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Cycles, Diversity and Competition in Rock-Paper-Scissors-Lizard-Spock Spatial Game Agent Simulations

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ABSTRACT
The emergence of complex spatial patterns in agent-based models is closely connected with the symmetries and relationships present between individual microscopic constituents. Games such as Rock-Paper-Scissors (RPS) have a closed cycle relationship amongst players which extends the symmetry. RPS and related games can be played by agents arranged on a spatial grid and have been shown to generate many complex spatial patterns. We consider the implications of extending the individual RPS game complexity to five-cycle games such as "Rock-Paper-Scissors-Lizard-Spock" that have competing cyclic reactions. We simulate large spatial systems using a reaction-rate formulation which are simulated for long run times to capture the dynamic equilibrium regime. We report on the stable and unstable phase mixtures that arise in these agent models and comment on the effects that drive them.

KEY WORDS
rock paper scissors lizard Spock; game theory; agents; spatial complexity; emergence.

1 Introduction

Spatial games as played by software agents have proved a useful platform for studies of complexity, with considerable activity reported very recently in the research literature [1]. The rock-paper-scissors (RPS) game [2] exhibits the closed 3-cyclic relationship of "rock blunts scissors cuts paper wraps rock" and can be played by spatial (software) agents against other agents within their neighbourhood. The rock-paper-scissors-lizard-Spock (RPSLS) variation [3] has five player states and two key competing relationship 5-cycles as explained in Section 2. The RPSLS [4] game was invented by Kass but was brought to popular attention through its appearance on an episode of the television show "The Big Bang Theory" [5]. RPSLS has considerably higher intrinsic agent-player complexity than RPS and this manifests itself in the spatial spiral patterns that emerge in simulated systems. Figure 1 shows a typical spatial pattern arising in these models. A system of spatial agents is initialised randomly and is subsequently evolved in simulation time according to microscopically simple probabilistic rules.

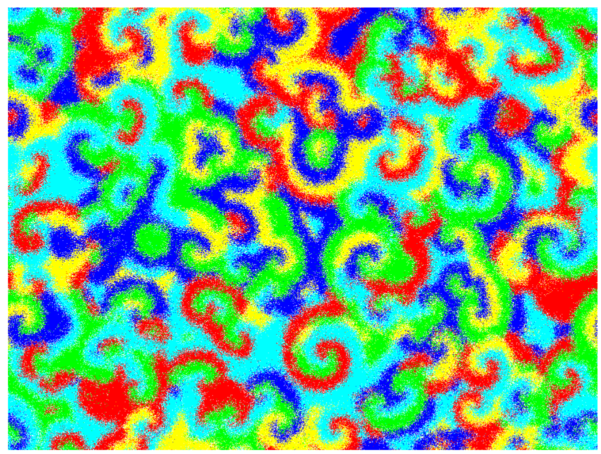

Figure 1: Snapshot configuration of the Rock, Paper, Scissors, Lizard, Spock! game on a 1024×768 spatial mesh, 2048 steps after a random start.

Gillespie formulated an approach for simulating discrete systems with stochastic rate equations [6, 7] and this method been developed further by several research groups including those of Reichenbach *et al*, Peltomaki *et al* and

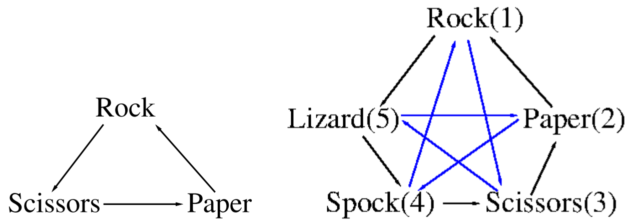

Figure 2: 3-Cycle Rock-Paper-Scissors (left) and 5-Cycle Rock-Paper-Scissors-Lizard-Spock (right).

Szabo, Solnaki *et al* [8]. The method has been used to investigate model phase diagrams [9]; the effect of cyclic dominance [10], effects of asymmetric mobility [11] and asymmetric exclusion processes [12] such as [13].

