## Arden’s Theorem:

If P and Q are two regular expressions over Σ, and if P does not contain ϵ , then the following equation in R given by R = Q + RP

has an unique solution i.e.,

$R = QP^*$

Let’s start by taking this equation as equation (i)

$R = Q + RP$ ......(i)

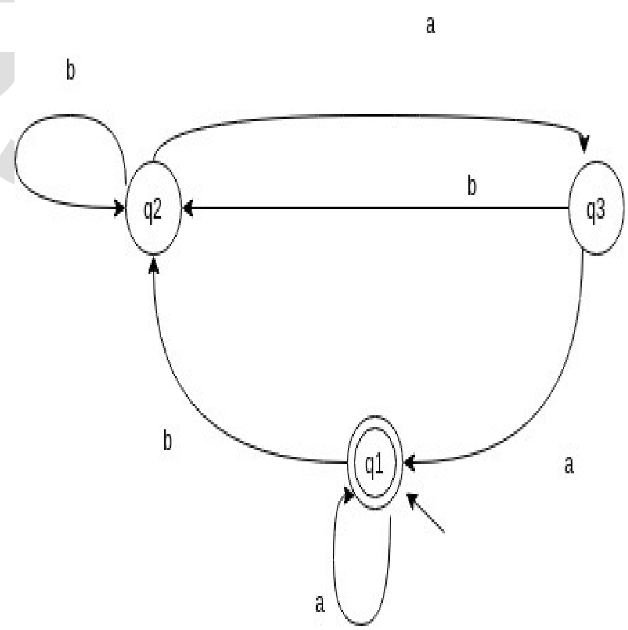
Now, replacing R by $R = QP^*$, we get,

$R = Q + QP^*P$

Taking Q as common,

$R = Q(\epsilon + P^*P)$
$R = QP^*$

## Example 1:




Initial and final state-q1
Step-01:

equations are
$q1=q1\ a + q3\ a + \epsilon$ ( 1
$q2= q1\ b + q2\ b + q3\ b$ (2
$q3= q2\ a$ (3

Bring state q2 in the form R = Q + RP.

$q2= q1\ b + q2\ b + (q2\ a)\ b$
$= q1\ b + q2(\ b + a\ b)$

by arden’s theorem

$q2=q1\ b\ (b+ab)^*$ (4
Using (3) in (1), we get-

$q1=q1\ a + q3\ a + \epsilon$

=q1a+(q2 a)a + ϵ

Using (4) in (1), we get-

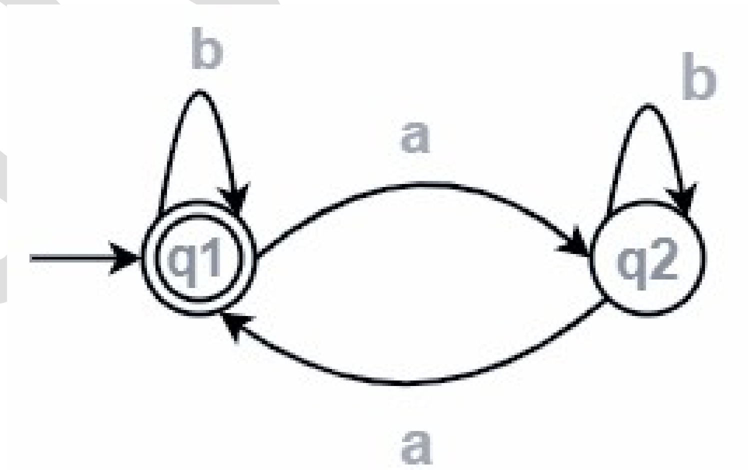
=q1 a+q1 b (b+ab)* a a + ϵ
=q1 (a+b (b+ab)* a a) + ϵ (5

Using Arden's Theorem in (5), we get-

q1= ϵ + q1 (a+b (b+ab)* a a)
q1=(a+b(b+ab)*aa)*

Thus, Regular Expression for the given DFA = (a+b(b+ab)*aa)*

## Example 2:




Step-01:

Form a equation for each state-

- q1 = ∈ + q1 b + q2 a (1)
- q2 = q1 a + q2 b (2)

Step-02:

Bring final state in the form R = Q + RP.

