regions of an image where the information of the interest in known to be located. Thus limiting search that has to be conducted in seeking the information. The knowledge base also can be quite complex such interrelated list of all major possible defects in a materials inspection problems or an image database containing high resolution satellite images of a region in connection with change detection application

Components of Image Processing System

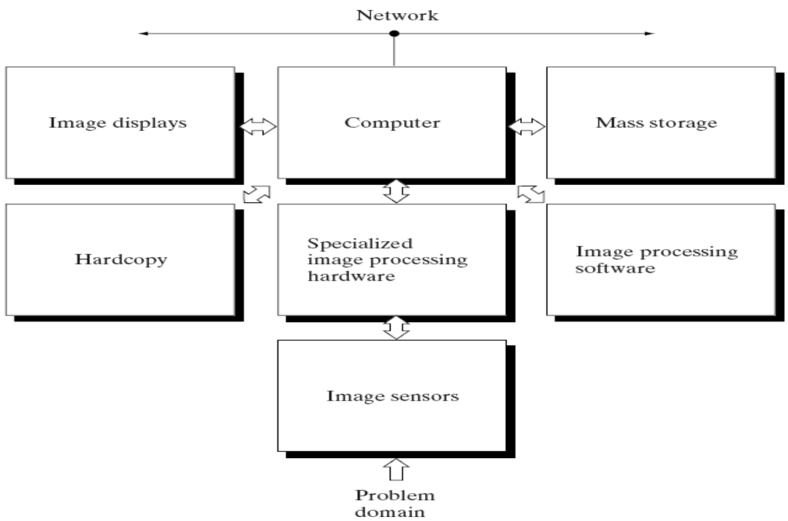


Image Sensors
With reference to sensing, two elements are required to acquire digital image. The first is a physical device that is sensitive to the energy radiated by the object we wish to image and second is specialized image processing hardware.

Specialized image processing hardware:
It consists of the digitizer just mentioned, plus hardware that performs other primitive operations such as an arithmetic logic unit, which performs arithmetic such addition and subtraction and logical operations in parallel on images.

Computer:
It is a general purpose computer and can range from a PC to a supercomputer depending on the application. In dedicated applications, sometimes specially designed computer are used to achieve a required level of performance.

Software
It consist of specialized modules that perform specific tasks a well designed package also includes capability for the user to write code, as a minimum, utilizes the specialized module. More sophisticated software packages allow the integration of these modules.

Mass storage
This capability is a must in image processing applications. An image of size 1024 x1024 pixels, in which the intensity of each pixel is an 8- bit quantity requires one megabytes of storage space if the image is not compressed.

Image processing applications falls into three principal categories of storage

i) Short term storage for use during processing

ii) On line storage for relatively fast retrieval
iii) Archival storage such as magnetic tapes and disks

Image displays
Image displays in use today are mainly color TV monitors. These monitors are driven by the outputs of image and graphics displays cards that are an integral part of computer system

Hardcopy devices
The devices for recording image includes laser printers, film cameras, heat sensitive devices inkjet units and digital units such as optical and CD ROM disk. Films provide the highest possible resolution, but paper is the obvious medium of choice for written applications.

Networking
It is almost a default function in any computer system in use today because of the large amount of data inherent in image processing applications. The key consideration in image transmission bandwidth.

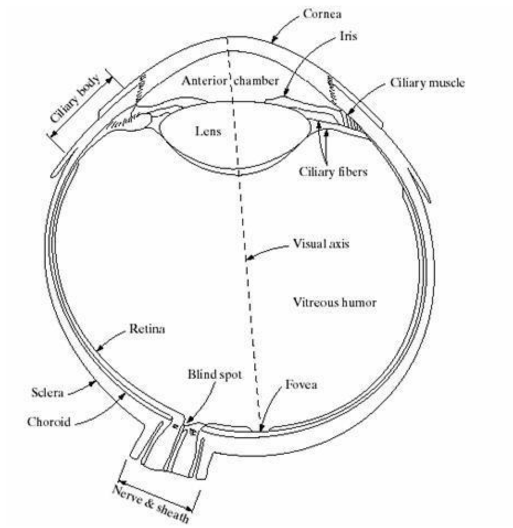
Elements of Visual Perception

Structure of the human Eye:
The eye is nearly a sphere with average approximately 20 mm diameter. The eye is enclosed with three membranes

a) **The cornea and sclera:** it is a tough, transparent tissue that covers the anterior surface of the eye. Rest of the optic globe is covered by the sclera
b) **The choroid:** It contains a network of blood vessels that serve as the major source of nutrition to the eyes. It helps to reduce extraneous light entering in the eye It has two parts

(1) Iris Diaphragms- it contracts or expands to control the amount of light that enters the eyes.
(2) Ciliary body

