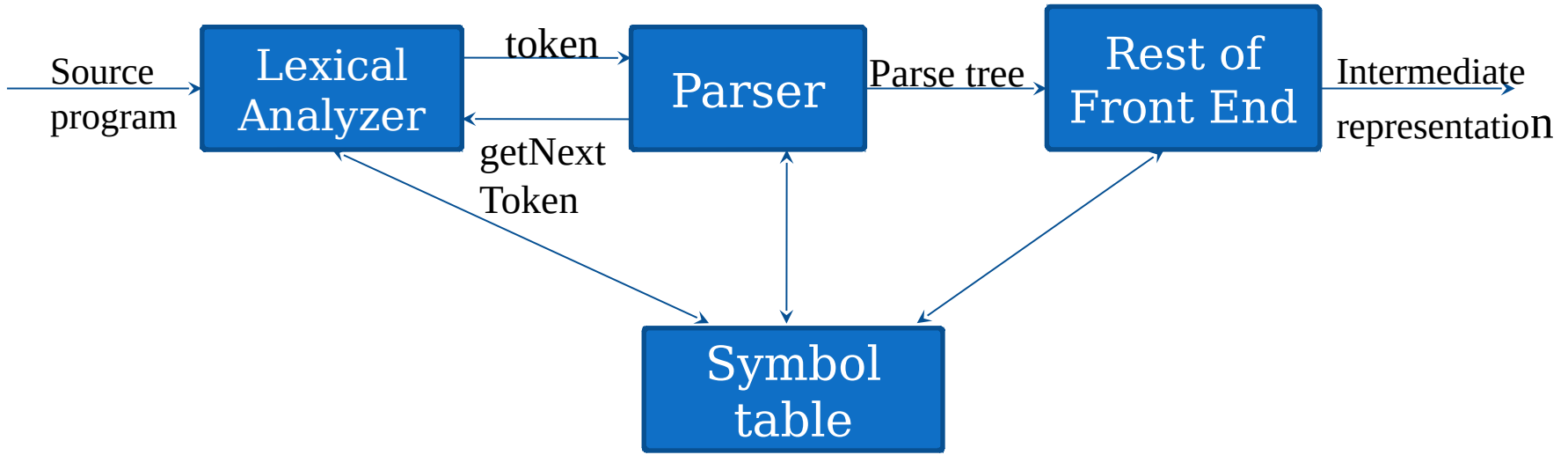


COMPILER DESIGN

UNIT 1

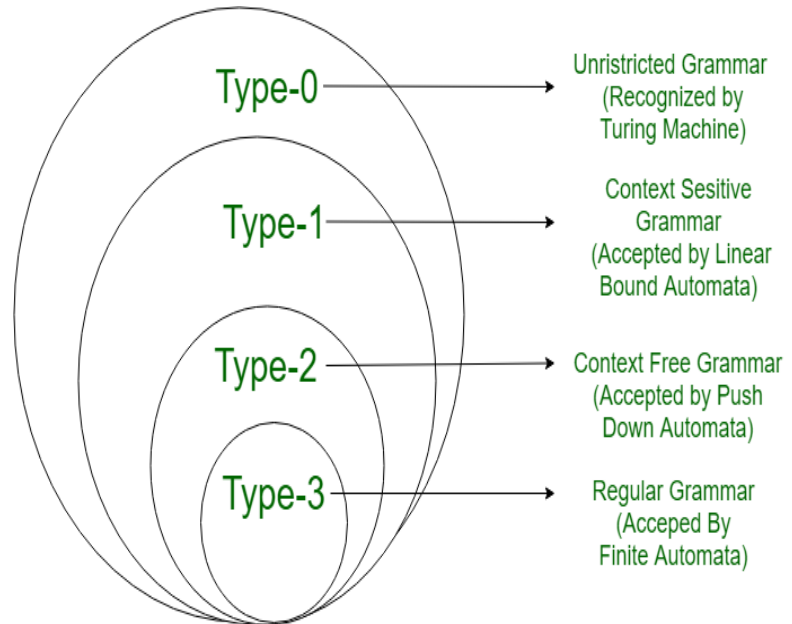
PART 4

The role of parser



According to [Chomsky hierarchy](#), grammars are divided of 4 types:

- Type 0 - unrestricted grammar.
- Type 1 - context sensitive grammar.
- Type 2 - context free grammar.
- Type 3 - Regular Grammar.



Type 0: Unrestricted Grammar

Type-0 grammars include all formal grammars.

Type 0 grammar language are recognized by turing machine.

These languages are also known as the Recursively Enumerable languages

For example,

- 1) $Sab \rightarrow ba$
 $A \rightarrow S.$
- 2) $S \rightarrow ACaB$
 $Bc \rightarrow acB$
 $CB \rightarrow DB$
 $aD \rightarrow Db$

Grammar Production in the form of

$$\alpha \rightarrow \beta$$

Where

$$\alpha \text{ is } (V + T)^* V (V + T)^*$$

V : Variables(Non terminal)

T : Terminals

(α is a string of terminals and nonterminals with at least one non-terminal)

$$\beta \text{ is } (V + T)^*$$

(β is a string of terminals and non-terminals)

In type 0 there must be at least one variable on Left side of production.

