Questions on Master's Method

Example-12 $T(n) = 4T(\frac{n}{2}) + 1$

$a=4, b=2, f(n) = 1=(n^0\log^0 n)$ $k=0, p=0$

$\log_2 4 =2 > k$

It is case 1

$T(n) = \theta(n^{\log_2 4}) = \theta(n^2)$

Example-13 $T(n) = 8T(\frac{n}{2}) + n^2$

$a=8, b=2, f(n) = n^2 =(n^2\log^0 n)$ $k=2, p=0$

$\log_2 8 =3 > k$

It is case 1

$T(n) = \theta(n^{\log_2 8}) = \theta(n^3)$

Example-14 $T(n) = 16T(\frac{n}{2}) + n^2$

$a=16, b=2, f(n) = n^2 =(n^2\log^0 n)$ $k=2, p=0$

$\log_2 16 =4 > k$

It is case 1

$T(n) = \theta(n^{\log_2 16}) = \theta(n^4)$

Example-15

$T(n) = T(\frac{n}{2}) + 1$

$a=1, b=2, f(n) = 1= (n^0\log^0 n)$ $k=0, p=0$

$\log_2 1 =0 = k$

It is case 2(2.1)

$T(n) = \theta(n^k \log^{p+1} n) = \theta(n^0 \log n) = \theta(\log n)$

Example-16 $T(n) = 2T(\frac{n}{2}) + n\log n$

$a=2, b=2, f(n) = n\log n = (n^1 \log^1 n)$ $k=1, p=1$

$\log_2 2 = 1 = k$

It is case 2(2.1)

$T(n) = \theta(n^k \log^{p+1} n) = \theta(n^1 \log^2 n) = \theta(n \log^2 n)$

Example-17

$T(n) = T(\frac{n}{2}) + n$

$a=1, b=2, f(n) = n\ (n^1 \log^0 n)$ $k=1, p=0$

$\log_2 1 = 0 < k$

It is case 3(3.1)

$T(n) = \theta(n^k \log^p n) = \theta(n^1 \log^0 n) = \theta(n)$

Example-18

$T(n) = 2T(\frac{n}{2}) + n^2$

$a=2, b=2, f(n) = n^2 = (n^2 \log^0 n)$ $k=2, p=0$

$\log_2 2 = 1 < k$

It is case 3(3.1)

$T(n) = \theta(n^k \log^p n) = \theta(n^2 \log^0 n) = \theta(n^2)$

Example-19

$T(n) = 2T(\frac{n}{2}) + n^2 \log n$

$a=2, b=2, f(n) = n^2 \log n = (n^2 \log^1 n)$

$k=2, p=1$

$\log_2 2 = 1 < k$

It is case 3(3.1)

$T(n) = \theta(n^k \log^p n) = \theta(n^2 \log^1 n) = \theta(n^2 \log n)$

Example-20

$T(n) = 4T(\frac{n}{2}) + n^3 \log^2 n$

$a=4, b=2, f(n) = n^3 \log^2 n$

$k=3, p=2$

$\log_2 4 = 2 < k$

It is case 3(3.1)

$T(n) = \theta(n^k \log^p n) = \theta(n^3 \log^2 n)$

Example-21

$T(n) = 2T(\frac{n}{2}) + n^3 / \log^3 n$

$a=2, b=2, f(n) = n^3 / \log^3 n$

$k=3, p= -3$

$\log_2 2 = 1 < k$

It is case 3(3.2)

$T(n) = \theta(n^k) = \theta(n^3)$

Special Cases of Master Method

Example-1 $T(n) = T\sqrt{n} + c$

Here, a=1 but b is also 1

Now, do some steps to apply master theorem

Step-1 Assume $n = 2^i$ $\Rightarrow$ $i = \log n$

$T(2^i) = T(2^{i/2}) + c$

Step-2 Assume $T(2^i) = S(i)$

$S(i) = S(i/2) + C$

Here, a=1, b=2, k=0 , p=0 & $\log_2 1 = 0 = k$

It is case2 (2.1)

$S(i) = \theta(i^k \log^{p+1} i) = \theta(i^0 \log^{0+1} i) = \theta(\log i)$

Step-3 $S(i) = \theta(\log i)$

$\Downarrow$

$T(2^i) = \theta(\log i)$

Step-4 $T(2^i) = \theta(\log i)$

$\mathbf{T(2^{\log n}) = \theta(\log \log n)}$

$\boxed{T(n) = \theta(\log \log n)}$

Example-2 $T(n) = 2T\sqrt{n} + \log n$

Now, do some steps to apply master theorem

Step-1 Assume $n = 2^i \Rightarrow i = \log n$

$T(2^i) = 2\,T(2^{i/2}) + \log 2^i$

Step-2 Assume $T(2^i) = S(i)$

$S(i) = 2\,S(i/2) + i$

Here, a=2, b=2, k=1 , p=0 & $\log_2 2 = 1 = k$

It is case2 (2.1)

$S(i) = \theta(i^k \log^{p+1} i) = \theta(i^1 \log^{0+1} i) = \theta(i * \log i)$

Step-3 $S(i) = \theta(i * \log i)$

$\Downarrow$

$T(2^i) = \theta(i * \log i)$

Step-4 $T(2^i) = \theta(i * \log i)$

$\mathbf{T(2^{\log n}) = \theta(\log n * \log \log n)}$

$$\boxed{T(n) = \theta(\log n * \log\log n)}$$