Periodicity property

The 2D-DFT of a function $f(x,y)$ is said to be periodic with a period N if

$$F(u,v) \longrightarrow F(u+pM, v+qN) \quad — ①$$

Proof :

$$F(u+pM, v+qN) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{\frac{-j2\pi x}{M}(u+pM)}\, e^{\frac{-j2\pi y}{N}(v+qN)} \quad \rightarrow ②$$

$$F(u+pM, v+qN) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{\frac{-j2\pi ux}{M}}\, e^{\frac{-j2\pi xpM}{M}}\, e^{\frac{-j2\pi yv}{N}}\, e^{\frac{-j2\pi yqN}{N}} \quad \rightarrow ③$$

By taking $e^{\frac{-j2\pi ux}{M}}$, $e^{\frac{-j2\pi vy}{N}}$

$$F(u+pM, v+qN) = \left[\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{\frac{-j2\pi ux}{M}}\, e^{\frac{-j2\pi vy}{N}}\right] e^{-j2\pi xp}\, e^{-j2\pi yq} \quad \rightarrow ④$$

we know

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{\frac{-j2\pi ux}{M}}\, e^{\frac{-j2\pi vy}{N}} \quad \rightarrow ⑤$$

⑤ in ④ we get

$$F(u+pM, v+qN) = F(u,v)\, e^{-j2\pi xp}\, e^{-j2\pi yq}$$

The $e^{-j2\pi xp}$, $e^{-j2\pi yq}$ values are always '1' for any integer value of x, p, q and y.

$$\therefore \quad F(u+pM, v+qN) = F(u,v) \times 1$$

$$F(u+pM, v+qN) = F(u,v)$$

Rotation property:

The rotation property states that if a function is rotated by the angle, its fourier transform also rotate by an amount.

$$f(m,n) \rightarrow f(r\cos\theta, r\sin\theta)$$

$$\text{DFT}\left[f(r\cos\theta, r\sin\theta)\right] \rightarrow F\left[R\cos\phi, R\sin\phi\right]$$

$$\text{DFT}\left[f(r\cos(\theta+\theta_0), r\sin(\theta+\theta_0))\right]$$

$$\rightarrow F\left[R\cos(\phi+\phi_0), R\sin(\phi+\phi_0)\right]$$