

# Linear Algebra Assignment II

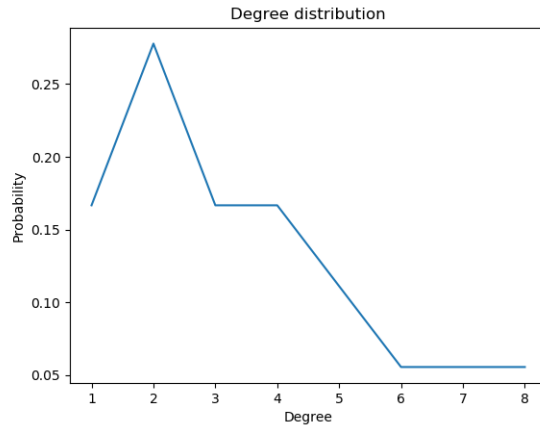
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## 1 Problem - I

### 1.1 Task I

For the given dataset, the graph plotted is shown below.



We can see this distribution somewhat similar to inverse gaussian distribution.

### 1.2 Task V

For the test data given, Min eigen value was found to be :

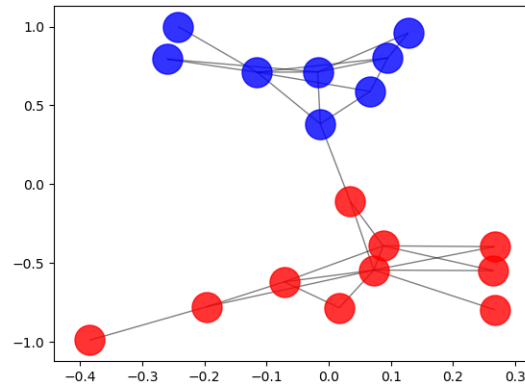
$$-1.3813844561213333e^{-16}$$

(which is close to zero)

and Eigen vector corresponding to the minimum eigen value is obtained as Transpose of  $[0.23570226039551526 \ 0.2357022603955154 \ 0.2357022603955153 \ 0.23570226039551526 \ 0.2357022603955154 \ 0.23570226039551562, \ 0.23570226039551528 \ 0.2357022603955153 \ 0.23570226039551626, \ 0.23570226039551626 \ 0.23570226039551626 \ 0.23570226039551642, \ 0.23570226039551634 \ 0.23570226039551628 \ 0.23570226039551628, \ 0.23570226039551628 \ 0.2357022603955163 \ 0.23570226039551614]$  , which is a scaled version of all 1's

### 1.3 Task VI

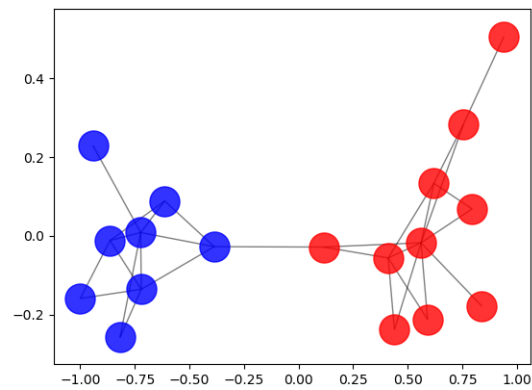
For the given test data, the following coloured plot is found



It is observed that, the points are being clustered into two different groups. The graph is being divided by the cut edge. The edge joining the two clusters is a cut edge (i.e. if it is removed the graph will have 2 connected components.)

### 1.4 Bonus I

For the given test data, the following coloured plot is found using **numpy**



No difference in clustering is observed.