The *lens* is made up of concentric layers of fibrous cells and is suspended by fibers that attach to the ciliary body. It contains 60 to 70% water, about 6% fat, and more protein than any other tissue in the eye.
The lens is colored by a slightly yellow pigmentation that increases with age. In extreme cases, excessive clouding of the lens, caused by the affliction commonly referred to as *cataracts*, can lead to poor color discrimination and loss of clear vision.
c) **Retina:** it is innermost membrane of the eye. When the eye is properly focused, light from an object outside the eye is imaged on the retina. There are various light receptors over the surface of the retina



The two major classes of the receptors are-
1) **Cones**- it is in the number about 6 to 7 million. These are located in the central portion of the retina called the fovea. These are highly sensitive to color. Human can resolve fine details with these cones because each one is connected to its own nerve end. Cone vision is called **photopic or bright light vision**
2) **Rods** – these are very much in number from 75 to 150 million and are distributed over the entire retinal surface. The large area of distribution and the fact that several roads are connected to a single nerve give a general overall picture of the field of view.They are not involved in the color vision and are sensitive to low level of illumination. Rod vision is called is **scotopic or dim light vision.**
The absent of reciprocators is called **blind spot**. Figure shows the density of rods and cones for a cross section of the right eye passing through the region of emergence of the optic nerve from the eye.

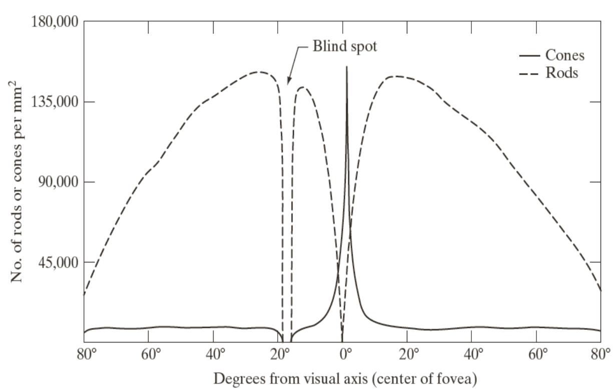
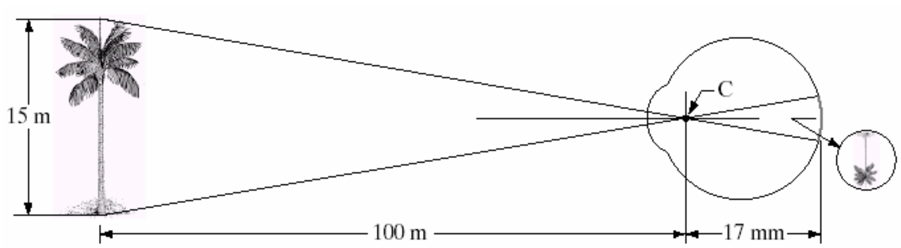


Image Formation in the Eye:

- The major difference between the lens of the eye and an ordinary optical lens in that the former is flexible.
- The shape of the lens of the eye is controlled by tension in the fiber of the ciliary body. To focus on the distant object the controlling muscles allow the lens to become thicker in order to focus on object near the eye it becomes relatively flattened.
- The distance between the center of the lens and the retina is called the focal length and it

varies from 17mm to 14mm as the refractive power of the lens increases from its minimum to its maximum**.**

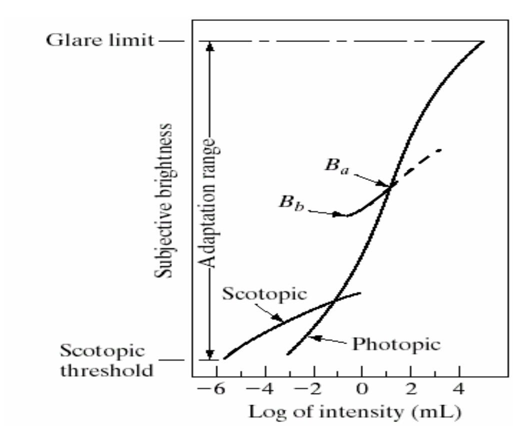
When the eye focuses on an object farther away than about 3m the lens exhibits its lowest refractive power. When the eye focuses on a nearly object. The lens is most strongly refractive. The retinal image is reflected primarily in the area of the fovea. Perception then takes place by the relative excitation of light receptors, which transform radiant energy into electrical impulses that are ultimately decoded by the brain.