Type - 1 Grammar

Type-1 grammars generate the context-sensitive languages. The language generated by the grammar are recognized by the [Linear Bound Automata](#)

For Example,

- 1) $S \rightarrow AB$
 $AB \rightarrow abc$
 $B \rightarrow b$
- 2) $AB \rightarrow AbBc$
 $A \rightarrow bcA$
 $B \rightarrow b$

In Type 1

I. First of all Type 1 grammar should be Type 0.

II. Grammar Production in the form of

$\alpha \rightarrow \beta$

$|\alpha| \leq |\beta|$

i.e count of symbol in α is less than or equal to β

Type - 2 Grammar

Type-2 grammars generate the context-free languages. The language generated by the grammar is recognized by a [Pushdown automata](#).

For example,

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow bB$

$B \rightarrow c$

Type-2 grammars generate the context-free languages.

In Type 2,

1. First of all it should be Type 1.
2. Left hand side of production can have only one variable.

$$|\alpha| = 1.$$

There is no restriction on β

.

Type - 3 Grammar

Type-3 grammars generate regular languages. Type-3 grammars must have a single non-terminal on the left-hand side and a right-hand side consisting of a single terminal or single terminal followed by a single non-terminal.

The productions must be in the form

$$X \rightarrow a \text{ or } X \rightarrow aY$$

where $X, Y \in N$ (Non terminal)

and $a \in T$ (Terminal)

The rule $S \rightarrow \epsilon$ is allowed if S does not appear on the right side of any rule.

Example

$$X \rightarrow \epsilon$$

$$X \rightarrow a \mid aY$$

$$Y \rightarrow b$$

$$A \rightarrow aB$$

$$A \rightarrow \epsilon$$

WHAT ARE CONTEXT FREE GRAMMARS?

- “Context Free Grammar are notation for specifying concrete syntax”
- The languages generated by context free grammars are known as the context free languages
- Inherently recursive structure of a programming language are defined by a context free grammar.

FORMAL DEFINITION OF CFG

CFG stands for context-free grammar. It is a formal grammar which is used to generate all possible patterns of strings in a given formal language. Context-free grammar G can be defined by four tuples as:

$$G = (V, T, P, S)$$

Where,

G is the grammar, which consists of a set of the production rule. It is used to generate the string of a language.

T is the final set of a terminal symbol. It is denoted by lower case letters.

V is the final set of a non-terminal symbol. It is denoted by capital letters.

P is a set of production rules, which is used for replacing non-terminals symbols (on the left side of the production) in a string with other terminal or non-terminal symbols (on the right side of the production).

S is the start symbol which is used to derive the string. We can derive the string by repeatedly replacing a non-terminal by the right-hand side of the production until all non-terminal have been replaced by terminal symbols.

EXAMPLE

Construct a CFG for a language $L = \{wcwR \mid \text{where } w \in (a, b)^*\}$.

Solution:

The string that can be generated for a given language is

$L \{c, aca, aacaa, bcb, abcba, bacab, abbcbbba, \dots\}$

The grammar could be:

1. $S \rightarrow aSa$ rule 1
2. $S \rightarrow bSb$ rule 2
3. $S \rightarrow c$ rule 3

Now if we want to derive a string "abbcbbba", we can start with start symbols.

1. $S \rightarrow aSa$
2. $S \rightarrow abSba$ from rule 2
3. $S \rightarrow abbSbba$ from rule 2
4. $S \rightarrow abbcbbba$ from rule 3

Thus any of this kind of string can be derived from the given production rules.

EXAMPLE

Construct a CFG for the language $L = a^n b^{2n}$ where $n \geq 1$.

Solution:

The string that can be generated for a given language is $\{abb, aabbbb, aaabbbbbbb....\}$.

The grammar could be:

1. $S \rightarrow aSbb \mid abb$

Now if we want to derive a string "aabbbb", we can start with start symbols.

1. $S \rightarrow aSbb$
2. $S \rightarrow aabbbb$

$$S \rightarrow 0S1$$

$$S \rightarrow 01$$

Construct the CFG for the language having any number of a's over the set $\Sigma = \{a\}$.

$L = \{a, aa, aaa, aa, \dots\}$

$S \rightarrow aS$

$S \rightarrow \epsilon$

WHAT IS BNF?

