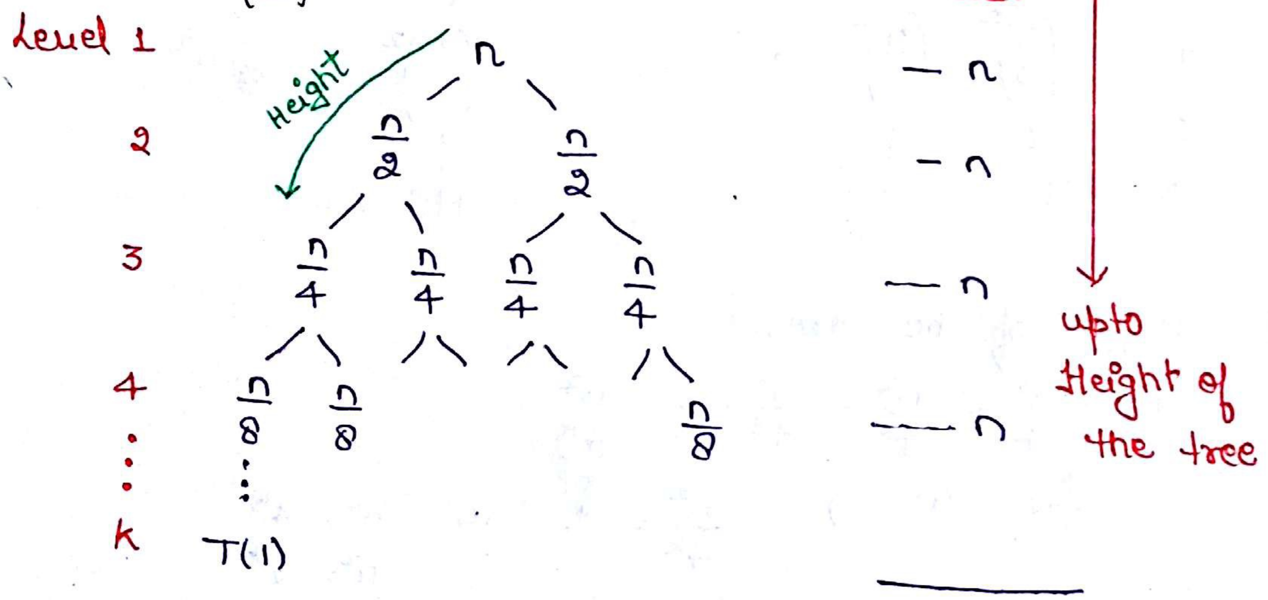
(c) 6

RECURSION TREE METHOD

Recursion tree method is pictorial representation of iteration method, which is in the form of a tree where at each level nodes are expanded.

Each node represents the cost of single subproblem somewhere in the set of recursive function invocation. We sum the cost within each level of the tree to obtain a set of per level cost and then we sum all the per level cost to determine the total cost.

Ques- $T(n) = 2T\left(\frac{n}{2}\right) + n$



$\Theta(n \cdot \log n)$

I. Height of the tree

Let the tree grows upto k levels

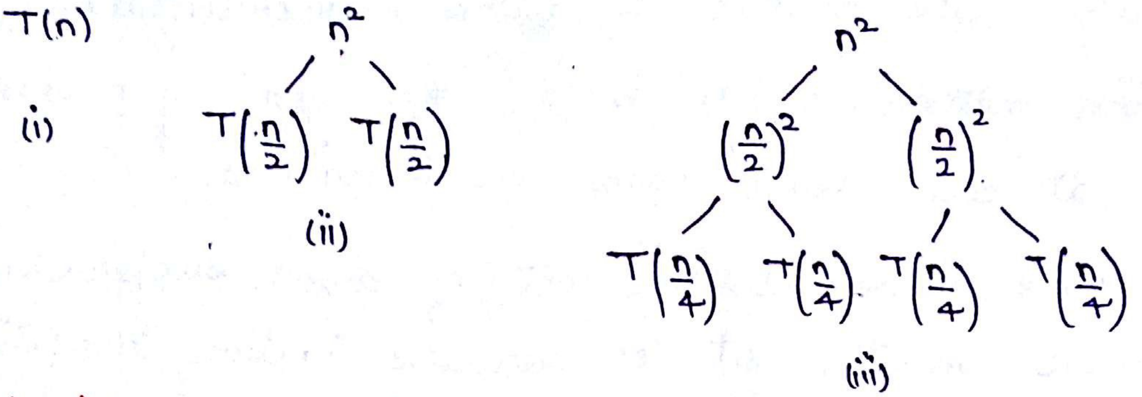
$\frac{n}{2^0}, \frac{n}{2^1}, \frac{n}{2^2}, \frac{n}{2^3} \ldots \frac{n}{2^k}$ – base condition