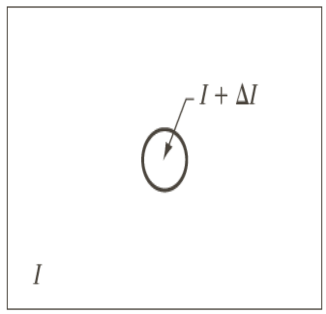
The geometry in Fig. illustrates how to obtain the dimensions of an image formed on the retina. For example, suppose that a person is looking at a tree 15 m high at a distance of 100 m. Letting *h* denote the height of that object in the retinal image, the geometry of Fig. yields 15/100 = h/17 or h = 2.55mm. The retinal image is focused primarily on the region of the fovea. Perception then takes place by the relative excitation of light receptors, which transform radiant energy into electrical impulses that ultimately are decoded by the brain.

Brightness Adaption and Discrimination:

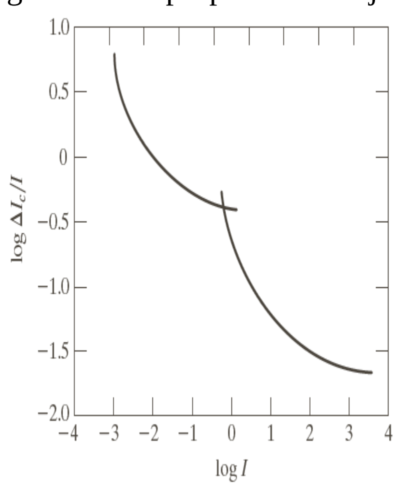
Digital image are displayed as a discrete set of intensities. The range of light intensity levels to which the human visual system can adopt is enormous- on the order of 1010 from scotopic threshold to the glare limit. Experimental evidences indicate that subjective brightness is a logarithmic function of the light intensity incident on the eye.



The curve represents the range of intensities to which the visual system can adopt. But the visual system cannot operate over such a dynamic range simultaneously. Rather, it is accomplished by change in its overcall sensitivity called brightness adaptation. For any given set of conditions, the current sensitivity level to which of the visual system is called brightness adoption level , Ba in the curve. The small intersecting curve represents the range of subjective brightness that the eye can perceive when adapted to this level. It is restricted at level Bb , at and below which all stimuli are perceived as indistinguishable blacks. The upper portion of the curve is not actually restricted. Whole simply raise the adaptation level higher than Ba. The ability of the eye to discriminate between change in light intensity at any specific adaptation level is also of considerable interest. Take a flat, uniformly illuminated area large enough to occupy the entire field of view of the subject. It may be a diffuser such as an opaque glass, that is illuminated from behind by a light source whose intensity, I can be varied. To this field is added an increment of illumination ΔI in the form of a short duration flash that appears as circle in the center of the uniformly illuminated field. If ΔI is not bright enough, the subject cannot see any perceivable changes.

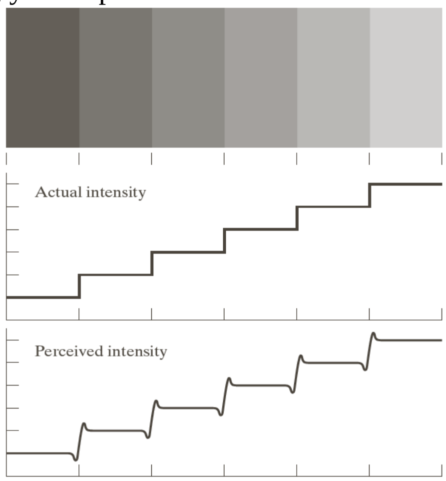


As ΔI gets stronger the subject may indicate of a perceived change. ΔIc is the increment of illumination discernible 50% of the time with background illumination I. Now, ΔIc /I is called the Weber ratio.
Small value means that small percentage change in intensity is discernible representing "good" brightness discrimination.
Large value of Weber ratio means large percentage change in intensity is required representing "poor brightness discrimination". Optical illusion In this the eye fills the non existing information or wrongly pervious geometrical properties of objects

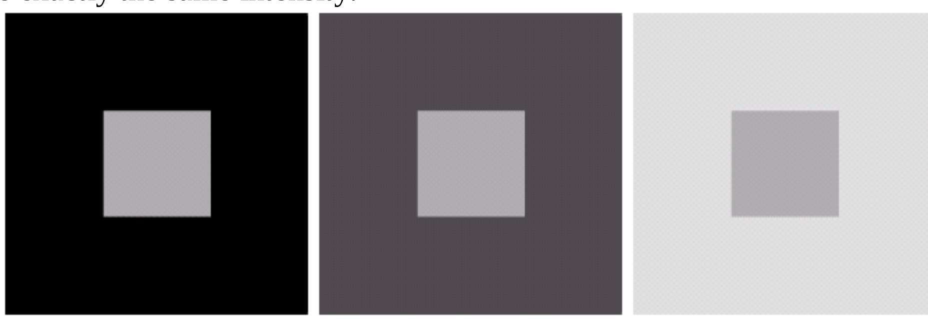


A plot of as a function of log *I* has the general shape shown in Fig. This curve shows that brightness discrimination is poor (the Weber ratio is large) at low levels of illumination, and it improves significantly (the Weber ratio decreases) as background illumination increases.