- It stands for Backus-Naur Form
- It is a formal, mathematical way to specify context-free grammars
-

`<number> ::= <digit> | <number> <digit>`

`<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9`

“::=” means “is defined as” (some variants use “:=” instead)

“|” means “or”

- Angle brackets mean a Non Terminal
- Symbols without angle brackets are Terminals

Example $123 \Rightarrow 12\ 3 \Rightarrow 1\ 2\ 3$

BNF for Expressions

```
<expression> ::= <expression> + <term>
                | <expression> - <term>
                | <term>
<term> ::= <term> * <factor>
          | <term> / <factor>
          | <factor>
<factor> ::= ( <expression> )
          | <variable>
          | <number>
```

```
1+2*3
a+b*c
1+2+3
1*2*3
1*(2+3*5)
```

```
E-> E+T | E-T | T
T-> T*F | T/F | F
F-> (E) | V | D
V-> a | b | z
D-> 0 | 1 | 9
```

There is the production for any grammar as follows:

```
S → aSa
S → bSb
S → c
```

In BNF, we can represent above grammar as follows:

```
S → aSa | bSb | c
```

```
1*2*3
```

EBNF is a few simple extensions to BNF which make expressing grammars more convenient

- “*” (The Kleene Star): means 0 or more occurrences
- “+” (The Kleene Cross): means 1 or more occurrences
- “[...]”: means 0 or 1 occurrences
- () Use of parentheses for grouping
- {}* used to show arbitrary sequence
- If you have a rule such as:
-

```
<real no> ::= <integer part>.<fraction part> | .<fraction part>
```

You can replace it with:

```
<real no> ::= [<integer part>].<fraction part>
```

- If you have a rule such as:

`<signed integer> ::= + <integer> | - <integer>`

`<integer> ::= <digit>`

`| <integer> <digit>`

You can replace it with:

`<signed integer> ::= [+|-]<digit>{ <digit>}*`

`<signed integer> ::= [+|-]{ <digit>}+`

- If you have a rule such as:

`<exp> ::= <term>`

`| <exp> + <term>`

`| <exp> - <term>`

You

can

replace

it

with:

`<exp> ::= <term>[(+|-)<term>]`

PARSE TREE

- Parse tree is the graphical representation of symbol. The symbol can be terminal or non-terminal.
- In parsing, the string is derived using the start symbol. The root of the parse tree is that start symbol.
- It is the graphical representation of symbol that can be terminals or non-terminals.
- Parse tree follows the precedence of operators. The deepest sub-tree is traversed first. So, the operator in the parent node has less precedence over the operator in the sub-tree.
- All leaf nodes have to be terminals.
- All interior nodes have to be non-terminals.
- In-order traversal gives original input string.

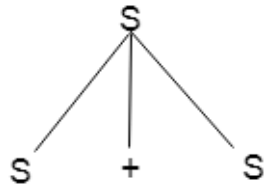
Production rules:

$S \rightarrow S + S \mid S * S$

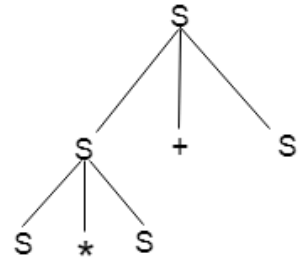
$S \rightarrow a \mid b \mid c$

$w = a * b + c$

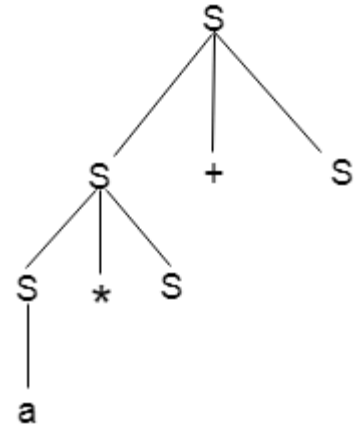
Step 1:



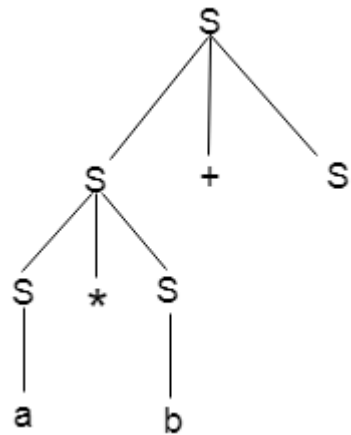
Step 2:



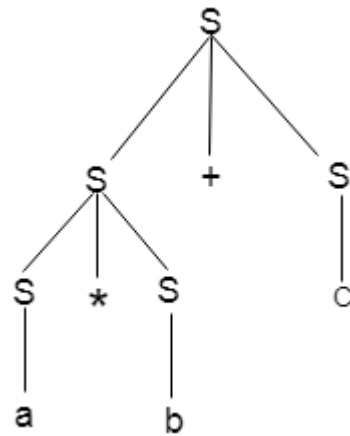
Step 3:



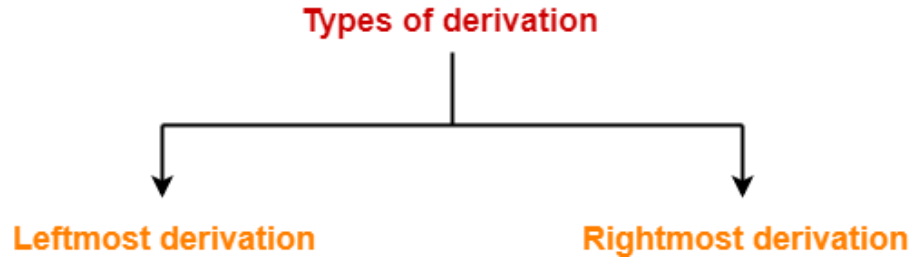
Step 4:



