Q60. Discuss about the two-dimensional Hadamard transform.

(or)

Determine the kernel coefficients of 2D Hadamard transforms for N = 8.

Ans:

One-dimensional kernel for Hadamard Transforms is,

$$g(x,u) = \frac{1}{N}(-1)^{\sum_{i=0}^{n-1} b_i(x)b_i(u)} \quad \ldots (1)$$

Where,

$$N = 2^n$$

Then, two-dimensional kernel for Hadamard Transform is given as,

$$g(x, y, u, v) = \frac{1}{N}(-1)^{\sum_{i=0}^{n-1} b_i(x)b_i(u)+b_i(y)b_i(u)} \quad \ldots (2)$$

Above equation can be written as,

$$g(x, y, u, v) = \frac{1}{N}(-1)^{\sum_{i=0}^{n-1} b_i(x)b_i(u)} \cdot \frac{1}{N}(-1)^{\sum_{i=0}^{n-1} b_i(y)b_i(v)} \quad \ldots (3)$$

From equation (3) it can be observed that $g(x, y, u, v)$ is separable and symmetric. Then equation (3) can be written as.

$$g(x, y, u, v) = g(x, u).\, g(y, v) \quad \ldots (4)$$

$\therefore$ Hadamard kernel for two dimensions is the product of two one-dimensional kernel hadamard transformation as shown in table below,

U \ X	0	1	2	3	4	5	6	7
0	+	+	+	+	+	+	+	+
1	+	−	+	−	+	−	+	−
2	+	+	−	−	+	+	−	−
3	+	−	−	+	+	−	−	+
4	+	+	+	+	−	−	−	−
5	+	−	+	−	−	+	−	+
6	+	+	−	−	−	−	+	+
7	+	−	−	+	−	+	+	−

Table (1): Hadamard Kernel Matrix for N = 8

V \ Y	0	1	2	3	4	5	6	7
0	+	+	+	+	+	+	+	+
1	+	–	+	–	+	–	+	–
2	+	+	–	–	+	+	–	–
3	+	–	–	+	+	–	–	+
4	+	+	+	+	–	–	–	–
5	+	–	+	–	–	+	–	+
6	+	+	–	–	–	–	+	+
7	+	–	–	+	–	+	+	–

Table (2): Hadamard Kernel Matrix for N = 8

Here +, – represent + 1 and – 1 and the term $\frac{1}{N}$ is omitted here for simplicity.

Note: Hadamard transforms can be applied for images with values of N other than integer power of 2, and maximum limit of N is 200 ($N = 200$).