SURVEYING

(VOLUME II)

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FIFTEENTH EDITION

(Thoroughly Revised and Enlarged)

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PREFACE TO THE TWELFTH EDITION

In the Twelfth Edition of the book, the subject matter has been thoroughly revised and updated. Many new articles and solved examples have been added. The entire book has been typeset using laser printer. The authors are thankful to Shri Mool singh Gahlot for the fine laser typesetting done by him.

JODHPUR
15th Aug. 1994

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PREFACE TO THE FIFTEENTH EDITION

In the Fifteenth Edition, the subject matter has been thoroughly revised, updated and rearranged. In each Chapter, many new articles have been added. *Four new* Chapters have been added at the end of the book : Chapter 13 on '*Field Astronomy*', Chapter 14 on '*Photogrammetric Surveying*', Chapter 15 on '*Electromagnetic Distance Measurement (EDM)*' and Chapter 16 on '*Remote Sensing*'. All the diagrams have been redrawn using computer graphics and the book has been computer type-set in a bigger format keeping in pace with the Modern trend. Account has been taken throughout of the suggestions offered by many users of the book and grateful acknowledgement is made to them. The authors are thankful to Shri M.S. Gahlot for the fine Laser type setting done by him. The Authors are also thankful Shri R.K. Gupta, Managing Director Laxmi Publications, for taking keen interest in publication of the book and bringing it out nicely and quickly.

JODHPUR
Independence Day
15th August, 2005

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A simple curve [Fig. 1.1 (*a*)] is the one which consists of a single arc of a circle. It is tangential to both the straight lines.

A compound curve [Fig. 1.1 (*b*)] consists of two or more simple arcs that turn in the same direction and join at common tangent points.

A reverse curve [Fig. 1.1 (*c*)] is the one which consists of two circular arcs of same or different radii, having their centres to the different sides of the common tangent. Both the arcs thus bend in different directions with a common tangent at their junction.

SIMPLE CURVES

1.2. DEFINITIONS AND NOTATIONS (Fig. 1.2)

1. *Back tangent.* The tangent (AT_1) previous to the curve is called the back tangent or first tangent.

2. *Forward tangent.* The tangent (T_2B) following the curve is called the forward tangent or second tangent.

3. *Point of intersection.* If the two tangents AT_1 and BT_2 are produced, they will meet in a point, called the point of intersection (P.I.) or vertex (V).

4. *Point of curve (P.C.).* It is the beginning of the curve where the alignment changes from a tangent to a curve.

5. *Point of tangency (P.T.).* It is end of the curve where the alignment changes from a curve to tangent.

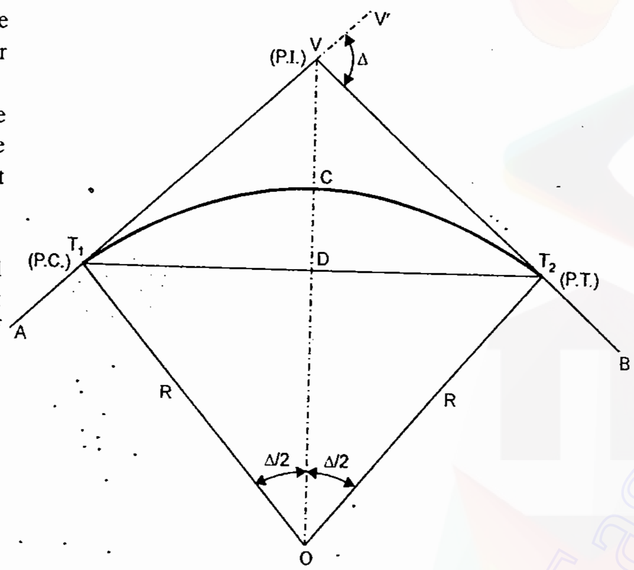

FIG. 1.2. PARTS OF A CIRCULAR CURVE.

6. *Intersection angle.* The angle $V'VB$ between the tangent AV produced and VB is called the intersection angle (Δ) or the *external deflection angle* between the two tangents.

7. *Deflection angle to any point.* The deflection angle to any point on the curve is the angle at P.C. between the back tangent and the chord from P.C. to point on the curve.

8. *Tangent distance* (*T*). It is the distance between P.C. to P.I. (also the distance from P.I. to P.T.).

9. *External distance* (*E*). It is distance from the mid-point of the curve to P.I.

10 *Length of curve* (*L*) It is the total length of the curve from P.C. to P.T.

11. *Long chord.* It is chord joining P.C. to P.T.

12. *Mid ordinate* (*M*). It is the ordinate from the mid-point of the long chord to the mid-point of the curve.

13. *Normal chord* (*C*). A chord between two successive regular stations on a curve.

14. *Sub-chord* (*c*). Sub-chord is any chord shorter than the normal chord.

15. *Right-hand curve.* If the curve deflects to the right of the direction of the progress of survey, it is called the right-hand curve.

16. *Left-hand curve.* If the curve deflects to the left of the direction of the progress of survey, if is called the left-hand curve.

1.3. DESIGNATION OF CURVE

The *sharpness* of the curve is designated either by its *radius* or by its *degree of curvature.* The former system is adopted in Great Britain while the later system is used in America, Canada, India and some other countries.

The degree of curvature has several slightly different definitions. According to the *arc definition* generally used in highway practice, the degree of the curve is defined as the central angle of the curve that is subtended by an arc of 100 ft length. According to the *chord definition* generally used in railway practice, the degree of the curve is defined as the central angle of the curve that is subtended by its chord of 100 ft length.

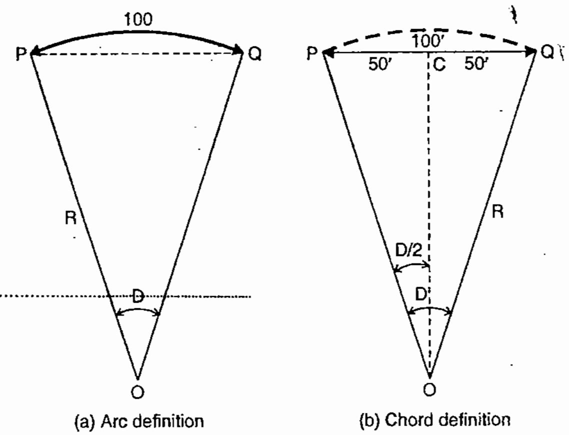

FIG. 1.3. DEGREE OF CURVE (FEET UNITS).

The relation between the radius (R) and degree of the curve (D) can easily by derived with reference to Fig. 1.3.

Arc definition. From familiar proportion [Fig. 1.3 (*a*)], we have

$$100 : 2\pi R = D° : 360°$$

or
$$R = \frac{360°}{D} \times \frac{100}{2\pi} = \frac{5729.578}{D} \text{ ft.} \qquad \ldots(1.1)$$

Thus, radius of 1° curve is 5729.578 ft.

To the first approximation, we have

$$R = \frac{5730}{D} \qquad \ldots[1.1\ (a)]$$

Chord definition. From triangle POC [Fig. 1.3 (*b*)],

$$\sin \tfrac{1}{2} D = \frac{50}{R} \qquad [1.1\ (b)]$$

Before a curve is set out, it is essential to locate the tangents, points of intersection (P.I.), point of the curve (P.C.) and point of tangency (P.T.).

Location of tangent. Before setting out the curve, the surveyor is always supplied with a working plan upon which the general alignment of tangent is known in relation to the traverse controlling the survey of that area. Knowing offsets to certain points on both the tangents, the tangents can be staked on the ground by the tape measurements. The tangents may then be set out by theodolite by trial and error so that they pass through the marks as nearly as possible. The total deflection angle (Δ) can then be measured by setting the theodolite on the P.I.

Location of tangent points. After having located the P.I. and measured Δ, the tangent length (T) can be calculated from equation 1.5, *i.e.*,

$$T = R \tan \frac{\Delta}{2}$$

The point T_1 (Fig. 1.2) can be located by measuring back a distance $VT_1 = T$ on the rear tangent.

Similarly, the point T_2 can be located by measuring a distance $VT_2 = T$ on the forward tangent.

Knowing the chainage of P.I., the chainage of point T_1 can be known by subtracting the tangent length from it. The length of the curve is then added to the chainage of T_1 to get the chainage of T_2. The tangent points must be located with greater precision.

Peg Interval. For the ease in calculations and setting out, it is essential that the pegs on the curve are at regular interval from the begining to the end. Such interval is known as *peg interval* and the chord joining two such adjacent pegs is known as the *full chord or normal chord.* The length of the normal chord is generally taken equal to 100 ft in English units or 20 metres in metric units, so that angle subtended by the normal chord at the centre is equal to the degree of the curve. The stations having the chainages in the multiples of chain lengths are known as *full stations*. Except by chance, the tangent points will not be full stations (*i.e.*, their chainages will not be multiples of full chains). The distance between the point T_1 and the first peg will be less than the length of the normal chord so that the first peg may be a full station. Thus, the first chord joining the point of curve T_1 to the first peg will be a *sub-chord*. Similarly, the last chord, joining the last peg on the curve and the tangent point T_2 will be a sub-chord. All other intermediate chords will be normal chords or units chords. Thus, if the chainage T_1 is n chains $+ m$ links, the first chord length will be the remaining portion of the chain length *i.e.*, (100 – m) links. Similarly, if the chainage of T_2 is n' chains $+ m'$ links, the last chord length will be m' links.

The length of the normal or unit chord should be so selected that there is no appreciable difference between the lengths of the chord and the arc. If the length of the chord is not greater than one-tenth of the radius, it will give sufficiently accurate results, the error being 8 mm in 20 m. For more accurate results, the length of normal chord should be limited to 1/20 of its radius so that the error is only 2 mm in 20 m.

Linear methods of Setting Out

Following are some of the linear methods for setting out simple circular curves :

(1) By ordinates or offsets from the long chord.

(2) By successive bisection of arcs.

(3) By offsets from the tangents.

(4) By offsets from chords produced (or by deflection distances).

Location of tangent points. If an angle measuring instrument is not available, the following procedure may be adopted for the location of tangent points (Fig. 1.4) :

(1) Produce two straights to meet at V.

(2) Select two inter-visible points E and G on the two straights, equidistant from V. VE and VG should be as long as possible.

(3) Join. EG, measure it and bisect it at F. Join VF and measure it.

From similar triangles, VEF and VT_1O we have

$$\frac{VT_1}{OT_1} = \frac{VF}{EF}$$

$$\therefore \quad VT_1 = T = \frac{VF}{EF} \cdot OT_1 = \frac{VF}{EF} \cdot R$$

Thus, the tangent poins T_1 and T_2 can be located by measuring VT_1 and VT_2 each equal to T along the straights.

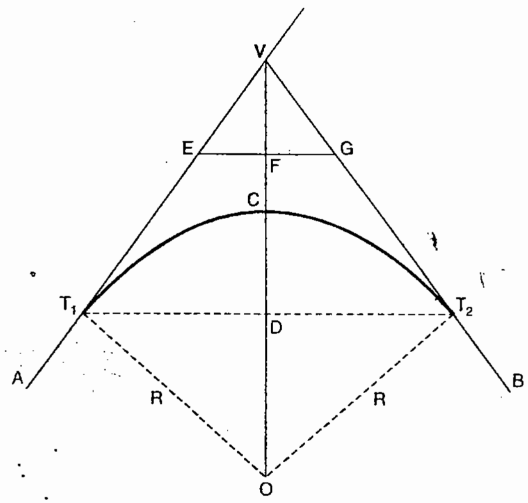

FIG. 1.4. LOCATION OF TANGENT POINTS.

1.6. BY ORDINATES FROM THE LONG CHORD : (Fig. 1.5)

Let R = Radius of the curve.

O_0 = Mid-ordinate.

O_x = Ordinate at distance x from the mid-point of the chord.

T_1 and T_2 = Tangent points.

L = Length of the long chord actually measured on the ground.

Bisect the long chord at point D.

From triangle OT_1D,

$$OT_1^2 = T_1D^2 + DO^2$$

or
$$R^2 = \left(\frac{L}{2}\right)^2 + (CO - CD)^2 = \left(\frac{L}{2}\right)^2 + (R - O_0)^2$$

$$\therefore \quad (R - O_0) = \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

$$T_1D^2 = DE(2R + DE)$$

or $$x^2 = O_x\,(2R + O_x)$$

Neglecting O_x in comparison to $2R$, we get

$$O_x = \frac{x^2}{2R} \quad \text{...(approximate)}$$

(ii) Perpendicular Offsets (Fig. 1.8)

Let $DE = O_x$ = Offset perpendicular to the tangent

$T_1D = x$, measured along the tangent

Draw EE_1 parallel to the tangent.

From triangle EE_1O, we have

$$E_1O^2 = EO^2 - E_1E^2$$

or $$(T_1O - T_1E_1)^2 = EO^2 - E_1E^2$$

or $$(R - O_x)^2 = R^2 - x^2$$

From which, $O_x = R - \sqrt{R^2 - x^2}$ ...(exact) ...(1.13)

The corresponding approximate expression for O_x may be obtained by expanding the term $\sqrt{R^2 - x^2}$. Thus,

$$O_x = R - R\left(1 - \frac{x^2}{2R^2} - \frac{x^4}{8R^4} \ldots\ldots\right)$$

Neglecting the other terms except the first two of the expansion

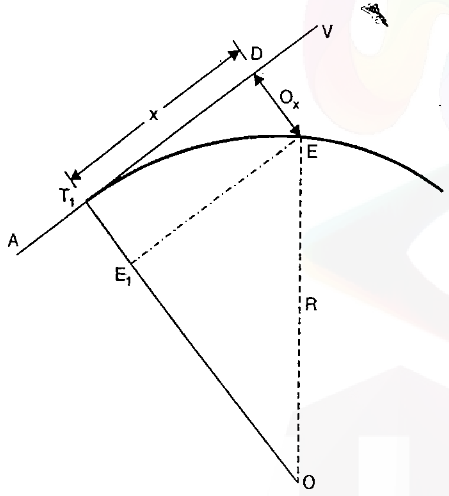

FIG. 1.8. SETTING OUT BY PERPENDICULAR OFFSETS

$$O_x = R - R + \frac{x^2}{2R}$$

or $$O_x = \frac{x^2}{2R} \quad \text{...[1.13 (a)]}$$

It should be noted that if the curve is set out by the approximate expression given above, the points on the curve will lie on a parabola and not on the arc of a circle. However, if the versed sine of the curve is less than one-eighth of its chord, the curve approximates very closely to a circle.

∵ To set out the curve, distances x_1, x_2, x_3....etc., are measured from the first tangent point along the tangent and the perpendicular offsets calculated above are erected with the help of an optical square at the corresponding points. When the distace x increases, the offsets become too large to set out accurately. In that case, the central position of the curve may be set out from a third tangent drawn through the apex of the curve.

The method is useful for small curves only.

1.9. BY DEFLECTION DISTANCES (OR OFFSETS FROM THE CHORDS PRODUCED)

The method is very much useful for long curves and is generally used on highway curves when a theodolite is not available.

Let $T_1A_1 = T_1A$ = initial sub-chord $= C_1$

A, B, D etc.= points on the curve

$AB = C_2$

$BD = C_3$ etc.

T_1V = Rear Tangent

$\angle A_1T_1A = \delta$ = deflection angle of the first chord

$A_1A = O_1$ = first offset

$B_2B = O_2$ = second offset

$D_2D = O_3$ = third offset, etc.

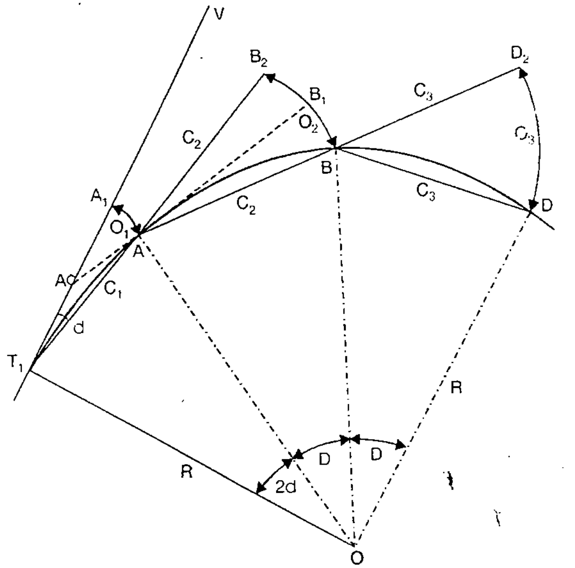

FIG. 1.9. SETTING OUT THE CURVE BY DEFLECTION DISTANCES.

Now, arc $A_1A = O_1 = T_1A\,.\,\delta$ ...(i)

Since T_1V is the tangent to the circle at T_1

$$\angle T_1OA = 2\angle A_1T_1A = 2\delta$$

$$\therefore \quad T_1A = R\,.\,2\delta$$

$$\delta = \frac{T_1A}{2R} \quad \text{...(ii)}$$

Substituting the value of δ in (i), we get

$$\text{Arc } A_1A = O_1 = T_1A\,.\,\frac{T_1A}{2R} = \frac{T_1A^2}{2R}$$

Taking arc T_1A = chord T_1A (very nearly), we get

$$O_1 = \frac{C_1^2}{2R} \quad \text{...[1.14 (a)]}$$

In order to obtain the value of the second offset O_2 for getting the point B on the curve, draw a tangent AB_1 to the curve at A to cut the rear tangent in A'. Join T_1A and prolong it to a point B_2 such that $AB_2 = AB = C_2$ = length of the second chord. Then $O_2 = B_2B$.

As from equation 1.14 (a), the offset B_1B from the tangent AB_1 is given by

$$B_1B = \frac{C_2^2}{2R} \quad \text{...(iii)}$$

Again, $\angle B_2AB_1 = \angle A'AT_1$ being opposite angles.

Since T_1A' and $A'A$ are both tangents, they are equal in length.

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Now $\frac{\angle T_1OA}{C_1} = \frac{180°}{\pi R}$

or $\angle T_1OA = 2\delta_1 = \frac{180°\, C_1}{\pi R}$

From which $\delta_1 = \frac{90 C_1}{\pi R}$ degrees

$= \frac{90 \times 60 C_1}{\pi R} = 1718.9 \frac{C_1}{R}$ minutes.

Similarly,

$\delta_2 = 1718.9 \frac{C_2}{R}$; $\delta_3 = 1718.9 \frac{C_3}{R}$,

or, in general,

$$\delta = 1718.9 \frac{C}{R} \text{ minutes} \quad ...(1.1)$$

where C is the length of the chord.

For the first chord T_1A, the deflection angle = its tangential angle

or $\Delta_1 = \delta_1$ ...(1)

For the second point B, let the deflection angle by = Δ_2.

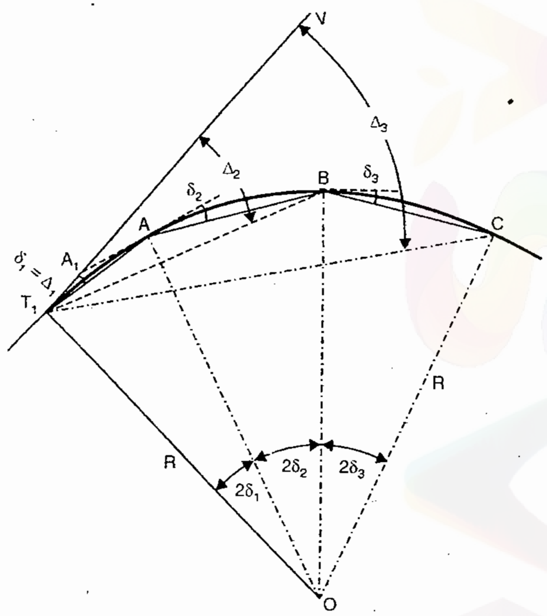

FIG. 1.10. RANKINE'S METHOD OF TANGENTIAL ANGLES.

Since δ_2 = tangential angle for the chord AB,

$\angle AOB = 2\delta_2$

$\therefore$ $\angle AT_1B$ = Half the angle subtended by AB at the centre = δ_2

Now $\Delta_2 = \angle VT_1B = \angle A_1T_1A + \angle AT_1B$

or $\Delta_2 = \delta_1 + \delta_2 = \Delta_1 + \delta_2$ ...(2)

Similarly, $\Delta_3 = \delta_1 + \delta_2 + \delta_3 = \Delta_2 + \delta_3$ ...(3)

and $\Delta_n = \delta_1 + \delta_2 + \ldots \delta_n = \Delta_{n-1} + \delta_n$ ...(1.16)

Hence, the deflection angle for any chord is equal to the deflection angle for the previous chord plus the tangential angle for that chord.

Check : Deflection angle of the long chord, *i.e.*,

$\angle VT_1T_2 = \Delta_n = \frac{\Delta}{2}$, where Δ is the intersection angle or the external deflection angle for the curve.

If the degree of the curve is equal to D for a 20 m chord,

$$\delta_2 = \delta_3 \ldots = \delta_{n-1} = \tfrac{1}{2} D$$

Similarly, if c and c' are the first and the last sub-chords

$$\delta_1 = \frac{c}{20} \cdot \frac{D}{2} = \frac{cD}{40}, \text{ where } c \text{ is metres} \; ; \; \delta_n = \frac{c'}{20} \cdot \frac{D}{2} = \frac{c'D}{40}$$

$\therefore$ $\Delta_1 = \delta_1 = \frac{cD}{40}$

$\Delta_2 = \Delta_1 + \delta_2 = \frac{cD}{40} + \frac{1}{2} D$

$\Delta_3 = \Delta_2 + \delta_3 = \frac{cD}{40} + \frac{1}{2} D + \frac{1}{2} D = \frac{cD}{40} + D$

$\Delta_n = \Delta_{n-1} + \delta_n = \frac{cD}{40} + (n-2)\frac{D}{2} + \frac{c'D}{40}$

Similarly, if the degree of the curve is equal to D for a 100 ft chord,

$\delta_1 = \frac{c \times D}{200}$; $\delta_2 = \delta_3 = \ldots \delta_{n-1} = \frac{D}{2}$

$\delta_n = \frac{c'D}{200}$.

Procedure for Setting out the Curve

(1) Set the theodolite at the point of curve (T_1). With both plates clamped to zero, direct the theodolite to bisect the point of intersection (V). The line of sight is thus in the direction of the rear tangent.

(2) Release the vernier plate and set angle Δ_1 on the vernier. The line of sight is thus directed along chord T_1A.

(3) With the zero end of the tape pointed at T_1 and an arrow held at a distance $T_1A = c$ along it, swing the tape around T_1 till the arrow is bisected by the cross-hairs. Thus, the first point A is fixed.

(4) Set the second deflection angle Δ_2 on the vernier so that the line of sight is directed along T_1B.

(5) With the zero end of the tape pinned at A, and an arrow held at distance $AB = C$ along it, swing the tape around A till the arrow is bisected by the cross-hairs, thus fixing the point B.

(6) Repeat steps (4) and (5) till the last point T_2 is reached.

Check : The last point so located must coincide with the point of tangency (T_2) fixed independently by measurements from the point of intersection. If the discrepancy is small, last few pegs may be adjusted. If it is more, the whole curve should be reset.

In the case of the left hand curve, each of the calculated values of the deflection angle (*i.e.* Δ_1, Δ_2 etc.) , should be subtracted from 360°. The angles so obtained are to be set on the vernier of theodolite for setting out the curve.

In the above method, three men are required : the surveyor to operate the theodolite, and two chainmen to measure the chord lengths with chain or tape. This method is most frequently used for setting out circular curves of large radius and of considerable length.

Field Notes

The record of deflection angles for various points is usually kept in the following form (next page) :

1.11. TWO THEODOLITE METHOD

In this method, two theodolites are used one at P.C. and the other at P.T. The method is used when the ground is unsuitable for chaining and is *based on the principle that the angle between the tangent and the chord is equal to the angle which that chord subtends in the opposite segment.*

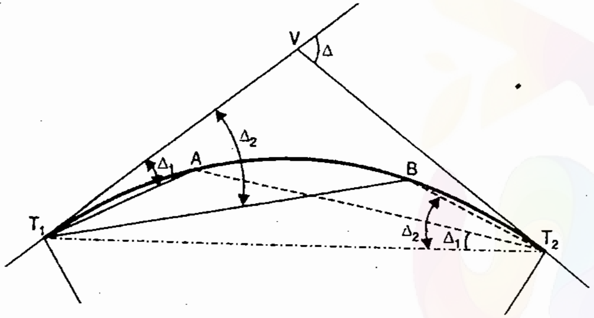

FIG. 1.12. TWO THEODOLITE METHOD.

Thus, in Fig. 1.12

$\angle VT_1A = \Delta_1$ = Deflection angle for A

But $\angle AT_2T_1$ is the angle subtended by the chord T_1A in the opposite segment.

$\therefore \quad \angle AT_2T_1 = \angle VT_1A = \Delta_1$

Similarly, $\quad \angle VT_1B = \Delta_2 = \angle T_1T_2B$

Hence the angle between the long chord and the line joining any point to T_2 is equal to the deflection angle to the point measured with respect to the rear tangent.

Method of Setting Out the Curve

(1) Set up one transit at P.C. (T_1) and the other at P.T.(T_2).

(2) Clamp both the plates of each transit to zero reading.

(3) With the zero reading, direct the line of sight of the transit at T_1 towards V. Similarly, direct the line of sight of the other transit at T_2 towards T_1 when the reading is zero. *Both the transits are thus correctly oriented.*

(4) Set the reading of each of the transits to the deflection angle for the first point A. The line of sight of both the theodolites are thus directed towards A along T_1A and T_2A respectively.

(5) Move a ranging rod or an arrow in such a way that it is bisected simultaneously by cross-hairs of both the instruments. Thus, point A is fixed.

(6) To fix the second point B, set reading Δ_2 on both the instruments and bisect the ranging rod.

(7) Repeat steps (4) and (5) for location of all the points.

The method is expensive since two instruments and two surveyors are required. However, the method is most accurate since each point is fixed independently of the others. An error in setting out one point is not carried right through the curve as in the method of tangential angles.

1.12. TACHEOMETRIC METHOD

By the use of a tacheometer, chaining may be completely dispensed with, though the method is much less accurate than Rankine's. In this method, a point on the curve is fixed by the deflection angle from the rear tangent and measuring, tacheometrically, the distance of that point from P.C. (T_1) and *not* from the precceding point as in Rankine's method. Thus, in this method also, each point is fixed independently of the others and the error in setting out one point is not carried right through the curve as in the Rankine's method.

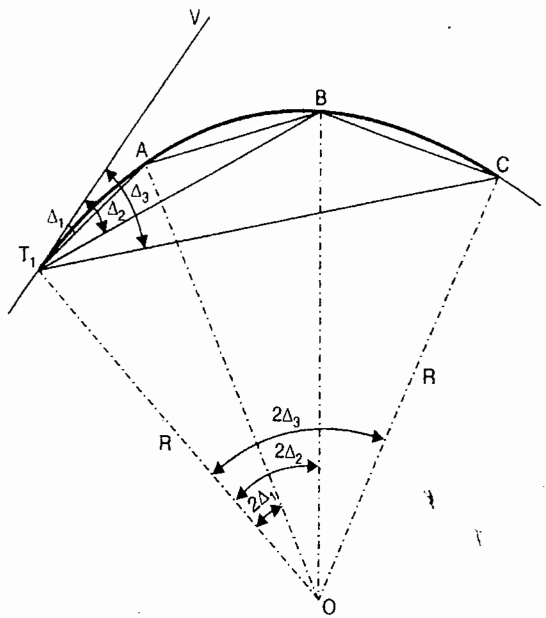

FIG. 1.13. SETTING OUT BY TACHEOMETRIC METHOD.

In Fig. 1.13, T_1A, T_1B, T_1C etc. are the *whole chords* joining point A, B, C etc. to the point of curvature (T_1).

Evidently $T_1A = L_1 = 2R \sin \Delta_1$

$T_1B = L_2 = 2R \sin \Delta_2$

$T_1C = L_3 = 2R \sin \Delta_3$ etc. etc.

and $\quad T_1T_2 = L_n = 2R \sin \Delta_n$

$= 2R \sin \dfrac{\Delta}{2} = L$

= length of the long chord.

Having known these lengths, the repective staff intercepts $s_1, s_2, s_3, \ldots, s_n$ can be calculated from the tacheometric formulae :

$L = \dfrac{f}{i} s + (f + d)$, when the line of sight is horizontal

or $\quad L = \dfrac{f}{i} s \cos^2 \theta + (f + d) \cos \theta$, when the line of sight is inclined.

Procedure for Setting Out the Curve

(1) Set the tacheometer at T_1 and sight the point of intersection (V) when the reading is zero. The line of sight is thus oriented along the rear tangent.

(2) Set the angle Δ_1 on the vernier, thus directing the line of sight along T_1A.

(3) Direct a staffman to move in the direction T_1A till the calculated staff intercept s_1 is obtained. The staff is generally held vertical. Thus, the first point A is fixed.

(4) Set the angle Δ_2 now, thus directing the line of sight along T_1B. Move the staff backward or forward along T_1B until the staff intercept s_2 is obtained, thus fixing the point B.

(5) Fix other points similarly.

Since the staff intercept increases with its distance from the tacheometer, accurate staff reading is not possible when the distances along the whole chords become too large. In that case, the curve is to be located by shifting tacheometer to the last point located on the curve.

Length of first sub-chord $(c) = 1060 - 1050.52 = 9.48$ m.

or more conveniently, $c = (53 + 00) - (52 + 52.6) = 47.4$ links $= 9.48$ m

Length of last sub-chord $(c') = 1314.94 - 1300 = 14.94$ m

or more conveniently, $c' = (65 + 74.7) - (65 + 00) = 74.7$ links $= 14.94$ m

Number of full chords $= \dfrac{1300 - 1060}{20} = \dfrac{240}{20} = 12$, each of 20 m length

Total number of chords $= 1 + 12 + 1 = 14$

Length of first offset $O_1 = \dfrac{c^2}{2R} = \dfrac{(9.48)^2}{2 \times 300} = 0.15$ m

where $R = 15$ chains $= 15 \times 20$ m $= 300$ m

Length of second offset $O_2 = \dfrac{C}{2R}(c + C) = \dfrac{20}{2 \times 300}(9.48 + 20) = 0.98$ m

$$O_3, O_4 = \ldots\ldots O_{12} = \frac{C^2}{2R} = \frac{(20)^2}{300} = 1.33 \text{ m}$$

Last offset $O_n = \dfrac{c'}{2R}(C + c') = \dfrac{14.94}{2 \times 300}(20 + 14.94) = 0.87$ m

Example 1.4. *Calculate the necessary data for setting out the curve of example 1.3 if it is intended to set out the curve by Rankine's method of tangential angles. If the theodolite has a least count of 20", tabulate the actual readings of deflection angles to be set out.*

Solution.

As calculated earlier

$$c = 9.48 \text{ m}$$
$$c' = 14.94 \text{ m}$$
$$C = 20 \text{ m}$$

The tangential angle $\delta = 1718.9 \dfrac{C}{R}$ min.

where $R = 15 \times 20 = 300$ m

δ_1 for the first chord $= 1718.9 \dfrac{9.48}{300} = 54.32$ min $= 54' \, 19''$

$$\delta_2 = \delta_3 \ldots \delta_{13} = \delta = 1718.9 \frac{20}{300} = 114.593 \text{ min} = 1° \, 54' \, 35.6''$$

δ_{14} for last chord $= 1718.9 \dfrac{14.94}{300} = 85.592$ min $= 1° \, 25' \, 35''$.

The deflection angles for various chords are as follows :

	(Deflection angle)			(Theodolite reading)		
	°	′	″	°	′	″
$\Delta_1 = \delta_1 =$	00	54	19.0	00	54	20
$\Delta_2 = \Delta_1 + \delta =$	02	48	54.6	02	49	00
$\Delta_3 = \Delta_2 + \delta =$	04	43	30.2	04	43	40
$\Delta_4 = \Delta_3 + \delta =$	06	38	5.8	06	38	00
$\Delta_5 = \Delta_4 + \delta =$	08	32	41.4	08	32	40
$\Delta_6 = \Delta_5 + \delta =$	10	27	17.0	10	27	20
$\Delta_7 = \Delta_6 + \delta =$	12	21	52.6	12	22	00
$\Delta_8 = \Delta_7 + \delta =$	14	16	28.2	14	16	20
$\Delta_9 = \Delta_8 + \delta =$	16	11	3.8	16	11	00
$\Delta_{10} = \Delta_9 + \delta =$	18	05	39.4	18	05	40
$\Delta_{11} = \Delta_{10} + \delta =$	20	00	15.0	20	00	20
$\Delta_{12} = \Delta_{11} + \delta =$	21	54	50.6	21	55	00
$\Delta_{13} = \Delta_{12} + \delta =$	23	49	26.2	23	49	20
$\Delta_{14} = \Delta_{13} + \delta_{14} =$	25	15	1.2	25	15	00

Check : $\Delta_{14} = \frac{1}{2} \Delta = \frac{1}{2}(50° \, 30') = 25° \, 15'$.

1.13. OBSTACLES TO THE LOCATION OF CURVES

CASE 1. WHEN THE P.I. IS INACCESSIBLE

When the point of intersection is inaccessible, a line is run (or traverse if necessary) to connect the two tangents. The deflection angles it makes with the tangents are measured with transit set-ups at its end. Its length is also measured very accurately. The deflection angle Δ of the curve is then equal to the sum of the two deflection angles. The distances from the ends of the line to the P.I. are calculated by the solution of the triangle. The *procedure* is as follows : (Fig. 1.14).

1. Select two intervisible points A and B and the two tangents so that line AB is moderately on the level ground. If the ground towards the P.I. is unsuitable for

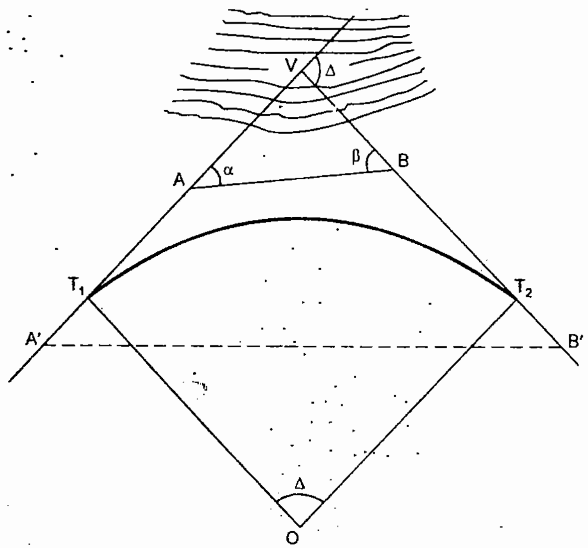

FIG. 1.14. CURVE LOCATION WHEN P.I. IS INACCESSIBLE.

CASE 5. WHEN BOTH P.C. AND P.I. ARE INACCESSIBLE (Fig. 1.17)

1. Select any point A on the rear tangent. Run any convenient line AB and measure its length. Measure angles VAB and VBA by the theodolite set-ups at A and B. Calculate AT_1 and the angle Δ as discussed in case (1) of § 1.13. Knowing the chainage of A and distance AT_1, calculate the chainage of T_1 and also of T_2.

2. Imagine the curve produced backward to C on the perpendicular offset AC. Then

$$\sin COT_1 = \frac{AT_1}{R}$$

or $\angle COT_1 = \alpha = \sin^{-1}\left(\frac{AT_1}{R}\right)$

and $AC = R(1 - \cos\alpha)$

$= R$ versin α

Thus, make the perpendicular offset AC at A, as calculated above.

3. From C, draw a chord CD parallel to AT_1, making $CD = 2\,AT_1$. *Then, point D will be on the curve.*

4. Set a theodolite at D and deflect from DC an angle equal to COT_1 (*i.e.*, $= \alpha$) for a tangent to the curve at D. Prepare a table of deflection angles with respect to D and set out the whole curve from it.

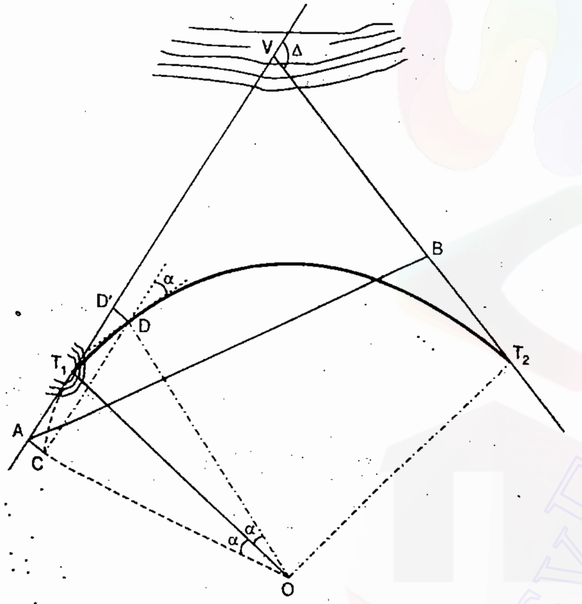

FIG. 1.17. BOTH P.C. AND P.I. INACCESSIBLE.

Alternative to step (4), adopt the procedure for setting out the curve from D as described in case 6 below.

CASE 6. WHEN THE COMPLETE CURVE CANNOT BE SET OUT FROM P.C.

In case of very long curves or obstructions intervening the line of sight, it may not be possible to set out the whole curve from one single set up of the instrument at P.C. In such a case, it is necessary to set up the instrument at one or more points along the curve. We will consider two cases of the set ups of the theodolite at intermediate points on the curve :

(*a*) When the P.C. is visible from the intermediate point.

(*b*) When the P.C. is not visible from the intermediate point.

CASE (*a*) When the P.C. is visible from the intermediate point.

First method :

Let C be the last point set out from the P.C. (T_1) and let its deflection angle be Δ_c. Assuming that T_1 is visible from C, the procedure for setting out the rest of the curve is as follows :

1. Shift the theodolite at C and set it there.

2. Set the vernier to read 0° and backsight on T_1 with the telescope inverted.

3. Transit the telescope. The line of sight is now directed along T_1C produced.

4. Unclamp the upper plate, and set the vernier to read deflection angle Δ_d to the forward point D as if it were located from T_1 and locate the point D as usual.

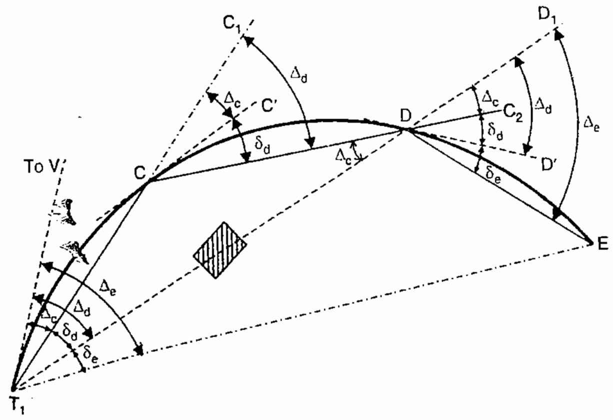

FIG. 1.18. SETTING OUT FROM INTERMEDIATE POINT

A careful study of Fig. 1.18 will reveal that when the angle Δ_d is on the circle, the line of sight is towards D. However, the proof is given below :

Let CC' be the tangent to the curve at C

$$\angle C_1CC' = \Delta_c$$

$$\delta_d = \angle C'CD = \text{tangential angle for chord } CD.$$

Since $\angle CT_1D$ is the angle that the chord CD substends in the opposite segment, we have

$$\angle CT_1D = C'CD = \delta_d$$

Hence $\angle VT_1D$ = deflection angle for $D = \angle VT_1C + \angle CT_1D = \Delta_c + \delta_d = \Delta_d$

But $\angle C_1CD = \angle C_1CC' + \angle C'CD = \Delta_c + \delta_d$

Hence $\angle VT_1D = \angle C_1CD$.

Thus when the instrument is transferred to any point on the curve and oriented as explained above, no new calculations are required for continuing the curve, but the previously calculated deflection angles can be used.

Second method to case (a) :

In the above method, it is assumed that the instrument is in good adjustment. If it is not, the curve can be set out as below :

(1) While the last point C is sighted from T_1, fix a point C_1 in the direction T_1C produced.

(2) Set the theodolite at C. Clamp both the plates with zero reading and bisect C_1 accurately. The instrument is thus correctly oriented.

(3) Release the vernier plate and set the vernier to deflection angle Δ_d to set out the point D.

Treating T_1D as the mid-ordinate of a curve whose radius is R and central angle 2θ, we have

$$T_1D = R - R\cos\theta = R(1-\cos\theta) = = R \text{ versin } \theta$$

or
$$R = \frac{T_1D}{\text{versin }\theta} = \frac{z\sin\alpha}{\text{versin }\theta} = \frac{z\sin\alpha}{(1-\cos\theta)} \quad \ldots(1.19)$$

From which R can be determined.

If the co-ordinates x and y are given, first calculate the angle α from the relation $\tan\alpha = \frac{y}{x}$ and then determine θ from equation 1.18. The radius R is then given by

$$R = \frac{T_1D}{\text{versin }\theta} = \frac{y}{\text{versin }\theta} = \frac{y}{1-\cos\theta} \quad \ldots[1.19\ (a)]$$

The tangent $T = R\tan\frac{\Delta}{2}$.

(2) PASSING A CURVE TANGENTIAL TO THREE LINES (Fig. 1.20)

Given three lines T_1A, AB, BT_2, their deviation angle α and β and the length of AB ($= d$), it is required to find the radius R of a curve that will be tangential to the three lines.

Let T_1, C and T_2 be the tangential points.

$$AC = AT_1 = R\tan\tfrac{1}{2}\alpha \quad \ldots(i)$$

$$BC = BT_2 = R\tan\tfrac{1}{2}\beta \quad \ldots(ii)$$

Adding (*i*) and (*ii*), we get

$$d = AB = AC + BC$$
$$= R\left(\tan\tfrac{1}{2}\alpha + \tan\tfrac{1}{2}\beta\right)$$

From which, $R = \dfrac{d}{\tan\frac{1}{2}\alpha + \tan\frac{1}{2}\beta}$ ...(1.20)

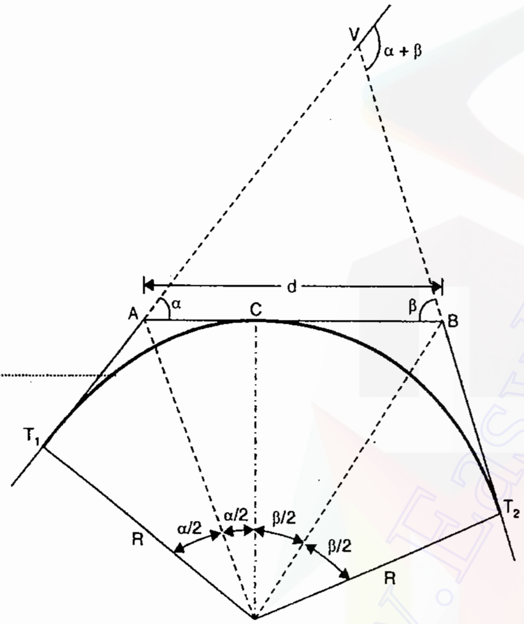

FIG. 1.20. CURVE TANGENTIAL TO THREE LINES

(3) SHIFTING FORWARD TANGENT PARALLEL OUTWARD : RADIUS UNCHANGED

Given the distance p by which the forward tangent is shifted outward, it is required to locate the new position of P.C. if the radius is unchanged.

Let T_1T_2 be the original curve and $T_1'T_2'$ be the new curve when the tangent VT_2 is shifted to a new position $V'T_2'$ parallel to itself by a distance p. Let O' be the new centre. In Fig. 1.21, the firm lines show the elements of the original curve, while

the dotted lines and the letters with dash correspond to the condition when the forward tangent is shifted.

Thus, T_2A = perpendicular distance = p

$$T_1T_1' = OO' = T_2T_2' = VV' = \frac{p}{\sin\Delta} \quad \ldots(i)$$

$\therefore$ Chainage of T_1' = chainage of $T_1 + \dfrac{p}{\sin\Delta}$

Thus, the new P.C. (T_1') can be located.

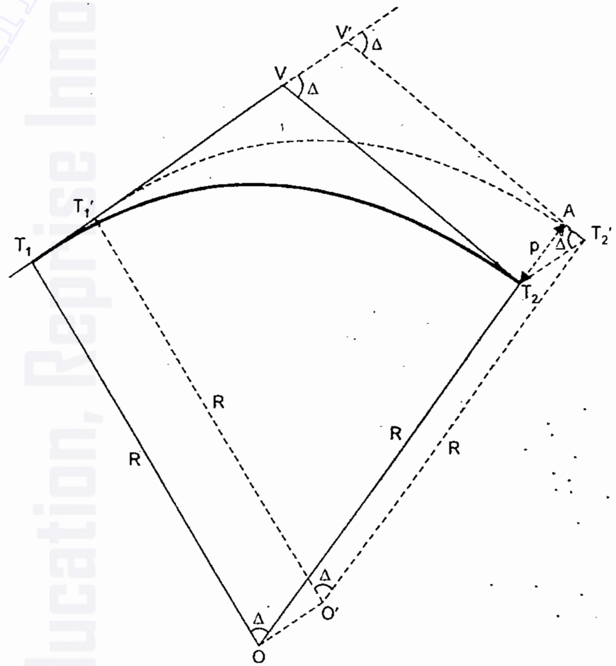

FIG. 1.21. SHIFTING FORWARD TANGENT OUTWARD (SAME RADIUS)

If, however, the tangent is shifted inward, equation (*i*) still holds good, but to find the chainage of T_1' the distance $\dfrac{p}{\sin\Delta}$ will have to be subtracted from chainage of T_1.

(4) SHIFTING FORWARD TANGENT PARALLEL OUT-WARD : RADIUS CHANGED

Given the distance p by which the forward tangent is shifted outward it is required to find the new radius R' without changing the position of P.C. (Fig. 1.22)

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(6) CHANGING THE DIRECTION OF FORWARD TANGENT : P.T. MOVED AHEAD

Given the angle θ by which the forward tangent VT_2 is rotated to a new position $V'T_2'$ without changing the P.C. , it is required to find the new radius (R') and the new P.T. (T_2')

In Fig. 1.24. the firm lines show the original elements while the dotted lines correspond to the case when the forward tangent is rotated through θ. Since the position of the P.C. is unchanged, the new centre (O) will lie on T_1O. Let T_2' be the new P.T.

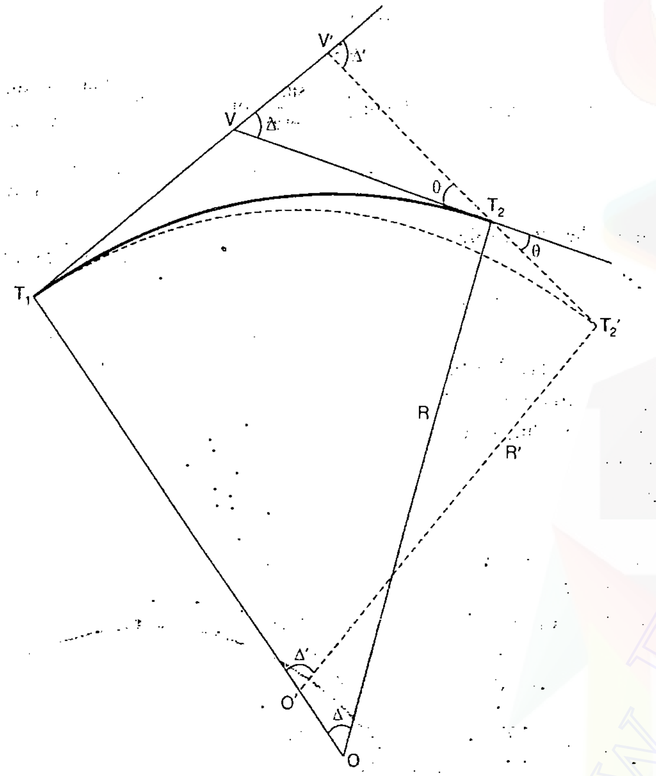

FIG. 1.24. CHANGING DIRECTION OF FORWARD TANGENT : P.C. UNCHANGED.

We have $\Delta' = \Delta + \theta$

By sine rule, $VV' = VT_2 \cdot \dfrac{\sin\theta}{\sin\Delta'} = T\dfrac{\sin\theta}{\sin\Delta'}$

Now $T' = T_1V' = T_1V + VV'$

or $$T' = T + T\frac{\sin\theta}{\sin\Delta'} = T\left(1 + \frac{\sin\theta}{\sin\Delta'}\right) \quad ...(1)$$

But $T' = R'\tan\dfrac{\Delta'}{2}$ and $T = R\tan\dfrac{\Delta}{2}$

Substituting these values, we get

$$R'\tan\frac{\Delta'}{2} = R\tan\frac{\Delta}{2}\left(1 + \frac{\sin\theta}{\sin\Delta'}\right)$$

or $$R' = R\frac{\tan\frac{\Delta}{2}}{\tan\frac{\Delta'}{2}}\left(1 + \frac{\sin\theta}{\sin\Delta'}\right) \quad ...(1.23)$$

Again, to locate the position of T_2' consider the triangle $VV'T_2$, from which

$$V'T_2 = VT_2 \cdot \frac{\sin\Delta}{\sin\Delta'} = T \cdot \frac{\sin\Delta}{\sin\Delta'}$$

Now $$T_2T_2' = V'T_2' - V'T_2 = T' - T\frac{\sin\Delta}{\sin\Delta'} = T\left(1 + \frac{\sin\theta}{\sin\Delta'}\right) - T\frac{\sin\Delta}{\sin\Delta'}$$

$$= T\left(1 - \frac{\sin\Delta - \sin\theta}{\sin\Delta'}\right)$$

Thus, T_2' can be located.

Example 1.5. *The following notes refer to setting out of a circular curve to the right, of 15 chains radius between two straights AB, BC and intersection B of which was inaccessible.*

Measurement ab = 6.21 chains from a in AB to b in BC

Theodolite at a : interior angle α = 23° 43′

Theodolite at b : interior anlge β = 25° 54′

Chainage of a = 29.059

Owing to obstructions it will be impossible to set out angles from the tangent at the first tangent point A beyond that to the peg at 31.0 chains on the curve, and the theodolite will be set up at this peg in order to continue the curve, to the second tangent point C.

Describe concisely the procedure of setting out from an intermediate peg on the curve, and show in tabular form the tangential angles to be set out at A and at 31.0 chains for peg at even chains on the curve giving also the nearest readings for a vernier reading to 20″. (U.L.)

Solution.

Given : $\alpha = 23° 43'$, $\beta = 25° 24'$, $R = 15$ chains

∴ $\Delta = \alpha + \beta = 23° 43' + 25° 54' = 49° 37'$

From triangle Bab, we have $Ba = ab\dfrac{\sin\beta}{\sin\Delta} = 6.21\dfrac{\sin 25° 54'}{\sin 49° 37'} = 3.561$ chains

Length of curve $= \dfrac{\pi R\Delta}{180°} = \dfrac{\pi \times 15 \times 49° 37'}{180°} = 12.989$ chains

Tangent length $T = BA = R\tan\frac{1}{2}\Delta = 15\tan\dfrac{49° 37'}{2} = 6.934$ chains

$$\theta = \tan^{-1}\frac{D}{L} = \tan^{-1}\frac{254.2}{153.04} = 58°\ 54'$$

Bearing of SP = S 58° 54′ W = 238° 54′

Bearing of PS = N 58° 54′ E

Length of $PS = D$ cosec 58° 54′ = 254.20 cosec 58° 54′ = 296.9 m

$\angle VPS$ = Bearing of PS – Bearing of PV = 58° 54′ – 40° = 18° 54′ = α

$\angle VSP$ = Bearing of SV – Bearing of SP

= (100° + 180°) – 238° 54′ = 41° 06′ = β

Total deflection angle Δ = 100° – 40° = 60° = α + β

From triangle VPS, we have

$$PV = PS \cdot \frac{\sin\beta}{\sin\Delta} = 296.9\,\frac{\sin 41°\ 06'}{\sin 60°} = 225.4 \text{ m}$$

Radius of the curve is given by

$$R = \frac{1146}{D} = \frac{1146}{4} = 286.5 \text{ m}$$

Tangent length $T = T_1V = R \tan\frac{1}{2}\Delta$ = 286.5 tan 30° = 165.4 m

$$\text{Length of the curve} = \frac{\Delta}{D} \times 20 = \frac{60}{4} \times 20 = 300 \text{ m}$$

Chainage of P =	1618.8 metres
Add length PV =	225.4
Chainage of V =	1844.2
Subtract tangent length =	165.4
Chainage of T_1 =	1678.8
Add length of curve =	300.0
Chainage of T_2 =	1978.8

Example 1.7. *Two straight lines PQ and QR on the centre-line of a proposed road on a rocky headland are to be connected by a circular curve of 600 ft radius. From the traverse notes, it is found that if the bearing of PQ is assumed to be N 0° 0′ E the bearing of QR will be N 48° 20′ E while if P be taken as the origin of co-ordinates, the latiude and departure of R will be + 725 ft and + 365 ft respectively.*

Determine the distance of the tangent points of the curve from the stations P and R.

Solution.

With reference to Fig. 1.26, we have

Δ = $\angle R'QR$ = 48° 20′

Latitude of $R = PR'$ = 725′

Departure of $R = PR''$ = 365′

Length of $PR = \sqrt{(725)^2 + (365)^2}$

= 811.71 ft.

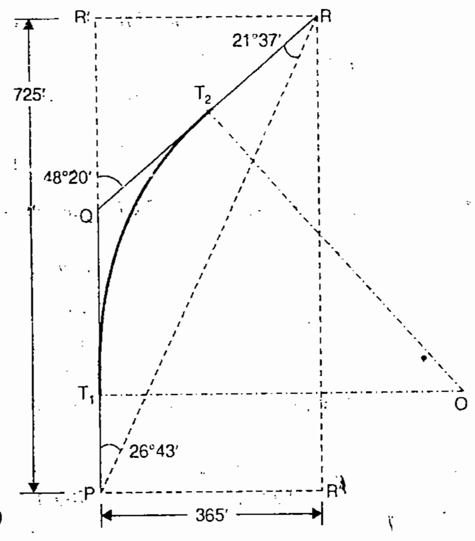

FIG. 1.26

Bearing of $PR = \tan^{-1}\frac{D}{L}$

$= \tan^{-1}\frac{365}{725}$ = N 26° 43′ E

∴ $\angle QPR$ = 26° 43′

$\angle RQP$ = 180° – 48° 20′ = 131° 40′

$\angle QRP$ = 48° 20′ – 26° 43′ = 21° 37′

From triangle QPR,

$$QP = \frac{PR}{\sin 48°\ 20'} \times \sin 21°\ 37'$$

$$= 811.71\,\frac{\sin 21°\ 37'}{\sin 48°\ 20'} = 400.30$$

and
$$QR = \frac{PR}{\sin 48°\ 20'} \times \sin 26°\ 43'$$

$$= 811.71\,\frac{\sin 26°\ 43'}{\sin 48°\ 42'} = 488.51$$

For the given circular curve, tangent distance is given by

$$T = QT_1 = QT_2 = R\ \tan\frac{\Delta}{2} = 600\ \tan\frac{48°\ 20'}{2} = 269.23$$

Distance $PT_1 = QP - QT_1$ = 400.30 – 269.23 = **131.07**

and Distance $RT_2 = QR - QT_2$ = 488.51 – 269.23 = **219.28**

Example 1.8. *Two straights T_1V and VT_2 are intersected by a third line AB. The angles VAB and VBA are measured to be 26° 24′ and 34° 36′, and the distance AB = 358 metres. Calculate the radius of the simple circular curve which will be tangential to the three lines T_1A, AB and BT_2 and the chainages of P.C. and P.T. if the chainage of V = 6857.3 metres.*

Solution. (Fig. 1.20)

Let the curve T_1CT_2 be tangential to three lines at T_1, C and T_2.

$\angle VAB$ = α = 26° 24′

$\angle VBA$ = β = 34° 36′

For the arc T_1C, central angle T_1OC = α = 26° 24′

Tangent $T_1A = AC = R \tan\frac{1}{2}\alpha = R$ tan 13° 12′ ...(1)

Similarly, for the arc CT_2, the central angle

T_2OC = β = 34° 36′

$$\angle ABV = \Delta - \angle BAV = 60° \, 30' - 41° \, 50' = 18° \, 40'$$

By sine rule, $AV = AB \dfrac{\sin ABV}{\sin \Delta} = 441 \dfrac{\sin 18° \, 40'}{\sin 60° \, 30'} = 162.2$ m

Let T_1 be the P.C. and T_2 be the P.T.

$$T_1V = VT_2 = R \tan \frac{\Delta}{2} = 200 \tan 30° \, 15' = 116.6 \text{ m.}$$

Length of curve $= \dfrac{\pi R \Delta}{180°} = \dfrac{\pi \times 200 \times 60° \, 30'}{180°} = 211.2$

Chainage of A = 4262.5 metres
Add length of AV = 162.2

Chainage of V = 4424.7
Subtract tangent length = 116.6

Chainage of T_1 = 4308.1
Add length of curve = 211.2

Chainage of T_2 = 4519.3

Since the chainage of the points on the curve is to be multiple of 20 m

Chainage of first point = 4320 m

Length of first sub-chord

$= 4320 - 4308.1 = 11.9$ m

Length of last sub-chord

$= 4519.3 - 4500 = 19.3$ m

Length of full chord = 20 m

Tangential angle δ_1 for first sub-chord

$$= \frac{1718.9 \, c'}{R} = \frac{1718.9 \times 11.9}{200}$$

$= 102'.27 = 1° \, 42' \, 16''.4$

Tangential angle δ for normal chord

$$= \frac{1718.9 \times 20}{200} = 171'.89$$

$= 2° \, 51' \, 53''.4$

Tangential angle for last sub-chord

$$= \delta_n = \frac{1718.9 \times 19.3}{200}$$

$= 165'.87 = 2° \, 45' \, 52''$

No. of full chords $= \dfrac{4500 - 4320}{20} = 9$

∴ Total number of chords

$= 1 + 9 + 1 = 11.$

Since it is a left hand curve, theodolite readings will be (360° – Deflection angle) as tabulated below

Point	*Chainage (m)*	*Chord Length (m)*	δ			Δ			*Actual theodolite Reading*			*Remarks*
			°	′	″	°	′	″	°	′	″	
T_1	4308.1								360	0	0	
1	4320	11.9	1	42	16.4	1	42	16.4	358	17	40	
2	4340	20	2	51	53.4	4	34	09.8	355	26	00	
3	4360	20	2	51	53.4	7	26	03.2	352	34	00	
4	4380	20	2	51	53.4	10	17	56.6	349	42	00	
5	4400	20	2	51	53.4	13	09	50.0	346	50	00	
6	4420	20	2	51	53.4	16	01	43.4	343	58	20	
7	4440	20	2	51	53.4	18	53	36.8	341	06	20	
8	4460	20	2	51	53.4	21	45	30.2	338	14	20	
9	4480	20	2	51	53.4	24	37	23.6	335	22	40	
10	4500	20	2	51	53.4	27	29	17.0	332	30	40	
T_2	4519.3	19.3	2	45	52.0	30	15	9.0	329	45	00	

Example 1.11. *On the basis of preliminary survey, it was proposed to connect two straights, having deflection angle of 110 °, by a circular curve of 400 metres radius. Later, however, it was decided to shift the forward tangent outward paralled to itself by a distance of 50 metres. Calculate (a) the new radius of the curve, and (b) chainages of new P.I. and P.T., if the position of original P.C. (chainage 9218.4 metres) is not to be changed.*

Solution. (Fig. 1.22).

Let V' and T_2' be the new P.I. and P.T., and R' be the new radius.

$$\angle VT_1T_2 = V'T_2'T_1 = \tfrac{1}{2}\Delta = 55°$$

$$T_2T_2' = \frac{p}{\sin \frac{1}{2}\Delta} = \frac{50}{\sin 55°} = 16 \text{ m}$$

$$VV' = \frac{p}{\sin \Delta} = \frac{50}{\sin 110°} = 53.2 \text{ m}$$

New tangent $\quad T' = T_1V + VV'$

or $\quad R' \tan \dfrac{\Delta}{2} = R \tan \dfrac{\Delta}{2} + 53.2$

or $\quad R' = \dfrac{400 \tan 55° + 53.2}{\tan 55°} = \dfrac{571.3 + 53.2}{1.428} = \mathbf{437.3 \ m}$

Length of the new curve $= \dfrac{\pi R' \Delta}{180°} = \dfrac{\pi \times 437.3 \times 110°}{180°} = 839.6$ m

4. The chainage at the point of intersection of the tangents to a railway curve is 3876 links, and the angle between them is 124°.

Find the chainage at the beginning and end of the curve if it is 40 chains radius, and calculating the angle which are required in order to set out this cure (*a*) with a theodolite, (*b*) with a chain and tape only.

5. The tangents to a railway meet at an angle of 148°. Owing to the position of a building, a curve is to be chosen that will pass near a point 10 metres from the point of intersection of the tangents on the bisector of the angle 148°. Calculate the suitable radius of the curve.

6. The intersection point *C* of two railway straights *ABC* and *CDE* is inaccessible and so convenient points *B* and *C* in the straights are selected giving $BD = 6.10$ chains, and $\angle CBD = 9° 24'$ and $\angle CDB = 10° 36'$and the forward chainage of $B = 90.50$ chains. The conditions of the site are such that it is decided to make *B* the first tangent point. Determine the radius of a circular curve to connect the straights, tabulate all data necesary to set out pegs at 1 chain intervals of through chainage and show that your calculations are checked.

7. Two straights of a proposed road deflect through an angle of 120°. Originally, they were to be connected by a curve of 520 metres radius. However, due to the revision of the scheme, the deflection angle is to be increased to 132°. Calculate the suitable radius of the curve such that the original starting point of the curve (P.C.) does not change.

8. On the basis of preliminary survey, it was proposed to connect two straights, having deflection angle of 112°, by a circular curve of 400 metres radius, and the direction of both the tangents were set out in the field. However, while setting out the curve, it was thought desirable to change the radius to 450 metres without changing the direction of the forward tangent. Calculate the distance by which the forward tangent must be shifted parallel to itself so that the point of curvature (P.C.) remains unaltered.

ANSWERS

1. (*a*) $O_1 = 3\frac{1}{3}$ links, O_2 ...... $O_{13} = 6\frac{2}{3}$ links, $O_{14} = 4\frac{1}{3}$ links
   (*b*) $\delta_1 = \delta_2 \ldots. = \delta_{13} = 1° 54'.6$, $\delta_{14} = 1° 25'.3$.
2. $O_o = 2$ m ; $O_5 = 1.88$ m ; $O_{10} = 1.50$ m ; $O_{15} = 0.88$ m.
4. 17.492 chains and 56.587 chains
   (*a*) $\delta_1 = 21'.83$; δ_2 to $\delta_{39} = 42'.97$; $\delta_{40} = 25'.22$
   (*b*) $O_1 = 0.32$ links; $O_2 = 1.88$ links; O_3 to $O_{39} = 2.5$ links; $O_{40} = 1.16$ links.
5. $R = 248.1$ m.
6. 18.61 chains ; $\delta_1 = 46' 11''$; $\delta_2 = 2° 18' 34''$.
7. 513.2 m.
8. 68.73 m.

2

Curve Surveying II : Compound and Reverse Curves

2.1. ELEMENTS OF A COMPOUND CURVE

In Fig. 2.1, T_1DT_2 is a *two centred compound curve* having two circular arcs T_1D and DT_2 meeting at a common point *D* known as the point of compound curvature (P.C.C.). T_1 is the *point of curve* (P.C.) and T_2 is the *point of tangency* (*P.T.*). O_1 and O_2 are the centres of the two arcs.

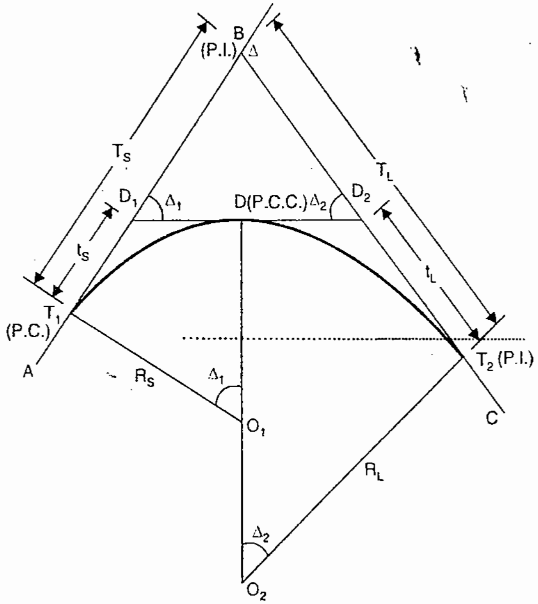

FIG. 2.1. TWO CENTRED COMPOUND CURVE.

R_S = the smaller radius (T_1O_1)

R_L = the longer radius (T_2O_2)

D_1D_2 = common tangent

Δ_1 = deflection angle between the rear and the common tangent

Δ_2 = deflection angle between the forward common and the forward tangent

Δ = total deflection angle

t_S = the length of the tangent to the arc (T_1D) having a smaller radius

t_L = the length of the tangent to the arc $D\,T_2$ having a longer radius

T_S = tangent distance $T_1\,B$ corresponding to the shorter radius

T_L = tangent distance BT_2 corresponding to the longer radius

From Fig. 2.1, we have

$$t_S = T_1D_1 = D_1D = R_S \tan \tfrac{1}{2}\Delta_1 \qquad \ldots[2.1\ (a)]$$

Required : Δ_1, Δ_2 and T_S

In Fig. 2.3, prolong the long curve T_2D to a point D' until it has a central angle $D'O_2T_2 = \Delta$. Its tangent $B'D'$ will then be parallel to the tangent BT_1.

Then, $$T_2B' = B'D' = R_L \tan \tfrac{1}{2}\Delta \quad ...(1)$$

Braw BP perpendicular to $D'B'$.

Prolong BT_1 to meet $D'O_2$ in Q. Draw O_1S perpendicular to $D'O_2$.

Then, $$D'Q = BP = BB' \sin\Delta = (T_2B' - T_2B) \sin\Delta$$
$$= (R_L \tan \tfrac{1}{2}\Delta - T_L) \sin\Delta \quad ...(2)$$

Also, $$B'P = BB' \cos\Delta = (R_L \tan \tfrac{1}{2}\Delta - T_L) \cos\Delta \quad ...(3)$$

Now, $$O_2S = O_2D' - D'Q - QS = R_L - BP - R_S \quad ...(4)$$

From triangle O_1O_2S, $$\cos\Delta_1 = \frac{O_2S}{R_L - R_S} \quad ...(5)$$
$$\Delta_2 = \Delta - \Delta_1 \quad ...(6)$$
$$O_1S = (R_L - R_S) \sin\Delta_1 \quad ...(7)$$

and $$T_S = T_1B = QB - QT_1 = D'P - O_1S$$
$$= D'B' + B'P - O_1S \quad ...(8)$$

Thus, Δ_1, Δ_2 and T_S are determind from (5), (6) and (8) above.

Case (4) : Given Δ, T_S, T_L and R_S.

Required : Δ_1, Δ_2 and R_L.

Refer Fig. 2.2,

As in case (2), we have

$$T_1B' = B'D' = R_S \tan \tfrac{1}{2}\Delta \quad ...(1)$$
$$T_2Q = (T_S - R_S \tan \tfrac{1}{2}\Delta) \sin\Delta \quad ...(2)$$

and $$B'P = B'B \cos\Delta = (T_S - R_S \tan \tfrac{1}{2}\Delta) \cos\Delta \quad ...(3)$$

Now $$O_1S = QD' = QP + PB' - B'D' = T_L + B'P - B'D' \quad ...(4)$$

Join DD' and prolong it to pass through T_2.

Evidently. $$\angle T_2D'Q = \tfrac{1}{2}\Delta_2$$

$\therefore$ $$\tan \tfrac{1}{2}\Delta_2 = \frac{T_2Q}{QD'} = \frac{BP}{QD'} \quad ...(5)$$
$$\Delta_1 = \Delta - \Delta_2 \quad ...(6)$$
$$R_L - R_S = O_1O_2 = \frac{O_1S}{\sin\Delta_2}$$

$\therefore$ $$R_L = R_S + \frac{O_1S}{\sin\Delta_2} \quad ...(7)$$

Thus, Δ_1, Δ_2 and R_L can be computed from (5), (6) and (7) above.

Case (5) : *Given* : T_S, T_L and R_L

Required : Δ_1, Δ_2 and R_S

Refer Fig. 2.3. As in case (3), we have

$$T_2B' = B'D' = R_L \tan \tfrac{1}{2}\Delta \quad ...(1)$$
$$D'Q = BP = (R_L \tan \tfrac{1}{2}\Delta - T_L) \sin\Delta \quad ...(2)$$

and $$B'P = BB' \cos\Delta = (R_L \tan \tfrac{1}{2}\Delta - T_L) \cos\Delta \quad ...(3)$$

Now $$O_1S = QT_1 = QB - T_1B = D'P - T_1B = D'B' + B'P - T_S \quad ...(4)$$

Join DT_1 and prolong it to pass through D'.

Evidently $\angle D'T_1Q = \tfrac{1}{2}\Delta_1$

$\therefore$ $$\tan \tfrac{1}{2}\Delta_1 = \frac{D'Q}{QT_1} = \frac{BP}{O_1S} \quad ...(5)$$
$$\Delta_2 = \Delta - \Delta_1 \quad ...(6)$$
$$R_L - R_S = O_1O_2 = \frac{O_1S}{\sin\Delta_1}.$$
$$R_S = R_L - \frac{O_1S}{\sin\Delta_1} \quad ...(7)$$

Thus, Δ_1, Δ_2 and R_S can be computed from (5), (6) and (7).

2.3. SETTING OUT COMPOUND CURVE

The compound curve can be set by method of deflection angles. The first branch is set out by setting the theodolite at T_1 (P.C.) and the second branch is set out by setting the theodolite at the point D (P.C.C.). The procedure is as follows :

(1) After having known any four parts, calculate the rest of the three parts by the formulae developed in § 2.2.

(2) Knowing T_S and T_L, locate points T_1 and T_2 by linear measurements from the point of intersection.

(3) Calculate the length of curves l_S and l_L. Calculate the chainage of T_1, D and T_2 as usual.

(4) For the first curve, calculate the tangential angles etc., for setting out the curve by Rankine's method.

(5) Set the theodolite at T_1 and set out the first branch of the curve as already explain.

(6) After having located the last point D (P.C.C) shift the theodolite to D and set it there. With the vernier set to $\left(360° - \frac{\Delta_1}{2}\right)$ reading, take a backsight on T_1 and plunge the telescope. The line of sight is thus oriented along T_1D produced and if the theodolite is now swung through $\frac{\Delta_1}{2}$, the line of sight will be directed along the common tangent DD_2. Thus the theodolite is correctly oriented at D.

$$\Delta_2 = \Delta - \Delta_1 = 93° - 53° 40' = 39° 20'$$

$$t_1 = T_1D_1 = R_1 \tan \frac{\Delta_1}{2} = 286.5 \tan 26° 50' = 144.9 \text{ m}$$

$$t_2 = T_2D_2 = R_2 \tan \frac{\Delta_2}{2} = 229.2 \tan 19° 40' = 81.9 \text{ m}$$

$$T_1 = T_1B = t_1 + (t_1 + t_2) \frac{\sin \Delta_2}{\sin \Delta}$$

$$= 144.9 + (144.9 + 81.9) \frac{\sin 39° 20'}{\sin 93°} = 288.9 \text{ m}$$

Length of the first arc = l_1

$$= \frac{\Delta_1}{D_1} \times 20 = \frac{53° 40'}{4°} \times 20 = 268.3 \text{ m}$$

Length of the second arc = l_2

$$= \frac{\Delta_2}{D_2} \times 20 = \frac{39° 20'}{5°} \times 20 = 157.3 \text{ m}$$

Chainage of P.I = 912.2 m = 45 + 610 (in 20 m units)
Subtract T_1 = 288.9 = 14 + 445

Chainage of P.C. = 623.3 = **31.165**
Add l_1 = 268.3 = 13.415

Chainage of P.C.C. = 891.6 = **44.580**
Add l_2 = 157.3 = 7.865

Chainage of P.T. = 1048.9 = **52.445**.

Example 2.3. *A compound curve is to consist of an arc of 36 chains followed by one of 48 chains radius and is to connect two straights which yield a deflection angle of 84° 30′. At the intersection point the chainage, if continued along the first tangent, would be 86 + 48 and starting point of the curve is selected at chainage 47 + 50. Calculate the chainage at the point of junction of the two branches and at the end of the curve.*

Solution. (Fig. 2.2).

Here, R_S, R_L, Δ and T_S are given. In order to calculate the chainages of various points, we will have to first determine Δ_1 and Δ_2.

$$T_S = 86.48 - 47.50 = 38.98 \text{ chains.}$$

As in Fig. 2.2, prolong the shorter arc to a point D' so that its central angle is equal to $\Delta = 84° 30'$. The tangent $D'B'$ will then be parallel to initial tangent BT_2. Draw BP perpendicular to $B'D'$.

Then $T_2Q = BP = BB' \sin \Delta$

$$= (T_1B - T_1B') \sin \Delta = (T_S - R_S \tan \tfrac{1}{2}\Delta) \sin \Delta$$

$$= (38.98 - 36 \tan 42° 15') \sin 84° 30' = 6.26 \text{ chains}$$

$$O_2S = O_2T_2 - T_2Q - QS = R_L - T_2Q - R_S$$

$$= 48 - 6.26 - 36 = 5.74 \text{ chains}$$

$$\cos \Delta_2 = \frac{O_2S}{R_L - R_S} = \frac{5.74}{48 - 36}$$

$\therefore$ $\Delta_2 = 61° 24.$

$\therefore$ $\Delta_1 = 84° 30' - 61° 24' = 23° 6'$

$$\text{Length of the first arc} = l_1 = \frac{\pi R_S \Delta_1}{180°} = \frac{\pi \times 36 \times 23° 6'}{180°} = 14.52 \text{ chains}$$

$$\text{Length of the second arc} = l_2 = \frac{\pi R_L \Delta_2}{180°} = \frac{\pi \times 48 \times 61° 24'}{180°} = 51.44 \text{ chains}$$

Chainage of P.C. = 47.50 chains
Add length of the first arc = 14.52

Chainage of P.C.C. = **62.02**
Add length of the second arc = 51.44

Chainage of end of curve = **113.46**

Example 2.4. *A compound curve is to connect two straights having a deflection angle of 90°. As determined from the plan, the lengths of the two tangents are 350 metres and 400 metres repectively. Calculate the lengths of the two arcs if the radius of the first curve is be 300 metres.*

Solution. (Fig. 2.2)

Given $T_S = 350$ m
$T_L = 400$ m
$\Delta = 90°$
$R_S = 300$ m

Required to find Δ_1, Δ_2 and R_L

$$T_1B' = B'D' = R_S \tan \tfrac{1}{2}\Delta = 300 \tan 45° = 300 \text{ m}$$

$$T_2Q = BP = (T_S - R_S \tan \tfrac{1}{2}\Delta) \sin \Delta = (350 - 300 \tan 45°) \sin 90° = 50 \text{ m}$$

$$B'P = BB' \cos 90° = \text{zero}$$

$$O_1S = QD' = QP + PB' - B'D' = T_L + 0 - 300 = 400 - 300 = 100 \text{ m}$$

$$\tan \tfrac{1}{2}\Delta_2 = \frac{T_2Q}{QD'} = \frac{BP}{QD'} = \frac{50}{100} = 0.5$$

$\therefore$ $\tfrac{1}{2}\Delta_2 = 26° 34'$

or $\Delta_2 = 53° 8'$

$$\Delta_1 = \Delta - \Delta_2 = 90° - 53° 8' = 36° 52'.$$

REVERSE CURVES

2.4. ELEMENTS OF A REVERSE CURVE

A reverse curve consists of two simple curves of opposite direction that join at a common tangent point called the point of reverse curvature (P.R.C.). They are used when the straights are parallel or include a very small angle of intersection and are frequently encountered in mountaineous countries, in cities, and in the layout of railway spur tracks and cross-over. The use of reverse curve should be avoided on highways and main railway lines where speeds are high for the following reasons :

(1) Sudden change of cant is required from one side of P.R.C. to the other.

(2) There is no opportunity to elevate the outer bank at P.R.C.

(3) The sudden change of direction is uncomfortable to passengers and is objectionable.

(4) Steering is dangerous in the case of highways and the driver has to be very cautious.

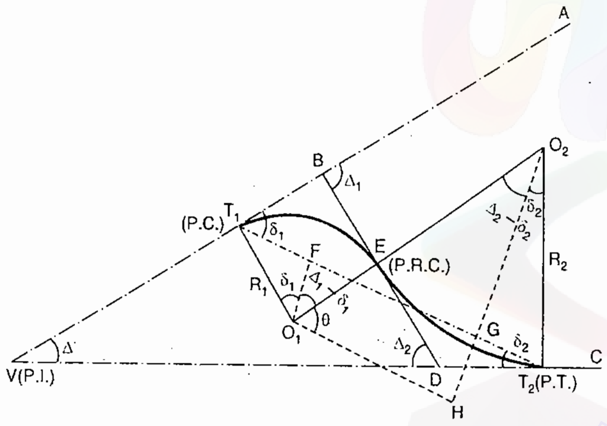

FIG. 2.5. REVERSE CURVE ($\Delta_1 > \Delta_2$)

It is definitely an advantage to separate to curves by either a short length of straight or a reversed spiral. The elements of a reverse curve are not directly determinate unless some condition or dimension is specified as, for example, equal radii ($R_1 = R_2$) or equal central angle ($\Delta_1 = \Delta_2$). Frequently, a common or equal radius is used for both parts of the curve in order to use largest radius possible.

Fig. 2.5 shows the general case of a reverse curve in which VA and VC are the two straights and $T_1E\,T_2$ is reverse curve. T_1 is the point of curvature (P.C.), E is the point of reverse curvature (P.R.C.) and T_2 is the point of tangency (P.T.). O_1 and O_2 are the centres of the two branches. BD is the common tangent.

Let R_1 = the smaller radius

R_2 = the greater radius

Δ_1 = central angle for the curve having smaller radius

Δ_2 = central angle for the curve having greater radius (Δ_1 is greater than Δ_2)

Δ = total deviation between the tangents

δ_1 = angle between tangent AV and the line T_1T_2 joining the tangent points

δ_2 = angle between tangent VC and the line T_2T_1 joining the tangent points.

Since E is the point of reverse curvature, the line O_1O_2 is perpendicular to the common tangent BD at E. Join T_1 and T_2 and drop perpendiculars O_1F and O_2G on it from

O_1 and O_2 respectively. Through O_1, draw O_1H parallel to T_1T_2 to cut the line O_2G produced in H.

Since T_1B and BE are tangents to the first arc, $\angle ABE = \Delta_1$. Similarly, since ED and DT_2 are tangents to the second arc, $\angle EDV = \Delta_2$.

From triangle BVD, $\quad \Delta_1 = \Delta + \Delta_2$

or $\quad \Delta = \Delta_1 - \Delta_2 \qquad (1) \qquad ...(2.4)$

From triangle T_1VT_2, $\quad \delta_1 = \Delta + \delta_2$

$\Delta = \delta_1 - \delta_2 \qquad (2)$

From (1) and (2), $\Delta_1 - \Delta_2 = \delta_1 - \delta_2 \qquad (3) \qquad ...(2.5)$

Since T_1O_1 is $\perp$ to T_1B and O_1F is $\perp$ to T_1T_2 we have

$$\angle T_1O_1F = \angle BT_1F = \delta_1$$

Similarly, $\angle T_2\,O_2G = \angle FT_2D = \delta_2$

Hence $\quad \angle FO_1E = \Delta_1 - \delta_1$ and $\angle EO_2G = \Delta_2 - \delta_2$

Since O_1F and O_2G are parallel, we have

$$\angle FO_1E = EO_2G$$

or $\quad (\Delta_1 - \delta_1) = (\Delta_2 - \delta_2) \qquad (3a)$

which is the same as obtained in (3).

Again, $\quad T_1F = R_1 \sin \delta_1$

$T_2G = R_2 \sin \delta_2$

and $\quad FG = O_1H = O_1O_2 \sin (\Delta_2 - \delta_2) = (R_1 + R_2) \sin (\Delta_2 - \delta_2)$

Hence $\quad T_1T_2 = T_1F + T_2G + FG$

or $\quad T_1T_2 = R_1 \sin \delta_1 + R_2 \sin \delta_2 + (R_1 + R_2) \sin (\Delta_2 - \delta_2) \qquad (4) \qquad ...(2.6)$

Again, $\quad O_1F = HG = R_1 \cos \delta_1$.

$O_2G = R_2 \cos \delta_2$

$O_2H = O_1O_2 \cos (\Delta_2 - \delta_2) = (R_1 + R_2) \cos (\Delta_2 - \delta_2) = (R_1 + R_2) \cos (\Delta_1 - \delta_1)$

$O_2H = O_1F + O_2G$

or $\quad (R_1 + R_2) \cos (\Delta_2 - \delta_2) = R_1 \cos \delta_1 + R_2 \cos \delta_2$

or $$\cos (\Delta_2 - \delta_2) = \cos (\Delta_1 - \delta_1) = \frac{R_1 \cos \delta_1 + R_2 \cos \delta_2}{R_1 + R_2} \qquad (5) \qquad ...(2.7)$$

In the above treatment, it has been assumed that Δ_1 is greater than Δ_2 so that $\Delta = \Delta_1 - \Delta_2$. In general, however, $\Delta = \pm (\Delta_1 - \Delta_2)$ according as the point of intersection occurs before or after the reverse curve.

2.5. RELATIONSHIPS BETWEEN VARIOUS PARTS OF A REVERSE CURVE

The various quantities involved in a reverse curve are Δ, Δ_1, Δ_2, δ_1, δ_2, R_1 and R_2. In order to co-relate these, three quantities and one condition equation (of either equal radius or equal central angle) must be known. We shall consider various cases of common occurrence.

or $$L^2 + R_1^2 \sin^2 \delta_1 + R_2^2 \sin^2 \delta_2 + 2R_1 R_2 \sin \delta_1 \sin \delta_2 - 2L (R_1 \sin \delta_1 + R_2 \sin \delta_2)$$
$$= R_1^2 + R_2^2 + 2R_1 R_2 - (R_1^2 \cos^2 \delta_1 + R_2^2 \cos^2 \delta_2 + 2R_1 R_2 \cos \delta_1 \cos \delta_2)$$

or $$L^2 - 2L(R_1 \sin \delta_1 + R_2 \sin \delta_2) + R_1^2 (\sin^2 \delta_1 + \cos^2 \delta_1) + R_2^2 (\sin^2 \delta_2 + \cos^2 \delta_2)$$
$$= R_1^2 + R_2^2 + 2R_1 R_2 - 2R_1 R_2 \cos \delta_1 \cos \delta_2 - 2R_1 R_2 \sin \delta_1 \sin \delta_2$$

or $$L^2 - 2L (R_1 \sin \delta_1 + R_2 \sin \delta_2) = 2R_1 R_2 - 2R_1 R_2 \cos (\delta_1 - \delta_2)$$

or $$L^2 - 2L(R_1 \sin \delta_1 + R_2 \sin \delta_2) = 4R_1 R_2 \sin^2 \left(\frac{\delta_1 - \delta_2}{2} \right) \qquad \text{...(2.11)}$$

Knowing R_1 (or R_2), we can calculate R_2 (or R_1) from the above equation. The angle O_2O_1H (= θ) and hence Δ_1 and Δ_2 can then be calculated.

CASE 4. PARALLEL STRAIGHTS

Given. The two radii R_1 and R_2 and the central angles.

Required. To calculate various elements.

Condition Equation $\quad \Delta_1 = \Delta_2$

In Fig. 2.8, let AT_1 and T_2C be two straights parallel to each other so that there is no point of intersection.

Let $\quad R_1$ = smaller radius

R_2 = larger radius

Δ_1 = central angle corresponding to R_1

Δ_2 = central angle corresponding to R_2

L = distance T_1T_2

v = perpendicular distance between the two straights

h = distance between the perpendiculars at T_1 and T_2

E = point of reverse curvature.

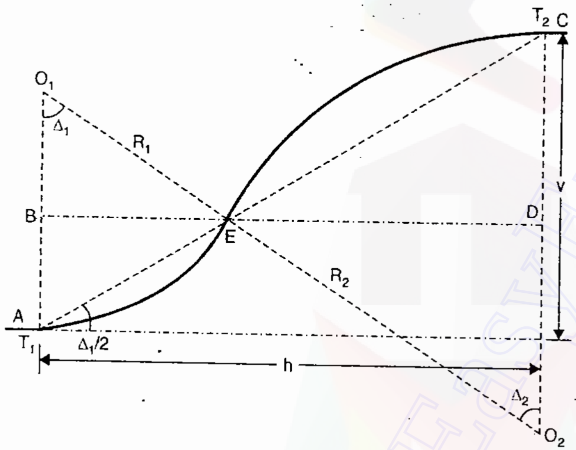

FIG. 2.8. REVERSE CURVE : PARALLEL TANGENTS.

Through E, draw a line BD parallel to the two tangents.

Since O_1T_1 and $O_2 T_2$ are parallel to each other, we have

$$\Delta_1 = \Delta_2$$
$$T_1B = O_1T_1 - O_1B = R_1 - R_1 \cos \Delta_1 = R_1(1 - \cos \Delta_1) = R_1 \text{ versin } \Delta_1$$
$$T_2D = O_2 T_2 - O_2D = R_2 - R_2 \cos \Delta_2 = R_2 - R_2 \cos \Delta_1$$
$$= R_2 (1 - \cos \Delta_1) = R_2 \text{ versin } \Delta_1$$
$$v = T_1B + DT_2 = R_1 \text{ versin } \Delta_1 + R_2 \text{ versin } \Delta_1$$
$$= (R_1 + R_2) \text{ versin } \Delta_1 = (R_1 + R_2)(1 - \cos \Delta_1) \qquad \text{...(2.12)}$$

Again, $\quad T_1E = 2 R_1 \sin \dfrac{\Delta_1}{2}$

$$T_2E = 2R_2 \sin \frac{\Delta_2}{2} = 2 R_1 \sin \frac{\Delta_1}{2}$$

$\therefore$ $$T_1T_2 = L = T_1E + ET_2 = 2R_1 \sin \frac{\Delta_1}{2} + 2R_2 \sin \frac{\Delta_1}{2} = 2 (R_1 + R_2) \sin \frac{\Delta_1}{2} \qquad \text{...(2.13)}$$

But $\quad \sin \dfrac{\Delta_1}{2} = \dfrac{v}{L}$

$\therefore$ $$L = 2(R_1 + R_2) \frac{v}{L}$$

From which, $$L = \sqrt{2v (R_1 + R_2)} \qquad \text{...(2.14)}$$

$$BE = R_1 \sin \Delta_1 \; ; \; ED = R_2 \sin \Delta_2 = R_2 \sin \Delta_1$$

$\therefore$ $$BD = h = (R_1 \sin \Delta_1 + R_2 \sin \Delta_1) \qquad \text{...(2.15)}$$
$$= (R_1 + R_2) \sin \Delta_1$$

Special case :

If $\quad R_1 = R_2 = R$, we have

$$v = 2R(1 - \cos \Delta_1) \qquad \text{...(2.12 a)}$$
$$L = 4 R \sin \frac{\Delta_1}{2} \qquad \text{...(2.13 a)}$$
$$L = \sqrt{4Rv} \qquad \text{...(2.14 a)}$$
$$h = 2R \sin \Delta_1 \qquad \text{...(2.15 a)}$$

Example 2.6. *Two parallel railway lines are to be connected by a reverse curve, each section having the same radius. If the lines are 12 meters apart and the maximum distance between tangent points measured parallel to the straights is 48 metres, find the maximum allowable radius.*

If however, both the radii are to be different, calculate the radius of the second branch if that of the first branch is 60 metres. Also, calculate the lengths of both the branches.

Solution. (Fig. 2.8)

(*a*) *Given* : $\quad h = 48$ m and $v = 12$ m

$$\tan \frac{\Delta_1}{2} = \frac{v}{h} = \frac{12}{48} = 0.25 \text{ m}$$

$\therefore$ $$\frac{\Delta_1}{2} = 14° \, 2' \text{ or } \Delta_1 = 28° \, 4'$$
$$\sin \Delta_1 = 0.47049$$

Now $\quad BE = R \sin \Delta_1$ and $ED = R \sin \Delta_1$

$\therefore$ $$BE + ED = h = R \sin \Delta_1 + R \sin \Delta_1 = 2R \sin \Delta_1$$

or $$R = \frac{h}{2 \sin \Delta_1} = \frac{48}{2 \times 0.47049} = \mathbf{51.1 \text{ m.}}$$

(*b*) Let R_1 and R_2 be the radii.

As calculated above, $\Delta_1 = 28° \, 4'$ and $\sin \Delta_1 = 0.47079$

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(*a*) Prepare a sketch giving all the distances necessary for pegging T_1, C and T_2 initially.

(*b*) Submit in a tabular form complete notes for setting out the curve by tangential angles, pegging, through chainages (*U.L.*)

4. What do you understand by the following forms of curves and where are they generally used ?

1. Lemniscate 2. Compound curve 3. Reverse curve

If in a compound curve the directions of two straights and one radius are known, how will you find out analytically the radius of the other curve ?

5. A railway siding is to be curved through a right angle. In order to avoid buildings, the curve is to be compound, the radius of the two branches being 8 chains and 12 chains. The distance from the intersection point of the end straights to the tangent point at which the arc of 8 chains radius leaves straight is to be 10.08 chains. Obtain the second tangent length, or distance from the intersection point to the other end of the curve, and the length of the whole curve. (*T.C.D.*)

6. A compound railway curve *ABC* is to have the radius of arc *AB* 600 metres and that of *BC* 400 metres. The intersection point *V* of the straights is located, and the intersection angle is observed to be 35° 6′. If the arc *AB* is to have a length of 200 metres, calculate the tangent distances *VA* and *VC*.

7. A curve of 300 m radius has been pegged out to connect two railway tangents having deflection angle = 15° 26′, and the chainage of the initial tangent point has been found to be 3841.7 metres. On further examination of the ground, it is decided to alter the radius to 450 metres. Calculate the chainage of the new initial and final tangent points, and the distances between the new and original curves at their mid-point.

ANSWERS

1. Chainage of P.C. = 153.845 chains.
   Chainage of P.C.C. = 166.227 chains.
   Chainage of P.T. = 171.250 chains.
2. R = 25.00 chains ; chainage of 1st tangent point = 138.188 ; chainage of P.R.C. = 151.859 ; chainage of 2nd tangent point = 169.602.
3. IA, 7.823 chains ; IB, 6.697 chains
   AT_1 = 7.068 chains ; BT_2 = 5.658 chains
   chainage of C, 166.638 chains ; T_2 = 179.046 chains.
5. 11.51 chains ; 16.85 chains
6. 176.3 m ; 145.7 m.
7. 3821.4 m; 3942.6 m ; 1.37 m.

3

Curve Surveying III : Transition Curves

3.1. GENERAL REQUIREMENTS

A *transition* or easement curve is a curve of varying radius introduced between a straight and a circular curve, or between two branches of a compound curve or reverse curve. In case of a highway, in order to hold the vehicle in the centre of the lane, the driver is required to move his steering almost instantly to the position necessary for the curve at the moment he passes the P.C. In doing so, the sudden impact of centrifugal force coupled with the inertia of the vehicle would cause the vehicle to sway outwards, and if this exceeds a certain value the vehicle may overturn. In case of railways, the side thrust is wholly taken by the pressure exerted by the rails on the flanges of the wheels thus causing wear of the rail in the region of the tangent point. To avoid these effects, a curve of changing radius must be introduced between the straight and the circular curve. The functions of a transition curve are :

(1) To accomplish gradually the transition from the tangent to the circular curve, so that the curvature is increased gradually from zero to a specified value.

(2) To provide a medium for the gradual introduction or change of the required super-elevation.

A transition curve introduced between the tangent and the circular curve should fulfil the following conditions :

(1) It should be tangential to the straight.

(2) It should meet the circular curve tangentially.

(3) Its curvature should be zero at the origin on straight.

(4) Its curvature at the junction with the circular curve should be the same as that of the circular curve.

(5) The rate of increase of curvature along the transition should be the same as that of increase of cant or super-elevation.

(6) Its length should be such that full cant or super-elevation is attained at the junction with the circular curve.

Super-elevation on Highways : Side Friction Factor

Side friction factor (f) is defined as the force transferred by friction parallel to the pavement per unit force normal to the pavement.

Consider Fig. 3.1 (b).

Let N = the sum of the forces normal to the pavement.

T = the sum of the forces parallel to the pavement transferred to it by friction.

f = side friction factor $= \frac{T}{N}$.

Resolving the forces P and W normal to the pavement,

$$N = P \sin\theta + W \cos\theta \qquad \ldots(i)$$

Resolving the forces P and W tangential to the pavement,

$$T = P \cos\theta - W \sin\theta \qquad \ldots(ii)$$

Now, $T = f N$

or $(P \cos\theta - W \sin\theta) = f(P \sin\theta + W \cos\theta)$

or $P(\cos\theta - f \sin\theta) = W(\sin\theta + f \cos\theta)$

or
$$\frac{P}{W} = \frac{(\sin\theta + f\cos\theta)}{\cos\theta - f\sin\theta} = \frac{\tan\theta + f}{1 - f\tan\theta}$$

But
$$\frac{P}{W} = \frac{v^2}{gR}$$

$\therefore$
$$\frac{v^2}{gR} = \frac{\tan\theta + f}{1 - f\tan\theta} \qquad \ldots(3.5)$$

Equation 3.5 represents an exact relationship between the various quantities involved and for the safe design of highways, the right hand side must be equal to or greater than the left hand side. The maximum value of f may be taken equal to 0.25 for average conditions.

It is evident from equation 3.5 that the centrifugal force is balanced by the sum of the effect of the super-elevation and the effect of friction. If the super-elevation is increased, more of the centrifugal force will be balanced by it and less friction will be required. It has yet not been agreed upon as to how much centrifugal force must be balanced by super-elevation and how much by side friction. There are two extreme methods :

(1) Method of maximum friction.

(2) Method of maximum super-elevation.

(1) Method of Maximum Friction

In this method, whole of the centrifugal force is balanced by the side friction till the maximum limit of the latter is reached. If the radius is still lesser, the rest of the centrifugal force is balanced by introducing the super-elevation. Thus, if R is the *minimum* radius for the standard velocity v on surface having no super-elevation or cant, side-slip will occur if the side thrust due to centrifugal force is greater than the adhesion between the tyres and the road surface. That is, if

$$\frac{W v^2}{gR} > f W$$

or
$$R = \frac{v^2}{fg} \qquad \ldots(3.6)$$

(The above equation could also be obtained by putting $\tan\theta = 0$ in Eq. 3.5)

If V' is in miles per hour, $g = 32.2$ ft/sec^2, and R' is in feet, we have

$$R' = \frac{V'^2}{14.97 f}$$

Taking average value of $f = 0.25$, we get

$$R' = 0.267\ V'^2 \qquad \ldots(3.6\ a)$$

Similarly, if V is in kilometer per hour, $g = 981$ cm/sec^2, R in metres and $f = 0.25$, we get

$$R = 0.03143\ V^2 \qquad \ldots(3.6\ b)$$

If R is to be provided lesser than that given by equation 3.6, super-elevation of appropriate value will have to be introduced till equation 3.5 is satisfied.

(2) Method of Maximum Super-elevation

In this method, whole of the centrifugal force is balanced by the super-elevation only till the maximum limit of the latter is reached. If the radius provided is still lesser, friction would be relied on to balance the rest of the centrifugal force. If R is the minimum radius for the standard velocity v, we have

$$\tan\theta = \frac{v^2}{gR} \qquad \ldots(3.1\ a)$$

or
$$R = \frac{v^2}{g\tan\theta} \qquad \ldots(3.7)$$

(The above equation could also be obtained by putting $f = 0$ in equation 3.5).

If R is to be provided lesser than that given by equation 3.7, friction would be relied on till equation 3.5 is satisfied.

3.2. LENGTH OF TRANSITION CURVE

The length of the transition curve should be such that the required super-elevation or cant is provided at a suitable rate. There are three methods for determining its length:

(a) First Method : By an Arbitrary Gradient

In this method, the super-elevation e is provided at an arbitray rate, say 1 in n. Then the length L of the transition curve is given by

$$L = ne \qquad \ldots(3.8)$$

The value of n may vary between 300 to 1200.

Let the rate of canting be 1 cm in n metres.

From equation 3.2 (b), $e = 1.18\,\frac{V^2}{R}$ cm where V in km/sec and R is in metres.

Substituting these values in equation 3.8, we get and $$L' = \frac{3.16}{R'}\left(\frac{R'}{0.267}\right)^{3/2} \approx 23\sqrt{R'} \text{ ft.} \quad \text{...(for roads)} \qquad \text{...(3.11 } b)$$

3.3. THE IDEAL TRANSITION CURVE : THE CLOTHOID

As stated earlier, the centrifugal force acting on a vehicle is given by

$$P = \frac{W v^2}{g r}$$

where r is the radius of curvature at any point on the curve.

If the centrifugal force P is to increase at a constant rate, P must vary with time. Again if the speed of the vehicle is constant, the distance l along the transition curve measured from the tangent point must varv with time. Hence, we have

$$P \propto l \propto \frac{W v^2}{g r}$$

But W, v and g are all constants.

Hence $$l \propto \frac{1}{r} \qquad \text{...(3.12)}$$

or $$l\, r = \text{constant} = LR$$

where L = total length of the curve, upto its end

R = radius of the curve at its end (*i.e*, minimum radius)

Also the cant from the equilibrium point of view is given by

$$e = 1.18\, \frac{V^2}{r} \qquad \text{(Eq. 3.2 } b)$$

where r is the radius of the curve.

If e is to increase at a contant rate, it is proportional to l.

$\therefore$ $$e \propto l \propto 1.18\,\frac{V^2}{r} \quad \text{or} \quad l \propto \frac{1}{r} \qquad \text{...(3.12)}$$

or $$l\, r = \text{constant} = LR.$$

Thus, the fundamental requirement of a transition curve is that its radius of curvature r at any point shall vary inversely as the distance (l) from the beginning of the curve. Such a curve is the *Clothoid or the Glover's sprial* and is known as the *ideal transition curve.*

Let T = tangent point = beginning of the transition cuve

TA = initial tangent

D = Point of junction of the transition and circular curve

B = any point on the curve at distance l along the curve

r = radius of the curve at any point B

ϕ = the inclination of the tangent to the transition curve at B to the initial tangent TA = deviation angle

Δ_S = spiral angle = the angle between the initial tangent and the tangent to the transition curve at the junction point D

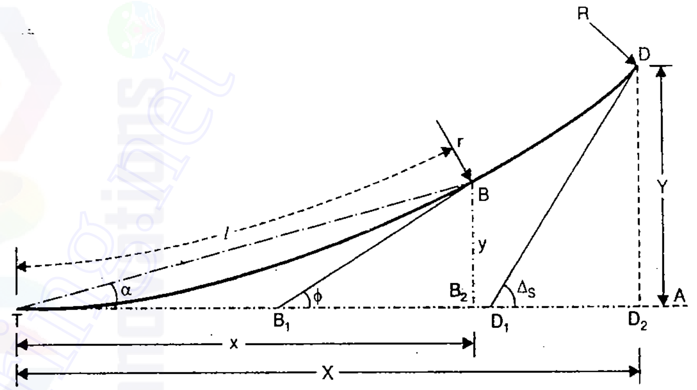

FIG. 3.2. THE IDEAL TRANSITION CURVE.

l = length of the curve from T to B

R = radius of the circular curve

L = total length of the transition curve

X = the x co-ordinate of D

Y = the y co-ordinate of D

$x = TB_2$ = x-co-ordinate of any point B on the transition curve

$y = BB_2$ = y-co-ordinate to any point B on the transition curve

We have $$l \,.\, r = L \,.\, R = \text{constant}$$

or $$\frac{1}{r} = \frac{l}{RL}$$

But $$\frac{1}{r} = \text{curvature} = \frac{d\phi}{dl}$$

$\therefore$ $$\frac{d\phi}{dl} = \frac{l}{RL}$$

or $$d\phi = \frac{l}{RL}\,.\, dl$$

Integrating, we get $$\phi = \frac{l^2}{2\,RL} + C$$

When $l = 0$, $\phi = 0$

$\therefore$ $C = 0$

Hence $$\phi = \frac{l^2}{2RL} \qquad \text{...(3.13)}$$

This is the intrinsic equation of the ideal transition curve.

Equation 3.13 can also be expressed in the form

$$l = \sqrt{2RL\phi} = K\sqrt{\phi} \qquad \text{...(3.13 } a)$$

This very closely resembles the expression

$$\tan\frac{\phi}{3}=\frac{\phi}{3}+\frac{\phi^3}{81}+\frac{\phi^5}{18225}\ ; \qquad \text{Hence}\quad \tan\alpha\approx\tan\frac{\phi}{3}$$

Since ϕ is very small (usually a small friction of a radian)

$$\alpha=\frac{\phi}{3} \qquad \text{...(3.16)}$$

$$=\frac{1}{3}\cdot\frac{l^2}{2RL}=\frac{l^2}{6RL}\ \text{radians} \qquad \text{...(3.16 } a)$$

$$=\frac{l^2}{6RL}\cdot\frac{180}{\pi}\times 60=\frac{1800\,l^2}{\pi RL}\ \text{minutes} \qquad \text{...(3.16 } b)$$

Accurate Relation between α and ϕ

We have
$$\tan\alpha=\frac{\phi}{3}+\frac{\phi^3}{105}+\frac{\phi^5}{5997}\ \ldots\ldots$$

Since
$$\alpha=\tan\alpha-\frac{\tan^3\alpha}{3}+\frac{\tan^5\alpha}{5}\ \ldots\ldots$$

it can be shown that
$$\alpha=\frac{\phi}{3}-\frac{8\phi^3}{2835}-\frac{32\phi^5}{467775}\ \ldots\ldots$$

This can be expressed as

$$\alpha=\frac{\phi}{3}-\delta \qquad \text{...(3.16 } c)$$

where
$$\delta=3.095\times10^{-3}\,\phi^3+2.285\times10^{-8}\,\phi^5 \qquad \text{...(3.16 } d)$$

(ϕ being in degrees and δ in seconds)

If it is required to find the value of α when ϕ is known, ϕ should be divided by 3 and a small correction δ, given in the Table 3.1 should be subtracted.

TABLE 3.1. VALUES OF δ

$\phi°$	δ ′	δ ″	$\phi°$	δ ′	δ ″	$\phi°$	δ ′	δ ″
1	0	0	16	0	13	31	1	32
2	0	0	17	0	15	32	1	41
3	0	0	18	0	18	33	1	51
4	0	0	19	0	21	34	2	2
5	0	0	20	0	25	35	2	13
6	0	0	21	0	29	36	2	24
7	0	1	22	0	35	37	2	37
8	0	2	23	0	38	38	2	50
9	0	2	24	0	43	39	3	4
10	0	3	25	0	48	40	3	18
11	0	4	26	0	54	41	3	33
12	0	5	27	1	1	42	3	49
13	0	7	28	1	8	43	4	6
14	0	8	29	1	16	44	4	24
15	0	10	30	1	25	45	4	42

MODIFICATION OF THE IDEAL TRANSITION CURVE : THE CUBIC SPIRAL

Neglecting all the terms of equation 3.15 *b*, except the first one, we get

$$y=\frac{l^3}{6RL} \qquad \text{...(3.17)}$$

which is the equation of the cubic spiral.

The approximation made here is

$$\sin\phi=\phi$$

or
$$\frac{dy}{dl}=\sin\phi=\phi=\frac{l^2}{2RL} \qquad \text{(From Eq. 3.13)}$$

$\therefore$
$$dy=\frac{l^2}{2RL}\cdot dl$$

Integrating,
$$y=\frac{l^3}{6RL}+c$$

where c = constant of integration = 0, since $y = 0$ when $l = 0$

$$\therefore \qquad y=\frac{l^3}{6RL} \qquad \text{...(3.17)}$$

The cubic spiral is set out by chords and offsets from the initial tangent. If, however, the curve is set out by deflection angles, we get

$$\alpha=\frac{\phi}{3}=\frac{l^2}{6RL}\ \text{radians.}$$

THE CUBIC PARABOLA

Neglecting all the terms of equation 3.14 except the first one, we get

$$x=l \qquad \text{...}(i)$$

Similarly, from equation 3.15 *b*, we have

$$y=\frac{l^3}{6RL} \qquad \text{...}(ii)$$

From (*i*) and (*ii*), we get

$$y=\frac{x^3}{6RL} \qquad \text{...(3.18)}$$

This is equation of the cubic parabola, which is also known as *Froude's transition curve*. The use of both the cartesian co-ordinates are made in setting out the curve.

