Master's Method

The Problem is divided into number of sub-problems each of size $\frac{n}{b}$ and need time f(n) to combine the solution, then running time T(n) can be:

$$T(n) = aT(\frac{n}{b}) + f(n)$$

Where a >= 1 , b>1 & $f(n)=\theta(n^k \log^p n)$

1- $\log_b a$
2- k (power of n)

Case-1: If $\log_b a > k$ then $\theta(n^{\log_b a})$

Case-2: If $\log_b a = k$

2.1 If $p > -1$ then $\theta(n^k \log^{p+1} n)$

2.2 If $p = -1$ then $\theta(n^k \log\log n)$

2.3 If $p < -1$ then $\theta(n^k)$

Case-3: If $\log_b a < k$

3.1 If $p >= 0$ then $\theta(n^k \log^p n)$

3.2 If $p < 0$ then $\theta(n^k)$

Cases when Master's Method Failed

1. $T(n) = 2^n \, T(\frac{n}{2}) + n^5$

   Here $a=2^n$ not a constant

   Master's Method will work when a & b are constants

2. $T(n) = 1.5 \, T(\frac{n}{2}) + n^2$

   Here $a=1.5$ is a fraction, a is a number of sub-problems and sub-problems can not be fraction

   Master's Method will not work with fraction values of a or b

3. $T(n) = 25 \, T(\frac{n}{2}) - n^3$

   F(n) can not be negative

Example-1

$T(n) = 2T(\frac{n}{2}) + 1$

$a=2, b=2, f(n) = 1(n^0 \log^0 n) \quad k=0, p=0$

$\log_2 2 = 1 > k$

It is case 1

$T(n) = \theta(n^{\log_2 2}) = \theta(n^1)$

Example-2

$T(n) = 4T(\frac{n}{2}) + n$

$a=4, b=2, f(n) = n\ (n^1 \log^0 n) \quad k=1, p=0$

$\log_2 4 = 2 > k$

It is case 1

$T(n) = \theta(n^{\log 2\ 4}) = \theta(n^2)$

Example-3

$T(n) = 8T(\frac{n}{2}) + n$

$a=8, b=2, f(n) = n\ (n^1 \log^0 n) \quad k=1, p=0$

$\log_2 8 = 3 > k$

It is case 1

$T(n) = \theta(n^{\log 2\ 8}) = \theta(n^3)$

Example-4

$T(n) = 9T(\frac{n}{3}) + 1$

a=9, b=3, f(n) = 1 ($n^0 \log^0 n$)   k=0, p=0

$\log_3 9 = 2 > k$

It is case 1

$T(n) = \theta(n^{\log 3\ 9}) = \theta(n^2)$

Example-5

$T(n) = 2T(\frac{n}{2}) + n$

a=2, b=2, f(n) = n ($n^1 \log^0 n$)   k=1, p=0

$\log_2 2 = 1 = k$

It is case 2(2.1)

$T(n) = \theta(n^k \log^{p+1} n) = \theta(n \log n)$

Example-6

$T(n) = 4T(\frac{n}{2}) + n^2$

a=4, b=2, f(n) = n^2 ($n^2 \log^0 n$)   k=2, p=0

$\log_2 4 = 2 = k$

It is case 2(2.1)

$T(n) = \theta(n^k \log^{p+1} n) = \theta(n^2 \log n)$

Example-7

$T(n) = 8T(\frac{n}{2}) + n^3$

a=8, b=2, f(n) = n^3 ($n^3 \log^0 n$)   k=3, p=0

$\log_2 8 = 3 = k$

It is case 2(2.1)

$T(n) = \theta(n^k \log^{p+1} n) = \theta(n^3 \log n)$

Example-8

$T(n) = 2T(\frac{n}{2}) + \frac{n}{logn}$

a=2, b=2, $f(n) = n\ (n^1 \log^{-1} n)$ k=1, p= -1

$\log_2 2 = 1 = k$

It is case 2(2.2)

$T(n) = \theta(n^k \log\log n) = \theta(n \log\log n)$

Example-9

$T(n) = 2T(\frac{n}{2}) + n / (\log^2 n)$

a=2, b=2, $f(n) = n\ (n^1 \log^{-2} n)$ k=1, p= -2

$\log_2 2 = 1 = k$

It is case 2(2.3)

$T(n) = \theta(n^k) = \theta(n)$

Example-10

$T(n) = 4T(\frac{n}{2}) + n^3$

a=4, b=2, $f(n) = n^3\ (n^3 \log^0 n)$ k=3, p=0

$\log_2 4 = 2 < k$

It is case 3(3.1)

$T(n) = \theta(n^k \log^p n) = \theta(n^3)$