

## CHAPTER 9

### SOME APPLICATIONS OF TRIGONOMETRY

#### 9.1 Introduction

In the previous chapter, you have studied about trigonometric ratios. In this chapter, you will be studying about some ways in which trigonometry is used in the life around you. Trigonometry is one of the most ancient subjects studied by scholars all over the world. As we have said in Chapter 8, trigonometry was invented because its need arose in astronomy. Since then the astronomers have used it, for instance, to calculate distances from the Earth to the planets and stars. Trigonometry is also used in geography and in navigation. The knowledge of trigonometry is used to construct maps, determine the position of an island in relation to the longitudes and latitudes.

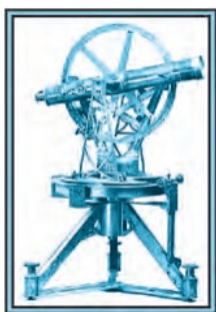


Figure 1: A Theodolite

#### 9.2 Heights and Distances

Let us consider Fig. 8.1 of previous chapter, which is redrawn below in Fig. 9.1.

In this figure, the line  $AC$  drawn from the eye of the student to the top of the minar is called the line of sight. The student is looking at the top of the minar. The angle  $BAC$ , so formed by the line of sight with the horizontal, is called the angle of elevation of the top of the minar from the eye of the student. Thus, the line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer. The angle of elevation of

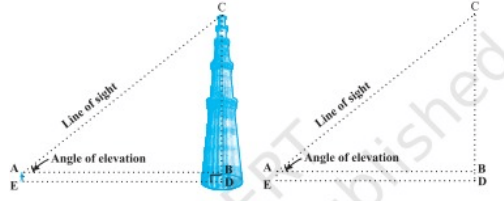


Figure 2: Line of sight and angle of elevation

the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object (see Fig. 9.2).

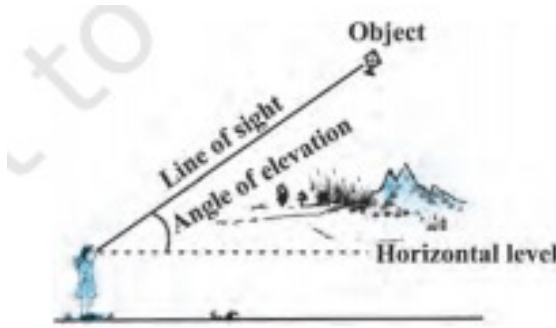


Fig. 9.2

Figure 3: Angle of elevation

Now, consider the situation given in Fig. 8.2. The girl sitting on the balcony is looking down at a flower pot placed on a stair of the temple. In this case, the line of sight is below the horizontal level. The angle so formed by the line of sight with the horizontal is called the angle of depression. Thus, the angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed (see Fig. 9.3).

Now, you may identify the lines of sight, and the angles so formed in Fig. 8.3. Are they angles of elevation or angles of depression? Let us refer to Fig. 9.1 again. If you want to find the height  $CD$  of the minar without actually measuring it, what information do you need? You would need to know the following: (i) the distance  $DE$  at which the student is standing from the foot of the minar (ii) the angle of elevation,  $\angle BAC$ , of the top of the minar (iii) the height  $AE$  of the student. Assuming that the above three conditions are known, how can we determine the height of the minar? In the figure,  $CD = CB + BD$ . Here,  $BD = AE$ , which is the height of the student.

which of the trigonometric ratios can we use? Which one of them has the two values that we have and the one we need to determine? Our search narrows down to using either  $\tan A$

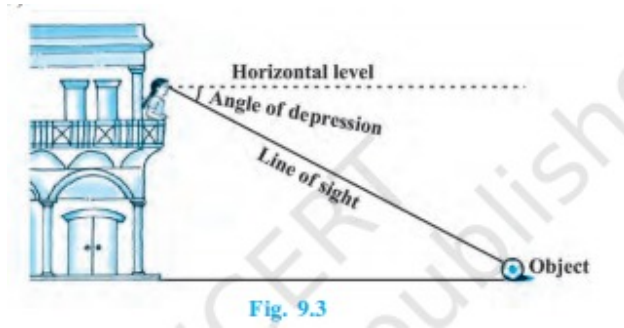


Figure 4: Angle of elevation

or  $\cot A$ , as these ratios involve  $AB$  and  $BC$ .

## Example 4

From a point  $P$  on the ground the angle of elevation of the top of a 10 m tall building is  $30^\circ$ . A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from  $P$  is  $45^\circ$ . Find the length of the flagstaff and the distance of the building from the point  $P$ . (You may take  $\sqrt{3} = 1.732$ )

### Solution:

In Fig. 9.7,  $AB$  denotes the height of the building,  $BD$  the flagstaff and  $P$  the given point. Note that there are two right triangles  $PAB$  and  $PAD$ . We are required to find the length of the flagstaff, i.e.,  $DB$  and the distance of the building from the point  $P$ , i.e.,  $PA$ .

Since we know the height of the building  $AB$ , we will first consider the right  $\triangle PAB$ .

$$\tan 30^\circ = \frac{AB}{AP}$$

i.e.,

$$\frac{1}{\sqrt{3}} = \frac{10}{AP}$$

Therefore,

$$AP = 10\sqrt{3}$$

i.e., the distance of the building from  $P$  is  $10\sqrt{3}$  m = 17.32 m.

Next, let us suppose  $DB = x$  m. Then  $AD = (10 + x)$  m.

Now, in right  $\triangle PAD$ ,

$$\tan 45^\circ = \frac{AD}{AP} = \frac{10 + x}{10\sqrt{3}}$$

Therefore,

$$1 = \frac{10 + x}{10\sqrt{3}}$$

$$x = 10(\sqrt{3} - 1)$$

So, the length of the flagstaff is 7.32 m.

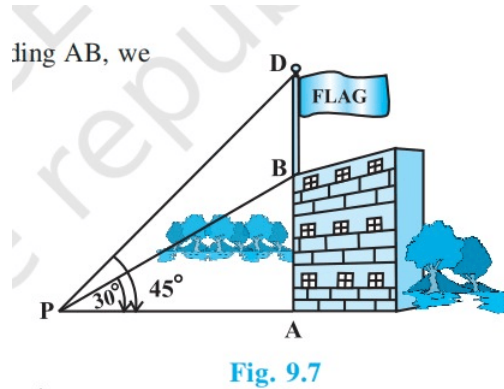


Figure 5: Example 4 diagram

## Example 5

The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is  $30^\circ$  than when it is  $60^\circ$ . Find the height of the tower.

**Solution:**

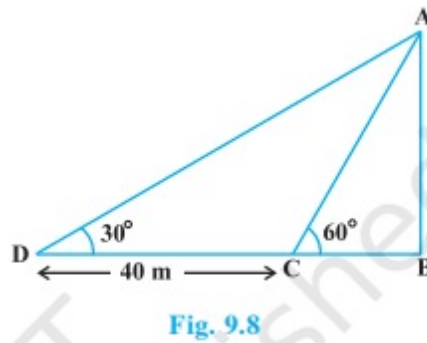


Figure 6: Example 5 diagram

Let the height of the tower be  $h$  and the shorter shadow be  $x$ .

$$\tan 60^\circ = \frac{h}{x} \Rightarrow h = x\sqrt{3}$$

$$\tan 30^\circ = \frac{h}{x + 40}$$

Substituting,

$$(x\sqrt{3})\sqrt{3} = x + 40 \Rightarrow 3x = x + 40 \Rightarrow x = 20$$

$$h = 20\sqrt{3}$$

## 9.3 Summary

1. The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer. with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.
2. The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.
3. The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.