The approximation $x = l$ corresponds to the assumption $\cos\phi = 1$. Thus, in cubic parabola two approximations are made, *viz*, $\cos\phi = 1$ and $\sin\phi = \phi$, while in cubic spiral, only one approximation *viz.*, $\sin\phi=\phi$, is made. Since the cosine series is less rapidly converging than the sine series, greater error is involved in the approximation $\cos\phi = 1$ than involved in the approximation $\sin\phi = \phi$. *Hence a cubic spiral is superior to a cubic parabola.* However, the cubic parabola is the most widely used transition curve owing to the ease with which it may be set out by rectangular co-ordinates.

OB = perpendicular to the shift tangent at B

A = point of intersection of the perpendicular OB with the original tangent

DE = line perpendicular to OA

Since the tangent DD_1 makes an angle Δ_S with the original tangent, $\angle BOD = \Delta_S$.

Now, arc $BD = R\Delta_S = R\dfrac{L}{2R} = \dfrac{L}{2}$, since $\Delta_S = \dfrac{L}{2R}$ from Eq. 3.13 c ...(i)

When CD is very nearly equal to BD, we have

$$CD = \frac{L}{2} \quad \ldots(3.20)$$

Hence the shift AB bisects the transition curve at C.

Again, $s = BA = EA - EB = Y - (OB - OE) = Y - R(1 - \cos \Delta_S)$

$$= Y - 2R \sin^2 \frac{\Delta_S}{2} = Y - 2R\frac{\Delta_S^2}{4}, \text{ where } \Delta_S \text{ is small.}$$

But $EA = DD_2 = Y = \dfrac{L^3}{6RL} = \dfrac{L^2}{6R}$ and $\Delta_S = \dfrac{L}{2R}$

$$\therefore \quad s = \frac{L^2}{6R} - \frac{2R}{4}\left(\frac{L}{2R}\right)^2$$

or

$$s = \frac{L^2}{6R} - \frac{L^2}{8R} = \frac{L^2}{24\,R} \quad \ldots(3.21)$$

Also, $CA = y$ co-ordinate of C when $l = \dfrac{L}{2}$

$$= \frac{l^3}{6RL} = \frac{\left(\frac{L}{2}\right)^3}{6RL} = \frac{L^2}{48R} = \frac{1}{2}s = \frac{1}{2}\cdot BA \quad \ldots(3.22)$$

Hence the transition curve bisects the shift.

Precise expression for shift (s)

From equation 3.15 (a), we have

$$Y = EA = K\left(\frac{\Delta_S^{3/2}}{3} - \frac{\Delta_S^{7/2}}{42} + \frac{\Delta_S^{11/12}}{1320} - \ldots\right)$$

Also, $EB = R(1 - \cos \Delta_S)$ where $\Delta_S = L/2R$ radians

Now, $s = EA - EB$

Substituting the values and expanding $\cos \Delta_S$, we get

$$s = \frac{L^2}{24R}\left(1 - \frac{\Delta_S^2}{48} + \frac{\Delta_S^4}{1320} - \ldots\right) \text{ where } \Delta_S \text{ is in radians.} \quad \ldots(3.21\ a)$$

When Δ_S is in degrees, the above expression reduces to

$$s = \frac{L^2}{24R}(1 - 1.08792 \times 10^{-5}\,\Delta_S^2 + 7.03 \times 10^{-11}\,\Delta_S^4 - \ldots) \quad \ldots(3.21\ b)$$

or

$$s = \frac{L^2}{24R}(1 - U) \quad \ldots(3.21\ c)$$

The values of U for different values of Δ_S are given in the Table 3.2.

TABLE 3.2. VALUES OF U FOR DIFFERENT VALUES OF Δ_S

Δ_S	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
0°	0.00000	000	000	000	000	000	000	001	001	001
1	0.00001	001	002	002	002	002	003	003	004	004
2	0.00004	005	005	006	006	007	007	008	009	009
3	0.00010	010	011	012	013	013	014	015	016	017
4	0.00017	018	020	020	021	022	023	024	025	026
5	0.00027	028	029	031	032	033	034	035	037	038
6	0.00039	040	042	043	045	046	047	049	050	052
7	0.00053	055	056	058	060	061	063	065	066	068
8	0.00070	071	073	075	077	079	080	082	084	086
9	0.00088	090	092	094	095	098	100	102	104	107
10	0.00109	111	113	115	118	120	122	125	127	129
11	0.00131	134	136	139	141	144	146	149	152	154
12	0.00157	159	162	164	167	170	173	175	178	181
13	0.00184	186	189	192	195	198	201	204	207	210
14	0.00213	216	219	222	225	229	232	235	238	241
15	0.00224	247	251	254	258	261	264	268	271	275
16	0.00278	282	285	289	292	296	299	203	307	310
17	0.00314	318	321	325	329	333	336	340	344	348
18	0.00352	356	260	364	368	372	376	380	384	388
19	0.00392	396	400	404	408	413	418	421	425	430
20	0.00434	438	442	447	451	456	460	465	469	474
21	0.00478	483	487	492	497	501	506	511	515	520
22	0.00525	530	535	539	544	549	554	559	563	568
23	0.00573	578	583	589	594	599	604	609	614	619
24	0.00624	629	634	640	645	650	665	661	666	672
25	0.00677	682	688	693	699	704	710	716	721	727
26	0,00732	738	744	749	755	761	766	772	778	784
27	0.00789	795	801	807	813	819	825	831	837	843
28	0.00849	855	861	867	873	879	885	891	898	904
29	0.00910	916	923	929	935	941	948	954	961	967
30	0.00973	980	986	993	999	$\overline{006}$	$\overline{013}$	$\overline{019}$	$\overline{026}$	$\overline{032}$
31	0.01039	046	052	059	066	073	079	086	093	100
32	0.01107	114	120	128	134	141	148	155	162	169
33	0.01177	184	191	198	205	212	219	227	234	241
34	0.01248	256	263	270	278	285	292	300	307	315
35	0.01322	330	337	344	352	360	368	375	383	390
36	0.01398	406	414	421	429	437	445	453	461	468
37	0.01476	484	492	500	408	516	524	532	540	548
38	0.01556	564	573	581	589	597	605	613	621	630
39	0.01639	647	655	644	672	681	689	697	706	714
40	0.01723	731	740	748	757	766	774	783	792	800
41	0.01809	818	827	835	844	853	862	871	880	889
42	0.01897	906	915	924	933	882	951	960	061	978
43	0.01988	997	$\overline{006}$	$\overline{015}$	$\overline{024}$	$\overline{034}$	$\overline{043}$	$\overline{052}$	$\overline{061}$	$\overline{071}$
44	0.02080	089	009	108	117	127	136	146	155	165
45	0.02174									

LENGTH OF LONG CHORD

To set out the curve on the ground, it is often necessary to know the length (C) of the long chord TD (Fig. 3.4), joining ends of the transition curve.

Evidently $\quad C = \sqrt{X^2 + Y^2}$

THE CUBIC SPIRAL

The co-ordinates of any point B are represented by

$$y = \frac{l^3}{6RL} \quad \text{where } l \text{ is measured along the curve.}$$

For the junction point D, $l = L$

$$\therefore \qquad Y = \frac{L^3}{6RL} = \frac{l^2}{6R}$$

The intrinsic equation of the curve is

$$\phi = \frac{l^2}{2RL}$$

$$\alpha = \frac{\phi}{3} = \frac{l^2}{6RL} \text{ radians} = \frac{1800\, l^2}{\pi RL} \text{ minutes} = \frac{573\, l^2}{RL} \text{ minutes}$$

$$\alpha_S = \frac{573\, L}{R} \text{ minutes}$$

Total tangent length $TV = AV + TA = (R + s) \tan \frac{\Delta}{2} + \frac{L}{2}\left(1 - \frac{s}{5R}\right)$

If, however Δ_S is very small, the total tangent length may be taken approximetly equal to $(R + s) \tan \frac{\Delta}{2} + \frac{L}{2}$.

THE CUBIC PARABOLA

The co-ordinates of any point B are respresented by

$$y = \frac{x^3}{6RL} \quad \text{where } x \text{ and } y \text{ are cartesian co-ordinates}$$

$$\tan \alpha = \frac{y}{x} = \frac{x^2}{6RL} = \frac{l^2}{6RL}$$

$$\alpha = \frac{l^2}{6RL} \text{ radians} = \frac{1800\, l^2}{\pi LR} \text{ minutes} = \frac{573\, l^2}{RL} \text{ minutes}$$

$$\alpha_S = \frac{1800\, L}{\pi R} = \frac{573\, L}{R} \text{ minutes}$$

$$\Delta_S = \frac{L}{2R} \text{ radians} = \frac{1719\, L}{R} \text{ minutes}$$

The co-ordinates of the junction point D are

$$X = L$$

and

$$Y = \frac{L^2}{6R} = 4s$$

Total tangent length $= AV + TA = (R + s) \tan \frac{\Delta}{2} + (X - R \sin \Delta_S)$

But $\qquad X = L$ and $\sin \Delta_S \approxeq \Delta_S = \frac{L}{2R}$ radians.

$$\therefore \text{ Total tangent length} = (R + s) \tan \frac{\Delta}{2} + \left(L - R\,\frac{L}{2R}\right) = (R + s) \tan \frac{\Delta}{2} + \frac{L}{2} \qquad \ldots(3.24)$$

Length of the Combined Curve

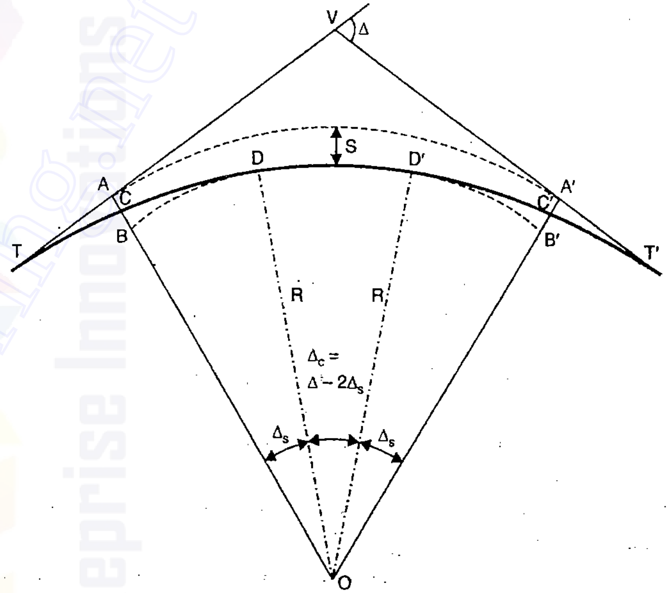

FIG. 3.5. THE COMBINED CURVE

In Fig. 3.5, DD' is the circular curve, and TD and $D'T'$ are the two transition curves at the two ends of the circular curve. If Δ is the total deflection angle between the original tangents and Δ_S is the spiral angle for each transition curve, we have

$$\Delta_C = \Delta - 2\Delta_S \quad \text{where} \quad \Delta_C = \text{central angle for the circular curve.}$$

Hence, the length of the circular curve

$$= \frac{\pi R \Delta_C}{180°} = \frac{\pi R (\Delta - 2\Delta_S)}{180°}$$

Total length of the combined curve

$$= \frac{\pi R (\Delta - 2\Delta_S)}{180°} + 2L \qquad \ldots(3.25)$$

The total length of the combined curve can also be approximately found by considering the circular arc $BDD'B'$ having the total central angle Δ.

$$\text{Total length} = \frac{\pi R \Delta}{180°} + \frac{L}{2} + \frac{L}{2} = \frac{\pi R \Delta}{180°} + L \qquad \ldots(3.25\ a)$$

3.5. COMPUTATIONS AND SETTING OUT

In order to make the computations for various quantities of the transition and circular curves, the data necessary are : (*i*) the deflection angle Δ between the original tangents, (*ii*) the radius R of the circular curve, (*iii*) the length L of the transition curve and (*iv*) the chainage of the point of the intersection (V).

the position of D by measuring the offset $DD_2 = \frac{L^2}{6R} = 4s$.

(7) To set out the circular curve, shift the theodolite to the junction point D. To orient the theodolite with reference to the common tangent DD_1, direct the line of sight towards DT with the reading equal to $(360° - \frac{2}{3}\Delta_S)$ for a right hand curve. Since $\angle DTV = \frac{1}{3}\angle DD_1V = \frac{1}{3}\Delta_S$, we have $\angle D_1DT = \frac{2}{3}\Delta_S$. When the theodolite is rotated in azimuth by an angle $\frac{2}{3}\Delta_S$ (till zero reading is obtained on the circle), the line of sight will be directed along DD_1. On transiting the theodolite now, the line of sight is directed along the tangent D_1D with reference to which the deflection angles of the circular curve have been calculated. When the line of sight is thus correctly oriented, the reading on the circle will be zero. To locate the first peg on the circular curve, the first deflection angle Δ_1 is set out on the curve as usual.

Set out the circular curve in the usual way till the junction point D' is reached, the position of which may be checked by measuring the offset $(= 4s)$ to the second tangent at the point.

(8) Set out the other transition curve from T' as before.

SETTING OUT BY TANGENT OFFSETS (Fig. 3.3)

(1) Locate the tangents point T as explained above and obtain its chainages.

(2) Calculate the offset y from the expression

$$y = \frac{l^3}{6RL}.$$

(3) Locate each peg by swinging the chord length from the preceding peg until required offset is obtained.

Cubic Parabola

(1) Locate the tangent point T as explained.

(2) Choose convenient values of co-ordinates x and calculate the corresponding values of y from the equation

$$y = \frac{x^3}{6RL}.$$

(3) Measure the abscissae (x) along the tangent TV and locate the points on the curve by setting out the respective offsets (y).

SETTING OUT BY FIXED ANGLES OF EQUAL CHORDS

Sometimes, it is not necessary to drive the pegs at even chainages along the transition curve. In that case, the calculations are simplified by using a fixed set of angles and calculating the corresponding length of chords required.

From equation 3.13, we have

$$\phi = \frac{l^2}{2RL} \qquad \ldots(i)$$

At $\qquad l = L, \quad \phi = \Delta_S$

$$\therefore \qquad \Delta_S = \frac{L^2}{2RL} \qquad \ldots(ii)$$

Dividing (i) and (ii), we have

$$\frac{\phi}{\Delta_S} = \frac{l^2}{L^2}$$

or
$$l = L\sqrt{\frac{\phi}{\Delta_S}} \qquad \ldots(3.26)$$

Let the first deviation angle be equal to ϕ_1 and the corresponding value of the chord $= l = c$. If after setting out n chords (each of length c), the deviation angle is ϕ_n, we have

$$l = nc = L\sqrt{\frac{\phi_n}{\Delta_S}}$$

or
$$\phi_n = \frac{\Delta_S\, n^2c^2}{L^2}$$

$$\therefore \qquad \phi_1 = \frac{\Delta_S\, c^2}{L^2}$$

Hence
$$\phi_n = n^2\,\phi_1 \qquad \ldots(3.27)$$

Thus, the length c for the first deviation angle can be calculated from the equation 3.26. For the equal chords, the subsequent deviation angles can calculated from equation 3.27. For example,

$$\phi_2 = n^2\phi_1 = (2)^2\,\phi_1 = 4\,\phi_1$$

$$\phi_3 = (3)^2\phi_1 = 9\,\phi_1$$

$$\phi_4 = (4)^2\,\phi_1 = 16\,\phi_1$$

and so on.

The last deviation angle will be Δ_S corresponding to a total length of $L = mc + c'$, where m = total number of chords each of length c and c' is the last sub-chord. Since the polar deflection angle $\alpha = \frac{1}{3}\phi$, we have

$$\alpha_1 = \frac{1}{3}\phi_1$$

$$\alpha_2 = \frac{1}{3}\phi_2 = \frac{4}{3}\phi_1$$

$$\alpha_3 = \frac{1}{3}\phi_3 = \frac{9}{3}\phi_1$$

$$\alpha_4 = \frac{1}{3}\phi_4 = \frac{16}{3}\phi_1$$

and so on.

With the help of Table 3.1, Table 3.4 can be prepared giving the polar deflection angles for equal chords, the chords length being selected so that first angle is $\alpha_1 = 1'$ (or $\phi_1 = 3'$).

(6) The first and the last transition curves and the two branches of the circular curves can be set out as explained earler.

(7) The offsets for the intermediate or common transition curve can be approximately calculated from the equation

$$y = \frac{4(s_1 - s_2)}{L^3} x^3.$$

(8) Locate the points G and G' (*i.e.*, the points in which the intermediate transition curve meet the two arcs) by setting out $\frac{L'}{2}$ from F_2 in each direction.

3.7. SPIRALLING REVERSE CURVES

In the case of reverse curve, the amount and *direction* of curvature changes from one value to the other and hence a reverse transition curve, as shown in Fig. 3.8, should be inserted between the two branches. The procedure for calculations is similar to that for a compound curve.

Let e_1 and e_2 be the required super-elevations calculated from the expression.

$$e = \frac{v^2}{gR} G$$

Greatest change of cant $= (e_1 + e_2)$

If n = rate of canting, the length L' of the reverse transition curve is given by

$$L' = n(e_1 + e_2).$$

Half of this will be provided to each side of P.R.C.

The distance EG between the tangents of the two shifted arcs $= GF + FE = s_1 + s_2$.

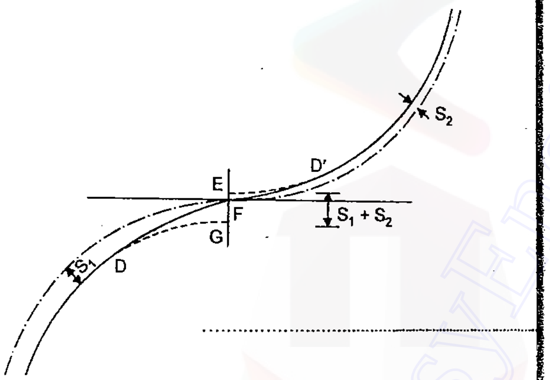

FIG. 3.8. SPIRALLING REVERSE CURVE.

The offsets to the transition curve may be calculated from the expression :

$$y = \frac{4(s_1 + s_2)}{L'^3} x^3.$$

3.8. BERNOULLI'S LEMNISCATE CURVE

Bernoulli's Lemniscate is commonly used in road work where it is required to have the *curve transitional throughout* having no intermediate circular curve. Since the curve is symmetrical and transitional throughout, the super-elevation or cant continuously increases till the apex is reached. This may, sometimes, be objectionable specially in railways. However, on highways, it is used in perference to the spiral for the following reasons :

(1) Its radius of curvature decreases more gradually.

(2) Its rate of increase of curvature diminishes towards the transition curve — thus fulfiling an essential condition.

(3) It corresponds to an autogenous curve of an automobile (*i.e.*, the path actually traced by an automobile when turning freely).

In Fig. 3.9, curve OA is the lemniscate, OB the clothoid and OC the cubic parabola. For small deviation angles (upto say 12°) there is little difference between the three, but for large angles, the cubic parabola leaves the other two curves ; its radius of curvature reaches a minimum value when $\phi \approx 24° \, 6'$ and starts to increase again. The clothoid and lemniscate are almost identical upto deviation angle of 60°, but after that the radius of curvature of lemniscate is greater than that of the clothoid. At deviation angle of 135°, the radius of curvature of the lemniscate is minimum and at a greater deviation angle it begins to increase again.

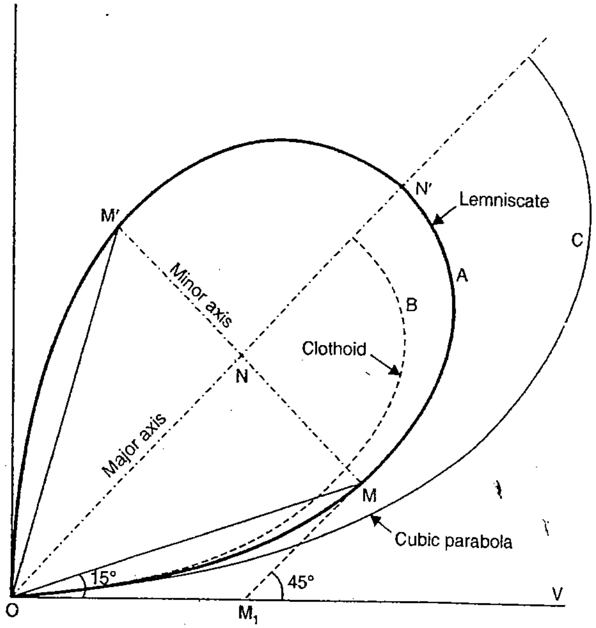

FIG. 3.9. VARIOUS TRANSITION CURVES.

Fig. 3.10 shows half the lemniscate curve in the first quadrant.

OV = initial tangent

OA = the major axis of the curve (or the polar ray making a polar deflection angle of 45° with the tangent)

P = any point on the curve

PP_1 = tangent to the curve at P

ϕ = angle between the tangent to the curve at P and the initial tangent = deviation angle

b = length OP of the polar ray

α = polar deflection angle

θ = angle between the polar ray PO and the tangent PP_1 to the curve at P.

The polar equation of Bernoulli's Lemniscate is

$$b = K \sqrt{\sin 2\alpha} \quad \ldots(3.28)$$

From the properties of polar co-ordinates,

$$\tan \theta = b \frac{d\alpha}{db}$$

But $\quad \dfrac{db}{d\alpha} = \dfrac{K \cos 2\alpha}{\sqrt{\sin 2\alpha}}$ from Eq. 3.28.

$$r_{min} = \frac{K}{3} = \frac{1}{3}(ON') = \frac{1}{3} \text{ major axis}$$

Length $\quad OMN' = 1.31115\ ON' = 1.31115\ K.$

Lemnisate Curve used Transitional Throughout

Fig. 3.11 shows the lemniscate curve used transitional throught out. Let T_1 and T_2 = tangent points.

M = apex of the curve

V = P.I.

Δ = total deflection angle of the tangents

VM = the apex distance

AMB = common tangent to the two branches of the lemniscate

$\angle VAM = \phi$ for the polar ray T_1M

α_n = polar deflection angle for T_1M.

Curve T_1M and MT_2 are two lemniscates symmetrical about VM. The curve is transitional throughout having no circular curve between the two branches. VM is the bisector of $\angle AVB$ and is common normal to the common tangent AMB.

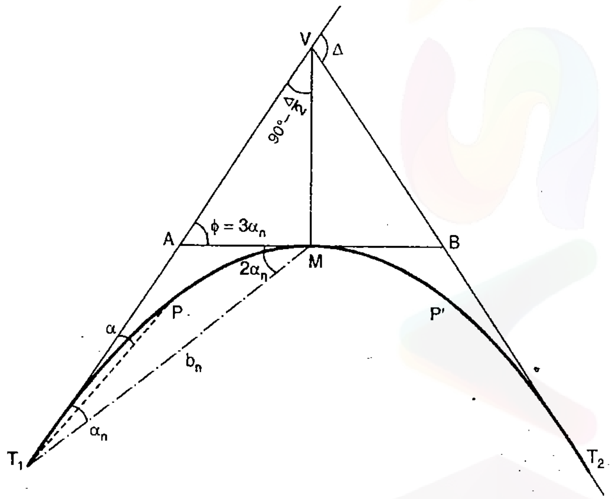

FIG. 3.11. LEMNISCATE CURVE TRANSITIONAL THROUGHOUT.

$$\angle AVB = (180° - \Delta)$$

$$\therefore \quad \angle AVM = \frac{1}{2}(180° - \Delta) = 90° - \frac{\Delta}{2}$$

$$\angle VMA = 90°$$

$$\angle VAM = \phi_n = 180° - \frac{1}{2}(180° - \Delta) = 90° - \frac{\Delta}{2}$$

But $\quad \phi_n = 3\,\alpha_n$

$$\therefore \quad \alpha_n = \frac{1}{3}\phi_n = \frac{1}{3}\frac{\Delta}{2} = \frac{\Delta}{6} \qquad ...(3.34)$$

Hence, for the curve to be transitional throughout, the maximum polar deflection angle must be equal to $\frac{1}{6}th$ of the deflection angle between the initial tangents.

Now, consider triangle T_1VM

$$\angle T_1VM = \frac{1}{2}(180° - \Delta) = 90° - \frac{\Delta}{2}$$

$$\angle T_1MV = 90° + 2\alpha_n = 90° + \frac{\Delta}{3}.$$

$$\angle VT_1M = \alpha_n = \frac{\Delta}{6}$$

Thus, all the three angles are known.

(*i*) *If the apex distance OM and the angle Δ are given*, the other two sides T_1V and T_1M can be calculated by sine rule. Knowing the tangent length $T_1V(= VT_2)$ the tangent points T_1 and T_2 can be located and the curve can be set out.

(*ii*) *If the minimum radius at end (M) and the angle Δ are given*, the length of the polar ray T_1M can be calculated from the equation 3.31, *i.e.*

$$b_n = 3\,r \sin 2\,\alpha_n$$

Knowing $T_1M = b_n$, the lengths T_1V and VM can be calculated by the sine rule. The tangent points T_1 and T_2 can then be located and the two branches can be set out by theodolite from T_1 and T_2.

For setting out the curve by deflection angles, a table giving various values of α and b may be prepared by assuming successive values of α and then calculating the values of b from the relation.

$$b = K\sqrt{\sin 2\,\alpha}.$$

Lemniscate as Transition Curve at the Ends of Circular Curve:

We have seen that for the lemniscate to be transitional throughout, the polar deflection angle should be $\frac{1}{6}$th of the deflection angle between the tangents. If, however, α_n is lesser than $\frac{1}{6}\Delta$, it is necessary to introduce circular curve between the two lemniscate curves.

Fig. 3.12 shows lemniscate curve TD, used as a transition at the beginning of a circular curve DD', D being the junction point where the lemniscate meets the curve tangentially.

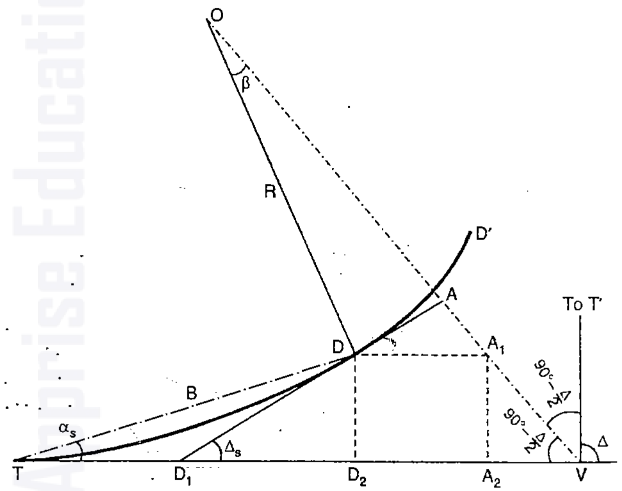

FIG. 3.12

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Shift $s = \frac{L^2}{24\,R} = \frac{(227)^2}{24(315)} = 6.82$ m

Total tangent length $= (R + s) \tan \Delta + \frac{L}{2} = (315 + 6.82) \tan 80° + \frac{227}{2} = 1938.6$ m.

Chainage of P.I.	= 42862.0
Deduct tangent length	= 1938.6
Chainage of T_1	= 40923.4
Add length of transition curve	= 227.0
Chainage of junction	= 41150.4
Add length of circular curve	= 212.8
Chainage of the other junction	= 41363.2
Add length of transition	= 227.0
Chainage of T_2	= 41590.2

Example 3.3. *Two Clothoid spirals for a road transition between two straights meet at a common tangent point. If the deflection angle between the straights is 30°, the chainage of P.I. 6387 metres and the maximum speed 120 km per hour, calculate the chainage of the tangent points and the point of compound curvature. The curve may be designed on the basis of comfort condition of centrifugal ratio (safety condition).*

Take $\alpha = 0.4$ *m/sec²/sec.*

Solution.

(*a*) *For the comfort condition*

$$L = \frac{v^3}{\alpha R} \qquad \text{But} \qquad L = 2R\Delta_S$$

$$\therefore \quad 2R\Delta_S = \frac{v^3}{\alpha R}$$

or $$R = \sqrt{\frac{v^3}{2\alpha\Delta_S}}$$

Here $$2\Delta_S = 30°$$

$$\Delta_S = \frac{30}{2} \times \frac{\pi}{180} = \frac{\pi}{12} \text{ radians}$$

$$V = 120 \text{ km per hour}$$

$$v = \frac{120 \times 1000}{3600} = \frac{100}{3} \text{ m/sec} \; ; \; \alpha = 0.4 \text{ m/sec}^3$$

$$R = \sqrt{\left(\frac{100}{3}\right)^3 \times \frac{1}{2 \times 0.4} \frac{12}{\pi}} = 420.5 \text{ m}$$

$$L = 2\,R\,\Delta_S = 2 \times 420.5\,\frac{\pi}{12} = 220.2 \text{ m.}$$

(*b*) *For the centrifugal ratio*

$$\frac{v^2}{gR} = \frac{1}{4}$$

or $$R = \frac{4v^2}{g} = \frac{4}{9.81}\left(\frac{100}{3}\right)^2 = 453.1 \text{ m}$$

$$L = 2R\,\Delta_S = 237.3 \text{ m}$$

The curve may, therefore, be designed on the safety condition having $L = 237.3$ m and $R = 453.1$ m = minimum radius of curvature for safety condition.

If X and Y are the co-ordinates at the end of the first clothoid, the tangent length is equal to $X + Y \tan \frac{\Delta_S}{2}$.

But $$X = L\left(1 - \frac{L^2}{40\,R^2}\right) = 237.3\left\{1 - \frac{(237.3)^2}{40\,(453.1)^2}\right\} = 235.66 \text{ m}$$

and $$Y = \frac{L^2}{6R}\left(1 - \frac{L^2}{56R^2}\right) = \frac{(237.3)^2}{6 \times 453.1}\left\{1 - \frac{(237.3)^2}{56(453.1)^2}\right\} = 20.7 \text{ m.}$$

$\therefore$ Total tangent length $= 235.66 + 20.7 \tan \frac{150}{2} = 238.3$ m

Chainage of P.I.	= 6387 metres
Subtract tangent length	= 238.3
Chainage of P.C.	= 6148.7
Add length of transition curve	= 237.3
Chainage of junction with the other clothoid	= 6386.0
Add length of transition curve	= 237.3
Chainage of P.T.	= 6623.3 metres.

Example 3.4. *A circular curve of 1000 m radius deflects through an angle of 40°. This curve is to be replaced by one of smaller radius so as to admit transition 200 m long at each end. The deviation of the new curve from the old at their mid-point is 1 m towards the intersection point.*

Determine the amended radius assuming that the shift can be calculated with sufficient accuracy on the old radius. Calculate the lengths of track to be lifted and of new track to be laid.

Solution.

In Fig. 3.13, let $T_1E_1\,T_1'$ be the old curve with radius R_1 and centre O_1. Let TET' be the new curve with TD and $D'T'$ as the transitions and DED' as the circular arc of radius R and centre O. Evidently AC is the shift of the new curve.

Spiral angle, $\Delta_S = \frac{L}{2R} \times \frac{180°}{\pi} = \frac{3}{2 \times 20} \times \frac{180°}{\pi} = 4° \ 17'.8.$

Central angle for the circular arc $= \Delta_C = \Delta - 2\Delta_s$

$= 40° \ 30' - 8° \ 35'.6 = 31° \ 54'.4$

Length of the circular arc $= \frac{\pi \ R \ \Delta_C}{180°} = \frac{\pi(20) \ 31° \ 54'.4}{180°} = 11.136$ chains

Length of the combined curve	$= 11.136 + (2 \times 3) = 17.136$ chains
Chainage of P.I.	$= 68 + 350$ chains
Subtract tangent length	$= 8 + 882$
Chainage of the beginning of the transition curve	$= 59 + 468$
Add length of transition curve	$= 3 + 000$
Chainage of the junction of the transition curve with the circular curve	$= 62 + 468$
Add length of circular curve	$= 11 + 136$
Chainage of the junction of the circular curve with the transition curve	$= 73 + 604$
Add length of transition curve	$= 3 + 000$
Chainage of end the transition curve	$= 76 + 604$ chains

Deflection angle (α) for the first transition curve

$$\alpha = \frac{1800 \ l^2}{\pi \ RL} = \frac{1800 \ l^2}{\pi \times 20 \times 3} = \frac{30}{\pi} \ l^2 \text{ minutes}$$

The various values of α are tabulated below :

Point	Chainage	*l* (Chains)	α		
			°	′	″
T	59 + 468	–		–	
1	59 + 500	0.032	0	0	0.6
2	60 + 000	0.532	0	2	42
3	60 + 500	1.032	0	10	10
4	61 + 000	1.532	0	22	25
5	61 + 500	2.032	0	39	26
6	62 + 000	2.532	1	1	13
D	62 + 468	3.000	1	25	56

Check : $\alpha_d = \alpha_S = \frac{1}{3} \Delta_S = 1° \ 25' \ 56''.$

Deflection angles for the circular curve :

$$\delta = 1718.9 \ \frac{c}{R} \text{ min.}$$

Length of first sub-chord	$= (63 + 000) - (62 + 468)$
	$= 0.532$ chains
Length of regular chord	$= 1$ chain
Length of the last sub-chord	$= (73 + 604) - (73) = 0.604$

The deflection angles are tabulated below :

Point	Chainage	δ			Δ		
		°	′	″	°	′	″
D	62 + 468		—			—	
1	63 + 000	0	45	44	0	45	44
2	64 + 0	1	25	57	2	11	41
3	65 + 0	1	25	57	3	37	38
4	66 + 0	1	25	57	5	03	35
5	67 + 0	1	25	57	6	29	32
6	68 + 0	1	25	57	7	55	29
7	69 + 0	1	25	57	9	21	26
8	70 + 0	1	25	57	10	47	23
9	71 + 0	1	25	57	12	13	20
10	72 + 0	1	25	57	13	39	17
11	73 + 0	1	25	57	15	05	14
D′	73 + 604	0	51	55	15	57	09

Check : $\Delta_{D'} = \frac{1}{2} \Delta_C = \frac{1}{2} (31° \ 54' \ 4'') = 15° \ 57' \ 12''.$

Deflection angles for the second transition curve :

The second transition curve will be set out from the point of tangency (T').

$$\alpha = \frac{1800 \ l^2}{\pi \ RL} = \frac{1800 \ l^2}{\pi \times 20 \times 3} = \frac{30}{\pi} \ l^2 \text{ minutes}$$

Point	Chainage	*l* (Chains)	α		
			°	′	″
D′	73 + 604	3.000	1	25	56
1	74 + 000	2.604	1	04	45
2	74 + 500	2.104	0	42	16
3	75 + 000	1.604	0	24	33
4	75 + 500	1.104	0	11	38
5	76 + 000	0.604	0	03	29
6	76 + 500	0.104	0	00	10
T′	76 + 604	0.000	0	00	00

of point with respect to an origin assumed at the point of tangency of the spiral with the main tangent. (U.L.)

5. The deflection angle between two straights forming the tangents of a highway curve is 48°. The curve is to consist of central circular arc with two equal transition spirals and the following conditions are to be satisfied:

(a) Radius of circular arc : 140 m

(b) External distance not greater than 16 m

(c) Tangent length not greater than 106 m.

It is proposed to adopt a spiral length of 80 m. Ascertain whether this length is suitable.

6. A transition curve is required for a circular curve of 400 m radius, the gauge being 1.5 m between rail centre and maximum super-elevation restricted to 12 cm. The transition is to be designed for a velocity such that no lateral pressure is imposed on the rails and the rate of gain of the radial acceleration is 30 cm/sec^3. Calculate the required length of transition curve and the design speed.

7. Two straights on the centre line of a proposed railway intersect at 1270.8 metres, the deflection angle being 38° 24′. It is proposed to put in a circular curve of 320 m radius with cubic parabolic transition curve 36 m long at each end. The combined curve is to be set out by the method of deflection angles with pegs at every 10 m through chainage on the transition curves and with pegs at every 20 m through chainage on the circular curve. Tabulate the data relative to the first two stations on the first transition curve and the junctions of the transition curve with circular etc.

ANSWERS

5. Tangent length = 103.24 m. , External distance = 15.54 m.
6. 46.4 m.
7. Deflection angles 3′ 47″ ; 17′26″ ; 1° 4′ 27″ ; 15° 58′ 48″.

4

Curve Surveying IV : Vertical Curves

4.1. GENERAL

A vertical curve is used to join two intersecting grade lines of rail-roads, highways or other routes to smooth out the changes in vertical motion. An abrupt change in the rate of the grade could otherwise subject a vehicle passing over it to an impact that would be either injurious or dangerous. The vertical curve, thus, contributes to the safety, comfort and appearance. Either a circular arc or a parabola may be used for this purpose, but for simplicity of calculation work, the latter is preferred and is invariably used. The parabolic curve also produces the best riding qualities, since the rate of change in grade is uniform throughout in a parabola having a vertical axis. This is proved as under.

The general equation of a parabola with a vertical axis can be written as

$$y = ax^2 + bx \qquad \ldots(i)$$

The slope of this curve at any point in given by $\dfrac{dy}{dx} = 2ax + b$ ...(ii)

The rate of change of slope or rate of change of grade (r) is given by

$$\frac{d^2y}{dx^2} = r = 2a = \text{constant} \qquad \ldots(iii)$$

Thus, the grade changes *uniformly throughout the curve,* which is a desired condition.

The Grade

The grade or gradient of a rail-road or highway is expressed in two ways :

(i) As a percentage : *e.g.* 2% or 3%

(ii) As 1 vertical in n horizontal (1 in n) : *e.g.* 1 in 100 or 1 in 400.

A grade is said to be *upgrade or* + ve grade when elevations along it increase, while it is said to be a downgrade or − ve grade when the elevations decrease along the direction of motion.

Rate of change of grade (r) : Equation (ii) gives the grade at any point on the curve. The gradient changes from point to point on the curve, but the rate of change of grade, given by equation, (iii) is constant in a parabola. For first class railways, the

(5) A downgrade ($-g_1\%$) followed by another downgrade ($-g_2\%$) : $g_2 > g_1$ (Fig. 4.5)

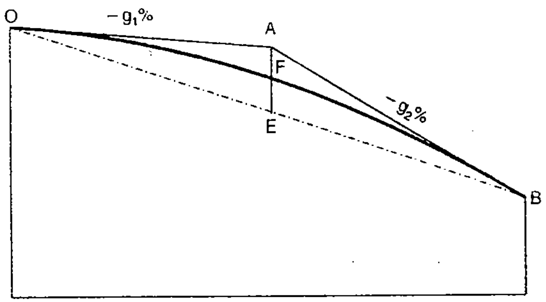

FIG. 4.5. SUMMIT OR CONVEX ($g_2 > g_1$).

(6) A downgrade ($-g_1\%$) followed by another downgrade ($-g_2\%$) : $g_1 > g_2$ (Fig. 4.6).

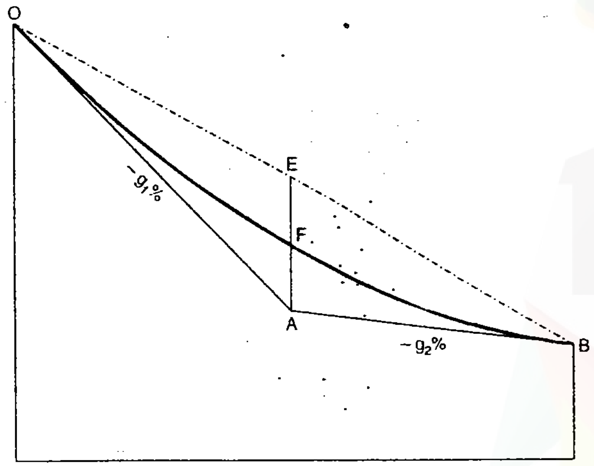

FIG. 4.6. SAG OF CONCAVE ($g_1 > g_2$).

4.3. LENGTH OF VERTICAL CURVE

The length of the vertical curve can be obtained by dividing the algebraic difference of the two grades by the rate of change of grade, due regard being paid to the sign of the grade. Thus,

$$\text{Length of curve } (L) = \frac{\text{Total change of grade}}{\text{Rate of change of grade}} = \frac{g_1 - g_2}{r} \text{ chains} \quad \ldots(4.1)$$

where $g_1 - g_2$ = Algebraic difference of the two grades (%)

r = Rate of change of grade (%) per chain

While substituting the numerical values of g_1 and g_2, due regard should be paid to the sign of the grade.

For example, if $g_1 = +1.2\%$ and $g_2 = -0.8\%$

and $r = 0.1\%$ per 20 m chain

$$L = \frac{g_1 - g_2}{r} = \frac{(+1.2) - (-0.8)}{0.1} = \frac{1.2 + 0.8}{0.1} = 20 \text{ chains} = 400 \text{ m}$$

In general practice, nearest number of L in chain lengths is adopted and $\frac{1}{2}L$ is set out to the either side of the apex. In case of highways, however, the minimum length of the curve is determined from the consideration of sight distance as discussed in § 4.5.

4.4. COMPUTATIONS AND SETTING OUT A VERTICAL CURVE

In vertical curves, all distances along the curve are measured *horizontally* and all offsets from the tangents to the curve are measured *vertically*. The length of the curve is thus its horizontal projection, without appreciable error since the curve is quite flat.

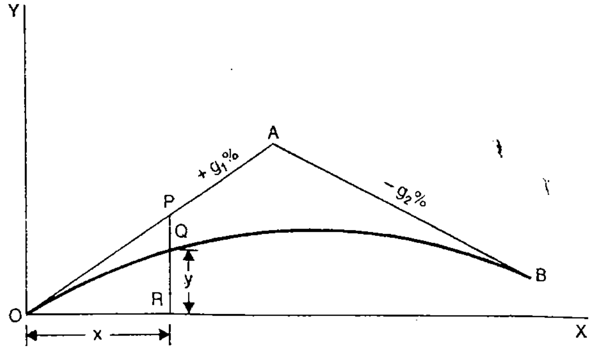

FIG. 4.7. THE PARABOLA.

In Fig. 4.7, let

OX and OY = The axes of the rectangular ordinates passing through the beginning (O) of the vertical curve

OA = Tangent having $+g_1\%$ slope

AB = Tangent having $-g_2\%$ slope

Q = Any point on the curve having co-ordinates (x, y)

Draw PQR, a vertical line through Q.

The equation of the parabola can be written as

$$y = ax^2 + bx \qquad \therefore \qquad \frac{dy}{dx} = 2ax + b$$

At $\quad x = 0, \quad \dfrac{dy}{dx} = +g_1$

$\therefore \quad g_1 = 2a(0) + b \quad$ or $\quad b = g_1$

Hence, the equation of the parbola is

$$y = ax^2 + g_1 x \quad \ldots(4.2)$$

Let $PQ = h$ = vertical distance between the tangent and the corresponding point Q on the curve
= Tangent correction

$\therefore \quad PQ = PR - QR$

The elevation of F can also be found by subtracting algebraically n^2k from the elevation of A. Thus.

Elevation of F = Elevation of $A - n^2 k = ne_1 - n^2k$.

(3) Compute the tangent corrections from the expression

$$h = kN^2$$

Thus
$$h_1 = 1\ k$$
$$h_2 = 4\ k$$
$$h_3 = 9\ k$$
$$\dots \quad \dots$$
$$h_N = (2n)^2\ k.$$

(4) Compute the elevation of the corresponding stations on the tangent OAC. Thus:

Elevation of tangent at any station (n')=elevation of point of tangency (O) + n'e₁

where n' is the *number* of that station from O.

(5) Find the elevations of the corresponding stations on the curve by adding algebraically the tangent corrections to the elevations of the corresponding stations.

If the value of k is positive, the tangent corrections are to be subtracted from grade elevations ; if it is negative, tangent corrections are additive.

The result may be tabulated as under :

Station	*Chainage*	*Tangent or grade elevation*	*Tangent correction*	*Elevation of the curve*	*Remarks*

Elevation by Chord Gradients

The chord gradient is the difference in elevation between the two ends of a chord joining two adjacent stations. Thus, in this method, the successive differences in elevation between the points on the curve are calculated and the elevation of each point is determined by adding the chord gradient to the elevation of the preceding point.

Consider two adjacent points P and Q of vertical curve having OA and BA as the initial tangents meeting at A. Through O, draw a horizontal line OQ_2. Through P and Q draw vertical line P_1PP_2 and Q_1QQ_2 shown in Fig. 4.9. Through P, draw a horizontal PQ_3.

Let e_1 and e_2 be the rises (or falls) of the tangents per chord length l.

$\therefore \quad P_1P_2 = e_1$ if P is the first station

P_1P = tangent correction, given by $h = kN^2 = 1\ k$

Difference in elevation between P and O

$= PP_2 = P_1P_2 - P_1P = e_1 - k$

where $k = \dfrac{e_1 - e_2}{4n}$

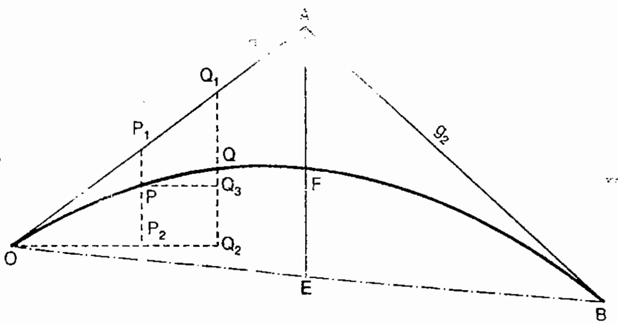

FIG. 4.9. METHOD OF CHORD GRADIENTS.

$\therefore$ *First chord gradient* $= e_1 - k$

Similarly, $Q_1Q_2 = 2\ e_1$

$Q_1Q = (2)^2\ k = 4k$

$Q_3Q_2 = PP_2 = e_1 - k$

Difference in elevation between Q and $P = QQ_3$

$$= Q_1Q_2 - Q_1Q - Q_3Q_2 = 2\ e_1 - 4k - (e_1 - k) = e_1 - 3\ k$$

$\therefore$ *Second chord gradient* $= e_1 - 3k$ ...(2)

Hence Nth chord gradient $= e_1 - (2N - 1)k$ ...(4.5)

Knowing the chord gradient for different points, their elevations can be easily calculated.

Thus, elevation of 1st station = Elevation of tangent point + First chord gradient

Elevation of 2nd station = Elevation of 1st station + Second chord gradient etc. etc.

Example 4.1. *A parabolic vertical curve is to be set out connecting two unifrom grades of + 0.8% and – 0.9%. The chainage and reduced level of point of intersection are 1664 metres and 238.755 m respectively. The rate of change of grade is 0.05% per chain of 20 m. Calculate the reduced levels of the various station pegs.*

Solution. (Fig. 4.8)

Total change of grade $= g_1 - g_2$

$= (+0.8) - (-0.9) = +1.7\%$

Rate of change of grade $= r = 0.05\%$ per chain

Length of the vertical curve $= \dfrac{1.7}{0.05} = 34$ chains

Length of the curve on either side of the apex

$= 17$ chains $= 340$ m

Chainage of the point of intersection = 1664 m

Chainage of the first tangent point = 1664 – 340 = 1324 m

Chainage of the second tangent point = 1664 + 340 = 2004 m

Station	Chainage	Tangent Elevation	Tangent Correction (–ve)	Curve Elevation	Remarks
28	1884	240.515	3.920	236.595	
29	1904	240.675	4.205	236.470	
30	1924	240.835	4.500	236.335	
31	1944	240.995	4.805	236.190	
32	1964	241.155	5.120	236.035	
33	1984	241.315	5.445	235.870	
34	2004	242.475	5.780	235.695	End of the curve

Example 4.2. *A – 1.0 percent grade meets a + 2.0 percent grade at station 470 of elevation 328.605 metres. A vertical curve of length 120 metres is to be used. The peges are to be fixed at 10 metres interval. Calculate the elevations of the points on the curve by (a) tangent corrections and (b) by chord gradients.*

If the pege are to be driven with their tops at the formation of the curve, calculate the staff readings required. given that height of collimation is 330.890.

Solution.

(a) *Tangent correction*

Total number of stations in 10 m unit $= \dfrac{120}{10} = 12$

Number of stations to each side of apex $= n = 6$

Change of elevation of first tangent per chord length of 10 m

$$\therefore \quad = e_1 = \frac{g_1}{100} \times 10 = \frac{-1.0}{100} \times 10 = -0.10 \text{ m}$$

Change of elevation of second tangent per chord length of 10 m

$$= e_2 = \frac{g_2}{100} \times 10 = \frac{+2.0}{100} \times 10 = +0.20 \text{ m}$$

Elevation of point of intersection $= 328.605$ m

Elevation of the beginning of curve $= 328.605 - ne_1$

$= 328.605 - (6)(-0.10)$

$= 329.205$ m

Elevation of the end of curve $= 328.605 + ne_2$

$= 328.605 + (6)(0.2)$

$= 329.805$ m

The tangent correction with respect to the first tangent is given by

$$h = kN^2$$

where $\quad k = \dfrac{e_1 - e_2}{4n} = \dfrac{(-0.10) - (0.20)}{4 \times 6} = \dfrac{-0.3}{24} = -\dfrac{1}{80}$

Hence $\quad h = -\dfrac{N^2}{80}$

Since the sign of k is negative, h will be additive to the tangent elevations to get the elevations on the curve.

For the first point, tangent elevation = elevation of the beginning of the curve + e_1

$= 329.205 - 0.10 = 329.105$

Tangent correction $= \dfrac{1}{80} = 0.0125$ m ≈ 0.010 m

(Since the readings can be taken upto an accuracy of the multiples of 0.005 m).

Elevation of first point $= 329.105 + 0.010 = 329.115$ m

Similarly, for the second point, tangent elevation

$= 329.205 - 0.2 = 329.005$

Tangent correction $= \dfrac{(2)^2}{80} = 0.050$ m

Elevation of second point $= 329.005 + 0.050 = 329.055$ m.

The values for other points along with the required staff reading are tabulated below. The required staff readings for the pegs are obtained by subtracting the elevations of the points from the height of collimation.

Station	Chainage	Tangent elevation	Tangent correction (+ ve)	Curve elevation	Ht. of collimation	Staff reading	Remarks
0	410	329.205	0	329.205	330.890	1.685	Beginning of the curve
1	420	329.105	0.010	329.115		1.775	
2	430	329.005	0.050	329.055		1.835	
3	440	328.905	0.115	329.020		1.870	
4	450	328.805	0.200	329.005		1.885	
5	460	328.705	0.315	329.020		1.870	
6	470	328.605	0.450	329.055		1.835	Vertex of the curve
7	480	328.505	0.615	329.120		1.770	
8	490	328.405	0.800	329.205		1.685	
9	500	328.305	1.015	329.320		1.570	
10	510	328.205	1.250	329.455		1.435	
11	520	328.105	1.515	329.620		1.270	
B	530	328.005	1.800	329.805		1.085	End of the curve

Check:

Elevation of mid-point of OB $= \frac{1}{2}(329.205 + 329.805) = 329.505$ m

Elevation of the vertex $= \frac{1}{2}(329.505 + 328.605) = 329.055$ m

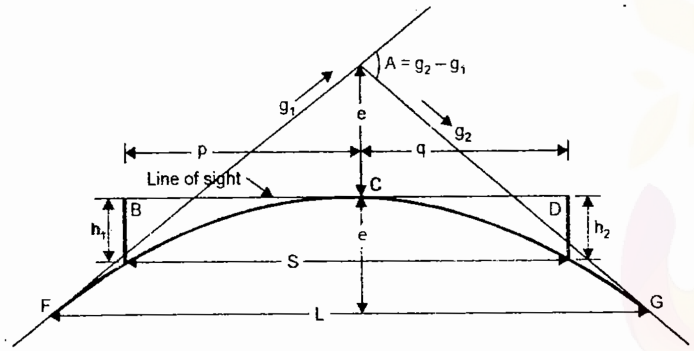

FIG. 4.10. SIGHT DISTANCE ($S < L$)

$$h = C x^2$$

From Fig. 4.10, when $x = L$, $h = \frac{(g_1 - g_2)}{100}\frac{L}{2}$

$$\therefore \qquad \frac{(g_1 - g_2)}{100}\frac{L}{2} = CL^2$$

or
$$C = \frac{g_1 - g_2}{200L} \qquad \ldots(2)$$

At C,
$$h = e = \frac{g_1 - g_2}{200L}\left(\frac{L}{2}\right)^2 = \frac{g_1 - g_2}{800}L = \frac{AL}{800} \qquad \ldots(4.6)$$

where $A = g_1 - g_2$ = algebraic difference in grades in percent.

Now, from Fig. 4.10, we have

$$h_1 = Cp^2 \text{ and } h_2 = Cq^2$$

$$\therefore \qquad S = p + q = \sqrt{\frac{h_1}{C}} + \sqrt{\frac{h_2}{C}} = \frac{1}{\sqrt{C}}(\sqrt{h_1} + \sqrt{h_2}) \qquad \ldots(4.7\ a)$$

$$S = \frac{14.14\sqrt{L}}{\sqrt{g_1 - g_2}}(\sqrt{h_1} + \sqrt{h_2}) \qquad \ldots(4.7)$$

To calculate the length L of the curve in terms of S, square Eq. 4.7. Thus,

$$S^2 = \frac{200\,L}{g_1 - g_2}(\sqrt{h_1} + \sqrt{h_2})^2$$

or
$$L = \frac{S^2(g_1 - g_2)}{200(\sqrt{h_1} + \sqrt{h_2})^2} \qquad \ldots(4.8)$$

The unit of L will be the same as the units of S and h.

Taking $h_1 = h_2 = h$ for *passing condition*, we have

$$L = \frac{S^2(g_1 - g_2)}{800\,h} \qquad \ldots[4.8\ (a)]$$

Taking $h_1 = 4.5$ ft and $h_2 = \frac{1}{2}$ ft (*stopping condition*),

we get
$$L = \frac{S^2(g_1 - g_2)}{1460} \text{ ft} \qquad \ldots[4.8\ (b)]$$

Taking $h_1 = 1.37$ m and $h_2 = 0.10$ m, we get

$$L = \frac{S^2(g_1 - g_2)}{297} \text{ metres.} \qquad \ldots[4.8\ (c)]$$

Example :

Let $g_1 = 1\%$; $g_2 = -1.5\%$

$h_1 = 1.37$ m ; $h_2 = 0.10$ m

and $S = 200$ metres (min.)

$$\therefore \qquad L = \frac{(200)^2(1 + 1.5)}{297} = 336 \text{ metres.}$$

Case 2. $S > L$

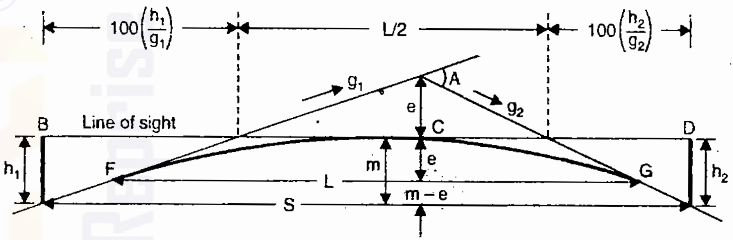

FIG. 4.11. SIGHT DISTANCE ($S > L$).

Fig. 4.11 shows the condition when the sight distance S is greater than L. Assuming scalar values for g_1 and g_2, we have

$$S = \tfrac{1}{2}L + 100\left(\frac{h_1}{g_1} + \frac{h_2}{g_2}\right) \qquad \ldots(1)$$

For the value of A making S a minimum, the rate of change in g_2 will be equal and opposite to the rate of change in g_1.

Setting the first derivative of S to zero, we get

$$\frac{h_1}{(g_1)^2} - \frac{h_2}{(g_2)^2} = 0$$

or
$$g_2 = \sqrt{\frac{h_2}{h_1}} \times g_1$$

$$\therefore \qquad A \text{ (scaler value)} = g_2 + g_1 = \sqrt{\frac{h_2}{h_1}}\,g_1 + g_1$$

or
$$A = \frac{\sqrt{h_1} + \sqrt{h_2}}{\sqrt{h_1}} \times g_1$$

or
$$g_1 = \frac{\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} \times A \qquad \ldots(2)$$

or $$\left(\frac{\Delta_L}{\Delta_S}\right)^2 = \frac{L^2}{S^2} \quad \text{...(6)}$$

From Eqs. (5) and (6), we get

$$L = \frac{S^2 . A}{8\,m} \quad \text{...(4.14)}$$

Taking $C = 4.2$ m (14 ft)

$h_1 = 1.8$ m (6.0 ft)

$h_2 = 0.45$ m (1.5 ft)

we get $$m = C - \frac{h_1 + h_2}{2} = 4.2 - \frac{1.8 + 0.45}{2} = 3.075$$

Hence from Eq. 4.14,

$$L = \frac{S^2 A}{24.6} \quad \text{...(4.15)}$$

and $$S = \sqrt{\frac{24.6\,L}{A}} \quad \text{...(4.16)}$$

Note. When the critical edge of the overhead stucture is not directly above the vertex of the vertical curve, Eqs. 4.12 and 4.16 are still valid, provided that the edge is not more than 60 m from the vertex.

PROBLEMS

1. A rising gradient of 1 in 50 on a proposed road meets a falling gradient of 1 to 40 and the reduced level of the intersection point is found from a longitudinal section to be 207.54 feet above O.D. These gradients are to be connected by a parabolic vertical curve, and a shifting distance of 1000 ft is to be provided over the summit, assuming that the line of sight is 3 ft 9 in. above the road surface. Calculate the necessary levels for setting out the vertical curve.

ANSWERS

1. Length of curve = 1500 ft.

5

Trigonometrical Levelling

5.1. INTRODUCTION

Trigonometrical levelling is the process of determining the differences of elevations of stations from observed vertical angles and known distances, which are assumed to be either horizontal or geodetic lengths at mean sea level. The vertical angles may be measured by means of an accurate theodolite and the horizontal distances may either be *measured* (in the case of plane surveying) or *computed* (in the case of geodetic observations).

We shall discuss the trigonometrical levelling under two heads:

(1) Observations for height and distances, and (2) Geodetical observations.

In the first case, the principles of the plane surveying will be used. It is assumed that the distances between the points observed are not large so that either the effect of curvature and refraction may be neglected or proper correction may be applied *linearly* to the calculated difference in elevation. Under this head fall the various methods of angular levelling for determining the elevations of particular points such as the top of chimney, or church spire etc.

In the geodetical observations of trigonometrical levelling, the distance between the points measured is geodetic and is large. The ordinary principles of plane surveying are not applicable. The corrections for curvature and refraction are applied in *angular measure* directly to the observed angles.

HEIGHTS AND DISTANCES

In order to get the difference in elevation between the instrument station and the object under observation, we shall consider the following cases :

Case 1 : Base of the object accessible.

Case 2 : Base of the object inaccessible : instrument stations in the same vertical plane as the elevated object.

Case 3 : Base of object inaccessible : instrument stations not in the same vertical plane as the elevated object.

5.2. BASE OF THE OBJECT ACCESSIBLE

Let it be assumed that the horizontal distance between the instrument and the object can be measured accurately (Fig. 5.1).

$$= (PP' - P'P'') + (AA' + A'A'')$$

$$= (D_1 \tan \alpha_1 - P'P'') + (D_2 \tan \beta_1 + A'A'')$$

If $D_1 = D_2 = D$, $P'P''$ and $A'A''$ will be equal.

Hence, $H_1 = D(\tan \alpha_1 + \tan \beta_1)$

The instrument is then shifted to O_2, midway between A and B, and the angles α_2 and β_2 are observed. Then the difference in elevation between B and A is

$$H_2 = D' (\tan \alpha_2 + \tan \beta_2)$$

where $D' = D_3 = D_4$

The process is continued till Q is reached.

5.3. BASE OF THE OBJECT INACCESSIBLE : INSTRUMENT STATIONS IN THE SAME VERTICAL PLANE WITH THE ELEVATED OBJECT

If the horizontal distance between the instrument and the object cannot be measured due to obstacles, etc., two instrument stations are used so that they are in the same vertical plane as the elevated object (Fig. 5.5).

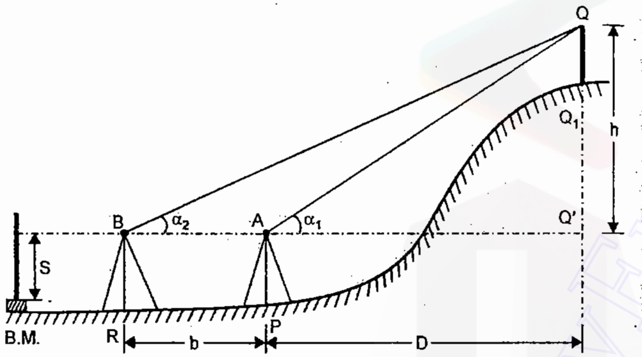

FIG. 5.5. INSTRUMENT AXES AT THE SAME LEVEL.

Procedure :

1. Set up the theodolite at P and level it accurately with respect to the altitude bubble.
2. Direct the telescope towards Q and bisect it accurately. Clamp both the plates. Read the vertical angle α_1.
3. Transit the telescope so that the line of sight is reversed. Mark the second instrument station R on the ground. Measure the distance RP accurately.

   Repeat steps (2) and (3) for both face observations. The mean values should be adopted.
4. With the vertical vernier set to zero reading, and the altitude bubble in the centre of its run, take the reading on the staff kept at the nearby B.M.
5. Shift the instrument to R and set up the theodolite there. Measure the vertical angle α_2 to Q with both face observations.
6. With the vertical vernier set to zero reading, and the altitude bubble in the centre of its run, take the reading on the staff kept at the nearby B.M.

In order to calculate the R.L. of Q, we will consider three cases : (a) when the instrument axes at A and B are at the same level, (b). when they are at different levels but the difference is small, and (c) when they are at very different levels.

(a) **Instrument axes at the same level** (Fig. 5.5)

Let $h = QQ'$

α_1 = angle of elevation from A at Q

α_2 = angle of elevation from B to Q

S = Staff reading on B.M. taken from both A and B, the reading being the same in both the cases

b = horizontal distances between the instrument stations.

D = horizontal distance between P and Q.

From triangle AQQ', $h = D \tan \alpha_1$ ...(1)

From triangle BQQ', $h = (b + D) \tan \alpha_2$ ...(2)

Equating (1) and (2), we get

$$D \tan \alpha_1 = (b + D) \tan \alpha_2$$

or $D (\tan \alpha_1 - \tan \alpha_2) = b \tan \alpha_2$

or
$$D = \frac{b \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} \quad ...(5.2)$$

$$h = D \tan \alpha_1 = \frac{b \tan \alpha_1 \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} = \frac{b \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)} \quad ...(5.3)$$

R.L. of Q = R.L. of B.M. $+ S + h$.

(b) **Instrument axes at different levels** [Fig. 5.6. and Fig. 5.7]

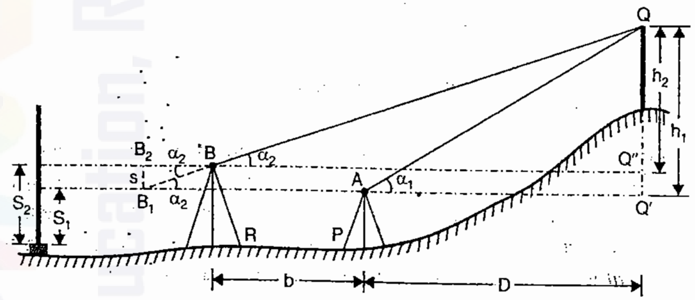

FIG. 5.6. INSTRUMENT AXES AT DIFFERENT LEVELS.

Figs. 5.6 and 5.7 illustrate the cases when the instrument axes are at different levels. If S_1 and S_2 are the corresponding staff readings on staff kept at B.M., the difference in levels of the instrument axes will be either $(S_2 - S_1)$ (if the axis at B is higher) or $(S_1 - S_2)$ (if the axis at A is higher). Let Q' be the projection of Q on horizontal line through A and Q'' be the projection on horizontal line through B. Let us derive the expression for Fig. 5.6. when S_2 is greater than S_1.

From triangle QAQ', $h_1 = D \tan \alpha_1$ ...(1)

From triangle BQQ'', $h_2 = (b + D) \tan \alpha_2$ ...(2)

Subtracting (2) from (1), we get

$$(h_1 - h_2) = D \tan \alpha_1 - (b + D) \tan \alpha_2$$

and $$h_2 = (b + D) \tan \alpha_2 \qquad \ldots(2)$$

Subtracting (1) from (2), we get

$$(h_2 - h_1) = s = (b + D) \tan \alpha_2 - D \tan \alpha_1$$

or $$D (\tan \alpha_1 - \tan \alpha_2) = b \tan \alpha_2 - s$$

$\therefore$ $$D = \frac{b \tan \alpha_2 - s}{\tan \alpha_1 - \tan \alpha_2} \qquad \ldots(3)$$

and $$h_1 = D \tan \alpha_1 = \frac{(b \tan \alpha_2 - s) \tan \alpha_1}{\tan \alpha_1 - \tan \alpha_2} = \frac{(b - s \cot \alpha_2) \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)} \qquad \ldots[5.5\ (b)]$$

From Fig. 5.9, we have

Height of station P above the axis at $B = h - r = b \tan \alpha - r$

Height of axis at A above the axis at $B = s = b \tan \alpha - r + h'$

where h' the height of the instrument at P.

Substituting this value of s in (3) and equation 5.5 (b), we can get D and h_1.

Now R.L. of Q = R.L. of $A + h_1$ = R.L. of $B + s + h_1$

= (R.L. of B.M. + back sight taken from B) $+ s + h_1$

where $$s = b \tan \alpha - r + h'.$$

5.4. BASE OF THE OBJECT INACCESSIBLE : INSTRUMENT STATIONS NOT IN THE SAME VERTICAL PLANE AS THE ELEVATED OBJECT

Let P and R be the two instrument stations *not* in the same vertical plane as that of Q. The procedure is as follows :

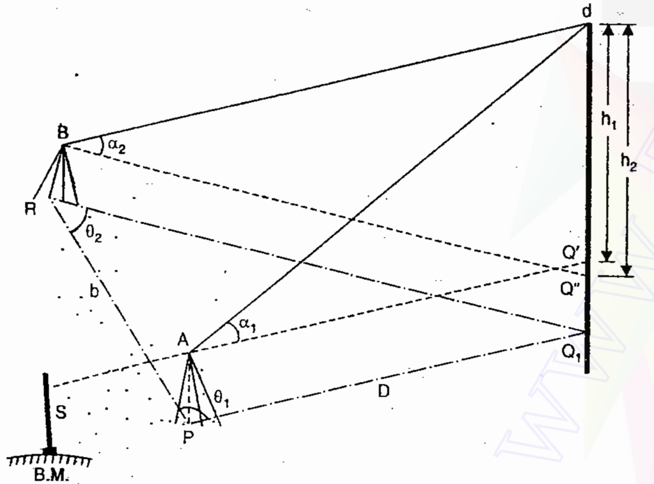

FIG. 5.10. INSTRUMENT AND THE OBJECT NOT IN THE SAME VERTICAL PLANE.

(1) Set the instrument at P and level it accurately with respect to the altitude bubble. Measure the angle of elevation α_1 to Q.

(2) Sight the point R with reading on horizontal circle as zero, and measure the angle RPQ_1 *i.e.*, the horizontal angle θ_1 at P.

(3) Take a back sight S on the staff kept at B.M.

(4) Shift the instrument to R and measure α_2 and θ_2 there.

In Fig. 5.10, AQ' is the horizontal line through A, Q' being the vertical projection of Q. Thus, AQQ' is a vertical plane. Similarly, BQQ'' is a vertical plane, Q'' being the vertical projection of Q on a horizontal line through B. PRQ_1 is a horizontal plane, Q_1 being the vertical projection of Q and R vertical projection of B on a horizontal plane passing through P. θ_1 and θ_2 are the horizontal angles, and α_1 and α_2 are the vertical angles measured at A and B respectively.

From triangle AQQ', $$QQ' = h_1 = D \tan \alpha_1 \qquad \ldots(1)$$

From triangle PRQ_1, $$\angle PQ_1R = 180° - (\theta_1 + \theta_2) = \pi - (\theta_1 + \theta_2)$$

From the sine rule,

$$\frac{PQ_1}{\sin \theta_2} = \frac{RQ_1}{\sin \theta_1} = \frac{RP}{\sin[\pi - (\theta_1 + \theta_2)]} = \frac{b}{\sin (\theta_1 + \theta_2)}$$

$\therefore$ $$PQ_1 = D = \frac{b \sin \theta_2}{\sin (\theta_1 + \theta_2)} \qquad \ldots(2)$$

and $$RQ_1 = \frac{b \sin \theta_1}{\sin (\theta_1 + \theta_2)} \qquad \ldots(3)$$

Substituting the value of D in (1), we get

$$h_1 = D \tan \alpha_1 = \frac{b \sin \theta_2 \tan \alpha_1}{\sin (\theta_1 + \theta_2)} \qquad \ldots(5.6)$$

$\therefore$ R.L. of Q = R.L. of B.M. $+ S + h_1$

As a check, $$h_1 = RQ_1 \tan \alpha_2 = \frac{b \sin \theta_1 \tan \alpha_2}{\sin (\theta_1 + \theta_2)}$$

If a reading on a B.M. is taken from B, the R.L. of Q can be known by adding h_2 to R.L. of B.

Example 5.1. *An instrument was set up at P and the angle of elevation to a vane 4 m above the foot of the staff held at Q was 9° 30′. The horizontal distance between P and Q was known to be 2000 metres. Determine the R.L. of the staff station Q, given that the R.L. of the instrument axis was 2650.38.*

Solution.

Height of vane above the instrument axis

$$= D \tan \alpha = 2000 \tan 9° 30' = 334.68 \text{ m}$$

Correction for curvature and refraction $= \dfrac{6}{7} \dfrac{D^2}{2R}$

or $$C = 0.06728\, D^2 \text{ m, when } D \text{ is in km}$$

apart. The horizontal angle measured at P between R and the top of the flat staff was 60° 30′ and that measured at R between the top of the flag staff and P was 68° 18′. The angle of elevation to the top of the flag staff was measured to be 10° 12′ at P. The angle of elevation to the top of the flag staff was measured to be 10° 48′ at R. Staff readings on B.M. when the instrument was at P = 1.965 m and that with the instrument at R = 2.055 m. Calculate the elevation of the top of the hill if that of B.M. was 435.065 m.

Solution. (Fig. 5.10)

Given : $b = 60$ m

$\theta_1 = 60° \; 30'$; $\theta_2 = 68° \; 18'$

$\alpha_1 = 10° \; 12'$; $\alpha_2 = 10° \; 18'$

$$\therefore \quad PQ_1 = D = \frac{b \sin \theta_2}{\sin (\theta_1 + \theta_2)}$$

and

$$h_1 = D \tan \alpha_1 = \frac{b \sin \theta_2 \tan \alpha_1}{\sin (\theta_1 + \theta_2)} = \frac{60 \sin 68° \; 18' \tan 10° \; 12'}{\sin (60° \; 30' + 68° \; 18')} = 12.87 \text{ m}$$

$\therefore$ R.L. of Q = (R.L. of instrument axis at P) + h_1

= (435.065 + 1.965) + 12.87 = **449.900 m.**

Check

$$h_2 = \frac{b \sin \theta_1 \tan \alpha_2}{\sin (\theta_1 + \theta_2)} = \frac{60 \sin 60° \; 30' \tan 10° \; 48'}{\sin (60° \; 30' + 68° \; 18')} = 12.78 \text{ m}$$

$\therefore$ R.L. of Q = R.L. of instrument axis at $R + h_2$ = (435.065 + 2.055) + 12.78

= **449.9 m.**

5.5. DETERMINATION OF HEIGHT OF AN ELEVATED OBJECT ABOVE THE GROUND WHEN ITS BASE AND TOP ARE VISIBLE BUT NOT ACCESSIBLE

(*a*) **Base line horizontal and in line with the object**

Let A and B be the two instrument stations, b apart. The vertical angles measured at A are α_1 and α_2, and those at B are β_1 and β_2, corresponding to the top (E) and bottom (D) of the elevated object. Let us take a general case of instruments at different heights, the difference being equal to s.

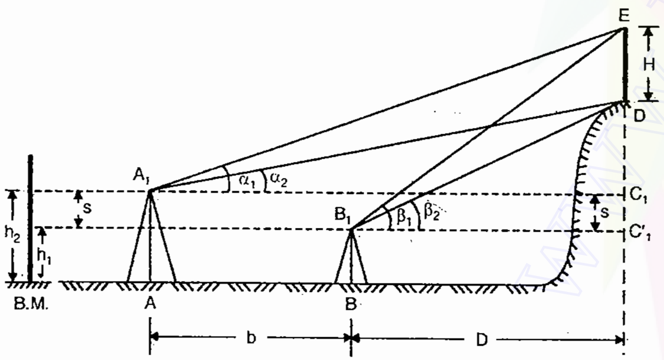

FIG. 5.11

Now $AB = b = C_1E \cot \alpha_1 - C_1'E \cot \beta_1 = C_1E \cot \alpha_1 - (C_1E + s) \cot \beta_1$

$\therefore \quad b = C_1E (\cot \alpha_1 - \cot \beta_1) - s \cot \beta_1$

$$\text{or} \quad C_1E = \frac{b + s \cot \beta_1}{\cot \alpha_1 - \cot \beta_1} \qquad \ldots(1)$$

Also, $AB = b = C_1D \cot \alpha_2 - C_1'D \cot \beta_2 = C_1 D \cot \alpha_2 - (C_1D + s) \cot \beta_2$

or $b = C_1D (\cot \alpha_2 - \cot \beta_2) - s \cot \beta_2$

$$\text{or} \quad C_1D = \frac{b + s \cot \beta_2}{\cot \alpha_2 - \cot \beta_2}$$

$$\therefore \quad H = C_1E - C_1D = \frac{b + s \cot \beta_1}{\cot \alpha_1 - \cot \beta_1} - \frac{b + s \cot \beta_2}{\cot \alpha_2 - \cot \beta_2} \qquad \ldots(5.7)$$

If heights of the instruments at A and B are equal, $s = 0$

$$\therefore \quad H = b\left[\frac{1}{\cot \alpha_1 - \cot \beta_1} - \frac{1}{\cot \alpha_2 - \cot \beta_2}\right] \qquad \ldots(5.7\ a)$$

Horizontal distance of the object from B

$EC_1' = D \tan \beta_1$ and $DC_1' = D \tan \beta_2$

$\therefore \quad EC_1' - DC_1' = H = D (\tan \beta_1 - \tan \beta_2)$

$$\text{or} \quad D = \frac{H}{\tan \beta_1 - \tan \beta_2} \qquad \ldots(5.7\ b)$$

where H is given by Eq. 5.7.

(*b*) **Base line horizontal but not in line with the object**

Let A and B be two instrument stations, distant b. Let α_1 and α_2 be the vertical angles measured at A, and β_1 and β_2 be the vertical angle measured at B, to the top (E) and bottom (D) of the elevated object. Let θ and φ be the horizontal angles measured at A and B respectively.

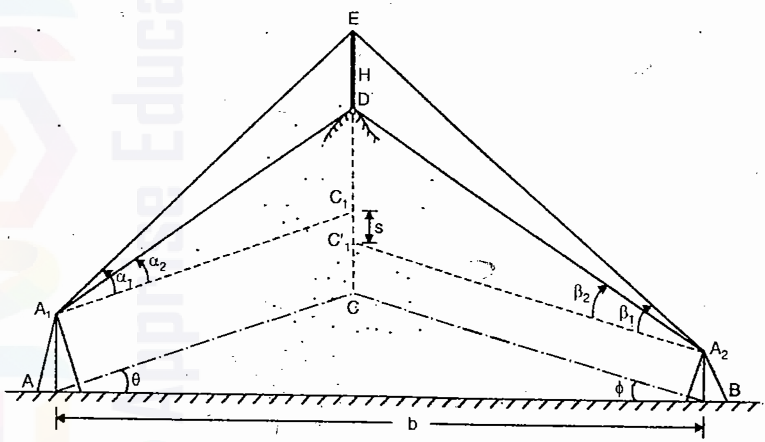

FIG. 5.12

At C : 13° 12′ 10″

Calculate (a) the height of station E above the line ABC

(b) the bearing of the line AE

(c) the horizontal distance between A and E :

Solution : Refer Fig. 5.14.

Given : $\alpha = 7° 13' 40''$;

$\beta = 10° 15' 00''$;

$\gamma = 13° 12' 10''$;

$b_1 = 314.12$ m;

and $b_2 = 252.58$ m

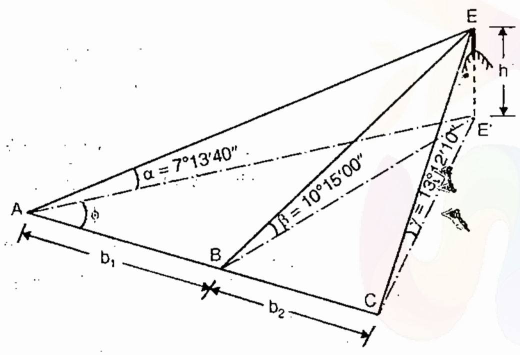

FIG. 5.14

Substituting the values in Eq. 5.10, we get

$$EE' = h = \left[\frac{b_1 b_2 (b_1 + b_2)}{b_1 (\cot^2 \gamma - \cos^2 \beta) + b_2 (\cot^2 \alpha - \cot^2 \beta)}\right]^{1/2}$$

$$= \left[\frac{314.12 \times 252.58\,(314.12 + 252.58)}{314.12(\cot^2 13° 12' 10'' - \cot^2 10° 15' 00'') + 252.58\,(\cot^2 7° 13' 40'' - \cot^2 10° 15' 00'')}\right]^{1/2}$$

$= 104.97$ m

$\therefore$ Height of E above $ABC = 104.97 + 1.4 =$ **106.37 m**

Also, From Eq. 5.9.

$$\cos \varphi = \frac{h^2 (\cot^2 \alpha - \cot^2 \beta) + b_1^2}{2\, b_1\, h \cot \alpha}$$

$$= \frac{(104.97)^2 (\cot^2 7° 13' 40'' - \cot^2 10° 15' 00'') + (314.12)^2}{2 \times 314.12 \times 104.97 \cot 7° 13' 40''}$$

$= 0.859205$

or $\varphi = 30° 46' 21''$

Hence bearing of $AE = 110° 16' 48'' - 30° 46' 21''$

$=$ **79° 30′ 27″**

Length $AE' = h \cot \alpha = 104.97 \cot 7° 13' 40''$

$=$ **827.70 m**

GEODETICAL OBSERVATIONS

5.7. TERRESTRIAL REFRACTION

The effect of refraction is to make the objects appear *higher* than they really are. In plane surveying where a graduated staff is observed either with horizontal line of sight or inclined line of sight, the effect of refraction is to decrease the staff reading and the correction is applied linearly to the observed staff reading. In trigonometrical levelling employed

for determining the elevations of widely distributed points, the correction is applied to the observed angles.

In Fig. 5.15, P and Q are the two points the difference in elevation between these being required.

Let O = centre of the earth

PO' = tangent to the level line through P = horizontal line at P

QO' = horizontal line at Q

$\angle P'PO' = \alpha_1$ = *observed* angle of elevation from P to Q (corrected for the difference in the heights of the signal and the instrument)

$\angle Q'QQ_2 = \beta_1$ = observed angle of depression from Q to P (corrected for the difference in the heights of the signal and the instrument).

r = angle of refraction or angular correction for refraction = $\angle P'PQ = \angle Q'QP$

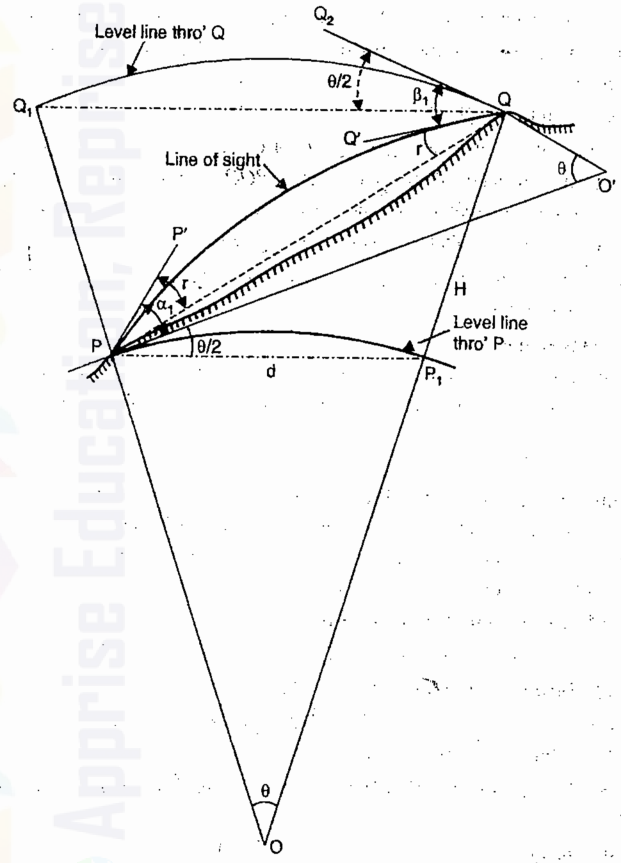

FIG. 5.15. TERRESTRIAL REFRACTION.

Angular correction of refraction $= m\theta = \dfrac{md}{R \sin 1''}$ seconds

Hence, combined angular correction $= \left[\dfrac{d}{2R \sin 1''} - \dfrac{md}{R \sin 1''} \right] = \dfrac{(1-2\,m)d}{2R \sin 1''}$ seconds ...(5.13)

The combined correction is positive for angles of elevation and negative for angles of depression.

5.8. AXIS SIGNAL CORRECTION (EYE AND OBJECT CORRECTION)

In order to observe the points from the theodolite station, signals of appropriate heights are erected at the points to be observed. The signals may or may not be of the same height as that of the instrument. If the height of the signal is not the same as that of the height of the instrument axis above the station, a correction known as the *axis signal correction* or *eye and object correction* is to be applied.

Let

h_1 = height of instrument at P, for observation to Q

h_2 = height of instrument at Q, for observation to P

s_1 = height of the signal at P, instrument being at Q

s_2 = height of the signal at Q, instrument being at P

d = horizontal distance between P and Q

α = observed angle of elevation uncorrected for the axis signal

β = observed angle of depression, uncorrected for axis signal

α_1 = angle of elevation corrected for axis signal

β_1 = angle of depression corrected for axis signal.

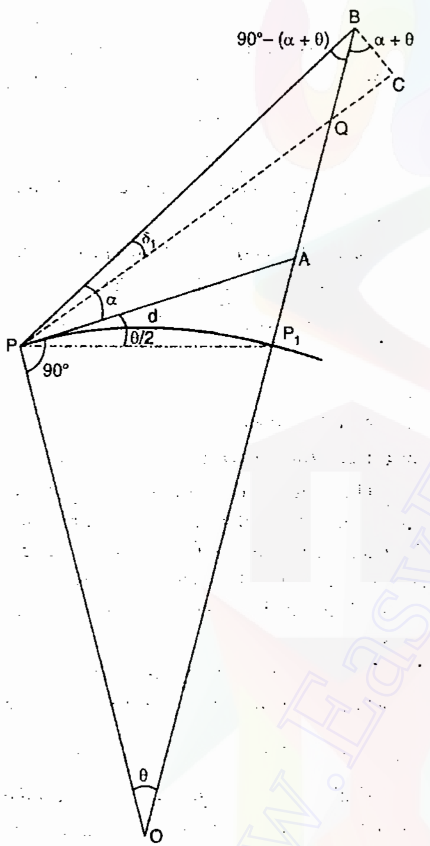

FIG. 5.16. AXIS SIGNAL CORRECTION.

In Fig. 5.16,

PA = horizontal line at P

Q = point observed

BQ = difference in the height of signal at Q and the height of instrument at P $= (s_2 - h_1)$

$\therefore$ $\angle BPA = \alpha$ = angle observed from P to Q

$\angle BPQ = \delta_1$ = axis signal correction (angular) at P.

At B, draw BC perpendicular to BP, to meet PQ produced in C.

For triangle PBO,

$$\angle BPO = \angle BPA + \angle APO = \alpha + 90° = 90° + \alpha$$

$$\angle POB = \theta$$

$$\therefore \quad \angle PBO = 180° - (90° + \alpha) - \theta \quad = 90° - (\alpha + \theta)$$

$$\therefore \quad \angle QBC = 90° - [90° - (\alpha + \theta)] = (\alpha + \theta)$$

The angle δ_1 is usually very small and hence $\angle BCQ$ can be approximately taken equal to 90°.

$$\therefore \quad BC = BQ \cos (\alpha + \theta) \text{ very nearly} = (s_2 - h_1) \cos (\alpha + \theta) \qquad ...(1)$$

For triangle PP_1B,

$$\angle BPP_1 = \alpha + \theta/2$$

$$\angle PBP_1 = 90° - (\alpha + \theta)$$

$$\angle PP_1B = 180° - [90° - (\alpha + \theta)] - (\alpha + \theta/2) = (90° + \theta/2)$$

Now $$\frac{PB}{\sin PP_1B} = \frac{PP_1}{\sin PBP_1}$$

or $$PB = PP_1 \frac{\sin PP_1B}{\sin PBP_1} = \frac{d \sin (90° + \theta/2)}{\sin [90° - (\alpha + \theta)]} = d \frac{\cos \theta/2}{\cos (\alpha + \theta)} \qquad ...(2)$$

From triangle PBC,

$$\tan \delta_1 = \frac{BC}{PB}$$

Substituting the value of BC from (1), and of PB from (2),

we get $$\tan \delta_1 = \frac{(s_2 - h_1) \cos (\alpha + \theta)}{d \dfrac{\cos \theta/2}{\cos (\alpha + \theta)}}$$

or $$\tan \delta_1 = \frac{(s_2 - h_1) \cos^2 (\alpha + \theta)}{d \cos \theta/2} \quad ...(\text{exact}) \qquad ...(5.14)$$

Usually, θ is small in comparison to α and may be ignored

$$\tan \delta_1 = \frac{(s_2 - h_1) \cos^2 \alpha}{d} \qquad ...[5.14\ (a)]$$

The correction is evidently *subtractive* for this case.

Similarly, if observations are taken from Q towards P, it can be proved that

$$\tan \delta_2 = \frac{(s_1 - h_2) \cos^2 \beta}{d} \quad (\text{additive}) \qquad ...(5.15)$$

The correction for axis signal is negative for angles of elevation and positive for angles of depression.

If, however, the vertical angle α (or β) is very small, we can take, with sufficient accuracy,

$$\tan \delta_1 = \delta_1 = \frac{s_2 - h_1}{d \sin 1''} \text{ seconds} \qquad ...[5.16\ (a)]$$

Equation 5.17 (*b*) or 5.17 (*c*) should be used only when θ is small. Otherwise equation 5.17 or 5.17 (*a*) should be used.

(*ii*) For angle of depression

In Fig. 5.18, let

β = observed angle of depression to P

β_1 = observed angle corrected for axis signal $= \beta + \delta_2$

or
$$= \beta + \frac{s_1 - h_2}{d \sin 1''} = \beta + \frac{\tan^{-1}(s_1 - h_2)\cos^2\beta}{d}$$

d = horizontal distance = arc QQ_1 = chord $QQ_1 \approx QB$

$\angle Q'QP = r = m\theta$

$Q_1P = H$ = difference in elevation between P and Q.

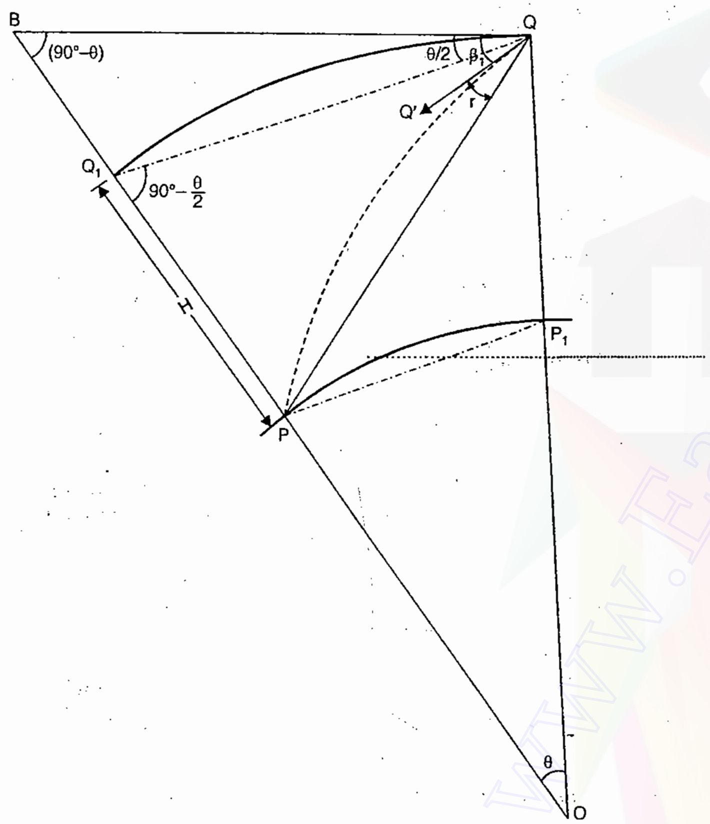

FIG. 5.18. DIFFERENCE IN ELEVATION BY SINGLE OBSERVATION : ANGLE OF DEPRESSION.

In triangle QPQ_1, $\angle PQQ_1 = \angle Q'QB + \angle Q'QP - \angle Q_1QB = \beta_1 + m\theta - \frac{\theta}{2}$ ...(1)

$$\angle PBQ = 90° - \theta$$

$$\therefore \quad \angle QQ_1P = (90° - \theta) - \frac{\theta}{2} = 90° - \frac{\theta}{2} \qquad ...(2)$$

and
$$\angle Q_1PQ = 180° - \left(90° - \frac{\theta}{2}\right) - \left(\beta_1 + m\theta - \frac{\theta}{2}\right) = 90° - (\beta_1 + m\theta - \theta) \qquad ...(3)$$

Now,
$$\frac{PQ_1}{\sin PQQ_1} = \frac{QQ_1}{\sin Q_1PQ}$$

or
$$PQ_1 = H = QQ_1 \frac{\sin PQQ_1}{\sin Q_1PQ}$$

or
$$PQ_1 = \frac{d \sin\left(\beta_1 + m\theta - \frac{\theta}{2}\right)}{\sin[90° - (\beta_1 + m\theta - \theta)]} = \frac{d \sin\left(\beta_1 + m\theta - \frac{\theta}{2}\right)}{\cos(\beta_1 + m\theta - \theta)} \quad ...\text{(exact)} \qquad ...(5.18)$$

$$= \frac{d \sin\left\{\beta_1 - (1 - 2m)\frac{d}{2R \sin 1''}\right\}}{\cos\left\{\beta_1 - (1 - m) - \frac{d}{R \sin 1''}\right\}} \qquad ...[5.18\ (a)]$$

where the quantities $(1 - 2m)\frac{d}{2R \sin 1''}$ and $(1 - m)\frac{d}{R \sin 1''}$ are in seconds.

Approximate Expressions

Equation 5.18 is the exact expression for the difference in elevation H. An approximate expression, however, can be had by considering $\angle PQ_1Q$ to be equal to 90° specially when θ is very small. Then

$$Q_1P = H = QQ_1 \tan PQQ_1 = d \tan\left(\beta_1 + m\theta - \frac{\theta}{2}\right) \qquad ...[5.18\ (b)]$$

$$= d \tan\left\{\beta_1 - (1 - 2m)\frac{d}{2R \sin 1''}\right\} \qquad ...[5.18\ (c)]$$

Application of corrections in linear measure

The difference in elevation between P and Q can also be obtained by applying the three corrections (*i.e.*, curvature, refraction and axis-signal) in linear measure.

Thus, axis-signal correction in linear measure $= s_2 - h_1$

$$\text{Curvature correction} = \frac{d^2}{2R}$$

$$\text{Refraction correction} = rd = m\theta \,.\, d = m\frac{d}{R}\,.\,d = \frac{md^2}{R}$$

$$\text{Combined correction for curvature and refraction} = \frac{d^2}{2R} - \frac{md^2}{R} = (1 - 2m)\frac{d^2}{2R}$$

If α is the observed angle, uncorrected for curvature, refraction and axis-signal, we have

$H = d \tan\alpha$ – (Ht. of signal – Ht. of instrument) + curvature correction – refraction correction

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$$H = \frac{d \sin\left(\frac{\alpha_1 + \beta_1}{2}\right)}{\cos\left\{\left(\frac{\alpha_1 + \beta_1}{2}\right) + \frac{\theta}{2}\right\}} \qquad \ldots(5.20)$$

If however, $\frac{\theta}{2}$ is small in comparison to $\frac{\alpha_1 + \beta_1}{2}$, it can be neglected. Then

$$H = d\,\frac{\sin\left(\frac{\alpha_1 + \beta_1}{2}\right)}{\cos\left(\frac{\alpha_1 + \beta_1}{2}\right)} = d \tan \frac{\alpha_1 + \beta_1}{2} \qquad \ldots[5.20\ (a)]$$

If, however, both α_1 and β_1 are the angles of depression, the expression for H can be obtained by changing the sign of α_1 in equation 5.20 :

$$H = \frac{d \sin\left(\frac{\beta_1 - \alpha_1}{2}\right)}{\cos\left(\frac{\beta_1 - \alpha_1}{2} + \frac{\theta}{2}\right)} \qquad \ldots(5.21)$$

If the value of H obtained from the above expression is positive, Q is higher than P. If H is negative, Q will be lower than P.

Thus, in general, the expression for H is

$$H = \frac{d \sin\left(\frac{\beta_1 \pm \alpha_1}{2}\right)}{\cos\left\{\frac{\beta_1 \pm \alpha_1}{2} + \frac{\theta}{2}\right\}}$$

Use plus sign when α_1 is the angle of elevation and minus sign when it is the angle of depression.

Example 5.8. *Correct the observed altitude for the height of signal, refraction and curvature from the following data :*

Observed altitude	$= +2°\,48'\,39''$
Height of instrument	$= 1.12$ m
Height of signal	$= 4.87$ m
Horizontal distance	$= 5112$ m
Co-efficient of refraction	$= 0.07$ m
R sin 1″	$= 30.88$ m.

Solution.

Given : $\alpha = +2°\,48'\,39''$; $h = 1.12$ m ; $s = 4.87$ m

$d = 5112$ m ; $m = 0.07$

The axis signal correction $= \delta = \dfrac{s - h}{d \sin 1''}$ seconds

$$= \frac{4.87 - 1.12}{5112 \sin 1''} = \frac{3.75 \times 206265}{5112} \left(\text{since } \sin 1'' = \frac{1}{206265}\right)$$

$= 151''.31 = 2'\,31''.31$ (subtractive)

The central angle $= \theta = \dfrac{d}{R \sin 1''} = \dfrac{5122}{30.88} = 165''.54$

Curvature correction $= \dfrac{\theta}{2} = 82''.77$ (additive)

Refraction correction $= r = m\theta = 0.07 \times 165.54$

$= 11''.59$ (subtractive)

Total correction $= \dfrac{\theta}{2} - \delta - r = 82''.77 - 151''.31 - 11''.59$

$= -80''.13 = 1'20''.13$ (subtractive)

Correct altitude $= 2°\,48'\,39'' - 1'\,20''.13 = 2°\,47'\,18''.87$

Example 5.9. *Find the R.L. of Q from the following observations:*

Horizontal distance between P and Q	$= 9290$ m
Angle of elevation from P to Q	$= 2°\,06'\,18''$
Height of signal at Q	$= 3.96$ m
Height of instrument at P	$= 1.25$ m
Co-efficient of refraction	$= 0.07$
R sin 1″	$= 30.88$ m
R.L. of P	$= 396.58$ m.

Solution.

Given : $d = 9290$ m ; $\alpha = +2°\,06'\,18'$; $s = 3.96$ m

$h = 1.25$ m ; $R \sin 1'' = 30.88$ m ; $m = 0.07$

Axis signal correction $= \delta = \dfrac{s - h}{d \sin 1''} = \dfrac{(3.96 - 1.25)}{9290 \sin 1''}$ seconds

$$= \frac{2.71 \times 206265}{9290} = 60''.17 \text{ (subtractive)}$$

$\alpha_1 = \alpha - \delta = 2°\,06'\,18'' - 60''.17 = 2°\,05'\,17''.83$

$\theta = \dfrac{d}{R \sin 1''}$ seconds $= \dfrac{9290}{30.88} = 300''.84 = 5'0''.84$

$\dfrac{\theta}{2} = 150''.42 = 2'30''.42$

$r = m\theta = 0.07 \times 300.84 = 21''.06$

Now, from equation 5.17, we have

$$H = \frac{d \sin\left(\alpha_1 - m\theta + \frac{\theta}{2}\right)}{\cos(\alpha_1 - m\theta + \theta)} = \frac{9290 \sin(2°\,5'\,17''.83 - 21''.06 + 2'\,30''.42)}{\cos(2°\,5'\,17''.83 - 21''.06 + 5'0''.84)}$$

$$= \frac{9290 \sin 2°\,7'27''.19}{\cos 2°\,9'\,57''.61} = 344.59 \text{ m}$$

R.L. of $Q =$ R.L. of $P + H = 396.58 + 344.59 =$ **741.17 m.**

Example 5.10. *Find the difference of levels of the points P and Q and the R.L. of P from the following data :*

Height of signal at Q = 4.07 m

Height of the instrument at P = 1.27 m

Height of the instrument at Q = 1.34 m

Calculate (a) the R.L. of Q, if that of P is 1248.65 m and (b) the average co-efficient of refraction at the time of observations.

Take R sin 1″ = 30.88 m.

Solution.

Given : $d = 33128$ m ; $\alpha = -6'\ 20''$; $\beta = -8'\ 10''$

$s_1 = 4.87$ m ; $s_2 = 4.07$ m ; $h_1 = 1.27$ m ; $h_2 = 1.34$ m

$$\text{Axis signal correction at } P = \delta_1 = \frac{s_2 - h_1}{d \sin 1''} = \frac{4.07 - 1.27}{33128 \sin 1''} = \frac{2.80 \times 206265}{33128}$$

$$= 17''.43 \quad (\text{additive to } \alpha)$$

$$\text{Axis signal correction at } Q = \delta_2 = \frac{s_1 - h_2}{d \sin 1''} = \frac{4.87 - 1.34}{33128 \sin 1''} = \frac{3.53 \times 206265}{33128}$$

$$= 21''.98 \quad (\text{additive to } \beta)$$

$$\therefore \quad \alpha_1 = \alpha + \delta_1 = 6'\ 20'' + 17''.43 = 6'\ 37''.43 \text{ (depression)}$$

$$\beta_1 = \beta + \delta_2 = 8'\ 10'' + 21''.98 = 8'\ 31''.98 \text{ (depression)}$$

$$\frac{\beta_1 - \alpha_1}{2} = \frac{1}{2}(8'\ 31''.98 - 6'\ 37''.43) = 57''.27$$

$$\frac{\beta_1 + \alpha_1}{2} = \frac{1}{2}(8'\ 31''.98 + 6'37''.43) = 7'34''.71$$

$$\theta = \frac{d}{R \sin 1''} = \frac{33128}{30.88} = 1072''.8 = 17'\ 52''.8$$

$$\frac{\theta}{2} = 8'\ 56''.4$$

$$r = \frac{\theta}{2} - \left(\frac{\beta_1 + \alpha_1}{2}\right) = 8'\ 56''.4 - 7'\ 34''.71 = 1'21''.69 = 81''.69$$

$$m = \frac{r}{\theta} = \frac{81.69}{1072.8} = \mathbf{0.0762}$$

The difference in elevation (H) is given by

$$H = \frac{d \sin\left(\frac{\beta_1 - \alpha_1}{2}\right)}{\cos\left(\frac{\beta_1 - \alpha_1}{2} + \frac{\theta}{2}\right)} = \frac{33128 \sin 57''.27}{\cos (57''.27 + 8'\ 56''.4)}$$

$$= \frac{33128 \sin 57''.27}{\cos 9'\ 53''.67} = 9.20 \text{ m}$$

$\therefore$ R.L. of θ = R.L. of $P + H = 1248.65 + 9.20 =$ **1257.85 m.**

Example 5.13. *In the trigonometrical measurement of the difference in level of two stations P and Q, 10480 m apart, the following data were obtained :*

Instrument at P, angle of elevation of Q = 0′ 15″

Instrument at Q, angle of depression of P = 3′ 33″

Height of instrument at P = 1.42 m

Height of instrument at Q = 1.45 m

Height of signal at P = 3.95 m

Height of signal at Q = 3.92 m

Find the difference in level between P and Q, and the curvature and refraction correction. Take R sin 1″ = 30.38 metres.

Solution.

Given : $d = 10480$ m ; $\alpha = +0'\ 15''$; $\beta = -3'\ 33''$

$h_1 = 1.42$ m ; $h_2 = 1.45$ m ; $s_1 = 3.95$ m ; $s_2 = 3.92$ m

$$\text{Axis signal correction at } P = \delta_1 = \frac{s_2 - h_1}{d \sin 1''} = \frac{3.92 - 1.42}{10480 \times \sin 1''}$$

$$= \frac{2.50 \times 206265}{10480} = 49''.30$$

This is subtractive since α is the angle of elevation.

$$\text{Axis signal correction at } Q = \delta_2 = \frac{s_1 - h_2}{d \sin 1''} = \frac{3.95 - 1.45}{10480 \sin 1''}$$

$$= \frac{2.5 \times 206265}{10480} = 49''.30$$

This is additive since β is the angle of depression.

$$\therefore \quad \alpha_1 = \alpha - \delta_1 = 0'\ 15'' - 49''.30 = -34''.30 \quad i.e., \ 34''.30 \text{ (dep.)}$$

$$\beta_1 = -(\beta + \delta_2) = -(3'\ 33'' + 49''.30) = -4'22''.30$$

$$\theta = \frac{d}{R \sin 1''} = \frac{10480}{30.80} = 339''.38 = 5'\ 39''.38$$

$$\frac{\theta}{2} = 2'\ 49''.69$$

$$\therefore \text{ Curvature correction} = \frac{\theta}{2} = \mathbf{2'\ 49''.69}$$

Also, from Eq. 5.12 (*a*),

$$r = \frac{\theta}{2} - \left(\frac{\beta_1 + \alpha_1}{2}\right)$$

$$= 2'\ 49''.69 - \left(\frac{4'\ 22''.30 + 34''.30}{2}\right) = 21''.39$$

Refraction correction = r = **21″.39**.

From Eq. 5.21, we have

Find the difference in level between P and Q, and the curvature and refraction corrections. Take $R \sin 1'' = 30.88$ m.

ANSWERS

1. 37.558 m ; 278.824 m.
2. 267.796.
3. 442.347.
4. 290.335 ; 33.9 m.
7. 606.55 m.
8. 44142 ft.
9. m=0.0693 ; H=89.13 m.
10. m= 0.0784 ; 342.48 m.

6

Hydrographic Surveying

6.1. INTRODUCTION

Hydrographic survey is that branch of surveying which deals with the measurement of bodies of water. It is the art of delineating the submarine levels, contours and features of seas, gulfs, rivers and lakes. It is used for :

(1) making nautical charts for navigation and determination of rocks, sand bars, lights and buoys ;

(2) making subaqueous investigations to secure information needed for the construction, development and improvement of port facilities ;

(3) measurement of areas subject to scour or silting and to ascertain the quantities of dredged material ;

(4) controlling and planning of engineering projects like bridges, tunnels, dams, reservoirs, docks and harbours ;

(5) establishing mean sea level and observation of tides ;

(6) determination of shore lines ; and

(7) measurement of discharge of rivers.

Horizontal and Vertical Control

The main operation in hydrographic surveying is to determine the depth of water at a certain point. The measurement of depth below the water surface is called *sounding*. Thus, to take the sounding, a vertical control is necessary and to locate the sounding (*i.e.*, the point where the sounding is taken), a horizontal control is necessary. The *horizontal control* may consist of either a triangulation or a traverse. For surveys of large extent, a second or third order triangulation may be used as the main control. For surveys of small extent, a transit-tape-traverse may be used. For small detached surveys, a control system may be developed by a combination of stadia and graphical triangulation procedures with plane table. In the case of a long narrow river, the horizontal control is established by running a single traverse line on one shore. If the width of body of water is more than 1 kilometre, traverse may be run on both the shores and may be connected at intervals.

When the soundings are recorded, it is essential to know the gauge reading. *i.e.*, the level of water which continously goes on changing. Tide or water-stage gauges are kept in operation to establish the common datum and to give the height of water for

Suggested system of marking poles and lead lines

The U.S. Coast and Geodetic survey recommends the following system of marking the poles and the lead lines :

Poles : Make a small permanent notch at each half foot. Paint the entire pole white and the spaces between the 2- and 3-, the 7- and 8- and the 12- and 13-ft marks black. Paint $\frac{1}{2}''$ red bands at the 5- and 10-ft marks, a $\frac{1}{2}''$ in black band at each of the other foot marks and $\frac{1}{4}''$ bands at the half foot marks. These bands are black where the pole is white and *vice-versa*.

