Greedy Algorithm

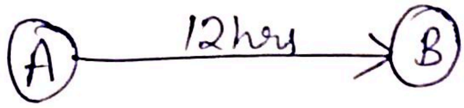
A Greedy Algorithm is a strategy that makes the optimal choice at each stage with hope of finding a global optimal.

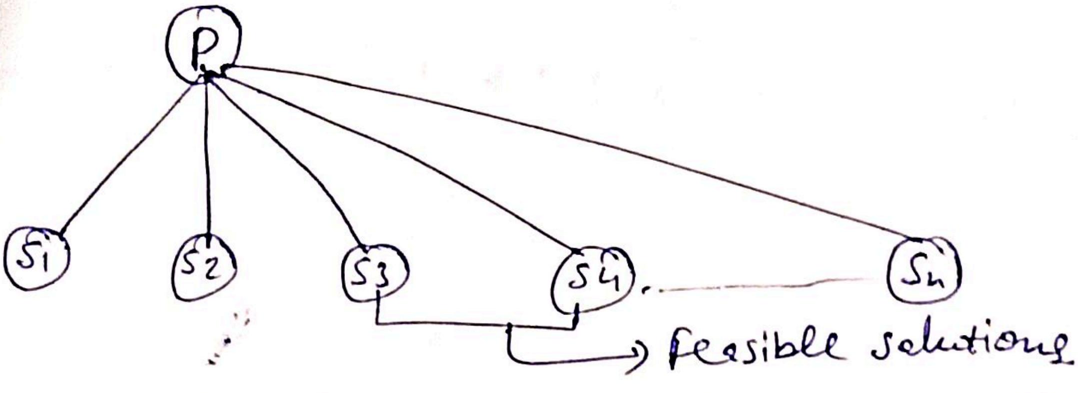
Optimization ⟶ Finding minimum optimal (Minimum Cost)

↓

Finding maximum optimal (Max job in limited time)

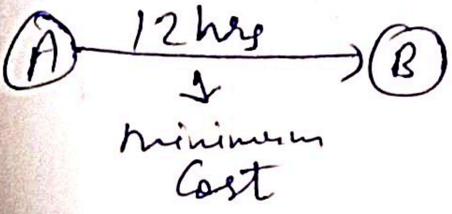
example:





There are $S_1, S_2, S_3, S_4, \ldots S_n$ ways to travel from (A) to (B) but with a condition of 12hrs traveling time, only S_3 & S_4 takes time 12 hrs to travel from (A) ⟶ (B). Therefore, S_3 & S_4 are feasible solutions

Now, Suppose one more Condition i.e. Cost is added with time to travel from (A) → (B)

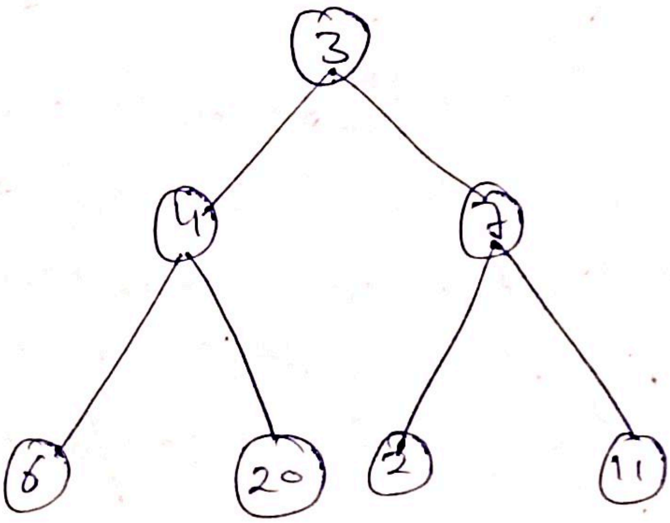


Only S_3 gives minimum cost, therefore S_3 is optimal Solution for this problem

"Optimal Solution is the best one from feasible Solution."

