1.2 THE SIMPLE FORM OF THE RADAR EQUATION

The radar equation relates the range of a radar to the characteristics of the transmitter, receiver, antenna, target, and environment. It is useful not just as a means for determining the maximum distance from the radar to the target, but it can serve both as a tool for understanding radar operation and as a basis for radar design. In this section, the simple form of the radar equation is derived.

If the power of the radar transmitter is denoted by P_t, and if an isotropic antenna is used (one which radiates uniformly in all directions), the *power density* (watts per unit area) at a distance R from the radar is equal to the transmitter power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius R, or

$$\text{Power density from isotropic antenna} = \frac{P_t}{4\pi R^2} \tag{1.3}$$

Radars employ directive antennas to channel, or direct, the radiated power P_t into some particular direction. The *gain* G of an antenna is a measure of the increased power radiated in the direction of the target as compared with the power that would have been radiated from an isotropic antenna. It may be defined as the ratio of the maximum radiation intensity from the subject antenna to the radiation intensity from a lossless, isotropic antenna with the same power input. (The radiation intensity is the power radiated per unit solid angle in a given direction.) The power density at the target from an antenna with a transmitting gain G is

$$\text{Power density from directive antenna} = \frac{P_t G}{4\pi R^2} \tag{1.4}$$

The target intercepts a portion of the incident power and reradiates it in various directions.

The measure of the amount of incident power intercepted by the target and reradiated back in the direction of the radar is denoted as the radar cross section σ, and is defined by the relation

$$\text{Power density of echo signal at radar} = \frac{P_t G}{4\pi R^2}\frac{\sigma}{4\pi R^2} \tag{1.5}$$

The radar cross section σ has units of area. It is a characteristic of the particular target and is a measure of its size *as seen by the radar*. The radar antenna captures a portion of the echo power. If the effective area of the receiving antenna is denoted A_e, the power P_r received by the radar is

$$P_r = \frac{P_t G}{4\pi R^2}\frac{\sigma}{4\pi R^2} A_e = \frac{P_t G A_e \sigma}{(4\pi)^2 R^4} \tag{1.6}$$

The maximum radar range R_{max} is the distance beyond which the target cannot be detected. It occurs when the received echo signal power P_r just equals the minimum detectable signal S_{min}. Therefore

$$R_{max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}}\right]^{1/4} \tag{1.7}$$

This is the fundamental form of the radar equation. Note that the important antenna parameters are the transmitting gain and the receiving effective area.

Antenna theory gives the relationship between the transmitting gain and the receiving effective area of an antenna as

$$G = \frac{4\pi A_e}{\lambda^2} \tag{1.8}$$

Since radars generally use the same antenna for both transmission and reception, Eq. (1.8) can be substituted into Eq. (1.7), first for A_e then for G, to give two other forms of the radar equation

$$R_{max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right]^{1/4} \tag{1.9}$$

$$R_{max} = \left[\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{min}}\right]^{1/4} \tag{1.10}$$