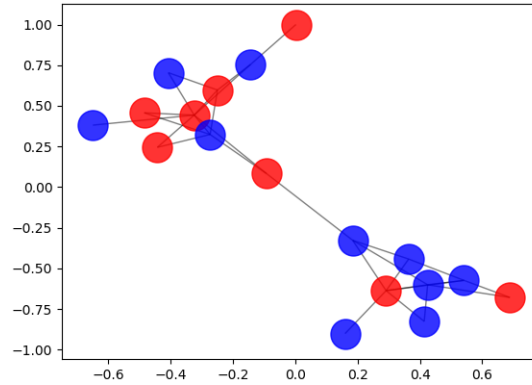
### 1.5 Bonus II

If we are asked to divide into more than 2 clusters, then we can do so by taking more than 1 eigen vectors. For example, we can take the eigen vectors corresponding to second smallest and third smallest eigen value. Now we can divide

the nodes into 4 clusters (according to sign of the corresponding entries of eigen vector elements) [i.e.  $++$ ,  $+-$ ,  $--$ ,  $-+$ ] In this way, if we take  $m$  eigen vectors, we can cluster the nodes into  $2^m$  clusters.

### 1.6 Bonus III

The below plot is observed if we take the second largest eigen vector instead of second smallest.



My observation is that the clustering is not perfect as it was with the second smallest eigen vector.

### 1.7 Bonus IV

The central edge of the graph is the edge that has one node in each of the clusters.(as observed)

## 2 Problem - II

### 2.1 Task II

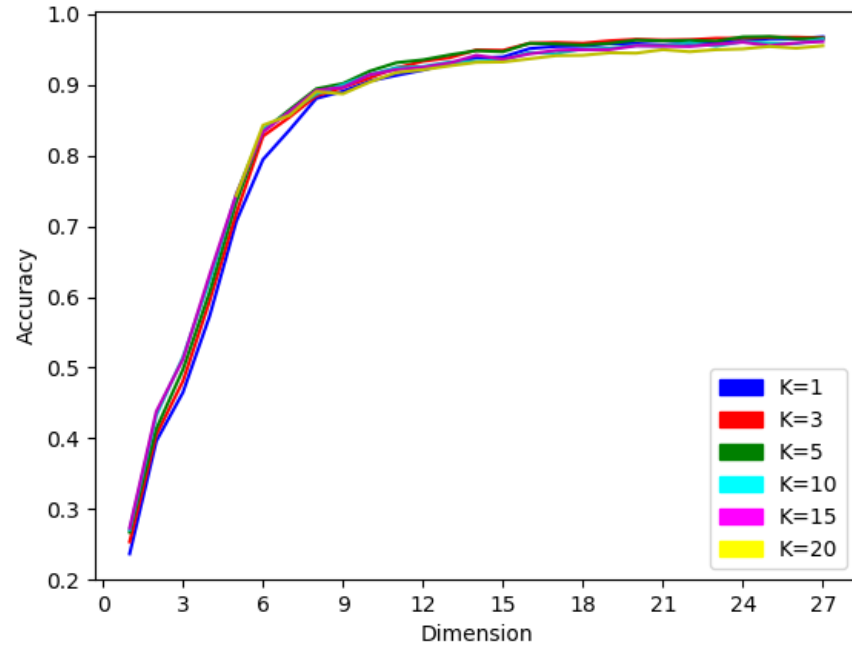
| M | K | Accuracy |
|---|---|----------|
| 1 | 1 | 0.2365   |
| 1 | 3 | 0.2535   |
| 2 | 1 | 0.396    |
| 2 | 3 | 0.405    |
| 3 | 1 | 0.465    |
| 3 | 3 | 0.481    |
| 4 | 1 | 0.573    |
| 4 | 3 | 0.5945   |
| 1 | 5 | 0.267    |