Lead Lines : A lead line is marked in feet as follow :

Feet	*Marks*
2, 12, 22 etc	Red bunting
4, 14, 24 etc.	White bunting
6, 16, 26 etc.	Blue bunting
8, 18, 28 etc.	Yellow bunting
10, 60, 110 etc.	One strip of leather
20, 70, 120 etc.	Two strips of leather
30, 80, 130 etc.	Leather with two holes
40, 90, 140 etc.	Leather with one hole
50	Star-shaped leather
100	Star-shaped leather with one hole.

The intermediate odd feet (1, 3, 5, 7, 9 etc.) are marked by white seizings.

(*iv*) **Sounding Machine**

Where much of sounding is to be done, a sounding machine as very useful. The sounding machine may either be hand driven or automatic. Fig. 6.3 shows a typical hand driven Weddele's sounding machine.

The lead weight is carried at the end of a flexible wire cord attached to the barrel and can be lowered at any desired rate, the speed of the drum being controlled by means of a brake. The readings are indicated in two dials — the outer dial showing the depth in feet and the inner showing tenths of a foot. A handle is used to raise the level which can be suspended at any height by means of a paul and ratchet. The sounding

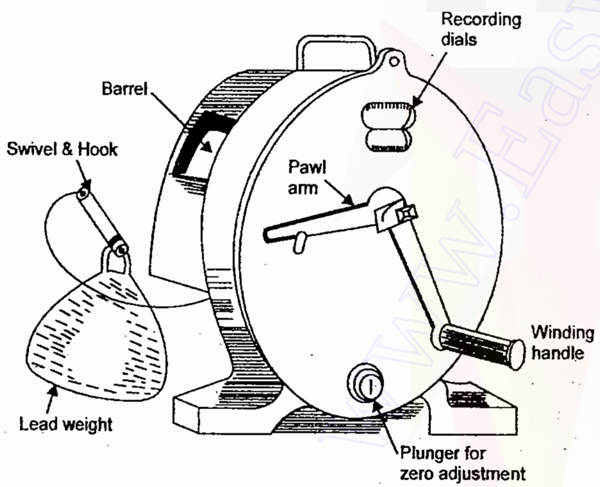

FIG. 6.3. WEDDELE'S SOUNDING MACHINE.

machine is mounted in a sounding boat and can be used up to a maximum depth of 100 ft.

(*v*) **Fathometer : Echo-sounding**

A fathometer is used for ocean sounding where the depth of water is too much, and to make a continuous and accurate record of the depth of water below the boat or ship at which it is installed. It is an *echo-sounding* instrument in which water depths are obtained by determining the time required for the sound waves to travel from a point near the surface of the water to the bottom and back. It is adjusted to read depth in accordance with the velocity of sound in the type of water in which it is being used. A fathometer may indicate the depth visually or indicate graphically on a roll which continuously goes on revolving and provide a virtual profile of the lake or sea.

The main parts of an echo-sounding apparatus are :

1. Transmitting and receiving oscillators.
2. Recorder unit.
3. Transmitter/Power unit

Fig. 6.4. illustrates the principle of echo-sounding. It consists in recording the interval of time between the emission of a sound impulse direct to the bottom of the sea and the reception of the wave or echo, reflected from the bottom. If the speed of sound in that water is v and the time interval between the transmitter and receiver is t, the depth h is given by

$$h = \frac{1}{2} vt \qquad \ldots(6.1)$$

Due to the small distance between the receiver and the transmitter, a slight correction is necessary in shallow waters. The error between the true depth and the recorded depth can be calculated very easily by simple geometry. If the error is plotted against the recorded depth, the true depth can be easily known. The recording of the sounding is produced by the action of a small current passing through chemically impregnated paper from a rotating stylus to an anode plate. The stylus is fixed at one end of a radial arm which revolves at constant speed. The stylus makes a record on the paper at the instants when the sound impulse is transmitted and when the echo returns to the receiver.

The record of depth is made by a stylus on a moving band of dry paper as shown in Fig.6.5

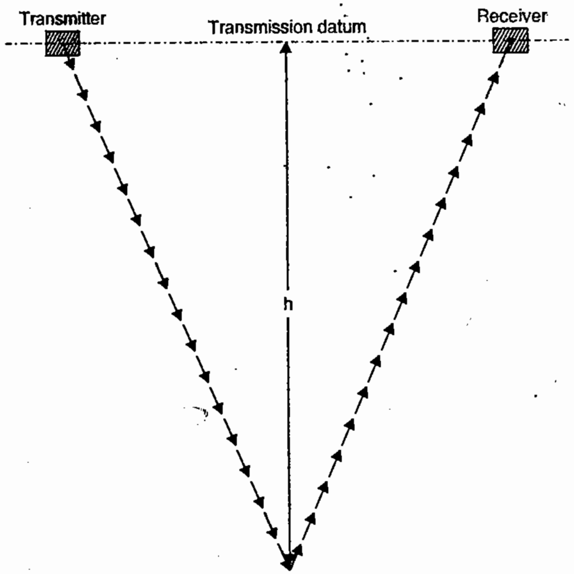

FIG. 6.4. PRINCIPLE OF ECHO-SOUNDING.

(c) By theodolite angles and EDM distances from the shore

(d) By microwave systems

(a) By conning the survey vessel

The process of keeping the survey vessel or boat on a known course is known as *conning* the vessel. The task of conning is mainly one of seamanship. One of the most common method of conning is to fix markers (poles, beacons etc.) on the shore, thus providing the 'ranges' along which the vessel is run. The method is suitable for work in rivers and open seas upto 5 km off shore.

Range. A range or range line is the line on which soundings are taken. They are, in general, laid perpendicular to the shore line and parallel to each other if the shore is straight or are arranged radiating from a prominent object when the shore line is very irregular.

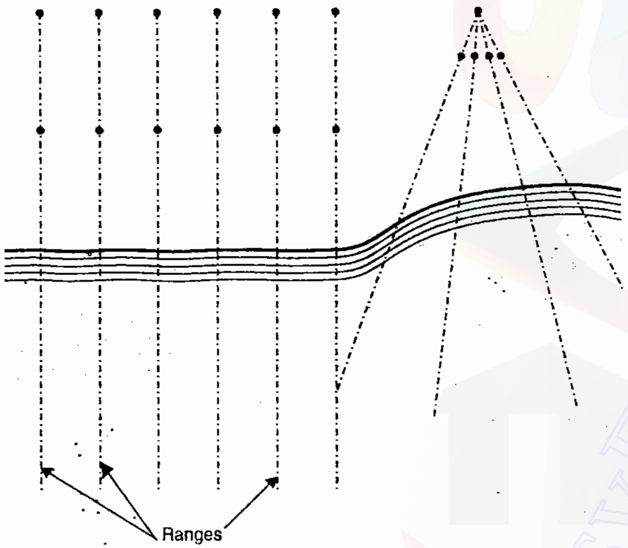

FIG. 6.6. RANGES.

Shore signals. Each range line is marked by means of signals erected at two points on it at a considerable distance apart. Signals can be constructed in a variety of ways. They should be readily seen and easily distinguished from each other. The most satisfactory and economic type of signal is a wooden tripod structure dressed with white and coloured signal of cloth. The position of the signals should be located very accurately since all the soundings are to be located with reference to these signals.

1. Location by Cross-Rope

This is the most accurate method of locating the soundings and may be used for rivers, narrow lakes and for harbours. It is also used to determine the quantity of materials removed by dredging, the soundings being taken before and after the dredging work is done. A single wire or rope is stretched across the channel etc. as shown in Fig. 6.7 and is marked by metal tags at appropriate known distance along the wire from a reference point or zero station on shore. The soundings are then taken by a

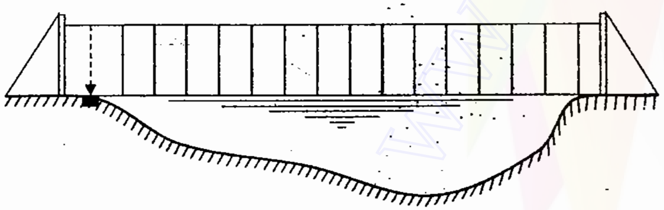

FIG. 6.7. SOUNDING FROM A GRADUATED LINE.

weighted pole. The position of the pole during a sounding is given by the graduated rope or line.

In another method, specially used for harbours etc., *a reel boat* is used to stretch the rope. The zero end of the rope is attached to a spike or any other attachment on one shore. The rope is wound on a drum on the reel boat. The reel boat is then rowed across the line of sounding, thus unwinding the rope as it proceeds. When the reel boat reaches the other shore, its anchor is taken ashore and the rope is wound as tightly as possible. If anchoring is not possible, the reel is taken ashore and spiked down. Another boat, known as the sounding boat, then starts from the previous shore and soundings are taken against each tag of the rope. At the end of the soundings along that line, the reel boat is rowed back along the line thus winding in the rope. The work thus proceeds.

2. Location by Range and Time Intervals

In this method, the boat is kept in range with the two signals on the shore and is rowed along it at constant speed. Soundings are taken at different time intervals. Knowing the constant speed and the total time elapsed at the instant of sounding, the distance of the total point can be known along the range. The method is used when the width of channel is small and when great degree of accuracy is not required. However, the method is used in conjunction with other methods, in which case the first and the last soundings along a range are located by angles from the shore and the intermediate soundings are located by interpolation according to time intervals.

(b) By observations with sextant or theodolite

3. Location by Range and One Angle from the Shore

In this method, the boat is ranged in line with the two shore signals and rowed along the ranges. The point where sounding is taken is fixed on the range by observation of the angle from the shore. As the boat proceeds along the shore, other soundings are also fixed by the observation of angles from the shore. Thus, in Fig. 6.8 (*a*), *B* is the instrument station, A_1A_2 is the range along which the boat is rowed and α_1, α_2, α_3 etc., are the angles measured at *B* to points 1, 2, 3 etc. The method is very accurate and very convenient for plotting. However, if the angle at the sounding point (say angle β) is less than 30°, the fix becomes poor. The nearer the intersection angle (β) is to a right angle, the better. If the angle diminishes to about

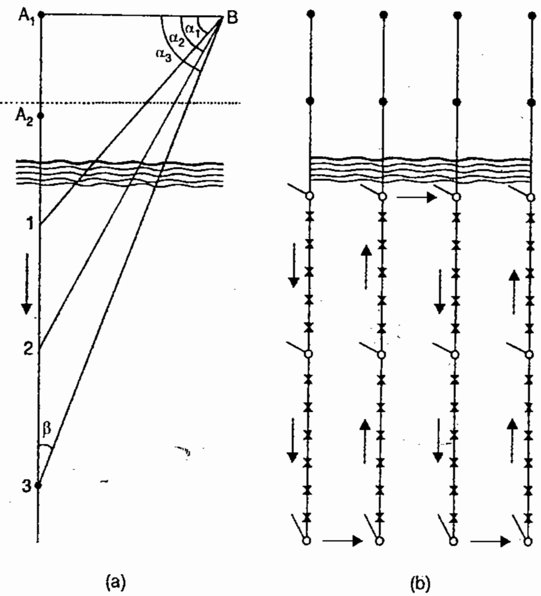

FIG. 6.8. LOCATION BY RANGE AND ONE ANGLE FROM THE SHORE

6. Location by Two Angles from the Boat

In this method, the position of the boat can be located by the solution of the three-point problem by observing the two angles subtended at the boat by three suitable shore objects of known position. The three-shore points should be well-defined and clearly visible. Prominent natural objects such as church spire, lighthouse, flagstaff, buoys etc, are selected for this purpose. If such points are not available, range poles or shore singals may be taken. Thus, in Fig. 6.11, *A*, *B* and *C* are the shore objects and *P* is the position of the boat from which the angles α and β are measured. Both the angles should be observed simultaneously with the help of two sextants, at the instant the sounding is taken. If both the angles are observed by surveyor alone, very little time should be lost in taking the observation. The angles on the circle are read afterwards. The method is used to take the soundings at isolated points. The surveyor has better control on the operations since the survey party is concentrated in one boat. If sufficient number of prominent points are available on the shore, preliminary work of setting out and erecting range signals is eliminated. The position of the boat is located by the solution of the three point problem either analytically or graphically.

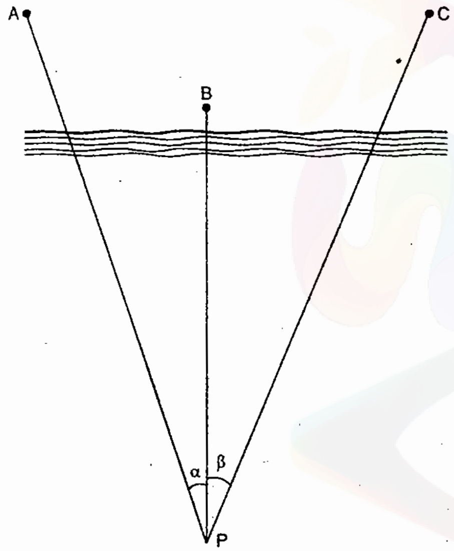

FIG. 6.11. LOCATION BY TWO ANGLES FROM THE BOAT.

7. Location by One Angle from the Shore and the other from the Boat.

This method is the combination of methods 5 and 6 described above and is used to locate the isolated points where soundings are taken. Two points *A* and *B* (Fig. 6.12) are chosen on the shore, one of the points (say *A*) is the instrument station where a theodolite is set up, and the other (say *B*) is a shore signal or any other prominent object. At the instant the sounding is taken at *P*, the angle α at *A* is measured with a theodolite while the angle β at the boat is measured with the help of a sextant. Knowing the distance *d* between the two points *A* and *B* by ground survey, the position of *P* can be located by calculating the two co-ordinates *x* and *y*.

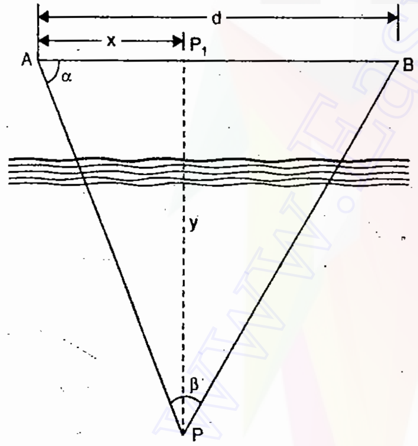

FIG. 6.12. LOCATION BY ONE ANGLE FROM THE SHORE AND THE OTHER FROM THE BOAT.

8. Location by Intersecting Ranges

This method is used when it is required to determine by periodical sounding at the same points, the rate at which silting or scouring is taking place. This is very essential on the harbours and reservoirs. The position of sounding is located by the intersection of two ranges, thus completely avoiding the angular observations. Suitable signals are erected at the shore. The boat is rowed along a range perpendicular to the shore and soundings are taken at the points in which inclined ranges intersect the range, as illustrated in Fig. 6.13. However, in order to avoid the confusion, a definite system of flagging the range poles is necessary. The position of the range poles is determined very accurately by ground survey.

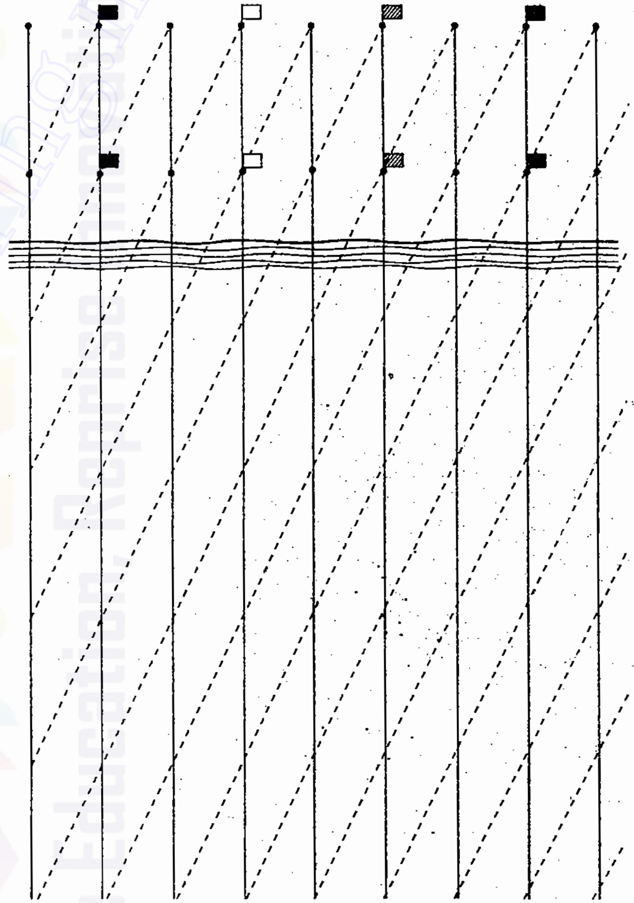

FIG. 6.13. LOCATION BY INTERSECTING RANGES.

D_B by the microwave system (see chapter 15), a fix is obtainable since the three sides of the triangle *ABP* are now known. It must be borne in mind that sloping distances are measured, and that '*line of sight*' conditions must be satisfied when selecting the shore stations. The transmissions should clear the sea surface by at least 3 m, and well conditioned triangles should be sought for accuracy of fix. With the best conditions, an accuracy of ± 0.1 m is claimed.

Fig. 6.16 (*a*) shows the master unit of Tellurometer MRD1 while Fig. 6.16 (*b*) shows the remote unit of Tellurometer MRD1, placed at the shore station To measure a single range, two instruments, one the *master*, being on the vessel, and second the *remote*, at the shore station, whilst a *third unit* the *master antenna* completes the basic system. The master unit contains all the required circuitory to produce two sets of range information. The *master antenna* unit is connected by cable to the master unit and the two can be upto 30 m apart.

6.6. REDUCTION OF SOUNDINGS

The *reduced soundings* are the reduced levels of the sub-marine surface in terms of the adopted datum. When the soundings are taken, the depth of water is measured with reference to the existing water level at that time. If the gauge readings are also taken at the same time, the soundings can be reduced to a common unvarying datum. The datum most commonly adopted is the '*mean level of low water of spring tides*' and is written either as L.W.O.S.T. (low water, ordinary spring tides) or M.L.W.S. (mean low water springs). For reducing the soundings, a correction equal to the difference of level between the actual water level (read by gauges) and the datum is applied to the observed soundings, as illustrated in the table given below :

Gauge Reading at L.W.O.S.T. = 3.0 m.

	Gauge (*m*)	*Distance* (*m*)	*Sounding* (*m*)	*Correction* (*m*)	*Reduced sounding* (*m*)	*Remarks*
8.00 A.M.	3.5	10	2.5	− 0.5	2.0	
		20	3.2		2.7	
		30	3.9		3.4	
		40	4.6		4.1	
8.10 A.M.	3.5	50	5.3	− 0.5	4.8	
		60	5.4		4.9	
		70	5.1		4.6	
		80	4.7		4.2	
		90	3.6		3.1	
8.20 A.M.	3.5	100	2.1	−0.5	1.6	

6.7. PLOTTING OF SOUNDINGS

The method of plotting the soundings depends upon the method used for locating the soundings. If the soundings have been taken along the range lines, the position of shore signals can be plotted and the sounding located on these in the plan. In the fixes

by angular methods also, the plotting is quite simple, and requires the simple knowledge of geometry. However, if the sounding has been located by two angles from the boat by observations to three known points on the shore, the plotting can be done either by the mechanical, graphical or the analytical solution of the three-point problem.

THE THREE POINT PROBLEM

Statement *: Given the three shore signals A, B and C, and the angles α and β subtended by AP, BP and CP at the boat P, it is required to plot the position of P (Fig. 6.17).*

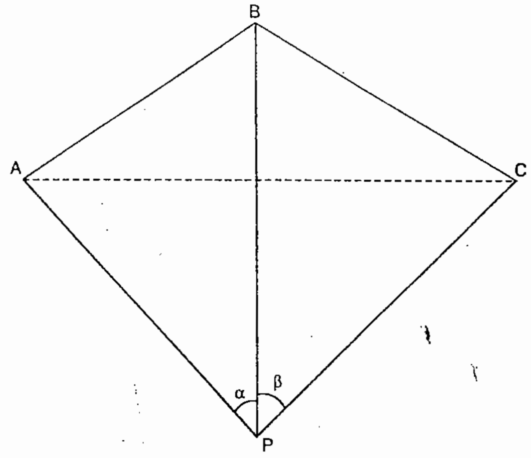

FIG. 6.17. THE THREE-POINT PROBLEM.

1. Mechanical Solution

(*i*) *By Tracing Paper*

Protract angles α and β between three radiating lines from any point on a piece of tracing paper. Plot the positions of signals *A*, *B*, *C* on the plan. Applying the tracing paper to the plan, move it about until all the three rays simultaneously pass through *A*, *B* and *C*. The apex of the angles is then the position of *P* which can be pricked through.

(ii) By Station Pointer : (Fig. 6.18)

The station pointer is a three-armed protractor and consists of a graduated circle with fixed arm and two movable arms to the either side of the fixed arm. All the three arms have bevelled or fiducial edges. The fiducial edge of the central fixed arm corresponds to the zero of the circle. The fiducial edges of the two moving arms can be set to any desired reading and can be clamped in position. They are also provided with verniers and slow motion screws to set the angle very precisely. To plot position of *P*, the movable arms are clamped to read the angles α and β very precisely. The station pointer is then moved on the plan till the three fiducial edges simultaneously touch *A*, *B* and *C*.

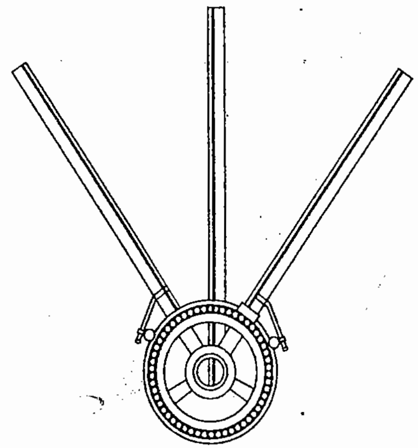

FIG. 6.18. STATION POINTER.

The centre of the pointer then represents the position of *P* which can be recorded by a prick mark.

Hence $\angle adb = \angle apd = \alpha$

and $\angle bpc = \angle bec = \beta$

The problem is, however, indeterminate, if the points A, B, C and P are concyclic.

3. Analytical Solution

In Fig. 6.22, let A, B, and C be the shore signals whose position is known. Let α and β be the observed angles at P.

Let $\angle BAP = x$; $\angle BCP = y$; $\angle ABC = z$

a = distance BC

b = distance AC

and c = distance AB.

Now $x + y = 360° - (\alpha + \beta + z) = \theta$ (say) ...(1) ...(6.5)

Since α, β and z are known, θ can be calculated.

From the triangle PAB,

$$PB = \frac{c}{\sin\alpha}\sin x$$

From the triangle PCB,

$$PB = \frac{a}{\sin\beta}\sin y$$

Equating the two, we get

$$c\cdot\frac{\sin x}{\sin\alpha} = a\cdot\frac{\sin y}{\sin\beta}$$

$$\sin y = \frac{c\sin x\sin\beta}{a\sin\alpha}$$

But $y = \theta - x$, from (1)

Hence, $$\sin(\theta - x) = \frac{c\sin x\sin\beta}{a\sin\alpha}$$

or $$\sin\theta\cos x - \cos\theta\sin x = \frac{c\sin x\sin\beta}{a\sin\alpha}$$

Dividing both the sides by $\sin\theta\sin x$, we get

$$\cot x - \cot\theta = \frac{c\sin\beta}{a\sin\alpha\sin\theta}$$

or $$\cot x = \cot\theta + \frac{c\sin\beta}{a\sin\alpha\sin\theta}$$

or $$\cot x = \cot\theta\left\{1 + \frac{c\sin\beta\sec\theta}{a\sin\alpha}\right\} \quad ...(2)$$

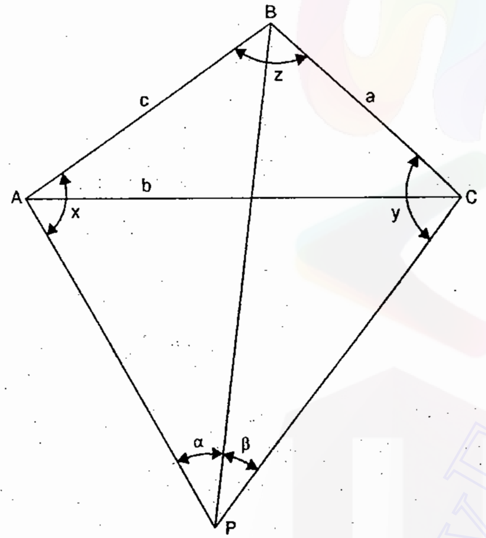

FIG. 6.22.

The value of x can, thus, be calculated from (2). Knowing the angle x, the angle y can be calculated from the relation $y = \theta - x$

Again, from ΔABP,

$$AP = \frac{c}{\sin\alpha}\sin\angle ABP = \frac{c}{\sin\alpha}\sin(180° - x - \alpha)$$

$$= \frac{c}{\sin\alpha}\cdot\sin(x + \alpha) \quad ...(6.6)$$

and $$BP = \frac{c}{\sin\alpha}\sin x$$

Similarly, from ΔBPC,

$$BP = \frac{a}{\sin\beta}\sin y \quad ...(6.7)$$

and $$CP = \frac{a}{\sin\beta}\sin CBP$$

$$= \frac{a}{\sin\beta}\sin(180° - y - \beta)$$

$$= \frac{a}{\sin\beta}\sin(y + \beta) \quad ...(6.8)$$

Calculating AP, BP and CP, the position of P can be plotted.

Three cases may arise according to the position of the boat (P) with respect to the ground signals A, B and C :

Case 1. When B and P are to the opposite sides of the line AC (Fig. 6.22).

Case 2. When B and P are to the same side of the line AC [Fig. 6.23 (*a*)].

Case 3. When P is within the triangle ABC [Fig. 6.23 (*b*)].

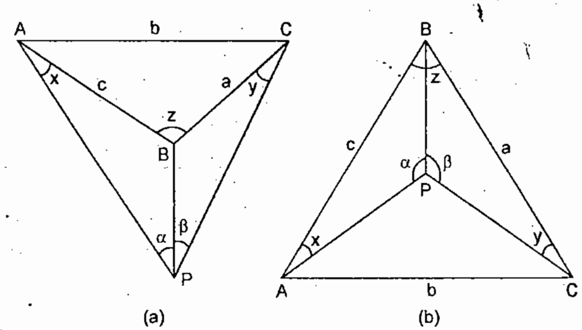

FIG. 6.23

In case (2), Fig. 6.23 (*a*), we have

$$x + y = z - (\alpha + \beta) = \theta \quad ...(6.9)$$

In case (3), Fig. 6.20 (*b*), we have

$$x + y = 360° - (\alpha + \beta + z) = \theta \quad ...(6.10)$$

Knowing the value of θ, the value of x can be calculated from Eq. (2) derived above.

Example 6.1. *A, B and C are three visible stations in a hydrographical survey. The computed sides of the triangle ABC are : AB, 1130 m ; BC, 1372 m ; and CA, 1889 m. Outside this triangle (and nearer to AC), a station P is established and its position is to be found by three point resection on A, B and C, the angles APB and BPC being respectively 42° 35′ and 54° 20′.*

Determine the distances PA and PC.

D. The tide at *A* is called the *superior lunar tide* or tide of moon's upper transit, while tide at *B* is called *inferior* or *antilunar tide.*

Now let us consider the earth's rotation on its axis. Assuming the moon to remain stationary, the major axis of lunar tidal equilibrium figure would maintain a constant position. Due to rotation of earth about its axis from west to east, once in 24 hours, point *A* would occupy successive positions *C*, *B* and *D* at intervals of 6 h. Thus, point *A* would experience regular variation in the level of water. It will experience high water (tide) at intervals of 12 h and low water midway between. This interval of 6 h variation is true only if moon is assumed stationary. However, in a lunation of 29.53 days the moon makes one revolution relative to sun from the new moon to new moon. This revolution is in the same direction as the diurnal rotation of earth, and hence there are 29.53 transits of moon across a meridian in 29.53 mean solar days. This is on the assumption that the moon does this revolution in a plane passing through the equator. Thus, the interval between successive transits of moon or any meridian will be 24 h, 50.5 m. Thus, the average interval between successive high waters would be about 12 h 25 m. The interval of 24 h 50.5 m between two successive transits of moon over a meridian is called the *tidal day*.

2. The Solar Tides

The phenomenon of production of tides due to force of attraction between earth and sun is similar to the lunar tides. Thus, there will be *superior solar tide* and an *inferior* or *anti-solar tide*. However, sun is at a large distance from the earth and hence the tide producing force due to sun is much less.

Let M_E = mass of earth

M_M = Mass of moon

M_S = mass of sun

D_M = mean distance from the centre of earth to the centre of the moon

D_S = mean distance from the centre of earth to the centre of the sun

R = radius of earth

K = constant of gravitation

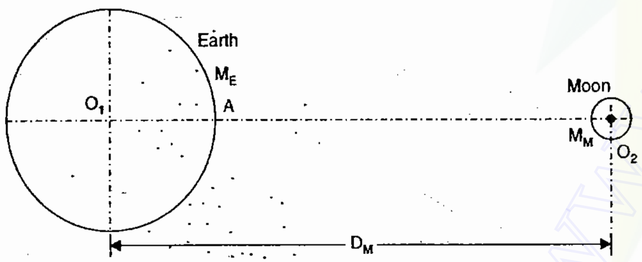

FIG. 6.25.

Consider point *A*, facing the moon. Tide producing force F_M of the moon on unit mass at *A* is given by

$$F_M = KM_M\left[\frac{1}{(D_M-R)^2} - \frac{1}{D_M^2}\right] = KM_M\left[\frac{(D_M^2 - D_M^2 - R^2 + 2D_MR)}{(D_M-R)^2\,D_M^2}\right]$$

$$= KM_M\left[\frac{R\,(2D_M - R)}{(D_M-R)^2\,D_M^2}\right]$$

Assuming radius of the earth *R* very small in comparsion to the distance between earth and moon, we have

$$F_M \approx KM_M\left(\frac{2R}{D_M^3}\right) \qquad ...(6.11)$$

Similarly, tide producing force F_S of the sun on unit mass at *A* is given by

$$F_S \approx KM_S\left(\frac{2R}{D_S^3}\right) \qquad ...(6.12)$$

Hence $$\frac{F_S}{F_M} = \frac{M_S}{M_M}\left(\frac{D_M}{D_S}\right)^3 \qquad ...(6.13)$$

Now mass of sun, $M_S = 331{,}000\ M_E$

Mass of moon, $M_M = \frac{1}{18} M_E$

D_S = 149,350,600 km ; D_M = 384,630 km

Substituting the values in Eq. 6.13, we get

$$\frac{F_S}{F_M} = 0.458 \qquad ...(6.14)$$

Hence solar tide = 0.458 lunar tide.

3. Combined effect : Spring and neap tides

Equation 6.14 shows that the solar tide force is less than half the lunar tide force. However, their combined effect is important, specially at the new moon when both the sun and moon have the same celestial longitude, they cross a meridian at the same instant. Assuming that both the sun and moon lie in the same horizontal plane passing through the equator, the effects of both the tides are added, giving rise to *maximum* or *spring tide of new moon*. The term 'spring' does not refer to the season, but to the springing or waxing of the moon. After the new moon, the moon falls behind the sun and crosses each meridian 50 minutes later each day. In after $7\frac{1}{2}$ days, the difference between longitude of the moon and that of sun becomes 90°, and the moon is in quadrature as shown in Fig. 6.26 (*b*). The crest of moon tide coincides with the trough of the solar tide, giving rise to the *neap tide of the first quarter*. During the neap tide, the high water level is below the average while the low water level is above the average. After about 15 days of the start of lunation, when full moon occurs, the difference between moon's longitude and of sun's longitude is 180°, and the moon is in opposition. However, the crests of both the tides coincide, giving rise to *spring tide of full moon*. In about 22 days after the start of lunation, the difference in longitudes of the moon and the sun becomes

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(*a*) AGE OF TIDE

In the equilibrium theory, the earth is assumed to be enveloped with sea of uniform depth. This condition is fulfiled only in Southern Ocean extending southwards from about 40° S latitude. Therefore, it is only in this portion of ocean where equilibrium figure may be developed. *Primary tide waves* are, therefore, generated there and *derivative* or *secondary waves* are propagated into the Pacific, Atlantic and Indian Oceans. These derivative waves proceed in a general north and south direction, though their direction is influenced by the form of coast lines, and intervention of land masses. The velocity of wave travel may exceed 1000 km per hour, though it is less in shallow water. The amplitude, *i.e.*, the vertical range from crest to trough, is not more than 60 to 90 cm. *Due to the direction of propagation of tide wave, high or low water occurs at different times at various places on the same meridian.* Thus, the greatest spring tide arrives several tides after transits at new or full moon. *The time which elapse between the generation of spring tide and its arrival at the place is called the age of the tide at that place.* The age of tide varies for different places, upto a maximum of 3 days, and is reckoned to the nearest $\frac{1}{4}$ day. It is obtained as the mean of several observations. The *age of the tide* is one of the non-harmonic constants and its values for different ports are published in section I of part II of the Admiralty Tide Table.

(*b*) LUNITIDAL INTERVAL

Lunitidal interval is the time interval that elapses between the moon's transits and the occurrence of the next high water. The value of lunitidal interval is found to vary because of existence of priming and lagging. The values of lunitidal interval can be *observed* and if they are plotted for a fortnight against the times of moon's transits, a curve such as shown in Fig. 6.28 is obtained. Such a curve has approximately the same form for each fortnight and hence may be used for the rough prediction of time of tide at a place. The time of transit of moon at Greenwich is given in the *Nautical Almanac*. The time of transit at the given place can be derived by adding 2 m for every hour of west longitude

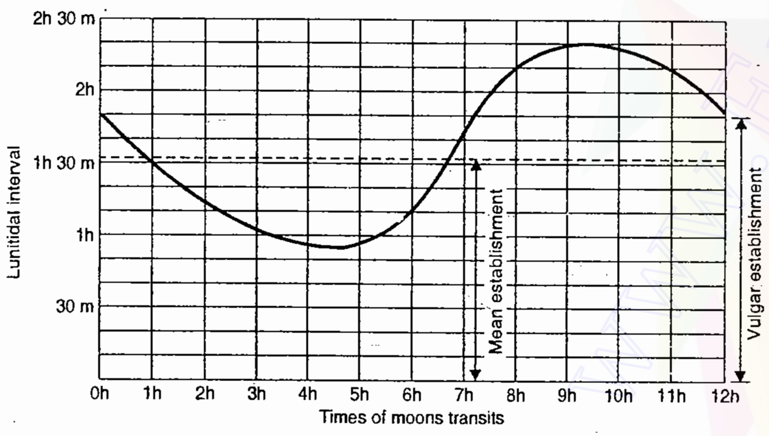

FIG. 6.28. LUNITIDAL INTERVAL.

and subtracting 2 m for every hour of east longitude of the place, to the time of transit at Greenwich. Knowing the time of moon's transit at the place, lunitidal interval is obtained from the curve (Fig. 6.28) and added to the time of preceding transit to know the approximate time of occurence of next high water at the place.

(*c*) MEAN ESTABLISHMENT

The average value of lunitidal interval at a place is known as its *mean establishment*, as shown by dotted line in Fig. 6.28. If the value of mean establishment is known, the lunitidal interval and hence the time of high water at a place can be estimated, provided the age of the tide at the place is also known. The procedure of determination is as follows :

1. Find from the charts, the age of tide and mean establishment for the place.

2. Knowing the hour of moon's transit at the place, on the day in question, determine the time of moon's transit on the day of generation of the tide (the day of generation of tide is equal to the day in question minus the age of the tide).

3. Corresponding to the time of transit of moon on the day of generation of tide (determined in step 2), find out the amount of priming or lagging correction from the table given below :

Hour of moon's transit	0	1	2	3	4	5	6	7	8	9	10	11	12
Correction in minutes	0	– 16	– 31	– 41	– 44	– 31	0	+31	+44	+41	+31	+16	0

4. Add algebraically the priming or lagging correction to the mean establishment to get the lunitidal interval for the day in question.

5. Add the lunitidal interval to the time of moon's transit on the day in question, to get the approximate time of high water.

Example 6.2. *Find the time of afternoon high water at a place with the following data* :

(*i*) *time of moon's transit on that day* = *4 h 40 m*

(*ii*) *mean establishment* = *3 h 10 m*

(*iii*) *age of tide* = *2 days.*

Solution : We know that moon falls behind the sun at the rate of 50 m per day. Hence at the birth of tide, 2 days earlier, the time of moon's transit = 4 h 40 m – 2 × 50 m = 3 h 0 m.

From the table, corresponding to the time of transit of 3 h 0 m the correction for priming = – 41 m.

∴ Lunitidal interval = mean establishment – correction

= 3 h 10 m – 41 m = 2 h 29 m

∴ Time of high water

= Time of moon's transit + Lunitidal interval

= 4 h 40 m + 2 h 29 m = 7 h 09 m = **7.09 P.M.**

1. Non-registering type of tide gauges

(*i*) Staff gauge (*ii*) Float gauge (*iii*) Weight gauge.

2. Self-registering type tide gauges.

Non-registering type tide gauges are those in which an attendant is required to take reading from time to time. In the self-registering type, no attendant is required.

1. Staff gauge. [Fig. 6.29 (*a*)]. This is the simplest type of gauge, which is firmly fixed in vertical position. The gauge consists of a board, about 15 cm to 25 cm broad, and of suitable height, having graduation to a least count of 5 to 10 cm. The zero of gauge is fixed at the predetermined level. Alternatively, the elevation of zero of the level may be determined by levelling. The staff is read directly, from some distance.

2. Float gauge. [Fig. 6.29 (*b*)]. On account of the wash of the sea, it may be difficult to read a staff gauge accurately. In that case a float gauge shown in Fig. 6.29 (*b*) may be used. It consists of a simple float with a graduated vertical rod, enclosed in a long wooden box of 30 cm ×30 cm square section. The box has few holes at the bottom through which water may enter and lift the float. The reading are taken through a slit window against some suitable index.

3. Weight gauge. The weight gauge, shown in Fig. 6.29 (*c*) consists of a weight attached to a wire or chain. The chain passes through a pulley, along the side of a graduated board. The weight is lowered to touch the water surface and the reading is taken against

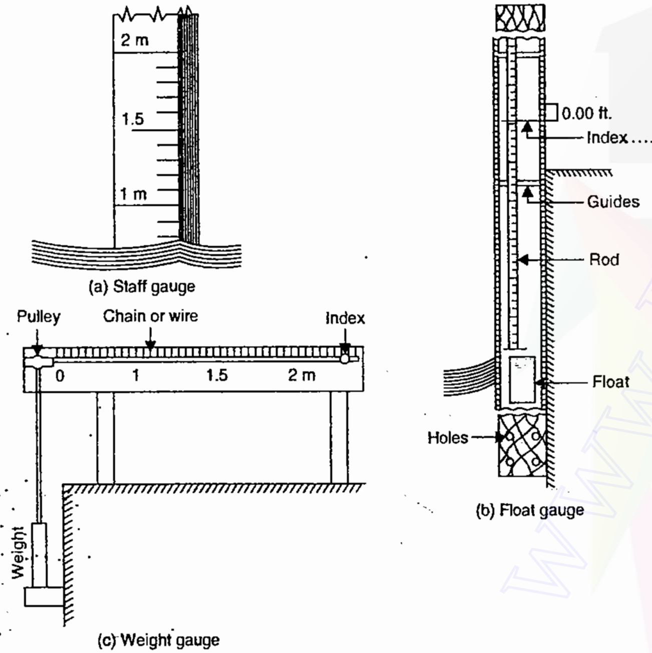

FIG. 6.29. NON-SELF REGISTERING TYPE TIDE GAUGES.

an *index* attached to the chain. The reduced, level of the water surface corresponding to the zero reading is determined earlier, by attaching the foot of the staff against the bottom of the suspended weight and taking its reading with a level, when the index of the chain is against zero reading.

Self-registering gauges

A self-registering gauge automatically registers the variation of water level with time. It essentially consists of a float protected from wind, waves etc. The float has attached to it a wire or cord which passes over a wheel (called the float wheel) and is maintained at constant tension by some suitable arrangement. The movement of the float is transferred to the wheel which reduces it through some gear system, and is finally communicated to a pencil attached to a lever. The movement of the pencil, corresponding to the movement of the float is recorded on a graph paper wound round a drum which is rotated at constant speed by some suitable clock-work. Thus graphical record of movement of the float with time is recorded automatically. Such a gauge is usually housed in a well constructed under a building so that effect of wind and other disturbances is reduced.

6.11. MEAN SEA LEVEL AS DATUM

For all important surveys, the datum selected is the mean sea level at a certain place. *The mean sea level may be defined as the mean level of the sea, obtained by taking the mean of all the height of the tide, as measured at hourly intervals over some stated period covering a whole number of complete tides,* The mean sea level, defined above shows appreciable variations from day to day, from month to month and from year to year. Hence the period for which observations should be taken depends upon the purpose for which levels are required. The daily changes in the level of sea may be more. The monthly changes are more or less periodic. The mean sea level in a particular month may be low while it may be high in some other months. Mean sea level may also show appreciable variations in its annual values. Due to variations in the annual values and due to greater accuracy needed in modern geodetic levelling, it is essential to base the mean sea level on observations extending over a period of about 19 years. During this period, the moon's nodes complete one entire revolution. The height of mean sea level so determined is referred to the datum of tide gauge at which the observations are taken. The point or place at which these observations are taken is known as a tidal station. If the observations are taken on two stations, situated say at a distance of 200 to 500 kms on an open coast, one of the station is called primary tidal station while the other is called secondary tidal station. Both the stations may then be connected by a line of levels.

PROBLEMS

1. In a harbour development scheme at the mouth of a tidal river, it has been found necessary to take soundings in order to buoy the navigation channel.

Explain clearly how you would determine the levels of points on the river bed and fix the positions of the soundings.

(a) by use of sextant in a boat :

(b) by use of the theodolite on the shore. (*U.L.*)

is attached to the main telescope. The instrument is generally mounted on an extension leg tripod. For ease in reading the vertical angles by the transitman, the vertical circle is sometimes graduated on the edge instead of the side. The centre point of the transit is definitely marked on the top of the telescope.

In places where a tripod cannot be used, suspension type mine transit is employed. Fig. 7.2 shows a typical *suspension theodolite* by Funnel Kassel. The instrument is supported on a bracket being screwed horizontally into adjacent mine timbers. The horizontal circle along with its verniers are on the top of the telescope and the vertical circle, and hence the use of auxiliary telescope is obviated. The horizontal circle is 9 cm ($3\frac{1}{2}$ in.) in diameter, the vertical circle 7 cm ($2\frac{3}{4}$ in.) in diameter and the whole instrument weighs only 5.5 lb. If required, the instrument can also be supported on tripod, and for this purpose the vertical circle and the telescope are provided with sensitive *reversion spirit levels.*

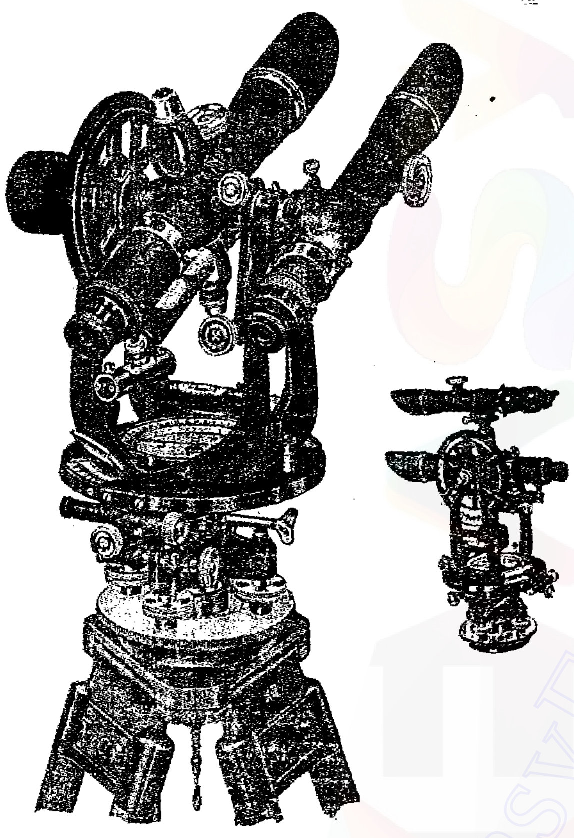

FIG. 7.1. MINING TRANSIT SHOWING THE AUXILIARY TELESCOPE IN BOTH POSITIONS.

The Correction for Side-Telescope Horizontal Angles

The side telescope is fitted at a slight distance away from the main telescope ; this eccentricity affects the horizontal angles measured with the auxiliary telescope.

Thus, in Fig. 7.3, O is the centre of the main telescope and C is the centre of the auxiliary telescope at distance OC from the main. The circle denotes the locus of the centre C of the auxiliary telescope when the instrument is revolved in azimuth for the measurement of the horizontal angle. A and B are the two points, and it is required to measure the true horizontal angle θ between these two subtended at the centre of the instrument. When the point A is sighted through the auxiliary telescope, the line of sight CA is tangential to the circle, reading on the horizontal circle being zero-zero. To sight the point B, the instrument is rotated in azimuth, so that the centre C of the auxiliary telescope comes to a position C', the line $C'B$ being the line of sight tangential to the

circle. The measured angle θ' is then the angle through which the line of sight has been rotated.

Evidently, the correct angle θ is given by the relation

$$\theta + \alpha = \theta' + \beta$$

or $$\theta = \theta' + (\beta - \alpha) \quad ...(i)$$

where $$\alpha = \sin^{-1}\frac{OC}{AO} = \tan^{-1}\frac{OC}{AC} \quad ...(ii)$$

and $$\beta = \sin^{-1}\frac{OC'}{OB} = \tan^{-1}\frac{OC'}{BC'} \quad ...(iii)$$

Thus, the correct angle θ can be computed by applying, algebraically, the correction $(\beta - \alpha)$ to the observed angle θ'. However, the observed angle can be directly equal to the true angle, if :

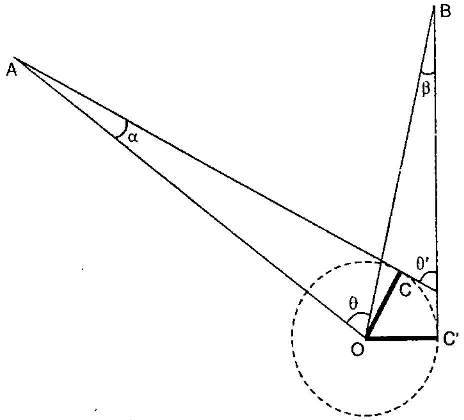

FIG. 7.3. CORRECTION FOR SIDE-TELESCOPE HORIZONTAL ANGLES.

(*i*) both the sights OA and OB are of equal length, thus making α and β equal;

(*ii*) by taking both face observations, one with telescope direct and the other with telescope reversed, the mean reading being used to give the true angle.

The above correction for horizontal angle is, however, not necessary if the auxiliary telescope is fitted on the top of the main telescope.

The Correction for Top-Telescope Vertical Angles

If the auxiliary telescope is fitted to the side of the main telescope, the horizontal angles need correction while the vertical angles do not need the correction. However, if the auxiliary telescope is fitted to the top of the main telescope, the vertical angle needs correction.

Thus, in Fig. 7.4, A is the point to be observed, B and C are the centres of the main and auxiliary telescopes, respectively. α' is the vertical angle measured with the aux-

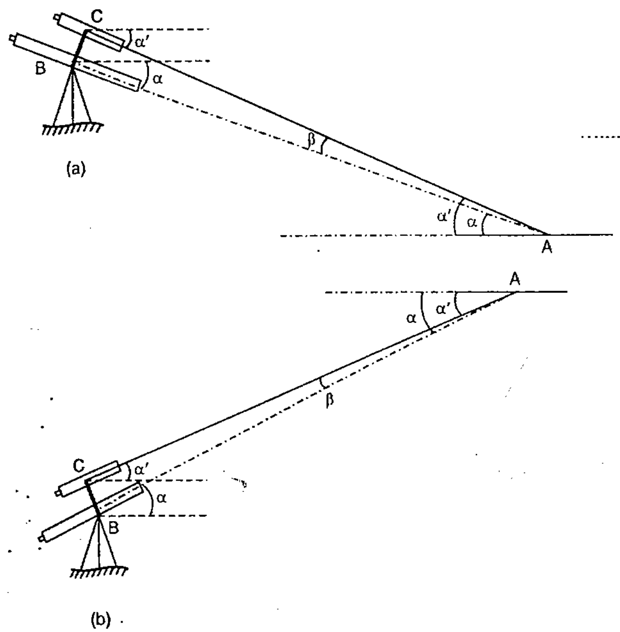

FIG. 7.4. CORRECTION FOR TOP TELESCOPE VERTICAL ANGLE.

of the instrument to the mark made on the plumb line. The height of point (H.P.) is considered to be positive when it is measured above the floor and is negative when measured below the roof.

The difference in height between the instrument station and the point sighted can then be calculated.

Thus in Fig. 7.5 (*a*), the difference in height (*H*) is given by

$$\text{H.I.} + H = \text{H.P.} + V \quad \text{or} \quad H = (\text{H.P.} - \text{H.I.}) + V \qquad \ldots(i)$$

In Fig. 7.5 (*b*) $\quad H = (\text{H.P.} + \text{H.I.}) + V \qquad \ldots(ii)$

In both the expressions, $V = L \sin \alpha \qquad \ldots(iii)$

where L is the measured inclined distance.

The horizontal distance D is given by

$$D = L \cos \alpha \qquad \ldots(iv)$$

7.5 TUNNEL ALIGNMENT AND SETTING OUT

Tunnels are usually constructed in the mountainous districts when a section of the road is subject to avalanches, and they not only protect the road, but serve as places of refuge for travellers. In cities, tunnels are sometimes employed for underground railways or roads to relieve the traffic congestion. Tunnels are also used in mining operations. They are sometimes constructed under rivers, where the construction of a bridge is considered undersirable or impracticable. In the case of mountainous railways, they are employed either to have the shortest route between two points or where the cost of cutting is expensive.

Tunnels are entered either on the level or by inclines. For the purpose of facilitating the construction of the operations, and for checking the accuracy of the alignment and levels, *vertical shafts* are often used. For proper drainage, a tunnel may be made slightly inclined to the horizontal, the gradient being in one direction if the tunnel is short, and in both the directions from the centre if it is long.

The setting out of a tunnel comprises four operations :

(*i*) Surface surveys or setting out

(*ii*) The connection of surface and underground surveys

(*iii*) Setting out underground

(*iv*) Level in tunnels.

Surface Alignment and Measurements

The centre line of the proposed tunnel should be accurately marked on the surface of the ground whenever it is possible. In the case of high snow-clad mountain ranges, this may not be possible. In such cases, the centre line must at least be set out over the contiguous shafts near the ends of the tunnel. The shaft at the extremities of the tunnel must be connected by triangulation or precise traverse with greatest possible care to ensure accuracy.

To set out the centre line on the surface, specially for a tunnel straight in plan, a suitable point is chosen on the centre line from which both the extremities can be commanded. An observatory is erected, and an instrument is erected in it, the instrument being centered exactly over the centre line. Two points in the preliminary setting out are taken as fixed, one at the observatory and the other being some conveniently situated point in the line. From the main station at the observatory, the line is set on suitable permanent objects at the ends of the tunnel and near to each shaft. In towns, the centre line is marked on the surface by driving spikes or wedges of iron, the centre line being marked on these with a steel punch. In order that these may not be replaced, measurements are taken from the corners of buildings or permanent marks. In the coverby, the centre line on the surface may be marked by stout pegs having brass nails driven into them, the exact line being marked on these with a steel punch.

The exact horizontal distance between two terminals of the tunnel is then measured. An accurate steel tape must be used, and all the corrections must be applied to the observed distance to get the correct distance. For very accurate results, the distance may be measured by the usual equipment used for base measurement in triangulation. The corrections for tension, temperature, grade, sag and absolute length are applied in the usual way to obtain the true horizontal length of the centre line. In case it is not possible to measure the distance directly due to obstacles etc. then length and direction of the centre line of the tunnel must be obtained by precise traversing or triangulation.

Transferring Surface Line Down Shafts and Setting Out Underground Line

After having fixed the centre line on the surface the setting out of underground line can be done by transferring surface line down the shafts wherever they are vertical. The points are selected in the centre line near the mouth of each shaft in a position clear of the works in connection with the sinking operations. A theodolite is then set over one of these points on the surface and the line of sight is directed towards the other point. The line is then set out accurately on two baulks of timber kept across the shaft perpendicular to the centre line and very near to the two edges of the shaft. From these marked points on the baulks, two plumb lines are suspended down the shaft [Fig. 7.6 (*a*)].

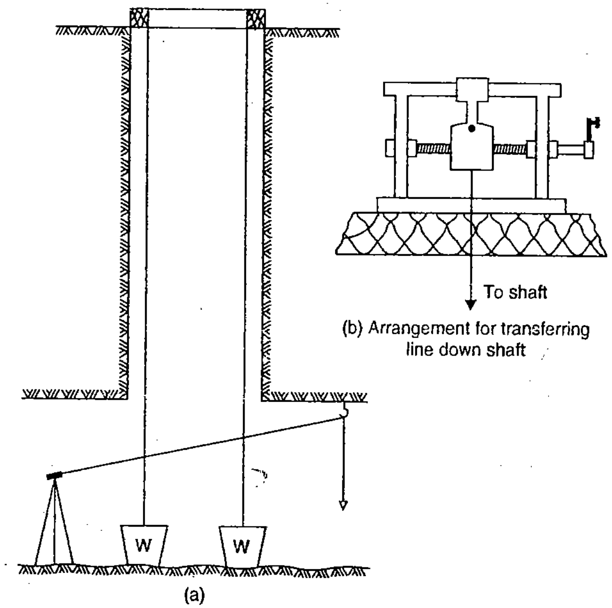

FIG. 7.6. TRANSFERRING SURFACE LINE UNDERGROUND.

The theodolite is then transferred underground and set exactly in line with the two suspended wires. The line joining these wires and hence the line of sight of that theodolite gives the direction of the centre line of the tunnel underground. The line is then set with

Solution. (Fig. 7.7)

$$\sin \alpha = 3.092 \frac{\sin 16' 12''}{4}$$

$$\therefore \quad \alpha = 948'' = 15' 48''$$

$\therefore$ Azimuth of $CA = 80° 40' 15'' + 15' 48'' = \mathbf{80° 56' 03''}$

and $x = AC \sin \alpha = AC.\ \alpha$, when α is in radians

$$= (4 + 3.902) \frac{948}{206265} = \mathbf{0.0363 \ m.}$$

7.6. SUSPENSION MINING COMPASS

It basically consists of a compass box connected with a suspension frame. The string of the suspension frame is set along the dip of the strata and its slope is measured with the help of a large diameter clinometer with plumb bob. Fennel Kessel manufactures two variations: (*i*) Kassel type and (*ii*) Freiberg type.

Fig. 7.9 shows the photograph of the Kassel type mining compass. The compass is connected by *hinges* with suspension frame which has the advantage of easy packing and taking less space in the container. The clamping screw of the knife edged magnetic needle is placed on the brim of the compass ring. The horizontal circle is divided at intervals of 1 degree and figured every 10 degrees. The clinometer, made from light metal, has diameter of 9.4 inch and is graduated to $\frac{1}{3}$ degree.

FIG. 7.10. FREIBERG TYPE MINING SUSPENSION COMPASS WITH CLINOMETER.

Freiberg type compass with clinometer is shown in Fig. 7.10. The functions of the mining compass of Freiberg type are exactly the same as of the Kassel type. Its mechanical features depart in two things from the Kassel type, *viz.*, the rigid connection of the compass suspension with the frame and the clamping screw to be placed centrically, under the compass box.

7.7. BRUNTON'S UNIVERSAL POCKET TRANSIT

Brunton's Universal pocket transit is one of the most convenient and versatile instrument for preliminary surveying on the surface or underground. It is suitable for forestry, geological and mining purposes, and for simple contour and tracing work. The main part of Brunton Pocket Transit is the magnetic compass with a 5 cm long magnetic needle pivoting on an agate cap. Special pinion arrangement provides for the adjustment of the local variation of the declination with a range of ± 30°. For accurate centring purposes a circular spirit bubble is built in. A clinometer connected with a tubular spirit bubble covers measurement of vertical angles within a range of ± 90° . Fig. 7.11 shows the photograph of Brunton Universal pocket transit along with box containing various accessories..

The Brunton pocket transit comprises a wide field of application for which it is equipped with the following special accessories :

1. ***Camera tripod*** for measurement of horizontal and vertical angles.
2. ***Plane table*** for using the compass as an alidade.
3. ***Protractor base plate*** for protracting work in the field or in the office.
4. ***Suspension plate*** for use of the instrument as a mining compass.
5. ***Brackets*** for suspension plate

Measurement of horizontal angles

Horizontal and vertical angles can be measured by using the camera tripod with the ball joint. For measuring horizontal angles, the compass box has to be screwed on the ball joint until the locking pin will fit into the socket which is imbedded in the compass case. For more precise centring, a plumb bob can be fastened at the plumb hook of the tripod. Accurate setting of the instrument is accomplished with a circular spirit bubble. The north end of the needle indicates magnetic bearing on the compass graduation.

Measurement of vertical angle

For measuring vertical angles, the compass has to be fitted in the ball joint. The observations have to be carried out with completely opened mirror by sighting through the hole of the diopter ring and the pointer. Before readings can be taken, the tubular bubble which is connected with the clinometer arm has to be centered by turning the small handle mounted at the back of the compass. Using the instrument in this vertical position, it is necessary to lock the needle to prevent the agate cap and the pivot from being damaged.

Use as a mining compass

Brunton compass can be fitted on the suspension plate and be used as mining compass. The compass is correctly positioned on the plate when the locking pin fits into the socket. Then, the North-South line of the compass is parallel to the longitudinal axis of the suspension plate.

For vertical angle measurements, the hook hinges have be fitted. The brackets prevent the suspension outfit from sliding along the rope. Before readings of vertical circle can be taken, accurate centring of the clinometer arm bubble is necessary.

Use with plane table

The compass in connection with the protector base plate can be used for protecting work in the field or in the office. The parallelism of the base plate edges and the line of sight of the compass is secured when the locking pin on the plate fits accurately into the socket. This combination gives the possibility to employ the compass as an alidade for minor plane table surveys.

7.8. MOUNTAIN COMPASS-TRANSIT

A mountain compass-transit (also known as compass theodolite) basically consists of a compass with a telescope. Both these are mounted on a levelling head which can be mounted on a tripod. For movement of the instrument about vertical axis, a clamp and tangent screw is used. For measurement of vertical angles, the telescope can rotate about the trunnion axis, provided with a clamp and slow motion screw. The instrument is levelled

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8.2. CLASSIFICATION OF TRIANGULATION SYSTEM

The basis of the classification of triangulation figures is the accuracy with which the length and azimuth of a line of the triangulation are determined. Triangulation systems of different accuracies depend on the extent and the purpose of the survey. The accepted grades of triangulation are :

(1) First order or Primary Triangulation
(2) Second order or Secondary Triangulation
(3) Third order or Tertiary Triangulation

(1) First-Order or Primary Triangulation

The first order triangulation is of the highest order and is employed either to determine the earth's figure or to furnish the most precise control points to which secondary triangulation may be connected. The primary triangulation system embraces the vast area (usually the whole of the country). Every precaution is taken in making linear and angular measurements and in performing the reductions. The following are the general specifications of the primary triangulation :

1.	Average triangle closure	: Less than 1 second
2.	Maximum triangle closure	: Not more than 3 seconds
3.	Length of base line	: 5 to 15 kilometres
4.	Length of the sides of triangles	: 30 to 150 kilometres
5.	Actual error of base	: 1 in 300,000
6.	Probable error of base	: 1 in 1,000,000
7.	Discrepancy between two measures of a section	: 10 mm $\sqrt{\text{kilometres}}$
8.	Probable error of computed distance	: 1 in 60,000 to 1 in 250,000
9.	Probable error in astronomic azimuth	: 0.5 seconds

(2) Second Order or Secondary Triangulation

The secondary triangulation consists of a number of points fixed within the framework of primary triangulation. The stations are fixed at close intervals so that the sizes of the triangles formed are smaller than the primary triangulation. The instruments and methods used are not of the same utmost refinement. The general specifications of the secondary triangulation are :

1.	Average triangle closure	: 3 sec
2.	Maximum triangle closure	: 8 sec
3.	Length of base line	: 1.5 to 5 km
4.	Length of sides of triangles	: 8 to 65 km
5.	Actual error of base	: 1 in 150,000
6.	Probable error of base	: 1 in 500,000
7.	Discrepancy between two measures of a section	: 20 mm $\sqrt{\text{kilometres}}$
8.	Probable error of computed distance	: 1 in 20,000 to 1 in 50,000
9.	Probable error in astronomic azimuth	: 2.0 sec.

(3). Third-Order or Tertiary Triangulation

The third-order triangulation consists of a number of points fixed within the framework of secondary triangulation, and forms the immediate control for detailed engineering and other surveys. The sizes of the triangles are small and instrument with moderate precision may be used. The specifications for a third-order triangulation are as follows :

1.	Average triangle closure	: 6 sec
2.	Maximum triangle closure	: 12 sec
3.	Length of base line	: 0.5 to 3 km
4.	Length of sides of triangles	: 1.5 to 10 km
5.	Actual error of base	: 1 in 75,0000
6.	Probable error of base	: 1 in 250,000
7.	Discrepancy between two measures of a section	: 25 mm $\sqrt{\text{kilometres}}$
8.	Probable error of computed distance	: 1 in 5,000 to 1 in 20,000
9.	Probable error in astronomic azimuth	: 5 sec.

8.3. TRIANGULATION FIGURES OR SYSTEMS

A *triangulation figure* is a group or system of triangles such that any figure has one side, and only one, common to each of the preceding and following figures. The common Figures or Systems are :

(1) Single chain of triangles [Fig. 8.1 (*a*)]
(2) Double chain of triangles [Fig. 8.1 (*b*)]
(3) Central point Figures [Fig. 8.1 (*c*)]
(4) Quadrilaterals [Fig. 8.1 (*d*)].

(i) Single chain of triangles :

This figure is used where a narrow strip of terrain is to be covered. Though the system is rapid and economical, it is not so accurate for primary work since the number of conditions to be fulfilled in the figure adjustment is relatively small. Also, it is not possible to carry the solution of triangles through the figures by two independent routes. If the accumulation of errors is not be become excessive, base lines must be introduced frequently.

(ii) Double chain of triangles :

It is used to cover greater area.

(iii) Centred figures :

Centred figures are used to cover area, and give very satisfactory results in flat country. The centred figures may be quadrilaterals, pentagons, or hexagons with central stations. The system provides the desired checks on the computations. However, the progress of work is slow due to more settings of the instrument.

(iv) Quadrilaterals :

The quadrilateral with four corner stations and observed diagonal forms the *best figures.* They are best suited for hilly country. Since the computed lengths of the sides can be the probable fraction error $\left(i.e., \dfrac{\delta a}{a}\right)$ in the side $\bar{a} = \pm \beta \sqrt{\cot^2 A + \cot^2 C}$

This is minimum when $\cot^2 A + \cot^2 C$ is minimum.

But $\qquad C = 180° - A - B = 180° - 2A$

$\therefore \qquad \cot^2 A + \cot^2 2A$ should be minimum.

Differentiating $\cot^2 A + \cot^2 2A$ with respect to A and equating it to zero, we get, after reduction,

$$4 \cos^2 A + 2 \cos^2 A - 1 = 0$$

From which $\quad A = 56° \ 14'$ approx.

Hence the best shape of a triangle is isosceles with base angles equal to 56° 14′. However, from practical considerations, an equilateral triangle is the most suitable. In general, however, triangles having an angle smaller than 30° or greater than 120° should be avoided.

Criterion of Strength of Figure

The strength of figure is a factor to be considered in establishing a triangulation system for which the computations can be maintained within a desired degree of precision. The U.S. Coast and Geodetic Survey has developed a very rapid and convenient method of evaluating the strength of a triangulation figure. The method is based on an expression for the square of the probable error (L^2), that would occur in the sixth place of the logarithm of any side, if the computations were carried from a known side through a single chain of triangles after the net had been adjusted for the side and angle conditions. The expression for L^2 is

$$L^2 = \frac{4}{3} d^2 \frac{D - C}{D} \Sigma \, [\delta_A^2 + \delta_A \delta_B + \delta_B^2] \qquad \ldots[8.1 \ (a)]$$

If R represents the terms in the equation affected by the shape of figure, then

$$R = \frac{D - C}{D} \Sigma \, [\delta_A^2 + \delta_A \delta_B + \delta_B^2] \qquad \ldots(8.1)$$

and

$$L^2 = \frac{4}{3} d^2 R \qquad \ldots[8.1 \ (b)]$$

where $\quad d$ = probable error of an observed direction, in seconds.

D = number of directions observed (forward and/or back), excluding those along the known or fixed line.

δ_A = difference per second in the sixth place of logarithm of the sine of the distance angle A of each triangle in the chain used.

δ_B = same as δ_A but for the distance angle B.

$\Sigma \, (\delta_A^2 + \delta_A \delta_B + \delta_B^2)$

= summation of values for the particular chain of triangles through which the computation is carried from the known line to the line required. The value of $\Sigma \, (\delta_A^2 + \delta_A \delta_B + \delta_B^2)$ for a triangle is given in Table 8.1.

The *distance angles* A and B of a triangle are the angles opposite to (*i*) known side and (*ii*) the side which is to be computed. The third angle of the triangle, which

TABLE 8.1

VALUES OF $\delta_A^2 + \delta_A \delta_B + \delta_B^2$

	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
0																							
10	428	359																					
12	359	295	253																				
14	315	253	214	187																			
16	284	225	187	162	143																		
18	262	204	168	143	126	113																	
20	245	189	153	130	113	100	91																
22	232	177	142	119	103	91	81	74															
24	221	167	134	111	95	83	74	67	61														
26	213	160	126	104	89	77	68	61	56	51													
28	206	153	120	99	83	72	63	57	51	47	43												
30	199	148	115	94	79	68	59	53	48	43	40	33											
35	188	137	106	85	71	60	52	46	41	37	33	27	23										
40	179	129	99	79	65	54	47	41	36	32	29	23	19	16									
45	172	124	93	74	60	50	43	37	32	28	25	20	16	13	11								
50	167	199	89	70	57	47	39	34	29	26	23	18	14	11	9	8							
55	162	115	86	67	54	44	37	32	27	24	21	16	12	10	8	7	5						
60	159	112	83	64	51	42	35	30	25	22	19	14	11	9	7	5	4	4					
65	155	109	80	62	49	40	33	28	24	21	18	13	10	7	6	5	4	3	2				
70	152	106	78	60	48	38	32	27	23	19	17	12	9	7	5	4	3	2	2	1			
75	150	104	76	58	46	37	30	25	21	18	16	11	8	6	4	3	2	2	1	1	1		
80	147	102	74	57	45	36	29	24	20	17	15	10	7	5	4	3	2	1	1	1	0	0	
85	145	100	73	55	43	34	28	23	19	16	14	10	7	5	3	2	2	1	1	0	0	0	0
90	143	98	71	54	42	33	27	22	19	16	13	9	6	4	3	2	1	1	1	0	0	0	0
95	140	96	70	53	41	32	26	22	18	15	13	9	6	4	3	2	1	1	0	0	0	0	
100	138	95	68	51	40	31	25	21	17	14	12	8	6	4	3	2	1	1	0	0	0		
105	136	93	67	50	39	30	25	20	17	14	12	8	5	4	2	2	1	1	0	0			
110	134	91	65	49	38	30	24	19	16	13	11	7	5	3	2	2	1	1	1				
115	132	89	64	48	37	29	23	19	15	13	11	7	5	3	2	2	1	1					
120	129	88	62	46	36	28	22	18	15	12	10	7	5	3	2	2	1						
125	127	86	61	45	35	27	22	18	14	12	10	7	5	4	3	2							
130	125	84	59	44	34	26	21	17	14	12	10	7	5	4	3								
135	122	82	58	43	33	26	21	17	14	12	10	7	5	4									
140	119	80	56	42	32	25	20	17	14	12	10	8	6										
145	116	77	55	41	32	25	21	17	15	13	11	9											
150	112	75	54	40	32	26	21	18	16	15	13												
152	111	75	53	40	32	26	22	19	17	16													
154	110	74	53	41	33	27	23	21	19														
156	108	74	54	42	34	28	25	22															
158	107	74	54	43	35	30	27																
160	107	74	56	45	38	33																	
162	107	76	59	48	42																		
164	109	79	63	54																			
166	113	86	71																				
168	122	98																					
170	143																						

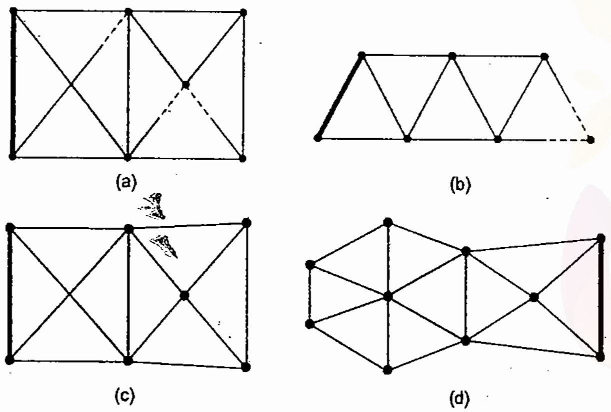

FIG. 8.2

$\therefore$ $C = (10 - 7 + 1) + (13 - 14 + 3) = 4 + 2 = 6$

$\frac{D - C}{D} = \frac{21 - 6}{21} = \frac{15}{21} = \mathbf{0.714}$

(*b*) $n = 11$

$s = 7$

$n' = 9$

$s' = 6$

$D = \{(11 \times 2) - 2\} - 2 = 18$

$C = (9 - 6 + 1) + (11 - 14 + 3) = 4$

$\frac{D - C}{D} = \frac{18 - 4}{18} = \mathbf{0.778}$

(*c*) $n = 13$

$s = 7$

$n' = 13$

$s' = 7$

$D = (13 \times 2) - 2 = 24$

$C = (13 - 7 + 1) + (13 - 14 + 3) = 7 + 2 = 9$

$\frac{D - C}{D} = \frac{24 - 9}{24} = \mathbf{0.625}.$

(*d*) $n = 19$

$s = 10$

$n' = 19$

$s' = 10$

$C = (19 \times 2) - 2 = 36$

$\therefore$ $D = (19 - 10 + 1) + (19 - 20 + 3) = 10 + 2 = \mathbf{12}$

$\frac{D - C}{D} = \frac{36 - 12}{36} = \frac{24}{36} = \mathbf{0.667}$

Example 8.3. *Compute the strength of the figure ABCD for each of the routes by which the length BD can be computed from the known side AC. All the stations were occupied, and all the angles were measured.*

Solution.

Here, D = the number of directions observed (not including the fixed side AC) = 10

n = total number of lines = 6

n' = total number of lines observed in both directions = 6

s = the number of stations = 4

s' = the number of stations occupied = 4

Hence $C = (n' - s' + 1) + (n - 2s + 3)$

$= (16 - 4 + 1) + (6 - 8 + 3) = 3 + 1 = 4$

and $\frac{D - C}{D} = \frac{10 - 4}{10} = 0.60$

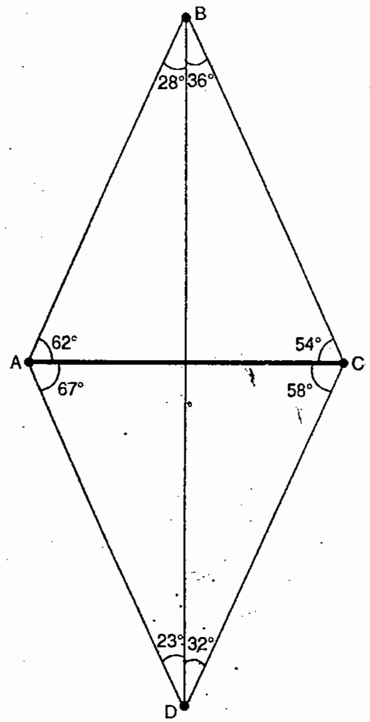

FIG. 8.3.

(a) Strength of figure by route 1 using Δs ACD and ADB.

Common side = AD

For triangle ACD, distance angles are 55° and 58°

$\therefore$ From Table 8.1, $\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 6$

For triangle ABD, distance angles are 28° and 129°

$\therefore$ $\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 12$

Hence $\Sigma(\delta_A^2 + \delta_A\delta_B + \delta_B^2) = 6 + 12 = 18$

$\therefore$ $R_1 = \frac{D - C}{D}\Sigma(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$

$= 0.6 \times 18 = 10.8 \approx 11.$

(b) Strength of figure by route 2 using Δs ACD and DCB

Common side = CD

For triangle ACD, distance angles = 55° and 67°

$\therefore$ $\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 4.6$

For triangle DCB, distance angles = 36° and 112°

$\therefore$ $\delta_A^2 + \delta_A\delta_B + \delta_B^2 = 6.6$

$\therefore$ $\Sigma(\delta_A^2 + \delta_A\delta_B + \delta_B^2) = 4.6 + 6.6 = 11.2$

and $R_2 = 0.6 \times 11.2 \approx 7$

(c) Strength of figure by route 3 using Δs ACB and ABD

Common side = AB

For triangle ACB, distance angles = 64° and 54°

it is necessary to raise both the instrument as well as the signal to overcome the curvature of the earth and to clear all the intervening obstructions. The height of the instrument as well as the signal depends upon the following factors :

1. The distance between the stations.
2. The relative elevation of stations.
3. The profile of the intervening ground.

1. The Distance between the Stations

If there is no obstruction due to intervening ground, the distance of the visible horizon from a station of known elevation above datum is given by

$$h = \frac{D^2}{2R}(1 - 2m) \qquad \ldots(8.2)$$

where h = height of the station above datum

D = distance to the visible horizon

R = mean radius of the earth

m = mean co-efficient of refraction

= 0.07 for sights over land, and = 0.08 for sights over sea.

If the values of D and R are substituted in proper units, the value of h corresponding to $m = 0.07$ is given by

$h = 0.574\ D^2$, where h is in feet and D is in miles

and $h = 0.06728\ D^2$, where h is in metres and D is in km.

2. Relative Elevation of Stations

If there is no obstruction due to intervening ground, the formula $h = \frac{D^2}{2R}(1 - 2m)$ may be used to get the necessary elevation of a station at distance, so that it may be visible from another station of known elevation.

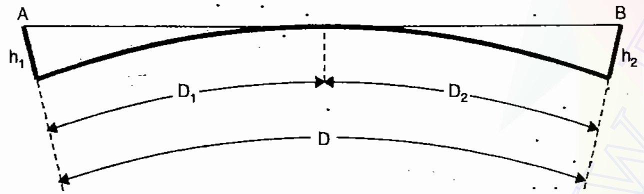

FIG. 8.4.

For example, let h_1 = known elevation of station A above datum

h_2 = required elevation of B above datum

D_1 = distance from A to the point of tangency

D_2 = distance from B to the point of tangency

D = the known distance between A and B.

Then, $h_1 = 0.06728\ D_1^2$

$$D_1 = \sqrt{\frac{h_1}{0.06728}} = 3.8553\sqrt{h_1} \qquad \ldots(i)\ [8.2\ (a)]$$

where D_1 is in km and h_1 is in metres.

Knowing D_1, D_2 is given by

$$D_2 = D - D_1 \qquad \ldots(ii)$$

Knowing D_2, h_2 is calculated from the relation

$$h_2 = 0.06728\ D_2^2 \text{ metres} \qquad \ldots(iii)\ [8.2\ (b)]$$

Thus, the required elevation h_2 is determined. If the actual ground level at B is known it can be ascertained whether it is necessary to elevate the station B above the ground, and if so, the required height of tower can be calculated. However, while making the above calculations, the line of sight should not graze the surface at the point of tangency but should be above it by 2 to 3 metres.

3. Profile of the Intervening Ground

In the reconnaissance, the elevations and positions of peaks in the intervening ground between the proposed stations should be determined. A comparison of their elevations should be made to the elevation of the proposed line of sight to ascertain whether the line of sight is clear off the obstruction or not. The problem can be solved by using the principles discussed in the factors (1) and (2) above, or by a solution suggested by Captain G.T. McCaw. The former method will be clear from the worked out examples.

Captain G.T. McCaw's Method

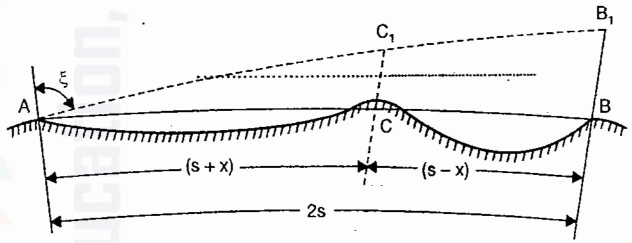

FIG. 8.5.

In Fig. 8.5, let h_1 = height of station A above datum

h_2 = height of station B above datum

h = height of line of sight at the obstruction C

$2s$ = distance between the two stations A and B

$(s + x)$ = distance of obstruction C from A

$(s - x)$ = distance of obstruction C from B

ζ = zenith distance from A to B

The height h of the line of sight at the obstruction is given by

$$h = \tfrac{1}{2}(h_2 + h_1) + \tfrac{1}{2}(h_2 - h_1)\frac{x}{s} - (s^2 - x^2)\operatorname{cosec}^2\zeta\left(\frac{1 - 2m}{2R}\right) \qquad \ldots(8.3)$$

Let c_1, d_1 and b_1 be the points in which a horizontal line through A cut the vertical lines through C, D and B respectively. The corresponding heights cc_1, dd_1 and bb_1 are given by

$$cc_1 = 0.06728\ (ce)^2 = 0.06728\ (7.18)^2 = 3.49 \text{ m}$$

$$dd_1 = 0.06728\ (ed)^2 = 0.06728\ (32.82)^2 = 72.47 \text{ m}$$

and $$bb_1 = 0.06728\ (eb)^2 = 0.06728\ (72.82)^2 = 356.77 \text{ m}$$

Now $$Bb = \text{Elev. of } B = 1160 \text{ m}$$

$\therefore$ $$Bb_1 = Bb - bb_1 = 1160 - 356.77 = 803.23 \text{ m}$$

Let AB be the line of sight.

Now from triangles Ac_1c_2, Ad_1d_2 and Ab_1B

$$c_1c_2 = Bb_1 \frac{Ac_1}{Ab_1} = 803.23 \times \frac{50}{130} = 308.93 \text{ m}$$

and $$d_1d_2 = Bb_1 \frac{Ad_1}{Ab} = 803.23 \times \frac{90}{130} = 556.08 \text{ m}$$

Elevation of line of sight at C = elevation of $c_2 = cc_1 + c_1c_2 = 3.49 + 308.93 = 312.42$ m

Elevation of line of sight at D = elevation of $d_2 = dd_1 + d_1d_2 = 72.47 + 556.08 = 628.55$ m

Elevation of $C = 308$ m and that of $D = 632$ m

Thus, the line of sight clears the peak C, but fails to clear the peak D by $632 - 628.55 = 3.45$ m $= d_2D$.

Let Ad_3 be the new line of sight, such that

$$Dd_3 = 3 \text{ metres (minimum)}$$

Hence $$d_2d_3 = d_3D + d_2D = 3 + 3.45 = 6.45 \text{ m}$$

Hence $$Bb_3 = d_2d_3 \frac{AB}{Ad_2} = 6.45 \times \frac{130}{90} = 9.32 \text{ m} \approx 9.5 \text{ m (say).}$$

Hence minimum height of scaffold at B = **9.5 m.**

Example 8.6. *The altitudes of two proposed stations A and B, 100 km apart, are respectively 420 m and 700 m. The intervening obstruction situated at C, 70 km from A has an elevation of 478 m. Ascertain if A and B are intervisible, and, if necessary, find by how much B should be raised so that the line of sight must nowhere be less than 3 m above the surface of the ground.*

Solution (Fig. 8.7).

Let $aceb$ be the visible horizon and a horizontal sight Ab_1 through A meet the horizon tangentially in e.

The distance Ae to the visible horizon from station A of an altitude 420 metres is given by

$$D = Ae = 3.8553\sqrt{h} = 3.8553\sqrt{420} = 79.01 \text{ km}$$

Now $$AC = 70 \text{ km and } AB = 100 \text{ km}$$

$\therefore$ $$ec = Ae - AC = 79.01 - 70 = 9.01 \text{ km}$$

and $$eb = AB - Ae = 100 - 79.01 = 20.99 \text{ km}$$

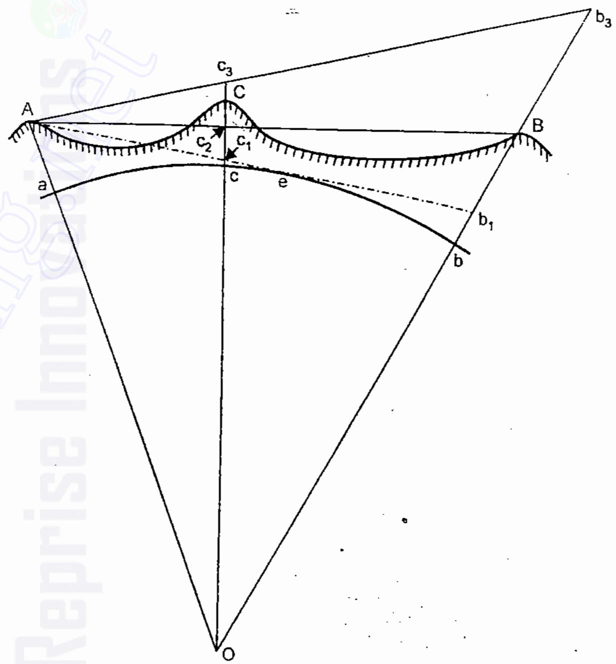

FIG. 8.7

The corresponding heights cc_1 and bb_1 are given by

$$cc_1 = 0.06728\ (ec)^2 = 0.06728\ (9.01)^2 = 5.46 \text{ m}$$

and $$bb_1 = 0.06728\ (eb)^2 = 0.06728\ (20.99)^2 = 29.64 \text{ m}$$

New $$Bb = \text{Elev. of } B = 700$$

$\therefore$ $$Bb_1 = Bb - bb_1 = 700 - 29.64 = 670.36 \text{ m}$$

Now, from similar triangles Ac_1c_2 and Ab_1B,

$$c_1c_2 = Bb_1 \frac{Ac_1}{Ab_1} = 670.36 \times \frac{70}{100} = 469.25 \text{ m}$$

$\therefore$ Elevation of line sight at C = elevation of c_2

$$= cc_1 + c_1c_2 = 5.46 + 469.25 = 474.71 \text{ m}$$

$\therefore$ Elevation of $C = 478$ m

Hence the line of sight fails to clear the peak by

$$c_2C = 478 - 474.71 = 3.29 \text{ m}$$

In order that the line of sight should at least be 3 m above the ground anywhere, the line of sight should be raised by $(3.29 + 3) = 6.29$ m.

That is, $$c_2c_3 = 6.29 \text{ m}$$

Luminous or Sun Signals

Sun signals are those in which the sun's ray's are reflected to the observing theodolite, either directly, as from a beacon, or indirectly from a signal target. They are generally used when the length of sight exceeds 30 km. The heliotrope and heliograph are the special instruments used as sun signals. The *heliotrope* consists of a plane mirror to reflect the sun's rays and a line of sight to enable the attendant to direct the reflected rays towards the observing station. The line of sight may be either telescopic or in the form of a sight vane with an aperture carrying crosswires. The heliotrope is centred over the station mark, and the line of sight is directed upon the distant station by the attendant at the heliotrope. Flashes are sent from the observing station to enable the direction to be established. Because of the motion of the sun, the heliotroper must adjust the mirror every minute on its axes. The reflected rays from a divergent beam have an angle equal to that subtended by the sun at the mirror *viz.* about 32 min. The base of the cone of the reflected rays has therefore, a diameter of about 10 metres in every kilometre of distance. Since the signal is visible from any point within this base, great refinement in pointing the heliotrope is unnecessary. However, in order that the signal may be visible, the error in alignment should be less than 16 minutes. Another form of heliotrope is the *'Galton Sun Signal'*

Night Signals

Night signals are used in observing the angles of a triangulation system at night. Various forms of night signals used are ;

(1) Various forms of oil lamps with reflectors or optical collimators for lines of sight less than 80 kilometres.

(2) Acetylene lamp designed by captain G.T. McCaw for lines of sight up to 80 kilometres.

Phase of Signals

Phase of signal is the error of bisection which arises from the fact that, under lateral illumination, the signal is partly in light and partly in shade. The observer sees only the illuminated portion and bisects it. *It is thus the apparent displacements of the signal.* The *phase correction* is thus necessary so that the observed angle may be reduced to that corresponding to the centre of the signal.

The correction can be applied under two conditions :

(*i*) When the observation is made on the bright portion.

(*ii*) When the observation is made on the bright line.

(i) When the observation is made on the bright portion.

Fig. 8.10 (*a*) shows the case when the observation is made on the bright portion *FD*.

Let A = position of the observer.

B = centre of the signal (in plan).

FD = visible portion of the illuminated surface.

AE = line of sight

E = mid-point of FD

β = phase correction

θ_1 and θ_2 = angles which the extremities of the visible portion make with AB.

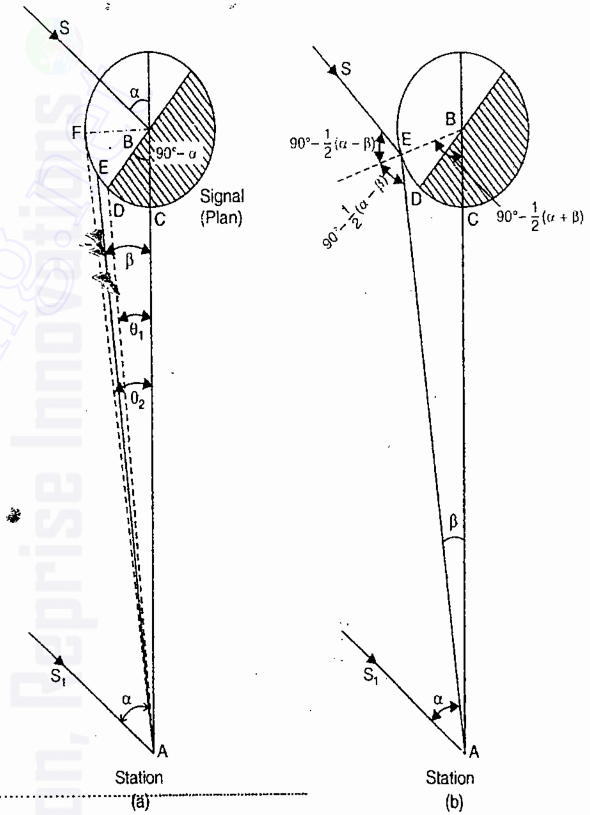

FIG. 8.10. PHASE CORRECTION.

α = the angle which the direction of sun makes with AB

r = radius of the signal

D = distance AB

The phase correction $\beta = \theta_1 + \frac{1}{2}(\theta_2 - \theta_1) = \frac{1}{2}(\theta_1 + \theta_2)$

But $\theta_2 = \dfrac{r}{D}$ radians

and $\theta_1 = \dfrac{r \sin (90° - \alpha)}{D} = \dfrac{r \cos \alpha}{D}$ radians

$\therefore \quad \beta = \frac{1}{2}\left\{\dfrac{r \cos \alpha}{D} + \dfrac{r}{D}\right\} = \dfrac{r\,(1 + \cos \alpha)}{2D}$

or $\beta = \dfrac{r \cos^2 \frac{1}{2}\alpha}{D}$ radians

$$= \frac{r \cos^2 \frac{1}{2}\alpha}{D \sin 1''} \text{ seconds} = \frac{206265\, r \cos^2 \frac{1}{2}\alpha}{D} \text{ seconds} \quad \ldots(8.4)$$

Forms of Base Measuring apparatus :

There are two forms of base measuring apparatus :

(A) *Rigid Bars* (B) *Flexible apparatus.*

(A) Rigid Bars

Before the introduction of invar tapes, rigid bars were used for work of highest precision. The rigid bars may be divided into two classes :

(*i*) ***Contact apparatus,*** in which the ends of the bars are brought into successive contacts. Example : The Eimbeck Duplex Apparatus.

(*ii*) ***Optical apparatus,*** in which the effective lengths of the bars are engraved on them and observed by microscopes. Example : The Colby Apparatus and the Woodward Iced Bar Apparatus.

The rigid bars may also be divided into the following classes depending upon the *way* in which the uncertainties of temperature corrections are minimised :

(*i*) *Compensating base bars,* which are designed to maintain constant length under varying temperature by a combination of two more metals. Example : The Colby Apparatus.

(*ii*) *Bimetallic non-compensating base bars,* in which two measuring bars act as a bimetallic thermometer. Example : The Eimbeck Duplex Apparatus (U.S. Coast and Geodetic Survey), Borda's Rod (French system) and Bessel's Apparatus (German system).

(*iii*) *Monometallic base bars,* in which the temperature is either kept constant at melting point of ice, or is otherwise ascertained. Example : The Woodward Iced Bar Apparatus, and Struve's Bar (Russian system)

The Colby Apparatus

This is a compensating and optical type rigid bar apparatus designed by Maj-Gen. Colby to eliminate the effect of changes of temperature upon the measuring appliance. The apparatus was employed in the Ordinance Survey and the Indian Surveys. All the ten bases of G.T. of Survey of India were measured with the Colby apparatus. The apparatus (Fig. 8.11) consists of two bars, one of steel and the other of brass, each 10 ft long and riveted together at the centre of their length. The ratio of coefficients of linear expansion of these metals having been determined as 3 : 5. Near each end of the compound bar, a metal tongue is supported by double conical pivots held in forked ends of the bars. The tongue projects on the side away from the brass rod. On the extremities of these tongues, two minute marks *a* and *a′* are put, the distance between them being exactly

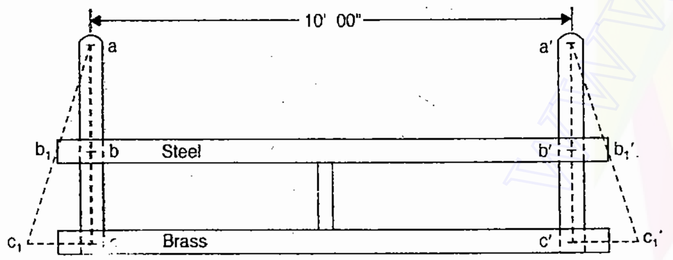

FIG. 8.11. THE COLBY APPARATUS

equal to 10′ 0″. The distance *ab* (or *a′b′*) to the junction with the steel is kept $\frac{3}{5}$ths of the distance *ac* (or *a′c′*) to the brass junction. Due to change in temperature, if the distance *bb′* of steel changes to b_1b_1' by an amount *x*, the distance *cc′* of brass will change to c_1c_1' by an amount $\frac{5}{3}x$ *thus unaltering the positions of dots a and a′*. The brass is coated with a spcial preparation in order to render it equally susceptible to change of temperature as the steel. The compound bar is held in the box at the middle of its length. A spirit level is also placed on the bar. In India, five compound bars were simultaneously employed in the field. The gap between the forward mark of one bar and the rear bar of the next was kept constant equal to 6″ by means of a *framework* based on the same principles as that of the 10′ compound bar. The framework consists of two microscopes, the distance between the cross-wires of which was kept exactly equal to 6″. To start with, the cross-wires of the first microscope of the *framework* were brought into coincidence with the platinum dot, let into the centre of the mark of the one extremity of the base line. The platinum dot *a* of the first compound bar was brought into the coincidence with the cross-hairs of the second microscope. The cross-hairs of the first microscope of the second framework (consisting two microscopes 6″ apart) are then set over the end *a′* of the first rod. The work is thus continued till a length of (10′ × 5 + 5 × 6″) = 52′ 6″ is measured at a time with the help of 5 bars and 2 frameworks. The work is thus continued till the end of the base is reached.

(B) Flexible Apparatus

In recent years, the use of flexible instruments has increased due to the longer length that can be measured at a time without any loss in accuracy. The flexible apparatus consists of (*a*) steel or invar tapes, and (*b*) steel and brass wires. The flexible apparatus has the following *advantages* over the rigid bars :

(*i*) Due to the greater length of the flexible apparatus, a wider choice of base sites is available since rough ground with wider water gap can be utilised.

(*ii*) The speed of measurement is quicker, and thus less expensive.

(*iii*) Longer bases can be used and more check bases can be introduced at closer intervals.

Steel Tapes

Steel tapes are semi-tempered bands of tough, flexible steel which has a thermal coefficient of expansion of very nearly 0.00000645 per degree Fahrenheit. The temperature of a steel tape cannot be measured with sufficient accuracy by mercurial thermometer in the day time. Accurate results can, however, be obtained if the measurements are made at night or on cloudy or even hazy days when there is little radiant heat. At these times the tape and air temperatures are nearly the same so that the temperature of the tape can be accurately determined and corrections applied.

Invar Tapes and Wires

The research of Dr. Guillaume, of the French Bureau of Weights and Measures, led to the discovery of *invar,* the least expansible steel alloy containing about 36% nickel. The coefficient of thermal expansion is the lowest of all the known metals and alloys and seldom exceeds 0.0000005 per dergree F. However, the temperature coefficient not

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for contact with the tripod heads. The tension applied should not be less than 20 times the weight of the tape.

Measurements by Steel and Brass Wires : Principle of Bimetallic Thermometer

The method of measurement by steel and brass wires is based on Jaderin's application of the principle of bimetallic thermometer to the flexible apparatus. The steel and brass wires are each 24 m long and 1.5 to 2.6 mm in diameter. The distance between the measuring tripods is measured first by the steel wire and then by the brass wire by Jaderin's method as explained above (Fig. 8.13) with reference to invar tape or wire. Both the wires are nickel plated to ensure the same temperature conditions for both. From the measured lengths given by the steel and brass wires, the temperature effect is eliminated as given below :

Let L_s = distance as computed from the absolute length of the steel wire.

L_b = distance computed from the absolute length of the brass wire.

α_s = co-efficient of expansion for steel.

α_b = co-efficient of expansion for brass.

D = corrected distance.

T_m = mean temperature during measurement.

T_s = Temperature at standardisation

$T = T_m - T_s$ = temperature increase.

Now $\quad D = L_s(1 + \alpha_s T) = L_b(1 + \alpha_b T) \quad$ ...(1)

or $\quad T(L_b\,\alpha_b - L_s\,\alpha_s) = L_s - L_b$

$$\therefore \quad T = \frac{L_s - L_b}{L_b\,\alpha_b - L_s\,\alpha_s} \quad \text{...(2)}$$

Substituting this value of T in (1) for steel wire, we get

$$D = L_s\left\{1 + \frac{\alpha_s(L_s - L_b)}{L_b\,\alpha_b - L_s\,\alpha_s}\right\}$$

$$\therefore \text{ Correction for steel wire} = D - L_s = +\frac{L_s\,\alpha_s(L_s - L_b)}{L_b\alpha_b - L_s\,\alpha_s}$$

$$\approx +\frac{\alpha_s(L_s - L_b)}{\alpha_b - \alpha_s} \text{ with sufficient accuracy.}$$

Similarly, correction for brass wire $= D - L_b \approx +\dfrac{\alpha_b(L_s - L_b)}{\alpha_b - \alpha_s}$

The corrections can thus be applied without measuring the temperature in the field. The method has however been superseded by the employment of invar tapes or wires.

8.8. CALCULATION OF LENGTH OF BASE : TAPE CORRECTIONS

After having measured the length, the correct length of the base is calculated by applying the following corrections :

1. Correction for absolute length 2. Correction for temperature
3. Correction for pull or tension 4. Correction for sag

5. Correction for slope 6. Correction for alignment
7. Reduction to sea level. 8. Correction to measurement in vertical plane

1. Correction for Absolute Length

If the *absolute length* (or actual length) of the tape or wire is not equal to its *nominal* or *designated length,* a correction will have to be applied to the measured length of the line. If the absolute length of the tape is greater than the nominal or the designated length, the measured distance will be too short and the correction will be additive. If the absolute length of the tape is lesser than the nominal or designated length, the measured distance will be too great and the correction will be subtractive.

Thus, $\quad C_a = \dfrac{L.\,c}{l} \quad$ ... (8.6)

where C_a = correction for absolute length

L = measured length of the line

c = correction per tape length

l = designated length of the tape

C_a will be of the same sign as that of c.

2. Correction for Temperature

If the temperature in the field is *more* than the temperature at which the tape was standardised, the length of the tape *increases,* measured distance becomes *less,* and the correction is therefore, *additive.* Similarly, if the temperature is *less,* the length of the tape *decreases,* measured distance becomes *more* and the correction is *negative.* The temperature correction is given by

$$C_t = \alpha(T_m - T_0)\,L \quad \text{...(8.7)}$$

where α = coefficient of thermal expansion

T_m = mean temperature in the field during measurement

T_0 = temperature during standardisation of the tape

L = measured length.

If, however, steel and brass wires are used simultaneously, as in Jaderin's Method, the corrections are given by

$$C_t \text{ (brass)} = \frac{\alpha_b(L_s - L_b)}{\alpha_b - \alpha_s} \quad \text{...[8.8 (a)]}$$

and

$$C_t \text{ (steel)} = \frac{\alpha_s(L_s - L_b)}{\alpha_b - \alpha_s} \quad \text{...[8.8 (b)]}$$

To find the new standard temperature T_0' which will produce the nominal length of the tape or band

Some times, a tape is not of standard or designated length at a given standard temperature T_0. The tape/band will be of the designated length at a new standard temperature T_0'.

Let the length at standard temperature T_0 be $l \pm \delta l$, where l is the designated length of the tape.

Let ΔT be the number of degrees of temperature change required to change the length of the tape by $= \delta l$

It should be noted that the Sag Correction is always negative. If however, the tape was standardised on catenary, and used on flat, the correction will be equal to 'Sag Correction for standard pull – sag correcion at the measured pull', and will be positive if the measured pull in the field is more than the standard pull.

For example, let the tape be standardised in catenary at 100 N pull.

If the pull applied in the field is 120 N, the Sag Correction will be = Sag Correction for 100 N pull – Sag Correction for 120 N pull

$$= \frac{l_1 W_1^2}{24\,(100)^2} - \frac{l_1 (W_1)^2}{24\,(120)^2} = \frac{l_1 W_1^2}{24}\left[\frac{1}{(100)^2} - \frac{1}{(120)^2}\right]$$

and is evidently positive

If the pull applied in the field is 80 N, the Sag Correction will be

$$= \frac{l_1 W_1^2}{24\,(100)^2} - \frac{l_1 W_1^2}{24\,(80)^2} = \frac{l_1 W_1^2}{24}\left[\frac{1}{(100)^2} - \frac{1}{(80)^2}\right]$$ *and is evidently negative.*

If, however the pull applied in the field is equal to the standard pull, no Sag Correction is necessary. See Example 8.11.

Equation 8.12 gives the Sag Correction when the ends of the tape are at the same level. If, however, the ends of the tape are not at the same level, but are at an inclination θ with the horizontal, the Sag Correction given is by the formula,

$$C_s' = C_s \cos^2\theta \left(1 + \frac{wl}{P}\sin\theta\right) \qquad ...[8.13\ (a)]$$

when tension P is applied at the higher end ;

and $$C_s' = C_s \cos^2\theta \left(1 - \frac{wl}{P}\sin\theta\right) \qquad ...[8.13\ (b)]$$

when tension P is applied at the lower end.

If, however, θ is small, we can have

$$C_s' = C_s \cos^2\theta \qquad ...(8.14)$$

irrespective of whether the pull is applied at the higher end or at the lower end. *It should be noted that equation 8.14 includes the corrections both for sag and slope, i.e. if equation 8.14 is used, separate correction for slope is not necessary.* See Example 8.13.

Normal Tension. Normal tension is the pull which, when applied to the tape, equalises the correction due to pull and the correction due to sag. Thus, at normal tension or pull, the effects of pull and sag are neutralised and no correction is necessary.

The correction for pull is $C_p = \dfrac{(P_n - P_0)\, l_1}{AE}$ (additive)

The correction for sag $C_{s1} = \dfrac{l_1\,(wl_1)^2}{24\,P_n^2} = \dfrac{l_1\, W_1^2}{24\, p_n^2}$ (subtractive)

where P_n = the *normal pull* applied in the field.

Equating numerically the two, we get

$$\frac{(P_n - P_0)\, l_1}{AE} = \frac{l_1 W_1^2}{24\, P_n^2}$$

$$P_n = \frac{0.204\, W_1 \sqrt{AE}}{\sqrt{P_n - P_0}} \qquad ...(8.15)$$

The value of P_n is to be determined by trial and error with the help of the above equation.

5. Correction for Slope or Vertical Alignment

The distance measured along the slope is always greater than the horizontal distance and hence the correction is always *subtractive.*

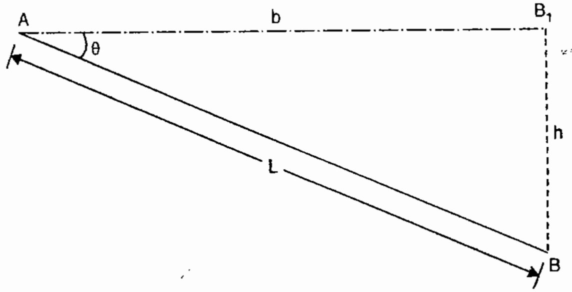

FIG. 8.15. CORRECTION FOR SLOPE.

Let

$AB = L$ = inclined length measured

AB_1 = horizontal length

h = difference in elevation between the ends

C_V = slope correction, or correction due to vertical alignment

Then $$C_V = AB - AB_1 = L - \sqrt{L^2 - h^2}$$

$$= L - L\left(1 - \frac{h^2}{2L^2} - \frac{h^4}{8L^4}\right) = \frac{h^2}{2L} + \frac{h^4}{8L^3} + ...$$

The second term may safely be neglected for slopes flatter than about 1 in 25.

Hence, we get $$C = \frac{h^2}{2L} \text{ (subtractive)} \qquad ...(8.16)$$

Let L_1, L_2 ....etc.= length of successive uniform gradients

h_1, h_2, ...etc.= differences of elevation between the ends of each.

The total slope correction $= \dfrac{h_1^2}{2L_1} + \dfrac{h_2^2}{2L_2} + ... = \Sigma \dfrac{h^2}{2L}$

If the grades are of uniform length L, we get total slope correction $= \dfrac{\Sigma h^2}{2L}$

If the angle (θ) of slope is measured instead of h, the correction is given by

$$C_V = L - L\cos\theta = L\,(1 - \cos\theta) = 2L\sin^2\frac{\theta}{2} \qquad ...(8.17)$$

Effect of measured value of slope θ

Usually, the slope θ of the line is measured instrumentally, with a theodolite. In that case the following modification should be made to the measured value of the slope. See Fig. 8.16.

Let h_1 = height of the instrument at A

h_2 = height of the target at B

α = measured vertical angle

Now correction of sag $= C_s = \dfrac{nl_1(wl_1)^2}{24\,P^2} = \dfrac{nl_1 W_1^2}{24\,P^2} = \dfrac{3 \times 10 \times (6.24)^2}{24\,(100)^2} = \mathbf{0.00487}$ **m.**

Example 8.10. *A steel tape 20 m long standardised at 55° F with a pull of 10 kg was used for measuring a base line. Find the correction per tape length, if the temperature at the time of measurement was 80 °F and the pull exerted was 16 kg. Weight of 1 cubic cm of steel = 7.86 g, Wt. of tape = 0.8 kg and E = 2.109 × 10^6 kg/cm^2. Coefficient of expansion of tape per 1°F = 6.2 × 10^{-6}.*

Solution. Correction for temperature $= 20 \times 6.2 \times 10^{-6}(80 - 55) = 0.0031$ m (additive)

Correction for pull $= \dfrac{(P - P_0)L}{AE}$

Now, weight of tape $= A\,(20 \times 100)(7.86 \times 10^{-3})$ kg $= 0.8$ kg (given)

$\therefore \quad A = \dfrac{0.8}{7.86 \times 2} = 0.051$ sq. cm

Hence, $\quad C_p = \dfrac{(16 - 10)\,20}{0.051 \times 2.109 \times 10^6} = 0.00112$ (additive)

Correction for sag $= \dfrac{l_1(wl_1)^2}{24\,P^2} = \dfrac{20(0.8)^2}{24\,(16)^2} = 0.00208$ m (subtractive)

$\therefore$ **Total correction** $= +0.0031 + 0.00112 - 0.00208 = \mathbf{+0.00214\ m}$

Example 8.11. *A nominal distance of 30 m was set out with a 30 m steel tape from a mark on the top of one peg to a mark on the top of another, the tape being in catenary under a pull of 100 N and at a mean temperature of 70 ° F. The top of one peg was 0.25 m below the top of the other. The top of the higher peg was 460 m above mean sea level. Calculate the exact horizontal distance between the marks on the two pegs and reduce it to mean sea level, if the tape was standardised at a temperature of 60° F in catenary under a pull of (a) 80 N. (b) 120 N and (c) 100 N.*

Take radius of earth = 6370 km.