Two phenomena clearly demonstrate that perceived brightness is not a simple function of intensity. The first is based on the fact that the visual system tends to undershoot or overshoot around the boundary of regions of different intensities. Figure 2.7(a) shows a striking example of this phenomenon. Although the intensity of the stripes is constant, we actually perceive a brightness pattern that is strongly scalloped near the boundaries fig. These seemingly scalloped bands are called *Mach bands*.



The second phenomenon, called *simultaneous contrast*, is related to the fact that a region's perceived brightness does not depend simply on its intensity, as Fig demonstrates. All the center squares have exactly the same intensity.

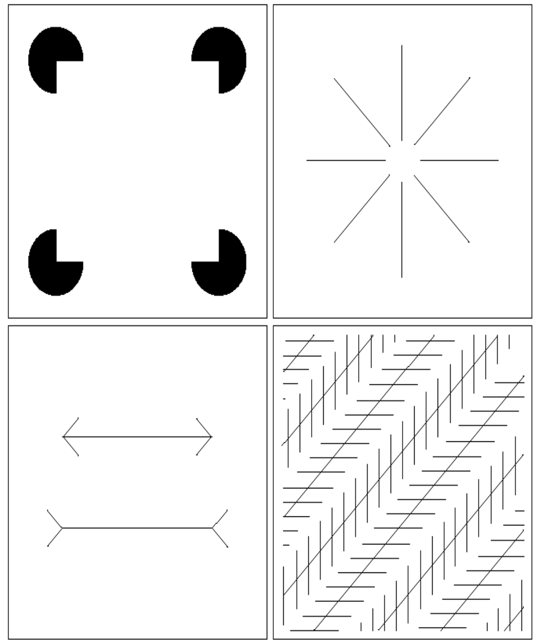


a b c

FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.

However, they appear to the eye to become darker as the background gets lighter.

Other examples of human perception phenomena are optical illusions, in which the eye fills in nonexisting information or wrongly perceives geometrical properties of objects. Figure shows some examples. In Fig.a. the outline of a square is seen clearly, despite the fact that no lines defining such a figure are part of the image.The same effect, this time with a circle, can be seen in Fig.(b); note how just a few lines are sufficient to give the illusion of a complete circle. The two horizontal line segments in Fig. (c) are of the same length, but one appears shorter than the other. Finally, all lines in Fig.(d) that are oriented at 45° are equidistant and parallel. Yet the crosshatching creates the illusion that those lines are far from being parallel. Optical illusions are a characteristic of the human visual system that is not fully understood.



a b
c d

FIGURE 2.9 Some well-known optical illusions.

Image Sensing and Acquisition:

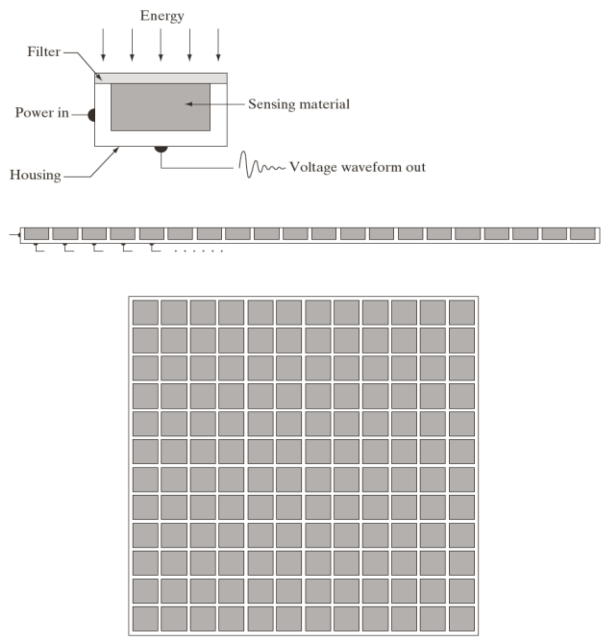
The types of images in which we are interested are generated by the combination of an "illumination" source and the reflection or absorption of energy from that source by the elements of the "scene" being imaged.

Depending on the nature of the source, illumination energy is reflected from, or transmitted through, objects. An example in the first category is light reflected from a planar surface. An example in the second category is when X-rays pass through a patient's body for the purpose of generating a diagnostic X-ray film. In some applications, the reflected or transmitted energy is focused onto a photo converter (e.g., a phosphor screen), which converts the energy into visible light. Electron microscopy and some applications of gamma imaging use this approach.