In this present work we extend the rate equation approach used for RPS-like games to the case of competing cyclic relationships. We add an additional probabilistic rate equation to allow modelling the of RPSLS system. For this paper we report on two dimensional spatial agent game systems.

This paper is structured as follows: in Section 2 we summarise key ideas for formulating a set of agents playing games like RPS or RPSLS. In Section 3 we describe how the probabilistic rate equations are established, including the competing cyclic relationships needed for agents to play RPSLS. We describe some metrics to apply to the model simulations to characterise their behaviour in Section 4 and present some systems measurements and emergent properties from the simulations in Section 5 alongwith some discussion of their implications in Section 6. We also offer some conclusions and suggested areas for further study.

2 Game Formulation

Games such as rock-paper-scissors are traditionally played by two or more players simultaneously [14] declaring their choice of the three possible entities and with a scored point based on the cyclic precedence rules below. For our purposes in this present work we focus only on two-simultaneous-player games although an agent plays the game against all its neighbouring agents in turn over time.

The traditional "rock-paper-scissors" rules are usually taken to be:

scissors cuts paper

paper wraps rock

rock bluntens scissors

with the verb in each case essentially denoting "beats," and the single resulting cycle is shown in figure 2(left).

The game was often played iteratively such as best out of three or five or some-such arrangement. As Kass has noted [3], playing RPS with opponents you knew well, often resulted in a stalemate from over-familiarity with their likely strategy. The game is therefore complicated and made considerably more "interesting" by increasing the number of strategy choices from three to five.

Kass describes the "rock-paper-scissors-Spock-lizard" rules as:

scissors cuts paper

paper covers rock

rock crushes lizard

lizard poisons Spock

Spock smashes scissors

scissors decapitates lizard

lizard eats paper

paper disproves Spock

Spock vapourises rock

rock crushes scissors

which describe two 5-cycles, which are shown in figure 2(right).

For the purposes of human players it is important the rules be easily remembered. It become difficult therefore to imagine a practical game played by humans of much more than Kass's five-entity game RPSLS. A computer however need have no such limitations and we can explore the implications of arbitrarily high cyclic games. To that end it is easier to develop a formulation based on numbered states: $1, 2, 3 \ldots$, which is a useful notation to analyse the cycles present.

The RPS Graph has a single circuit or loop of length 3. Each node in the graph has an in-degree of 1 and an out-degree of 1, and this a total degree of 2. The RPSLS graph has richer behaviour, with each node having a total degree of 4 consisting of 2 inputs and 2 outputs. There are 12 circuits, the two longest being of length 5:

```
1 3 2 1            1 5 4 1
1 3 2 4 1          1 5 2 1
1 3 5 4 1          1 5 2 4 1
1 3 5 2 1          3 2 4 3
1 3 5 2 4 1        3 5 4 3
1 5 4 3 2 1        3 5 2 4 3
```

where we have numbered the nodes: rock=1; paper=2; scissors=3; Spock=4; lizard=5.

Obviously a single node and a 2-node system cannot sustain a sensible cyclic relation at all. As we have seen a 3-node system has one and only one cycle. The number of possible cycles grows rapidly with the number of player strategies or states.

3 Spatial Game Formulation

We consider a model system with a number of Q states or player strategies, labelled $V, A, B, C, ...$ where we use V to denote a vacancy or empty site and the letters $A, B, C, ...$ to label the number $N_s \equiv Q-1$ of different species of player agent. This notation conveniently maps easily and directly onto the $1, 2, 3...Q$ notation used to describe the cycles in the previous section. Following Reichenbach *et al* [15], we first consider the probabilistic reaction equations:

$$\begin{aligned} AE &\rightarrow AV \text{ with rate } \mu \\ BA &\rightarrow BV \text{ with rate } \mu \\ CB &\rightarrow CV \text{ with rate } \mu \\ DC &\rightarrow DV \text{ with rate } \mu \\ ED &\rightarrow EV \text{ with rate } \mu \end{aligned} \quad (1)$$

and:

$$xV \rightarrow xx \text{ with rate } \sigma \quad (2)$$

where we use x to denote any $x \in \{A, B, C, ...\}$. In addition species can move around by exchanging positions with neighbours using:

$$xy \rightarrow yx \text{ with rate } \epsilon \quad (3)$$

where $x, y \in \{A, B, C, ...\}, x \neq y$. In each case the arrow denotes an update rule that can be followed when a site and a neighbouring site are selected at random.

