Iteration Method

It means to expand the recurrence and express it as a summation of terms of n and initial condition.

1.

$$T(n) = \begin{cases} 1 & n=0 \\ T(n-1)+1 & n>0 \end{cases}$$

$T(n-2)=T(n-3)+1$

$T(n) = T(n-1) + 1$……………..1^{st} time

$= T(n-2) + 1 + 1$…………..2^{nd} time

$= T(n-3) + 1 + 1 + 1$……….3rd time

$\Big\downarrow$ k times

$= T(n-k) + k$

Assume $n-k = 0$

$k=n$

$= T(n-n) + n$

$= T(0)+n$

$T(n) = 1+n$….exact answer

$T(n)=O(n)$

2.

$$T(n) = \begin{cases} 1, & n=0 \\ \mathbf{T(n-1) + n, \; n>1} \end{cases}$$

T(n) = T(n-2) + n-1………………..n^2

T(n) = T(n-1) + **n ………….1ST TIME**

= T(n-2) + n-1 + n…………2ND TIME

= T(n-3) + n-2 + n-1 + n …………….3RD TIME

↓

= T(n-k) + n-(k-1) + n-(k-2) + ... + n-1 + n

assume n-k = 0 so k = n and T(0) = 1

= 1+ 1 + 2 +3+ ... + n

= 1+ n(n+1)/2

= 1+n^2/2 +n/2….exact answer

we take the dominating term which is n^2*1/2 therefor 1/2 = big O

T(n)=O(n^2)

3.

$$\mathbf{T(n)} = \begin{cases} 1, & n=0 \\ \mathbf{T(n-1) + logn \;, \; n>0} \end{cases}$$

T(n) = T(n-1) + logn……………….1st time

=T(n-2) + log(n-1)+ logn……………2nd time

= T(n-3) + log(n-2) + log(n-1) + logn…………3rd time

↓

$= T(n-k) + \log(n-(k-1)) + \log(n-(k-2)) + \ldots + \log(n-1) + \log n$

Assume $n-k = 0$ so $k = n$ and $T(0) = 1$

$= 1 + \log 1 + \log 2 + \ldots + \log n$

$= 1 + \log(1*2*3*\ldots\ldots*n)$

$= 1 + \log n!$

$= 1 + \log n^n$

$=1+n\log n$…exact answer

$= O(n\log n)$

4.

$$T(n) = \begin{cases} 1, & n=0 \\ T(n-2) + n^2, & n>0 \end{cases}$$

$T(n-2) = T(n-4) + (n-2)^2$

$T(n) = T(n-2\ 2*1) + n^2$ ……………1^{st} time

$=T(n-4\ 2*2) + (n-2\ 2*1)^2 + n^2$ …………2^{nd} time

$= T(n-6\ 2*3\ ) + (n-4\ 2*2\ )^2 + (n-2\ 2*1\ )^2 + n^2$ ………..3^{rd} time

$= T(n-8\ \ 2*4\ ) + (n-6\ 2*3\ \ )^2 + (n-4\ 2*2\ \ )^2 + (n-2\ 2*1\ \ )^2 + n^2$

$\downarrow$

$= T(n-2k\ ) + (n-2(k-1))^2 + (n-2(k-2))^2 + \ldots. + (n-0)^2$

Assume $n-2k=0$

$n=2k$

$k=n/2$

$=T(0)+2^2 + 4^2 + \ldots\ldots\ldots\ldots.. + n^2$

$=1+ (2*1)^2+(2*2)^2 + (2*3)^2 + \ldots + (2*n/2)^2$

$= 1 + 2^2 (1^2 + 2^2 + 3^2 + \ldots\ldots. + (n/2)^2)$
$= 1 + 2^2 [n/2\ (n/2+1)\ (2n/2+1)]/6$
$= O\ (n^3)$