CIVIL ENGINEERING

ESE TOPICWISE

CONVENTIONAL SOLVED PAPER-I

1995-2020

IES MASTER
PUBLICATION

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CONTENT

UNIT-1

STRENGTH OF MATERIALS

SYLLABUS

Basics of strength of materials, Types of stresses and strains, Bending moments and shear force, concept of bending and shear stresses;

Elastic constants, Stress, Plane stress, Strains, Plane strain, Mohr's circle of stress and strain. Elastic theories of failure. Principal Stresses, Bending, Shear and Torsion.

CONTENTS

CHAPTER 1

STRENGTH OF MATERIALS

Q-1: ***A cylindrical piece of steel 80 mm dia and 120 mm long is subjected to an axial compressive force of 50,000 kg. Calculate the change in the volume of the piece if bulk modulus = 1.7 × 10^6 kg/cm^2 and Poisons' ratio = 0.3.***

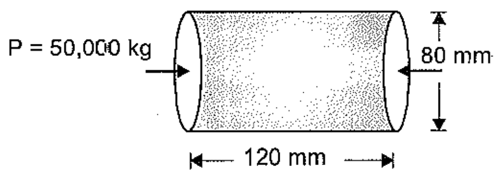
[10 Marks, ESE-1997]

Sol: **Given:**

Axial compressive load = 50,000 kg

Bulk modulus (k) = $1.7 \times 10^{+6}$ kg/cm^2

Passion's ratio (μ) = 0.3



Determine:

We know that
$$\frac{\Delta V}{V} = \varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu(\sigma_y)}{E} - \frac{\mu(\sigma_z)}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\mu(\sigma_z)}{E} - \frac{\mu(\sigma_x)}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\mu(\sigma_x)}{E} - \frac{\mu\sigma_y}{E}$$

In our case,
$$\sigma_y = 0 \; ; \; \sigma_z = 0$$

$$\sigma_x = \frac{-50000 \text{ kg}}{\frac{\pi}{4}(8)^2 \text{ cm}^2} = -995.2 \text{ kg/cm}^2$$ {(-ve) because its compressive}

Also, we know,
$$E = 3k(1-2\mu) = 3 \times 1.7 \times 10^6 \times (1 - 2 \times 0.3)$$
$$= 2.04 \times 10^6 \text{ kg/cm}^2$$

$\Rightarrow$
$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{-995.2}{2.04 \times 10^6} = -4.878\,4 \times 10^{-4}$$ [(–) because comp.]

$$\varepsilon_y = -\frac{\mu\sigma_x}{E} = 0.3 \times 4.878 \times 10^{-4} = 1.4635 \times 10^{-4}$$

$$\varepsilon_z = -\frac{\mu\sigma_x}{E} = 1.4635 \times 10^{-4}$$

$\Rightarrow$
$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_2 = -1.951 \times 10^{-4}$$

$$\Rightarrow \qquad \frac{\Delta V}{V} = -1.951 \times 10^{-4}$$

$$\Delta V = -1.951\times10^{-4}\times\frac{\pi}{4}(8)^2\times12\text{ cm}^3 = -0.1176\text{ cm}^3$$

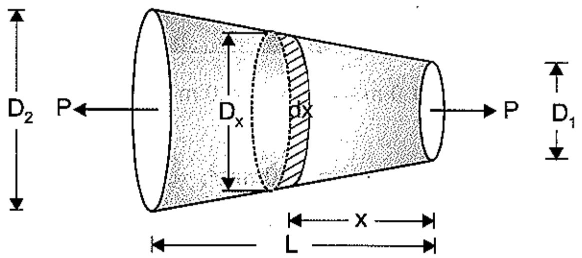
Change in vol. is (–)ve

$\Rightarrow$ There is volume reduction of 0.1176 cm^3

Q-2: ***A steel rod, circular in cross-section, tapers from 30 mm diameter to 15 mm diameter over a length of 600 mm. Find how much its length will increase under a pull of 20 kN if Young's modulus of elasticity = 200 kN/mm². Derive the formula used.***

[15 Marks, ESE-1998]

Sol:



For deriving the expression of elongation for tapered beam, we assume a tapered beam of

Length = L, Small end Dia = D_1, Larger end dia = D_2

$\therefore \quad D_x = D_1+\left(\frac{D_2-D_1}{L}\right)x$, {where D_x is Dia at any distance x from smaller end}

i.e., $D_x = D_1 + kx$, where $k = \left(\frac{D_2-D_1}{L}\right)$

Change in the length of element of length dx = $d(\Delta L)$

$$d(\Delta L) = \frac{Pdx}{AE}$$

So net elongation
$$\int d(\Delta L) = \int_0^L \frac{Pdx}{AE} = \int_0^L \frac{Pdx}{\frac{\pi}{4}(D_1+kx)^2 E}$$

$$\Rightarrow \qquad \Delta L = \int_0^L \frac{4Pdx}{\pi(D_1+kx)^2 E} = \frac{4P}{\pi E}\int_0^L \frac{dx}{(D_1+kx)^2} = \frac{4P}{\pi E}\left[\frac{-1}{k(D_1+kx)}\right]_0^L$$

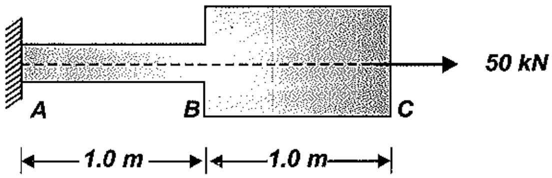
$$\Rightarrow \qquad \boxed{\Delta L = \frac{4PL}{\pi E D_1 D_2}}$$

Values given, $P = 20\text{ kN} = 20\times10^3\text{ N}$

$D_1 = 15$ mm, $D_2 = 30$ mm, $L = 600$ mm, $E = 200\text{ kN/mm}^2 = 200\times10^3\text{ N/mm}^2$

$$\therefore \qquad \Delta L = \frac{4\times20\times10^3\times600}{\pi\times15\times30\times200\times10^3} = 0.1697\text{mm}$$

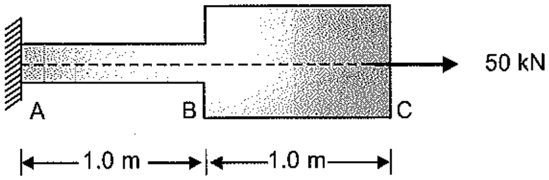
Q-3: ***Draw the diagram of normal forces, stresses and displacements along the length of the stepped bar ABC shown in figure.***



cross-sectional area over AB = 100 mm²; and area over BC = 200 mm²; modulus of elasticity = 200 kN/mm²

[10 Marks, ESE-1998]

Sol:



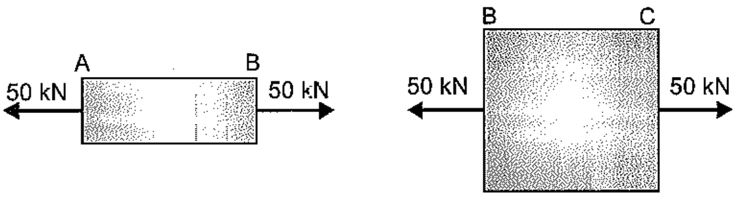
Given:

$$A_{AB} = 100 \text{ mm}^2, \quad A_{BC} = 200\text{mm}^2, \quad E = 200 \text{ kN/mm}^2$$

Draw

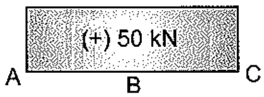
Diagram of normal forces normal stresses & displacement → along the length

- **FBD for AB and BC,**



- **Normal force diagram**

Normal force diagram will be constant equal to 50 kN (tension)



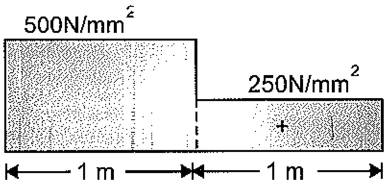
Normal Force Diagram

- **Normal stress diagram**

$$\text{Stress, } \sigma_{AB} = \left(\frac{50 \times 1000}{100}\right) = 500 \text{N/mm}^2$$

$$\sigma_{BC} = \frac{50 \times 1000}{200} = 250 \text{N/mm}^2$$

The normal stress diagram have discontinuity at interface as shown below



Normal stress Diagram

- **Displacement diagram**

We know that displacement , $\delta = \dfrac{PL}{AE}$ or $\dfrac{\sigma L}{E}$

For portion AB, σ = 500 N/mm² and E = 2 × 10⁵ N/mm².

$$\therefore \quad \delta = \left(\frac{500\times L}{2\times10^5}\right)\text{meter, where L is meter}$$

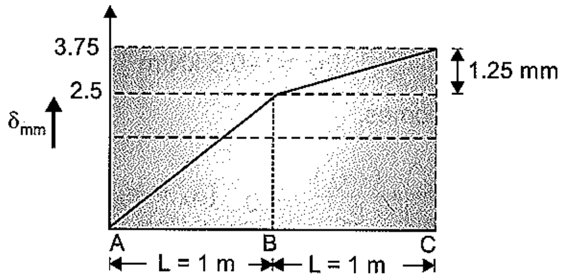
$$= \left(\frac{500\times L\times1000}{2\times10^5}\right)\text{mm, where L in meter}$$

$$\delta = (2.5\ L)\ \text{mm}$$

For portion BC, σ = 250 N/mm²

$$\therefore \quad \delta = \left(\frac{250\times L\times1000}{2\times10^5}\right) = (1.25L)\text{mm, where L in meter}$$

So, Displacement curve is as shown below,



Q-4: (a) ***The given figure shows three metal cubes A, B and C, of side 100 mm in direct contact, resting on a rigid base and confined in the x-coordinate direction between two rigid end-plates.***

If the upper face of the centre cube (cube B) is subjected to uniform compressive stress of 0.5 kN/mm², compute for cube B, the following:

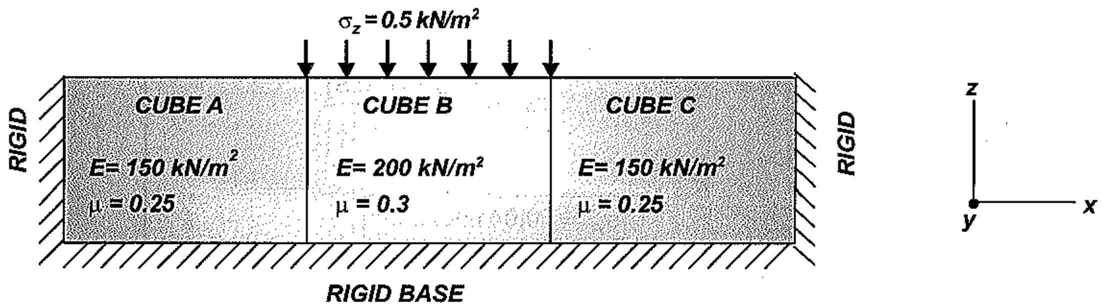
(i) ***The direct stress in the x-direction (σ_A).***

(ii) ***The direct strains in the three coordinate directions x, y and z.***

(iii) ***The volumetric strain.***

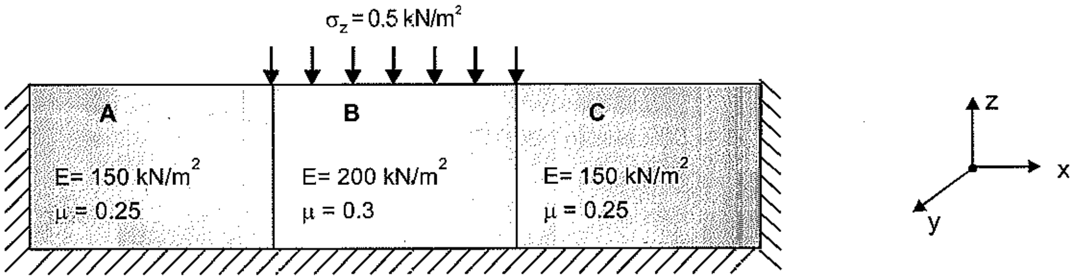
The elastic properties for the three cubes A, B and C are given in figure.

(b) ***State all assumptions made.***



[30 + 10 = 40 Marks, ESE-1999]

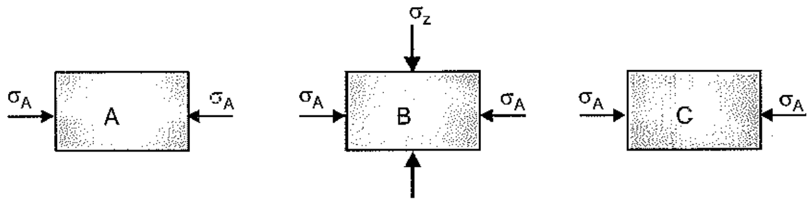
Sol: (a)



(i) It is given that cubes are confined in X-direction

$$(\Delta_A + \Delta_B + \Delta_C) \text{ in x-direction} = 0$$

where Δ_A, Δ_B, Δ_C, are change in length of cubes A, B and C in x-direction.



$\Rightarrow$ $[\sigma_Z = -0.5 \text{ kN/m}^2]$

Hence equation (i) becomes,

$$\left(\frac{-\sigma_A}{E_A}\right)+\left(-\frac{\sigma_A}{E_B}-\mu_B\frac{\sigma_Z}{E_B}\right)+\frac{(-\sigma_A)}{E_C} = 0 \quad [\Delta_A = \epsilon_A \times L \ ; \ \Delta_B = \epsilon_B \times L ; \Delta_C = \epsilon_c \times L$$

As side of each cube is same $\Rightarrow \epsilon_A + \epsilon_B + \epsilon_C = 0]$

$$\Rightarrow \quad \frac{-\sigma_A}{E_A}-\frac{\sigma_A}{E_B}-\frac{\mu_B\sigma_Z}{E_B}-\frac{\sigma_A}{E_C} = 0$$

Since, $E_A = E_C$

$$\therefore \quad \frac{-\sigma_A}{E_A}-\frac{\sigma_A}{E_B}-\mu_B\frac{\sigma_Z}{E_B}-\frac{\sigma_A}{E_A} = 0$$

$$\Rightarrow \quad \frac{-2\sigma_A}{E_A}-\frac{\sigma_A}{E_B}-\mu_B\frac{\sigma_z}{E_B} = 0$$

$$\Rightarrow \quad \sigma_A\left[\frac{2}{E_A}+\frac{1}{E_B}\right] = \frac{-\mu_B\sigma_z}{E_B}$$

$$\Rightarrow \quad \sigma_A = \frac{-\mu_B\sigma_z}{\left[2\frac{E_B}{E_A}+1\right]}$$

$$\therefore \quad \sigma_A = \frac{-0.3\times(-0.5)}{\left(\frac{2\times200}{150}+1\right)}$$

So, $\boxed{\sigma_A = 0.041 \text{kN/mm}^2}$

$\Rightarrow$ Direct stress in x-direction for Cube B = 0.041 kN/mm² (compressive)

Direct strain in three co-ordinate directions for cube B are given by

(ii) $$\epsilon_x = \frac{-0.041}{E_B} + \frac{0.3\times0.5}{E_B} = 5.45\times10^{-4}$$

$$\epsilon_y = \frac{0.3\times0.041}{E_B} + \frac{0.3\times0.5}{E_B} = 8.11\times10^{-4}$$

$$\epsilon_z = \frac{-0.5}{E_B} + \frac{0.3\times0.041}{E_B} = -2.44\times10^{-3}$$

(iii) Volumetric strain for cube

$$\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= 5.45\times10^{-4} + 8.11\times10^{-4} - 2.44\times10^{-3}$$

$$= -1.084\times10^{-3} \text{ (Ans.)}$$

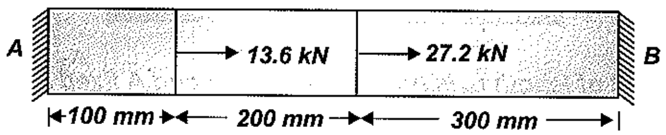
(b) Assumption:

(i) Material is homogeneous, isotropic and linear elastic i.e., hooke's law is valid.

(ii) The deformations are small such that principle of superposition can be applied

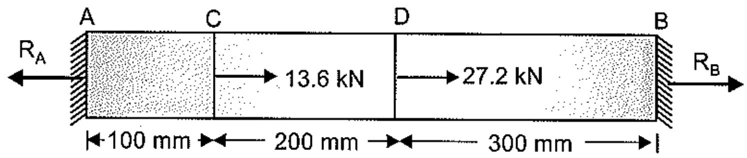
(iii) Cubes faces are frictionless

Q-5: ***A prismatic bar is fastened between two rigid walls at A and B and subjected to loads as shown in Fig. below, Determine the reactions at the supports.***

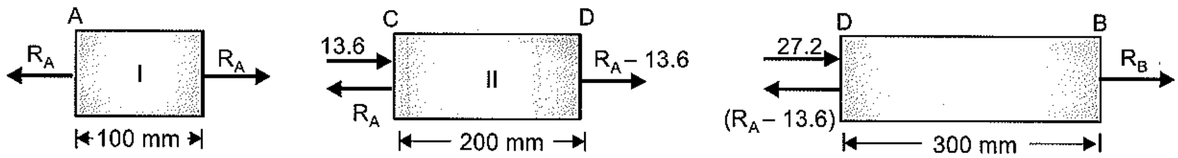


[10 + 10 + 10 + 10 = 40 Marks, ESE-1999]

Sol:



Split the whole bar into 3 small bars and show the free body diagram.



Consider part III and we write force equilibrium,

$$R_A - 13.6 - 27.2 = R_B$$

$\therefore$ $$\boxed{R_A - R_B = 40.8} \quad \text{...(i)}$$

Since both A and B are fixed supports so, we, apply the computability equation.

$$\Delta_{AC} + \Delta_{CD} + \Delta_{DB} = 0$$

$$\frac{R_A\times100}{A\times E} + \frac{(R_A-13.6)\times200}{A\times E} + \frac{R_B\times300}{AE} = 0$$

$\Rightarrow$ $$R_A + 2R_A - 27.2 + 3R_B = 0$$

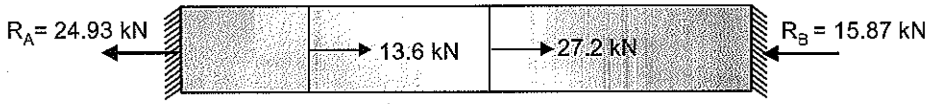
$$\Rightarrow \qquad 3R_A + 3R_B = 27.2$$

$$\Rightarrow \qquad R_A + R_B = 9.06 \qquad \ldots\text{(ii)}$$

$\therefore$ Solving above (ii) equation we get

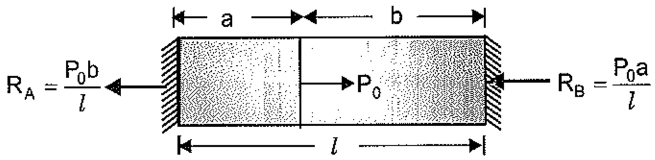
$$R_A = 24.9335 \text{ kN}$$

$$R_B = -15.87 \text{ kN}$$



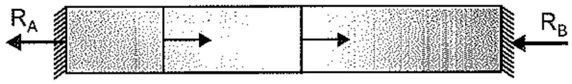
Alternatively:

We know that,



So,
$$R_A = 13.6\times\frac{500}{600}+27.2\times\frac{300}{600}=24.93\,\text{kN}$$

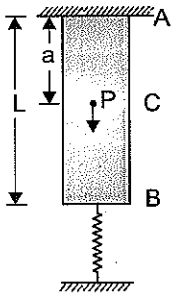
$$R_B = 13.6\times\frac{100}{600}+27.2\times\frac{300}{600}=15.87\,\text{kN}$$



Q-6: ***A floating column AB of length L, is modelled with top end A fixed and bottom end yielding under column load. The yielding support can be visualized as an axial spring of stiffness k. If an axial force P is applied at a distance 'a' from the top, determine reactions at the ends A and B, and displacement at the end B.***

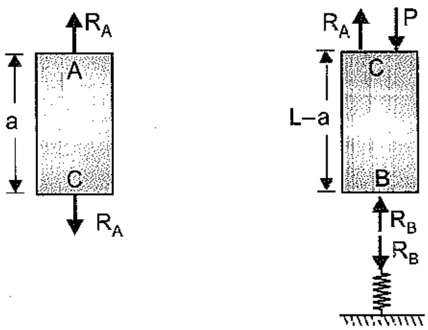
[10 Marks, ESE-2003]

Sol:



In the above figure movement of point B downwards will be equal to, compression in spring. Using this compatibility condition, the solution can be obtained.

The free body diagram of the structure will be as shown below.



Net elongation in column is,

$$\Delta_B = \Delta_{AC} + \Delta_{CB} = \frac{R_A a}{AE} + \frac{(-P+R_A)(L-a)}{AE}$$

From equilibrium of point BC, $P - R_A = R_B$

Now, from compatibility condition, net elongation in bar = compression in spring.

$$\frac{R_A a}{AE} + \frac{(-P+R_A)(L-a)}{AE} = \frac{P-R_A}{K}$$

$$R_A\left[\frac{a}{AE} + \frac{L-a}{AE} + \frac{1}{K}\right] = \frac{P}{K} + \frac{P(L-a)}{AE}$$

$$\Rightarrow \quad R_A\left[\frac{L}{AE} + \frac{1}{K}\right] = P\left[\frac{1}{k} + \frac{L-a}{AE}\right]$$

$$\boxed{R_A = \frac{P\,[AE + k\,(L-a)]}{kL + AE}}$$

$$R_B = P - R_A = P - P\left[\frac{AE+kL}{AE+kL} - \frac{ka}{AE+kL}\right]$$

$$\boxed{R_B = \frac{Pak}{AE+kL}}$$

and $$\boxed{\text{Compression of spring } = \frac{R_B}{K} = \frac{Pa}{KL+AE}}$$

Alternatively:

By Castigliano's theorem, $\dfrac{\partial U}{\partial R_B} = -\Delta_C$

$$U = U_{AC} + U_{CB} = \frac{R_A^2 a}{2AE} + \frac{R_B^2(L-a)}{2AE}$$

$$U = \frac{(P-R_B)^2 a}{2AE} + \frac{R_B^2(L-a)}{2AE}$$

$$\frac{\partial U}{\partial R_B} = \frac{-2(P-R_B)a}{2AE} + \frac{2R_B(L-a)}{2AE}$$

$$\Rightarrow \quad -\Delta_C = \frac{-(P-R_B)a}{AE} + \frac{R_B(L-a)}{AE}$$

$$\Rightarrow \quad \frac{-R_B}{K} = -\frac{(P-R_B)a}{AE} + \frac{R_B(L-a)}{AE}$$

$$\Rightarrow \quad \frac{R_B}{K} = \frac{(P-R_B)a}{AE} - \frac{R_B(L-a)}{AE}$$

$$\Rightarrow \quad R_B\left[\frac{1}{K}+\frac{a}{AE}+\frac{L-a}{AE}\right] = \frac{Pa}{AE}$$

$$R_B\left(\frac{1}{K}+\frac{L}{AE}\right) = \frac{Pa}{AE}$$

$$\Rightarrow \quad R_B = \left[\frac{Pak}{kL+AE}\right]$$

Q-7: ***In a tensile test, a test piece of 25 mm diameter is tested over a gauge length of 125 mm. The elongation over this length is 0.0875 mm under a pull of 68725 N. In a torsion test, a test piece was made of the same material and of same diameter, and it twisted 0.025 radians over a length of 250 mm at a torque of 0.3068 kN-m. Find Poisson's ratio and the three elastic modulii of the test piece material.***

[15 Marks, ESE-2007]

Sol:

Elongation = 0.0875 mm;

Under P = 68725 N

$$\frac{Pl}{AE} = \Delta \quad \text{... (i)}$$

Angle of twist, θ = 0.25 radian

l = 250 mm

$$\frac{Tl}{GJ} = \theta \quad \text{... (ii)}$$

From 1:

$$E = \frac{Pl}{A\Delta} = \frac{68725 \times 125 \text{ Nmm}}{\frac{\pi}{4}(25)^2\text{mm}^2 \times 0.0875 \text{ mm}} = 2.00 \times 10^5 \text{ N/mm}^2$$

From 2:

$$G = \frac{Tl}{J\theta} = \frac{0.3068 \times 10^6 \text{ Nmm} \times 250 \text{ mm}}{\frac{\pi(25)^4}{32}\text{mm}^4 \times 0.025} = 0.800 \times 10^5 \text{ N/mm}^2$$

$$G = \frac{E}{2(1+\mu)} = 0.8 \times 10^5$$

$$\frac{2\times10^5}{2(1+\mu)} = 0.8 \times 10^5$$

$$(1+\mu) = \frac{1}{0.8} = 1.25$$

$$\Rightarrow \quad \boxed{\mu = 0.25}$$

Thus,

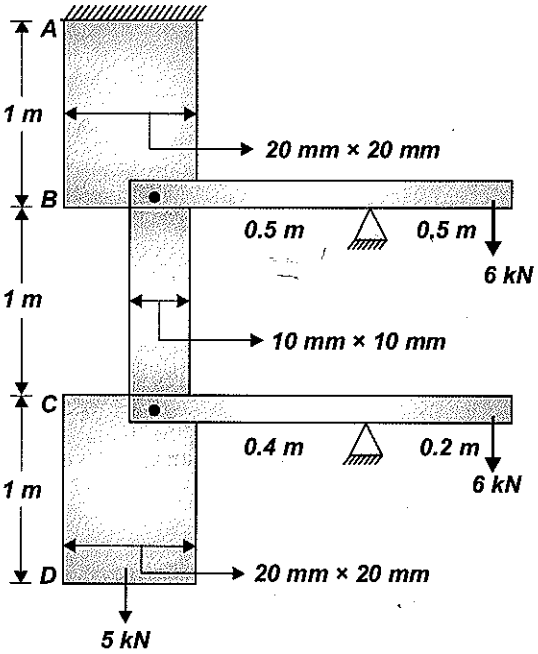
$$\boxed{E = 2\times10^5 \text{ N/mm}^2}$$

$$\boxed{G = 0.8\times10^5 \text{ N/mm}^2}$$

$$K = \frac{E}{3(1-2\mu)} = \frac{2\times10^5}{3\times(1-0.5)} = \frac{2}{1.5}\times10^5$$

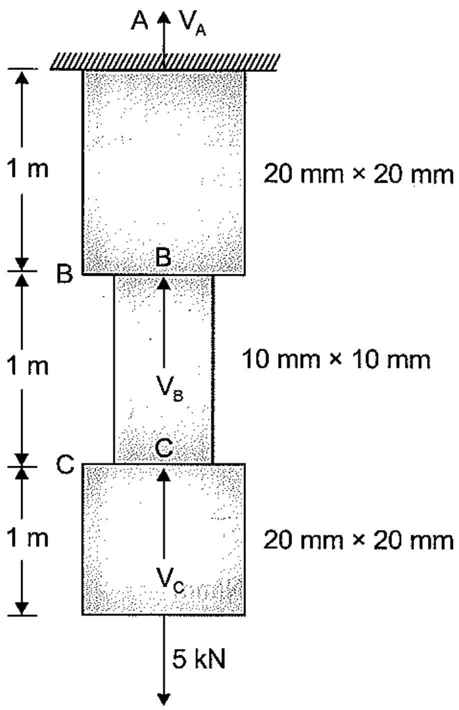
$$\boxed{K = 1.33\times10^5 \text{ N/mm}^2}$$

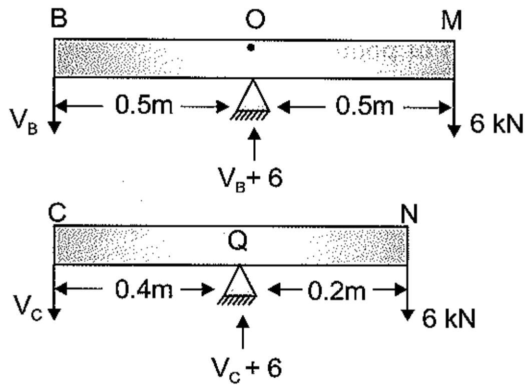
Q-8: ***A stepped vertical steel bar ABCD is fixed at the top end A. Each segment of the bar AB, BC and CD is 1 m long and has cross-sections 20 mm × 20 mm, 10 mm × 10 mm and 20 mm × 20 mm respectively. A 5 kN load is applied directly at D and 6 kN loads are applied on the levers attached to the stepped bar at B and C as shown in the figure. Find the vertical displacement of D and the change in volume of the bar. E = 2 × 10^5 MPa and Poisson's ratio, μ = 0.25. Connections between the levers and bar at B and C are hinged.***



[15 Marks, ESE-2009]

Sol:





Determination of V_B and V_C

Considering beam BOM,

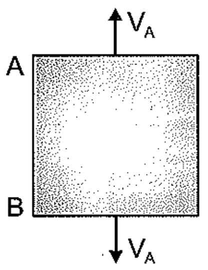
$$\sum M\Big|_{\text{about O}} = 0$$

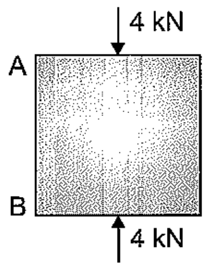
$$\Rightarrow \qquad V_B \times 0.5 = 6 \times 0.5 \quad \Rightarrow \boxed{V_B = 6\text{kN}}$$

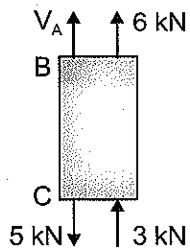
Considering beam CQN $\qquad \Sigma M|_{\text{about } Q} = 0$

$$\Rightarrow \qquad V_C \times 0.4 = 6 \times 0.2 \quad \Rightarrow \boxed{V_C = 3\text{kN}}$$

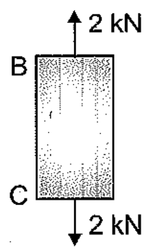
Free body diagram of ABCD

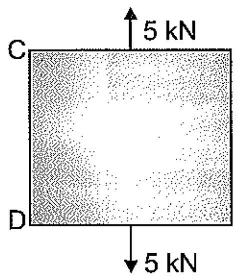


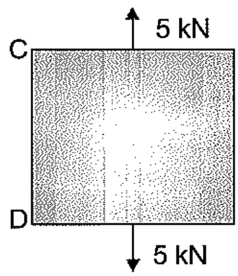




For equilibrium
$V_A + 6 = 2$
$\Rightarrow \; V_A = -4\text{kN}$
i.e, opposite of assumed direction







Deflection of point D

$$\delta_D = -\Delta_{AB} + \Delta_{BC} + \Delta_{CD}$$

$$= \frac{-(4\times10^3)\times(1\times10^3)}{(20\times20)\times(2\times10^5)} + \frac{(2\times10^3)\times(1\times10^3)}{(10\times10)\times(2\times10^5)} + \frac{(5\times10^3)\times(1\times10^3)}{(20\times20)\times(2\times10^5)}$$

$$= -0.05 + 0.1 + 0.0625 = 0.1125 \text{ mm.}$$

Volumetric strain determination

$$\frac{\Delta V_{AB}}{V_{AB}} = \frac{-\sigma}{E} + \frac{\mu\times\sigma}{E} + \frac{\mu\times\sigma}{E} \qquad \text{where } \sigma = \frac{4\times10^3}{20\times20} \text{N/mm}^2$$

$$\Delta V_{AB} = \frac{\sigma}{E}(2\mu - 1)V_{AB}$$

$$= \frac{4\times10^3}{20\times20} \times \frac{1}{2\times10^5} \; (2 \times 0.25 - 1) \times 20 \times 20 \times 1000$$

$$\Delta V_{AB} = -10\text{mm}^3$$

$$\Delta V_{BC} = \frac{\sigma}{E}(1-2\mu)V_{BC}\ ; \quad \sigma = \frac{2\times10^3}{10\times10}\text{N/mm}^2$$

$$\Delta V_{BC} = \frac{2\times10^3}{10\times10\times2\times10^5}\ (1-2\times0.25)\times10\times10\times1000$$

$$\therefore \quad \Delta V_{BC} = 5\text{mm}^3$$

$$\frac{\Delta V_{CD}}{V_{CD}} = \frac{\sigma}{E}(1-2\mu)$$

$$\Delta V_{CD} = \frac{\sigma}{E}(1-2\mu)V_{CD}\ ; \quad \sigma = \frac{5\times10^3}{20\times20}\text{N/mm}^2$$

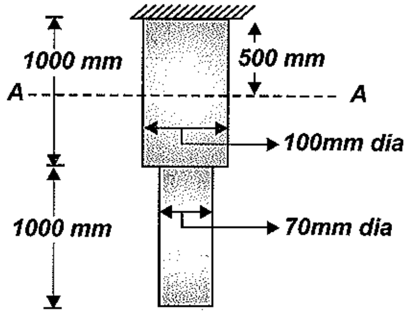
$$\therefore \quad \Delta V_{CD} = \frac{5\times10^3}{20\times20}\times\frac{1}{2\times10^5}\times(1-2\times0.25)\times20\times20\times1000$$

$$\therefore \quad \Delta V_{CD} = 12.5\text{ mm}^3$$

Hence total change of stepped bar, ABCD, = $\Delta V_{AB} + \Delta V_{BC} + \Delta V_{CD}$

$$= -10 + 5 + 12.5 = +7.5\text{ mm}^3 \text{ (increases)}$$

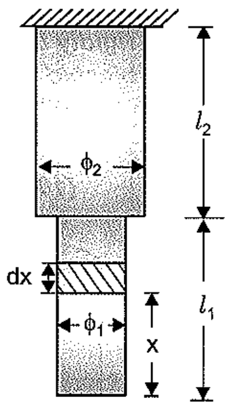
Q-9: ***A stepped bar with circular cross-section and supported at top, hangs vertically under its own weight. Dimensions of the bar are as shown in the figure below. Calculate the elongation of the bar under its own weight. What is the change in diameter of the bar at section AA shown in the figure?***

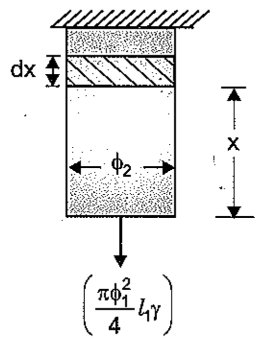


E = 2 × 10⁵ N/mm², density γ = 8 × 10⁻⁵ N/mm³ and Poisson's ratio μ = 0.2.

[20 Marks, ESE-2010]

Sol:





$$\text{Elongation of smaller dia bar} = \int_0^{l_1}\frac{\left(\frac{\pi\phi_1^2}{4}\text{x}\,\gamma\right)dx}{\frac{\pi\phi_1^2}{4}\times E} = \int_0^{l_1}\frac{\gamma x\,dx}{E} = \frac{\gamma}{E}\left.\frac{x^2}{2}\right|_0^{l_1} = \frac{\gamma l_1^2}{2E}$$

$$\text{Elongation of larger dia bar} = \int_0^{l_2} \frac{\left(\frac{\pi\phi_1^2}{4}\gamma l_1 + \frac{\pi\phi_2^2}{4}x\gamma\right)dx}{\frac{\pi\phi_2^2}{4}E}$$

$$= \frac{1}{E}\int_0^{l_2}\left(\frac{\gamma\phi_1^2 l_1}{\phi_2^2} + \gamma x\right)dx = \frac{\gamma\phi_1^2 l_1 l_2}{\phi_2^2 E} + \frac{\gamma l_2^2}{2E}$$

$$\text{Total elongation} = \frac{\gamma l_1^2}{2E} + \frac{\gamma l_2^2}{2E} + \frac{\gamma\phi_1^2 l_1 l_2}{\phi_2^2 E}$$

$$= \frac{8.0\times10^{-5}}{2\times10^5\times2}\left[(1000)^2 + (1000)^2 + 2\times(1000)(1000)\times\left(\frac{70}{100}\right)^2\right]$$

$$\boxed{\text{Total elongation} = 5.96\times10^{-4}\text{ mm}}$$

Change in dia at A-A

$$\text{Lateral strain} = \frac{-\mu\sigma}{E} = \frac{\Delta\phi}{\phi_2}$$

$$\Delta\phi = \frac{-\mu\sigma}{E}\phi_2$$

Stress at A-A

$$\sigma = \frac{\gamma\frac{\pi}{4}\left(\phi_1^2 l_1 + \phi_2^2\frac{l_2}{2}\right)}{\frac{\pi\phi_2^2}{4}} = \gamma\left[\frac{\phi_1^2}{\phi_2^2}l_1 + \frac{l_2}{2}\right]$$

$$\sigma = 8\times10^{-5}\left[\left(\frac{70}{100}\right)^2\times1000 + 500\right]$$

$$\sigma = 0.0792\text{ N/mm}^2$$

$$\Rightarrow \quad \Delta\phi = -\frac{0.2\times0.0792}{2\times10^5}\times100$$

$$\Delta\phi = 7.92\times10^{-6}\text{ mm (decrease)}$$

Q-10: ***A compound bar consists of a circular rod of steel of diameter 25 mm, rigidly fitted into a copper tube of internal diameter 25 mm and thickness 2.5 mm. If the bar is subjected to an axial load of 100 kN, find the stresses developed in the two materials. Given, $E_S = 2\times10^5$ N/mm² and $E_{cu} = 1.2\times10^5$ N/mm².***

[10 Marks, ESE-2011]

Sol:

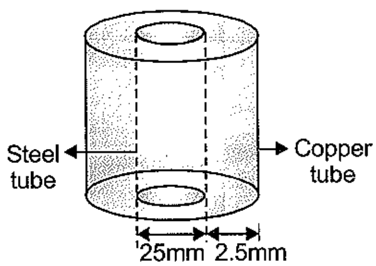
Diameter of steel bar = 25 mm

External Diameter of copper tube = 25 + 2 × 2.5 = 30mm

Bar is subjected to load (P) = 100 kN

$E_s = 2\times10^5$ N/mm²

$E_{cu} = 1.2\times10^5$ N/mm²



$$\text{Cross sectional area of steel }(A_s) = \frac{\pi}{4}\times25^2 = 490.875\text{mm}^2$$

Cross sectional area of copper (A_c) = $\frac{\pi}{4}(30^2 - 25^2) = 215.985 \text{mm}^2$

Let the external force resisted by steel and copper are by P_S and P_C respectively.

$\Rightarrow$ $$P = P_S + P_C \quad \text{...(i)}$$

and due to compatibility $$\Delta_S = \Delta_C$$

$\Rightarrow$ $$\frac{P_S L_S}{A_S E_S} = \frac{P_C L_C}{A_c E_{Cu}}$$

$\Rightarrow$ $$\frac{P_s}{490.875 \times 2 \times 10^5} = \frac{P_c}{215.985 \times 1.2 \times 10^5} \quad [\because L_s = L_c]$$

$\Rightarrow$ $$\boxed{P_S = 3.787 P_c} \quad \text{...(ii)}$$

By substituting it into eq^n (i)

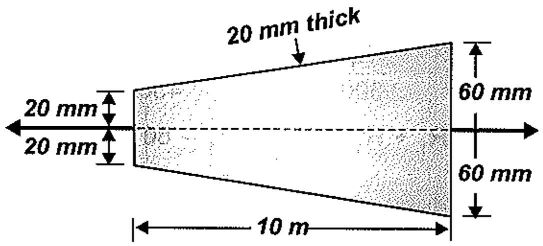
$$P_s + P_C = P \Rightarrow 4.787\ P_c = P \Rightarrow P_c = \frac{P}{4.787} = 20.89 \text{ kN}$$

$\therefore$ $$P_S = 79.11 \text{ kN}$$

$\therefore$ $$\text{Stress in steel bar, } p_s = \frac{P_s}{A_s} = \frac{79.11 \times 10^3}{\frac{\pi}{4} \times 25^2} = 161.16 \text{ N/mm}^2$$

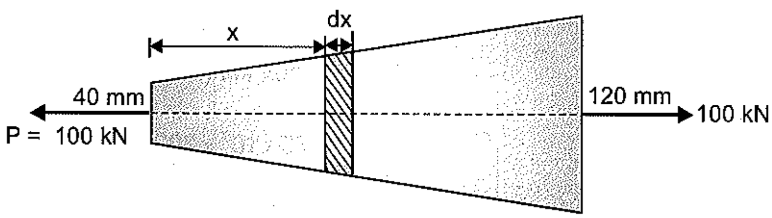
$$\text{Stress in copper tube, } p_C = \frac{P_c}{A_c} = \frac{20.89 \times 10^3}{215.985} = 96.72 \text{ N/mm}^2$$

Q-11: ***Compute the total elongation caused by an axial load of 100 kN applied to a flat bar 20 mm thick, tapering from a width of 120 mm to 40 mm in a length of 10 m as shown in fig. Assume E = 200 GPa***



[20 Marks, ESE-2013]

Sol:



$$t = 20 \text{ mm} ; L = 10 \text{ m}$$

$$B_x = 40 + \left(\frac{120 - 40}{10}\right)x$$

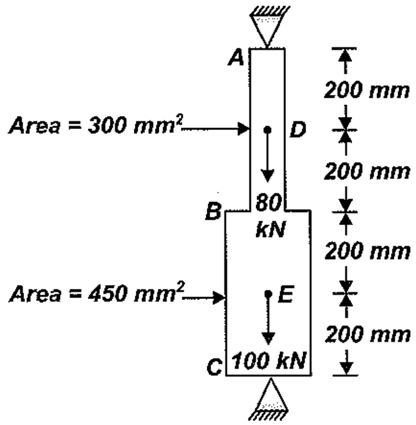
$$B_x = 40 + 8x \quad [\text{x is in meter } B_x \text{ in mm}]$$

$$d\Delta = \frac{P\,dx}{(40 + 8x) \times t \quad E}$$

$$\Delta = \int_0^{10} \frac{P}{tE}\left(\frac{1}{40+8x}\right)dx = \frac{P}{tE}\frac{\ln(40+8x)}{8}\Big|_0^{10}$$

$$\frac{P}{8tE}\left[\ln\frac{120}{40}\right] = \frac{P\log_e 3}{8tE} = \frac{100\times10^3\times\log_e 3}{8\times20\times2\times10^5}\text{m} = 3.433\times10^{-3}\text{ mm}$$

Q-12: ***A non-prismatic rod is attached between two supports as shown in figure. It is subjected to axial loads of 80 kN and 100 kN. Determine the support reactions and elongations. Take E = 200 GPa.***



[25 Marks, ESE-2016]

Sol: ∵ The rod is supported between two unyielding supports

∴ Net elongation of the whole bar will be zero i.e.,

$$\delta_{AD}+\delta_{DB}+\delta_{BE}+\delta_{EC} = 0$$

$$\frac{R_1\times200}{300\times E}+\frac{(R_1-80)\times200}{300\times E}+\frac{(R_1-80)\times200}{450\times E}+\frac{(R_1-180)\times200}{450\times E} = 0$$

$$\frac{R_1}{300}+\frac{R_1-80}{300}+\frac{R_1-80}{450}+\frac{R_1-180}{450} = 0$$

$$1.5\,(R_1+R_1-80)+(R_1-80+R_1-180) = 0$$

$$R_1 = 76\text{ kN}$$

$$R_{AD} = 76\text{ kN}$$

$$R_{DB} = -4\text{ kN (compressive)}$$

$$R_{BE} = -4\text{ kN (compressive)}$$

$$R_{EC} = -104\text{ kN (compressive)}$$

$$\delta_{AD} = \frac{76\times10^3\times200}{300\times2\times10^5} = 0.253\text{ mm}$$

$$\delta_{DB} = \frac{-4\times10^3\times200}{300\times2\times10^5} = -0.013\text{ mm}$$

$$\delta_{BE} = \frac{-4\times10^3\times200}{450\times2\times10^5} = -0.0088\text{ mm}$$

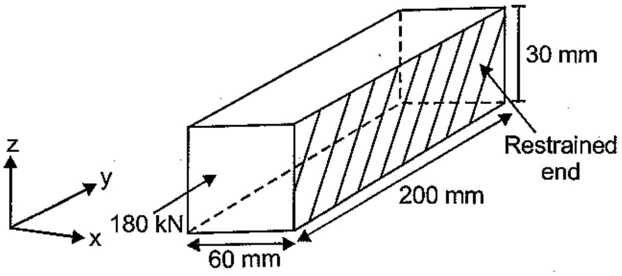
$$\delta_{EC} = \frac{-104\times10^3\times200}{450\times2\times10^5} = -0.231\text{ mm}$$

Q-13: ***A rectangular bar of 30mm × 60mm cross-section and 200mm length is restrained from expansion along its 30mm × 200mm sides by surrounding material. Find the change in dimensions and volume when a compressive force of 180 kN acts in axial direction.***

$E = 2 \times 10^5$ MPa; $\mu = 0.3$

[12 Marks, ESE-2017]

Sol: **Given:** P = 180 kN



So, Axial thrust $= \dfrac{180 \times 1000}{60 \times 30} = 100 \text{ MPa} = \sigma_y \text{(compressive)}$

Strain in the x-direction is restrained. Hence $\varepsilon_x = 0$ Let stress in x direction is σ_x.

So, $$\varepsilon_x = \left(\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}\right) = \frac{\sigma_x}{E} - \frac{0.3(-100)}{E} = 0$$

$\Rightarrow$ $$\sigma_x = -30 \text{ Mpa}$$

So, $$\sigma_x = 30 \text{ Mpa (c)}$$

$$\sigma_y = 100 \text{ MPa (c)}$$

$$\sigma_z = 0$$ [Since the material is free in z-direction]

Strain in the Different Directions

$$\varepsilon_x = 0$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}$$

$$\varepsilon_y = \left(-\frac{100}{E} - \frac{(-0.3 \times 30)}{E}\right)$$

$$\varepsilon_y = \frac{\Delta y}{y} = \frac{(-100+9)}{E} = -\frac{91}{E}$$ [i.e., compression]

So, Change in length in y-direction $= \left(-\dfrac{91}{E} \times 200\right)$ mm

$$= -\frac{91}{2 \times 10^5} \times 200 \text{ mm}$$

$$= -0.091 \text{ mm}$$ [i.e., reduction in length]

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} \quad (\text{where } \sigma_z = 0)$$

$$= -\frac{0.3}{E}[-30 - 100] = \frac{39}{E}$$ [tension]

So, Change in length in z-direction $= 30 \times \frac{39}{2 \times 10^5} = 0.00585$ mm [i.e., increase in length]

$$\text{Volumetric strain} = \varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \left(-\frac{91}{E} + \frac{39}{E}\right) = -\frac{52}{E}$$

So, $$\text{Change in volume} = -\frac{52}{2 \times 10^5} \times 30 \times 200 \times 60 = -93.6 \text{ mm}^3$$

Here, negative sign means reduction in volume.

Q-14: ***A very long steel drill pipe got stuck in hard clay at an unknown depth. The drill pipe was applied a large upward force and observed that the drill pipe came out elastically by 500 mm. It was also observed that there was elongation of 0.04 mm in a gauge length of 200 mm. Estimate the depth of hard clay bed. Following consideration may be taken into account:***

Resistance offered by all materials/elements may be taken as zero.

[12 Marks, ESE-2018]

Sol: **Given that:**

Field data:

Drill pipe came out elastically $(\Delta L) = 500$ mm

Observed data:

Elongation, $(\delta l) = 0.04$ mm

Gauge length, $(l) = 200$ mm

To find out: Depth of hard clay bed

Let the force applied be 'P' and area of pipe be 'A'.

During field situation,

Let the length of pipe from ground level upto the clay bed be 'L'

Then, $$\delta L = \frac{PL}{AE}$$

or $$\boxed{\frac{\delta L}{L} = \frac{P}{AE}} \quad \text{...(i)}$$

But, for a gauge length $(l) = 200$ mm

Extension, $(\delta l) = 0.04$ mm

Then, $$\delta l = \frac{Pl}{AE}$$

$$\boxed{\frac{\delta l}{l} = \frac{P}{AE}} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{\delta L}{L} = \frac{\delta l}{l}$$

$$\therefore \quad L = \delta L\left(\frac{l}{\delta l}\right)$$

$$\Rightarrow \quad L = 0.5 \times \left(\frac{200}{0.04}\right)$$

$$\boxed{L = 2500 \text{ m}}$$

Hence, the depth of the hard clay bed will be 2500 m.

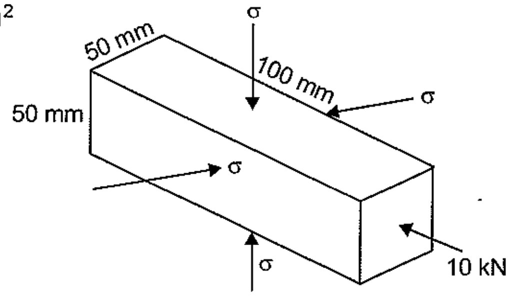
Q-15: ***A square bar (50 mm × 50 mm cross-section) of 100 mm length is subjected to an axial compressive load of 10 kN. Calculate the change in volume of the bar, if all lateral strains of the bar are prevented by a uniform pressure on its four lateral faces. Calculate this pressure value and the change in volume. Also calculate the value of bulk modulus K and shear modulus G for the material of bar. Following parameters may be used if required.***

(i) $E = 2 \times 10^5\ N/mm^2$

(ii) $\mu = 0.25$

[20 Marks, ESE-2018]

Sol: (i) Square bar having cross-section area = 50 × 50 mm²



$$\text{Length} = 100\text{ mm}$$

$$\text{Axial compressive load} = 10\text{ kN}$$

$$E = 2 \times 10^5\ N/mm^2$$

$$\mu = 0.25$$

Stress in axial direction,

$$\sigma_{axial} = \frac{-10 \times 10^3}{2500} = -4\ N/mm^2$$

Given: Lateral strains are prevented by applying a uniform pressure on all lateral faces.

Let uniform pressure is 'σ'

$$\varepsilon_{lateral} = 0$$

$$0 = \frac{-\sigma}{E} - \frac{\mu(-\sigma)}{E} - \frac{\mu(-4)}{E}$$

$$-\sigma[1-\mu] + 4\mu = 0$$

$$\sigma = \frac{4\mu}{1-\mu} = \frac{4 \times 0.25}{1-0.25} = \frac{4}{3}\text{ MPa (comp.)}$$

$$\varepsilon_{axial} = \frac{-4}{E} - \frac{\mu(-\sigma)}{E} - \frac{\mu(-\sigma)}{E}$$

$$\varepsilon_{axial} = \frac{-4 + 2 \times 0.25 \times \frac{4}{3}}{2 \times 10^5} = \frac{-4 + \frac{2}{3}}{2 \times 10^5} = \frac{-10}{6 \times 10^5}$$

$$\boxed{\varepsilon_{axial} = -1.67 \times 10^{-5}}$$

Then,
$$\frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\frac{\delta V}{(50)^2 \times 100} = -1.67 \times 10^{-5} \quad (\varepsilon_y = \varepsilon_z = 0)$$

$$\boxed{\delta V = -4.17\ mm^3}$$

–ve sign implies that volume decreases

(ii) From, E, μ, K G relation

$$E = 2G(1+\mu)$$

$$\Rightarrow \quad G = \frac{E}{2(1+\mu)} = \frac{2 \times 10^5}{2(1.25)}$$

$$\Rightarrow \quad \boxed{G = 0.8 \times 10^5 \text{ N/mm}^2}$$

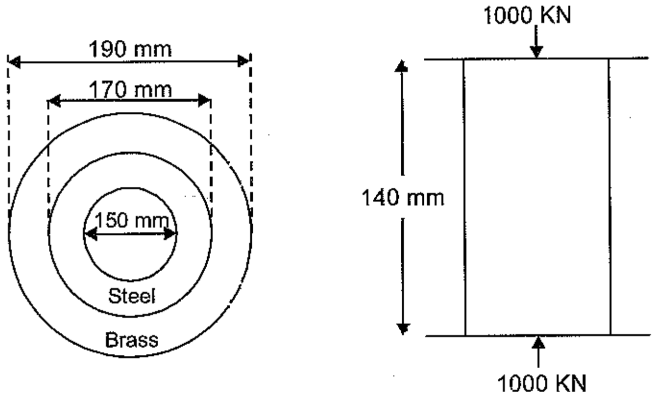
$$E = 3K(1-2\mu)$$

$$\boxed{K = 1.33 \times 10^5 \text{ N/mm}^2}$$

Q-16: ***A compound tube consists of a steel tube 150 mm internal diameter and 170 mm external diameter and a brass tube of 170 mm internal diameter and 190 mm external diameter. The two tubes are of the same length. The compound tube carries an axial load of 1000 kN. Find the stresses and the load carried by each tube and the amount it shortens. Length of each tube is 140 mm. Take E for steel as 2 × 10^5 N/mm^2 and for brass as 1 × 10^5 N/mm^2.***

[8 Marks, ESE-2019]

Sol: **Given data:**



$$P = 1000 \text{ kN}$$

$$L = 140 \text{ mm}$$

$$E_S = 2 \times 10^5 \text{ N/mm}^2$$

$$E_b = 1 \times 10^5 \text{ N/mm}^2$$

$$A_S = \frac{\pi}{4}(170^2 - 150^2) = 5026.55 \text{ mm}^2$$

$$A_b = \frac{\pi}{4}(190^2 - 170^2) = 5654.867 \text{ mm}^2$$

Using equilibrium equation

$$P_S + P_b = 1000 \qquad ..(i)$$

Using compatibility equation

$$\Delta_S = \Delta_b$$

$$\frac{P_S l_S}{A_S E_S} = \frac{P_b l_b}{A_b E_b}$$

$$\frac{P_S \times 140}{5026.55 \times 2 \times 10^5} = \frac{P_b \times 140}{5654.867 \times 1 \times 10^5}$$

$$P_S = 1.778 P_b \qquad ..(ii)$$

(i) From eqn. (i) and (ii), we get

$$P_S = 640 \text{ KN}$$

$$P_b = 360 \text{ KN}$$

(ii) So stress in steel $\sigma_S = \frac{P_S}{A_S} = \frac{640\times10^3}{5026.55} = 127.32\text{N/mm}^2$

Stress in bras $\sigma_b = \frac{P_b}{A_b} = \frac{360\times10^3}{5654.867} = 63.66\text{N/mm}^2$

(iii) $\Delta_b = \Delta_S = \frac{P_s l_S}{A_S E_S} = \frac{640\times10^3\times140}{5026.55\times2\times10^5} = 0.089\text{mm}$

Q-17: ***On a steel bar specimen of 15 mm diameter and 150 mm gauge length, when tested as a tensile test specimen, a force of 15 kN produces an extension of 0.063 mm. When the specimen of same diameter and same length is tested under torsion, a twisting moment of 6.94 Nm produces an angular twist of 0.15°. Determine the Poisson's ratio of the material of the bar.***

[4 Marks, ESE-2019]

Sol: **Given data:**

Diameter (D) = 15 mm

Gauge length (L) = 150 mm

Load (P) = 15 kN

Axial deflection (Δ) = 0.063 mm

Torsion (T) = 6.94 N-m

Twisting angle (θ) = 0.15° = 2.618 × 10^{-3} rad.

$$\text{Axial Deflection } (\Delta) = \frac{Pl}{AE}$$

$$0.063 = \frac{15\times10^3\times150}{\frac{\pi}{4}\times15^2\times E}$$

$$E = 202.1\times10^3 \text{ N/mm}^2$$

Using Torsion Formula

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\Rightarrow \quad \frac{6.94\times10^3}{\frac{\pi}{32}\times15^4} = \frac{G\times2.618\times10^{-3}}{150}$$

$$\Rightarrow \quad G = 80\times10^3 \text{ N/mm}^2$$

We know,

$$E = 2G(1+\mu)$$

$$202.1\times10^3 = 2\times80\times10^3(1+\mu)$$

$$\mu = 0.263$$

Q-18: ***Three vertical rods carry a tensile load of 100 kN. Area of cross-section of each rod is 500 mm². Their temperature is raised by 60°C and the load is now so adjusted that they extend equally. Determine the load shared by each. The outer two rods are of steel and the middle one is of brass.***

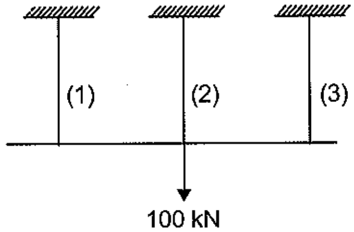
$$E_S = 2E_B = 210\text{GPa}. \quad \alpha_S = 11\times10^{-6}/^\circ C; \quad \alpha_B = 18\times10^{-6}/^\circ C$$

[12 Marks, ESE-2019]

Sol: **Given:** Area, A = 500 mm²

Change in temperature, $\Delta T = 60°C$

And Elongation of Bar (1), (2) and (3)



$$\Delta_1 = \Delta_2 = \Delta_3$$

$$E_s = 2E_B = 210 \text{ GPa}$$

$$\alpha_s = 11 \times 10^{-6}/°C$$

$$\alpha_B = 18 \times 10^{-6}/°C$$

Let the load shared by outer steel bars be 'P_S' and that shared by middle brass bar is 'P_B'

So, under equilibrium,

$$2P_S + P_B = 100 \text{ kN} \quad \text{...(i)}$$

Now, Elongation of outer steel bars,

$$\Delta_1 = \Delta_3 = \Delta_{(load)} + \Delta_{(rise\ in\ temperature)}$$

$$= \frac{P_S \times l}{AE_S} + l\alpha_s\Delta T \quad [l_1 = l_2 = l_3 = l]$$

Also elongation of inner brass bar

$$\Delta_2 = \Delta_{(load)} + \Delta_{(rise\ in\ temperature)}$$

$$\Delta_2 = \frac{P_B l}{AE_B} + l\,\alpha_B\Delta T$$

According to question

$$\Delta_1 = \Delta_2$$

$$\Rightarrow \quad \frac{P_S \times l}{AE_s} + l\alpha_s\Delta T = \frac{P_B l}{AE_B} + l\alpha_B\Delta T$$

$$\Rightarrow \quad \frac{P_S}{500 \times 210 \times 10^3} + 11 \times 10^{-6} \times 60 = \frac{P_B}{500 \times 105 \times 10^3} + 18 \times 10^{-6} \times 60$$

$$\Rightarrow \quad P_S - 2P_B = 44100 \text{ N} \quad \text{...(ii)}$$

On solving equation (i) and equation (ii), we get

$$\boxed{\begin{aligned} P_S &= 48.82 \text{ kN} \\ P_B &= 2.36 \text{ kN} \end{aligned}}$$

Q-19: ***Calculate the modulus of rigidity and bulk modulus of a cylindrical bar of diameter 30 mm and of length 2.0 m if the longitudinal strain in a bar during a tensile stress is six times the lateral strain. Find the change in the volume, when the bar is subjected to a hydrostatic pressure of 120 N/mm². Take E = 1 × 10⁵ N/mm².***

[12 Marks, ESE-2019]

Sol: **Given data:**

Diameter, (D) = 30 mm

Length, (L) = 2 m

$E = 1 \times 10^5 \text{ N/mm}^2$

Hydrostatic pressure = 120 N/mm²

Longitudinal strain = 6 time lateral strain

We know,

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\mu = \frac{1}{6}$$

Again $$K = \frac{E}{3(1-2\mu)} = \frac{1\times10^5}{3\left(1-2\times\frac{1}{6}\right)}$$

$$\boxed{K = 0.5\times10^5\ \text{N/mm}^2}$$

$$G = \frac{E}{2(1+\mu)} = \frac{1\times10^5}{2\left(1+\frac{1}{6}\right)}$$

$$\boxed{G = 0.429\times10^5\ \text{N/mm}^2}$$

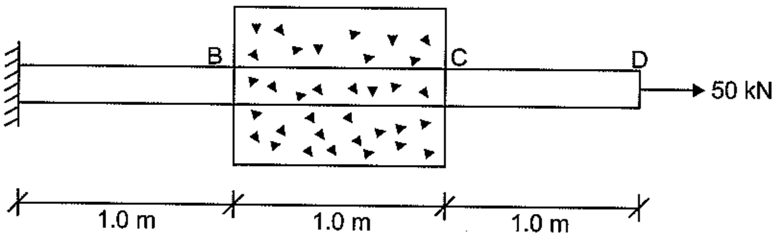
Bulk modulus is given, $$K = \frac{p}{\frac{\Delta V}{V}}$$

$$\frac{\Delta V}{V} = \frac{p}{K} = \frac{120}{0.5\times10^5}$$

$\Rightarrow$ $$\Delta V = \frac{120}{0.5\times10^5}\times\frac{\pi}{4}\times30^2\times2000$$

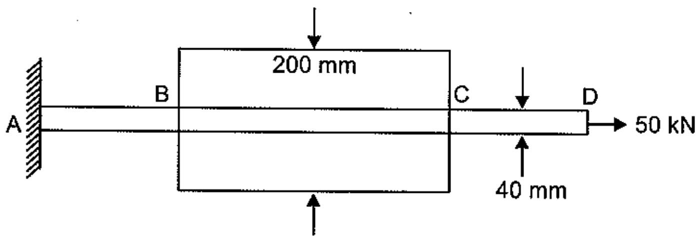
$\Rightarrow$ $$\boxed{\Delta V = 3392.92\ \text{mm}^3}$$

Q-20: ***A semi-composite steel bar as shown in Figure is loaded at free end with an axial load of 50 kN. Determine the axial stiffness of the system and extension of the free end. Diameter of steel bar is 40 mm, outer diameter of the concrete portion is 200 mm. Modulus of elasticity of steel = 200 GPa, Modulus of elasticity of concrete = 20 GPa. Central portion of the bar is embedded with concrete.***

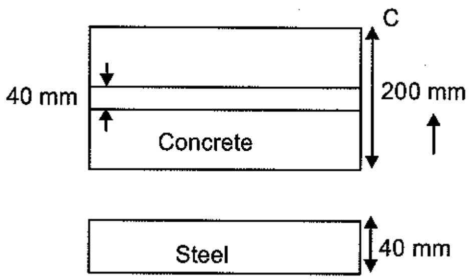


[12 Marks, ESE-2020]

Sol: Given data.