|    |    |        |
|----|----|--------|
| 1  | 10 | 0.27   |
| 1  | 15 | 0.273  |
| 2  | 5  | 0.4135 |
| 2  | 10 | 0.4325 |
| 2  | 15 | 0.438  |
| 3  | 5  | 0.4975 |
| 3  | 10 | 0.515  |
| 3  | 15 | 0.512  |
| 4  | 5  | 0.607  |
| 4  | 10 | 0.6235 |
| 4  | 15 | 0.6315 |
| 5  | 1  | 0.707  |
| 5  | 3  | 0.7185 |
| 5  | 5  | 0.7345 |
| 5  | 10 | 0.745  |
| 5  | 15 | 0.7455 |
| 5  | 20 | 0.7435 |
| 6  | 1  | 0.7945 |
| 6  | 3  | 0.8275 |
| 6  | 5  | 0.835  |
| 6  | 10 | 0.8375 |
| 6  | 15 | 0.8335 |
| 6  | 20 | 0.843  |
| 7  | 1  | 0.8365 |
| 7  | 3  | 0.8545 |
| 7  | 5  | 0.865  |
| 7  | 10 | 0.8615 |
| 7  | 15 | 0.8625 |
| 7  | 20 | 0.857  |
| 8  | 1  | 0.881  |
| 8  | 3  | 0.8845 |
| 8  | 5  | 0.8945 |
| 8  | 10 | 0.887  |
| 8  | 15 | 0.8925 |
| 8  | 20 | 0.8905 |
| 9  | 1  | 0.891  |
| 9  | 3  | 0.8965 |
| 9  | 5  | 0.902  |
| 9  | 10 | 0.9005 |
| 9  | 15 | 0.8965 |
| 9  | 20 | 0.8875 |
| 10 | 1  | 0.9055 |
| 10 | 3  | 0.91   |
| 10 | 5  | 0.9195 |
| 10 | 10 | 0.9145 |

|    |    |        |
|----|----|--------|
| 10 | 15 | 0.9145 |
| 10 | 20 | 0.904  |
| 11 | 1  | 0.9135 |
| 11 | 3  | 0.924  |
| 11 | 5  | 0.9315 |
| 11 | 10 | 0.9245 |
| 11 | 15 | 0.9215 |
| 11 | 20 | 0.9175 |
| 12 | 1  | 0.921  |
| 12 | 3  | 0.9335 |
| 12 | 5  | 0.9355 |
| 12 | 10 | 0.926  |
| 12 | 15 | 0.925  |
| 12 | 20 | 0.9215 |
| 13 | 1  | 0.9305 |
| 13 | 3  | 0.9385 |
| 13 | 5  | 0.9425 |
| 13 | 10 | 0.9325 |
| 13 | 15 | 0.931  |
| 13 | 20 | 0.927  |
| 14 | 1  | 0.9385 |
| 14 | 3  | 0.9495 |
| 14 | 5  | 0.9485 |
| 14 | 10 | 0.9335 |
| 14 | 15 | 0.942  |
| 14 | 20 | 0.932  |
| 15 | 1  | 0.9395 |
| 15 | 3  | 0.949  |
| 15 | 5  | 0.947  |
| 15 | 10 | 0.9355 |
| 15 | 15 | 0.9365 |
| 15 | 20 | 0.9325 |
| 16 | 1  | 0.9515 |
| 16 | 3  | 0.959  |
| 16 | 5  | 0.959  |
| 16 | 10 | 0.946  |
| 16 | 15 | 0.944  |
| 16 | 20 | 0.937  |
| 17 | 1  | 0.9545 |
| 17 | 3  | 0.96   |
| 17 | 5  | 0.958  |
| 17 | 10 | 0.9445 |
| 17 | 15 | 0.949  |
| 17 | 20 | 0.9415 |
| 18 | 1  | 0.955  |

|    |    |        |
|----|----|--------|
| 18 | 3  | 0.959  |
| 18 | 5  | 0.9565 |
| 18 | 10 | 0.951  |
| 18 | 15 | 0.95   |
| 18 | 20 | 0.942  |
| 19 | 1  | 0.9585 |
| 19 | 3  | 0.9625 |
| 19 | 5  | 0.959  |
| 19 | 10 | 0.952  |
| 19 | 15 | 0.95   |
| 19 | 20 | 0.9455 |
| 20 | 1  | 0.9585 |
| 20 | 3  | 0.9645 |
| 20 | 5  | 0.9635 |
| 20 | 10 | 0.956  |
| 20 | 15 | 0.9555 |
| 20 | 20 | 0.945  |
| 21 | 1  | 0.9565 |
| 21 | 3  | 0.9635 |
| 21 | 5  | 0.9625 |
| 21 | 10 | 0.9555 |
| 21 | 15 | 0.955  |
| 21 | 20 | 0.95   |
| 22 | 1  | 0.959  |
| 22 | 3  | 0.964  |
| 22 | 5  | 0.963  |
| 22 | 10 | 0.9595 |
| 22 | 15 | 0.9545 |
| 22 | 20 | 0.947  |
| 23 | 1  | 0.9605 |
| 23 | 3  | 0.966  |
| 23 | 5  | 0.962  |
| 23 | 10 | 0.955  |
| 23 | 15 | 0.9575 |
| 23 | 20 | 0.95   |
| 24 | 1  | 0.963  |
| 24 | 3  | 0.966  |
| 24 | 5  | 0.968  |
| 24 | 10 | 0.9615 |
| 24 | 15 | 0.9605 |
| 24 | 20 | 0.951  |
| 25 | 1  | 0.9655 |
| 25 | 3  | 0.9675 |
| 25 | 5  | 0.9685 |
| 25 | 10 | 0.9595 |