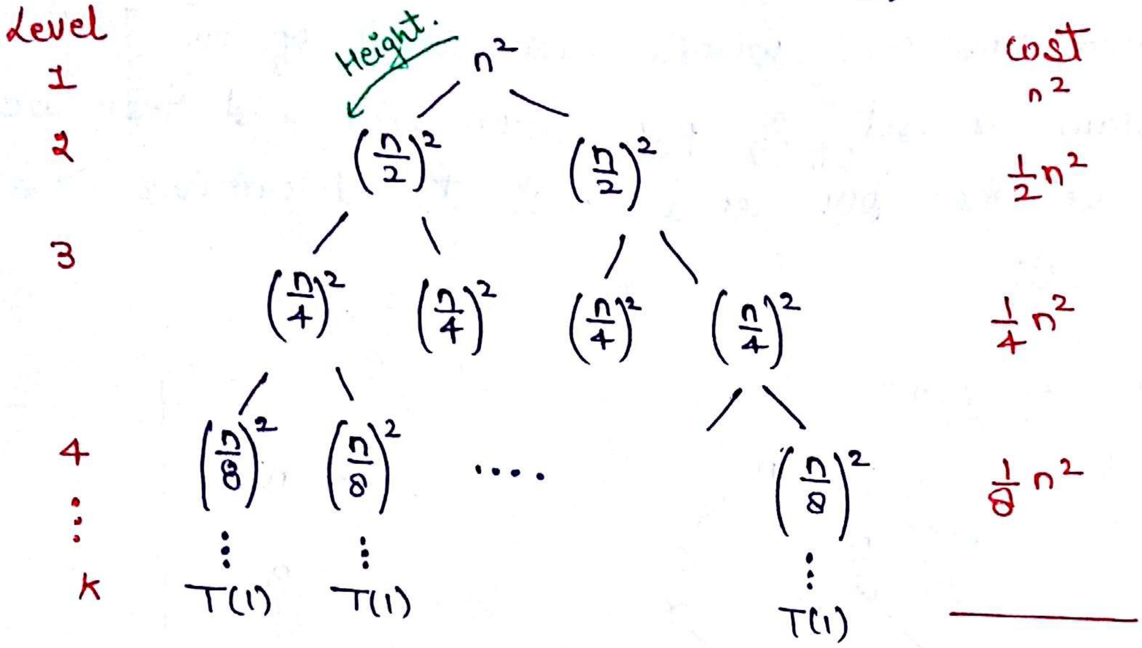
$\therefore \frac{n}{2^k} = 1 \Rightarrow k = \log_2 n$ – Height of the tree

Cost at each level $= n$

Overall cost $= \Theta(n \log n)$

Ques- $T(n) = 2T\left(\frac{n}{2}\right) + n^2$





I. Height of the tree.

$$n^2 + \frac{n^2}{4} + \frac{n^2}{4^2} + \frac{n^2}{4^3} \cdots \frac{n^2}{4^k}$$

for $T(1) \Rightarrow \quad \frac{n^2}{4^k} = 1 \quad$ ie $\quad n^2 = 4^k$

$$n^2 = 2^{2k}$$

$$\log n^2 = \log_2 2^{2k}$$

$$2\log n = 2k\log_2 2$$

Height of the tree $\leftarrow k = \log n$

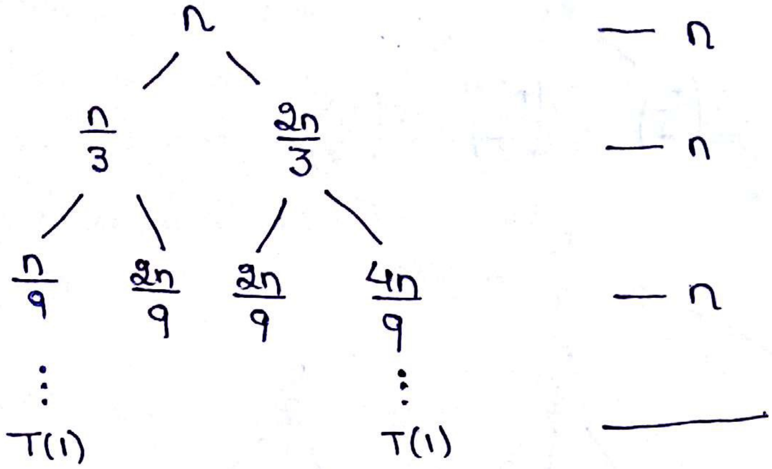
$$T(n) \leq n^2 + \frac{1}{2}n^2 + \frac{1}{4}n^2 + \frac{1}{8}n^2 \cdots \text{ logn times.}$$

$$\leq \sum_{i=0}^{\log n}\left(\frac{1}{2^i}\right)n^2 \qquad \sum_{i=0}^{\infty} a^i = \frac{a}{1-r}$$