Density of tape = 7.86 g/cm^3

Section of tape = 0.08 sq. cm.

Co-efficient of expansion = 6 × 10^{-6} per 1° F

Young's modulus = 2 × 10^7 N/cm^2

Solution.

(*i*) Correction for standardisation ... *nil.*

(*ii*) Correction for slope $= \dfrac{h^2}{2L} = \dfrac{(0.25)^2}{2 \times 30} = 0.0010$ m (subtractive)

(*iii*) Temperature correction $= L\alpha\,(T_m - T_0) = 30 \times 6 \times 10^{-6}\,(70 - 60)$
$= 0.0018$ m (additive)

(*iv*) Tension correction $= \dfrac{(P - P_0)\,L}{AE}$

(*a*) When $P_0 = 80$ N, tension correction $= \dfrac{(100 - 80)\,30}{0.08 \times 2 \times 10^7} = 0.0004$ m (additive).

(*b*) When $P_0 = 120$ N, tension correction $= \dfrac{(100 - 120)\,30}{0.08 \times 2 \times 10^7} = 0.0004$ m (subtractive)

(*c*) When $P_0 = 100$ N, tension correction = zero

(*v*) Sag correction $= \dfrac{LW^2}{24\,P^2}$

Now mass of tape per metre run

$= (0.08 \times 1 \times 100) \times \dfrac{7.86}{1000}$ kg $= 0.06288$ kg/m

$\therefore$ Weight of tape per metre run $= 0.06288 \times 9.81 = 0.6169$ N/m

$\therefore$ Total weight of tape $= 0.6169 \times 30 = 18.51$ N.

(*a*) When $P_0 = 80$ N

Sag correction $= \dfrac{30 \times (18.51)^2}{24\,(80)^2} - \dfrac{30\,(18.51)^2}{24\,(100)^2} = 0.0669 - 0.04283 = 0.02407$ (additive).

(*b*) When $P_0 = 120$ N

Sag correction $= \dfrac{30\,(18.51)^2}{24\,(120)^2} - \dfrac{30\,(18.51)^2}{24\,(100)^2} = 0.02974 - 0.04283$

≈ -0.0131 m (*i.e.*, subtractive).

(*c*) When $P_0 = 100$ N $= P$, sag correction is zero.

Final correction

(*a*) Total correction $= -0.0010 + 0.0018 + 0.0004 + 0.02407$ m $= \mathbf{+0.02527}$ m.

(*b*) Total correction $= -0.0010 + 0.0018 - 0.0004 - 0.0131$ m $= \mathbf{-0.0127}$ m.

(*c*) Total correction $= -0.0010 + 0.0018 + 0 + 0 = \mathbf{+0.0008}$ **m.**

Example 8.12. *It is desired to find the weight of the tape by measuring its sag when suspended in catenary with both ends level. If the tape is 20 m long and the sag amounts to 20.35 cm at the mid span under a tension of 100 N, what is the weight of the tape?*

Solution. (Fig. 8.14). From expression (2) of sag, we have

$$h = \frac{wl_1\,d_1}{8P}$$

But $\quad h = 20.35$ cm (given)

Taking $\quad l_1 = d_1$ (approximately), we get

$$h = \frac{wl_1^2}{8P}$$

or $\quad w = \dfrac{8Ph}{l_1^2} = \dfrac{8 \times 100}{20 \times 20} \times \dfrac{20.35}{100}$ N/m $= \mathbf{0.407\ N/m}$

Mass of tape $= \dfrac{0.407}{9.81} = 0.0415$ **kg/m = 41.5** g/m.

$$\sec^2 \psi \frac{d\psi}{dx} = \frac{w}{H}\frac{ds}{dx} \quad \ldots(3)$$

Now, from the elemental triangle [Fig. 3.40 (*c*)]

$$\frac{ds}{dx} = \sec \psi$$

$$\therefore \quad \sec^2 \psi \cdot \frac{d\psi}{dx} = \frac{w}{H} \sec \psi$$

or

$$\sec \psi \cdot \frac{d\psi}{dx} = \frac{w}{H} \quad \ldots(4)$$

Let x' be half the length of tape, and ψ' be the inclination of tangent at the end. Integrating Eq. (4) from O to B, we get

$$\int_0^{\psi'} \sec \psi \, d\psi = \int_0^{x'} \frac{w}{H} dx$$

$$\therefore \quad \left[\log_e (\sec \psi + \tan \psi)\right]_0^{\psi'} = \frac{w}{H} x'$$

or

$$x' = \frac{H}{w}\left(\log_e \frac{\sec \psi' + \tan \psi'}{1+0}\right)$$

or

$$x' = \frac{H}{w} \log_e (\sec \psi' + \tan \psi') \quad \ldots(5)$$

Again, resolving vertically for one-half of the tape,

$$T \sin \psi' = W \quad \text{or} \quad \sin \psi' = \frac{W}{T}.$$

$$\therefore \quad \cos \psi' = \sqrt{1 - \sin^2 \psi'} = \frac{\sqrt{T^2 - W^2}}{T}$$

Also,

$$\tan \psi' = \frac{W}{\sqrt{T^2 - W^2}}$$

Substituting the values in Eq. (5), we get

$$x' = \frac{H}{w} \log_e \left[\frac{T}{\sqrt{T^2 - W^2}} + \frac{W}{\sqrt{T^2 - W^2}}\right] = \frac{H}{w} \log_e \left(\frac{T+W}{\sqrt{T^2 - W^2}}\right)$$

$$= \frac{H}{w} \log_e \sqrt{\frac{T+W}{T-W}} = \frac{1}{2}\frac{H}{w} \log_e \frac{T+W}{T-W}$$

The total horizontal distance $= 2x'$

$$= \frac{H}{w} \log_e \frac{T+W}{T-W} \quad \text{(Hence proved)}$$

Example 8.15. *A field tape, standardised at 18°C measured 100.0056 m. Determine the temperature at which it will be exactly of the nominal length of 100 m. Take* $\alpha = 11.2 \times 10^{-6}$ *per °C.*

Solution : Given $\delta l = 0.0056$ m ; $T_0 = 18°$ C

New standard temperature $T_0' = T_0 \pm \dfrac{\delta l}{l\alpha} = 18° - \dfrac{0.0056}{100 \times 11.2 \times 10^{-6}} = 18° - 5° = 13°$ C

Example 8.16. *A distance AB measures 96.245 m on a slope. From a theodolite set at A, with instrument height of 1.400 m, staff reading taken at B was 1.675 m with a vertical angle of 4° 30′ 40″. Determine the horizontal length of the line AB. What will be the error if the effect were neglected.*

Solution : Given $h_1 = 1.400$ m; $h_2 = 1.675$ m; $\alpha = 4° 30' 20''$; $l = 96.245$ m

$$\delta\alpha'' = \frac{206265\,(h_1 - h_2)\cos\alpha}{l} = \frac{206265\,(1.400 - 1.675)\cos 4° 30' 20''}{96.245}$$

$$= -588'' = -0° 09' 48''$$

$$\therefore \quad \theta = \alpha + \delta\alpha = 4° 30' 20'' - 0° 09' 48'' = 4° 20' 32''$$

$\therefore$ Horizontal length $L = l \cos\theta = 96.245 \cos 4° 20' 32'' = 95.966$ m.

If the effect were neglected, $L = 96.245 \cos 4° 30' 40'' = 95.947$ m

$\therefore$ Error = **0.019 m**

Example 8.17. *(a) Calculate the elongation at 400 m of a 1000 m mine shaft measuring tape hanging vertically due to its own mass. The modulus of elasticity is* 2×10^5 *N/mm²*, *the mass of the tape is 0.075 kg/m and the cross-sectional area of the tape is 10.2 mm².*

(b) If the same tape is standardised as 1000.00 m at 175 N tension, what is the true length of the shaft recorded as 999.126 m ?

Solution

(*a*) Taking $M = 0$, we have

$$s_x = \frac{mgx}{2AE}(2l - x) = \frac{0.075 \times 9.81 \times 400\,(2000 - 400)}{2 \times 10.2 \times 2 \times 10^5} = \mathbf{0.115\ m}$$

(*b*)

$$s = \frac{gx}{AE}\left[M + \frac{m}{2}(2l - x) - \frac{P_0}{g}\right]$$

Here $x = 999.126$, $M = 0$ and $P_0 = 175$

$$\therefore \quad s = \frac{9.81 \times 999.126}{10.2 \times 2 \times 10^5}\left[0 + \frac{0.075}{2}(2 \times 1000 - 999.126) - \frac{175}{9.81}\right]$$

$$= \mathbf{0.095\ m}$$

8.9. MEASUREMENT OF HORIZONTAL ANGLES

Instrument. The instruments for geodetic survey require great degree of refinement. In earlier days of geodetic surveys, the required degree of refinement was obtained by making greater diameter of the horizontal circles. The greater theodolite of *Ordinance Survey* has a diameter of 36″. These large diameter theodolites were replaced by the micrometer theodolites (similar in principle to the old 36″and 24″ instruments) such as the Troughton and Simm's 12″ or the Parkhurst 9″. However, more recently the tendency has been to replace the micrometer theodolites by others of the double reading type (glass arc) such as the Wild, Zeiss and Tavistock having diameters of $5\frac{1}{2}''$ and 5″ respectively.

Telescope power : 65 ×

Clear objective glass aperture : 60 mm (2.36″)

Azimuth (horizontal) circle on glass : 360°

Diameter of scale : 250 mm (9.84″)

Interval between divisions : 2′

Direct reading to 0.1″

Elevation (vertical) circle on glass 360°

Diameter of scale : 145 mm (5″.71)

Interval between divisions : 4′

Direct readings to 0.2″

Setting circle, for telescope angle of sight

Interval of division 1°

Scale reading microscope interval 10′

Angles can be estimated to 1′

Sensitivity of suspension level 1″

of elevation circle level 5″

of Horrebow level (both)– 2″

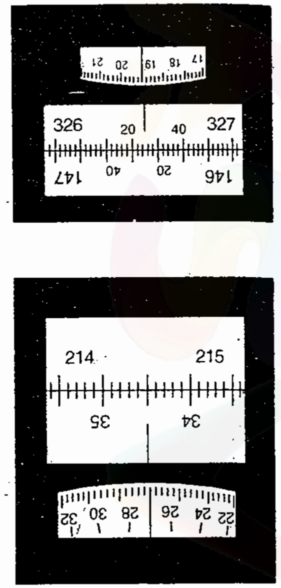

FIG. 8.27.

The vertical and azimuth circles are both equipped with a reading micrometer which gives automatically the arithmetic mean of two diametrically opposed readings. Fig. 8.27 shows the examples of circle readings.

The eye-piece is equipped with the so-called longitude micro-meter for accurate recording of a star's transit. The reversal of the horizontal axis and telescope is carried out by a special hydraulic arrangement which ensures freedom from vibration. Electrical light- ing, to illuminate both circles and field, is built into the body.

Methods of Observation of Horizontal Angles

There are two general methods of observing angles in triangulation :

(1) The Repetition method, and

(2) The Direction method, or reiteration method, or the method of series.

In the *direction method*, the several angles at a station are measured in terms of the direction of their side from that of an initial station. In the *repetition method,* each angle is measured independently by multiplying it mechanically on the circle, the result being obtained by dividing the multiple angle by the number of repetitions. The repetition method is adopted when a repetition theodolite, *i.e.*, the vernier theodolite with a slow motion screw for the lower plate is available. The direction theodolites are equipped with optical micrometers and are much more accurate. Therefore, the repetition method is confined to secondary and tertiary work only while the direction method is used for the primary work.

(A) The method of Repetition

To measure the angle *PQR* at the station *Q*, the following procedure is followed (Fig. 8.28) :

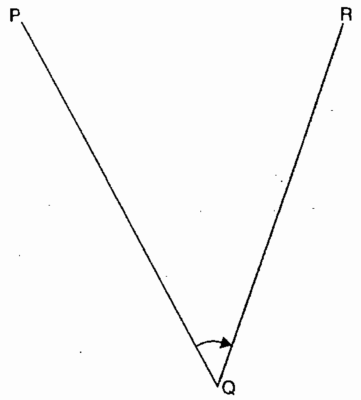

FIG. 8.28

(1) Set the instrument at *Q* and level it. With the help of upper clamp and tangent screw, set 0° reading on vernier *A*. Note the reading of vernier *B*.

(2) Loose the lower clamp and direct the telescope towards the point *P*. Clamp the lower clamp and bisect point *P* accurately by lower tangent screw.

(3) Unclamp the upper clamp and turn the instrument *clockwise* about the inner axis towards *R*. Clamp the upper clamp and bisect *R* accurately with the upper tangent screw. Note the reading of verniers *A* and *B* to get the approximate value of the angle *PQR*.

(4) Unclamp the lower clamp and turn the telescope clockwise to sight *P* again. Bisect *P* accurately by using the lower tangent screw. *It should be noted that the vernier reading will not be changed in this operation since the upper plate is clamped to the lower.*

(5) Unclamp the upper clamp, turn the telescope clockwise and sight *R*. Bisect *R* accurately by upper tangent screw.

(6) Repeat the process until the angle is repeated the required number times (usually 3). The average angle with the face left will be equal to final the reading divided by three.

(7) Change face and make 3 more repetitions as described above. Find the average angle with face right, by dividing the final reading by three.

(8) The average horizontal angle is then obtained by taking the average of the two angles obtained with face left and face right.

Sets by Method of Repetition for High Precision

For measuring an angle to the highest degree of precision, several *sets* of repetitions are usually taken. There are two methods of taking a single set :

(*a*) *First method.* (1) Keeping the telescope normal throughout, measure the angle *clockwise* by 6 repetitions. Obtain the *first value* of the angle by dividing the final reading by 6. (2) Invert the telescope and measure the angle *counter-clockwise* by 6 repetitions. Obtain the *second value* of the angle by dividing the final reading by 6. (3) Take the mean of the first and second value to get the average value of the angle by *first set.*

Take as many sets in this way as may be desired. For first order work, five or six sets are usually required. The final value of the angle will be obtained by taking the mean of the values obtained by different sets.

(*b*) *Second method.* (1) Measure the angle clockwise by six repetitions, the first three with the telescope normal and the last three with telescope inverted. Find the *first value* of the angle by dividing the final by six.

(2) Without altering the reading obtained in the sixth repetition, measure the explement of the angle (*i.e.*, 360° – *PQR*) *clockwise* by six repetitions, the first three with telescope inverted and the last three with telescope normal. Take the reading which should theoretically

due to the eccentricity of the station is generally known as *'reduction of centre.'* The distance between the true station and the satellite station is determined either by method of trigonometrical levelling or by triangulation. Satellite stations should be avoided as far as possible in primary triangulation.

In Fig. 8.30, let A, B, C = triangulation stations

S = satellite station for B.

$d = BS$ = eccentric distance between B and S, determined by trigonometrical levelling or by triangulation.

$\theta = \angle ASC$ = observed angle at a S.

α = True angle at B.

$\gamma = \angle CSB$ = observed angle at S.

$\beta_1 = \angle SAB$.

$\beta_2 = \angle SCB$.

$AC = b$, $AB = c$ and $BC = a$

O = point of intersection of lines AB and CS.

(1) The angles CAB and ACB are known by observations to B from A and C respectively. The length of the side AC is known by computations from the adjacent triangle. The sides AB and BC can then be calculated by applying sine rule to the triangle ABC.

Thus, $$BC = a = \frac{b \sin CAB}{\sin ABC} \qquad \ldots(1)$$

and $$AB = c = \frac{b \sin ACB}{\sin ABC} \qquad \ldots(2)$$

In the above expressions, $\angle ABC$ may be taken equal to $180° - \angle BAC - \angle BCA$, *at the first instance* to calculate the sides AB and BC.

(2) Knowing the sides AB and BC, and the eccentric distance SB, triangles ABS and CBS can be solved by sine rule to get the values of the angles β_1 and β_2 respectively.

Thus, from triangle ABS $$\sin \beta_1 = \frac{SB \sin ASB}{BC} = \frac{d \sin(\theta + \gamma)}{a}$$

And, from triangle CBS, $$\sin \beta_2 = \frac{SB \sin BSC}{BC} = \frac{d \sin \gamma}{a}$$

Since BS is very small in comparsion to BA and BC, the angles β_1 and β_2 are extremely small, and we may write

$$\beta_1 \text{ (seconds)} = \frac{\sin \beta_1}{\sin 1''} = \frac{d \sin(\theta + \gamma)}{c \sin 1''} = \frac{d \sin(\theta + \gamma)}{c} \times 206265 \qquad \ldots[8.27\ (a)]$$

and $$\beta_2 \text{ (seconds)} = \frac{\sin \beta_2}{\sin 1''} = \frac{d \sin \gamma}{a \sin 1''} = \frac{d \sin \gamma}{a} \times 206265 \qquad \ldots[8.27\ (b)]$$

(3) After having calculated the angles β_1 and β_2, the observed angle θ at S is reduced to that at B as follows :

$$\angle ABC = \alpha = \angle AOC - \beta_2 = (\beta_1 + \theta) - \beta_2 = \theta + \beta_1 - \beta_2 \qquad \ldots(1)$$

$$= \theta + \frac{d \sin(\theta + \gamma)}{c \sin 1''} - \frac{d \sin \gamma}{a \sin 1''}$$

The above expression for the true angle α does not cover all the four possible cases corresponding to the four positions of the satellite station S, as shown by S_1, S_2, S_3 and S_4 in Fig. 8.31 (*a*)].

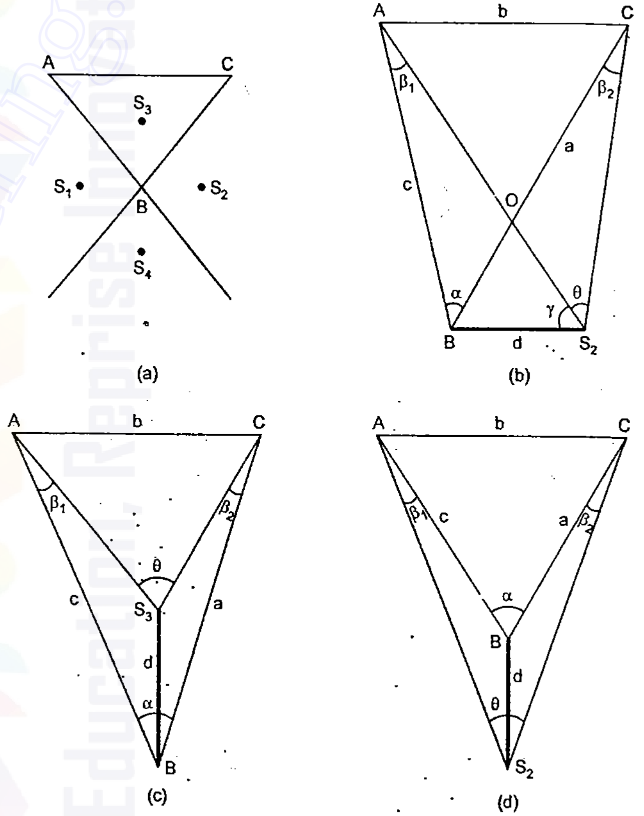

FIG. 8.31

Case I. Position S_1 to the left of B [Fig. 8.31 (*a*) and Fig. 8.30]

The true angle $\alpha = \theta + \beta_1 - \beta_2 \qquad \ldots(1)$

Case II. Position S_2 to the right of B [Fig. 8.31 (*b*)]

The true angle $\alpha = \angle AOC - \beta_1 = (\theta + \beta_2) - \beta_1 = \theta - \beta_1 + \beta_2 \qquad \ldots(2)$

Case III. Position S_3 between AC and B [Fig. 8.31(*c*)]

The true angle $\alpha = \theta - \beta_1 - \beta_2 \qquad \ldots(3)$

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$$\therefore \quad AB = AC\,\frac{\sin ACB}{\sin ABC} = 4248.5\,\frac{\sin 60°\,26'\,12''}{\sin 60°\,15'\,22''} = 4256.1 \text{ m}$$

and $$BC = AC\,\frac{\sin CAB}{\sin ABC} = 4248.5\,\frac{\sin 59°\,18'\,26''}{\sin 60°\,15'\,22''} = 4207.7 \text{ m}$$

Now from $\Delta\,ABS$, $\sin \beta_1 = BS\,\dfrac{\sin ASB}{AB}$

Since β_1 is extremely small, we have

$$\beta_1 = \frac{\sin \beta_1}{\sin 1''} = BS\,\frac{\sin ASB}{AB} \times 206265 \text{ seconds}$$

$$= 12.2\,\frac{\sin 30°\,20'\,30''}{4256.1} \times 206265 \text{ seconds} = 298.67 \text{ seconds} = 4'\,58''.67$$

Similarly, from ΔCBS,

$$\sin \beta_2 = BS\,\frac{\sin BSC}{BC} \quad \text{or} \quad \beta_2 = BS\,\frac{\sin BSC}{BC} \times 206265 \text{ seconds}$$

$$= 12.2\,\frac{\sin 29°\,45'\,6''}{4207.7} \times 206265 \text{ seconds}$$

$$= 296.78 \text{ seconds} = 4'\,56''.78$$

Now the correct angle $ABC = \angle ASC + \beta_1 + \beta_2$

$$= (30°\,20'\,30'' + 29°\,45'\,6'') + 4'\,58''.67 + 4'\,56''.78 = \mathbf{60°\,15'\,31''.45}$$

Example 8.20. *From a satellite station S, 5.8 metres from the main triangulation station A, the following directions were observed :*

A	*0°*	*0'*	*0 "*
B	*132°*	*18'*	*30 "*
C	*232°*	*24'*	*6 "*
D	*296°*	*6'*	*11 "*

The lengths AB, AC and AD were computed to be 3265.5 m, 4022.2 m and 3086.4 m respectively

Determine the directions of AB, AC and AD.

Solution. (Fig. 8.33)

The correction to any direction is given by

$$\beta = \frac{d \sin \theta}{D \sin 1''} \text{ seconds} \qquad \ldots(8.28)$$

(*a*) *For the line AB* :

θ = angle reduced to the direction SA

$= 132°\,18'\,30''$

$d = AS = 5.8$ m

$D = AB = 3265.5$

$$\therefore \quad \beta = \frac{5.8 \sin 132°18'30''}{3265.5} \times 206265 \text{ seconds}$$

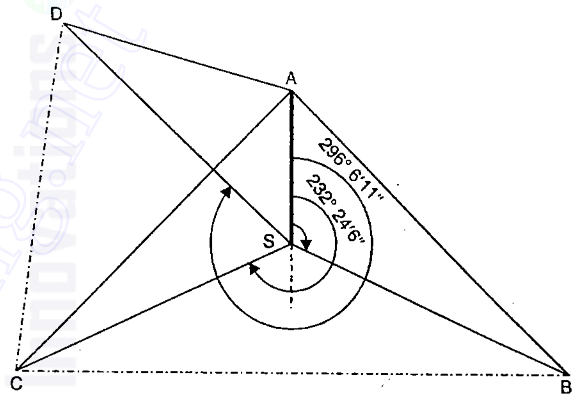

FIG. 8.33

$= +\,270''.9 = +\,4'\,30''.9$

$\therefore$ Direction of AB = direction of $SB + \beta$

$= 132°\,18'\,30'' + 4'\,30''.9$

$= \mathbf{132°\,23'\,0''.9}$

(*b*) *For the line AC* :

θ = angle reduced to the direction S

$= 232°\,24'\,6''$

$D = AC = 4022.2$ m

$$\therefore \quad \beta = \frac{5.8 \sin 232°\,24'\,6''}{4022.2} \times 206265 \text{ seconds}$$

$= -\,235.7$ seconds

$= -\,3'\,55''.7$

$\therefore$ Direction of AC = Direction of $SC + \beta$

$= 232°\,24'\,6'' - 3'\,55''.7$

$= \mathbf{232°\,20'\,4''.3}$

(*c*) *For the line AD* :

θ = angle reduced to the direction $SA = 296°\,6'\,11''$

$D = AD = 3086.4$ m

$$\beta = \frac{5.8 \sin 296°\,6'\,11''}{3086.4} \times 206265 \text{ seconds}$$

$= -\,348.1$ seconds

$= -\,5'\,48''.1$

$\therefore$ Direction of AD = direction of $SD + \beta$

PROBLEMS

1. How do you determine the intervisibility of triangulation stations?

Two triangulation stations A and B are 40 km apart and have elevations of 178 m and 175 m respectively. Find the minimum height of signal required at B so that the line of sight may not pass nearer the ground than 3 metres. The intervening ground may be assumed to have a uniform elevation of 150 metres.

2. The altitudes of two proposed stations A and B, 80 km apart are respectively 225 m and 550 m. The intervening obstructions situated at C, 40 km from A has an elevation of 285 m. Ascertain if A and B are intervisible, and if necessary, find by how much B should be raised so that the line of sight must nowhere be less than 3 m above the surface of the ground.

3. The altitudes of two proposed triangulation stations A and C, 65 miles apart, are respectively 703 ft and 3520 ft above sea level datum, while the heights of two eminences B and D on the profile between A and C are respectively 1170 and 2140 ft, the distance AB and AD being respectively 24 miles and 45 miles.

Ascertain if A and C are intervisible and, if necessary, determine a suitable height for a scaffold at C, given that A is a ground station. The earth's mean radius may be taken as 3960 miles, and coefficient of refraction 0.07. (*U.L.*)

4. What is meant by a satellite station and reduction to centre ? Derive expression for reducing the angles measured at the satellite stations to centre.

5. What is meant by the eccentricity of signal ? How would you correct the observation when made upon an eccentric signal ?

6. On occupying a ground station A of a triangulation survey, it was evident that some elevation of the theodolite would be neccessary, in order to sight the signals at adjacent stations: P on the left and Q on the right. It was found, however, that these stations could be seen from a ground station B, south-west of A, so that AB approximately bisects the angle PBQ.

Where upon, B was adopted as a false station and the distance AB was carefully measured, being 2.835 m, while the angles PBA and ABQ were observed to be 28° 16′ 35″ and 31° 22′ 20″ respectively. The side PQ was computed to be 994.87 metres in the adjacent triangle, and when A was under observation, the interior angles at P and Q were found to have mean value of 62° 34′ 15″ and 57° 39′ 20″ respectively. Determine accurately the magnitude of the angle PAQ.

7. Directions are observed from a satellite station S, 62.18 m from staton C, with the following results :

A, 0° 0′ 0″ ; B, 71° 54′ 32″; C, 296° 12′ 2″.

The approximate lengths of AC and BC are respectively 8041 m and 10864 m. Calculate the angle ACB.

8. In a quadrilateral $ABCD$ in clockwise order, forming part of a triangulation, a church spire was observed as the central station O. Accordingly, a satellite station S was selected 6.71 metres from O, and inside the triangle BOC. The following table gives the approximate distance from the central station and the angles observed from S.

Observed station	*Horizontal angle at S measured clockwise from O*	*Distance (metres)*
A	30° 45′ 30″	OA = 5532
B	98° 32′ 00″	OB = 6789
C	210° 10′ 40″	OC = 3914
D	320° 14′ 15″	OD = 4670
O	360° 00′ 00″	

Calculate the four central angles at O.

9. Discuss the effect of phase in sighting a sun signal and show with sketches how it may be eliminated or reduced.

Derive formulae for the correction to be applied to cylindrical signals (*a*) when the bright portion is bisected and (*b*) when the bright line is bisected. (*U.L*)

10. What is meant by 'base net' ? Explain how you would extend a base line.

11. (*a*) What are the principal objects to be kept in view in selecting the ground for a base line in large survey ? Enumerate in sequence the operations necessary before the measurement of the base line commences. State the correction to be applied in base line measurements.

(*b*) Explain how you would prolong a given base line. (*U.L.*)

12. Show that in base line measurement with tapes and wires in flat catenary with supports at different levels, the total correction will be $-(x+c)$, where x is the parabolic approximation for sag between the level supports and c, the level or slope correction taken permissibly to the first approximation. (*U.L.*)

13. Find the sag correction for 30 m steel tape under a pull of 80 N in three equal spans of 10 m each. Mass of one cubic cm of steel $= 7.86 \text{ g/cm}^3$. Area of cross-section of the tape = 0.10 sq. cm.

14. A steel tape is 30 m long at a temperature of 65°F when lying horizontally on the ground. Its sectional area is 0.082 sq. cm, its mass 2 kg and coefficient of expansion 65×10^{-7} per 1° F. The tape is stretched over three equal spans. Calculate actual length between the end graduations under the following conditions : temp. 85° F, pull 180 N. Take $E = 2.07 \times 10^7 \text{ N/cm}^2$.

15. A 30 m steel tape was standardized on the flat and was found to be exactly 30 m under no pull at 66° F. It was used in catenary to measure a base of 5 bays. The temperature during the measurement was 92° F and pull exerted during the measurement was 100 N. The area of cross-section of the tape was 0.08 sq. cm. The specific mass of steel is 7.86 g/cm^3.

$$\alpha = 0.0000063 \text{ per } 1° \text{ F and } E = 2.07 \times 10^7 \text{ N/cm}^2.$$

Find the true length of the line.

16. A base line for a triangulation is to be measured with a steel tape. Give a complete list of the necessary apparatus with sketches and describe how you would carry out the measurment. Give approximate dimensions of the tape you would use. What kind of steel should it be made of ? Give your reasons. Write down a complete list of corrections which must be applied to the measued length, indicating whether these corrections are additive or subractive. (*A.M.I.C.E.*)

17. A copper transmission line, $\frac{1}{2}$ in. in diameter, is stretched between the two points, 1000 ft apart, at the same level, with a tension of $\frac{1}{2}$ ton, when the temperature is 90° F. It is neccessary to define its limiting positions when the temperature varies. Making use of the corrections for sag, temperature, and elasticity normally applied to base line measurements by tape in catenary, find the tension at a temperature of 14° F and the sag in the two cases. Young's modulus for copper $10 \times 10^6 \text{ lb/in}^2$, its density 555 lb/ft^3 and its coefficient of linear expeansion 9.3×10^{-6} per ° F. (*U.L.*)

18. A nominal distance of 100 ft was set out with a 100 ft steel tape from a mark on the top of one peg to a mark on the top of another, the tape being in catenary under a pull of 20 lb and at a mean temperature of 70° F. The top of one peg was 0.56 ft below the top of the other. The tape has been standardized in catenary under a pull of 25 lb at a temperature of 62° F.

Calculate the exact horizontal distance between the marks on the two pegs and reduce it to mean sea level. The top of the higher peg was 800 ft above mean sea level.

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errors. *Accidental error represent the limit of precision in the determination of a value. They obey the laws of chance and, therefore, must be handled according to the mathematical laws of probability.*

The theory of errors that is discussed in this chapter deals only with the accidental errors after all the known errors are eliminated and accounted for.

9.2. DEFINITIONS

The following are some of the terms which shall be used :

1. **Independent Quantity.** An observed quantity may be classified as (*i*) *independent* and (*ii*) *conditioned.* An independent quantity is the one whose value is independent of the values of other quantities. It bears no relation with any other quantity and hence change in the other quantities does not affect the value of this quantity. Example : reduced levels of several bench marks.
2. **Conditioned Quantity.** A conditioned quantity is the one whose value is dependent upon the values of one or more quantities. Its value bears a rigid relationship to some other quantity or quantities. *It is also called a dependent quantity.* For example, in a triangle ABC, $\angle A + \angle B + \angle C = 180°$. In this *conditioned equation,* any two angles may be regarded as independent and the third as dependent or conditioned.
3. **Direct Observation.** An observation is the numerical value of a measured quantity, and may be either direct or indirect. A *direct observation* is the one made directly on the quantity being determined, *e.g.,* the measurement of a base, the single measurement of an angle etc.
4. **Indirect Observation.** An indirect observation is one in which the observed value is deduced from the measurement of some related quantities, *e.g.,* the measurement of angle by repetition (a multiple of the angle being measured.)
5. **Weight of an Observation.** The weight of an observation is a number giving an indication of its precision and trustworthiness when making a comparison between several quantities of different worth. Thus, if a certain observation is of weight 4, it means that it is four times as much reliable as an observation of weight 1. When two quantities or observations are assumed to be equally reliable, the observed values are said to be of equal weight or of unit weight. Observations are called weighted when different weights are assigned to them. Observations are required to be weighted when they are made with unequal care and under dissimilar conditions. Weights are assigned to the observations or quantities observed in direct proportion to the number of observations.
6. **Observed Value of a Quantity.** An observed value of a quantity is the value obtained when it is corrected for all the known errors.
7. **True Value of Quantity.** The true value of a quantity is the value which is absolutely free from all the errors. The true value of a quantity is indeterminate since the true error is never known.
8. **Most Probable Value.** The most probable value of a quantity is the one which has more chances of being *true* than has any other. It is deduced from the several measurements on which it is based.
9. **True Error.** A true error is the difference between the true value of a quantity and its observed value.

10. **Most Probable Error.** The most probable error is defined as that quantity which added to, and subtracted from, the most probable value fixes the limits within which it is an even chance the true value of the measured quantity must lie.
11. **Residual Error.** A residual error is the difference between the most probable value of a quantity and its observed value.
12. **Observation Equation.** An observation equation is the relation between the observed quantity and its numerical value.
13. **Conditioned Equation.** A conditioned equation is the equation expressing the relation existing between the several dependent quantities.
14. **Normal Equation.** A normal equation is the one which is formed by multiplying each equation by the co-efficient of the unknown whose normal equation is to be found and by adding the equations thus formed. As the number of normal equations is the same as the number of unknowns, the most probable values of the unknown can be found from these equations.

9.3. THE LAWS OF ACCIDENTAL ERRORS

Investigations of observations of various types show that accidental errors follow a definite law, the *law of probability*. This law defines the occurrence of errors and can be expressed in the form of equation which is used to compute the probable value or the probable precision of a quantity. The most important features of accidental errors which usually occur are :

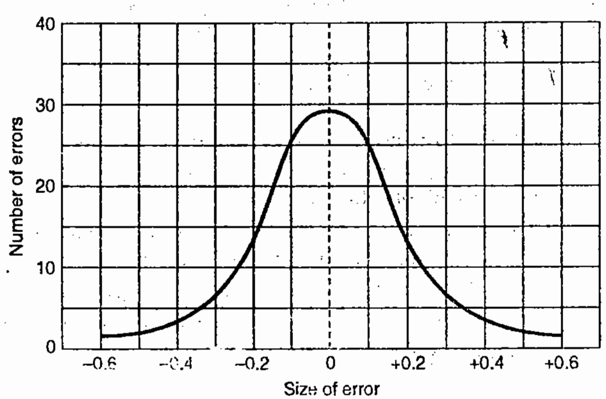

FIG. 9.1. PROBABILITY CURVE.

(*i*) Small errors tend to be more frequent than the large ones ; that is they are the *most probable*.

(*ii*) Positive and negative errors of the same size happen with equal frequency; that is, they are *equally probable.*

(*iii*) Large errors occurs infrequently and are impossible.

Probability Curve. The theory of probability describes these features by stating that the relative frequencies of errors of different extents can be represented by a curve as shown in Fig. 9.1.

This curve, called the *curve of error* or *probability curve*, forms the basis for the mathematical derivation of theory of errors.

The formula for probable error is difficult to derive. It is stated here categorically: *Probable error* of a single measurement is given by

$$E_s = \pm\, 0.6745 \sqrt{\frac{\Sigma v^2}{n-1}} \qquad ...(9.1)$$

If n is large and e is kept small by making precise measurement, $\frac{\Sigma e}{n}$ becomes practically infinitesimal with respect to M.

Hence $x \approx M$ ...(9.4)

Thus, the arithmetic mean is the true value where the number of observed value is very large.

Let $r_1, r_2, r_3.....r_n$ be the residuals (*i.e.* the difference between the mean values and the observed values). Thus,

$$\left.\begin{aligned} M - V_1 &= r_1 \\ M - V_2 &= r_2 \\ M - V_3 &= r_3 \\ &\cdots\cdots \\ M - V_n &= r_n \end{aligned}\right] \qquad ...(4)$$

Adding the above,

$$nM - \Sigma V = \Sigma r$$

or
$$M = \frac{\Sigma V}{n} + \frac{\Sigma r}{n}$$

Under the preceding conditions and by preceding equation

$$M = \frac{\Sigma V}{n}$$

and hence
$$\frac{\Sigma r}{n} = 0 \qquad ...(5)$$

Hence the sum of the residuals equals zero and the sum of plus residual equals the sum of the minus residuals.

Let N be any other value of the unknown other than the arithmetic mean. We have

$$\left.\begin{aligned} N - V_1 &= r_1' \\ N - V_2 &= r_2' \\ N - V_3 &= r_3' \\ &\cdots\cdots \\ N - V_n &= r_n' \end{aligned}\right] \qquad ...(6)$$

Squaring equation (4) and adding, we get

$$\Sigma r^2 = nM^2 + \Sigma V^2 - 2M\,\Sigma V \qquad ...(7)$$

Similarly, squaring equations (6) and adding, we get

$$\Sigma r'^2 = nN^2 + \Sigma V^2 - 2N\,\Sigma V \qquad ...(8)$$

Substituting $nM = \Sigma V$ in equation (7), we get

$$\Sigma r^2 = M\Sigma V - 2M\Sigma V + \Sigma V^2 = \Sigma V^2 - M\Sigma V$$

$$= \Sigma V^2 - \frac{\Sigma V^2}{n}, \text{ by putting } M = \Sigma\frac{V}{n}$$

or
$$\Sigma V^2 = \Sigma r^2 + \frac{\Sigma V^2}{n} \qquad ...(9)$$

Substituting ΣV^2 of equation (9) in equation (8), we get

$$\Sigma r'^2 = nN^2 + \left(\Sigma r^2 + \frac{\Sigma V^2}{n}\right) - 2N\Sigma V = \Sigma r^2 + n\left(N^2 - 2N\frac{\Sigma V}{n} + \frac{\Sigma V^2}{n^2}\right) = \Sigma r^2 + n\left(N - \frac{\Sigma V}{n}\right)^2$$

As $\left(N - \frac{\Sigma V}{n}\right)^2$ is always positive, Σr^2 is less than $\Sigma r'^2$. *That is, the sum of the squares of the residuals found by the use of the arithmetic mean is a minimum.* This is, thus, the fundamental law of least squares.

9.5. LAWS OF WEIGHTS

From the method of least squares the following laws of weights are established :

(1) ***The weight of the arithmetic mean of the measurements of unit weight is equal to the number of observations.***

For example, let an angle A be measured six times, the following being the values:

$\angle A$	Weight	$\angle A$	Weight
30° 20′ 8″	1	30° 20′ 10″	1
30° 20′ 10″	1	30° 20′ 9″	1
30° 20′ 7″	1	30° 20′ 10″	1

$\therefore$ Arithmetic mean $= 30° 20' + \frac{1}{6}(8'' + 10'' + 7'' + 10'' + 9'' + 10'') = 30° 20' 9''$.

Weight of arithmetic mean = number of observations = 6.

(2) ***The weight of the weighted arithmetic mean is equal to the sum of the individual weights.***

For example, let an angle A be measured six times, the following being the values:

$\angle A$	Weight	$\angle A$	Weight
30° 20′ 8″	2	30° 20′ 10″	3
30° 20′ 10″	3	30° 20′ 9″	4
30° 20′ 6″	2	30° 20′ 10″	2

Sum of the individual weights = 2 + 3 + 2 + 3 + 4 + 2 = 16

Weighted arithmetic mean $= 30° 20' + \frac{1}{16}[8'' \times 2) + (10'' \times 3)$
$+ (6'' \times 2) + (10'' \times 3) + (9'' \times 4) + (10'' \times 2)] = 30° 20' 9''$.

Weight of the weighted arithmetic mean = 16

(3) ***The weight of algebraic sum of two or more quantities is equal to the reciprocal of the sum of reciprocals of individual weights.***

For example let $\alpha = 42° 10' 20''$, weight 4
$\beta = 30° 40' 10''$, weight 2

Sum of reciprocals of individual weights $= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$\therefore$ Weight of $\alpha + \beta\ (= 72° 50' 30'') = \dfrac{1}{\frac{1}{4}+\frac{1}{2}} = \dfrac{1}{\frac{3}{4}} = \dfrac{4}{3}$

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(a) *Probable error (p.e.) of single observation of unit weight*

$$= E_s = \pm 0.6745 \sqrt{\frac{\Sigma wv^2}{n-1}} \quad ...(9.7)$$

(b) *Probable error of single observation of weight w*

$$= \frac{\text{p.e. of single observation of unit weight}}{\sqrt{\text{weight}}} = \frac{E_s}{\sqrt{w}} = \pm 0.6745 \sqrt{\frac{\Sigma wv^2}{w(n-1)}} \quad ...(9.8)$$

(b) *Probable error of weighted arithmetic mean*

$$= \pm 0.6745 \sqrt{\frac{\Sigma wv^2}{\Sigma w \times (n-1)}} \quad ...(9.9)$$

Case 3. Probable Error of Computed Quantities

The probable error of computed quantities follow the following *laws* depending upon the relation between the computed quantity and the observed quantity.

1. *If a computed quantity is equal to sum or difference of the observed quantity plus or minus a constant, the probable error of the computed quantity is the same as that of the observed quantity.*

Let x = observed quantity; y = computed quantity ; k = a constant

Such that $y = \pm x \pm k$

Then $e_y = e_x$ ...(9.10)

where e_x = probable error of the observed quantity

e_y = corresponding probable error of the computed quantity

For example, Let $\angle A + \angle B = 90°$

$\angle B = 46° 30' 20''$

p.e. in observation of $\angle B = \pm 0''.4$

Then, the p.e. in observation of $\angle A = \pm 0''.4$

Now $\angle A = 90° - \angle B = \angle 90° - 46° 30' 20'' = 43° 29' 40''$

and probable value of $\angle A = 43° 29' 40'' \pm 0''.4$.

2. *If a computed quantity is equal to an observed quantity multiplied by a constant, the p.e. of computed quantity is equal to the p.e. of observed quantity multiplied by the constant.*

Let x = observed quantity ; y = computed quantity ; k = a constant

Such that $y = kx$

Then $e_y = ke_x$ ...(9.11)

For example, let $A = 4.6\ B$

$B = 2.2$ (observed) ; p.e. in $B = \pm 0.02$

Then $A = 4.6\ B = 4.6 \times 2.2 = 10.12$

and p.e. in observation of $A = k \times$ (p.e. of B)

$= 4.6 \times (\pm 0.02) = \pm 0.092$

Hence probable value of $A = 10.12 \pm 0.092$.

3. *If a computed quantity is equal to the sum of two or more observed quantities, the p.e. of the computed quantity is equal to the square root of sum of the square of p.e.'s of observed quantities.*

Let x_1, x_2, x_3 ....be the observed quantities

y = computed quantity

Such that $y = x_1 + x_2 + x_3$ ...

Then $e_y = \sqrt{e_{x1}^2 + e_{x2}^2 + e_{x3}^2 ......}$ ...(9.12)

where e_y = p.e. of the computed quantity

e_{x1}, e_{x2}, e_{x3} ....etc = p.e. of the observed quantities.

For example, let $A + B + C = 180°$

$A = 30° 30' 12'' \pm 0''.2$

$B = 68° 45' 48'' \pm 0''.6$

$C = 80° 44' 00'' \pm 0''.4$

To determine the probable error of the summation.

Now $y = A + B + C = 180°$

$\therefore$ $e_y = \sqrt{e_a^2 + e_b^2 + e_c^2} = \sqrt{(0.2)^2 + (0.6)^2 + (0.4)^2}$

$= \sqrt{0.56} = \pm 0''.75$ = p.e. of the summation.

4. *If a computed quantity is a function of an observed quantity, its probable error is obtained by multiplying the p.e. of the observed quantity with its differentiation with respect to that quantity.*

Let x = observed quantity ; y = computed quantity

Such that $y = f(x)$.

Then $e_y = \frac{dy}{dx} e_x$ ...(9.13)

For example let $A = 4.6\ B$

$B = 2.2$ (observed)

p.e. of $B = \pm 0.02$

Now $A = 4.6\ B$

$\therefore$ $\frac{dA}{dB} = 4.6$

$\therefore$ $e_a = 4.6\ e_b = 4.6\ (\pm 0.02) = \pm 0.092$

which is the same as found by rule 2.

5. *If a computed quantity is a function of two more observed quantities, its probable error is equal to the square root of summation of the squares of the p.e. of the observed quantity multiplied by its differentiation with respect to that quantity.*

Let y = computed quantity

x_1, x_2, x_3 etc = observed quantities

Such that $y = f(x_1, x_2, x_3$ etc.)

$\therefore \quad c_2 = 4c_1 = 3''.56$ and $c_3 = \frac{25}{4} c_1 = 5''.55$

Check : $c_1 + c_2 + c_3 = 0''.89 + 3''.56 + 5''.55 = 10''$

Hence the corrected angles are

$$\alpha = 78° 12' 12'' + 0''.89 = 78° 12' 12''.89$$
$$\beta = 136° 48' 30'' + 3''.56 = 136° 48' 33''.56$$

and $$\gamma = 144° 59' 08'' + 5''.55 = 144° 59' 13''.55$$

Sum $= 360° 00' 00''.00$

Example 9.4. *An angle A was measured by different persons and the following are the values :*

Angle		*Number of measurements*
65° 30′ 10″	...	2
65° 29′ 50″	...	3
65° 30′ 00″	...	3
65° 30′ 20″	...	4
65° 30′ 10″	...	3

Find the most probable value of the angle.

Solution. As stated earlier, the most probable value of an angle is equal to its weighted arithmetic mean.

$$65° 30' 10'' \times 2 = 131° 00' 20''$$
$$65° 29' 50'' \times 3 = 196° 29' 30''$$
$$65° 30' 00'' \times 3 = 196° 30' 00''$$
$$65° 30' 20'' \times 4 = 262° 01' 20''$$
$$65° 30' 10'' \times 3 = 196° 30' 30''$$

$$\text{Sum} = 982° 31' 40''$$
$$\Sigma \text{ weight} = 2 + 3 + 3 + 4 + 3 = 15$$

$\therefore$ Weighted arithmetic mean $= \frac{982° 31' 40''}{15} = 65° 30' 6''.67$

Hence most probable value of the angle = **65° 30′ 6″.67**

Example 9.5. *Adjust the following angles closing the horizon :*

$A = 110° 20' 48''$	*wt.* 4
$B = 92° 30' 12''$	*wt.* 1
$C = 56° 12' 00''$	*wt.* 2
$D = 100° 57' 04''$	*wt.* 3

Solution. Sum of the observed angles $= 360° 00' 04''$

$\therefore$ Error $= + 4''$

Total correction $= - 4''$

This error of 4″ will be distributed to the angles in an inverse proportion to their weights.

Let c_1, c_2, c_3 and c_4 be the corrections to the observed angles A, B, C and D respectively.

$$\therefore \quad c_1 : c_2 : c_3 : c_4 = \frac{1}{4} : \frac{1}{1} : \frac{1}{2} : \frac{1}{3}$$

or $$c_1 : c_2 : c_3 : c_4 = 1 : 4 : 2 : \frac{4}{3} \quad \ldots(1)$$

Also $c_1 + c_2 + c_3 + c_4 = 4'' \quad \ldots(2)$

From (1), we have

$$c_2 = 4c_1 \; ; \quad c_3 = 2c_1 \quad \text{and} \quad c_4 = \frac{4}{3} c_1$$

Substituting these values of c_2, c_3 and c_4 in (2), we get

$$c_1 + 4c_1 + 2c_1 + \frac{4}{3} c_1 = 4$$

or $$c_1 \left(1 + 4 + 2 + \frac{4}{3} \right) = 4$$

$$\therefore \quad c_1 = \frac{4 \times 3}{25} = \frac{12}{25} = 0''.48$$

Hence $$c_2 = 4c_1 = 1''.92$$
$$c_3 = 2c_1 = 0''.96$$
$$c_4 = \frac{4}{3} c_1 = 0''.64$$

Hence the corrected angles are

$$A = 110° 20' 48'' - 0''.48 = 110° 20' 47''.52$$
$$B = 92° 30' 12'' - 1''.92 = 92° 30' 10''.08$$
$$C = 56° 12' 00'' - 0''.96 = 56° 11' 59''.04$$

and $$D = 100° 57' 04'' - 0''.64 = 100° 57' 03''.36$$

Sum $= 360° 00' 00''.00$

9.8. NORMAL EQUATIONS

A normal equation is the one which is formed by multiplying each equation by the coefficient of the unknown whose normal equation is to be found and by adding the equation thus formed. As the number of normal equations is the same as the number of unknowns, the most probable values of the unknowns can be found from the equations.

Consider a round of angles observed at a central station, the horizon closing with three angles x, y and z, which are geometrically fixed by the condition equation

$$x + y + z = 360° = -d \text{ (say)}$$

If all the angles are of equal weight, the error e in the round will be $(x + y + z + d)$. The most probable value of each angle can then be obtained by applying a correction of $\frac{1}{3} e$ to each observed angle.

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$$9x + 9y + 3z - 12 = 0$$
$$2x + 4y + 4z - 12 = 0$$
$$5x + y + 4z - 21 = 0$$

$\therefore$ Normal equation for y is $16x + 14y + 11z - 45 = 0$ ...(II)

Similarly, the coefficients of z are 1, 2 and 4. Hence

$$3x + 3y + z - 4 = 0$$
$$2x + 4y + 4z - 12 = 0$$
$$20x + 4y + 16z - 84 = 0$$

$\therefore$ Normal equation for z is $25x + 11y + 21z - 100 = 0$ ...(III)

Hence the normal equations for x, y and z are

$$35x + 16y + 25z - 123 = 0 \quad ...(I)$$
$$16x + 14y + 11z - 45 = 0 \quad ...(II)$$
$$25x + 11y + 21z - 100 = 0 \quad ...(III)$$

(*b*) The normal equation of an unknown quantity is formed by multiplying each equation by the algebraic co-efficient of that quantity in that equation and the weight of that equation, and adding the result.

Thus, in equations (1), (2) and (3) the product of coefficients of x and weight of respective equations are : (3×2), (1×3) and (5×1). Hence

$$18x + 18y + 6z - 24 = 0 \text{ (from 1)}$$
$$3x + 6y + 6z - 18 = 0 \text{ (from 2)}$$
$$25x + 5y + 20z - 105 = 0 \text{ (from 3)}$$

Normal equation for x is $46x + 29y + 32z - 147 = 0$ ...(I *a*)

Similarly, the product of coefficient of y and weight of each equation, in the original equations are (3×2), (2×3) and (1×1) respectively. Hence

$$18x + 18y + 6z - 24 = 0 \text{ (from 1)}$$
$$6x + 12y + 12z - 36 = 0 \text{ (from 2)}$$
$$5x + y + 4z - 21 = 0 \text{ (from 3)}$$

$\therefore$ Normal equation for y is $29x + 31y + 22z - 81 = 0$ ...(II *a*)

And, the product of coefficient of z and weight of each equation, in the origional equations are (1×2), (2×3) and (4×1) reespectively. Hence

$$6x + 6y + 2z - 8 = 0 \text{ (from 1)}$$
$$6x + 12y + 12z - 36 = 0 \text{ (from 2)}$$
$$20x + 4y + 16z - 84 = 0 \text{ (from 3)}$$

$\therefore$ Normal equation for z is $32x + 22y + 30z - 128 = 0$ ...(III *a*)

Hence the normal equations for x, y and z are as follows :

$$46x + 29y + 32z - 147 = 0 \quad ...(I\ a)$$
$$29x + 31y + 22z - 81 = 0 \quad ...(II\ a)$$
$$32x + 22y + 30z - 128 = 0 \quad ...(III\ a)$$

9.9. DETERMINATION OF THE MOST PROBABLE VALUES

As defined earlier, the most probable value of a quantity is the one which has more chances of being true than has any other. It is deduced from the several measurements on which it is based. In practice, the following cases may arise of which the most probable value may be required to be determined :

1. Direct observations of equal weights.
2. Direct observations of unequal weights.
3. Indirectly observed quantities involving unknowns of equal weights.
4. Indirectly observed quantities involving unknowns of unequal weights.
5. Observation equations accompanied by condition equation.

Case 1. Direct Observations of Equal Weights

As stated earlier, *the most probable value of the directly observed quantity of equal weights is equal to the arithmetic mean of the observed values.*

Thus, if $V_1, V_2, V_3, ... V_n$ is the observed value of a quantity of equal weight, and M is the arithmetic mean, then

$$M = \frac{V_1 + V_2 + V_3 + ... V_n}{n} = \text{most probable value} \quad ...(9.16)$$

Case 2. Direct Observations of Unequal Weights

As proved earlier, *the most probable value of an observed quantity of unequal weights is equal to the weighted arithmetic mean of the observed quantities.*

Thus, if V_1, V_2, V_3 etc. are the observed quantities with weights w_1, w_2, w_3 etc. and N is the most probable value of the quantity, we have

$$N = \frac{w_1 V_1 + w_2 V_2 + w_3 V_3 + ... w_n V_n}{w_1 + w_2 + w_3 + ... w_n} \quad ...(9.16)$$

Case 3 and 4. Indirectly Observed Quantities Involving Unknowns of Equal Weights or Unequal Weights.

When the unknowns are independent of each other, their most probable values can be found by forming the normal equations for each of the unknown quantities, and treating them as simultaneous equations to get the values of the unknowns. The rules for forming the normal equations have already been discussed. See examples 9.7, 9.8 9.9, 9.10 and 9.11 for illustration.

Case 5. Observation Equations Accompained by Condition Equation

When the observation equations are accompanied by one or more condition equations, the latter may be reduced to an observation equation which will eliminate one of the unknowns. The normal equation can then be formed for the remaining unknowns. There is also another

Hence the normal equations are

$$6A + 4B = 517° 23' 7''.2 \quad \ldots(I)$$

$$4A + 7B = 581° 39' 18''.9 \quad \ldots(II)$$

To solve these for A and B, multiply I by 2 and II by 3. Thus,

$$12A + 8B = 1034° 46' 14''.4 \quad \ldots(1)$$

$$12A + 21B = 1744° 57' 56''.7 \quad \ldots(2)$$

Subtracting (1) from (2), we get

$$13B = 710° 11' 42''.3$$

$$B = \mathbf{54° 37' 49''.4}$$

Substituting value of B in (1), we get

$$A = \mathbf{49° 48' 38''.3}.$$

Example 9.10. *The following are mean values observed in the measurement of three angles α, β and γ at one station ;*

$$\alpha = 76° 42' 46''.2 \text{ with weight } 4$$

$$\alpha + \beta = 134° 36' 32''.6 \text{ with weight } 3$$

$$\beta + \gamma = 185° 35' 24''.8 \text{ with weight } 2$$

$$\alpha + \beta + \gamma = 262° 18' 10''.4 \text{ with weight } 1$$

Calculate the most probable value of each angle. (U.L.)

Solution.

To form the normal equation for unknown, multiply each equation by the coefficient of that unknown and also by the weight of the equation, and take the sum of the resulting equations.

Thus, forming normal equation for α for we have

$$4\alpha = 306° 51' 04''.8$$

$$3\alpha + 3\beta = 403° 49' 37''.8$$

$$\alpha + \beta + \gamma = 262° 18' 10''.4$$

$$\therefore \quad 8\alpha + 4\beta + \gamma = 972° 58' 53''.0 \quad \ldots\text{(Normal equation for } \alpha)$$

Forming normal equation for β, we have

$$3\alpha + 3\beta = 403° 49' 37''.8$$

$$2\beta + 2\gamma = 371° 10' 49''.6$$

$$\alpha + \beta + \gamma = 262° 18' 10''.4$$

$$\therefore \quad 4\alpha + 6\beta + 3\gamma = 1037° 18' 37''.8 \quad \ldots\text{(Normal equation for } \beta)$$

Forming normal equation for γ we have

$$2\beta + 2\gamma = 371° 10' 49''.6$$

$$\alpha + \beta + \gamma = 262° 18' 10''.4$$

$$\therefore \quad \alpha + 3\beta + 3\gamma = 633° 29' 00''.0 \quad \ldots\text{(Normal equation for } \gamma)$$

Hence the three normal equations are

$$8\alpha + 4\beta + \gamma = 972° 58' 53''.0 \quad \ldots(1)$$

$$4\alpha + 6\beta + 3\gamma = 1037° 18' 37''.8 \quad \ldots(2)$$

$$\alpha + 3\beta + 3\gamma = 633° 29' 00''.0 \quad \ldots(3)$$

Solving the above three equations simultaneously for α, β and γ we get

$$\alpha = \mathbf{76° 42' 46''.17}$$

$$\beta = \mathbf{57° 53' 46''.13}$$

$$\gamma = \mathbf{127° 41' 38''.26}$$

9.10. ALTERNATIVE METHOD OF DIFFERENCES

The above *direct method* of solving the normal equations is very laborious since it involves large numbers. In order to make them as small as possible, we can solve the equation by *method of differences*. A set of values is assumed for the most probable values of the unknown quantities and the most probable series of errors are determined by normal equations. The errors so found are then added algebraically to the observed values to get the most probable values of the measurements. The procedure for the solution of the problem is as follows :

(1) Let k_1, k_2, k_3 etc. be the corrections (or the residual errors) to the observed values.

(2) Replace the observation equations by equations in terms of k_1, k_2, k_3 etc., to express the discrepancy between the observed results and those given by the assumed values, *always subtracting the latter from the former.*

(3) Form the normal equations in terms of k_1, k_2, k_3, etc. and solve them to get k_1, k_2, k_3 etc.

(4) Add these algebraically to the quantities to get their most probable values.

Example 9.11. *The following observations of three angles A, B and C were taken at one station :*

$$A = 75° 32' 46''.3 \text{ with weight } 3$$

$$B = 55° 09' 53''.2 \text{ with weight } 2$$

$$C = 108° 09' 28''.8 \text{ with weight } 2$$

$$A + B = 130° 42' 41''.6 \text{ with weight } 2$$

$$B + C = 163° 19' 22''.5 \text{ with weight } 1$$

$$A + B + C = 238° 52' 9''.8 \text{ with weight } 1$$

Determine the most probable value of each angle.

Solution.

Let k_1, k_2, k_3 be the most probable correction to A, B and C. Then the most probable values of A, B and C are :

$$A = 75° 32' 46''.3 + k_1 \quad \ldots(1)$$

$$B = 55° 09' 53''.2 + k_2 \quad \ldots(2)$$

$$C = 108° 09' 28''.8 + k_3 \quad \ldots(3)$$

$$A + B = 130° 42' 39''.5 + k_1 + k_2 \text{ by adding (1) and (2).} \quad \ldots(4)$$

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Solution.

The condition equation is

$$A + B + C = 180°$$

From which $C = 180° - (A + B)$

Thus, the third unknown C can be eliminated writing one more observation equation;

$$C = 180° - (A + B) = 58° \, 01' \, 16''$$

or $$A + B = 180° - 58° \, 01' \, 16'' = 121° \, 58' \, 44''$$

Hence, the new observation equations are :

$$A = 68° \, 12' \, 36''$$
$$B = 53° \, 46' \, 12''$$

and $$A + B = 121° \, 58' \, 44''$$

Normal equation for A :

$$A = 68° \, 12' \, 36''$$
$$A + B = 121° \, 58' \, 44''$$

$$2A + B = 190° \, 11' \, 20''$$

Normal equation for B :

$$B = 53° \, 46' \, 12''$$
$$A + B = 121° \, 58' \, 44''$$

$$A + 2B = 175° \, 44' \, 56''$$

Hence, the normal equations are :

$$2A + B = 190° \, 11' \, 20'' \quad ...(1)$$
$$A + 2B = 175° \, 44' \, 56'' \quad ...(2)$$

Solving these, we get

$$A = \mathbf{68° \, 12' \, 34''.7}$$
$$B = \mathbf{53° \, 46' \, 10''.6}$$

$\therefore$ $C = 180° - (A + B) = 180° - (68° \, 12' \, 34''.7 + 53° \, 46' \, 10''.6) = \mathbf{58° \, 1' \, 14''.7}$

Alternative Solution

$$A = 68° \, 12' \, 36''$$
$$B = 53° \, 46' \, 12''$$
$$C = 58° \, 01' \, 16''$$

$\therefore$ $A + B + C = 180° \, 0' \, 04''$

$\therefore$ Total correction $= -4''$

Since the weight of each of the observations is equal, the corrections will be equally divided.

Hence corrected A (most probable values of A)

$$= 68° \, 12' \, 36'' - 1''.33 = \mathbf{68° \, 12' \, 34''.67}$$
$$B = 53° \, 46' \, 12'' - 1''.33 = \mathbf{53° \, 46' \, 10''.67}$$

and $$C = 58° \, 01' \, 16'' - 1''.33 = \mathbf{58° \, 01' \, 14''.67}$$

Example 9.14. *The angles of a triangle ABC were recorded as follows :*

$A = 77° \, 14' 20''$ weight 4

$B = 49° \, 40' 35''$ weight 3

$C = 53° \, 04' 52''$ weight 2

Give the corrected values of the angles. (*K.U.*)

Solution.

The condition equation is

$$A + B + C = 180° \quad \text{or} \quad C = 180° - (A + B)$$

Thus, the unknown C can be eliminated by forming one more observation equation in terms of the two unknowns A and B :

$$C = 180° - (A + B) = 53° \, 4' \, 52''$$

or $$A + B = 180° - 53° \, 4' \, 52'' = 126° \, 55' \, 8''$$

Hence the observation equations are :

$$A = 77° \, 14' \, 20'' \quad \text{(weight 4)}$$
$$B = 49° \, 40' \, 35'' \quad \text{(weight 3)}$$
$$A + B = 126° \, 55' \, 08'' \quad \text{(weight 2)}$$

Normal equation for A :

$$4A = 308° \, 57' \, 20''$$
$$2A + 2B = 253° \, 50' \, 16''$$

$$6A + 2B = 562° \, 47' \, 36''$$

or $$3A + B = 281° \, 23' \, 48''$$

Normal equation for B :

$$3B = 149° \, 01' \, 45''$$
$$2A + 2B = 253° \, 50' \, 16''$$

$$2A + 5B = 402° \, 52' \, 01''$$

Hence, the normal equations are :

$$3A + B = 281° \, 23' \, 48''$$

and $$2A + 5B = 402° \, 52' \, 01''$$

Solving the above simultaneously for A and B, we get

$$A = \mathbf{77° \, 14' \, 23''}$$
$$B = \mathbf{49° \, 40' \, 39''}$$
$$C = 180° - (A + B) = 180° - (77° \, 14' \, 23'' + 49° \, 40' \, 39'') = \mathbf{53° \, 4' \, 58''}$$

Hence the three normal equations for k_1, k_2, k_3 are :

$$5\,k_1 + 2\,k_2 + 2\,k_3 = -6$$
$$2\,k_1 + 4\,k_2 + 2\,k_3 = -6$$
$$2\,k_1 + 2\,k_2 + 6\,k_3 = -6$$

Solving these simultaneously for k_1, k_2 and k_3 we get

$$k_1 = -0''.63 \quad ; \quad k_2 = -0''.95 \quad ; \quad k_3 = -0''.47$$

Hence the most probable values of the angles are

$AOB = 83° 42' 28''.75 - 0''.63 =$ **83° 42′ 28″.12**

$BOC = 102° 15' 43''.26 - 0''.95 =$ **102° 15′ 42″.31**

$COD = 94° 38' 27''.22 - 0''.47 =$ **94° 38′ 26″.75**

$DOA =$ **79° 23′ 22″.82**

9.11. METHOD OF CORRELATES

Correlates or correlatives are the unknown multiples or independent constants used for finding most probable values of unknowns. We have already studied the method of normal equations for finding the most probable values of quantities from observations involving condition equations. The direct method of normal equations was used for simple cases while the 'method of differences' or 'corrections' was used for reducing the arithmetical work. The condition equation was used to eliminate one of the unknown thus giving one more observation equation. However, the method of normal equations become more tedious when the number of conditions are more. In that case, the method of correlates may be used.

In the method of correlates, all the condition equations are collected. To this is added one more equation of condition imposed by the theory of least squares, *i.e.*, the sum of the squares of the residual errors should be minimum.

Suppose, for example, the angles A, B, C, D are measured at a station closing the horizon, the observed values of angles, A, B, C, D may be of weights w_1, w_2, w_3 and w_4 respectively.

Let E be the total residual error in the summation of the four angles such that

$$A + B + C + D - 360° = E$$

Let e_1, e_2, e_3 and e_4 be the corrections to be applied to the observed angles. Then, we have one *equation of condition* :

$$\Sigma e = e_1 + e_2 + e_3 + e_4 = E \qquad \ldots(1)$$

Further the *least square condition* requires that

$$\Sigma(we^2) = w_1\,e_1^2 + w_2\,e_2^2 + w_3\,e_3^2 + w_4\,e_4^2 = \text{a minimum} \qquad \ldots(2)$$

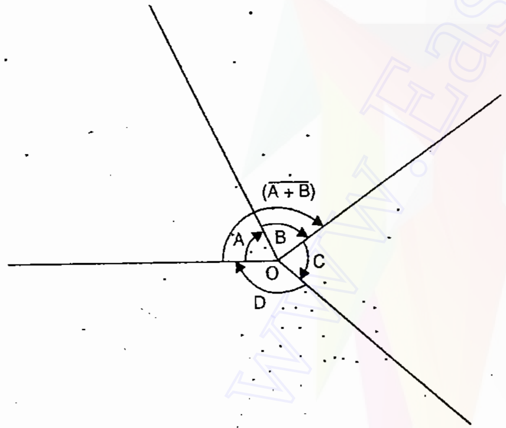

FIG. 9.2. METHOD OF CORRELATES.

Thus, we get two condition equations. Differentiating these two equations, we get

$$\Sigma(\delta e) = \delta e_1 + \delta e_2 + \delta e_3 + \delta e_4 = 0 \qquad \ldots(3)$$

and

$$\Sigma(we\,\delta e) = w_1\,e_1\,\delta e_1 + w_2\,e_2\,\delta e_2 + w_3\,e_3\,\delta e_3 + w_4\,e_4\,\delta e_4 = 0 \qquad \ldots(4)$$

Multiply equation (3) by a correlative $-\lambda_1$ and add the result to equation (4). Thus,

$$-\lambda_1\,\delta e_1 - \lambda_1\,\delta e_2 - \lambda_1\,\delta e_3 - \lambda_1\,\delta e_4 = 0$$

$$w_1\,e_1\,\delta e_1 + w_2\,e_2\,\delta e_2 + w_3\,e_3\,\delta e_3 + w_4\,e_4\,\delta e_4 = 0$$

$$\therefore \quad \delta e_1(w_1\,e_1 - \lambda_1) + \delta e_2(w_2\,e_2 - \lambda_1) + \delta e_3(w_3\,e_3 - \lambda_1) + \delta e_4(w_4\,e_4 - \lambda_1) = 0 \qquad \ldots(5)$$

Since $\delta e_1, \delta e_2, \delta e_3$ are definite quantities and are independent of each other, their coefficient must vanish independently, or

$$\lambda_1 = w_1\,e_1 = w_2\,e_2 = w_3\,e_3 = w_4\,e_4$$

From which

$$\left.\begin{aligned} e_1 &= \frac{\lambda_1}{w_1} \\ e_2 &= \frac{\lambda_1}{w_2} \\ e_3 &= \frac{\lambda_1}{w_3} \\ e_4 &= \frac{\lambda_1}{w_4} \end{aligned}\right] \qquad \ldots(6)$$

Equation (6) shows that the corrections to be applied are inversely proportional to the weights.

To find the value of the correlative λ_1, substitute these values of e_1, e_2, e_3 and e_4 in equation (1). Thus,

$$\frac{\lambda_1}{w_1} + \frac{\lambda_1}{w_2} + \frac{\lambda_1}{w_3} + \frac{\lambda_1}{w_4} = E$$

or

$$\lambda_1\left(\frac{1}{w_1} + \frac{1}{w_2} + \frac{1}{w_3} + \frac{1}{w_4}\right) = E \qquad \ldots(7) \quad \ldots(9.17)$$

From equation (7), the value of λ_1 can be calculated since w_1, w_2, w_3, w_4 and E are known. Knowing the value of λ_1, the corrections e_1, e_2, e_3, e_4 can be calculated from equation (6). These corrections, when applied to the observed angles, will give the most probable values of the angles.

In the above treatment, only one condition equation [*i.e.* Σ (angle) = 360°] was imposed and, therefore, there was only one correlative λ_1. However, if there are more than one condition equations, the first equation (in the form of equation 3 obtained after differentiation) is multiplied by $-\lambda_1$, second by $-\lambda_2$, third by $-\lambda_3$ and so on, and these are added to equation 4 (obtained from the least square principles) to get pairs of equations such as equation (5) and $\lambda_1, \lambda_2, \lambda_3$ etc. can be calculated.

For example, in addition to the individual angles A, B, C and D, angle $(\overline{A+B})$ was also measured with weight w_5. Let e_5 be the correction to be applied to $(\overline{A+B})$. Let E' be the error of closure between the combined angle $(\overline{A+B})$ and the summation of angles A and B, such that

$$(\overline{A+B}) - A - B = E'$$

From the least square condition, Σwe^2 = a minimum

$$\therefore \quad 4e_1^2 + 3e_2^2 + 2e_3^2 = \text{a minimum} \quad \ldots(2)$$

Differentiating (1) and (2), we get

$$\delta e_1 + \delta e_2 + \delta e_3 = 0 \quad \ldots(3)$$

and $\quad 4\, e_1\, \delta e_1 + 3\, e_2\, \delta e_2 + 2\, e_3\, \delta e_3 = 0 \quad \ldots(4)$

Multiplying (3) by $-\lambda$ and adding it to (4), we get

$$\delta e_1(4e_1 - \lambda) + \delta e_2(3e_2 - \lambda) + \delta e_3(2e_3 - \lambda) = 0 \quad \ldots(5)$$

Since the coefficients of $\delta e_1, \delta e_2, \delta e_3$ must vanish independently, we have

$$\left.\begin{array}{lll} 4\, e_1 - \lambda = 0 & \text{or} & e_1 = \dfrac{\lambda}{4} \\ 3\, e_2 - \lambda = 0 & \text{or} & e_2 = \dfrac{\lambda}{3} \\ 2\, e_3 - \lambda = 0 & \text{or} & e_3 = \dfrac{\lambda}{2} \end{array}\right]$$

Substituting these values of e_1, e_2 and e_3 in (1), we get

$$\frac{\lambda}{4} + \frac{\lambda}{3} + \frac{\lambda}{2} = 13'' \qquad \text{or} \qquad \lambda\left(\frac{13}{12}\right) = 13''$$

or $\quad \lambda = +12'' \quad$ and $\quad e_1 = \dfrac{\lambda}{4} = \dfrac{12}{4} = +3''$

$$e_2 = \frac{\lambda}{3} = \frac{12}{3} = +4'' \quad \text{and} \quad e_3 = \frac{\lambda}{2} = \frac{12}{2} = +6''$$

Hence the corrected angles are

$$A = 77°\ 14'\ 20'' + 3'' = \mathbf{77°\ 14'\ 23''}$$

$$B = 49°\ 40'\ 35'' + 4'' = \mathbf{49°\ 40'\ 39''}$$

$$C = 53°\ \ 4'\ 52'' + 6'' = \mathbf{53°\ 4'\ 58''}$$

Note. This example was solved by the two methods of normal equations. It can be seen that the method of correlates applied above to solution of the same problem is very much easier since the computations are very much reduced.

Example 9.18. *Solve example 9.15 by method of correlates.*

Solution.

AOB =	85° 42′ 28″.75	wt. 3
BOC =	102° 15′ 43″.26	wt. 2
COD =	94° 38′ 27″.22	wt. 4
DOA =	79° 23′ 24″.77	wt. 2
Sum	= 360° 00′ 03″.00	

Hence, the total correction $E = 360° - (360°\ 0'\ 3'') = -3''$.

Let e_1, e_2, e_3 and e_4 be the individual corrections to the four angles respectively. Then, by the condition equation, we get

$$e_1 + e_2 + e_3 + e_4 = -3'' \quad \ldots(1)$$

Also, from the least square principle, $\Sigma(we^2)$ = a minimum

Hence $\quad 3e_1^2 + 2e_2^2 + 4e_3^2 + 2e_4^2 = $ a minimum $\quad \ldots(2)$

Differentiating (1) and (2), we get

$$\delta e_1 + \delta e_2 + \delta e_3 + \delta e_4 = 0 \quad \ldots(3)$$

and $\quad 3\, e_1\, \delta e_1 + 2\, e_2\, \delta e_2 + 4\, e_3\, \delta e_3 + 2\, e_4\, \delta e_4 = 0 \quad \ldots(4)$

Multiplying equation (3) by $-\lambda$ and adding it to (4), we get

$$\delta e_1(3e_1 - \lambda) + \delta e_2\,(2e_2 - \lambda) + \delta e_3(4e_3 - \lambda) + \delta e_4\,(2e_4 - \lambda) = 0 \quad \ldots(5)$$

Since the coefficients of $\delta e_1, \delta e_2, \delta e_3$ and δe_4 must vanish independently, we have

$$\left.\begin{array}{lll} 3\, e_1 - \lambda = 0 & \text{or} & e_1 = \dfrac{\lambda}{3} \\ 2\, e_2 - \lambda = 0 & \text{or} & e_2 = \dfrac{\lambda}{2} \\ 4\, e_3 - \lambda = 0 & \text{or} & e_3 = \dfrac{\lambda}{4} \\ 2\, e_4 - \lambda = 0 & \text{or} & e_4 = \dfrac{\lambda}{2} \end{array}\right] \quad \ldots(6)$$

Substituting these values in (1), we get

$$\frac{\lambda}{3} + \frac{\lambda}{2} + \frac{\lambda}{4} + \frac{\lambda}{2} = -3'' \quad \text{or} \quad \lambda\left(\frac{19}{12}\right) = -3'' \quad \text{or} \quad \lambda = -\frac{3 \times 12}{19}$$

Hence

$$e_1 = -\frac{1}{3} \cdot \frac{3 \times 12}{19} = -\frac{12}{19} = -0.63''$$

$$e_2 = -\frac{1}{2} \cdot \frac{3 \times 12}{19} = -\frac{18}{19} = -0.95''$$

$$e_3 = -\frac{1}{4} \cdot \frac{3 \times 12}{19} = -\frac{9}{19} = -0.47''$$

$$e_4 = -\frac{1}{2} \cdot \frac{3 \times 12}{19} = -\frac{18}{19} = -0.95''$$

$$\text{Sum} = -3.0''$$

Hence the corrected angles

AOB =	83° 42′ 28″.75 − 0″.63 =	83° 42′ 28″.12
BOC =	102° 15′ 43″.26 − 0″.95 =	102° 15′ 42″.31
COD =	94° 38′ 27″.22 − 0″.47 =	94° 38′ 26″.75
DOA =	79° 23′ 23″.77 − 0″.95 =	79° 23′ 22″.82
	Sum	= 360° 00′ 00″.00

This example was also solved by the method of differences of normal equations. The method of correlates is however, much more easier.

Solution.

Error of closure = (8.164 + 6.284 + 5.626) − 19.964 = 20.074 − 19.964 = 0.11 m

Total correction = − 0.11 m

Let e_1, e_2, e_3 and e_4 be the corrections to the observed quantities taken in order. Hence we have condition equation :

$$e_1 + e_2 + e_3 + e_4 = -0.11 \text{ m} \quad \ldots(1)$$

Also, from least square condition, $\Sigma(we^2)$ = a minimum

$$2\,e_1^2 + 2\,e_2^2 + 3\,e_3^2 + 3\,e_4^2 = \text{a minimum} \quad \ldots(2)$$

Differentiating (1) and (2), we get

$$\delta e_1 + \delta e_2 + \delta e_3 + \delta e_4 = 0 \quad \ldots(3)$$

and
$$2\,e_1\,\delta e_1 + 2\,e_2\,\delta e_2 + 3\,e_3\,\delta e_3 + 3\,e_4\,\delta e_4 = 0 \quad \ldots(4)$$

Multiplying equation (3) by − λ and adding it to (4), we get

$$\delta e_1(2e_1 - \lambda) + \delta e_2(2e_2 - \lambda) + \delta e_3(3e_3 - \lambda) + \delta e_4(3e_4 - \lambda) = 0$$

Since the co-efficients of δe_1, δe_2, δe_3 and δe_4 must vanish independently, we get

$$2\,e_1 - \lambda = 0 \quad \text{or} \quad e_1 = \frac{\lambda}{2}$$

$$2\,e_2 - \lambda = 0 \quad \text{or} \quad e_2 = \frac{\lambda}{2}$$

$$3\,e_3 - \lambda = 0 \quad \text{or} \quad e_3 = \frac{\lambda}{3}$$

$$3\,e_4 - \lambda = 0 \quad \text{or} \quad e_4 = \frac{\lambda}{3}$$

Substituting the values of e_1, e_2, e_3 and e_4 in (1), we get

$$\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{3} + \frac{\lambda}{3} = -0.11$$

or
$$\lambda = -0.11 \times \frac{3}{5} = -0.066 \text{ m}$$

Hence
$$e_1 = \frac{\lambda}{2} = -0.033 \text{ m}$$

$$e_2 = \frac{\lambda}{2} = -0.033 \text{ m}$$

$$e_3 = \frac{\lambda}{3} = -0.022 \text{ m}$$

$$e_4 = \frac{\lambda}{3} = -0.022 \text{ m}$$

$$\text{Total} = -0.110 \text{ m}$$

Hence the corrected levels are

B = 8.164 − 0.033 = 8.131 above A

C = 6.284 − 0.033 = 6.251 above B = 14.382 above A

D = 5.626 − 0.022 = 5.604 above C = 19.986 above A

Check : Level of A above D = − 19.964 − 0.022 = − 19.986

9.12. TRIANGULATION ADJUSTMENTS

In a triangulation system, all the measured angles should be corrected so that they satisfy :

(*i*) Conditions imposed by the station of observation, known as the *station adjustment;* and

(*ii*) Conditions imposed by the figure, known as the *figure adjustment*.

The most accurate method is that of least squares, and the most rigid application follows when the entire system is adjusted in one mass, all the angles being simultaneously involved. The process is exceedingly laborious, even in nets comprising few figures. As such, it is always convenient to break it into three parts which are each adjusted separately.

(*i*) Single angle adjustment. (*ii*) Station adjustment.

and (*iii*) Figure adjustment.

(1) Single Angle Adjustment

Generally, several observations are taken for a single angle. The corrections to be applied are inversely proportional to the weight and directly proportional to the square of probable errors. In the case of the measurement of the angle with equal weights, the most probable value is equal to the arithmetic mean of the observations. In the case of the weighted observations, the most probable value of the angle is equal to the weighted arithmetic mean of the observed angles. See examples 9.2, 9.3, 9.4 and 9.5.

(2) Station Adjustment

Station adjustment is the determination of the most probable values of two or more angles measured at a station so as to satisfy the condition of being geometrically consistent. There are three cases of station adjustment :

(*i*) when the horizon is closed with angles of equal weights

(*ii*) when the horizon is closed with angles from unequal weights

(*iii*) when several angles are measured at a station individually, and in combination.

Case 1. When the horizon is closed with angles of equal weights.

In Fig. 9.3, angles A, B and C have been measured and the horizon is closed. Hence $A + B + C$ should be equal to 360°. If this condition is not satisfied, the error is distributed *equally* to all the three angles.

Case 2. When the horizon is closed with angles of unequal weights.

If the angles observed are of unequal weight, discrepancy is distributed among the angles inversely as the respective weights.

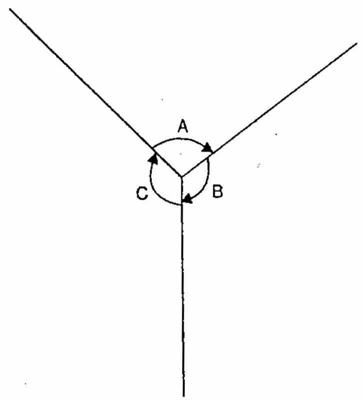

FIG. 9.3

$$A = 38° 25' 20'' + k_1 \quad ...(1)$$
$$B = 32° 36' 12'' + k_2 \quad ...(2)$$
$$C = 48° \ 9' 16'' + k_3 \quad ...(3)$$
$$A + B = 71° 01' 32'' + k_1 + k_2 \text{ by adding (1) and (2).} \quad ...(4)$$
$$A + B + C = 119° 10' 48'' + k_1 + k_2 + k_3 \text{ by adding (1), (2) and (3).} \quad ...(5)$$
$$B + C = 80° 45' 28'' + k_2 + k_3 \text{ by adding (2) and (3).} \quad (6)$$

Substituting these values in the observation equations, we get

$k_1 = 0$ weight 1

$k_2 = 0$ weight 1

$k_1 + k_2 = -3$ weight 2

$k_1 + k_2 + k_3 = -5$ weight 1

$k_2 + k_3 = 0$ weight 2

Normal equation for k_1 :

$$k_1 = 0$$
$$2k_1 + 2k_2 = -6$$
$$k_1 + k_2 + k_3 = -5$$

$$\therefore \quad 4k_1 + 3k_2 + k_3 = -11$$

Normal equation for k_2 :

$$k_2 = 0$$
$$2k_1 + 2k_2 = -6$$
$$k_1 + k_2 + k_3 = -5$$
$$2k_2 + 2k_3 = 0$$

$$\therefore \quad 3k_1 + 6k_2 + 3k_3 = -11$$

Normal equation for k_3 :

$$k_1 + k_2 + k_3 = -5$$
$$2k_2 + 2k_3 = 0$$

$$k_1 + 3k_2 + 3k_3 = -5$$

Hence the three normal equations are

$$4k_1 + 3k_2 + k_3 = -11 \quad ...(I)$$
$$3k_1 + 6k_2 + 3k_3 = -11 \quad ...(II)$$
$$k_1 + 3k_2 + 3k_3 = -5 \quad ...(III)$$

Solving these simultaneously for k_1, k_2 and k_3 we get

$$k_1 = -2''.29$$
$$k_2 = -1''.24$$
$$k_3 = +1''.88$$

Hence the most probable values of the angles are

$$A = 38° 25' 20'' - 2''.29 = \mathbf{38° 25' 17''.71}$$
$$B = 32° 36' 12'' - 1''.24 = \mathbf{32° 36' 10''.76}$$
$$C = 48° \ 9' 16'' + 1''.88 = \mathbf{48° 9' 17''.88}$$

9.13. FIGURE ADJUSTMENT

The determination of the most probable values of the angles involved in any geometrical figure so as to fulfil geometrical conditions is called the figure adjustment. The figure adjustment, therefore, involves one or more condition equations. We have already discussed the simple cases of condition equations by the method of normal equations and also by the method of correlates. When the condition equations are more, the method of correlates is much simpler.

The triangulation system mainly consists of the following geometrical figures :

(*i*) triangles

(*ii*) quadrilaterals

(*iii*) polygons with central figure.

We shall discuss the adjustments of all the three figures separately in detail.

9.14. ADJUSTMENT OF A GEODETIC TRIANGLE

A triangle is the basic figure of any triangulation system. All the three angles of a triangle are to corrected. The following are the *general rules* for applying the corrections to the observed angles.

Let A, B and C be the observed angles

e_1, e_2 and e_3 be the corresponding corrections

e = the total correction (equal to the discrepancy)

n_1, n_2 and n_3 = number of observations for angles A, B and C respectively

w_1, w_2 and w_3 = relative weights for A, B and C

E_1, E_2 and E_3 = probable error of A, B and C.

Rule 1. Equal corrections. *If all the angles are of equal weight, the discrepancy is distributed equally to all the three angles.*

i.e., $$e_1 = e_2 = e_3 = \frac{1}{3}e$$

Rule 2. Inverse weight corrections. *If all the angles are of unequal weight, the discrepancy is distributed to all the angles in inverse proportion to the weights.*

i.e., $$e_1 : e_2 : e_3 = \frac{1}{w_1} : \frac{1}{w_2} : \frac{1}{w_3}$$

Hence $$e_1 = \frac{\frac{1}{w_1}}{\frac{1}{w_1} + \frac{1}{w_2} + \frac{1}{w_3}} e$$

$$R = 20889000 \text{ ft.}$$

$$\sin 1'' = \frac{\pi}{180 \times 60 \times 60} = \frac{\pi}{648000}$$

$$\in = \frac{A}{(20889000)^2} \cdot \frac{648000}{\pi} \text{ seconds}$$

If Δ is the area in sq. miles,

$$\in = \frac{\Delta\,(5280)^2}{(20889000)^2} \cdot \frac{648000}{\pi} \text{ seconds}$$

or $$\in = \frac{\Delta}{76} \text{ seconds (approximately)} \qquad ...(1)$$

If S = area in square km, we have

$$\in = \frac{S \times 0.386}{76} \qquad \text{(since 1 sq. km.} = 0.386 \text{ sq. mile)}$$

or $$\in = \frac{S}{197} \text{ seconds} \qquad ...(2)$$

The above expression can also be obtained independently by substituting $R = 6370$ km in Eq. 9.19.

Thus, $$\in = \frac{648000\, S}{\pi\,(6370)^2} = \frac{S}{179} \text{ seconds}$$

Hence, generalising the expression for $\in$ we get

$$\left.\begin{aligned} \in &= \frac{A}{76} \text{ seconds, when } A \text{ is in sq. miles} \\ \text{and} \quad \in &= \frac{A}{197} \text{ seconds, when } A \text{ is in sq. km} \end{aligned}\right] \qquad ...(9.20)$$

Knowing the spherical excess, the discrepancy in the observed angles is given by

$$e = 180° + \in - (A + B + C) \qquad ...(9.21)$$

This discrepancy is to be distributed to the angle (A, B and C) as per rules already discussed.

In the calculation of the spherical excess ($\in$) , the area (S) of the spherical triangle is involved. This area cannot be accurately determined unless the angles are accurately known which, in turn, can be known only if the spherical excess is known. Hence, in the first approximation, the area S is calculated by treating the triangle as a plane triangle, and using the observed angles.

Thus $$S = \tfrac{1}{2}\, ab . \sin C \qquad ...(9.22)$$

or $$= \tfrac{1}{2}\, c^2 \frac{\sin A \sin B}{\sin C} \qquad ...(9.23)$$

where c is the known side and A, B and C are the *observed angles*.

Knowing the area, the spherical excess can be calculated and the corrected angles can be computed.

COMPUTATION OF THE SIDES OF A SPHERICAL TRIANGLE

The sides of the spherical triangle can be calculated by one of the following three methods :

1. By spherical trigonometry
2. By Delambre's method
3. By Legendre's method

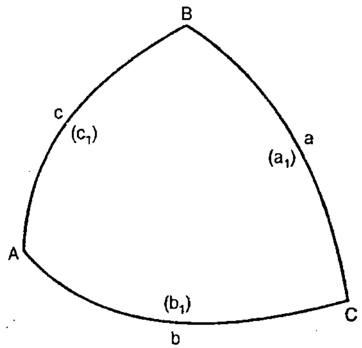

FIG. 9.5.

1. By Spherical Trigonometry

Knowing the length of one side and the three adjusted angles, the lengths of the other two sides can be calculated by the formulae of spherical trigonometry.

Let A, B and C = adjusted angles of the spherical triangle

$a = BC$, $b = AC$ and $c = AB$,

$BC = a$ = known side.

a_1 = angle subtended by side BC at the centre of the sphere

b_1 = angle subtended by side CA at the centre of the sphere

c_1 = angle subtended by side AB at the centre of the sphere

The computations are done in the following steps :

Step 1. Calculate the central angle a_1 of the side $BC\ (= a)$

$$\text{arc} = R \times \text{central angle}$$

or $$\text{central angle} = \frac{\text{arc}}{R}$$

or $$a_1 = \frac{180°\, a}{\pi R}, \text{ where } a_1 \text{ is in degrees}$$

and R is the radius of the earth.

Step 2. Knowing a_1 calculate the central angles b_1 and c_1 by the sine rule.

$$\sin b_1 = \sin a_1 \cdot \frac{\sin B}{\sin A}$$

and $$\sin c_1 = \sin a_1 \cdot \frac{\sin C}{\sin A}$$

Step 3. Knowing the central angles b_1 and c_1, calculate the corresponding lengths of the arcs $CA\ (=b)$ and $AB (=c)$ by the relations

$$b = \frac{\pi R b_1}{180°}$$

and $$c = \frac{\pi R c_1}{180°}$$

$$= \frac{1.528 \times 4}{1.528 + 0.937 + 1.6} = \frac{1.528 \times 4}{4.065} = 1''.51 \text{ (– ve)}$$

correction to $B = \dfrac{K_2}{K_1 + K_2 + K_3} \cdot e = \dfrac{0.937 \times 4}{4.065} = 0''.92$ (–ve)

correction to $C = \dfrac{K_3}{K_1 + K_2 + K_3} \cdot e = \dfrac{1.6 \times 4}{4.065} = 1''.57$ (–ve)

Check : Total correction = $1''.51 + 0''.92 + 1''.57 = 4''.00$

Hence the corrected values of the angles are

$A = 56°\ 12'\ 34''.5 - 1''.51 = 56°\ 12'\ 32''.99$

$B = 68°\ 36'\ 14''.5 - 0''.92 = 68°\ 36'\ 13''.58$

$C = 55°\ 11'\ 15'' \quad - 1''.57 = 55°\ 11'\ 13''.43$

Sum = $180°\ 00'\ 00''.00$

9.15. ADJUSTMENT OF CHAIN OF TRIANGLES

Let us consider a chain of triangles *ABC, ACD, DCE* etc. as shown in Fig. 9.7. The numbers 1, 2, 3 etc. represent the angle numbers and not their values. All the angles have been measured with equal precision. The adjustment is done into two steps :

(*i*) Station adjustment.

(*ii*) Figure adjustment.

(*i*) **Station Adjustment**

Since all the angles at a station have been measured, their sum must be equal to 360°. Hence we get the following condition equations :

$\angle 1 + \angle 2 + \angle 3 = 360°$ ...(1)

$\angle 4 + \angle 5 = 360°$ ...(2)

$\angle 6 + \angle 7 + \angle 8 + \angle 9 = 360°$ ...(3)

$\angle 10 + \angle 11 + \angle 12 = 360°$ ...(4)

$\angle 13 + \angle 14 = 360°$ ...(5)

The discrepancy denoted by each of the angles should be distributed equally to the component angles since all the angles have been measured with equal precision.

(*ii*) **Figure Adjustment**

After having adjusted the individual angles, each triangle is taken separately for figure adjustment. The sum of the three angles in each triangle should be equal to

Thus in triangle *ABC*,

$\angle 1 + \angle 4 + \angle 6 = 180°$ ...(6)

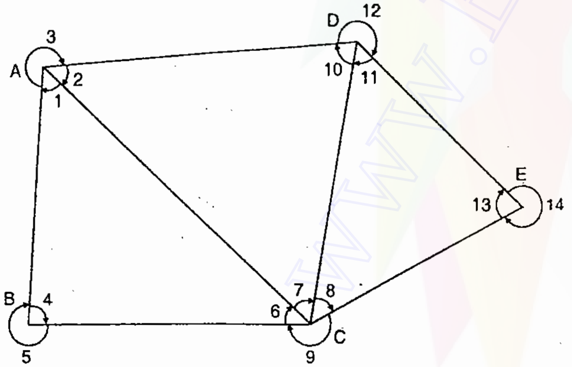

FIG. 9.7.

Thus in triangle *ACD*, $\angle 2 + \angle 7 + \angle 10 = 180°$ ...(7)

Thus in triangle *CDE*, $\angle 8 + \angle 11 + \angle 13 = 180°$ ...(8)

If the angles are of equal weight, the discrepancy is distribued equally to all the three angles. If the angles are weighted, the discrepancy is distributed in inverse proportion to their weights.

9.16. ADJUSTMENT OF TWO CONNECTED TRIANGLES

Fig 9.8 shows two connected triangles *ACD* and *BCD*. The angles A, B, C_1, C_2, D_1 and D_2 have been measured. The summation angles $C\ (= ACB)$ and $D (= ADB)$ have also been measured. Thus, there are eight angles. There are four independent condition equations that must be satisfied by the adjusted values of the angles. These equations are called the *angle equations* and are as follows :

$\angle A + \angle C_1 + \angle D_1 = 180°$ ...(1)

$\angle B + \angle C_2 + \angle D_2 = 180°$ ...(2)

$\angle C_1 + \angle C_2 = C$ ...(3)

$\angle D_1 + \angle D_2 = D$ ...(4)

There are total eight unknowns, out of which C_1, C_2, D_1 and D_2 must be regarded as the independent unknowns, and the remaining four as the dependent ones since they can be easily obtained from the condition equations.

The solution can be obtained either by means of normal equations or by correlates.

If the method of the normal equation is adopted, the four unknowns A, B, C and D can be expressed in terms of the independent unknowns, C_1, C_2, D_1 and D_2 . Thus,

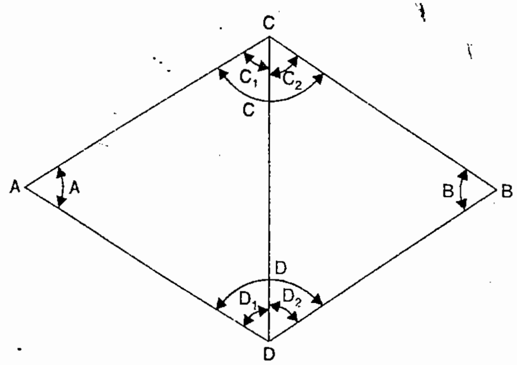

FIG. 9.8. TWO CONNECTED TRIANGLES

$\angle A = 180° - (\angle C_1 + \angle D_1)$ ...(*a*)

$\angle B = 180° - (\angle C_2 + \angle D_2)$ ...(*b*)

$\angle C = \angle C_1 + \angle C_2$ ...(*c*)

and $\angle D = \angle D_1 + \angle D_2$ ...(*d*)

From the new observation equations so formed in terms of C_1, C_2, D_1 and D_2 the normal equations can be formed in terms of the differences (or corrections) and their values can be known.

Example 9.24 illustrates the procedure for the adjustment.

Example 9.24. *The following are the measured values of equal weight for two connected triangles ACD and BCD (Fig. 9.8) :*

A	*68° 12′ 24″*
B	*52° 28′ 46″*

Hence, the normal equations are :

$$3k_1 + k_2 + k_3 = -4 \quad \ldots(i)$$
$$k_1 + 3k_2 + k_4 = -6 \quad \ldots(ii)$$
$$k_1 + 3k_3 + k_4 = -5 \quad \ldots(iii)$$
$$k_2 + k_3 + 3k_4 = -7 \quad \ldots(iv)$$

Solving these equations, we get

$$k_1 = -0''.46$$
$$k_2 = -1''.48$$
$$k_3 = -1''.15$$
$$k_4 = -1''.08$$

Hence, the corrected values of the angles are :

$$C_1 = 62°\ 18'\ 40'' - 0''.46 = 62°\ 18'\ 39''.54$$
$$C_2 = 65°\ 57'\ 51'' - 1''.48 = 65°\ 57'\ 49''.52$$
$$D_1 = 49°\ 28'\ 59'' - 1''.15 = 49°\ 28'\ 57''.85$$
$$D_2 = 61°\ 33'\ 28'' - 1''.08 = 61°\ 33'\ 26''.92$$

Also $\quad A = 180° - (C_1 + D_1) = 180° - 111°\ 47'\ 37''.39 = 68°\ 12'\ 22''.61$

$B = 180° - (C_2 + D_2) = 180° - 127°\ 31'\ 16''.44 = 52°\ 28'\ 43''.56$

$C = C_1 + C_2 \qquad = 128°\ 16'\ 29''.06$

$D = D_1 + D_2 \qquad = 111°\ 02'\ 24''.77$

Check : Sum $\quad A + B + C + D \qquad = 360°\ 00'\ 00''.00$

9.17. ADJUSTMENT OF A GEODETIC QUADRILATERAL

In a geodetic quadrilateral, all the eight angles ($\theta_1, \theta_2, \ldots. \theta_8$) shown in Fig. 9.9 are measured *independently*. If the size of the quadrilateral is small, it may be taken to be a plane quadrilateral. However, if the size is large, the spherical excess of each triangle can be calculated separately and a correction of $\frac{1}{3}$ spherical excess may be applied to each angle of the triangles, thus giving the plane angles. There are three methods of adjusting a geodetic quadrilateral

1. Rigorous method of least squares.
2. Approximate method.
3. Method of equal shifts. (See § 9.20)

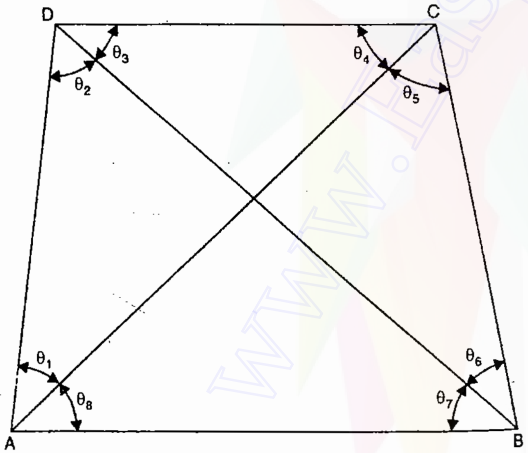

FIG. 9.9. GEODETIC QUADRILATERAL

1. ADJUSTMENT OF QUADRILATERAL BY METHODS OF LEAST SQUARES

In Fig. 9.9, $\theta_1, \theta_2, \theta_3, \ldots. \theta_8$ are the eight corner angles. The theodolite is set up only at the four stations *A, B, C* and *D* and *not* at the intersection of the diagonals. If we imagine to stand at the intersection of the diagonals and see the sides *AD, DC, CB,* and *BA* in turn, then the angles to the left are known as *left angles* and angles to the right are known as *right angles*. Thus θ_1, θ_3, θ_5 and θ_7 are the left angles while θ_2, θ_4, θ_6 and θ_8 are the right angles.

The conditions to be fulfilled by the adjusted values of the angles are :

(*i*) **Angle equations**

$$\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 + \theta_7 + \theta_8 = 360° \quad \ldots(1)$$
$$\theta_1 + \theta_2 = \theta_5 + \theta_6 \quad \ldots(2)$$
$$\theta_3 + \theta_4 = \theta_7 + \theta_8 \quad \ldots(3)$$

(*ii*) **Side equations**

In addition to the three angle equations, a geodetic quadrilateral (or any other figure) must also fulfil the side equation so that the figure is closed. Even if the angle equations are satisfied, the quadrilateral may not be closed as shown in Fig. 9.10, by drawing all the lines parallel to those of Fig. 9.9.

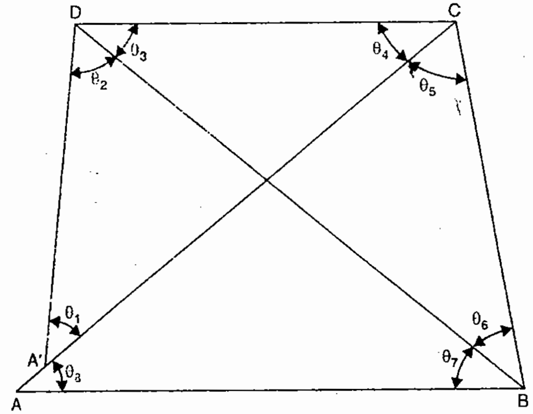

FIG. 9.10. GEODETIC QUADRILATERAL NOT FULFILLING THE SIDE EQUATION.

Let us consider Fig. 9.9 again, which is a closed figure.

From triangle *ADC*,

$$DC = AD \frac{\sin \theta_1}{\sin \theta_4}$$

$$= AB \frac{\sin \theta_7}{\sin \theta_2} \cdot \frac{\sin \theta_1}{\sin \theta_4} \qquad \text{since } AD = AB \frac{\sin \theta_7}{\sin \theta_2}$$

$$= BC \frac{\sin \theta_5}{\sin \theta_8} \cdot \frac{\sin \theta_7}{\sin \theta_2} \cdot \frac{\sin \theta_1}{\sin \theta_4} \qquad \text{since } AB = BC \frac{\sin \theta_5}{\sin \theta_8}$$

$$= DC \frac{\sin \theta_3 \sin \theta_5 \sin \theta_7 \sin \theta_1}{\sin \theta_6 \sin \theta_8 \sin \theta_2 \sin \theta_4} \qquad \text{since } BC = DC \frac{\sin \theta_3}{\sin \theta_6}$$

Hence $\sin \theta_1 \cdot \sin \theta_3 \cdot \sin \theta_5 \sin \theta_7 = \sin \theta_2 \cdot \sin \theta_4 \cdot \sin \theta_6 \cdot \sin \theta_8$

Taking log of both the sides, and denoting the left angles by θ_L and right angles by θ_R , we get

$$\Sigma \log \sin \theta_L = \Sigma \log \sin \theta_R \ ; \quad \text{or simply} \quad \Sigma \log \sin L = \Sigma \log \sin R \quad \ldots(4) \ \ldots(9.24)$$

where *L* denotes left angles and *R* denotes the right angles.

Thus, we have four condition equations.

$$e_1 = \lambda_1 + \lambda_2 + f_1\,\lambda_4 \qquad ...(3a)$$
$$e_2 = \lambda_1 + \lambda_2 - f_2\,\lambda_4 \qquad ...(3b)$$
$$e_3 = \lambda_1 + \lambda_3 + f_3\,\lambda_4 \qquad ...(3c)$$
$$e_4 = \lambda_1 + \lambda_3 - f_4\,\lambda_4 \qquad ...(3d)$$
$$e_5 = \lambda_1 - \lambda_2 + f_5\,\lambda_4 \qquad ...(3e)$$
$$e_6 = \lambda_1 - \lambda_2 - f_6\,\lambda_4 \qquad ...(3f)$$
$$e_7 = \lambda_1 - \lambda_3 + f_7\,\lambda_4 \qquad ...(3g)$$
$$e_8 = \lambda_1 - \lambda_3 - f_8\,\lambda_4 \qquad ...(3h)$$

which are the same as found earlier.

2. ADJUSTMENT OF QUADRILATERAL BY APPROXIMATE METHOD

This method is generally used for adjusting a quadriateral of moderate size or of minor importance. The method gives fairly accurate results. However, the least square condition is not satisfied by this method.

Refer to Fig. 9.9 ; the condition equations are

$$\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 + \theta_7 + \theta_8 = 360° \qquad ...(1)$$
$$\theta_1 + \theta_2 = \theta_5 + \theta_6 \qquad ...(2)$$
$$\theta_3 + \theta_4 = \theta_7 + \theta_8 \qquad ...(3)$$

and $\Sigma \log \sin L = \Sigma \log \sin R \qquad ...(4)$

In the above, $\theta_1, \theta_2, ...\theta_8$ are the angles adjusted for spherical excess if necessary.

All the four equations are satisfied in the following steps :

(1) Find the sum of $\theta_1, \theta_2,....,\theta_8$ and subtract it from 360°. Correct each angle by distributing one-eighth of the discrepancy so that equation 1 is satisfied.

(2) From the values of the angles so obtained, find the difference between $(\theta_1 + \theta_2)$ and $(\theta_5 + \theta_6)$. Correct each angle by one fourth of the discrepancy. If $(\theta_1 + \theta_2)$ is less than $(\theta_5 + \theta_6)$, the sign of corrections to θ_1 and θ_2 is positive and that for θ_5 and θ_6 is negative. Similarly, if $(\theta_1 + \theta_2)$ is more than $(\theta_5 + \theta_6)$ the sign of correction to θ_1 and θ_2 is negative and that for θ_5 and θ_6 is positive. Thus equation (2) is satisfied.

(3) Similarly, find the difference between $(\theta_3 + \theta_4)$ and $(\theta_7 + \theta_8)$. Correct each angle by one-fourth of the discrepancy. If $(\theta_3 + \theta_4)$ is less than $(\theta_7 + \theta_8)$, the sign of corrections to θ_3 and θ_4 is positive and that for θ_7 and θ_8 is negative and *vice versa.*

Thus equation (3) is satisfied.

(4) The adjusted values of $\theta_1, \theta_2, \theta_3,....,\theta_8$ are then tested for side equation. Find the log sine of angles $\theta_1, \theta_2, ..., \theta_8$. Take the sum of log sine of left angles, and also of log sine of right angles. Find the difference between Σ log sin L and Σ log sin R, and find the discrepancy.

Let m be the discrepancy

i.e. $m = \Sigma \log \sin L - \Sigma \log \sin R$

Let $f_1, f_2, f_3, ..., f_8$ be the tabular differences 1″ for log sin θ_1, log sin θ_2,...log sin θ_8.

Find Σf^2.

i.e. $\Sigma f^2 = f_1^2 + f_2^2 + f_3^2 + ... f_8^2$

Then the correction to angle $\theta_1 = \dfrac{f_1}{\Sigma f^2}\, m$

Then the correction to angle $\theta_2 = \dfrac{f_2}{\Sigma f^2}\, m$

... ...

Correction to angle $\theta_8 = \dfrac{f_8}{\Sigma f^2}\, m$

If Σ log sin L is greater than Σ log sin R, the corrections to left angles are negative and corrections to right angles are positive and *vice versa* so that Eq. (4) is satisfied.