Pros: Simple, easy to implement, run fast

Cons: Do not always yield optimal solution



Maximization

Greedy: 3+7+11=21

Actual: 3+4+20=27

Minimization

Greedy: 3+4+6=13

Actual: 3+7+2=12

```
Algo Greedy (a, n)
{
  Solution = 0;
  for i to n do
  {
    x = Select (a)
    if feasible (solution, x)
    then Solution Union (Solution, x)
  }
  return Solution;
}
```

Applications of Greedy Algorithm

1- Activity Selection Problem
2- Huffman Coding
3- Job Sequencing Problem
4. Fractional knapsack Problem
5- Finding Minimum Spanning Tree
6. Single Source Shortest path.

Fractional Knapsack Problem

$n = 7$

$m = 15$

Objects O	1	2	3	4	5	6	7
Profits P	10	5	15	7	6	18	3
weights w	2	3	5	7	1	4	1

Knapsack with Capacity = 15
or
Container
or
Bag

It is a Optimization (Maximization) Problem

This knapsack problem is for those objects which are divisible

$0 \leq x \leq 1$

Constraint

$\sum x_i w_i \leq m$

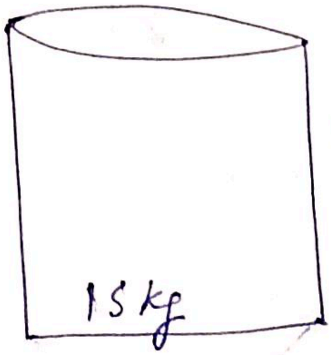
	1	2	3	4	5	6	7
$\frac{P}{W}$	5	1.3	3	1	6	4.5	3

x (1, 2/3, 1, 0, 1, 1, 1)

x_1 x_2 x_3 x_4 x_5 x_6 x_7

objective

$\max \sum x_i p_i$



$15 - 1 = 14$

$14 - 2 = 12$

$12 - 4 = 8$

$8 - 5 = 3$

$3 - 1 = 2$

$2 - 2 = 0$

$\sum x_i w_i =$

Greedy-5

$1 \times 2 + \frac{2}{3} \times 3 + 1 \times 5 + 0 \times 7 + 1 \times 1 + 1 \times 4 + 1 \times 1$

$= 2 + 2 + 5 + 0 + 1 + 4 + 1$

$= 15$ kg

Profit

$= 1 \times 10 + \frac{2}{3} \times 5 + 1 \times 15 + 1 \times 6 + 1 \times 18 + 1 \times 3$

$\sum x_i P_i = 10 + \frac{10}{3} + 15 + 6 + 18 + 3 = 54.6$

Greedy knapsack

{

for $i=1$ to n;

Compute p_i/w_i; $\rightarrow O(n)$

Sort objects in non-increasing order of p/w $\rightarrow O(n\log n)$

for $i=1$ to n

if ($m>0$ && $w_i \le m$)

$M = M - w_i$;

$P = P + p_i$; $\Big\}\ O(n)$

else break;

if ($m>0$)

$P = P + p_i \left(\frac{m}{w_i}\right)$; — $O(1)$

}

$$\boxed{O(n\log n)}$$

$m = 15 \quad n = 7$

Objects	1	2	3	4	5	6	7
Profits	10	5	15	7	6	18	3
weights	2	3	5	7	1	4	1
P_i / w_i	5	5/3 (1.6)	3	1	6	4.5	3

Object no: 5 1 6 3 7 2 4

2
7
3
6
1
5

$m = \cancel{15}\ \cancel{14}\ \cancel{12}\ \cancel{8}\ \cancel{3}\ 2$

$P = 6 + 10 + 18 + 15 + 5\left(\frac{2}{3}\right)$

$= 55.3$

$m = 15$ $n = 5$

objects	1	2	3	4	5
P	2	28	25	18	9
w	1	4	5	3	3
P/w	2	7	5	6	3

$\cancel{2}, \cancel{4}, \cancel{3}, 5, 1$

5
3
4
2

$m = \cancel{15}\ \cancel{11}\ \cancel{8}\ \cancel{3}\ 0$

$P = 28 + 18 + 25 + 9$

$=$