FBD of BC



$$A_c = \frac{\pi}{4}(200^2 - 40^2)$$
$$= 30159.29\ \text{mm}^2$$
$$A_s = \frac{\pi}{4}\times40^2 = 1256.64\ \text{mm}^2$$

Using compatibility equation

$$(\Delta_s)_{BC} = (\Delta_C)_{BC}$$

$$\Rightarrow \quad \frac{P_s L}{A_s E_s} = \frac{P_c L}{A_C E_C}$$

$$\Rightarrow \quad \frac{P_s \times L}{\frac{\pi}{4} \times 40^2 \times 200} = \frac{P_c \times L}{\frac{\pi}{4}(200^2 - 40^2) \times 20}$$

$$\Rightarrow \quad \boxed{2.4 P_s = P_C} \quad \text{... (ii)}$$

again $\quad P_S + P_C = 50 \quad$ ... (ii)

From equation (i) and (ii)

$$P_s + 2.4 P_s = 50$$

$$\boxed{P_S = 14.706 \text{ kN}}$$

$$\boxed{P_C = 35.294 \text{ kN}}$$

Deformation $\quad \Delta = \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$

$$\Delta = \frac{50 \times 10^3 \times 1000}{1256.64 \times 2 \times 10^5} + \frac{14.706 \times 10^3 \times 1000}{1256.64 \times 2 \times 10^5} + \frac{50 \times 10^3 \times 1000}{1256.64 \times 2 \times 10^5}$$

$$\Delta = 0.1989 + 0.0585 + 0.1989$$

$$\boxed{\Delta = 0.456 \text{ mm}}$$

(ii) Axial stiffness = required force for unit displacement

Assume P force required for unit displacement force in BC portion of steel section

$$= \frac{14.706}{50} \times P$$

$$= 0.294 \text{ P}$$

Again, $\quad \Delta_{AB} + \Delta_{BC} + \Delta_{CD} = 1$

$$\frac{PL}{AE} + \frac{0.294 \, PL}{AE} + \frac{PL}{AE} = 1$$

$$\frac{2.294 \, PL}{AE} = 1$$

$$\boxed{P = \frac{AE}{2.294 \, L}}$$

$$P = \frac{1256.64 \times 2 \times 10^5}{2.294 \times 1000}$$

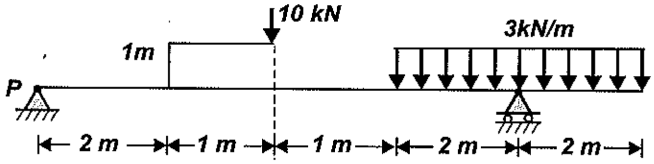
$$P = 109558.84 \text{ N/mm}$$

$$\boxed{\text{So, Axial stiffness} = 1.095 \times 10^5 \text{ N/mm}}$$

CHAPTER 2

SHEAR FORCE AND BENDING MOMENT

Q-1: ***Draw bending moment and shear force diagrams for the beam loaded as shown in figure.***



[15 Marks, ESE-1995]

Sol: There are two approaches to draw BMD

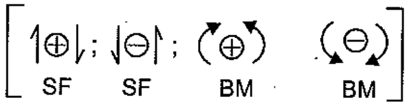
(i) Analytical approach (ii) Graphical approach

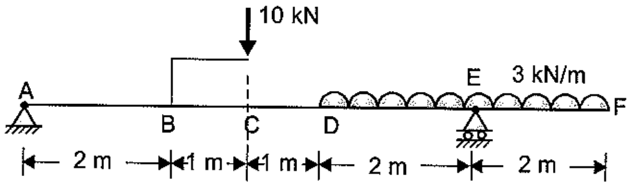
In Analytical approach, we write down equation of BM of different segment of beam and Plot these to obtain the BMD.

In the graphical approach we make use of the relationship between BM, SF and loading and plot the BMD.

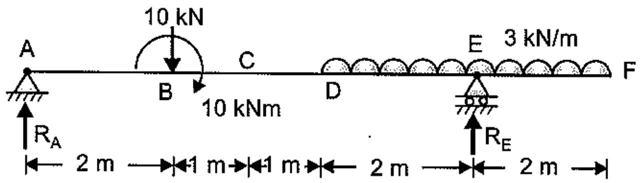
In this problem for illustration purpose we will draw the BMD by both the approaches.

Analytical approach





The above loading is equivalent to that shown below



From $\Sigma F_V = 0$

$$R_A + R_E = 10 + 12 = 22$$

From $\Sigma M_A = 0$

$$R_E \times 6 - 3\times4\times6 - 10\times2 - 10 = 0$$

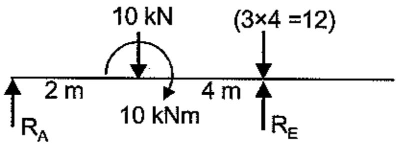
$\Rightarrow$ $$R_E = \frac{102}{6} = 17$$

$\Rightarrow$ $$R_E = 17 \text{ kN}$$

$$R_A = 5 \text{ kN}$$

X will always be measured from left support

Alternatively: reaction can be calculated as



$$\Rightarrow R_A = 10\times\frac{4}{6} - \frac{10}{6} + 0$$

$$R_E = 10\times\frac{2}{6} - \frac{10}{6} + 12$$

Note: This loading diagram is a applicable only for calculating reactions

For Span AB [0 ≤ x < 2m]

$$\boxed{V = 5}$$

$$\boxed{M = 5x}$$

at x = 0, V = 5; at x = 0; M = 0
at x = 2, V = 5; at x = 2; M = 10
BM variation is linear

For Span BD [2 ≤ x < 4m]

$$\boxed{V = 5 - 10 = -5}$$

$$\boxed{M = 5x - 10(x-2) + 10 = -5x + 30}$$

at x = 2; V = – 5; at x = 4; V = – 5
at x = 2; M = 20; at x = 4; M = 10
BM variation is linear

For Span DE [4 < x ≤ 6]

$$\boxed{V = 5 - 10 - 3(x-4) = -3x + 7}$$

$$M = 5x - 10(x-2) + 10 - \frac{3(x-4)^2}{2}$$

$$= -5x + 30 - 1.5x^2 - 24 + 12x$$

$$\boxed{M = -1.5x^2 + 7x + 6}$$

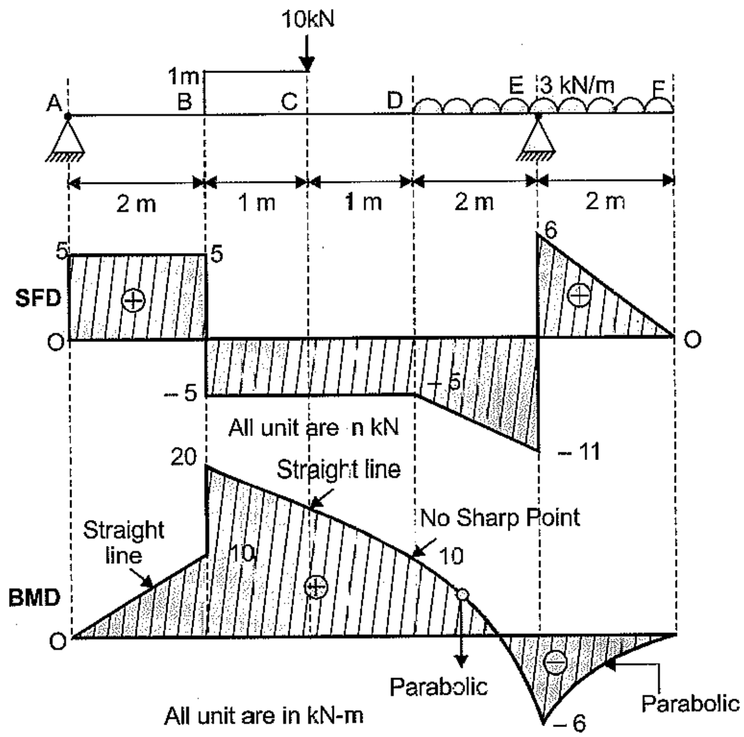
at x = 4; V = – 5; at x = 6; V = – 11
at x = 4; M = 10; at x = 6; M = – 6
Variation of BM is parabolic, slope of BMD is SF = – 3x + 7
⇒ as x increases slope increases & it is (–)ve

For Span EF [6 < x ≤ 8]

$$V = 3[8 - x] = 24 - 3x$$

$$M = \frac{-3(8-x)^2}{2} = -1.5(8-x)^2$$

at x = 6; V = 6; at x = 8; V = 0
at x = 6; M = 6; at x = 8; M = 0
Variation is parabolic, slope of BM = 24 – 3x is (+)ve in the range of x = 6 to x = 8 and as x increase, slope decreases, slope becomes 0 at x = 8



GRAPHICAL APPROACH

Conceptual background:

1. Sign convention 

i.e., on the left side of section considered,
Downward loading = (–ve)
Upward loading = (+ve)

2. $\dfrac{dV}{dx}$ [i.e., slope of SFD] = w [load intensity]

3. $\dfrac{dM}{dx}$ [i.e. slope of BMD] = V [shear force]

4. $V_2 - V_1 = \int_1^2 w\,dx$ i.e., difference of SF between two points (2) and (1) [2 at higher value of x]

   = Area under loading diagram between (1) and (2)

5. $M_2 - M_1 = \int_1^2 V\,dx$ i.e., difference of BM between two points (2) and (1) = area under SFD between (1) & (2)

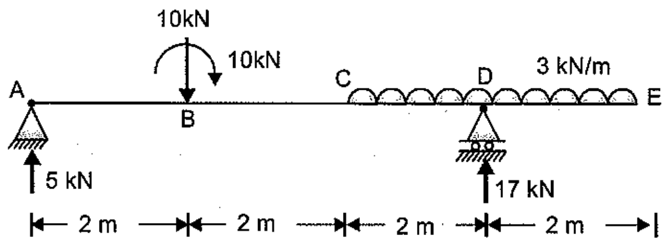
6. Upward point loading leads to sudden jump in SFD by an amount equal to the point loading

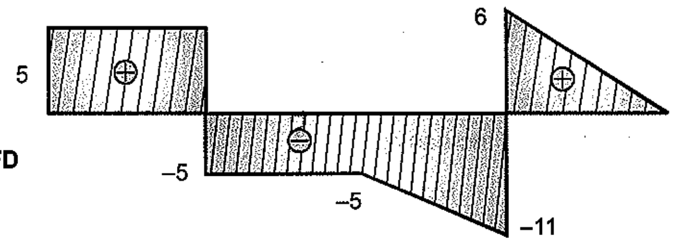
7. Clockwise concentrated couple leads to jump in BMD equal to the amount of concentrated couple.

8. x is always measured from left to right.

9. If loading is nth order curve, SFD will be (n + 1) order curve and BMD will be (n + 2) order curve.

SFD





SFD

- At A, there is upward point load ⇒ SFD jump up by 5 kN
- A to B, load intensity is zero ⇒ Slope of SFD = 0
- At B, there is downward point load of 10 kN ⇒ SFD jumps down by 10kN
- B to C, load intensity = 0 ⇒ Slope of SFD = 0
- C to D, load intensity = – 3 kN/m {(–ve) because of sign convention}

  ⇒ Slope of SFD is (–)ve and constant, equal to –3kN/m

$$V_D - V_C = -3 \times 2$$

$$\Rightarrow \quad V_D - (-5) = -6$$

$$\Rightarrow \qquad v_D = -11$$

- At D there is upward point load of 17 kN

$\Rightarrow$ SFD jumps up by 17 kN

$$V_{Final} - (-11) = 17$$

$$\Rightarrow \qquad V_{Final} = 6$$

D to E load intensity is (–)ve and constant {(–ve) because of sign convention}

$$\Rightarrow \qquad V_E - V_D = -3 \times 2$$

$$V_E - 6 = -6$$

$$\Rightarrow \qquad V_E = 0$$

BMD

At A, BM = 0 {$\because$ moment to the left of A = 0}

A to B

SF is (+)ve and constant

$\Rightarrow$ Slope of BMD is (+)ve and constant, $M_B - M_A$ = Area under SFD b/w A to B

$$\Rightarrow \qquad M_B - 0 = 5 \times 2$$

$$\Rightarrow \qquad M_B = 10 \quad \text{\{just to the left of B\}}$$

At B; There is clockwise concentrated couple of 10 kNm

$\Rightarrow$ BMD Jump up by 10 kNm $\Rightarrow M_B = 10 + 10 = 20$ {just to the right of B}

B to C

SF is (–)ve and constant

$\Rightarrow$ Slope of BMD is (–)ve and constant

$$M_C - M_B = \text{Area under SFD between B and C}$$

$$M_C - (20) = -5 \times 2$$

$$M_C = 10 \text{ kNm}$$

C to D

SF is (–)ve and increasing

$\Rightarrow$ Slope of BMD is (–)ve and increasing

$$M_D - M_C = \text{Area under SFD between D and C}$$

$$M_D - 10 = \frac{-5-11}{2} \times 2 = -16$$

$$\boxed{M_D = -6}$$

SFD is Ist order curve $\Rightarrow$ BMD is 2nd order curve i.e., parabolic

D to E

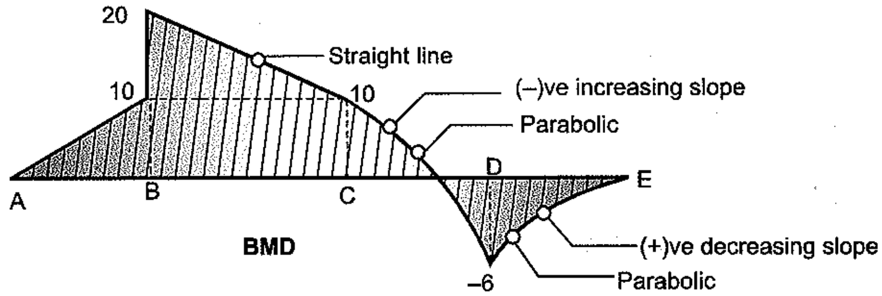
SF is (+)ve and decreasing

$\Rightarrow$ Slope of BMD is (+)ve and decreasing

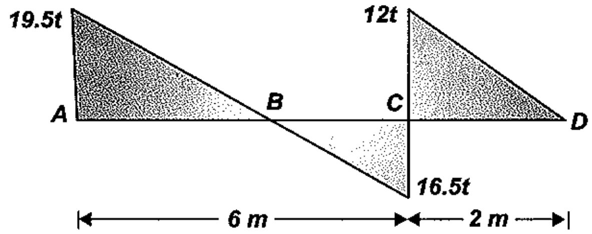
$$M_E - M_D = \text{Area under SFD between D to E}$$

$$M_E - (-6) = \frac{1}{2}\times 6\times 2 = 6$$

$$\Rightarrow \quad M_E = 0$$

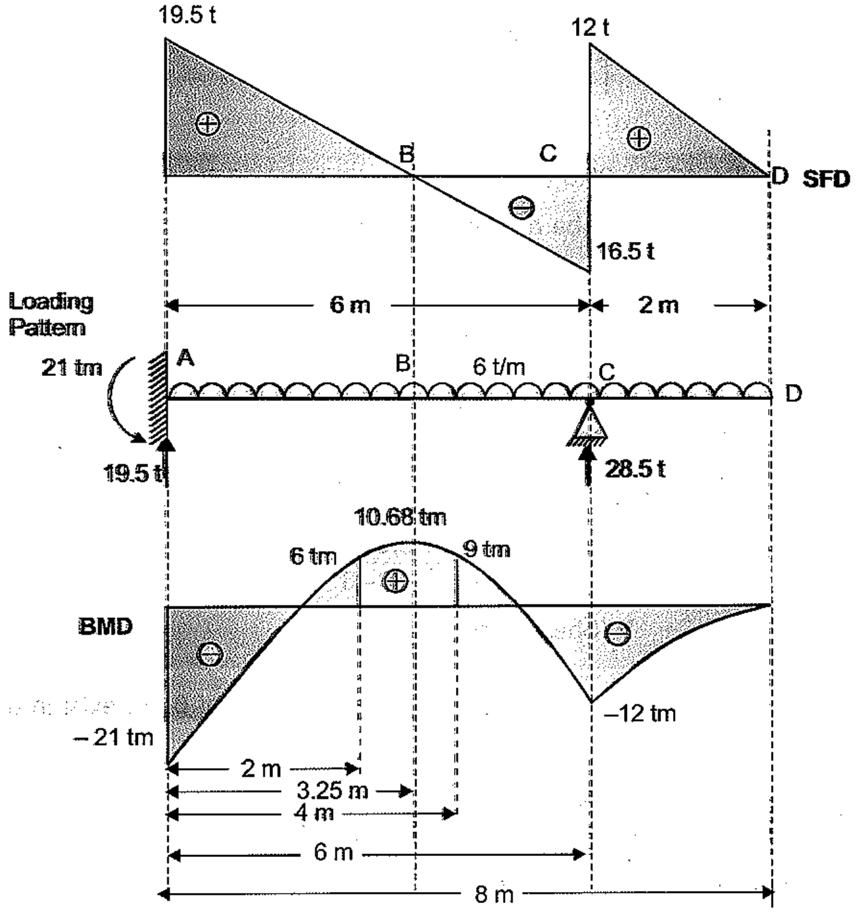


Q-2: ***Shear force diagram of a beam is shown in the figure below Identify the location and nature of supports. Draw loading and bending moment diagram showing values of bending moment every 2m and also showing position and magnitude of maximum bending moment.***



[15 Marks, ESE-1996]

Sol:



Conceptual background

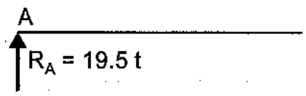
Two points we keep in our mind:

(a) By moving from left to right if any S.F.D. suddenly drops then it means that there is a point load in downward direction and if it suddenly jumps then it means that there will be an upward force and so, there will be a possibility of support.

(b) $\frac{dV}{dx} = W$ (Load intensity)

Step 1: At point A,

S.F.D. jumps upward by 19.5t thus there will be a reaction of 19.5t. R_A = 19.5t



Step 2: Between point A and C

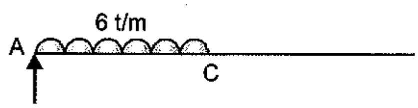
Finding the equation of line passing through (0, 19.5t) and (6. –16.5t)

$$\frac{y-19.5t}{x-0} = \frac{19.5t+16.5t}{-6}$$

$$\Rightarrow \quad y = -(6t)x + (19.5\ t)$$

i.e., $$V \text{ (Shear force)} = -6x + 19.5$$

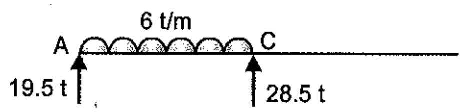
and we know that $\frac{dV}{dx}$ = load intensity = –6 (–ve sign indicates downward load)



Step 3: At point C.

Sudden jump of (12 + 16.5)= 28.5 t takes places

This means that again there will be a support which gives a reaction of 28.5t at point C.



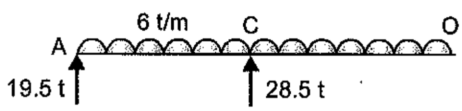
Step 4: Between point C and D.

Finding equation of line passing through (6, 12t) and (8, 0)

$$\frac{y-0}{(x-8)} = \frac{12-0}{6-8} \Rightarrow y = \frac{12(x-8)}{-2} \Rightarrow y = -6x + 48$$

So, $$V \text{ (Shear force)} = -6x + 48$$

$$\frac{dV}{dx} = W = -6 \text{ (u.d.l. of 6 t/m exist in downward direction)}$$



Now, we have to check the structure for $\Sigma M = 0$

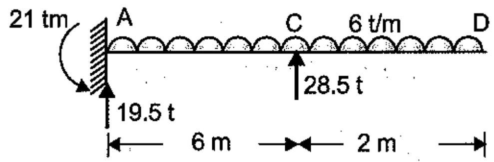
Take an anticlockwise moment about point A.

$$M_A = \left(28.5\times6-6\times\frac{8^2}{2}\right) = -21\text{ tm} \neq 0$$

⇒ Moment equilibrium does not exist. To ensure moment equilibrium without disturbing the force equilibrium, we have to have a concentrated couple acting any where in the beam equal to 21 tm anticlockwise.

For simplicity, let us assume that there is a fixed support at A where the fixed end moment is 21 tm anticlockwise

Step 5: Hence an acceptable loading diagram can be as shown below



Bending moment between point A and C

$$\boxed{M = (19.5)x-21-\frac{6x^2}{2}} \quad \text{.... (i)}$$

At x = 0 $\qquad M|_0 = -21\text{ tm}$

At x = 2 $\qquad M|_2 = 19.5\times2-21-\frac{6\times2^2}{2} = 6\text{tm}$

At x = 4 $\qquad M|_4 = (19.5)4-21-\frac{6\times4^2}{2} = 9\text{tm}$

At x = 6 $\qquad M|_6 = (19.5)6-21-\frac{6\times6^2}{2} = -12\text{tm}$

Bending moment variation between point C and D.

$$M = -\frac{6(8-x)^2}{2} \quad \text{(x measured from A only)} \quad \text{... (ii)}$$

At x = 6, M = – 12 tm

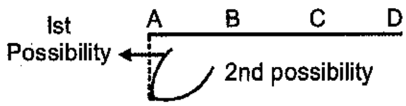
At x = 8, M = 0

Step 6: Sketching bending moment

BMD can be drawn by plotting equation (i) and (ii)

Conceptual background

Now question arises as to how to draw $M = (19.5)x - 21-\frac{6x^2}{2}$ from the end A, since we have two ways to draw a parabola as shown below.



To solve this ambiguity we just go to the S.F.D. diagram, since we know that $\frac{dM}{dx} = V$

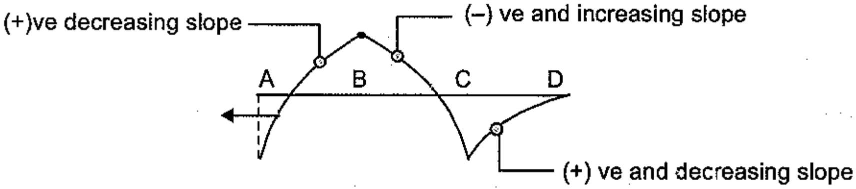
And between point A and B → V is + ve and decreasing

So, $\frac{dM}{dx}$ between A and B → (+ ve) and decreasing.

In both the possibilities the slope is (+ ve) but in 2nd possibility slope is increasing.

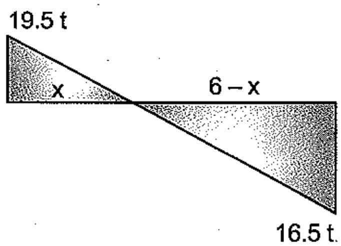
So, we will go for the Ist possibility.

Between point B and C, Shear force = (– ve) and increasing. So, $\frac{dM}{dx}$ = (– ve) and increasing



And this way we can draw the B.M.D. of complex variations easily

Obviously M will be maximum when $\frac{dM}{dx} = 0$ i.e., V = 0



For portion AC, $V = -6x + 19.5$

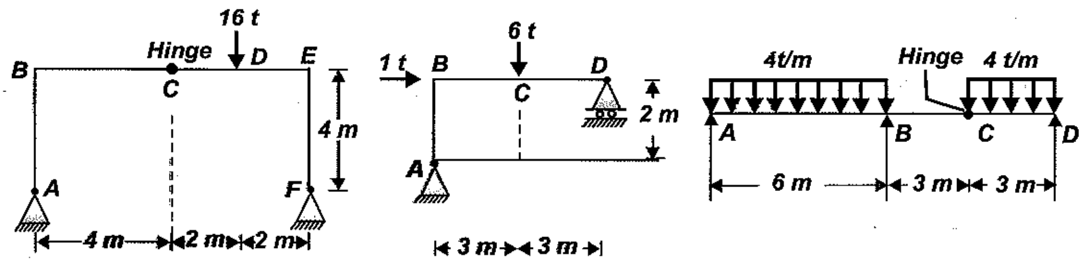
$\Rightarrow \quad x = \frac{19.5}{6}$

$\Rightarrow \quad x = 3.25 \text{ m}$

$\therefore \quad M|_{3.25} = 19.5 \times 3.25 - 21 - \frac{6 \times 3.25^2}{2} = 10.68 \text{ tm}$

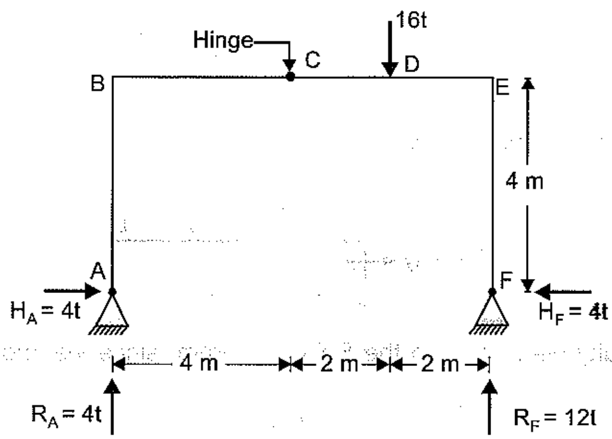
The final B.M.D and loading diagram has been drawn at the beginning.

Q-3: ***Analyse the following structures (frames & beam) shown in figure and draw bending moment diagrams.***



[8 + 8 +9 = 25 Marks, ESE-1996]

Sol: (i)



Calculation of Reaction:

$$R_A + R_F = 16\text{ t} \quad \text{... (i) equation } \Sigma F_y = 0$$
$$H_A = H_F \quad \text{... (ii) equation } \Sigma F_x = 0$$

$BM|_C = 0,$

$\Rightarrow$ $R_A \times 4 = H_A \times 4$

$\Rightarrow$ $R_A = H_A$ ... (iii)

and $BM|_C = 0$

$\Rightarrow$ $R_F \times 4 = H_F \times 4 + 16 \times 2$

$\Rightarrow$ $R_F = H_F + 8$ ... (iv)

$\therefore$ By solving (i), (ii), (iii) and (iv),

$$R_A + R_F = 16\text{ t}$$

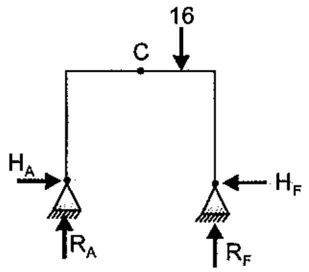
$\Rightarrow$ $H_A + H_A + 8 = 16$

$\Rightarrow$ $(H_A = 4\text{ t})$ and $(H_F = 4\text{t})$

$$R_A = 4\text{ t and } R_F = 12\text{ t}$$

Direction of each reactions are shown in figure above.

Alternative way to calculation of reaction



$$R_A = \frac{16\times 2}{8} = 4\,;\, R_F = 16\times\frac{6}{8} = 12$$

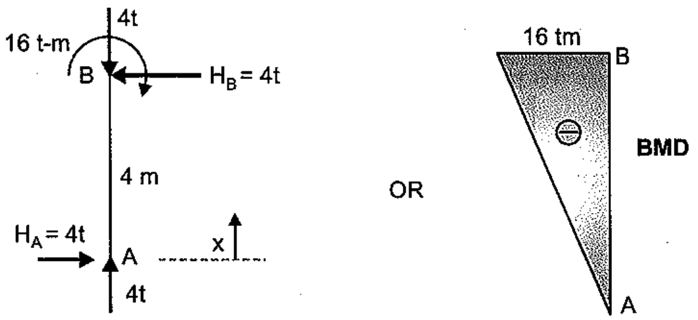
BM at C= 0

From left, $R_A \times 4 - H_A \times 4 = 0$

$\Rightarrow$ $H_A = R_A = 4$

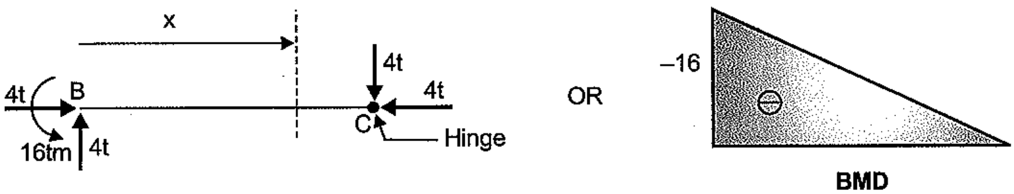
Bending moment analysis

For Part AB



B.M. = (–4x) t-m $\rightarrow$ Linear variation

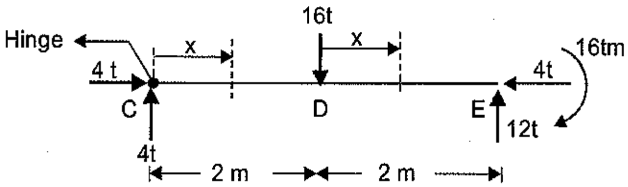
For Part BC

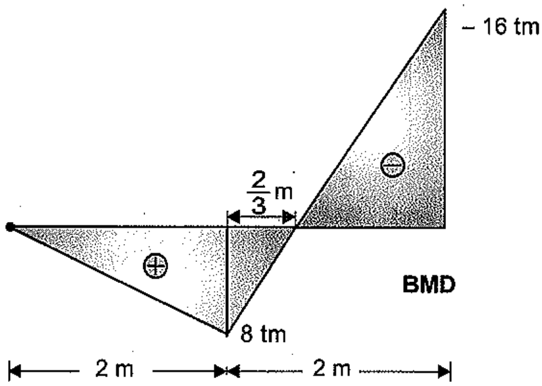


$$(\text{B.M.}) = -16 + 4x \text{ (linear variation)}, \text{B.M}|_{\text{at point B}} = -16$$

$$\text{B.M.}|_{\text{at point C}} = -16 + 4 \times 4 = 0$$

For Part CE





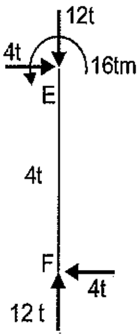
Moment for part CD = 4x $\Rightarrow$ At point D, B.M = 8

Moment for part DE, = 4(2 + x) – 16x = 8 – 12x, where $0 \le x \le 2$

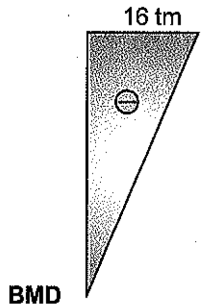
At point E = 8 – 24 = – 16

B.M. will be zero at, $x = \left(\frac{8}{12}\right) = \left(\frac{2}{3}\right)$m from D.

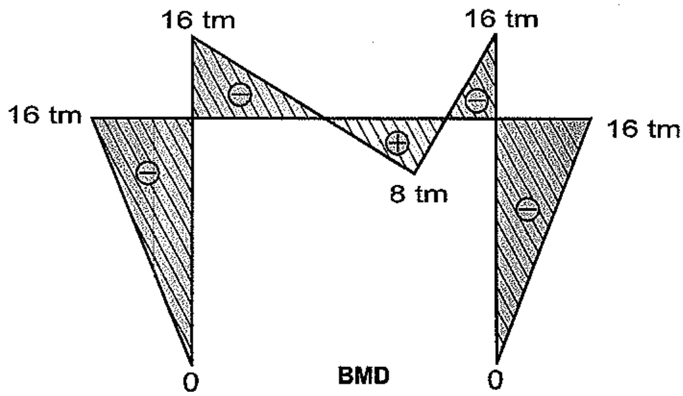
For Part EF



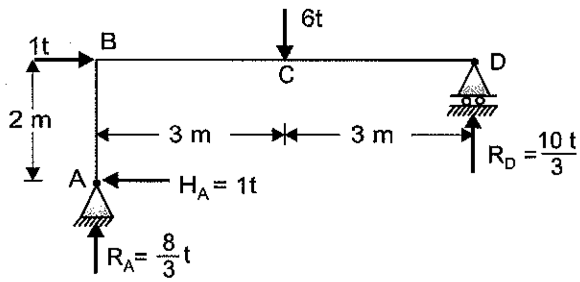
OR



Plotting by combining the various part BMD



(ii)



Reaction Calculation

From $\Sigma F_V = 0$, $\boxed{R_A + R_D = 6}$ ... (i)

From $\Sigma F_H = 0$, $\boxed{H_A = 1}$ ... (ii)

From $\Sigma M_D = 0$, $R_A \times 6 + H_A \times 2 = 6 \times 3$

$\Rightarrow$ $R_A \times 6 = 18 - 2 = 16$

$\Rightarrow$ $R_A = \dfrac{16}{6} = \left(\dfrac{8}{3}\right)t$

$\therefore$ $R_D = 6 - \dfrac{8}{3} = \dfrac{10}{3}t$

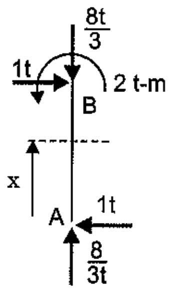
Bending Moment

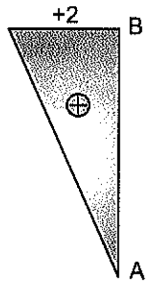
- **For part AB**

B.M. variation for part AB = $(1 \times x)$

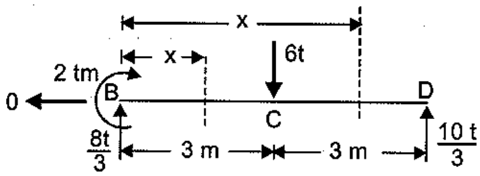
At A, BM = 0

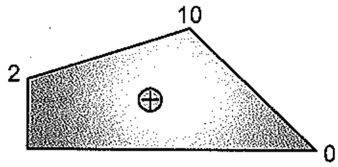
and At B, BM = 2 tm





- **For part BD,**





- **For part BC,**

$$BM = \frac{8x}{3} + 2 ; 0 < x \leq 3$$

So, at x = 3, i.e., C, $BM = \dfrac{8 \times 3}{3} + 2 = 10$ tm

- **For part CD**

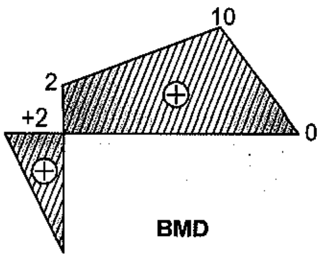
$$BM = 2 + \frac{8x}{3} - 6(x - 3)$$

$$BM = 20 - \frac{10}{3}x \; ; \; 3 \leq x \leq 6$$

at x = 3; BM = 10

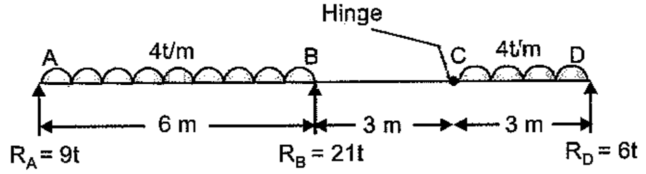
at x = 6; BM = 0

By combining we get



BMD

(iii)



Reaction Calculation

$$\boxed{R_A + R_B + R_D = 4\times 9 = 36} \quad \text{... (i)}$$

BM at C = 0 $\Rightarrow$ $R_D \times 3 = \dfrac{4\times3\times3}{2}$ $\Rightarrow$ $\boxed{R_D = 6}$

$\Rightarrow$ $$\boxed{(R_A + R_B) = 30} \quad \text{... (ii)}$$

Again, $M_C = 0$, $\Rightarrow$ $R_A \times 9 + R_B \times 3 = 4 \times 6 \times 6$

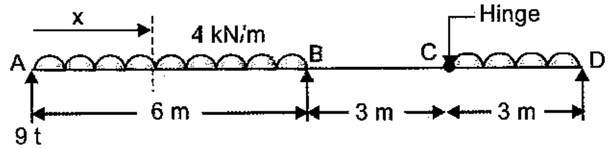
$\Rightarrow$ $3R_A + R_B = 48$

From (ii), $R_A + R_B = 30$

$2R_A = 18$

$\therefore$ $\boxed{R_A = 9}$; $\boxed{R_B = 21}$

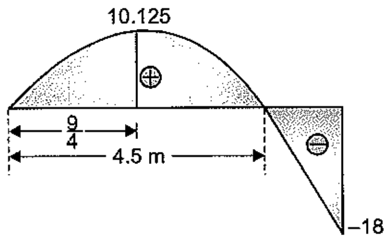
Bending moment diagram



Span AB

$$BM = 9x - \frac{4x^2}{2} = (9x - 2x^2) \;;\; 0 \le x \le 6$$

- $M|_B = 9 \times 6 - 2 \times 36 = 54 - 72 = -18$
- $\dfrac{dM}{dx} = 9 - 4x$
- For BM to be maximum

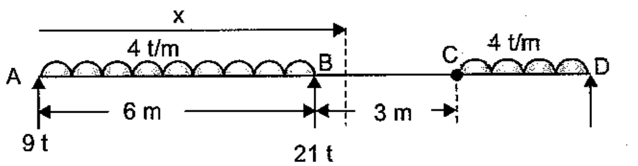


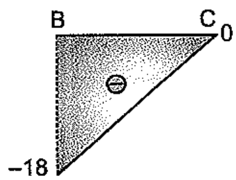
$$9 - 4x = 0 \Rightarrow x = \left(\frac{9}{4}\right)$$

$$\therefore \quad M|_{max} = 9\times\frac{9}{4} - \frac{4}{2}\times\left(\frac{9}{4}\right)^2 = \left(\frac{81}{8}\right) = 10.125 \text{ t-m}$$

- where BM = 0 $\Rightarrow$ $9x - 2x^2 = 0$ $\Rightarrow$ $x = \dfrac{9}{2} = 4.5\text{m}$
- From x = 0 to x = 9/4, BM slope is (+)ve and decreasing and from x = 9/4 to x = 6, BM slope is (–)ve and increasing

Bending moment for part BC



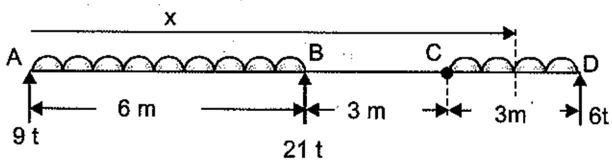


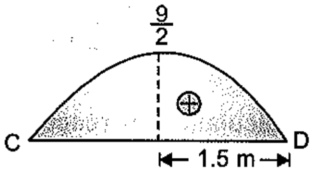
$B.M = 9x + 21(x - 6) - 4 \times 6 \times (x - 3) = 6x - 54 \; ; \; 6 \le x < 9$

at $x = 9$, $BM = 0$

at $x = 6$, $BM = -18$

Bending moment for part CD





$$BM = \frac{-4(12-x)^2}{2} + 6(12-x)$$

$$M = -2(12-x)^2 + 6(12-x) \; ; \quad 9 \le x \le 12$$

at $x = 9$, $M = 0$

at $x = 12$, $M = 0$

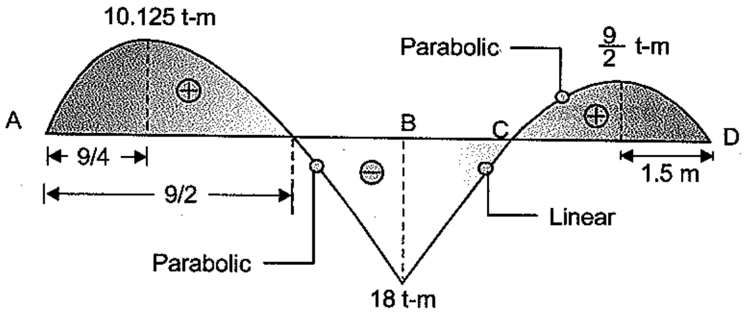
$$\frac{dM}{dx} = +4(12-x) - 6 = 42 - 4x$$

$\frac{dM}{dx} = 0$; at $x = \frac{42}{4} = 10.5$

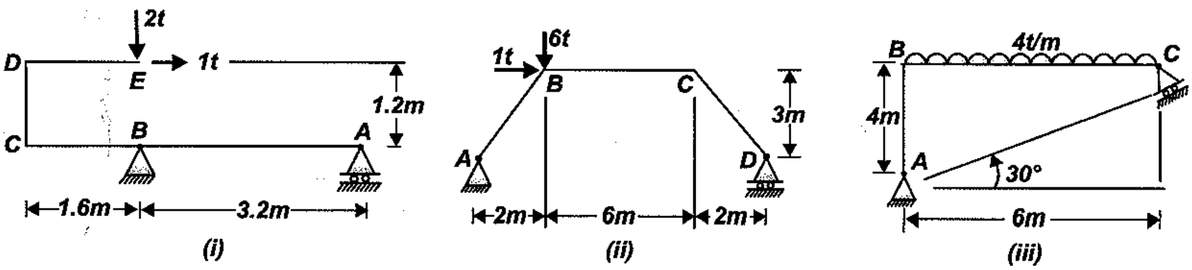
$$M_{max} = -2(1.5)^2 + 6 \times 1.5 = 4.5 \text{ tm}$$

From x = 9 m to x = 10.5 m, Slope of BM is (+)ve and decreasing and from x= 10.5 to 12 m slope of BM is (–)ve and increasing

So, By combining we get,

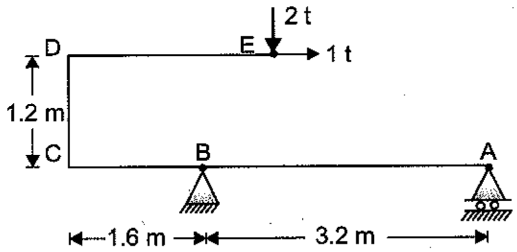


Q-4: ***Analyse the rigid jointed frames shown in figure and draw bending moment diagrams***

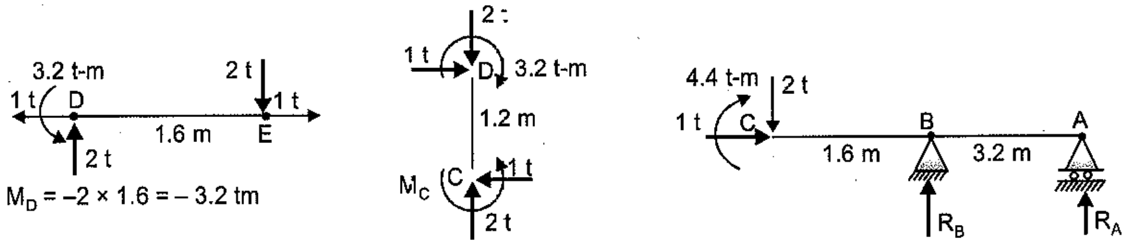


[8 + 8 + 9 = 25 Marks, ESE-1997]

Sol: (i)



Calculation of reaction and drawing FBD for each segment



Calculation for M_c: $M_C = 3.2 \text{ tm} + 1 \times 1.2 = 4.4\text{tm}$

Calculation of unknown reactions (R_A and R_B)

$$\boxed{R_B + R_A = 2t} \quad \text{... (i)}$$

$$\boxed{H_B = 1t} \quad \text{... (ii)}$$

$\Sigma M_B = 0$

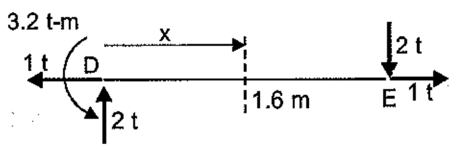
$\therefore \quad R_A \times 3.2 = 4.4 - 2t \times 1.6 \Rightarrow R_A \times 3.2 = (4.4 - 3.2)$

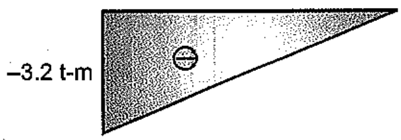
$\Rightarrow \quad R_A = \dfrac{3}{8} t = 0.375\,t$

$\therefore \quad R_B = 2 - \dfrac{3}{8} = \left(\dfrac{13}{8}\right) t = 1.625\,t$

B.M.D. Diagram for each case

For DE

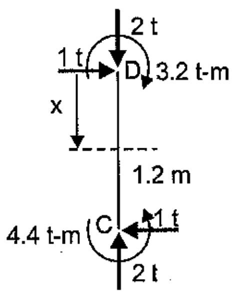


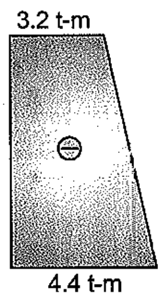


(Sign has been taken by looking at the beam from bottom)

Equation: BM = $-3.2 + 2x$; $0 < x \leq 1.6$

For DC

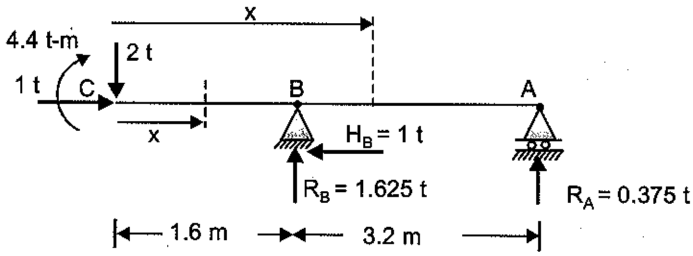


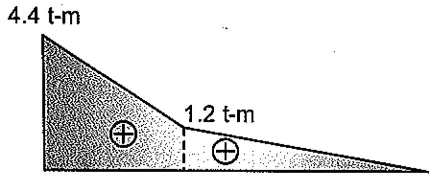


(Sign has been taken by looking at DC from right)

$$B.M.\Big|_{equation} = -(3.2+x)$$

For Beam part (CBA)





(Sign has been taken by looking at the beam from bottom)

For CB part

$$M = (4.4 - 2x)\ ;\ 0 < x \le 1.6$$

$$\therefore \quad M_B = 4.4 - 2 \times 1.6 = 4.4 - 3.2 = 1.2 \text{ t-m}$$

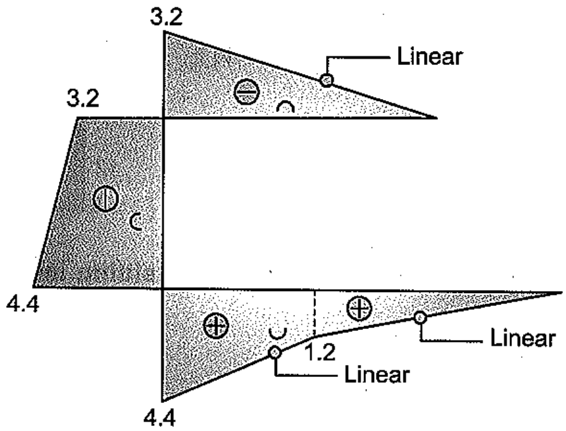
For BA part

$$M = 0.375\,(4.8 - x)\ ;\ 1.6 < x \le 4.8$$

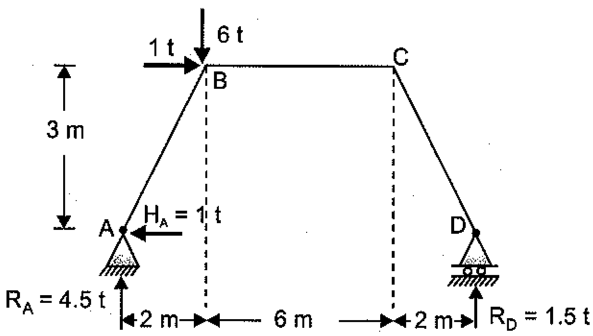
$$M_B = 0.375\,(4.8 - 1.6) = 1.2$$

By combining all segments we get, the combined BMD for all segments

At this stage one should note that the sign given to BM could be different for different position of observer. The purpose of giving sign is to determine on which side there would be tension in the members. To avoid this confusion, we should show the curvature instead of giving sign to BMD. Hence, BMD can be drawn as below.



(ii)



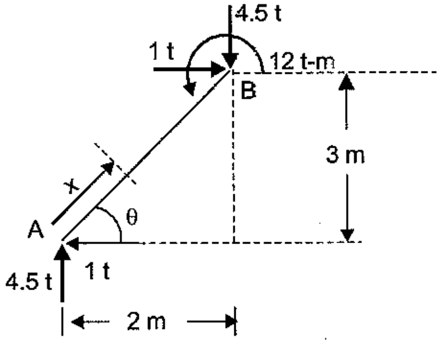
Calculation of Reaction:

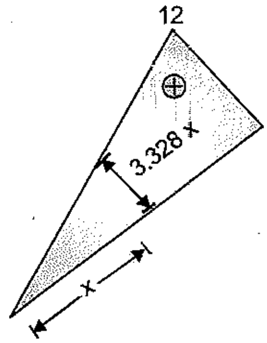
$$\boxed{R_A + R_D = 6} \quad \text{... (i)}$$

$$\boxed{H_A = 1t} \quad \text{... (ii)}$$

$\sum M_D = 0, \quad (R_A \times 10) + (1 \times 3) - (6 \times 8) = 0, \quad \Rightarrow \boxed{R_A = 4.5\text{ t}} \quad \therefore \quad \boxed{R_D = 1.5\text{ t}}$

B.M. for part AB





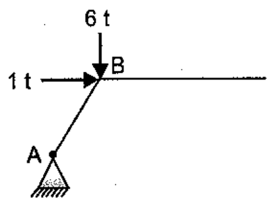
[Sign to BMD is given by looking at BM while sitting inside the frame]

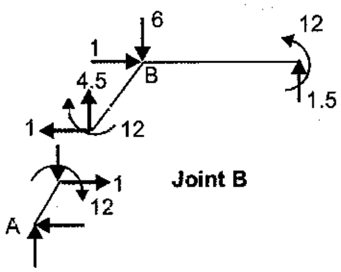
M_B Calculation: $\quad M_B = (4.5 \times 2) + (1 \times 3) = 9 + 3 = 12$ t-m (As shown in figure)

$$M = (4.5x\cos\theta) + (1x\sin\theta) \quad \{M = \text{BM at 'x' distance from A in part AB}\}$$

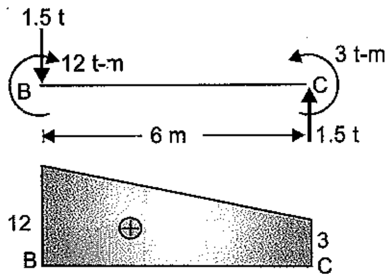
$$= 4.5 \times x \times \frac{2}{\sqrt{13}} + 1 \times x \times \frac{3}{\sqrt{13}} = (3.3282x) \text{ t-m}$$

For part BC





Joint B



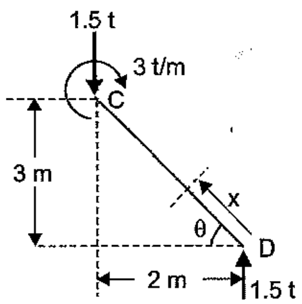
[sign to BMD is given by looking at BM while sitting inside the frame]

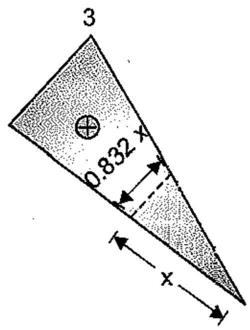
$M_x = +12 - 1.5x; \quad 0 \le x \le 6$

$\Rightarrow \quad M_B = 12, \quad \Rightarrow \quad M_C = 12 - 9 = 3$t-m

M_x shows linear variation with x

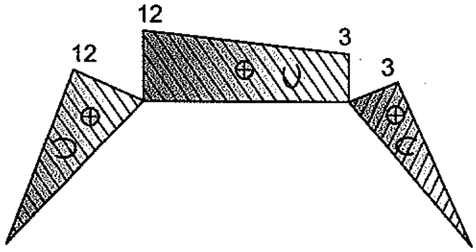
For part CD



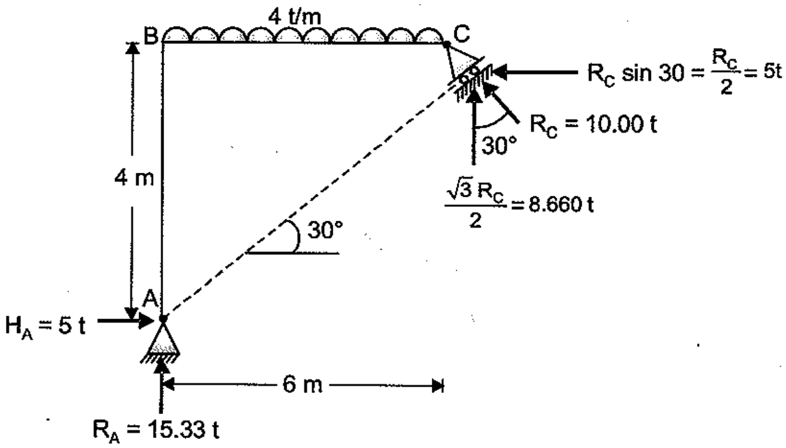


$$\text{B.M}|_{eqh} = 1.5x\cos\theta = \left(\frac{1.5 \times 2x}{\sqrt{13}}\right) = (0.832x)$$

Combining the various parts we have the combined BMD as follows



(iii)



Calculation of Reaction:

$$R_A + \frac{\sqrt{3}}{2}R_C = 24 \quad \text{... (i)}$$

$$H_A = \frac{R_C}{2} \quad \text{... (ii)}$$

$\Sigma M_C = 0$, So, $R_A \times 6 - H_A \times 4 - 4 \times 6 \times 3 = 0$

$$\Rightarrow \quad R_A \times 6 - 4 \times \frac{R_C}{2} = 72$$

$$\Rightarrow \quad \boxed{6R_A - 2R_C = 72} \quad \text{... (iii)}$$

$\therefore$ Solving (i) and (iii) $R_A = 15.34$ t, $\therefore$ $R_C = 10.00$ t and $\Rightarrow$ $H_A = 5.00$ t

Now,

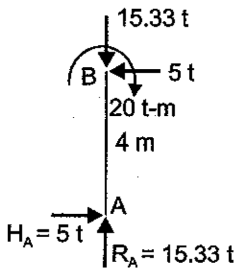
B.M calculation for part AB

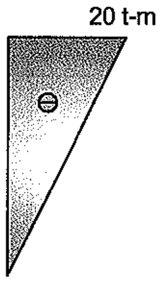
x from A

$$M_x = -5x \; ; \; 0 \le x \le 4$$

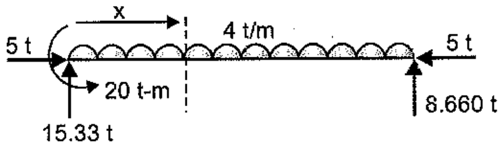
$$\Rightarrow \quad M_A = 0$$

$$M_B = -20$$





B.M. for part BC



So,
$$BM = -20 + 15.33\,x - \frac{4x^2}{2}$$
$$M = -2x^2 + 15.33x - 20$$
$$\frac{dM}{dx} = -4x + 15.33$$
$$\frac{dM}{dx} = 0 \text{ at } x = \frac{15.33}{4} = 3.8325 \text{ m}$$
$$M_{max} = \{-2(3.8325)^2 + 15.33\,(3.8325) - 20\}$$
$$M_{max} = 9.376 \text{ tm}$$

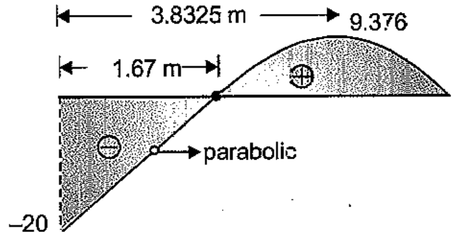
- Upto $x = 3.8325$ m, slope of BMD is (+)ve and decreasing and beyond that it is (–)ve and increasing

  BM will be zero when

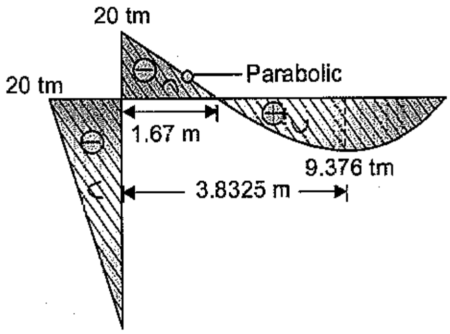
  $-2x^2 + 15.33x - 20 = 0$

  $\Rightarrow \quad x = 1.67$ m

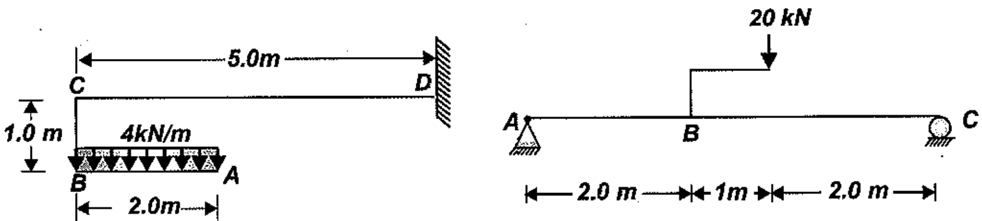
  So,



So, by combining the BMD of all segments we have



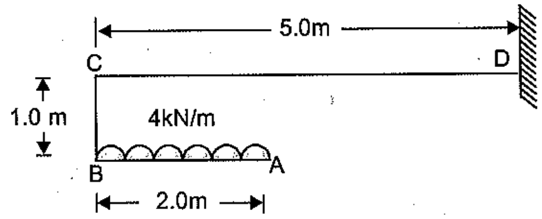
Q-5: ***Draw the bending moment and shearing force diagrams for the members shown in figure.***



(i) *(ii)*

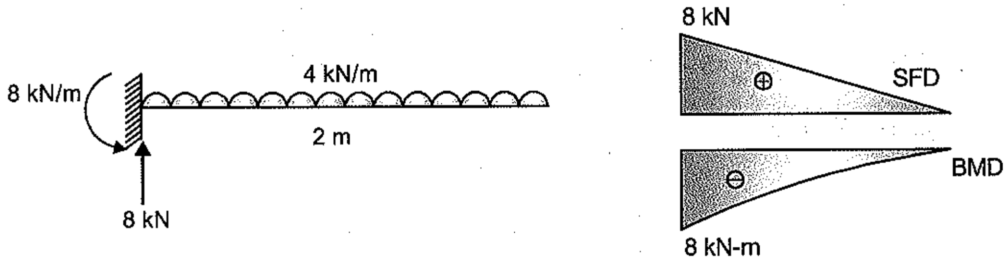
[10 + 10 = 20 Marks, ESE-1998]

Sol: (i)

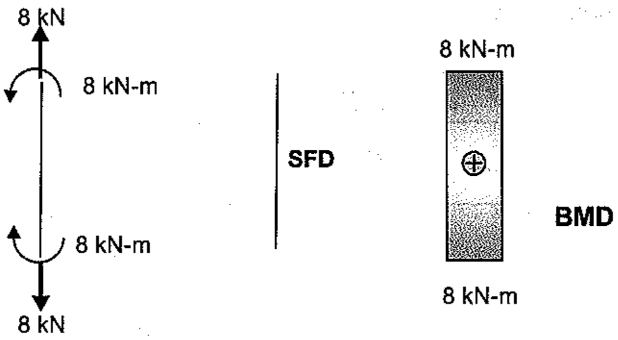


Free Body diagram, SFD and BMD of different segments:

For part BA

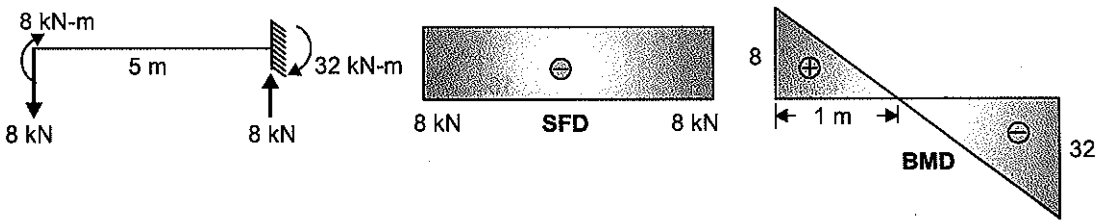


For part CB

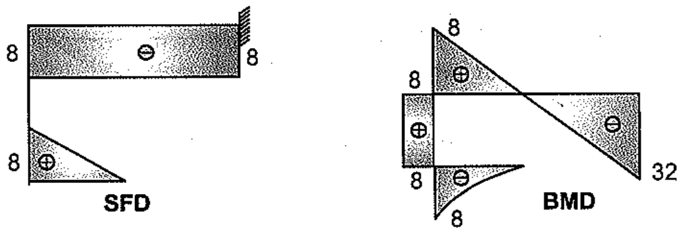


Sign to BMD given by looking at beam from right side

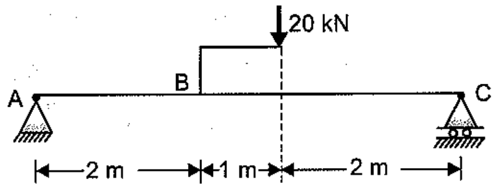
For part CD



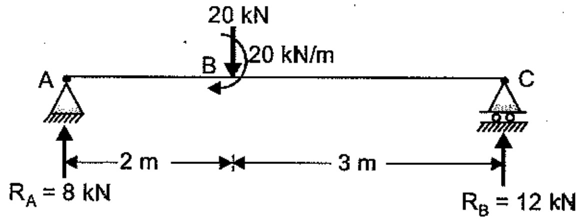
By combining all these three.



(ii)



Eliminating the lever arm from point B and providing equivalent force and B.M. at point B, we have,

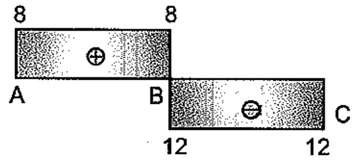


Calculation of reaction

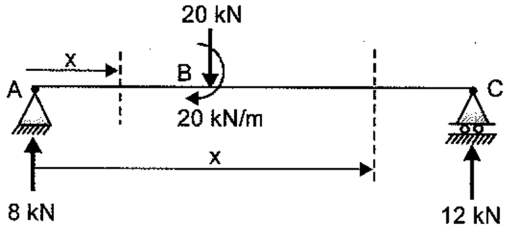
$$R_A + R_B = 20 \text{ kN}$$

$\sum M_A = 0 \Rightarrow R_B \times 5 - (20 \times 2) - 20 = 0 \Rightarrow \boxed{R_B = \frac{20\times3}{5} = 12\text{kN}}$ $\therefore$ $\boxed{R_A = 8\text{ kN}}$

S.F.D. Diagram



Bending moment Diagram

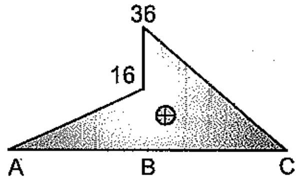


For part AB,

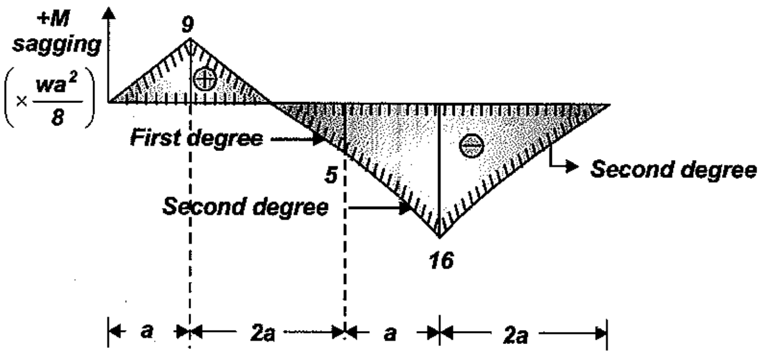
$$B.M.|_{eq} = 8x \text{ where } 0 \le x \le 2 \quad \text{So, } B.M.|_B = 16$$

For part BC,

$$B.M.|_{eq} = B.M.|_B = 36$$



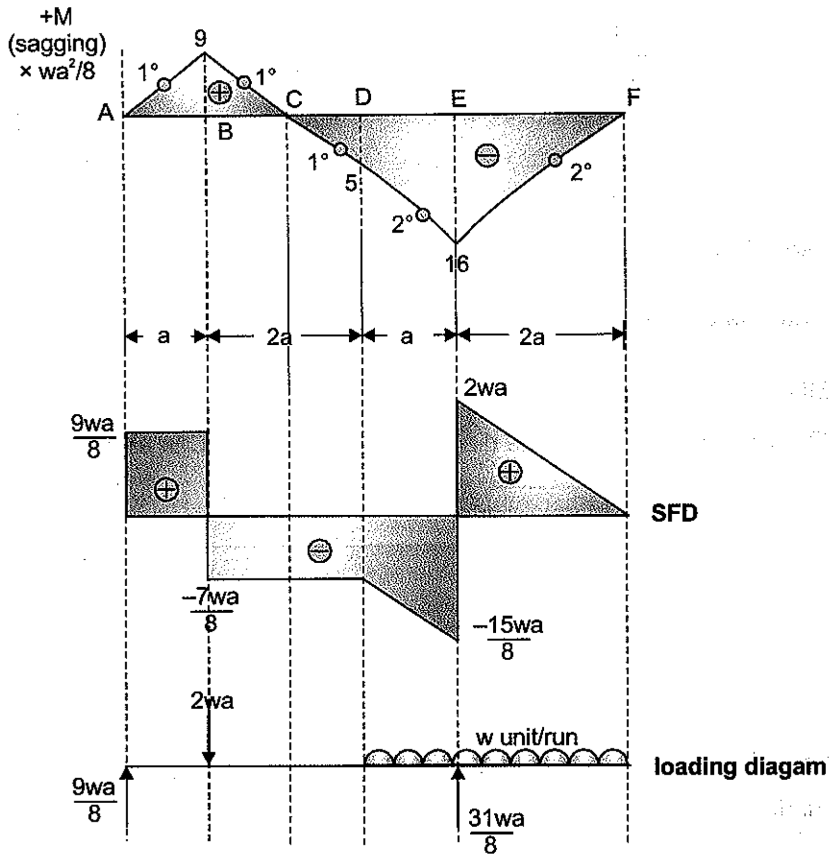
Q-6:



Draw the shear force, load, deflected shape diagrams corresponding to the bending moment diagram given in the above figure. Specify the values at all change of load positions and at all points of maximum moments.