The idea is simple: Incoming energy is transformed into a voltage by the combination of input electrical power and sensor material that is responsive to the particular type of energy being detected.

The output voltage waveform is the response of the sensor(s), and a digital quantity is obtained from each sensor by digitizing its response. In this section, we look at the principal modalities for image sensing and generation.

Figure 2.12 shows the three principal sensor arrangements used to transform illumination energy into digital images.The idea is simple: Incoming energy is transformed into a voltage by the combination of input electrical power and sensor material that is responsive to the particular type of energy being detected.The output voltage waveform is the response of the sensor(s), and a digital quantity is obtained from each sensor by digitizing its response. In this section, we look at the principal modalities for image sensing and generation.



a
b
c

FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Image Acquisition Using a Single Sensor:

The components of a single sensor. The most familiar sensor of this type is the photodiode, which is constructed of silicon materials and whose output voltage waveform is proportional to light. The use of a filter in front of a sensor improves selectivity.

For example, a green (pass) filter in front of a light sensor favors light in the green band of the color spectrum. As a consequence, the sensor output will be stronger for green light than for other components in the visible spectrum.

In order to generate a 2-D image using a single sensor, there has to be relative displacements in both the x- and y-directions between the sensor and the area to be imaged. Figure 2.13 shows an arrangement used in high-precision scanning, where a film negative is mounted onto a drum whose mechanical rotation provides displacement in one dimension. The single sensor is mounted on a lead screw that provides motion in the perpendicular direction. Since mechanical motion can be controlled with high precision, this method is an inexpensive (but slow) way to obtain high-resolution images. Other similar mechanical arrangements use a flat bed, with the sensor moving in two linear directions. These types of mechanical digitizers sometimes are referred to as *microdensitometers*.

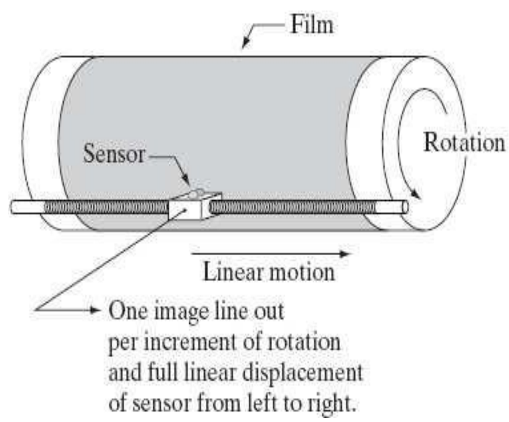


Image Acquisition Using Sensor Strips:

- ✓ A geometry that is used much more frequently than single sensors consists of an in-line arrangement of sensors in the form of a sensor strip, shows.
- ✓ The strip provides imaging elements in one direction. Motion perpendicular to the strip provides imaging in the other direction. This is the type of arrangement used in most flat bed scanners.
- ✓ Sensing devices with 4000 or more in-line sensors are possible.
- ✓ In-line sensors are used routinely in airborne imaging applications, in which the imaging system is mounted on an aircraft that flies at a constant altitude and speed over the geographical area to be imaged.
- ✓ One- dimensional imaging sensor strips that respond to various bands of the electromagnetic spectrum are mounted perpendicular to the direction of flight.
- ✓ The imaging strip gives one line of an image at a time, and the motion of the strip completes the other dimension of a two-dimensional image.
- ✓ Lenses or other focusing schemes are used to project area to be scanned onto the sensors.
- ✓ Sensor strips mounted in a ring configuration are used in medical and industrial imaging to obtain cross-sectional ("slice") images of 3-D objects