Step 5:



LEFTMOST, RIGHTMOST DERIVATIONS



- A **left-most derivation** of a sentential form is one in which rules transforming the left-most nonterminal are always applied
- A **right-most derivation** of a sentential form is one in which rules transforming the right-most nonterminal are always applied

Consider the following grammar-

$$S \rightarrow aB / bA$$
$$S \rightarrow aS / bAA / a$$
$$B \rightarrow bS / aBB / b$$

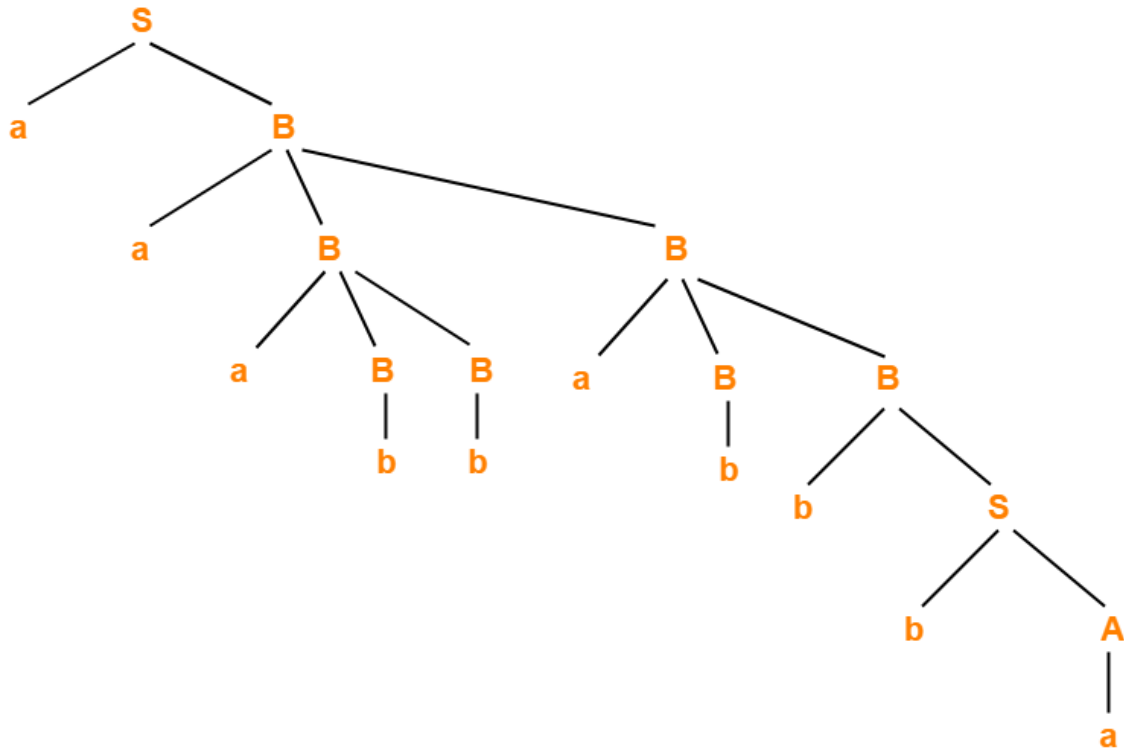
(Unambiguous Grammar)

Let us consider a string $w = aaabbabbba$

Now, let us derive the string w using leftmost derivation.

Leftmost Derivation-

$S \rightarrow aB \rightarrow aaBB$	(Using $B \rightarrow aBB$)
$\rightarrow aaaBBB$	(Using $B \rightarrow aBB$)
$\rightarrow aaabBB$	(Using $B \rightarrow b$)
$\rightarrow aaabbB$	(Using $B \rightarrow b$)
$\rightarrow aaabbaBB$	(Using $B \rightarrow aBB$)
$\rightarrow aaabbabB$	(Using $B \rightarrow b$)
$\rightarrow aaabbabbS$	(Using $B \rightarrow bS$)
$\rightarrow aaabbabbbA$	(Using $S \rightarrow bA$)
$\rightarrow aaabbabbba$	(Using $A \rightarrow a$)



Leftmost Derivation Tree

Rightmost Derivation-

Consider the following grammar-

$S \rightarrow aB / bA$

$S \rightarrow aS / bAA / a$

$B \rightarrow bS / aBB / b$

(Unambiguous Grammar)

Let us consider a string $w = aaabbabbba$

Now, let us derive the string w using rightmost derivation.