The system thus has a total density $\rho = 1-v = a+b+c+...$ and is the fraction of sites occupied by non-vacancies. We model a lattice of interacting agents of total size $N = L \times L$, which is initially populated by a uniform and random mix of the Q different states. This model is evolved using the rate equations in time and we can measure a number of bulk quantities as described in

We have introduced a new rate equation so as to separately model the inner blue cycle as shown in Figure 2.

$$\begin{aligned} AC &\rightarrow AV \text{ with rate } \alpha \\ BD &\rightarrow BV \text{ with rate } \alpha \\ CE &\rightarrow CV \text{ with rate } \alpha \\ DA &\rightarrow DV \text{ with rate } \alpha \\ EB &\rightarrow EV \text{ with rate } \alpha \end{aligned} \quad (4)$$

The processes and rate equations are parameterised by: μ, α which control the (two) selection rates at which one species consumes another; σ which controls the reproduction process when a species expands into vacant sites; and the diffusion constant ϵ which governs how quickly species can move around the spatial model system. For the $d = 2$ dimensional systems and the work reported here we use $\sigma = \mu = \alpha = 1, \epsilon = 2d + 1 = 5$ as base vales, with some parameter scans done in α, ϵ. Once a site and neighbour are randomly chosen, the process is to be followed is determined stochastically by normalising the rates so that the probability of forward selection for example is $\frac{\mu}{\mu+\alpha+\sigma+\epsilon}$, and so forth.

In principle one could contrive experiments with different values of $\mu_{A,B}, \mu_{B,C}, ...$ and similarly for $\alpha_{A,C}, \alpha_{B,D}, ...$ We have for simplicity set all these to a single μ or α value.

Reichenbach *et al* [15] and also Peltomaki *et al* [16] have investigated variations in ϵ for the three and four state models. In a prior work [17] we investigated the simpler RPS model with up to $Q = 14$ states or fixed rates In this present work we focus on the effects of varying the rates while fixing the number of states $Q = 6$ as appropriate for the RPSLS game.

We simulate the model on a square lattice, with most results quoted on systems of $N = 1024 \times 1024$ sites, for simulation times of ≈ 16384 steps. A simulation step is defined as on average carrying out one attempt to update each and every randomly chosen site. This algorithm avoids sweeping effects or correlation pattern artifacts that would be artificially introduced if we simply swept through sites to update in index order.

The key point to note is that although we have five player strategies in a game like RPSLS, we use six states in the spatial model. We are free to set the number of vacancies to zero but it turns out the vacancies add thermal noise to the model system and considerably speed up its dynamics [17].

In Figure 3 we show some snapshots of a typical RPSLS model configuration as it evolves on a logarithmic time scale. Following a uniform random initialisation with an equal fraction of agents of all player strategies (and vacancies) present, the system goes through two distinct temporal stages. During the first transient stage agents start to organise themselves into cooperative or competitive regions according to the two cyclic RPSLS rules and larger spatial structures - patches and waves and interleaving layers start to form. The second phase shows the development of competing spiral structures that grow in spatial extent with time and is characterised by a dynamic equilibrium reached amongst the agent players as they follow the rate equations.