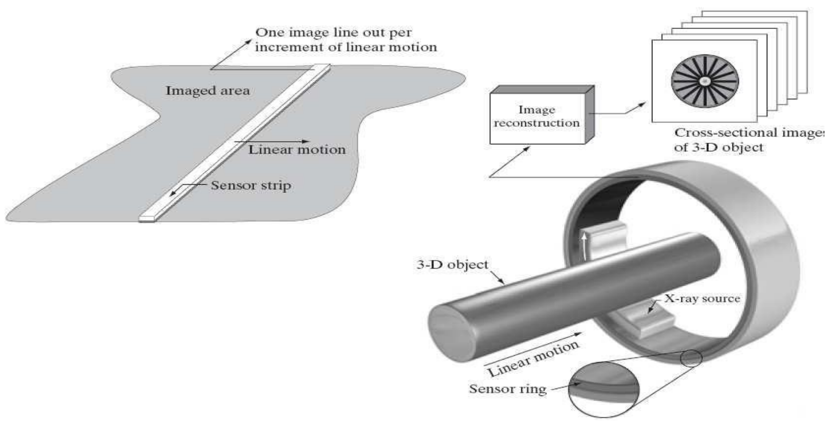
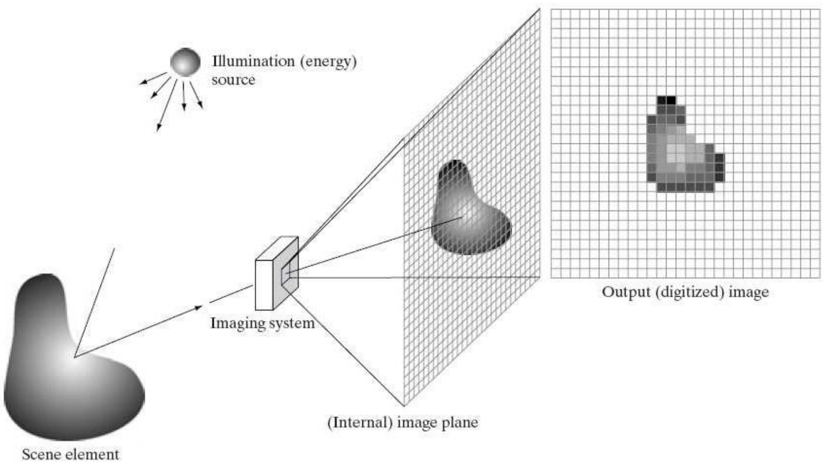


Image Acquisition Using Sensor Arrays*:*

The individual sensors arranged in the form of a 2-D array. Numerous electromagnetic and some ultrasonic sensing devices frequently are arranged in an array format. A typical sensor for these cameras is a CCD array.

CCD sensors are used widely in digital cameras and other light sensing instruments. The response of each sensor is proportional to the integral of the light energy projected onto the surface of the sensor, a property that is used in astronomical and other applications requiring low noise images. Noise reduction is achieved by letting the sensor integrate the input light signal over minutes or even hours. The two dimensional, its key advantage is that a complete image can be obtained by focusing the energy pattern onto the surface of the array. Motion obviously is not necessary, as is the case with the sensor arrangements

This figure shows the energy from an illumination source being reflected from a scene element, but, as mentioned at the beginning of this section, the energy also could be transmitted through the scene elements. The first function performed by the imaging system is to collect the incoming energy and focus it onto an image plane. If the illumination is light, the front end of the imaging system is a lens, which projects the viewed scene onto the lens focal plane. The sensor array, which is coincident with the focal plane, produces outputs proportional to the integral of the light received at each sensor. Digital and analog circuitry sweep these outputs and convert them to a video signal, which is then digitized by another section of the imaging system



A Simple Image Formation Model:

An image is a two-dimensional function of the form $f(x, y)$. The value or amplitude of f at spatial coordinates (x, y) is a positive scalar quantity whose physical meaning is determined by the source of the image. When an image is generated from a physical process, its intensity values are proportional to energy radiated by a physical source.
Hence,

$$0 < f(x, y) < \infty$$

The function $f(x, y)$ may be characterized by two components: (1) the amount of source illumination incident on the scene being viewed, and (2) the amount of illumination reflected by the objects in the scene. Appropriately, these are called the *illumination* and *reflectance* components and are denoted by $i(x, y)$ and $r(x, y)$, respectively. The two functions combine as a product to form :

$$f(x, y) = i(x, y)\, r(x, y)$$

where

$$0 < i(x, y) < \infty \text{ and } 0 < r(x, y) < \infty$$

Reflectance is bounded by 0 (total absorption) and 1 (total reflectance)
Let the intensity (gray level) of a monochrome image at any coordinates (x0, y0) be denoted by

$$l = f(x_o, y_o)$$

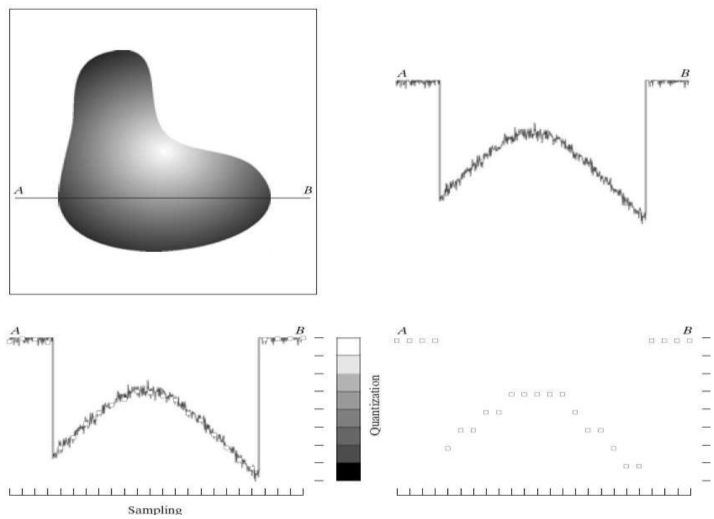
l lies in the range $L_{min} \leq l \leq L_{max}$

$L_{min} = i_{min} r_{min}$ *and* $L_{max} = i_{max} r_{max}$

The interval [L_{min}, L_{max}] is called the *gray* (or *intensity*) *scale*. Common practice is to shift this interval numerically to the interval [0, L-1], where l = 0 is considered black and l = L-1is considered white on the gray scale. All intermediate values are shades of gray varying from black to white.

