* HISTOGRAM EQUALISATION (OR) HISTOGRAM LINEARIZATION :-

Let us consider r is original image & S is processed image

So that $S = T(r)$ — ①.

In histogram equalisation, we consider 'r' & 's' as random variables.

The transformation function in ① should satisfy the following conditions :

(i) r limit is from $0 < r < 1$

(ii) $T(r)$ should be a single valued & monotonically increasing function.

(iii) The transformation should be continuous & differentia

* If 'r' limit is from [0 1] then black & white image.

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The probability density function of transformed gray levels for ① is obtained as :

$$P_s(s)ds = P_r(r)\,dr$$

$$P_s(s) = P_r(r)\frac{dr}{ds} \quad \text{— ②}$$

From ①,

$$S = T(r) = \int_0^r P_r(w)\,dw$$

Differentiating the eqn, we get

$$\frac{d(s)}{dr} = \frac{d}{dr}\left(\int_0^r P_r(w)\,dw\right)$$ where w is dummy variable

$$\frac{ds}{dr} = P_r(r)$$

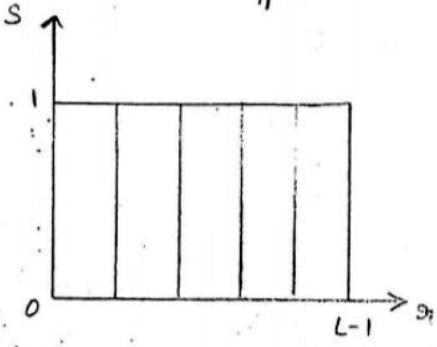
From (2),

$$P_s(s) = P_r(r) \cdot \frac{1}{P_r(r)}$$

$$\boxed{P_s(s) = 1}$$

It means for all intensity levels the o/p is '1'

$P_s(s) = 1$ represents uniform Equalisation.



Drawback :-

By using Histogram Equalisation we can't get accurate manipulations.

* Example :-

perform Histogram Equalisation for the image

$$\begin{bmatrix} 4 & 4 & 4 & 4 \\ 5 & 4 & 3 & 4 \\ 3 & 3 & 4 & 5 \\ 4 & 5 & 2 & 5 \end{bmatrix}_{4\times4}$$

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Solution :-

The max value of image = 5

∴ we need a minimum of 3 bits to represent the number 5 (101). There are 8 possible gray levels from 0 to 7.

The histogram of i/p image is given below:

Gray level	0	1	2	3	4	5	6	7
No. of pixels	0	0	1	3	8	4	0	0

Step-1 :- Compute the cumulative sum of above values:

Gray level	0	1	2	3	4	5	6	7
No. of pixels	0	0	1	3	8	4	0	0
Cumulative Sum	0	0	1	4	12	16	16	16

Step-2 :- Divide the cumulative sum obtained in step-1 by total no. of pixels.

In this case, the total no. of

Gray level	0	1	2	3	4	5	6	7
No. of pixels	0	0	1	3	8	4	0	0
Cumulative sum	0	0	1	4	12	16	16	16
Total no. of pixels	0/16	0/16	1/16	4/16	12/16	16/16	16/16	16/16

Step-3 :- Multiply the result obtained in step-2 by the max gray level value which is 7 in this case.

Gray level	0	1	2	3	4	5	6	7
No. of pixels	0	0	1	3	8	4	0	0
Cumulative Sum	0	0	1	4	12	16	16	16
Total no. of pixels	0/16	0/16	1/16	4/16	12/16	16/16	16/16	16/16
Multiplying the result by 7.	0	0	$7/16 \cong 1$	2	5	7	7	7

Step-4 :- Mapping of gray level by one-to-one correspondence.

original gray level	Histogram equalised values
0	0
1	0
2	1
3	2
4	5
5	7
6	7
7	7

The original image & the histogram equalised images are shown side by side

$$\begin{bmatrix} 4 & 4 & 4 & 4 \\ 5 & 4 & 3 & 4 \\ 3 & 3 & 4 & 5 \\ 4 & 5 & 2 & 5 \end{bmatrix} \xrightarrow[\text{Equalisation}]{\text{Histogram}} \begin{bmatrix} 5 & 5 & 5 & 5 \\ 7 & 5 & 2 & 5 \\ 2 & 2 & 5 & 7 \\ 5 & 7 & 1 & 7 \end{bmatrix}$$

original image

Histogram equalised image