Due to the fulfilment of the side equation, the values of the angles will be changed, thus disturbing the condition equations (1), (2) and (3). In case more accuracy is required, both the adjustments are repeated.

Example 9.25. *The following are the observed values of eight angles of a Geodetic quadrilateral after they have been corrected for spherical excess. Adjust the quadrilateral. The observations may be assumed to be of equal weight.*

$\theta_1 = 71° 26' 03''.59$

$\theta_2 = 53° 39' 54''.60$

$\theta_3 = 31° 18' 10''.53$

$\theta_4 = 23° 35' 52''.03$

$\theta_5 = 89° 40' 10''.42$

$\theta_6 = 35° 25' 47''.08$

$\theta_7 = 14° 18' 02''.87$

$\theta_8 = 40° 36' 00''.15$

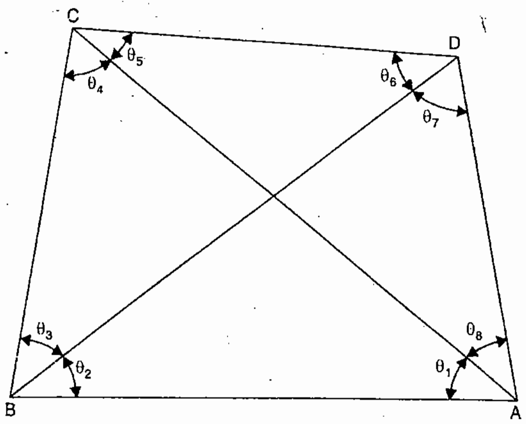

FIG. 9.11.

(*a*) **Solution by method of least squares**

The angle equations are :

$$\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 + \theta_7 + \theta_8 = 360°$$
$$\theta_1 + \theta_2 = \theta_5 + \theta_6$$
$$\theta_3 + \theta_4 = \theta_7 + \theta_8$$

Sum of the observed angles = 360° 00′ 01″.27

$\therefore \quad E_1 = 360° - \Sigma\theta = -1''.27$

$\theta_1 + \theta_2 = 125° 05' 58''.19$

$\theta_5 + \theta_6 = 125° 05' 57''.50$

(*b*) **Solution by Approximate Method**

The quadrilateral is adjusted by approximate method in the following steps :

Step 1. $\Sigma\theta = 360° \ 00' \ 01''.27$

$\therefore \quad E_1 = 360° - \Sigma\theta = -01''.27$

Distributing this equally to all the eight angles, the correction to each angle $= -\frac{1}{8} \times 1''.27 = -0''.16$ (approximately). Hence the corrected angles are

$\theta_1 = 71° \ 26' \ 03''.43$
$\theta_2 = 53° \ 39' \ 54''.44$
$\theta_3 = 31° \ 18' \ 10''.37$
$\theta_4 = 23° \ 35' \ 51''.87$
$\theta_5 = 89° \ 40' \ 10''.26$
$\theta_6 = 35° \ 25' \ 46''.92$
$\theta_7 = 14° \ 18' \ 02''.71$
$\theta_8 = 40° \ 36' \ 00''.00$ (approx.)

Sum $= 360° \ 00' \ 00''.00$

Step 2. $\theta_1 + \theta_2 = 125° \ 05' \ 57''.87$

$\theta_5 + \theta_6 = 125° \ 05' \ 57''.18$

$\therefore \quad E_2 = 125° \ 05' \ 57''.87 - 125° \ 05' \ 57''.18 = -0''.69$

Distributing this equally to all the four angles, correction to each angle $= \frac{1}{4} \times 0''.69 = 0''.17$ approximately. The correction will be negative to θ_1 and θ_2 and positive to θ_5 and θ_6

Hence the corrected angles are

$\theta_1 = 71° \ 26' \ 03''.43 - 0''.17 = 71° \ 26' \ 03''.26$
$\theta_2 = 53° \ 39' \ 54''.44 - 0''.17 = 53° \ 39' \ 54''.27$

Sum $= 125° \ 05' \ 57''.53$

$\theta_5 = 89° \ 40' \ 10''.26 + 0''.17 = 89° \ 40' \ 10''.43$
$\theta_6 = 35° \ 25' \ 46''.92 + 0''.17 = 35° \ 25' \ 47''.09$

Sum $= 125° \ 05' \ 57''.52$

Step 3.

$\theta_3 + \theta_4 = 54° \ 54' \ 02''.24$

$\theta_7 + \theta_8 = 54' \ 54' \ 02''.71$

$\therefore \quad E_3 = 54° \ 54' \ 02''.71 - 54° \ 54' \ 02''.24 = 0''.47$

Distributing this equally to all the four angles, the correction to each angle $= \frac{1}{4} (0''.47) = 0''.12$ approximately. The correction will be positive to θ_3 and θ_4 and negative to θ_7 and θ_8. Hence the corrected angles are

$\theta_3 = 31° \ 18' \ 10''.37 + 0''.12 = 31° \ 18' \ 10''.49$
$\theta_4 = 23° \ 35' \ 51''.87 + 0''.12 = 23° \ 35' \ 51''.99$

Sum $= 54° \ 54' \ 02''.48$

$\theta_7 = 14° \ 18' \ 02''.71 - 0''.12 = 14° \ 18' \ 02''.59$
$\theta_8 = 40° \ 36' \ 00''.00 - 0''.12 = 40° \ 35' \ 59''.88$

Sum $= 54° \ 54' \ 02''.47$

Step 4.

The log sines of the corrected angles are tabulated below:

Left				*Right*			
	Angle	*Log sine*	*f*		*Angle*	*Log sine*	*f*
θ_1	71° 26′ 03″.26	9.9767895	7.0	θ_2	53° 39′ 54″.27	9.9061018	15.5
θ_3	31° 18′ 10″.49	9.7156380	34.6	θ_4	23° 35′ 51″.99	9.6024004	48.2
θ_5	89° 40′ 10″.43	9.9999928	0.1	θ_6	35° 25′ 47″.09	9.7632065	29.6
θ_7	14° 18′ 02″.59	9.3927157	82.6	θ_8	40° 35′ 59″.88	9.8134300	24.0
	Sum	39.0851360			Sum	39.0851387	

Hence $m = \Sigma \log \sin L - \Sigma \log \sin R = -27$

This correction will be distributed to each angle in proportion to $\frac{f}{\Sigma f^2} m$. The correction will be positive for left angles and negative for right angles.

From the above table, $\Sigma f^2 = 12114$

Correction for $\theta_1 = +7 \times \frac{27}{12114} = +0''.016$

Similarly Correction for $\theta_3 = +0''.077$

Correction for $\theta_5 =$ zero

Correction for $\theta_7 = +0''.184$

Correction for $\theta_2 = -0''.035$

Correction for $\theta_4 = -0''.107$

Correction for $\theta_6 = -0''.066$

Correction for $\theta_8 = -0''.053$

Hence the corrected angles are

In this first column under e, the correction $e_1, e_2, e_3, ...e_{12}$ are entered. In the second column under λ_1, the coefficients of $e_1, e_2, e_3, e_4, ...e_{12}$ in equation (1) are entered. In third column under λ_2, the coefficients of $e_1, e_2, e_3, ..., e_{12}$ in equation (2) are entered. In the fourth column under λ_3, the coefficients of $e_1, e_2, ... e_{12}$ in equation (3) are entered. Similarly, in the fifth, sixth and seventh columns under λ_4, λ_5 and λ_6, the coefficients $e_1, e_2, ...e_{12}$ of in equation (4), (5) and (6) respectively are entered.

The table is thus completed.

The equations for $e_1, e_2, ..., e_{12}$ are then obtained by multiplying by $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ by the coefficients under them and in horizontal line with, $e_1, e_2, ...e_{12}$ respectively. Thus,

$$\begin{aligned}
e_1 &= \lambda_1 + f_1\,\lambda_6 & e_7 &= \lambda_4 + f_7\,\lambda_6 \\
e_2 &= \lambda_1 - f_2\,\lambda_6 & e_8 &= \lambda_4 - f_8\,\lambda_6 \\
e_3 &= \lambda_2 + f_3\,\lambda_6 & e_9 &= \lambda_1 + \lambda_5 \\
e_4 &= \lambda_2 - f_4\,\lambda_6 & e_{10} &= \lambda_2 + \lambda_5 \\
e_5 &= \lambda_3 + f_5\,\lambda_6 & e_{11} &= \lambda_3 + \lambda_5 \\
e_6 &= \lambda_3 - f_6\,\lambda_6 & e_{12} &= \lambda_4 + \lambda_5
\end{aligned} \qquad ...(9.27)$$

The equations are the same as derived earlier.

Substituting the values of $e_1, e_2, e_3, ...e_{12}$, in equation (1), (2), (3), (4), (5) and (6), we get the following equations for $\lambda_4, \lambda_5\ \lambda_1, \lambda_2, \lambda_3$, and λ_6 :

$$\begin{aligned}
3\,\lambda_1 + \lambda_5 + \lambda_6\,(f_1 - f_2) &= E_1 & &...(i) \\
3\,\lambda_2 + \lambda_5 + \lambda_6\,(f_3 - f_4) &= E_2 & &...(ii) \\
3\,\lambda_3 + \lambda_5 + \lambda_6\,(f_5 - f_6) &= E_3 & &...(iii) \quad ...(9.28) \\
3\,\lambda_4 + \lambda_5 + \lambda_6\,(f_7 - f_8) &= E_4 & &...(iv) \\
\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + 4\,\lambda_5 &= E_5 & &...(v) \\
\lambda_1\,(f_1 - f_2) + \lambda_2\,(f_3 - f_4) + \lambda_3(f_5 - f_6) + \lambda_4(f_7 - f_8) + \lambda_6\Sigma(f^2) &= E_6 & &...(vi)
\end{aligned}$$

Solving theses equation simultaneously, we get $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and λ_6. Substituting these values in equations for $e_1, e_2, ...e_{12}$ we get the values of the corrections and hence the corrected angles can be known.

Approximate Solution for $\lambda_1, \lambda_2, ...\lambda_6$ by Dale's Method

The equation (*i*) to (*vi*) for $\lambda_1, \lambda_2, ...\lambda_6$ given above can be solved by the *method of successive approximations* suggested by Prof. J.B. Dale. The solution is done in the following steps :

Step 1.

Neglecting the terms for λ_6, add equations (*i*), (*ii*), (*iii*) and (*iv*)

$$3\,\lambda_1 + 3\,\lambda_2 + 3\,\lambda_3 + 3\,\lambda_4 + 4\,\lambda_5 = E_1 + E_2 + E_3 + E_4 \qquad ...(a)$$

Step 2.

Multiply equation (*v*) by (3). Thus

$$3\,\lambda_1 + 3\,\lambda_2 + 3\,\lambda_3 + 3\,\lambda_4 + 12\,\lambda_5 = 3E_5 \qquad ...(b)$$

Step 3.

Subtracting Eq. (*a*) from Eq. (*b*), we get

$$\lambda_5 = \frac{3E_5 - (E_1 + E_2 + E_3 + E_4)}{8} \qquad ...(c)...(9.29)$$

(**Note.** If there are n sides of the polygon, the denominator will be $2n$).

Step 4.

Substituting this value of λ_5 in equation (*i*), (*ii*), (*iii*) and (*iv*) and *still ignoring the terms for* λ_6, we get

$$\lambda_1 = \frac{E_1 - \lambda_5}{3} \qquad \lambda_3 = \frac{E_3 - \lambda_5}{3}$$

$$\lambda_2 = \frac{E_2 - \lambda_5}{3} \qquad \lambda_4 = \frac{E_4 - \lambda_5}{3} \qquad ...(9.30)$$

Step 5.

Substituting these approximate values of $\lambda_1, \lambda_2, \lambda_3$ and λ_4 in Eq. (*vi*), we get

$$\lambda_6 = \frac{E_6 - \Sigma\lambda\,(f_L - f_R)}{\Sigma f^2} \qquad ...(9.31)$$

Thus the approximate values of $\lambda_1, \lambda_2, ...\lambda_6$ are known.

Step 6.

For the second approximation, let

$$\begin{aligned}
E_1' &= E_1 - \lambda_6\,(f_1 - f_2) \\
E_2' &= E_2 - \lambda_6\,(f_3 - f_4) \\
E_3' &= E_3 - \lambda_6\,(f_5 - f_6) \\
E_4' &= E_4 - \lambda_6\,(f_7 - f_8)
\end{aligned}$$

Then $$\lambda_5 = \frac{3E_5 - (E_1' + E_2' + E_3' + E_4')}{8} \qquad ...[9.29\ (a)]$$

Step 7. Finally :

$$\lambda_1 = \frac{E_1' - \lambda_5}{3} \qquad \lambda_3 = \frac{E_3' - \lambda_5}{3}$$

$$\lambda_2 = \frac{E_2' - \lambda_5}{3} \qquad \lambda_4 = \frac{E_4' - \lambda_5}{3} \qquad ...[9.30\ (a)]$$

Step 8.

$$\lambda_6 = \frac{E_6' - \Sigma\lambda\,(f_L - f_R)}{\Sigma f^2} \qquad ...[9.31\ (a)]$$

If this value of λ_6 differs appreciably from its previous value, repeat steps (6), (7) and (8) till no appreciable change is effected by repeating the process.

The method of solution has been explained fully in example 9.26.

9.19. ADJUSTMENTS OF GEODETIC TRIANGLES WITH CENTRAL STATION BY METHOD OF LEAST SQUARES

Let ABC be a geodetic triangle with P as the central station. $\theta_1, \theta_2, ... \theta_9$ are the observed angles corrected for the spherical excess, if any.

$$3\lambda_1 + \lambda_4 + \lambda_5 (f_1 - f_2) = E_1$$
$$3\lambda_2 + \lambda_4 + \lambda_5 (f_3 - f_4) = E_2$$
$$3\lambda_3 + \lambda_4 + \lambda_5 (f_5 - f_6) = E_3$$
$$\lambda_1 + \lambda_2 + \lambda_3 + 3\lambda_4 = E_4$$
$$\lambda_1 (f_1 - f_2) + \lambda_2 (f_3 - f_4) + \lambda_3 (f_5 - f_6) + \lambda_6 \Sigma (f)^2 = E_5$$

Solving these equations simultaneously, we get $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5. Substituting these values in equations for $e_1, e_2, ... e_9$, we can get the values of the corrections, and hence the corrected angles can be known.

Example 9.26. *The following are the measured angles of a quadrilateral ABCD with the central point E:*

Triangle	*Central Angle*	*L.H. Angle*	*R.H. Angle*
AEB	*59° 03′ 10″*	*61° 00′ 54″*	*59° 56′ 06″*
BEC	*118° 23′ 50″*	*32° 03′ 54″*	*29° 32′ 06″*
CED	*60° 32′ 05″*	*56° 28′ 01″*	*62° 59′ 49″*
DEA	*122° 00′ 55″*	*28° 42′ 00″*	*29° 17′ 00″*

Adjust the quadrilateral. (U.L.)

Solution.

Fig. 9.14 shows the quadrilateral in which the L.H. angles have been denoted by odd numbers and R.H. angles by even numbers.

$\theta_1 =$ 61° 00′ 54″
$\theta_2 =$ 59° 56′ 06″
$\theta_3 =$ 32° 03′ 54″
$\theta_4 =$ 29° 32′ 06″
$\theta_5 =$ 56° 28′ 01″
$\theta_6 =$ 62° 59′ 49″
$\theta_7 =$ 28° 42′ 00″
$\theta_8 =$ 29° 17′ 00″
$\theta_9 =$ 59° 03′ 10″
$\theta_{10} =$ 118° 23′ 50″
$\theta_{11} =$ 60° 32′ 05″
$\theta_{12} =$ 122° 00′ 55″

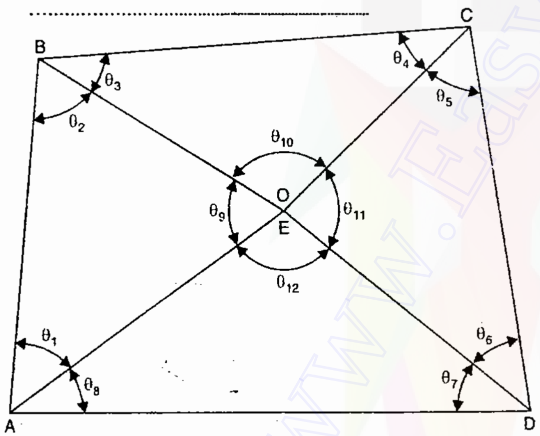

FIG. 9.14

The condition equations are

$$\theta_1 + \theta_2 + \theta_9 = 180°$$
$$\theta_3 + \theta_4 + \theta_{10} = 180°$$
$$\theta_5 + \theta_6 + \theta_{11} = 180°$$
$$\theta_7 + \theta_8 + \theta_{12} = 180°$$
$$\theta_9 + \theta_{10} + \theta_{11} + \theta_{12} = 360°$$
$$\Sigma \log \sin L = \Sigma \log \sin R$$

$$E_1 = 180° - (\theta_1 + \theta_2 + \theta_9) = 180° - 180° 00' 10'' = -10''$$
$$E_2 = 180° - (\theta_3 + \theta_4 + \theta_{10}) = 180° - 179° 59' 50'' = +10''$$
$$E_3 = 180° - (\theta_5 + \theta_6 + \theta_{11}) = 180° - 179° 59' 55'' = +5''$$
$$E_4 = 180° - (\theta_7 + \theta_8 + \theta_{12}) = 180° - 179° 59' 55'' = +5''$$
$$E_5 = 360° - (\theta_9 + \theta_{10} + \theta_{11} + \theta_{12}) = 360° - 360° = 0$$
$$E_6 = \Sigma \log \sin R - \Sigma \log \sin L$$

The values of log sine of the angles are tabulated below :

	Left				*Right*		
	Angle	*Log sine*	*f*		*Angle*	*Log sine*	*f*
θ_1	61° 00′ 54″	9.9418823	11.67	θ_2	59° 56′ 06″	9.9372458	12.18
θ_3	32° 03′ 54″	9.7249972	33.63	θ_4	29° 32′ 06″	9.6928074	37.15
θ_5	56° 28′ 01″	9.9209407	13.93	θ_6	62° 59′ 49″	9.9498691	10.73
θ_7	28° 42′ 00″	9.6814434	38.45	θ_8	29° 17′ 00″	9.6894232	37.57
	Sum	39.2692636			Sum	39.2693455	

$$\Sigma f^2 = 5995$$

$\therefore$ $E_6 = 39.2693455 - 39.2692636 = +819$

Hence if $e_1, e_2, ... e_{12}$ are the corrections to $\theta_1, \theta_2, ... \theta_{12}$ we get the following equations :

$$e_1 + e_2 + e_9 = -10 \quad ...(1)$$
$$e_3 + e_4 + e_{10} = +10 \quad ...(2)$$
$$e_5 + e_6 + e_{11} = +5 \quad ...(3)$$
$$e_7 + e_8 + e_{12} = +5 \quad ...(4)$$
$$e_9 + e_{10} + e_{11} + e_{12} = 0 \quad ...(5)$$
$$11.67\, e_1 - 12.18\, e_2 + 33.63\, e_3 - 37.15\, e_4 + 13.93\, e_5 - 10.73\, e_6 + 38.45 e_7 - 37.57 e_8 = +819 \quad ...(6)$$

To form the equations, we prepare a table such as given in § 9.16

The following equations are obtained :

$e_1 = \lambda_1 + 11.67\, \lambda_6$	$e_7 = \lambda_4 + 38.45\, \lambda_6$
$e_2 = \lambda_1 - 12.18\, \lambda_6$	$e_8 = \lambda_4 - 37.57\, \lambda_6$
$e_3 = \lambda_2 + 33.63\, \lambda_6$	$e_9 = \lambda_1 + \lambda_5$
$e_4 = \lambda_2 - 37.15\, \lambda_6$	$e_{10} = \lambda_2 + \lambda_5$
$e_5 = \lambda_3 + 13.93\, \lambda_6$	$e_{11} = \lambda_3 + \lambda_5$
$e_6 = \lambda_3 - 10.73\, \lambda_6$	$e_{12} = \lambda_4 + \lambda_5$

TABLE 9.1

METHOD OF LEAST SQUARES (EXAMPLE 9.26)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Triangle	*Central Angle*	*Correction*	*L.H. angle*	*Log sine*	*Increase for 1″*	*Correction of angle*	*Correction of Log sine*
AEB	59° 03′ 10″	− 4″.14	61° 00′ 54″	9.9418823	11.67	− 1″.29	− 15
BEC	118° 23′ 50″	+ 2″.66	32° 03′ 54″	9.7249972	33.63	+ 8″.53	+ 287
CED	6° 32′ 05″	+ 0″.69	56° 28′ 01″	9.9209407	13.93	+ 3″.85	+ 54
DEA	122° 00′ 55″	+ 0″.79	28° 42′ 00″	9.6814434	38.45	+ 7″.32	+ 281
	360° 00′ 00″			39.2692636			+ 607 − 212 + + 819

(1)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Triangle	*R.H. angle*	*Log sine*	*Increase for 1″*	*Correction of angle*	*Correction of Log sine*	*Sum of angles*	*Total Correction*
AEB	59° 56′ 06″	9.9372458	12.18	− 4″.56	− 56	180° 00′ 10″	− 10″
BEC	29° 32′ 06″	9.6928074	37.15	− 1″.19	− 44	179° 59′ 50″	+ 10″
CED	62° 59′ 49″	9.9498691	10.73	+ 0″.47	+ 5	179° 59′ 55″	+ 5″
DEA	29° 17′ 00″	9.6894232	37.57	− 3″.12	− 117	179° 59′ 55″	+ 5″
		39.2693455 39.2692636 ← 819			− 212		

The method of equal shifts indicates that any shift which is necessary to satisfy the local equation should be the same for each triangle of the polygon. Similarly, any shift necessary to satisfy the side equation should be the same for each triangle.

Solution of Example 9.26 by method of equal shifts

To illustrate the method of equal shifts, we will work out Example 9.26 by this method.

The solution is done in the following steps. See Table 9.2 and 9.3.

Step 1.

Fill up columns (1), (2), (4), (9), (14) and (15) of Table 9.2. Column (15) gives the total corrections to be applied to each triangle.

Step 2.

One-third of the corrections of column (15) will be the correction for the corresponding central angle. In column 16 (Table 9.3) for 1st trial correction for central angles, fill up these $\frac{1}{3}$rd corrections with appropriate signs. The sum of column (16) comes out to be + 3″.34. To satisfy the station equation, the sum of all the four central angles should be 360°. Since the sum of the observed central angles was zero, the sum of the corrections of column (16) should be zero. To make it zero, apply a correction of $-\frac{3.34}{4} = -0''.84$ to each 1st trial corrections and enter the second trial corrections so obtained in column (17). The sum of corrections of column (17) is − 0″.02 which is distributed arbitrarily, and final corrections are entered in column (18). The sum of column (18) should be equal to zero. *Thus, the station equation is satisfied.* Complete column (19) by applying the correction to the corresponding central angles.

Step 3.

Fill up column (20) which is the difference of columns (15) and (18). This remaining correction is to be distributed equally to left and right hand angles of each triangle. Thus, columns (21) and (26) are completed.

Step 4.

From the seven-figure log tables, fill up columns (5), (6), (10) and (11). The sum of log sines of *unadjusted angles* shows a shift of 819 from right to the left. Let us first asertain the actual shift after applying the correction to the left and right angles as per columns (21) and (26). Knowing the difference for 1″, find the corresponding differences for the corrections of column (21) and (26), and thus fill up columns (22) and (27) respectively. For example, the log sine difference for 1″ of left angles of triangle *AEB* is 11.67. Hence the difference for − 2″.92 will be = 11.67 × (− 2″.92) = − 34. Enter these differences in columns (22) and (27) to the seventh figure only and not the decimal part.

Step 5.

Take the sum of column (22) and add it to the sum of column (5). Thus, we get $39.2692636 + \overline{211} = 39.2692847$. Similarly, take the sum of column (27) and add it to sum column (10). Thus, we get $39.2693455 + \overline{203} = 39.2693658$. Hence the total shift required $= 39.2693658 - 39.2692847 = 811$ from right to left. Hence the left hand angles will have to be increased and the right hand angles will have to be decreased.

Step 6.

Take the sum of columns (6) and (11). The Σ (log sine diff. for 1″) for all the eight angles $= 97.68 + 98.13 = 195.81$. This shows that if we make a shift of 1″ in all the angles (increasing left angles and decreasing right angles), the total change in log sine angles will be 195.81. In other words, 195.81 corresponds to 1″ of shift. Hence the total of 811 {in the log (sine of angles) value} corresponds to $\frac{811}{195.81} = 4''.15$. Hence increase each left angle by 4″.15 and decrease each right angle by 4″.15 and complete columns (23) and (28). Columns (25) and (30) then be obtained by applying the corrections to the corresponding angles.

Step 7.

As a check, complete columns (24) and (29), by multiplying corrections by the corresponding differences of 1″ in the log sine. Add the sum of column (24) to the sum of column (5) and the sum of column (29) to the sum of column (10). Comparing the results so obtained, we observe that the required correction is 10 which is negligible since it causes an error of $\frac{10}{195.81} = 0.005''$ in each angle. Columns (7) and (8) correspond to column (23) and (24). Similarly, columns (12) and (13) correspond to columns (28) and (29).

Step 8.

Finally, complete column (31) by taking the sums of columns (19), (25) and (30).

PROBLEMS

1. An angle has been measured under different field conditions, with results as follows :

28° 24′ 20″	28° 24′ 00″
28° 24′ 40″	28° 24′ 40″
28° 24′ 40″	28° 24′ 20″
28° 25′ 00″	28° 24′ 40″
28° 24′ 20″	28° 25′ 20″

Find (*i*) the probable error of single observation, (*ii*) probable error of the mean.

2. The following values were recorded for a triangle *ABC*, the individual measurements being uniformly precise :

$A = 62° 28' 16''$; 6 obs.

$B = 56° 44' 36''$; 8 obs.

$C = 60° 45' 56''$; 6 obs.

Find the correct values of the angles. (*B.U.*)

3. At a station *O* in a triangulation survey, the following results were obtained :

Angle	Observed Values °	′	″	Weight
AOB	67	14	32.4	1.1
BOC	75	36	21.5	1.0
COD	59	56	02.0	0.7
DOE	83	24	17.1	0.8
EOA	73	48	45.0	1.4

The weights are proportional to the reciprocals of the squares of the probable errors. Adjust the angles. (*R.T.C.*)

4. The observations closing the horizon at a station are

$A = 24° 22' 18''.2$ Weight 1

$B = 30° 12' 24''.4$ Weight 2

$A + B = 54° 34' 48''.6$ Weight 3

$C = 305° 25' 13''.9$ Weight 2

$B + C = 335° 37' 38''.0$ Weight 3

Find the most probable values of the angles *A*, *B* and *C* (*P.U.*)

5. Adjust the angles α and β, observations of which give

$\alpha = 20° 10' 10''$ weight 6

$\beta = 30° 20' 30''$ weight 4

$\alpha + \beta = 50° 30' 50''$ weight 2 (U.B.)

6. The following values of angles were measured at a station :

$a = 20° 10' 14''$ weight 2

$b = 30° 15' 20''$ weight 2

$c = 42° 02' 16''$ weight 3

$a + b = 50° 25' 37''$ weight 3

$b + c = 72° 17' 34''$ weight 3

$a + b + c = 92° 27' 52''$ weight 1

Find the most probable values of the angles *a*, *b* and *c*.

7. *A*, *B*, *C*, *D* form a round of angles at a station so that $A + B + C + D = 360°$.

Their observed values were

$A = 76° 24' 40''$; $B = 82° 14' 25''$

$C = 103° 37' 50''$; $D = 97° 43' 15''$

The angle $B + C$ was also separately measured *twice* and found to average 185° 52′ 20″. Find the probable values of each of the four angles if all six measurements were of equal accuracy. (*U.L.*)

on the maps, intended for purpose of navigation, to show peaks and hill tops, along the coast. The relief or elevations may also be indicated by *tinting*. The area lying between two selected contours is coloured by one tint, that between the two others by another tint and so on.

10.3. CONTOURS AND CONTOUR INTERVAL

The system now in general use for representing the form of the surface is that employing contour lines. The elevations of the contours are known definitely, and hence the elevation of any point on ground may be derived from the map. At the same time, this system makes the form or relief apparant to the eye. Thus, this system fulfils both purposes discussed earlier.

The vertical distance between two consecutive contours is called the *contour interval*. The contour interval is kept constant for a contour plan, otherwise the general appearance of the map will be misleading. The horizontal distance between two points on two consecutive contours is known as the *horizontal equivalent* and depends upon the steepness of the ground. The choice of proper contour interval depends upon the following considerations:

(i) ***The nature of the ground.*** The contour interval depends upon whether the country is flat or highly undulated. A contour interval chosen for flat ground, will be highly unsuitable for undulated ground. For very flat ground, a small interval is necessary. If the ground is more broken, greater contour interval should be adopted, otherwise the contours will come too close to each other.

(ii) ***The scale of the map.*** The contour interval should be inversely proportional to the scale. If the scale is small, the contour interval should be large. If the scale is large, the contour interval should be small.

(iii) ***Purpose and extent of the survey.*** The contour interval largely depends upon the purpose and the extent of the survey. For example, if the survey is intended for detailed design work or for accurate earth work calculations, small contour interval is to be used. The extent of survey in such cases will generally be small. In the case of location surveys, for lines of communications, for reservoirs and drainage areas, where the extent of survey is large, a large contour interval is used.

(iv) ***Time and expense of field and office work.*** If the time available is less, greater contour interval should be used. If the contour interval is small, greater time will be taken in the field survey, in reduction and in plotting the map.

Considering all these aspects, the contour interval for a particular contour plan is selected. This contour interval is kept constant in that plan, otherwise it will mislead the general appearance of the ground. Table 10.1 suggests some suitable values of contour interval. Table 10.2 suggests the values of contour interval for various purposes.

For general topographical work, the general rule that may be followed is as follows:

$$\text{Contour interval (metres)} = \frac{25}{\text{No. of cm per km}}$$

or

$$\text{Contour interval (feet)} = \frac{50}{\text{No. of inches per mile}}$$

TABLE 10.1

Scale of map	*Type of ground*	*Contour interval (metres)*
Large (1 cm = 10 m or less)	Flat	0.2 to 0.5
	Rolling	0.5 to 1
	Hilly	1, 1.5 or 2
Intermediate (1 cm = 10 to 100 m)	Flat	0.5, 1 or 1.5
	Rolling	1, 1.5 or 2
	Hilly	2, 2.5 or 3
Small (1 cm = 100 m or more)	Flat	1, 2 or 3
	Rolling	2 to 5
	Hilly	5 to 10
	Mountainous	10, 25 or 50

TABLE 10.2

Purpose of survey	*Scale*	*Contour interval (metres)*
1. Building sites	1 cm = 10 m or less	0.2 to 0.5
2. Town planning schemes, reservoirs etc.	1 cm = 50 m to 100 m	0.5 to 2
3. Location surveys	1 cm = 50 m to 200 m	2 to 3

10.4. CHARACTERISTICS OF CONTOURS

The following characteristic features may be used while plotting or reading a contour plan or topographic map :

1. Two contour lines of different elevations cannot cross each other. If they did, the point of intersection would have two different elevations, which is absurd. However, contour lines of different elevations can intersect only in the case of an overhanging cliff or a cave.

2. Contour lines of different elevations can unite to form one line only in the case of a vertical cliff.

3. Contour lines close together indicate steep slope. They indicate a gentle slope if they are far apart. If they are equally spaced, uniform slope is indicated. A series of straight, parallel and equally spaced contour represent a plane surface.

4. A contour passing through any point is perpendicular to the line of steepest slope at that point. This agrees with (2) since the perpendicular distance between contour lines is the shortest distance.

5. A closed contour line with one or more higher ones inside it represents a hill. Similarly, a closed contour line with one or more lower ones inside it indicates a depression without an outlet.

6. Two contour lines having the same elevation cannot unite and continue as one line. Similarly, a single contour cannot split into two lines. This is evident because a

B.M. is not nearby, fly-levelling may be performed to establish a Temporary Bench Mark (T.B.M.) in that area. Having known the height of the instrument, the staff reading is calculated so that the bottom of the staff is at an elevation equal to the value of the contour. For example, if the height of the instrument is 101.80 metres, the staff reading to set a point on the contour of 100.00 metres will be 1.80 m. Taking one contour at a time (say 100.0 m contour), the staff man is directed to keep the staff on the points on the contour so that readings of 1.80 m are obtained every time. In Fig. 10.1, the dots represent the points determined by this method.

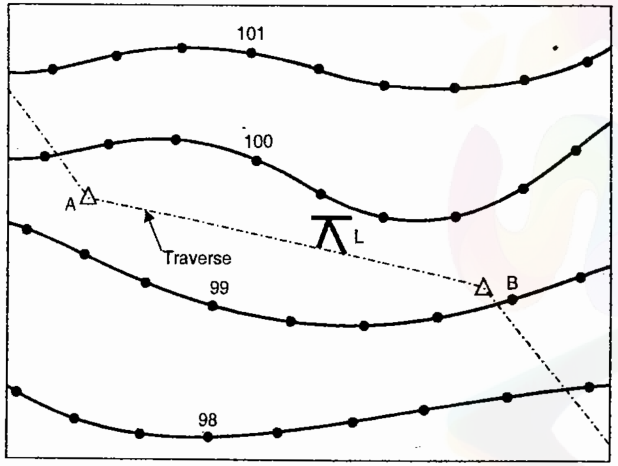

FIG. 10.1

If a hand level is used, slightly different procedure is adopted in locating the points on the contour. A ranging pole having marks at every decimetre interval may be used in conjunction with any type of hand level, preferably an Abney Clinometer. To start with, a point is located on one of the contours, by levelling from a B.M. The starting point must be located on the contour which is a mean of those to be commanded from that position. The surveyor then holds the hand level at that point and directs the rod man till the point on the rod corresponding to the height of the instrument above the ground is bisected. To do this conveniently, the level could be held against a pole at some convenient height, say, 1.50 m. If the instrument (*i.e.* the hand level) is at 100 m contour, the reading of the rod to be bisected at each point of 100.5 m, the rod reading to be bisected with the same instrument position will be $(1.50 - 0.5) = 1.0$ m. The work can thus be continued. The staff man should be instructed to insert a lath or twig at the points thus located. The twig must be split to receive a piece of paper on which the R.L. of the contour should be written.

(*ii*) ***Horizontal control.*** After having located the points on various contours, they are to be surveyed with a suitable control system ; the system to be adopted depends mainly on the type and extent of area. For small area, chain surveying may be used and the points may be located by offsets from the survey lines. In a work of larger nature, a traverse may be used. The traverse may use a theodolite, or a compass or a plane table as the principal instrument.

In the direct method, two survey parties generally work simultaneously—one locating the points on the contours and the other surveying those points. However, if the work is of a small nature, the points may be located first and then surveyed by the same party. Thus in Fig. 10.1, the points shown by dots are surveyed with respect to points *A* and *B* which may be tied by a traverse shown by chain-dotted lines.

(*b*) **Indirect method.**

In this method, some guide points are selected along a system of straight lines and their elevations are found. The points are then plotted and contours are drawn by interpolation. These guide points are not, except by coincidence, points on the contours to be located. While interpolating, it is assumed that slope between any two adjacent guide points is uniform. The following are some of the indirect methods of locating the ground points :

(1) ***By squares*** (Fig. 10.2). The method is used when the area to be surveyed is small and the ground is not very undulating. The area to be surveyed is divided into a number of squares. The size of the square may vary from 5 to 20 m depending upon the nature of the contours and contour interval. The elevations of the corners of the square are then determined by means of a level and a staff. The contour lines may then be drawn 'by interpolation.' It is not necessary that the squares may be of the same size. Sometimes rectangles are also used in place of squares. When there are appreciable breaks in the surface between corners, guide points in addition to those at corners may also be used. The squares should be as big as practicable, yet small enough to conform to the inequalities of the ground and to the accuracy required. The method is also known as *spot levelling*.

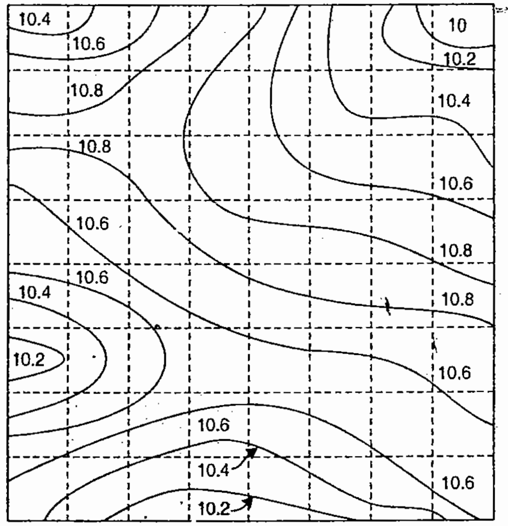

FIG. 10.2.

(2) ***By cross-sections.*** In this method, cross-sections are run transverse to the centre line of a road, railway or canal etc. The method is most suitable for route survey. The spacing of the cross-section depends upon the character of the terrain, the contour interval and the purpose of the survey. The cross-sections should be more closely spaced where the contours curve abruptly, as in ravines or on spurs. The cross-section and the points can then be plotted and the elevation of each point is marked. The contour lines are then interpolated on the assumption that there is uniform slope between two points on two adjacent contours. Thus, in Fig. 10.3, the points marked with dots are the points actually surveyed in the field while the points marked × on the first cross-section are the points interpolated on contours.

The same method may also be used in *direct method* of contouring with a slight modification. In the method described above, points are taken *almost* at regular intervals on a cross-section. However, the contour points can be located directly on the cross-section, as in the direct method. For example, if the height of the instrument is 101.80 m and if it is required to trace a contour of 100 m on the ground, the levelling staff readings placed on all such points are 1.80 m, and all these points will be on 100 m contour.

Keep the tracing cloth on the line in such a way that point *A* may lie on a *parallel* representing an elevation of 99.2 metres. Now rotate the tracing cloth on drawing in such a way that point *B* may lie on a parallel representing 100.5 metres. The points at which the parallels representing 99.5 (point *x*), 100.0 (point *y*) and 100.5 (point *z*) may now be pricked to get the respective positions of the contour points on the line *AB*.

Second method.

The second method is illustrated in Fig. 10.6. A line *XY* of any convenient length is taken on a tracing cloth and divided into several parts, each representing any particular interval, say 0.2 m. On a line perpendicular to *XY* at its midpoint, a pole *O* is chosen and radial lines are drawn joining the pole *O* and the division on the line *XY*. Let the bottom radial line represent an elevation of 97.0. If required, each fifth radial line representing one metre interval may be made dark. Let it be required to interpolate contours of 98, 99, 100 and 101 metres elevation between two points *A* and *B* having elevations of 97.6 and 101.8 metres. Arrange the tracing cloth on the line *AB* in such away that the points *A* and *B* lie simultaneously on radial lines representing 97.6 and 101.8 metres respectively. The points at which radial lines of 98, 99, 100 and 101 metres intersect *AB* may then be pricked through.

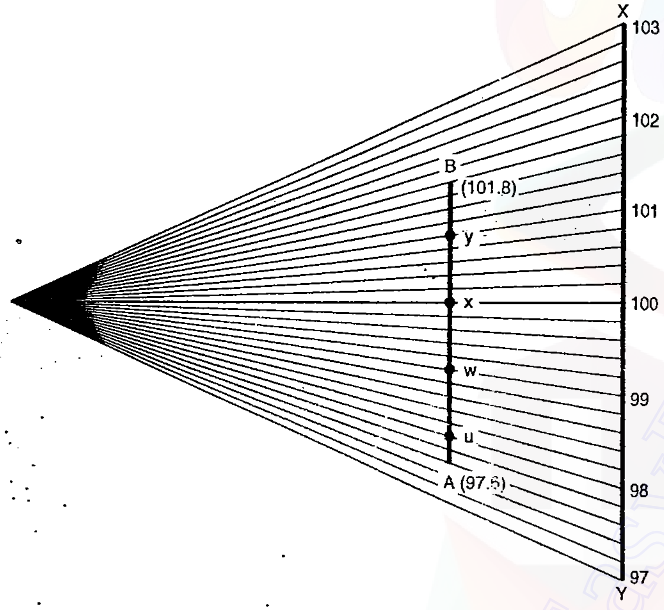

FIG. 10.6.

Contour drawing. After having interpolated the contour points between a network of guide points, smooth curve of the contour lines may be drawn through their corresponding contour points. While drawing the contour lines, the fundamental properties of contour lines must be borne in mind. The contour lines should be inked in either black or brown. If the contour plan also shows some of the features like roads etc., it is preferable to use brown ink for contour so as to distinguish it clearly from rest of the features. The value of the contours should be written in a systematic and uniform manner.

11

Route Surveying

11.1. INTRODUCTION

Surveys along a comparatively narrow strip of territory for the location, design and construction of any route of transportation, such as highways and railroads, aqueducts, canals and flumes, pipe-line for water, sewage, oil and gas, cableways and belt conveyors and power, telephone and telegraph transmission line is called *route surveying*. Route surveying includes all field work and requisite calculations, together with maps, profiles and other drawings.

Route surveys are done with two main aims : (1) determining the best *general route* between the termini and (2) fixing the alignment grades and other details of the *selected route*. Engineering principles require that the route be chosen in such a way that the project may be constructed and operated with the greatest economy and utility.

A comprehensive route survey consists of the following sequence of surveys :

(1) *Reconnaissance* of the terrain between the termini.

(2) *Preliminary surveys* over one or more locations along the general route recommended in the reconnaissance report.

(3) *Location survey*.

(4) *Construction survey*.

11.2. RECONNAISSANCE SURVEY

A reconnaissance survey is a rapid but thorough examination of an area or a strip of territory between the termini of the project to determine which of the several possible routes may be worthy of a detailed survey. Reconnaissance survey is the most important of the series of surveys mentioned above. A considerable amount of time and expense may be saved on unusable instrument surveys if the most desirable line is obtained during the reconnaissance. A very thorough and exhaustive examination of the whole area should be made to ensure that no possible route has been overlooked. Each route may be studied, and all but the most desirable eliminated. The work of reconnaissance must be entrusted to a very experienced engineer. The *locating engineer* should be endowed with a rare combination of technical thoroughness, business, judgement and visionary foresight, to enable him to select a route that will satisfy the future demand, in so far as this may be economically justified.

engineer : (1) the transit party, (2) the level party, and (3) the topography or cross-section party. The locating engineer is directly responsible for the location of the line, the prosecution of the work, the conduct of the survey, and all the other relating matters.

The *transit party* usually consists of four to seven men—chief of party, transitman, and two to four helpers. The survey work consists in *open traversing* with a transit along the selected route. In the case of highways, the traverse is usually run by the deflection angles, while in the case of railways, it is run by the method of back angles. The azimuths of the first and the last lines of the traverse are determined by astronomical observations. In the case of long traverse, azimuths are taken at about 20 km intervals. The *transitman* reads and records all angles, bearings and distances. Unless a special topography party is organised for the purpose, the transitman also records the topography notes that may have influence in determining the final location, such as the position and bearing of streams, drainage structures, property lines, intersecting roads or railroads and pipe lines. The plane table can sometimes be used to advantage particularly in rough country which is comparatively free from bushes, trees, or other obstructions to sightings. The transit-stadia method may also be used for making a preliminary survey. The transit-stadia method is rapid and economical, though less precise. If such a method is adopted, alignment, elevation and topographic details are carried in one operation by single party.

The *level party* includes three men : level man, rod man and note keeper. Sometimes, the level man himself records the data. The level party does important jobs : it establishes bench marks along the proposed route at regular and convenient places, (*ii*) it runs a longitudinal section of the traverse lines. The bench marks so obtained form the vertical control for the survey. Bench marks should be set on permanent objects and be carefully described in the notes. The elevations of the ground at all stakes on the transit line, points of change in slope and at intersections with roads, streams, railways etc. are determined. The levelling work is checked by taking observations on existing permanent bench marks and G.T.S bench marks. Each day's work is plotted from the profile level notes.

The *topography or cross-section* consists of three or four men —level man, rod man, chain man and recorder. The instruments used are ordinarily a hand level, rod and tape. If the ground is not very abrupt, an engineer's level is sometimes substituted for the hand level. However, a hand level is often used instead of the engineer's level with greater speed and with an accuracy within 0.05 m.

The topographer should prepare his note-book in advance with the alignment and elevation of the centre line points that he expects to cover the next day. This information can be supplied by the transit and level parties from their previous work. Cross-sections are set out at every 30 m stations, at right angles to the traverse lines and on either side of it by an optical square. In hilly and mountainous country, the distance between the cross-sectional lines may be reduced to 10 metres, while in flat or level country, it may be increased to 100 m. The topographer also records and sketches the natural and artificial features of the topography. The extent of the topography included in the notes will depend on the character of the ground being traversed, the speed and accuracy of the topo-grapher and his ability to visualise the features affecting the choice of the final line. At places where a change in horizontal direction is necessary, a wider strip of topography may enable the *locating engineer* to choose a more desirable final alignment when studying

the finished map. The most important features to be noted in taking of topography are the contours, streams and general character of the land traversed.

Paper location. The transit line of the preliminary survey is a series of straight lines, and no horizontal curves are introduced on this survey. After the survey is complete, a complete map is prepared on a scale of 1 cm=40 m or 1 cm=50 m. On this map, the final line can be located to include as much of the tangent lines as thought to be advisable, with horizontal curves introduced at points where changing in direction appear feasible. This new line drawn on the map is known as the *paper location*, and it can be located anywhere within the strip that has been given to all the features affecting the location. The factors affecting the choice of the *paper location* are : (1) minimum gradients and curvature, (2) balancing (equalization) of earth work, (3) heavy earthwork, (filling or cutting), (4) suitable crossing for rivers etc. The paper location of the alignment and grade line form a basis for the final location of the line. Several trials are necessary before the final location is marked on the preliminary survey map, fulfilling all the major requirements of the best location. In order to avoid excessive erasing of the pencil lines during the trial and error process of paper location, it is advisable to use a fine silk thread and needle. The first step is usually to stick pins at the controlling points and to stretch the thread from point to point. Portion of the thread lines between controlling points are then shifted back and forth until the line appears to fulfill the conditions of a good alignment. For each trial position of the thread-line location, a corresponding profile may be made from the topographic map and grade lines are then laid out with a thread on the profile. An adjustment of grade lines on the profile may require a corresponding shift in location on the map. When the thread on the map has been adjusted to show the final location of the centre line, the portions of the thread between pins represent tangents which are to be connected by curves. The line of location including curves, may now be *drawn* on the map to replace the thread lines and the so called *paper location* is completed.

11.4. LOCATION SURVEY

The location survey is the ground location of proposed line marked on the map, *i.e.*, it consists in laying out the *paper location* on the ground. The main purpose of location survey is to make minor improvements on the line as may appear desirable on the ground, and to fix up the final grades. The line as finally located on the ground is called the *field location*. The tangents, curves and drainage structure are established by means of a continuous transit survey, taking into account whatever adjustments in the line and grade appear to be advisable. Profile levels are run over the centre line, bench marks are established, and profile made which shows the ground line and the grade line. All other lines needed in construction are established with reference to the centre line. Cross-section notes are taken in order that the quantity of earth work may be computed. The notes are taken at every full station and at intermediate points along the line where the ground slope changes abruptly. These notes are taken by means of a rod and level on a line perpendicular to the centre line. On curves, the notes are taken on a radial line at the point of observations. Great care should be exercised to take the observations at 90° to the centre line to prevent serious error in the earthworth calculations, where the cut or fill is likely to be large. All important features in the close proximity of the located lines are also surveyed. The boundaries of private properties, with names of owners, are surveyed with ranges 10 mm, 0.5 in. and 0.02 ft. The smallest interval on the drum is 0.2 mm, 0.01 in. and 0.001 ft. respectively.

12.3. COLLIMATORS

Collimators are used as reference marks in instrument workshops and for optical tooling. A telescope can be converted into a collimator by focussing to infinity and fitting an eye-piece lamp in place of the standard eye-piece.

A collimator manufactured by Otto Fennel can be levelled by means of four levelling screws. Similarly, Wild manufactures a *workshop* collimator for permanent mounting. However, there are T 2, T 3 or even T4 telescopes *without eye-piece* but with built-in reticule illumination. Special reticule patterns are available. Pointing to a collimator is like pointing to a perfect target at infinity. Neither the position of the instrument nor the distance from the collimator have any influence on the measurement or its acuracy. For interchangeability between eye-pieces and eye-piece accessories, the eye-piece mount has a bayonet fastening. Wild eye-piece No. 53 of 40X is useful for optical tooling and laboratory measurements, as well illuminated targets can be pointed with high accuracy. Because only the centre of field of view is used, the fall-off in image quality around the edge with very high magnification has no influence on the measurements.

Auto-collimation

Auto-collimation is the process of making the telescope line of sight perpendicular to a plane mirror. It is used in *optical tooling* and laboratory work for alignment and measuring small deflections. Wild theodolites converted into autocollimators are often used for measurements in laboratories and workshops. Autocollimation offers many advantages and is particularly suitable for the exact definition of reference directions and planes, the determination of minute angular changes and deviations, the setting out and checking of perpendiculars, the calibration of angle measuring devices etc. Fig. 12.6 shows an autocollimation eye piece, fitted to NA2 level, for setting machine parts and instrument components precisely vertical. With the autocollimation eyepiece fitted, the telescope magnification is 24 ×. Fig. 12.7 shows autocollimation eyepiece fitted to T1000 theodolite, which converts it into an autocollimator for measuring tasks in laboratories and industry. Fig. 12.8 shows different versions of autocollimation eyepieces fitted to T1 telescope. The autocollimation eye-piece (1) simply interchanges with the telescope eyepiece. *An autocollimation prism and autocollimation mirror are available*. For steep and vertical sights, there is the *diagonal autocollimation eyepiece* (3). A collimator provides a perfect reference target at infinity and can be obtained by fitting the eyepiece lamp (2) to the telescope. This is particularly useful in instrument workshops and laboratories as well as for *optical tooling*.

For observations, Wild GAS 1 autocollimation mirror is attached to the object. The telescope is focussed to infinity and then pointed to the mirror. A reflected image of the cross-pairs is seen in the field of view. By turning either the telescope or object with mirror, the reticule cross and its reflected image are made to coincide—auto-collimation. The line of sight is then at right angles to the mirror. The mirror is front silvered optically plane, 50 mm in diameter housed in stable titanium housing. Three tapped holes with M 4 thread allow mirror to be attached and adjusted to various mounts.

Angular deviations of the mirror from a preset line of sight can easily be measured. For this, read both circles, turn the telescope to achieve autocollimation and read the circles again. The difference in the readings gives the angular deviations in Hz and V. Autocollimation is independent of distance ; the mirror can even be directly in front of the objective.

If a reference for horizontal angles only is required, or if the theodolite has to be moved and its height changed, the *Wild GAP 1 Autocollimation prism* is used instead of a mirror. It is particularly suitable for machine assembly and alignment, and for checking the parallelism of rollers in steel and paper mills.

12.4. OPTICAL PLUMMETS : ZENITH AND NADIR PLUMMETS

Optical plummets are optical devices used for precise centring. We will discuss here two such devices :

(*i*) Telescope roof plummet (*ii*) ZNL Zenith and Nadir plummet

1. Telescope roof plummet

Fig. 12.9 shows the photograph of the telescope roof plummet. This special optical plummet is used for rapid centring under roof markers in mines and tunnels. It fits the telescope of the T-2 theodolite with the instrument in face right position. Centring accuracy is 1 to 2 mm in 10 m.

2. ZNL Zenith and Nadir plummet (Fig. 12.10)

The Wild ZNL zenith and nadir plummet is used for upward and downward plumbing with 1:30000 accuracy in construction, mining and industry. A separate instrument with detachable tribach, the ZNL interchanges with forced-centring against Wild T-1 and T2 accessories. Fig. 12.10 (*a*) shows the position of the ZNL for zenith (or upward) plumbing while Fig. 12.10 (*b*) shows the position for Nadir (or downward) plumbing.

Wild has two ***automatic plummets*** : the ZL Automatic Zenith plummet and the NL automatic Nadir plummet. These are the plummets of highest precision used for industry, deformation measurements, mining and precise construction. They define the plumb line with 1:200000 accuracy. They interchange with Wild T-1 and T-2 theodolites.

12.5. OBJECTIVE PENTAPRISM

Fig. 12.11 shows the photograph of objective pentaprism by Wild. The pentaprism turns the line of sight through 90°. It is used for plumbing up and down, transferring directions to different levels, and for setting out. The plumbing accuracy is 1:70000.

12.6. TELEMETER

'Telemeter' is a special device which is attached to the objective end of the telescope, to measure directly the horizontal distance. A corresponding counter weight is attached to the telescope near its eye-piece end. The method of measurement is similar to tacheometry, except that the observations are taken on a horizontal rod. Thus, the method use a horizontal base which is variable.

Fig. 12.12 shows special horizontal stadia rod with vernier, used with the telemeter. The image of the vernier is obtained which is displaced with respect to the main graduations of the rod by the amount of deflection. For field measurement over the rod kept at the point under observation, the image of the vernier is first brought into the middle of the field of view. Then the micrometer drum is rotated until a graduation on the vernier

We shall consider here the following two models of electronic theodolites manufactured by M/s Wild Heerbrugg Ltd.

(*i*) Wild T-1000 electronic theodolite

(*ii*) Wild T-2000 and T-2000 S electronic theodolite

12.8.2. WILD T-1000 'THEOMAT'

Wild electronic theodolites are known as 'Theomat'. Fig 12.13 shows the photograph of Wild T-1000 electronic theodolite. Although it resembles a conventional theodolite (*i.e.*, optical theodolite) in size and weight, the T-1000 works with electronic speed and efficiency. It measures electronically and opens the way to electronic data aquisition and data processing. It has 30 × telescope which gives a bright, high-contrast, erect image. The coarse and fine focusing ensures that the target is seen sharp and clear. Pointing is fast and precise, even in poor observing conditions. The displays and reticle plate can be illuminated for works in mines and tunnels and at night.

The theodolite has two control panels, each with key-board and two liquid-crystal displays. It can be used easily and quickly in both positions. Fig. 12.14 shows the control panel of T-1000. The LCDs with points and symbols present the measured data clearly and unambiguously. The key-board has just six multifunction keys. The main operations require only a single keystroke. Accepted keystrokes are acknowledged by a beep. Colour-coding and easy-to-follow key sequences and commands make the instrument remarkably easy to use.

The theodolite has an absolute electronic-reading system with position-coded circles. There is no initialization procedure. Simply switch on and read the results. Circle reading is instantaneous. The readings up-date continuously as the instrument is turned. Readings are displayed to 1″. The standard deviation of a direction measured in face left and face right is 3″.

The theodolite has practice-tested *automatic index*. A well-damped pendulum compensator with 1″ setting accuracy provides the reference for T-1000 vertical circle readings. The compensator is built on the same principles as the compensator used in Wild automatic levels and optical theodolites. Thus with T-1000, one need not rely on a plate level alone. Integrated circuits and microprocessors ensure a high level to performance and operating comfort. Automatic self-checks and diagonostic routines makes the instrument easy to use.

T-1000 theodolite has electronic clamp for circle setting and repetition measurements. Using simple commands, one can set the horizontal circle reading to zero or to any value. The theodolite can be operated like a conventional theodolite using any observing procedure, including the repetition method. In addition to the conventional clockwise measurements, horizontal circle readings can be taken counter-clock-wise. Horizontal-collimation and vertical index errors can be determined and stored permanently. The displayed circle readings are corrected automatically. Displayed heights are corrected for earth curvature and mean refraction.

Keys			Command
DIST			Distance measurement
REC			Recording
ALL			Measurement and recording
DSP	Hz		Display Hz-circle and Hz-distance
SET	TRK		Tracking
SET	SET	Hz	Set horizontal-circle reading to zero

FIG. 12.15. TYPICAL COMMANDS IN T-1000 ELECTRONIC-THEODOLITE (WILD HEERBRUGG)

As stated earlier, the whole instrument is controlled from the key-board. Fig. 12.15 gives details of typical commands obtained by pressing corresponding keys. Fig. 12.16 gives typical display values obtained by pressing different keys. The power for T-1000 theodolite is obtained from a small, rechargeable 0.45 Ah Ni Cd battery which plugs into the theodolite standards.

Left display	Right display	Shows
Hz 137̣34̣52	V 91̣37̣56	Horizontal circle and vertical circle
Hz 137̣34̣54	◿ 118̣542	Horizontal circle and horizontal distance
V 91̣37̣55	◿ 3̣375	Vertical circle and height differences
V 91̣37̣54	◿ 118̣597	Vertical circle and slope distance

FIG. 12.16. TYPICAL-DISPLAYS ON THE PANELS OF T-1000

Wild T-1000 theodolite is fully compatible. It is perfectly modular, having the following uses :

(*i*) It can be used alone for angle measurement only.

(*ii*) It combines with Wild *Distomat* for angle and distance measurement.

(*iii*) It connects to GRE 3 data terminal for automatic data aquisition.

(*iv*) It is compatible with Wild theodolite accessories.

(*v*) It connects to computers with RS 232 interface.

Fig. 12.17 depicts diagrammatically, all these functions.

'Distomat' is a registered trade name used by Wild for their *electro-magnetic distance measurement* (EDM) instruments (see chapter 15). Various models of distomats, such as DI-1000, DI-5, DI-5S, DI-4/4L etc. are available, which can be fitted on the top of the telescope of T-1000 theodolite. The telescope can transit for angle measurements in both the positions. No special interface is required. With a Distomat fitted to it, the theodolite takes both angle and distance measurements. Wild DI-1000 distomat is a miniaturized EDM, specially designed for T-1000. It integrates perfectly with the theodolite to form the ideal combination for all day-to-day work. Its range is 500 m on to 1 prism and 800 m to 3 prisms, with a standard deviation of 5 mm + 5 ppm. For larger distances, DI-5S distomat can be fitted, which has a range of 2.5 km to 1 prism and 5 km to 11 prisms. For very long distances, latest long-range DI-3000 distomat, having a range of 6 km to 1 prism and maximum range of 14 km in favourable conditions can be fitted. Thus, with a distomat, T-1000 becomes *electronic total station.*

The T-1000 theodolite attains its full potential with the GRE 3 data terminal. This versatile unit connects directly to the T-1000. Circle readings and slope distances are transferred from the theodolite. Point numbering, codes and information are controlled from the GRE 3.

12.8.3. WILD T-2000 THEOMAT

Wild T-2000 Theomat (Fig. 12.18 *a*) is a high precision electronic angle measuring instrument. It has micro-processor controlled angle measurement system of highest accuracy. Absolute angle measurement is provided by a dynamic system with opto-electronic scanning the computer. Prompt messages and information can be transferred to the T-2000 displays. Of particular interest is the possibility of measuring objects by intersection from two theodolites (Fig. 12.23).

Two T 2000 type instruments can be connected to the Wild GRE 3 Data Terminal. Using the Mini-RMS program, co-ordinates of intersected points are computed and recorded. The distance between any pair of object points can be calculated and displayed. For complex applications and special computations, two or more T 2000 or T 2000 S can be used with the *Wild-Leitz RMS 2000 Remote Measuring System.*

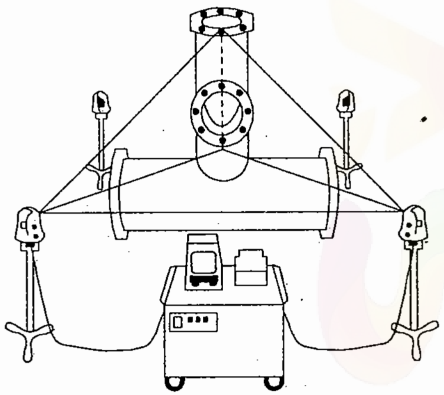

FIG. 12.23. RMS (REMOTE MEASURING SYSTEM) INTERSECTION METHOD.

12.8.4. WILD T 2000 S 'THEOMAT'

Wild T 2000 S [Fig. 12.18 (*b*)] combines the pointing accuracy of a *special telescope* with the precision of T 2000 dynamic circle measuring system. This results in angle measurement of the highest accuracy. The telescope is *panfocal* with a 52 mm obejctive for an exceptionally bright, high contrast image. It focuses to object 0.5 m from the telscope. The focusing drive has coarse and fine movements.

Magnification and field of view vary with focusing distance. For observations to distant targets, the field is reduced and magnification increased. At close range, the field of view widens and magnification is reduced. This unique system provides ideal conditions for observation at every distance. With the standard eye-piece, magnification is 43 × with telescope focused to infinity. Optional eye-pieces for higher and lower magnification can also be fitted.

Stability of the line of sight with change in focusing is a special feature of the T 2000 S telescope. It is a true alignment telescope for metrology, industry and optical tooling industry. T 2000 S can also be fitted with a special target designed for pointing to small targets.

A special target can also be built into the telescope at the intersection of the horizontal and vertical axes. The target is invaluable for bringing the lines of sight of two T 2000 S exactly into coincidence. This is the usual preliminary procedure prior to measuring objects by the RMS intersection method.

For fatigue-free, maximum-precision auto-collimation measurements, the telescope is available with an auto-collimation eye-piece with negative reticle (green cross).

Like T 2000, the T 2000 S takes all Wild Distomats. It can also be connected the GRE 3 Data Terminal.

13

Field Astronomy

13.1. DEFINITIONS OF ASTRONOMICAL TERMS

1. The Celestial Sphere. The millions of stars that we see in the sky on a clear cloudless night are all at varying distances from us. Since we are concerned with their relative distances rather than their actual distance from the observer, it is exceedingly convenient to picture the stars as distributed over the surface of an imaginary spherical sky having its centre at the position of the observer. This imaginary sphere on which the stars appear to lie or to be studded is known as the *Celestial Sphere.* The radius of the celestial sphere may be of any value – from a few thousand metres to a few thousand kilometres. Since the stars are very distant from us, the centre of the earth may be taken as the centre of the celestial sphere.

2. The Zenith and Nadir. The Zenith (Z) is the point on the upper portion of the celestial sphere marked by plumb line above the observer. It is thus the point on the celestial sphere immediately above the observer's station. The *Nadir* (*Z'*) is the point on the lower portion of the celestial sphere marked by the plumb line below the observer. It is thus the point on the celestial sphere vertically below the observer's station.

3. The Celestial Horizon. (also called *True* or *Rational horizon* or *geocentric horizon*). It is the great circle traced upon the celestial sphere by that plane which is perpendicular to the Zenith–Nadir line, and which passes through the centre of the earth. (*Great circle* is a section of a sphere when the cutting plane passes through the centre of the sphere).

4. The Terrestrial Poles and Equator. *The terrestrial poles* are the two points in which the earth's axis of rotation meets the earth's sphere. The *terrestrial equator* is the great circle of the earth, the plane of which is at right angles to the axis of rotation. The two poles are equidistant from it.

5. The Celestial Poles and Equator. If the earth's axis of rotation is produced indefinitely, it will meet the celestial sphere in two points called the *north and south celestial poles* (*P* and *P'*). The *celestial equator* is the great circle of the celestial sphere in which it is intersected by the plane of terrestrial equator.

6. The Sensible Horizon. It is a circle in which a plane passing through the point of observation and tangential to the earth's surface (or perpendicular to the Zenith–Nadir line) intersects with celestial sphere. The line of sight of an accurately levelled telescope lies in this plane.

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22. Equinoctial Points. The points of the intersection of the ecliptic with the equator are called the equinoctial points. The declination of the sun is zero at the equinoctial points. The *Vernal Equinox* or the *First Point of Aries* (♈) is the point in which the sun's declination changes from south to north, and marks the commencement of spring. It is a fixed point on the celestial sphere. The *Autumnal Equinox* or the First Point of Libra (♎) is the point in which the sun's declination changes from north to south, and marks the commencement of autumn. Both the equinoctial points are six months apart in time.

23. The Ecliptic. Ecliptic is the great circle of the heavens which the sun appears to describe on the celestial sphere with the earth as a centre in the course of a year. The plane of the ecliptic is inclined to the plane of the equator at an angle (called the *obliquity*) of about 23° 27′, but is subjected to a diminution of about 5″ in a century.

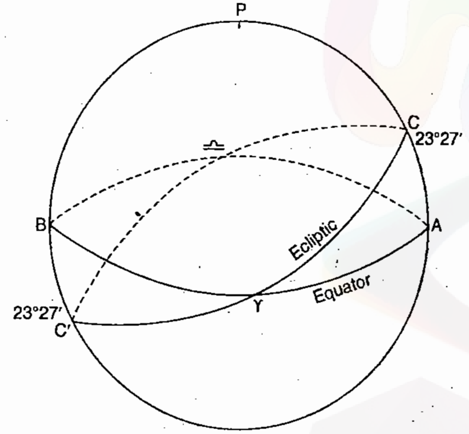


FIG. 13.3. THE ECLIPTIC.

24. Solastices. Solastices are the points at which the north and south declination of the sun is a maximum. The point *C* (Fig. 13.3) at which the north declination of the sun is maximum is called the *summer solastice*, while the point *C′* at which south declination of the sun is maximum is known as the *winter solastice*. The case is just the reverse in the southern hemisphere.

25. North, South, East and West Directions. The north and south points correspond to the projection of the north and south poles on the horizon. The *meridian line* is the line in which the observer's meridian plane meets horizon plane, and the north and south points are the points on the extremities of it. The direction *ZP* (in plan on the plane of horizon) is the direction of north, while the direction *PZ* is the direction of south. The *east-west line* is the line in which the prime vertical meets the horizon, and east and west points are the extremities of it. *Since the meridian plane is perpendicular to both the equatorial plane as well as horizontal plane, the intersections of the equator and horizon determine the east and west points* (see Fig. 13.1).

13.2. CO-ORDINATE SYSTEMS

The position of a heavenly body can be specified by two spherical co-ordinates, *i.e.*, by two angular distances measured along arcs of two great circles which cut each other at right angles. One of the great circle is known as the primary circle of the reference and the other as the *secondary circle* of reference. Thus in Fig. 13.4, the position of the point *M* can be specified with reference to the plane *OAB* and the line *OA*, *O* being the origin of the co-ordinates. If a plane is passed through *OM* and perpendicular to the plane of *OAB*, it will cut the latter in the line *OB*. The two spherical co-ordinates of the point *M* are, therefore, angles *AOB* and *BOM* at the centre *O*, or the arcs *AB* and *BM*. In practical astronomy, the position of a celestial body can be specified by the following systems of co-ordinates :

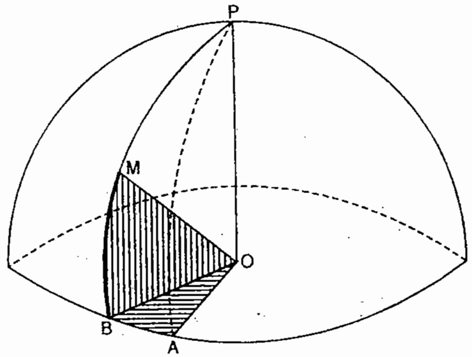


FIG. 13.4

1. The *horizon system*
2. The *independent equatorial system*
3. The *dependent equatorial system*
4. The *celestial latitude and longitude system.*

The *horizon system* is dependent on the position of the observer. The *independent equatorial system* is independent of the position of the observer and the positions apply to observers anywhere on the earth. In the *dependent equatorial system*, one of the great circle of reference is independent of the position of the observer while the other great circle perpendicular to the former is dependent on the position of the observer. There is yet another system of co-ordinates, known as the *celestial system*, in which the position of a body is specified by the *celestial latitude* and the *celestial longitude.*

1. THE HORIZON SYSTEM (ALTITUDE AND AZIMUTH SYSTEM)

In the horizon system, the horizon is the plane of reference and the co-ordinates of a heavenly body are (*i*) the *azimuth* and (*ii*) the *altitude*. This system is necessitated by the fact that we can measure only horizontal and vertical angles with the engineer's transit. The two great circles of reference are the horizon and the observer's meridian, the former being the primary circle and the latter the secondary circle.

In Fig. 13.5, *M* is the heavenly body in the Eastern part of the celestial sphere, *Z* is the observer's zenith and *P* is the celestial pole. Pass a vertical circle (*i.e.*, a great circle through *Z*) through *M* to intersect the horizon plane at *M′*. The first co-ordinate of *M* is, then, the *azimuth* (*A*) which is the angle between the observer's meridian and the vertical circle through the body. The azimuth can either be measured as the angular distance along the horizon, measured from the meridian to the foot of the vertical circle through the point. It is also equal to the angle at the zenith between the meridian and the vertical circle through *M*. The other co-ordinate of *M* is the *altitude* (α) which is the angular distance measured above (or below) the horizon, measured on the vertical circle through the body. Similarly, Fig. 13.6 shows the position (*M*) of the body in the Western part of the celestial sphere. It should be noted that, in the Northern hemisphere, the azimuth is always measured from the *north* either eastward, or westward, depending

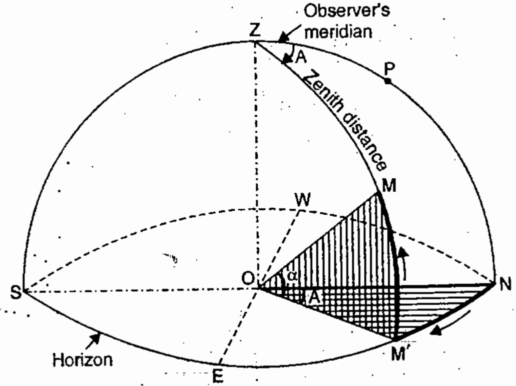


FIG. 13.5 BODY IN THE EASTERN PART OF THE CELESTIAL SPHERE.

ridian and the declination circle of the body. *In the northern hemisphere, the hour angle is always measured from the south and towards west upto the declination circle.* Its value varies from 0 ° to 360 °. If H varies from 0 ° to 180 °, the star is in the western hemisphere, otherwise in the eastern hemisphere. Fig. 13.9 shows the plan on the plane of the equator, illustrating how the hour angle is measured *westward* for two positions of the observer. The other co-ordinate is the declination, as in the second system. Thus, in Fig. 13.8, SM' is the hour angle, and M_1M is the declination of the celestial body (M), M' and M_1 being the projections of M on the horizon and equator respectively.

4. THE CELESTIAL LATITUDE AND LONGITUDE SYSTEM

In this system of the co-ordinates, the primary plane of reference is the ecliptic. The second plane of reference is a great circle passing through the First Point of Aries and perpendicular to the plane of the ecliptic. The two co-ordinates of a celestial body are (*i*) the celestial latitude and (*ii*) the celestial longitude.

The *celestial latitude* of a body is the arc of great circle perpendicular to the ecliptic, intercepted between the body and the ecliptic. It is positive or negative depending upon whether measured north or south of the ecliptic. The *celestial longitude* of a body is the arc of a ecliptic intercepted between the great circle passing through the First Point of Aries and the circle of the celestial latitude passing through the body. It is measured *eastwards* from 0° to 360°. Thus, in Fig. 13.10, M_1M is the celestial latitude (north) and ΥM_1 is the celestial longitude for the heavenly body (M).

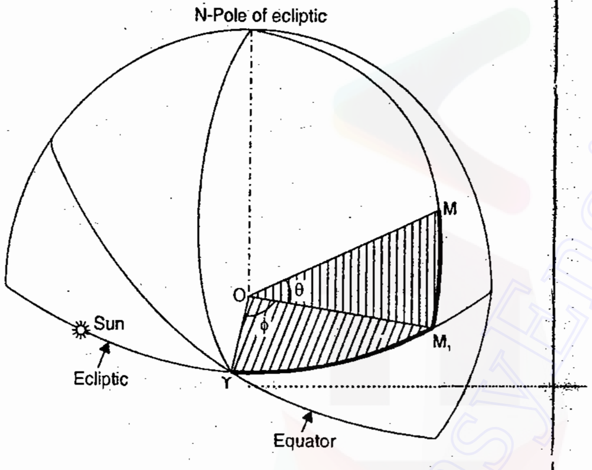


FIG. 13.10. THE CELESTIAL LATITUDE AND LONGITUDE.

Comparison of the Systems. As stated earlier, the azimuth and altitude of a star are not constant but are continuously changing due to diurnal motion. On the other hand, the right ascension and declination of a star are constant, because the reference point, the First Point of Aries, partakes of the diurnal motion of the stars. However, there is no instrument which can measure right ascension and declination of the star directly. The azimuth and the altitude of a star can be directly measured with the help of a theodolite. Knowing the hour angle and the azimuth of a star, its right ascension and declination can be computed from the solution of the astronomical triangle provided the instant of time at which the body was in a certain position (*i.e.*, the hour angle) is also determined. Thus, both the systems are necessary — the first one for the direct field observations and the second one for the computations required in respect of the preparation of the star catalogues.

13.3. THE TERRESTRIAL LATITUDE AND LONGITUDE

We have discussed the various systems of co-ordinates to establish the position of a heavenly body on the celestial sphere. In order to mark the position of a point on the earth's surface, it is necessary to use a system of co-ordinates. The terrestrial latitudes and longitudes are used for this purpose.

The terrestrial *meridian* is any great circle whose plane passes through the axis of the earth (*i.e.*, through the north and south poles). Terrestrial equator is the great circle whose plane is perpendicular to the earth's axis. The *latitude* θ of a place is the angle subtended at the centre of the earth north by the arc of meridian intercepted between the place and the equator. The latitude is north or positive when measured above the equator, and is south or negative when measured below the equator. The latitude of a point upon the equator is thus 0 °, while at the North and South Poles, it is 90 °N and 90 ° S latitude respectively. The *co-latitude* is the complement of the latitude, and is the distance between the point and pole measured along the meridian.

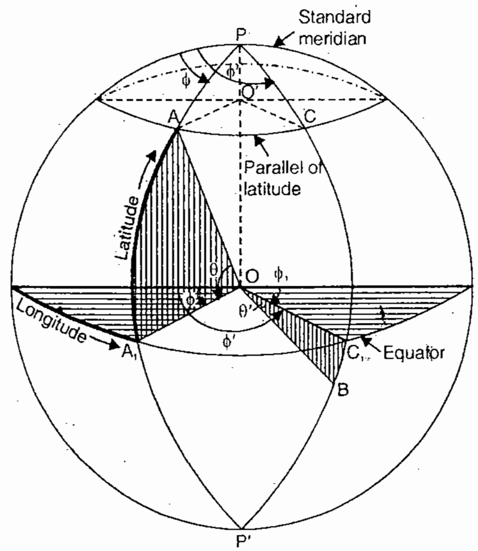


FIG. 13.11. THE TERRESTRIAL LATITUDE AND LONGITUDE.

The *longitude* (φ) of a place is the angle made by its meridian plane with some fixed meridian plane arbitrarily chosen, and is measured by the arc of equator intercepted between these two meridians. The prime meridian universally adopted is that of Greenwich. The longitude of any place varies between 0 ° to 180 °, and is reckoned as φ ° east or west of Greenwich. All the points on meridian have the same longitude.

The Parallel of Latitude

The *parallel of latitude* through a point is a small circle in which a plane through that point, and at right angles to the earth's axis, intersects the earth's surface. *All the points on the parallel of latitude have the same latitude.* The meridians are great circles of the same diameter while the parallel of a latitude are small circles, and are of different diameters depending upon the latitude of the place through which the parallel of the latitude is drawn. Due to this reason *a degree of longitude* has got different values at different latitudes — higher the latitude smaller the value. At the equator, a degree of longitude is equivalent to a distance of about 69 miles. However, a degree of latitude has the constant value of 69 miles everywhere.

To find the distance between two points A and C on a parallel of latitude, consider Fig. 13.11 in which θ = latitude of A = latitude of C, φ = longitude of A, and φ' = longitude of C. The angular radius PA of the parallel of latitude = 90 ° − θ.

$$\sin\frac{A}{2}=\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b\sin c}} \qquad ...(13.4)$$

$$\cos\frac{A}{2}=\sqrt{\frac{\sin s\sin(s-a)}{\sin b\sin c}} \qquad ...(13.5)$$

$$\tan\frac{A}{2}=\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)}} \qquad ...(13.6)$$

where $s=\frac{1}{2}(a+b+c)$

4. Similarly,

$$\sin\frac{a}{2}=\sqrt{\frac{-\cos S\cos(S-A)}{\sin B\sin C}} \qquad ...(13.7)$$

$$\cos\frac{a}{2}=\sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B\sin C}} \qquad ...(13.8)$$

$$\tan\frac{a}{2}=\sqrt{\frac{-\cos S\cos(S-A)}{\cos(S-B)\cos(S-C)}} \qquad ...(13.9)$$

where $S=\frac{1}{2}(A+B+C)$

5.
$$\tan\tfrac{1}{2}(a+b)=\frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c \qquad ...(13.10)$$

$$\tan\tfrac{1}{2}(a-b)=\frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c \qquad ...(13.11)$$

6.
$$\tan\tfrac{1}{2}(A+B)=\frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C \qquad ...(13.12)$$

$$\tan\tfrac{1}{2}(A-B)=\frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C \qquad ...(13.13)$$

THE SOLUTION OF RIGHT-ANGLED SPHERICAL TRIANGLE

The relationships of right-angled spherical triangle are very conveniently obtained from *'Napier's rules of circular parts'*.

In [Fig. 13.14 (*a*)], *ABC* is a spherical triangle right-angled at *C*. Napier defines the circular parts as follows :

(*i*) the side *a* to one side of the right-angle,

(*ii*) the side *b* to the other side of the right-angle,

(*iii*) the complement (90° – *A*) of the angle *A*,

(*iv*) the complement (90° – *c*) of the side *c*,

and (*v*) the complement (90° – *B*) of the angle *B*.

These five parts are supposed to be arranged round a circle [Fig. 13.14 (*b*)] in order in which they stand in the triangle. Thus, starting with the side *a*, we have, *in order*, *b*, 90° – *A*, 90° – *c* and 90° – *B*. Then, if any part is considered as the *'middle part'* the two parts adjacent to it as *'adjacent parts'*, and the remaining two as *'opposite parts'*, we have the following rules by Napier :

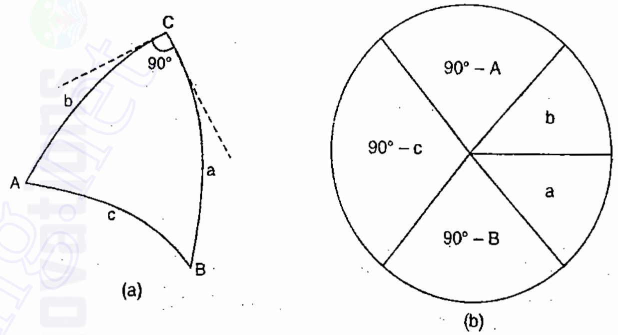


FIG. 13.14. NAPIER'S RULES OF CIRCULAR PARTS.

sine of middle part = product of *tangents* of the *adjacent* parts ...(*i*)

and ***sine of middle part*** = product of *cosines* of opposite parts ...(*ii*)

Thus, $\sin b=\tan a\tan(90°-A)$

and $\sin b=\cos(90°-B)\cos(90°-c)$

By choosing different parts in turn as the middle parts, we can obtain all the possible relationships between the sides and angles.

THE SPHERICAL EXCESS

The spherical excess of a spherical triangle is the value by which the sum of three angles of the triangle exceeds 180°.

Thus, spherical excess $E=(A+B+C-180°)$ ...(13.14)

Also, $\tan^2\frac{1}{2}E=\tan\frac{1}{2}s\tan\frac{1}{2}(s-a)\tan\frac{1}{2}(s-b)\tan\frac{1}{2}(s-c)$ ...(13.15)

In geodetic work the spherical triangles on the earth's surface are comparatively small and the spherical excess seldom exceeds more than a few seconds of arc. The spherical excess, in such case, can be expressed by the approximate formula

$$E=\frac{\Delta}{R^2\sin 1''}\text{ seconds} \qquad ...[13.15\ (a)]$$

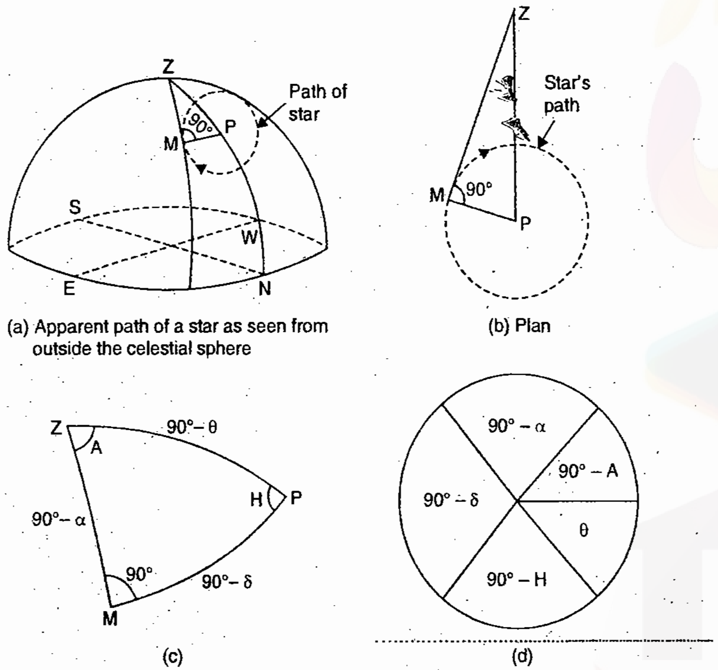
where *R* is the radius of the earth and Δ is the area of triangle expressed in the same linear units as *R*.

In order to prove the above expression for the spherical excess, let us consider the spherical triangle *ABC* [Fig. 13.14 (*c*)] which is formed by three great circles. These three great circles divide the whole sphere in eight divisions—the four in one hemisphere being similar to the other four in the other hemisphere because of symmetry.

Let Δ = area *ABC* ; Δ_1 = area *ACD*

Δ_2 = area *CDE* ; Δ_3 = area *BCE*

The star is said to be at *eastern elongation*, when it is at its greatest distance to the east of the meridian, and at *western elongation*, when it is at its greatest distance to the west of the meridian. Fig. 13.16 (*a*) and (*b*) show the star *M* at its eastern elongation.



(a) Apparent path of a star as seen from outside the celestial sphere (b) Plan

(c) (d)

FIG. 13.16. STAR AT ELONGATION.

If the declination (δ) and the latitude of the place of observation is known, the azimuth (*A*), hour angle (*H*) and the altitude (α) of the body can be calculated from the Napier's rule [Fig. 13.16 (*c*) and (*d*)]. The five parts taken in order are, the two sides $(90° - \alpha)$, $(90° - \delta)$ and the complements of the rest of the three parts, *i.e.*, $(90° - H)$, $[90° - (90° - \theta)] = \theta$ and $(90° - A)$.

Thus, sine of middle part = product of tangents of adjacent parts.

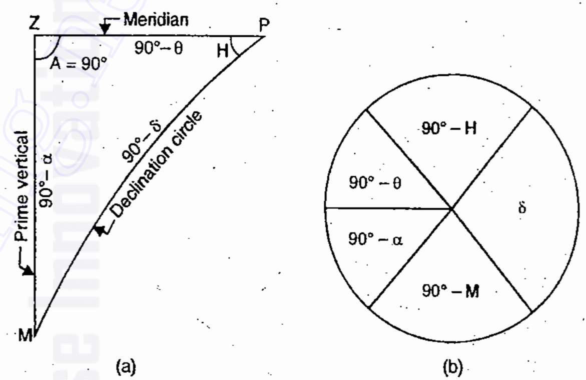
$$\sin (90° - H) = \tan (90° - \delta) \tan \theta \quad \text{or} \quad \cos H = \frac{\tan \theta}{\tan \delta} = \tan \theta \, . \cot \delta \qquad \ldots(13.19)$$

Similarly,
$$\sin \theta = \cos (90° - \delta) \, . \cos (90° - \alpha) \quad \text{or} \quad \sin \alpha = \frac{\sin \theta}{\sin \delta} = \sin \theta \, . \operatorname{cosec} \delta \qquad \ldots(13.20)$$

and
$$\sin (90° - \delta) = \cos (90° - A) \cos \theta \quad \text{or} \quad \sin A = \frac{\cos \delta}{\cos \theta} = \cos \delta \, . \sec \theta \qquad \ldots(13.21)$$

STAR AT PRIME VERTICAL

When the star is on the prime vertical of the observer, the astronomical triangle is evidently right-angled at *Z*.



(a) (b)

FIG. 13.17. STAR AT PRIME VERTICAL.

If the declination (δ) and the latitute (θ) of the place of observation are known, the altitude (α) and the hour angle (*H*) can be calculated by Napier's rule. The five parts taken in order are : the two sides $(90° - \theta)$ and $(90° - \alpha)$, and the complements of the rest of the three parts, *i.e.*, $(90° - M)$, $90° - (90° - \delta) = \delta$ and $(90° - H)$.

Now sine of middle part = product of cosine of opposite parts.

$$\therefore \quad \sin \delta = \cos (90° - \theta) \cos (90° - \alpha) = \sin \theta \sin \alpha \quad \therefore \quad \sin \alpha = \frac{\sin \delta}{\sin \theta} = \sin \delta \operatorname{cosec} \theta \qquad \ldots(13.22)$$

$$\text{And} \quad \sin (90° - H) = \tan (90° - \theta) \tan \delta \quad \text{or} \quad \cos H = \frac{\tan \delta}{\tan \theta} = \tan \delta \cot \theta \qquad \ldots(13.23)$$

STAR AT HORIZON

If a star (*M*) is at horizon, its altitude will be zero and the zenith distance will be equal to 90°.

If the latitude θ and the declination δ are known, the azimuth *A* and the hour angle *H* can be calculated by putting α = zero in equations 13.17 *a* and 13.18 *a*.

Thus,
$$\cos A = \frac{\sin \delta}{\cos \theta} = \sin \delta \sec \theta \qquad \ldots(13.24)$$

and
$$\cos H = - \tan \delta \tan \theta \qquad \ldots(13.25)$$

STAR AT CULMINATION

A star is said to culminate or transit when it crosses the observer's meridian. Each star crosses a meridian twice in its one revolution around the pole – the two culminations

$$ZP = Z\,M_2 + M_2\,P$$

or $\quad (90° - \theta) = (90° - \alpha) + p$, where p = polar distance $= M_2\,P$

or $\quad \theta = \alpha - p \qquad \ldots(2)$

Similarly, if the star is north of the zenith but below the pole, as at M_3, we have

$$ZM_3 = ZP + PM_3$$

or $\quad (90° - \alpha) = (90° - \theta) + p$, where p = polar distance $= M_3\,P$

or $\quad \theta = \alpha + p \qquad \ldots(3)$

The above relations form the basis for the usual observation for latitude.

3. The Relation between Right Ascension and Hour Angle.

Fig. 13.22 shows the plan of the stellar sphere on the plane of the equator. M is the position of the star and $\angle SPM$ is its westerly hour angle. H_M. ♈ is the position of the First Point of Aries and angle SP♈ is its westerly hour angle. $\angle$♈PM is the right ascension of the star. Evidently, we have

∴ ***Hour angle of Equinox = Hour angle of star + R.A. of star.***

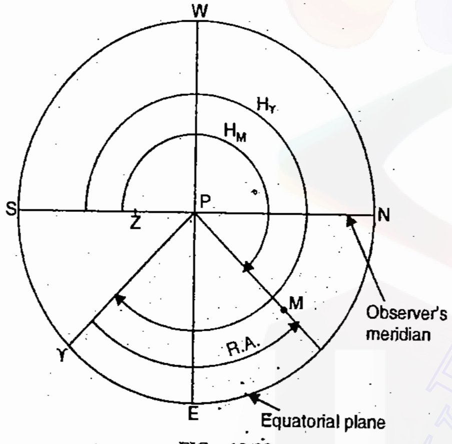


FIG. 13.22

Example 13.1. *Find the difference of longitude between two places A and B from their following longitudes :*

(1) *Longitude of A = 40° W*
Longitude of B = 73° W

(2) *Long. of A = 20° E*
Long. of B = 150° E

(3) *Longitude of A = 20° W*
Longitude of B = 50° W

(4) *Long. of A = 40° E*
Long. of B = 150° W

Solution.

(1) The difference of longitude between A and $B = 73° - 40° = $ **33°**

(2) The difference of longitude between A and $B = 150° - 20° = $ **130°**

(3) The difference of longitude between A and $B = 20° - (-50°) = $ **70°**

(4) The difference of longitude between A and $B = 40° - (-150°) = $ **190°**

Since it is greater than 180°, it represents the obtuse angular difference. The acute angular difference of longitude between A and B, therefore, is equal to $360° - 190° = $ **170°**.

Example 13.2. *Calculate the distance in kilometers between two points A and B along the parallel of latitude, given that*

(1) *Lat. of A, 28° 42′ N ; longitude of A, 31° 12′ W*
Lat. of B, 28° 42′ N ; longitude of B, 47° 24′ W

(2) *Lat. of A, 12° 36′ S ; longitude of A, 115° 6′ W*
Lat. of B, 12° 36′ S ; longitude of B, 150° 24′ E.

Solution.

The distance in nautical miles between A and B along the parallel of latitude = difference of longitude in minutes × cos latitude.

(1) Difference of longitude between A and $B = 47° 24' - 31° 12' = 16° 12' = 972$ minutes

∴ Distance = 972 cos 28° 42′ = 851.72 nautical miles
= 851.72 × 1.852 = **1577.34 km.**

(2) Difference of longitude between A and B
$= 360° - \{115° 6' - (-150° 24')\} = 94° 30' = 5670$ min.

∴ Distance = 5670 cos 12° 36′ = 5533.45 nautical miles
= 5533.45 × 1.852 = **10,247.2 km.**

Example 13.3. *Find the shortest distance between two places A and B, given that the longitudes of A and B are 15° 0′ N and 12° 6′ N and their longitudes are 50° 12′ E and 54° 0′ E respectively. Find also the direction of B on the great circle route. Radius of earth = 6370 km.*

Solution.

In Fig. 13.23, the positions of A and B have been shown.

In the spherical triangle ABP,

$b = AP = 90°$ – latitude of $A = 90° - 15° 0' = 75°$

$a = BP = 90°$ – latitude of B
$= 90° - 12° 6' = 77° 54'$

$P = \angle APB$ = difference of longitude
$= 54° 0' - 50° 12' = 3° 48'$.

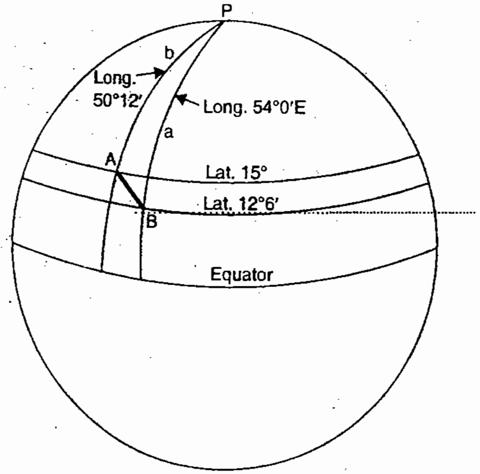


FIG. 13.23.

The shortest distance between two points is the distance along the great circle passing through the two points.

Knowing the two sides one angle, the third side AB ($= p$) can be easily computed by the cosine rule.

Thus $\quad \cos P = \dfrac{\cos p - \cos a \cos b}{\sin a \sin b}$

or $\quad \cos p = \cos P \sin a \sin b + \cos a \cos b$

$= \cos 3° 48' \sin 77° 54' \sin 75° + \cos 77° 54' \cos 75° = 0.94236 + 0.05425 = 0.99661$

∴ $p = AB = 4° 40' = 4°{\cdot}7$

Now, $\quad$ arc ≈ radius × central angle $= \dfrac{6370 \times 4°{\cdot}7 \times \pi}{180°} = 522.54$ km.

Hence distance $AB = $ **522.54 km.**

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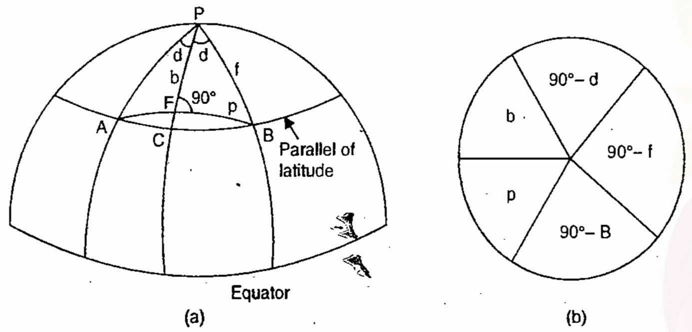

FIG. 13.25.

Distance $FB = p$ can be calculated by Napier's rule for the circular parts shown in Fig. 13.25 (*b*).

$\therefore$ sine middle part = Product of cosines of opposite parts

$$\therefore \quad \sin p = \cos(90° - d)\cos(90° - f) = \sin d \sin f = \sin d \sin(90° - l) = \sin d \cos l$$

or $$FB = p = \sin^{-1}(\sin d \cos l)$$

Hence $$AB = 2\,FB = 2\,p = 2\sin^{-1}(\sin d \cos l) \text{ radians.}$$

$\therefore$ Distance AB along great circle = radius × angle at the centre of earth $= R \times 2p$

$$= 2R\sin^{-1}(\sin d \cos l) \qquad \text{(Proved).}$$

The greatest distance between the great circle AFB and the parallel of latitude ACB will evidently be along CF (since $\angle F = 90°$).

The distance $PF = b$ can be found by Napier's rule.

sin middle part = product of tangents of adjacent parts

or $$\sin(90° - d) = \tan b \tan(90° - f)$$

or $$\cos d = \tan b \cot f = \tan b \cot(90° - l) = \tan b \tan l$$

$$\therefore \quad \tan b = \cos d \cot l \quad \text{or} \quad b = PF = \tan^{-1}(\cos d \cot l)$$

Now $$CF = CP - PF$$

But $$CP = (90° - l) = \frac{\pi}{2} - l \text{ radians}$$

$$\therefore \quad CF = \left(\frac{\pi}{2} - l\right) - \tan^{-1}(\cos d \cot l) \text{ radians}$$

$\therefore$ Distance along CF = Radius × angle at the centre $= R\left\{\left(\frac{\pi}{2} - l\right) - \tan^{-1}(\cos d \cot l)\right\}$ **Ans.**

Example 13.6. *Find the zenith distance and altitude at the upper culmination of the stars from the following data :*

(*a*)	*Declination of star* = 42° 15′ N	*Latitude of observer* = 26° 40′ N
(*b*)	*Declination of star* = 23° 20′ N	*Latitude of observer* = 26° 40′ N
(*c*)	*Declination of star* = 65° 40′ N	*Latitude of observer* = 26° 40′ N

Solution. (Fig. 13.18)

(*a*) Since the declination of the star is greater than the latitude of the observer ($\delta > \theta$), the upper culmination of the star occurs to the north side of zenith, *i.e.*, between Z and P.

Hence zenith distance at upper culmination $= ZA = ZP - AP$

$$= (90° - \theta) - (90° - \delta) = (\delta - \theta) = 42°\,15' - 26°\,40' = \mathbf{15°\,35'}$$

$\therefore$ Altitude of the star at upper culmination $= 90° - 15°\,35' = \mathbf{74°\,25'}$.

(*b*) Since the declination of the star is lesser than the latitude of the observer, the upper culmination of the star occurs at the south side of the zenith.

$\therefore$ Zenith distance of the star at upper culmination $= ZA_1 = A_1P - ZP$

$$= (90° - \delta) - (90° - \theta) = \theta - \delta = 26°\,40' - 23°\,20' = \mathbf{3°\,20'}$$

$\therefore$ Altitude of the star at the upper culmination $= 90° - 3°\,20' = \mathbf{86°\,40'}$.

(*c*) Fig. 13.19, $\delta = 65°\,40'$ N ; $90° - \theta = 90° - 26°\,40' = \mathbf{63°\,20'}$

Since the declination of the star is greater than the co-latitude, the star is circumpolar, and will never set. The upper culmination will occur at the north side of zenith, *i.e.*, between Z and P.

$\therefore$ Zenith distance at the upper culmination $= ZA_2 = ZP - A_2P$

$$= (90° - \theta) - (90° - \delta) = \delta - \theta = 85°\,40' - 26°\,40' = \mathbf{39°}.$$

$\therefore$ Altitude of the star at the upper culmination $= 90° - 39° = \mathbf{51°}$.

Example 13.7. *Find the zenith distance and altitude at the lower culmination for a star having declination = 85° 20′ if the latitude of the place of observation = 46° 50′.*

Solution.

$$\delta = 85°\,20' \;;\; 90° - \theta = 90° - 46°\,50' - 43°\,10'$$

Since the declination of the star is greater than the co-latitude of the place, it is circumpolar and will not set.

In Fig. 13.19, let A_1 be the lower culmination of a circumpolar star M_1. Its zenith distance at the lower culmination $= ZA_1 = ZP + PA_1$

$$= (90° - \theta) + (90° - \delta) = 180° - \delta - \theta = 180° - 85°\,20' - 46°\,50' = \mathbf{47°\,50'}$$

The altitude of the star $= 90° - 47°\,50' = \mathbf{42°\,10'}$.

Example 13.8. *A star having a declination of 56° 10′ N has its upper transit in the zenith of the place. Find the altitude of the star at its lower transit.*

Solution. (Fig. 13.18)

Let M be the star having A and B as its upper and lower transits. Since the upper culmination is at the zenith, Z and A coincide.

Hence zenith distance of star = zero

and Polar distance of the star $= AP = ZP$ = co-latiude of place

$$\therefore \quad 90° - \delta = 90° - \theta \quad \text{or} \quad \theta = \delta = 56°\,10'$$

At the lowest transit of the star at B, its zenith distance $= ZB = ZP + PB$

$$= (90° - \theta) + (90° - \delta) = 180° - \theta - \delta = 180° - 2\delta = 180° - 112°\,20' = 67°\,40'$$

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$$\cos A = \frac{\cos PM - \cos PZ \cdot \cos PM}{\sin PZ \cdot \sin ZM} = \frac{\cos 98° 30' - \cos 40° \cos 67° 11'}{\sin 40° \sin 67° 11'}$$

$$= \frac{-0.14781 - 0.29687}{0.59250} = -0.75051.$$

Since $\cos A$ is negative, the value of A is between 90° and 180°

$\therefore$ $\cos (180° - A) = -\cos A = 0.75051$

$\therefore$ $(180° - A) = 41° 22'$ or $A = 138° 38'$

$\therefore$ Azimuth of star = **138° 38′ E.**

Example 13.12. *Determine the hour angle and declination of a star from the following data :*

(i) Altitude of the star = 22° 36′

(ii) Azimuth of the star = 42° W

(iii) Latitude of the place of observation = 40° N.

Solution. (Fig. 13.26)

Since the azimuth of the star is 42° *W*, the star is in the western hemi-sphere. In the astronomical ΔPZM, we have

PZ = co-latitude = 90° − 40° = 50° ; ZM = co-altitude = 90° − 22° 36° = 67 24′; angle A = 42°

Knowing the two sides and the included angle, the third side can be calculated from the cosine formula (Eq. 13.2 *a*).

Thus, $\cos PM = \cos PZ \cdot \cos ZM + \sin PZ \cdot \sin ZM \cdot \cos A$

$= \cos 50° \cdot \cos 67° 24' + \sin 50° \cdot \sin 67° 24' \cdot \cos 42°$

$= 0.24702 + 0.52556 = 0.77258$

$\therefore$ $PM = 39° 25'$

$\therefore$ Declination of the star = δ = 90° − PM = 90° − 39° 25′ = **50° 35′ *N*.**

Similarly, knowing all the three sides, the hour angle H can be calculated from Eq. 13.2.

$$\cos H = \frac{\cos ZM - \cos PZ \cdot \cos PM}{\sin PZ \cdot \sin PM} = \frac{\cos 67° 24' - \cos 50° \cdot \cos 39° 25'}{\sin 50° \cdot \sin 39° 25'}$$

$$= \frac{0.38430 - 0.49659}{0.48640} = -0.23086$$

$\therefore$ $\cos (180° - H) = 0.23086$ $\therefore$ $180° - H = 76° 39'$

$\therefore$ H = **103° 21′.**

Example 13.13. *Determine the hour angle and declination of a star from the following data :*

(1) Altitude of the star = 21° 30′

(2) Azimuth of the star = 140° E

(3) Latitude of the observer = 48° N.

Solution

Refer Fig. 13.27. Since the azimuth of the star is 140° *E*, it is in eastern hemi-sphere.

In the astronomical triangle ZPM, we have

$ZM = 90° - \alpha = 90° - 21° 30' = 68° 30'$; $ZP = 90° - \theta = 90° - 48° = 42°$; $A = 140°$

Knowing the two sides and the included angle, the third side can be calculated by the cosine rule (Eq. 13.2 *a*).

Thus $\cos PM = \cos ZM \cos ZP + \sin ZM \sin ZP \cos A$

$= \cos 68° 30' \cos 42° + \sin 68° 30' \sin 42° \cos 140°$

$= 0.27236 - 0.47691 = -0.20455$

$\therefore$ $\cos (180° - PM) = 0.20455$ or $180° - PM = 78° 12'$

$\therefore$ $PM = 101° 48'$

$\therefore$ Declination of the star = 90° − 101° 48′ = − 11° 48′= − **11° 48′** S

Again, knowing all the three sides, the angle H_1 can be calculated from the cosine formula, (Eq. 13.2). Thus

$$\cos H_1 = \frac{\cos MZ - \cos ZP \cdot \cos MP}{\sin ZP \sin MP} = \frac{\cos 68° 30' - \cos 42° \cos 101° 48'}{\sin 42° \sin 101° 48'}$$

$$= \frac{0.36650 + 0.15198}{0.65499} = 0.79161 \quad \therefore \quad H_1 = 37° 40'$$

But H_1 is the angle measured in the eastward direction.

$\therefore$ Hour angle of the star = 360° − H_1 = 360° − 37° 40′ = **322° 20′.**

Example 13.14. *Calculate the sun's azimuth and hour angle at sunset at a place in latitude 42° 30′ N, when its declination is (a) 22°12′ N and (b) 22° 12′ S.*

Solution

Let us consider the astronomical triangle ZPM, where M is the position of the sun. Since the sun is on the horizon at its setting, it altitude is zero, and hence ZM = 90°.

Also, $ZP = 90° - 42° 30' = 47° 30'$

(*a*) $PM = 90° - 22° 12' = 67° 48'$

From the triangle ZPM, we get by cosine rule

$\cos PM = \cos ZP \cdot \cos ZM + \sin ZP \cdot \sin ZM \cdot \cos A$

But $\cos ZM = \cos 90° = 0$ and $\sin ZM = \sin 90° = 1$

$\therefore$ $\cos A = \dfrac{\cos PM}{\sin ZP} = \dfrac{\cos 67° 48'}{\sin 47° 30'}$ Hence $A = 59° 10'$

Hence azimuth of the sun at setting = **59° 10′ West.**

Again, from the cosine rule, we get

$\cos ZM = \cos ZP \cdot \cos PM + \sin ZP \cdot \sin PM \cdot \cos H$

But $\cos ZM = \cos 90° = 0$

Hence $\cos H = -\cot ZP \cdot \cot PM = -\cot 47° 30' \cot 67° 48'$

or $\cos (180° - H) = +\cot 47° 30' \cot 67° 48'$

$\therefore$ $180° - H = 68° 03'$ or $H = 180° - 68° 03' = 111° 57'$

Hence sun's hour angle at sunset = 111° 57′ = $7^h 27^m 48^s$.

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the sun's declination changes from north to south. The points at which sun's declinations are a maximum are called *solstices*. The point at which the north declination of sun is maximum is called the *summer solstice*, while the point at which the south declination of the sun is maximum is known as the *winter solstice*.

The Earth's Orbital Motion Round the Sun — The Seasons

The earth moves *eastward* around the sun once in a year in a path that is *very nearly* a huge circle with a radius of about 93 millions of miles. *More accurately*, the path is described as an *ellipse*, one focus of the ellipse being occupied by the sun. The

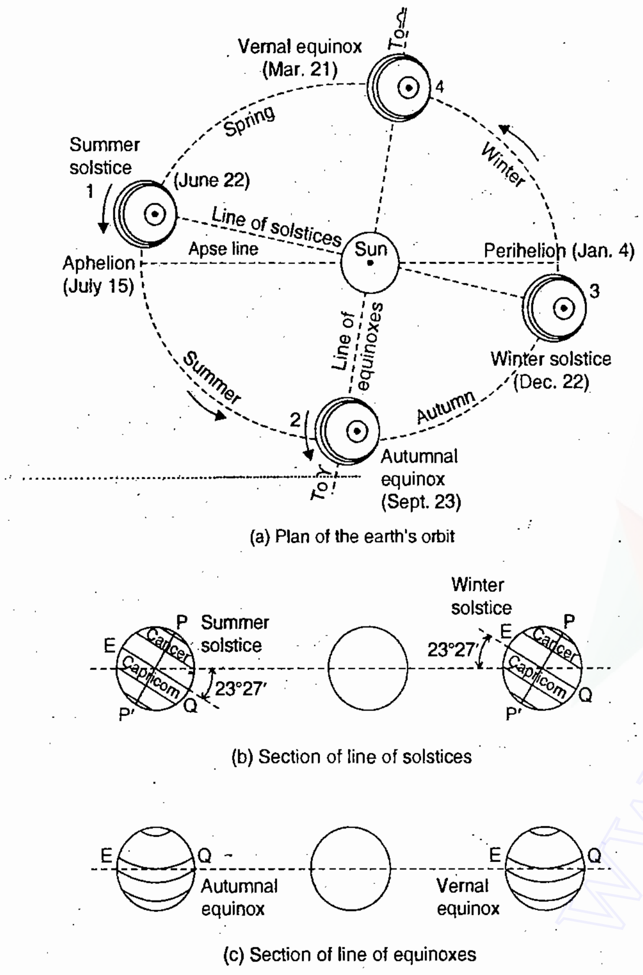

FIG. 13.28. EFFECT OF EARTH'S ANNUAL MOTION.

earth is thus at varying distances from the sun. The orbit lies (very nearly) in one *plane*. The apparent path of the sun is in the same plane. The plane passes through the centre of the celestial sphere and intersects it in a great circle called the *ecliptic*. The plane of the ecliptic is inclined at about 23° 27′ to that of the equator. Hence, the *axis of the earth is inclined to the plane of the ecliptic at an angle of 66° 33′, and remains practically parallel to itself throughout the year*. The inclination of the axis of the earth round its orbit causes variations of seasons. Fig. 13.28 shows the diagrammatic plan and sections of earth's orbit.