$S \rightarrow aB$

$\rightarrow aaBB$ (Using $B \rightarrow aBB$)

$\rightarrow aaBaBB$ (Using $B \rightarrow aBB$)

$\rightarrow aaBaBbS$ (Using $B \rightarrow bS$)

$\rightarrow aaBaBbbA$ (Using $S \rightarrow bA$)

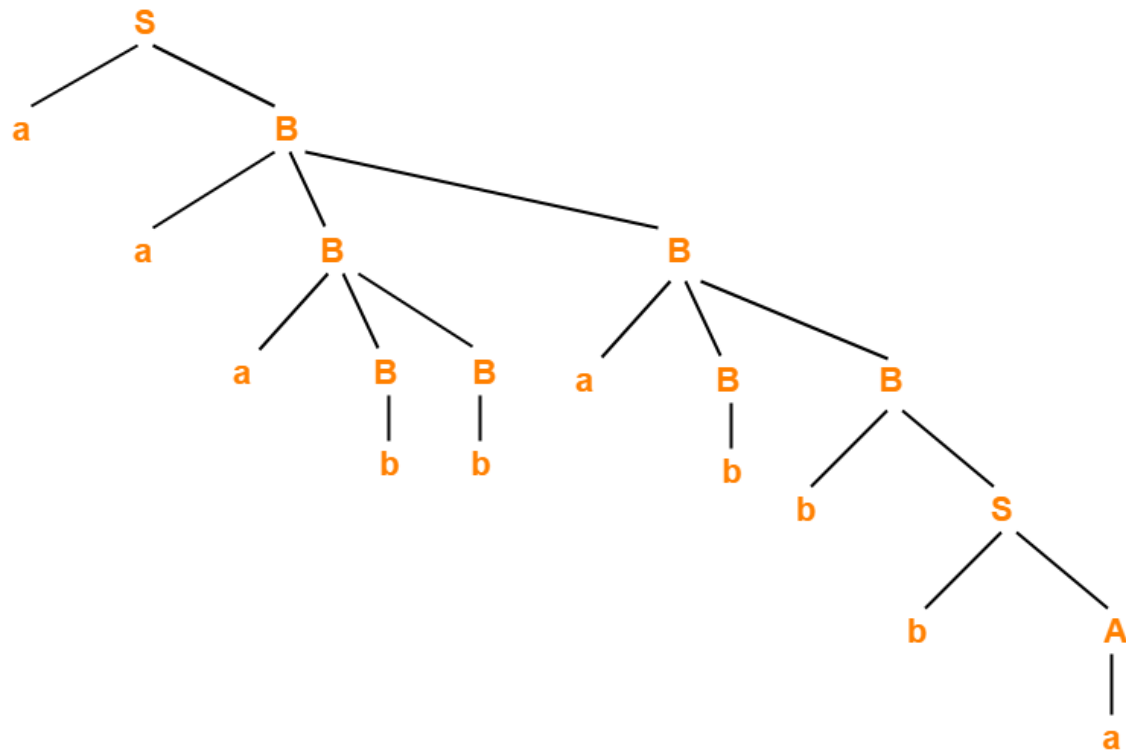
$\rightarrow aaBaBbba$ (Using $A \rightarrow a$)

$\rightarrow aaBabbba$ (Using $B \rightarrow b$)

$\rightarrow aaaBBabbba$ (Using $B \rightarrow aBB$)

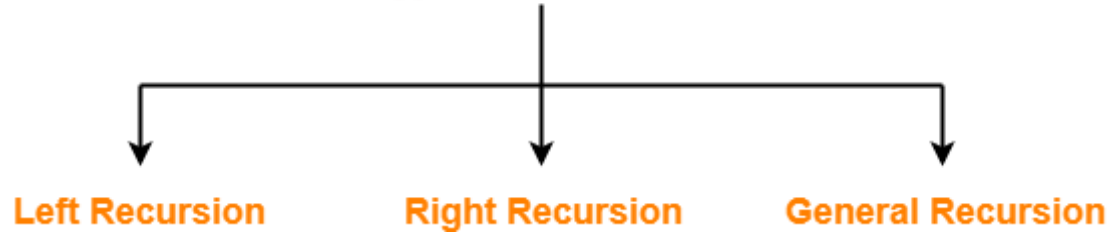
$\rightarrow aaaBbabbba$ (Using $B \rightarrow b$)

$\rightarrow aaabbabbba$ (Using $B \rightarrow b$)



Rightmost Derivation Tree

Types of Recursion



1. Left Recursion
2. Right Recursion
3. General Recursion

General Recursion-

- The recursion which is neither left recursion nor right recursion is called as general recursion.

Example-

$$S \rightarrow aSb / \epsilon$$

Right Recursion-

- A production of grammar is said to have **right recursion** if the rightmost variable of its RHS is same as variable of its LHS.
- A grammar containing a production having right recursion is called as Right Recursive Grammar.

Example-

$$S \rightarrow aS / \epsilon$$

(Right Recursive Grammar)

Left Recursion-

- A production of grammar is said to have **left recursion** if the leftmost variable of its RHS is same as variable of its LHS.
- A grammar containing a production having left recursion is called as Left Recursive Grammar.