Shown alongside the configuration snapshots are the location of the vacancies. Tracking the 'V' sites gives insight into where the current spatial regions of activity and fluc-

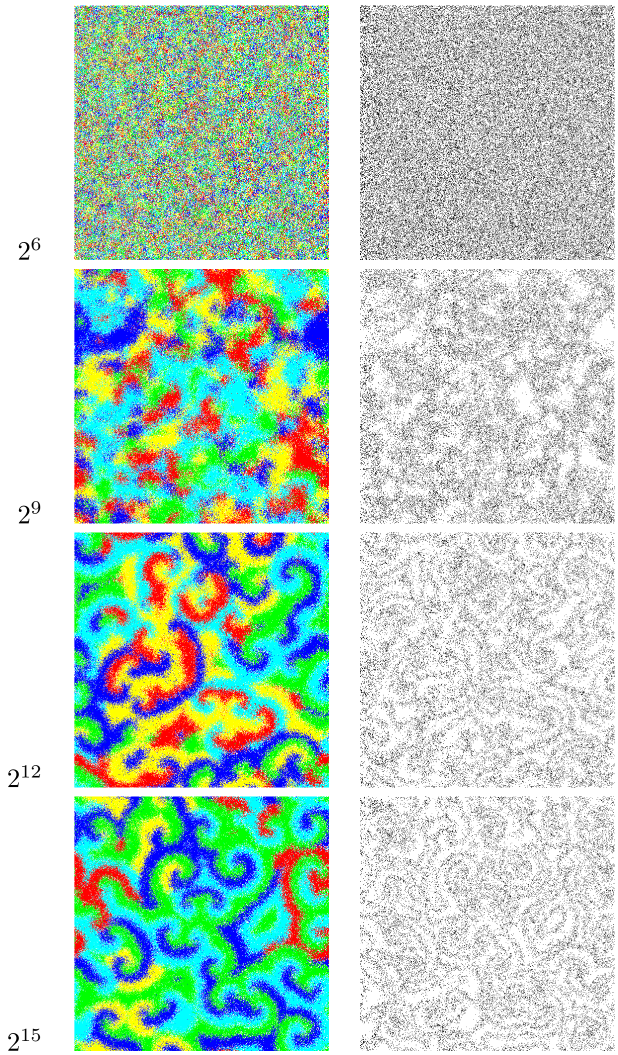

Figure 3: RPSLS agents (left) and distribution of vacancies (right) at times: 64, 512, 4096, 32768 for a 512×512 player mesh. Red=Rock; Yellow=Paper; Blue=Scissors; Green=Spock; Cyan=Lizard.

tuations are. In a previous work the vacancies were shown to be crucial in determining the symmetry of an RPS-like game. In the figure here we see vacancies are more spread out and are not just located at domain boundaries, but penetrate quite deeply into the spirals, waves and layers of interacting agents.

4 Metrics

There are a number of bulk properties we can measure on the system to characterise its behaviour. Simple population traces against time can be plotted, and in particular the number of species extinctions needs to be tracked. We define N_{Ex} as the number of extinctions that have occurred since the system was initialised. Similarly population fractions N_V is the number of vacancies and $N_1, N_2, ...N_Q$ can also be tracked.

A simple count of the like-like species bonds or nearest neighbour relationships is:

$$N_{\text{same}} = \frac{1}{N.d} \sum_{i=1}^{N} \sum_{j=1}^{d} 1 : s_i = s_j \tag{5}$$

where N is the number of agent cells or sites in the d-dimensional lattice and each $s_i = 0, 1, 2, ...Q$, where we use $Q = 6$ as the number of possible states, including vacancies.

Similarly $N_{\text{diff}} = 1 - N_{\text{same}}$ is the fraction of possible bonds in the system that are different. An energy formulation would link N_{same} to an energy function through some like-like coupling term. We expect this to relax to a steady state value for a given rate equation parameter set.

The game playing rate equations are track-able via the selection or neutral fractions. We define:

$$N_{\text{sel}} = \frac{1}{N.d} \sum_{i=1}^{N} \sum_{j=1}^{d} 1 : s_i = x; s_j = x - 1, \forall x \tag{6}$$

so it measures the fraction of agents in the system that apply the selection game playing rule. We can split this into N^{μ}_{sel} and N^{α}_{sel} for the two separate rules cycles we employ in the RPSLS game:

$$\begin{aligned} N^{\mu}_{\text{sel}} &= \frac{1}{N.d} \sum_{i=1}^{N} \sum_{j=1}^{d} 1 : s_i = x; s_j = x - 1, \forall x. \\ N^{\alpha}_{\text{sel}} &= \frac{1}{N.d} \sum_{i=1}^{N} \sum_{j=1}^{d} 1 : s_i = x; s_j = x - 2, \forall x. \end{aligned} \tag{7}$$

where the μ rule applies to successive player strategies around the RPSSL loop and the α rule skips a value.

