* FIRST ORDER DERIVATIVE (OR) GRADIENT MASKING (OR) PRE-WITT MASKING

* Image differentiation enhances edges & other discontinuities & de-emphasizes areas with slowly varying intensities.

By using Gradient masking we find out the vertical & horizontal thick values only.

Gradient function, $\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$

$$f(x,y) = \left[\left| \frac{\partial f}{\partial x} \right|^2 + \left| \frac{\partial f}{\partial y} \right|^2 \right]$$

* Let us consider an image,

ω_1	ω_2	ω_3
ω_4	ω_5	ω_6
ω_7	ω_8	ω_9

Differentiation is nothing but difference b/w previous & present images.

$$\frac{\partial f}{\partial x} = \omega_7 - \cancel{\omega_4} + \omega_8 - \cancel{\omega_5} + \omega_9 - \cancel{\omega_6} + \cancel{\omega_4} - \omega_1 + \cancel{\omega_5} - \omega_2 + \cancel{\omega_6} - \omega_3$$

$$= \omega_7 + \omega_8 + \omega_9 - (\omega_1 + \omega_2 + \omega_3)$$

$$\therefore \frac{\partial f}{\partial x} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\frac{\partial f}{\partial y} = \omega_3 - \cancel{\omega_2} + \omega_6 - \cancel{\omega_5} + \omega_9 - \cancel{\omega_8} + \cancel{\omega_2} - \omega_1 + \cancel{\omega_5} - \omega_4 + \cancel{\omega_8} - \omega_7$$

$$= \omega_3 + \omega_6 + \omega_9 - (\omega_1 + \omega_4 + \omega_7)$$

$$\therefore \frac{\partial f}{\partial y} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

From the definition, one-dimensional function $f(x)$ is

In x-direction, $\frac{\partial f}{\partial x} = f(x+1) - f(x)$.

In y-direction, $\frac{\partial f}{\partial y} = f(y+1) - f(y)$

Two-dimensional function $f(x,y)$ is

In x-direction, $\frac{\partial f}{\partial x} = f(x+1, y) - f(x,y)$

In y-direction, $\frac{\partial f}{\partial y} = f(x, y+1) - f(x,y)$

$$\therefore \nabla f(x,y) = f(x+1, y) + f(x, y+1) - 2f(x,y).$$

Eg:- Take an image as
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

Its equivalent is

$$\begin{bmatrix} 0 & 0 & 0 & & \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 & 8 \\ & 1 & 2 & 3 & 4 \\ & & & & \end{bmatrix}$$

$$\Rightarrow \text{abs}\left[\Sigma\begin{bmatrix}0&0&0\\0&1&2\\0&5&6\end{bmatrix} * \begin{bmatrix}-1&-1&-1\\0&0&0\\1&1&1\end{bmatrix} + \Sigma\begin{bmatrix}0&0&0\\0&1&2\\0&5&6\end{bmatrix} * \begin{bmatrix}-1&0&1\\-1&0&1\\-1&0&1\end{bmatrix}\right]$$

$$\Rightarrow \begin{bmatrix}0&0&0\\2&2&2\\6&6&6\end{bmatrix} + \begin{bmatrix}0&0&0\\-3&0&3\\-11&0&11\end{bmatrix}$$

$$\Rightarrow 24 + 0 = 24$$

$=$

[Negative values are not taken]