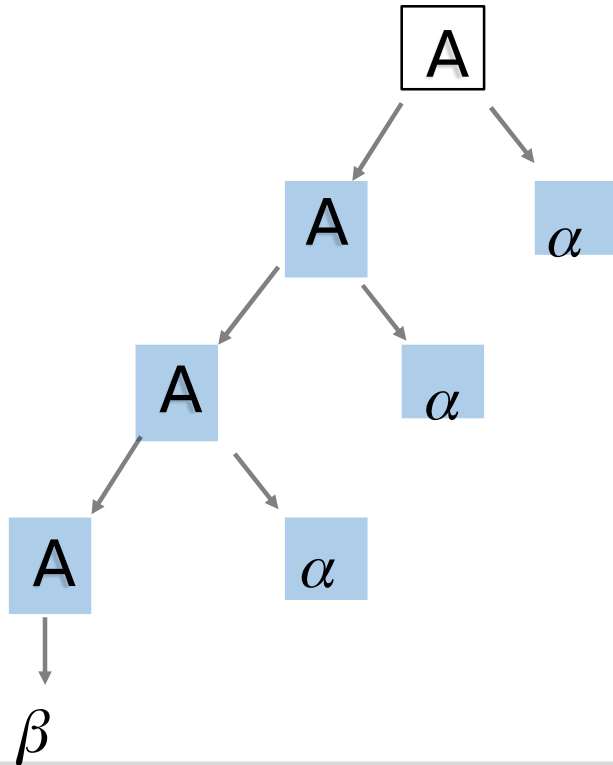
Example-

$$S \rightarrow Sa / \epsilon$$

(Left Recursive Grammar)

LEFT RECURSION

$$A \rightarrow A\alpha/\beta$$

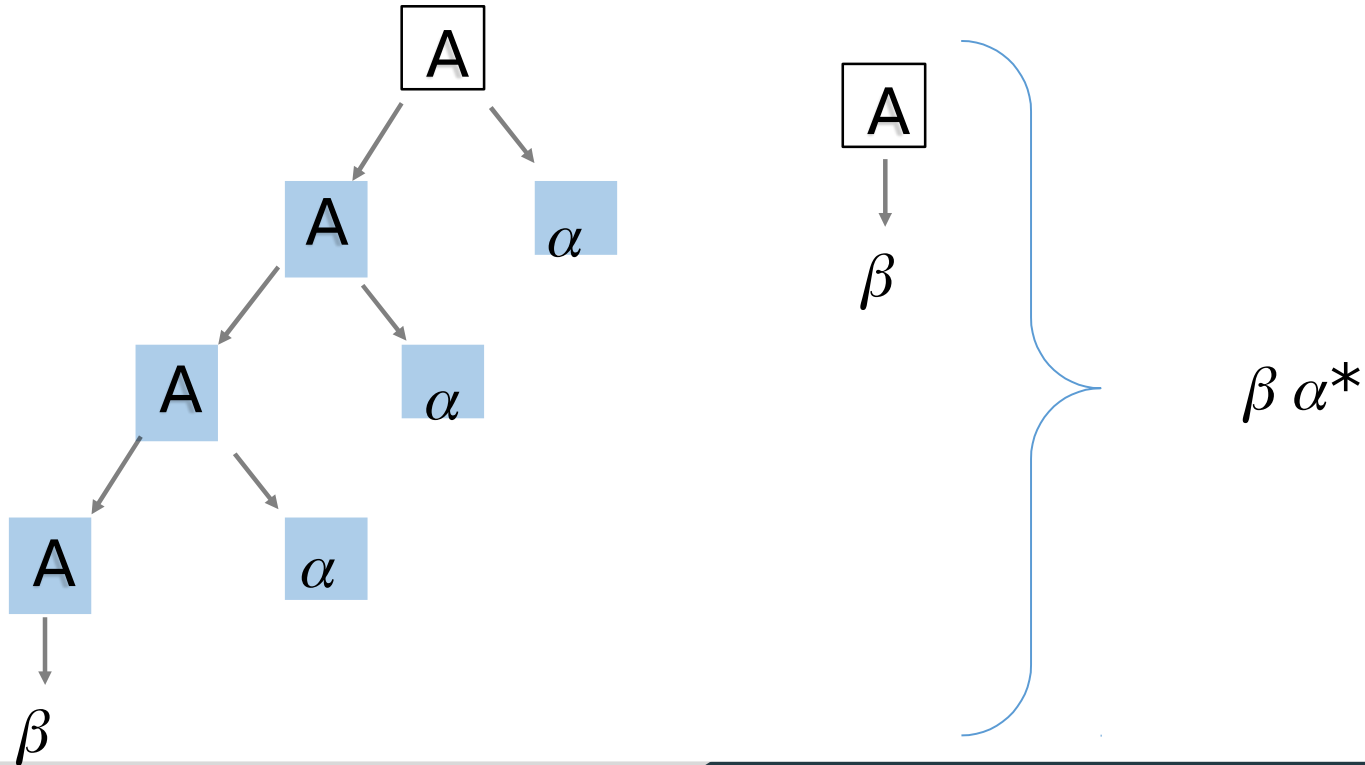


Infinite
problem

Top down parser doesn't allow
in
Left Recursion bcz of it's infinite
problem.
So... we need to eliminate this
problem.


