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**IIITB COMET**
F O U N D A T I O N

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## 1971/1.

Prove that the following assertion is true for $n = 3$ and $n = 5$, and that it is false for every other natural number $n > 2$:

If $a_1, a_2, ..., a_n$ are arbitrary real numbers, then

1. $(a_1 - a_2)(a_1 - a_3)...(a_1 - a_n) + (a_2 - a_1)(a_2 - a_3)...(a_2 - a_n)$
   $+ ... + (a_n - a_1)(a_n - a_2)...(a_n - a_{n-1}) \geq 0$

## 1971/2.

Consider a convex polyhedron $P_1$ with nine vertices $A_1, A_2, ...., A_9$; let $P_i$ be the polyhedron obtaianed $P_i$ by a translation that moves vertex $A_1$ to $A_i (i = 2, 3, ..., 9)$. Prove that at least two of the polyhedra $P_1, P_2, ..., P_9$ have an interior point in common.

## 1971/3.

Prove that the set of integers of the form $2^k - 3(k = 2, 3, ....)$ contains an infinite subset in which every two members are relatively prime.

## 1971/4.

All the faces of tetrahedron $ABCD$ are acute-angled triangles. We consider all closed polygonal paths of the form $XYZX$ defined as follows: $X$ is a point on edge $AB$, distinct from $A$ and $B$; similarly $Y, Z$ are interior points of edges $BC, CD$ Prove :

1. If $\angle DAB + \angle BCD \neq \angle CDA + \angle ABC$, then among the polygonal paths there is none of minimal lenght.

2. If $\angle DAB + \angle BCD = \angle CDA + \angle ABC$, then there are infinitely many shortest polygonal paths, there is none of minimal length.

3. $\angle DAB + \angle BCD = \angle CDA + \angle ABC$, then there are infinitely many shortest polygonal paths, their common length being $2AC sin(\alpha/2)$, where $\alpha = \angle BAC + \angle CAD + \angle DAB$

## 1971/5.

Prove that for every natural number $m$, there exists a finite set $S$ of points in a plane with the following property For every point $A$ in $S$, there are exactly $m$ points in $S$ which are at unit distance from $A$.

## 1971/6.

Let $A = (a_{ij})$, $i, j = 1, 2, \ldots, n$ be a square matrix whose elements are non-negative integers. Suppose that whenever an element $a_{ij} = 0$, the sum of the elements in the $i$th row and the $j$th column is $\geq n^2 2$.