We can also define N_{neut} as the remaining neutral fraction to whom the game playing rules did not apply.

5 Results

The metrics described above show the different processes that dominate during the transient and dynamic equilibrium phases observed.

Figure 4 shows how the measurement metrics typically vary with simulation time - the data is for a single run

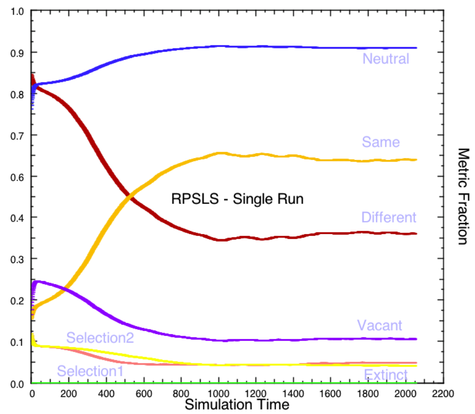

Figure 4: Metrics for a single 1024×1024 RPSLS spatial game showing the transient and dynamic equilibrium time regimes.

and therefore shows the unaveraged temporal fluctuations as spatial agent domains wax and wane. Providing no agent player-species die out, the metric time traces tend towards well defined dynamic equilibrium values that are controlled solely by the rate equation parameters and are independent of the initial model starting conditions. We measure these long-term values – averaged over between around 100 separately initialised model runs. We can thus produce surface plots showing the various fractional metrics, with two-parameter scans on the x-y axes of the diffusion parameter ϵ and the (outer cycle) selection parameter μ Results shown are for parameter values: $\mu = \sigma = 1$ on a 1024×1024 model lattice run for 8192 equilibration steps followed by 8192 measurement steps with α, ϵ varying as shown on the axes.

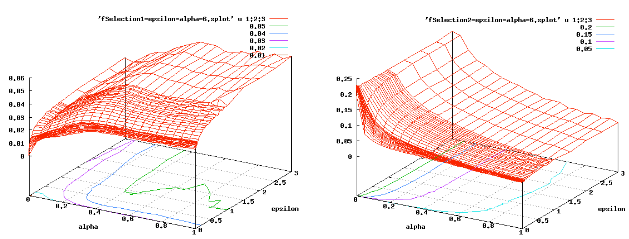

Figure 5: Fraction of Agents that have played the Selection 1 (μ, left) and 2 (α, right) RPSLS rules.

Figure 5 shows how the two selection rate equations contrast with varying diffusion mobility ϵ and by adjusting the inner cycle RPSLS interaction rate α. The two selection processes work in opposite senses but combine cause a subtle cross over effect around $\alpha^* = 0.4$ which is seen in the neutral bond count metric as shown in Figure 6.

This effect is also apparent in the surface plot of the fraction of different bonds, in Figure 7 where again a crease in

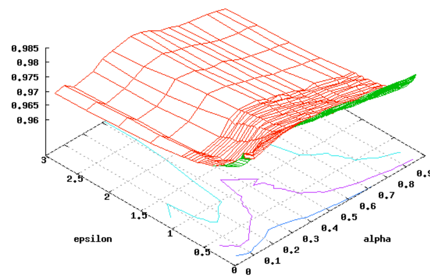

Figure 6: Fraction of Neutral Agent-Agents.

