Ans:

Smoothing Methods in Frequency Domain

The image output in the frequency domain of an image $f(x, y)$ is given by,

$$G(u, v) = H(u, v).F(u, v)$$

Where, $H(u, v)$ and $F(u, v)$ are the Fourier transforms of the filter response function and input image.

The response function of the filter is responsible for the smoothing and sharpening of image and hence is carefully selected.

Smoothing Methods in Frequency Domain: In smoothing method, the high frequency components of an image are weakened (attenuated) by low pass filters with a transfer function. To achieve smoothing, consider that $H(u, v)$ is the filter transfer function for function given $G(u, v)$ by attenuating high frequency components of $F(u, v)$.

2. Butterworth Low Pass Filter

The response of the Butterworth low pass filter is given as,

$$H(u, v) = \frac{1}{1 + \left[\frac{D(u, v)}{D_0}\right]^{2n}}$$

BLPF is of n^{th} order and D_0 is the distance from centre of circle.

(i) BLPF cannot clearly attenuate all the high frequency components.

(ii) For smooth filtering it is must to redefine the cutoff frequency point such that $H(u, v)$ is optimum.

$$H(u, v) = 0.5 \text{ when } D(u, v) = D_0$$

The BLPF of order 2 has medium ringing and negative values and is suited for low pass filtering and desired ringing.

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