1. T(n-2) = T(n-3)*(n-2) $\qquad$ T(0)=1

T(n) = T(n-1)*n……………….1 time

= T(n-2)*(n-1)*n…………..2 time

= T(n-3)*(n-2)*(n-1)*(n-0)……….3 time

$\downarrow$

= T(n-k)*(n-(n-1))*(n-(k-2))………(n-1)*n

n-k = 0

k = n

= T(0)*1*2*3*4……(n-2)*(n-1)*n

= n*(n-1)*(n-2)*(n-3)*….*3*2*1

T(n) = n!

= $\Omega(2^n)$

= $O(n^n)$

2. $T(n) = T(n-1) + \frac{1}{n}$ $\qquad$ T(0) = 1

$T(n) = T(n-1) + \frac{1}{n}$…………..1st time

$= T(n-2) + \frac{1}{n-1} + \frac{1}{n}$……….2nd time

$= T(n-3) + \frac{1}{n-2} + \frac{1}{n-1} + \frac{1}{n}$…..3rd time

$\downarrow$

$= T(n-k) + \frac{1}{n-(k-1)} + \frac{1}{n-(k-2)} + .. + \frac{1}{n}$

n-k = 0

$k = n$

$T(n) = T(0) + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} \dots \dots \dots + \frac{1}{n}$

$T(n) = 1 + \log n$

$T(n) = O(\log n)$

3. $T(n/2^2) = 2T(n/2^3) + 4n/2^2$ $\quad\quad T(1) = 4$

$T(n/2) = 2T(n/2^2) + 4n/2$

$T(n) = 2T(n/2) + 4n$.....1^{st} time

$= 2 * 2T(n/2^2) + 4n + 4n$ ....2^{nd} time

$= 2^3T(n/2^3) + 3*4n$ ....3^{rd} time

$\downarrow$

$= 2^k T(n/2^k) + k*4n$

$n/2^k = 1$

$k = \log n$

$T(n) = 2^{\log n}\, T(n/2^{\log n}) + \log n * 4n$

$= 4n + 4n\log n$

$T(n) = O(n \log n)$

4. $T(n) = T(n/2) + n$ $\quad\quad T(1) = 1$

$T(n) = T(n/2^2) + n/2 + n$

$= T(n/2^3) + n/2^2 + n/2 + n$

$\downarrow$

$= T(n/2^k) + n/2^{k-1} + n/2^{k-2} + \ldots + n$

$n/2^k = 1$

$k = \log n$

$T(n) = T(n / 2^{\log n}) + n/(2^{\log n-1}) + n/2^{\log n-2} + \ldots\ldots.. + n$

$= T(1) + 2 + 2^2 + 2^3 + \ldots\ldots\ldots\ldots + 2^{\log n}$

$= 1 + [2+2^2 + \ldots\ldots\ldots.. + 2^{\log n}]$

$a+ar+ar^2ar^3+\ldots\ldots\ldots\ldots\ldots+ar^k=a(r^{k+1}-1)/(r-1)$

$a = 2 \quad r = 2$

$= 1 + 2(2^{\log n}-1)/(2-1)$

$= 1+ 2(n-1)$

$= 2n-1$

$=O(n)$