Linear Algebra Assignment I

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1 Problem - I

1.1 Bonus question

Yes, we can tell the maximum amount of potion that can be prepared out of the given ingredients. If we take each ingredient in maximum i.e. m_1, m_2, m_3, m_4, than the quantity of each according to various ratios being prepared would be maximum in total.

2 Problem - II

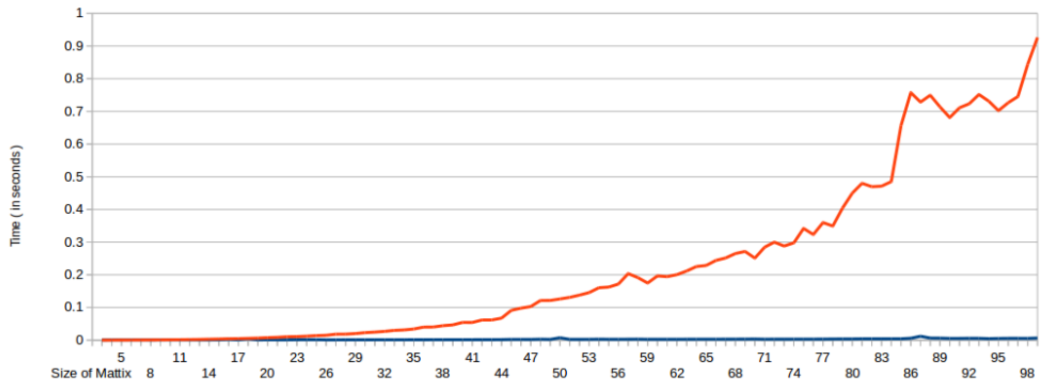
2.1 Second Part

The time in below table is in seconds.

Size of matrix	Time taken by Numpy implementation	Time talken by my Implementation
3	0.000607	0.000114
4	0.000728	0.000188
5	0.000646	0.000247
6	0.000644	0.000375
7	0.000683	0.000504
8	0.000674	0.00071
9	0.000714	0.000896
10	0.000786	0.001198
11	0.000803	0.001566
12	0.000814	0.001878
13	0.000807	0.002232
14	0.000815	0.002758
15	0.00087	0.003351
16	0.000883	0.004015
17	0.019521	0.007293
18	0.024492	0.005709
19	0.000938	0.005935
20	0.000959	0.006843

21	0.000934	0.007798
22	0.001049	0.009011
23	0.001011	0.010676
24	0.001031	0.011573
25	0.001071	0.012815
26	0.001114	0.014194
27	0.001101	0.019182
28	0.001155	0.017298
29	0.001341	0.021355
30	0.001179	0.022208
31	0.001617	0.024467
32	0.001293	0.027317
33	0.001248	0.029046
34	0.001383	0.031659
35	0.001388	0.034162
36	0.005602	0.038022
37	0.001468	0.040147
38	0.001502	0.046256
39	0.001465	0.047125
40	0.001543	0.050121
41	0.001548	0.053845
42	0.001678	0.061607
43	0.001664	0.066089
44	0.002103	0.066367
45	0.001876	0.072375
46	0.001809	0.07655
47	0.001843	0.085555
48	0.001875	0.08573
49	0.00218	0.097427
50	0.001987	0.0953
51	0.00197	0.100692
52	0.002036	0.114387
53	0.002084	0.125097
54	0.002886	0.156338
55	0.002758	0.16381
56	0.00273	0.490499
57	0.003634	0.478637
58	0.003976	0.330869
59	0.004812	0.201828
60	0.003082	0.209823
61	0.003162	0.220393
62	0.00485	0.201896
63	0.002757	0.214648
64	0.008934	0.222989
65	0.002881	0.2405

66	0.00297	0.26233
67	0.003086	0.254403
68	0.003158	0.260414
69	0.003198	0.273681
70	0.003211	0.288618
71	0.003354	0.306609
72	0.00338	0.364858
73	0.003907	0.372463
74	0.005319	0.412921
75	0.004027	0.406063
76	0.004302	0.466005
77	0.004197	0.464257
78	0.004367	0.463425
79	0.003622	0.411501
80	0.004459	0.423162
81	0.003892	0.437871
82	0.003516	0.46641
83	0.004218	0.525641
84	0.003916	0.491401
85	0.004281	0.619799
86	0.004911	0.595451
87	0.005017	0.613381
88	0.005179	0.641346
89	0.005381	0.676259
90	0.005351	0.970028
91	0.007573	1.091042
92	0.007832	1.063807
93	0.008227	0.944528
94	0.006603	0.921807
95	0.00728	0.940351
96	0.006795	0.904228
97	0.005947	0.853793
98	0.006229	0.870714
99	0.006324	0.920782



The red line in above figure denotes the time taken by My implementation of the inverse.
The blue line represents the time taken by the numpy implementation of the inverse.

As we can see from the table and figure above that upto very small matrices (size ≤ 7) my implementation was performing better. Then after that the time difference increased rapidly. We can conclude that the implementation is nowhere near the numpy.linalg.inv implementation of calculating inverse in performance.

numpy.linalg.inv(A) calls numpy.linalg.solve(A,I) , where I is the identity matrix and solves using laplack's LU Factorization. [1]

2.2 Third Part

Yes, We can find the inverse. We know that $(A^{-1})^T = (A^T)^{-1}$

Hence, if we convert the matrix A into its transpose. And the compute the inverse using the row operations. Then we will get inverse matrix of A^T. As we know that this inverse matrix will be equal to $(A^T)^{-1}$, which will be again equal to $(A^{-1})^T$ So, if we calculate the transpose of it, we would get the matrix A^{-1}

So doing column operations is like executing row operations on the transpose of the matrix and then converting the inverse matrix to its transpose.

2.3 Bonus Question

If we have only two out of the three operations at our disposal, we can explore the following possibilities:-

1. We can construct the operation MULTIPLY using sub-operations.
   "MULTIPLY C r1" will be same as "MULTIPLY AND ADD C-1 r1 r1"
   In this case , we can see that a MULTIPLY operation is being converted to MULTIPLY AND ADD operation. There is not much difference in complexity of the operations. So there will not be any overhead in this case.

2. We can construct the operation SWAP using the following sub-operations.
   Row1 -> Row1 + Row2 : MULTIPLY 1 WITH Row2 AND ADD TO ROW1
   Row2 -> (-1)Row2 : MULTIPLY -1 with Row2
   Row2 -> Row2 + Row1 : MULTIPLY 1 WITH Row1 AND ADD TO Row2
   Row1 -> Row1 - Row2 : MULTIPLY -1 WITH Row2 AND ADD TO Row1

   In this case, we can see that there is much difference in complexity. A simple SWAP operation of 0(1) complexity is being converted into suboperations that require traversal of a complete row , now the complexity of the operation is O(x), where x is the number of columns.

References

1. https://stackoverflow.com/questions/16569613/how-does-numpy-linalg-inv-calculate-the-inverse-of-an-orthogonal-matrix