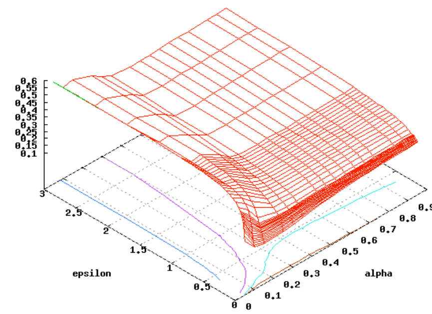

Figure 7: Fraction of Different Agent-Agent bonds.

the smoothly varying surface is seen at around $\alpha = 0.4$

The surface plots are all shown on the same horizontal scales although different vertical scales are used as the metric fractions do differ in their scales of interest as seem in Figure 4. Some simple contouring projections are shown underneath each surface parametric plot.

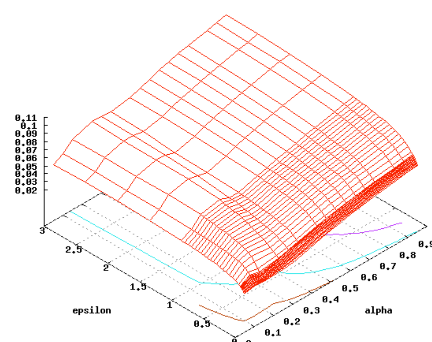

Figure 8: Fraction of Vacancies.

Figure 8 shows the long-term trend in the count of vacant sites in the RPSLS model. This is seen to drop monotonically with decreasing agent mobility, but to rise with greater activity from the inner cycle α rate equation. Once again there is a crease in the surface arising around $\alpha = 0.4$.