[25 Marks, ESE-2000]

Sol:



Analyzing BMD for different segments

- **Part AB:** AB has +ve and constant slope

  So, shear force between AB is (+)ve and constant and there is no loading between AB.

$$\text{Slope of BMD} = \left(\frac{dM}{dx}\right) = \frac{9wa^2}{8a} = \frac{9wa}{8}$$

$$\text{SF in AD} = \frac{9wa}{8}$$

- **Point B:** At point B sudden change in slope of BMD occurs (i.e., a kink exists in BMD)

  This implies that shear force changes suddenly $\Rightarrow$ there is point load at B

$$\text{Change in shear} = \left(\frac{dM}{dx}\right)_{\text{final}} - \left(\frac{dM}{dx}\right)_{\text{initial}}$$

$$\left(\frac{dM}{dx}\right)_{\text{final}} = \frac{\frac{-5wa^2}{8} - \frac{9wa^2}{8}}{2a} = \frac{-7wa}{8}, \left(\frac{dM}{dx}\right)_{\text{initial}} = \frac{9wa}{8}$$

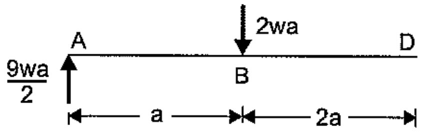
So,
$$\Delta V = \frac{-7wa}{8} - \frac{9wa}{8} = -2wa$$

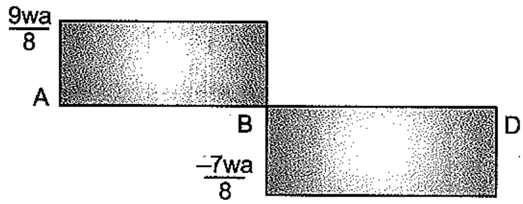
This means that a point load of 2wa acts downward at point B.

- **For Part BCD**

$$\text{BMD slope} = \text{constant} = \frac{-7wa}{8}$$

$$\Rightarrow \quad \text{SF} = \frac{-7wa}{8}$$



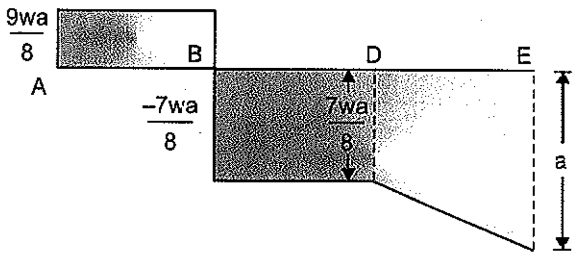


- **For Part DE:** Slope of BMD is (–ve) and increasing.

So, shear force is also (–ve) and increasing $\left[\because \dfrac{dM}{dx} = V\right]$

as BMD is parabolic, hence SF is linear

⇒ The variation of shear force diagram between D and C is as shown follow



Suppose SFD gives the ordinate α at Point E

Area under the S.F.D. between points D and E is

$$= -\left[\frac{\frac{7wa}{8}+\alpha}{2} \times a\right]$$

Since we know that, $$\frac{dM}{dx} = V$$

$$\therefore \quad \int_{M_D}^{M_E} dM = \int_D^E V dx$$

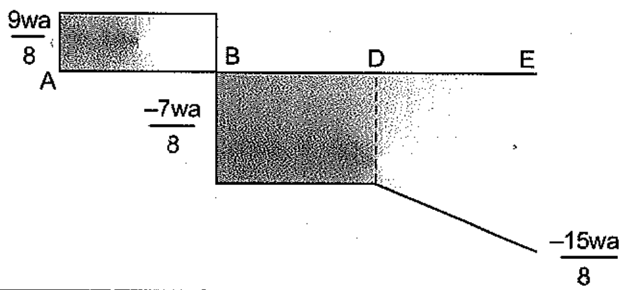
$$M_E - M_D = \frac{-7wa^2}{16} - \frac{1}{2}\alpha a$$

$$\Rightarrow \quad \frac{-16wa^2}{8} + \frac{5wa^2}{8} = \frac{-7wa^2}{16} - \frac{1}{2}\alpha a$$

$$\Rightarrow \quad \frac{1}{2}\alpha a = \frac{-7wa^2}{16} + \frac{32wa^2}{16} - \frac{10wa^2}{16}$$

$$\Rightarrow \quad \frac{1}{2}\alpha a = \frac{15wa^2}{16} \quad \therefore \quad \alpha = \frac{15wa}{8}$$

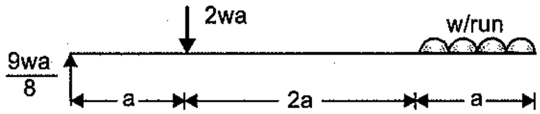
So, the S.F.D. looks like



Since $\left(\dfrac{dV}{dx}\right) = w$ So, $\dfrac{\frac{15wa}{8}+\frac{7wa}{8}}{a} = w \Rightarrow \dfrac{-wa}{a} = -w$

Means a uniformly distributed downward load act between points B and E of w unit/unit run

So, loading diagram upto point E is as shown below



For Point E: Again there is a kink at point E $\Rightarrow$ point load exist

$$(\text{Slope})^{BMD}_{\text{at point F}} = +ve, (\text{Slope})^{BMD}_{E} = (-ve)$$

So, change in shear $= \left.\dfrac{dM}{dx}\right|_F - \left.\dfrac{dM}{dx}\right|_E = (+ve)$

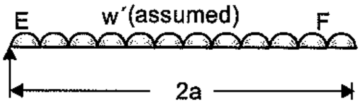
$\Rightarrow$ There is an upward point load at E

But the problem at this point is that we can't determine the slope $\left.\dfrac{dM}{dx}\right|_F$ and $\left.\dfrac{dM}{dx}\right|_E$ from the given B.M.D.

So, we assume that the slope of B.M.D. at F = 0

$\Rightarrow$ $\dfrac{dM}{dx} = V = 0$ at F $\Rightarrow$ No load at F.

and as there is 2nd degree curve between E and F, there will be udl in EF.



Bending moment should be equal from either side.

So, $M_E = \dfrac{-w'\times(2a)^2}{2} = \dfrac{-16w'a^2}{8}$

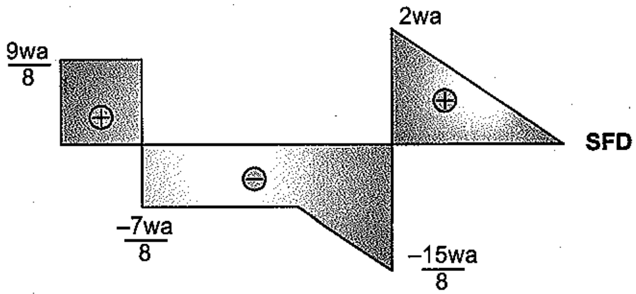
$\Rightarrow$ $w' = w$

and shear force at $E = w' \times 2a = 2wa$

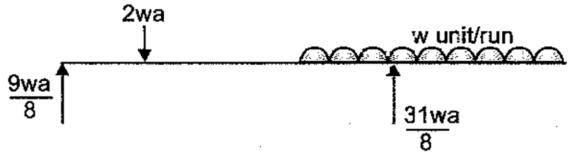
[Note:- Right side downward shear force is +ve]

$$\text{Change in SF} = \text{point load} = 2wa - \left(-\frac{15wa}{8}\right) = \frac{31wa}{8}$$

So, S.F.D. becomes



and loading diagram becomes



Check for the moment equilibrium:

$$\Sigma M_E = \frac{9wa}{8} \times 6a - 2wa \times 5a - 3wa \times 1.5a + \frac{31wa}{8} \times 2a = 0$$

$\Rightarrow$ Beam is in moment equilibrium also.

Conceptual background

As loading diagram has been drawn from S.F.D., the force equilibrium will be satisfied. But we have to check for the moment equilibrium. As we know if there is no moment equilibrium, a couple will exist in the span or at the end. Since a couple leads to a jump in a BMD which is not shown in the BMD of problem so we may avoid checking the moment equilibrium in this problem.

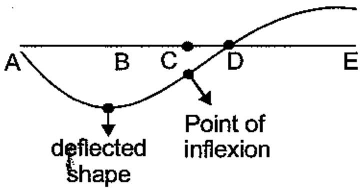
Determining the deflected shape

Since $$M = \frac{EId^2y}{dx^2}$$

its mean that if M is (+ve) $\Rightarrow \frac{d^2y}{dx^2} > 0$ So, shape will be $\cup$

if $M = 0$ $\frac{d^2y}{dx^2} = 0$ So, point of inflection

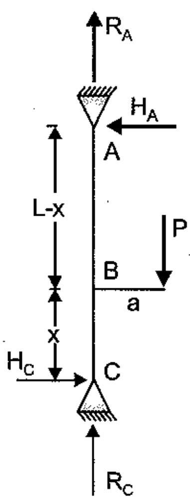
if $M < 0$ $\frac{d^2y}{dx^2} < 0$ So, shape will be $\cap$

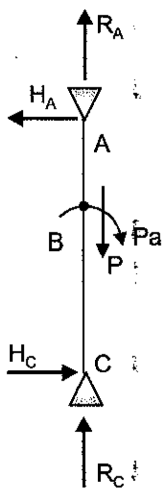


Q-7: ***A column of length L, with the ends fixed in position but free to rotate, is provided with a bracket which carries a vertical load 'p' at a distance 'a' from the axis of the column. Due to improper connection, the bracket slides down. Draw thrust, bending and shear force diagrams when the bracket is at a height 'x' from the base.***

[10 Marks, ESE-2002]

Sol:





$$\Sigma F_y = 0 \quad \therefore \quad \boxed{R_A + R_C = P} \quad \text{... (i)}$$

$$\Sigma F_x = 0 \quad \therefore \quad \boxed{H_A = H_C} \quad \text{... (ii)}$$

$$\Sigma M_A = 0 \quad \therefore \quad \boxed{H_C \times L = Pa} \quad \text{... (iii)}$$

Compatibility

$$\Delta_{AB} + \Delta_{BC} = 0$$

$$\Rightarrow \quad \frac{R_A(L-x)}{A \times E} - \frac{R_C x}{AE} = 0$$

$$\Rightarrow \quad \boxed{R_A(L-x) = R_C x} \quad \text{... (iv)}$$

We have four equation and four unknowns, by solving then we get

$$R_A = \frac{Px}{L}; \qquad R_C = P\left(\frac{L-x}{L}\right); \qquad H_A = H_C = \frac{Pa}{L}$$

Axial Thrust Diagram

For part AB $\rightarrow$ tensile force = $\frac{P(x)}{L} \rightarrow (+ve)$

For part BC $\rightarrow$ compressive force = $P\left(\frac{L-x}{L}\right) \rightarrow (-ve)$

Shear Force Diagram

Shear force is constant throughout the column = $\frac{Pa}{L}$

Bending Moment Diagram

In part AB, $\quad M = \frac{Pa}{L} \times z \quad$ where $0 < z < L - x$

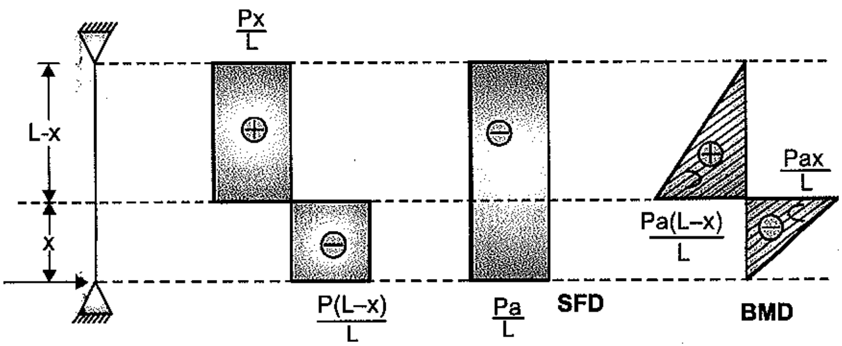
So, $\quad M|_B = \frac{Pa}{L} \times (L-x)$

And, for part BC, (z is taken from point C)

$$M = \frac{-PaZ}{L} \quad 0 < z < x \quad \text{[sign to BMD given by looking from right]}$$

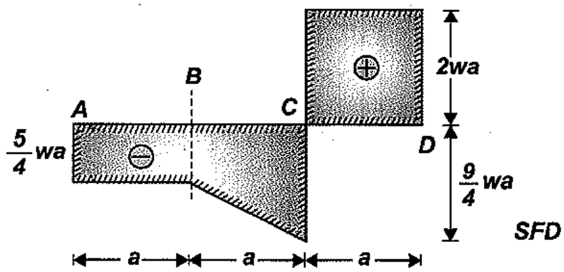
$$\therefore \quad M|_B = \frac{-Pax}{L}$$

So, The various Diagrams are as follows:



Axial thrust diagram

Q-8:



For the shear force diagram of a loaded beam shown in the above figure, draw the corresponding load and bending moment diagrams.

[20 Marks, ESE-2002]

Sol: **Analysis of Loading Diagram**

At A: SF is $-\frac{5Wa}{4}$ $\Rightarrow$ Downward loading at A equal to $\frac{5Wa}{4}$.

A to B: SF = constant. $\Rightarrow$ No load intensity in AB.

B to C: $\frac{dV}{dx} = -\frac{\frac{-9Wa}{4}-\left(-\frac{5Wa}{4}\right)}{a} = -W$ $\Rightarrow$ Downward udl of W.

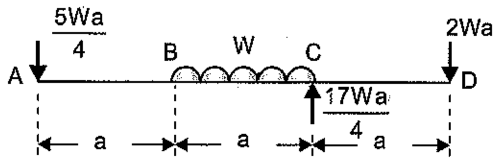
At C: Sudden jump up of SFD by

$$2Wa - \left(-\frac{9Wa}{4}\right) = \frac{17Wa}{4}$$

C to D: SFD = constant $\Rightarrow$ No loading in CD

At D: SFD falls down by 2Wa $\Rightarrow$ Point load downward by 2Wa at D.

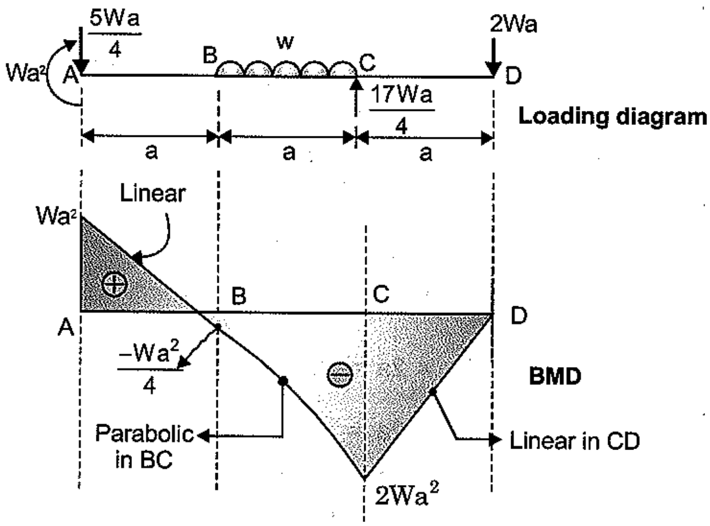
Based as above analysis, probable loading diagram is



Loading derived from SFD always satisfies the force equilibrium but moment equilibrium may not be satisfied.

$$\Rightarrow \quad \Sigma M_A) = (2Wa \times 3a) - \left(\frac{17\,Wa}{4}\times 2a\right) + \left(Wa\times\frac{3a}{2}\right) = -Wa^2 \neq 0$$

To make $\Sigma M_A) = 0$ there must be a concentrated moment some where in AD of clockwise nature and of magnitude Wa^2. Let the concentrated moment be applied at A. Hence the possible loading diagram and the corresponding BMD are



Justification

BMD

At A: BM is sagging (+) = Wa^2

A to B: $\dfrac{dM}{dx} = SF = -\dfrac{5Wa}{4}$ $\Rightarrow M_B = Wa^2 - \dfrac{5Wa}{4} \times a = -\dfrac{Wa^2}{4}$

B to C: $\dfrac{dM}{dx}$ = (–)ve and increasing

$M_C - M_B$ = Area under SFD between C and B.

$$M_C - \left(-\frac{Wa^2}{4}\right) = \frac{-\frac{9Wa}{4} - \frac{5Wa}{4}}{2} \times a$$

$$M_C + \frac{Wa^2}{4} = -\frac{14Wa^2}{8}$$

$$M_C = -\frac{16Wa^2}{8} = -2Wa^2$$

$M_C = -2Wa^2$

Since loading is udl BMD will be parabolic

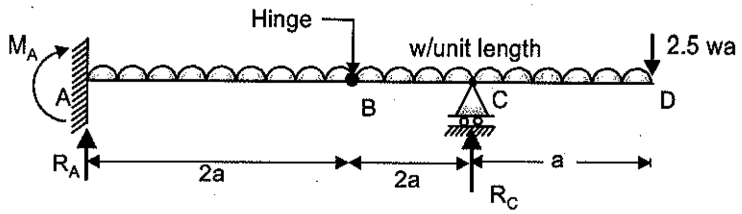
C to D: $\dfrac{dM}{dx} = 2Wa = SF$ = constant $\Rightarrow$ Slope is BMD is (+)ve & constant

At D: BM = 0

Q-9: ***A continuous beam ABCD of total length '5a' is fixed at A and has a simple support at the point C distance '4a' from A. The portion CD of length 'a' is an overhang. The beam carries uniformly distributed load w per unit length throughout its length and a concentrated load of magnitude 2.5 wa at the free end D. Draw the shear force and bending moment diagrams, if there is a hinge at the mid-span point B of the span AC. Clearly mark the position of points of contraflexure.***

[10 Marks, ESE-2003]

Sol:



Finding reactions

$$R_A + R_C = 5wa+2.5wa=7.5wa$$

i.e., $$R_A + R_C = 7.5\ wa \quad \text{... (i)}$$

$M_B = 0$ for forces on the left side of internal hinge

$$\Rightarrow \quad M_A + R_A \times 2a - \frac{w\times(2a)^2}{2} = 0$$

$$\Rightarrow \quad M_A + 2R_A a = 2wa^2 \quad \text{... (ii)}$$

from right hand side, $$R_C \times 2a = (2.5wa\times 3a)+\frac{w\times(3a)^2}{2}$$

$$R_C \times 2a = 7.5wa^2+4.5wa^2$$

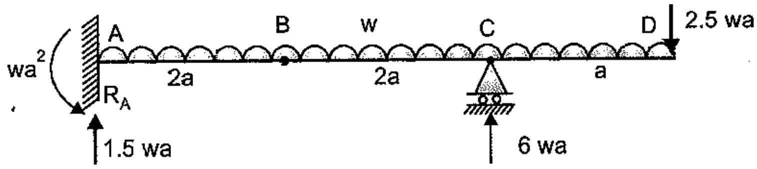
$$R_C \times 2a = 12wa^2$$

$$\Rightarrow \quad \boxed{R_C = 6wa} \quad \text{... (iii)}$$

By putting the value of (iii) into equation (i) $\boxed{R_A = 1.5\ wa}$

$$\therefore \quad M_A = 2wa^2-2\times1.5wa\times a=2wa^2-3wa^2=-wa^2 \quad \text{... from (ii)}$$

Hence the loading diagram with their support reactions is as show below



Shear force and Bending moment diagram

For Part AC

S.F. $\boxed{\text{Shear force }(V) = 1.5wa - wx}$ [Variation linear with slope -w]

So, $$V|_B = 1.5wa-w\times 2a=-0.5wa$$

$$V|_C = 1.5wa-w\times 4a=-2.5wa$$

Shear force will be zero at point given by

$$1.5wa = wx$$

$$\Rightarrow \quad x = 1.5\ a$$

$$\boxed{M = (1.5wa)x-wa^2-\frac{wx^2}{2}}$$

$$M|_{x=2a} = (1.5wa)2a-wa^2-\frac{w\times(2a)^2}{2} = 3wa^2-wa^2-2wa^2=0$$

$$M|_{x=4a} = (1.5wa)\times 4a-wa^2-w\times\frac{(4a)^2}{2} = 6wa^2-wa^2-8wa^2=-3wa^2$$

And, $$M|_{x=1.5a} = (1.5wa)1.5a-wa^2-\frac{w(1.5a)^2}{2}=0.125wa^2$$

$$\frac{dM}{dx} = 1.5\ wa - wx$$

$$\left.\frac{dM}{dx}\right|_{x=4a} = 1.5wa-w\times 4a=-2.5wa$$

For part CD (x taken from point A)

$$V = 2.5\,wa + w(5a - x) = 7.5\,wa - wx$$

$$V|_C = 7.5\,wa - w(4a) = 3.5\,wa$$

And,

$$M = -\left[2.5\,wa\,(5a - x) + \frac{w\,(5a - x)^2}{2}\right]$$

$$M|_C = -\left[2.5wa\,(5a - 4a) + \frac{w\,(5a - 4a)^2}{2}\right] = -\left[2.5\,wa^2 + \frac{wa^2}{2}\right] = -3wa^2$$

$$\frac{dM}{dx} = -[2.5\,wa\,(-1) + w\,(5a - x)\,(-1)] = 7.5\,wa - wx$$

$$\left.\frac{dM}{dx}\right|_C = 7.5\,wa - 4wa = 3.5\,wa$$

Hence kink formation in B.M.D. at point C.

Finding contraflexure points

$$M = 0 \Rightarrow \quad 1.5\,wax - wa^2 - \frac{wx^2}{2} = 0$$

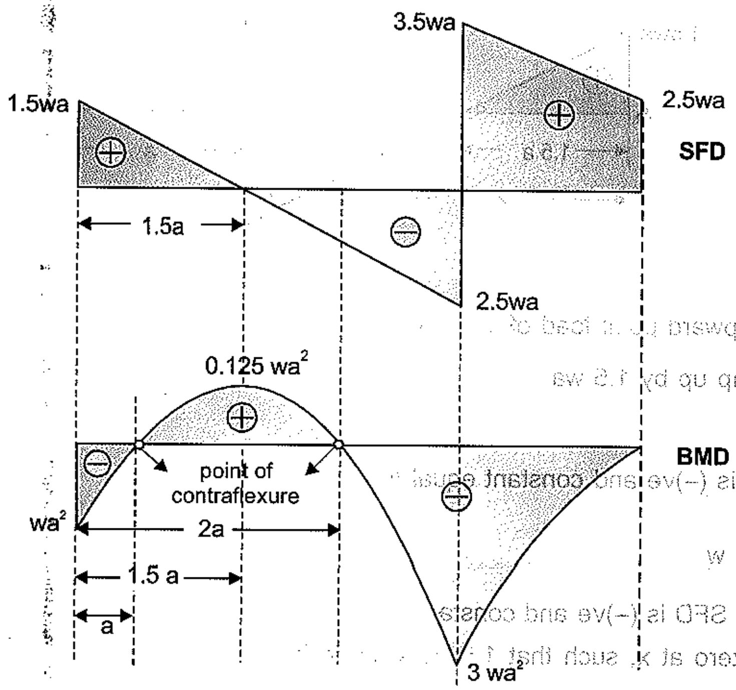
$$\Rightarrow \quad 3\,wax - 2\,wa^2 - wx^2 = 0$$

$$\Rightarrow \quad wx^2 - 3wax + 2wa^2 = 0$$

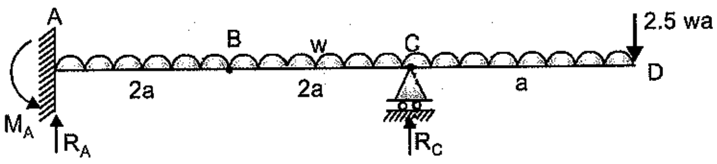
$$\Rightarrow \quad x^2 - 3\,ax + 2a^2 = 0$$

$$x = 2a,\ a$$

Hence the BMD and SFD can be drawn as shown below.



Alternative solution:



From $\Sigma F_V = 0$

$$R_A + R_C = 5wa + 2.5\ wa = 7.5\ wa$$

At hinge B,
$$M_B = 0$$

$\Rightarrow$ $$R_C \times 2a = 2.5\ wa\ (3a) + w \times 3a \times 1.5a$$

$\Rightarrow$ $$R_C = \frac{7.5\ wa^2}{2a} + \frac{4.5\ wa^2}{2a} = \frac{12wa^2}{2a} = 6wa$$

$$\boxed{R_C = 6wa}$$

$\Rightarrow$ $$\boxed{R_A = 1.5\ wa}$$

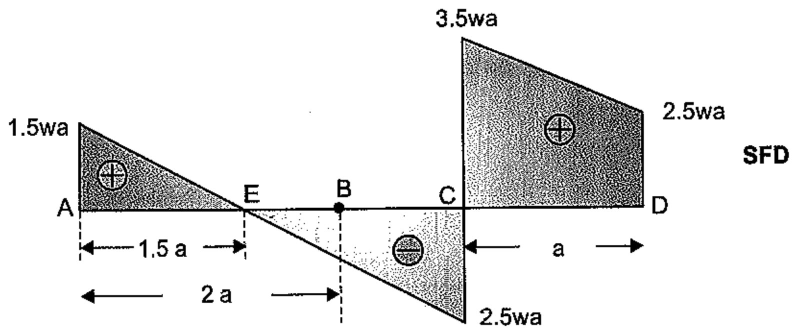
Also $M_B = 0$

$\Rightarrow$ $$-M_A + R_A \times 2a - w \times 2a \times a = 0$$

$\Rightarrow$ $$M_A = 1.5\ wa \times 2a - 2wa^2$$
$$M_A = 3wa^2 - 2wa^2 = wa^2$$

$\Rightarrow$ $$\boxed{M_A = wa^2}$$

Drawing SFD



Description

At A there is upward point load of 1.5 wa

$\Rightarrow$ SFD jump up by 1.5 wa

A to C

Load intensity is (–)ve and constant equal to –w

$\Rightarrow$ $$\frac{dV}{dx} = -w$$

$\Rightarrow$ Slope of SFD is (–)ve and constant

SF becomes zero at x, such that 1.5 wa – wx = 0

$\Rightarrow$ x = 1.5 a

At C

$$V_C - V_A = \text{Area under loading diagram between A and C}$$

$\Rightarrow$ $$V_C - 1.5\ wa = -4wa$$

$\Rightarrow$ $$V_C = -2.5\ wa$$

At C, There is upward point load of 6wa

$\Rightarrow$ SFD jumps up by 6wa

$\Rightarrow$ SF just to the right of C = 6wa – 2.5 wa = 3.5 wa

C to D

$$\frac{dV}{dx} = -w$$

$\Rightarrow$ Slope of SFD is (–)ve and constant equal to -w

$$V_D - V_C = \text{Area under loading diagram between C and D}$$

$$V_D - 3.5\ wa = -w \times a$$

$$V_D = 2.5\ wa$$

At D, there is downward point load of 2.5 wa

$\Rightarrow$ SFD jumps down by 2.5 wa

Drawing BMD

At A there is anticlockwise moment of $M_A = wa^2$

$\Rightarrow$ BMD jumps down by wa^2

A to E

SF, is (+)ve and decreasing

$\Rightarrow$ $\frac{dM}{dx} = v$ is (+)ve and decreasing

as SFD is linear, BMD is parabolic

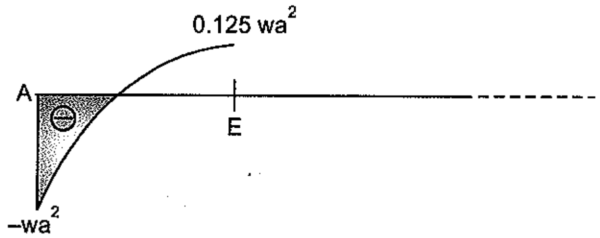
BM is max at E

$$M_E - M_A = \text{Area under SFD between A to E}$$

$\Rightarrow$ $$M_E - (-wa^2) = \frac{1}{2} \times 1.5\ wa \times 1.5\ a$$

$$M_E = \frac{2.25}{2} wa^2 - wa^2 = \frac{0.25\ wa^2}{2}$$

$$\boxed{M_E = 0.125\ wa^2}$$



E to C

SF is (–)ve and increasing

$\Rightarrow \quad \frac{dM}{dx} = V$ is (–) ve and increasing

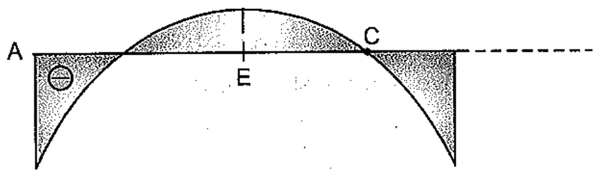
$\Rightarrow$ Slope of BMD is (-ve) and increasing

As SFD is linear, BMD will be parabolic

$$M_C - M_E = \frac{1}{2} \times (-2.5\ wa) \times 2.5\ a = -3.125\ wa^2$$

$$\Rightarrow \quad V_C = -3.125\ wa^2 + 0.125\ wa^2$$

$$M_C = -3wa^2$$



C to D

SF is (+)ve and decreasing,

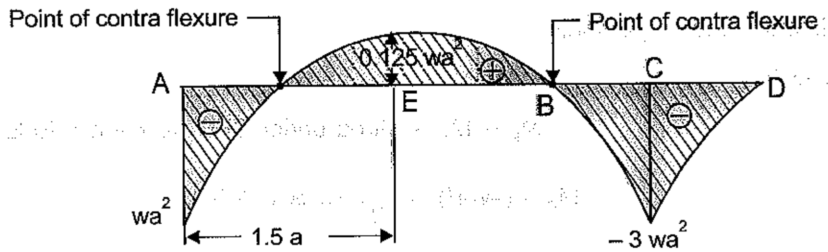
$\Rightarrow \quad \frac{dM}{dx} = V$ is (+)ve and decreasing

$\Rightarrow$ Slope of BMD is (+)ve and decreasing, as SFD is linear, BMD is parabolic

$$M_D - M_E = \frac{3.5\ wa + 2.5\ wa}{2} \times a$$

$$M_D - (-3wa^2) = 3wa^2$$

$$\Rightarrow \quad M_D = 0$$



To locate the point of contraflexure

We have

$$1.5\ wa.x - wa^2 - \frac{wx^2}{2} = 0$$

$$3wax - 2wa^2 - wx^2 = 0$$

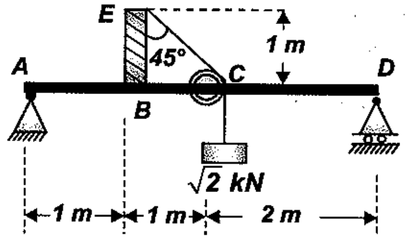
$$x^2 - 3ax + 2a^2 = 0$$

$$(x - a)\ (x - 2a) = 0$$

$$\Rightarrow \quad x = a,\ x = 2a$$

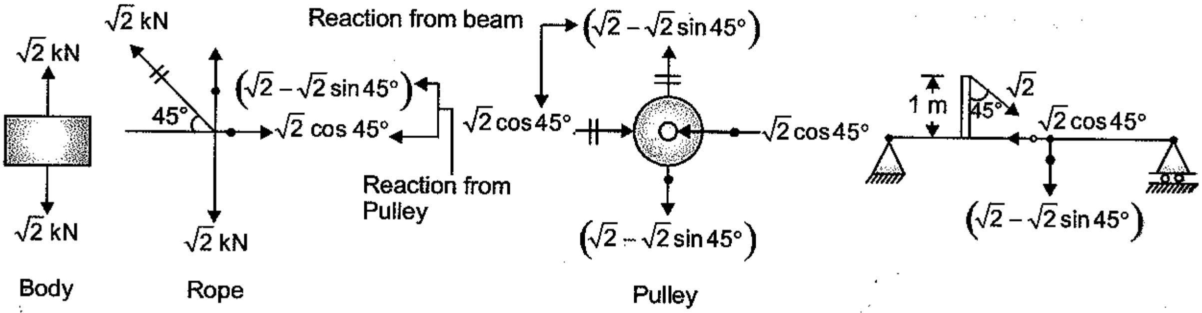
Thus from end A, points of contraflexure are at a distance of a and 2a

Q-10: ***A beam ABCD is hinged at 'A' and simply supported at D. A vertical sturt BE, 1 m long is fixed at B, 1m from A and a frictionless pulley is attached to the beam at 'C', 2 m from A. A flexible string carries a load of $\sqrt{2}$ kN and passes over the pulley and is attached to the strut at 'E' as shown in the fig. Draw BMD, SFD and axial force diagram of beam.***



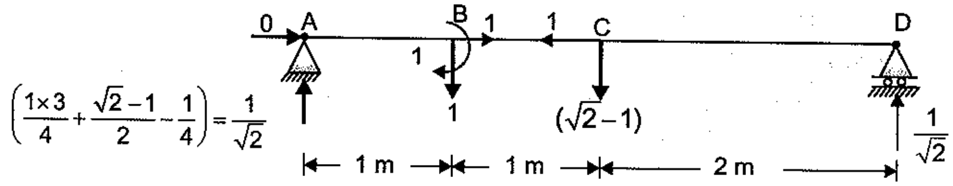
[10 Marks, ESE-2004]

Sol: The free body diagram of the above the structure is



(Reaction on beam should be equal and opposite that from pulley)

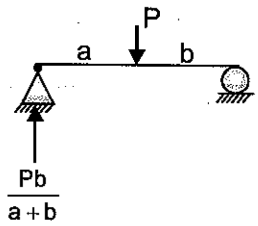
Hence net force on beam is shown below



$$\left(\frac{1\times 3}{4}+\frac{\sqrt{2}-1}{2}-\frac{1}{4}\right)=\frac{1}{\sqrt{2}}$$ $$\frac{1}{\sqrt{2}}$$

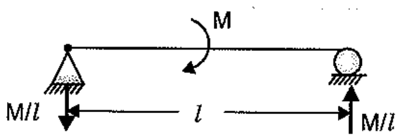
Note: We have taken the effect of individual loads on reactions and by principle of superposition added them to obtain the net reactions.

$1\times\frac{3}{4}$ has been obtained from



$\frac{\sqrt{2}-1}{2}$ has been obtained from symmetry of loading

$\frac{1}{4}$ has been obtained from



Consider part AB

$$SF = V_A = +\ 0.707 \text{ kN}$$

$$BM = 0.707 \times x \text{ (Linear variation)}$$

$$B.M.|_{x=1m} = 0.707$$

and axial thrus = 0

Consider part BC, (x is taken from point A)

$$SF = 0.707 - 1 = -0.293 \text{ kN}$$

$$BM = 0.707x - 1(x - 1) + 1 = 2 - 0.293x$$

Since there is a couple at point *B*, so first the BMD will jump up by an amount 1 kN-m and then follow the above equation

$$B.M.|_{x=1} = 1.707$$

and $$B.M._{x=2} = 1.414 \text{ kNm}$$

and Axial thrust = 1 kN (compressive)

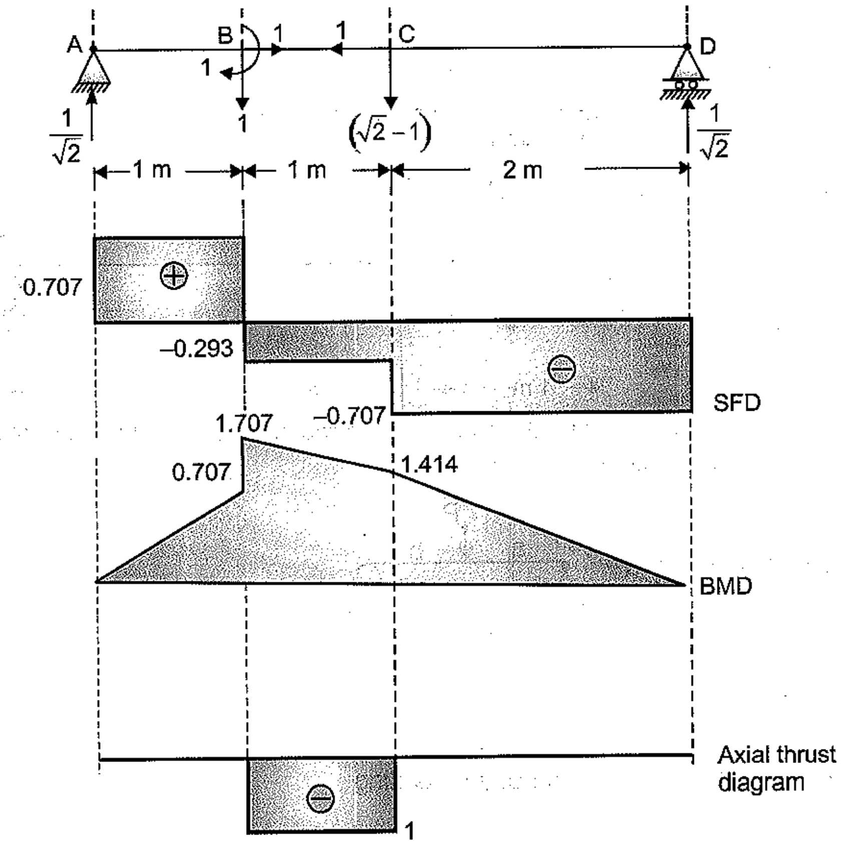
Consider part CD: (*x* is taken from right end A)

$$SF = -0.707 \text{ kN}$$

$$BM = 0.707(4 - x)$$

$$B.M.|_{x=2} = 0.707 \times (4 - 2) = 1.414 \text{ kNm}$$

Axial thrust = 0



Alternatively:

Justification

SFD

- At A $\longrightarrow$ Sudden Rise by $\frac{1}{\sqrt{2}}$ kN due to upward reaction.

- A to B $\longrightarrow$ Constant [No load intensity].
- At B $\longrightarrow$ Sudden drop by 1 kN [due to point load].
- B to C $\longrightarrow$ Constant [No load intensity].
- At C $\longrightarrow$ Sudden drop by $\left(\sqrt{2}-1\right)$ [due to point load].
- C to D $\longrightarrow$ Constant (no load intensity).
- At D $\longrightarrow$ Sudden rise by $\frac{1}{\sqrt{2}}$ [due to upward reaction].

BMD

- At A $\longrightarrow$ BM = 0
- A to B $\longrightarrow$ $\frac{dM}{dx}$ = constant = 0.707 [because SF = 0.707 constant].

From left $M_B - 0$ = Area under SFD between A to B.

$$M_B = 0.707 \times 1 = 0.707 \text{ kNm}$$

Due to concentrated clockwise moment BM jumps up by 1 kNm.

$\Rightarrow$ BM at B to the right of B = 1 + 0.707 = 1.707 kNm

B to C:

$$\frac{dM}{dx} = -0.293 = \text{SF in BC}$$

$$M_C - M_B = -0.293 \times 1 = -0.293$$

$$M_C - 1.707 = -0.293$$

$$M_C = 1.414 \text{ kNm}$$

C to D:

$$\frac{dM}{dx} = -0.707 = \text{SF in CD} \Rightarrow M_D - M_C = -0.707 \times 2 = -1.414$$

$$\Rightarrow \quad M_D - 1.414 = -1.414$$

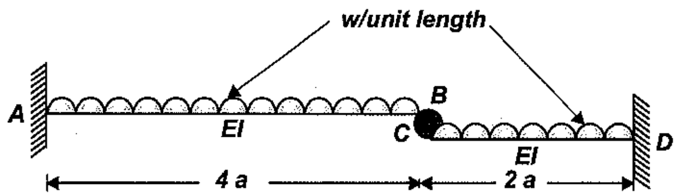
$$\Rightarrow \quad M_D = 0$$

$$\text{BM at D} = 0$$

Axial Force Diagram

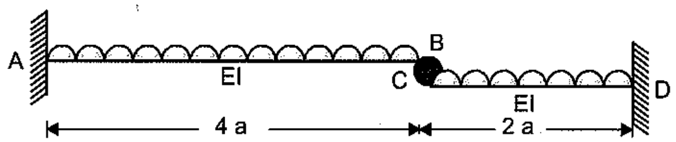
Axial force (compressive) only exists in BC.

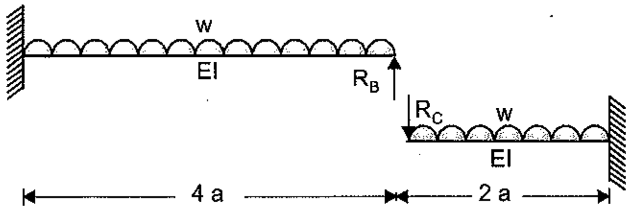
Q-11: ***The free end B of a cantilever AB, '4a' long is supported from below on the free end C of the cantilever CD '2a' long through a roller as shown in figure. The two beams have uniform and equal flexural rigidities and carry a u.d.l. of intensity 'w'/unit length. Draw the B.M. diagram for AB and CD.***



[10 Marks, ESE-2004]

Sol:





Since end B is resting freely on C, there would not be any moment at B or C.

$$R_B = R_C = R \text{ (say)}$$

From compatibility condition

Downward deflection of point B = Downward deflection of point C

$$\text{Downward deflection of point B} = \frac{w\times(4a)^4}{8EI} - \frac{R\times(4a)^3}{3EI}$$

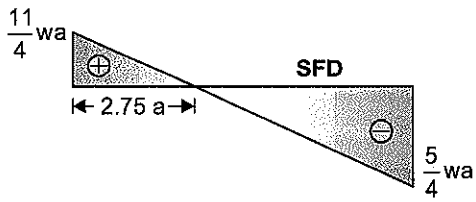
$$\text{Downward Deflection of point C} = \frac{w\times(2a)^4}{8EI} + \frac{R\times(2a)^3}{3EI}$$

$$\therefore \quad \frac{256\,wa^4}{8EI} - \frac{64Ra^3}{3EI} = \frac{w\times16a^4}{8EI} + \frac{R\times8a^3}{3EI}$$

$$\Rightarrow \quad \frac{240a^4w}{8EI} = \frac{72Ra^3}{3EI}$$

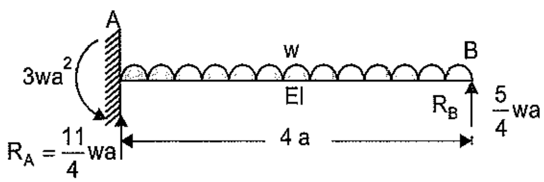
$$\Rightarrow \quad R = \frac{240\times3wa}{8\times72}$$

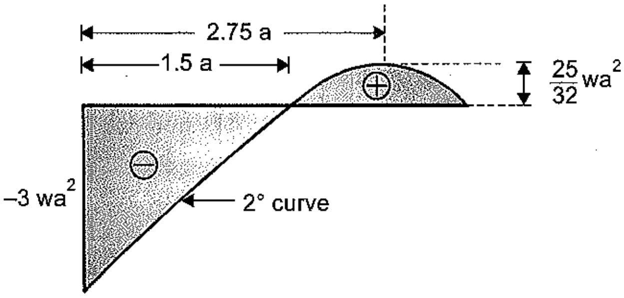
$$\Rightarrow \quad \boxed{R = \frac{5}{4}wa}$$



Now we can draw the B.M.D. for each part

PART AB:





$$R_A = 4\,wa - \frac{5}{4}aw = \frac{11aw}{4}$$

$$\text{Moment about point A} = \frac{5}{4}aw\times4a - w\times4a\times\frac{4a}{2} = -3wa^2$$

Hence B.M.D. equation (x taken from point A)

$$M = -3wa^2 + \frac{11wa}{4}x - \frac{wx^2}{2}$$

$$\frac{dM}{dx} = \frac{11wa}{4} - wx$$

$$\frac{dM}{dx} = 0 \text{ when } x = \frac{11a}{4} = 2.75\,a$$

$$M_{max} = -3wa^2 + \frac{11wa}{4}(2.75\,a) - \frac{w}{2}(2.75a^2) = \frac{25\,wa^2}{32}$$

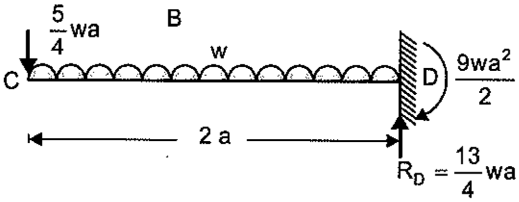
$$M = 0 \text{ is given by}$$

$$-12a^2 + 11ax - 2x^2 = 0$$

$$2x^2 - 11ax + 12a^2 = 0$$

$$x = \frac{11a \pm \sqrt{121a^2 - 96a^2}}{4} = \frac{11a \pm 5a}{4} = 4a, 1.5\,a$$

For part CD, (x is taken from LHS)



$$R_D = \frac{5}{4}wa + 2wa = \frac{13\,wa}{4}$$

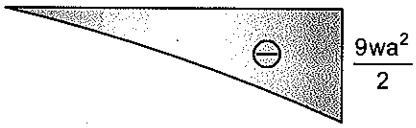
$$M = \frac{-5}{4}wax - \frac{wx^2}{2}$$

$$M|_{2a} = \frac{-5}{4}\times w \times a \times 2a - \frac{w\times(2a)^2}{2} = \frac{-5}{2}wa^2 - 2wa^2 = \frac{-9\,wa^2}{2}$$

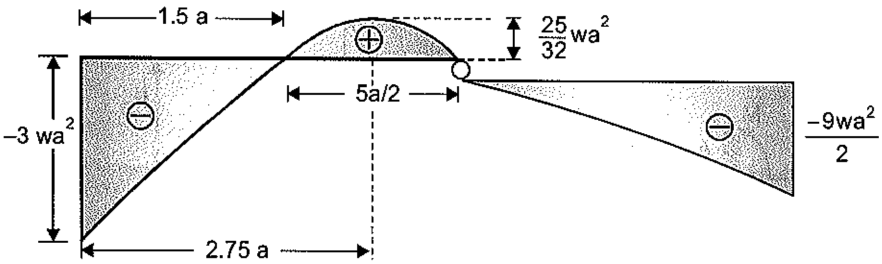
$$M|=0 \text{ when } x=0 \text{ and } x = \frac{-5}{2}a \text{ (not possible)}$$

$$\frac{dM}{dx} = \frac{-5}{4}wa - wx$$

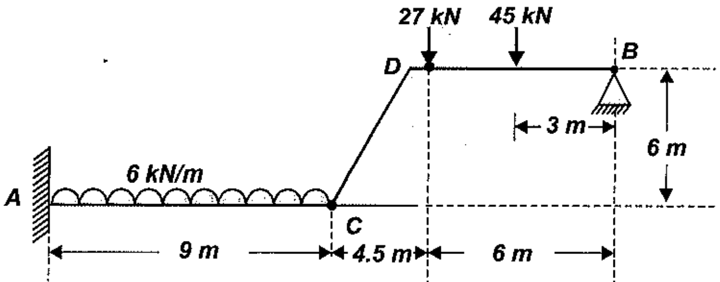
Since, with increase in x, slope of BMD remains (–)ve but increases hence the BMD for part CD will be



Hence, by combining the BMD.

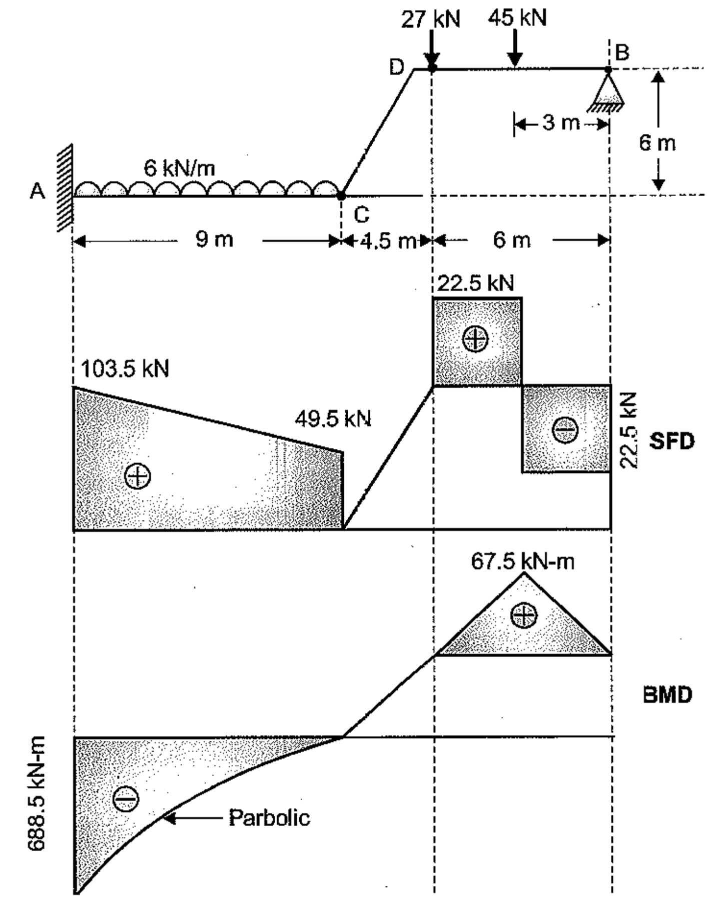


Q-12: ***Determine all the reaction components at suspports A (fixed) and B (hinged). Joints C and D are hinged. Effect of axial forces in various beam parts: AC, CD, DB need not be considered as shown in figure.***

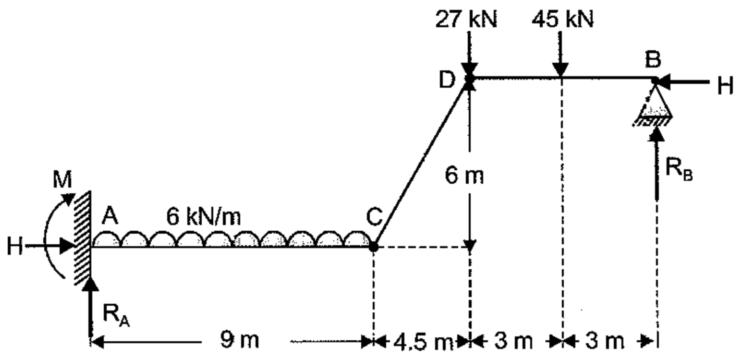


[20 Marks, ESE-2005]

Sol:



Calculation of reactions: R_A and R_B



BM at D = 0 {from right} $\Rightarrow$ $R_B \times 6 = 45 \times 3$

$$\therefore \quad \boxed{R_B = \frac{45 \times 3}{6} = 22.5 \text{kN}}$$

$$\Sigma F_y = 0$$

$$\therefore \quad R_A + R_B = 27 + 45 + 6 \times 9$$

$$\therefore \quad \boxed{R_A = 126 - 22.5 = 103.5 \text{ kN}}$$

Calculation of horizontal reaction

BM at C = 0 (from right)

$$R_B \times 10.5 - 27 \times 4.5 - 45 \times 7.5 + H \times 6 = 0$$

$$\Rightarrow \quad 22.5 \times 10.5 - 27 \times 4.5 - 45 \times 7.5 + H \times 6 = 0$$

$$\boxed{H = 37.125 \text{ kN}}$$

Calculation of moment at point A

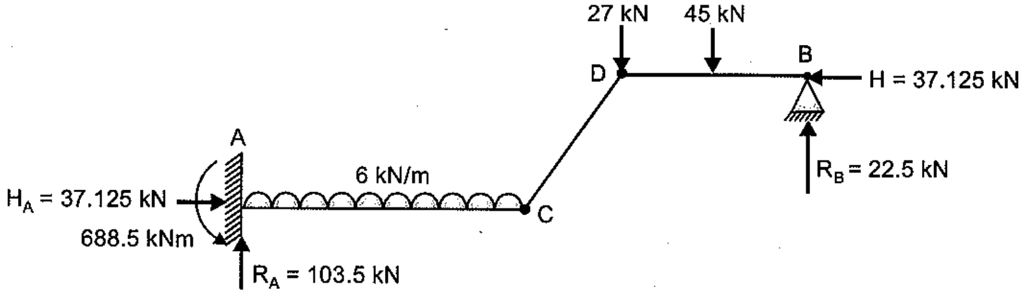
BM at C = 0 (from left)

$$R_A \times 9 - 6 \times \frac{9^2}{2} + M = 0$$

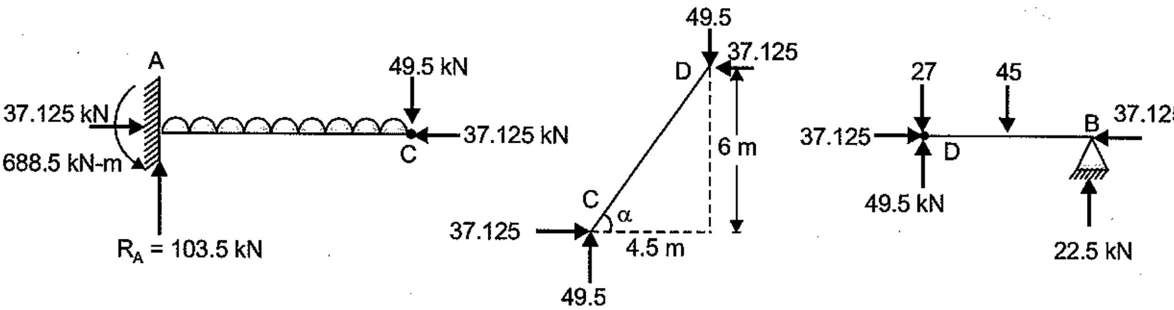
$$\Rightarrow \quad 103.5 \times 9 - 6 \times \frac{9^2}{2} + M = 0$$

$$\Rightarrow \quad M = -688.5 \text{ kN-m}$$ (–)ve sign [mean opposite to assumed direction]

Hence final force diagram becomes



Drawing the F.B.D. for each member, we have

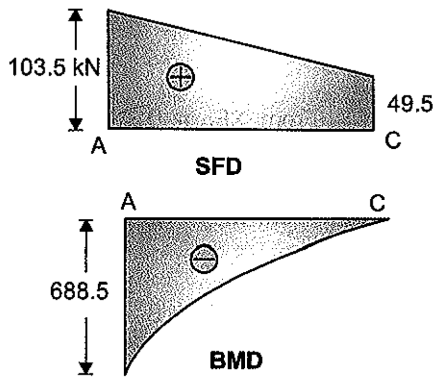


Note: For part CD, the resultant of 37.125 and 49.5 will pass through the rod itself, means the rod CD only transfers the axial force. We can check, the resultant will pass through the rod itself by,

$$\left(\frac{37.125}{49.5}\right) = \frac{3}{4} \quad \text{and} \quad \left(\frac{4.5}{6}\right) = \frac{3}{4}$$

Shear force and Bending moment diagram

Between Part A and C



$$\text{Shear force} = R_A - 6x = 103.5 - 6x$$

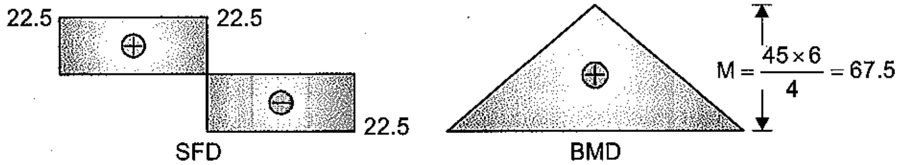
So, $$V|_C = 103.5 - 6 \times 9 = 49.5 \text{ kN}$$

and $$BM = -688.5 + 103.5x - \frac{6x^2}{2}$$

$$B.M.|_C = 0$$

$$\frac{dM}{dx} = V = 103.5 - 6x \text{ i.e., (+ve) \& decreasing}$$

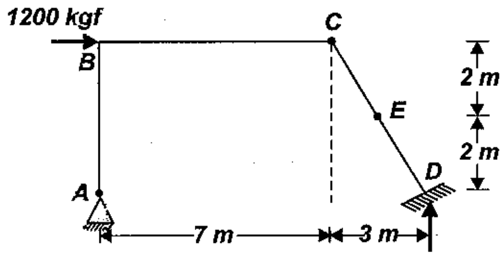
BMD. & SFD of part DB is same as simply supported beam.



$$M = \frac{45\times6}{4} = 67.5$$

By combining the SFD and BMD of different parts we can draw SFD and BMD of whole structure.

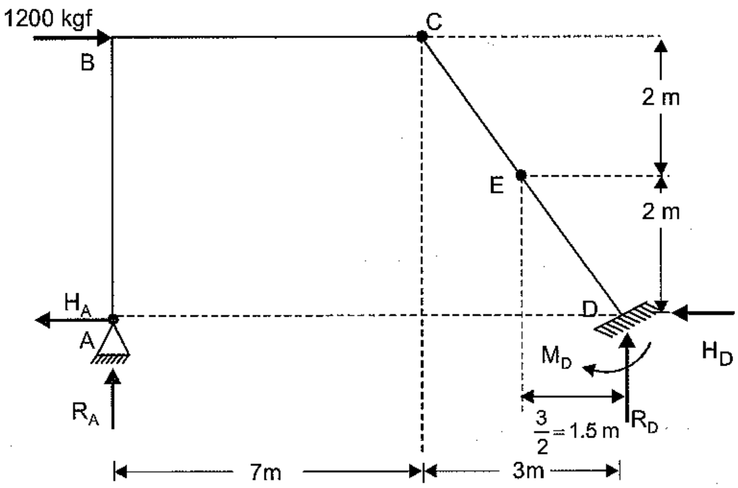
Q-13: ***A portal frame ABCD as shown in the figure has hinged end at A and fixed end at D. Vertical leg AB = 4 m, horizontal BC = 7 m. Joint C and mid point E of inclined leg CD may be acting as hinges A and D are at same level.***



Calculate the components of reactions at A and D when a horizontal force of 1200 kgf is applied at joint B.

[20 Marks, ESE-2007]

Sol:



From $\Sigma F_H = 0$

$$1200 - H_A - H_D = 0 \quad \text{... (i)}$$

From $\Sigma F_V = 0$

$$R_A + R_D = 0 \quad \text{... (ii)}$$

From $\Sigma M_A = 0$

$$1200 \times 4 - R_D \times 10 + M_D = 0 \quad \text{... (iii)}$$

From BM at E = 0 from right

$\Rightarrow \quad H_D \times 2 - R_D \times 1.5 + M_D = 0 \quad$ ... (iv)

From BM at C = 0 (from right) [Sign to BMD given by seeing from right]

$\Rightarrow \quad H_D \times 4 - R_D \times 3 + M_D = 0 \quad$ ... (v)

From (iv) and (v)

$$\boxed{M_D = 0}$$

$\Rightarrow$ From (iii) $\quad \boxed{R_D = 480 \text{ kgf}}\ (\uparrow)$

$\Rightarrow$ From (ii) $\quad \boxed{R_A = -480 \text{ kgf}}\ (\downarrow)$

From (iv) $\quad H_D = \dfrac{1.5\,R_D}{2}$

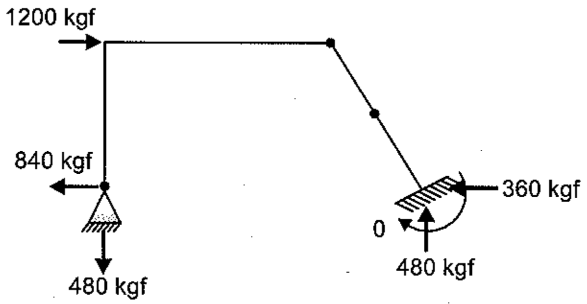
$\Rightarrow \quad H_D = \dfrac{1.5}{2} \times 480 = 360 \text{ kgf}$

$$\boxed{H_D = 360 \text{ kgf}}\ (\leftarrow)$$

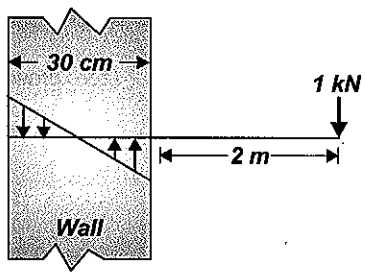
From (i) $\quad H_A = 1200 - 360 = 840$ kfg

$$\boxed{H_A = 840 \text{ kgf}}\ (\leftarrow)$$

Thus the reaction are as shown below.