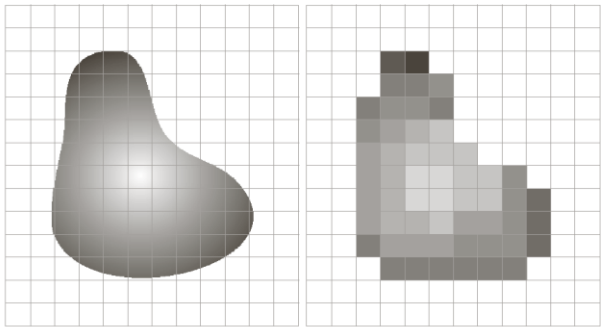
Image Sampling and Quantization:

To create a digital image, we need to convert the continuous sensed data into digital form. This involves two processes: *sampling* and *quantization*. A continuous image, f(x, y), that we want to convert to digital form. An image may be continuous with respect to the x- and y-coordinates, and also in amplitude. To convert it to digital form, we have to sample the function in both coordinates and in amplitude. Digitizing the coordinate values is called *sampling*. Digitizing the amplitude values is called *quantization*.



The one-dimensional function shown in Fig. 2.16(b) is a plot of amplitude (gray level) values of the continuous image along the line segment AB. The random variations are due to image noise. To sample this function, we take equally spaced samples along line AB, The location of each sample is given by a vertical tick mark in the bottom part of the figure. The samples are shown as small white squares superimposed on the function. The set of these discrete locations gives the sampled function. However, the values of the samples still span (vertically) a continuous range of gray-level values. In order to form a digital function, the gray-level values also must be converted (*quantized*) into discrete quantities. The right side gray-level scale divided into eight discrete levels, ranging from black to white. The vertical tick marks indicate the specific value assigned to each of the eight gray levels. The continuous gray levels are quantized simply by assigning one of the eight discrete gray levels to each sample. The assignment is made depending on the vertical proximity of a sample to a vertical tick mark. The digital samples resulting from both sampling and quantization.

When a sensing array is used for image acquisition, there is no motion and the number of sensors in the array establishes the limits of sampling in both directions. Quantization of the sensor outputs is as before. Figure 2.17 illustrates this concept. Figure 2.17(a) shows a continuous image projected onto the plane of an array sensor. Figure 2.17(b) shows the image after sampling and quantization. Clearly, the quality of a digital image is determined to a large degree by the number of samples and discrete intensity levels used in sampling and quantization.



a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Representing Digital Images

The result of sampling and quantization is matrix of real numbers. Assume that an image f(x,y) is sampled so that the resulting digital image has M rows and N Columns. The values of the coordinates (x,y) now become discrete quantities thus the value of the coordinates at origin become (x,y) =(0,0).The matrix can be represented in the following form as well.

$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0, N-1) \\ f(1,0) & f(1,1) & \cdots & f(1, N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1, N-1) \end{bmatrix}$$

Each element of this matrix is called an *image element*, *picture element*, *pixel*, or *pel*.
Traditional matrix notation to denote a digital image and its elements:

$$\mathbf{A} = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

The sampling process may be viewed as partitioning the x-y plane into a grid with the coordinates of the center of each grid being a pair of elements from the Cartesian products Z2 which is the set of all ordered pair of elements (Zi, Zj) with Zi and Zj being integers from Z. Hence f(x,y) is a digital image if gray level (that is, a real number from the set of real number R) to each distinct pair of coordinates (x,y). This functional assignment is the quantization process.

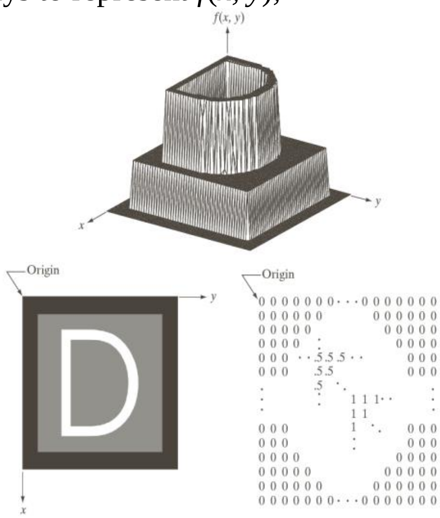
If the gray levels are also integers, Z replaces R, and a digital image become a 2D function whose coordinates and the amplitude value are integers. Due to processing storage and hardware consideration, the number of gray levels typically is an integer power of 2. $L=2^K$ Then, the number b, of bits required to store a digital image is

$$b=M * N * K$$

When M=N The equation become $b=N^2*K$

When an image can have 2^k gray levels, it is referred to as "k- bit". An image with 256 possible gray levels is called an "8-bit image (because $256=2^8$).