ELIMINATION OF LEFT RECURSION

$$A \rightarrow A\alpha/\beta$$



ELIMINATION OF LEFT RECURSION

$$A \rightarrow A\alpha | \beta$$


$$A \rightarrow \beta A'$$
$$A' \rightarrow \alpha A' | \epsilon$$

ELIMINATION OF LEFT RECURSION

$$A \rightarrow \beta \alpha^*$$

$$\begin{aligned} A &\rightarrow \beta A' \\ A' &\rightarrow \alpha A' \mid \epsilon \end{aligned}$$



$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \beta_1 \mid \beta_2$$

$$A \rightarrow \beta_1 A' \mid \beta_2 A'$$

$$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \epsilon$$

EXAMPLE : IMMEDIATE LEFT RECURSION

$$\begin{array}{ccccccc}
 E & \rightarrow & E & + & T & | & \epsilon \\
 \underbrace{} & & \underbrace{} & & \underbrace{} & & \underbrace{} \\
 A & & A & & \alpha & & \beta
 \end{array}$$

$$\begin{array}{l}
 E \rightarrow TE \\
 \rightarrow +TE \mid \epsilon
 \end{array}$$

$$A \rightarrow A\alpha \mid \beta$$

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' \mid \epsilon$$

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EXAMPLE

$$\begin{array}{c}
 T \rightarrow T * F \mid \\
 \text{F} \\
 A \quad \quad \alpha \quad \beta \\
 A
 \end{array}$$

$$\begin{array}{l}
 T \rightarrow FT \\
 , \\
 T' \rightarrow *FT' \mid \epsilon
 \end{array}$$

$$A \rightarrow A\alpha \mid \beta$$

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' \mid \epsilon$$

$$A \rightarrow Abc \mid A B \quad \mid d \mid fB$$

$$A \rightarrow A\alpha_1 A\alpha_2 \quad \beta_1 \beta_2$$

$$B \rightarrow g$$

UPDATED GRAMMAR

$$A \rightarrow d A' \mid f B A'$$

$$A' \rightarrow bc A' \mid B A' \mid \epsilon$$

$$B \rightarrow g$$

$$A \rightarrow A\alpha_1 \mid \alpha_2 \mid \beta_1 \mid \beta_2$$

$$A \rightarrow \beta_1 A' \mid \beta_2 A'$$

$$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \epsilon$$

$$A \rightarrow Abc \mid A B \mid d$$

$$A \rightarrow A \alpha_1 A \alpha_2 \beta_1$$

$$B \rightarrow g$$

UPDATED GRAMMAR

$$A \rightarrow d A'$$

$$A' \rightarrow bc A' \mid BA' \mid \epsilon$$

$$B \rightarrow g$$

$$T \rightarrow T * F \mid F$$

$T \rightarrow T * F \mid F$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$S \rightarrow A$

$A \rightarrow Ad \mid Ae \mid aB \mid ac$

$B \rightarrow bBc \mid f$

$$T \rightarrow T * F$$

$$T \rightarrow T / F$$

$$T \rightarrow a$$

$$T \rightarrow T * F \mid T / F \mid a$$

$$A \rightarrow A \alpha_1 A \alpha_2 \mid \beta$$

$$T \rightarrow a T'$$

$$T' \rightarrow *F T' \mid /FT' \mid$$

$$\epsilon$$

$$A \rightarrow A\alpha \mid \beta$$


$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \epsilon$$

INDIRECT LEFT RECURSION

$S \rightarrow Aa \mid b$

$A \rightarrow Ac \mid Sd \mid f$



$A \quad \alpha \quad \beta$

A

Case 1: order of Non Terminal- S ,A

Case 2: order of Non Terminal- A, S

1. $E \rightarrow E + T \mid T$

$T \rightarrow T \times F \mid F$

$F \rightarrow \text{id}$

2. $S \rightarrow (L) \mid a$

$L \rightarrow L, S \mid S$

1. $A \rightarrow Ba / Aa / c$

$B \rightarrow Bb / Ab / d$

1. $E \rightarrow TE'$

$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow xFT' \mid \epsilon$$

$$F \rightarrow id$$

EXAMPLE: INDIRECT LEFT RECURSION

CASE 1: $S \rightarrow Aa \mid b$
 $A \rightarrow Ac \mid Sd \mid f$

For S

- There is no immediate left recursion
- Don't enter in the inner loop

For A

- Replace $A \rightarrow Sd$ with $S \rightarrow Aa \mid b$
 $A \rightarrow Ac \mid Aad \mid bd \mid f$

$$A \rightarrow A\alpha \mid \beta$$

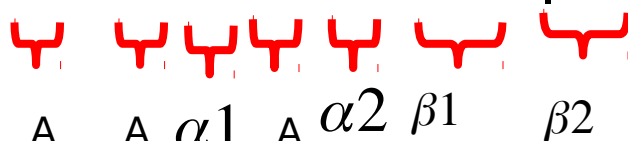
$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' \mid \epsilon$$