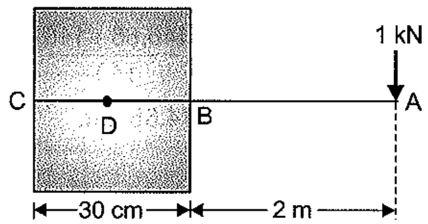


Q-14: ***A cantilever beam of clear span 2 m built into a wall of 30 cm thickness. The beam carries a concentrated load of 1 kN at the free end. Assume that the pressure exerted by the wall on the beam is linearly varying along the 30 cm thickness of the wall into which the beam is embedded. Find the pressure distribution exerted by the wall on the beam.***

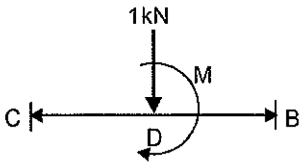


[15 Marks, ESE-2008]

Sol:

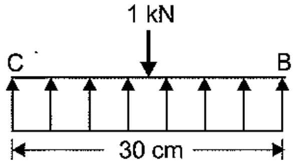


First we transfer the net effect of 1kN force acting at point 'A' to the midpoint 'D' of the part of beam which is built into the wall,

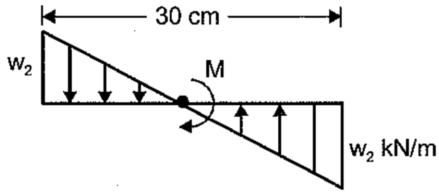


$$M = 1\times\left[2+\frac{30}{2}\times\frac{1}{100}\right] = 2.15\text{kN/m}$$

Since this part of beam (CB) is uniformly supported within the wall. Hence, the wall will develop reaction force to balance the unbalanced force as shown below,



Balancing vertical load
(Fig. A)



Assuming linear variation as per question
(Fig. B)

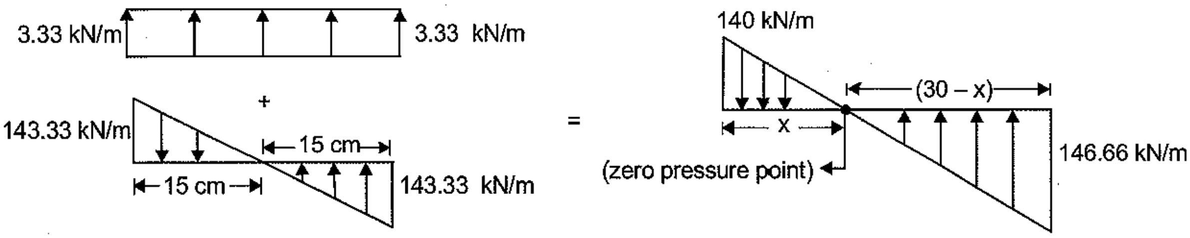
For figure A, $w_1\times 0.30 = 1 \quad \Rightarrow w_1 = \frac{10}{3} = 3.33\text{kN/m}$

For figure B, $2\times\left\{\frac{1}{2}\times w_2\times\frac{l}{2}\right\}\times\frac{2\left(\frac{l}{2}\right)}{3} = 2.15$

$$\Rightarrow \quad \left(\frac{w_2 l}{2}\right)\times\left(\frac{l}{3}\right) = 2.15$$

$$\Rightarrow \quad w_2 l^2 = 2.15\times 6 \Rightarrow w_2 = \frac{2.15\times 6}{0.3^2} = 143.33\text{ kN/m}$$

Hence by superimposing these two free body diagrams we can get the pressure diagram exerted by the wall on the embedded part of the beam.



Finding location of zero pressure point,

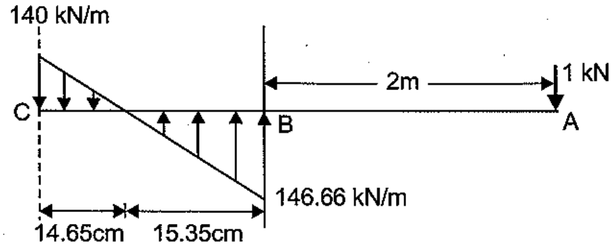
From similar triangles,

$$\frac{140}{x} = \frac{146.66}{30-x} \Rightarrow \frac{30-x}{x} = \frac{146.66}{140}$$

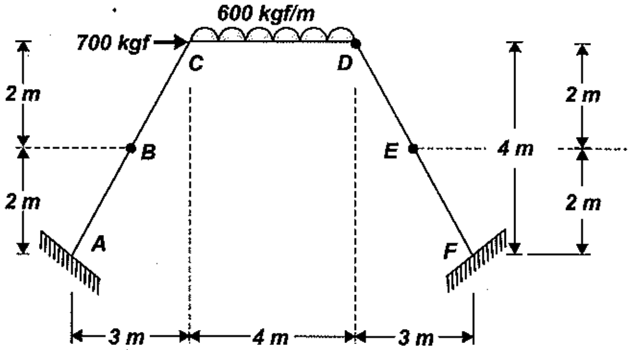
$$\Rightarrow \quad \frac{30}{x} = \frac{146.66+140}{140}$$

$$\Rightarrow \quad x = 14.65\text{ cm.}$$

Hence the final diagram is,



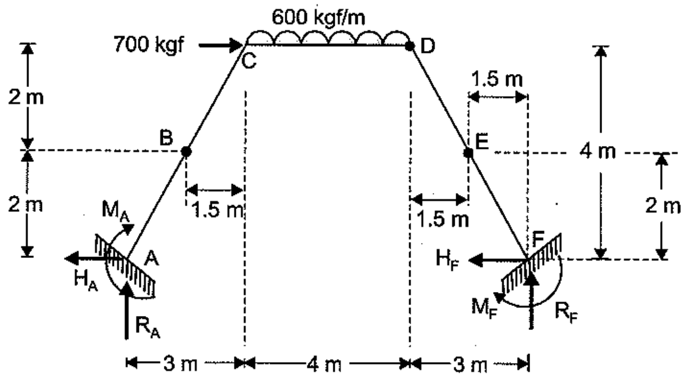
Q-15:



A portal frame ABCDEF with inclined legs has hinges at B, D and E as shown in the above figure. Joint at C is monolithic. Supports at A and F are fixed. Calculate all components of reactions.

[20 Marks, ESE-2009]

Sol:



From $\Sigma F_V = 0$

$$R_A + R_F = 600 \times 4 = 2400 \text{ kgf} \qquad \text{... (i)}$$

From $\Sigma F_H = 0$

$$H_A + H_F = 700 \qquad \text{... (ii)}$$

From $\Sigma M_F = 0$

$$(R_A \times 10) + (M_A) + (H_A \times 0) + (700 \times 4) - (600 \times 4 \times 5) + M_F = 0$$

$$10\ R_A + M_A + M_F = 9200 \qquad \text{... (iii)}$$

From BM at E from right = 0

$\Rightarrow$ $H_F \times 2 - R_F \times 1.5 + M_F = 0$ [sign to BM given by looking from right] ... (iv)

From BM at D from right = 0

$$\Rightarrow \quad H_F \times 4 - R_F \times 3 + M_F = 0 \qquad \text{... (v)}$$

$\Rightarrow$ From (iv) and (v), $\boxed{M_F = 0}$

From BM at B from left = 0

$(H_A \times 2) + (R_A \times 1.5) + (M_A) = 0$ [sign to BM given by looking from right] ... (vi)

Now from (v) $$H_F = \frac{3}{4}R_F$$

From (i) $$H_F = \frac{3}{4}(2400 - R_A)$$

From (ii) $$H_A = 700 - H_F = 700 - \frac{3}{4}(2400 - R_A)$$

From (iii) $$M_A = (9200 - 10\,R_A) \qquad [\because M_F = 0]$$

Now from (vi)

$$2\left[700 - \frac{3}{4}(2400 - R_A)\right] + (R_A \times 1.5) + (9200 - 10\,R_A) = 0$$

$$\Rightarrow \quad 1400 - 3600 + 1.5\,R_A + 1.5\,R_A - 10\,R_A + 9200 = 0$$

$$\Rightarrow \quad \boxed{R_A = 1000\text{ kgf}}$$

$$\Rightarrow \quad \boxed{M_A = 9200 - 10000 = -800\text{ kgfm}}$$

$$H_A = 700 - \frac{3}{4}(2400 - 1000)$$

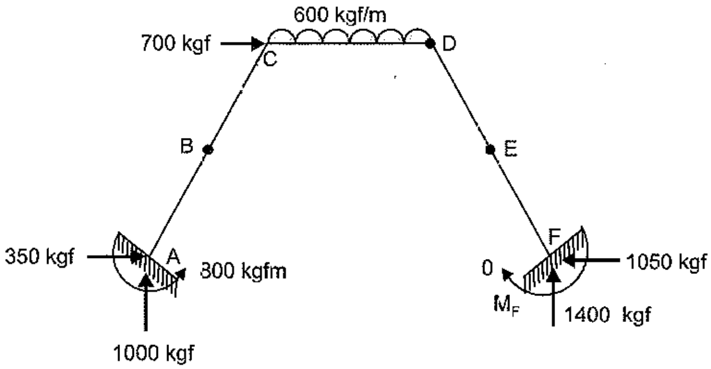
$$\Rightarrow \quad \boxed{H_A = -350\text{ kgf}}$$

$$\Rightarrow \quad \boxed{H_F = \frac{3}{4}(2400 - 1000) = 1050\text{ kfg}}$$

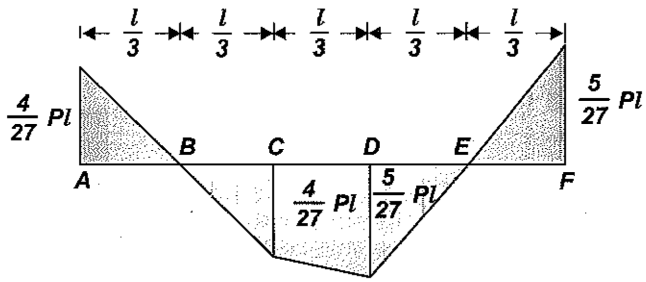
$$\boxed{R_F = \frac{4}{3}H_F = 1400\text{ kgf}}$$

$$\boxed{M_F = 0}$$

Hence the reactions are as shown below.



Q-16: ***Bending moment diagram of a balanced cantilever beam is shown below. Draw the beam and find the loads acting on the beam.***



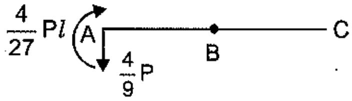
[10 Marks, ESE-2010]

Sol: **At point *A***

BMD has sudden jump, so a clockwise moment exist at point *A* as shown

Between point A and C

BMD is a straight line

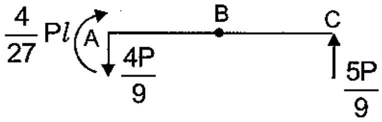


$$\therefore \quad \frac{dM}{dx} = \frac{\left\{-\frac{4}{27}-\frac{4}{27}\right\}Pl}{\frac{2}{3}l} = -\frac{4}{9}P \text{ (constant)}$$

We know that, $\quad V = \frac{dM}{dx} \quad \therefore \; V = -\frac{4}{9}P$

$\Rightarrow$ SF is constant between A and C and at point A, SF (–)ve, this implies that there is (↓) force of 4P/9 at A and no load intensity between A and C.

At point C



$$\left.\frac{dM}{dx}\right|_{\text{L.H.S. of C}} = -\frac{4}{9}P$$

$$\left.\frac{dM}{dx}\right|_{\text{R.H.S. of C}} = \frac{M_D - M_C}{\frac{l}{3}} = \left(\frac{\frac{-5}{27}+\frac{4}{27}}{\frac{l}{3}}\right)Pl = -\left(\frac{P}{9}\right)$$

$\Rightarrow$ SF from left $= \frac{-4P}{9}$ and SF from right $= \frac{-P}{9}$

Hence at C, SF changes from $\frac{-4P}{9}$ to $\frac{-P}{9}$

$\Rightarrow$ There is upward load of $\left(\frac{-P}{9}+\frac{4P}{9}\right) = \frac{3P}{9}$ at C

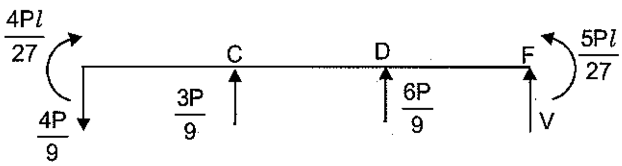
Hence, at point C an upward force exist of magnitude $= \frac{3P}{9}$

Between point C and D

The slope is constant hence, $V = \frac{dM}{dx}$ = constant

Therefore, no loading exist between C and D.

At point D



$$\left.\frac{dM}{dx}\right|_{\text{L.H.S. of D}} = \frac{-P}{9}$$

$$\left.\frac{dM}{dx}\right|_{\text{R.H.S. of D}} = \frac{\left(\frac{5}{27}+\frac{5}{27}\right)Pl}{\frac{2l}{3}} = \frac{5}{9}P$$

$\Rightarrow$ SF change from $\frac{-P}{9}$ to $\frac{5P}{9}$

$\Rightarrow$ There is an upward point load of $\frac{6P}{9}$ at D

Between point D and F

Since $\frac{dM}{dx}$ constant, hence no loading will exist in span DF.

At point F

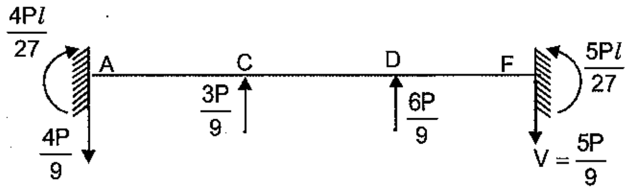
Calculation of vertical force V at point E.

For force equilibrium $V+\frac{6P}{9}+\frac{3P}{9}-\frac{4P}{9}=0$

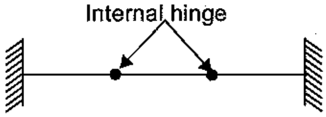
$\because \quad V=-\frac{5P}{9}$

There exists a (+ve) BM at F equal to $\frac{5Pl}{27}$

Hence the loading diagram is as shown.

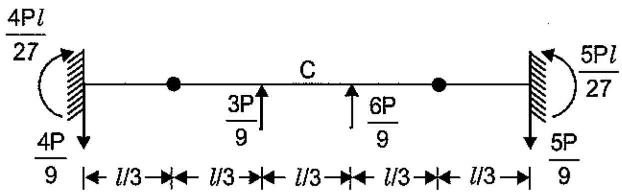


But it is given that the beam is a balanced cantilever beam.

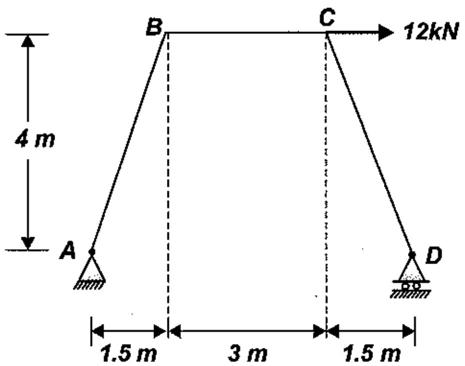


Balanced cantilever beam

As BM is zero at B and E. Hence the final beam with loading on it will look like

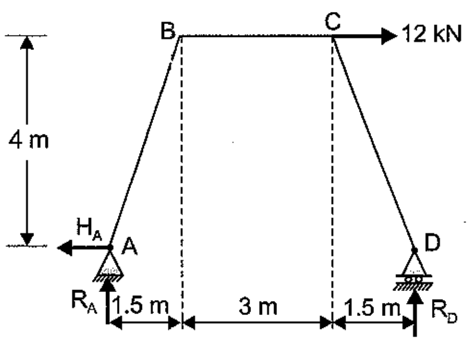


Q-17: ***Find the reactions at supports A and D of the structure shown below. Draw BM diagram indicating BM values at salient points. Supports A and D are at the same level.***



[10 Marks, ESE-2010]

Sol:

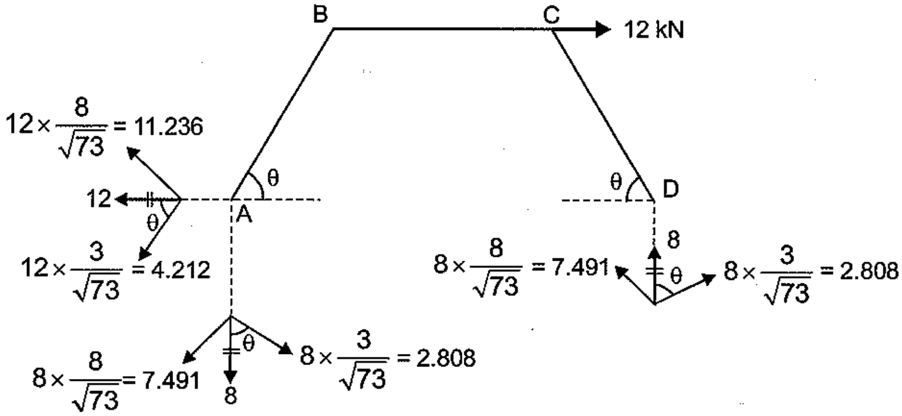


$$\boxed{H_A = 12\text{kN}} \qquad \{\text{from } \Sigma F_x = 0\}$$

$$R_A = -\frac{12 \times 4}{6} \text{ kN}$$

$$\Rightarrow \quad \boxed{R_A = -8 \text{ kN}}$$

$$\boxed{R_D = -R_A = 8 \text{ kN}} \qquad \{\text{from } \Sigma F_y = 0\}$$

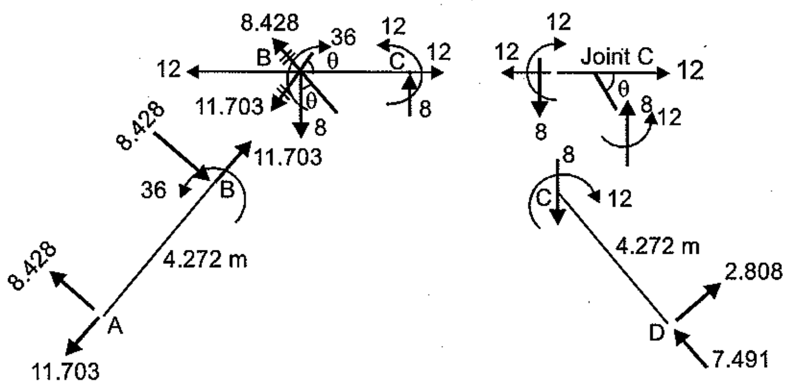


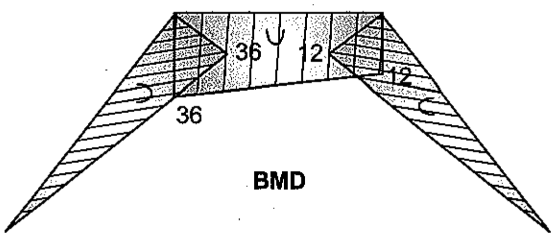
$$\tan\theta = \frac{4}{1.5} = \frac{8}{3}$$

$$\sin\theta = \frac{8}{\sqrt{73}}$$

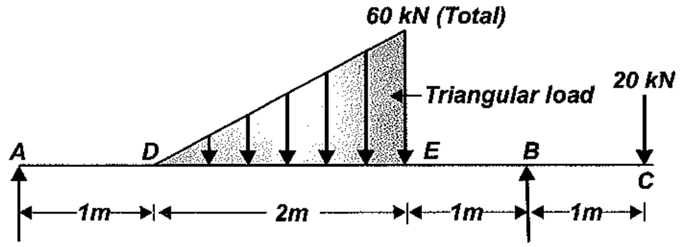
$$\cos\theta = \frac{3}{\sqrt{73}}$$

Hence FBD will look line



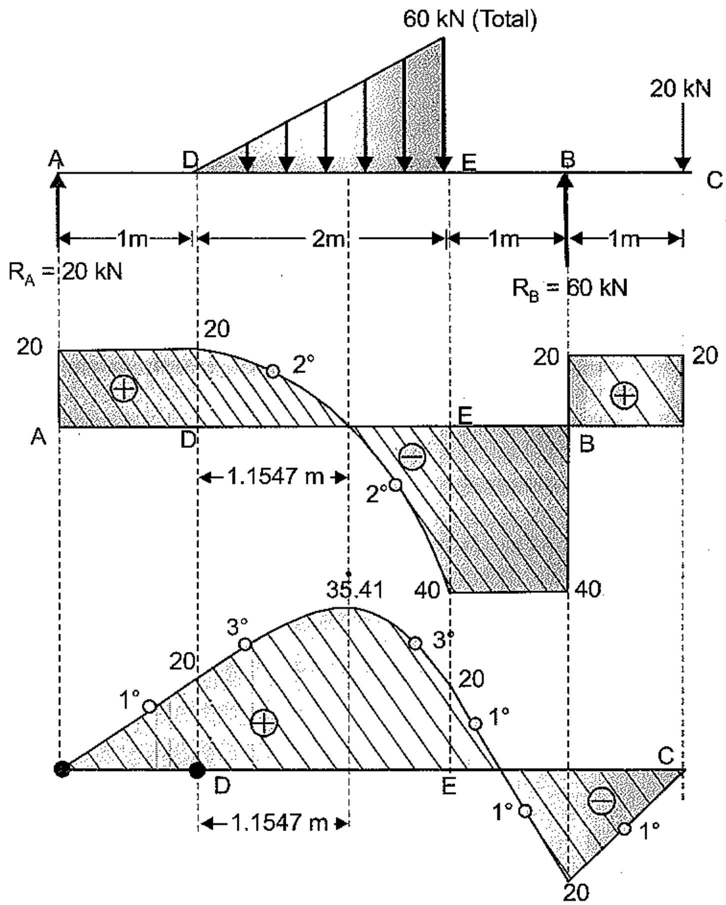


Q-18: ***Draw the bending moment and shear force diagrams for overhanging beam shown in the figure below. Indicate the significant values including the point of contraflexure.***



[10 Marks, ESE-2011]

Sol:



Determination of Reaction

$$R_A + R_B = 60 + 20 = 80 \text{ kN} \quad \text{...(i)}$$

$$\Sigma M_B = 0$$

$$\Rightarrow \quad R_A \times 4 - 60 \times \left(\frac{2}{3} + 1\right) + 20 \times 1 = 0$$

$$\Rightarrow \quad R_A \times 4 \text{ m} = 100 - 20$$

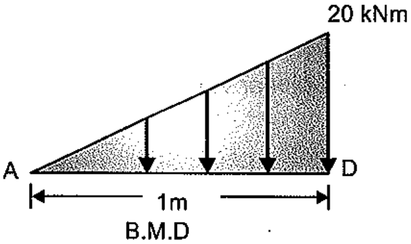
$$\Rightarrow \quad R_A = 20 \text{ kN and } R_B = 60 \text{ kN}$$

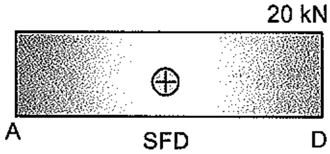
Determination of intensity of linear variation of load:

$$\frac{1}{2} \times W_E \times 2 = 60 \qquad \therefore \quad W_E = 60 \text{ kN/m}$$

$\Rightarrow$ Load intensity at a distance x from D = $(W_x) = (30x)$

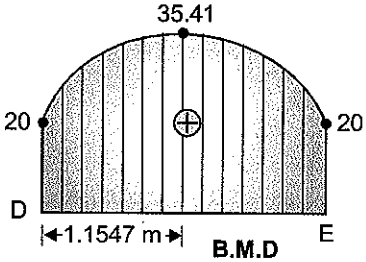
For part AD (x from left end)





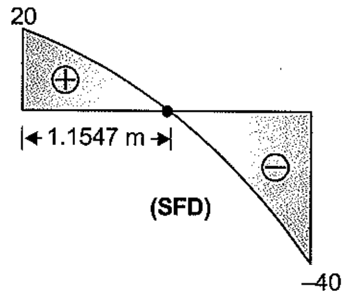
$$B.M = R_A x = 20x \text{ (linear variation)}$$
$$S.F. = 20 \text{ kN}$$
$$\therefore \quad B.M\,|_{x=1m} = 20 \text{ KN-m}$$

For part DE (x from point D)



$$S.F = 20 - \left(\frac{1}{2} \times W_x \times x\right)$$
$$SF = (20 - 15x^2) \text{ (parabolic)}$$
$$\text{Slope of SF} = -30x$$

$\Rightarrow$ Slope of SF is (–) and increasing with x



$$B.M.D = 20(x+1) - \left(\frac{1}{2} \times W_x \times x\right) \times \frac{x}{3}$$
$$= 20(x+1) - \left(\frac{1}{2} \times 30x \times x\right) \times \frac{x}{3}$$
$$= 20(x+1)(5x^3)$$
$$= 20x + 20 - 5x^3 \text{ (Cubic)}$$
$$B.M|_{x=0} = 20 \text{ KN-m}$$
$$B.M|_{x=2} = 40 + 20 - 40 = 20 \text{ KN-m}$$
$$S.F|_{x=0} = 20 \qquad [\because SF = 20 - 15x^2]$$
$$S.F|_{x=2} = -40 \text{ kN}$$

Finding location of point where S.F = 0

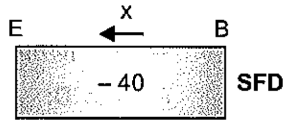
$$20 - 15x^2 = 0 \Rightarrow x = \sqrt{\frac{20}{15}} = 1.1547 \text{ m}$$

Since bending moment will be maximum at this point

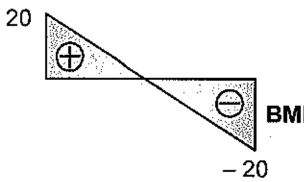
$$\Rightarrow \quad B.M|_{x=1.1547m} = 20 \times 1.1547 + 20 - 5 \times (1.1547)^3 = 35.4 \text{ KN-m}$$

Hence the SFD and BMD can be drawn

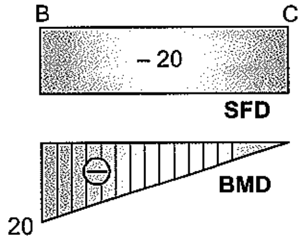
For part EB (x from point B)



$$S.F = 20 - 60 = -40$$
$$B.M = -20(1+x) + 60x$$
$$= (40x - 20) \text{ (Linear)}$$
$$\therefore \quad B.M|_{x=0} = -20 \text{ KN-m}$$
$$B.M|_{x=1m} = 20 \text{ KN-m}$$



For port BC (x taken from point C)



$$S.F = -20 \text{ kN}$$

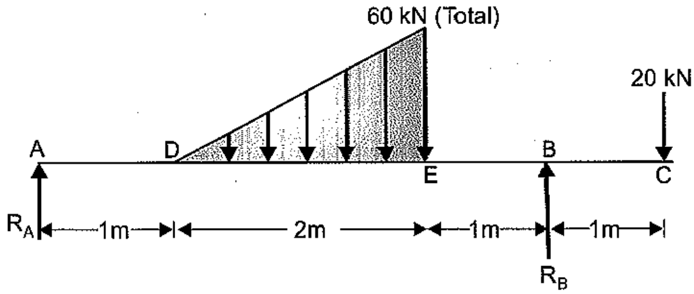
$$B.M = -20x \text{ (linear variation)}$$

at $x = 0$, B.M = 0

at $x = 1$, B.M = – 20 kN-m

Hence by combining all these diagrams we can get the desired BMD and SFD which is drawn at the start of this question.

Alternatively:



Let the max load intensity of triangular load be w kN/m

$$\Rightarrow \quad \frac{1}{2}\times 2\times w = 60 \text{ kN}$$

$$w = 60 \text{ kN/m}$$

Calculation of Reaction

From $\Sigma Fv = 0$

$$R_A + R_B = 20 + 60 = 80$$

From $\Sigma M_A = 0$

$$R_B \times 4 - 20 \times 5 - 60\left(1+2\times\frac{2}{3}\right) = 0$$

$$R_B = \frac{100+60\left(\frac{7}{3}\right)}{4} = \frac{240}{4} = 60 \text{ kN}$$

$$\Rightarrow \quad \boxed{R_B = 60 \text{ kN}}$$

$$\boxed{R_A = 20 \text{ kN}} \qquad [\because R_A + R_B = 80]$$

Shear force diagram

At A: There is an upward point load of 20 kN

$\Rightarrow$ SF jumps up by 20 kN

At A to D:

Load intensity = 0

$$\Rightarrow \quad \frac{dV}{dx} = 0$$

$\Rightarrow$ Slope of SFD = 0

$\Rightarrow$ SFD is constant in the region A to D

D to E

Load intensity is (–)ve and increasing

$\Rightarrow \quad \frac{dV}{dx}$ is (–)ve and increasing

$\Rightarrow$ Slope of SFD is (–)ve and increasing

$$V_E - V_D = \text{Area under load intensity diagram}$$

$$= -60 \times \frac{1}{2} \times 2 = -60$$

$$\Rightarrow \quad V_E - (20) = -60$$

$$\Rightarrow \quad \boxed{V_E = -40\,\text{kN}}$$

Since loading is linear SFD is parabolic

E to B

Load intensity = 0

$\Rightarrow \quad \frac{dV}{dx} = 0$

$\Rightarrow$ Slope of SFD is constant

$\Rightarrow$ SFD is constant in the region E to B

A to B

There is an upward point load of 60 kN

$\Rightarrow$ SFD jumps up by 60 kN

$$\Rightarrow \quad V_{E_R} - V_{E_L} = 60$$

$$V_{E_R} = 60 + (-40) = 20 \text{ kN}$$

[V_{E_R} = SF at E to the right of E]
[V_{E_L} = SF at E to the left of E]

B to C

Load intensity = 0

$\Rightarrow \quad \frac{dV}{dx} = 0$

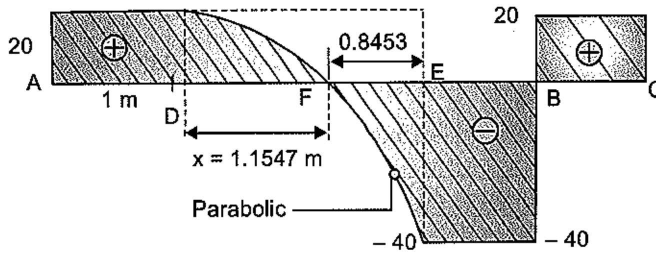
$\Rightarrow$ SFD slope = 0

$\Rightarrow$ SF is constant in the region B to C

At C

There is a downward point load of 20 kN

$\Rightarrow$ SFD jumps down by 20 kN



SFD

Let SF be zero at a distance x from D

$$\Rightarrow \quad 20-\frac{1}{2}\times x\times 30x = 0$$

$$x = \sqrt{\frac{40}{30}} = \frac{2}{\sqrt{3}} = 1.1547 \text{ m}$$

Bending moment diagram

At A, BM = 0

A to D

Shear force in (+)ve and constant

$\Rightarrow$ $\frac{dM}{dx}$ is (+)ve and constant

$\Rightarrow$ Slope of BM is (+)ve and constant

$M_D - M_A = (20) \times 1$ [$\because$ $M_D - M_A$ = Area under SFD betweem A to D]

$M_D - 0 = 0$

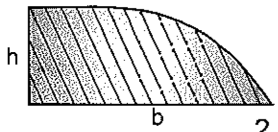
$\boxed{M_D = 20 \text{ kNm}}$

D to F

Shear force is (+)ve and decreasing

$\Rightarrow$ $\frac{dM}{dx}$ is (+)ve and decreasing

$\Rightarrow$ Slope of BM is (+)ve and decreasing



Area of parabola $= \frac{2}{3}bh$

$M_F - M_D = \frac{2}{3}\times 20\times 1.1547 = 15.396$ [$\because$ $M_F - M_D$ = Area under SFD between D to F]

$M_F - 20 = 15.396$

$\Rightarrow$ $\boxed{M_F = 35.396 \text{ kNm}}$

As SFD is parabolic, BMD will be cubic.

F to E

SF is (–)ve and increasing

$\Rightarrow$ $\frac{dM}{dx}$ is (–)ve and increasing

$$M_E - M_F = \text{Area under SFD b/w F to E}$$

$$= -\left\{\left[\frac{1}{3}\times(60\times 2)\right]-\left[20\times 2-\frac{2}{3}\times 20\times 1.1547\right]\right\}$$

$$= -\left\{40-40+\frac{2}{3}\times 20\times 1.1547\right\} = -15.396$$

$$\Rightarrow \quad M_E - 35.396 = -15.396$$

$$\Rightarrow \quad \boxed{M_E = 20 \text{ kNm}}$$

As SFD in parabolic, BMD will be cubic

E to B

SF is (–)ve and constant

$\Rightarrow \quad \dfrac{dM}{dx}$ is (–)ve and constant

$\Rightarrow$ Slope of BMD is (–)ve and constant

$$M_B - M_E = \text{Area under SFD between E to B}$$

$$\Rightarrow \quad M_B - M_E = -40 \times 1$$

$$M_B = 20 - 40 = -20$$

$$\boxed{M_B = -20 \text{ kNm}}$$

B to C

Shear force is (+)ve and constant

$\Rightarrow \quad \dfrac{dM}{dx}$ is (+)ve and constant

$\Rightarrow$ Slope of BMD is (+)ve and constant

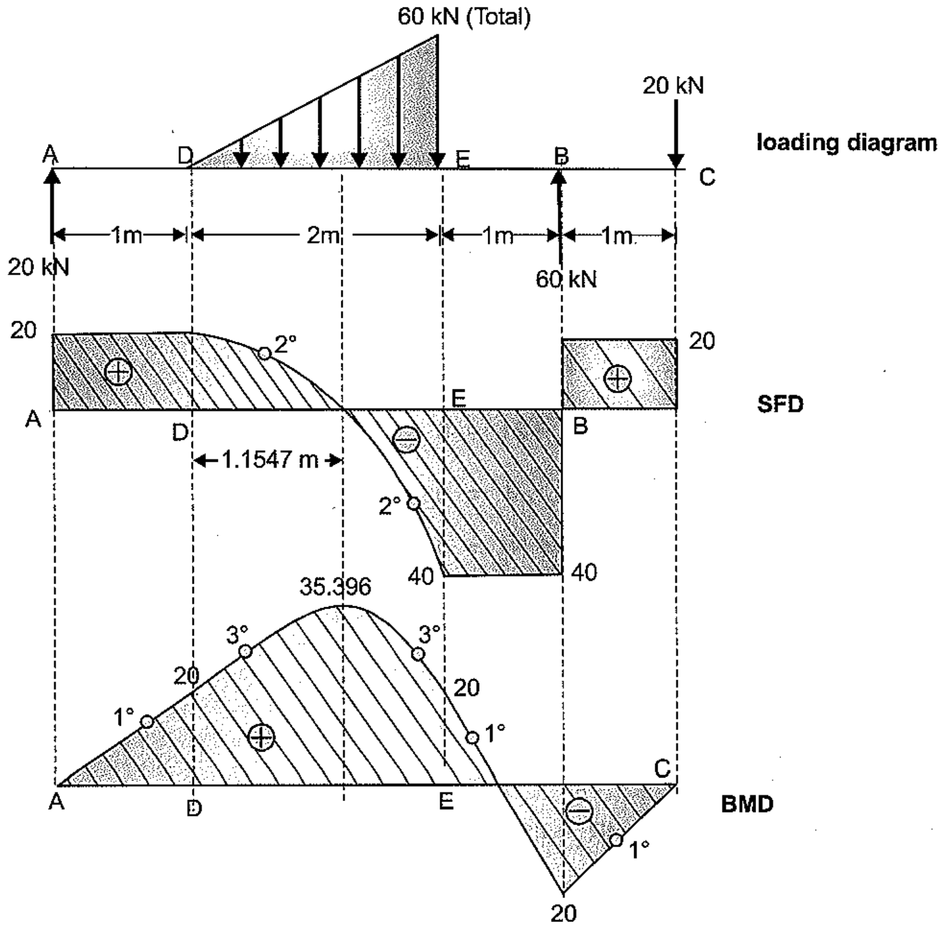
$$M_C - M_B = \text{Area under SFD between B to C}$$

$$M_C - M_B = 20 \times 1$$

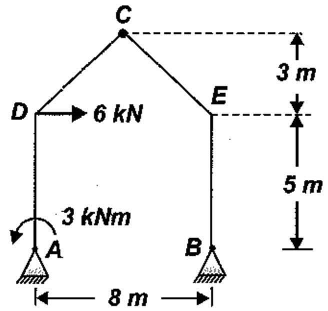
$$M_C - (-20) = 20$$

$$\Rightarrow \quad \boxed{M_C = 0}$$

The BMD and SFD is thus as shown below



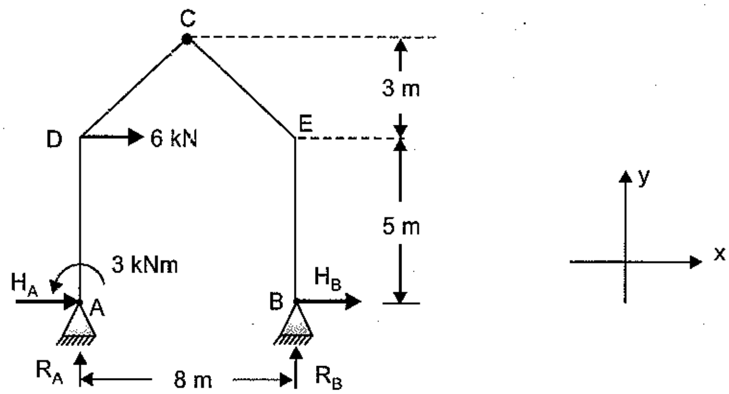
Q-19:



Find all the support reactions and draw BM diagram for the frame shown above. Frame has hinged supports at A & B and internal hinge at C.

[15 Marks, ESE-2012]

Sol:



$$\Sigma F_y = 0$$

$\Rightarrow$ $$R_A + R_B = 0 \quad \text{... (i)}$$

$$\Sigma F_x = 0$$

$\Rightarrow$ $$H_A + H_B + 6 = 0 \quad \text{... (ii)}$$

BM at C = 0

$\Rightarrow$ $$H_A \times 8 + 6 \times 3 + 3 - R_A \times 4 = 0$$

$$8H_A - 4R_A + 21 = 0 \quad \text{... (iii)}$$

Also, $$H_B \times 8 + R_B \times 4 = 0 \quad \text{.... (iv)}$$

$$8H_B + 4\,R_B = 0$$

$\Rightarrow$ From (ii), (iii) and (iv)

$$8H_A + 8H_B + 48 = 0$$

$$4R_A - 21 - 4\,R_B + 48 = 0$$

$$4R_A - 4R_B = -27 \quad \text{... (A)}$$

From (A) and (i) $$4R_A + 4R_B = 0$$

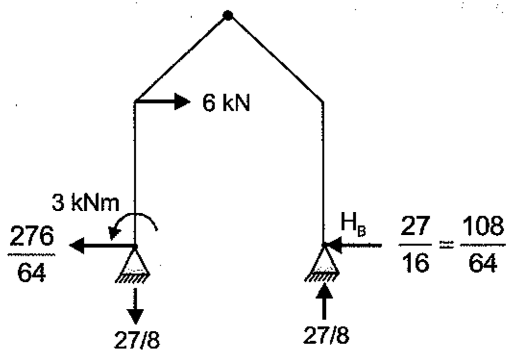
$$4R_A - 4R_B = -27$$

$$\boxed{R_A = -\frac{27}{8}}$$

$\Rightarrow$ $$\boxed{R_B = \frac{27}{8}}$$

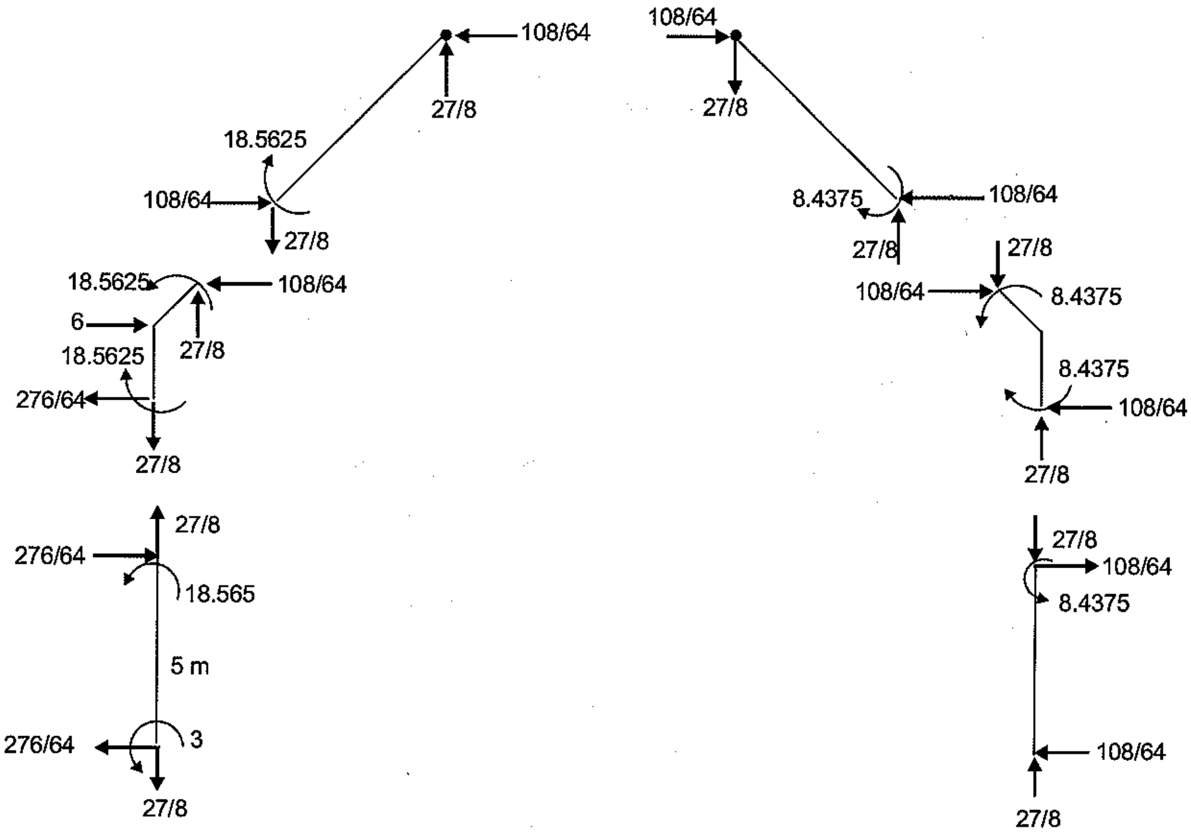
$$H_B = -\frac{R_B}{2} \quad \text{from (iv)}$$

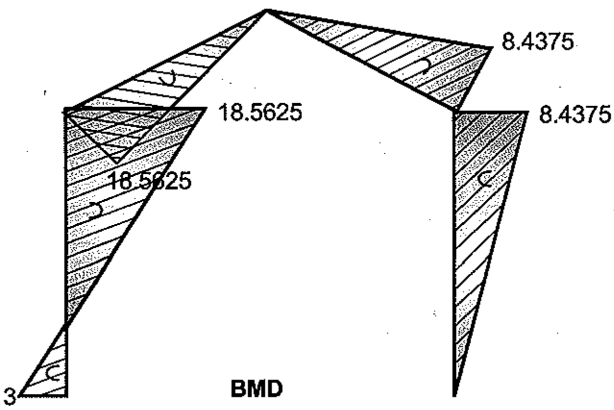
$\Rightarrow$ $$\boxed{H_B = -\frac{27}{16}}$$



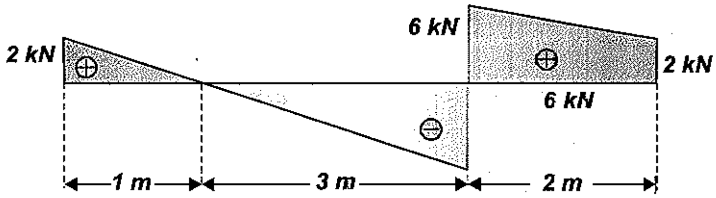
$$H_A = \frac{4R_A - 21}{8} = \frac{4\times\left(-\frac{27}{8}\right)-21}{8} = \frac{-108-168}{64} = \frac{-276}{64}$$

The free body diagram for the frame is as shown below



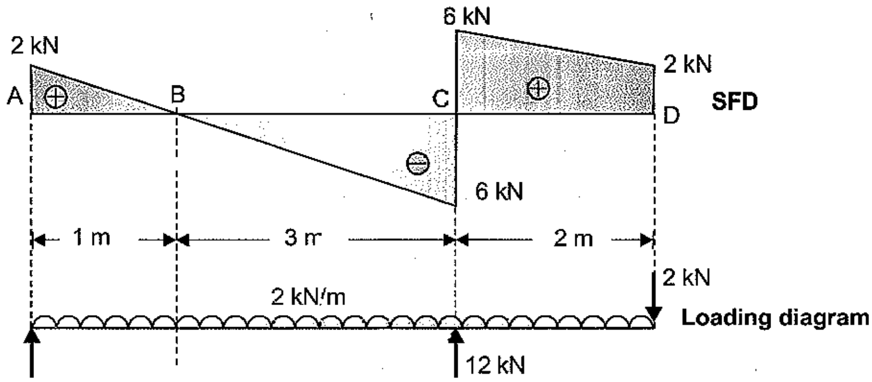


Q-20: Draw the BMD of a beam from the SFD of the beam



[4 Marks, ESE-2013]

Sol:

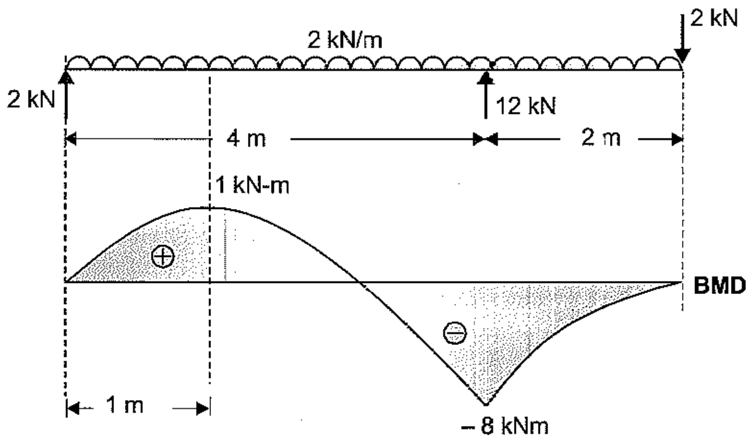


Loading diagram satisfies the force equilibrium. Let us check the moment equilibrium

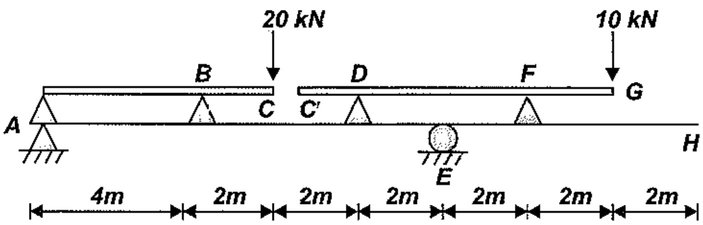
$$\Sigma M_A) = 2 \times 6 - 12 \times 4 + 2 \times 6 \times 3 = 12 - 48 + 36 = 0$$

⇒ Moment equilibrium is satisfied

⇒ Loading diagram shown above is final

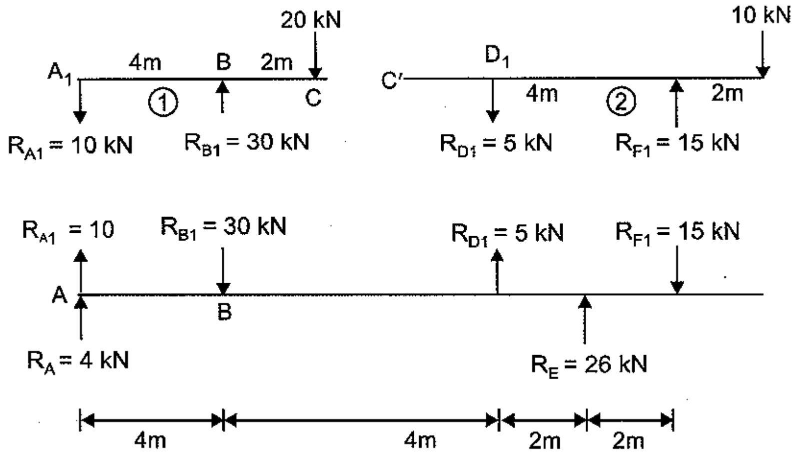


Q-21: A girder and floor beam constrcution subjected to concentrated loads is shown in Figure. Draw the free body diagrams and determine support reactions.



[12 Marks, ESE-2017]

Sol:



In beam (1), taking moment about point $A_1 = 0$

So, $$R_{B_1} \times 4 = 20 \times 6$$

$\Rightarrow$ $$R_{B_1} = 30\text{ kN}(\uparrow)\text{ (upward)}$$

So, $$R_{A_1} = 20 - 30\ (\Sigma fv = 0, R_{A1} + R_{B1} = 20\text{kN})$$
$$= 10\text{ kN}(\downarrow)\text{ (downward)}$$

In beam (2), taking moment about points $D_1 = 0$

then, $$R_{F_1} \times 4 = 10 \times 6$$

$\Rightarrow$ $$R_{F_1} = 15\text{ kN}(\uparrow)\text{ (upward)}$$

So, $$R_{D_1} = 10 - 15\ (\Sigma fv = 0, R_{D1} + R_{f1} = 10\text{kN})$$
$$= -5\text{ kN}(\downarrow)\text{ (downward)}$$

Now in girder taking moment about point A = 0

So, $$R_E \times 10 - 30 \times 4 + 5 \times 8 - 15 \times 12 = 0$$

So, $$R_E = 26\text{ kN (upward)}$$

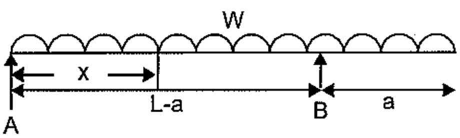
From $\sum F_v = 0$

$$10 - 30 + 5 - 15 + R_A + 26 = 0$$
$$R_A = 4\text{ kN (upward)}$$

Q-22: ***A beam of span L carries a uniformly distributed load ω per unit length on its whole span. It has one simple support at its left end and other support is at a distance of a from the other end. Find the value of a so that the maximum bending moment for the beam is as small as possible. Find also the maximum bending moment for this condition.***

[20 Marks, ESE-2019]

Sol:



$$R_A + R_B = WL$$

Taking moment about A.

$$(L-a)R_B = WL\times\frac{L}{2}$$

$$R_B = \frac{WL^2}{2(L-a)}$$

$$R_A = \frac{wL\,(L-2a)}{2(L-a)}$$

Moment in span AB (x from A)

$$M_x = \frac{WL(L-2a)}{2(L-a)}x-\frac{Wx^2}{2}$$

For minimum bending moment in span AB

$$\frac{dM_Y}{d_x} = 0$$

$$\frac{WL(L-2a)}{2-(L-a)}-\frac{2Wx}{2} = 0$$

$$x = \frac{L(L-2a)}{2(L-a)}$$

For value a so that maximum bending moment for the beam is as small as possible.

Maximum +ve bendig moment = Maximum –ve bending moment

$$\frac{WL^2(L-2a)^2}{4(L-a)^2}-\frac{WL^2(L-2a)^2}{2\times4(L-a)^2} = \frac{Wa^2}{2}$$

$$\Rightarrow \quad \frac{L^2(L-2a)^2}{4(L-a)^2} = a^2$$

$$\Rightarrow \quad \frac{L\,(L-2a)}{2\,(L-a)} = a$$

$$\Rightarrow \quad L^2-2aL = 2aL-2a^2$$

$$\Rightarrow \quad a^2-2aL+\frac{L^2}{2} = 0$$

$$a = 1.707\ L,\ 0.292L$$

$$a = 1.707 \text{ is not acceptable}$$

$$a = 0.292\ L$$

Maximum bendig moment

$$M_{max} = \frac{Wa^2}{2} = \frac{W\times(0.292L)^2}{2}$$

$$= 0.042\ WL^2$$

CHAPTER 3

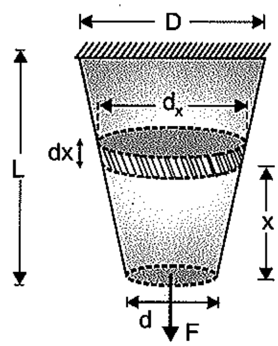
DEFLECTION OF BEAM

Q-1: ***A vertical tapered rod of 'L' has its diameter varying linearly from 'd' at lower end to 'D' at the upper end which has fixed support. Young's modulus of the material is 'E' show that the elongation of the rod at its lower end when subjected to a longitudinal force 'F' is given by***

$$\delta = \frac{4FL}{\pi dDE}$$

[10 Marks, ESE-1996]

Sol: Diameter of tapered beam at any distance x from the lower end



$$d_x = d + \left(\frac{D-d}{L}\right)x$$

Suppose $\frac{D-d}{L} = k$

$\therefore$ $d_x = d + kx$

Suppose width of the small element is (dx).

So, change in length $\delta\ell = \frac{Fdx}{A_x.E}$ {A_x = area of C/S at x from lower end}

So, total increase in length $= \int \delta l = \int_0^L \frac{Fdx}{A_x E} = \int_0^L \frac{Fdx}{\frac{\pi}{4}(d+kx)^2 E} = \int_0^L \frac{4F\ dx}{\pi(d+kx)^2 E} = \frac{4F}{\pi E}\int_0^L \frac{dx}{(d+kx)^2}$

Let $z = d + kx \Rightarrow dz = kdx \Rightarrow dx = \frac{dz}{k}$

When $x = 0$, $z = d$

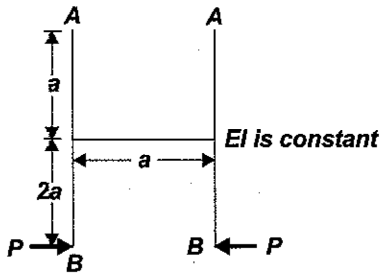
When $x = L$, $z = d + kL = d + \left(\frac{D-d}{L}\right)\times L = D$

$$\therefore \quad \frac{4F}{\pi E}\int_d^D \frac{dz}{k.z^2} = \frac{4F}{k\pi E} = \left|\frac{-1}{z}\right|_d^D = \frac{4F}{k\pi E}\left\{\frac{1}{d} - \frac{1}{D}\right\}$$

$$\delta = \frac{4F}{k\times\pi E}\times\frac{D-d}{Dd} = \frac{4F}{\left(\frac{D-d}{L}\right)\times\pi E}\times\frac{(D-d)}{Dd}$$

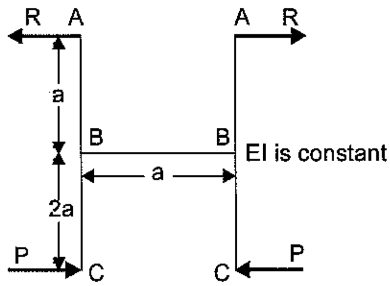
$$\delta = \frac{4F}{\pi DdE}$$

Q-2: ***In the structure shown in the above figure, calculate the distance by which the points A move away under the action of forces P.***



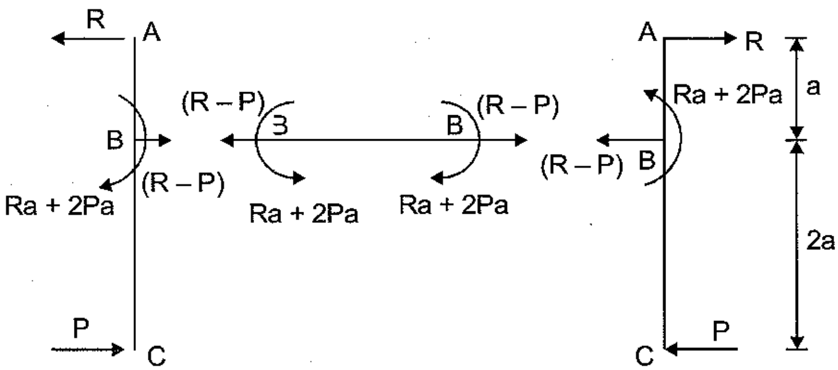
[10 Marks, ESE-2000]

Sol: If distance between two points are to be found out and both points can move, we will apply equal and opposite imaginary forces (R) at both points (as shown in figure).



$$\Delta_{AA} = \left.\frac{\partial U}{\partial R}\right|_{R=0}$$

F.B.D.



$$U = \int \frac{M^2 dx}{2EI} = 2U_{AB} + 2U_{BC} + U_{BB}$$

$$= 2\times\int_0^a \frac{R^2x^2dx}{2EI} + 2\int_0^{2a}\frac{P^2x^2}{2EI}dx + \int_0^a \frac{(Ra+2Pa)^2}{2EI}dx + \frac{(R-P)^2a}{2AE}$$

Note: U_{BB} constitute's energy due to bending and due to axial force. If in the question it is mentioned that the beam is axially rigid then only we can ignore the contribution of axial energy.

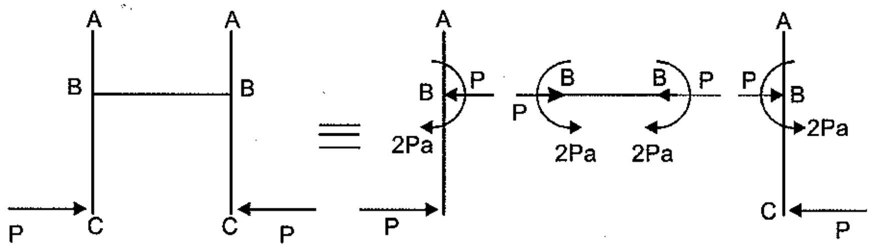
$$U = \frac{R^2a^3}{3EI} + \frac{8P^2a^3}{3EI} + \int_0^a \frac{R^2a^2 + 4P^2a^2 + 4PRa^2}{2EI}dx + \frac{(R-P)^2}{2AE}a$$

$$\left.\frac{\partial U}{\partial R}\right|_{R=0} = 0+0+0+0+\frac{4Pa^3}{2EI}+0+0-\frac{Pa}{AE}$$

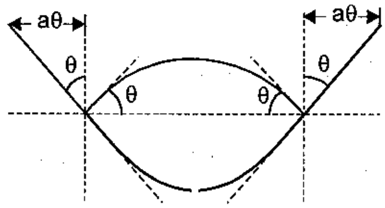
$$\therefore \quad \left.\frac{\partial U}{\partial R}\right|_{R=0} = \frac{4Pa^3}{2EI} - \frac{Pa}{AE}$$

$$\therefore \quad \Delta_{AA} = \left.\frac{\partial U}{\partial R}\right|_{R=0} = \frac{2Pa^3}{EI} - \frac{Pa}{AE}$$

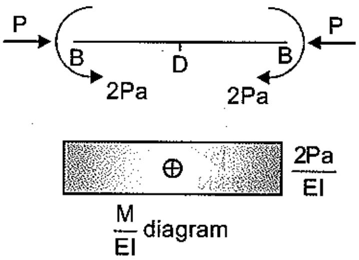
2nd Method :



Since BB will always remain perpendicular to AC, hence the deflection is shown as below.



Consider B-B section,



From moment area method,

$$\theta = \frac{2Pa}{EI} \times \frac{a}{2} = \frac{Pa^2}{EI}$$

And from the deflected shape.

$$\text{Total deflection of AA} = 2a\theta$$

$$\therefore \quad \text{Deflection of AA} = \frac{2Pa^3}{EI}$$

Since BB is not axially rigid and compressed by a compressive force P. Hence compression in member $B = \frac{Pa}{AE}$

So, $\quad \text{Deflection of AA} = \frac{2Pa^3}{EI} - \frac{Pa}{AE}$ **Ans.**

Q-3: ***The equation for the deflected shape of a beam carrying a uniformly distributed load and simply suppoted at the ends is given below :***

$$y = \frac{1}{EI}\left[-2x^3 + \frac{x^4}{6} + 36\,x\right]$$

Determine the load carried by the beam and draw the B.M.D. and S.F.D. The unit of EI is kN-m^2.

[14 Marks, ESE-2001]

Sol:

$$y = \frac{1}{EI}\left(-2x^3 + \frac{x^4}{6} + 36x\right)$$

$$\frac{dy}{dx} = \frac{1}{EI}\left(-6X^2+\frac{4x^3}{6}+36\right)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{1}{EI}\left[-12x+\frac{12x^2}{6}\right]$$

$$\therefore \quad \frac{EId^2y}{dx^2} = -12x + 2x^2$$

$$\Rightarrow \quad \boxed{\frac{EId^2y}{dx^2} = 2x^2-12x} \quad \ldots (i)$$

We know that, $\frac{EId^2y}{dx^2} = M$

So, $M = 2x^2 - 12x \quad \Rightarrow \quad \frac{dM}{dx}=4x-12=V_x \quad \Rightarrow \quad \frac{dV_x}{dx}=4=W$

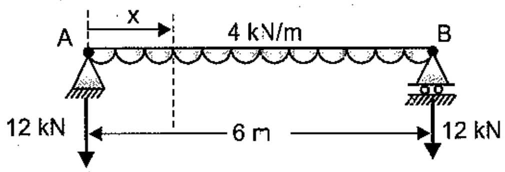
$\Rightarrow$ Beam carries a udl of 4 kN/m.

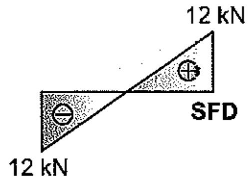
(+)ve value of udl means upward loading.

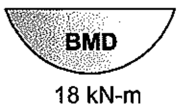
Finding the position where $V_x = 0 \Rightarrow 4x - 12 = 0 \Rightarrow x = 3m$

Since we know that if we have S.S.B. with udl the S.F. will be zero at the half of the length, so,

$$\frac{L}{2} = 3m \quad \Rightarrow \quad L = 6\ m$$







$$\text{Shear force} = (-12 + 4x)$$

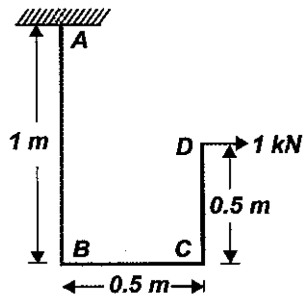
$$V_A = -12\ kN, \quad V_B = 12\ kN$$

$$\text{Bending moment, } M_x = -12x+\frac{4x^2}{2}-12x-2x^2$$

$$\therefore \quad M_A = 0, \quad M_B = 0$$

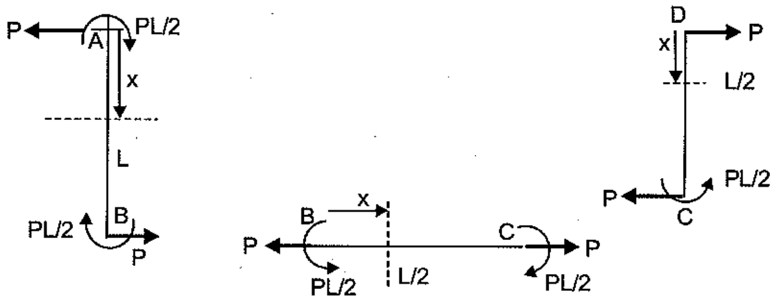
and Maximum B.M. $= -12 \times 3 + 2 \times 3^2 = -18$ kN-m occurs at $x = 3$ m

Q-4: ***Find the horizontal displacement of the end D of the bent rod of circular cross-section 20 mm diameter. A horizontal force of 1 kN acts at D as shown in figure. Consider only bending energy $E = 2 \times 10^5$ MPa.***



[10 Marks, ESE-2004]

Sol: The Free body diagram for the above loading is given by



where P = 1 kN
L = 0.5 m

From castigliano's theorem

$$\frac{\partial U}{\partial P} = \Delta$$

$$U = U_{AB} + U_{BC} + U_{CD}$$

$$U_{AB} = \int_0^L \frac{M^2 dx}{2EI} = \int_0^L \frac{\left(\frac{PL}{2} - Px\right)^2 dx}{2EI}$$ [sign to BMD in AB given by looking from left]

Let $\frac{PL}{2} - Px = \phi$

$$d\phi = -Pdx$$

at $x = 0,\ \phi = \frac{PL}{2}$

at $x = L,\ \phi = \frac{-PL}{2}$

$$\Rightarrow \quad U_{AB} = \int_{PL/2}^{-PL/2} \frac{-(\phi)^2 d\phi}{P \times 2EI} = -\frac{1}{2PEI}\left[\frac{\phi^3}{3}\right]_{+PL/2}^{-PL/2}$$

$$U_{AB} = \frac{-1}{2PEI}\left[\frac{1}{3}\right]\left[\frac{-P^3L^3}{8} - \frac{P^3L^3}{8}\right] = \frac{+2P^3L^3}{8 \times 3 \times 2PEI} = \frac{P^2L^3}{24\,EI}$$

$$U_{BC} = \int_0^{L/2} \frac{M^2 dx}{2EI} = \int_0^{L/2} \frac{(PL/2)^2 dx}{2EI} = \frac{P^2L^2}{8\,EI} \times \frac{L}{2} = \frac{P^2L^3}{16\,EI}$$

$$U_{CD} = \int_0^{L/2} \frac{M^2 dx}{2EI} = \int_0^{L/2} \frac{(-Px)^2 dx}{2EI} = \frac{P^2}{2EI}\frac{(L/2)^3}{3} = \frac{P^2L^3}{48\,EI}$$

$$U = U_{AB} + U_{BC} + U_{CD}$$

$$U = \frac{P^2L^3}{EI}\left[\frac{1}{24} + \frac{1}{16} + \frac{1}{48}\right] = \frac{P^2L^3}{8EI}$$

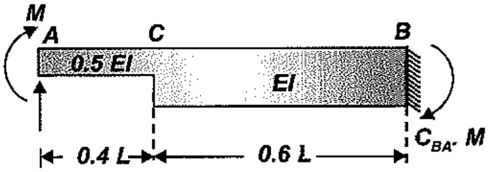
$$\frac{\partial U}{\partial P} = \frac{PL^3}{4EI} = \Delta$$

By putting the values we have

$$\Delta = \frac{(1 \times 10^3\ N)(1 \times 10^3\ mm)^3}{4 \times \left(2 \times 10^5\ N/mm^2\right)\left(\frac{\pi}{64}(20\ mm)^4\right)}$$

$$\boxed{\Delta = 159.155\ mm}$$

Q-5: ***A stepped beam A, as shown in the figure, changes cross-section at C such that EI of AC is half that of CB.***



Solve alternatively by moment area method

End A is simply supported and B is fixed. If on application of a moment 'M' at end A, moment induced at B is C_{BA}. M, determine the value of C_{BA}.