|    |    |        |
|----|----|--------|
| 25 | 15 | 0.9555 |
| 25 | 20 | 0.9545 |
| 26 | 1  | 0.9655 |
| 26 | 3  | 0.9675 |
| 26 | 5  | 0.9655 |
| 26 | 10 | 0.9585 |
| 26 | 15 | 0.9595 |
| 26 | 20 | 0.952  |
| 27 | 1  | 0.968  |
| 27 | 3  | 0.9665 |
| 27 | 5  | 0.966  |
| 27 | 10 | 0.965  |
| 27 | 15 | 0.9615 |
| 27 | 20 | 0.9555 |
| 28 | 1  | 0.9695 |
| 28 | 3  | 0.9655 |
| 28 | 5  | 0.9665 |
| 28 | 10 | 0.964  |
| 28 | 15 | 0.9635 |
| 28 | 20 | 0.953  |
| 29 | 1  | 0.9695 |
| 29 | 3  | 0.9665 |
| 29 | 5  | 0.9685 |
| 29 | 10 | 0.9625 |
| 29 | 15 | 0.9595 |
| 29 | 20 | 0.9505 |



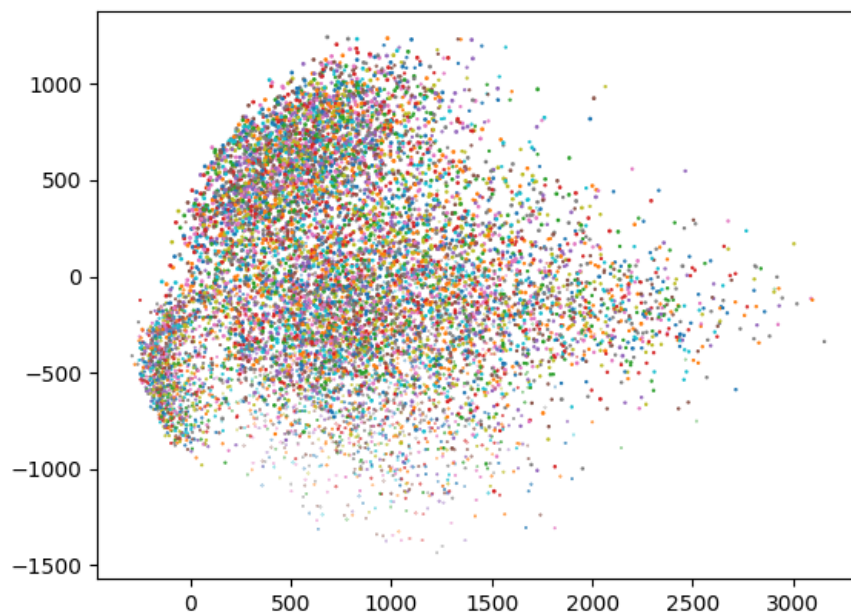
The above graph represents the data that is reported in above table.

By observing the above accuracy plots, I choose  $\mathbf{M=20}$  and  $\mathbf{K=3}$  for the model.



## 2.2 Bonus I

The vectors, when projected to 2D space resulted in the below figure:



No observation could be made from above figure.

## 2.3 Bonus II

### References

1. <https://stackoverflow.com/questions/16569613/how-does-numpy-linalg-inv-calculate-the-inverse-of-an-orthogonal-matrix>