As previously mentioned, the earth's orbit is an ellipse with the sun at one of its foci. The earth is thus at varying distances from the sun. The earth is at a point nearest the sun (called the *perihelion* of the earth's orbit) on about January 4 and at a point farthest from the sun (called the *aphelion* of the earth's orbit) on about July 5. The earth's rate of angular movement around the sun is greatest at perihelion and least at aphelion.

In position I, the earth is in that part of the orbit where the northern end of the axis is pointed towards the sun. The sun appears to be farthest north on about June 22, and at this time the days are longest and nights are shortest. The summer begins in the northern hemisphere. This position of the earth is known as the summer *solstice*. In position 2 (Sept. 23), the sun is in the plane of the equator. The nights are equal everywhere. The instant at which this occurs is called the *Autumnal Equinox*. The axis of the earth is perpendicular to the line joining the earth and the sun. In position 3, the earth is in that part of the orbit where the northern end of axis is pointed away from the sun. The sun appears to be farthest south (Dec. 22) and at this time winter begins in the northern hemisphere. The days are shortest and nights are longest. The position of the earth is known as the *winter solstice*. In position 4 (March 21), the sun is again in the plane of the equator. The day and night are equal everywhere. The instant at which this occurs is called the *Vernal Equinox*. The line of the equinoxes is the intersection of the planes of the ecliptic and the equator, and is at right angles to the line of solstices.

Fig. 13.29 (*b*) shows the sun's apparent positions at different seasons. Let us study this in conjunction with Fig. 13.29 (*a*). Thus, on Fig. 13.29 (*a*), we shall trace the annual motion of the sun, while on Fig. 13.29 (*b*), we shall trace the apparent diurnal paths of the sun at different seasons. As is clear from Fig. 13.29 (*a*), the sun's declination changes daily as it progresses along the ecliptic. Due to the change in the declination, its apparent path of each day is different from that of the day before. *The apparent path thus ceases to be circular and all the daily paths taken together will give rise to one continous spiral curve*. However, for explanation purposes, we shall assume that throughout each day, the sun's declination is constant – retaining the same value it has at sunrise. On this assumption the sun's daily paths will consist of a series of parallels instead of a spiral as illustrated in Fig. 13.29 (*b*).

On 21st March, the sun is at ♈ [Fig. 13.29 (*a*)] and its declination is zero. The sun's daily path on this day will be along the equator rising at E and setting at W of the horizon. Its hour angle at E will be $EPZ = 90°$ when it rises. At W, it will again have an hour angle of 90° when it sets. *Thus, day and night will be of equal duration*. On that day, the meridian altitude SB of the sun is equal to the co-latitude. As the sun

It is, therefore, a measure of the angle through which the earth has rotated since the equinox was on the meridian. *The local sidereal time is, thus, equal to the right ascension of the observer's meridian.*

Since the sidereal time is the hour angle of the firsr point of Aries, *the hour angle of a star is the sidereal time that has elapsed since its transit.* In Fig 13.30, M_1 is the position of a star having SPM_1 ($= H$) as its hour angle measured *westward* and ΥPM_1 is its right ascension (R.A.) measured *eastward*. $SP\Upsilon$ is the hour angle of Υ and hence the local sidereal time.

Hence, we have $$SPM_1 + M_1P\Upsilon = SP\Upsilon$$

or ***star's hour angle + star's right ascension = local sidereal time*** ...(1)

If this sum is greater than 24 hours, deduct 24 hours, while if it is negative *add*, 24 hours,

In Fig. 13.30 (*b*), the star M_2 is in the other position. ΥPM_2 is its Right Ascension (eastward) and ZPM_2 is its hour angle (westward). Evidently,

$$ZPM_2 \text{ (exterior)} + \Upsilon PM_2 - 24^h = SP\Upsilon = \text{L . S . T .}$$

or ***star's hour angle + star's right ascension*** $- 24^h =$ L . S . T .

This supports the preposition proved with reference to Fig. 13.30 (*a*). The relationship is true for all positions of the star.

When the star is on the meridian, its hour angle is zero. Hence equation 1 reduces to

Star's right ascesion = local sidereal time at its transit.

A sidereal clock, therefore, records the right ascension of stars as they make their upper transits.

The hour angle and the right ascension are generally measured in *time* in preference to angular units. Since one complete rotation of celestial sphere through 360° occupies 24 hours, we have

$$24 \text{ hours} = 360° \quad ; \quad 1 \text{ hour} = 15°$$

The difference between the local sidereal times of two places is evidently equal to the difference in their longitudes.

2. Solar Apparent Time

Since a man regulates his time with the recurrence of light and darkness due to rising and setting of the sun, the sidereal division of time is not suited to the needs of every day life, for the purposes of which the sun is the most convenient time measurer. A *solar day* is the interval of time that elapes between two successive *lower* transits of the sun's centres over the meridian of the place. The lower transit is chosen in order that the date may change at mid-night. The solar time at any instant is the hour angle of the sun's centre reckoned westward from 0^h to 24^h. This is called the *apparent solar time*, and is the time indicated by a sun-dial. Unfortunately, the apparent solar day is not of constant length throughout the year but changes. Hence our modern clocks and chronometers cannot be used to give us the apparent solar time. The non-uniform length of the day is due to two reasons :

(1) The orbit of the earth round the sun is not circular but elliptical with sun at one of its foci. The distance of the earth from the sun is thus variable. In accordance with the law of gravitation, the apparent angular motion of the sun is not uniform – it moves faster when is nearer to the earth and slower when away. Due to this, the sun reaches the meridian sometimes earlier and sometimes later with the result that the days are of different lengths at different seasons.

(2) The apparent diurnal path of the sun lies in the ecliptic. Due to this, even though the eastward progress of the sun in the ecliptic were uniform, the time elapsing between the departure of a meridian from the sun and its return thereto would vary because of the obliquity of the ecliptic.

The sun changes its right ascension from 0^h to 24^h in one year, advancing eastward among the stars at the rate of about 1° a day. Due to this, the earth will have to turn nearly 361° about its axis to complete one solar day, which will consequently be about 4 minutes longer than a sidereal day. Both the obliquity of the ecliptic and the sun's unequal motion cause a *variable rate of increase* of the sun's right ascension. If the rate of change of the sun's right ascension were uniform, the solar day would be of constant length throughout the year.

3. Mean Solar Time

Since our modern clocks and chronometers cannot record the variable apparent solar time, a *fictitious sun* called the *mean sun* is imagined to move at a uniform rate along the equator. The motion of the mean sun is the average of that of the true sun in its right ascension. It is supposed to start from the vernal equinox at the same time as the true sun and to return the vernal equinox with the true sun. The *mean solar day* may be defined as the interval between successive transit of the mean sun. The mean solar day is the average of all the apparent solar days of the year. The mean sun has the constant rate of increase of right ascension which is the average rate of increase of the true sun's right ascension.

The *local mean noon* (L.M.N.) is the instant when the mean sun is on the meridian. The mean time at any other instant is given by the hour angle of the mean sun reckoned westward from 0 to 24 hours. For civil purposes, however, it is found more convenient to begin the day at midnight and complete it at the next midnight, dividing it into two periods of 12 hours each. Thus, the zero hour of the mean day is at the *local mean midnight* (L.M.M.). The *local mean time* (L.M.T.) is that reckoned from the local mean midnight. The difference between the local mean time between two places is evidently equal to the difference in the longitudes.

From Fig. 13.30 (*a*) if M_1 is the position of the sun, we have

Local sidereal time = R.A. of the sun + hour angle of the sun ...(1)

Similarly, Local sidereal time = R.A. of the mean sun + hour angle of the mean sun ...(2)

The hour angle of the sun is zero at its upper transit. Hence

Local sidereal time of apparent noon = R.A. of the sun ...(3)

Local sidereal time of mean noon = R.A. of the mean sun ...(4)

B and *C* will coincide only at equinoxes and solstices. Between the equinox to solstice, *C* will be in advance of *B*, and any given meridian will (as the earth rotates in the direction of the arrow) overtake first the true sun *A* and then the mean sun. *That is, apparent noon will precede mean noon and hence the equation of time will be additive.* Similarly, between the solstice to equinox, *C* will be behind *A* and the equation of time is subtractive. In Fig. 13.33, the curve *A-A* denotes the equation of time due to the obliquity of ecliptic. *It may be noted that the equation of time due to this reason vanishes four times in a year — at equinoxes and solstices.* Fig. 13.31 (*b*) shows the plan, on equatorial plane, of the positions of the true and mean sun at different parts of the year.

Thus, to conclude, *the equation of time due to obliquity of the ecliptic is due to the fact that the uniform motion along the ecliptic does not represent uniform motion in the right ascension.*

2. Ellipticity or the Eccentricity of the Orbit

Let us now neglect the obliquity of ecliptic so that the orbit of the sun is in the equator, and its apparent path is elliptical as shown in Fig. 13.32. At the Perihelion (December 31), the true sun (*A*) and the mean sun (*C*) start at the same instant. The mean sun (*C*) rotates with uniform rate while the true sun (*A*) moves with the greater angular velocity since it is nearer the earth at Perihelion. Due to this, the true sun precedes the mean sun. Now, since the earth rotates from west to east (*i.e.*, in the same direction as that of the motion of the sun along its orbit indicated by the arrow), any meridian at a place on it will overtake the mean sun before the true sun. The mean noon will thus occur before the apparent noon, the mean time will exceed the apparent time and hence the *equation of time will be negative*. After 90° from the Perihelion, the true sun, though ahead of the mean sun, will have decrease in its angular velocity so that the distance between the sun and the mean sun goes on decreasing. At the Aphelion (July 1), both the suns meet and the equation of time becomes zero. Between December 31 to July 1, equation of time thus remains negative. After July 1, the true sun has lesser angular velocity than the uniform velocity of the mean sun, and the mean sun precedes the true sun. The apparent noon will thus occur earlier than the mean noon at a particular meridian, the apparent time exceeds the mean time, and the *equation of time becomes positive*. After about 90° (October 1) from the Aphelion, the gap between the mean sun and true sun gradually reduces due to gradual increase in the angular velocity of the true sun, till both the suns reach perihelion at the same

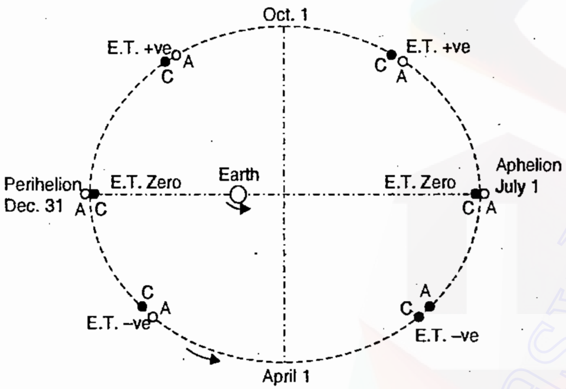

FIG. 13.32. EFFECT OF ELLIPTICITY OF THE ORBIT.

instant. The equation of time is thus positive from July 1 to December 31. In Fig. 13.33, the curve *B-B* denotes the equation of time due to ellipticity of the orbit.

The Final Curve for Equation of Time

In Fig. 13.33, the curve *C-C* shows the final equation of time obtained by combining the curves *A-A* and *B-B*. It will be seen that the equation of time vanishes four times a year, on or about April 16, June 14, September 2, and December 25. From December 25 till April 16, it is negative having a maximum value of about $14^m\,20^s$ on February 12. From April 16 to June 14 it is positive, having its maximum value of about $3^m\,44^s$ on May 15. From June 14 to September 2, it is again negative with a maximum value of $6^m\,24^s$ on July 27. Between September 2 and December 25, it is again positive, attaining its greatest positive value for the year 1951, about $16^m\,23^s$ on November 4.

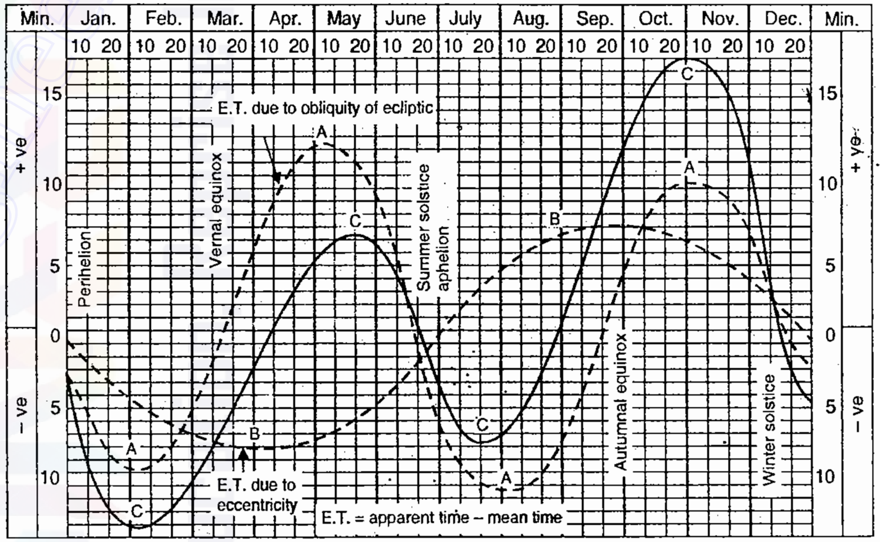

FIG. 13.33. THE EQUATION OF TIME : THE CORRECTION TO BE ADDED TO THE MEAN TIME TO OBTAIN APPARENT TIME.

4. Standard Time

We have seen that the local mean time at a particular place is reckoned from the lower transit of the mean sun. Thus, at different meridians there will be different local mean times. In order to avoid confusion arising from the use of different local mean time it is necessary to adopt the mean times on a particular meridian as the standard time for the whole of the country. Such a *standard meridian* lies an exact number of hours from Greenwich. The mean time associated with the standard meridian is known as the *standard time*. The difference between standard time and local mean time at any place is that due to the difference of longitude between the given place and the standard meridian used. For places east of the standard meridian, local mean time is later (or greater) than

$$L.M.T. = \text{Standard M.T.} \pm \text{Difference in the longitudes} \left(\frac{E}{W}\right) \quad ...(1)$$

$$L.A.T. = \text{Standard A.T.} \pm \text{Difference in the longitudes} \left(\frac{E}{W}\right) \quad ...(2)$$

$$L.S.T. = \text{Standard S.T.} \pm \text{Difference in the longitudes} \left(\frac{E}{W}\right) \quad ...(3)$$

Use (+) sign if the meridian of place is to the east of the standard meridian, and (−) sign if it to the west of the standard meridian.

If the local time is to be found from the given Greenwich time, we have

$$\textit{L.M.T. = G.M.T.} \pm \textit{Longitude of the place} \left(\frac{E}{W}\right)$$

Example 13.20. *The standard time meridian in India is 82° 30′ E. If the standard time at any instant is 20 hours 24 minutes 6 seconds, find the local mean time for two places having longitudes (a) 20° E, (b) 20° W.*

Solution

(*a*) The longitude of the place = 20° *E*

Longitude of the standard meridian = 82° 30′ *E*

∴ Difference in the longitudes = 82° 30′ − 20° = 62° 30′, *the place being to the west of the standard meridian.*

Now 62° of longitude $= \frac{62°}{15}$ h $= 4^h\ 8^m\ 0^s$

30′ of longitude $= \frac{30}{15}$ m $= 0^h\ 2^m\ 0^s$

Total $= 4^h\ 10^m\ 0^s$

Now L.M.T. = Standard time − Difference in longitude (*W*)

$= 20^h\ 24^m\ 6^s - 4^h\ 10^m\ 0^s = 16^h\ 14^m\ 6^s$ past midnight $= 4^h\ 14^m\ 6^s$ P.M.

(*b*) Longitude of the place = 20° *W*

Longitude of the standard meridian = 82° 30′ *E*.

∴ Difference in the longitude = 20° + 82° 30′ = 102° 30′, the meridian of the place being to the west to the standard meridian.

Now 102° of longitude $= \frac{102}{15}$ h $= 6^h\ 48^m\ 0^s$

30′ of longitude $= \frac{30}{15}$ m $= 0^h\ 2^m\ 0^s$

Total $= 6^h\ 50^m\ 0^s$

Standard time $= 20^h\ 24^m\ 6^s$

Subtract the difference in longitude $= 6^h\ 50^m\ 0^s$

∴ Local mean time $= 13^h\ 34^m\ 6^s$ past mid-night $= 1^h\ 34^m\ 6^s$ P. M.

Example 13.21. *Find the G.M.T. corresponding to the following L.M.T.*

(a) $9^h\ 40^m\ 12^s$ A.M. at a place in longitude 42° 36′ W.

(b) $4^h\ 32^m\ 10^s$ A.M. at a place in longitude 56° 32′ E.

Solution.

(*a*) Longitude of the place is 42° 36′ *W*

Now $42° = \frac{42}{15}$ h $= 2^h\ 48^m\ 0^s$

$36' = \frac{36}{15}$ m $= 0^h\ 2^m\ 24^s$

Total $= 2^h\ 50^m\ 24^s$

Now since the place is to the west of Greenwich, the Greenwich time will be more.

∴ G.M.T. = L.M.T. + Longitude (*W*)

L.M.T. $= 9^h\ 40^m\ 12^s$ (A.M.)

Add the longitude $= 2^h\ 50^m\ 24^s$

∴ G.M.T. $= 12^h\ 30^m\ 36^s$

or G.M.T. $= 0^h\ 30^m\ 36^s$ (P.M.)

(*b*) Longitude of the place = 56° 32′ *E*

Now $56° = \frac{56}{15}$ h $= 3^h\ 44^m\ 0^s$

$32' = \frac{32}{15}$ m $= 0^h\ 2^m\ 8^s$

Total $= 3^h\ 46^m\ 8^s$

Since the place is to the east of Greenwich, the Greenwich time will be lesser than the local time.

∴ G.M.T. = L.M.T. − Longitude (*E*)

L.M.T. $= 4^h\ 32^m\ 10^s$ (A.M.)

Subtract longitude $= 3^h\ 46^m\ 8^s$

G.M.T. $= 0^h\ 46^m\ 2^s$ (A.M.)

Example 13.22. *Given the Greenwich civil time (G.C.T.) as $6^h\ 40^m\ 12^s$ P.M. on July 2, 1965, find the L.M.T. at the places having the longitudes (a) 72° 30′ E, (b) 72° 30′ W, and (c) 110° 32′ 30″ E.*

$$\text{E.T. at G.M.N.} = 5^m\ 10.65^s$$

$$\text{Time interval after G.M.N.} = 4^h\ 34^m\ 40^s = 4.578^h$$

(The above time interval is approximate, since it has been calculated by subtracting G.M.N. from the G.A.T. while actually the G.M.N. should be subtracted from G.M.T. which is not known at present).

$\therefore$ Increase for 4.578^h @ 0.22^s per hour $= (4.578 \times 0.22)^s = 1.01^s$

$\therefore$ E.T. at observation $= 5^m\ 10.65^s + 1.01^s = 5^m\ 11.66^s$

Now G.A.T. of observation $= 16^h\ 34^m\ 40^s$

Add E.T. $= 0^h\ 5^m\ 11.66^s$

G.M.T. of observation $= 16^h\ 39^m\ 51.66^s$

Deduct longitude in time $= 1^h\ 22^m\ 0^s$

L.M.T. of observation $= 15^h\ 17^m\ 51.66^s$

13.9.3. CONVERSION OF MEAN TIME INTERVAL TO SIDEREAL TIME INTERVAL AND VICE VERSA

The tropical year: A year is the period of earth's revolution about the sun, from some determinate position back again to the same position. The reference point chosen for the use of man is the first point of Aries (♈). The year so chosen is the *tropical year* or *the solar year*. A *Sidereal year* is the time taken by the earth in making one complete revolution round the sun with reference to a fixed star.

The first point of Aries has a retrograde motion westwards through an arc of 50.22″ per year. The retrograde motion of the first point of Aries is due to the attraction of the moon and the sun which causes the direction of the axis of the earth alter its position very gradually in such a way that earth arrives at the position of the vernal equinox a little earlier each year. This phenomenon is known as the *Precession of Equinoxes*. Due to the precession of Equinoxes, therefore, the earth does not revolve by 360° round the sun from the positions of vernal equinox to vernal equinox, but revolves through (360° – 50″.22)

The sun advances among the stars in the same direction — west to east — as the earth revolves about the axis. Any given meridian, therefore, crosses the first point of Aries exactly once oftener than it does the sun, in the course of a tropical year. According to Bassel, there are 365.2422 mean solar days in a tropical year, and in the same period there are 366.2422 sidereal days.

Thus, we have the relation

365.2422 mean solar day = 366.2422 sidereal days

or $\quad 1 \text{ mean solar day} = 1 + \dfrac{1}{365.2422} \text{ sidereal days} = 24^h\ 3^m\ 56.56^s \text{ sidereal time}$ ...(I)

Thus, the mean solar day is $3^m\ 56.56^s$ longer than the sidereal day.

Hence 1 hour mean solar time $= 1^h + 9.8565^s$ sidereal time

1 minute mean solar time $= 1^m + 0.1642^s$ sidereal time

1 second mean solar time $= 1^s + 0.0027^s$ sidereal time

Thus, to convert the mean solar time to the sidereal time, we will have to add a correction of 9.8565^s per hour of mean time. This correction is called the *acceleration*.

To get the concept how a mean solar day is of a longer time interval than the sidereal time, let us study Fig. 13.34.

Let C be the centre of the earth and O be the position of the observer at noon of its meridian at the date of the equinox. Let C_1 be the position of the earth's centre the next day. After the earth makes one complete rotation (with reference to ♈), the observer will be at O_1 and *the sidereal time will be the same as it was the day before when he was at O*. However, the solar day is the time interval between two successive transits of the centre of the sun over the meridian. *In order that the sun transits the observer's meridian, the earth will have to revolve additionally by the arc O_1O'*. The time taken for this additional rotation is 3 minutes 56.66 seconds.

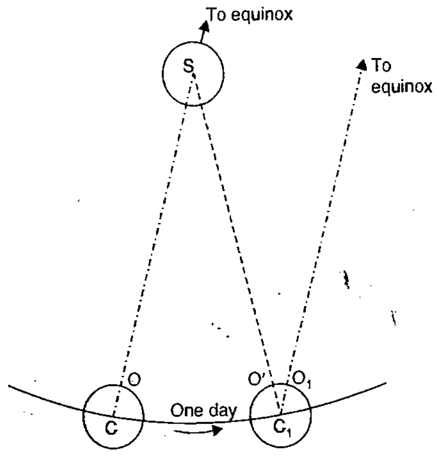

FIG. 13.34

Thus, we have

366.2422 sidereal days = 365.2422 solar days.

To convert sidereal time into mean time, we have

$$1 \text{ sidereal day} = \frac{365.2422}{366.2422} \text{ mean solar day} = 1 - \frac{1}{366.2422} \text{ mean solar day}$$

or $\quad 1$ sidereal day $= 23^h\ 56^m\ 4.09^s$ mean solar time

$\therefore \quad 1^h$ sidereal time $= 1^h - 9.8296^s$ mean solar time

1^m sidereal time $= 1^m - 0.1638^s$ mean solar time

1^s sidereal time $= 1^s - 0.0027^s$ mean solar time

Thus, *to convert 1 hour sidereal time to the mean solar time, a correction of 9.8296 seconds per hour will have to be subtracted from the sidereal time.* This correction is called the *retardation*.

Example 13.25. *Convert 4 hours 20 minutes 30 seconds of mean solar time into equivalent interval of sidereal time.*

Solution.

To convert the mean solar time to the sidereal time, we will have to first calculate the acceleration at the rate of 9.8565^s per hour of mean time.

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13.9.5. GIVEN THE LOCAL MEAN TIME AT ANY INSTANT, TO DETERMINE THE LOCAL SIDEREAL TIME

At a given meridian, let us have two clocks, one showing the mean time and the other the sidereal time. At the local mean mid-night, the mean time in the mean clock will be zero. At that time (*i.e.* L.M.M.) the L.S.T. can easily be computed if the G.S.T. at G.M.M. is known. If the place is to the west of the Greenwich, the sidereal clock will have a gain over the mean time at L.M.M. at the rate of 9.8565 seconds per hour, as discussed in § 13.9.4 above. At any other instant at the given meridian, the mean clock will show the time that has elapsed since the lower transit of the sun over the meridian. This mean time interval can be easily converted into sidereal time interval as discussed in § 13.9.3 above. Thus, the L.S.T. at L.M.T. will be equal to L.S.T. at L.M.M. plus the sidereal time interval. Hence *the rules for finding the L.S.T. at L.M.T. are:*

(*a*) From the given G.S.T. at G.M.M., calculate L.S.T. at L.M.M.

(*b*) Convert the given L.M.T. (or mean time interval) into sidereal time interval since L.M.M.

(*c*) L.S.T. at L.M.T.= L.S.T. at L.M.M. + S.I. from L.M.M.

Example 13.28. *Find the L.S.T. at place in longitude 85° 20′ E at $6^h\,30^m$ P.M., G.S.T. at G.M.N. being $6^h\,32^m\,12^s$.*

Solution.

Longitude = 85° 20′ *E*

	h	m	s
$85° = \frac{85}{15}$ h =	5	40	0
$20' = \frac{20}{15}$ m =	0	1	20
∴ Longitude in hours =	5	41	20 *E*

Since the place is to the east of Greenwich, let us calculate the loss of sidereal time for $5^h\,41^m\,20^s$ of longitude.

$5^h \times 9.8565^s = 49.283$ seconds

$41^m \times 0.1642^s = 6.732$ seconds

$20^s \times 0.0027^s = 0.054$ second

Total = 56.069 seconds

L.S.T. at L.M.N. = G.S.T. at G.M.N. – retardation

$= 6^h\,32^m\,12^s - 56.069^s = 6^h\,31^m\,15.931^s$ ...(1)

Now, L.M.T. = $6^h\,30^m$ P.M.

∴ M.T. interval from L.M.N. = $6^h\,30^m$

Let us convert it into sidereal time interval by adding the acceleration to the mean time interval.

Thus, $6^h \times 9.8565^s = 59.139$ seconds

$30^m \times 0.1642^s = 4.926$ seconds

Total acceleration $= 64.065^s = 1^m\,4.065^s$

∴ Sidereal Time Interval = Mean time interval + acceleration since L.M.N.

$= 6^h\,30^m + 1^m\,4.065^s = 6^h\,31^m\,4.065^s$

Now L.S.T. at L.M.N. $= 6^h\,31^m\,15.931^s$

Add S.I. since L.M.N. $= 6^h\,31^m\,4.065^s$

∴ L.S.T. at L.M.T. $= 13^h\,02^m\,19.996^s = 1^h\,02^m\,19.996^s$ P.M.

13.9.6. GIVEN THE LOCAL SIDEREAL TIME, TO DETERMINE THE LOCAL MEAN TIME

If the G.S.T. at G.M.M. is given, the L.S.T. at L.M.M. can be calculated as discussed earlier. The L.S.T. at L.M.M. can then be subtracted from L.S.T. to get the number of sidereal hours, minutes and seconds past midnight. This sidereal time interval can then converted into the mean time interval by subtracting the retardation at the rate of 9.8296^s per hour of S.I. thus obtaining the L.M.T. *The rules are, therefore :*

(*a*) Find the L.S.T. at L.M.M. from the known G.S.T. at G.M.M.

(*b*) Subtract L.S.T. at L.M.M. from the L.S.T. at get the S.I.

(*c*) Convert the S.I. into mean time interval, thus getting L.M.T.

Example 13.29. *The local sidereal time at a place (Longitude 112° 20′ 15″ W) is $18^h\,28^m\,12^s$.*

Calculate the corresponding L.M.T. given that G.S.T. at G.M.M. is $8^h\,10^m\,28^s$ on that day.

Solution :Let us first convert the longitude into time units :

	h	m	s
$112° = \frac{112}{15}$ h =	7	28	0
$20' = \frac{20}{15}$ m =	0	1	20
$15'' = \frac{15}{15}$ s =	0	0	1
∴ Longitude =	7	29	21

Since the place has west longitude,

L.S.T. at L.M.M.= G.S.T. at G.M.M. + acceleration.

Let us calculate the acceleration at the rate of 9.8565^s per hour.

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(*c*) Convert this sidereal interval into mean time interval by subtracting the retardation at the rate of 9.8296^s per hour of sidereal interval.

(*d*) The mean time interval obtained in (*c*) is thus the G.M.T. at the instant under consideration. Compute the L.M.T. by allowing for the difference of longitude.

We shall work out example 13.29 by the alternative method.

Example 13.31. *Solve example 13.29 by the alternative method.*

Solution

Longitude $= 112° 20' 15'' W = 7^h 29^m 21^s W$

	h	m	s
L.S.T.	= 18	28	12
Add longitude	= 7	29	21
∴ G.S.T. at the instant	= 25	57	33
G.S.T. at G.M.M.	= 8	10	28
∴ S.I. since G.M.M.	= 17	47	05

Let us now convert this S.I. in mean time interval by subtracting the retardation.

$17^h \times 9.8296 = 167.103$ seconds

$47^m \times 0.1638 = 7.699$ seconds

$5^s \times 0.0027 = 0.014$ seconds

Total retardation $= 174.816$ seconds $= 2^m\ 54.816^s$

∴ Mean time interval = S.I. – retardation

$= 17^h\ 47^m\ 05^m - 2^m\ 54.816^s$

∴ G.M.T. $= 17^h\ 44^m\ 10.184^s$

Subtract longitude $= 7^h\ 29^m\ 21^s$

L.M.T. $= 10^h\ 4^m\ 49.184^s$

13.9.9. TO DETERMINE THE L.M.T. OF TRANSIT OF A KNOWN STAR ACROSS THE MERIDIAN, GIVEN G.S.T. OF G.M.N.

We have already seen that when a star transits or culminates across the meridian, the R.A. of the star, expressed in time, is the sidereal time. In the Nautical Almanac, the astronomical co-ordinates of all the stars in terms of Right Ascension and declination are given. Thus, knowing the R.A., the L.S.T. at the time of transit of the star is known. The problem is now to convert the L.S.T. into the L.M.T. by the method described in § 13.9.6 or in §13.9.8. The following are the steps :

(*a*) Find the R.A. of the star from the N.A. This is then the L.S.T. at the time of the transit of the star.

(*b*) From the known value of G.S.T. of G.M.M. or (G.M.N.), calculate the L.S.T. of L.M.M. (or L.M.N.).

(*c*) Subtract this L.S.T. of L.M.M. from the L.S.T. of the transit of the star to get the S.I. that has elapsed since L.M.M.

(*d*) Convert this S.I. to mean time interval which, then, gives the L.M.T. at the transit of the star.

Example 13.32. *What will be the L.M.T.'s of upper and following lower transit at a place in longitude 162° 30′ 15″ W of a star whose R.A. is $22^h\ 11^m\ 30^s$, if the G.S.T. of previous G.M.N. is $10^h\ 30^m\ 15^s$.*

Solution.

		h	m	s
Longitude :	$162° = \frac{162}{15}$ h =	10	48	0
	$30' = \frac{30}{15}$ m =	0	2	0
	$15'' = \frac{15}{15}$ s =	0	0	1
		10	50	1

Since the place is to the west, we will have to add the acceleration to get the L.S.T. at L.M.N.

$10^h \times 9.8565^s = 98.565$ seconds

$50^m \times 0.1642^s = 8.210$ seconds

$1^s \times 0.0027^s = 0.003$ second

Total acceleration $= 106.778$ seconds $= 1^m\ 46.778^s$

	h	m	s
G.S.T. of G.M.N.	= 10^h	30^m	1.5^s
Add acceleration	=	1	46.778
∴ L.S.T. of L.M.N.	= 10	32	01.778
Now R.A. of star = L.S.T.	= 22	11	30
Subtract L.S.T. of L.M.N.	= 10	32	1.778
∴ S.I. since L.M.N.	= 11	39	28.222

Let us now convert this S.I. into mean time interval by subtracting retardation.

Example 13.34. *The G.M.T. of transit of the first point of Aries (♈) on March 2 is $13^h\ 21^m\ 54^s$. Find the L.M.T. of transit of the first point of Aries on the same day at a place (a) Longitude 40° 30′E (b) 40° 30′ W.*

Solution

Longitude = 40° 30′ *E*

	h	m	s
$40° = \frac{40}{15}$ h =	2	40	0
$30' = \frac{30}{15}$ m =	0	2	0
=	2	42	0

Gain of sidereal clock at the rate of 9.8296^s per hour of longitude :

$2^h \times 9.8296^s = 19.659$ seconds

$42 \times 0.1638^s = 6.880$ seconds

Total ∴ = 26.539 seconds

	h	m	s
(a) G.M.T. of transit of ♈ =	13	21	54
Add the correction for eastern longitude =	0	0	26.539
∴ L.M.T. of transit of ♈ =	13^h	22^m	20.539^s

	h	m	s
(b) G.M.T. of transit of ♈ =	13	21	54
Subtract the correction for the western longitude =	0	0	26.539
∴ L.M.T. of transit of ♈ =	13	21	27.461

13.9.11. GIVEN THE L.S.T. AT ANY PLACE, TO DETERMINE THE CORRESPONDING L.M.T. IF THE G.M.T. OF TRANSIT OF THE FIRST POINT OF ARIES ON THE SAME DAY IS ALSO GIVEN

We know that L.S.T. at any instant is the time interval that has elapsed since the transit of ♈ on the meridian. This L.S.T. can be converted into equivalent number of mean hours by subtracting the retardation at the rate of 9.8296^s per sidereal hour. Also, from the known G.M.T. of transit of ♈, the L.M.T. of transit of ♈ can be calculated. This L.M.T. is nothing but the time shown by the mean clock when the sidereal clock shows 0^h. Therefore, the L.M.T. at the instant under consideration can be obtained by

adding the mean hours (corresponding to the given L.S.T.) to the L.M.T. at the time of transit of ♈. *The steps therefore are* :

(1) From the known G.M.T. of transit ♈, calculate the L.M.T. of transit of ♈ by method discussed in §13.9.10.

(2) Convert the given L.S.T. to mean hours.

(3) Add (1) and (2) to get the L.M.T. corresponding to the given L.S.T.

Example 13.35. *The local sidereal time at a place (longitude 50° 30′E) on 17th May, 1948 is $11^h\ 30^m\ 12^s$. Find the corresponding L.M.T. given that the G.M.T. of transit of ♈ on the 17th May, 1948 is $7^h\ 12^m\ 28^s$.*

Solution

Longitude = 50° 30′ *E*

	h	m	s
$50° = \frac{50}{15}$ h =	3	20	0
$30' = \frac{30}{15}$ m =	0	2	0
Total =	3	22	0

The correction at the rate 9.8296 per hour of longitude is

$3^h \times 9.8296 = 29.489$ seconds

$22^m \times 0.1638 = 3.604$ seconds

Total correction = 33.093 seconds

	h	m	s
G.M.T. at transit of ♈ =	7^h	12^m	28^s
Add the correction =	0	0	33.093
∴ L.M.T. at transit at ♈ =	7	13	1.093 ...(1)

L.M.T.= $11^h\ 30^m\ 12^s$, and may be converted to mean hours by subtracting the retardation.

$11^h \times 9.8296 = 108.126$ seconds

$30^m \times 0.1638 = 4.914$ seconds

$12^s \times 0.0027 = 0.032$ seconds

Total retardation = 113.072 seconds= $1^m\ 53.072^s$

Mean hours = Sidereal hours – Retardation = $11^h\ 30^m\ 12^s - 1^m\ 53.072^s = 11^h\ 28^m\ 18.928^s$ ...(2)

Adding (1) and (2), we get

L.M.T. = $7^h\ 13^m\ 1.093^s + 11^h\ 28^m\ 18.928^s$ = $\mathbf{18^h\ 41^m\ 20.021^s}$.

13.9.14. TO FIND THE LOCAL SIDEREAL TIME OF ELONGATION OF A STAR

We have already seen in § 13.8 (Fig. 13.30) that

Star's hour angle + star's right ascension = Local Sidereal Time.

Thus, to get the L.S.T. of elongation of the star, add the westerly hour angle (or subtract the easterly hour angle) to the R.A. of the star at its elongation. If the result is more than 24^h, 24^h are deducted, while if the result is negative, 24 hours are added to it.

Example 13.38. *Find the L.S.T. at which β Ursae Minor is will elongate on the evening at a place in latitude 50° 30′ N given that the R.A. of the star is $14^h\,50^m\,52^s$ and its declination is + 74° 22′.*

Solution

The right ascension and the declination of the star are given. Let us first calculate its hour angle at elongation. When the star is at elongation, we have, from Eq. 13.19,

$$\cos H = \frac{\tan\theta}{\tan\delta} = \frac{\tan 50° 30'}{\tan 74° 22'}$$

$$\log\ \tan\ 50° 30' = \bar{1}.0838955$$
$$\log\ \tan\ 74° 22' = \bar{1}.5531022$$

$\therefore$ $\log\ \cos\ H = \bar{1}.5307933$

$\therefore$ $H = 70° 9' 18''$

or $H = 4^h\,40^m\,37.2^s$

Add R.A. = 14 50 52.0

$\therefore$ L.S.T. = $19^h\,31^m\,29.2^s$

Example 13.39. *If the G.S.T. of G.M.N. is $14^h\,30^m\,28.25^s$, what will be the H.A. of a star of R.A. $23^h\,20^m\,20^s$ at a place in longitude 120° 30′ W at 2.05 A.M. G.M.T. the same day ?*

Solution

We know that, L.S.T. = R.A. of star + Hour angle of the star.

From the above relation, the hour angle of the star can very easily be found out by subtracting R.A. of the star from the L.S.T. of the event. The only problem, therefore, is to calculate the L.S.T. corresponding to the given L.M.T., given the G.S.T. of G.M.N.

Let us first calculate the L.S.T. of L.M.N.

Longitude = 120° 30′ W = $8^h\,2^m$ W.

Since the place is to the west, we have to add the acceleration at the rate of 9.8565 per hour of longitude to the G.S.T. of G.M.N. to get the L.S.T. of L.M.N.

Now $8^h \times 9.8565 = 78.85$ seconds

$2^m \times 0.1642 = 0.33$ second

Total acceleration = 79.18 seconds

G.S.T. of G.M.N. = $14^h\,30^m\,28.25^s$

Add acceleration = 79.18^s

$\therefore$ L.S.T. of L.M.N. = $14^h\ 31^m\ 47.43^s$ ...(1)

Now G.M.T. = $2^h\ 5^m\ 0^s$

Subtract longitude = $8^h\ 2^m\ 0^s$

$\therefore$ L.M.T. of the event = $18^h\,3^m\,0^s$ (previous day).

L.M.N. (day of given G.S.T. of G.M.N.) = $12^h\ 0^m\ 0^s$

Subtract L.M.T. of event (previous day) = $18^h\ 3^m\ 0^s$

$\therefore$ Mean time interval between the event = $17^h\ 57^m\ 0^s$ and the L.M.N.

Let us convert this mean time interval to the sidereal time interval by adding acceleration at the rate of 9.8565^s per mean hour.

Thus $17^h \times 9.8565 = 167.56$ seconds

$57^m \times 0.1642 = 9.36$ seconds

Total acceleration = 177.92 seconds = $2^m\ 57.92^s$

$\therefore$ S.I. between the event and L.M.N.= $17^h\ 57^m\ 0^s + 2^m\ 57.92^s = 17^h\ 59^m\ 57.92^s$ (before L.M.N.)

Now L.S.T. of L.M.N. = $14^h\,31^m\,47.43^s$

Subtract S.I. = $17^h\,59^m\,57.92^s$

L.S.T. of event = $20^h\,31^m\,49.51^s$ ...(2)

Now H.A. = L. S. T. – R. A.

$= (20^h\,31^m\,49.51^s) - (23^h\,20^m\,20^s) + 24^h = 21^h\,11^m\,29.51^s$

(**Note.** 24^h have been added to make the hour angle positive).

Example 13.40. *Find the R.A. of the mean sun at 5.30 A.M. on July 28, 1964 in a place in longitude 75° 28′ W, and also the R.A. of the meridian of the place, given that G.S.T. at G.M.M on the given date is $20^h\,15^m\,32.58^s$.*

Solution.

We know that, L.S.T. = R.A. of the star + hour angle of the star.

Here, the mean sun is fictitious star.

Hence L.S.T. = R.A.M.S. + hour angle of the mean sun.

But hour angle of mean sun = L.M.T. + 12 hours

(since L.M.T. is measured from the lower transit).

$$\Delta_0'' = \Delta'_{1/2} - \Delta'_{-1/2} = 1083.9 - 1067.2 = +16''.7$$

$$\Delta'_{3/2} = f_2 - f_1 = +1100''.3$$

$$\Delta_1'' = \Delta'_{3/2} - \Delta'_{1/2} = 1100''.3 - 1083''.9 = +16''.4$$

Putting the values in the Bessel's formula, we get

$$f_n = f_0 + n\,\Delta'_{1/2} + \frac{n(n-1)}{4}(\Delta''_0 + \Delta_1'') = -16°\,14'\,24''.0 + 0.2917\,(+1083''.9)$$

$$-\frac{0.2917\,(0.2917-1)}{4} \times (16''.7 + 16''.4)$$

$$= -16°\,14'\,24''.0 + 316''.15 - 1''.71 = -16°\,9'\,9''.56.$$

Note. (1) The four dates from which the interpolation is done should be so selected that the instant lies between the two middle dates.

(2) The value of the declination by the approximate method (linear interpolation) will be equal to $-16°\,14'\,24''.0 + 0.2917\ (1083''.9) = -16°\,9'\,7''.85$.

13.11. INSTRUMENTAL AND ASTRONOMICAL CORRECTIONS TO THE OBSERVED ALTITUDE AND AZIMUTH

(A) INSTRUMENTAL CORRECTIONS

The angle measuring instruments used in astronomical observations are theodolite and sextant. For precise work, a theodolite having a least count of 1″ (or less) is used. The theodolite should be in prefect adjustments. However, following are some of the instrumental corrections that are generally applied to the observed altitude and azimuth.

(*a*) Corrections for Altitudes

(1) Correction for Index Error. If the vertical circle verniers do not read zero when the line of sight is horizontal, the vertical angles measured will be incorrect. The error is known as the *index error*. The index error can be eliminated by taking both face observations. However, it may sometimes not be practicable to take both face observations when the altitude of a star or the sun is to be observed. In such a case, the correction for the index error is necessary.

The index error may be determined as follows :

(*i*) Set the theodolite on firm ground and level it accurately with reference to altitude bubble.

(*ii*) Bisect a well-defind object such as a church spire (or a chimney top) with the telescope normal (face left). Observe the vertical angle α_1.

(*iii*) Change the face and bisect the same object again with telescope reversed (face right). Observe the vertical angle α_2.

Let the index error be e.

∴ Correct vertical angle will be

$$\alpha = (\alpha_1 + e) \quad \text{and} \quad \alpha = (\alpha_2 - e)$$

$$\therefore \quad \alpha = \frac{(\alpha_1 + e) + (\alpha_2 - e)}{2} = \frac{\alpha_1 + \alpha_2}{2}$$

Thus, the correct vertical angle is the mean of the two observed angles.

Hence $\quad e = (\alpha - \alpha_1)$

For example, let $\alpha_1 = 4°\,15'\,8''$ and $\alpha_2 = 4°\,15'\,16''$

∴ Mean vertical angle $= \alpha = 4°\,15'\,12''$

Hence, the index error correction for face left observation $= +4''$

Hence, the index error correction for face right observation $= -4''$

The index error correction is said to be +ve or −ve according as this amount is to be added to or subtracted from the observed altitude.

(2) Correction for Bubble Error. If the altitude bubble does not remain central while the observations are made, the correction for bubble error is essential. The correction for bubble error is given by

$$C = \frac{\Sigma O - \Sigma E}{n} \times v \ \text{seconds} \qquad \ldots(13.27)$$

where C = correction for bubble error in seconds, to be applied to the mean altitude observed.

ΣO = the sum of readings of the object glass end of the bubble.

ΣE = the sum of readings of the eye-piece end of the bubble.

n = the number of bubble ends read (= 2 when single face observation is taken, and 4 when both face observations are made).

v = angular value of one division of the bubble in seconds.

If ΣO is greater than ΣE, the correction is positive, otherwise negative.

(*b*) Correction for Azimuths

Since most astronomical observations require the line of sight to be elevated through a large vertical angle, it is important that the horizontal axis shall be truly horizontal. To fulfill this, it is most important that (1) the instrument is accurately levelled so that the vertical axis is truly vertical and (2) the trunnion axis is exactly perpendicular to the vertical axis. If the vertical axis is not truly vertical (*i.e.* if the bubble does not preserve a central position through a series of observations), the trunnion axis will be inclined even though the instrument is in perfect adjustment. The error due to the inclination of the trunnion axis cannot be eliminated. However, its inclination can be determined by means of a striding level with a sensitive bubble tube.

Correction for Trunnion Axis Dislevelment. The bubble readings on the striding level will show whether the trunnion axis is truly horizontal or not. If not, each horizontal direction should be corrected for trunnion axis dislevelment. It can be shown that the correction to be applied to the azimuth of a low point with respect to a high point, caused by an inclination of the trunnion axis of the transit is given by

$$c = b \tan \alpha \ \text{seconds}$$

where c = correction to the azimuth

b = inclination of the horizontal axis of the transit with respect to the horizontal, in seconds

α = vertical angle to the high point.

The angle of deviation of the ray from its direction on entering the earth's atmosphere to its direction at the surface of the earth is called the *refraction angle* of correction. The refraction correction is always subtractive to the observed altitude. The magnitude of refraction depends upon the following:

(*i*) the density of air

(*ii*) the temperature

(*iii*) the barometric pressure

and (*iv*) the altitude.

It is constant for all bodies and does not depend upon the distance of the body from the observer.

At a pressure of 29.6 inches of mercury and a temperature of 50° F, the correction for refraction can be calculated from the following formula :

Correction for refraction (in seconds)

$$= 58'' \cot \alpha = 58'' \tan z \qquad \ldots(13.30)$$

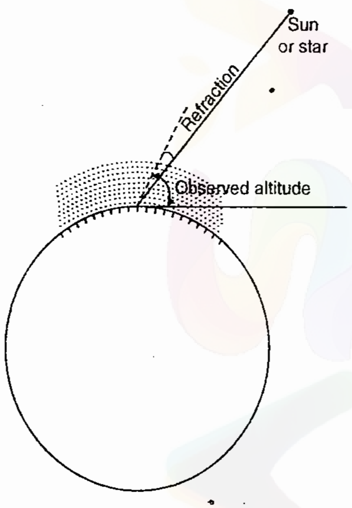

FIG. 13.36. REFRACTION.

where α = the apparent altitude of the heavenly body

z = the apparent zenith distance of the heavenly body.

The correction for refraction is always subtractive.

The values of mean refraction for different altitudes are given in Chamber's Mathematical Tables corresponding to barometer pressure, temperature of external air and temperature of thermometer attached to barometer.

The refraction correction for low altitudes is uncertain and hence observation for precise determination should never be taken on a celestial body which is nearer the horizon. The refraction, however, does not affect the azimuth.

3. Correction for Dip of the Horizon. The angle of the dip is the angle between the true and visible horizon. When the observations are taken with the help of a sextant at the sea, the altitude of the star or sun is measured from the visible horizon of the sea. Owing to the curvature of the earth, the visible horizon is below the true horizon. Hence, the angle of dip (*i.e.* the angle between the two horizons) must be subtracted from the observed altitude of the body.

In Fig. 13.37,

A = position of the observer

$AB = h$ = Height of the observer above sea level

S = position of the sun or star

AD = visible horizon

AC = true horizon

$\angle SAD = \alpha'$ = observed altitude of the sun or star

$\angle SAC = \alpha$ = true altitude of the sun or star

$\angle CAD = \beta$ = angle of dip

R = radius of the earth

Now, $BO = R$; $AO = (R + h)$

$\therefore \quad AD = \sqrt{(R+h)^2 - R^2}$

$\angle CAD = \angle AOD = \beta$

$$\therefore \quad \tan \beta = \frac{AD}{OD} = \frac{\sqrt{(R+h)^2 - R^2}}{R} = \sqrt{\frac{h\,(2R+h)}{R^2}} \quad \ldots\text{(exact)} \quad \ldots[13.31\ (a)]$$

$$\text{or} \quad \tan \beta = \sqrt{\frac{2h}{R}} \quad \ldots\text{(approximately)}\ldots[13.31\ (b)]$$

If β is small, we many have

$$\tan \beta = \beta \text{ (radians)} = \sqrt{\frac{2h}{R}} \quad \ldots(13.31)$$

The correction for dip is always subtractive.

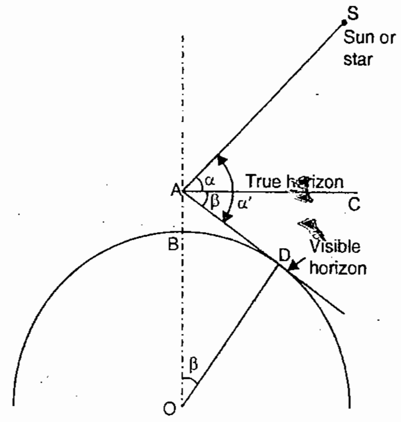

FIG. 13.37. DIP OF THE HORIZON.

4. Correction for Semi-diameter. The semi-diameter of the sun or star is half the angle subtended at the centre of the earth, by the diameter of the sun or the star. Since the distance of the sun from the earth is not constant throughout the year, the semi-diameter varies from 15′ 46″ in July to 16′ 18″ in January. Its value at its mean distance from the earth is 16′ 1″.18. The Nautical Almanac gives the values of sun's semi-diameter for every day in the year.

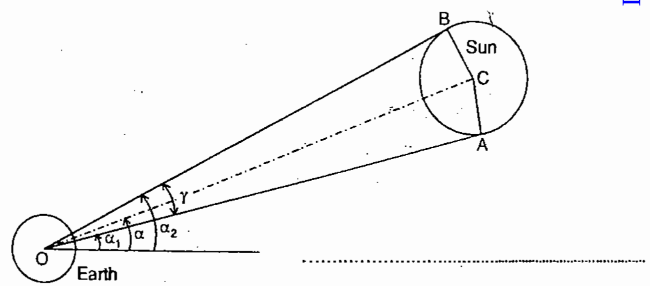

FIG. 13.37. (*a*) CORRECTION FOR SEMI-DIAMETER.

As the sun is large, its centre cannot be sighted precisely, and it is customary to bring the cross-hairs tangent to the sun's image. When the horizontal cross-hair is brought tangent to the lower edge of the sun, the sight is said to be taken at sun's lower limb [Fig. 13.38 (*a*)]. Similarly, when the horizontal cross-hair is brought tangent to the upper edge of the sun, the sight is said to be taken at sun's upper limb [Fig. 13.38 (*b*)]. Figs. 13.38 (*c*) and 13.38 (*d*) illustrate the observations taken to sun's right limb and left limb respectively.

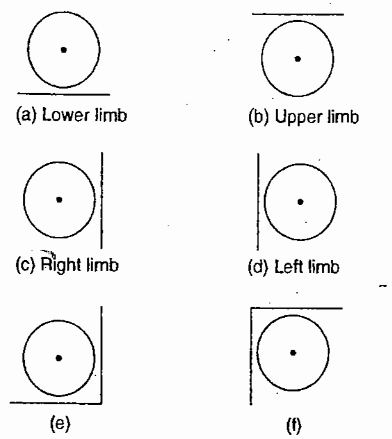

FIG. 13.38. OBSERVATION TO SUN.

In Fig. 13.37 (*a*), OA is the ray corresponding to the lower limb of the sun. The observed altitude α_1 is evidently lesser than the correct altitude α. Similarly, OB is the ray corresponding to observations. The difference between the chronometer time and the time determined from the observation gives *chronometer correction* and should be added algebraically to the reading of the watch to give the true time at the instant. *The correction is positive when the chronometer is slow and negative when it is fast.*

The following are some of the methods usually employed for the determination of time :

(1) By meridian observation of a star or the sun. (By transit of a star or sun)

(2) By ex-meridian altitude of a star or the sun.

(3) By equal altitudes of star or the sun.

1 (a) TIME BY MERIDIAN TRANSIT OF A STAR

The application of this method requires a knowledge of the local longitude and a previous determination of the direction of the meridian. This forms the most direct method of obtaining local time and is used for primary field determinations. The basis of the method is the fact that when a star transits the meridian, its hour angle is zero and local sidereal time is equal to the right ascension of the star.

In Fig. 13.39, ZP is the observer's meridian and M is the position (in general) of a star.

$\angle SP\Upsilon$ = Local sidereal time

$\angle SPM$ = Hour angle (H) of the star (measured westward)

$\angle \Upsilon PM$ = R.A. of the star.

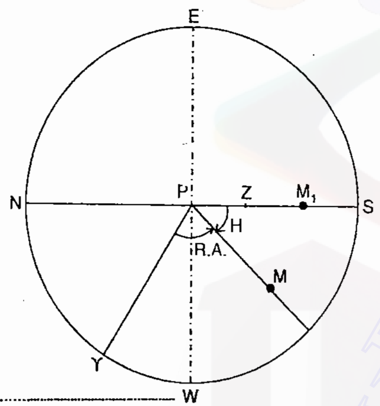

FIG. 13.39

Evidently, $\angle SP\Upsilon = \angle SPM + \angle \Upsilon PM$

or L.S.T. = Hours angle + R.A.

M_1 is the position of the star when it crosses the meridian, and its hour angle (H) is zero. Thus,

$$\text{L.S.T.} = \text{R.A.}$$

The right ascensions of various stars are given in the Ephemeris for the date.

The star is observed with a theodolite, the line of sight being directed along the known direction of the meridian. The chronometer is read at the instant the star transits across the vertical wire. The chronometer error is then determined by comparing the true sidereal time (equal to the right ascension) of the star with the sidereal time kept by the watch or chronometer. If the chronometer is keeping Greenwich sidereal time, it is necessary to apply only the local longitude to the right ascension of the star to obtain the true Greenwich sidereal time. If the chronometer keeps the local mean time, the local sidereal time determined above is converted into local mean time by method discussed earlier and the error of the chronometer is determined. Generally, the chronometer error is found in this way on two different days and average daily rate of error during the period is found by dividing the change in the error by the number of days elapsed.

1. (b) TIME BY MERIDIAN TRANSIT OF THE SUN

When the sun is observed on the meridian of the place at upper transit, its hour angle is zero and the L.A.T. is 12 hours. The transit of the sun is observed with a theodolite and the times at which the east and west limbs of the sun pass the vertical hair are noted by means of the chronometer. The mean of the two readings gives the mean time at the local apparent noon. If only one limb is observed, allowance must be made for the time that the semi-diameter takes to cross the meridian. From the Nautical Almanac, we can find the G.M.T. of G.A.N. for the given date, from which the L.M.T. of L.A.N. may be found. This L.M.T. of L.A.N. can then be compared with the chronometer time at the instant of the observation to give the error of the chronometer.

Error in the Observations of the Meridian Transit of Star or Sun

The method of meridian transit of a star or the sun, though simple, is not very much used because it is impracticable to secure that the instrumental line of sight lies exactly in the plane of the meridian. The observed times are subject to the following three principal corrections :

(i) The Azimuth Correction

If the instrument is in accurate adustment, but the direction of the meridian is in error, the line of sight set out along the meridian will pass through the zenith of the observer and not through the celestial pole. The correction is given by

$$\textit{Azimuth correction} = e \sin z \sec \delta$$

where e = error of azimuth in *seconds of time*

z = zenith distance

δ = declination of the star.

e is considered positive if the line of sight it too far east when the telescope is pointed south, and is negative if the line of sight is too far west. It can be shown that if the latitude of the place is 30° and the polar distance of a star is 40°, *an error of 1 minute of arc in the direction of the meridian will make the time of transit wrong by two seconds.* The method, therefore, requires the meridian to be set out very accurately.

The error is very great if the polar distance of the star is small, and is least for those that transit near the zenith.

(ii) The Level Correction

If the horizontal axis is not perfectly horizontal, the line of sight may depart considerably at high altitudes. Due to this, the transit will be observed either too soon or too late according to the direction of tilt of the transverse axis. The correction is given by :

$$\textit{Level correction} = b \cos z \sec \delta$$

where b = inclination of the horizontal axis in seconds of arc (determined by the readings of the striding level) and is positive when the left (or west) end of the axis is higher

z = zenith distance

δ = declination of the star.

than 2^m or 3^m in the error, the resulting error in computing the declination will not exceed 2″ or 3″, and recalculations are not necessary if observations are made with small instrument. If greater discrepancy is found between the correct and the chronometer time, the former is used for a better interpolation of δ and the computation of H is repeated with the new value. Also, a knowledge of the latitude of the place is essential for the computation of H. The precision in the knowledge of the latitude of the place depends upon the precision in the observation of altitude and also upon the time at which observation is made. When the sun is near the prime vertical, the effect of an error in latitude is small.

The error of the watch on local mean time is then equal to the difference between the time of observation by watch and the time of observation as determined by calculations. The observation is often combined with the observation of the sun for azimuth, the watch readings and altitude readings being common to both.

Booking of Field Observations

The field observations are usually entered in the field book in the following form: (Table 13.1).

TABLE 13.1

Star observed	Face	Vertical Angle												Time			Mean of time		
		A			B		Mean			Mean vertical Angle									
		°	′	″	′	″	°	′	″	°	′	″		h	m	s	h	m	s
α -ophiuchi W	L	38	30	20	30	40	38	30	30					7	21	11			
	R	37	26	30	26	10	37	26	20					7	27	30			
	R	36	30	40	30	20	36	30	30					7	32	20			
	L	35	50	10	50	00	35	50	5	37	4	21		7	38	05	7	29	46.5

3. (a) TIME BY EQUAL ALTITUDE OF A STAR

In this method, a star is observed at the same altitude on opposite sides of the meridian. The mean of the two chronometer times at which a star attains equal altitudes east and west of the meridian is evidently the chronometer time of transit, since the two observations are clearly made at equal intervals of time before and after the Star's meridian transit. The method is, therefore, very simple and accurate and is used when the direction of the meridian is not accurately known. The altitude of the star need not be determined and, therefore, no correction is required for refraction. The observations must be made when the star is near the prime vertical so that its altitude changes rapidly. When the star crosses the meridian, its hour angle is equal to zero and its right ascension

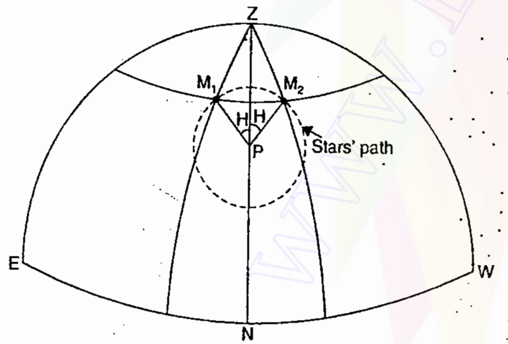

FIG. 13.40. TIME BY EQUAL ALTITUDE.

is therefore the local sidereal time. The local sidereal time so obtained may be converted to local mean time which can then be compared with the mean time of the chronometer during the observations, and the error of the chronometer can be known.

To make the observations, the following steps are necessary :

(1) Set up the instrument on firm ground and level it accurately.

(2) Compute the approximate altitude of the star and set it on the vertical circle.

(3) Follow the motion of the star in azimuth with the vertical cross-hair by means of horizontal tangent screw.

(4) Note the chronometer time (T_1) when the star crosses the horizontal hair.

(5) Turn the instrument in azimuth and again follow the star when the star approaches the same altitude to the other side of the meridian.

(6) Note the chronometer time (T_2) when the star crosses the horizontal hair.

Mean time of transit of the star $= \frac{1}{2}(T_1 + T_2)$

It is very important to note that during the above observations the face of the theodolite is not changed. However, the altitude bubble must be accurately centred by means of clip screws prior to each observation. For accurate results, a series of observations are made on the same star.

In Fig. 13.40, the dotted circle shows the daily path of the star round the pole. M_1 is the position of the star of the east of the meridian ZP and M_2 is its position to the west of the meridian when it attains the same altitude as at M_1.

The method has the following advantages :

(1) Since the actual altitude of the star is not required the instrumental errors–such as index error, collimation error, errors due to graduations etc. are not involved.

(2) No knowledge is required of latitude, declination, or even azimuth.

The method has, however, the following disadvantages :

(1) A long interval of time elapses between the two observations – sometimes several hours.

(2) The precision of the result depends upon the refraction having the same value for both observations. Due to long interval of time, the refraction may change appreciably, thus affecting the result.

However, the time between the two observations can be reduced if the declination of the selected star is nearly equal to the latitude. To eliminate the uncertainties of refraction near the horizon, the star should have an altitude of something more than 45°.

The Error due to Slight Inequality in the Altitudes of Two Corresponding Observations:

In Fig. 13.40,

ZM_1 = zenith distance of first observation = z

ZP = co-latitude = c

PM_1 = polar distance = p

ZPM_1 = hour angle = H

$M_1ZP = A$ = azimuth of the star.

Solution

Let us first convert the G.S.T. of G.M.M. into L.S.T. of L.M.M.

Longitude = $4^h\ 30^m\ E$

Loss in the sidereal time at the rate of 9.8565^s per hour of longitude is :

$$4^h \times 9.8565 = 39.43 \text{ seconds}$$
$$30^m \times 0.1642 = 4.93 \text{ seconds}$$

Total retardation $= 44.36$ seconds

$\therefore$ L.S.T. of L.M.M. = G.S.T. of G.M.M. – Retardation

$= 14^h\ 38^m\ 12^s - 44.36^s = 14^h\ 37^m\ 27.64^s$

Now L.S.T. of observation = R.A. of the star = $7^h\ 36^m\ 21.24^s$

$\therefore$ S.I. = L.S.T. of observation – L.S.T. of L.M.M.

$= (7^h\ 36^m\ 21.24^s - 14^h\ 37^m\ 27.64^s) + 24^h = 16^h\ 58^m\ 53.6^s$.

Let us now convert the S.I. into mean time interval by subtracting the retardation at the rate of 9.8296 seconds per hour of sidereal time.

$$16^h \times 9.8296 = 157.27 \text{ seconds}$$
$$58^m \times 0.1638 = 9.49 \text{ seconds}$$
$$53.6^s \times 0.0027 = 0.14 \text{ second}$$

Total retardation $= 166.90$ seconds $= 2^m\ 46.90^s$

$\therefore$ Mean time interval since L.M.M. = S.I. – Retardation

$= 16^h\ 58^m 53.6^s - 2^m\ 46.90^s = 16^h\ 56^m\ 6.7^s$

Standard time shown by chronometer

$= 5^h\ 56^m\ 8.86^s$ P.M. $= 17^h\ 56^m\ 8.86^s$ since L.M.M.

$\therefore$ Local time of chronometer

$= 17^h\ 56^m\ 8.86^s$ – Difference of longitude

$= 17^h\ 56^m\ 8.86^s - 1^h = 16^h\ 56^m\ 8.86^s$

(Since the place of observation is at longitude 1^h to the west of standard meridian).

$\therefore$ Chronometer error = **2.16 seconds (Fast).**

Example 13.45. *The following notes refer to an observation for time made on a star on Feb. 18, 1965 :*

Latitude of the place	$= 36°\ 30'\ 30''$ *N*
Mean observed altitude of the star	$= 30°\ 12'\ 10''$
R.A. of star	$= 5^h\ 18^m\ 12.45^s$
Declination of the star	$= 16°\ 12'\ 18''.4$

This star is to the east of the meridian.

Mean sidereal time observed by sidereal chronometer $= 1^h\ 2^m\ 5.25^s$

Find the error of the chronometer.

Solution. The hour angle of the star is determined from the following formula :

$$\tan\frac{H}{2} = \sqrt{\frac{\sin(s-c)\sin(s-p)}{\sin s\,.\,\sin(s-z)}}\ ; \quad \text{where} \quad s = \tfrac{1}{2}(z+c+p)$$

$$z = 90° - \alpha = 90° - 30°\ 12'\ 10'' = 59°\ 47'\ 50''$$
$$p = 90° - \delta = 90° - 16°\ 12'\ 18''.4 = 73°\ 47'\ 41''.6$$
$$c = 90° - \theta = 90° - 36°\ 30'\ 30'' = 53°\ 29'\ 30''$$

$$2s = 187°\ 05'\ 01''.6$$
$$s = 93°\ 32'\ 30''.8$$

$(s-c) = 40°\ 3'\ 0''.8$; $(s-p) = 19°\ 44'\ 49''.2$; $(s-z) = 33°\ 44'\ 40''.8$

$$\log\sin(s-c) = \bar{1}.8085208$$
$$\log\sin(s-p) = \bar{1}.5287565$$
$$\log\operatorname{cosec} s = 0.0008302$$
$$\log\operatorname{cosec}(s-z) = 0.2553212$$

$\log\tan^2\frac{H}{2} = \bar{1}.5934287$; $\log\tan\frac{H}{2} = \bar{1}.7967144$

$\therefore$ $\frac{H}{2} = 32°\ 3'\ 17''.6$ or $H = 64°\ 6'\ 35''.2 = 4^h\ 16^m\ 26.3^s$

Since the star is to the east of the meridian, the westerly hour angle

$= 24^h - 4^h\ 16^m\ 26.3^s = 19^h\ 43^m\ 33.7^s$

R.A. of the star $= 5^h\ 18^m\ 12.45^s$

Add hour angle $= 19^h\ 43^m\ 33.70^s$

$\therefore$ L.S.T. of observation $= 25^h\ 01^m\ 46.15^s = 1^h\ 01^m\ 46.15^s$

Sidereal time by chronometer $= 1^h\ 2^m\ 5.25^s$

$\therefore$ Error of chronometer $=$ **19.1^s** (fast).

Example 13.46. *The mean observed altitude of the sun, corrected for refraction, parallax and level was 36° 14′ 16″.8 at a place in latitude 36° 40′ 30″ N and longitude 56° 24′ 12″ E. The mean watch time of observation was* $15^h\ 49^m\ 12.6^s$ *the watch being known to be about* 3^m *fast on L.M.T. Find the watch error given the following :*

Declination of the sun at the instant of observation $= +17°\ 26'\ 42''.1$.

G.M.T. of G.A.N. $= 11^h\ 56^m\ 22.8^s$.

$\therefore$ L.M.T. of transit of the star as shown by the chronometer = $7^h\ 47^m\ 02^s$ P.M. ...(2)

Chronometer error = **11.52 seconds (Fast)**

13.13. TIME OF RISING OR SETTING OF A HEAVENLY BODY

In Fig. 13.41, *SEN* is the horizon and *M* is the position of a star when it is rising. It is required to find the time of rising and setting of the star.

The spherical triangle *PMN* is right-angled at *N*, since the plane of the observer's meridian is perpendicular to the horizon.

$\therefore$ $\cos MPN = \cos MP \cdot \tan PN$

Now $\angle ZPM = H$ = hour angle of the star at its rising

$MP = \delta$ = declination of the star

$PN = \theta$ = altitude of the pole = latitude of the observer

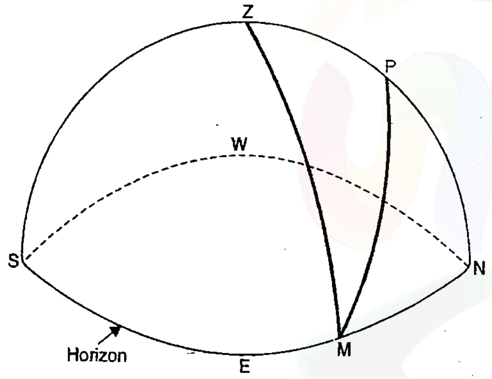

FIG. 13.41. RISING AND SETTING OF STAR.

$\angle MPN = 180° - H$

Hence $\qquad \cos H = -\tan \delta \tan \theta$

Knowing the declination of the star and the latitude of the place, its hour angle can be known. Then,

L.S.T. of rising of star = R.A. of the star + Hour angle.

Thus, the local sidereal time of the rising of the star can be known, and this can be converted into L.M.T., if desired.

The hour angle of setting will obviously be the same as that of rising. In the above treatment, we have neglected the effect of refraction, which amounting as it does to about 36′ on the horizon, will cause stars to be just visible when they are really 36′ below the horizon.

Length of Day and Night :

The hour angle *H* of the sunrise or sun-set is given by

$\cos H = -\tan \delta \tan \theta$ where δ is the declination of the sun.

If the change in the declination δ of the sun is ignored

Length of the day = twice hour angle in time units $= \dfrac{2H}{15}$

Similarly, length of the night $= 2\left(\dfrac{180° - H}{15}\right)$

The equation $\cos H = -\tan \delta \tan \theta$ can be used to determine the length of the day at different places and at different times.

(1) At a place at equator, $\theta = 0$

$\therefore \qquad \cos H = 0 \quad$ or $\quad H = 0° \quad$ and $\quad H = 90°$

$\therefore$ Length of day (or night) $= \dfrac{2H}{15} = 12^h$

Hence for all values of δ, the days are always equal to the nights at equator.

(2) At the time of equinox, the sun is at equator and hence $\delta = 0$

$\therefore \qquad \cos H = 0 \quad$ or $\quad H = 0° \quad$ and $\quad H = 90°$

$\therefore$ Length of day (or night) $= \dfrac{2H}{15} = 12^h$

Hence for all values of θ (i.e., at all the places on the earth) the day is equal to the night.

(3) If $\qquad \delta = 90° - \theta$; $\qquad \cos H = -1$ or $H = 180°$

$\therefore$ Length of day $= \dfrac{2 \times 180°}{15} = 24°$ (*i.e.* the sun does not set).

(4) If $\qquad \delta = -(90° - \theta)$; $\qquad \cos H = 1$ and $H = 0$

$\therefore$ Length of the day $= 0^h$

Hence the sun does not rise at all.

The Duration of Twilight

Twilight is the subdued light which separates night from day. When the sun sets below the horizon, the darkness does not come instantaneously because the sun's rays still illuminate the atmosphere above us. The particles of vapour etc. in the atmosphere reflect the light and scatter it in all directions. As the sun sinks down, the intensity of the diffused light diminishes. Observations have shown that the diffused light is received so long as the sun does not sink 108° below the horizon. To find the duration of twilight at particular place, we must, therefore, find the time the sun takes to alter its zenith distance from 90° to 108° in the evening, or from 108° to 90° in the morning.

With our previous notations, we have

$$\cos 108° = \sin \delta \sin \theta + \cos \delta \cos \theta \cos H' \qquad ...(1)$$

where $\qquad H'$ = hour angle of the end of twilight.

If H is the hour angle of the sunset we have $\cos H = -\tan \delta \tan \theta \qquad ...(2)$

From the above two equations, H and H' can be calculated for given values of δ and θ.

Hence duration of twilight $= H' - H$.

13.14. THE SUN DIALS

The sun dial enables the time to be fixed accurately enough for ordinary purposes of life, though the precision obtained is much less than that obtained by the methods already discussed. The sun dial gives apparent solar time from which mean time may be obtained. It is useful particularly in places where there are no means available for checking watch or clock times.

A sun dial essentially consists of :

(*i*) a straight edge, called the *stile* or *gnomon* of the dial and

(*ii*) the graduated circle on which the shadow of the gnomon falls.

azimuth or true bearing of any line from the station, so that the azimuths of all the survey lines meeting there may be derived. The determination of the direction of the true meridian or of the azimuth of a line is most important to the surveyor. There are several methods of determining the direction of the true meridian, but preference is given to such methods as will allow a set of observations to be taken so that (*i*) instrumental errors may be eliminated, by taking face left and face right observations and (*ii*) interval or time between the observations may not be too great.

Reference mark

In order to determine the azimuth of a star or other celestial body, it is frequently necessary to have a *reference mark* (R.M.) or *referring object* (R.O.). When steller observations are taken, the reference mark should be made to imitate the light of a star as nearly as possible. The reference mark may be a triangulation station or it may consist of a lantern or an electric light placed in a box or behind a screen, through which a small circular hole is cut to admit the light to the observer. The diameter of the hole should not be more than 1 cm. The mark should preferable be so far from the instrument that the focus of the telescope will not have to be altered when changing from the star to the mark. A distance of about a mile is quite satisfactory.

The following are some of the principal methods of determining the azimuth or the direction of the true meridian :

1. By observations on star at equal altitudes.
2. By observations on a circumpolar star at elongation.
3. By hour angle of star or the sun.
4. By observation of Polaris.
5. By ex-meridian observations on sun or star.

1 (*a*) OBSERVATIONS ON THE STARS AT EQUAL ALTITUDES

The simplest method of determining the direction of the celestial pole is probably that observing at star at equal altitudes. In this method, the knowledge of the latitude or local time is not necessary, and no calculations are involved. However, the duration of the work is a great inconvenience, extending from four to six hours at night. Also the effects of atmospheric refraction may vary considerably during the interval, affecting the vertical angles to an unknown extent.

The method is based on the fact that if the angle subtended between the reference mark and a star is measured in two positions of equal altitude, the angle between the mark and the meridian is given by half the algebraic sum of the two observed angles.

The dotted circle in Fig. 13.40 represents the circular path of a star round the pole, and it is required to determine the direction of the centre P of this circle. M_1 and M_2 are the two positions of the star at equal altitude, and all that the observer has to do to get his true meridian is to bisect the angle between M_1 and M_2

Thus, in Fig. 13.43, R is the reference mark (R.M.) and O is the position of the instrument station through which the direction of the true meridian is to be established. M_1 and M_2 are two positions of a star at equal altitudes. The field observations are taken in the following steps :

(1) Set the instrument at O and level it accurately.

(2) Sight the R.M. with the reading $0° 0' 0''$ on the horizontal circle.

(3) Open the upper clamp and turn the telescope clockwise to bisect accurately the star at position M_1. Clamp both horizontal as well as vertical circle.

(4) Read the horizontal angle θ_1 as well as the altitudes α of the star.

(5) When the star reaches the other side of the meridian, follow it through the telescope, by unclamping the upper clamp, and bisect it when it attains the same altitude. In this observation, the telescope is turned in azimuth and the vertical circle reading remains unchanged. Read the angle θ_2.

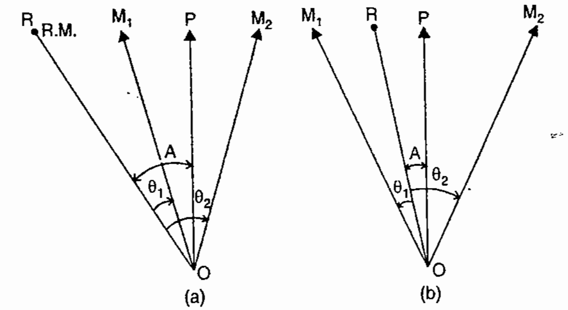

FIG. 13.43. AZIMUTH BY EQUAL ALTITUDES.

Let A be the azimuth of the line OR, *i.e.* the angle between the true meridian and the reference object. Since the direction of the meridian is midway between the two positions of the star, the azimuth of the line may be determined according as both the positions of the star are to the same side of R or to the different sides of R.

Case I : Both positions of the star to the same side [Fig. 13.43 (a)].

$$\theta_1 = \angle ROM_1 \; ; \qquad \theta_2 = \angle ROM_2$$

$$A = \text{azimuth} = \angle ROP, \text{ (where } P \text{ is the position of the pole)}$$

$$= \theta_1 + \frac{\theta_2 - \theta_1}{2} = \frac{\theta_1 + \theta_2}{2}$$

Hence the azimuth of the line is equal to half the sum of the two observed angles.

Knowing the azimuth of the line OR, the azimuth of any other line through O can be determined by measuring the horizontal angle between OR and that line. Also if it is required to set out the direction of the true meridian, and angle equal to $\frac{\theta_1 + \theta_2}{2}$ can be set out from the line OR.

Case II. Both positions of the star are on opposite sides of the line. [Fig. 13.43 (b)].

$$\text{Azimuth} = A = \angle M_1OP - \angle M_1OR = \tfrac{1}{2}\angle M_1OM_2 - \angle M_1OR = \tfrac{1}{2}(\theta_1 + \theta_2) - \theta_1 = \frac{\theta_2 - \theta_1}{2}$$

Hence the azimuth of the line is equal to half the difference of the two observed angles.

In the observations taken above, it is assumed that the instrument is in perfect adjustment. If it is not so, it is necessary to take at least four observations (two with face left and two with face right) to eliminate the instrumental errors. The position M_1 of the star is observed with both the faces, and the position M_2 is also observed with both the faces, and the mean is taken. However, in the duration that elapses between two face observations of M_1, the position and altitude of the star slightly changes and this should be properly east of the meridian, it is said to be in eastern elongation. When it is at its greatest distance to the west of the meridian, it is said to be in western elongation. In this position, the star's diurnal circle is tangent to the vertical circle to the star.

Figs. 13.45 (*a*) and 13.45 (*b*) show two views of the stars at elongation. M_1 is the position of the star at its eastern elongation, and M_2 is the position of the star as its western elongation. In this position, the vertical circle of star makes its greatest angle with the plane of the meridian. The vertical through M_1 (or M_2) is tangential to the diurnal path of the star shown by dotted circle. Evidently, therefore, $\angle ZM_1P$ is a right angle. Also, when the star is at western elongation (position M_2), $\angle ZM_2P$ is a right angle.

At the instant of elongation of the star, its motion is vertical and it is in a favourable position for observations upon its azimuth because its horizontal movement is very slight for some time before and some time after it arrives M_1 (or M_2). When the star is in eastern elongation (M_1), it appears to move vertically downwards, and when it is in western elongation, it appears to move vertically upwards at the instant of elongation. It is clear from the figure that the points M_1 and M_2 will always be at a greater altitude than the celestial pole P. However, greater the declination of the star, more nearly will be the altitude of M_1 and M_2 approach that of P.

Prior to making the field observations, it is necessary to *calculate* the time at which the star will elongate. This can be done as follows:

(*i*) The hour angle (H) of the star can be calculated from equation 13.19

$$\cos H = \frac{\tan\theta}{\tan\delta} = \tan\theta\cot\delta$$

(*ii*) Calculate the local sidereal time of elongation :

L.S.T. (of elongation) = R. A. $\pm H$

Use plus sign for western elongation and minus sign for eastern elongation.

(*iii*) Convert this L.S.T. to mean time by method discussed earlier.

Thus, the mean time of elongation of the star is known. At least 15 to 20 minutes before the time of elongation, the instrument is set up and carefully levelled. Five minutes before the time of elongation, a pointing is made on the reference mark. The upper clamp is then unclamped and the star is sighted. The star is then followed in azimuth. At the time of elongation, the star stops moving horizontally, and appears to move vertically along the vertical hair. This will take place exactly at the time calculated above. The horizontal circle reading gives the angle that the star makes with the reference line. To this, if we add the azimuth of the star, the azimuth of the survey line can very easily be known.

The azimuth of the star at its elongation can be calculated from Eq. 13.21:

$$\sin A = \frac{\cos\delta}{\cos\theta} = \cos\delta\,.\,\sec\theta.$$

However, in order to eliminate the error, at least two observations should be made — one with face left a few minutes before the elongation and other with the face right a few minutes after the elongation. If more time is taken between these two sets of readings, the azimuth will not be correct. In general, the observations should not be extended beyond five minutes on either side of the time of elongation and during this time as many readings between the R.M. and the star as are possible should be taken.

The following table gives the time after the moment of elongation when the aizmuth changes by 5″ for a place in latitude 30°:

Polar distance of the star	*Time after moment of elongation before azimuth changes by 5″*
10°	3 min. 33 sec.
15°	3 min. 7 sec.
20°	2 min. 35 sec.
30°	2 min. 11 sec.

As there will be a corresponding and nearly equal period before elongation, it follows that for stars having 20° polar distance, 5 min. and 10 seconds can be the maximum time to the observer before the azimuth can change by 5″ in that period. For a star whose polar distance is 10°, the corresponding time is 7 min. 6 sec. *The nearer the star is to the pole the greater the length of time available for the observations.* In ordinary observations, a surveyor uses a 20″ theodolite so as to determine the azimuth within 20″. Hence, it will be sufficiently accurate if he takes two observations of the star, one with the face left and the other with the face right, not exactly at the time of elongation, but one just before and the other just after the elongation.

However, for very accurate results, it is better to apply the following correction to the value of azimuth (A) of the star from the formula for elongation.

$$\textit{correction (in seconds)} = 1.96\tan A\sin^2\delta\,(t_E - t)^2 \qquad \text{...(13.35)}$$

where $t_E - t$ is the sidereal interval in minutes between the time of observation and that of elongation. The above formula is applied only when $(t_E - t)$ does not exceed 30 minutes.

The Effect of an Error in the Latitude

For the calculation of the azimuth, the declination (δ) of the star and the latitude (θ) of the place of observation must be accurately known. The declination is taken from the star almanac. Let us now study the effect of an error in the latitude on the determination of the azimuth

Let y = error in the latitude and x = corresponding error in the azimuth.

We have $\sin A = \dfrac{\cos\delta}{\cos\theta}$ or $\sin A\,.\,\cos\theta = \cos\delta$ ...(1)

Putting the actual values of A and θ in the above expression, we get

$$\sin(A + x)\cos(\theta + y) = \cos\delta$$

Expanding $\sin(A + x)$ and $\cos(\theta + y)$, and replacing $\sin x$, $\sin y$ by x and y respectively, and $\cos x$, $\cos y$ by unity, we get

$$(\sin A + x\cos A)(\cos\theta - y\sin\theta) = \cos\delta \qquad \text{...(2)}$$

Subtracting (1) from (2), and neglecting the term having the product of small quantities x and y, we get

$$x\cos A\cos\theta - y\sin A\sin\theta = 0$$

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The correction is evidently zero at culmination.

(4) AZIMUTH BY OBSERVATIONS ON POLARIS OR CLOSE CIRCUMPOLAR STAR

The *most precise* determination of azimuth may be made by measuring the horizontal angle between the R.M. and a close circumpolar star. The chronometer time of each observation is noted very precisely. From the corrected chronometer times the hour angle of the circumpolar star can then be obtained as discussed earlier. The azimuth of the star can then be calculated by the solution of the astronomical triangle. Since the close circumpolar stars move very slowly in azimuth and errors in the observed times will thus have a small effect upon the computed azimuths, it is evident that only such stars should be chosen for primary or precise work.

Since Polaris (α Ursae Minoris) is the brightest circumpolar star, it is used in preference to others whenever practicable. In general, however, the observations on close circumpolar stars have the following advantages :

(1) Since the motion in the azimuth is very slow, the number of observations may be increased materially and greater accuracy may be secured.

(2) Observations may be made at any convenient time, without calculating the time of elongation or waiting for the time of elongation.

(3) If observations are made on the bright pole star, it is usually possible to sight the star during the twilight when no artificial illumination for the R.M. and for the instrument is necessary.

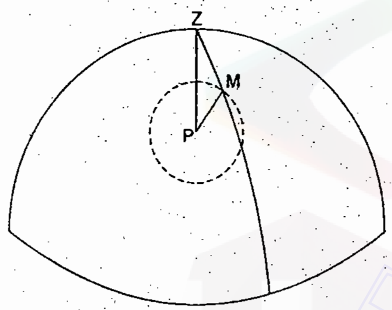


FIG. 13.47. OBSERVATIONS TO POLARIS.

In Fig. 13.47, P is the pole, Z is the zenith of the observer and M is the position of the close circumpolar star. The dotted circle shows the diurnal path of the polar star.

The hour angle H ($\angle ZPM$) is known from the observed chronometer time.

$\angle MZP = A$ = azimuth of the pole star (to be computed)

PM = polar distance = co-declination (known)

ZP = co-latitude $= c = 90° - \theta$ (known)

The azimuth (A) is given by

$$\tan A = \frac{\sin H}{\cos\theta\tan\delta - \sin\theta\cos H}$$

or

$$\tan A = \sec\theta\,.\,\cot\delta\sin t\,.\left(\frac{1}{1-a}\right) \qquad ...(13.40)$$

where

$$a = \tan\theta\cot\delta\cos H \qquad ...(13.41)$$

The values of $\log\dfrac{1}{1-a}$ are tabulated for different values of A in the *Special Publication 14*, United States Coast and Geodetic Survey.

The value to be taken for the hour angle is that corresponding to the mean of corrected chronometer timings of n observations. However, for the accurate results, the

curvature of the path of the star should be taken into consideration, and the calculated azimuth should be corrected by the following amount :

$$\text{Curvature correction for one set} = \frac{\tan A \sin^2\delta}{n}\,\Sigma\,\frac{2\sin^2\frac{1}{2}\Delta t}{\sin 1''} \qquad ...(13.42)$$

where n = number of the observations in one set

Δt = angular equivalent of the sidereal time interval (in seconds) between the individual observation and the mean of the set.

For the most accurate work, the striding level should also be observed. If the horizontal axis is inclined during a pointing on the star or the R.M., the horizontal circle reading should be corrected by :

$$\text{Level correction} = \frac{d}{2n}(\Sigma W - \Sigma E)\tan\alpha \qquad ...(13.43)$$

where d = value of one division of the striding level

ΣW and ΣE = sum of west and east reading of the bubble end, reckoned from centre of bubble in direct and reversed position

α = altitude of star or R.M.

Programme of observations

The field observations are arranged in the following steps :

(1) With the face left, point twice the R.M. Read both the verniers of the horizontal circle at each pointing.

(2) With the face left, point twice the star and read both the verniers of the horizontal circle at each pointing. Note the timing of each pointing.

(3) Change the face. Read twice on the star with face right and note the time and the angles.

(4) Read twice upon the R.M. with face right.

Alternative programme of field observations

1. Set the instrument over the instrument mark. With both the plates clamped to zero, sight the R.M.
2. Turn the telescope in azimuth and bisect the star. Note the chronometer time.
3. Read the striding level and reverse it.
4. Read the circle.
5. Intersect the star again and note the time.
6. Read the striding level.
7. Read the circle.
8. Point to R.M. and read the circle.

5. (a) AZIMUTH BY EX-MERIDIAN OBSERVATIONS ON STAR

The determination of azimuth by ex-meridian observation of a star or sun is the method most commonly used by a surveyor except for the determination of primary standard. The observations are the same as that for the determination of time, and the two determinations may be combined if the watch times of the altitudes are also recorded. Knowing the latitude

5. Turn to the R.M., reverse the face and take another sight on the R.M.

6. Take two more observations of the sun precisely in the same way as in steps (3) and (4) above, but this time with the sun in quadrants 2 and 4. Note the time of each observation.

7. Finally bisect the R.M. to see that the reading is zero.

During the above four observations (two with face left and two with face right), the sun changes its position considerably, and accurate results cannot be obtained by averaging the measured altitudes and the times. However, the time taken between the first two readings, with the sun in quadrants 1 and 3, is very little and hence the measured altitudes and the corresponding times can be averaged to get one value of the azimuth. Similarly, the altitudes and the timings of the last two readings, with the sun in quadrants 2 and 4, can be averaged to get another value of the azimuth. The two values of azimuths so obtained (one with face left and the other with face right) can be averaged to get the final value of the azimuth.

For very precise work, the altitude readings should be corrected for the inclination, if any, of the trunnion axis as discussed earlier.

The reduction is performed in the same manner as for the corresponding star observation. The correct value of sun's declination can be computed by knowing the time of observation, by the methods discussed earlier.

The Effect of an Error in Latitude upon the Calculated Azimuth

Let y = error in co-latitude (c)

and x = the corresponding error in the calculated value of azimuth.

We know that $\cos p = \cos c \cos z + \sin c \sin z \cdot \cos A$ ...(1)

Hence $\cos p = \cos (c + y) \cos z + \sin (c + y) \sin z \cdot \cos (A + x)$ ...(2)

Subtracting these two and making the approximations that

$\sin x = x$, $\sin y = y$, $\cos x = 1$ and $\cos y = 1$, we get

$\cos z \cdot y \sin c + \sin z \sin c \cos A - \sin z (\sin c + y \cos c) \times (\cos A - x \sin A) = 0$

or $\cos z \cdot y \sin c - y \sin z \cos c \cos A + x \sin z \sin c \sin A = 0$

(neglecting the terms having product of x and y)

$$\therefore \quad x = \frac{-\cos z \sin c + \sin z \cos c \cos A}{\sin z \cdot \sin c \cdot \sin A}$$

which gives on simplification, $x = \dfrac{-\cot H}{\sin c} \cdot y$ ...(13.45)

It is clear from the above formula that for a given value of y, x is maximum when $\cot H$ is maximum, *i.e.*, when H is minimum. *Hence at all times near noon, the error in azimuth produced by a defective knowledge of the latitude is very much increased.* The error is least at 6 A.M. or 6 P.M. The error also increases with increase in the value of θ, and is the greatest near the pole.

The Effect of an Error in the Sun's Declination upon the Calculated Azimuth

Let y = error in the co-declination (p) of the sun.

x = corresponding error in calculated value of A.

Then $x = (\text{cosec } c \cdot \text{cosec } H) \cdot y$ ...(13.46)

For a given value of y, x is maximum at times near to noon, and is least at 6 A.M. and at 6 P.M.

Also, x increases as the latitude of the place increases. *This method becomes unreliable in arctic or antarctic regions where the given value of y produces very great error in the azimuth.*

The Effect of an Error in the Measured Altitude

Let y = error in the co-altitude (z)

x = corresponding error in the calculated value of azimuth

Then $x = -(\cot M \cdot \text{cosec } z)\ y$; where M = parallactic angle ZMP. ...(13.47)

The value of x is infinitely great when $M = 0°$ or $180°$, *i.e.* when the sun is on the meridian. *Hence, in this case also, it is concluded that the resulting error in azimuth is very great if the observations are made near noon.* The error is however, small if angle M is near 90°.

Example 13.48. *A star was observed at western elongation at a station A in latitude 54° 30′ N and longitude 52° 30′ W. The declination of the star was 62° 12′ 21″ N and its right ascension 10^h 58^m 36^s, the G.S.T. of G.M.N. being 4^h 38^m 32^s. The mean observed horizontal angle between the referring object B and the star was 65° 18′ 42″. Find (a) the altitude of star at elongation, (b) the azimuth of the line AP and (c) the local mean time of elongation.*

Solution

(a) *Altitude of the star, its hour angle and azimuth.*

Since the star is observed at elongation, the angle ZMP of the astronomical triangle ZMP is a right angle. Hence, from Napier's rule for circular parts.

$$\sin \alpha = \frac{\sin \theta}{\sin \delta} = \frac{\sin 54° 30'}{\sin 62° 12' 21''} \quad ...(1)$$

or $\alpha = 66° 58' 6''.7$

$\therefore$ Hence the altitude of the star $= \mathbf{66° 58' 6''.7}$.

Also, $$\sin A = \frac{\cos \delta}{\cos \theta} = \frac{\cos 62° 12' 21''}{\cos 54° 30'} \quad ...(2)$$

or $A = \mathbf{53° 25'}$

and $$\cos H = \frac{\tan \theta}{\tan \delta} = \frac{\tan 54° 30'}{\tan 62° 12' 21''}$$

or $H = 42° 21' 20'' = \mathbf{2^h\ 49^m\ 25.3^s}$ ...(3)

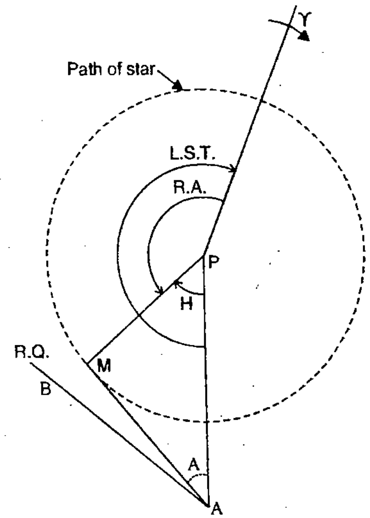


FIG. 13.51. STAR AT WESTERN ELONGATION.

To convert this L.S.T. to L.M.T., let us first find the L.S.T. of L.M.M. from the given value of G.S.T. at G.M.M.

$$\text{Longitude} = 5^h\ 40^m\ 18^s\ W$$

Acceleration for this at the rate of 9.8565 seconds per hour of longitude is

$$5^h \times 9.8565 = 49.28 \text{ seconds}$$
$$40^m \times 0.1642 = 6.57 \text{ seconds}$$
$$18^s \times 0.0027 = 0.05 \text{ second}$$

$$\text{Total correction} = 55.90 \text{ seconds}$$

$\therefore$ L.S.T. at L.M.M. = G.S.T. at G.M.M. + acceleration

$$= 4^h\ 58^m\ 23.84^s + 55.90^s = 4^h\ 59^m\ 19.74^s$$

Now S.I. between the L.M.M. and elongation

= L.S.T. – L.S.T. at L.M.M.

$$= 9^h\ 8^m\ 42.4^s - 4^h\ 59^m\ 19.74^s = 4^h\ 09^m\ 22.66^s$$

This may be converted to mean time interval by subtracting the retardation at the rate of 9.8296 seconds per sidereal hour.

$$\therefore \quad 4^h \times 9.8296 = 39.32 \text{ seconds}$$
$$9^m \times 0.1638 = 1.47 \text{ seconds}$$
$$22.66^s \times 0.0027 = 0.06 \text{ second}$$

$$\text{Total retardation} = 40.85 \text{ seconds}$$

Mean time interval = S.I. – retardation

$$= 4^h\ 09^m\ 22.66^s - 40.85^s = \mathbf{4^h\ 8^m\ 41.81^s}$$

Fig. 13.52 shows the relative positions, in plan, of the observer (*Z*), the pole (*P*), the star (*M*), the ♈, and referring object (R.O.).

Example 13.50. *At a place (Latitude 35° N, Longitude 15° 30′ E), the following observations were taken on a star :*

Observed angle between the R.M. and star = 36° 28′ 18″ (clockwise)

R.A. of star :	*$10^h\ 12^m\ 6.3^s$*
Declination of star :	*20° 6′ 48″.4*
G.M.T. of observation :	*$19^h\ 12^m\ 28.6^s$*
G.S.T. of G.M.M. :	*$10^h\ 12^m\ 36.2^s$*

Calculate the true bearing of the reference mark.

Solution

Here, the observations have been taken for the hour angle of the star to calculate the azimuth of the line. From the observed chronometer time (G.M.T.) let us first calculate the hour angle of the star.

G.S.T. of G.M.M. = $10^h\ 12^m\ 36.2^s$.

Since the place has western longitude, let us subtract the retardation from the given G.S.T. of G.M.M. to calculate the L.S.T. of L.M.M.

$$\text{Longitude} = 15°\ 30'\ E = 1^h\ 2^m\ E$$

$$1^h \times 9.8656 = 9.87 \text{ seconds}$$
$$30^m \times 0.1642 = 4.93 \text{ seconds}$$

$$\text{Total} = 14.80 \text{ seconds}$$

$\therefore$ L.S.T. of L.M.M. = $10^h\ 12^m\ 36.2^s - 14.80^s = \mathbf{10^h\ 12^m\ 21.4^s}$

Now G.M.T. of observation = $19^h\ 12^m\ 28.6^s$

Add east longitude = $1^h\ 2^m$

$\therefore$ L.M.T. of observation = $20^h\ 14^m\ 28.6^s$

Convert this L.M.T. into S.I. by adding the acceleration at the rate of 9.8656 per hour.

$$20^h \times 9.8656 = 197.13 \text{ seconds}$$
$$14^m \times 0.1642 = 2.30 \text{ seconds}$$
$$28.6^s \times 0.0027 = 0.79 \text{ second}$$

$\therefore$ Total = 200.22 seconds = $3^m\ 20.22^s$

$\therefore$ S.I. = Mean time + acceleration

$$= 20^h\ 14^m\ 28.6^s + 3^m\ 20.22^s = 20^h\ 17^m\ 48.82^s$$

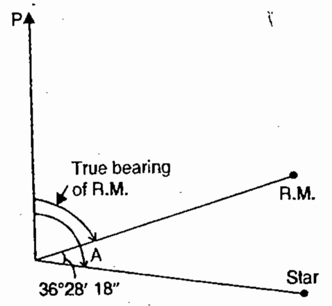


FIG. 13.53

$\therefore$ L.S.T. of observation = L.S.T. of L.M.M. + S.I.

$$= 10^h\ 12^m\ 21.4^s + 20^h\ 17^m\ 48.82^s$$
$$= 30^h\ 30^m\ 10.22^s$$

Subtract R.A. of star = $10^h\ 12^m\ 6.30^s$

$\therefore$ Hour angle of the star = $20^h\ 18^m\ 3.92^s$ = 304° 30′ 58″.8 (westerly)

$\therefore$ Smallest hour angle in arc (*i.e.* easterly hour angle)

$$= H_1 = 360° - H = 360° - 304°\ 30'\ 58''.8 = 55°\ 29'\ 1''.2 \qquad \text{...(1)}$$

Thus the hour angle is known to us.

The value of the azimuth (*A*) of the star is calculated from the following expression:

$$\tan A = \tan H \cdot \cos B \cdot \operatorname{cosec}(B - \theta) \qquad \text{(Eq. 13.37)}$$

where $\tan B = \tan \delta \sec H$ (Eq. 13.38) = tan 20° 6′ 48″.4 . sec 55° 29′ 1″.2

$$\therefore \quad B = 32°\ 52'\ 27''$$

date when G.S.T. of G.M.M. in $3^h\ 12^m\ 12^s$. Calculate the approximate direction, east or west of the meridian, and the altitude, at which the telescope should be pointed to locate the star so that exact observations may be made on it.

Solution. In order to calculate the hour angle of the star, let us first compute the L.S.T. of observation of the star.

G.M.T. of observation = $17^h\ 5^m\ 0^s$

To covert it into S.I., add the acceleration at the rate of 9.8656 seconds per hour.

$17^h \times 9.8656 = 167.56$ seconds

$5^m \times 0.1642 = 0.82$ second

Total = 168.38 seconds = $2^m\ 48.38^s$

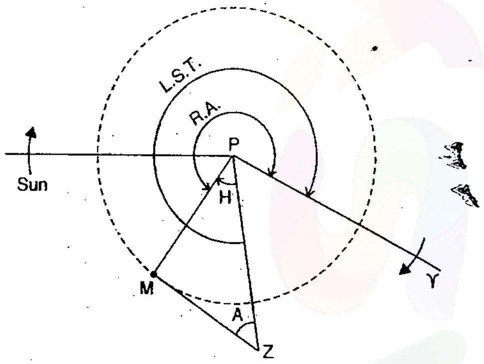


FIG. 13.54

$\therefore$ S.I. = G.M.T. + acceleration

$= 17^h\ 5^m + 2^m\ 48.38^s = 17^h\ 7^m\ 48.38^s$

$\therefore$ G.S.T. of observation = G.S.T. of G.M.M. + S.I.

$= 3^h\ 12^m\ 12^s + 17^h\ 7^m\ 48.38^s$

$= 20^h\ 20^m\ 0.38^s$

Add west longitude $= 1^h\ 18^m$

$\therefore$ L.S.T. of observation $= 21^h\ 38^m\ 0.38^s$

Subtract R.A. of star $= 16^h\ 23^m\ 30.0^s$

$\therefore$ H.A. of star $= 5^h\ 14^m\ 30.38^s = 78°\ 37'\ 36''$

In Fig. 13.54, M is the position of the star at the instant of observation, in relation to the sun and ϒ. Z is zenith of the observer and P is the pole.

$\therefore$ PM = co-declination = 90° − 29° 52′ = 60° 08′ = p

PZ = co-latitude = 90° − 52° 8′ = 37° 52′ = c

Now, from the astronomical triangle ZPM,

$$\cos H = \frac{\sin \alpha - \sin \delta \sin \theta}{\cos \delta \cdot \cos \theta} = \frac{\cos z - \cos p \cos c}{\sin p \cdot \sin c}$$

or $\cos z = \cos H \cdot \sin p \sin c + \cos p \cos c$

$= \cos 78°\ 37'\ 36'' \cdot \sin 60°\ 08' \cdot \sin 37°\ 52' + \cos 60°\ 08' \cdot \cos 37°\ 52'$

From which $z = 60°\ 7'\ 32''$

$\therefore$ Altitude of star = 90° − z = 29° 52′ 28″

Also by rule, $\dfrac{\sin A}{\sin p} = \dfrac{\sin H}{\sin z}$

$$\therefore \quad \sin A = \sin p \cdot \frac{\sin H}{\sin z} = \sin 60°\ 08' \cdot \frac{\sin 78°\ 37'\ 36''}{\sin 60°\ 7'\ 32''}$$

A = 78° 38′ 56″ (west).

Example 13.54. *Find the azimuth of the line QR from the following ex-meridian observations for azimuth.*

	Object	*Face*	*Altitude Level*	
			O	*E*
1	*Q*	*L*	...	...
2	*Sun*	*L*	*5.4*	*4.6*
3	*Sun*	*R*	*5.2*	*4.8*
4	*R*	*R*	...	...

	Horizontal Circle		*Vertical Circle*	
	A	*B*	*C*	*D*
1.	*30° 12′ 20″*	*210° 12′ 10″*	...	...
2.	*112° 20′ 30″*	*292° 20′ 20″*	*24° 30′ 20″*	*24° 30′ 40″*
3.	*293° 40′ 40″*	*113° 40′ 30″*	*25° 00′ 00″*	*25° 1′ 00″*
4.	*211° 50′ 30″*	*31° 50′ 20″*	...	...

Latitude of station Q = 36° 48′ 30″ N ; Longitude of station Q = $4^h\ 12^m\ 32^s$ E

Declination of the sun at G.M.N. = 1° 32′ 16″.8 N decreasing 56″.2 per hour

Mean of L.M.T. of two observations = $4^h\ 15^m\ 30^s$ P.M. by watch ; watch 4 seconds slow at noon, gaining 0.8 seconds per day.

The value of level division = 15″

Correction for horizontal parallax = 8″.76

Correction for refraction = 57″ cot (apparent altitude).

Solution.

Mean horizontal angle = $\frac{1}{2}$ [(112° 20′ 25″ − 30° 12′ 15″) + (293° 40′ 35″ − 211° 50′ 25″)]

= $\frac{1}{2}$ [(82° 8′ 10″ + 81° 50′ 10″)] = 81° 59′ 10″

Mean observed altitude = mean of the four vernier readings = 24° 45′ 30″

$$\text{Level correction} = + \frac{\Sigma O - \Sigma E}{4} \times \text{value of the one level division}$$

$$= + \frac{10.6 - 9.4}{4} \times 15'' = + 4''.5$$

$\therefore$ Apparent altitude = 24° 45′ 30″ + 4″.5 = 24° 45′ 34″.5

Refraction correction = − 57″ cot 24° 45′ 34″.5 = 1′ 6″.7

Correction for parallax = + 8″.77 cos 24° 45′ 34″.5 = 7″.8

$\therefore$ True altitude = 24° 45′ 34″.5 − 1′ 6″.7 + 7″.8 = 24° 44′ 35″.6

$EM_2 = \delta_2$ = declination of the star.

Now $EZ = ZM_2 + EM_2$

or $\theta = (90° - \alpha_2) + \delta_2$ or $\theta = z + \delta_2$

Hence *latitude = zenith distance + declination.*

Case 3. When the star is between the zenith and the pole.

M_3 is the position of the star when it is between the zenith and the pole.

$NM_3 = \alpha_3$ = altitude of the star

$ZM_3 = (90° - \alpha_3) = z_3$ = zenith distance of the star

$EM_3 = \delta_3$ = declination of the star

Now $EM = EM_3 - ZM_3$

or $\theta = \delta_3 - (90° - \alpha_3) = \delta_3 - z_3$

Hence *latitude = declination – zenith distance.*

Case 4. When the star is between the pole and the horizon.

M_4 is the position of the star when it is between the pole and the horizon.

$NM_4 = \alpha_4$ = altitude of the star

$ZM_4 = (90° - \alpha_4) = z_4$ = zenith distance of the star

$FM_4 = \delta_4$ = declination of the star.