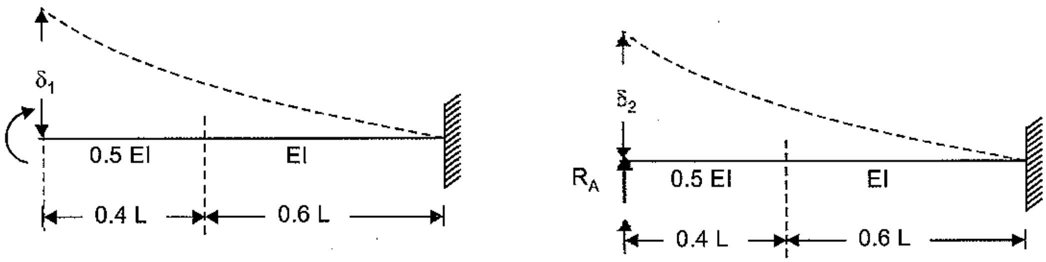
[20 Marks, ESE-2007]

Sol:

Let the reaction at A be R_A

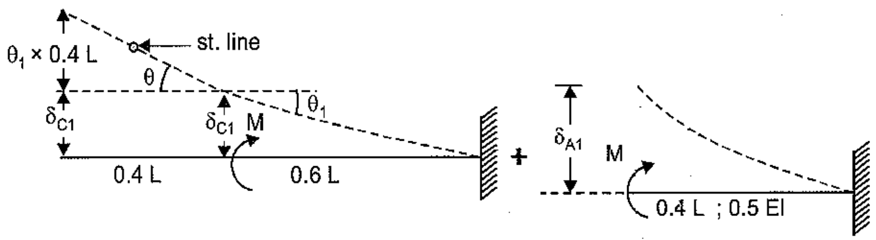
Net vertical deflection at A = 0 [compatibility condition]

i.e.



$$\delta_1 + \delta_2 = 0 \quad \text{[Compatibility condition]}$$

Calculation of δ_1



$$\Rightarrow \delta_1 = \delta_{c_1} + \theta_1 \times 0.4\,L + \delta_{A_1}$$

$$\delta_{c_1} = \frac{M(0.6\,L)^2}{2EI} = \frac{0.18\,ML^2}{EI}$$

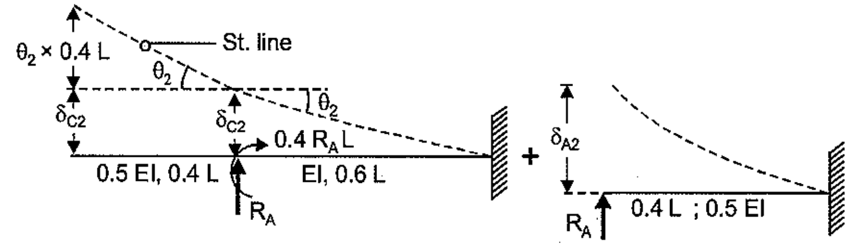
$$\theta_1 = \frac{M(0.6\,L)}{EI} = \frac{0.6\,ML}{EI}$$

$$\delta_{A_1} = \frac{M(0.4\,L)^2}{2(EI \times 0.5)} = \frac{0.16\,ML^2}{EI}$$

$\Rightarrow$
$$\delta_1 = \frac{0.18\,ML^2}{EI} + \frac{0.6\,ML}{EI} \times 0.4\,L + \frac{0.16\,ML^2}{EI}$$

$$\boxed{\delta_1 = \frac{0.58\,ML^2}{EI}}$$

Calculation of δ_2



$$\Rightarrow \delta_2 = \delta_{c_2} + \theta_2 \times 0.4\,L + \delta_{A_2}$$

$$\delta_{c_2} = \frac{R_A\,(0.6\,L)^3}{3EI} + \frac{(0.4\,R_A L)\,(0.6L)^2}{2EI} = \frac{0.144\,R_A L^3}{EI}$$

$$\theta_2 = \frac{R_A\,(0.6\,L)^2}{2EI} + \frac{(0.4\,R_A L)\,(0.6\,L)}{EI} = \frac{0.42\,R_A L^2}{EI}$$

$$\delta_{A_2} = \frac{R_A\,(0.4L)^3}{3\,(0.5\,EI)} = \frac{0.0427\,R_A L^3}{EI}$$

$$\Rightarrow \qquad \delta_2 = \frac{0.3546\,R_A L^3}{EI}$$

$\Rightarrow$ From compatibility condition

$$\frac{0.58\,ML^2}{EI} + \frac{0.3546\,R_A\,L^3}{EI} = 0$$

$$\Rightarrow \qquad R_A = \frac{-1.635\,M}{L}$$

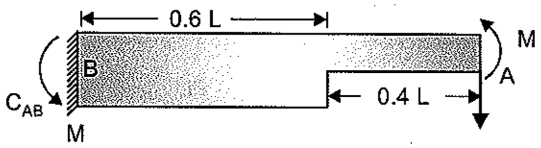
$\Rightarrow$ From $\Sigma M_B = 0$

$$M + \left(\frac{-1.635\,M}{L}\right) \times L + C_{BA}\,.\,M = 0$$

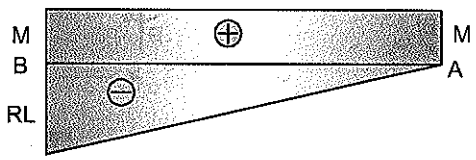
$$\Rightarrow \qquad M\,[C_{BA} + 1 - 1.635] = 0$$

$$\Rightarrow \qquad \boxed{C_{BA} = +0.635}$$

Alternatively:

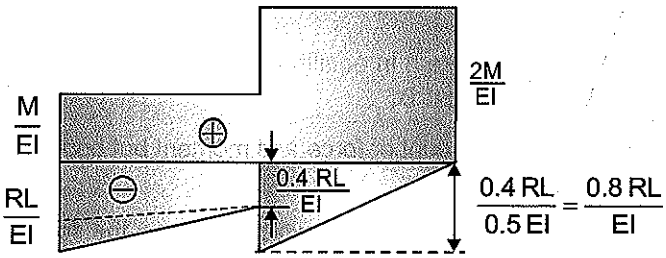


$$\frac{(1 + C_{AB})}{L} M = R$$



BMD

hence



$\frac{\mathbf{M}}{\mathbf{EI}}$ **diagram**

$\delta_{A/B} = 0$ hence moment of $\frac{M}{EI}$ diagram about A should be zero

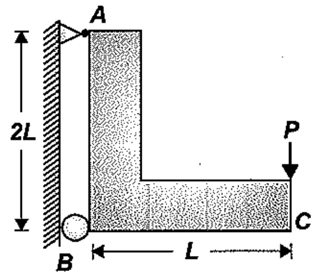
$$\left[\frac{2M}{EI}0.4L\right]\times 0.2\,L + \left[\frac{M}{EI}\times 0.6L\right]\times 0.7\,L + \left[-\frac{0.8RL}{EI}\times\frac{1}{2}\times 0.4L\right]\times\frac{2}{3}\times 0.4\,L$$

$$+\left[-\frac{0.4}{EI}\times 0.6\,L\right]\times 0.7\,L + \left[\frac{1}{2}\times\left(-\frac{0.6\,RL}{EI}\right)\times 0.6\,L\right]\times 0.8L = 0$$

$$\frac{0.58\,ML^2}{EI} - 0.3546\frac{RL^3}{EI} = 0$$

$$\Rightarrow \quad R = 1.6356\frac{M}{L} = \frac{(1+C_{AB})}{L}M \Rightarrow C_{AB} = 0.6356$$

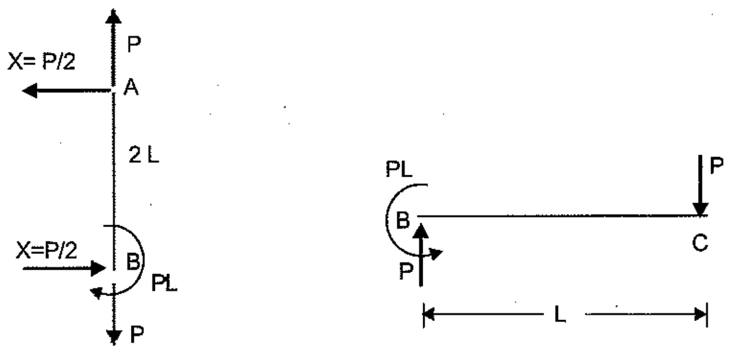
Q-6: ***A simple frame ABC has circular cross-section with cross-sectional area 'a' and moment of inertia I. The frame is supported and loaded as shown in the figure. Obtain the deflection under the load P considering the effects of both the bending and axial forces. Young's modulus is E for the material.***



[15 Marks, ESE-2008]

Sol: Circular cross-section with cross-sectional area = a

Moment of inertia = I



FBD of part AB **FBD of part BC**

Step for drawing FBD

- Consider part BC, to make it in equilibrium an upward force 'P' and a moment 'PL" anticlockwise has to be applied on point B.
- For part AB the same amount of force and moment but opposite in direction will generate as shown in figure.
- Since there is no any other agent which balance the force P generate at the point B in the downward direction, a force has to be applied of magnitude P at point A.

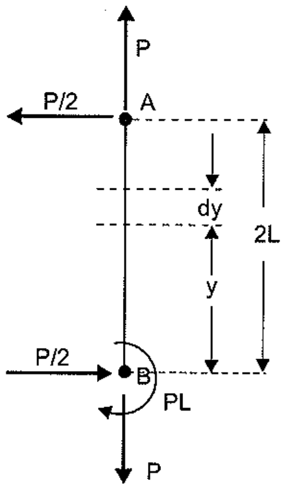
- There is also a moment of magnitude PL operating at the point B. To balance it, an unknown force of magnitude X is applied at point A.
- Application of force X at point 'A' disturbed the force equilibrium, hence another force of same magnitude i.e, X has to be applied at point B in opposite direction.

Determination of unknown force' 'X'

$$(X) \times 2L = PL \Rightarrow X = P/2$$

There are two methods to determine the deflection of point C.

Method I: (Energy method):

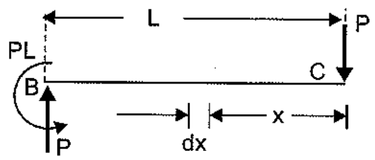


$$U_{AB} = \int\frac{M^2dy}{2EI}+\int\frac{P^2dy}{2AE} = \int_0^{2L}\frac{\left(PL-\frac{P}{2}y\right)^2 dy}{2EI}+\int_0^{2L}\frac{P^2dy}{2aE}$$

$$= P^2\int_0^{2L}\frac{\left(L-\frac{y}{2}\right)^2 dy}{2EI}+P^2\int_0^{2L}\frac{dy}{2aE}$$

$$= P^2\left[\left.\frac{\left(L-\frac{y}{2}\right)^3}{3\times\left(-\frac{1}{2}\right)2EI}\right|_0^{2L}+\frac{2L}{2aE}\right]$$

$$= P^2\left[0+\frac{L^3}{3\times\frac{1}{2}\times 2EI}+\frac{2L}{2aE}\right]$$



$$= P^2\left[\frac{L^3}{3EI}+\frac{L}{aE}\right]$$

$$\Rightarrow U_{AB} = \frac{P^2L^3}{3EI}+\frac{P^2L}{aE}$$

$$U_{BC} = \int_0^L\frac{M^2dx}{2EI} = \int_0^L\frac{(-Px)^2 dx}{2EI} = \frac{P^2}{2EI}\int_0^L x^2dx = \frac{P^2}{2EI}\left|\frac{x^3}{3}\right|_0^L = \frac{P^2L^3}{6EI}$$

So, $U = U_{AB} + U_{BC} = \dfrac{P^2L^3}{3EI}+\dfrac{P^2L}{aE}+\dfrac{P^2L^3}{6EI}$

Hence, Deflection under the load $= \dfrac{\delta U}{\delta P} = \dfrac{2PL^3}{3EI}+\dfrac{2PL}{aE}+\dfrac{2PL^3}{6EI}$

$$\Rightarrow \frac{\delta U}{\delta P} = \frac{PL^3}{EI}+\frac{2PL}{aE}$$

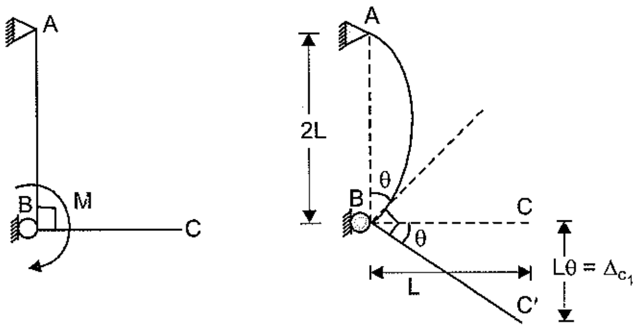
i.e., $\boxed{\text{Deflection} = \dfrac{PL^3}{EI}+\dfrac{2PL}{aE}}$

(+)ve value of deflection means it is in the direction of P.

Method-II:

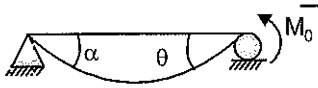
Deflection of point C due to individual forces

Due to moment at point B



(Deflected shape)

From the figure $\quad \Delta_{C_1} = L\theta = L \times \dfrac{M(2L)}{3EI} = \dfrac{2ML^2}{3EI}$

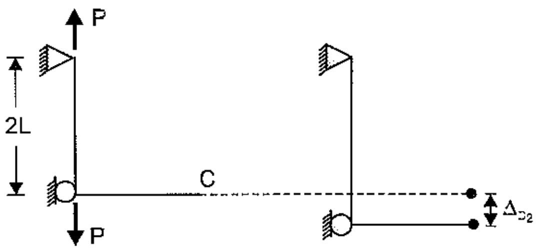


$\because \quad \theta = \dfrac{M_0 L}{3EI}; \quad \alpha = \dfrac{M_0 L}{6EI}$

Since $\quad M = PL$

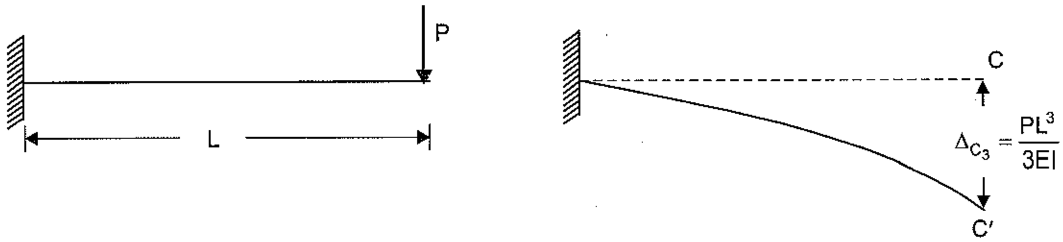
$$\Delta_{C_1} = \left\{\frac{2PL^3}{3EI}\right\}$$

Due to Axial force acting at point B,



$$\Delta_{C_2} = \frac{PL}{aE} = \frac{P(2L)}{aE} = \frac{2PL}{aE}$$

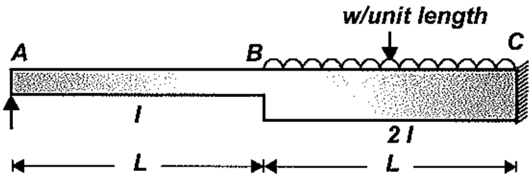
Due to vertical force acting on point C



Hence, the net deflection of point C (Δ_c) = $\Delta_{C_1} + \Delta_{C_2} + \Delta_{C_3}$

$$\boxed{\Delta_C = \frac{2PL^3}{3EI} + \frac{2PL}{aE} + \frac{PL^3}{3EI} = \frac{PL^3}{EI} + \frac{2PL}{aE}}$$

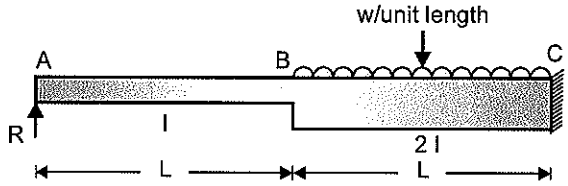
Q-7:



A stepped beam ABC, simply supported at A and fixed at C as shown in the above figure carries a uniformly distributed load of intensity 'W' per unit length over BC. Determine the vertical reaction at A using moment area or energy method.

[20 Marks, ESE-2009]

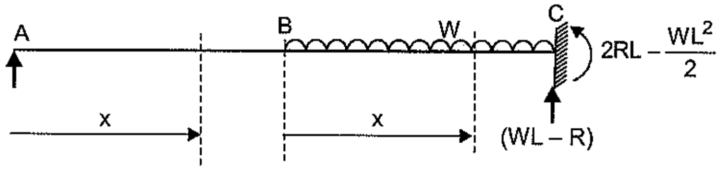
Sol:



Energy method

Let the reaction at point A = R

Strain energy stored in the beam,



$$U = \int \frac{M^2 dx}{2EI} = U_{AB} + U_{BC}$$

$$U_{AB} = \int_0^L \frac{(Rx)^2 dx}{2EI} = \frac{R^2}{2EI} \times \left[\frac{x^3}{3}\right]_0^L = \frac{R^2L^3}{6EI}$$

$$U_{BC} = \int_0^L \frac{\left[R(L+x) - \frac{WX^2}{2}\right]^2}{2EI(2I)} \times dx = \int_0^L \frac{\left[R(L+x) - \frac{WX^2}{2}\right]^2 dx}{4EI}$$

$$U = \frac{R^2L^3}{6EI} + \int_0^L \frac{\left[R(L+x) - \frac{WX^2}{2}\right]^2 dx}{4EI}$$

$$\frac{\partial U}{\partial R} = \frac{2RL^3}{6EI} + \int_0^L \frac{\left\{\frac{\partial}{\partial R}\left[R(L+x) - \frac{WX^2}{2}\right]^2\right\} dx}{4EI}$$

$$\frac{\partial U}{\partial R} = \frac{2RL^3}{6EI} + \int_0^L \frac{2\left[R(L+x) - \frac{WX^2}{2}\right] \times (L+x)dx}{4EI}$$

$$\frac{\partial U}{\partial R} = \frac{2RL^3}{6EI} + \int_0^L \left(RL + Rx - \frac{Wx^2}{2}\right)\frac{(L+x)dx}{2EI}$$

$$\frac{\partial U}{\partial R} = \frac{RL^3}{3E1} + \frac{1}{2EI}\int_0^L \left(RL^2 + RLx + RxL + Rx^2 - \frac{WLx^2}{2} - \frac{Wx^3}{2}\right)dx$$

$$\frac{\partial U}{\partial R} = \frac{RL^3}{3EI}+\frac{1}{2EI}\left(RL^2x+\frac{2RLx^2}{2}+\frac{Rx^3}{3}-\frac{WLx^3}{6}-\frac{Wx^4}{8}\right)_0^L$$

$$\frac{\partial U}{\partial R} = \frac{RL^3}{3EI}+\frac{1}{2EI}\left[RL^2x+RLx^2+\frac{Rx^3}{3}-\frac{WL^3x}{6}-\frac{Wx^4}{8}\right]_0^L$$

$$\frac{\partial U}{\partial R} = \frac{RL^3}{3EI}+\frac{1}{2EI}\left(RL^3+RL^3+\frac{RL^3}{3}-\frac{WL^4}{6}-\frac{WL^4}{8}\right)$$

$$\frac{\delta U}{\partial R} = \frac{RL^3}{3EI}+\frac{7RL^3}{6EI}-\frac{7WL^4}{48EI}$$

Since the point is supported at A, Hence $\frac{\delta U}{\partial R}=0$

$$\Rightarrow \qquad \frac{RL^3}{3EI}+\frac{7RL^3}{6EI} = \frac{7WL^4}{48EI}$$

$$\Rightarrow \qquad \boxed{R = \frac{7WL^4}{48EI}\times\frac{6EI}{9L^3}=\left(\frac{7}{72}WL\right)}$$

+ve sign indicates the reaction is in assumed direction.

Moment Area Method

Suppose the reaction at A is R.

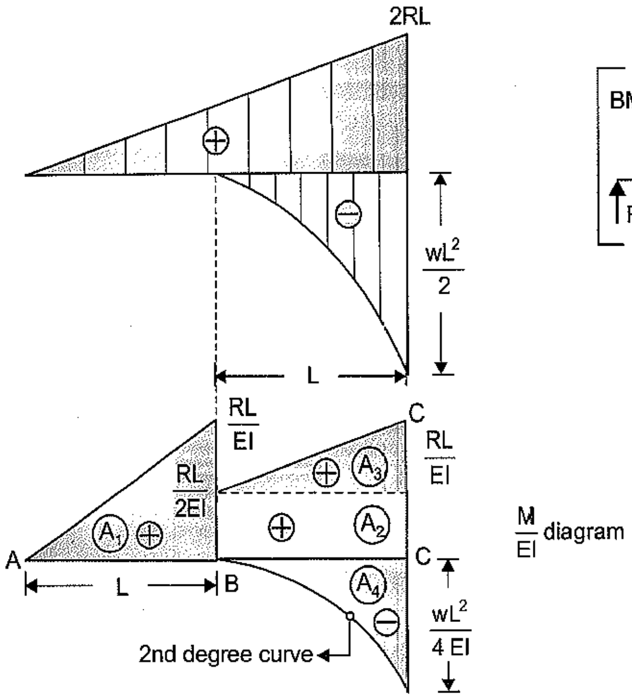
Hence the reaction at point C is (wL – R)

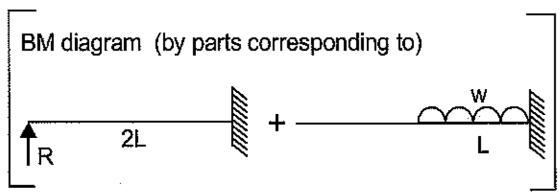
i.e., $V_A = R\ (\uparrow)$

and $V_C = wL - R\ (\uparrow)$

The concept behind the question to solve by moment area method is δ_{AC} = 0 and according to the theorem,

δ_{AC} = Moment of area of $\frac{M}{EI}$ diagram between *A* and *C* about point *A*.





Moment of area of $\frac{M}{EI}$ diagram between A and C about A,

$\Rightarrow \quad A_1x_1 + A_2x_2 + A_3x_3 - A_4x_4$

$$\Rightarrow \quad \left(\frac{1}{2}\times\frac{RL}{EI}\times L\times\frac{2L}{3}\right)+\left(\frac{RL}{2EI}\times L\times\left(L+\frac{L}{2}\right)\right)+\left(\frac{1}{2}\times\frac{RL}{2EI}\times L\times\left(L+\frac{2L}{3}\right)\right)-\left(\frac{wL^2}{4EI}\times\frac{L}{3}\times\left(2L-\frac{L}{4}\right)\right)$$

$$\Rightarrow \quad \frac{RL^3}{3EI}+\frac{3RL^3}{4EI}+\frac{5RL^3}{12EI}-\frac{7wL^4}{48EI}$$

$$\Rightarrow \quad \frac{16RL^3+36RL^3+20RL^3-7wL^4}{48EI} = \delta_{AC}=0$$

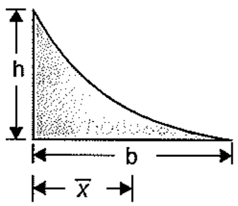
Since A is supported,

hence $\delta_A = 0$

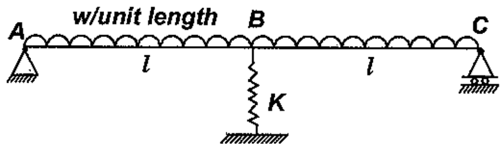
$$\Rightarrow \quad 72\,RL^3 = 7wL^4$$

$$\Rightarrow \quad \boxed{R = \frac{7}{72}wL} \quad \textbf{Ans.}$$

Note: For parabolic portion $A = \frac{bh}{3}$ and $\bar{x}=\frac{b}{4}$.

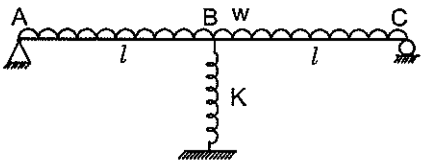


Q-8: ***Find the reaction at B for the beam shown below. K is the stiffness of spring at B. Beam is of constant section. use energy method of analysis.***

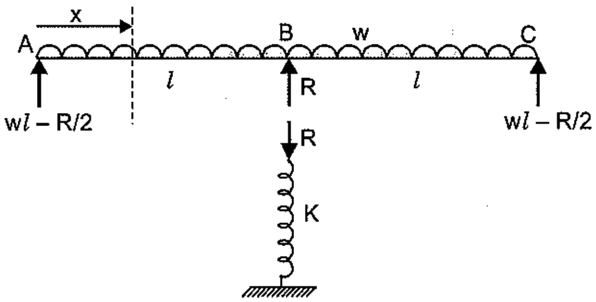


[10 Marks, ESE-2010]

Sol:



The free body diagram can be shown as below



Downward deflection of beam ABC $= \dfrac{-\partial U}{\partial R} = \dfrac{R}{K}$

Strain energy of beam $= 2\displaystyle\int_0^l \frac{\left[(wl - R/2)x - \frac{wx^2}{2}\right]^2 dx}{2EI}$

$$\frac{R}{k} = -\frac{\partial U}{\partial R} = \frac{-2}{2EI}\int_0^l \left[(wl - R/2)\,x - \frac{wx^2}{2}\right](-x)dx$$

$$\Rightarrow \quad \frac{R}{k} = \frac{1}{EI}\left[(wl - R/2)\frac{x^3}{3} - \frac{wx^4}{8}\Big|_0^l\right]$$

$$\Rightarrow \quad \frac{R}{k} = \frac{1}{EI}\left[\frac{(wl - R/2)l^3}{3} - \frac{wl^4}{8}\right]$$

$$\Rightarrow \quad \frac{R}{k} = \frac{1}{EI}\left[\frac{5wl^4}{24} - \frac{Rl^3}{6}\right]$$

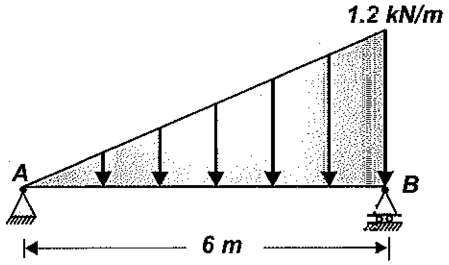
$$R\left[\frac{1}{k} + \frac{l^3}{6EI}\right] = \frac{5wl^4}{24\,EI}$$

$$R = \frac{5wl^4}{24\,EI\left[\frac{1}{k} + \frac{l^3}{6EI}\right]}$$

R = Reaction at B

Q-9: ***A beam AB of 6 m span carries a distributed load of varying intensity as shown in the figure below. Calculate the deflection at the centre of the beam.***

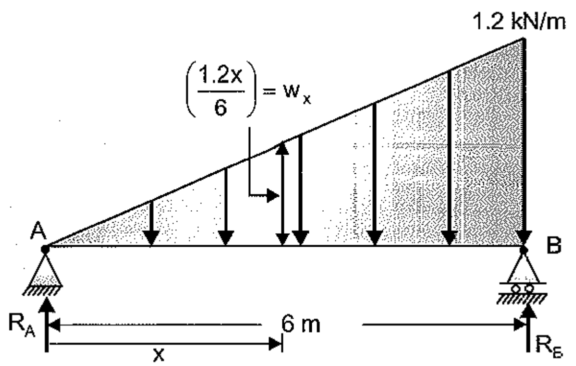
Load intensity at support B = 1.2 kN/m, EI = 420 × 10^9 N-mm².



Note: In actual ESE question EI is given to be 420× 10^6 N-mm² but it seems to be incorrect, hence we have solved this question taking EI = 420 × 10^9 N-mm²

[16 Marks, ESE-2011]

Sol:



EI = 420 × 10^9 N-mm²

$\Sigma F_V = 0$

$$R_A + R_B = \frac{1}{2}\times 6\times 1.2 = 3.6\text{ kN}$$

$\Sigma M_B = 0$

$$\Rightarrow \quad R_A \times 6 - \left(\frac{1}{2}\times 6\times 1.2\right)\times\frac{6}{3} = 0$$

$$\Rightarrow \quad \boxed{R_A = 1.2\text{ kN}}$$

$$\Rightarrow \quad \boxed{R_B = 2.4\text{ kN}}$$

$$EI\frac{d^2y}{dx} = R_A.x - \frac{1}{2}\times w_x\times x\times\frac{x}{3}$$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} = 1.2\,x - \frac{1.2}{36}x^3$$

$$\Rightarrow \quad EI\frac{dy}{dx} = 1.2\frac{x^2}{2} - \frac{1.2}{36}\frac{x^4}{4} + C_1$$

$$\Rightarrow \quad EI\,y = 1.2\frac{x^3}{6} - \frac{1.2}{36}\frac{x^5}{20} + C_1x + C_2$$

End conditions

$$\text{At } x = 0,\ y = 0 \Rightarrow C_2 = 0$$

$$\text{At } x = 6\text{ m},\ y = 0 \Rightarrow 1.2\times\frac{6^3}{6} - \frac{1.2}{36}\times\frac{6^5}{20} + 6C_1 = 0$$

$$\Rightarrow \quad C_1 = -5.04$$

$$EI\,y = \frac{1.2\,x^3}{6} - \frac{1.2}{36}\frac{x^5}{20} - 5.04\,x$$

Deflection at mid span $\Rightarrow$ at $x = 3$ m, let $y = \delta$

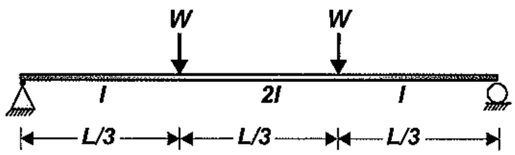
$$\Rightarrow \quad EI\delta = 1.2\times\frac{3^3}{6} - \frac{1.2}{36}\times\frac{3^5}{20} - 5.04\times 3$$

$$\Rightarrow \quad EI\delta = -10.125$$

$$\Rightarrow \quad \delta = \frac{-10.125}{EI}\text{kNm}^3 = \frac{-10.125\times 10^{12}\text{ Nmm}^3}{420\times 10^9\text{ Nmm}^2} = -24.107\text{ mm}$$

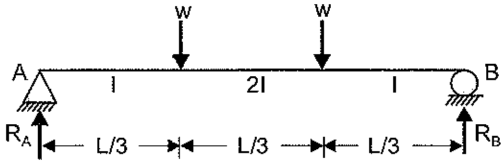
(–ve) sign indicates downward deflection

Q-10: ***A simply supported beam carries two point loads W each at its one-third sections as shown in figure. Determine the maximum deflection at its mid-span and slope at an end using the conjugate beam method.***



[15 Marks, ESE-2014]

Sol:



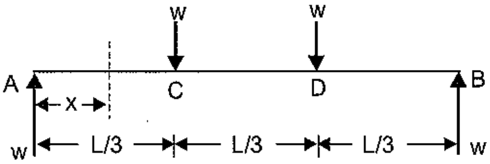
R_A = support reaction at A

R_B = support reaction at B

from symmetry we can easily find that $R_A = R_B = W$

To find deflection and slope using conjugate beam method, we have to draw $\frac{M}{EI}$ diagram for given beam and then apply this $\frac{M}{EI}$ diagram as loading on conjugate beam.

B.M.D. of a given beam



For $\quad 0 < x < \frac{L}{3}$

$$\text{B.M.} = w.x$$

at $\quad x = \frac{L}{3}$

$$\text{B.M.} = w.\frac{L}{3}$$

For $\quad \frac{L}{3} < x < \frac{2L}{3}$

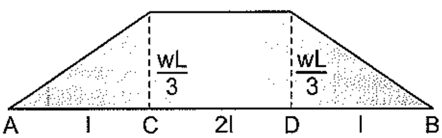
$$\text{B.M.} = w.x - w.\left(x - \frac{L}{3}\right) = \frac{wL}{3}(\text{constant})$$

For $\quad \frac{2L}{3} < x < L$

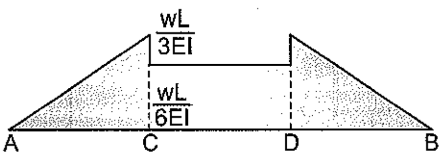
$$\text{B.M.} = wx - w\left(x - \frac{L}{3}\right) - w.\left(x - \frac{2L}{3}\right) = w.\frac{L}{3} - wx + \frac{2wL}{3} = wL - wx$$

$\therefore$ At $x = L$; BM = 0

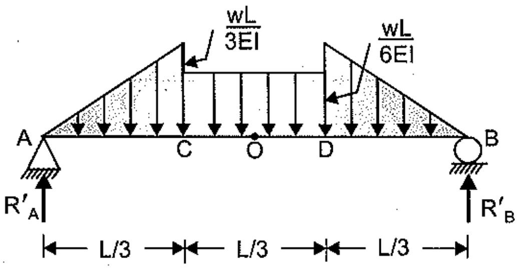
$\therefore$ BMD will be



$\therefore$ $\frac{M}{EI}$ diagram will be



For simply supported beam, conjugate beam is also simply supported. Therefore conjugate beam with loading is



Now, deflection in given beam at any point is equal to bending moment in conjugate beam at that point and slope at any point in given beam is equal to shear force in conjugate beam at that point.

Now, $$R'_A + R'_B = \frac{1}{2}\times\frac{L}{3}\times\frac{wL}{3EI}+\frac{wL}{6EI}\times\frac{L}{3}+\frac{1}{2}\times\frac{wL}{3EI}\times\frac{L}{3}$$

From symmetry, $$R'_A = R'_B$$ $\left[\begin{array}{l} R'_A = \text{reaction at A in conjugate beam} \\ R'_B = \text{reaction at B in conjugate beam}\end{array}\right]$

$\therefore$ $$R'_A = R'_B = \frac{wL^2}{12EI}$$

Shear force at A is also equal to R'_A

$\therefore$ $$\theta_A = R'_A = \frac{wL^2}{12EI}$$

Similarly, $$\theta_B = R'_B = \frac{wL^2}{12EI}$$

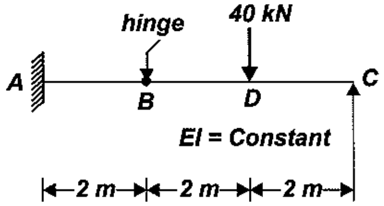
Now mid-span deflection in a given beam is equal to bending moment at mid-span in a conjugate beam.

$$\delta_0 = R'_A\times\left(\frac{L}{3}+\frac{L}{6}\right)-\frac{1}{2}\times\frac{L}{3}\times\frac{wL}{3EI}\times\left(\frac{L}{9}+\frac{L}{6}\right)-\frac{wL}{6EI}\times\frac{L}{6}\times\frac{L}{12}$$

$\therefore$ $$\delta_0 = \frac{wL^2}{12EI}\times\frac{L}{2}-\frac{wL^2}{18EI}\times\frac{5L}{18}-\frac{wL^2}{36EI}\times\frac{L}{12}$$

$$\delta_0 = 0.0239\frac{wL^3}{EI}\ (\downarrow)$$

Q-11: ***A beam fixed at one end and simply supported at the other end is having a hinge at B as shown in the figure.***

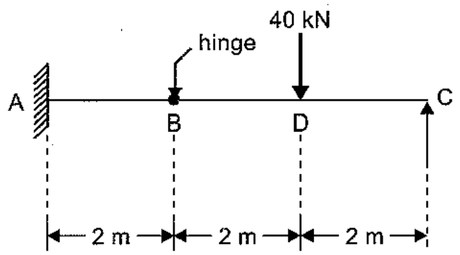


Determine the deflection (a) under the load and (b) at the hinge B.

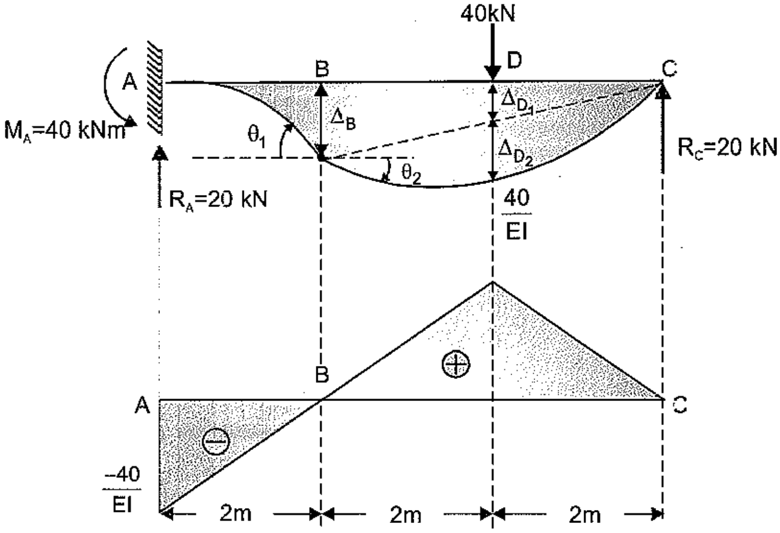
Use Moment Area Method.

[10 Marks, ESE-2015]

Sol:



Since there is an internal hinge in the beam, moment area method can't be directly applied. Deflected shape of the beam is as follows:



$$\Sigma M_B) = -R_C \times 4 + 40 \times 2 = 0$$

$\Rightarrow$ $$R_C = 20 \text{ kN}$$

$\Rightarrow$ $$R_A = 40 - R_C = 40 - 20 = 20 \text{ kN}$$

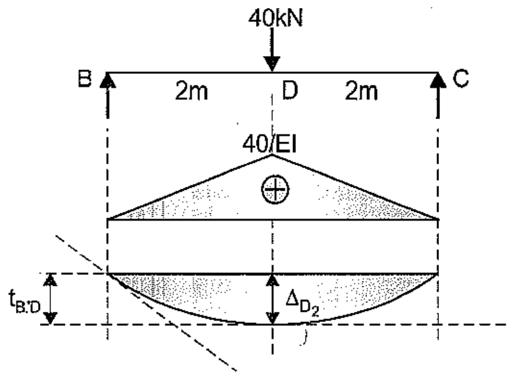
$\Rightarrow$ $$M_A = 40 \text{ kNm}$$

Now, deflection at D, $\Delta_D = \Delta_{D_1} - \Delta_{D_2}$

$$\Delta_B = -\frac{1}{\cancel{2}} \times \cancel{2} \times \frac{40}{EI} \times \frac{2}{3} \times 2 = \frac{-160}{3EI}$$

By similar triangles, $$\Delta_{D_1} = \frac{\Delta_B}{2} = \frac{-80}{3EI}$$

Now, part BC will be analyzed as



$$t_{B/D} = \frac{1}{2}\times 2\times\frac{40}{EI}\times\frac{2}{3}\times 2 = \frac{160}{3EI}$$

$$\Delta_{D_2} = -t_{B/D} = \frac{-160}{3EI}$$

$\Rightarrow$
$$\Delta_D = \frac{-80}{3EI}-\frac{160}{3EI}=\frac{-240}{3EI}\text{ (downwards)} = \frac{-80}{EI}$$

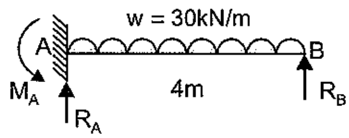
At hinge B, deflection,
$$\Delta_B = \frac{-160}{3EI}\text{(downwards)}$$

Q-12: ***A propped cantilever beam of length 4 m is subjected to UDL of 30 kN/m over the entire length of the span. If the flexural rigidity of the beam is 2 × 10⁴ kN-m², what would be the rotation at the propped support of the beam? Also determine the moment developed at the fixed support.***

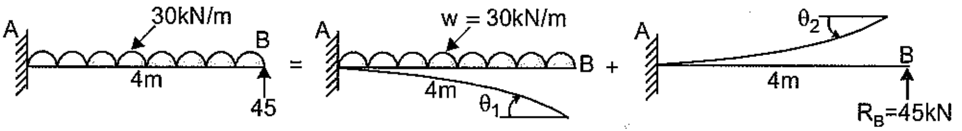
[5 Marks, ESE-2015]

Sol:

$$EI = 2\times 10^4\text{ kN-m}^2$$



$$R_B = \frac{3wL}{8}=\frac{3\times 30\times 4}{8}= 45\text{ kN}$$



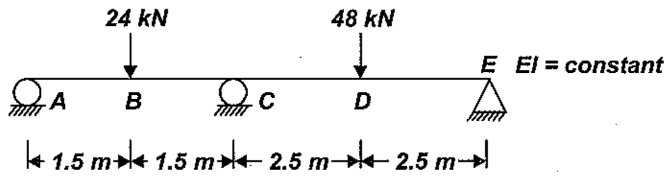
$$\theta_1 = \frac{wL^3}{6EI}=\frac{30\times 4^3}{6\times 2\times 10^4} = \frac{2}{125}\text{rad (clockwise)}$$

$$\theta_2 = \frac{R_BL^2}{2EI}=\frac{45\times 4^2}{2\times 2\times 10^4}=\frac{9}{500}\text{radians (anticlockwise)}$$

$$\theta_B = \theta_2-\theta_1=\frac{1}{500} = 0.002\text{ radians (anticlockwise)}$$

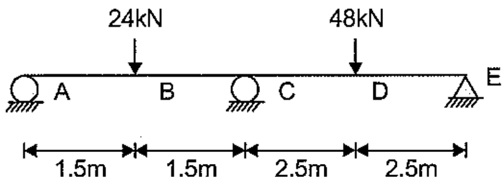
$$M_A = 30\times 4\times\frac{4}{2}-45\times 4 = 60\text{ kNm (Hogging)}$$

Q-13: ***Analyze a 2 span beam shown in figure using the method of consistent deformations and draw shear force and bending moment diagrams. Treat reaction at A as redundant.***

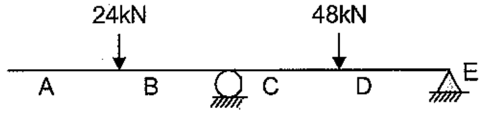


[20 Marks, ESE-2016]

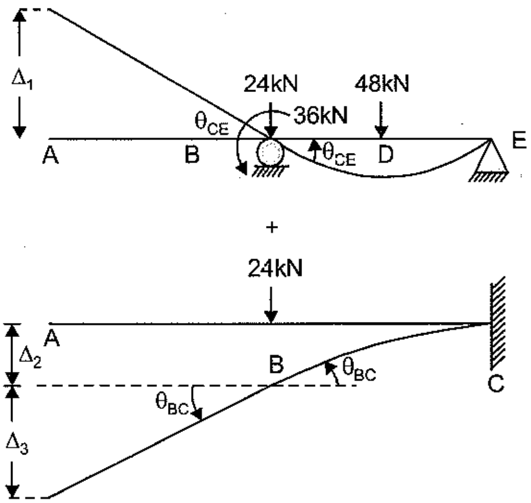
Sol:



Taking reaction at A as redundant



Upward deflection at A, $\Delta = \Delta_1 - \Delta_2 - \Delta_3$



$$\Delta_1 = \theta_{CE} \times 3$$

$$\theta_{CE} = \frac{48 \times 5^2}{16EI} - \frac{36 \times 5}{3EI} = \frac{15}{EI}$$

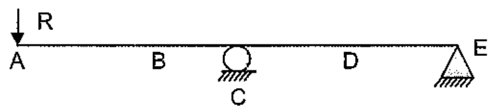
$$\Delta_1 = \frac{15}{EI} \times 3 = \frac{45}{EI}$$

$$\Delta_2 = \frac{24 \times 1.5^3}{3EI} = \frac{27}{EI}$$

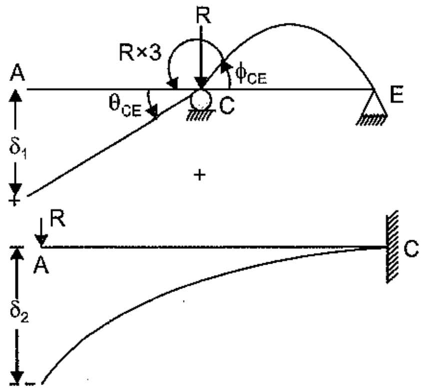
$$\Delta_3 = \theta_{BC} \times 1.5 = \frac{24 \times 1.5^2}{2EI} \times 1.5 = \frac{40.5}{EI}$$

$$\therefore \quad \Delta = \frac{45}{EI} - \frac{27}{EI} - \frac{40.5}{EI} = \frac{-22.5}{EI} = \frac{22.5}{EI} \text{ (downwards)}$$

Applying redundant reaction at A.



Downward deflection at A = $\delta_0 = \delta_1 + \delta_2$



$$\delta_1 = \theta_{CE} \times 3$$

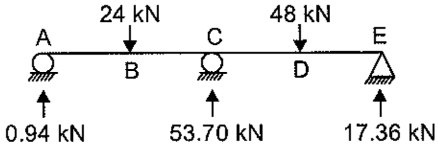
$$\theta_{CE} = \frac{(R \times 3) \times 5}{3EI} = \frac{5R}{EI}$$

$$\delta_1 = \frac{5R}{EI} \times 3 = \frac{15R}{EI}$$

$$\delta_2 = \frac{R \times 3^3}{3EI} = \frac{9R}{EI}$$

$$\delta = \delta_1 + \delta_2 = \frac{24R}{EI} \text{ downwards}$$

From compatibility condition, $\Delta + \delta = 0$

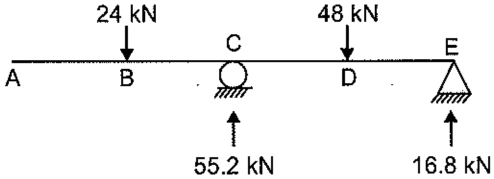


$$\Rightarrow \quad \frac{24R}{EI} + \frac{22.5}{EI} = 0$$

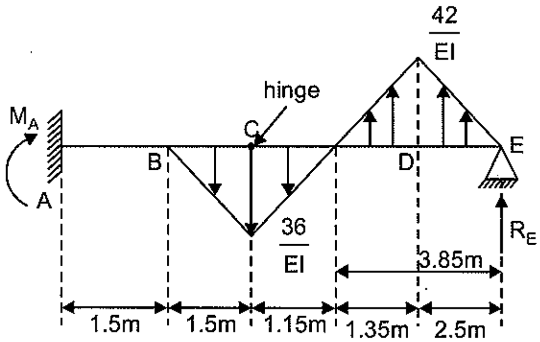
$$\Rightarrow \quad R = -0.94 \text{ kN}$$

Alternate method (conjugate beam method)

Taking reaction at A as redundant.



Conjugate beam:



BM at C from Right side = 0

$$\Rightarrow \quad \frac{1}{2} \times \frac{36}{EI} \times 1.15 \times \frac{1}{3} \times 1.15 = \frac{1}{2} \times \frac{42}{EI} \times 1.35 \times \left(1.15 + \frac{2}{3} \times 1.35\right) + \frac{1}{2} \times \frac{42}{EI} \times 2.5 \times \left(2.5 + \frac{1}{3} \times 2.5\right) + R_E \times 5$$

$$\Rightarrow \quad 7.935 = 58.1175 + 175 + 5R_E$$

$$\Rightarrow \quad R_E = \frac{-45.04}{EI}$$

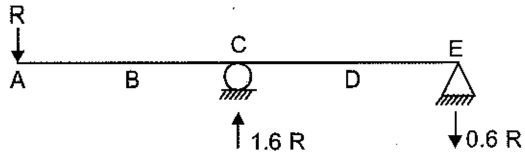
BM at A = M_A

$$M_A = \frac{-1}{2} \times \frac{36}{EI} \times 1.5 \times \left(1.5 + \frac{2}{3} \times 1.5\right) - \frac{1}{2} \times \frac{36}{EI} \times 1.15 \times \left(3 + \frac{1}{3} \times 1.15\right) + \frac{1}{2} \times \frac{42}{EI} \times 1.35 \times \left(4.15 + \frac{2}{3} \times 1.35\right)$$
$$+ \frac{1}{2} \times \frac{42}{EI} \times 2.5 \times \left(5.5 \times \frac{2.5}{3}\right) - \frac{45.04}{EI} \times 8$$

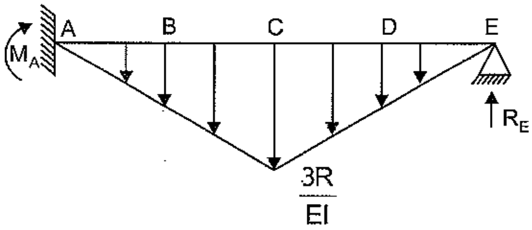
$$M_A = \frac{-67.5}{EI} - \frac{70.035}{EI} + \frac{143.17}{EI} + \frac{332.5}{EI} - \frac{360.32}{EI}$$

$$M_A = \frac{-22.185}{EI} \quad \Rightarrow \quad \Delta_A = \frac{22.185}{EI} \text{(downward)}$$

Applying redundant reaction at A



Conjugate beam:



BM at C from Right side = 0

$$\Rightarrow \quad \frac{1}{2}\times\frac{3R}{EI}\times 5\times\frac{1}{3}\times 5 = 5\times R_E$$

$$\Rightarrow \quad R_E = \frac{2.5}{EI}$$

$$M_A = \frac{-1}{2}\times\frac{3R}{EI}\times 3\times\frac{2}{3}\times 3-\frac{1}{2}\times\frac{3R}{EI}\times\times\left(3+\frac{5}{3}\right)+\frac{2.5}{EI}\times 8$$

$$M_A = \frac{-9R}{EI}-\frac{35R}{EI}+\frac{20R}{EI}=\frac{-24R}{EI}$$

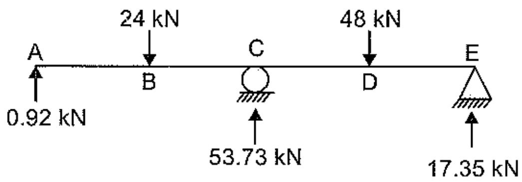
$$\Rightarrow \quad \delta_A = \frac{24R}{EI}\text{(downward)}$$

From compatibility condition,

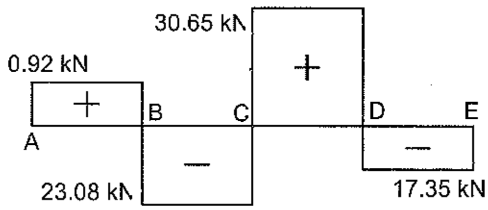
$$\delta_A+\Delta_A = 0$$

$$\Rightarrow \quad \frac{22.185}{EI}+\frac{24R}{EI} = 0$$

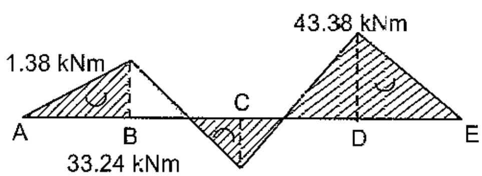
$$\Rightarrow \quad R = -0.92 \text{ kN}$$



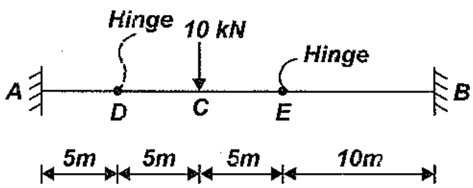
Shear force diagram,



Bending moment diagram



Q-14: ***Find the deflection at C for the beam as shown in Figure. EI = constant. Use conjugate beam method.***

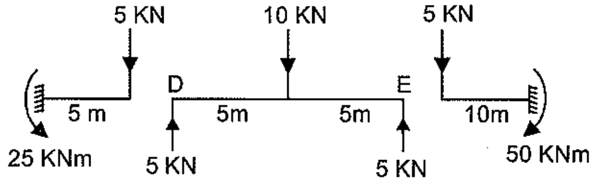


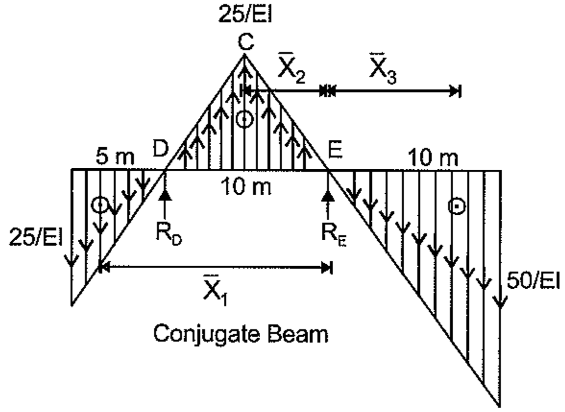
[20 Marks, ESE-2017]

Sol: (Sign convention): (+), (−)

⊕ M/EI Diagram means ↑ loading in conjugate beam and ⊖ Ve $\frac{M}{EI}$ mean ↓ loading in conjugate beam,

⊖ BM in conjugate beam means ↓ deflection ⊕ BM mean ↑ deflection





Conjugate Beam

$$X_1 = \left(10+\frac{2}{3}\times 5\right)m$$

$$\bar{X}_2 = 5 \text{ m}$$

$$\bar{X}_3 = \frac{2}{3}\times 10 \text{ m}$$

In conjugate beam fixed end A and B is converted to the free and D and E of internal hinge is converted to pin support.

Calculation of reaction at D, taking moment about point E = 0.

So $$R_D\times 10-\frac{1}{2}\times\frac{25}{EI}\times 5\times\left(10+\frac{2}{3}\times 5\right)+\left(\frac{1}{2}\times\frac{25}{EI}\times 10\times 5\right)+\left(\frac{1}{2}\times\frac{50}{EI}\times 10\times\frac{2}{3}\times 10\right)=0$$

$$\Rightarrow \quad R_D\times 10-\frac{2500}{3EI}+\frac{625}{EI}+\frac{5000}{3EI}=0$$

$$\Rightarrow \quad R_D = -\frac{145.83}{EI}$$

So taking moment about point C

(Bending moment at any point in conjugage beam is equal to deflection at that point in real beam)

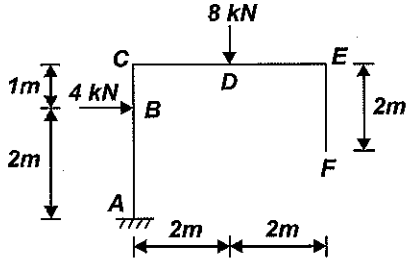
$$\therefore \quad \Delta_C = \frac{-145.83}{EI}\times 5+\left(\frac{1}{2}\times\frac{25}{EI}\times 5\times\frac{5}{3}\right)\times 5-\frac{1}{2}\times\frac{25}{EI}\times 5\times\left(5+\frac{2}{3}\times 5\right)$$

$$= -\frac{729.15}{EI}+\frac{104.167}{EI}-\frac{520.83}{EI} = -\frac{1145.81}{EI}$$

$$\boxed{\Delta_c = \frac{1145.81}{EI}(\downarrow)}$$

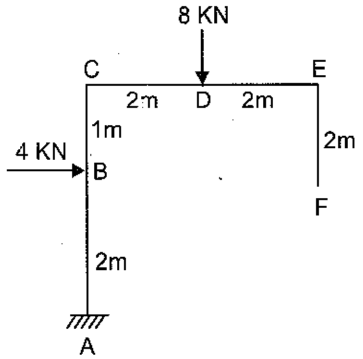
As per the sign convention of conjugate beam ⊖ BM in conjugate beam mean ↓ deflection.

Q-15: ***Determine the horizontal deflection at F for the frame shown in Figure. Take EI = constant.***

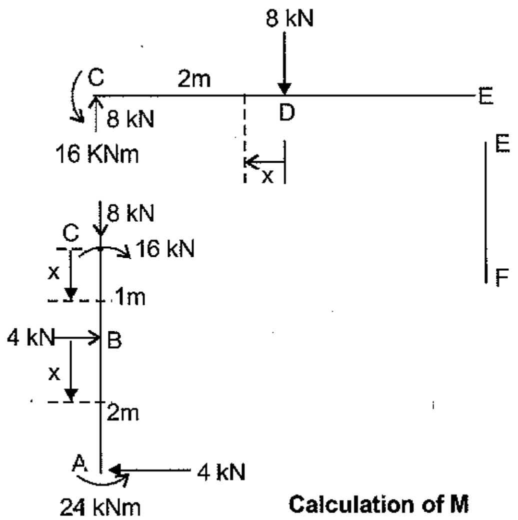


[20 Marks, ESE-2017]

Sol:

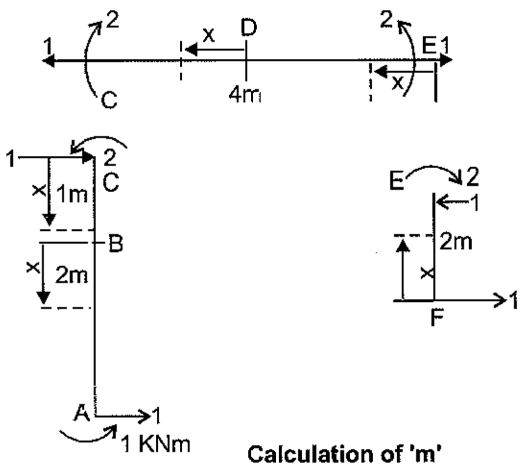


Using Unit Load Method:



Calculation of M

Applying unit load at point F in rightward direction.



Calculation of 'm'

Member	Range of x	I	M	m
FE	0 – 2	I	0	x
ED	0 – 2	I	0	2
DC	0 – 2	I	–8x	2
CB	0 – 1	I	–16	2 – x
BA	0 – 2	I	–16 – 4x	2 – (1 + x) = 1 – x

So, $$\Delta_F = \int_0^2 \frac{0\times x dx}{EI} + \int_0^2 \frac{0\times 2dx}{EI} + \int_0^2 \frac{-8x\times 2}{EI}dx + \int_0^1 \frac{-16\times(2-x)}{EI}dx + \int_0^2 \frac{-(16+4x)(1-x)}{EI}dx$$

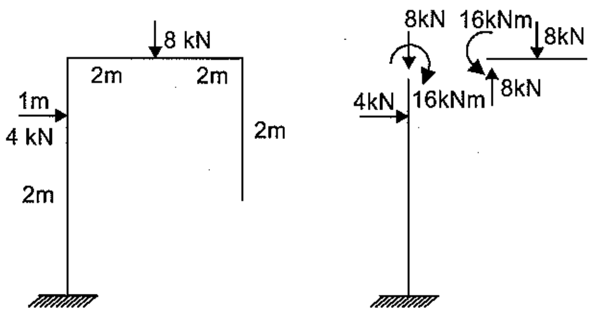
$$= \frac{-32}{EI} - \frac{24}{EI} + \frac{8}{3EI} = \frac{-160}{3EI}$$

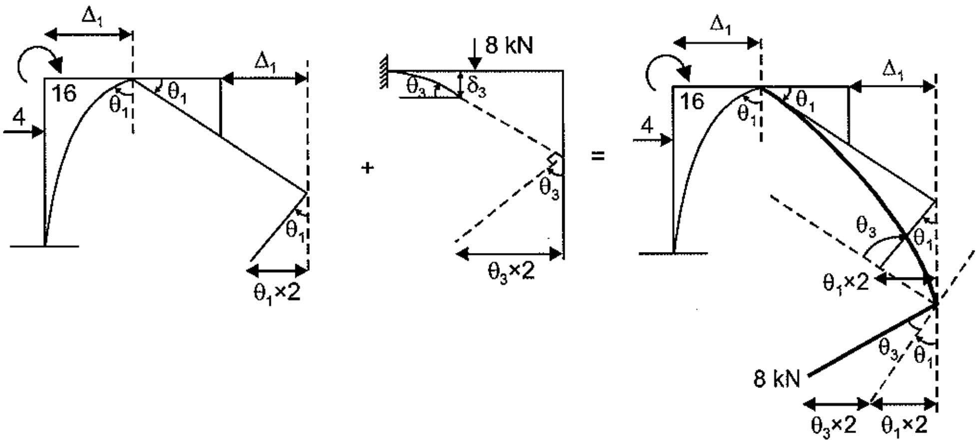
Δ_f negative mean that deflection is opposite to the applied unit load ie. leftwards.

So, deflection at point F is $\frac{160}{3EI}$ leftward.

$$\Delta_F = \frac{160}{3EI}(\leftarrow)$$

Alternate solution:





$$\Delta_{FH} = \Delta_1 - (\theta_1 + \theta_3)\times 2$$

$$\Delta_{FH'} = (\theta_1 + \theta_3)\times 2$$

$$\theta_1 = \frac{16 \times 3}{EI} + \frac{4(2)^2}{2EI}$$

$$\theta_3 = \frac{8 \times (2)^2}{2EI}$$

$$\Rightarrow \quad \Delta_{FH'} = (\theta_1 + \theta_3) \times 2$$

$$= \left[\frac{48}{EI} + \frac{8}{EI} + \frac{16}{EI}\right] \times 2$$

$$= \frac{72}{EI} \times 2 = \frac{144}{EI}$$

$$\Delta_1 = \frac{16(3)^2}{2EI} + \frac{4(2)^3}{3EI} + \frac{4(2)^2}{2EI} \times 1$$

$$= \frac{90.66}{EI}$$

$$\Delta_{FH} = \frac{90.66}{EI} - \frac{144}{EI} = -\frac{53.33}{EI}$$

CHAPTER 4

TRANSFORMATION OF STRESS AND STRAIN

Q-1: ***The principal stresses at a point in an elastic material are 1.5σ (tensile), σ (tensile) and 0.5 σ (compressive). The elastic limit in tension is 210 MPa and μ = 0.3. What would be the value of σ at failure when computed by the different theories of failure?***

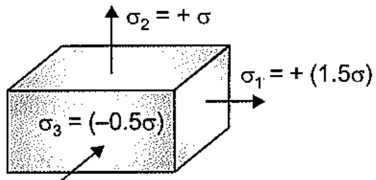
[15 Marks, ESE-1995]

Sol: **Given data:**

$\sigma_1 = +1.5\sigma$; $\sigma_2 = +\sigma$; $\sigma_3 = -(0.5)\sigma$ *

Elastic limit in tension (f_y) = 210 MPa.