In an attempt to understand the symmetry relationships that

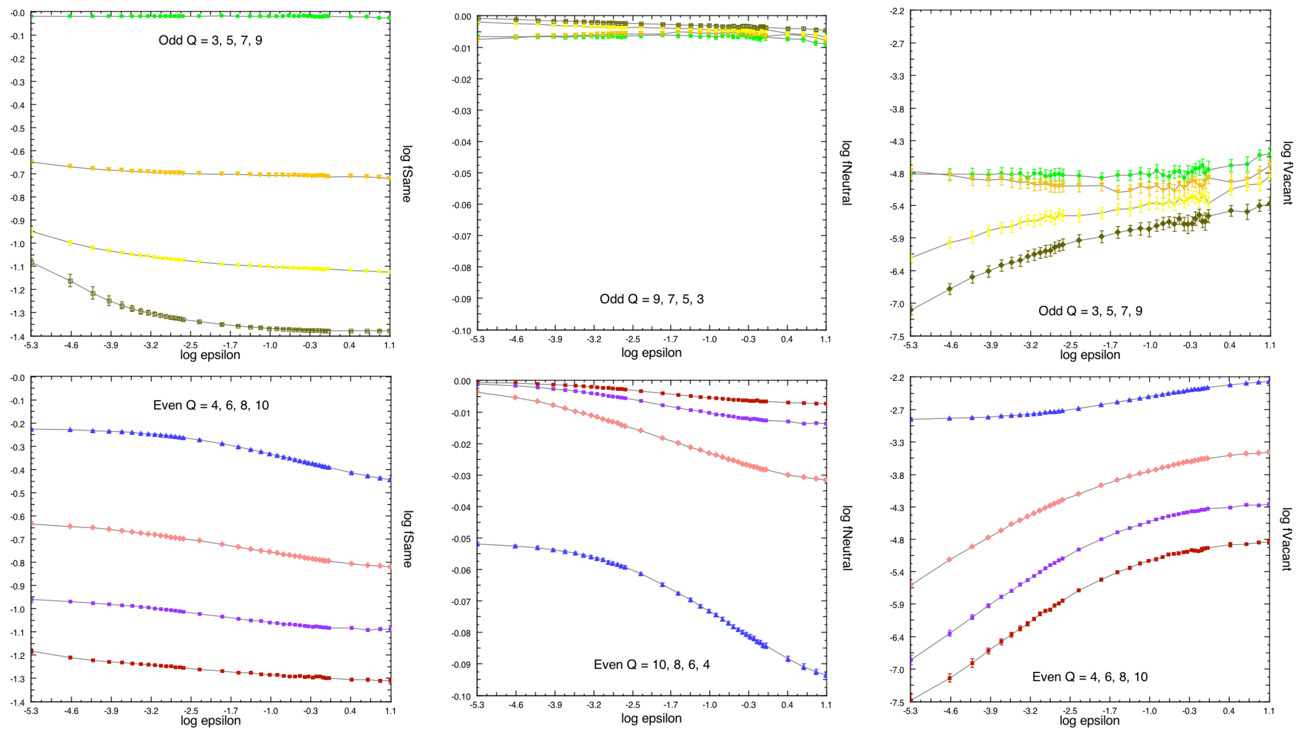

Figure 9: Fraction of Same, Neutral bonds and vacancies for odd/even Q in 1024×1024 RPSLS mesh. Q values as stated are from top to bottom.

arise from the competing phases we attempt to generalise the simulation by simulating systems with different numbers of initial states Q. We track some of the measurements for fixed parameter values: $\mu = \sigma = 1; \alpha = 0; \epsilon = 5$ on a 1024×1024 model lattice run for 8192 equilibration steps followed by 8192 measurement steps. Averages were made over 100 independent runs with error bars computed from the standard deviations over these 100 samples. Setting $\alpha = 0$ simplifies the selection rate process and allows a sensible comparison to be made between different Q values.

Figure 9 shows the fractions of same-bonds; neutral bonds and of vacant sites in the RPSLS model played on 1024×1024 cells, measured over 8192 steps, with an additional initial 8192 steps discarded for equilibration from a random start. Measurements are shown for various odd and even numbers of initial states Q. There are a number of interesting effects apparent.

Most prominently there is considerable difference in qualitative behaviour for even and odd numbers of player agents. For the RPSLS case of even $Q = 6$ we see how there is a broad separation of the fraction of Like-Like agents, and Vacant site curves. The case of odd-Q shows much higher variances in the measured points. We speculate this is due to the frustrations encountered by agents arranged in an odd-Q system. For even-Q it is possible for agents to arrange themselves in alternating layers. So although the rate equations drive the system to seek like-like agents, failing that, agents can arrange to be next to a non competing species in the case of Even Q. This is not possible in the case e of odd-Q and greater spatial fluctuations arise as more player win-lose situations arise.

6 Discussion & Conclusions

We have explored the spatial agent game of “rock, paper, scissors, lizard, Spock” and have shown it exhibits considerably more complex spatial behaviour than simpler single-cycle games such as RPS and its variations. In particular it exhibits spiral patterns as seen in other unrelated spatial agent models [18].

We found that the model system exhibits a transient phase following random initialisation that is inevitably followed by a period of dynamic equilibrium. Although domain coarsening occurs during the dynamic equilibrium phase it is characterised by unchanging average values for the various metrics we have discussed. The long term values of these metrics - and their variances can then be used to characterise the system as whole for the particular rate equation parameters chosen.

A study of the agent mobility suggests further evidence for the importance of the vacancy fraction in the spatial system. We have also investigated competing effects between the two RPSLS game player cycles and have found preliminary evidence for a phase transition at a non-trivial value of rate parameter $\alpha^* \approx 0.4$

There is further evidence for the symmetry relationships and consequences of even/odd numbers of player species. We put forward the hypothesis that this is due to frustrations whereby in an even Q system agents can treat "my enemy's enemy as my friend" and arrange themselves in a less directly competitive spatial arrangement than is forced in an odd Q situation.

In the work here we were able to simulate large enough systems in two dimensions that the extinctions regime could be avoided, However, experience shows that if the system is too small, and is simulated for long enough then species extinctions do occur and apply a shock to the system which must then relax back to a dynamic equilibrium. This makes it more computationally difficult to investigate three dimensional systems, although we hoe to study these to look for dimensional dependence on the α^*.

We have employed very simple geometries for the spatial system with short range neighbourhood player interactions, but models such as the RPSLS are likely also to show interesting effects over longer range interaction geometries such as small-world links [19] and shortcuts across the spatial playing ground.

In summary the RPSLS spatial model is an exciting platform for further investigations of spatial agent complexity and emergence and holds some promise as a tool for investigating predator-prey species diversity and coexistence [20].

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