Now PN = altitude of the pole = latitude of the place = θ

$= NM_4 + PM_4 = \alpha_4 + (PF - FM_4)$

$= \alpha_4 + (90° - \delta_4) = (90° - z_4) + (90° - \delta_4) = 180° - (z_4 + \delta_4)$

Hence *latitude = 180° –(zenith distance + declination).*

1. (b) LATITUDE BY MERIDIAN ALTITUDE OF THE SUN

The altitude of the sun at local apparent noon (meridian passage) may be measured by placing the line of sight of the transit in the plane of the meridian and observing the altitude of the upper or lower limb of the sun when it is on the vertical cross hair. The observed altitude is then corrected for instrumental errors, refraction, parallax and semi-diameter. The mean time of observation should also be noted. The declination of the sun continually changes, and hence a correct knowledge of mean time and longitude of the place of observation is essential in order to compute the value of declination at the instant of observation. Knowing the altitude and the declination of the sun at the instant of observation, the latitude can be computed as follows (Fig. 13.56).

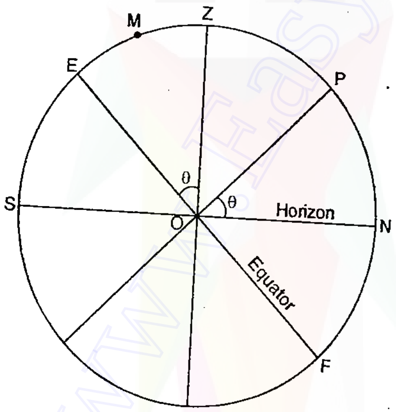


FIG. 13.56. MERIDIAN ALTITUDE OF THE SUN.

In Fig. 13.56, M is the position of the sun.

$SM = \alpha$ = meridian altitude of the sun (corrected).

$ZM = 90° - \alpha = z$ = meridian zenith distance of the sun.

$EM = \delta$ = declination of the sun.

Then latitude $= \theta = EZ = ZM + EM$

$= (90° - \alpha) + \delta = z + \delta$

or *latitude = zenith distance + declination.*

In the above expression, δ is positive or negative according as the sun is to north or south of the equator.

If the direction of the meridian is not known, the maximum altitude of the sun is observed and may be taken as the meridian altitude. This is not strictly true, due to sun's changing declination. However, the difference between the maximum altitude and the meridian altitude is usually a fraction of a second, and may be entirely neglected for observations made with the engineer's transit or the sextant.

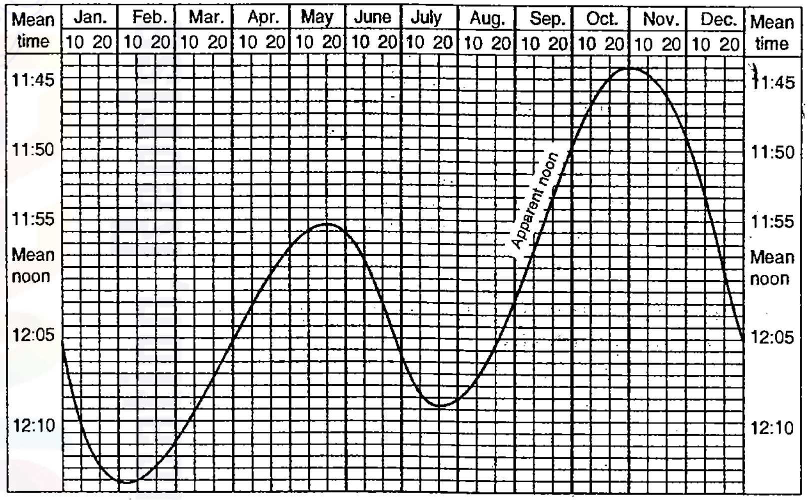


FIG. 13.57. MEAN TIME OF APPARENT NOON.

In order that the observer may be well ready for taking the observations at the meridian transit, standard time or the watch time of local apparent noon must be known. The standard time of local apparent noon varies throughout the year. Fig. 13.57 shows graphically the local mean time of the local apparent noon. The standard time can be known by applying a correction for the difference in longitude between the local meridian and the standard meridian. The observer should be ready to begin observing at this time.

(2) LATITUDE BY ZENITH PAIR OBSERVATIONS OF STARS

This method is an improvement over the previous method to get more precise results. The errors of observation, refraction and instrument can be effectively reduced by making observations upon two stars which culminate at approximately equal latitudes on opposite sides of observer's zenith. The altitude of one star at its culmination is observed first.

Substituting the value of m in equation (i), we get

$$\sin\delta = \sin\alpha \,.\, \sec(\theta - n) \,.\, \sin n$$

or
$$\cos(\theta - n) = \sin\alpha \,.\, \sin n \,.\, \operatorname{cosec}\delta \qquad \ldots(iv) \quad \ldots(13.50)$$

Thus, the value of n is obtained from equation (iii), and then substituted in equation (iv) to get the value of θ. For the use of the method of computation of θ, see example 13.60.

(5) LATITUDE BY PRIME VERTICAL TRANSIT

As defined earlier, the prime vertical is a plane at right angles to the meridian, running truly east and west. A star, having polar distance less than 90° and greater than the co-latitude of the place, will cross the prime vertical twice in a sidereal day. The field work, therefore, consists in measuring the time interval between east and west transits of the star. The best stars for observations are those that cross the prime vertical near the zenith.

Thus in Fig. 13.60 (a), S, W, N and E are the south, west, north and east points on the horizon. Z is the zenith of the observer, and P the pole. The dotted circle shows the path of a circumpolar star, WZE is the plane of the prime vertical passing through the west-east points and hence perpendicular to the meridian at Z. M_1 and M_2 are the east and west transits of the star across the prime vertical. Half the time that elapses between the two transits M_1 and M_2 in sidereal hours represents the angle M_1PZ (H).

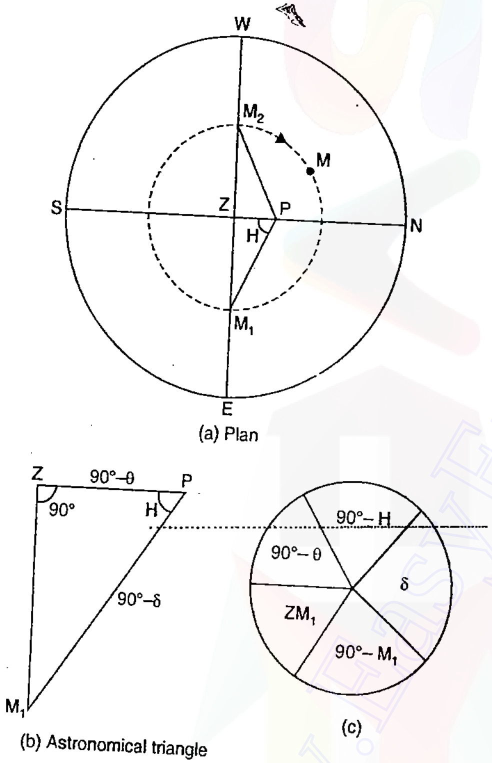


FIG. 13.60. LATITUDE BY PRIME VERTICAL TRANSIT

From the right angled triangle M_1PZ

$M_1P = 90° - \delta$ (known)

$\angle M_1PZ = H$ (known)

$ZP = (90° - \theta)$, to be computed.

From the Napier's rule for the right-angled triangle, [Fig. 13.60 (c)].

sine of the middle part = product of tangents of adjacent parts

$$\therefore \quad \sin(90° - H) = \tan(90° - \theta)\tan\delta$$

or
$$\cos H = \cot\theta \tan\delta$$

or
$$\tan\theta = \tan\delta \,.\, \sec H$$

where H = half the interval of time between the east and the west transits expressed in angular measure.

Since the altitude is not measured in this method, the errors due to uncertainty in the value of refraction is largely eliminated. Also, the exact knowledge of local time is not required since we have to simply measure the interval of sidereal hours that elapses between the two transits. However, the approximate local time of prime-vertical transits must be known. To take the time readings, the instrument has to be directed towards the direction of prime vertical, first to the east side and then to the west side, and measure the time when transit occurs, *i.e.*, where the star crosses the vertical cross-hair.

The effect of an Error in the Determination of the Time Interval

Let y = error in the determination of the *time interval*

and x = corresponding error in the latitude.

Then
$$x = y\,\frac{\sin 2\theta}{2}\sqrt{\frac{\tan^2\theta}{\tan^2\delta} - 1} \qquad \ldots(13.51)$$

From the above relationship between the two errors, we draw the following conclusions:

(1) If $\delta = \theta$, x is very small. However, the star would pass through the zenith and observations cannot be made.

(2) If $\delta = 0$, the star would pass through E and W points, the interval between the transits will be exactly 12 hours whatever may be the position of the observer and hence the determination cannot be made. The value of x will be great for very small value of δ.

Hence the stars observed should be as high up on the prime vertical as is consistent with an exact determination of the time of transit.

The effect of an Error in the Direction of Prime Vertical

The error in the setting out of the direction of the prime vertical has very little effect in the latitude of the place for ordinary engineering purposes. If the eastern transit occurs earlier due to the wrong direction of the prime vertical, the western transit will also take place correspondingly earlier, though not exactly by the same amount. In a latitude of 30°, even if the prime vertical is set out by 1° out of its true position, the resulting error in latitude determination will be less then 1″ for observations on a star having declination = 20°.

Striding Level Correction to Prime Vertical Determinations

For the prime vertical determinations, the instrument must be in perfect adjustment. If the transverse axis of the instrument is inclined by a certain value, the resulting error in the determination will be equal to this value. Hence striding level should always be used when taking the vertical observations.

Thus, in Fig. 13.61, if the transverse axis is inclined, ECW is the circle upon which observations are made instead of the true prime vertical EZW. The star is then observed to the transit at the point M on the inclined prime vertical. The observed angle $MPC = H$.

(7) LATITUDE BY CIRCUM-MERIDIAN ALTITUDE OF STAR OR THE SUN

The *circum-meridian observations* are the observations of stars or the sun taken near to the meridian. The method is used for very accurate determination of latitude by observing the circum-meridian altitudes at noted times of each of the several stars for a few minutes before and after transit and reducing them to the meridian altitude. The errors due to erroneous value of refraction, personal error and those due to instruments are very much reduced by observing an equal number of north and south stars in pairs of similar altitude. Accurate chronometer time and its error is also essential to calculate the hour angle of the individual stars. The observation of each star is commenced about 10^m before the computed time of transit and is continued for about 10^m after transit. Equal number of the face right and face left observations are necessary on a particular star. However, both face observations are not taken if observations are adequately paired on north and south stars.

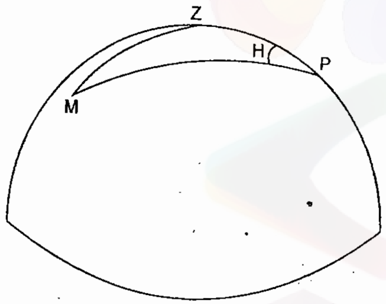


FIG. 13.63

In Fig. 13.63, let

$z = MZ$ = zenith distance of star M, corrected for refraction

$p = MP$ = polar distance

$c = PZ$ = co-latitude

$H = \angle MPZ$ = Hour angle

From the astronomical triangle MPZ, we get

$$\cos z = \cos c \cos p + \sin c \,.\, \sin p \cos H \qquad \ldots(1)$$

Let x = correction to be applied to the observed z to get the meridian zenith distance when the star is on meridian.

Then meridian zenith distance $= z - x$.

Again, when the star is on the meridian, its zenith distance

$$= MZ = MP - ZP = p - c.$$

Hence
$$z - x = p - c \qquad \ldots(2)$$

Writing $\cos H = 1 - 2\sin^2 \dfrac{H}{2}$ in (1), we get

$$\cos z = \cos c \cos p + \sin c \sin p - 2 \sin c \,.\, \sin p \,.\, \sin^2 \frac{H}{2}$$

or
$$\cos z = \cos (c - p) - 2 \sin c \,.\, \sin p \,.\, \sin^2 \frac{H}{2}$$

or
$$\cos z - \cos (p - c) = - 2 \sin c \,.\, \sin p \,.\, \sin^2 \frac{H}{2}$$

Substituting $p - c = z - x$ from (2), we get

$$\cos z - \cos (z - x) = - 2 \sin c \,.\, \sin p \sin^2 \frac{H}{2}$$

$$\therefore \quad \sin \frac{x}{2} \sin \left(z - \frac{x}{2} \right) = \sin c \,.\, \sin p \sin^2 \frac{H}{2}$$

From which
$$\sin \frac{x}{2} = \frac{\sin c \,.\, \sin p \,.\, \sin^2 \frac{H}{2}}{\sin \left(z - \frac{x}{2} \right)}$$

Since x is small, we can replace $\sin \frac{x}{2}$ by $\frac{x}{2} \sin 1''$, if x is measured in seconds of arc.

$$\therefore \quad x = \frac{\sin c \,.\, \sin p}{\sin \left(z - \frac{x}{2} \right)} \cdot \frac{2 \sin^2 \frac{H}{2}}{\sin 1''}$$

Also, putting $\sin \left(z - \dfrac{x}{2} \right) = \sin (z - x) = \sin (p - c)$ (approximately)

we get
$$x = \frac{\sin c \,.\, \sin p}{\sin (p - c)} \cdot \frac{2 \sin^2 \frac{H}{2}}{\sin 1''} \qquad \ldots(3)$$

But $\sin c = \cos \theta$; $\sin p = \cos \delta$

and $\sin (p - c) = \sin$ (meridian zenith distance) $= \cos$ (meridian altitude) $= \cos h$

where h = meridian altitude.

Then equation (3) reduces to

$$x = \frac{\cos \theta \cos \delta}{\cos h} \cdot \frac{2 \sin^2 \frac{H}{2}}{\sin 1''} \qquad \ldots(4) \quad (13.56)$$

But $h = \alpha + x$, where α = observed altitude

Hence
$$h = \alpha + \frac{\cos \theta \cos \delta}{\cos h} \cdot \frac{2 \sin^2 \frac{H}{2}}{\sin 1''} \qquad \ldots(6)$$

or
$$h = \alpha + Bm \qquad \ldots(13.57)$$

where
$$B = \frac{\cos \theta \,.\, \cos \delta}{\cos h} \quad \ldots[13.57\ (a)] \qquad \text{and} \qquad m = \frac{2 \sin^2 \frac{H}{2}}{\sin 1''} \quad \ldots[13.57\ (b)]$$

(H *is in arc measure*)

The factor m is usually taken from the tables.

If a series of observations are made upon the same star, the factor B is the same for each observation.

In the factor B, the value θ to be used is the approximate value deduced from the map or determined from the meridian observations. Similarly, h is the meridian altitude computed from the approximate latitude and the known declination of the star.

Let $\alpha_1, \alpha_2, \alpha_3 \ldots$ = circum-meridian altitudes of the same star

$m_1, m_2, m_3 \ldots$ = corresponding value of m.

Then $h_1 = \alpha_1 + Bm_1$; $h_2 = \alpha_2 + Bm_2$; $h_3 = \alpha_3 + Bm_3$ etc. etc.

Hence $h_0 = \alpha_0 + Bm_0$

Since the sun is to the south of the latitude,

$$\theta = \delta + z = 22° \; 18' \; 47''.05 + 45° \; 32' \; 37''.45 = 67° \; 51' \; 25''.5.$$

Example 13.58. *A star of declination 46° 45′ 33″ (south) is to be observed at lower and upper transit at a place in approximate latitude 80° south. Find the approximate apparent altitudes at which the star should be sighted in order that accurate observations may be made upon it.*

Solution

In Fig. 13.64, P' is the south pole and Z is the zenith of the observer. EO is the equator, and NS is the horizon, N and S being north and south points on it. M_1 is the position of the star at its upper transit and M_2 is the position at lower transit.

α_1 = apparent altitude at upper transit (north)

α_2 = apparent altitude at lower transit (south)

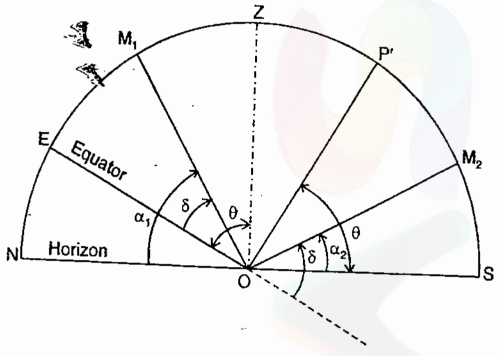


FIG. 13.64

Now $\alpha_1 = NOM_1 = NOZ - M_1OZ = 90° - (EOZ - EOM_1)$

$$= 90° - (\theta - \delta) = 90° - \theta + \delta = 90° - 80° + 46° \; 45' \; 30'' = \mathbf{56° \; 45' \; 30'' \; N}.$$

Similarly, $\alpha_2 = SOM_2 = P'OS - P'OM_2 = \theta - (90° - \delta) = \theta - 90° + \delta$

$$= 80° - 90° + 46° \; 45' \; 30'' = \mathbf{36° \; 45' \; 30'' \; S}.$$

Example 13.59. *The following data relate to an observation of latitude by zenith pair. Calculate the latitude.*

Star	*Declination*	*Observed altitude at transit*
M_1	20° 25′ 48″ S	48° 18′ 12″ N
M_2	79° 30′ 52″ S	47° 54′ 6″ S

Solution.

In Fig. 13.64, M_1 and M_2 denote the two stars ; P' is the south pole.

From the observations to star M_1 :

Latitude $= \theta = EOZ = NOZ - NOE = 90° - (NOM_1 - EOM_1) = 90° - \alpha_1 + \delta_1$ ...(1)

where α_1 = altitude of star M_1 and δ_1 = declination of the star M_1

From the observations to star M_2 :

Latitude $\theta = P'OS = 90° - (\delta_2 - \alpha_2) = 90° - \delta_2 + \alpha_2$ ...(2)

where α_2 = altitude of star M_2 and δ_2 = declination of star M_2

Hence average latitude $= \frac{1}{2}\left[(90° - \alpha_1 + \delta_1) + (90° - \delta_2 + \alpha_2)\right] = 90° - \frac{\alpha_1 - \alpha_2}{2} + \frac{\delta_1 - \delta_2}{2}$

In the above expression, α_1 and α_2 are the observed altitudes. These two altitudes are not exactly equal, and hence there will be little difference in the refraction correction for the two altitudes.

Taking into account the refraction correction, we have

$$\alpha_1' \text{ (corrected)} = \alpha_1 - r_1 \;; \qquad \text{and } \alpha_2' \text{ (corrected)} = \alpha_2 - r_2$$

where r_1 and r_2 are the refraction corrections.

$$\text{Hence average latitude} = 90° - \frac{\alpha_1 - \alpha_2}{2} + \frac{\delta_1 - \delta_2}{2} + \frac{r_1 - r_2}{2}$$

Here $r_1 = 58'' \cot \alpha_1 = 58'' \cot 48° \; 18' \; 12'' = 51''.68$

$r_2 = 58'' \cot \alpha_2 = 58'' \cot 47° \; 54' \; 6'' = 52''.41$

Substituting the values, we get

$$\text{Average latitude} = 90° - \frac{48° \; 18' \; 12'' - 47° \; 54' \; 6''}{2} + \frac{20° \; 25' \; 48'' - 79° \; 30' \; 52''}{2} + \frac{51''.68 - 52''.41}{2}$$

$$= 90° - 24' \; 6'' - 59° \; 5' \; 4'' - 0''.36 = \mathbf{30° \; 30' \; 49''.64}$$

It will be seen here that if the effect of refraction is *assumed* to be cancelled, the latitude will be 30° 30′ 50″. The effect of refraction is thus extremely small, and may be almost neglected if latitude is required to an accuracy of nearest 1″.

Example 13.60. *The altitudes of a star were observed at its upper and lower culmination at a place in north latitude and corrected for refraction. The values obtained are as follows:*

Star : α Aldebaran

Altitude at lower culmination = 18° 36′ 40″

Altitude at upper culmination = 59° 48′ 20″

Find the latitude of the place and the declination of the star.

Solution. (Fig. 13.58)

$$\text{The latitude } \theta = \frac{\alpha_1 + \alpha_2}{2} = \frac{18° \; 36' \; 40'' + 59° \; 48' \; 20''}{2} = 39° \; 12' \; 30''$$

Declination of the star $= EA = EZ + ZA = EZ + (ZN - AN) = \theta + (90° - \alpha_1)$

$= 90° + \theta - \alpha_1 = 90° + 39° \; 12' \; 30'' - 59° \; 48' \; 20'' = \mathbf{69° \; 24' \; 10''}$.

Check : Declination $= EA = EP - AP = 90° - AP = 90° - BP = 90° - (\theta - \alpha_2)$

$= 90° - 39° \; 12' \; 30'' + 18° \; 36' \; 40'' = \mathbf{69° \; 24' \; 10''}$.

Example 13.61. *A star was observed for latitude determination, and its corrected altitude is 40° 36′ 30″. The declination of the star is 10° 36′ 40″ and hour angle is 46° 36′ 20″. Compute the latitude of the place of observation.*

Solution. (Fig. 13.59). The latitude of the place is computed from the formula

$$\sin \alpha = \sin \theta \sin \delta + \cos \theta \; . \cos \delta \; . \cos H \qquad ...(1)$$

To solve this equation for θ, let $\sin \delta = m \sin n$ and $\cos \delta \cos H = m \cos n$.

Then, by reduction, the value of n is given by

$$\tan n = \tan \delta \sec H = \tan 10° \; 36' \; 40'' \sec 46° \; 36' \; 20''$$

or $n = 15° \; 15' \; 12''$

$\tan\theta = \tan\delta \,.\, \sec H$ ...(1)

Let us first calculate the hour angle (H) of the star at its prime vertical transit.

Interval between the passage across prime vertical $= 9^h\,22^m\,6^s$ meantime.

To convert it into sidereal time interval add acceleration at the rate of 9.8565 seconds per hour of meantime.

$$9^h \times 9.8565 = 88.71 \text{ seconds}$$
$$22^m \times 0.1642 = 3.61 \text{ seconds}$$
$$6^s \times 0.0027 = 0.02 \text{ second}$$

Total acceleration = 92.34 seconds $= 1^m\,32.34^s$

$\therefore$ Sidereal time interval $= 9^h\,22^m\,6^s + 1^m\,32.34^s$

$= 9^h\,23^m\,38.34^s = 140°\,54'\,35''.1$

$\therefore$ H = half the time interval $= 70°\,27'\,17''.55$

Hence $\tan\theta = \tan 15°\,20'\,48'' \sec 70°\,27'\,17''.55$

$\therefore$ $\theta = 39°\,20'\,25''.6$

Since the trunnion axis is inclined, let us correct the value,

Error due to striding level $= \dfrac{N-S}{2} \times d = \dfrac{16-11}{6} \times 16 = 40''$

As the north end of the axis is higher, the correction is additive.

Hence correct $\theta = 39°\,20'\,25''.6 + 40'' = \mathbf{39°\,21'\,5''.6}$.

Example 13.64. *In longitude 7° 20′ W, an observation for latitude was made on Polaris on a certain day. The mean of the observed latitude was 48° 36′ 40″ and the average of the local mean times, $20^h\,24^m\,50^s$. The readings of the barometer and thermometer were 30.42 inches and 58° F respectively. Find the latitude, given the following:*

R.A. of Polaris $= 1^h\,41^m\,48.64^s$

Declination of Polaris $= 88°\,58'\,28''.26$

G.S.T. of G.M.M. $= 16^h\,48^m\,20.86^s$.

Solution

(a) Calculation of polar distance.

From Chamber's Mathematical Tables (page 431)

Mean refraction for 48° 36′ 40″	= 51″
Correction for 58°F temp.	= − 1″
Correction for barometer	= + 1″
∴ Refraction correction	= 51″ (subtractive)

$\therefore$ True altitude = observed altitude − refraction $= 48°\,36'\,40'' - 51'' = 48°\,35'\,49''$.

(b) Calculation of hour angle (H).

The hour angle can be calculated by subtracting the R.A. from L.S.T.

Longitude $= 7°\,20'\,W = 0^h\,29^m\,20^s\,W$

Acceleration at the rate of 9.8565 seconds per hour of longitude :

$$29^m \times 0.1642 = 4.76 \text{ seconds}$$
$$20^s \times 0.0027 = 0.05 \text{ seconds}$$

$\therefore$ Acceleration = 4.81 seconds

$\therefore$ L.S.T. of L.M.M. = G.S.T. of G.M.M. + acceleration

$= 16^h\,48^m\,20.86^s + 4.81^s = 16^h\,48^m\,25.67^s$

L.M.T. of observation $= 20^h\,24^m\,50^s$

To convert it into sidereal interval, add acceleration at the rate of 9.8565 seconds per mean hour.

$$20^h \times 9.8565 = 197.13 \text{ seconds}$$
$$24^m \times 0.1642 = 3.94 \text{ seconds}$$
$$50^s \times 0.0027 = 0.14 \text{ second}$$

Total acceleration = 201.21 seconds $= 3^m\,21.21^s$

Sidereal interval since L.M.M. = Meantime interval + acceleration.

$= 20^h\,24^m\,50^s + 3^m\,21.21^s$.

$= 20^h\,28^m\,11.21^s$

Add L.S.T. of L.M.M. $= 16^h\,48^m\,25.67^s$

$\therefore$ L.S.T. $= 37^h\,16^m\,36.88^s - 24^h$

$= 13^h\,16^m\,36.88^s$

Deduct R.A. of Polaris $= 1^h\,41^m\,48.64^s$

$\therefore$ Hour angle $(H) = 11^h\,34^m\,46.24^s = 173°\,42'\,3''.6$.

Now, latitude $\theta = \alpha - p\cos H + \frac{1}{2}\sin 1''\,p^2\sin^2 H \,.\, \tan\alpha$.

p = polar distance $= 90° - 88°\,58'\,28''.26 = 1°\,1'\,31''.74 = 3691''.74$

First correction $= p\cos H = 3691''.74 \cos 173°\,42'\,3''.6 = -3669''.5 = -1°\,1'\,9''.5$

Second correction $= \frac{1}{2}\sin 1''\,p^2\sin^2 H \,.\, \tan\alpha$

$= \frac{1}{2} \times \dfrac{1}{206265}(3691.74)^2 \sin^2(173°\,42'\,3''.6)\tan 48°\,35'\,49'' = +0''.5$.

(*Note.* The above calculations for first and second corrections may be done with a five figure log-table if the answer is required to the nearest 1″.)

Hence $\theta = 48°\,35'\,49'' - (-1°\,1'\,9''.5) + 0''.5 = \mathbf{49°\,36'\,59''}$ **N.**

Chronometer is a very delicate instrument. The main difficulty arises from the fa that its rate while being transported, and while it is stationary is not the same. Hen the *travelling rate* of the chronometer should also be ascertained for precise determination Suppose it is required to determine the difference in longitude between two stations A an B, the chronometer being regulated to give the time of station A. The *'rate'* of the chronomete *i.e.*, the amount by which it gains or loses in 24 hours is found at A. The chronomete is then transported to the station B of unknown longitude and its error is determined wi reference to this meridian. *If the chronometer runs perfectly, the two watch correctio will differ by just the difference in longitude.*

The method is now not used by surveyors except where wireless or telegraphic communicatio are not available. However, it is still used for the determination of longitude at sea.

(2) LONGITUDE BY ELECTRIC TELEGRAPH

If the two places are connected by an electric telegraph, the longitude can be determine very accurately by sending telegraphic signals in opposite directions for the chronomete times (local). Let A and B be the stations, A being to the east of B.

Let t_1 = local time of A at which the signal is sent from A to B.

and t_2 = local time of B at which the signal is received at B.

If the transmission time is neglected, the difference in longitude (ϕ) is given b $\phi = t_1 - t_2$, t_1 being greater than t_2.

If, however, s is the time of transmission, $(t_1 + s)$ is the actual local time of A corresponding to the local time t_2 at B. Hence the difference in longitude is

$$\phi = (t_1 + s) - t_2 = (t_1 - t_2) + s \qquad ...(1)$$

Similarly, let a signal be sent in the reverse direction from B to A.

Let t_2' = local time of B at which the signal is sent from B to A,

t_1' = local time of A at which the signal is received.

If the transmission time is neglected, we get

$$\phi = t_1' - t_2'.$$

If, however, s is the time taken in transmitting the signal $(t_2' + s)$ is the actual local of B corresponding to the local time t_1' of A. Hence the difference in longitude is

$$\phi' = t_1' - (t_2' + s) = t_1' - t_2' - s$$

By averaging the two results, we get

$$\text{Difference in longitude} = \frac{1}{2}\left\{ (t_1 - t_2 + s) + (t_1' - t_2' - s) \right\} = \frac{1}{2}\left\{ (t_1 - t_2) + (t_1' - t_2') \right\}.$$

(3) LONGITUDE BY WIRELESS SIGNALS

The advent of wireless signals has rendered the carrying of the time of the reference meridian comparatively easy and most accurate. Time signals are now sent out from various wireless stations at stated intervals, and the surveyor, by their aid, may check his chronometer in almost any part of the world. A list of wireless signals, their times and durations of emission together with their wave lengths and type of signals, is given in the *Admiralty list of wireless signals*, which is published annually ; and changes or any corrections are notified in the weekly *Notices to Mariners*. Greenwich meantime signals are sent and usually

continue, for a period of five minutes. The signals are rhythmic and consist of a series of 61 Morse dots to the minute, the beginning and end of each minute being denoted by a dash, which is counted as zero of the series which follows.

PROBLEMS

1. At a point A in latitude 50° N, a straight line is ranged out, which runs due east of A. This straight line is prolonged for 60 Nautical miles to B. Find the latitude of B, and if it be desired to travel due North from B so as to meet the 50° parallel again at C, find the angle ABC at which we must set out, and the distance BC. (U.L.)

2. The R.A. of a star being $20^h\,24^m\,13.72^s$, compute the L.M.T. of its culmination at Madras (Long. 80° 14′ 19″.5 E) on Sept. 6, the G.S.T. at 0^h G.M.T. on that date being $22^h\,57^m\,06.95^s$

3. Find the L.S.T. at a station in longitude 76° 20′ E at 9.30 A.M. (Indian Zone Time) on August 10 on that date at G.M.M. The R.A. of mean sun is $9^h\,13^m\,30.9^s$. (G.U.)

4. From the N.A., it is found that on the date of observation, G.S.T. of G.M.N. is $3^h\,14^m\,26^s$. Taking retardation as 9.85 sec. per hour of longitude, find the L.M.T. in a place 75° W, when the local sidereal time is $5^h\,20^m\,0^s$. (B.U.)

5. Find the local mean time at which β Leonis made its upper transit on 1st May 1940 at a place 60° E. Given R.A. of β Leonis on 1st May was $11^h\,46^m\,02^s$ and G.S.T. of G.M:N was $9^h\,23^m\,23^s$. (B.U.)

6. Find the R.A. of the meridian of Bombay at 4.30 P.M. Given : Longitude of Bombay 72° 48′ 46″.8 East ; G.S.T. at G.M.M.= $10^h\,10^m\,40.73^s$ on that day.
(*Note*: R.A. of a place = L.S.T.)

7. What are the systems of co-ordinates employed to locate position of a heavenly body ? Why it is necessary, to have several systems instead of one ?

8. Explain the systems of time reckoning known as sidereal apparent solar and mean solar time, and show how they differ from each other. (I.R.S.E.)

9. What is equation of time ? Show, by means of sketches, that it vanishes four times a year.

10. Explain with aid of sketches how the quantities of the following groups are related to each other:
    (*i*) The R.A. of a star, the hour angle of the star at any instant and the sidereal time at that instant.
    (*ii*) Equation of time, apparent time and mean time.
    Show that the equation of time vanishes four times in a year. (A.M.I.E.)

11. (*a*) Explain the following terms :
    (*i*) Equation of time , (*ii*) Celestial sphere, (*iii*) Parallax, and (*iv*) Sidereal time.
    (*b*) An observation was made on Dec. 30, 1919 in longitude 82° 17′ 30″ E; the meridian altitude of the sun's lower limb was 40° 15′ 13″. The sun was on the south of the observer's zenith. Calculate the approximate latitude of the place. Correction for refraction 1′ 10″ ; for parallax = 6″.9 ; correction for semi-diameter 16′ 17″.5. Declination of star at G.A.N.= 23° 13′ 15″. decreasing at the rate of 9″.17 per hour (B.U.)

12. What are 'parallax' and 'refraction' and how do they affect the measurement of vertical angles in astronomical work ?
    Give rough values of the corrections necessary when measuring a vertical angle of 45°. (A.M.I.C.E.)

13. In longitude 60° W, an observation was made on β Tauri, whose R.A. was $5^h\,21^m\,59.48^s$. If the hour angle of the star was $9^h\,15^m\,8^s$, find the local mean time of observation.
    Given G.S.T. at G.M.N. = $14^h\,46^m\,39.53^s$.

29. A meridian altitude of the lower limb of the sun is taken on 5th Nov. 1934 in latitude N, longitude 78° 25′ W. Given the observed altitude = 47° 18′ 44″, parallax = 6″, refraction = 53″.6; declination of the sun at mid-night 4/5 Nov. 1934 = *S* 15° 24′ 27″.4 with an hourly variation of 46″.23 increasing; semi-dia. 16′ 9″.5.

Calculate the latitude of the observer's station, the equation of time at mid-night 4/5 Nov. 1934 is $+16^m\,21.5^s$ with an hourly variation -0.044^s. (*A.I.M.E.*)

Answers

1. Latitude of $B = 49°\ 59'\ 22''.6$; $\angle ABC = 88°\ 48'\ 40''$; $BD = 0.624$ Nautical miles.
2. $21^h\ 24^m\ 28.48^s$.
3. $6^h\ 19^m\ 30.3^s$.
4. $2^h\ 4^m\ 24.31^s$.
5. $2^h\ 22^m\ 54.49^s$ P.M..
6. $2^h\ 42^m\ 35.51^s$
11. (*b*) N 26° 17′ 7″.91.
12. + 6″ ; – 57″
13. $23^h\ 45^m\ 54.29^s$.
14. $5^h\ 4^m$ P.M. nearly.
16. $10^h\ 45^m\ 23.27^s$; 32° 26′ 16″ *N*.
18. Chronometer slow 2.5^s
19. 56.67^s slow.
20. $\theta = 19°\ 30'\ 22''.6$; L.M.T.'s : $9^h\ 52^m\ 40^s$ P.M. for M_1 ; $10^h\ 06^m\ 10^s$ P.M. for M_2
21. Choronometer slow 28^s
22. 112° 43′ 56″
23. 24° 2′ 8″ from south.
24. 4° 24′ 12″.
25. $7^h\ 58^s\ 19.13^s$: 180° 45′ 7″.75 from S point.
26. (*a*) West (*b*) $0^h\ 57^m\ 27.6^s$ Jan. 2; 315° 56′ 38″
27. 55° 10′ 40″ ; $8^h\ 12^m\ 34.2^s$ P.M.
28. 46° 03′ 36″ *N*.
29. 26° 47′ 56″.7 *N*.

14

Photogrammetric Surveying

14.1. INTRODUCTION

Photogrammetric surveying or photogrammetry is the science and art of obtaining accurate measurements by use of photographs, for various purposes such as the construction of planimetric and topographic maps, classification of soils, interpretation of geology, acquisition of military intelligence and the preparation of composite pictures of the ground. The photographs are taken either from the air or from station on the ground. *Terrestrial photogrammetry* is that branch of photogrammetry wherein photographs are taken from a fixed position on or near the ground. *Aerial photogrammetry* is that branch of photogrammetry wherein the photographs are taken by a camera mounted in an aircraft flying over the area. Mapping from aerial photographs is the best mapping procedure yet developed for large projects, and are invaluable for military intelligence. The major users of aerial mapping methods are the civilian and military mapping agencies of the Government.

The conception of using photographs for purposes of measurement appears to have originated with the experiments of Aime Laussedat of the Corps of Engineers of the French Army, who in 1851 produced the first measuring camera. He developed the mathematical analysis of photographs as perspective projections, thereby increasing their application to topography. Aerial photography from balloons probably began about 1858. Almost concurrently (1858), but independently of Laussedat, Meydenbauer in Germany carried out the first experiments in making critical measurements of architectural details by the intersection method on the basis of two photographs of the building. The ground photography was perfected in Canada by Capt. Deville, then Surveyor General of Canada in 1888. In Germany, most of the progress on the theoretical side was due to Hauck.

In 1901, Pulfrich in Jena introduced the stereoscopic principle of measurement and designed the *sterecomparator*. The *stereoautograph* was designed (1909) at the Zeiss workshops in Jena, and this opened a wide field of practical application. Scheimpflug, an Australian captain, developed the idea of double projector in 1898. He originated the theory of perspective transformation and incorporated its principles in the *photoperspectograph*. He also gave the idea of radial triangulation. His work paved the way for the development of aerial surveying and aerial photogrammetry.

In 1875, Oscar Messter built the first aerial camera in Germany and J.W. Bagloy and A. Brock produced the first aerial cameras in U.S.A. In 1923, Bauersfeld designed the *Zeiss stereoplanigraph*. The optical industries of Germany, Switzerland, Italy and France,

(6) On the top of the box, a *telescope* is fitted. The telescope can be rotated in a vertical plane, about a horizontal axis, and is fitted with vertical arc with verniers, clamp, and slow motion screw. The line of sight of the telescope is set in the same vertical plane as the optical axis of camera.

14.4. DEFINITIONS (Fig. 14.4)

Camera Axis. Camera axis is the line passing through the centre of the camera lens perpendicular both to the camera plate (negative) and the picture plane (photograph). The optical axis coincides with the camera axis in a camera free from manufacturing imperfections.

Picture Plane. Picture plane is the plane perpendicular to the camera axis at the focal distance in front of the lens. It is represented by the positive contact print or *photograph* taken from a plate or film.

Principal Point. Principal point (k or k') is defined by the intersection of the camera axis with either the picture plane (positive) or the camera plate (negative).

Focal Length. Focal length (f) is the perpendicular distance from the centre of the camera lens to either the picture plane or the camera plate. It satisfies the following relation

$$f = \frac{uv}{u + v}$$

where u and v are conjugate object and image distances.

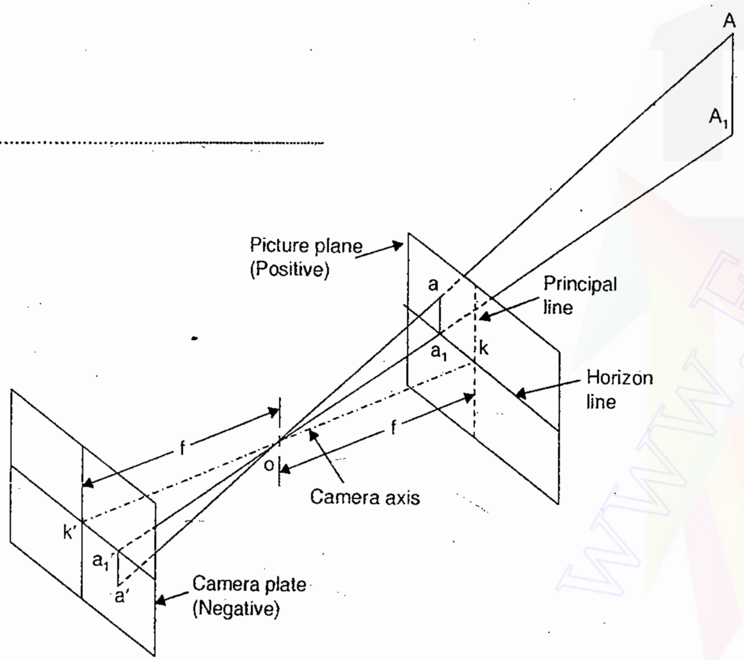


FIG. 14.4

Focal Plane (Image Plane). Focal plane is the plane (perpendicular to the axis of the lens) in which images of points in the object space of the lens are focused.

Nodal Point. Nodal point is either of two points on the optical axis of a lens (or a system of lenses) so located that when all object distances are measured from one point and all image distances are measured from the other, they satisfy the simple lens relation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

Also a ray emergent from the second point is parallel to the ray incident at the first.

Perspective Centre. Perspective centre is the point of origin or termination of bundles of perspective rays. The two such points usually associated with a survey photograph are the interior perspective centre and the exterior perspective centre. In a distortionless lens camera system, one perspective centre encloses the same angles as the other, and in a perfectly adjusted lens camera system, the interior and exterior centres correspond to the rear and front nodal points, respectively.

Principal Distance. When the contact prints from original negatives are enlarged (or reduced) before their use in the compilation of subsequent maps, the value of the focal length (f) of the camera is not applicable to the revised prints. The changed value of f, holding the same geometrical relations, is known as the *principal distance*. In other words, it is the perpendicular distance from the internal perspective centre to the plane of a particular finished negative or print. This distance is equal to the calibrated focal length corrected for both the enlargement or reduction ratio and the film (or paper) shrinkage (or expansion) and maintains the same perspective angles as the internal perspective centre to points on the finished negative or print as existed in the camera at the moment of exposure. This is a geometrical property of each particular finished negative or print.

Principal Plane. Principal plane is plane which contains principal line and the optical axis. It is, therefore, perpendicular to the picture plane and the camera plate.

Print. A print is a photographic copy made by projection or contact printing from a photographic negative or from a transparent drawing as in blue-printing.

Fiducial Mark. A fiducial mark is one of two, three or four marks, located in contact with the photographic emulsion in a camera image plane to provide a reference line or lines for the plate measurement of images.

Fiducial Axis. Opposite fiducial marks define a reference line. Two pairs of opposite fiducial marks define two reference lines that intersect at 90°. These two lines are referred to as the x and y axes or the fiducial axes.

Film Base. Film base is a thin, flexible, transparent sheet of cellulose nitrate, cellulose acetate or similar material, which is coated with a light sensitive emulsion and used for taking photographs.

14.5. HORIZONTAL AND VERTICAL ANGLES FROM TERRESTRIAL PHOTOGRAPH

The horizontal and vertical angles to various points in a photograph can easily be found analytically, graphically or instrumentally. Fig. 14.5 (a) shows two points A and B photographed with camera axis horizontal so that the picture plane is vertical and the horizon line is horizontal. The image of the ground points A and B appear at a and b respectively

$a_1 k_1 b_1$ and $a_2 k_2 b_2$ can be drawn at perpendicular distances of $2f$ from P and Q respectively. On each photograph, the x-co-ordinates of points a and b are scaled by a pair of proportionate dividers set for a 2 to 1 ratio, and transferred to the photograph traces, as shown by the positions a_1, b_1 and a_2 and b_2 respectively in both the photographs taken with the camera axes horizontal at the time of exposure. Join Pa_1 and Pb_1 and prolong them. Similarly, join Qa_2 and Qb_2 and prolong them to intersect the corresponding lines in A and B respectively, thus giving horizontal positions of A and B.

Camera Position by Resection. To fix the positions of the camera stations, a separate ground control is necessary. However, the camera station can also be located by three point resection if the positions of three prominent points (which may be photogrammetric triangulation stations) are known and they are also photographed.

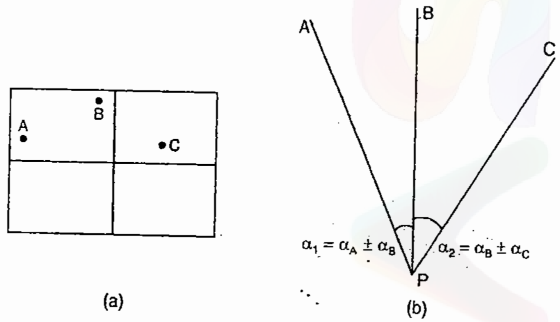


FIG. 14.7. CAMERA POSITION BY THREE POINT RESSECTION.

Thus, in Fig. 14.7. (a), let A, B and C be the three stations photographed. From § 14.5, the angles to A, B and C can be determined either graphically or analytically and hence angles α_1 ($= \alpha_A \pm \alpha_B$) and α_2 ($= \alpha_B \pm \alpha_C$) are known. If these angles are known graphically, a tracing paper resection on the plotted positions of A, B and C (on the map) will fix the map position of the camera station (P). If, however, the angles α_1 and α_2 are known analytically, the values may be set off by a three armed protractor for a graphical resection, or the values may be used to solve the three-point problem analytically for determining the position of the camera station.

Azimuth of a line from Photographic Measurement. The magnetic bearing or azimuth of the principal vertical plane is given by the reading of the cylindrical scale at its intersection with the vertical hair on the photograph. The horizontal angles of the lines with the principal plane can be calculated as discussed in § 14.5.

Thus, in Fig. 14.8(a), a, b and c are the positions of the three points A, B and C. The horizontal angles α_A, α_B and α_C (Fig. 14.8 b) can be determined. If ϕ is the azimuth of the principal plane (or the camera azimuth), we have

$$\phi_B = \text{azimuth of } B = \phi + \alpha_B$$

$$\phi_C = \text{azimuth of } C = \phi + \alpha_C$$

$$\phi_A = \text{azimuth of } A = \phi - \alpha_A + 360°$$

In general, therefore, we have

$$\textit{Azimuth of line} = \textit{camera azimuth} + \alpha$$

Due regard must be given to the algebraic sign of α. It may be considered positive when measured to the right of ok and negative when measured to the left. If the azimuth

calculated from the above relation comes out to be negative, 360° must be added to the result.

Orientation of Picture Traces

The accuracy in the plotted positions of various points depends upon the correct orientation and placing of picture traces on the plan. The two conditions that are to be fulfilled are: (1) the picture trace should be perpendicular to the line joining the plotted position (O) of the station and the principal point (k), and (2) the principal point (k) should be at the focal distance from O. When enlargements are used, the enlarged focal length should be laid down.

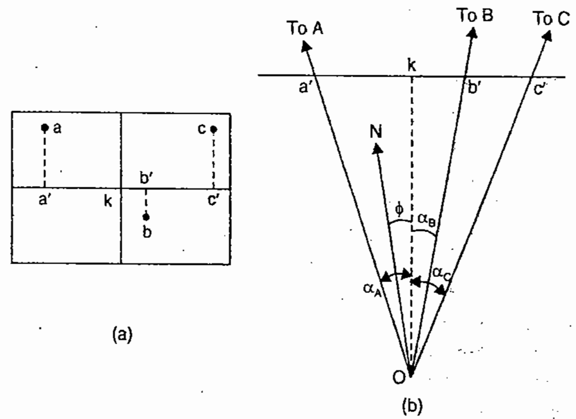


FIG. 14.8. AZIMUTH OF LINES FROM PHOTOGRAPHIC MEASUREMENT

In the case of photo-theodolite used for the photographic surveying, the bearing of the principal plane is known. In that case, the principal plane is laid at the known bearing, the principal point (k) is marked at a distance (f) from the camera station (O) and the picture trace is drawn perpendicular to that of the principal plane.

If, however, the photograph includes any point whose position is known on the plane, the orientation may be performed with respect to it as follows : (Fig. 14.9).

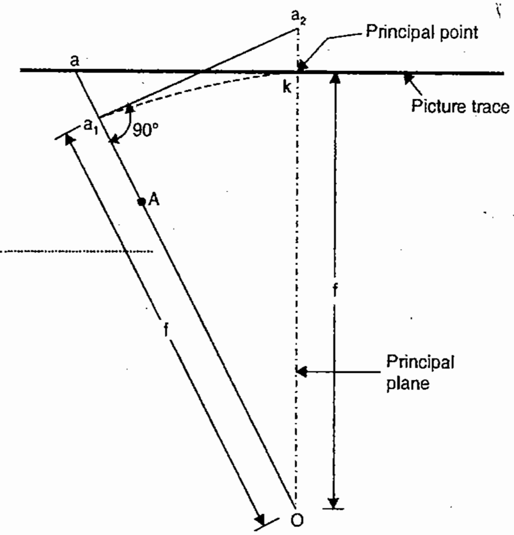


FIG. 14.9. ORIENTATION OF PICTURE TRACE FROM KNOWN POSITION OF POINT

Let A be the known position (on the plane) of the point and O be the known position of the camera station. Let ka be the distance (on the photographs) of the point A from the principal plane. Join OA and produce it. With O as the centre and radius equal to f ($= oa_1$), draw an arc. At a_1, draw a line a_1a_2 perpendicular to oa_1, making a_1a_2 equal to the photographic distance ak. Join a_2o, cutting the arc in k. Thus, the position of the principal point and that of the principal plane is known. Through k, draw ka perpendicular to ok, thus giving us position of the picture trace.

$$\therefore \quad \tan \alpha_a \cdot \tan \alpha_b = \frac{x_a x_b}{f^2}$$

Now
$$\tan \theta = \tan(\alpha_a + \alpha_b) = \frac{\tan \alpha_a + \tan a_b}{1 - \tan \alpha_a \tan \alpha_b} = \frac{\dfrac{x_a}{f} + \dfrac{x_b}{f}}{1 - \dfrac{x_a \cdot x_b}{f^2}}$$

$$\tan \theta\,(f^2 - x_a \cdot x_b) = f\,(x_a + x_b) \qquad \text{or} \qquad f^2 - \frac{(x_a + x_b)\,f}{\tan \theta} - x_a x_b = 0$$

which gives,

$$f = \frac{\dfrac{x_a + x_b}{\tan \theta} + \sqrt{\dfrac{(x_a + x_b)^2}{\tan^2 \theta} + 4\, x_a x_b}}{2} = \frac{x_a + x_b}{2 \tan \theta} + \sqrt{\frac{(x_a + x_b)^2}{4 \tan^2 \theta} + x_a \cdot x_b} \qquad \ldots(14.5)$$

Thus, the value of f can be calculated.

Example 14.1. *Three points A, B and C were photographed and their co-ordinates with respect to the lines joining the collimation marks on the photograph are :*

Point	*x*	y
a	*– 35.52 mm*	*+ 21.43 mm*
b	*+ 8.48 mm*	*– 16.38 mm*
c	*+ 48.26 mm*	*+ 36.72 mm*

The focal length of the lens is 120.80 mm. Determine the azimuths of the lines OB and OC, if that of OA is 354° 30′. The axis of the camera was level at the time of the exposure at the station O.

Solution

Fig. 14.8 shows the position of the points.

$$\tan \alpha_a = \frac{x_a}{f} = \frac{-35.52}{120.80} \qquad \therefore \quad \alpha_a = -16° 23'$$

$$\tan \alpha_b = \frac{x_b}{f} = \frac{+8.48}{120.80} \qquad \therefore \quad \alpha_b = +4° 0'$$

$$\tan \alpha_c = \frac{x_c}{f} = +\frac{48.26}{120.80} \qquad \therefore \quad \alpha_c = +21° 47'$$

Azimuth of camera axis $= \phi = \phi_a - \alpha_a = 354° 30' - (-16° 23') = \mathbf{10° 53'}$

Azimuth of $B = \phi + \alpha_b = 10° 53' + 4° = \mathbf{14° 53'}$

Azimuth of $C = \phi + \alpha_c = 10° 53' + 21° 47' = \mathbf{32° 40'}$.

Example 14.2. *Photographs of a certain area were taken from P and Q, two camera stations, 100 m apart. The focal length of the camera is 150 mm. The axis of the camera makes an angle of 60° and 40° with the base line at stations P and Q respectively. The image of a point A appears 20.2 mm to the right and 16.4 mm above the hair lines on the photograph taken at P and 35.2 mm to the left on the photograph taken at Q.*

Calculate the distance PA and QA and elevation of point A, if the elevation of the instrument axis at P is 126.845 m.

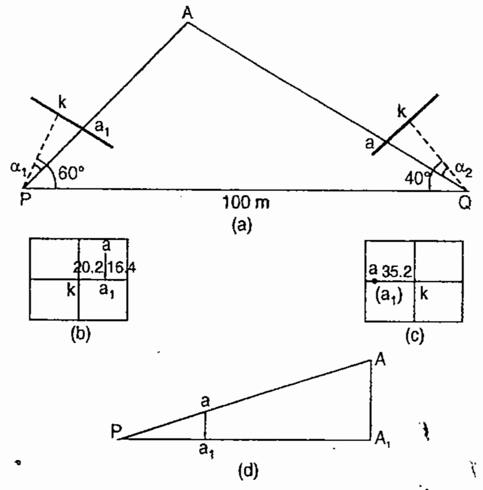


FIG. 14.13

Solution

Fig. 14.13 (a) shows the position of the ground point A with respect to the stations P and Q and the picture traces. Fig. 14.13 (*b*) shows the photograph taken at P and Fig. 14.13 (*c*) shows the photograph taken at Q, with the positions of a properly marked.

From the photograph at P,

$$\alpha_1 = \tan^{-1} \frac{ka_1}{f} = \tan^{-1} \frac{20.2}{150} = 7° 40'$$

$$\therefore \quad \angle APQ = 60° - \alpha_1 = 60° - 7° 40' = 52° 20'$$

From the photograph at Q,

$$\alpha_2 = \tan^{-1} \frac{ka}{f} = \tan^{-1} \frac{35.2}{150} = 13° 12'$$

$$\therefore \quad \angle AQP = 40° - \alpha_2 = 40° - 13° 12' = 26° 48'$$

$$\therefore \quad \angle PAQ = 180° - 52° 20' - 26° 48' = 100° 52'$$

From the triangle APQ,

$$AP = PQ \cdot \frac{\sin AQP}{\sin PAQ} = 100 \cdot \frac{\sin 26° 48'}{\sin 100° 52'} = \mathbf{45.9\ m}$$

and
$$AQ = PQ \cdot \frac{\sin APQ}{\sin PAQ} = 100 \cdot \frac{\sin 52° 20'}{\sin 100° 52'} = \mathbf{80.6\ m}$$

Calculation of R.L. of A

From the photograph at P,

$$Pa_1 = \sqrt{x_a^2 + f^2} = \sqrt{(20.2)^2 + (150)^2} = 151.33 \text{ mm.}$$

Let A_1 be the projection of A on the horizontal line Pa_1 drawn through P (Fig. 14.15 *d*). Then from the similar triangles,

$$\frac{AA_1}{aa_1} = \frac{PA}{Pa_1}$$

$$\therefore \quad AA_1 = aa_1 \cdot \frac{PA_1}{Pa_1} = \frac{16.4 \times 45.9}{151.53} = 4.975 \text{ m}$$

$$\therefore \quad \text{R.L. of } A = \text{R.L. of instrument axis} + AA_1 = 126.845 + 4.975 \text{ m} = \mathbf{131.820\ m}.$$

Fig. 14.17 (*a*) shows a lens forming the image *I* of an object *O*. The ray *OA*, which meets the lens near its top, emerges with its direction changed slightly downward, such as *O AA' I*. The ray *OC*, which meets the lens near its bottom, emerges with its direction changed slightly upward, such as *OCC'I*. In between these, there must be a ray that emerges parallel to its original direction, such as *OBB'I*. Let *OB* extended cut the optical axis at point *N*, and *IB'* extended cut the optical axis at *N'*. In an ideal lens, the cardinal points *N* and *N'* will be common for all the rays of an object, *i.e.*, they will meet at the common point *N* and *N'* as shown in Fig. 14.17 (*b*). These two points (*N* and *N'*) are known as the *front nodal point* and the *rear nodal point* respectively. *If a ray of light is directed at the front nodal point, it is refracted by the lens system that it appears to emerge from the rear nodal point without having undergone any change in direction.*

Fig. 14.17 (*d*) shows the optical diagram of a simple lens system composed of four elements with an air space between the two doublets. *CNN'C'* is the optical axis which pierces the two principal planes of the doublet at the two nodal points *N* and *N'*. The distance *N'C* between the rear nodal point and the plane of infinite focus is called the focal length of the lens system.

In photogrammetric computations it is often convenient to eliminate the distance *NN'* and to superimpose *N'* and *N* as shown in Fig. 14.17 (*c*). Under this condition, each ray is a straight line and the image is an identical representation of the object to the scale *f/H*. An actual lens system is designed to approach this ideal as closely as possible.

The Shutter **:** The camera shutter controls the interval of time during which light is allowed to pass through the lens. Since the aircraft moves at a high speed, a fast speed shutter is required to prevent blurring of the image caused by camera vibrations and the forward motion of the aircraft. The shutter speed generally varies from 1/100 second to 1/1000 second. There are three types of shutters used in aerial cameras :

(*a*) Between-the-lens type (*b*) Focal plane type (*c*) Louvre type.

In the *between-the-lens type*, the shutter is fixed in the space between the elements of the lens system, the space being equal to the fraction of an inch. With this type of shutter, the film is exposed only during the interval the shutter is open. The *focal plane type* shutters operate near the focal plane of the camera. These types of shutters permit higher shutter speeds and are provided in the cameras used for military operations. The film is progressively exposed throughout the time of passage of slit across the focal plane. This type of shutter is not useful for mapping purposes since it includes a distortion in the scale of the photograph in the direction of the movement of the shutter and position errors in the relationship of object points on photographs. The *louvre type* shutters are usually employed for large lens aperture with high speed. It consists of a number of metal strips about 5 mm wide supported on a metal frame and is placed either in front of the lens or at its back.

The Diaphragm **:**

A diaphragm is placed between the lens elements and acts as a physical opening of the lens system. It consists of a series of leaves which can be rotated to increase or decrease the size of the opening to restrict the size of the bundle of rays to pass through the lens. If the diaphragm opening is larger, the shutter speed has to be greater.

The Filter **:**

A filter consists of a piece of coloured glass placed in front of the lens. It filters the stray light (blue and violet) in the atmosphere caused by haze and moisture. It also protects the lens from the flying particles in the atmosphere.

(*ii*) Camera Cone :

The camera cone supports the entire lens assembly including the filter. At the top of it are provided the collimation marks which define the focal plane of the camera. The cone is made up of the material having low co-efficient of thermal expansion so that the collimation marks and the lens system are held in the same relative positions at operational temperatures. The elements of *interior orientation* are fixed by the relative positions of the lens, the lens axis, the focal plane and the collimation marks.

(*iii*) **The Focal Plane :**

The collimation marks are provided at the upper surface of the cone. The focal plane is provided exactly above the collimation marks. It is kept at such a distance from the near nodal point that best possible image is obtained.

(*iv*) **The Camera Body :**

The camera body is the part of the camera provided at the top of the cone. Sometimes, it forms the integral part of the cone in which case they act as an integral part to preserve the interior orientation once the camera is calibrated.

(*v*) **The Drive Mechanism :**

The camera drive mechanism is housed in the camera body and is used for (*i*) winding and tripping the shutter (*ii*) operating the vacuum system for flattening the film, and (*iii*) winding the film. It may be either operated manually or automatically.

(*vi*) **Magazine :**

A magazine holds the exposed and unexposed films and houses the film flattening device at the focal plane. The power operation of the movable parts of the magazine is supplied from the drive mechanism. The film is flattened at the focal plane either by inserting a piece of optical glass in the focal plane opening or by applying a vacuum to ribbed plate criss-crossed with tiny grooves and provided to the back of the film.

14.10. DEFINITIONS AND NOMENCLATURE

1. Vertical Photograph. A vertical photograph is an aerial photograph made with the camera axis (or optical axis) coinciding with the direction of gravity.

2. Tilted photograph. A tilted photograph is an aerial photograph made with the camera axis (or optical axis) *unintentionally* tilted from the vertical by a small amount, usually less than 3° (Fig. 14.18).

3. Oblique Photograph. An oblique photograph is an aerial photograph taken with the camera axis directed *intentionally* between the horizontal and the vertical. If the apparent horizon is shown in the photograph, it is said to be *high oblique*. If the apparent horizon is not shown, it is said to be *low oblique*.

O = perspective centre or the rear nodal point of the camera lens (or the exposure station)
k = principal point
K = ground principal point
Ok = principal distance
t = angle of tilt = $\angle kon$ = angular deviation of the photograph perpendicular from the plumb line
n = photo-nadir or photo plumb point
N = ground nadir or ground plumb point
noN = plumb line or vertical line through the perspective centre
i = isocentre
I = ground isocentre
nik = principal line
h = horizon point
$i_1\,i\,i_2$ = axis of tilt = isometric parallel

Relation Between Principal Point, Plumb Point and Isocentre :

From Figs. 14.18 and 14.19,

(1) nk = distance of the nadir point from the principal point

$$\frac{nk}{kO} = \tan t \qquad \text{or} \qquad nk = kO \,.\, \tan t = f \tan t \qquad \ldots(14.6)$$

since $kO = f$ = principal distance

(2) ki = distance of the isocentre from the principal point

$$\frac{ki}{kO} = \tan\frac{t}{2} \qquad \text{or} \qquad ki = kO \,.\, \tan\frac{t}{2} = f\tan\frac{t}{2} \qquad \ldots(14.7)$$

(3) kh = distance along the principal line, from the principal point to the horizon point

$$\frac{kh}{kO} = \cot t \qquad \text{or} \qquad kh = kO \,.\, \cot\, t = f \cot t. \qquad \ldots(14.8)$$

14.11. SCALE OF A VERTICAL PHOTOGRAPH

Since a photograph is the perspective projection, the images of ground points are displaced where there are variations in the ground elevation. Thus, in Fig. 14.20 (a) the images of two points A and A_0, vertically above each other, are displaced on a *vertical* photograph and are represented by a and a_0 respectively. Due to this displacement, there is no uniform scale between the points on such a photograph, except when the ground points have the same elevation. If the elevation of points vary, the scale of the vertical photograph will vary from point to point on the photograph.

Let us first take the case when the ground is horizontal, *i.e.*, all the points are having the same elevation, such as shown in Fig. 14.20 (*a*).

Let $$S = \text{scale} = \frac{\text{map distance}}{\text{ground distance}}$$

From Fig. 14.20 (*a*), $$S = \frac{ka}{KA} = \frac{Ok}{KA} = \frac{f}{H-h} = \frac{f}{H-h} \qquad \ldots(14.9)$$

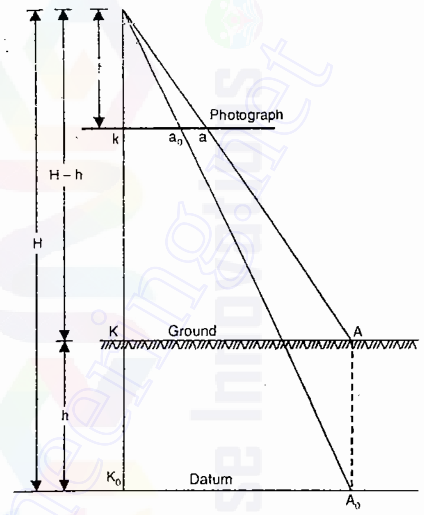


FIG. 14.20. (*a*) SCALE OF A VERTICAL PHOTOGRAPH

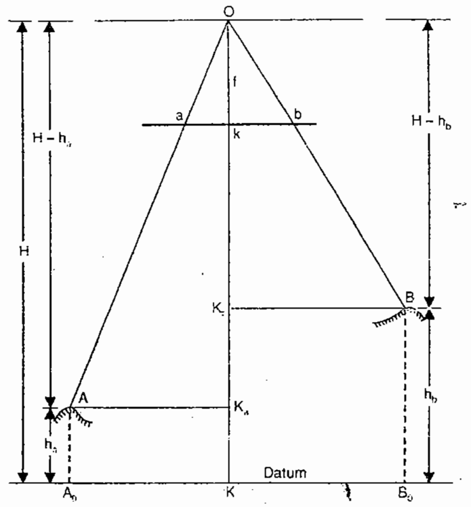


FIG. 14.20. (*b*) SCALE OF A VERTICAL PHOTOGRAPH.

where H = height of the exposure station (or the air plane) above the mean sea level
f = focal length of the camera
h = height of the ground above mean sea level

Let us now take the case when the points are not having the same elevation, as represented in Fig. 14.20 (*b*).

Let A and B be two points having elevations h_a and h_b respectively above mean sea level. They are represented by a and b respectively on the map. k is the principal point of the vertical photograph taken at height H above mean sea level.

The scale of the photograph at the elevation h_a is evidently equal to the ratio $\frac{ak}{AK_a}$.

From similar triangles, $$\frac{aK}{AK_a} = \frac{Ok}{OK_a} = \frac{f}{H-h_a}$$

Hence the scale of the photograph at the elevation h_a is equal to $\frac{f}{H-h_a}$.

Similarly, the scale of the photograph at the elevation h_b is equal to the ratio $\frac{bk}{BK_b}$.

From similar triangles, $$\frac{bk}{BK_b} = \frac{Ok}{OK_b} = \frac{f}{H-h_b}.$$

Hence the scale of the photograph at the height h_b is equal to $\frac{f}{H-h_b}$.

In general, therefore, the scale of the photograph is given by

As proved in the previous article, the ground length L is given by

$$L^2 = (X_a - X_b)^2 + (Y_a - Y_b)^2$$

Substituting the values of X_a, X_b, Y_a, Y_b as obtained in the previous article, we get

$$L^2 = \left[\frac{H-h_a}{f}x_a - \frac{H-h_b}{f}x_b\right]^2 + \left[\frac{H-h_a}{f}y_a - \frac{H-h_b}{f}y_b\right]^2 \qquad \ldots(14.16)$$

In the above expression, the ground distance L, and elevations h_a and h_b are known quantities. The photographic co-ordinates (x_a, y_a), (x_b, y_b) can be measured. The only unknown is H. Collecting the terms for H, the equation takes the quadratic form

$$pH^2 + qH + r = 0$$

where p, q and r are the numbers obtained after substituting the values of the known quantities. The value of H is then obtained by

$$H = \frac{-q + \sqrt{q^2 - 4pr}}{2p}$$

The computation of H by the solution of quadratic equation is rather very tedious and time consuming. Alternately, the value of H can be determined by successive approximations as follows :

Step 1 :

The first approximate value of H is obtained from the scale relationship

$$\frac{f}{H_{approx.} - h_{ab}} = \frac{ab}{AB} = \frac{l}{L} \qquad \ldots(14.17)$$

where h_{ab} = average elevation of line AB

$AB = L$ = known ground distance

$ab = l$ = measured photographic distance

Step 2 :

The approximate value of H so obtained is used for calculating the co-ordinates (X_a, Y_a) and (X_b, Y_b). Using these co-ordinates, the approximate value of H and the elevations h_a and h_b, the length of the line is computed. Length is then compared with the actual distance to get a more correct value of H. Thus,

$$\frac{H - h_{ab}}{H_{approx.} - h_{ab}} = \frac{\text{correct } AB}{\text{computed } AB} \qquad \ldots(14.18)$$

Step 3 :

With this value of H, step 2 is repeated till the computed length of AB and the correct length agree within necessary precision, usually 1 in 5000.

14.14. RELIEF DISPLACEMENT ON A VERTICAL PHOTOGRAPH

We have seen that if the photograph is truly vertical and the ground is horizontal, and if other sources of errors are neglected, the scale of the photograph will be uniform. Such a photograph represents a true *orthographic projection* and hence the true map of the terrain. In actual practice, however, such conditions are never fulfilled. When the ground is not horizontal, the scale of the photograph varies from point to point and is not constant.

Since the photograph is the perspective view, the ground relief is shown in perspective on the photograph. Every point on the photograph is therefore, displaced from their true orthographic position. This displacement is called *relief displacement*.

Thus, in Fig. 14.22, A, B and K are three ground points having elevations h_a, h_b and h_k above datum. A_0, B_0 and K_0 are their *datum positions* respectively, when projected vertically downwards on the datum plane. On the photograph, their positions are a, b and k respectively, the points k being chosen vertically below the principal point. If the datum points A_0, B_0 and K_0 are imagined to be photographed along with the ground points, their positions will be a_0, b_0 and k respectively. As is clear from the figure, the points a and b are *displaced outward* from their datum photograph positions, the displacement being along the corresponding radial lines from the principal point. The radial distance aa_0 is the *relief displacement* of A while bb_0 is the relief displacement of B. The point k has not been displaced since it coincides with the principal point of the photograph.

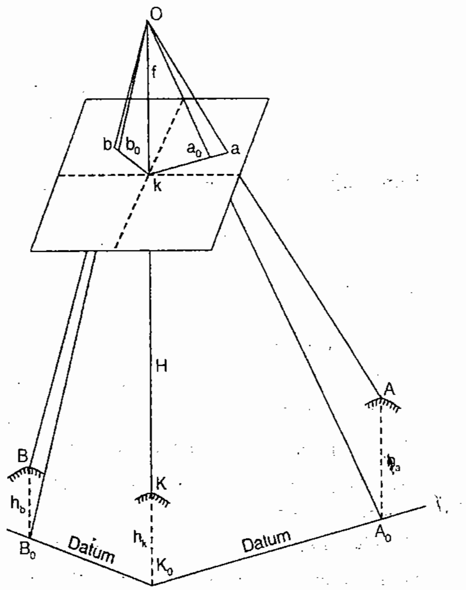


FIG. 14.22. RELIEF DISPLACEMENT ON VERTICAL PHOTOGRAPH.

To calculate the amount of relief displacement, consider Fig. 14.23 which shows a vertical section through the photograph of Fig. 14.22 along the line ka.

In Fig. 14.23,

Let r = radial distance a from k

r_0 = radial distance of a_0 from k

$R = K_0A_0$

Then, from similar triangles,

$$\frac{f}{H-h} = \frac{r}{R}, \text{ from which } r = \frac{Rf}{H-h} \qquad \ldots(1)$$

Also $$\frac{f}{H} = \frac{r_0}{R}, \text{ from which } r_0 = \frac{Rf}{H} \qquad \ldots(2)$$

Hence the relief displacement (d) is given by

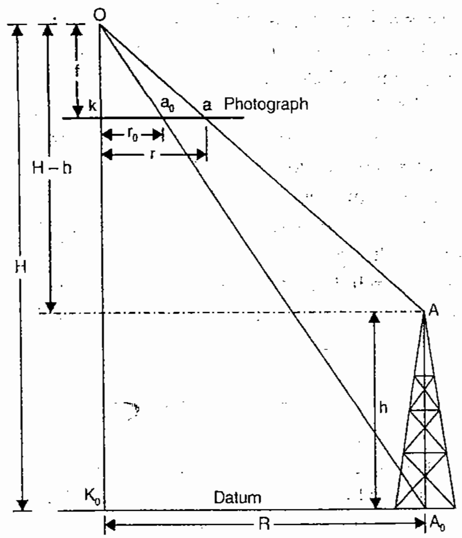


FIG. 14.23. CALCULATION OF RELIEF DISPLACEMENT.

$$\therefore \quad \frac{8.65 \text{ cm}}{2000 \text{ m}} = \frac{20 \text{ cm}}{(H-500) \text{ m}}$$

or
$$(H-500) = \frac{20 \times 2000}{8.65} = 4624 \text{ m}$$

$$\therefore \quad H = 4624 + 500 = 5124 \text{ m}$$

$$\therefore \quad S_{800} = \frac{20 \text{ cm}}{(5124-800) \text{ m}} = \frac{1 \text{ cm}}{216.2 \text{ m}}$$

Hence S_{800} is **1 cm = 216.2 cm.**

Example 14.7. *A section line AB appears to be 10.16 cm on a photograph for which the focal length is 16 cm. The corresponding line measures 2.54 cm on a map which is to a scale 1/50,000. The terrain has an average elevation of 200 m above mean sea level. Calculate the flying altitude of the aircraft, above mean sea level, when the photograph was taken.*

Solution.

The relation between the photo scale and map scale is given by

$$\frac{\text{Photo scale}}{\text{Map scale}} = \frac{\text{Photo distance}}{\text{Map distance}}$$

Here, map scale $= \dfrac{1}{50,000}$; Let the photo scale be $\dfrac{1}{n}$

$$\therefore \quad \frac{1/n}{1/50,000} = \frac{10.16}{2.54}$$

$$\therefore \quad \frac{1}{n} = \frac{10.16}{2.54} \times \frac{1}{50,000} = \frac{1}{12,500} \quad \text{or} \quad n = 12,500$$

Again,
$$S_{200} = \frac{1}{n} = \frac{f}{(H-h)} \quad \text{or} \quad \frac{1}{12,500} = \frac{(16/100) \text{ m}}{(H-200) \text{ m}}$$

or
$$(H-200) = \frac{16}{100} \times 12500 = 2000 \text{ m}$$

Hence $H = 2000 + 200 = \mathbf{2200}$ **m.**

Example 14.8. *Two points A and B having elevations of 500 m and 300 m respectively above datum appear on the vertical photograph having focal length of 20 cm and flying altitude of 2500 m above datum. Their corrected photographic co-ordinates are as follows:*

Point	*Photographic Co-ordinates*	
	x (cm)	*y (cm)*
a	*+ 2.65*	*+ 1.36*
b	*– 1.92*	*+ 3.65*

Determine the length of the ground line AB.

Solution

The ground co-ordinates are given by

$$X_a = \frac{H-h_a}{f} \cdot x_a = \frac{2500-500}{20} \times (+2.65) = +265 \text{ m}$$

$$Y_a = \frac{H-h_a}{f} \, y_a = \frac{2500-500}{20} \times (+1.36) = +136 \text{ m}$$

$$X_b = \frac{H-h_b}{f} \cdot x_b = \frac{2500-300}{20} \times (-1.92) = -211.2 \text{ m}$$

$$Y_b = \frac{H-h_b}{f} \cdot y_b = \frac{2500-300}{20} \times (+3.65) = +401.5 \text{ m}$$

$$(X_a - X_b)^2 = (265 + 211.2)^2 = 22.677 \times 10^4 \text{ m}^2$$

$$(Y_a - Y_b)^2 = (136 - 401.5)^2 = 7.049 \times 10^4 \text{ m}^2$$

Hence
$$AB = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2} = \sqrt{(22.677 + 7.049)\,10^4} = \mathbf{545 \text{ m}}.$$

Example 14.9. *The ground length of a line AB is known to be 545 m and the elevations of A and B are respectively 500 m and 300 m above m.s.l. On a vertical photograph taken with a camera having focal length of 20 cm include the images a and b of these points, and their photographic co-ordinates are :*

(x_a = + 2.65 cm, y_a = + 1.36 cm); (x_b = – 1.92 cm, y_b = + 3.65 cm).

The distance ab scaled directly from the photograph is 5.112 cm. Compute the flying height above the mean sea level.

Solution

From the scale relationship, the approximate height can be calculated from

$$\frac{f}{H_{approx.} - h_{ab}} = \frac{ab}{AB}$$

Here,
$$h_{ab} = \tfrac{1}{2}(500 + 300) = 400 \text{ m}$$

$$\therefore \quad \frac{20 \text{ (cm)}}{(H_{approx.} - 400) \text{ m}} = \frac{5.112 \text{ (cm)}}{545 \text{ (m)}}$$

$$\therefore \quad H_{approx.} - 400 = \frac{20 \times 545}{5.112} \quad \text{or} \quad H_{approx.} = 400 + 2132.2 = 2532.2 \text{ m}$$

Using this approximate height, the ground co-ordinates of A and B are calculated from Eq. 14.14.

$$X_a = \frac{H-h_a}{f} \cdot x_a = \frac{2532.2-500}{20} \times 2.65 = +269.3 \text{ m}$$

$$Y_a = \frac{H-h_a}{f} \cdot y_a = \frac{2532.2-500}{20} \times 1.36 = +138.2 \text{ m}$$

$$X_b = \frac{H-h_b}{f} \cdot x_b = \frac{2532.2-300}{30} \times (-1.92) = -214.3 \text{ m}$$

$$Y_b = \frac{H-h_b}{f} \cdot y_b = \frac{2532.2-300}{20} \times (3.65) = +407.3 \text{ m}$$

The ground length based on the approximate height is given by

$$L = \sqrt{(269.3 + 214.3)^2 + (138.2 - 407.3)^2} = 553.4 \text{ m}$$

The actual ground length is 545 m. The second approximate height is calculated as follows :

Let N and M be the points on on and om extended, at heights of h above datum. Thus N, M and A have the same elevation. The triangle NMA is in a horizontal plane. From the similar triangles $om'a$ and ONA, we get

$$\frac{m'a}{NA} = \frac{Om'}{ON}$$

But $\quad Om' = On - m'n = f \sec t - mn \sin t \quad ; \quad ON = ON_0 - NN_0 = H - h$

$$\frac{m'a}{NA} = \frac{\text{Map distance}}{\text{Ground distance}} = \text{scale at a point whose elevation is } h = S_h$$

Substituting the values in (1), we get

$$S_h = \frac{f \sec t - mn \sin t}{H - h} \qquad \text{...(2)}$$

In the above expression mn is the distance along the principal line, between the photo nadir and the foot of the perpendicular from the point under consideration. To find its value, let us consider the system of co-ordinates axes illustrated in Fig. 14.26.

Let the photographic co-ordinates of the image a be x and y. Let s be the angle of swing and θ be the angle between the y-axis and the principal line. If the y-axis be rotated to the position of the principal line, the new axis (or y'-axis) will be inclined to the original axis by an angle θ given by

$$\theta = 180° - s \qquad \text{...(14.21)}$$

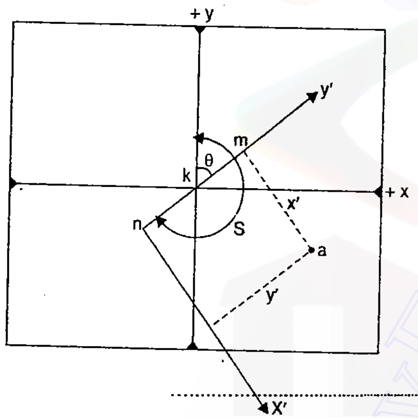


FIG. 14.26. CO-ORDINATE AXES THROUGH PLUMB POINT.

As in analytic geometry, the angle θ is considered to be *positive* when the rotation is in the *counter-clockwise* direction and *negative* when it is in the clockwise direction. Thus, the angle θ in Fig. 14.26. is negative. Let the new x-axis (or x'-axis) be selected through the nadir point n. The distance kn is equal to $f \tan t$ (see Eq. 14.6). The new co-ordinates (x', y') of the point a with reference to the x' and y' axis are given by

$$x' = x \cos \theta + y \sin \theta \qquad \text{...[14.22 (a)]}$$

$$y' = -x \sin \theta + y \cos \theta + f \tan t \qquad \text{...[14.22 (b)]}$$

The distance nm is therefore equal to y'. Substituting this in (2), we get

$$S_h = \frac{f \sec t - y' \sin t}{H - h} \qquad \text{...(14.23)}$$

It is clear that the co-ordinates y' is the same for the points on the line ma. *Hence the scale, which is the linear function of y, is constant for all the points on a line perpendicular to the principal line.*

For finding the scale at a given point on the photograph by Eq. 14.23, the following data is essential :

(1) focal length (2) tilt (3) height of flight

(4) height of the point (5) swing

and (6) the position of the point which respect to principal line.

14.16. COMPUTATION OF LENGTH OF LINE BETWEEN POINTS OF DIFFERENT ELEVATIONS FROM MEASUREMENTS ON A TILTED PHOTOGRAPH

In Fig. 14.25, triangles $m'ma$ and NMA are in horizontal planes. From scale relationship, we have $\quad S_h = \dfrac{am}{AM}$

But $\quad am = x'$ (from Fig. 14.26) ; $AM = X$; $S_h = \dfrac{f \sec t - y' \sin t}{H - h}$

Substituting the values, we get

$$AM = X = \frac{H - h}{f \sec t - y' \sin t} \cdot x' \qquad \text{...[14.24 (a)]}$$

Similarly, from scale relationships, we have $S_h = \dfrac{m'm}{NM}$

But $\quad m'm = mm \cos t = y' \cos t \quad ; \quad NM = Y \quad ;$ and $\quad S_h = \dfrac{f \sec t - y' \sin t}{H - h}$

Substituting the values, we get

$$NM = Y = \frac{H - h}{f \sec t - y' \sin t} \cdot y' \cos t \qquad \text{...14.24 [(b)]}$$

Thus, the ground co-ordinates (x, y) of any point are known.

If there are two points A and B, their ground co-ordinates (X_a, Y_a) and (X_b, Y_b) can be calculated, and the length L of the line AB computed from the expression

$$L = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2} \qquad \text{..[14.24 (c)]}$$

14.17. DETERMINATION OF FLYING HEIGHT FOR A TILTED PHOTOGRAPH

If the images of two points A and B having different known elevations and known length between them appear on a tilted photograph, the elevation or height H of the exposure station can be determined exactly in the same way as discussed in § 14.13. The method is outlined in the following steps :

Step 1 :

From the photographic co-ordinates (x_a, y_a), (x_b, y_b), calculate the photographic length ab (or scale it directly from the photograph). From the existing maps, or by another source, the ground length of AB is known. Calculate the approximate flying height from the scale relationship :

$$\frac{f}{H_{approx.} - h_{ab}} = \frac{ab}{AB} \quad ; \quad \text{where } h_{ab} = \text{average elevation of } AB.$$

Step 2 :

Using the approximate H so obtained, and the photographic co-ordinates, compute the ground co-ordinates (X_a, Y_a), (X_b, Y_b) by solution of equations [14.24 (a)] and [14.24 (b)].

and b are the images of two points on the tilted photograph, along its principal line, while a' and b' are the corresponding positions on the vertical photograph. Since i is the point of rotation, d_a and d_b represent the displacements of the points a and b with respect to a' and b' respectively. Let α be the inclination of the ray Oa with Ok. Similarly, β is the inclination of the ray Ob to Ok.

Thus d_a = tilt displacement of a with respect to a'

or $d_a = ia' - ia$

But $ia' = n'a' - n'i = f \tan (t + \alpha) - f \tan t/2$ and $ia = ka + ki = f \tan \alpha + f \tan t/2$

Hence $d_a = f \tan (t + \alpha) - f \tan t/2 - f \tan \alpha - f \tan t/2$

or $$d_a = f [\tan (t + \alpha) - \tan \alpha - 2 \tan t/2] \quad \text{...[14.25](a)}$$

Similarly, $d_b = ib - ib'$

$ib = kb - ki = f \tan \beta - f \tan t/2$; $ib' = n'b' + n'i = f \tan (\beta - t) + f \tan t/2$

$\therefore$ $d_b = f \tan \beta - f \tan t/2 - f \tan (\beta - t) - f \tan t/2$

or $$d_b = f [\tan \beta - \tan (\beta - t) - 2 \tan t/2] \quad \text{...[14.25(b)]}$$

In the above expressions, the angles α and β can be found by the relations :

$$\tan \alpha = \frac{ka}{f}, \text{ and } \tan \beta = \frac{kb}{f}.$$

It can be shown that equations [14.25 (a,b)] can be represented by the approximate formula

$$d = \frac{(ia)^2 \sin t}{f} \quad \text{...(14.26)}$$

It is quite clear from the figure that the tilt displacement of a point on the upward half of a tilted photograph is *inward* (such as for point a) while the tilt displacement of a point on the downward or nadir point half is *outward* (such as for b)

Equations 14.25 give the tilt displacements for the points on the principal line. The tilt displacement of a point not lying on the principal line is greater than that of a corresponding point on the principal line.

Let I = angle measured at the isocentre from the principal line to the point.

d_u = displacement of the point on the upward half of the tilted photograph.

d_d = displacement of the point on the downward half of the tilted photograph.

In Fig. 14.28 (plan), the point q is not on the principal line while point a is on the principal line. qq' is the displacement of q while aa' is the displacement of point a. Since both q and a are equidistant from the axis of tilt, we have

$$qq' = aa' \sec I$$

where I is the angle at the isocentre from the principal line to the point q.

Hence the ratio of the tilt displacement of a point not on the principal line to that of a point on the principal line is equal to the secant of the angle at the isocentre from the principal line to the point.

Thus, the expressions for d_u and d_d can be written as :

$$d_u = f \sec I [\tan (t + \alpha) - \tan \alpha - 2 \tan t/2] \quad \text{...[14.27(a)]}$$

$$d_d = f \sec I [\tan \beta - \tan (\beta - t) - 2 \tan t/2] \quad \text{...[14.27(b)]}$$

In Fig. 14.28 (plan), $p'q'r's'$ represents a square on the vertical photograph. The corresponding displaced points on the tilted photographs are p, q, r and s. Since the tilt displacements are always radial from the isocentre, the corresponding figure $p\,q\,r\,s$ becomes a rhombus.

14.19. RELIEF DISPLACEMENT ON A TITLED PHOTOGRAPH

It has been shown in § 14.14 that the relief displacement on a vertical photograph is radial from the principal point. The points are displaced radially outward from their datum photograph positions. Fig. 14.29 shows the relief displacement on a tilted photograph.

A, B and N are ground points, and A_0, B_0, N_0 are their corresponding datum positions. N and N_0 being vertically below the nadir point n. A and B are imaged at a and b respectively, a_0 and b_0 are the datum photograph positions of A_0 and B_0. i is the isocentre and k is the principal point. The plane $O\,N\,N_0\,A_0\,A$ is a vertical plane since it contains the plumb line ON. The points n, a_0 and a lie on the same vertical plane. Since the points n, a_0 and a also lie on the photograph, they are in the same line, *i.e.*, a_0 and a lie on a radial line from the nadir point. Similarly, the point n, b_0 and b are on the same line, and b_0 and b are radial from the nadir point. *Thus, on a tilted photograph, the relief displacement is radial from the nadir point.* The amount of relief displacement on a tilted photograph depends upon : (i) flying height, (ii) distance of the image from the nadir point, (iii) elevation of the ground point, and (iv) position of the point with respect to the principal line and to the axis of tilt. In the case of near vertical photograph, where the tilt is less than 3°, the relief displacement can be calculated from equations 14.19 with the modification that the radial distances r and r' are measured from the nadir point and *not* from the principal point.

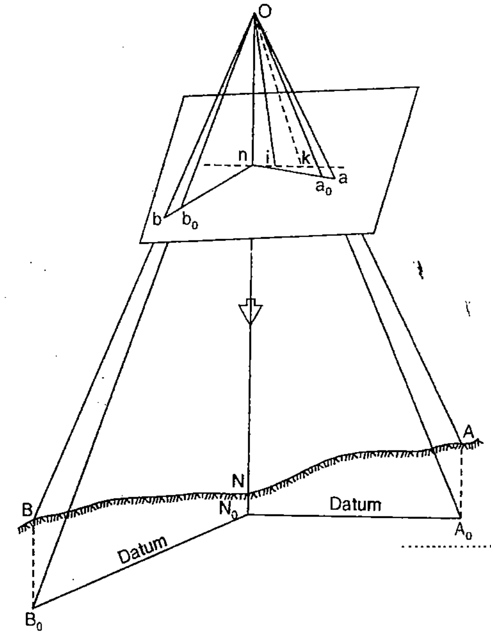


FIG. 14.29. RELIEF DISPLACEMENT ON A TILTED PHOTOGRAPH.

Thus $$d = \frac{rh}{H} \quad \text{...[14.28(a)]} \qquad \text{and} \qquad d = \frac{r_0 h}{H - h} \quad \text{...[14.28(b)]}$$

where d is the relief displacement on a tilted photograph, each other by 55 to 65 per cent. Fig. 14.31 (*a*) shows three successive flight lines. Fig. 14.31 (*b*) shows the vertical section containing the flight line and showing the overlap. Since the overlap is more than 50 per cent, alternate photographs will overlap one another by 10 to 30 per cent. When photographs are taken with this overlap, the entire area may be examined stereoscopically. The overlap between adjacent flight lines is known as *lateral overlap or sidelap*. The sidelap amounts to about 15 to 35 per cent. Fig. 14.31 (*c*) shows the vertical section taken normal to the three flight lines of Fig. 14.31 (*a*).

The number of individual photographs required to cover a given area increases with the increase in the overlap and sidelap, thus increasing the amount of work both in the field as well as in the office.

Reasons for Overlap

The following are some of the reasons for keeping overlap in the photographs :

To tie the different prints together accurately, it is desirable that the principal point of each print should appear on the edges of as many adjacent strips as possible.

(2) The distortions caused by the lens and by the tilt, and the relief displacements are more pronounced in the outer part of the photograph than near the centre of each photograph. If the overlap is more than 50%, these distortions and displacements can be overcome quite effectively while constructing the maps.

(3) In order to view the pairs of photographs stereoscopically, only the overlapped portion is useful. Hence the overlap should at least be 50% .

(4) Due to the overlap, each portion of the territory is photographed three to four times. Hence any picture distorted by excessive tilt or by cloud shadows etc. can be rejected without the necessity of a new photograph.

(5) If the flight lines are not maintained straight and parallel, the gaps between adjacent strips will be left. These gaps can be avoided by having sidelap.

(6) In the stereoscopic examination, objects can be viewed from more than one angle if sufficient overlap is provided.

Fig. 14.32 shows a photographic flight with an automatic aerial camera, the overlap of successive vertical photographs being 60%.

EFFECTIVE COVERAGE OF THE PHOTOGRAPH

The amount of overlap and sidelap to be used in flight planning depends upon the effective coverage of each photograph. The relation between the separation of flight lines and the separation between photographs must be arranged to give the greatest area to each stereopair.

The effective coverage of each photograph depends upon (*i*) size of format or focal plane opening, (*ii*) focal length and (*iii*) angular coverage of the lens. The effective angular coverage of the lens with the 12 in. (30.4 cm) focal length is represented by a cone the apex of which lies at the front nodal point and the apex angle of which is about 60°. In general, the effective coverage with a 12 in. lens will embrace more than 9″ × 9″ format size, and hence the entire photograph is usable (Fig. 14.33 *a*). The effective angular coverage with a 6″ (15.2 cm) wide angle lens is a cone of rays the apex of which is about 86°. A sizeable portion of the 9″ × 9″ format is not usable, and the useful

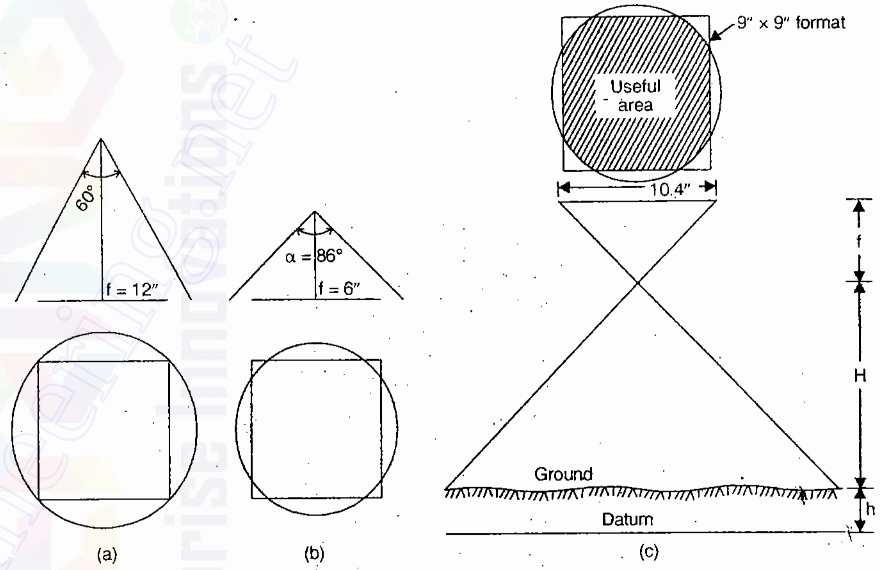


FIG. 14.33. ANGULAR COVERAGE.

circle at the negative plane is equal to $2f \tan \frac{\alpha}{2} = 11.2''$ approximately. Due to errors in directing the camera and in following the flight lines, this should be reduced to at least 10.4 in., as shown in Fig. 14.33 (*c*).

The effective area of overlap between the two photographs is that bound by the overlapping circles representing the effective coverage of the photographs. Since the stereomodels must fit each other, the useful steroareas must be assumed to be rectangles having a width equal to the interval B between exposures. the two longer sides of this rectangle pass through the principal points of the photographs. The stereoareas is shown cross hatched, and the largest rectangle possible is drawn within this area.

Let W = distance between the flight strips

A_s = stereo-areas (*i.e.*, area of the rectangle).

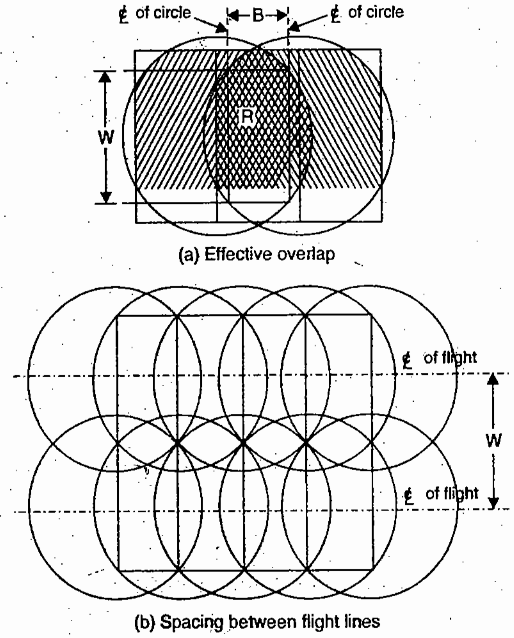


FIG. 14.34. FLIGHT LINES AND INTERVALS.

Now net length covered by each photograph $= L = (1 - P_l)\, sl$

$\therefore$ Number of photographs in each strip is given by

$$N_1 = \frac{L_1}{L} + 1 = \frac{L_1}{(1 - P_l)\, sl} + 1 \qquad \ldots[14.31(a)]$$

Similarly, net width covered by each photograph $= W = (1 - P_w)\, sw$

Hence the number of the strips required are given by

$$N_2 = \frac{L_2}{W} + 1 = \frac{L_2}{(1 - P_w)\, sw} + 1 \qquad \ldots[14.31(b)]$$

Thus, the number of photographs required is

$$N = N_1 \times N_2 = \left\{ \frac{L_1}{(1 - P_l)\, sl} + 1 \right\} \times \left\{ \frac{L_2}{(1 - P_w) sw} + 1 \right\} \qquad \ldots(14.31)$$

INTERVAL BETWEEN EXPOSURES

The time interval between the exposures can be calculated if the ground speed of the airplane and the ground distance (along the direction of flight between exposures are known.

Let V = ground speed of the airplane (km/hour)

L = ground distance covered by each photograph in the direction of flight $= (1 - P_l)\, sl$ in km

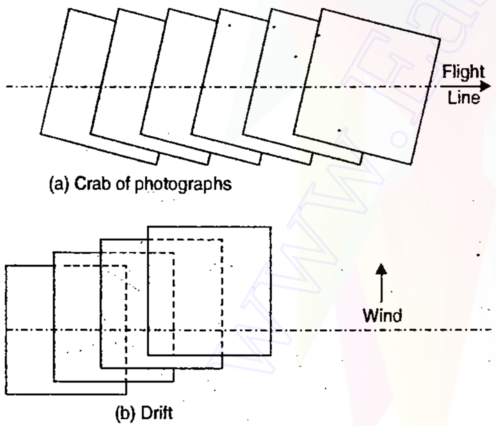
T = time interval between the exposures.

Then $$T = \frac{3600\, L}{V} \qquad \ldots(14.32)$$

The exposures are regulated by measuring the time required for the image of a ground point to pass between two lines on a ground-glass plate of the view-finder. Usually, however, the interval is not calculated, but the camera is tripped automatically by syncronising the speed of a moving grid in the view-finder with the speed of the passage of images across a screen.

CRAB AND DRIFT

Crab. Crab is the term used to designate the angle formed between the flight line and the edges of the photograph in the direction of flight, as shown in Fig. 14.35 (*a*). At the instant of exposure, if the focal plane of the camera is not square with the direction of flight, the crab is caused in the photograph. The arrangements are always made to rotate the camera about the vertical axis of camera mount. Crabbing should be eliminated since it reduces effective coverage of the photograph.



(a) Crab of photographs

(b) Drift

FIG. 14.35. CRAB AND DRIFT

Drift. Drift is caused by the failure of the photograph to stay on the predetermined flight line. If the aircraft is set on its course by compass without allowing for wind velocity, it will drift from its course, and the photographs shall be as shown in Fig. 14.35 (*b*). If the drifting from the predetermined flight line is excessive, reflights will have to be made because of serious gapping between adjacent flight lines.

COMPUTATION OF FLIGHT PLAN

For the computation of the quantities for the flight plan, the following data is required:

1. Focal length of the camera lens
2. Altitude of the flight of the aircraft
3. Size of the area to be photographed
4. Size of the photograph
5. Longitudinal overlap
6. Lateral overlap
7. Position of the outer flight lines with respect to the boundary of the area
8. Scale of the flight map
9. Ground speed of aircraft.

Knowing the above, the amount of film required can be calculated before hand, the flight lines can be delineated on the map and the time interval between exposures can be determined if an intervalometer is to be used.

Example 14.13. *The scale of an aerial photograph is 1 cm=100 m. The photograph size is 20 cm × 20 cm. Determine the number of photographs required to cover an area of 100 sq. km if the longitudinal lap is 60% and the side lap is 30 %.*

Solution.

Here $l = 20$ cm ; $w = 20$ cm ; $P_l = 0.60$; $P_w = 0.30$

$$s = \frac{H\,(\text{m})}{f\,(\text{cm})} = 100 \quad (i.e.\ \ 1 \text{ cm} = 100 \text{ m})$$

$\therefore$ The actual ground length covered by each photograph is

$$L = (1 - P_l)\, sl = (1 - 0.6)100 \times 20 = 800 \text{ m} = 0.8 \text{ km}$$

Actual ground width covered by each photograph is

$$W = (1 - P_w) sw = (1 - 0.3)100 \times 20 = 1400 \text{ m} = 1.4 \text{ km}$$

$\therefore$ Net ground area covered by each photograph is

$$a = L \times W = 0.8 \times 1.4 = 1.12 \text{ sq. km.}$$

Hence number of photographs required is

$$N = \frac{A}{a} = \frac{100}{1.12} = 90$$

Example 14.14. *The scale of an aerial photography is 1 cm = 100 m. The photograph size is 20 cm × 20 cm. Determine the number of photographs required to cover an area 10 km × 10 km, if the longitudinal lap is 60% and the side lap is 30%.*

Solution.

Here $L_1 = 10$ km ; $L_2 = 10$ km

The *basic control* consists in establishing the basic ~~net work of triangulation~~ stations, traverse stations, azimuth marks, bench marks etc.

The *photo control* consists in establishing the horizontal positions or elevations of the images of some of the identified points on the photographs, with respect to the basic control.

Each of these controls introduces *horizontal* control as well as *vertical* control and is known as basic horizontal control, basic vertical control, horizontal photo control, and vertical photo control respectively. The elevation of a vertical photo control point is determined by carrying a line of levels from a basic control bench mark to the point, and then carrying to the original bench mark or to a second bench mark for checking. The horizontal photo control points are located with respect to the basic control by third order or fourth-order triangulation, third order traversing, stadia traversing, trigonometric traversing, substance-bar traversing etc. etc., depending upon the accuracy required. Vertical photo control may be established by third-order leveling, fly levelling, transit-stadia levelling or precision barometric altimetry etc., depending upon the desired accuracy.

The photo control can be established by two methods :

(*i*) Post-marking method

(*ii*) Pre-marking method.

In the *post-marking* method, the photo control points are selected after the aerial photography. The distinct advantage of this method is in positive identification and favourable location of points.

In the pre-marking method, the photo-control points are selected on the ground first, and then included in the photograph. The marked points on the ground can be identified on the subsequent photograph. If the control traverse or triangulation station or bench marks are to be incorporated in the photo-control net work, they are marked with paint, flags etc. in such a way that identification on the photographs becomes easier. The selected control points should be sharp and clear in plan.

14.23. RADIAL LINE METHOD OF PLOTTING (ARUNDEL'S METHOD)

The radial line plot, often called *photo-triangulation* is the most accurate means of plotting a planimetric map from aerial photographs without the use of expensive instruments.

As discussed earlier, the displacement of image due to relief is radial from the principal point of vertical photograph. Hence the angles measured on the photograph at the principal point are true horizontal angles, independent of the height of the object and the flying height. The vertical photograph in space can thus be considered as an angle-measuring device similar to a transit or a plane table. Also, on tilted photographs, angles measured at the isocentre are true horizontal angles independent of tilt, provided that all objects photographed have the same elevation. On a near-vertical photograph, the isocentre is very near to the principal point. Hence the angles measured in the vicinity of these points are very nearly equal to the true horizontal angles, independent of tilt or elevation.

Thus, the radial line method is based on the following perspective properties of a vertical or near vertical photograph :

1. The displacements in a photograph due to ground relief and tilt are, within the limit of graphical measurement, radial from the principal point of the photograph.

2. The images near the principal point are nearly free from errors of tilt, and they are shown in their true orthographic positions, regardless of ground relief.
3. The position of a point included in properly overlapping photographs may be located on the map where three rays from three known points intersect.

Principles of Radial Line Resection and Intersection

Before discussing the procedure for preparing planimetric map from aerial photographs, let us study the principle of radial line resections and intersections. The principle can be best illustrated by the following two problems :

(*a*) To locate the principal point of photographs on a map.

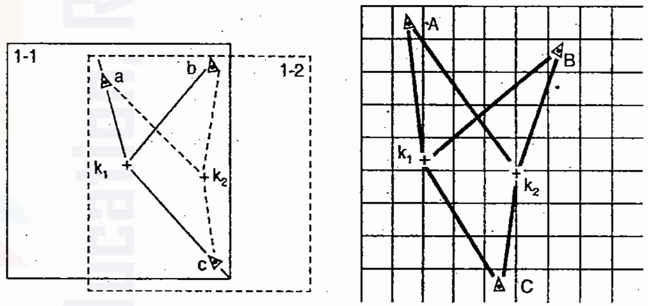
(*b*) To transfer images from a photograph to a map.

(*a*) TO LOCATE THE PRINCIPAL POINT OF PHOTOGRAPHS ON A MAP

A map represents the true horizontal positions of all points at the map scale which is uniform. The map position of principal point of vertical photograph can be located either by (*i*) three point resection or by (*ii*) two point resection.

(*i*) **Map position of principal point by three point resection**

Let *a, b, c* be three photo-control points appearing in both the photographs (Fig. 14.36 *a*), and *A, B* and *C* be their map positions already known by ground survey. It is required to know the map position of the principal points k_1 and k_2 by 3-point resection.



(a) Photographs (b) Map sheet

FIG. 14.36. LOCATION OF PRINCIPAL POINTS BY 3-POINT RESECTION.

On photograph No. 1 draw rays k_1a, k_1b and k_1c. Evidently, angles ak_1b and bk_1c are the true horizontal angles. Similarly, on photograph No. 2, draw rays k_2a, k_2b and k_2c. A piece of tracing paper is put on photograph No. 1, and the rays are traced. The tracing paper is now put on the map sheet and is oriented in such a way that all the three rays simultaneously pass through the plotted positions *A, B* and *C*. The point of intersection of the three rays is the true map position of the principal point k_1. The principal point k_2 of the second photograph can also be located in a similar manner. A three armed protractor can also be used in the place of a tracing paper.

photograph. The line joining the principal points then gives the direction of flight, which can be marked.

(2) Marking the Photographs

Before plotting the map control, each photograph is *marked* by selecting some points on it and drawing radial lines to them from the principal points. For this purpose, let us consider three consecutive photographs 1-1, 1-2 and 1-3 of the first strip, as shown in Fig. 14.40.

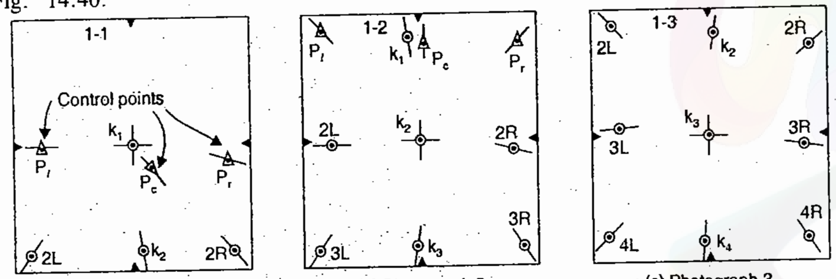


FIG. 14.40. MARKING THE PHOTOGRAPHS

On photograph No. 1, k_1 is its principal point while k_2 is the transferred principal point of photograph No. 2. The images of three ground-control points are identified and marked with needle points at P_l, P_c and P_r. Each of the control-points are enclosed in small triangles drawn with a soft coloured pencil or ink. In addition to three control-points, two additional *pass points* $2L$ and $2R$ are selected at the edge of photograph No. 1 and approximately in line with the transferred principal point k_2 of the second photograph. Thus, on photograph No. 1, in addition to its principal point k_1, there are six more points marked: P_l, P_c, P_r, $2L$, k_2 and $2R$. Short radial lines from the principal point of photograph No. 1 are then drawn through each of these six points. This completes the markings of photograph No. 1.

On photograph No. 2, k_2 is its principal point, while k_1 and k_3 are the transferred principal points of photograph Nos. 1 and 3 respectively. The ground control point P_l, P_c and P_r appear at its edge and nearly in line with k_1. The pass points $2L$ and $2R$ appear approximately in line with k_2. In addition to these, two additional pass points $3L$ and $3R$ are selected at its edge and approximately in line with the transferred principal points k_3 of photograph No. 3. Short radial lines from the principal point of photograph No. 2 are then drawn through each of the points marked on it. This completes the marking of photograph No. 2.