$\mu = 0.3$



(Macroscopic View of a Point)

Determine: σ at failure when computed by different theories of failure.

(i) **Maximum principle stress theory:** As per this theory for no failure maximum principal Stress should be less than yield stress under uniaxial loading.

So, $$(\sigma_1 = 1.5\sigma) \le f_y$$

$$\therefore \quad 1.5\sigma \le f_y \quad \Rightarrow \quad \sigma \le \frac{f_y}{1.5} \quad \Rightarrow \quad \sigma \le \left(\frac{210}{1.5}\right) = 140.00$$

$$\therefore \quad \sigma \le 140 \text{ MP}_a$$

(ii) **Maximum principal strain theory:** As per this theory, for no failure maximum principal strain should be less than yield strain under uniaxial loading

i.e., $$\varepsilon_{max} \le \frac{\sigma_y}{E}$$

Among $(\varepsilon_x, \varepsilon_y, \varepsilon_z)$, ε_x will be maximum because σ_1 is maximum

$$\therefore \quad \varepsilon_x = \frac{1.5\sigma}{E} - \frac{\mu\sigma}{E} + \frac{\mu(0.5\sigma)}{E} = \left(\frac{1.5 - 0.3 + 0.15}{E}\right)\sigma = \frac{1.35\sigma}{E}$$

$$\Rightarrow \quad \frac{1.35\sigma}{E} \le \frac{210}{E} \quad \Rightarrow \quad \sigma \le \frac{210}{1.35} \quad \Rightarrow \quad \sigma \le 155.55 \text{ MPa.}$$

(iii) **Maximum shear stress theory:** For no failure, maximum shear stress should be less than or equal to maximum shear stress under uniaxial loading.

Since we have 3–D case,

$$\text{Maximum shear stress} = \text{maximum}\left\{\frac{|\sigma_1 - \sigma_3|}{2}, \frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}\right\}$$

Maximum shear stress under uniaxial loading: $\tau_y = \frac{f_y}{2}$

∴ From this theory

$$\text{Maximum}\left\{\frac{|\sigma_1-\sigma_3|}{2},\frac{|\sigma_1-\sigma_2|}{2},\frac{|\sigma_2-\sigma_3|}{2}\right\} \le \frac{f_y}{2}$$

$$\Rightarrow \quad \frac{\sigma_1-\sigma_3}{2} \le \frac{210}{2} \Rightarrow \frac{(1.5\sigma)-(-0.5\sigma)}{2} \le 105$$

$$\Rightarrow \quad 2\sigma \le 210 \Rightarrow \sigma \le 105 \text{ MPa.}$$

(iv) Maximum strain energy theorem: For no failure, maximum strain energy absorbed at a point should be less than or equal to total strain energy per unit volume under uniaxial loading, when material is subjected to stress upto elastic limit.

$$\text{Total strain energy} = \frac{\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)}{2E}$$

$$\text{Total strain energy per unit volume under uniaxial loading} = \frac{f_y^{\ 2}}{2E}$$

∴ According to this theory, $\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3) \le f_y^2$

$$\Rightarrow \quad (1.5\sigma)^2+(\sigma)^2+(-0.5\sigma)^2-2\times0.3(1.5\sigma\times\sigma-\sigma\times0.5\sigma-1.5\sigma\times0.5\sigma) \le 210^2$$

$$\Rightarrow \quad \sigma^2 \le \frac{210^2}{3.35}$$

$$\Rightarrow \quad \sigma \le 114.73 \text{ MPa}$$

(v) Maximum distortion energy theory: For no failure, maximum shear strain energy in a body should be less than maximum shear strain energy due to uniaxial loading.

$$\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \le f_y^2$$

$$\Rightarrow \quad \frac{1}{2}\left[(1.5\sigma-\sigma)^2+(\sigma+0.5\sigma)^2+(-0.5\sigma-1.5\sigma)^2\right] \le 210^2$$

$$\Rightarrow \quad 0.25\sigma^2+2.25\sigma^2+4\sigma^2 \le 2\times210^2$$

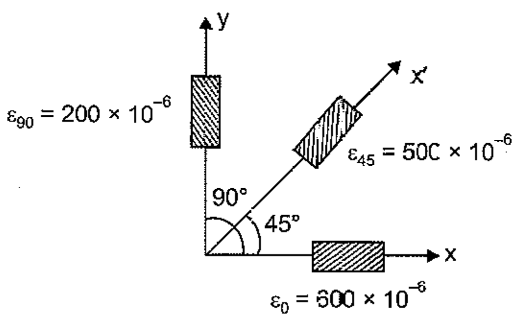
$$\Rightarrow \quad \sigma^2 \le \frac{2\times210^2}{6.5} \qquad \therefore \sigma \le \sqrt{\frac{2\times210^2}{6.5}}$$

$$\Rightarrow \quad \sigma \le 116.487 \text{ MPa}$$

Q-2: ***The strain measurements from a rectangular strain rosette were $e_0 = 600\times10^{-6}$, $e_{45} = 500\times10^{-6}$ and $e_{90} = 200\times10^{-6}$. Find the magnitude and direction of principal strains. If $E = 2\times10^5$ N/mm² and $\mu = 0.3$ find the principal stresses.***

[10 Marks, ESE-1995]

Sol:



We know that $$\varepsilon'_x = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \quad \text{... (i)}$$

and $$\varepsilon_{max}/_{min} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad \text{... (ii)}$$

Thus to determine the principal strain, we need normal strain in two mutually perpendicular direction and shear strain (γ_{xy}) associated with these directions.

From (i)

$$\varepsilon_{45} = \frac{\varepsilon_0 + \varepsilon_{90}}{2} + \frac{\varepsilon_0 - \varepsilon_{90}}{2}\cos(2\times 45^\circ) + \frac{\gamma_{xy}}{2}\sin(2\times 45^\circ) \qquad [\gamma_{xy} = \gamma_{0-90}]$$

$$500 \times 10^{-6} = \frac{600+200}{2}\times 10^{-6} + \frac{600-200}{2}\times 10^{-6}\times\cos 90^\circ + \frac{\gamma_{xy}}{2} \qquad \begin{bmatrix} \because \cos 90 = 0 \\ \sin 90 = 1 \end{bmatrix}$$

$$\Rightarrow \boxed{\gamma_{xy} = 200\times 10^{-6}}$$

From (ii)

$$\Rightarrow \varepsilon_{max}/_{min} = \frac{\varepsilon_0 + \varepsilon_{90}}{2} \pm \sqrt{\left(\frac{\varepsilon_0 - \varepsilon_{90}}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left(\frac{600+200}{2}\times 10^{-6}\right) \pm \sqrt{\left(\frac{600-200}{2}\right)\times 10^{-12} + \left(\frac{200}{2}\right)^2 \times 10^{-12}}$$

$$= [400 \pm \sqrt{(200)^2 + (100)^2}]\times 10^{-6} = [400 \pm 223.607]\times 10^{-6}$$

$$\Rightarrow \varepsilon_{max} = 623.607 \times 10^{-6} = \text{major principal strain}$$

$$\varepsilon_{min} = 176.393 \times 10^{-6} = \text{minor principal strain}$$

Also, we know that,

$$\tan 2\theta_P = \frac{\gamma_{xy}/2}{\varepsilon_x - \varepsilon_y/2} = \frac{200}{600-200} - \frac{200}{400} = \frac{1}{2}$$

$$\Rightarrow \theta_P = 13.282^\circ \text{ or } 103.282^\circ$$

One of these angles will be associated with major principal strain and other with minor principal strain.

To determine which of the angle is associated with major principal strain, let us put the value of θ_P in strain transformation eq.

$$\Rightarrow \varepsilon'_x = \frac{\varepsilon_0 + \varepsilon_{90}}{2} + \frac{\varepsilon_0 - \varepsilon_{90}}{2}\cos 2\theta_P + \frac{\gamma_{xy}}{2}\sin 2\theta_P$$

$$= \left[\frac{600+200}{2} + \frac{600-200}{2}\cos 2(13.282^\circ) + \frac{200}{2}\sin 2(13.482)\right]\times 10^{-6}$$

$$\Rightarrow \varepsilon'_x = 623.607 \times 10^{-6}$$

$$\Rightarrow \theta_P = 13.282^\circ \text{ is associated with major principal strain}$$

i.e., direction of major principal strain is at 13.282° in anticlockwise direction from ε_0 strain direction and hence direction of minor principal strain is at 103.282° in anticlockwise direction from ε_0 strain direction.

Calculation of principal stresses:

$$\frac{\sigma_{max}}{E} - \frac{\mu(\sigma_{min})}{E} = \varepsilon_{max}$$

$$\frac{\sigma_{min}}{E} - \frac{\mu\,\sigma_{max}}{E} = \varepsilon_{min}$$

$$\Rightarrow \quad \sigma_{max} - 0.3\,\sigma_{min} = 623.607 \times 10^{-6} \times 2 \times 10^5 \text{ N/mm}^2$$

$$\Rightarrow \quad \sigma_{max} - 0.3\,\sigma_{min} = 124.72 \text{ N/mm}^2 \quad \ldots \text{(i)}$$

$$\sigma_{min} - 0.3\,\sigma_{max} = 176.393 \times 10^{-6} \times 2 \times 10^5 \text{ N/mm}^2 \quad \ldots \text{(ii)}$$

$$\Rightarrow \quad 0.3\,\sigma_{min} - 0.09\,\sigma_{max} = 35.279 \times 0.3 \quad \ldots \text{(iii)}$$

From (i) + (iii)

$$(10.09)\sigma_{max} = 124.7 + 35.279 \times 0.3$$

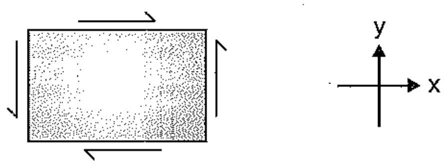
$$\Rightarrow \quad \boxed{\sigma_{max} = 148.685 \text{ N/mm}^2}$$

$$\Rightarrow \quad \boxed{\sigma_{min} = 79.885 \text{ N/mm}^2}$$

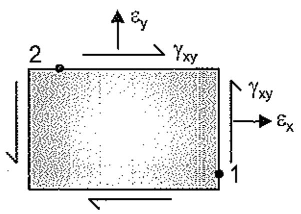
Alternative approach (Mohr circle approach):

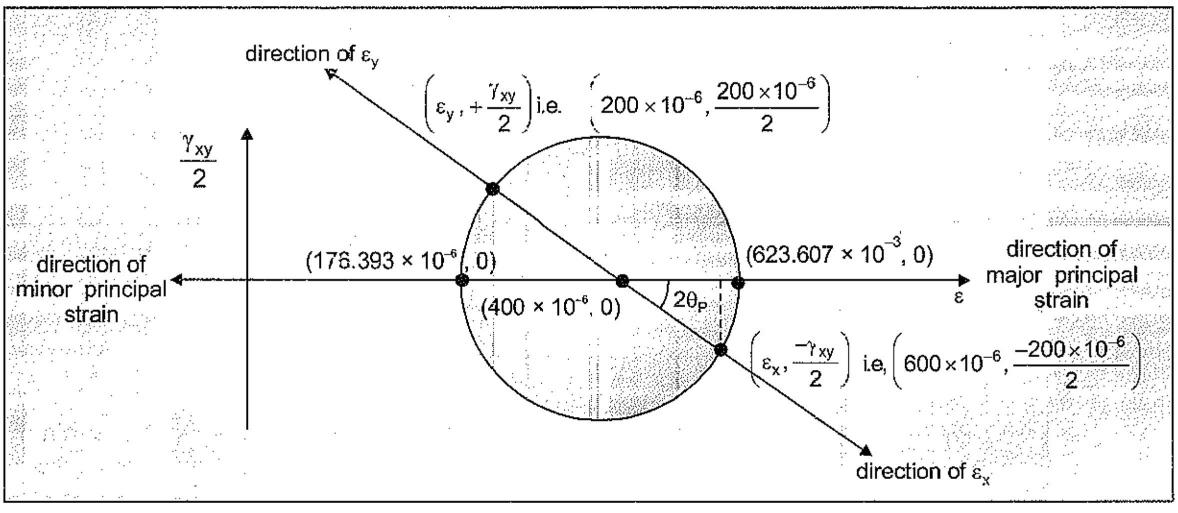
If we use Mohr transformation, we will not have to check which of the two angles 13.28° and 103.28° corresponds to major principal strain

By analytical approach we have found that γ_{xy} is (+)ve. This implies that it is associated with (+)ve shear stress as shown below.



Hence strains are shown as





$$\varepsilon_{max} = (400 \times 10^{-6}) + R$$

$$\varepsilon_{min} = (400 \times 10^{-6}) - R \qquad \text{[R = radius of circle]}$$

$$R = \left\{\sqrt{(600-400)^2 + \left(\frac{-200-0}{2}\right)^2}\right\} \times 10^{-6} = 223.607 \times 10^{-6}$$

$$\Rightarrow \quad \varepsilon_{max} = 623.607 \times 10^{-6}$$

$$\varepsilon_{min} = 176.393 \times 10^{-6}$$

$$\tan 2\theta_P = \frac{100}{600-400} = \frac{1}{2}$$

$$\sigma_P = 13.28°$$

⇒ Major principal strain is at 13.28° in anticlockwise direction from the direction of ε_x and minor principal strain is $13.28 + \frac{180}{2}$ = 103.28° in anticlockwise direction from the direction of ε_x.

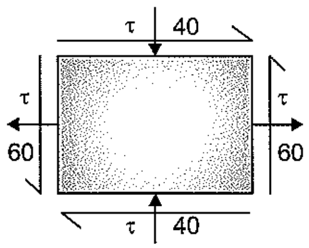
Q-3: ***What is Mohr's circle of stress?***

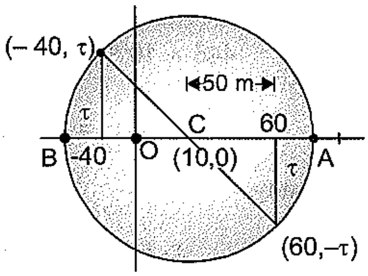
At a point in an elastic material, a direct tensile stress of 60 kN/mm² and a direct compressive stress of 40 kN/mm² are applied on planes at right angles to each other. If the maximum principal stress is limited to 75 N/mm² (tensile), find the shear stress that may be allowed on the planes. Also determine the minimum principal stress and the maximum shear stress.

[10 Marks, ESE-1998]

Sol: We know that state of stress at a point in a body can be expressed as a point on the circumference of a circle. Thus a circle is drawn in such a way that every point on the circumference of it represents state of stress on various inclined planes passing through the point. This circle is called a Mohr circle of stress.

Mohr circle of stress is used in the transformation of stresses i.e. to find out stress on any inclined plane by knowing the stress on two mutually perpendicular plane.





Mohr circle

If max principal stress is taken corresponding to point A then,

Maximum principal stress = Radius + OC = OA

$$\sqrt{\tau^2+50^2}+10 \le 75\,\text{N/mm}^2, \; \sqrt{\tau^2+50^2} \le 65\,\text{N/mm}^2, \quad \tau^2+50^2 \le 65^2, \quad \Rightarrow \quad \tau \le 41.533\,\text{N/mm}^2$$

If maximum principal stress is taken corresponding to point B then,

$$10 - \sqrt{\tau^2+(50)^2} \ge -75$$

$$\Rightarrow \quad \tau^2 + (50)^2 \le (85)^2$$

$$\tau \le 68.74\ \text{N/mm}^2$$

Thus from the above two cases,

$$\tau \leq 41.533 \text{ N/mm}^2$$

and hence Minimum principal stress $= -\text{Radius} + OC = -\sqrt{\tau^2+50^2}+10$

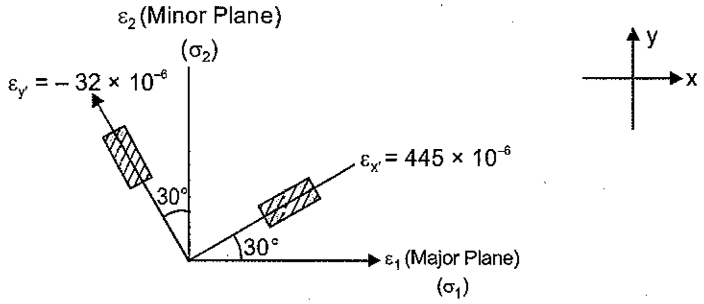
$$= -65 + 10 = -55 \text{ N/mm}^2$$

Maximum shear stress $= \text{Radius} = \sqrt{\tau^2+100^2} = (65)\text{N/mm}^2$

Q-4: ***In a stressed member, two strain gauges are fixed such that they are inclined at 30° to the known directions of the principal stresses. The strains measured in these two gauges are + 445 × 10⁻⁶ and – 32 × 10⁻⁶ respectively. E = 2.1 × 10⁵ N/sq. mm and Poisson's ratio (v) = 0.3. Determine the magnitude of the principal stress.***

[18 Marks, ESE-2001]

Sol:



Finding principal strains:

We know that
$$\varepsilon' = \frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta+\frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\gamma_{xy} = 0 \quad [\because \text{ Shear strain associated with principal axes} = 0]$$

So,
$$\varepsilon'_x = \frac{\varepsilon_1+\varepsilon_2}{2}+\frac{\varepsilon_1-\varepsilon_2}{2}\cos(2\times 30°)$$

$$\Rightarrow \quad 445 \times 10^{-6} = \frac{\varepsilon_1+\varepsilon_2}{2}+\frac{\varepsilon_1-\varepsilon_2}{4}$$

$$\Rightarrow \quad 3\varepsilon_1+\varepsilon_2 = 1780\times 10^{-6} \quad \text{... (i)}$$

And from the property.

$$\Rightarrow \quad \varepsilon_1+\varepsilon_2 = \varepsilon'_x+\in'_y$$

$$\Rightarrow \quad \varepsilon_1+\varepsilon_2 = 413\times 10^{-6} \quad \text{... (ii)}$$

By solving both there equation we get $\varepsilon_1 = +6.835\times 10^{-4}$, $\varepsilon_2 = -2.705\times 10^{-4}$

Now, we know that,

$$\varepsilon_1 = \frac{\sigma_1}{E}-\frac{\mu\sigma_2}{E}$$

and
$$\varepsilon_2 = \frac{\sigma_2}{E}-\frac{\mu\sigma_1}{E}$$

$$\therefore \quad (6.835\times 10^{-4}\times 2.1\times 10^5) = \sigma_1-\mu\sigma_2$$

$$\Rightarrow \quad \sigma_1 - 0.3\sigma_2 = 143.535 \quad \text{... (iii)}$$

Similarly $$\sigma_2 - 0.3\sigma_1 = -56.80 \quad \text{... (iv)}$$

By solving (iii) and (iv) $\sigma_1 = 139.005\ \text{N/mm}^2$ $\sigma_2 = -15.0984\ \text{N/mm}^2$

Alternative method

By using Mohr circles method

Generally we require three strain gauges to determine principal strains. In this question 2-strain gauges has been given and inclination of principal plane with the strain measuring planes given.

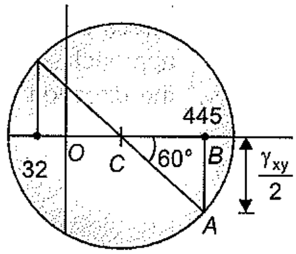
Hence, first we draw Mohr circle with,

$$\varepsilon_x = 445 \times 10^{-6}$$

$$\varepsilon_y = -32 \times 10^{-6}$$

and $$\gamma_{xy} = \text{unknown.}$$

Since principle plane is inclined at 30°. So in the Mohr circle, the corresponding angle will be 60°



$$\text{Co-ordinate of } C \equiv \left[\left\{\frac{445-32}{2}\right\}, 0\right] = (206.5, 0)$$

$$\tan 60° = \frac{(\gamma_{xy}/2)}{BC} \Rightarrow \left(\frac{\gamma_{xy}}{2}\right) = BC \tan 60°$$

$$\Rightarrow \quad \frac{\gamma_{xy}}{2} = (445 - 206.5)\tan 60° = 413.094$$

$$\therefore \quad \text{Radius of Mohr's circle} = \sqrt{\left(\frac{\gamma_{xy}}{2}\right)^2 + BC^2} = \sqrt{413.094^2 + (445-206.5)^2} = 477$$

$$\therefore \quad \text{Major principle strain} = \varepsilon_1 = OC + \text{Radius} = (206.5 + 477) \times 10^{-6} = 683.5 \times 10^{-6}$$

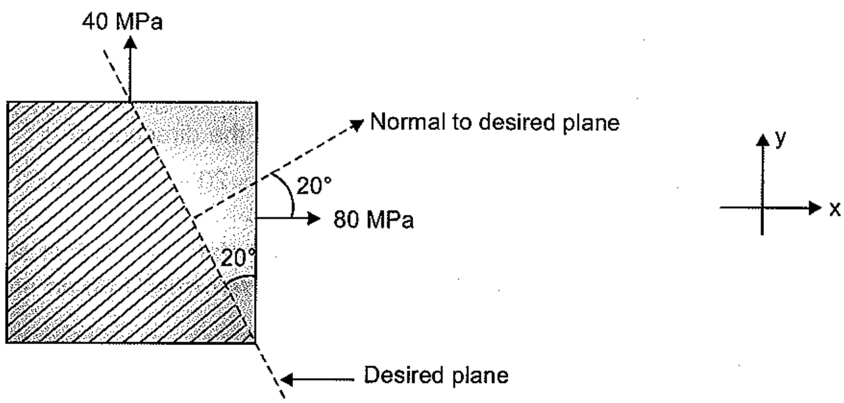
$$\varepsilon_2 = OC - R = (206.5 - 477) \times 10^{-6} = -270.5 \times 10^{-6}$$

Principle stress determination will be same as in 1st method.

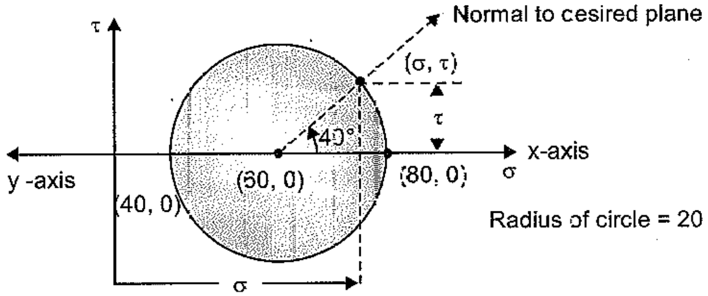
Q-5: ***The principal tensile stresses at a point are : σ_x = 80 MPa and σ_y = 40 MPa. Find the magnitudes of normal, tangential and the resultant stress on a plane at 20° with the major principal plane. What is angle of obliquity of the resultant stress with the major principal plane? Show by means of a sketch how normal stress (σ) and tangential stress (σ_t) act on the triangular element.***

[10 Marks, ESE-2005]

Sol: The stress element is as shown below



Using Mohr circle approach

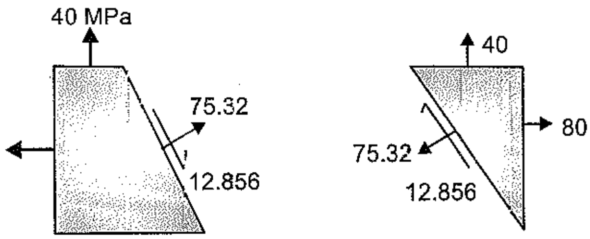


$$\text{Normal stress on desired plane} = \sigma = 60 + R\cos 40° = 60 + 20\cos 40° = 75.32\text{ MPa}$$

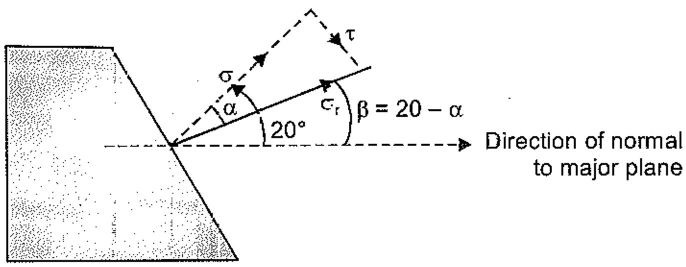
$$\text{Shear stress on desired plane} = \tau = 20\sin 40° = 12.856\text{ MPa}$$

$$\text{Resultant stress on the plane} = \frac{\sqrt{(\sigma.A)^2 + (\tau.A)^2}}{A} = \sqrt{\sigma^2 + \tau^2} = 76.41\text{ MPa}$$

Stress on the element are thus as shown below. As the shear stress is (+)ve in Mohr circle approach, this means that the shear will give clockwise rotation about the centre of element. Hence the direction of shear stress is as shown below.



Angle of obliquity (α) is actually the angle between resultant stress and the normal to the plane



$$\tan\alpha = \frac{\tau}{\sigma} = \frac{12.856}{75.32}$$

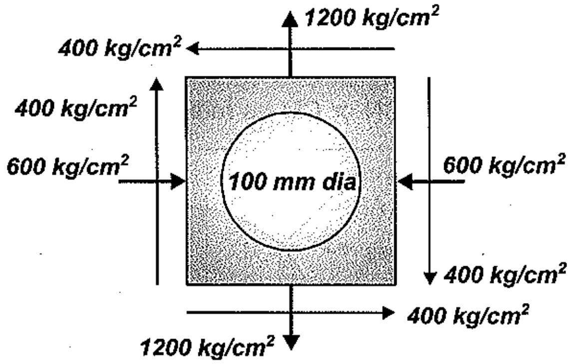
$$\alpha = 9.686°$$

However in the question we have been asked to find out the angle of obliquity w.r.t major principal plane i.e., angle between the normal to the major plane and the direction of σ_r

Thus angle of obliquity w.r.t major plane = $\beta = 20 - \alpha = 20 - 9.656 = 10.314°$

Q-6: ***A circle of 100 mm diameter is inscribed on a steel plate before it is stressed and then the plate is loaded as shown in the figure. Then the circle is deformed into an ellipse. Determine the major and minor axes of the ellipse and their directions.***

$E = 2.1 \times 10^6$ kg/cm^2; $1/m = 0.28$



[10 Marks, ESE-2006]

Sol: **Given:** The circle is deformed into an ellipse.

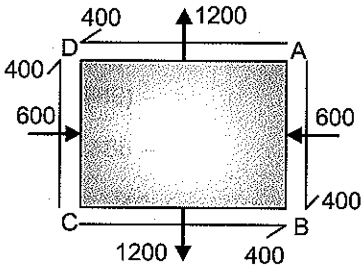
Determine: Major and minor axes of the ellipse and their directions.

Note: To solve this question two important things are necessary to understand.

(i) The direction of major and minor axes is in the direction of major and minor principal stresses. Because the maximum and minimum longitudinal strains take place along the major and minor principal planes.

(ii) To determine the length of major and minor axes, we have to determine strain in that direction.

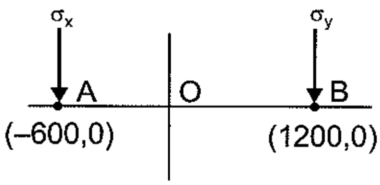
Step 1: Determining direction and magnitude of principal planes.



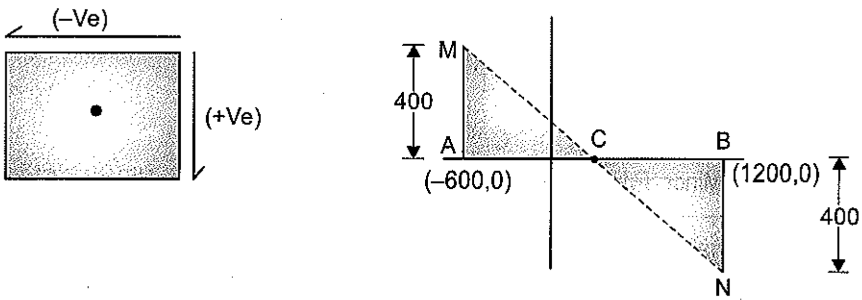
Conceptual background:

(Steps for drawing Mohr circle)

(a) Locating σ_x and σ_y

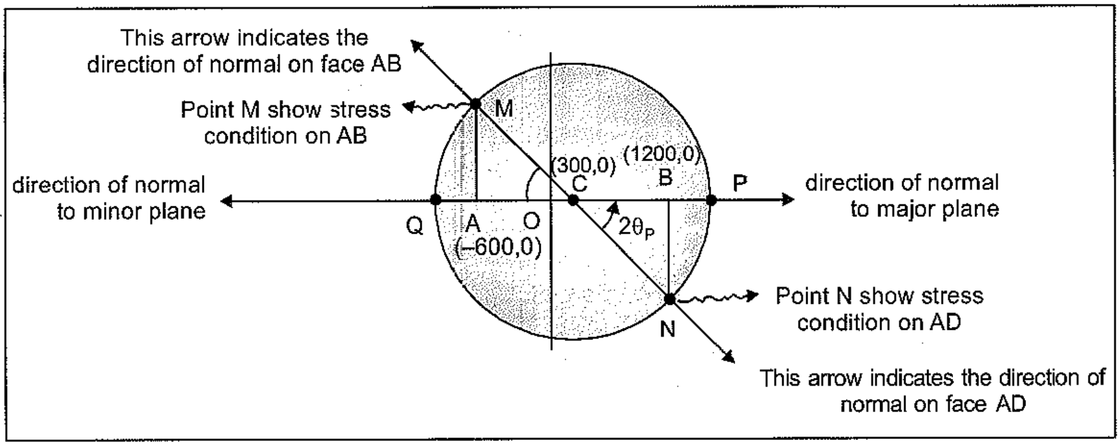


(b) Drawing shear stress as per convention for mohr circle



(c) In previous figure co-ordinate of centre 'C' is $\left(\frac{1200-600}{2}, 0\right) \equiv (300, 0)$

Considering C as centre and CN or CM as radius draw the Mohr's circle as shown below.



⇒ Determination of σ_{max} and σ_{min} with their directions.

$$\text{Radius} = \sqrt{BC^2 + BN^2} = \sqrt{900^2 + 400^2} = 984.89$$

Hence $\sigma_{max} = OP = OC + CP = OC + \text{radius}$

$= 300 + 984.89 = 1284.89 \text{ kg/cm}^2$

and, $\sigma_{min} = OQ = -(CQ - CO) = -(\text{Radius} - CO)$

$= -(984.89 - 300)$

$\sigma_{min} = -684.89 \text{ kg/cm}^2$

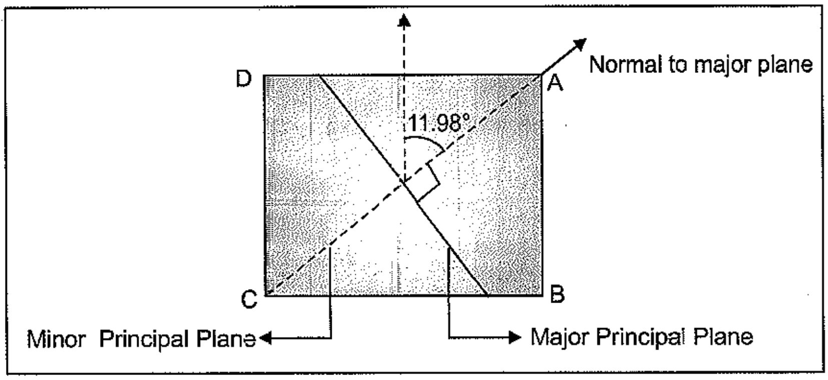
Direction, $\tan 2\theta_P = \frac{BN}{BC} = \frac{400}{900} = \frac{4}{9}$

∴ $2\theta_P = 23.96°$

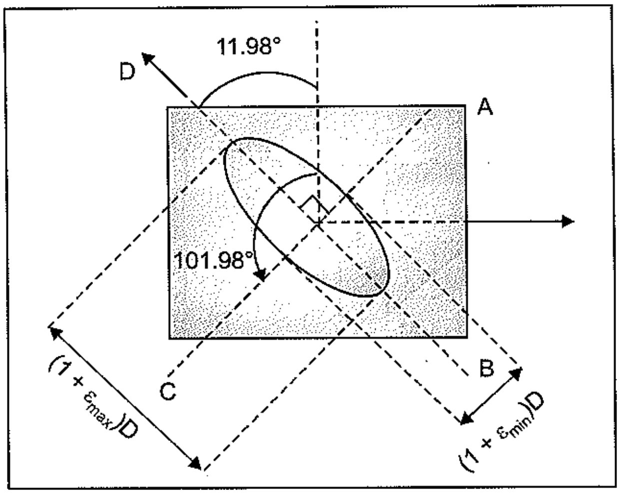
⇒ $\theta_P = 11.98°$

Major principal planes inclined at an angle 11.98° in anticlockwise direction from normal to face AD

Minor principal planes inclined at an angle (90 + 11.98°) = 101.98° in anticlockwise direction from normal to face AD



Since major axes of ellipse will be in the direction of major principal stress and minor axes of ellipse in the direction of minor principal stress. Hence the deformed shape of circle becomes as shown in figure below.



Since, $\epsilon_{max} = \dfrac{\sigma_{max}}{E} - \dfrac{\mu\sigma_{min}}{E} = \dfrac{1284.89}{2.1\times10^6} - \dfrac{0.28\times(-684.89)}{2.1\times10^6} = 7.0317\times10^{-4}$

and $\epsilon_{min} = \dfrac{\sigma_{min}}{E} - \dfrac{\mu\sigma_{max}}{E} = \dfrac{-684.89}{2.1\times10^6} - \dfrac{0.28\times1284.89}{2.1\times10^6} = -4.974\times10^{-4}$

Hence major axis of ellipse $= (1+\varepsilon_{max})D = \left[\left(1+7.0317\times10^{-4}\right)\times100\right]\text{mm} = 100.0703 \text{ mm}$

Minor axis of ellipse $= (1+\varepsilon_{min})\,D = \left(1-4.974\times10^{-4}\right)\times100 = 99.95 \text{ mm}$

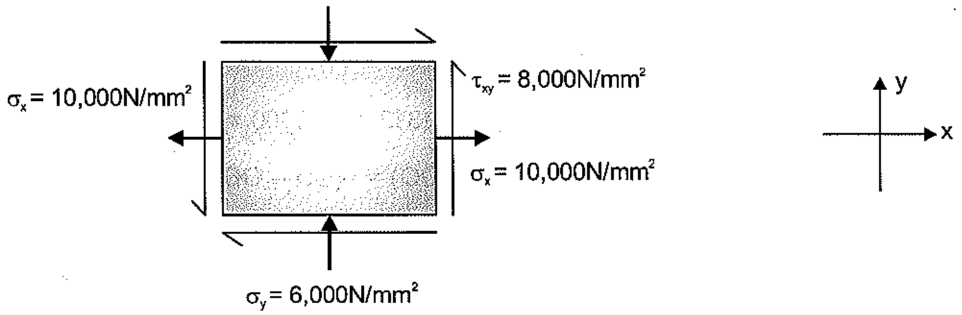
Q-7: ***The stresses in a flat steel plate in a condition of plane stress are:***

σ_x = 10,000 N/mm²; σ_y = –6000 N/mm²; τ_{xy} = 8000 N/mm²

Find the magnitude and orientation of the principal stresses in the plane of the plate.

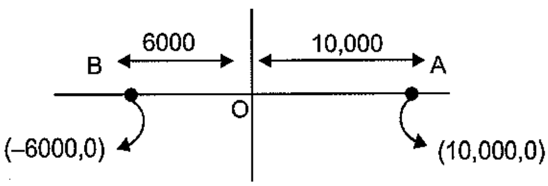
[15 Marks, ESE-2006]

Sol:

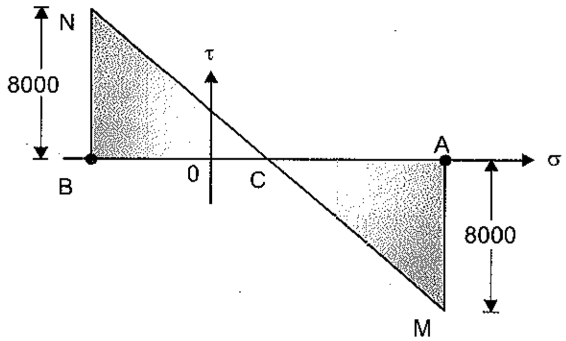


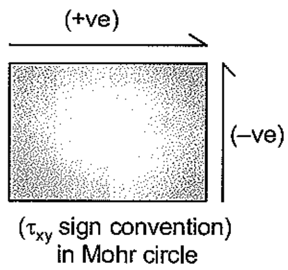
Drawing Mohr circle:

Step 1: Locating σ_x and σ_y



Step 2: Drawing parallel lines to the y-axis from point A and B upto 8000 (τ_{xy}) units and match the points M and N as shown below.





Step 3: Finding the co-ordinate of centre (C)

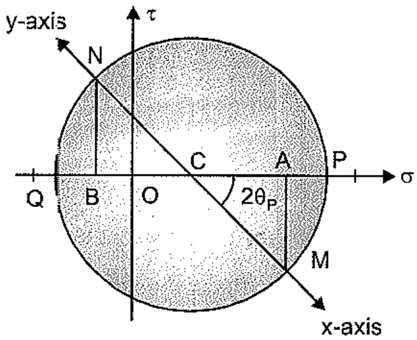
Since C is the middle point of point A (10,000,0) and point B (–6000,0).

$$\therefore \qquad \text{Centroid} = (C) = (2000,0)$$

$$\therefore \qquad \text{Radius (R)} = \sqrt{CA^2 + AM^2} = \sqrt{(10000-2000)^2 + (8000)^2}$$

$$= 8000\sqrt{2} = 11313.71$$

Step 4: Assuming C as a centre, drawing circle with radius CM.



Now we have drawn Mohr's circle

$$\therefore \qquad \text{Major principal stress } (\sigma_1) = OC + CP = 2000 + \text{Radius}$$

$$\therefore \qquad \sigma_1 = 11313.71 + 2000 = 13313.71 \text{ N/mm}^2$$

$$\text{Minor principal stress} = (OQ) = -(CQ - OC) = -(\text{Radius} - OC) = OC - \text{radius}$$

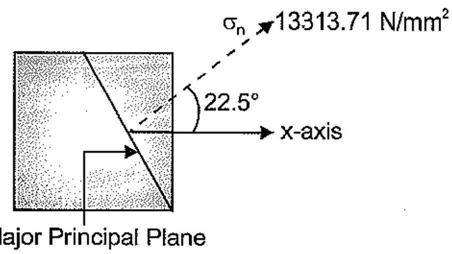
$$= 2000 - 11313.79 = -9313.71 \text{ N/mm}^2$$

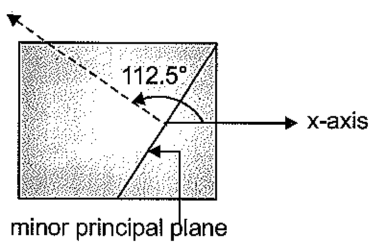
Orientation

$$\tan 2\theta_P = \frac{8000}{8000}$$

$$\therefore \qquad 2\theta_P = 45°, 225°$$

$$\therefore \qquad \theta_P = 22.5°, 112.5°$$





Alternatively:

Analytical method: $\sigma_x = 10{,}000\,\text{N/mm}^2$, $\sigma_y = -6000\,\text{N/mm}^2$, $\tau_{xy} = 8000\,\text{N/mm}^2$

$$\therefore \quad \sigma_1/\sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\therefore \quad \sigma_1/\sigma_2 = \frac{10{,}000 - 6000}{2} \pm \sqrt{\left[\frac{16000}{2}\right]^2 + 8000^2}$$

$$= 2000 \pm \sqrt{8000^2 + 8000^2} = 2000 \pm 8000\sqrt{2}$$

$$= 13313.71 \text{ N/m}^2 \quad \text{and,} = -9313.71 \text{ N/mm}^2$$

$\therefore$ Major principal stress, $(\sigma_1) = 13{,}313.71$ N/mm^2

Minor principal stress, $(\sigma_2) = -9313.71$ N/mm^2

Orientation, $$\frac{\sin 2\theta_P}{\cos 2\theta_P} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\therefore \quad \tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\therefore \quad \tan 2\theta_P = \frac{2 \times 8000}{16000} = 1$$

$$\therefore \quad \theta_P = 22.5^\circ\ ;\ 112.5^\circ$$

At this stage we have to ascertain which θ_p corresponds to major principal stress and which one corresponds to minor principal stress.

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

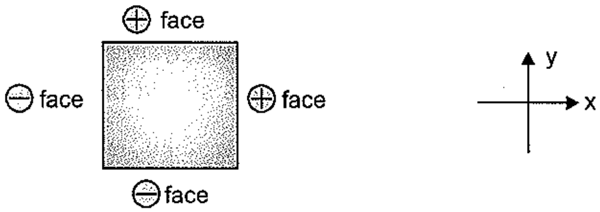
taking $\theta_P = 22.5^\circ$

$$\sigma = \frac{1000 - 6000}{2} + \frac{10000 + 6000}{2}\cos 45^\circ + 8000 \sin 45^\circ$$

$= 13313.71$ i.e., major principal stress

$\Rightarrow$ $\theta_P = 22.5^\circ$ corresponds to major principal stress and 112.5° corresponds to minor principal stress

τ_{xy} is (+)ve in analytical method because we have the sign convention that shear stress on (+)ve face in (+) co-ordinate direction is (+)ve.



Q-8: ***Describe any 'five' Theories of Failure (Elastic) giving necessary formulae and comparing with the values obtained from direct tension test.***

[15 Marks, ESE-2007]

Sol: Under uniaxial loading prediction of failure is easy, but when loading is complicated predicting failure becomes complicated and we need to check the various possible modes of failure.

To define these modes, various theory of failure are used.

(i) Maximum principle stress theory:

- It is also called Rankine's theory or **lame's theory** or maximum stress theory.
- As per this, for no failure, maximum principal stress should be less than yield stress under uniaxial loading.

if σ_1 = Max^m principal stress

then $\sigma_1 \leq f_y$

For design purpose, $\sigma_1 \leq \dfrac{f_y}{F.O.S}$

- This theory is suitable for brittle material and not suitable for ductile material.
- It is not suitable for hydrostatic loading
- It is not suitable in case of pure shear.

(ii) Maximum principal strain theory:

- It is also called Saint Venant theory
- As per this, for no failure, maximum principal strain should be less than yield strain under uniaxial loading.

For no failure

(a) In 1-D condition, $\varepsilon_{max} \leq \dfrac{f_y}{E}$ and for design $\varepsilon_{max} \leq \dfrac{f_y/FOS}{E}$

(b) In 2-D condition, $\varepsilon_{max} \leq \dfrac{f_y}{E}$ i.e., $\left(\dfrac{\sigma_1 - \mu\sigma_3}{E}\right) \leq \dfrac{f_y}{E}$ and for design purpose, $\dfrac{\sigma_1 - \mu\sigma_3}{E} \leq \dfrac{f_y/FOS}{E}$

(c) In 3-D condition, $\epsilon_{max} \leq \dfrac{f_y}{E}$ i.e, $\dfrac{\sigma_1 - \mu(\sigma_2 + \sigma_3)}{E} \leq \dfrac{f_y}{E}$ and for design purpose $\sigma_1 - \mu(\sigma_2 + \sigma_3)$ $\leq \dfrac{f_y}{FOS}$

- It is not applicable for hydrostatic loading for engineering material.
- It can't be applied for pure shear.
- It overestimates the elastic strength of ductile material.

(iii) Maximum shear stress theory:

- It is also called Tresca or Gusset or Coulomb theory.
- As per this, for no failure, maximum shear stress should be less than or equal to max^m shear stress under uniaxial loading i.e., $\left\{\dfrac{f_y}{2}\right\}$

For 3-D condition absolute max^m shear stress should be less than or equal to $\left\{\dfrac{f_y}{2}\right\}$,

i.e., max $\{|\sigma_1 - \sigma_3|, |\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|\} < f_y$, where σ_1, σ_2, σ_3, are principal stresses

For 2-D condition

$$\frac{|\sigma_1 - \sigma_3|}{2} \leq \frac{f_y}{2}$$

- This theory is applicable for ductile material
- This method gives most conservative design out of various theories of failure.
- It is not suitable for hydrostatic loading.

(iv) Maximum strain energy theory:

- It is also called Beltrami Haigh's theory.
- As per this, for no failure, maximum strain energy per unit volume absorbed at a point should be less than or equal to total strain energy per unit volume under uniaxial loading, when material is subjected to stress upto elastic limit.

If $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses and $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are the principal strains.

then $$U \le \frac{fy^2}{2E}$$

For 3D condition

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \le \frac{f_y^2}{2E}$$

For 2D condition

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\right] \le \frac{f_y^2}{2E}$$

- This theory is most frequently used for thin shells which is under plane stress condition.
- It is applicable for ductile material and not suitable for brittle material.

(v) Maximum shear strain energy theory

- It is also called distortion energy theory/Huber-Henky/Vonmises theory
- As per this, for no failure maximum shear strain energy in a body should be less than maximum shear strain energy due to uniaxial loading.

Distortion energy is given by $\frac{1}{12G}[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2]$

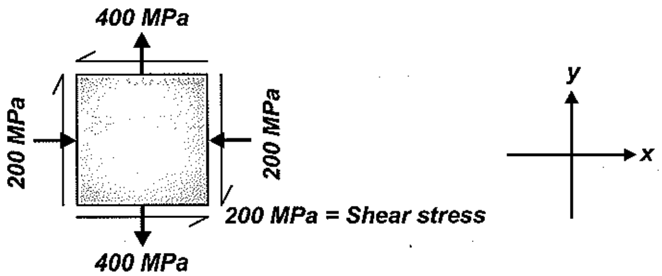
Hence for no failure,

$$\frac{1}{12G}[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2] \le \frac{1}{12G}[f_y^2 + f_y^2]$$

$$\Rightarrow \quad \frac{1}{2}[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2] \le f_y^2$$

- It is not applicable for brittle material
- This theory is most appropriate for case of pure shear in ductile material.
- It can't be applied in hydrostatic condition

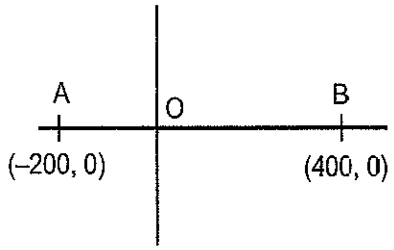
Q-9: ***Given the state of stress shown in the figure below, find the (i) principal stresses, and (ii) maximum shearing stresses and the associated normal stresses. Calculate the principal planes and maximum shear stress plane.***



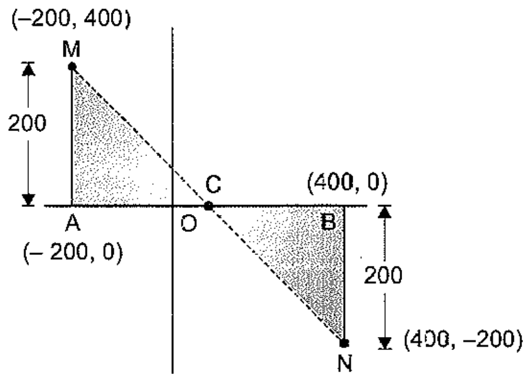
[10 Marks, ESE-2007]

Sol: **Drawing Mohr circle,** (σ_x = – 200, σ_y = 400)

Step-1: Plotting the point

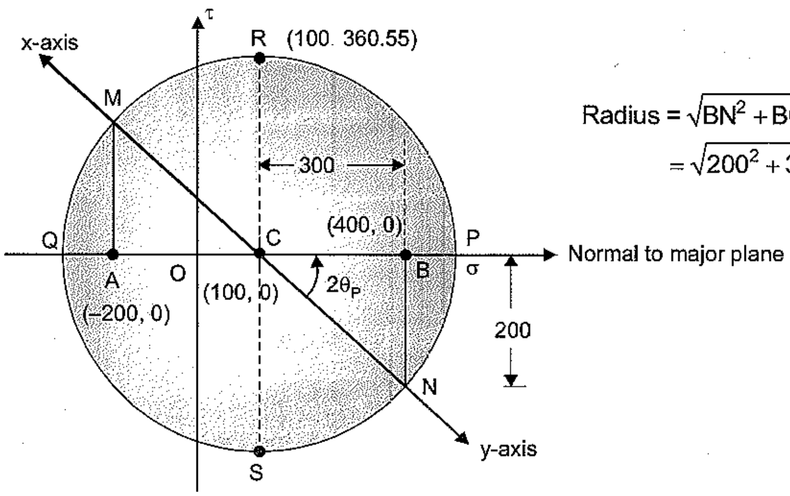


Step 2: Showing shear stress



Step 3: Finding the co-ordinate of centre 'C' $\equiv \left(\dfrac{400-200}{2}, 0\right) = (100, 0)$

Step 4: Draw the circle considering C as centre and CM or CN as radius.



$$\text{Radius} = \sqrt{BN^2 + BC^2}$$
$$= \sqrt{200^2 + 300^2} = 360.55$$

Determination of principal stresses and the planes.

σ_{major} = OP = OC + CP = OC + Radius = 100 + 360.55 = 460.55 MPa

σ_{minor} = OQ = – (CQ – CO) = – (Radius – 100) = – (360.55 – 100) = –260.55 MPa

Inclination of principal planes

(for σ_{major}) $\tan 2\theta_P = \dfrac{200}{300}$ $\therefore$ $2\theta_P = 33.69°$ $\Rightarrow$ $\theta_P = 16.845°$

(for σ_{minor}) inclination = $90 + \theta_P = 106.845°$

$\Rightarrow$ Normal to the major plane makes an angle of 16.845° with y-axis in anticlockwise direction. Similarly normal to minor plane makes an angle of 106.845° in anticlokwise direction from y axis.

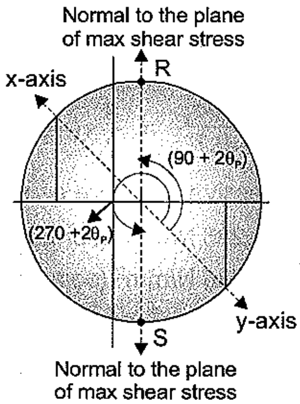
τ_{max} corresponds to point R and S

$\Rightarrow$ Max shear stress = 360.55 MPa

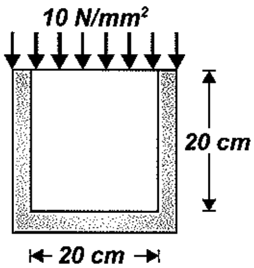
Normal stress on the plane of max shear stress = 100 MPa

Normal to the plane of max shear stress makes an angle of $\frac{90+2\theta_P}{2}$ and $\frac{270+2\theta_P}{2}$ in anticlockwise direction from y-axis

i.e., 61.845° and 151.845°

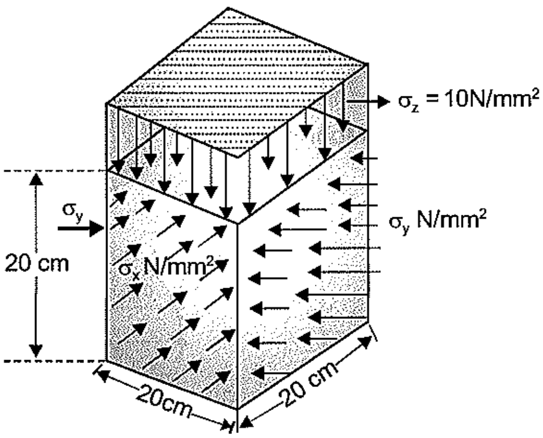


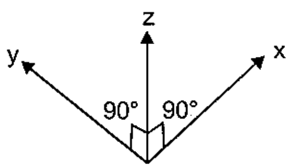
Q-10: ***A homogeneous isotropic material of size 20 cm × 20 cm × 20 cm is placed inside a rigid box of internal dimensions 20 cm × 20 cm × 20 cm. The top of the box is open and the material inside is subjected to a uniform compressive stress of 10 N/mm² from the top. Assuming there is no friction between the surface of contact, find the stresses on the side faces of the material. Find also the maximum shear stress, the normal stress on the plane of maximum shear stress and the change of volume of the material. Young's modulus E is 1 × 10⁵ N/mm² and Poisson's ratio μ = 0.3 for the material.***



[15 Marks, ESE-2008]

Sol:





When we apply σ_z (10N/mm²) stress uniformly distributed to the top of the given cube, the cube tend to push the wall in lateral direction, i.e, x and y. Since the wall is rigid, reaction force generated on the xz face and yz face of the cube is in such a way that the net strain in x and y direction becomes zero.

$$\varepsilon_x = \varepsilon_y = 0$$

Since cube is geometrically symmetric and the force σ_z is also applied symmetrically, so $\sigma_x = \sigma_y$.

$$\varepsilon_x = 0$$

$$\Rightarrow \quad \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} = 0$$

$$\Rightarrow \quad \frac{\sigma_x(1-\mu)}{E} = \frac{\mu\sigma_z}{E} \qquad [\because \sigma_x = \sigma_y]$$

$$\therefore \quad \sigma_x = \left\{\frac{\mu\sigma_z}{1-\mu}\right\} = \frac{0.3\sigma_z}{0.7} = \frac{3}{7}\times(-10) = -4.286\ \text{N/mm}^2$$

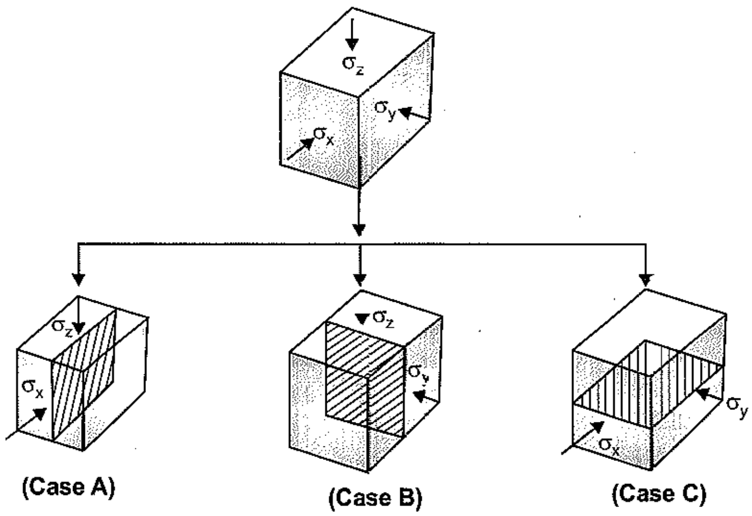
(–ve) sign of σ_x indicates it is compressive in nature as shown above.

$$\therefore \quad \sigma_x = 4.286\ \text{N/mm}^2\ \text{(compressive)}$$

$$\sigma_y = 4.286\ \text{N/mm}^2\ \text{(compressive)}$$

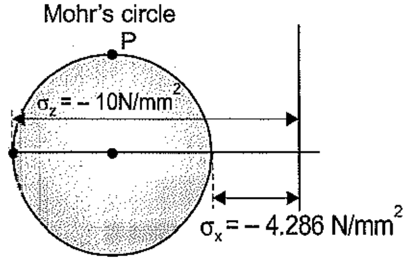
$$\sigma_z = 10\ \text{N/mm}^2\ \text{(compressive)}$$

Since principal stress analysis can be done in plane stress condition,
Hence, consider



In each case we find the maximum shear stress,

For case A: Mohr's circle

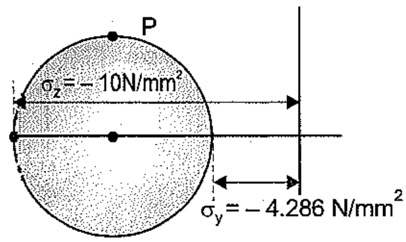


$$\text{Radius} = \frac{10-4.286}{2} = 2.857\ \text{unit}$$

Hence, τ_{max} = (y-coordinate of point p on Mohr circle)

$$= 2.857\ \text{N/mm}^2$$

For case B: Mohr's circle

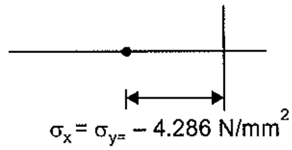


$$\text{Radius} = \frac{10-4.286}{2} = 2.857\ \text{unit}$$

Hence, $\tau_{max} = 2.857\ \text{N/mm}^2$

For case C: Mohr's circle

Since $\sigma_x = \sigma_y$, Mohr's circle will be the point



$\therefore$ Radius = 0

$\therefore$ $\tau = 0$

Hence in case 'A' and case 'B' $\tau_{max} = 2.857$ N/mm^2

and the normal stress at the same plane is the x - coordinate of point P = – (4.286 + radius)

$$= -(4.286 + 2.857) = -7.143 \text{ N/mm}^2 \text{ (compressive)}$$

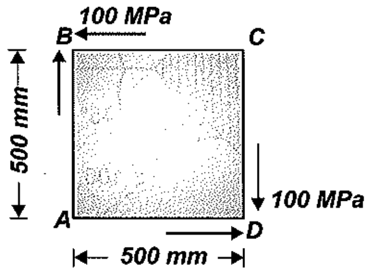
$$\text{Volumetric strain, } \frac{\Delta V}{V} = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

$$\therefore \quad \frac{\Delta V}{V} = \frac{-4.286 - 4.286 - 10}{1 \times 10^5}(1 - 2 \times 0.3)$$

$$\therefore \quad \frac{\Delta V}{20 \times 20 \times 20} = -7.424 \times 10^{-5}$$

$\therefore$ $\Delta V = -0.59392$ cm^3. (–ve sign indicates that there is vol. reduction)

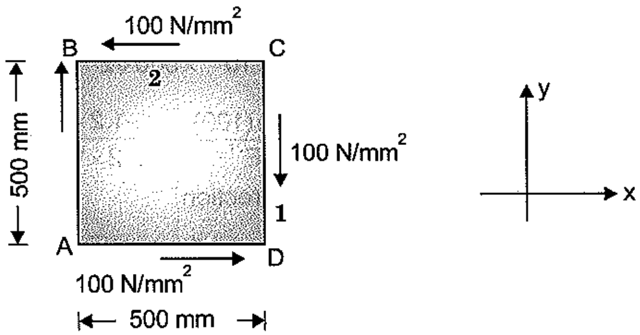
Q-11: ***A square plate of side 500 mm is subjected to pure shear of intensity 100 MPa as shown in the figure.***



Young's modulus of the material is 2 × 10⁵ MPa and Poisson's ratio is 0.2. Find the principal stresses, their directions and the change in lengths of the diagonals of the plate.

[10 Marks, ESE-2012]

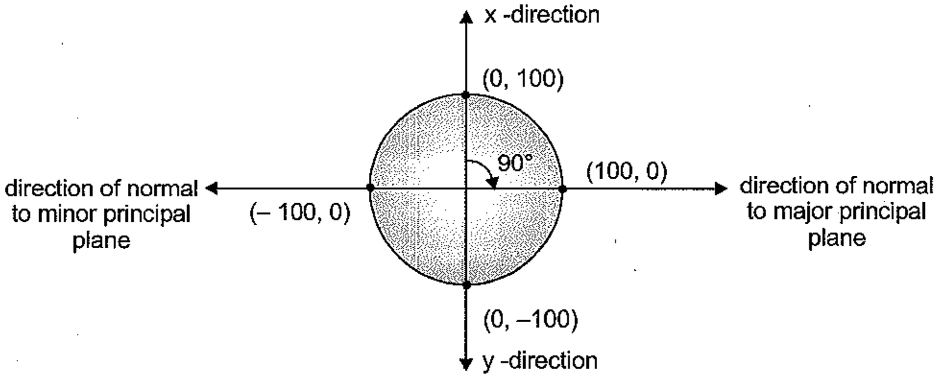
Sol:



$$E = 2 \times 10^5 \text{ MPa}$$

$$\mu = 0.2$$

The above figure is the case of pure shear. In pure shear case, the Mohr circle has centre at origin and radius equal to shear stress magnitude.



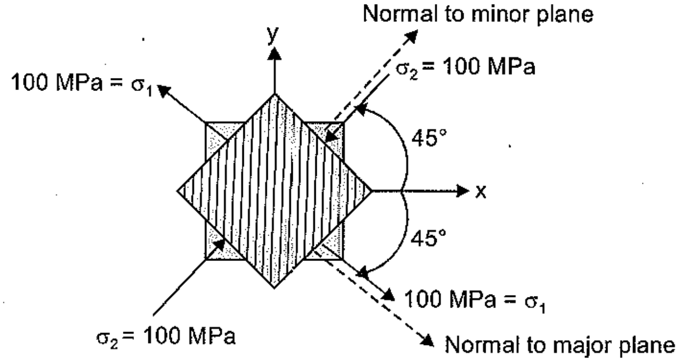
$$\text{Major principal stress} = \boxed{100\ \text{N/mm}^2 = \sigma_1}$$

$$\text{Minor principal stress} = \boxed{-100\ \text{N/mm}^2 = \sigma_2}$$

$$\text{Direction of major principal stress} = \frac{90}{2} = 45° \text{ in clockwise direction from x-direction}$$

and direction of minor principal stress is 45° in anticlockwise direction to x-direction.