$$\leq \left(\frac{1}{1-1/2}\right)n^2$$

$\therefore \quad T(n) \leq 2n^2 \Rightarrow \quad T(n) = \Theta(n^2)$

Ques-7 $T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n$



For Smaller Subtree

$$n, \frac{n}{3}, \frac{n}{3^2}, \frac{n}{3^2} \cdots \frac{n}{3^k}$$

For $T(1) \Rightarrow \frac{n}{3^k} = 1$

$$n = 3^k$$

$$\boxed{\log_3 n = k} \rightarrow \text{Height}$$

$$\therefore T(n) = \Omega(n \cdot \log_3 n) = \Omega\left(n \cdot \frac{\log_2 n}{\log_2 3}\right)$$

$$\approx \Omega(n \log_2 n)$$

For larger sub-tree

$$n, \frac{n}{3/2}, \frac{n}{(3/2)^2}, \frac{n}{(3/2)^3} \cdots\cdots \frac{n}{(3/2)^k}$$

For $T(1) \Rightarrow \frac{n}{(3/2)^k} = 1$

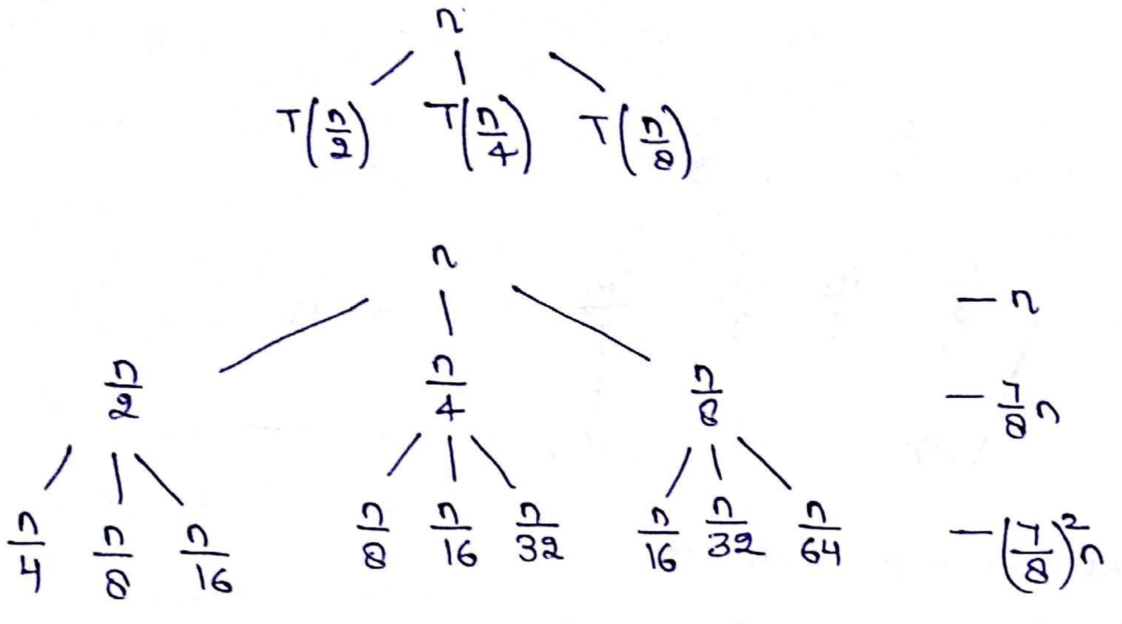
$$n = \left(\frac{3}{2}\right)^k$$

$$\boxed{k = \log_{3/2} n} \rightarrow \text{Height}$$

$$T(n) = \Theta(n \cdot \log_{3/2} n)$$

$$= \Theta(n \log_2 n)$$

Ques- $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right) + n$



$$T(n) \leq n + \frac{7}{8}n + \left(\frac{7}{8}\right)^2 n + \cdots$$

$$\leq n\left[1 + \frac{7}{8} + \left(\frac{7}{8}\right)^2 + \cdots\right]$$

$$\leq n \cdot \frac{1}{1 - \frac{7}{8}}$$

$$\leq 8n$$

$$T(n) = \Theta(n)$$

$\sum_{i=0}^{\infty} (\frac{1}{2})^i n = \frac{1}{1-7/8} n = 8n$

5. T(n) = T(n/10) + T(9n/10) + n

   Answers: **O (n log $_{10/9}$ n)**

   Ω (n log $_{10}$ n)

   Θ (n log $_{10/9}$ n)

6. T(n) = T(n/5) + T(4n/5) + n

   Answers: **O (n log $_{5/4}$ n)**

   Ω (n log $_{5}$ n)

   Θ (n log $_{5/4}$ n)