On photograph No. 3, k_3 is its principal point, while k_2 and k_4 are then transferred principal points of photograph Nos. 2 and 4 respectively. The pass points $2L$ and $2R$ appear at its upper edge and in line with k_2. The pass points $3L$ and $3R$ appear in line with k_3. In addition to these, two additional pass points $4L$ and $4R$ are selected at its lower edge and approximately in line with the transferred principal point k_4. Short radial lines from the principal point of photograph No. 3 are then drawn through each of the point marked on it. This completes the marking photograph No. 3.

Each of the succeeding photograph is marked in a similar manner until other ground control points are reached. The end photograph of the strip must include at least one control point.

(3) Plotting the Map Control : The data of the separate photographs are combined into a map showing correct relative locations of the selected points and the control points with the help of a sheet of transparent film base (cellulose acetate) or a good quality tracing paper which exhibits very small changes in its dimensions with changing atmospheric conditions.

The plotted positions of ground control points P_l, P_c and P_r chosen on photographs 1 and 2 are known on the base map. The tracing is stretched on the base map and these control points are transferred by pricking through with a needle. Photograph No. 1 is then slid under the tracing and is oriented in such a way that the radial lines through points P_l, P_c and P_r of the photograph pass through the plotted control points P_l, P_c, P_r on the tracing. In this position, all the rays and points are traced. The principal point k_1 and the transferred principal point k_2 are also traced.

Photograph No. 2 is then slid under the tracing and is oriented in such a way that rays previously drawn on the tracing pass through the corresponding points on the photograph, keeping the traced flight line $k_1 k_2$ coinciding with flight line $k_2 k_1$ on the photograph. Thus, photograph No. 2 is correctly oriented. In this position, all rays and points are traced. In this manner, each of the successive photograph is slid under the tracing, oriented and the rays traced till another ground control point is reached.

Fig. 14.41 shows the plotting of the map control on the tracing. It will be observed that at each of the pass points, there will appear three intersecting rays. The position of each of the points is located on the tracing at the point of intersection of the three rays. This point of intersection may not appear to coincide with the corresponding point on the photograph, because of the displacements due to ground relief. Sometimes, due to errors of plotting, the three rays may not intersect at a point, but may form a small triangle of error. In that case, the centre of the triangle is taken as the position of the point.

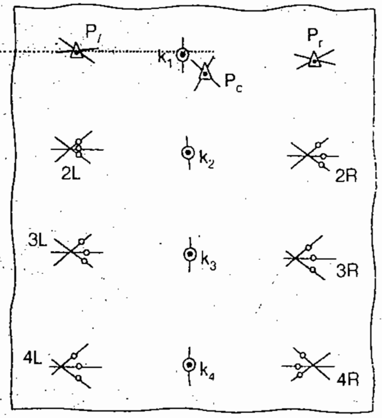


FIG. 14.41. COMPILATION OF MAP CONTROL.

The plotting work is thus continued till the next ground control point is reached. In a perfect map control work, the image of the control point, as located by the intersecting rays, will be the plotted position of the point traced from the base map.

In case, the plotted position of the ground control point does not coincide with its traced position, as usually is the case, the lines of flight, or the positions of the principal points are adjusted as shown in Fig. 14.42. P is the position of the ground control point as located by the intersection of rays, and P' is the corresponding position as traced from the base map. Thus, the total error is $P'P$ in magnitude as well as direction. Each of the photo points, and their positions are then transferred to the map by inserting sharp metal pins through the central holes of the studs.

If the effect of tilt is more in a photograph, its templet will not fit the assembly. Before it can be used, it will need to be rephotographed and the tilt effect removed. The important advantage of the slotted templet method is that any fault due to wrong position of the slot or other sources is mechanically detected when the templets do not fit.

STEREOSCOPY AND PARALLAX

14.24. STEREOSCOPIC VISION

The *depth perception* is the mental process of determining relative distance of objects from the observer from the impressions received through the eyes. Due to binocular vision, the observer is able to perceive the spatial relations, *i.e.*, the three dimensions of his field of view.

The impression of depth is caused mainly due to three reasons : (1) relative apparent size of near and far objects, (2) effects of light and shade, and (3) viewing of an object simultaneously by two eyes which are separated in space. Out of these, the third one is the most important. Each eye views an object from a slightly different position, and by a physiological process the two separate images combine together in the brain enabling us to see in three dimensions.

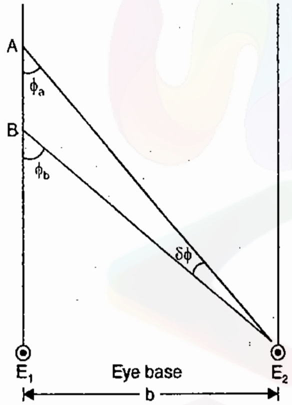


FIG. 14.44. ANGLES OF PARALLAX.

Angle of Parallax (or Parallactic Angle)

In normal binocular vision, the apparent movement of a point viewed first with one eye and then with the other is known as *parallax*. Since an object is viewed simultaneously by two eyes, the two rays of vision converge at an angle upon the object viewed. The angle of parallax or the parallactic angle is the angle of convergence of the two rays of vision. In Fig. 14.44, *A* and *B* are two objects in the field of view, and are being viewed by the two eyes represented in space by the positions, E_1 and E_2 . $E_1 E_2 = b$ is known as the *eye base*. The angle E_1AE_2 is the angle of parallax (φ_a) of object *A*, and the angle E_1BE_2 is the angle of parallax (φ_b) of object *B*. The object *B*, for which the parallactic angle φ_b is greater, will be judged to be nearer the observer than the object *A* for which the parallactic angle φ_a is smaller. The measure of the distance *BA* is evidently provided by the difference in the parallactic angles of *A* and *B*. This difference, *i.e*, $\varphi_b - \varphi_a (= \delta\varphi)$ is termed as the *differential parallax*.

Stereoscopic Fusion

The principles of stereoscopic vision can readily be applied to photogrammetry. An aerial camera takes a series of exposures at regular intervals of time. If a pair of photographs is taken of an object from two slightly different positions of the camera and then viewed by an apparatus which ensures that the left eye sees only the left-hand picture and the right eye is directed to the right hand picture, the two separate images of the object will fuse together in the brain to provide the observer with a spatial impression. This is known as a *stereoscopic fusion*. The pair of two such photographs is known as *stereopair*. Two

devices are used for viewing stereopairs : the *stereoscope* and the *anaglyph*. To illustrate the phenomenon of stereoscopic fusion, let us conduct an experiment (see Figs. 14.45 and 14.46) described below.

Fig. 14.45 shows two pairs of dots near the top edge of a sheet of paper. The

FIG. 14.45

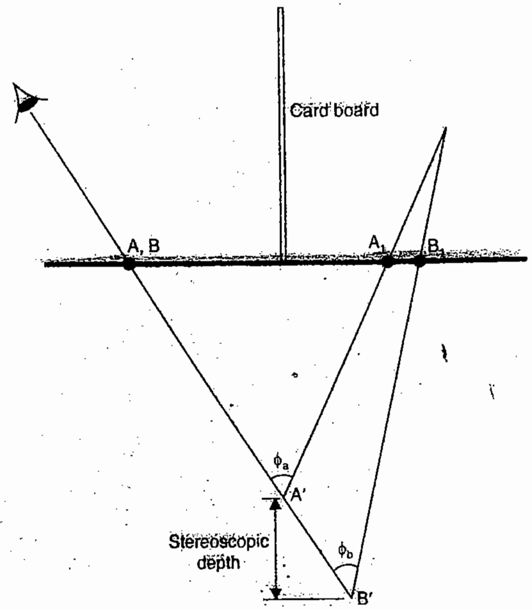


FIG. 14.46. STEREOSCOPIC FUSION.

distance between dots *A* and A_1 is less than the dots *B* and B_1. Place a piece of cardboard between *AB* and $A_1 B_1$, in the plane perpendicular to the sheet so that the left dots *A*, *B* are seen with the left eye and the right dots A_1 , B_1 are seen with the right eye. By staring hard, it will be observed that *A* and A_1 fuse together to form a single dot which appears closer than the fused image of *B* and B_1 (Fig. 14.46).

The apparent difference in level is known as *stereoscopic depth* and depends on the spacing between the dots. The spacing between the dots is called the *parallax difference*.

Clues to Depth Perception

As stated earlier, the depth perception is the mental process of determining relative distance of objects from the observer from the impression received through the eyes. Numerous impressions are received that serve as *clues to depth*, and the following clues are important from photogrammetry point of view :

(1) Head parallax (2) Accommodation
(3) Convergence (4) Retinal disparity.

(*1*) ***Head Parallax*** : Head parallax is the apparent relative movement of object at different distances from the observer when the observer moves.

The distance between the nodal point of the lens and the plane of the photograph depends upon the focal length of the lens. The two photographs can be brought so close to the eyes that proper convergence can be maintained without causing the photographs to interfere with each other as shown in Fig. 14.49. Since the photographs are very close to the eyes, the images occupy larger angular dimensions and therefore appear enlarged. Fig. 14.50 shows a lens stereoscope.

The lens stereoscope is apt to cause eye strain as accommodation is not in sympathy with convergence and the axes of the eyes are forced out their normal condition of visions. Most lens stereoscopes are however, quite small and can be slipped in one's pocket, this type being called a *pocket stereoscope*. Because of larger size, mirror stereoscope is not so portable as is the pocket stereoscope.

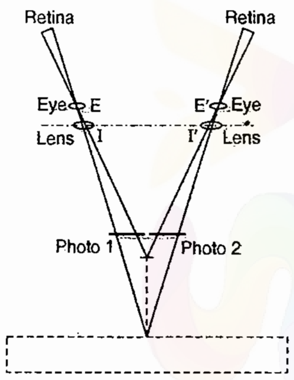


FIG. 14.49. OPTICAL DIAGRAM OF LENS STEREOSCOPE.

14.25. PARALLAX IN AERIAL STEREOSCOPIC VIEWS

Parallax of a point is the displacement of the image of the point on two successive exposures.

The difference between the displacements of the images of two points on successive exposures is called the difference in parallax between the two points.

In Fig. 14.51, two points A (lower) and B (higher) are being photographed by the two positions O and O' of an aerial camera. If the plane is moving at a speed of 200 km/hour and if the exposures are taken at an interval of 20 seconds, the lens centre moves a ground distance of about 1110 metres between the two exposures. Suppose that between the two exposures, the image of the lower object A has moved a distance 6.05 cm across the focal plane of the camera, and

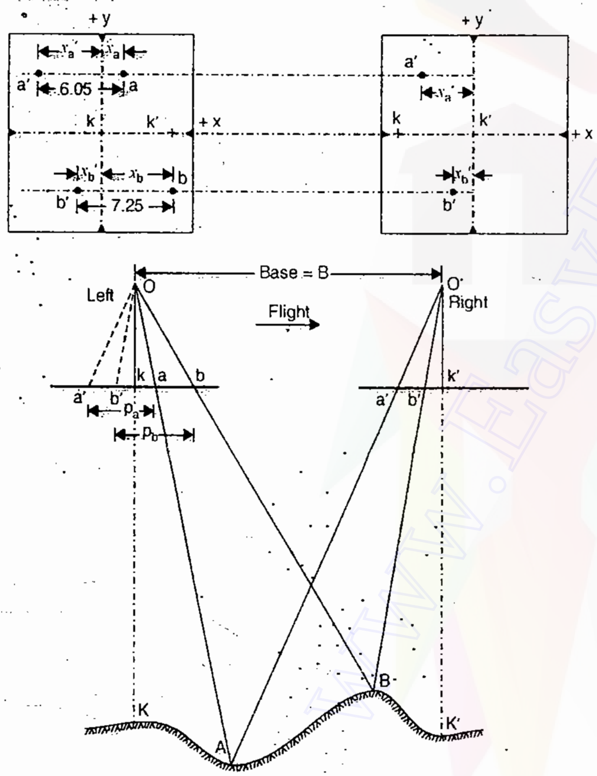


FIG. 14.51. PARALLAX.

the image of the higher object B has moved a distance of 7.25 cm. Then the parallax of the lower point is 6.05 cm and that of the higher point is 7.25 cm.

In the left photograph, a and b are the images of the two points. k' is the *transferred* principal point of the right photograph. Both the images a and b are to the right of the y-axis of the left photograph. In the right photograph, a' and b' are the images of the same points, both the images being to the left of the y-axis. Thus the images (a, b) of the points have moved to (a', b') between the two exposures. The movement aa' (shown on the left photographs) is the parallax of A, and bb' is the true parallax of B. The parallax of the higher point is more than the parallax of the lower point. Thus, each image in a changing terrain elevation has a slightly different parallax from that of a neighboring image. *This point-to-point difference in parallax exhibited between points on a stereopair makes possible the viewing of the photographs stereoscopically to gain an impression of a continuous three dimensional image of a terrain.*

The following are the ideal conditions for obtaining aerial stereoscopic views of the ground surface :

(1) two photographs are taken with sufficient overlap.

(2) the elevation of the camera positions remains the same for the two exposures.

(3) the camera axis is vertical so that the picture planes lie in the same horizontal plane.

Algebraic Definition of Parallax : As defined earlier the displacement of the image of a point on two successive exposures is called the parallax of the point. On a pair of overlapping photographs, the parallax is thus equal to the x-coordinate of the point measured on the left-hand photograph (or previous photograph) minus the x-coordinate of the point measured on the right-hand photograph (or next photograph). Thus

$$p = x - x' \qquad \ldots(14.23)$$

Thus, x-axis passes through the principal point and is *parallel* to the flight line, while the y-axis passes through the principal point and is perpendicular to the line of flight. In general, however, the flight-line x-axis is usually very close to the collimation mark x-axis, because of the effort made to eliminate drift and crab at the time of photography.

Thus, in Fig. 14.51, the parallax of points A and B are given by

$$p_a = x_a - x'_a \qquad \text{and} \qquad p_b = x_b - x'_b$$

In substituting the numerical values of x and x', their proper algebraic sign must be taken into consideration. Thus, in Fig. 14.51, if $x_a = 2.55$ cm, $x'_a = -3.50$ cm, $x_b = -4.05$ cm and $x'_b = -3.20$ cm, we have

$$p_a = +2.55 - (-3.50) = 6.05 \text{ cm}$$

$$p_b = +4.05 - (-3.20) = 7.25 \text{ cm.}$$

14.26. PARALLAX EQUATIONS FOR DETERMINING ELEVATION AND GROUND CO-ORDINATES OF A POINT

Let A be a point whose ground co-ordinates and elevation are to be found by parallax measurement.

Evidently, the parallax p_1 for the bottom of the flagstaff is given by

$$p_1 = x_1 - x'_1 \qquad \ldots(1)$$

Similarly, the parallax p_2 for the top of the flagstaff is given by

$$p_2 = x_2 - x'_2 \qquad \ldots(2)$$

Hence the difference in parallax (Δp) of top and bottom points is given by

$$\Delta p = p_2 - p_1 = (x_2 - x'_2) - (x_1 - x'_1) \qquad \ldots(3)$$

From equation 14.35, the elevation of any point is given by

$$h = H - \frac{Bf}{p}$$

Hence, for the top and bottom of flagstaff, we get

$$h_1 = H - \frac{Bf}{p_1} \quad \text{and} \quad h_2 = H - \frac{Bf}{p_2}$$

$\therefore$ Difference in elevation (Δh) is given by,

$$\Delta h = h_2 - h_1 = \left(H - \frac{Bf}{p_2}\right) - \left(H - \frac{Bf}{p_1}\right) = \frac{Bf}{p_1} - \frac{Bf}{p_2}$$

or

$$\Delta h = \left(\frac{p_2 - p_1}{p_1 p_2}\right) Bf \qquad \ldots(14.37)$$

or

$$\Delta h = \frac{\Delta p}{p_1 p_2} Bf \qquad \ldots[14.37(a)]$$

Now $\Delta p = p_2 - p_1$ or $p_2 = p_1 + \Delta p$

Hence, we have

$$\Delta h = \frac{\Delta p}{p_1 (p_1 + \Delta p)} \cdot Bf \qquad \ldots(14.38)$$

Mean Principal Base (b_m) : The distance between the principal point of a photograph and the position of transferred principal point of its next photograph obtained under fusion through stereoscope is called *principal base*. Thus, in Fig. 14.51,

$kk' = b$ = principal base of left photograph

and $k'k = b'$ = principal base of right photograph.

It should be noted that b and b' will not be equal since the elevation of ground positions of the principal points (K and K') are not the same.

The *mean principal base* is the mean value of the principal bases of the photographs.

Thus,

$$b_m = \frac{b + b'}{2}$$

If the ground principal points (K and K') have the same elevation, then under ideal conditions, $b_m = b$.

Now, in Fig. 14.53, let the datum pass through the bottom A_1 of the flagstaff (*i.e.* $h_1 = 0$). Assuming the ground to be now the datum plane, the ground principal points K and K' will be at the same elevation, and the parallax of the principal points (*i.e.*, the principal base) will be equal to b. If H is the height of camera above the datum (*i.e.* above A_1 now), the general relationship between b and B is given by

$$\frac{B}{b} = \frac{H}{f} \quad \text{or} \quad B = \frac{Hb}{f} = s \times b \qquad \ldots(14.39)$$

where s is the scale of the photograph at datum elevation. Substituting this value of air base in equation 14.38, we get

$$\Delta h = \frac{\Delta p}{p_1 (p_1 + \Delta p)} \cdot Hb$$

Since K, K' and A_1 are all at the same elevation, their parallaxes are the same. Hence p_1 = parallax of principal points $= b$

Hence, we get the parallax equation

$$\Delta h = \frac{H \Delta p}{b + \Delta p} = \frac{H \Delta p}{p_1 + \Delta p} \qquad \ldots(14.40)$$

While using equations 14.40, the following assumptions must always be kept in mind:

(1) The vertical control point (*i.e.*, point A_1) and the two ground principal points lie at the same elevation.

(2) The flying height (H) is measured above the elevation of the control point and not sea level (unless the control point happens to lie at sea level).

In practical applications, the mean principal base (b_m) is used in place of b, and flying height above the average terrain is taken as the value of H.

Alternative form of Parallax Equation for Δh

We have, $p_1 = \dfrac{fB}{H - h_1}$ and $p_2 = \dfrac{fB}{H - h_2}$

$$\therefore \quad \Delta p = p_2 - p_1 = fB\left(\frac{1}{H - h_2} - \frac{1}{H - h_1}\right) = fB\,\frac{h_2 - h_1}{(H - h_1)(H - h_2)}$$

But $\Delta h = h_2 - h_1$ and $h_2 = \Delta h + h_1$

$$\therefore \quad \Delta p = fB\,\frac{\Delta h}{(H - h_1)(H - \Delta h - h_1)}$$

or

$$\Delta p\,(H - h_1)^2 - \Delta p\,(H - h_1)\,\Delta h = fB\,\Delta h$$

or

$$\Delta h\,[(H - h_1)\,\Delta p + fB] = (H - h_1)^2\,\Delta p$$

or

$$\Delta h = \frac{(H - h_1)^2\,\Delta p}{(H - h_1)\,\Delta p + fB}$$

Dropping the suffix of h, we get

$$\Delta h = \frac{(H - h)^2\,\Delta p}{(H - h)\,\Delta p + fB} \qquad \ldots[14.41(a)]$$

where h is the elevation of lower point above datum.

Putting $fB = Hb$, we get

$$\Delta h = \frac{(H - h)^2\,\Delta p}{(H - h)\,\Delta p + bH} \qquad \ldots(14.41)$$

It should be noted that the above equation is in its most general form. Eq. 14.40 is the special form of this, and can be obtained by putting $h = 0$ (*i.e.*, lower point at datum) in Eq. 14.41). Thus

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Let $H = 8000$ m ; $b_0 = 100$ mm

and h be increased by 10 m for the interval $(H - h) = 8000$ m to $(H - h) = 3000$ m, and then by 5 m for interval $(H - h) = 3000$ to $(H - h) = 1500$ m.

When $h = 0$, $H - h = 8000$ m and $\Sigma \Delta p = 0$

When $h = 10$, $H - h = 7990$ m and $\Sigma \Delta p = \dfrac{100 \times 10}{7990} = 0.125$ mm

When $h = 1000$ m, $H - h = 7000$ m and $\Sigma \Delta p = \dfrac{100 \times 1000}{7000} = 14.286$ mm

When $h = 5000$ m, $H - h = 3000$ and $\Sigma \Delta p = \dfrac{100 \times 5000}{3000} = 166.667$ mm.

Thus, the values of $\Sigma \Delta p$ for the different values of h can be found and a master parallax table between $(H - h)$ and $\Sigma \Delta p$ can be prepared. Such a table will, however, be useful for direct computations only if $H = 8000$ and $b_0 = 100$ mm. The values of $\Sigma \Delta p$ in the table may, however, be adapted to other conditions also if they are multiplied by a constant K *such that*

$$K = \frac{b_0 \text{ (photo)} \times H \text{ (photo)}}{100 \times 8000} \quad \ldots(14.46)$$

Example 14.17. *A photogrammetric survey is carried out to a scale of 1 : 20000. A camera with a wide angle lens of f = 150 mm was used with 23 cm × 23 cm plate size for a net 60% overlap along the line of flight. Find the error in height given by an error of 0.1 mm in measuring the parallax of the point.*

Solution. Scale $= \dfrac{f}{H}$

$$\therefore \quad \frac{1}{20,000} = \frac{150/1000 \text{ (m)}}{H \text{(m)}}$$

or $$H = \frac{150}{1000} \times 20,000 = 3000 \text{ m}$$

The length of the air base is given by

$$B = \left(1 - \frac{p_l}{100}\right) ls = (1 - 0.6)\,\frac{23}{100} \times 20,000 = 1840 \text{ m}$$

From equation 3.41, we have

$$dh = \frac{(H - h)^2}{Bf} \cdot dp$$

Corresponding to the datum elevation, the error dh for $dp = 0.1$ mm is

$$dh = \frac{(3000 - 0)^2}{1840 \times 150} \times 0.1 = \mathbf{3.26 \text{ m.}}$$

Example 14.18. *In a pair of overlapping vertical photographs, the mean distance between two principal points both of which lie on the datum is 6.375 cm. At the time of photography, the air-craft was 600 m above the datum. The camera has a focal length of 150 mm. In the common overlap, a tall chimney 120 m high with its base in the datum surface is observed. Determine difference of parallax for top and bottom of chimney.*

Solution.

$$s = \text{Scale of the photograph for datum elevation} = \frac{f}{H} = \frac{150/1000}{600} = \frac{1}{4000}$$

For the datum elevation, we have, from Eq. 14.39, $\dfrac{B}{b} = \dfrac{H}{f}$

or $$B = \frac{H}{f}\, b = s \times b = 4000 \times \frac{6.375}{100} = 255 \text{ m}$$

The parallaxes for the top and the bottom of the chimney are calculated from Eq. 14.35, *i.e.*

$$p = \frac{Bf}{H - h}$$

For the bottom of the chimney, $h = 0$ (since the bottom of the chimney is the datum), and hence

$$p_1 = \frac{255 \times 150 \text{ (mm)}}{600} = 63.75 \text{ mm}$$

For the top of the chimney, $h = 120$ m

$$\therefore \quad p_2 = \frac{255 \times 150 \text{ (mm)}}{(600 - 120)} = 79.69 \text{ mm}$$

Hence difference of parallax is given by

$$\Delta p = (p_2 - p_1) = 76.69 - 63.75 = \mathbf{15.94 \text{ mm}}$$

Check. From equation 14.40,

$$\Delta h = \frac{H \Delta p}{b + \Delta p} = \frac{600 \text{ m} \times 15.94 \text{ (mm)}}{63.75 \text{ (mm)} + 15.94 \text{ (mm)}} = 120.09 \text{ m} \approx 120 \text{ m.}$$

which is the same as the given height of the chimney.

Example 14.19. *A flag pole appears in two successive photographs taken at an altitude of 2000 m above datum. The focal length of the camera is 120 mm and the length of the air base is 200 m. The parallax for the top of the pole is 52.52 mm and for the bottom is 48.27 mm. Find the difference in elevation of top and bottom of the pole.*

Solution

The difference in elevation between two points is given by equation 14.37, *i.e.*

$$\Delta h = \left(\frac{p_2 - p_1}{p_1\, p_2}\right) Bf = \left(\frac{52.52 - 48.27}{52.52 \times 48.27}\right) \times 200 \times 120 = \mathbf{44.2 \text{ m.}}$$

Example 14.20. *A pair of photographs was taken with an aerial camera from an altitude of 5000 m above m.s.l. The mean principal base measured is equal to 90 mm. The difference in parallax between two points is 1.48 mm. Find the difference in height between the two points if the elevation of the lower point is 500 m above datum. What will be the difference in elevation if the parallax difference is 15.5 mm ?*

Solution

(*a*) $$\Delta p = 1.48 \text{ mm.}$$

Since Δp is extremely small, Δh will also be small. Hence approximate formula [Eq. 14.42 (*a*)] can be used to calculate Δh.

is so oriented that it is parallel to the flight line. Move the right mark and make the fused dot to touch the ground point. Take the micrometer reading. Shift the bar bodily, put the left mark over the image c and move the right mark so that the fused mark again rests on the ground. Note the micrometer reading. The difference between the two readings gives the value Δp.

Thus in Fig. 14.56 when point a is fused, the separation of the marks is lesser and the point is higher as is clear from the two intersecting rays OaA and $O'a'A$ in the lower part of the diagram. Similarly, when c is fused, the separation of the marks is increased, and the point is lower as is clear from the two intersecting rays OcC and $O'c'C$.

The difference in elevation is then found by Equation 14.41, *i.e.*

$$\Delta H = \frac{(H-h)^2 \, \Delta p}{(H-h)\,\Delta p + b_m H} \qquad ..(14.41)$$

where b_m is the mean principal base.

14.29. RECTIFICATION AND ENLARGEMENT OF PHOTOGRAPHS

Rectification is the process of rephotographing an aerial photograph so that the effects of tilt are eliminated. The rectification of tilted photograph taken from a given exposure station in space transforms the photograph into an *equivalent vertical photograph* taken from the same exposure station. Often the equivalent vertical photograph is enlarged or reduced as part of the process.

Fig. 14.57 (a) shows a photograph of an area taken with air camera vertical. The intersecting roads appear on the photo in their true positions. Fig. 14.57 (b) shows the distorted appearance of the roads on a titled photograph. Fig. 14.57 (c) shows the appearance of a rectified print. The roads are restored to their true shape, though the print is no longer square.

If the photograph is to be magnified, the principal distance of the photograph is changed so that the following equation is satisfied :

$$p = mf \qquad ...(14.47)$$

where p = principal distance of the rectified photograph

f = focal length of the camera lens;

m = magnification factor.

If m is greater than unity, it denotes enlargement while if it is less than unity, it denotes an actual reduction of the photograph.

Various photographs are taken at different heights due to imperfect control in maintaining the aircraft perfectly at one altitude. The *purpose of magnification is to bring these to the same*

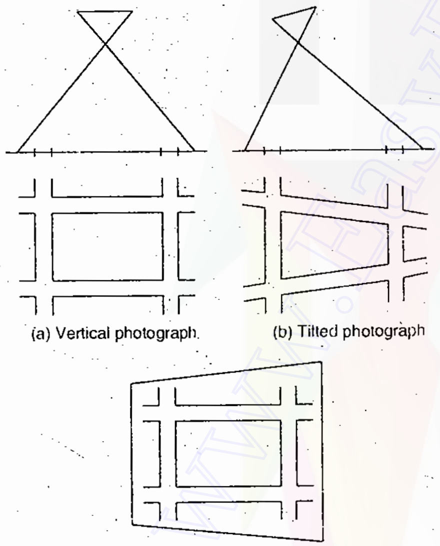


FIG. 14.57. RECTIFICATION OF TILTED PHOTOGRAPHS.

scale at a particular elevation—either at the datum elevation or the average elevation of the terrain.

Fig. 14.58 shows a photograph with a tilt t at the exposure station O and flying height h. The negative and the photograph are parallel to each other. The rectified enlargement $b'\,k'\,a'$ is inclined at an angle t with the negative, and its principal distance is $p = mf$. The horizontal plane of the rectified photograph is known as the *easel plane*. It is to be noted that for the rectified enlargement of the photograph, the negative should be placed at a distance f from the lens. The lens will project the images from the negative in the proper direction, but since the distance from the lens to the negative is equal to the focal length of the lens, the projected bundles of the rays will be parallel to one another and they would never come to focus the enlargement plane. This is shown in Fig. 14.59 (a). This gives the condition that the entire negative must be placed behind the focal plane of the lens used in the rectifier.

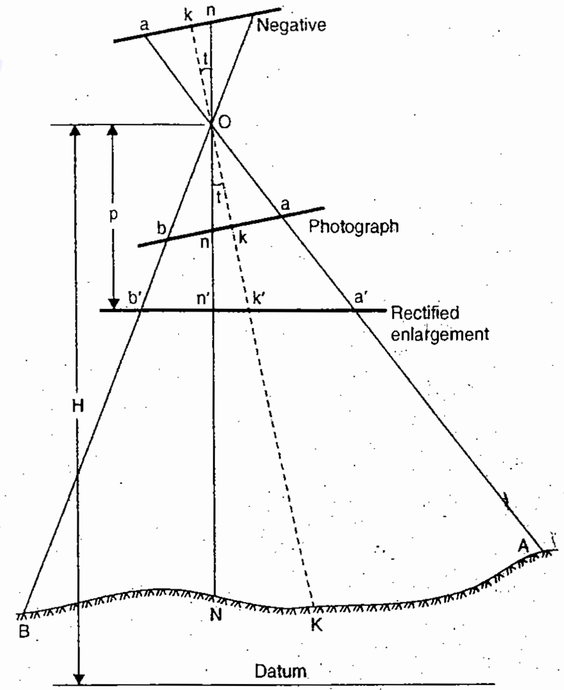


FIG. 14.58. RECTIFICATION AND ENLARGEMENT.

Scheimpflug Condition

As discussed in the previous paragraph if the negative plane is placed at the focal plane of the lens, the image cannot be focused. This is illustrated in Fig. 14.59 (a). If, however, the negative plane is placed beyond the focal plane, at a distance q from the lens and r is the corresponding position of the enlargement, the following two conditions are to be satisfied simultaneously :

$$\frac{1}{q} + \frac{1}{r} = \frac{1}{F} \quad \text{and} \quad r = mq \qquad ...(14.48)$$

where F is the focal length of the lens of the rectifier and m is the magnification.

The relationships stated above are for a vertical photograph. However, these apply also for a tilted photograph. These conditions are shown in Fig. 14.59 (c). x and x' are the conjugate distances for the point a, while y and y' are conjugate distances for the point b. Hence, we have

$$\frac{1}{x} + \frac{1}{x'} = \frac{1}{F} \quad \text{and} \quad \frac{1}{y} + \frac{1}{y'} = \frac{1}{F}$$

A stereoscopic plotting instrument has four general components:

(1) a projection system (2) a viewing system
(3) a measuring system (4) a tracing system.

It is beyond the scope of the present book to illustrate fully the theory and working of the various plotting machines. However a brief description of the multiplex plotter is given below.

The Multiplex Plotter

The multiplex is probably the most widely used of any type of plotting machine. The equipment includes a reduction printer, a set of projectors mounted in series on a horizontal bar and a tracing table which provides both a floating mark and a tracing pencil to draw the map. The reduction printer produces reduced pictures on small glass plates. The 23 cm × 23 cm (or 9″ × 9″) size is thus reduced to a size 4 cm × 4 cm on the glass plates called *diapositives*.

Fig. 14.61 shows a pair of multiplex projectors forming a stereoscopic model. Each projector consists of a light source, a plate holder for the diapositive plate, and a lens which transmits the rays coming from the diapositive plate into the open space below the projector. The spatial model is obtained by projecting one photograph of an overlapping pair in red light and the other in blue-green light, and by observing the combination of colours through spectacles containing one red and one blue-green lens. This model, in fact, has three dimensions, and is not to be considered as virtual stereoscopic image as seen in a simple stereoscope. This method of viewing is called the *anaglyph system* of viewing reflected light and fulfills the condition of stereoscopic viewing.

Provision is made to move each of the projector in the directions of the *X,Y,Z co-ordinate axes, and also to rotate the projector about each of these axes. The X*-motion is parallel to the supporting bar, the *Y*-motion is perpendicular to the supporting bar and in the horizontal direction, while the Z-motion is perpendicular to the supporting bar and in the vertical direction. These six motions of each projector, independent of the others, make it possible to orient each projector in exactly in the same relation to the control points on the drawing table below, which the camera in the air had to the same corresponding actual ground points.

The *tracing table* contains a circular white disc with a pinhole in its centre. A light bulb below the disc provides a small pin point of light. This illuminated pin point is visible from the projectors and forms the measuring mark or floating mark in the spatial model. The disc can be raised or lowered so that the floating mark rests on the ground of the model. The tracing pencil point vertically below the floating mark gives position of the point on the map sheet. The tracing pencil traces pencil traces on the plotting sheet the horizontal movements of the floating mark. The disc is raised or lowered by means of a screw on the centre post at the back of the tracing stand. On the left post of the tracing stand is a millimetre scale on which is read the height of the disc above the drawing table which may be considered as the datum plane. The elevation of any point in the spatial model can be found by reading the vertical scale of the tracing stand after the floating mark has been set on the given point.

To trace a specific contour line on the map sheet, the disc is raised or lowered to give the correct reading (recorded in mm on the scale) corresponding to the desired elevation of the contour line. The operator moves the tracing table (with the pencil in the raised position) until the measuring mark comes into apparent contact with the model surface. and then lowers the pencil on the map sheet. By examining the stereoscopic view of the model slightly ahead of the measuring mark, the tracing table is moved in such a direction as to keep the measuring mark in contact with the surface of the model at all times. The line traced by pencil below the floating mark is the contour line since the floating mark was moving at a fixed elevation. When one contour line has been traced, the disc can be set to another height and the next contour can be traced in a similar manner.

The actual apparatus consists of a series of projectors (and not only two) mounted on a horizontal bar. The filters of the projectors are alternately red and blue-green. When the first true model has been placed in position, by viewing through the first two projectors and orienting them properly, the second true model is established by adjusting the third projector. The orientation and adjustments are done by means of ground-control points plotted on the drawing table. Thus each of the successive projectors can be oriented. This procedure is called *aerial triangulation* or *bridging* and is extended till other ground control appears. The proper adjustments are made throughout the series of projectors before drawing of the map is begun.

PROBLEMS

1. Define the following :
   (*i*) Air base, (*ii*) Tilt displacement.
   (*iii*) Principal point. (*iv*) Isocentre. (*v*) Isometric parallel.
2. Describe with sketches the field work of a survey with phototheodolite. Explain how you would plot the survey.
3. What is tilt distortion ? Prove that, in a tilted photograph, tilt distortion is radial from the isocentre.
4. Describe the various steps involved in the combination of vertical air photographs by the principal point radial line method.
5. Vertical photographs were taken from height of 3048 m, the focal length of the camera lens being 15.24 cm. If the prints were 22.86 × 22.86 cm and the overlap 60%, what was the length of the air base ? What would be the scale of the print ? (R.U.)
6. (*a*) Derive the parallax equation for determining heights from a pair of vertical photographs.
   (*b*) Two ground points *A* and *B* appear on a pair of overlapping photographs which have been taken from a height of 3650 m above mean sea-level. The base lines as measured on the two photographs are 89.5 mm and 90.5 mm respectively. The mean parallax bar readings for *A* and *B* are 29.32 mm and 30.82 mm. If the elevation of *A* above mean sea-level is 230.35 m, compute the elevation of *B*.
7. Two objects *A* and *B* whose elevations are 500 m and 1500 m respectively above mean sea-level are photographed from certain height with the axis of the camera vertical. The coordinates expressed in mm of the corresponding photo-images *a* and *b* are:

Point	*x* co-ordinate	*y* co-ordinate
a	+ 200	+ 150
b	– 320	– 300

The focal length = 200 mm and length *AB* = 44227 m. Find the height of the camera station (R.U.)

range EDM instruments (such as Distomats) commonly used in surveying use *modulated infra-red waves*.

Properties of electromagnetic waves

Electromagnetic waves, though extremely complex in nature, can be represented in the form of periodic sinusoidal waves shown in Fig. 15.1. It has the following properties:

1. The waves completes a *cycle* in moving from identical points *A* to *E* or *B* to *F* or *D* to *H*.

2. The number of times the wave completes a cycle in one second is termed as *frequency* of the wave. The frequency is represented by *f* hertz (*Hz*) where 1 hertz (*Hz*) is one cycle per second. Thus, if the frequency *f* is equal to 10^3 Hz, it means that the waves completes 10^3 cycles per second.

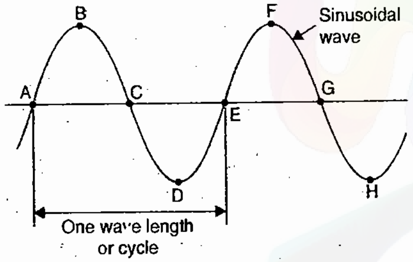


FIG 15.1 PERIODIC SINUSOIDAL WAVES.

3. The length traversed in one cycle by the wave is termed as *wave length* and is denoted by λ (metres). Thus the *wave length* of a wave is the distance between two identical points (such as *A* and *E* or *B* and *F*) on the wave.

4. The *period* is the time taken by the wave to travel through one cycle or one wavelength. It is represented by *T* seconds.

5. The *velocity* (*v*) of the wave is the distance travelled by in one second.

The frequency, wavelength and period can all vary according to the wave producing source. However, the velocity *v* of an electromagnetic wave depends upon the medium through which it is travelling. The velocity of wave in a vacuum is termed as *speed of light*, denoted by symbol *c*, the value of which is presently known to be 299792.5 km/s. For simple calculations, it may be assumed to be 3×10^8 m/s.

The above properties of an electromagnetic wave can be represented by the relation,

$$f = \frac{c}{\lambda} = \frac{1}{T} \qquad \text{...(15.1)}$$

Another property of the wave, known as *phase* of the wave, and denoted by symbol φ, is a very convenient method of identifying fraction of a wavelength or cycle, in EDM. One cycle or wave-length has a phase ranging from 0° to 360°. Various points *A*, *B*, *C* etc. of Fig. 15.1 has the following phase values :

Point →	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Phase $\phi°$	0	90	180	270	360 (or 0)	90	180	270

Fig. 15.2 gives the electromagnetic spectrum. The type of electromagnetic wave is known by its wavelength or its frequency. However, all these travel with a velocity *approximately* equal to 3×10^8 m/s. *This velocity forms the basis of all electromagnetic measurements.*

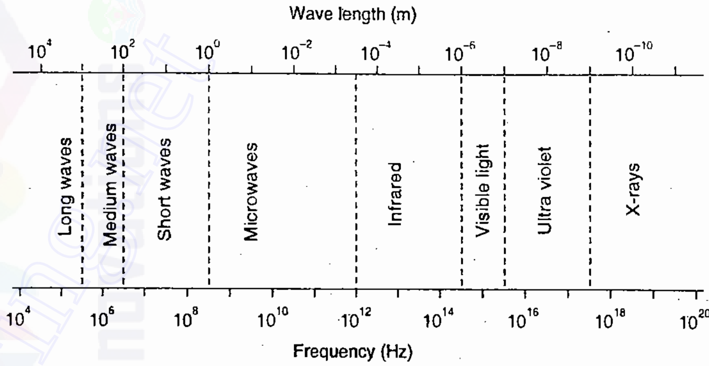


FIG. 15.2 ELECTROMAGNETIC SPECTRUM.

Measurement of transit times

Fig. 15.3 (*a*) shows a survey line *AB*, the length *D* of which is to be measured using EDM equipment placed at ends *A* and *B*. Let a transmitter be placed at *A* to propagate electromagnetic waves towards *B*, and let a receiver be placed at *B*, along with a timer. If the timer at *B starts* at the instant of transmission of wave from *A*, and *stops* at the instant of reception of incoming wave at *B*, the *transit time* for the wave from *A* and *B* in known.

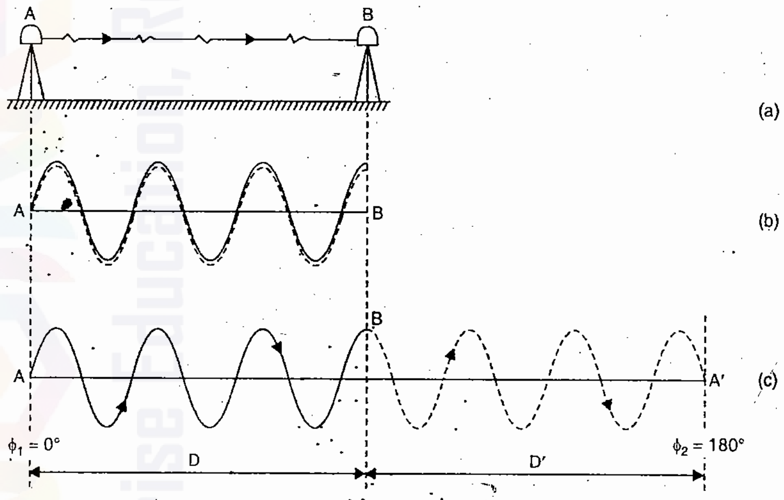


FIG. 15.3. MEASUREMENT OF TRANSIT TIME.

From this transit time, and from the known velocity of propagation of the wave, the distance *D* between *A* and *B* can be easily computed. However, this transit time is of the order of 1×10^{-6} s which requires very advanced electronics. Also it is extremely difficult to start the timer at *B* when the wave is transmitted at *A*. Hence a reflector varies in proportion to the amplitude of the modulating wave. *Frequency modulation* is used in all microwave EDM instruments while *amplitude modulation* is done in *visible light instruments* and infrared instruments using higher carrier frequencies.

15.4. TYPES OF EDM INSTRUMENTS

Depending upon the type of carrier wave employed, EDM instruments can be classified under the following three heads :

(*a*) Microwave instruments

(*b*) Visible light instruments

(*c*) Infrared instruments.

For the corresponding frequencies of carrier waves, reader may refer back to Fig. 15.2. It is seen that all the above three categories of EDM instruments use short wavelengths and hence higher frequencies.

1. Microwave instruments

These instruments come under the category of long range instruments, where in the carrier frequencies of the range of 3 to 30 GHz (1 GHz = 10^9) enable distance measurements upto 100 km range. *Tellurometer* come under this category.

Phase comparison technique is used for distance measurement. This necessitates the erection of some form of *reflector* at the remote end of the line. If *passive reflector* is placed at the other end, a weak signal would be available for phase comparison. Hence an electronic signal is required to be erected at the reflecting end of the line. This instrument, known as *remote instrument* is identical to the *master instrument* placed at the measuring end. The *remote instrument* receives the transmitted signal, amplifies it and transmits it back to the master in exactly the phase at which it was received. This means that microwave EDM instruments require two instruments and two operators. Frequency modulation is used in most of the microwave instruments. The method of varying the measuring wavelength in multiplies of 10 is used to obtain an unambiguious measurement of distance. The microwave signals are radiated from small aerials (called *dipoles*) mounted in front of each instrument, producing directional signal with a beam of width varying from 2° to 20°. Hence the alignment of master and remote units is not critical. Typical maximum ranges for microwave instruments are from 30 to 80 km, with an accuracy of ± 15 mm to ± 5 mm/km.

2. Visible light instruments

These instruments use visible light as carrier wave, with a higher frequency, of the order of $.5 \times 10^{14}$ Hz. Since the transmitting power of carrier wave of such high frequency falls off rapidly with the distance, the range of such EDM instruments is lesser than those of microwave units. A *geodimeter* comes under this category of EDM instruments.

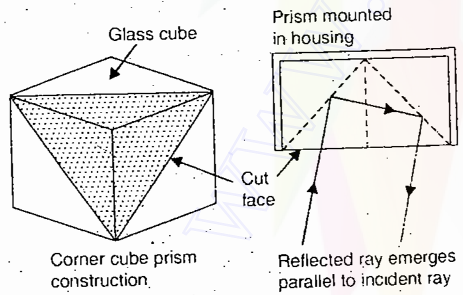


FIG. 15.5. CORNER CUBE PRISM

The carrier, transmitted as light beam, is concentrated on a signal using lens or mirror system, so that signal loss does not take place.

Since the beam divergence is less than 1°, accurate alignment of the instrument is necessary. *Corner-cube prisms*, shown in Fig. 15.5 are used as reflectors at the remote end. These prisms are constructed from the corners of glass cubes which have been cut away in a plane making an angle of 45° with the faces of the cube.

The light wave, directed into the cut-face is reflected by highly silvered inner surfaces of the prism, resulting in the reflection of the light beam along a parallel path. This is obtainable over a range of angles of incidence of about 20° to the normal of the front face of the prism. *Hence the aligment of the reflecting prism towards the main EDM instrument at the receiver (or transmitting) end is not critical.*

The advantage of visible light EDM instruments, over the microwave EDM instruments is that only one instrument is required, which work in conjunction with the inexpensive corner cube reflector. *Amplitude modulation* is employed, using a form of electro-optical shutter. The line is measured using three different wavelengths, using the same carrier in each case. The EDM instrument in this category have a range of 25 km, with an accuracy of ± 10 mm to ± 2 mm/km. The recent instruments use pulsed light sources and highly specialised modulation and phase comparison techniques, and produce a very high degree of accuracy of ± 0.2 mm to ± 1 mm/km with a range of 2 to 3 km.

3. Infrared instruments

The EDM instruments in this group use near infrared radiation band of wavelength about 0.9 μ m as carrier wave which is easily obtained from gallium arsenide (Ga As) infrared emitting diode. These diodes can be very easily directly *amplitude modulated* at high frequencies. Thus, modulated carrier wave is obtained by an inexpensive method. Due to this reason, there is predominance of infrared instruments in EDM. Wild Distomats fall under this category of EDM instruments.

The power output of the diodes is low. Hence the range of these instruments is limited to 2 to 5 km. However, this range is quite sufficient for most of the civil engineering works. The EDM instruments of this category are very light and compact, and these can be theodolite mounted. This enables angles and distances to be measured simultaneously at the site. A typical combination is Wild DI 1000 infra-red EDM with Wild T 1000 electronic theodolite ('Theomat'). The accuracy obtainable is of the order of ± 10 mm, irrespective of the distance in most cases.

The carrier wavelength in this group is close to the visible light spectrum. Hence infrared source can be transmitted in a similar manner to the visible light system using geometric optics, a lens/mirror system being used to radiate a highly collimated beam of angular divergence of less than 15′. Corner cube prisms are used at the remote end, to reflect the signal.

Electronic tacheometer, such as Wild TC 2000 'Tachymat' is a further development of the infrared (and laser) distance measurer, which combines theodolite and EDM units. Microprocessor controlled angle measurement give very high degree of accuracy, enabling horizontal and vertical angles, and the distances (horizontal, vertical, inclined) to be automatically displaced and recorded.

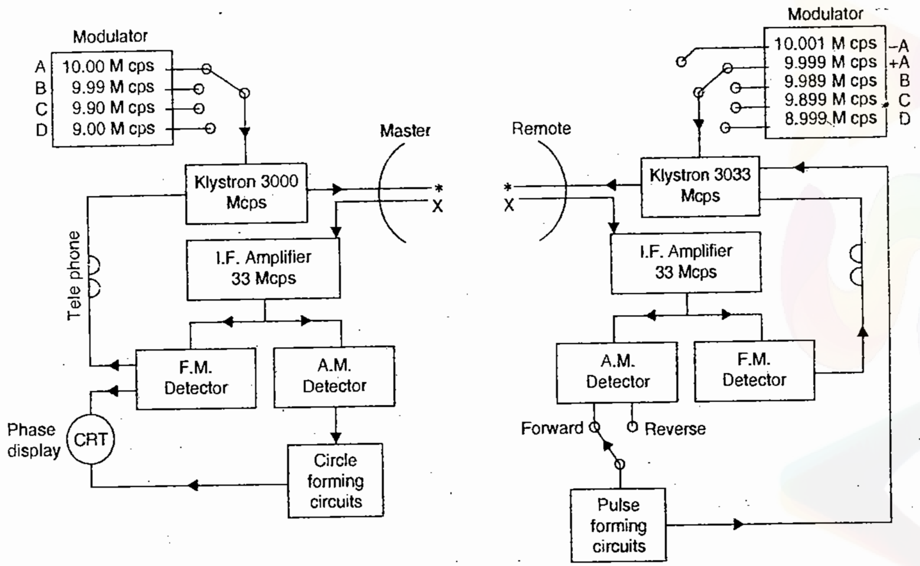


FIG. 15.9 BLOCK DIAGRAM OF THE TELLUROMETER SYSTEM.

frequencies, *i.e.* 3033 – 3000=33 Mc.s. (known as *intermediate frequency*) is obtained by an electrical 'mixer', and is used to provide sufficient sensitivity in the internal detector circuits at each instrument. In addition to the carrier wave of 3033 Mc.s., a crystal at the remote station is generating a frequency of 9.999 Mc.s. This is *heterodyned* with the incoming 10 Mc.s. to provide a 1 k c.p.s. signal. The 33 Mc.s. intermediate frequency signal is amplitude modulated by 1 k c.p.s. signal. The amplitude modulated signal passes to the amplitude demodulator, which detects the 1 k c.p.s. frequency. At the pulse forming circuit, a pulse with a repitition frequency of 1 k c.p.s. is obtained. This pulse is then applied to the klystron and frequency modulates the signal emitted, *i.e.*, 3033 Mc.s. modulated by 9.999 Mc.s. and pulse of 1 k c.p.s. This signal is received at the master station. A further compound heterodyne process takes place here also, where by the two carrier frequencies subtract to give rise to an intermediate frequency of 33 Mc.s. The two *pattern frequencies* of 10 and 9.999 Mc.s. also subtract to provide 1 k c.p.s *reference frequency* as amplitude modulation. The *change in the phase between this and the remote 1 k c.p.s. signal is a measure of the distance* . The value of phase delay is expressed in time units and appear as a *break* in a circular trace on the oscilloscope cathode ray tube.

Four low frequencies (*A*, *B*, *C* and *D*) of values 10.00, 9.99,9.90 and 9.00 Mc.p.s. are employed at the master station, and the values of phase delays corresponding to each of these are measured on the oscilloscope cathode ray tube. The phase delay of *B*, *C* and *D* are subtracted from *A* in turn. The *A* values are termed as 'fine readings' and the *B*, *C*, *D* values as 'coarse readings'. The oscilloscope scale is divided into 100 parts. The wavelength of 10 Mc.s. pattern wave as approximately 100 ft. (30 m) and hence

each division of the scale represents 1 foot on the two-way journey of the waves or approximately 0.5 foot on the length of the line. The final readings of *A*, *A* – *B*, *A* – *C* and *A* – *D* readings are recorded in millimicro seconds (10^{-9} seconds) and are converted into distance readings by assuming that the velocity of wave propagations as 299,792.5 km/sec. It should be noted that the success of the system depends on a property of the *heterodyne process*, that the phase difference between two heterodyne signal is maintained in the signal that results from the mixing.

15.7. WILD 'DISTOMATS'

Wild Heerbrugg manufacture EDM equipment under the trade name 'Distomat', having the following popular models :

1. Distomat DI 1000
2. Distomat DI 5S
3. Distomat DI 3000
4. Distomat DIOR 3002
5. Tachymat TC 2000 (Electronic tacheometer)

1. Distomate DI 1000

Wild Distomat DI 1000 is very small, compact EDM, particularly useful in building construction, civil engineering construction, cadastral and detail survey, particularly in populated areas where 99% of distance measurements are less than 500 m. It is an EDM that makes the tape redundant. It has a range of 500 m to a single prism and 800 m to three prisms (1000 m in favourable conditions), with an accuracy of 5 mm + 5 ppm. It can be fitted to all Wild theodolites, such as T 2000, T 2000 S, T 2 etc.

The infra-red measuring beam is reflected by a prism at the other end of the line. In the five seconds that it takes, the DI 1000 adjusts the signal strength to optimum level, makes 2048 measurements on two frequencies, carries out a full internal calibration, computes and displays the result. In the tracking mode 0.3 second updates follow the initial 3- second measurement. The whole sequence is automatic. One has to simply point to the reflector, touch a key and read the result.

The Wild modular system ensures full compatibility between theodolites and Distomats. The DI 1000 fits T 1, T 16 and T 2 optical theodolites, as shown in Fig. 15.10 (*a*). An optional key board can be used. It also combines with Wild T 1000 electronic theodolite and the Wild T 2000 informatics theodolite to form fully electronic *total station* [Fig. 15.10 (*b*)]. Measurements, reductions and calculations are carried out automatically. The DI 1000 also connects to the GRE 3 data terminal [Fig. 15.10 (*c*)]. If the GRE 3 is connected to an electronic theodolite with DI 1000, all information is transferred and recorded at the touch of a single key. The GRE can be programmed to carry out field checks and computations.

When DI 1000 distomat is used separately, it can be controlled from its own key board. There are only three keys on the DI 1000, each with three functions, as shown Fig. 15.11. Colour coding and a logical operating sequence ensure that the instrument is easy to use. The keys control all the functions. There are no mechanical switches. The liquid-crystal display is unusually large for a miniaturized EDM. Measured distances are presented clearly and unambiguously with appropriate symbols for slope, horizontal distance, height and setting out. In test mode, a full check is provided of the display, battery power and return signal strength. An audible tone can be activated to indicate return of signal. Scale (ppm) and additive constant (mm) settings are displayed at the start of each measurement.

or (c) connected on-line to a computer for remote control and real-time processing results. The following important operations can be achieved on moving objects:

(a) Offshore surveys. DI 3000 can be mounted on electronic theodolite for measuring to ships, dredgers and pipe laying barges, positioning oil rigs, controlling docking manoeuvres etc. (Fig. 15.16).

(b) Controlling objects on rails. DI 3000 can be connected on-line to computer for controlling the position of cranes, gantries, vehicles, machinery on rails, tracked equipments etc. (Fig. 15.17).

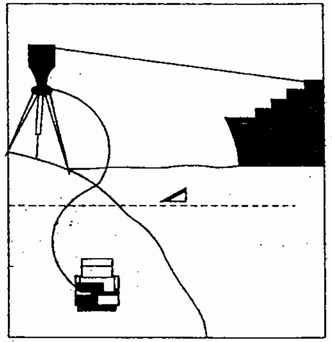


FIG. 15.16.

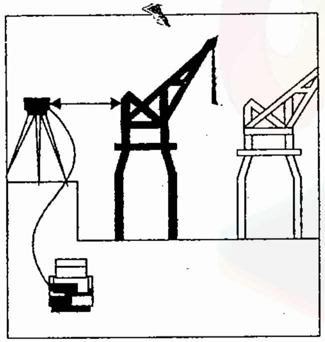


FIG. 15.17

(c) Monitoring movements in deformation surveys. DI 3000 can be connected with GRE 3 or computer for continuous measurement to rapidly deforming structures, such as bridges undergoing load tests (Fig. 15.18).

(d) Positioning moving machinery. DI 3000 can be mounted on a theodolite for continuous determination of the position of mobile equipment. (Fig. 15.19).

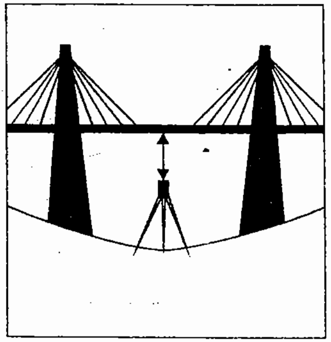


FIG. 15.18.

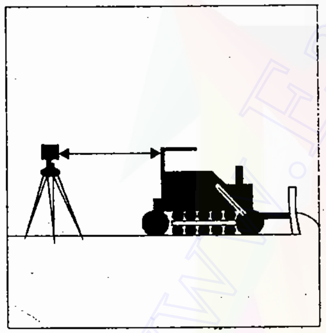


FIG. 15.19

The DI 3000 is also ideal all-round EDM for conventional measurements in surveying and engineering : control surveys, traversing, trigonometrical heighting, breakdown of GPS

networks, cadastral, detail and topographic surveys, setting out etc. It combines with Wild optical and electronic theodolites. It can also fit in a yoke as stand-alone instrument.

Fig. 15.20 shows a view of DI 3000 distomat, with its control panel, mounted on a Wild theodolite. The large easy to read LCD shows measured values with appropriate signs and symbols. An acoustic signal acknowledges key entries and measurement. With the DI 3000 on an optical theodolite, reductions are via the built in key board. For cadastral, detail, engineering and topographic surveys, simply key in the vertical circle reading. The DI 3000 displays slope and horizontal distance and height difference. For traversing with long-range measurements, instrument and reflector heights can be input the required horizontal distance. The DI 3000 displays the amount by which the reflector has to be moved forward or back. All correction parameters are stored in the non-volatile memory and applied to every measurements. Displayed heights are corrected for earths curvature and mean refraction.

4. Distomat DIOR 3002

The DIOR 3002 is a special version of the DI 3000. It is designed specifically for distance measurement without reflector. Basically, DIOR 3002 is also time pulsed Infra-red EDM. When used without reflectors, its range varies from 100 m to 250 m only, with a standard deviation of 5 mm to 10 mm. The interruptions of beam should be avoided. However, DIOR 3002, when used with reflectors have a range of 4 km to 1 prism; 5 km to 3 prisms and 6 km to 11 prisms.

Although, the DIOR 3002 can fitted on any of the main Wild theodolites, the T 1000 electronic theodolite is the most suitable. When used without reflectors, it can carry the following operation.

(i) Profile and cross-sections (Fig. 15.21). DIOR 3002 with an electronic theodolite, can be used for measuring tunnel profiles and cross-sections, surveying stopes, caverns, interior of storage tanks, domes etc.

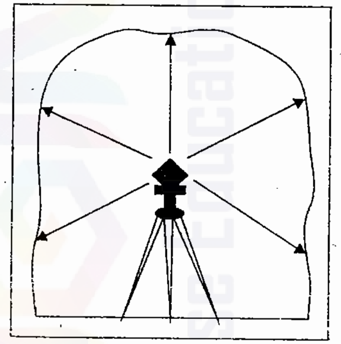


FIG. 15.21

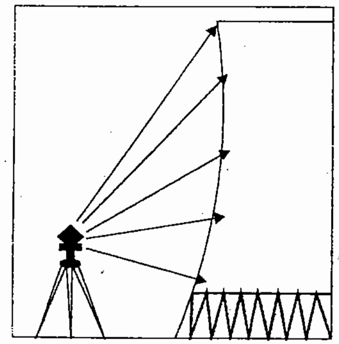


FIG. 15.22

(ii) Surveying and monitoring buildings, large objects quarries, rock faces, stock piles (Fig. 15.22). DIOR 3002 with a theodolite and data recorder can be used for measuring and monitoring large objects, to which access is difficult, such as bridges, buildings, cooling towers, pylons, roofs, rock faces, towers, stock piles etc.

Setting out can be fully automated with GRE 3 data terminal. The bearings and distances to the points to be set out are computed from the stored coordinates and transferred automatically to the TC 2000 *total station*.

Differences in Hz and V. For locating targets and for real time comparisons of measurements in deformation and monitoring surveys, it is advantageous to display angular differences in the horizontal and vertical planes between a required direction and the actual telescope pointing.

15.8. TOTAL STATION

A total station is a combination of an electronic theodolite and an electronic distance meter (EDM). This combination makes it possible to determine the coordinates of a reflector by aligning the instruments cross-hairs on the reflector and simultaneously measuring the vertical and horizontal angles and slope distances. A micro-processor in the instrument takes care of recording, readings and the necessary computations. The data is easily transferred to a computer where it can be used to generate a map. Wild, 'Tachymat' TC 2000, described in the previous article is one such *total station* manufactured by M/s Wild Heerbrugg.

As a teaching tool, a total station fulfills several purposes. Learning how to properly use a total station involves the physics of making measurements, the geometry of calculations, and statistics for analysing the results of a traverse. In the field, it requires team work, planning, and careful observations. If the total station is equipped with data-logger it also involves interfacing the data-logger with a computer, transferring the data, and working with the data on a computer. The more the user understands how a total station works, the better they will be able to use it.

Fundamental measurements : When aimed at an appropriate target, a total station measures three parameters (Fig. 15.31)

1. The *rotation* of the instrument's optical axis from the instrument north in a horizontal plane : i.e. *horizontal angle*

2. The *inclination* of the optical axis from the local vertical i.e. *vertical angle*.

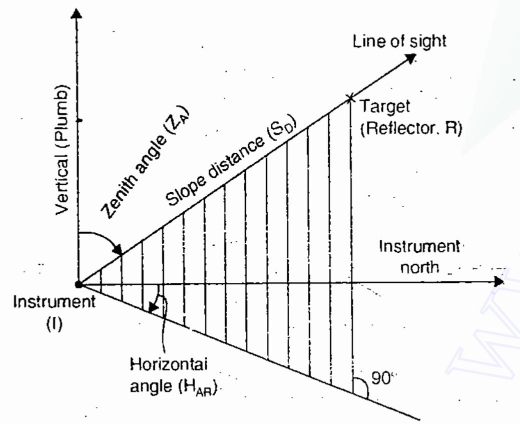


FIG. 15.31. FUNDAMENTAL MEASUREMENTS MADE BY A TOTAL STATION

3. The distance between the instrument and the target i.e. *slope distance*

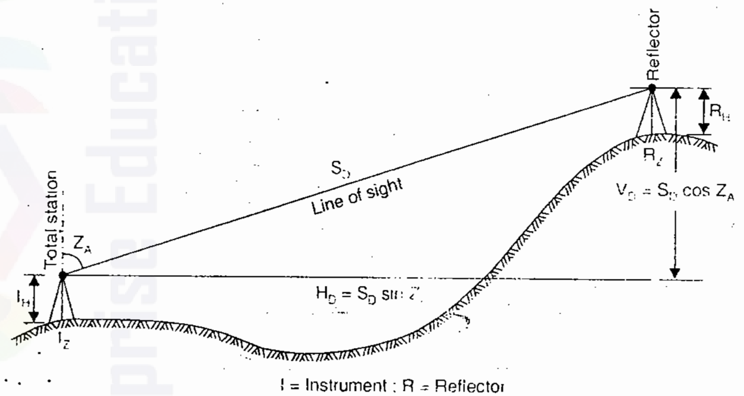
All the numbers that may be provided by the total station are derived from these three *fundamental measurements*

1. Horizontal Angle

The horizontal angle is measured from the zero direction on the horizontal scale (or *horizontal circle*). When the user first sets up the instrument the choice of the zero direction is made — this is *Instrument North*. The user may decide to set zero (North) in the direction of the long axis of the map area, or choose to orient the instrument approximately to True, Magnetic or Grid North. The zero direction should be set so that it can be recovered if the instrument was set up at the same location at some later date. This is usually done by sighting to another benchmark, or to a distance recognizable object. Using a magnetic compass to determine the orientation of the instrument is not recommended and can be very inaccurate. Most total stations can measure angle to at least 5 seconds, or 0.0013888°. The best procedure when using a Total Station is to set a convenient "north" and carry this through the survey by using backsights when the instrument is moved.

2. Vertical Angle : The vertical angle is measured relative to the local vertical (plumb) direction. The vertical angle is usually measured as a *zenith angle* (0 ° is vertically up, 90° is horizontal, and 180° is vertically down), although one is also given the option of making 0 ° horizontal. The zenith angle is generally easier to work with. The telescope will be pointing downward for zenith angles greater than 90 ° and upward for angles less than 90 °.

Measuring vertical angles requires that the instrument be exactly vertical. It is very difficult to level an instrument to the degree of accuracy of the instrument. Total stations contain an internal sensor (the vertical compensator) that can detect small deviations of the instrument from vertical. Electronics in the instrument then adjust the horizontal and



S_D = slope distance; V_D = Vertical distance between telescope and reflector; H_D = Horizontal distance; Z_A = Zenith angle; I_H = Instrument height; R_H = Reflector height; I_Z = Ground elevation of total station; R_Z = Ground elevation of reflector.

FIG 15.32 GEOMETRY OF THE INSTRUMENT (TOTAL STATION) AND REFLECTOR.

now want to calculate the X- (or East) and Y- (or North) coordinates. The zero direction set on the instrument is instrument north. This may not have any relation on the ground to true, magnetic or grid north. The relationship must be determined by the user. Fig. 15.33 shows the geometry for two different cases, one where the horizontal angle is less than 180° and the other where the horizontal angle is greater than 180°. The sign of the coordinate change [positive in Figure 15.33 (*a*) and negative in Fig. 15.33 (*b*)] is taken care of by the trigonometric functions, so the same formula can be used in all cases. Let us use symbol E for easting and N for northing, and symbol I for the instrument (i.e total station) and R for the reflector. Let R_E and R_N be the easting and northing of the reflector and I_E and I_N be the easting and northing of the instrument (i.e. total station)

From inspection of Fig. 15.33 the coordinates of the relector relative to the total station are

$dE =$ Change in Easting $= H_D \sin H_{AR}$

$dN =$ Change in Northing $= H_D \cos H_{AR}$

where H_D is the horizontal distance and H_{AR} is the horizontal angle measured in a clockwise sense from instrument north. In terms of fundamental measurments (i.e. equation 1) this is the same as

$$dE = S_D \sin Z_A \sin H_{AR} \quad \text{...(15.9)}$$

$$dN = S_D \cos (90° - Z_A) \cos H_{AR} = S_D \sin Z_A \cos H_{AR} \quad \text{...(15.10)}$$

If the easting and northing coordinates of the instrument station are known (in grid whose north direction is the same as instrument north) then we simply add the instrument coordinates to the change in easting and northing to get the coordinates of the reflector. The coordinates of the ground under the reflector, in terms of fundamental measurments are :

$$R_E = I_E + S_D \sin Z_A \sin H_{AR} \quad \text{...(15.11)}$$

$$R_N = I_N + S_D \sin Z_A \cos H_{AR} \quad \text{... (15.12)}$$

$$R_Z = I_Z + S_D \cos Z_A + (I_H - R_H) \quad \text{...(15.13)}$$

where I_E, I_N, and I_Z are the coordinates of the total station and R_E, R_N, R_Z are the coordinates of the ground under the reflector. These calculations can be easily done in a spreadsheet program.

All of these calculations can be made within a total station, or in an attached electronic notebook. Although it is tempting to let the total station do all the calculations, it is wise to record the three fundamental measurements. This allows calculations to be checked, and provides the basic data that is needed for a more sophisticated error analysis.

16

Remote Sensing

16.1. INTRODUCTION

Remote sensing is broadly defined as science and art of collecting information about objects, area or phenomena from distance without being in physical contact with them. In the present context, the definition of remote sensing is restricted to mean the process of acquiring information about any object without physically contacting it in any way regardless of whether the observer is immediately adjacent to the object or millions of miles away. Human eye is perhaps the most familiar example of a remote sensing system. In fact, sight, smell and hearing are all rudimentary forms of remote sensing. However, the term remote sensing is restricted to methods that employ electromagnetic energy (such as light, heat, microwave) as means of detecting and measuring target characteristics. Air craft and satellites are the common *platforms* used for remote sensing. Collection of data is usually carried out by highly sophisticated *sensors* (i.e. camera, multispectral scanner, radar etc.). The information carrier, or communication link is the electromagnetic energy. Remote sensing data basically consists of wave length intensity information by collecting the electromagnetic radiation leaving the object at specific wavelength and measuring its intensity. Photo interpretation can at best be considered as the *primitive form* of remote sensing. Most of the modern remote sensing methods make use of the reflected infrared bands, thermal infrared bands and microwave portion of the electromagnetic spectrum.

Classification of remote sensing

Remote sensing is broadly classified into two categories

(*i*) Passive remote sensing and (*ii*) Active remote sensing

Passive remote sensing : It uses sun as a source of EM energy and records the energy that is naturally radiated and/or reflected from the objects.

Active remove sensing : It uses its own source of EM energy, which is directed towards the object and return energy is measured.

16.2. HISTORICAL SKETCH OF REMOTE SENSING

Remote sensing became possible with the invention of camera in the nineteenth century. Astronomy was one of the first fields of science to exploit this technique. Although, it was during the first World War that free flying aircrafts were used in a remote sensing role, but the use of remote sensing for environmental assessment really became established after the second World War. It not only proved the value of aerial photography in land

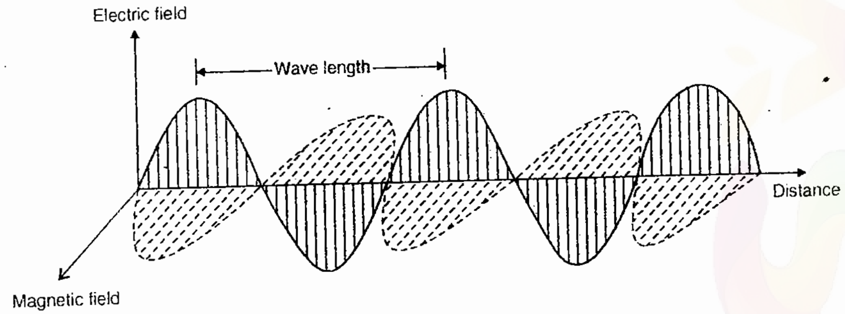


FIG. 16.2

Electromagnetic energy consists of *photons* having particle like properties such as energy and momentum. The EM energy is characterised in terms of velocity c ($\approx 3 \times 10^8$ m/s), wave length λ and frequency f. These parameters are related by the equation :

$$\lambda = \frac{c}{f} \qquad \ldots(16.1)$$

where λ = wave length, which is the distance between two adjacent peaks. The wave lengths sensed by many remote sensing systems are extremely small and are measured in terms of micro meter (μm or 10^{-6} m) or nanometer (nm or 10^{-9} m)

f = frequency, which is defined as the number of peaks that pass any given point in one second and is measured in Hertz (Hz).

The *amplitude* is the maximum value of the electric (or magnetic) field and is a measure of the amount of energy that is transported by the wave.

Wave theory concept explains how EM energy propagates in the form of a wave. However, this energy can only be detected when it interacts with the matter. This interaction suggests that the energy consists of many discrete units called *photons* whose energy (Q) is given by :

$$Q = h.f = \frac{h.c}{\lambda} \qquad \ldots(16.2)$$

where h = Plank's constant $= 6.6252 \times 10^{-34}$ J-s

The above equation suggests that shorter the wave length of radiation, more is the energy content.

16.4.2. ELECTROMAGNETIC SPECTRUM

Although *visible light* is the most obvious manifestation of EM radiation, other forms also exist. EM radiation can be produced at a range of wave lengths and can be categorised according to its position into discrete regions which is generally referred to *electro-magnetic spectrum*. Thus the electromagnetic spectrum is the continuum of energy that ranges from meters to nano-meters in wave length (Fig. 16.3) travels at at the speed of light and propagates through a vacuum like the outer space (Sabine, 1986). All matter radiates a

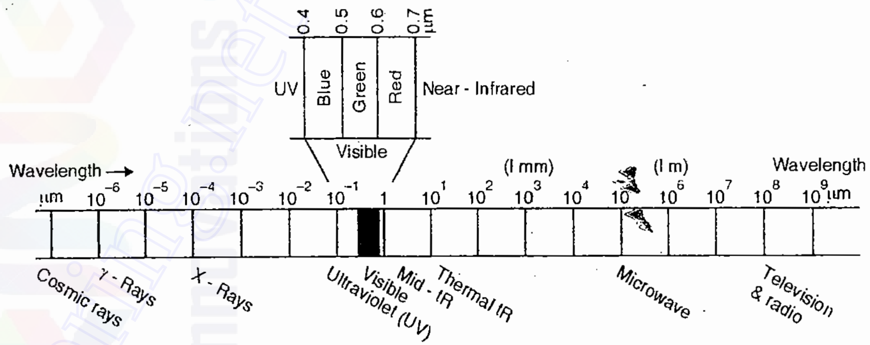


FIG. 16.3. ELECTROMAGNETIC SPECTRUM

range of electromagnetic energy, with the peak intensity shifting toward progressively shorter wave length at an increasing temperature of the matter. In general, the wave lengths and frequencies vary from shorter wavelength-high frequency cosmic waves to long wave length-low frequency radio waves (Fig. 16.3 and Table 16.1).

TABLE 16.1. ELECTROMAGNETIC SPECTRAL REGIONS (SABINE, 1987)

Region	*Wave length*	*Remarks*
1. Gamma ray	< 0.03 nm	Incoming radiation is completely absorbed by the upper atmosphere and is not available for remote sensing
2. X-ray	0.03 to 3.0 nm	Completely absorbed by atmosphere. Not employed in remote sensing
3. Ultraviolet	0.3 to 0.4 µ m	Incoming wavelengths less than 0.3 µ m are completely absorbed by ozone in the upper atmosphere
4. Photo graphic UV band	0.3 to 0.4 µm	Transmitted through atmosphere. Detectable with film and photodetectors, but atmospheric scattering is severe
5. Visible	0.4 to 0.7 µm	Images with film and photo detectors. Includes reflected energy peak of earth at 0.5 µm.
6. Infrared	0.7 to 1.00 µm	Interaction with matter varies with wave length. Atmospheric transmission windows are separated.
7. Reflected IR band	0.7 to 3.0 µm	Reflected solar radiation that contains information about thermal properties of materials. The bands from 0.7 to 0.9 µm is detectable with film and is called the photographic IR band.
8. Thermal IR	3 to 5 µ m	Principal atmospheric windows in the 8 to 14 µm thermal region. Images at these wavelengths are acquired by optical mechanical scanners and special vidicon systems but not by film. Microwave 0.1 to 30 cm longer wavelength can penetrate clouds, fog and rain. Images may be acquired in the active or passive mode
9. Radar	0.1 to 30 cm	Active form of microwave remote sensing. Radar images are acquired at various wavelength bands.
10. Radio	> 30 cm	Longest wavelength portion of electromagnetic spectrum. Some classified radars with very long wavelengths operate in this region.

C_1 = First radiation constant $= 3.742 \times 10^{16}$ W/m^2

C_2 = Second radiation constant $= 1.4388 \times 10^{-2}$ mK

It enables to assess the proportion of total radiant exitance within selected wave length.

16.5. EM RADIATION AND THE ATMOSPHERE

In remote sensing, EM radiation must pass through atmosphere in order to reach the earth's surface and to the sensor after reflection and emission from earth's surface features. The water vapour, oxygen, ozone, CO_2, aerosols, etc. present in the atmosphere influence EM radiation through the mechanism of (*i*) scattering, and (*ii*) absorption.

Scattering

It is unpredictable diffusion of radiation by molecules of the gases, dust and smoke in the atmosphere. Scattering reduces the image contrast and changes the spectral signatures of ground objects. Scattering is basically classified as (*i*) selective, and (*ii*) non-selective, depending upon the size of particle with which the electromagnetic radiation interacts. The *selective scatter* is further classified as (*a*) *Rayleigh's scatter*, and (*b*) *Mies scatter*.

Rayleigh's scatter: In the upper part of the atmosphere, the diameter of the gas molecules or particles is much less than the wave length of radiation. Hence haze results on the remotely sensed imagery, causing a bluish grey cast on the image, thus reducing the contrast. Lesser the wave length, more is the scattering.

Mie's scatter : In the lower layers of atmosphere, where the diameter of water vapour or dust particles approximately equals wave length of radiation, Mie's scatter occurs.

Non-selective scatter : Non-selective scattering occurs when the diameter of particles is several times more (approximately ten times) than radiation wavelength. For visible wave lengths, the main sources of non-selective scattering are pollen grains, cloud droplets, ice and snow crystals and raindrops. It scatters all wave length of visible light with equal efficiency. It justifies the reason why cloud appears white in the image.

Absorption

In contrast to scattering, atmospheric absorption results the effective loss of energy as a consequence of the attenuating nature of atmospheric constituents, like molecules of ozone, CO_2 and water vapour. Oxygen absorbs in the ultraviolet region and also has an absorption band centered on 6.3 μ m. Similarly CO_2 prevents a number of wave lengths reaching the surface. Water vapour is an extremely important absorber of EM radiation within infrared part of the spectrum.

Atmospheric windows

The amount of scattering or absorption depends upon (*i*) wave length, and (*ii*) composition of the atmosphere. In order to *minimise* the effect of atmosphere, it is essential to choose the regions with high *transmittance*.

The wavelengths at which EM radiations are partially or wholly transmitted through the atmosphere are known as atmospheric windows and are used to acquire remote sensing data.

Typical *atmospheric windows* on the regions of EM radiation are shown in Fig. 16.4.

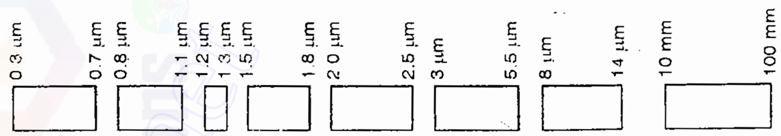


FIG. 16.4 ATMOSPHERIC WINDOWS

The sensors on remote sensing satellites must be designed in such a way as to obtain data within these well defined *atmospheric windows*.

16.6. INTERACTION OF EM RADIATION WITH EARTH'S SURFACE

EM energy that strikes or encounters matter (object) is called *incident radiation*. The EM radiation striking the surface may be (*i*) reflected/scattered, (*ii*) absorbed, and/or (*iii*) transmitted. These processes are not mutually exclusive — EM radiations may be partially reflected and partially absorbed. Which processes actually occur depends on the following factors (1) wavelength of radiation (2) angle of incidence, (3) surface roughness, and (4) condition and composition of surface material.

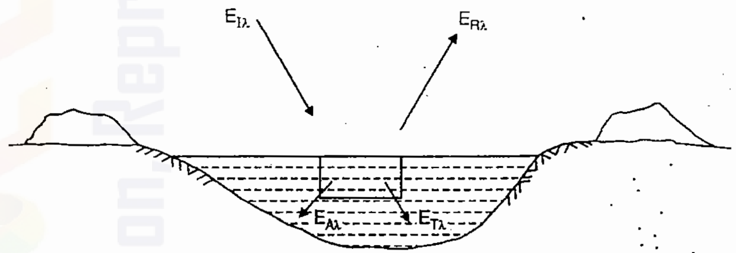


FIG. 16.5. INTERACTION MECHANISM

Interaction with matter can change the following properties of incident radiation:
(*a*) Intensity (*b*) Direction (*c*) Wave length (*d*) Polarisation, and (*e*) Phase.

The science of remote sensing detects and records there changes.

The energy balance equation for radiation at a given wave length (λ) can be expressed as follows.

$$E_{I\lambda} = E_{R\lambda} + E_{A\lambda} + E_{T\lambda} \qquad \ldots(16.6)$$

where $E_{I\lambda}$ = Incident energy; $E_{R\lambda}$ = Reflected energy

$E_{A\lambda}$ = Absorbed energy; $E_{T\lambda}$ = Transmitted energy.

The proportion of each fraction $(E_{R\lambda}/E_{A\lambda}/E_{T\lambda})$ will vary for different materials depending upon their composition and condition. Within a given features type, these proportions will vary at different wave lengths, thus helping in discrimination *of different objects*. Reflection, scattering, emission are called surface phenomenon because these are determined by the properties of surface, viz. colour, roughness. Transmission and absorption are called volume

Indian Remote Sensing Satellites (IRS)

1. Satellite for Earth Observation (SEO-I), now called Bhaskara-I was the first Indian remote sensing satellite launched by a soviet launch vehicle from USSR in June, 1979, into a near circular orbit.

2. SEO-II, (Bhaskara II) was launched in Nov. 1981 from a Soviet cosmodrome.

3. India's first semi-operational remote sensing satellite (IRS) was launched by the Soviet Union in Sept. 1987.

4. The IRS series of satellites launched by the IRS mission are : IRS IA, IRS IB, IRS IC, IRS ID and IRS P4.

16.8. SENSORS

Remote sensing sensors are designed to record radiations in one or more parts of the EM spectrum. Sensors are electronic instruments that receive EM radiation and generate an electric signal that correspond to the energy variation of different earth surface features. The signal can be recorded and displayed as numerical data or an image. The strength of the signal depends upon (*i*) Energy flux, (*ii*) Altitude, (*iii*) Spectral band width, (*iv*) Instantaneous field of view (IFOV), and (*v*) Dwell time.

A *scanning system* employs detectors with a narrow field of view which sweeps across the terrain to produce an image. When photons of EM energy radiated or reflected from earth surface feature encounter the detector, an electrical signal is produced that varies in proportion to the number of photons.

Sensors on board of Indian Remote sensing satellites (IRS)

1. Linear Imaging and Self Scanning Sensor **(LISS I)**

This payload was on board IRS 1A and 1B satellites. It had four bands operating in visible and near IR region.

2. Linear Imaging and Self Scanning Sensor **(LISS II)**

This payload was on board IRS 1A and 1B satellites. It has four bands operating in visible and near IR region.

3. Linear Imaging and Self Scanning Sensor **(LISS III)**

This payload is on board IRS 1C and 1D satellites. It has three bands operating in visible and near IR region and one band in short wave infra region.

4. Panchromatic Sensor **(PAN)**

This payload is on boards IRS 1C and 1D satellites. It has one band.

5. Wide Field Sensor **(WiFS)**

This payload is on boards IRS 1C and 1D satellites. It has two bands operating in visible and near IR region.

6. Modular Opto-Electronic Scanner **(MOS)**

This payload is on board IRS P3 satellite.

7. Ocean Colour Monitor **(OCM)**

This payload is on board IRS P4 satellite. It has eight spectral bands operating in visible and near IR region.

8. Multi Scanning Microwave Radiometer **(MSMR)**

This payload is on board IRS 1D satellite. This is a passive microwave sensor.

16.9. APPLICATIONS OF REMOTE SENSING

Remote sensing affords a practical means for accurate and continuous monitoring of the earth's natural and other resources and of determining the impact of man's activities on air, water and land. The launch of IRS 1C satellite (Dec. 1995) with state of art sensors provided a new dimension and further boosted the applications of space-base remote sensing technology for natural resources management. With the unique combinations of payload, the IRS-1C has already earned the reputation as the '*Satellite for all applications*' IRS-1C/1D carry three imaging sensors (LISS-III, PAN and WiFS) characterised by different resolutions and coverage capabilities. These three imaging sensors provide image data for virtually all levels of applications ranging from cadastral survey to regional and national level mapping. The LISS-III data with 21.2- 23.5 m resolution has significantly improved separability amongst various crops and vegetation types, leading to identification of small fields and better classification accuracy. The frequent availability of data from WiFS payload has helped in monitoring *dynamic phenomena* like vegetation, floods, droughts, forest fire etc.. A major benefit of the multi-sensor IRS-1C/1D payload is the capability to merge the multi spectral LISS-III data, with high resolution PAN imagery. This merger of multispectral and high resolution data facilitates detailed land cover classification and delineation of linear and narrow roads/lanes, structures, vegetation types and parcels of land.

A summary of RS applications is given below, discipline wise.

1. Agriculture

(*i*) Early season estimation of total cropped area

(*ii*) Monitoring crop condition using crop growth profile.

(*iii*) Identification of crops and their coverage estimation in multi-cropped regions.

(*iv*) Crop yield modelling

(*v*) Cropping system/crop rotation studies

(*vi*) Command area management

(*vii*) Detection of moisture stress in crops and quantification of its effect on crop yield

(*viii*) Detection of crop violations

(*ix*) Zoom cultivation—desertification

2. Forestry

(*i*) Improved forest type mapping

(*ii*) Monitoring large scale deforestation, forest fire

(*iii*) Monitoring urban forestry

(*iv*) Forest stock mapping

(*v*) Wild life habitat assessment

3. Land use and soils

(*i*) Mapping land use/cover (level III) at 1 : 25000 scale or better

4. What do you understand by electro-magnetic spectrum ? State the wave length regions, along with their uses, for remote sensing applications.

5. Explain the interaction mechanism of EM radiation with earth's surface, stating the basic interaction equation.

6. Write a note on remote sensing observation platforms

7. Write a note on various types of sensors used for remote sensing in India.

8. Write a detailed note on applications of remote sensing.

Appendix - A

ADDITIONAL EXAMPLES USEFUL FOR COMPETETIVE EXAMINATIONS

Example A-1. *What are the elements of a simple circular curve ? Two straight lines PQ and QR intersect at chainage (375 + 12), the angle of intersection being 110°. Calculate the chainage of tangent points of a right handed circular curve of 400 m radius.*

(U.P.S.C. Engg. Services Exam, 1983)

Solution

Various elements of simple circular curve are : (*i*) length of the curve (*ii*) tangent length (*iii*) length of long chord, (*iv*) apex distance, and (*v*) mid-ordinate.

Angle of deflection

$$\Delta = 180° - 110° = 70°$$

Tangent length $T = R \tan \Delta/2$

$$= 400 \tan 70°/2 = 280.08 \text{ m}$$

Length of circular curve, $l = \dfrac{\pi R \Delta}{180°}$

$$= \frac{\pi\,(400)\,(70°)}{180°} = 488.69 \text{ m.}$$

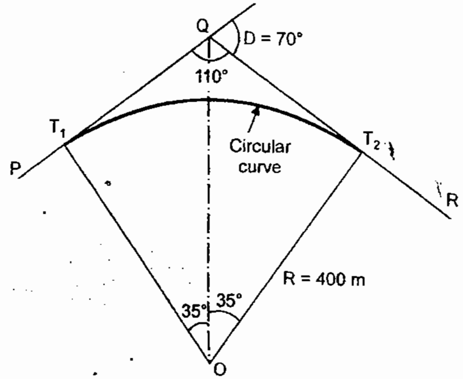


FIG. A-1.

Taking the length of chain as 30 m and length of link as 0.2 m,

Chainage of point of intersection Q

$$= 375 \times 30 + 12 \times 0.2 = 11252.40$$

$\therefore$ Chainage of point of curve, $T_1 = 11252.40 - 280.08 = 10972.32$ m

Chainage of point of tangency, $T_2 = 10972.32 + 488.69 =$ **11461.01 m**

Example A-2. *Two parallel railway lines are to be connected by a reverse curve, each section having the same radius. If the centre lines are 8 m apart, and the maximum distance between tangent points is 32 m, find the maximum allowable radius that can be used.*

(U.P.S.C. Engg. Services Exam.. 1985)

Solution : Given Distance $T_1 T_2 = L = 32$ m ; $v = 8$ m

We have the special case of $R_1 = R_2 = R$.

Hence from Eq. 2.37 (*a*), $L^2 = 4\,R\,v$

$$\therefore \quad R = \frac{L^2}{4\,v} = \frac{(32)^2}{4 \times 8} = \mathbf{32\ m}$$

vertical curve and the R.L. of the tangent points. Assume that the eye level of the driver is 1.125 m above the road surface.

(U.P.S.C. Engg. Services Exam. 1989)

Solution : Given $g_1 = +1.5\%$; $g_2 = -0.5\%$; $S = 300$ m ; $h_1 = 1.125$ m

Let us assume h_2 = height of obstruction = 0.1 m

$$L = \frac{S^2(g_1 - g_2)}{200(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{(300)^2(1.5 + 0.5)}{200(\sqrt{1.125} + \sqrt{0.1})^2} = 474.73 \simeq \mathbf{475\ m} \text{ (say)}$$

R.L. of summit = 75 m

$$\therefore \text{ R.L. of point of commencement} = 75 - \frac{1.5}{100}\left(\frac{475}{2}\right) = \mathbf{71.44\ m}$$

$$\text{R.L. of point of tangency} = 75 - \frac{0.5}{100}\left(\frac{475}{2}\right) = \mathbf{73.81\ m}$$

Example A-6. *Calculate the sun's azimuth and hour angle at sunset at a place in latitude 40° N when its declination is 20° N.*

(U.P.S.C. Engg. Services Exam, 1990)

Solution

Consider astronomical triangle *ZPM*, where *M* is the position of the sun at horizon and *P* is the north pole.

ZP = co-latitude = 90° − θ = 90° − 40° = 50°

ZM = 90° since the sun is at horizon at its setting

MP = 90° − δ = 90° − 20° = 70°

From cosine formula,

$$\cos A = \frac{\cos PM - \cos MZ \cos ZP}{\sin MZ \sin ZP}$$

$$= \frac{\cos 70° - \cos 90° \cos 50°}{\sin 90° \sin 50°}$$

$$= \frac{\cos 70°}{\sin 50°} = 0.4464756$$

$$\therefore \quad A = 63°.482215 = \mathbf{63°\ 28'\ 56''}$$

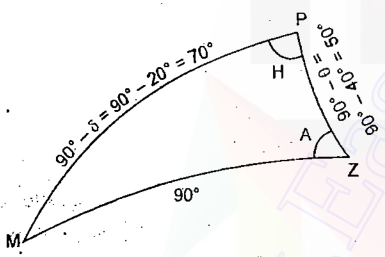


FIG. A-3

$$\text{Also} \quad \cos H = \frac{\cos MZ - \cos MP \cos ZP}{\sin MP \sin ZP} = \frac{\cos 90° - \cos 70° \cos 50°}{\sin 50° \sin 70°} = -0.3054073$$

or $\quad H = 170°.78267 = \mathbf{107°\ 46'\ 58''}$

Example A-7. *Two tangents intersect at chainage 1200 m, the deflection angle being 40°. Compute the data for setting out a 400 m radius curve by deflection angles and offsets. Take 30 m chord lengths in the general approach.*

(U.P.S.C. Engg. Services Exam, 1990)

Solution. Given $\Delta = 40°$; $R = 400$ m.

$$\text{Length of back tangent} = R \tan\frac{\Delta}{2} = 400 \tan 20° = 145.59 \text{ m}$$

$$\text{Length of the curve} = \frac{\pi R \Delta}{180°} = \frac{\pi(400)(40°)}{180°} = 279.25 \text{ m}$$

Chainage of point of intersection = 1200 m (given)

∴ Chainage of tangent T_1 = 1200 − 145.59 = 1054.41 m

Chainage of tangent T_2 = 1054.41 + 279.25 m = 1333.66

∴ Length of first subchord = 1080 − 1054.41 = 25.59 m

Length of last subchord = 1333.66 − 1320 = 13.66 m

Length of regular chord = 30 m

$$\text{Number of full chords} = \frac{279.25 - (25.59 + 13.66)}{30} = 8$$

Total No. of chords = 1 + 8 + 1 = 10

(1) Computation of Rankine's deflection angles

In general, $\quad \delta = 1718.9 \dfrac{C}{R}$ minutes

$$\therefore \quad \delta_1 = 1718.9 \times \frac{25.59}{400} = 109.97 \text{ minutes} = 1°49'58''$$

$$\delta_2 \text{ to } \delta_9 = 1718.9 \times \frac{30}{400} = 128.92 \text{ minutes} = 2°8'55''$$

$$\delta_{10} = 1718.9 \times \frac{13.66}{400} = 58.7 \text{ minutes} = 0°\ 58'\ 42''$$

∴ $\Delta_1 = \delta_1 = 1°49'58''$

$\Delta_2 = \Delta_1 + \delta_2 = 1°\ 49'\ 58'' + 2°\ 08'\ 55'' = 3°\ 58'\ 53''$

$\Delta_3 = \Delta_2 + \delta_3 = 3°\ 58'\ 53'' + 2°\ 08'\ 55'' = 6°\ 07'\ 48''$

$\Delta_4 = \Delta_3 + \delta_4 = 6°\ 07'\ 48'' + 2°\ 08'\ 55'' = 8°\ 16'\ 43''$

$\Delta_5 = \Delta_4 + \delta_5 = 8°\ 16'\ 43'' + 2°\ 08'\ 55'' = 10°\ 25'\ 38''$

$\Delta_6 = \Delta_5 + \delta_6 = 10°\ 25'\ 38'' + 2°\ 08'\ 55'' = 12°\ 34'\ 33''$

$\Delta_7 = \Delta_6 + \delta_7 = 12°\ 34'\ 33'' + 2°\ 08'\ 55'' = 14°\ 43'\ 28''$

$\Delta_8 = \Delta_7 + \delta_8 = 14°\ 43'\ 28'' + 2°\ 08'\ 55'' = 16°\ 52'\ 23''$

$\Delta_9 = \Delta_8 + \delta_9 = 16°\ 52'\ 23'' + 2°\ 08'\ 55'' = 19°\ 01'\ 18''$

$\Delta_{10} = \Delta_9 + \delta_{10} = 19°\ 01'\ 18'' + 0°\ 58'\ 42'' = 20°\ 00'\ 00''$ (check)

(2) Computation of offsets from chords produced

In general, $\quad O_0 = \dfrac{C_n}{2R}(C_{n-1} + C_n)$ ...(2.14 *c*)

$$\therefore \quad O_1 = \frac{C_1}{2R}(0 + C_1) = \frac{C_1^2}{2R} = \frac{(25.59)^2}{2 \times 400} = 0.82 \text{ m}$$

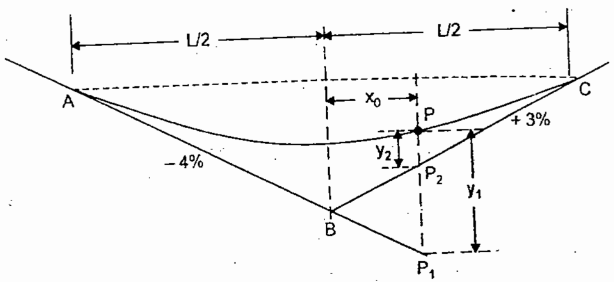


FIG. A-5

$$y_2 = PP_2 = 260 - 252.23 = 7.77 \text{ m}$$

Now
$$\frac{y_1}{y_2} = \frac{\left(\frac{L}{2} + x_0\right)^2}{\left(\frac{L}{2} - x_0\right)^2} \quad \text{or} \quad \frac{9.52}{7.77} = \left(\frac{\frac{L}{2} + 25}{\frac{L}{2} - 25}\right)^2$$

or
$$\frac{\frac{L}{2} + 25}{\frac{L}{2} - 25} = 1.1069 \quad \text{or} \quad \frac{L}{2} + 25 = 1.1069\,\frac{L}{2} - 27.672$$

or
$$0.1069\,\frac{L}{2} = 52.6725 \quad \text{From which} \quad \frac{L}{2} = 492.73 \quad \text{or} \quad L \simeq \mathbf{985.5\ m}$$

Example A-10. *A vertical photograph was taken from 3200 m above mean sea level with a camera of focal length 120 mm. It contained two points 'a' and 'b' corresponding to ground points A and B. Calculate the horizontal length AB, as well as the average scale along line ab from the following data :*

Photo points	*Elevation above msl (m)*	*Photo coordinates*	
		x (mm)	*y (mm)*
a	640	*+ 19.50*	*– 14.60*
b	780	*+ 26.70*	*+ 10.80*

(*U.P.S.C.Engg. Services Exam., 1997*)

Solution

The ground co-ordinates are given by

$$X_a = \frac{H - h_a}{f}\,x_a = \frac{3200 - 640}{120} \times (+19.50) = +416 \text{ m}$$

$$Y_a = \frac{H - h_a}{f}\,y_a = \frac{3200 - 640}{120} \times (-14.60) = -311.47 \text{ m}$$

$$X_b = \frac{H - h_b}{f}\,x_b = \frac{3200 - 780}{120} \times (+26.70) = +538.45 \text{ m}$$

$$Y_b = \frac{H - h_b}{f}\,y_b = \frac{3200 - 780}{120} \times (+10.80) = +217.80 \text{ m}$$

$$\therefore \text{ Length} \quad AB = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2}$$

$$= \sqrt{(416 - 538.45)^2 + (-311.47 - 217.80)^2} = \mathbf{543.25\ m}$$

Average scale $S_{av} = \dfrac{f}{H - h_{av}}$ where $h_{av} = \dfrac{640 + 780}{2} = 710$ m

$$\therefore \quad S_{av} = \frac{120 \text{ mm}}{(3200 - 710) \text{ m}} = \frac{1 \text{ mm}}{20.75 \text{ m}}$$

$\therefore$ Average scale is **1 mm = 20.75 m**

Example A-11. *Two tangents interest at chainage 50.60 (50 chains and 60 links), the deflection angle being 61°. Calculate the necessary data for setting out a circular highway curve of 20 chains radius to connect the two tangents by the method of offsets from the chord. Take peg interval equal to 100 links with length of the chain being 20 metres (100 links).*

(*U.P.S.C. Engg. Services Exam, 1998*)

Solution :

Given $R = 20 \times 20 = 400$ m; $\Delta = 61°$

Chainage of point of intersection V

$= 50.60$ chains

$= 50 \times 20 + 0.6 \times 20 = 1012$ m

Length of tangent $T = R \tan \Delta/2$

$= 400 \tan 30.5° = 235.62$ m.

$\therefore$ Chainage of $T_1 = 1012 - 235.62$

$= 776.38$ m

Length of curve

$$= \frac{\pi R \Delta}{180°} = \frac{\pi\,(400)\,61°}{180°} = 425.86 \text{ m}$$

$\therefore$ Chainage of T_2

$= 776.38 + 425.86 = 1202.24$ m

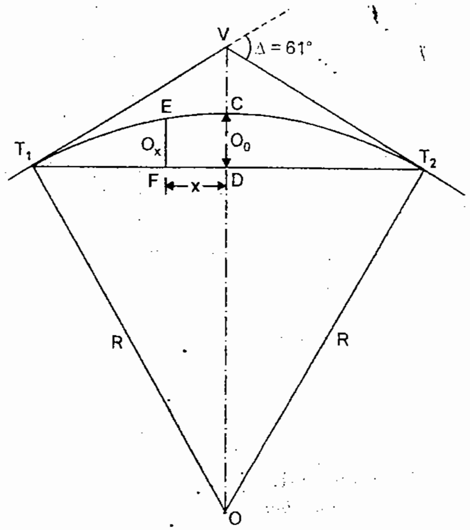


FIG. A-6

Let us set out the curve by means of offsets from the long chord.

Length of long chord $T_1T_2 = 2R \sin \Delta/2$

$= 2 \times 400 \sin 30.5° = 406.03$ m

Central ordinate $\quad O_0 = R - \sqrt{R^2 - (L/2)^2} = 400 - \sqrt{(400)^2 - (406.03/2)^2} = 55.35$ m

$$O_x = \sqrt{R^2 - x^2} - (R - O_0) = \sqrt{(400)^2 - x^2} - (400 - 55.35).$$

or $\quad O_x = \sqrt{160000 - x^2} - 344.65$ m

Taking x at a peg interval of 20 m, values of O_x can be computed as tabulated below.

or
$$h_1 = \frac{1.0179 \times 9.81\,(304.692)\,(300.000)}{8 \times 5000} = \mathbf{22.819\ m}$$

When the temperature falls to $-12°$ C,

Contraction of wire, $\delta l = l\,\alpha\,\Delta T = 304.692 \times 1.7 \times 10^{-5}\,[32-(-12)] = 0.228$ m

$\therefore$ Adjusted length of wire $= 304.692 - 0.228 = 304.464$ m

$$\therefore \quad \text{Now sag } h_2 = \frac{1.0179 \times 9.81\,(304.464)\,(300.00)}{8 \times 5000} = \mathbf{22.802\ m}$$

Since $h \propto \frac{1}{P}$,
$$P_2 = P_1\left(\frac{h_1}{h_2}\right) = 5000\left(\frac{22.819}{22.802}\right) = \mathbf{5003.7\ N}$$

Note : A change of 3.7 N in tension will expand the tape by
$$\delta l = \frac{l\,\Delta P}{AE} = \frac{304.692 \times 3.7}{\pi\,(6)^2 \times 7 \times 10^4} = 0.0001 \text{ m which is negligible.}$$

Example A-16. *The details given below refer to the measurement of the first 30 m bay of a base line. Determine the correct length of the bay reduced to mean sea level.*

With the tape hanging in catenary at a tension of 95 N and at a mean temperature of 13° C the recorded length was 29.9821 m. The difference in height between the ends was 0.40 m and the site was 500 m above m.s.l.

The tape had previously been standardised in catenary at a tension of 70 N and at a temperature of 15° C and the distance between the zeros was 29.9965 m. Take the following values : $R = 6367.3$ km; mass of tape $= 0.0191$ kg/m; sectional area of tape $= 3.63$ mm^2; $E = 2.1 \times 10^5$ N/mm^2 and temperature coefficient of expansion of tape $= 12 \times 10^{-6}$ per °C.

Solution

1. *Correction for standardisation*

The tape is 29.9965 m at 70 N and 15° C

$c = (29.9965 - 30.000) = -0.0035$ per 30 m

2. *Correction for temperature*

$$c_t = \alpha\,(T_m - T_0)\,l = 12 \times 10^{-6}\,(13-15) \times 30 = -0.0007 \text{ m}$$

3. *Correction tension*

$$c_p = \frac{(P-P_0)\,l}{A\,E} = \frac{(95-70).30}{3.63 \times 2.1 \times 10^5} = +0.0010 \text{ m}$$

4. *Correction for slope*

$$c_v = -\frac{h^2}{2L} = -\frac{(0.40)^2}{2 \times 30} = -0.0027 \text{ m}$$

5. *Correction for sag*

$$c_s = -\frac{(mg)^2\,l^3}{24}\left[\frac{1}{P^2} - \frac{1}{P_0^2}\right] = -\frac{(0.0191 \times 9.81)^2\,(30)^3}{24}\left[\frac{1}{(95)^2} - \frac{1}{(70)^2}\right] = +0.0037 \text{ m}$$

6. *Correction for reduction to m.s.l*

$$c_{msl} = -\frac{lh}{R} = -\frac{30 \times 500}{6367 \times 10^3} = -0.0024 \text{ m}$$

$\therefore$ Correct length $= 29.9821 - 0.0035 - 0.0007 + 0.0010 - 0.0027 + 0.0037 - 0.0024$

$= \mathbf{29.9775\ m}$

Example A-17. *A steel tape has the following specifications.*

(i) Mass = 0.5 kg *(ii) cross-sectional area = 2 mm^2*

(iii) Young's modulus = 20×10^{10} N/m^2 *(iv) length at 20° C and 50 N = 30.005 m*

(v) Coefficient of linear expansion = 11×10^{-6} per °C

It is to be used in catenary but in order to reduce the number of corrections to be applied to the measured lengths, it is suggested that

(a) the standard temperature be adjusted so that the actual length is equal to the nominal length of 30.000 m

(b) the tape be used at a tension such that the effects of sag and tension will be compensating.

(N.B. : The acceptable tension will be in the region of 100 N)

Solution

(a) *New standard temperature* : Desired $c_t = 30.005 - 30.000 = 0.005$ m

But $c_t = \alpha\,(T_m - T_0)\,l$

$$\therefore \quad \Delta T = \frac{c_t}{l\,\alpha} = \frac{0.005}{30 \times 11 \times 10^{-6}} = 15.2°\ C$$

Hence to contract the tape by 0.005 m, the temperature would be needed to be reduced by 15.2° C.

$\therefore$ Near standard temperature $= T_0 - 15.2° = 20° - 15.2° = \mathbf{4.8°\ C}$

(b) *Normal tension* (P_n)

$$c_p = \frac{P_n - P_0}{AE}\,l$$

$$c_s = \frac{(mg\,l)^2\,l}{24\,P_n^2}$$

Equating the two,
$$\frac{(mgl)^2\,l}{24\,P_n^2} = \frac{(P_n - P_0)\,l}{AE}$$

or
$$\frac{AE\,(mgl)^2}{24} = P_n^3 - P_n^2\,P_0$$

or
$$P_n^3 - 50\,P_n^2 - \frac{(0.5 \times 9.81)^2 \times 2 \times 2 \times 10^5}{24} = 0$$

or
$$P_n^3 - 50\,P_n^2 = 400984$$

Let us solve this by trial and error, in the tabular form shown below :

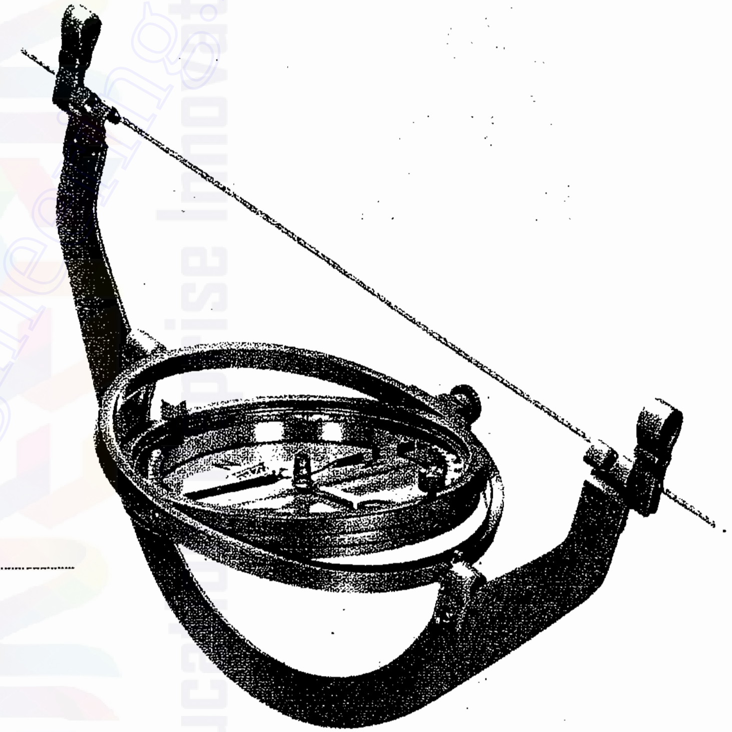


FIG. 7.9. KESSEL TYPE MINING COMPASS

(TO FACE PAGE 208)