$\because$ If a plane makes an angle of θ_P with principal plane, in Mohr's circle it makes and angle $2\theta_P$



Change in lengths of diagonals

$$\text{Diagonal length} = \sqrt{500^2 + 500^2} = 500\sqrt{2}\ \text{mm}$$

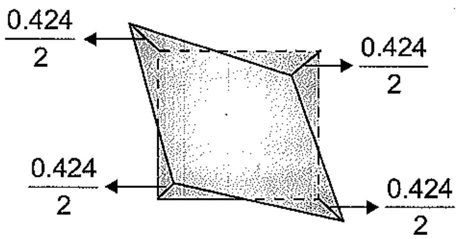
$$\text{Strain in the major stress direction} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} = \frac{100}{2\times10^5}(1+0.2) = 6\times 10^{-4}$$

$\Rightarrow$ Extension in the major stress direction $= 6\times10^{-4}\times500\sqrt{2} = 0.424$ mm

$$\varepsilon_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E} = \frac{-100}{E} - \frac{(0.2)(100)}{E} = \frac{-1.2\times100}{2\times10^5} = -6\times10^{-4}$$

ε_2 = Strain in minor stress direction

$\therefore$ Contraction in minor stress direction $= -6\times10^{-4}\times500\sqrt{2} = -0.424$ mm



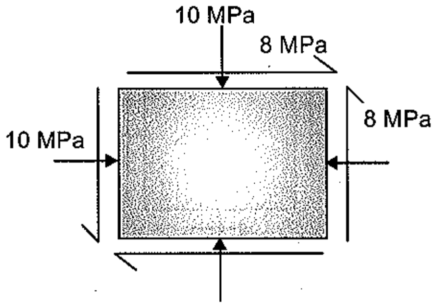
Q-12: ***In a 2-D body, normal stress σ_x = –10 MPa, σ_y = –10 MPa, and shear stress τ_{xy} = 8 MPa. Draw the Mohr circle and determine the principal stresses, principal planes, maximum shear stresses and their planes.***

[10 Marks, ESE-2014]

Sol: Given,

$$\sigma_x = -10 \text{ MPa}$$
$$\sigma_y = -10 \text{ MPa}$$
$$\tau_{xy} = 8 \text{ MPa}$$

∴ Stress element will be like

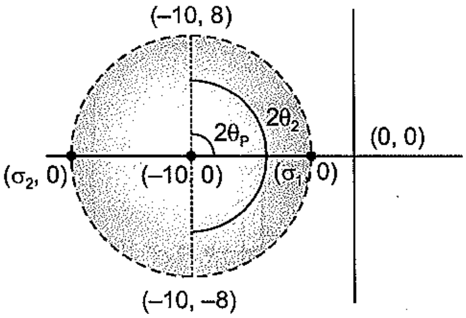


To draw Mohr circle for this stress element, we have to find centre and radius of Mohr circle.

$$\text{Centre, C} = \left(\frac{\sigma_x + \sigma_y}{2}, \frac{\tau_{xy} + \sigma_{yx}}{2}\right) = \left(\frac{-10-10}{2}, \frac{8-8}{2}\right) = (-10,\ 0)$$

$$\text{Radius, R} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-10+10}{2}\right)^2 + 8^2} = 8 \text{ unit}$$

Therefore, Mohr circle will be



It can easily be seen from the Mohr circle that principal stresses σ_1 and σ_2 are

$$\sigma_1 = -10 + 8 \qquad [\because \text{ radius of mohr circle} = 8 \text{ unit}]$$
$$= -2 \text{ MPa}$$
$$\sigma_2 = -10 - 8 = -18 \text{ MPa}$$

From Mohr circle

$$2\theta_{P_1} = 90° \qquad \begin{bmatrix}\theta_{P_1} \\ \theta_{P_2}\end{bmatrix} = \text{orientation of principal planes}$$

∴ $$\theta_{P_1} = 45°$$

and $$\theta_{P_2} = \theta_{P_1} + 90° \qquad \left[\because \text{Principal planes are perpendicular to each other}\right]$$
$$= 135°$$

Again maximum shear stress,

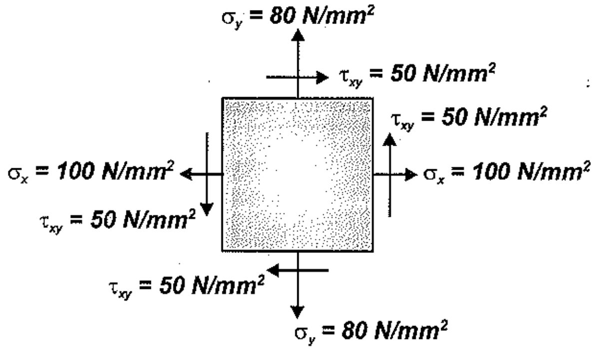
$$\tau_{max} = 8\text{MPa} \qquad \text{[From Mohr circle]}$$

and it is inclined at $\theta_1 = 0$ [θ_2 is shown in mohr circle]

and $2\theta_2 = 180°$

$\therefore$ $\theta_2 = 90°$

Q-13: ***A plane element of a body is subjected to stresses as shown in the figure.***



Find the factor of safety as per the following theories, if the yield stress given for the material is 200 N/mm² and Poisson's ratio = 0.3:

(i) Maximum principal stres thoery

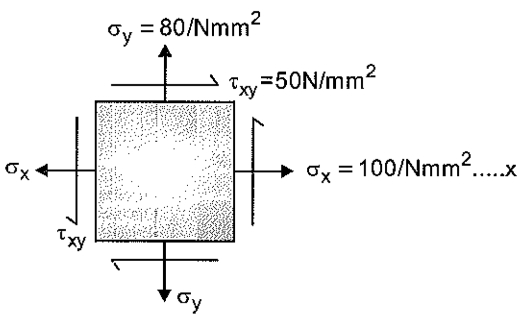
(ii) Maximum principal strain theory

(iii) Maximum shear stress theory

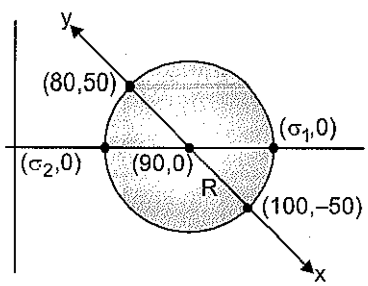
(iv) Maximum strain energy theory.

[15 Marks, ESE-2015]

Sol: Given element:



Representing in Mohr circle:



$$R = \sqrt{(100-90)^2 + (-50-0)^2} = 50.99 \simeq 51$$

$$\Rightarrow \quad \sigma_1 = 90 + R = 90 + 51 = 141 \text{ N/mm}^2 = \text{Max. Principal stress}$$

$$\sigma_2 = 90 - R = 90 - 51 = 39 \text{ N/mm}^2 = \text{Min. Principal stress}$$

$$f_y = 200 \text{ N/mm}^2$$

$$\mu = 0.3$$

(i) As per Max. Principal stress theory,

$$\sigma_{max} = \frac{f_y}{FOS}$$

$$\Rightarrow \quad 141 = \frac{200}{FOS}$$

$$\Rightarrow \quad FOS = 1.418$$

(ii) As per Max. Principal strain theory,

$$\varepsilon_{max} = \frac{\sigma_{max}}{E} - \mu\frac{\sigma_{min}}{E} = \frac{(f_y/FOS)}{E}$$

$$\Rightarrow \quad \frac{141}{\cancel{E}} - \frac{0.3 \times 39}{\cancel{E}} = \frac{200}{\cancel{E} \times FOS}$$

$$\Rightarrow \quad FOS = 1.547$$

(iii) As per Max. shear stress theory,

$$\tau_{max} = \max\left\{\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_1|}{2}, \frac{|\sigma_2|}{2}\right\} = \frac{f_y/FOS}{2}$$

$$\Rightarrow \quad \max\left\{\frac{141-39}{2}, \frac{141}{2}, \frac{39}{2}\right\} = \frac{200}{2 \times FOS}$$

$$\Rightarrow \quad \max\{51, 70.5, 19.5\} = \frac{100}{FOS}$$

$$\Rightarrow \quad FOS = \frac{100}{70.5} = 1.418$$

(iv) As per Max. strain energy theory,

$$U_{max} = \frac{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2}{2E} = \frac{(f_y/FOS)^2}{2E}$$

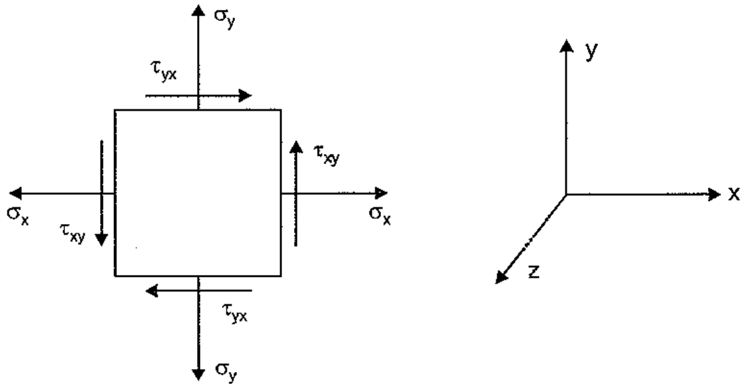
$$\Rightarrow \quad 141^2 + 39^2 - 2 \times 0.3 \times 141 \times 39 = \left(\frac{200}{FOS}\right)^2$$

$$\Rightarrow \quad FOS = 1.487$$

Q-14: ***Explain with appropriate figures the failure criteria based on maximum shear stress theory. Also draw its Mohr circle representation.***

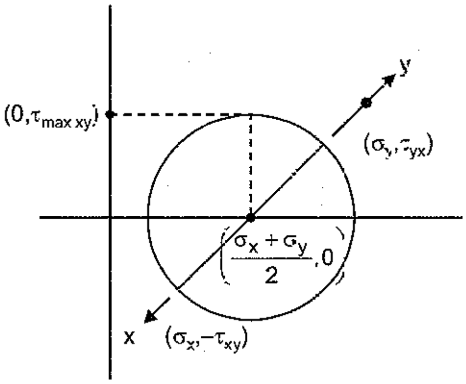
[15 Marks, ESE-2016]

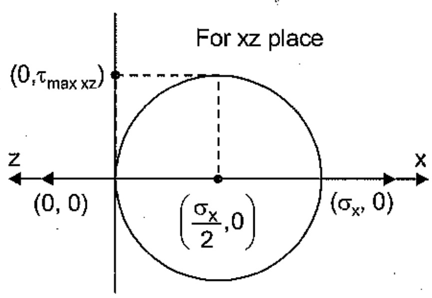
Sol: Maximum shear stress theory: As per this theory, for the safety of a material, the maximum shear stress induced in the material subjected to a combination of loads must be less than the maximum shear stress in the material subjected to uniaxial loading corresponding to yielding condition.

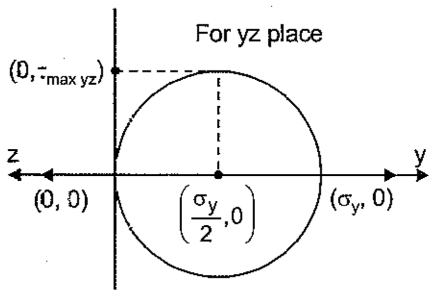


Consider a material subject to general loading condition in x-y plane only.

The state of stress can be represented by mohr circle as:







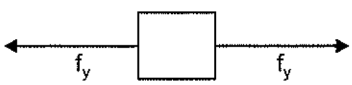
$\therefore$ Maximum shear stress (absolute),

$$\tau_{max,abs} = \text{maximum of}\begin{cases}\tau_{max\,xy}\\ \tau_{max\,yz}\\ \tau_{max\,zx}\end{cases}$$

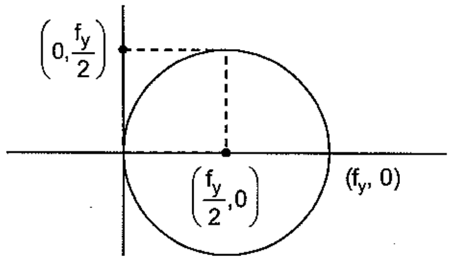
$$= \text{maximum of}\begin{cases}\sqrt{\left(\dfrac{\sigma_x-\sigma_y}{z}\right)^2+\tau_{xy}^2}\\ \dfrac{\sigma_x}{z}\\ \dfrac{\sigma_y}{z}\end{cases}$$

For condition of uniaxial loading under yielding condition, state of stress is represented as:

$f_y \rightarrow$ yielding stress



Mohr circle representation for the state of stress :



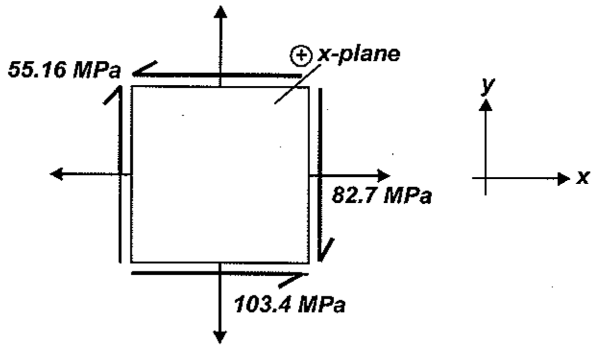
$$\tau_{max} = \frac{f_y}{2}$$

$\therefore$ As per maximum shear stress theory,

$$\tau_{max,abs} \leq \frac{(f_y/FOS)}{2}$$

$$\Rightarrow \quad \text{Maximum of} \begin{cases} \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \\ \frac{\sigma_x}{2}, \\ \frac{\sigma_y}{2} \end{cases} \leq \frac{1}{2}\left(\frac{f_y}{FOS}\right)$$

Q-15: ***A plane element in a body is subjected to stresses as shown in Figure. Determine the principal stresses and maximum shear stresses and the planes on which they act. Indicate them in separate sketches.***



[12 Marks, ESE-2017]

Sol: As per the sign convention i.e. tensile stress = +ve

Compressive stress = –ve

and shear stress on (+) x–plane in

(–) coordinate direction = (–)ve

We have,

Given that:

$$\sigma_y = +103.4 \text{ MPa}$$

$$\sigma_x = 82.7 \text{ MPa}$$

$$\tau_{xy} = -55.16 \text{ MPa}$$

Then $$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{103.4 + 82.7}{2} \pm \sqrt{\left(\frac{82.7 - 103.4}{2}\right)^2 + (-55.16)^2}$$

$$= 93.05 \pm 56.12$$

So, $$\sigma_1 = 149.170 \text{ MPa}$$

$$\sigma_2 = 36.93 \text{ MPa}$$

$$\tau_{\max\,(\text{in plane})} = \frac{\sigma_1 - \sigma_2}{2} = \frac{149.17 - 36.93}{2} = 56.12 \text{ MPa}$$

$$\tau_{\max\,(\text{abs.max})} = \max\left[\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_1|}{2}, \frac{|\sigma_2|}{2}\right] = 74.585 \text{ MPa}$$

For orientation:

$$\tan 2\theta_p = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \left(\frac{2x(-55.160)}{82.7 - 103.4}\right) = 5.33$$

$$2\theta_p = 79.37°$$

$\Rightarrow$ $$\theta_p = 39.68°$$

So, $\theta_{p_1} = 39.68°$; $\theta_{p_2} = 129.68°$ maximum principal plane orientation

$\theta_{s_1} = 39.68 + 45° = 84.68°$; $\theta_{s_2} = 174.68°$ max. shear plane orientation

Check for the maximum principal plane

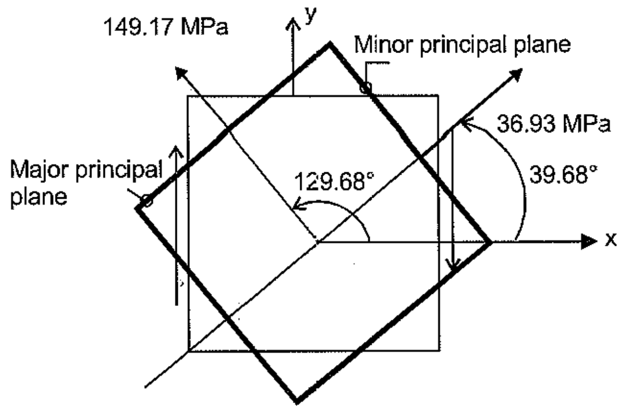
$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Putting $\theta = 39.68°$

$\Rightarrow$ $$\sigma_x' = \frac{103.4 + 82.7}{2} + \frac{82.7 - 103.4}{2}\cos(79.37) - 55.16\sin(79.37)$$

$$= 36.93 \text{ MPa} = \sigma_2 \text{ (minor principal stress)}$$

So, the plane at angle $\theta = 39.68°$ is the minor principal plane.



Orientation of max shear plane

By putting $\theta_s = 84.68°$

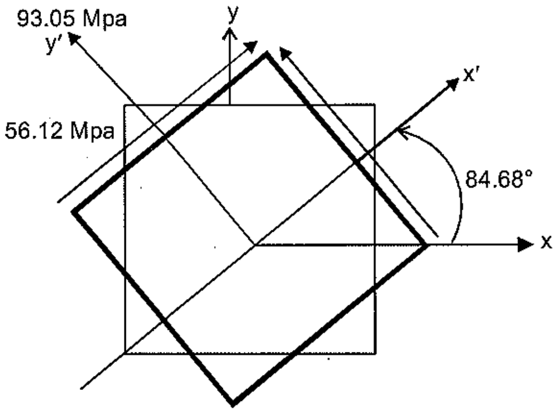
$$\tau_{x'y'} = \frac{-(\sigma_x - \sigma_y)}{20}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{(82.7 - 103.4)}{2}\sin(2\times 84.68) + (-55.16)\cos(2\times 84.68)$$

$$= 56.12$$

(+) value of $\tau_{x'y}$ means shear stress on (+) x' plane in (+) y' direction.

Normal stress on the plane of max shear stress $= \frac{\sigma_1 + \sigma_2}{2} = 93.05$ MPa



Q-16: ***In a steel flat plate a state of plane stress available. Calculate σ_y and principal stresses if σ_x = 140 N/mm², μ = 0.25, τ_{xy} = 40 N/mm², ϵ_z = -3.6×10^{-4} and E = 2×10^5 N/mm².***

[20 Marks, ESE-2018]

Sol: **Given that:** Plane stress condition

$$\sigma_x = 140 \text{ N/mm}^2$$

$$\mu = 0.25$$

$$\tau_{xy} = 40 \text{ N/mm}^2$$

$$\epsilon_z = -3.6 \times 10^{-4}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

To calculate:

(i) σ_y

(ii) Principal stresses

According to the given question, for plane stress condition,

$\sigma_z = \tau_{zx} = \tau_{zy} = 0$ (if z-axis is chosen perpendicular to the face on which no stress is acting).

Hence, the remaining plane stress components are σ_x, σ_y and τ_{xy}

Calculation for 'σ_y'

From Hooke's law,

$$\epsilon_z = \frac{-\mu}{E}(\sigma_x + \sigma_y)$$

On substituting the given values in the above equation, we get

$$\therefore \quad -3.6 \times 10^{-4} = \frac{-0.25}{2 \times 10^5}(140 + \sigma_y)$$

Hence, $\boxed{\sigma_y = 148 \text{ N/mm}^2}$

Calculation for principal stresses

As,

$$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1/2} = \frac{140 + 148}{2} \pm \sqrt{\left(\frac{140 - 148}{2}\right)^2 + (40)^2}$$

$$= (144 \pm 40.2) \text{ N/mm}^2$$

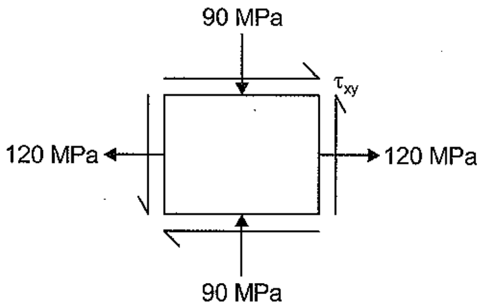
$$\boxed{\sigma_{1/2} = 184.2, 103.8 \text{ N/mm}^2}$$

Hence, the principal stresses are 184.2 N/mm^2 and 103.8 N/mm^2.

Q-17: ***Direct stresses of 120 MN/m^2 in tension and 90 MN/m^2 in compression are applied to an elastic material at a certain point on planes at right angles to each other. If the maximum principal stress is not to exceed 150 MN/m^2 in tension, to what shearing stress can the material be subjected? What is then the maximum resulting shearing stress in the material? Also find the magnitude of the other principal stress and its inclination to 120 MN/m^2 stress.***

[8 Marks, ESE-2019]

Sol:



(i) Given:

$$\sigma_1 \le 150 \text{ MPa}$$

$$\Rightarrow \quad \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \le 150$$

$$\Rightarrow \quad \frac{120 - 90}{2} + \frac{1}{2}\sqrt{(120 + 90)^2 + 4\tau_{xy}^2} \le 150$$

$$\tau_{xy} \le 84.85 \text{ MPa}$$

(ii) Maximum shear stress (τ_{max}) $= \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\times\tau_{xy}^2}$

$$= \frac{1}{2}\sqrt{(120+90)^2 + 4\times 84.85^2}$$

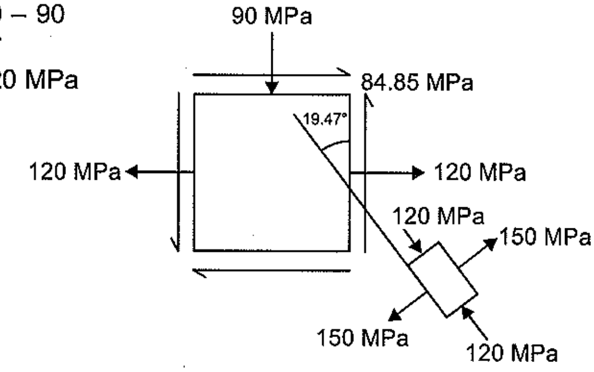
$$= 135 \text{ MPa}$$

(iii) Other principal stress

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

$$150 + \sigma_2 = 120 - 90$$

$$\sigma_2 = -120 \text{ MPa}$$



Again we know,

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\Rightarrow \quad \tan 2\theta_P = \frac{2\times 84.85}{120-(-90)}$$

$$\theta_P = 19.47°$$

So, inclination of minor principal stress with 120 MPa is ($19.47° + 90 = 109.47°$) in anti-clockwise direction

Q-18: ***What combination of Principal stresses will give the same factor of safety for failure by yielding according to the maximum shear stress theory and distortion energy theory. Consider only a two dimensional case.***

[10 Marks, ESE-2019]

Sol: Let the major and minor principal stresses are (σ_1) and (σ_2) respectively. As 2-dimensional condition is assumed, so $\sigma_3 = 0$.

Here two cases arise:

Case-I: The two principal stresses are unlike stresses.

Case-II: The two principal stresses are like stresses.

Case-I:

Maximum shear stress theory:

$$\text{Maximum of } \left[\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_1}{2}\right|, \left|\frac{\sigma_2}{2}\right|\right] = \frac{\left(\frac{f_y}{FOS}\right)}{2}$$

$$\Rightarrow \quad \left(\frac{\sigma_1 - \sigma_2}{2}\right) = \frac{\left(\frac{f_y}{FOS}\right)}{2} \quad \text{...(i)}$$

Maximum distortion energy theory:

$$\frac{1}{12G}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] = \frac{\left(\frac{f_y}{FOS}\right)^2}{6G} \quad \text{...(ii)}$$

Putting, (f_y/FOS) value from Eq. (i) to Eq. (ii), we get

$$\frac{1}{12}\left[(\sigma_1-\sigma_2)^2+(\sigma_2)^2+(\sigma_1)^2\right] = \frac{(\sigma_1-\sigma_2)^2}{6}$$

$$\Rightarrow \quad (\sigma_1-\sigma_2)^2+\sigma_1^2+\sigma_2^2 = 2(\sigma_1-\sigma_2)^2$$

$$(\sigma_1-\sigma_2)^2-\sigma_1^2-\sigma_2^2 = 0$$

$$\Rightarrow \quad \boxed{2\sigma_1\sigma_2 = 0}$$

$\because$ The product of principal stresses is zero, which means either σ_1 or σ_2 or both are zero.

Therefore, it can not result, in a two dimensional stress condition. Hence, Case II will exist.

Case-II:

Maximum shear stress theory:

$$\text{Maximum of}\left[\frac{|\sigma_1-\sigma_2|}{2},\frac{|\sigma_1|}{2},\frac{|\sigma_2|}{2}\right] = \frac{\left(\frac{f_y}{FOS}\right)}{2}$$

$$\Rightarrow \quad \left(\frac{\sigma_1}{2}\right) = \frac{\left(\frac{f_y}{FOS}\right)}{2} \qquad \text{...(iii)}$$

Putting, value of $\left(\frac{f_y}{FOS}\right)$ from Eq. (iii) to Eq. (ii), we have

$$\frac{1}{12}\left[(\sigma_1-\sigma_2)^2+(\sigma_2)^2+(\sigma_1)^2\right] = \frac{(\sigma_1)^2}{6G}$$

$$\Rightarrow \quad (\sigma_1-\sigma_2)^2+(\sigma_1)^2+(\sigma_2)^2 = 2(\sigma_1)^2$$

$$\Rightarrow \quad 2\sigma_2^2-2\sigma_1\sigma_2 = 0$$

$$\Rightarrow \quad \boxed{\sigma_2 = \sigma_1}$$

Hence, both stress should be equal and like stresses

Q-19: ***A bar of length 1.2 m, diameter 40 mm is subjected to an axial tensile load of 130 kN and a twisting moment of 600 N.m. If the same material yielded at an axial stress of 200 N/mm^2, determine the safety factor associated with the bar, considering***

(i) Principal stress failure theory

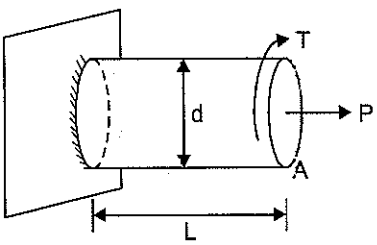
(ii) Maximum shear stress theory

(iii) Distortional strain energy theory

Take E = 200 GPa and μ = 0.25

[20 Marks, ESE-2020]

Sol:



Given, $L = 1.2$ m, $d = 40$ mm

$$T = 600 \text{ N-m}, \ P = 130 \text{ kN}$$
$$E = 200 \text{ GPa}, \ \mu = 0.25$$
$$\sigma_y = 200 \text{ N/mm}^2$$

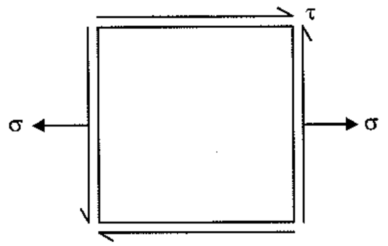
⇒ Normal stress (σ) due to axial load

$$\sigma = \frac{P}{A} = \frac{130\times10^3}{\frac{\pi}{4}\times40^2} = 103.45 \text{ MPa}$$

⇒ Maximum shear stress due to torsion

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16\times600\times10^3}{\pi\times40^3} = 47.74 \text{ MPa}$$

→ Critical stress element is at outer fiber of bar



$$\sigma_{major} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = 122.11 \text{MPa}$$

$$\sigma_{minor} = -18.66 \text{ MPa}$$

$$\tau_{max} = \tau_{abs,\,max} = \frac{\sigma_{major} - \sigma_{minor}}{2}$$

$$\Rightarrow \quad \tau_{abs,\,max} = 70.39 \text{ MPa}$$

Now,

(i) As per principal stress theory:

$$\sigma_{major} = \frac{f_y}{FOS}$$

$$\Rightarrow \quad FOS = \frac{f_y}{\sigma_{major}} = \frac{200}{122.11}$$

$$\boxed{FOS = 1.6378}$$

(ii) As per maximum shear stress theory:

$$\tau_{max} = \frac{f_y/2}{FOS}$$

$$\Rightarrow \quad \boxed{FOS = 1.42}$$

(iii) As per maximum distortional strain energy theory:

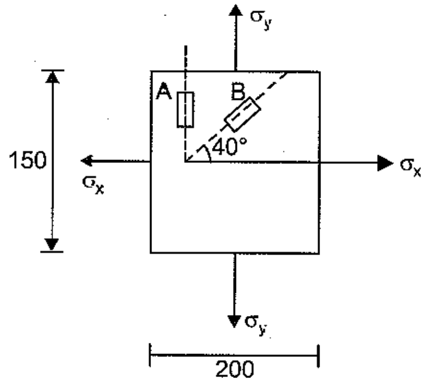
$$\frac{1}{12G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] = \frac{(f_y/FOS)^2}{6G}$$

$$\Rightarrow \quad \frac{1}{2}\left[(122.11+18.66)^2+(-18.66-0)^2+(0-122.11)^2\right] = \left(\frac{200}{FOS}\right)^2$$

$$\Rightarrow \quad \boxed{FOS = 1.51}$$

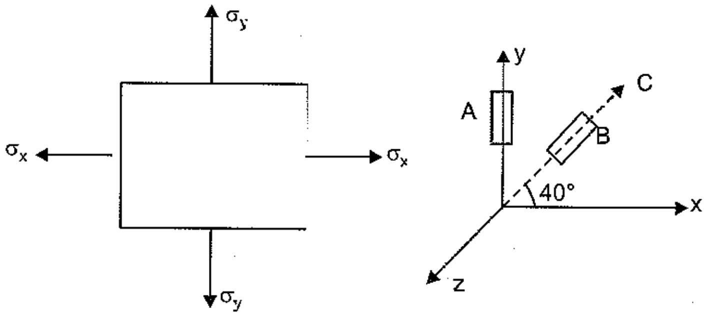
Q-20: ***A 200 mm × 150 mm × 10 mm aluminium plate is subjected to uniform bi-axial stresses σ_x and σ_y. Two strain gauge A and B are attached to the surface of the plate as shown in the Figure. If readings in strain gauges are $\epsilon_A = 200 \times 10^{-6}$ and $\epsilon_B = 285 \times 10^{-6}$, what are the values of σ_x and σ_y ? What is the reduction in thickness of the plate as a result of stresses ?***

Take Young's modulus E = 75 GPa and Poisson's ratio ν = 0.33.



[12 Marks, ESE-2020]

Sol:



$$\varepsilon_y = \varepsilon_A = 200 \times 10^{-6}$$

$$\varepsilon_B = 285 \times 10^{-6}$$

- As x and y planes are principal planes, hence shear strain on these planes will be zero.

$$\gamma_{xy} = 0$$

$$\Rightarrow \quad \varepsilon_B = \frac{\varepsilon_x + \varepsilon_y}{2} + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos(80°) + \frac{\gamma_{xy}}{2}\sin(80°)$$

$$\Rightarrow \quad 285 \times 10^{-6} = \frac{\varepsilon_x + 200 \times 10^{-6}}{2} + \left(\frac{\varepsilon_x - 200 \times 10^{-6}}{2}\right) \times \cos 80°$$

$$\Rightarrow \quad \varepsilon_x = 344.85 \times 10^{-6}$$

As we know

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

$$\Rightarrow \quad (344.85 \times 10^{-6}) \times 75 \times 10^3 = \sigma_x - \frac{\sigma_y}{3}$$

$$\Rightarrow \quad \sigma_x - \frac{\sigma_y}{3} = 25.86 \qquad \text{... (i)}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$$

$$(200 \times 10^{-6}) \times 75 \times 10^3 = \sigma_y - \frac{\sigma_x}{3}$$

$$\Rightarrow \quad -\frac{\sigma_x}{3} + \sigma_y = 15 \quad \text{... (ii)}$$

From (i) and (ii)

$$\sigma_x = 34.71 \text{ MPa}$$

$$\sigma_y = 26.57 \text{ MPa}$$

Reduction in thickness

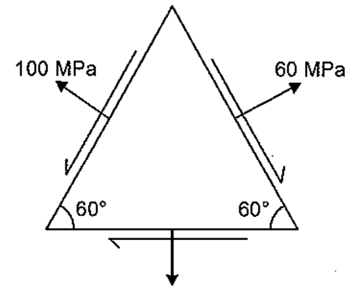
$$\Delta t = |\varepsilon_z| \times t$$

$$\Rightarrow \quad \Delta t = \left| -\frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} \right| \times t$$

$$\Delta t = \left| -\frac{34.71}{3 \times 75 \times 10^3} - \frac{26.57}{3 \times 75 \times 10^3} \right| \times 10$$

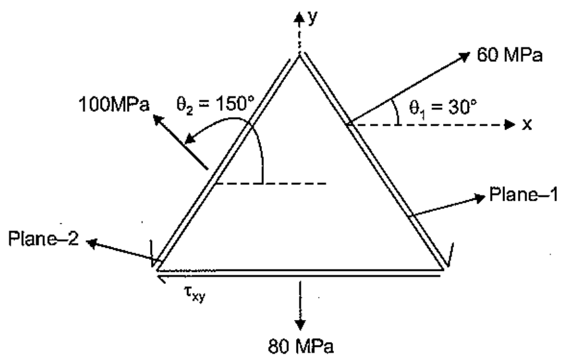
$$\Delta t = 2.72 \times 10^{-3} \text{ mm}$$

Q-21: ***In a strained body the normal stresses on three planes inclined as shown in Figure are 60 MPa (Tensile), 80 MPa (Tensile) and 100 MPa. Determine the shear stresses acting on these planes. Also find the principal stresses.***



[12 Marks, ESE-2020]

Sol:



Assuming, σ_x is normal stress on x-plane and τ_{xy} is shear stress on x-plane

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_1 = 60 = \frac{\sigma_x + 80}{2} + \left(\frac{\sigma_x - 80}{2}\right)\cos(2 \times 30) + \tau_{xy} \sin(2 \times 30°)$$

$$\Rightarrow \quad \left(0.5+0.5\times\frac{1}{2}\right)\sigma_x + \tau_{xy}\times\frac{\sqrt{3}}{2} + 80\left(0.5-0.5\times\frac{1}{2}\right) - 60 = 0$$

$$\Rightarrow \quad \frac{3}{4}\sigma_x + \frac{\sqrt{3}}{2}\tau_{xy} - 40 = 0 \qquad \text{....(i)}$$

$$\sigma_2 = 100 = \frac{\sigma_x+80}{2} + \left(\frac{\sigma_x-80}{2}\right)\cos(2\times150°) + \tau_{xy}\sin(2\times150°)$$

$$\frac{3}{4}\sigma_x - \frac{\sqrt{3}}{2}\tau_{xy} - 80 = 0 \qquad \text{.... (ii)}$$

From (i) and (ii)

$$\sigma_x = 80 \text{ MPa}$$

$$\boxed{\tau_{xy} = -23.09 \text{ MPa}}$$

Here negative sign means direction of shear stress is opposite as we assumed.

Hence, shear stress acting on y-plane = –23.09 MPa

→ On plane – 1

$$\tau_1 = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta_1 + \tau_{xy}\cos 2\theta_1$$

$$\tau_1 = -\left(\frac{80-80}{2}\right)\sin(2\times30°) - 23.09\times\cos(2\times30°)$$

$$\boxed{\tau_1 = -11.545 \text{ MPa}}$$

→ On plane – 2

$$\tau_2 = -\left(\frac{80-80}{2}\right)\sin(2\times150°) - 23.09\times\cos(2\times150°)$$

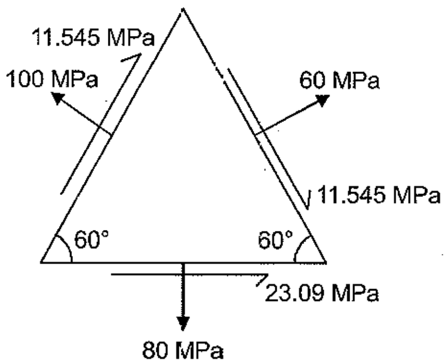
$$\boxed{\tau_2 = -11.545 \text{ MPa}}$$

Now,

$$\sigma_{major/minor} = \frac{\sigma_x-\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{major} = 80 + 23.09 = 103.09 \text{ MPa}$$

$$\sigma_{major} = 56.91 \text{ MPa}$$



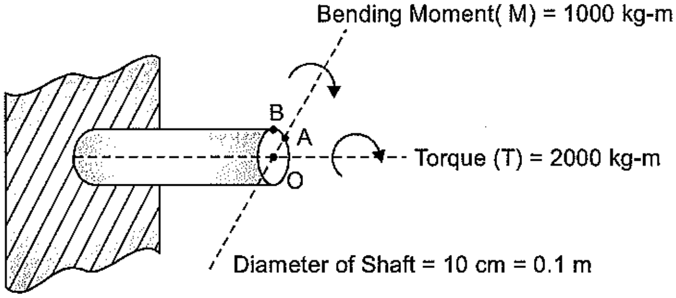
CHAPTER 5

COMBINED STRESS

Q-1: ***A steel shaft is subjected to a torque of 2000 kg-m and a bending moment of 1000 kgm. Diameter of the shaft = 10 cm. Calculate maximum, minimum principal stresses and maximum shear stress in the shaft at its surface.***

[15 Marks, ESE-1996]

Sol:



Determine:

(a) Maximum/Minimum principal stress at surface

(b) Maximum shear stress at surface

Shear stress due to torsion

We know that, $\frac{\tau}{r} = \frac{T}{I_P}$ $\quad \therefore \quad \tau_{max} = \frac{T \times r}{I_P} = \frac{T \times \frac{D}{2}}{\frac{\pi}{32} D^4} = \frac{16T}{\pi D^3}$

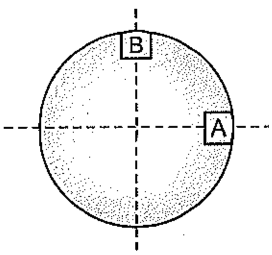
$$\therefore \qquad \tau_{max} = \frac{16T}{\pi D^3}$$

Bending stress due to Bending moment

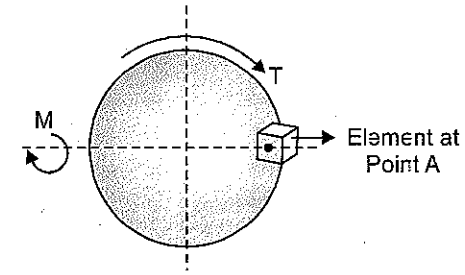
We know that $\quad \frac{f}{y} = \frac{M}{I} \quad \Rightarrow \quad f_{max} = \frac{M}{I} \times \frac{D}{2} = \frac{M}{\frac{\pi}{64} D^4} \times \frac{D}{2} = \frac{32M}{\pi D^3}$

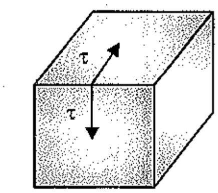
$$\therefore \qquad f_{max} = \frac{32M}{\pi D^3}$$

There are two position at the surface where we have to check for the maximum/minimum principal stress and maximum shear stress. There possible points are A &B. as shown in figure.

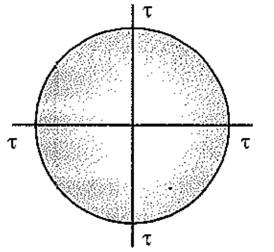


For point A





Element A will be in pure shear



Mohr circle for pure shear

The element at point A posses only shear stress. Bending stress will be zero because elements lies on neutral axis.

Hence with the help of Mohr circle for element A in pure shear.

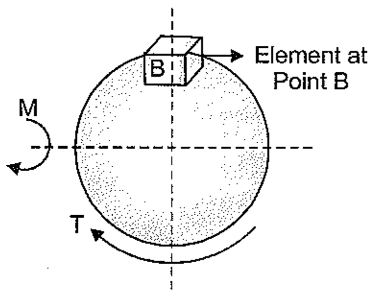
Major principal stress = τ

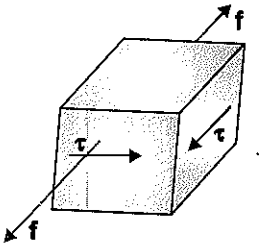
Minor principal stress = $-\tau$

and Maximum shear stress = τ

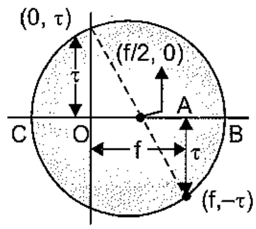
$$\text{where } \tau = \frac{16T}{\pi D^3} = \frac{16 \times 2000 \times 10^2}{\pi(10)^3} \text{ kg/cm}^2 = 1019.08 \text{ kg/cm}^2$$

For point B





Drawing Mohr's circle for this element



Mohr circle

Element at point B posses shear stress as well as normal stress due to bending as shown in fig.

$$\text{Radius of Mohr circle} = \sqrt{\tau^2 + \left(\frac{f}{2}\right)^2}$$

$$\text{Major \& minor principal stress} = \frac{f}{2} \pm \text{Radius of mohr circle}$$

$$= \frac{f}{2} + \sqrt{\tau^2 + \left(\frac{f}{2}\right)^2} \text{ and } \frac{f}{2} - \sqrt{\tau^2 - \left(\frac{f}{2}\right)^2}$$

$$= \frac{16M}{\pi D^3} + \sqrt{\left(\frac{16}{\pi D^3}\right)^2 (M^2 + T^2)} \text{ and } \frac{16M}{\pi D^3} - \sqrt{\left(\frac{16}{\pi D^3}\right)^2 (M^2 + T^2)}$$

$$\sigma_1 / \sigma_2 = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

$$\sigma_1 / \sigma_2 = \frac{16}{\pi \times (0.1)^3}\left[1000 \pm \sqrt{1000^2 + 2000^2}\right]$$

$$= 1.64895 \times 10^7 \text{ kg/m}^2 \quad \text{and,} \quad -6.2984 \times 10^6 \text{ kg/cm}^2$$

$$= 1648.95 \text{ kg/cm}^2 \text{ and } -629.84 \text{ kg/cm}^2$$

$\Rightarrow$ Major principal stress $= 1648.95$ kg/cm^2

Minor principal stress $= -629.84$ kg/cm^2

and, Maximum shear stress, = Radius of Mohr circle $= \sqrt{\tau^2 + \left(\frac{f}{2}\right)^2}$

$$= \frac{16}{\pi D^3}\sqrt{M^2+T^2} = 1139.4 \text{ kg/cm}^2$$

So, when we compare, the two location (A) and (B),

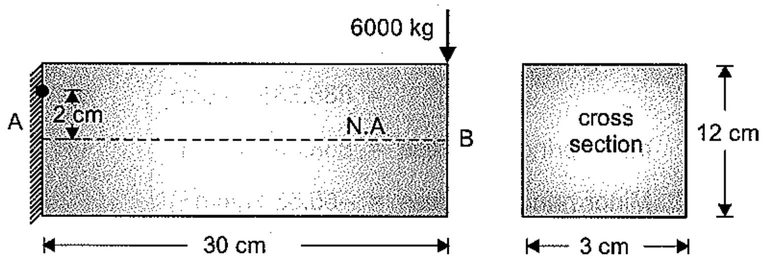
we will get $\sigma_1/\sigma_2 = 1648.12$ kg/cm^2, $-$ 629.524 kg/cm^2

$\therefore$ $(\tau)_{max} = 1139.4$ kg/cm^2 **Ans.**

Q-2: ***A horizontal steel cantilever is fixed at end A and its free end B supports a vertical load of 6000 kg. The cross-section of the cantilever is rectangular, 3 cm wide, 12 cm deep. AB = 30 cm. Calculate magnitude, nature and direction of principal stresses at end A at 2 cm above the neutral axis.***

[15 Marks, ESE-1997]

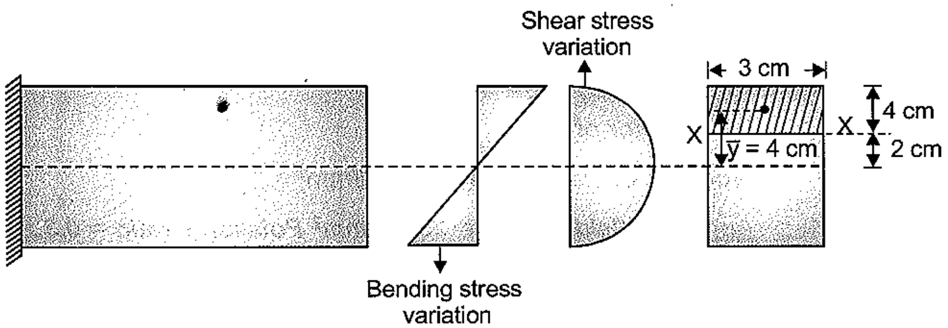
Sol:



Determine: Magnitude, nature & direction of principal stresses at end A at 2 cm above the neutral axis.

There are two types of stress generated at the point, 2 cm above N.A at end A

(i) Bending stress (ii) Shear stress



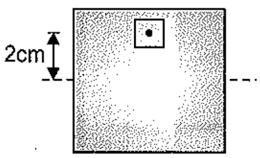
Bending stress calculation at X – X level (2 cm above from Neutral axes)

$$f = \frac{M}{I} \times y = \left[\frac{(6000 \times 30)}{\frac{3 \times 12^3}{12}} \times 2\right] = 833.33 \text{ kg/cm}^2$$

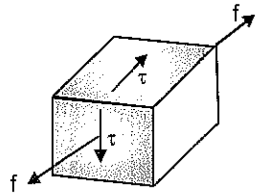
Shear stress calculation at X – X level

$$\tau = \frac{VA\bar{y}}{Ib} = \frac{6000\times(4\times3)\times4}{\frac{3\times12^3\times3}{12}} = 222.22\,kg/cm^2$$

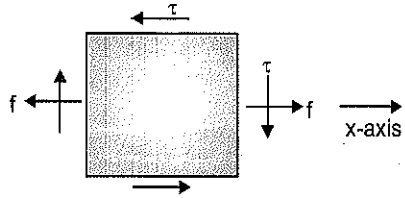
Stress representation on the element at any point 2 cm above the N.A. at point A.



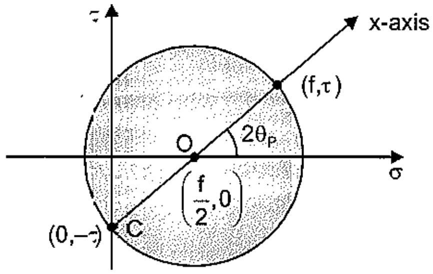
View of small element



Macroscopic view of element (3D- view)



2D -view



By Drawing Mohr circle, Radius $= \sqrt{\tau^2 + f^2/4}$

$$\text{Radius} = \sqrt{222.22^2 + \frac{833.33^2}{4}}$$

Radius $= 472.22$ kg/cm²

$\therefore$ $\sigma_1/\sigma_2 = 472.22 + \frac{833.33}{2}$ or, $-472.22 + \frac{833.33}{2}$

$= 888.885$ kg/cm² or, -55.55 kg/cm²

Major principal stress $= 888.885$ kg/cm²

Minor principal stress $= -55.55$ kg/cm²

Angle of inclination, $\sin 2\theta_P = \frac{222.22\,kg/cm^2}{472.22\,kg/cm2}$ $\therefore$ $2\theta_P = 28.07°$; $208.07°$

$\Rightarrow$ $\theta_P = 14.03°$ and $104.03°$

Normal to major principal plane makes a clockwise angle of 14.03° with the x-axis and normal to minor principal plane makes an angle of 104.03° clockwise with x-axis.

Q-3: ***A solid steel circular shaft is required to carry a torque of 40 kN.m and a bending moment of 20 kN.m. Determine the size of the shaft by any five theories of failure.***

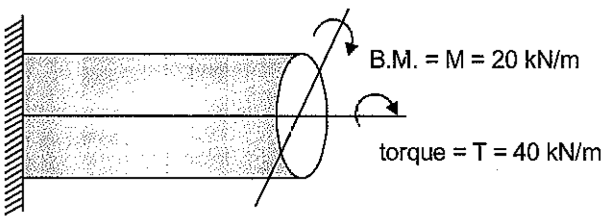
Factor of safety = 2.0

Modulus of elasticity = 200 kN/mm²

Yield stress = 250 N/mm² Poisson's ratio = 0.3.

[15 Marks, ESE-1998]

Sol:



Factor of safety (F) = 2.0, E = 200 kN/mm^2,

Yield stress = f_y = 250 N/mm^2, μ = 0.3

We know that when a shaft is subjected to combined bending and torsion, the magnitude of principle stresses is given by,

$$\sigma_1 / \sigma_2 = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2+T^2}\right]$$

(i) According to maximum principle stress theory.

$$\sigma_1 = \frac{16}{\pi D^3}\left[M+\sqrt{M^2+T^2}\right] \le \frac{fy}{F.O.S.}$$

$$\Rightarrow \quad \frac{16}{\pi D^3}\left[20+\sqrt{20^2+40^2}\right]\times 10^6 \le \frac{250}{2}$$

$$\Rightarrow \quad \frac{16}{\pi D^3}\left[20+\sqrt{20^2+40^2}\right] \le \frac{250}{2\times 10^6}$$

$$\Rightarrow \quad \left\{\frac{16\left[20+\sqrt{20^2+40^2}\right]\times 2\times 10^6}{250\pi}\right\}^{1/3} \le D$$

$$\boxed{D \ge 138.156 \text{ mm}}$$

(ii) According to maximum strain theory.

$$\epsilon_{max} = \frac{\sigma_1-\mu\sigma_2}{E} \le \left(\frac{fy}{E}\right)\times\frac{1}{F.O.,S.}$$

$$\Rightarrow \quad \frac{16}{\pi D^3}\left[M+\sqrt{M^2+T^2}\right]-\mu\frac{16}{\pi D^3}\left[M-\sqrt{M^2+T^2}\right] \le f_y\times\frac{1}{F.O.S.}$$

$$\Rightarrow \quad \frac{16}{\pi}\left\{M+\sqrt{M^2+T^2}-\mu M+\mu\sqrt{M^2+T^2}\right\}\times\frac{F.O.S.}{fy} \le D^3$$

$$\Rightarrow \quad \frac{16}{\pi}\left\{20(1-0.3)+(1+0.3)\sqrt{20^2+40^2}\right\}\times\frac{10^6\times 2}{250} \le D^3$$

$$\boxed{D \ge 143.24 \text{ mm}}$$

(iii) Maximum shear stress theory:

$$\tau_{max} \le \left(\frac{f_y}{2}\right)\cdot\frac{1}{F.O.S.}$$

and, $\tau_{max} = \dfrac{\sigma_1-\sigma_2}{2}$ $\quad\therefore\quad \dfrac{\sigma_1-\sigma_2}{2} \le \dfrac{f_y}{2\cdot(F.O.S.)} \quad\Rightarrow\quad \sigma_1-\sigma_2 \le \dfrac{f_y}{(F.O.S.)}$

$$\frac{16}{\pi D^3}\left[M+\sqrt{M^2+T^2}-M+\sqrt{M^2+T^2}\right] \le \frac{250}{2} \quad\Rightarrow\quad \frac{16}{\pi D^3}\left[2\times\sqrt{M^2+T^2}\right] \le 125$$

$$\Rightarrow \quad \frac{16\times 2}{125\pi}\left[\sqrt{40^2+20^2}\right]\times 10^6 \le D^3 \quad\Rightarrow\quad \boxed{D \ge 153.88\text{mm}}$$

(iv) Maximum strain energy theory:

$$\sigma_1^2+\sigma_2^2-2\mu\sigma_1\sigma_2 \le \left[\frac{f_y}{F.O.S.}\right]^2$$

$$\Rightarrow \quad \left(\frac{16}{\pi D^3}\right)^2\left[\left(M+\sqrt{M^2+T^2}\right)^2+\left(M-\sqrt{M^2+T^2}\right)^2-\left\{2\times0.3\times\left(M+\sqrt{M^2+T^2}\right)\times\left(M-\sqrt{M^2+T^2}\right)\right\}\right]\le\left(\frac{f_y}{F.O.S.}\right)^2$$

$$\Rightarrow \quad \left(\frac{16}{\pi D^3}\right)^2\left[\left(20+\sqrt{20^2+40^2}\right)^2+\left(20-\sqrt{20^2+40^2}\right)^2-2\times0.3\times\{20^2-(20^2+40^2)\}\right]\times10^{12}\le\left(\frac{f_y}{F.O.S.}\right)^2$$

$$\Rightarrow \quad 10^{12}\times\frac{16^2\times2^2}{\pi^2\times250^2}\left[2\left(20^2+20^2+40^2\right)+2\times0.3\times40^2\right]\le D^6 \qquad \left[\begin{array}{l}\text{using}, (a+b)(a-b)=a^2-b^2\\ (a+b)^2+(a-b)^2=2(a^2+b^2)\end{array}\right.$$

$$\Rightarrow \quad \boxed{D\ge145.68\text{ mm}}$$

(v) Maximum shear strain energy theory :

$$\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2 \le \left(\frac{f_y}{F.O.S.}\right)^2$$

$$\Rightarrow \quad \left(\frac{16}{\pi D^3}\right)^2\left[\left(M+\sqrt{M^2+T^2}\right)^2+\left(M-\sqrt{M^2+T^2}\right)^2-\left(M^2-M^2-T^2\right)\right] \le \left(\frac{fy}{F.O.S.}\right)^2$$

$$\Rightarrow \quad \left(\frac{16}{\pi D^3}\right)^2\left[2\left(M^2+M^2+T^2\right)+T^2\right] \le \left(\frac{250}{2}\right)^2$$

$$\Rightarrow \quad \frac{256}{\pi^2\times D^6}\left[2\times\left(2\times20^2+40^2\right)+40^2\right]\times10^{12} \le (125)^2$$

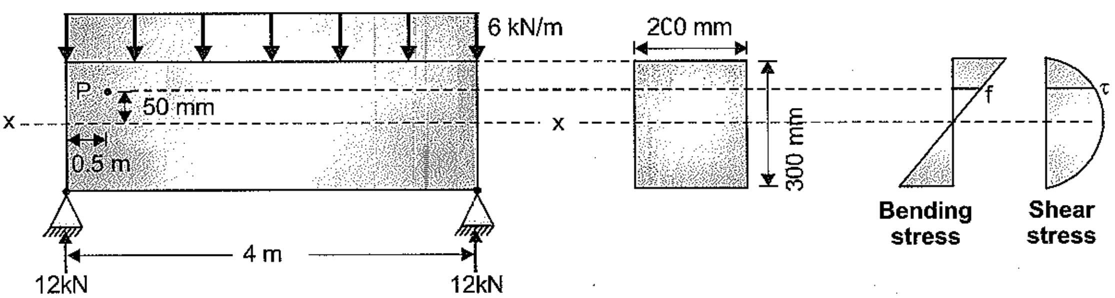
$$\boxed{D\ge148.269\text{ mm}}$$

Note that max shear stress theory gives most conservative results.

Q-4: ***A simply supported beam of rectangular cross section of size 200 × 300 mm (deep) supports a uniformly distributed load of 6.0 kN/m over an effective span of 4.0 m. Calculate the magnitude and direction of principal stresses at a point located 0.50 m from the left support and 50 mm above the neutral axis.***

[15 Marks, ESE-2000]

Sol:



Calculate the magnitude and direction of principal stresses. at P

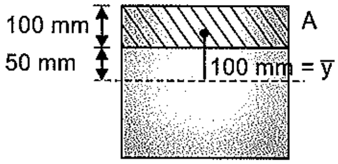
Calculation of bending moment at the section passing though point P

$$M = 12x-\frac{6x^2}{2}=12x-3x^2$$

$$\therefore \quad M\Big|_{x=0.5} = 12\times0.5-3\times0.5^2=6-0.75=5.25\text{ kN-m}$$

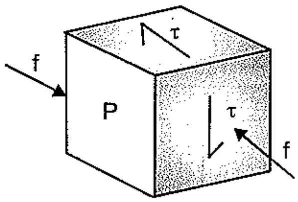
So, Bending stress, $f = \frac{M}{I} \times y \left[\frac{5.25 \times 10^6}{\left(\frac{200 \times 300^3}{12}\right)} \times 50 \right] \text{N/mm}^2 = 0.5833 \text{ N/mm}^2 \text{ (Comp)}$

Calculation of shear stress at the same section,

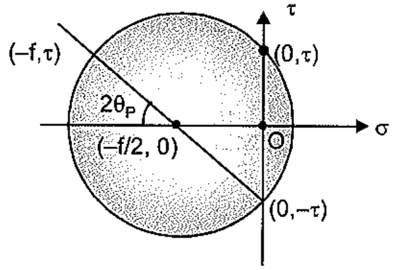


$V = 12 - 6x \quad \therefore \quad V|_{x=0.5\text{ m}} = 12 - 6 \times 0.5 = 9 \text{ kN}$

So, Shear stress, $\tau = \frac{VA\bar{y}}{Ib} = \left[\frac{9 \times 10^3 \times (200 \times 100) \times 100}{200 \times \frac{300^3}{12} \times 200} \right] = 0.2 \text{ N/mm}^2$



macroscopic view of element at point A



$$R = \text{radius of Mohr circle} = \sqrt{\tau^2 + \left(\frac{f}{2}\right)^2} = \sqrt{(0.2)^2 + \left(\frac{0.5833}{2}\right)^2}$$

$$= \sqrt{0.04 + 0.0850597} = 0.3557 \text{ N/mm}^2$$

Major principal stress and minor principal stress are σ_1 and σ_2

$$\sigma_1/\sigma_2 = -\frac{f}{2} \pm R$$

$$\sigma_1 = \frac{-0.5833}{2} - 0.3557 = -0.647 \text{ N/mm}^2$$

$$\sigma_2 = -\frac{0.5833}{2} + 0.3557 = 0.064 \text{ N/mm}^2$$

$$\tan 2\theta_P = \frac{\tau}{\left(\frac{f}{2}\right)} = \frac{2\tau}{f} = \frac{2 \times 0.2}{0.5833} = 0.685753$$

$\therefore \quad 2\theta_P = 34.44° \Rightarrow \theta_P = 17.22°$

$\Rightarrow$ θ_P for major principal stress = 17.22°

θ_P for minor principal stress = 107.22°

$\Rightarrow$ Normal to major plane makes clockwise angle of 17.22° from x-axis and normal to minor plane makes an angle of 107.22° in clockwise direction from x-axis.

Note: Max. principal stress is magnitude wise more than minor principal stress.

Q-5: ***A prismatic beam ABC is simply supported at A & B. AB = 20 m, BC = 1 m, C is free end. The entire beam is uniformly loaded with 10 kN/m. The cross-section of the beam is I with following particulars:***

Flanges: ***Width = 150 mm***

Thickness = 10 mm

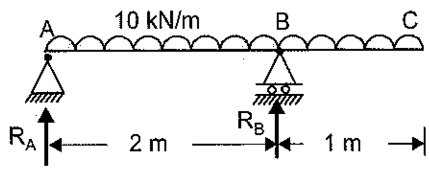
Web: ***Thickness = 10 mm***

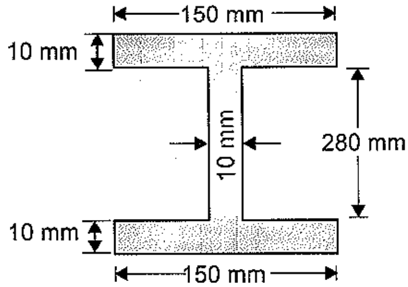
Overall depth of beam = 300 mm

Determine the maximum value of principal stress occurring anywhere in the beam. Specify the location.

[20 Marks, ESE-2002]

Sol: Determining the maximum value of principal stress occurring anywhere in the beam.





Reaction determination

From $\Sigma F_V = 0$, $R_A + R_B = 30$ ... (i)

$\Sigma M_B = 0$

$$\Rightarrow \quad 2R_A - \frac{10 \times 2 \times 2}{2} + 10 \times 1 \times \frac{1}{2} = 0$$

$$2R_A = 20 - 5 = 15$$

$$\Rightarrow \quad \boxed{R_A = 7.5}$$

$$\Rightarrow \quad \boxed{R_B = 30 - 7.5 = (22.5)\text{ kN}}$$

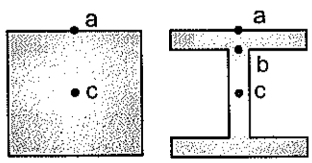
Conceptual background

To find out maximum normal stress (principal stress) anywhere in the beam, section that need to be investigated are

(a) Maximum bending moment location

(b) Maximum shear force location

Normally for rectangular section point 'a' should be investigated at location of maximum bending moment and point 'c' for the maximum shear location.



But in case of I-section both point 'a' and 'b' should be investigated at maximum B.M. section and 'b' and 'c' should be investigated at maximum shear stress location.

Evaluation of SFD and BMD

For Part AB:

$$\text{Shear force, } V = (7.5 - 10x)$$

$$V|_A = 7.5\text{kN};\ V|_B = 7.5 - 10\times2 = (-12.5\text{kN})$$

$$V = 0 \qquad \text{When } 7.5 - 10x = 0 \Rightarrow x = 0.75 \text{ m from A}$$

$$\text{Bending moment, } M = 7.5x - 5x^2$$

$$M|_A = 0 \text{ and } M|_B = 7.5\times2 - 5\times2^2 = -5\text{kN/m}$$

Location of M_{max} occurs at $\frac{dM}{dx} = 0$

$\Rightarrow \quad 7.5 - 10x = 0 \quad \Rightarrow \quad x = 0.75$ m

$\therefore \quad \boxed{M_{max} = M|_{x=0.75} = 7.5\times0.75 - 5\times(0.75)^2 = 2.81\text{kN-m}}$

For Part BC: (x taken from C)

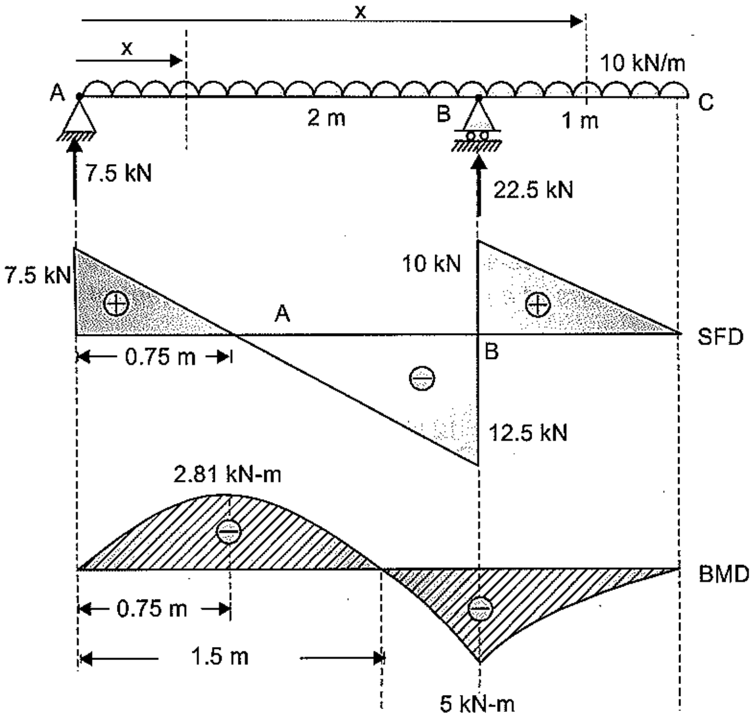
$$\text{Shear force} = V = 10(3 - x)$$

$\Rightarrow \quad V_C = 0; \quad V_B = 10$ kN

$$\text{Bending moment, } M = \frac{-10(3-x)^2}{2} \Rightarrow M_B = -5 \text{ (i.e at } x = 2)$$

$$M_C = 0 \quad \text{(i.e., at } x = 3)$$

Drawing S.F.D. and B.M.D.

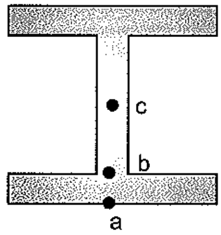


At point B (x = 2m from left end)

$$\text{B.M.} = -5 \text{ kNm and S.F.} = -12.5 \text{ kN}$$

$$\therefore \quad f = \frac{M}{I}\times y = \frac{5\times10^6}{8.14\times10^7}y = (6.1425\times10^{-2})y$$

and
$$q = \frac{VA\bar{y}}{Ib} = \left\{\frac{12.5\times10^3\times A\times\bar{y}}{8.14\times10^7\times b}\right\}$$



Now for I-section we will have to check for three points.