There are three basic ways to represent $f(x, y)$,



a
b c

FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).

Dynamic range of an imaging system to be the ratio of the maximum measurable intensity to the minimum detectable intensity level in the system. As a rule, the upper limit is determined by *saturation* and the lower limit by *noise*

The difference in intensity between the highest and lowest intensity levels in an image is called Contrast level of an image.

Spatial and Intensity Resolution:

Spatial resolution is the smallest discernible details are an image. *spatial resolution* can be stated in a number of ways, with *line pairs per unit distance*, and *dots* (*pixels*) *per unit distance.* Suppose a chart can be constructed with vertical lines of width w with the space between the also having width W, so a line pair consists of one such line and its adjacent space thus. The width of the line pair is 2w and there is 1/2w line pair per unit distance resolution is simply the smallest number of discernible line pair unit distance.

Dots per unit distance are a measure of image resolution used commonly in the printing and publishing industry. In the U.S., this measure usually is expressed as *dots per inch* (dpi). To give you an idea of quality, newspapers are printed with a resolution of 75 dpi, magazines at 133 dpi, glossy brochures at 175 dpi, and the book page at which you are presently looking is printed at 2400 dpi.

Intensity resolution refers to smallest discernible change in gray levels.

Measuring discernible change in gray levels is a highly subjective process reducing the number of bits R while repairing the spatial resolution constant creates the problem of false contouring .it is caused by the use of an insufficient number of gray levels on the smooth areas of the digital image . It is called so because the rides resemble top graphics contours in a map. It is generally quite visible in image displayed using 16 or less uniformly spaced gray levels.

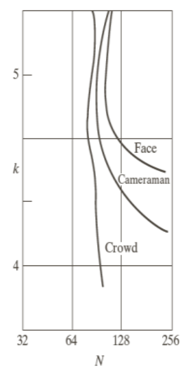
iso Preference Curves

To see the effect of varying N and K simultaneously, these pictures are taken having little, mid level and high level of details.

a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

Different image were generated by varying N and k and observers were then asked to rank the results according to their subjective quality. Results were summarized in the form of iso-preference curve in the N-k plane.



The iso-preference curve tends to shift right and upward but their shapes in each of the three image categories are shown in the figure. A shift up and right in the curve simply means large values for N and k which implies better picture quality The result shows that iso-preference curve tends to become more vertical as the detail in the image increases. The result suggests that for image with a large amount of details only a few gray levels may be needed. For a fixed value of N, the perceived quality for this type of image is nearly independent of the number of gray levels used.

Image Interpolation:

Interpolation is a basic tool used extensively in tasks such as zooming, shrinking, rotating, and geometric corrections. All these tasks are called resampling methods.
Interpolation is the process of using known data to estimate values at unknown locations.

Suppose that an image of 500 X 500 size pixels has to be enlarged 1.5 times to 750 X750 pixels. A simple way to visualize zooming is to create an imaginary 750 X750 grid with the same pixel spacing as the original, and then shrink it so that it fits exactly over the original image. Obviously, the pixel spacing in the shrunken 750 X750 grid will be less than the pixel spacing in the original image.

To perform intensity-level assignment for any point in the overlay, we look for its closest pixel in the original image and assign the intensity of that pixel to the new pixel in the 750 X750 grid.When we are finished assigning intensities to all the points in the overlay grid, we expand it to the original specified size to obtain the zoomed image. The method is called *nearest neighbor interpolation* because it assigns to each new location the intensity of its nearest neighbor in the original image.

A more suitable approach is *bilinear interpolation*, in which we use the four nearest neighbors to estimate the intensity at a given location. Let (x, y) denote the coordinates of the location to which we want to assign an intensity value and let $v(x, y)$ denote that intensity value. For bilinear interpolation, the assigned value is obtained using the equation

$$v(x, y) = ax + by + cxy + d$$

where the four coefficients are determined from the four equations in four unknowns that can be written using the four nearest neighbors of point (x, y).

The next level of complexity is *bicubic interpolation*, which involves the sixteen nearest neighbors of a point.The intensity value assigned to point (x, y) is obtained using the equation

$$v(x, y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$$