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EXAMPLE

Remove immediate left recursion from A

$$A \rightarrow A c \mid A a d \mid b d \mid f$$


$$\begin{array}{ccccccc} A & A & \alpha 1 & A & \alpha 2 & \beta 1 & \beta 2 \end{array}$$

$$A \rightarrow b d A' \mid f A$$

$$A' \rightarrow c A' \mid a d A' \mid \epsilon$$



$$S \rightarrow A a \mid b$$

$$A \rightarrow b d A' \mid f A'$$

$$A' \rightarrow c A' \mid a d A' \mid \epsilon$$

$$A \rightarrow A \alpha 1 \mid A \alpha 2 \mid \beta$$

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha 1 A' \mid \alpha 2 A' \mid \epsilon$$

EXAMPLE: INDIRECT LEFT RECURSION

CASE
2:

$$S \rightarrow Aa \mid b$$
$$A \rightarrow \underbrace{A} \underbrace{c} \mid \underbrace{Sd} \mid f$$

A α β

A
Remove immediate left recursion from A

$$A \rightarrow SdA' \mid fA$$

$$A' \rightarrow cA' \mid \epsilon$$

$$A \rightarrow A\alpha \mid \beta$$

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' \mid \epsilon$$

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EXAMPLE

For S : replace A by $A \rightarrow SdA' \mid fA'$ in $S \rightarrow Aa \mid b$

$$S \rightarrow Aa \mid b$$

$$S \rightarrow SdA' \mid fA'a \mid b$$

$$S \rightarrow \underbrace{SdA'}_A \mid \underbrace{fA'a}_\alpha \mid \underbrace{\quad}_\beta$$

$$S \rightarrow fA'aS' \mid bS'$$

$$S' \rightarrow dA'aS' \mid \epsilon$$



$$A \rightarrow A\alpha/\beta$$

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A'/\epsilon$$

$$A \rightarrow SdA' \mid fA'$$

$$A' \rightarrow cA' \mid \epsilon$$

$$S \rightarrow fA'aS' \mid bS'$$

$$S' \rightarrow dA'aS' \mid \epsilon$$

LEFT FACTORING

- In left factoring it is not clear which of two alternative productions to use to expand a nonterminal A.

i.e. if $A \longrightarrow \alpha\beta_1 \mid \alpha\beta_2$

- We don't know whether to expand A to $\alpha\beta_1$ or to $\alpha\beta_2$
- To remove left factoring for this grammar replace all A productions containing α as prefix by

$$A \longrightarrow \alpha A'$$

$$A' \longrightarrow \beta_1 \mid \beta_2$$

$S \rightarrow abc \mid ab$

A --> Br

B --> Cd

C --> At

YACC(yet another compiler compiler)

- YACC generates C code for syntax analyzer or parser.
- YACC uses grammar rules that allow it to analyze token from LEX and create syntax tree.
- YACC issues a warning message whenever a conflict occurs.
- YACC takes a default action when there is a conflict-

Conflict can be:

- shift reduce conflict
- Reduce reduce conflict

Structure of YACC

Definations

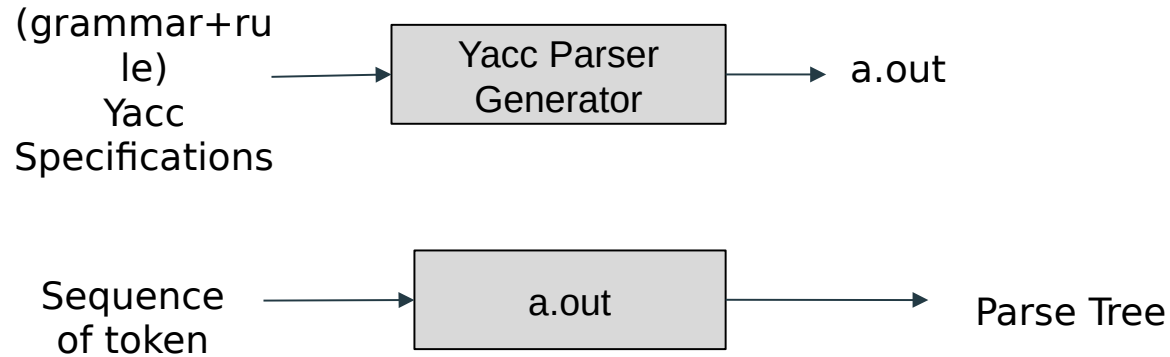
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Rules

%%

Subroutines (supporting C routines)

YACC(yet another compiler compiler)



YACC(yet another compiler compiler)