Using Arden's Theorem in (2), we get-

q2 = q1 a b* .......(3)

Using (3) in (1), we get-

$q1 = \in + q1\ b + q1\ a\ b^*\ a$

$q1 = \in + q1\ (b + a\ b^*\ a)$ .......(4)

Using Arden’s Theorem in (4), we get-

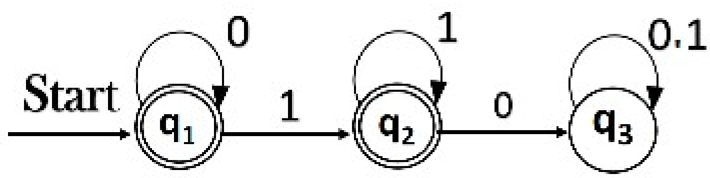
$q1 = \in (b + a\ b^*\ a)^*$

$q1 = (b + a\ b^*\ a)^*$

Thus, Regular Expression for the given DFA = $(b + a\ b^*\ a)^*$

## Example 3:

Construct the regular expression for the given DFA




Solution:

Let us write down the equations

$q1 = q1\ 0 + \varepsilon$

Since q1 is the start state, so ε will be added, and the input 0 is coming to q1 from q1 hence we write

State = source state of input × input coming to it

Similarly,

$q2 = q1\ 1 + q2\ 1$

$q3 = q2\ 0 + q3\ (0+1)$

Since the final states are q1 and q2, we are interested in solving q1 and q2 only. Let us see q1 first

$q1 = q1\ 0 + \varepsilon$

We can re-write it as

$q1 = \varepsilon + q1\ 0$

Which is similar to R = Q + RP, and gets reduced to R = OP*.

Assuming R = q1, Q = ε, P = 0

We get

$q1 = \varepsilon.(0)^*$
$q1 = 0^* \quad (\varepsilon.R^* = R^*)$

Substituting the value into q2, we will get

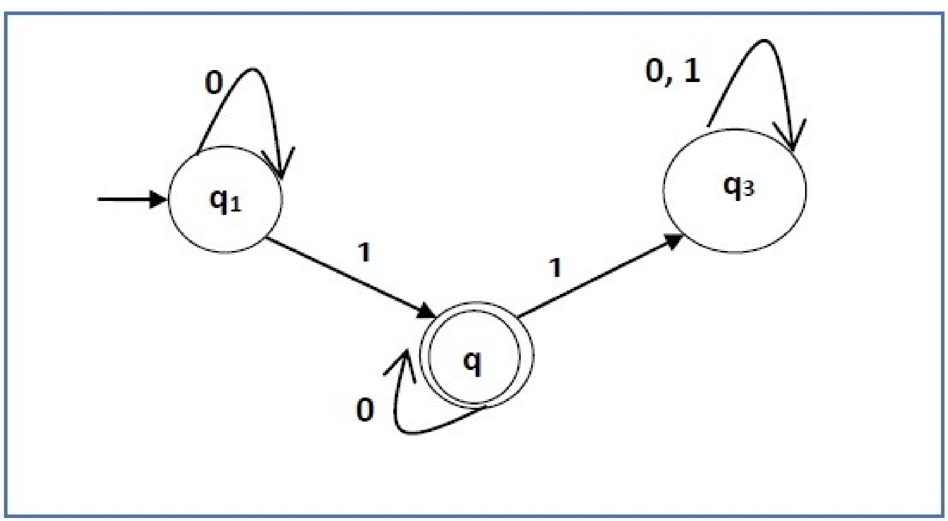
$q2 = 0^*\ 1 + q2\ 1$
$q2 = 0^*\ 1\ (1)^* \quad (R = Q + RP \rightarrow Q P^*)$

The regular expression is given by

$r = q1 + q2$
$= 0^* + 0^*\ 1.1^*$
$r = 0^* + 0^*\ 1^+ \quad (1.1^* = 1^+)$

## Example 4:

Construct a regular expression



corresponding to the automata given below −

Here the initial state is q1 and the final state is q2

Now we write down the equations −

q1 = q10 + ε

q2 = q11 + q20

q3 = q21 + q30 + q31

Now, we will solve these three equations −

q1 = ε0* [As, εR = R]

So, q1 = 0*

q2 = 0*1 + q20

So, q2 = 0*1(0)* [By Arden’s theorem]

Hence, the regular expression is 0*10*.