For Point 'a':

$$f = (6.1425 \times 10^{-2}) \times 150 = 9.213 \text{ N/mm}^2$$

$$\tau = 0$$

So, $\Rightarrow$ Normal stress = Principal stress = **9.213 N/mm²**

For Point 'b':

$$f = (6.1425 \times 10^{-2}) \times 140 = 8.599 \text{ N/mm}^2 \cong 8.6 \text{ N/mm}^2$$

$$\tau = \frac{VA\bar{y}}{Ib} = \frac{12.5\times10^3\times(10\times150)\times(145)}{8.14\times10^7\times(10)} = 3.34 \text{ N/mm}^2$$

$$\sigma_1/\sigma_2 = \frac{\sigma_x+\sigma_y}{2}\pm\sqrt{\frac{(\sigma_x-\sigma_y)^2}{4}+\tau_{xy}^2} = \frac{8.6+0}{2}\pm\sqrt{\frac{(8.6-0)^2}{4}+(3.34)^2}$$

$$= \mathbf{9.74 \text{ and } -1.14 \text{ N/mm}^2}$$

Determination at point 'c'

(Bending stress) $f = 0$

$$\tau = \frac{VA\bar{y}}{Ib} = \frac{12.5\times10^3\times[150\times10\times145+140\times10\times70]}{8.14\times10^7\times10} = 4.845 \text{ N/mm}^2$$

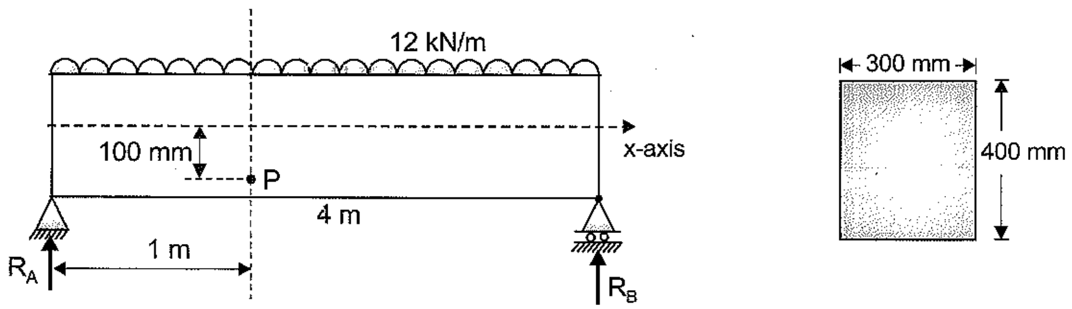
According to this σ_1/σ_2 = + 4.845, – 4.845 N/mm²

$\Rightarrow$ Maximum value of principal stress = 9.74 N/mm² and it occur at section B at the junction of flange and web

Q-6: ***A simply supported beam is 4 metres long and carries a uniformly distributed load of 12 kN/m over its entire length. The cross-section of the beam is 300 × 400 mm deep. Find the principal stresses and their directions at a point 100 mm below the neutral axis and at 1/4 of the span.***

[15 Marks, ESE-2003]

Sol:



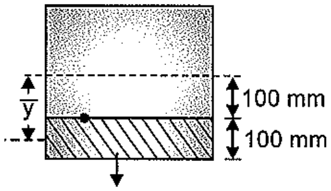
From symmetry of loading

$$R_A = R_B = 24 \text{ kN}$$

$$\Rightarrow \quad \text{SF at P} = 24 - 12 \times 1 = 12 \text{ kN}$$

Bending moment at point, P $= 24\times1-12\times1\times\frac{1}{2}=18$ kN-m

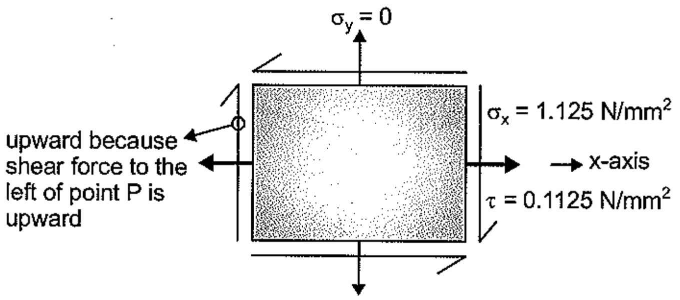
$\therefore$ Bending stress $= \frac{M}{I}\times y = \left[\frac{18\times10^6\times12}{300\times400^3}\times100\right]N/mm^2 = 1.125$ N/mm² (tension)

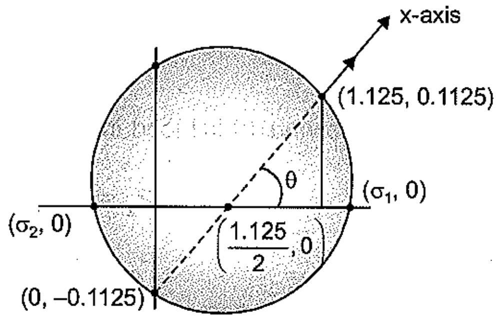


Area = A = 300 × 100 = 30000 mm²
$\bar{y}$ = 150 mm

and, Shear stress, $\tau = \frac{VA\bar{y}}{Ib} = \frac{12\times10^3\times100\times300\times150}{\frac{300\times400^3}{12}\times300} = 0.1125$ N/mm²

Diagrammatically showing the stress condition,





Mohr circle

$$\text{Radius} = \sqrt{0.1125^2+\left(\frac{1.125}{2}\right)^2} = 0.5736$$

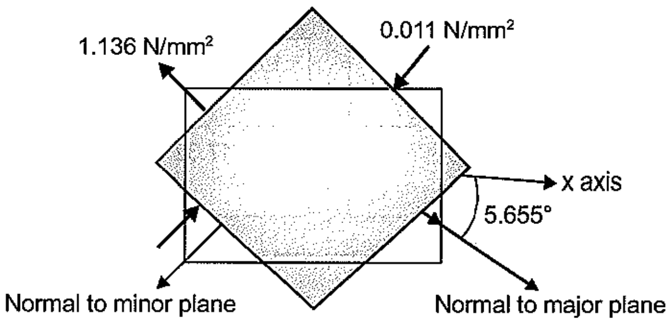
$$\therefore \quad \sigma_1/\sigma_2 = \left(\frac{1.125}{2}+0.5736\right), \left(-0.5736+\frac{1.125}{2}\right) = 1.136, -0.011 \text{ N/mm}^2$$

$\Rightarrow$ Major principal stress = 1.136 N/mm²

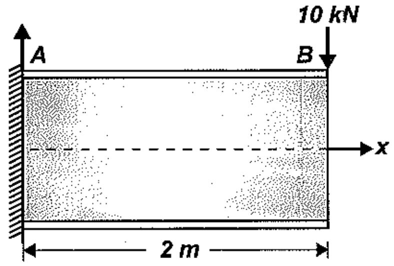
Minor principal stress = – 0.011 N/mm²

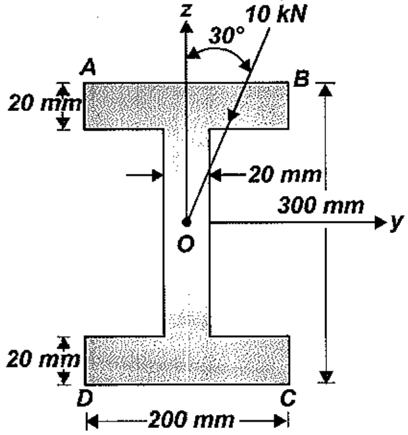
$$\tan 2\theta_P = \frac{0.1125}{\frac{1.125}{2}} \Rightarrow 2\theta_P = 11.3099°; 191.3099°$$

$\Rightarrow$ Normal to major principal plane makes an angle of 5.655° with x-axis in clockwise direction & normal to minor principal stress makes an angle of 95.655° in clockwise direction with x-axis



Q-7: ***A cantilever beam AB of I section has a span of 2 m and supports an inclined load of 10 kN at the free end B. The load passes through the centroid of the I section and makes an angle 30° with the vertical in the plane of the cross-section as shown in figure. Find the normal stresses at the corners A, B, C and D of the cross-section at the fixed end and shear stress component in the vertical direction at the centroid of the cross-section. The dimensions of the cross-section are given in the figure.***



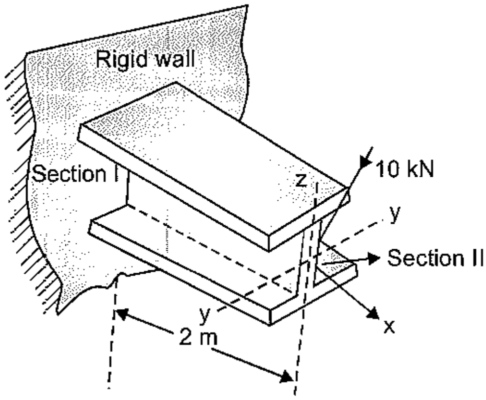


[20 Marks, ESE-2004]

Sol: Resolving 10 kN load in y and z direction,

$$P_z = 10\cos 30° = 10\times\frac{\sqrt{3}}{2} = 8.66\,\text{kN}$$

$$P_y = 10\sin 30° = 5\text{ kN}$$



Fixed end moments M_y and M_z are,

$$M_z = 5\times 2 = 10\text{ kNm}$$

$$M_y = 8.66\times 2 = 17.32\text{ kNm}$$

Moment of inertia about Y axis,

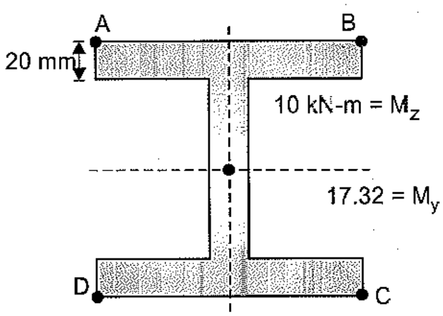
$$I_y = \left(\frac{200\,(300)^3}{12} - \frac{180\,(260)^3}{12}\right)$$

$$\Rightarrow \quad I_y = 1.8636\times 10^8\text{ mm}^4$$

Moment of inertia about Z-axis

$$I_z = \left(2\times\frac{20\,(200)^3}{12} + \frac{260\,(20)^3}{12}\right) = 2.684\times 10^7\text{ mm}^4$$

At Section-I where we have to calculate stresses at A, B, C and D



Assuming tension as positive and compression as negative, we will calculate the normal stresses.

$$\frac{M_y \times Z_{max}}{I_y} = \frac{17.32\times10^6\times150}{1.8636\times10^8} = 13.94\ \text{N/mm}^2$$

$$\frac{M_y \times Y_{max}}{I_z} = \frac{10\times10^6\times100}{2.684\times10^7} = 37.258\ \text{N/mm}^2$$

$$\sigma_A = 13.94 - 37.258 = -23.32\ \text{N/mm}^2\ \text{(compression)}$$

$$\sigma_B = +13.94 + 37.258 = 51.198\ \text{N/mm}^2\ \text{(tensile)}$$

$$\sigma_C = -13.94 + 37.258 = 23.32\ \text{N/mm}^2\ \text{(tensile)}$$

$$\sigma_D = -13.94 - 37.258 = -\ 51.198\ \text{Nmm}^2\ \text{(compressive)}$$

Shear stress component in vertical direction at centroid of x-sec is given

$$\tau = \frac{VA\bar{y}}{I_{yy}b}$$

$$V = 10\cos30^\circ = 5\sqrt{3}$$

$$A\bar{y} = A,\bar{y}_1 + A_2\bar{y}_2 = 200\times20\times140 + 130\times20\times65 = 729000\ \text{mm}^3$$

$$I_{yy} = 1.8636 \times 108\ \text{mm}^4$$

$$b = 20\ \text{mm}$$

$$\tau = \frac{VA\bar{y}}{Ib} = \frac{8.66\times10^3\times[20\times200\times140+130\times20\times65]}{1.8636\times10^8\times20} = 1.69\text{N}/\text{mm}^2$$

Q-8: ***A cantilever beam with circular cross-section of radius 100 mm is subjected to a uniformly distributed load over the entire span. It is given that the deflected shape of the beam has a maximum curvature of 1.018592 × 10^{-6} mm^{-1} and a maximum shear force of 1 kN. Find the intensity of loading on the beam and its span.***

$$E = 2 \times 10^5\ MPa$$

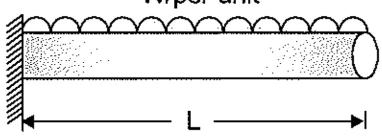
[10 Marks, ESE-2009]

Sol: Let the intensity of loading and length of span be W and L respectively.

Maximum curvature = 1.018592 × 10^{-6}/mm

Maximum shear force = 1 kN

E = 2 × 10^5 N/mm^2



We know that, $\frac{M}{EI} = \frac{d^2y/dx^2}{1+\left(\frac{dy}{dx}\right)^{3/2}}$, where y = deflection.

For linear Ist order analysis, $\frac{dy}{dx} \cong 0$. Hence $\frac{M}{EI} = \frac{d^2y}{dx^2}$

$$\therefore \quad \text{Maximum curvature} = \frac{M_{max}}{EI}$$

$$\Rightarrow \quad \frac{M_{max}}{2\times10^5\times\frac{\pi}{64}\times(100\times2)^4} = 1.018592\times10^{-6}$$

$$\Rightarrow \quad M_{max} = 16 \times 10^6 \text{ N-mm}$$

$$\Rightarrow \quad M_{max} = 16 \text{ kN-m}$$

Maximum bending moment in cantilever shown above occurs at the fixed end of magnitude $WL^2/2$

$$\Rightarrow \quad \frac{WL^2}{2} = 16 \quad \therefore \quad WL^2 = 32 \quad \text{... (i)}$$

Maximum shear force occurs at the fixed end of magnitude = WL

$$\Rightarrow \quad WL = 1 \quad \text{... (ii)}$$

Dividing eq^n (i) by eq^n (ii)

$$\therefore \quad L = 32 \text{ m}$$

and
$$W = \frac{1}{32} \text{ kN/m} = \frac{1000}{32} \text{N/ m} = 31.25 \text{N/m}$$

Q-9: ***A solid circular shaft of diameter 50 mm is subjected to pure bending of 3.5 kN-m. Find the maximum twisting moment that can be applied on this shaft such that the material of the shaft does not yield. Use Tresca's theory (maximum shear stress theory) of failure. The yield stress of the material in uniaxial tension is 400 N/mm?***

[15 Marks, ESE-2009]

Sol: Maximum bending stress developed at the surface, (f_{max})

$$f_{max} = \frac{M}{I} \times y_{max} = \frac{M}{\frac{\pi}{64} \times D^4} \times \frac{D}{2}$$

$$\boxed{f_{max} = \frac{32M}{\pi D^3}}$$

Suppose maximum twisting moment applied be T, then maximum shear stress induced will be,

$$\tau_{max} = \frac{T}{I_P} \times r_{max} = \frac{T}{\frac{\pi}{32} \times D^4} \times \frac{D}{2}$$

$$\boxed{\tau_{max} = \frac{16\,T}{\pi D^3}}$$

Max shear stress under combined action of bending and twisting is

$$\Rightarrow \quad \frac{T_{ec} \cdot r}{J} = \frac{T_{eq} \times D/2}{\frac{\pi D^4}{32}} = \frac{16 T_{eq}}{\pi D^3}$$

$$\Rightarrow \quad T_{eq} = \sqrt{M^2 + T^2}$$

$$\Rightarrow \quad \frac{16\,T_{eq}}{\pi D^3} = \frac{16\sqrt{M^2 + T^2}}{\pi D^3}$$

As per Tresca theory

$$\Rightarrow \quad \frac{16}{\pi D^3}\sqrt{M^2 + T^2} \le \frac{\sigma_y}{2}$$

D = 50 mm, M = 3.5 kN-m and σ_y = 400 N/mm^2

$$\frac{16}{\pi(50)^3}\times\sqrt{(3.5\times10^6)^2+T^2} \le \frac{400}{2}$$

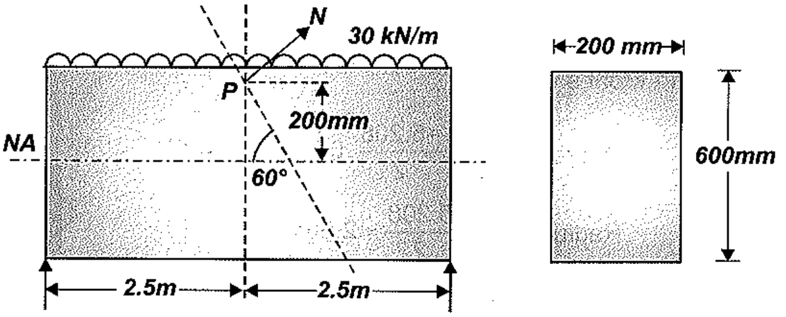
$$(3.5\times10^6)^2+T^2 \le \left[\frac{400\times\pi\times50^3}{32}\right]^2$$

$$T^2 \le 1.18457\times10^{13}$$

$$T \le 3.4417\times10^6 \text{ N-mm}$$

$\therefore$ $\boxed{\text{Maximum twisting moment} = 3.44 \text{ kN-m}}$

Q-10: ***A simply supported beam of span 5m and cross-section 200 mm × 600 mm is subjected to a uniformly distributed load of 30 kN/m including self-weight. A plane inclined at 60° to the axis of the beam is passing through a point P located on the central cross-section of the beam and 200 mm above the neutral axis. Find the normal stress and shear stress on the inclined plane at point P.***



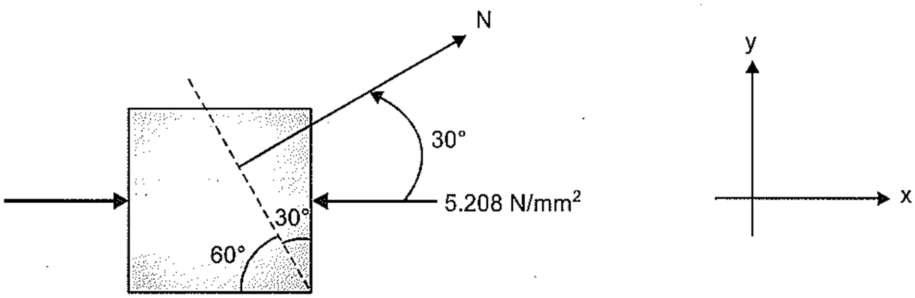
[10 Marks, ESE-2010]

Sol: BM at section at mid span = $\frac{wl^2}{8}=\frac{30\,(5)^2}{8}=93.75\,\text{kNm}$

Bending stress at point P = $\frac{My}{I}=\frac{93.75\times10^6\times200}{\frac{200(600)^3}{12}}$ = 5.208 N/mm^2

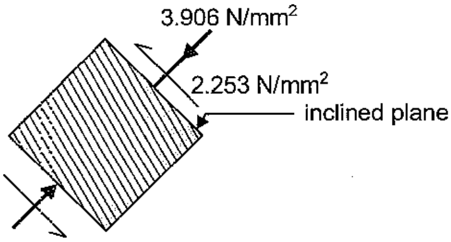
Shear stress at point P = $\frac{VA\bar{y}}{Ib}$ = 0 (Since shear force at mid span = 0)

The element at P

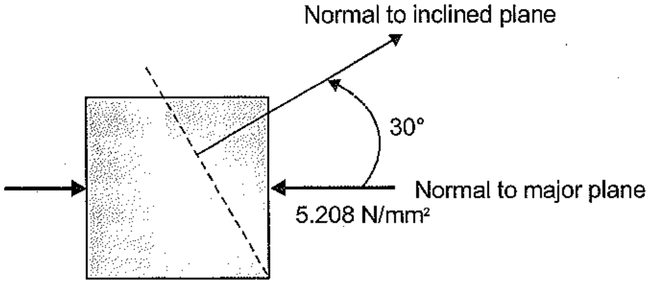


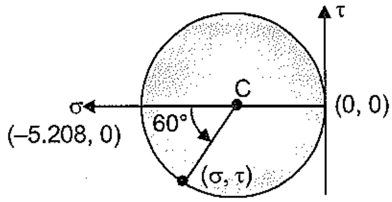
Normal stress on the inclined plane = $\frac{\sigma_x}{2}+\frac{\sigma_x}{2}\cos60°+0=\frac{-5.208}{2}\left(1+\frac{1}{2}\right)$ = –3.906 N/mm^2

Shear stress = $\frac{-\sigma_x}{2}\sin60°=\frac{+5.208}{2}\times\frac{\sqrt{3}}{2}$ = +2.255 N/mm^2

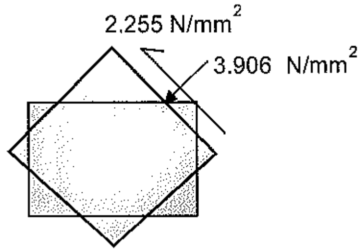


Alternative Method





As normal to inclined plane is at an angle of 30° anticlockwise with normal to major plane, it will make an angle of 60° (↺) in Mohr's circle.



$$\text{Centre, } C = \left(\frac{-5.208}{2}, 0\right) = (-2.604, 0)$$

$$\text{Radius, } R = \frac{5.208}{2} = 2.604$$

$$\Rightarrow \quad \tau = R \sin 60° = -2.255 \text{ N/mm}^2$$

$$\sigma = -\{2.604 + R\cos 60°\} = -3.906 \text{ N/mm}^2$$

Note: (-ve) sign of shear stress in Mohr's circle implies that it will give an anticlockwise rotation about the centre of the element.

Q-11: ***At a certain cross-section, a circular shaft 90 mm in diameter is subjected to a BM of 3 kNm and twisting moment of 6 kNm. Find the principal stresses induced in the section using maximum normal stress theory.***

[15 Marks, ESE-2014]

Sol: **Given:**

$$\text{Diameter of shaft, } d = 90 \text{ mm}$$

$$\text{Bending moment, } M = 3 \text{ kNm} = 3 \times 10^6 \text{ Nmm}$$

$$\text{Twisting moment, } T = 6 \text{ kNm} = 6 \times 10^6 \text{ Nmm}$$

Let principal stresses are σ_1 and σ_2.

We know that, for combined bending and torsion, principal stresses are given by

$$\sigma_1, \sigma_2 = \frac{16}{\pi d^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

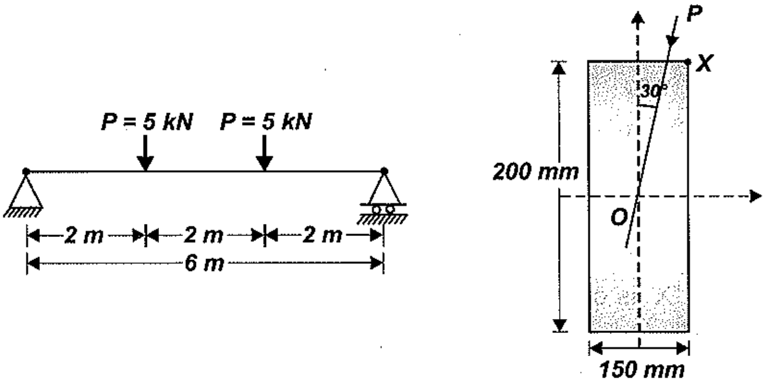
$$\therefore \quad \sigma_1, \sigma_2 = \frac{16}{\pi \times 90^3}\left[3 \times 10^6 \pm \sqrt{(3 \times 10^6)^2 + (6 \times 10^6)^2}\right]$$

$$\sigma_1, \sigma_2 = \frac{16}{\pi \times 90^3}\left[3 \pm \sqrt{45}\right] \times 10^6 = \frac{16}{\pi \times 90^3}[3 \pm 6.71] \times 10^6$$

$$\therefore \quad \sigma_1 = \frac{16\times(3+6.71)\times10^6}{\pi\times90^3} = 67.84 \text{ N/mm}^2$$

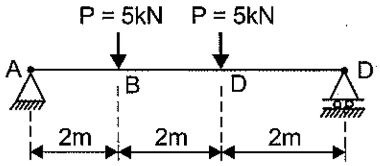
$$\sigma_2 = \frac{16\times(3-6.71)\times10^6}{\pi\times90^3} = -25.92 \text{ N/mm}^2$$

Q-12: ***A rectangular beam 150 mm wide and 200 mm deep, is simply supported on a span of 6 m. Two loads of 5 kN each are applied to the beam, each load being 2 m from the supports as shown in the figures. The plane of loads make an angle of 30° with the vertical plane of symmetry. Find the direction of neutral axis and the bending stress at a point marked 'X'.***

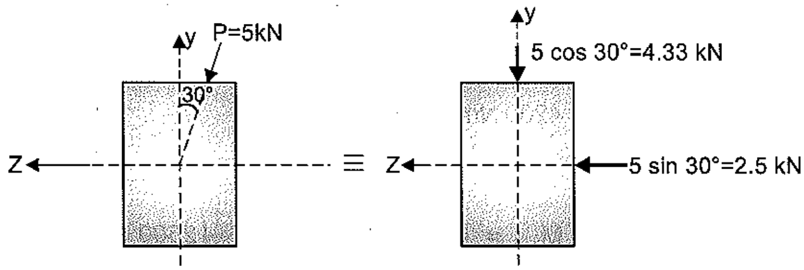


[20 Marks, ESE-2015]

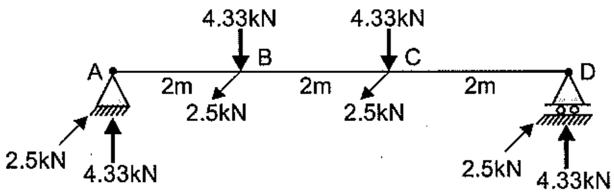
Sol: The given beam is



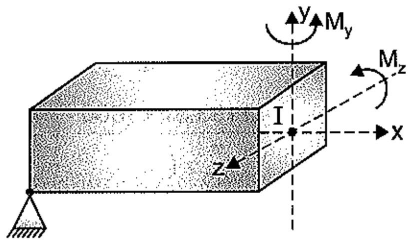
The loading can be shown as



The resolved loading on the beam can be shown as:

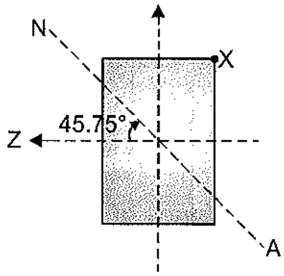


Max. BM will occur at mid-span of the beam.



[Here, M_y & M_z are the reaction moments]

Calculation of M_z & M_y:



$$M_z = 4.33 \times 3 - 4.33 \times 1 = 8.66 \text{ kNm}$$

$$M_y = 2.5 \times 3 - 2.5 \times 1 = 5 \text{ kNm}$$

$$\sigma_x = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y} \quad \text{(in Ist quadrant)}$$

$$= \frac{-8.66\,y}{150 \times \frac{200^3}{12}} + \frac{5z}{200 \times \frac{150^3}{12}} = -\frac{433}{5000} y + \frac{4}{45} z$$

For neutral axis, $\sigma_x = 0$

$\Rightarrow$ $y = 1.026\,z$

$\Rightarrow$ $\tan\phi = 1.026$

$\Rightarrow$ $\phi = 45.75°$

Bending stresses at "X" is calculated as

$$\sigma_x = \frac{-M_z y}{I_z} - \frac{M_y z}{I_y} = \frac{-8.66 \times 10^6 \times 100}{150 \times \frac{200^3}{12}} - \frac{5 \times 10^6 \times 75}{200 \times \frac{150^3}{12}} = -15.327 \text{ N/mm}^2$$

Q-13: ***A cylindrical shaft of 75mm diameter is subjected to a maximum bending moment of 2500 Nm and a twisting moment of 4200 Nm. Find the maximum principal stresses developed in the shaft. If the yield stress of the shaft material is 360 N/sq. mm, determine the factor of safety of the shaft using maximum shear stress theory.***

[20 Marks, ESE-2017]

Sol: Given that:

$d = 75$ mm ; $M = 2500$ Nm ; $T = 4200$ Nm; $f_y = 360$ N/mm^2

So, equivalent B.M.

$$M_{1/2} = \frac{1}{2}\left(M \pm \sqrt{M^2 + T^2}\right) = \frac{1}{2}\left(2500 \pm \sqrt{2500^2 + 4200^2}\right)$$

$$= \frac{1}{2}(2500 \pm 4887.74)$$

So, $M_1 = 3693.87$ Nm

$M_2 = -1193.87$ Nm

Maximum principal stress is equal to max bending stress due to equivalent max bending moment.

So, Max principal stress $= \sigma_1 = \dfrac{M_1}{z} = \dfrac{3693.87 \times 1000}{\frac{\pi}{32} \times (75)^3} = 89.23$ N/mm^2 (major principal stress)

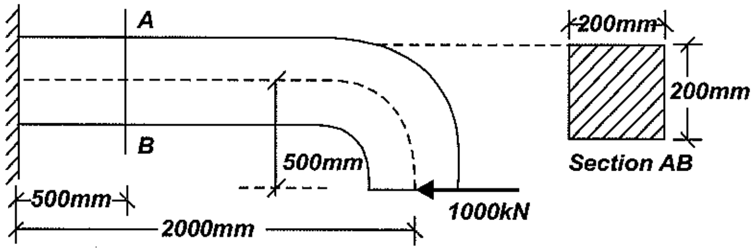
Min principal stress $= \sigma_2 = \dfrac{-1193.87 \times 1000}{\frac{\pi}{32} \times (75)^3} = -28.84$ N/mm^2 (minor principal stress)

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{89.23 - (-28.84)}{2} = 59.035 \text{ N/mm}^2$$

For max shear stress theory,

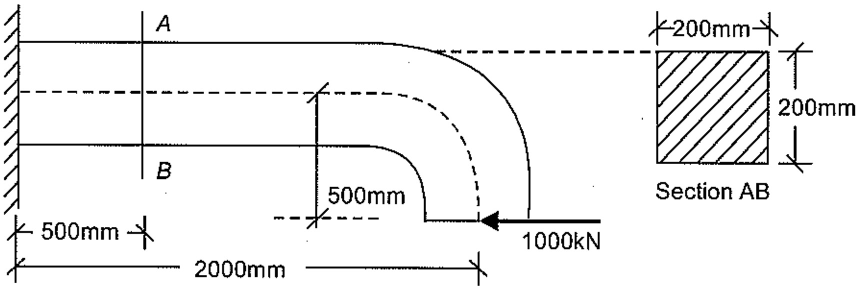
$$\text{Factor of safety} = \frac{f_y/2}{\tau_{max}} = \frac{360}{2 \times 59.035} = \boxed{3.049}$$

Q-14: ***A bracket has been loaded as shown in figure. Compute the top and bottom fibre stress at section A-B.***

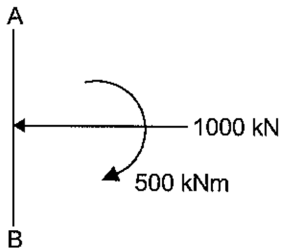


[12 Marks, ESE-2018]

Sol:



Stresses acting on section-AB



Calculation for the top fibre stress

$$= \frac{-P}{A} + \frac{M}{z}$$

Since,

$$z = \frac{I}{y} = \frac{bd^3}{12(d/2)} = \frac{bd^2}{6}$$

$$z = \frac{(200)^3}{6} = 1.333 \times 10^6 \text{ mm}^3$$

$$= \frac{(-1000 \times 10^3)\,N}{(200 \times 200)} + \frac{(500 \times 10^6)}{(1.333 \times 10^6)} = -25 + 375 = 350 \text{ N/mm}^2.$$

Calculation for the bottom fibre stress

$$= \frac{-P}{A} - \frac{M}{z} = \frac{-(1000 \times 10^3)\,N}{(200 \times 200)} - \frac{(500 \times 10^6)}{(1.333 \times 10^6)}$$

$$= -25 - 375 = -400 \text{ N/mm}^2$$

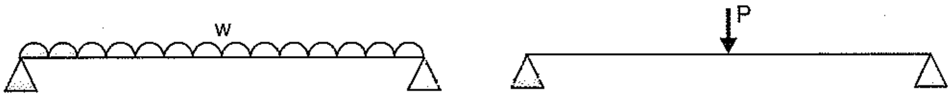
CHAPTER 6

BENDING STRESS IN BEAM

Q-1: ***Compare the strain energy of a beam simply supported at its ends loaded with a uniformly distributed load with that of the same beam loaded with a central concentrated load and each having the same value of the maximum bending stress.***

[10 Marks, ESE-1998]

Sol:

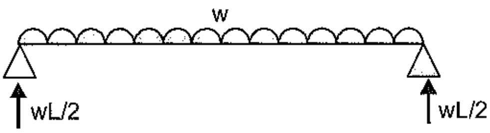


Each having the same maximum value of bending stress.

Compare the strain energy =

Strain energy calculation for udl

$$\text{Bending moment}\Big|_x = \frac{wLx}{2} - \frac{wx^2}{2}$$



$$M_{max} = \left(\frac{wL^2}{8}\right)$$

$$f_{max} = \frac{M}{I} \times y_{max} = \frac{wL^2}{8I} \times \frac{D}{2} = \frac{wL^2 D}{16I}$$

Strain energy of beam with udl

$$U = 2\int_0^{L/2} \frac{M^2 dx}{2EI} = 2\int_0^{L/2} \frac{\left(\frac{wL}{2}x - \frac{wx^2}{2}\right)^2 dx}{2EI} = \frac{w^2}{4EI}\int_0^{L/2} \left(Lx - x^2\right)^2 dx$$

Solving the integral part:

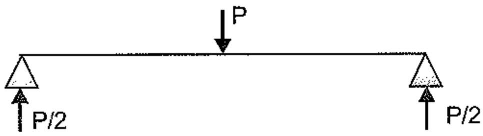
$$\int_0^{L/2} \left(Lx - x^2\right)^2 dx = \int_0^{L/2} \left(L^2x^2 + x^4 - 2Lx^3\right) dx = \left[\frac{L^2x^3}{3} + \frac{x^5}{5} - \frac{2L \cdot x^4}{4}\right]_0^{L/2}$$

$$= \left[\frac{L^5}{3\times 8} + \frac{L^5}{5\times 2^5} - \frac{2L^5}{4\times 2^4}\right] = \left[\frac{L^5}{24} + \frac{L^5}{160} - \frac{L^5}{32}\right] = \frac{L^5}{60}$$

$$\therefore \quad U = \frac{w^2L^5}{240EI}$$

Strain energy calculation point load

$$B.M.\Big|_{max} = \frac{P}{2}\cdot\frac{L}{2} = \frac{PL}{4}$$



$$\therefore \qquad f_{max} = \frac{PL}{4}\times\frac{1}{I}\times y_{max}$$

$$= \frac{PLD}{8I} \qquad \left\{y_{max} = \frac{D}{2}\right\}$$

And, Strain energy stored,
$$U = 2\int_0^{L/2}\frac{\left(\frac{P}{2}x\right)^2 dx}{2EI} = \frac{P^2}{4EI}\int_0^{L/2} x^2\,dx = \frac{P^2}{4EI}\times\left|\frac{x^3}{3}\right|_0^{L/2}$$

$$= \frac{P^2}{12EI}\times\left(\frac{L^3}{8}\right) = \frac{P^2L^3}{96EI}$$

Since maximum bending stress is same in both the cases hence

$$\frac{wL^2D}{16\,I} = \frac{PLD}{8I}$$

$$\Rightarrow \qquad 36\ wL = 2P$$

$$\therefore \qquad \boxed{P = \frac{wL}{2}}$$

And comparing strain energy,

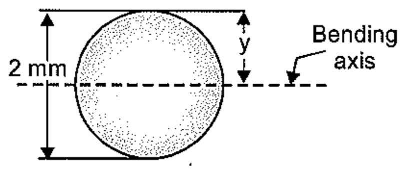
$$\frac{\text{strain energy stored}|_{udl}}{\text{strain energy stored}|_{\text{point load}}} = \frac{\frac{w^2L^5}{240EI}}{\frac{P^2L^3}{96EI}} = \frac{w^2L^5}{P^2L^3}\times\frac{96}{240} = \frac{w^2L^5\times 96}{\frac{w^2L^2}{4}\times L^3\times 240} = \frac{96\times 4}{240} = \frac{8}{5}$$

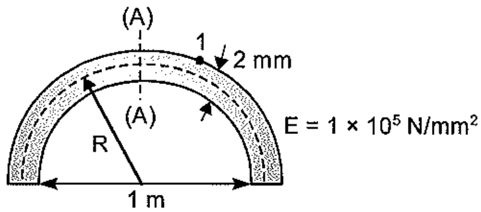
$$\boxed{\frac{U|_{udl}}{U|_{\text{point load}}} = \frac{8}{5}}$$

Q-2: ***A wire of 2 mm diameter is wound round a circular shaft of 1 metre diameter. What are the maximum normal & shear stresses in the wire? Specify the planes of their occurrence. E = 1 × 10^5 N/mm^2. Assume no applied tensile pull on the wire.***

[10 Marks, ESE-2002]

Sol:





Section A – A

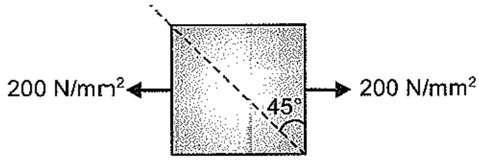
We know that from flexure formula

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

$$\Rightarrow \qquad f_{max} = \frac{Ey}{R} = \frac{1\times 10^5\times 1\text{mm}}{0.5\times 1000\text{ mm}}\ \text{N/mm}^2$$

$$\Rightarrow \qquad f_{max} = 200\ \text{N/mm}^2$$

Bending stress will give max normal stress of 200 N/mm².



Point 1

$$\text{Max normal stress} = 200 \text{ N/mm}^2$$

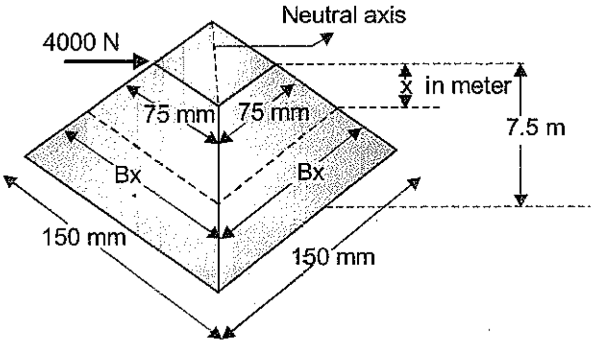
$$\text{Max shear stress} = \frac{200-0}{2} = 100 \text{ N/mm}^2$$

Max normal stress will occurs an shown on the element. Max shear stress will occur at 45° to max principal plane

Q-3: ***A vertical flag staff has a height of 7.5 m above ground and is made up of square section throughout. Its dimensions being 75 mm × 75 mm at the top, changing uniformly to 150 mm × 150 mm at the ground. A horizontal force of 4000 N is applied at the top, the location of this loading being along a diagonal of the section. Find the maximum stress due to this loading and its location.***

[10 Marks, ESE-2006]

Sol:



Width of square section at any distance x,

$$B_x = 75 + \frac{150-75}{7.5}x$$

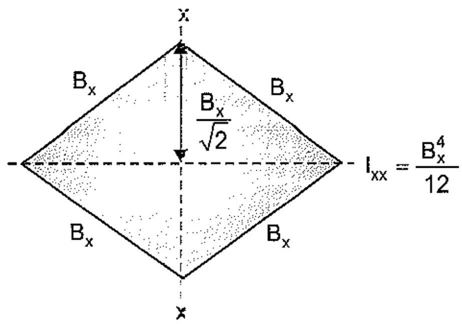
$$\Rightarrow \quad B_x = (75 + 10x) \text{ mm, where x in meter}$$

Bending moment at any distance x, is,

$$M_x = (4000x) \text{ Nm} = (4 \times 10^6 x)\text{N-mm}$$

Section modulus $\quad Z_x = \dfrac{I_{xx}}{y_{max}}$

$$I_{xx} = \frac{B_x^4}{12}$$



and $\quad y_{max} = \dfrac{B_x}{\sqrt{2}}$

$$Z_{xx} = \frac{\frac{B_x^4}{12}}{\frac{B_x}{\sqrt{2}}} = \frac{\sqrt{2}B_x^3}{12} = \frac{B_x^3}{6\sqrt{2}}$$

So, $$f_{x,\,max} = \frac{M_x}{Z_x} = \frac{4000 \times x \times 10^3}{\frac{B_x^{\,3}}{6\sqrt{2}}} = \frac{4000 \times 10^3 x \times 6\sqrt{2}}{(75+10x)^3}$$

(where $f_{x,\,max}$ in N/mm² and x in meter.)

For, $f_{x,\,max}$ to be maximum, $\frac{d(f_{x,\,max})}{dx} = 0$

$$\Rightarrow \quad 6\sqrt{2} \times 4 \times 10^6 \times \left[\frac{1(75+10x)^3 - 3(75+10x)^2 \times 10x}{(75+10x)^6}\right] = 0$$

$$\Rightarrow \quad (75 + 10x) - 30x = 0$$

$$\Rightarrow \quad 75 = 20x$$

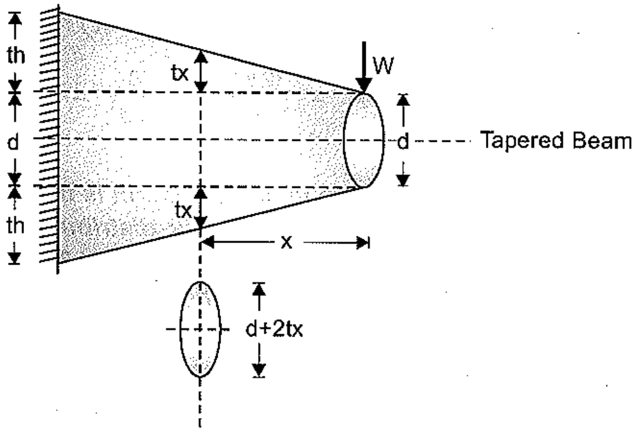
$$\therefore \quad x = \frac{75}{20} = 3.75\text{m}$$

Hence, $$f_{x,\,max}\big|_{x=3.75m} = \frac{4 \times 10^6 \times 6\sqrt{2} \times 3.75}{(75+37.5)^3} = 89.39\text{N/mm}^2$$

Q-4: ***A cantilever, circular in section, has a uniform taper with the horizontal. The diameter of the cantilever at the free end is 'd' and it tapers @ t units/m length on each side. A vertical load W acts at the free end of the cantilever, find an expression for the distance of the section from the free end at which the bending stress is maximum. The length of the cantilever is h units.***

[10 Marks, ESE-2006]

Sol:



Circular Cross-Section

$$d_x = d + 2tx$$

where d_x is the diameter at some distance x from the right end.

Bending stress determination,

$$f_x = \frac{M_x}{I} \times y = \frac{Wx}{I} \times y$$

$$\therefore \quad f_{x,max} = \frac{Wx}{I} \times y_{max} = \left\{\frac{Wx}{Z_x}\right\}$$

where, $$Z_x = \frac{I_{xx}}{y_{max}} = \frac{\pi}{32} d_x^{\,3}$$

$$\therefore \qquad f_{x,\,max} = \frac{Wx \times 32}{\pi d_x^{\,3}}$$

Hence,
$$\boxed{f_{x,\,max} = \frac{32Wx}{\pi(d+2tx)^3}}$$

We have to determine the value of 'x' for which this maximum bending stress $f_{x,\,max}$ is max^m.

Hence,
$$\frac{d\,(f_{x,\,max})}{dx} = 0$$

$$\Rightarrow \qquad \frac{32W}{\pi}\frac{d}{dx}\left[\frac{x}{(d+2tx)^3}\right] = 0$$

$$\frac{1\cdot(d+2tx)^3 - [3(d+2tx)^2 \times 2t \times x]}{(d+2tx)^6} = 0$$

$$\Rightarrow \qquad (d + 2tx) - 6tx = 0$$

$$\Rightarrow \qquad d - 4t\,x = 0$$

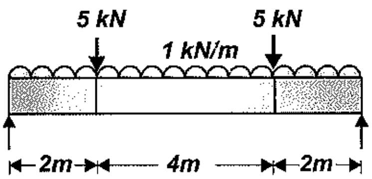
$$\Rightarrow \qquad d = 4tx$$

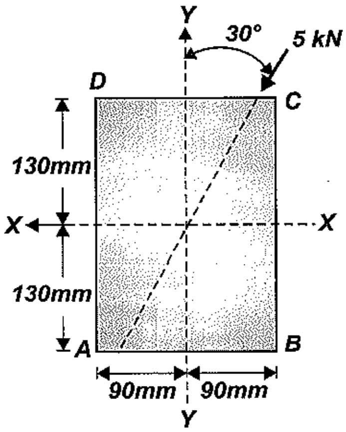
$$\therefore \qquad x = \frac{d}{4t}$$

and,
$$f_{x,\,max}\big|_{x=d/4t} = \frac{32\times W}{\pi}\times\frac{d/4t}{\left(d+2t\cdot\frac{d}{4t}\right)^3} = \frac{32W}{\pi}\times\frac{d/4t}{\left(\frac{3d}{2}\right)^3}$$

$$= \frac{32W}{\pi}\times\frac{d}{4t}\times\frac{8}{27d^3} = \frac{64W}{27\,\pi d^2 t}$$

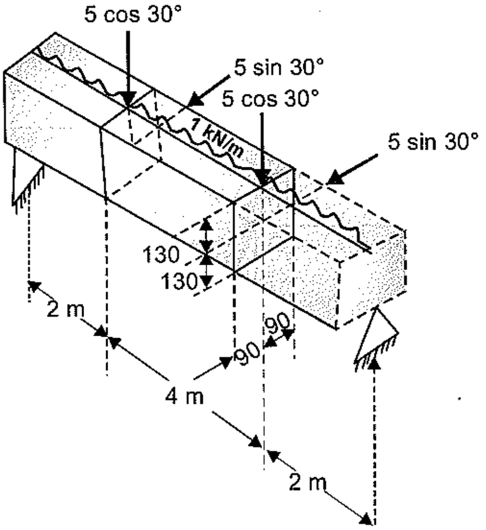
Q-5: ***A rectangular steel beam 180 mm × 260 mm in cross-section is used as a simply supported beam over a span of 8 m and is subjected to a uniformly distributed load of 1 kN/m and also a concentrated load of 5 kN at 2 m from each support inclined at 30° to the vertical axis. Determine the bending stresses at four corners of the beam and location of the neutral axis of the cross-section.***



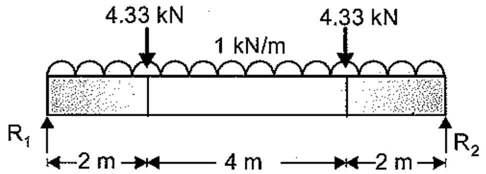


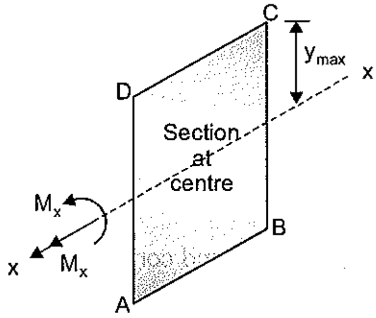
[20 Marks, ESE-2006]

Sol: After breaking the force into their components:



Consider vertical Loading





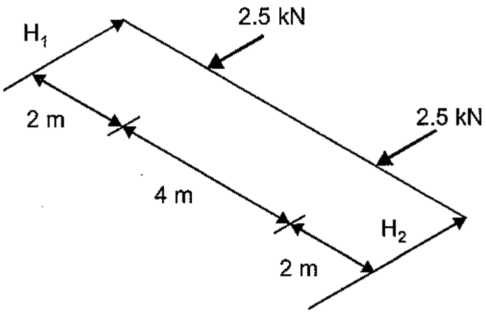
Since beam load is symmetrical hence, $R_1 = R_2 = 4.33 + \frac{8}{2} = 8.33$ kN

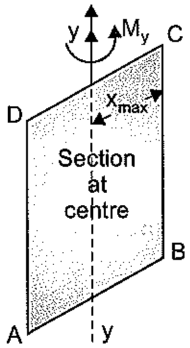
Maximum bending moment will occur at the centre due to symmetrical loading,

i.e., $x = 4$ m.

$$M_x = 8.33 \times 4 - 1 \times 4 \times 2 - 4.33 \times 2 = 16.66 \text{ kN-m}$$

Consider horizontal Loading



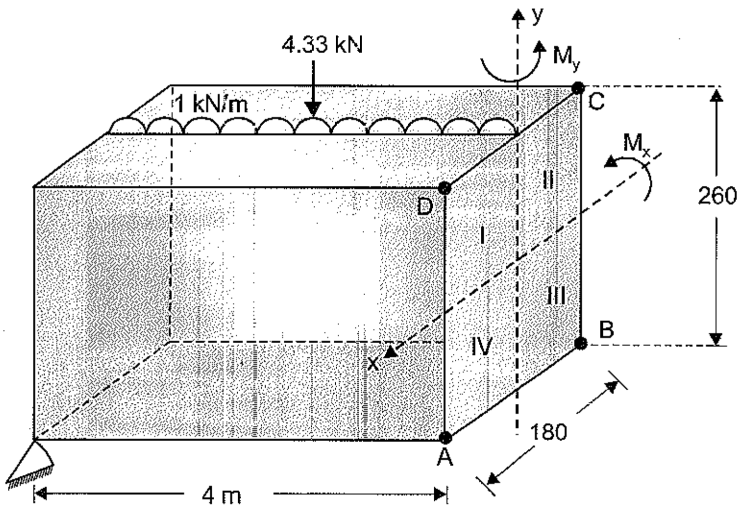


Due to symmetry $H_1 = H_2 = 2.5$ kN

Maximum bending moment again will occur at centre,

$$M_y = \text{B.M.}|_{\text{at centre}} = 2.5 \times 4 - 2.5 \times 2 = 5 \text{ kN-m}$$

Thus the BM at the mid span section is as shown below



$M_x = +16.66$ kNm
$M_y = +5$ kNm

Bending stress at a general point is given by

$$\sigma = -\frac{M_x y}{I_{xx}} + \frac{M_y x}{I_{yy}}$$ [in this equation; M_x,M_y, x and y have algebraic sign]

Since, M_x, and M_y are both (+)ve i.e orientation of M_x and M_y are in (+)ve co-ordinate direction (as per right hand thumb rule)

Note: The expression of σ can be writing by writing the value of σ for any point in Ist quadrant. Take tension (+)ve and compression (–)ve

Hence,

$$I_{xx} = \frac{180\,(260)^3}{12} = 2.6364 \times 10^8 \text{ mm}^4$$

$$I_{yy} = \frac{260\,(180)^3}{12} = 1.2636 \times 10^8 \text{ mm}^4$$

$\Rightarrow$ $$\sigma = \frac{-16.66 \times 10^6 \times y}{2.6364 \times 10^8} + \frac{5 \times 10^6 x}{1.2636 \times 10^8}$$

$$\boxed{\sigma = -0.0632\, y + 0.0396\, x} \quad \text{... (i)}$$

By putting the value of co-ordinates of point A, B, C and D in above equation (i) we can get the stress with its sign.

(+)ve sign = Tension
(–)ve sign = Compression

For point A

$x = 90$ mm
$y = -130$ mm
$\sigma = -0.0632 \times (-130) + 0.0396 \times (90)$
$\sigma = +8.216 + 3.564 = 1178$ N/mm² (tension)

For Point B

$x = -90$ mm
$y = -130$ mm
$\Rightarrow$ $\sigma = 8.216 - 3.564 = 4.652$ N/mm² (tension)

For Point C

$$x = -90 \text{ mm}$$

$$y = +130 \text{ mm}$$

$\Rightarrow$ $$\sigma = -8.216 - 3.564 = -11.78 \text{ N/mm}^2 \text{ (compression)}$$

For Point D

$$x = 90 \text{ mm}$$

$$y = 130 \text{ mm}$$

$\Rightarrow$ $$\sigma = -8.216 + 3.564 = -4.652 \text{ N/mm}^2 \text{ (Compression)}$$

Note that these values can also be obtained by simply noting the nature of stresses generated at any point due to M_x and M_y

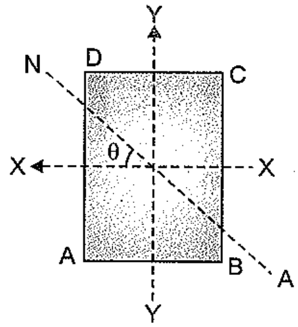
For example at B, M_x will produce tension and M_y will produce compression.

But for writing the equation for N.A the approach discussed is better because we can get the equation of N.A, by simply equating σ to zero

$\Rightarrow$ Equation of N.A in given by

$$y = \frac{0.0396}{0.0632}x$$

$$\boxed{y = 0.6266\,x}$$



$\tan\theta = 0.6266$

$\Rightarrow \theta = 32.07°$

It can be verified that as point C and D both have compression and A and B both has tension hence C and D are both on one side of N.A and so are A and B.

Q-6: ***A circular log of timber has diameter D. Determine the dimension of the strongest rectangular section, one can cut from this log.***

[10 Marks, ESE-2011]

Sol: We know that, section having larger Z will result more moment, where z = flexural strength

Hence $$D^2 = x^2 + y^2 \quad \text{.... (from figure)}$$

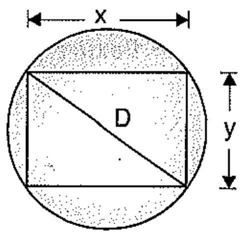
Flexural strength of rectangular section is given by

$$Z = \frac{I}{y} = \frac{xy^3/12}{y/2} = \frac{xy^2}{6} = \left\{\frac{x(D^2 - x^2)}{6}\right\}$$

$$\left[\begin{array}{l} \because x^2 + y^2 = D^2 \\ \Rightarrow y = (D^2 - x^2)^{1/2} \end{array}\right]$$

for Z to be maximum, $$\frac{dZ}{dx} = 0$$

$$\frac{d}{dx}\left[\frac{(x)(D^2 - x^2)}{6}\right] = 0$$



$\Rightarrow$ $$(D^2 - x^2) + (-2x)(x) = 0$$

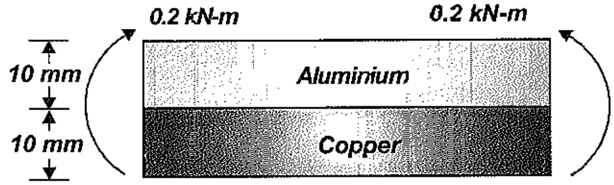
$\Rightarrow$ $$D^2 - x^2 - 2x^2 = 0$$

$\Rightarrow$ $$D^2 - 3x^2 = 0$$

$$\Rightarrow \quad x = \frac{D}{\sqrt{3}} \Rightarrow y = (D^2 - x^2)^{1/2} = \left(D^2 - \frac{D^2}{3}\right)^{1/2} = \sqrt{\frac{2}{3}}D$$

Hence for strongest section, $\boxed{x = \frac{D}{\sqrt{3}}, \quad y = \sqrt{\frac{2}{3}}\,D}$

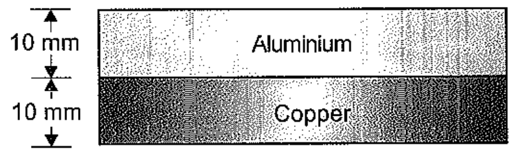
Q-7:



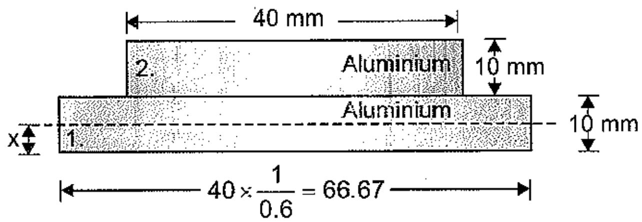
A strip of copper 40 mm wide and 10 mm thick is bonded with another strip of aluminium of same size to form a bimetallic strip of 40 mm × 20 mm. The strip is subjected to a pure bending moment of 0.2 kN-m as shown in the above figure. Calculate the radius of curvature of the strip and the maximum tensile and compressive stresses. $E_C = 1 \times 10^5$ MPa, $E_{al} = 0.6 \times 10^5$ MPa.

[20 Marks, ESE-2012]

Sol:



Transformed section for the assembly is given as below



$$\frac{1}{0.6} = \frac{E_c}{E_{al}} = \text{width modification factor}$$

Location of N.A in transformed section (x)

$$x = \frac{A_1\bar{x}_1 + A_2\bar{x}_2}{A_1 + A_2} \qquad \text{[C.G of the x-section]}$$

$$= \frac{66.67 \times 10 \times 5 + 40 \times 10 \times 15}{66.67 \times 10 + 40 \times 10} = 8.75 \text{ mm}$$

Moment of inertia about N.A of transformed section

$$= \frac{66.67\,(8.75)^3}{3} + \frac{66.67\,(10 - 8.75)^3}{3} + \frac{40\,(10)^3}{12} + 40 \times 10 \times (20 - 5 - 8.75)^2$$

$$= 33889.635 \text{ mm}^4$$

From flexure formula

$$\frac{M}{I} = \frac{f}{y} = \frac{E_{Al}}{R}$$

$$\Rightarrow \quad R = \frac{E_{Al}\,I}{M} = \frac{0.6 \times 10^5 \text{ N/mm}^2 \times 33889.635 \text{ mm}^4}{0.2 \times 10^3 \text{ Nmm}} \text{ mm}$$

$$R = 10.1669 \text{ m}$$

Max compressive stress occurs at top and Max tensile stress occurs at bottom.

$$\text{Stress at top in transformed section} = \frac{My_T}{I} = \frac{0.2\times10^6\times(20-8.75)}{33889.635} = 66.392\ \text{N/mm}^2$$

$$\text{Stress at bottom in transformed section} = \frac{66.392}{(20-8.75)}\times 8.75 = 51.638\ \text{N/mm}^2$$

$$\text{Actual stress at top} = 66.392 \times 1 = 66.392\ \text{N/mm}^2$$

$$\left[\text{Width modification factor} = \frac{E_{Al}}{E_{Al}} = 1\right]$$

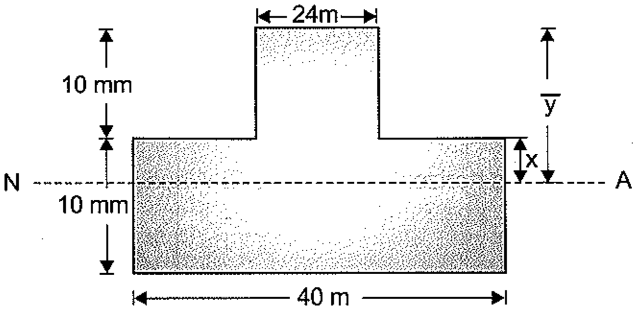
$$\text{Actual stress at bottom} = 51.638\times\frac{1}{0.6} = 86.064\ \text{N/mm}^2$$

$$\left[\text{Width modification factor of copper} = \frac{E_{cu}}{E_{Al}} = \frac{1}{0.6}\right]$$

Alternative Method:

Since the bimetallic structure is non-homogenous in nature, hence the flexure formula $\left(\frac{M}{I}=\frac{f}{y}=\frac{E}{R}\right)$ can't be applied directly. For application of flexural formula we will use transformed equivalent section concept as below,

In this case, equivalent width of aluminium = $40 \times \frac{E_{al}}{E_{cu}} = \frac{40\times0.6\times10^5}{1\times10^5} = 24\text{m}$



Neutral axis of the above cross-section is nothing but its centroidal axes.

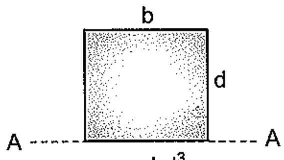
$$24\times10\times(5+x)+x\times40\times\frac{x}{2} = 40(10-x)\times\left(\frac{10-x}{2}\right)$$

$$\Rightarrow \quad 1200 + 240x + 20x^2 = 20\,(100 + x^2 - 20x)$$

$$\Rightarrow \quad 1200 + 240x + 20x^2 = 2000 + 20x^2 - 400x$$

$$\Rightarrow \quad 640x = 2000 - 1200$$

$$\Rightarrow \quad x = \frac{800}{640} = 1.25\text{m}$$



$$I_{AA} = \frac{bd^3}{3}$$

$$\text{Hence } I_{xx} = \frac{40\,(10-1.25)^3}{3}+\frac{24\,(10+1.25)^3}{3}+\frac{(40-24)\times1.25^3}{3} = 20333.33\ \text{mm}^4$$

$\therefore$ We know that, (flexural formula)

$$\frac{M}{I} = \frac{f}{y}=\frac{E}{R}$$

$$\therefore \quad \frac{M}{EI} = \frac{I}{R}$$

$$\therefore \qquad R = \frac{EI}{M} = \frac{20333.33 \times 1 \times 10^5}{0.2 \times 10^6} = 10166.67 \text{ mm}$$

Hence $\boxed{R = 10.166\text{m}}$

Due to sagging moment as given, compressive stress develop at the upper part of neutral axis and tensile stress develop at the bottom part of neutral axis.

$$\sigma_{top} \text{ (in transformed section)} = \frac{M}{I} \times y_{top} = \frac{0.2 \times 10^6}{20333.33} \times 11.25 = 110.65 \text{ N/mm}^2$$

$$\sigma_{bottom} \text{ (in transformed section)} = \frac{M}{I} \times y_{bottom} = \frac{0.2 \times 10^6}{20333.33} \times 8.75 = 86.06 \text{ N/mm}^2$$

The corresponding stress in original strip are,

$$\sigma_{Al} \text{ (at top edge of original section)} = 110.65 \times \frac{Eal}{Ec} = 110.65 \times \frac{0.6 \times 10^5}{1 \times 10^5}$$

$$\therefore \qquad \sigma_{Al} = 66.39 \text{ N/mm}^2 \text{ (Compressive)}$$

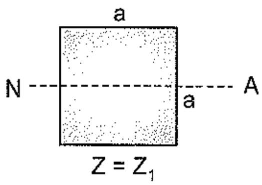
and σ_{cu} (at bottom sedge of original section) $= 86.066 \text{ N/mm}^2 \times \frac{E_{cu}}{E_{cu}} = 86.066 \text{ N/mm}^2$ (Tensile)

Q-8: ***A beam of square section of the side 'a' is placed such that (i) two sides are horizontal (ii) one diagonal is horizontal. Find the ratio of moments of resistance of the section in two positions for same permissible bending stress***

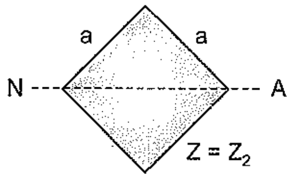
[4 Marks, ESE-2013]

Sol:

Case I: When two sides horizontal



Case II: When one diagonal horizontal



$$\Rightarrow \qquad \frac{(M.R)_1}{(M.R)_2} = \frac{\sigma \times Z_1}{\sigma \times Z_2} = \frac{Z_1}{Z_2} \qquad \text{[where, } \sigma = \text{permissible bending stress]}$$

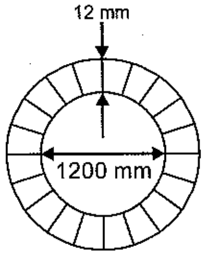
$$\Rightarrow \qquad \frac{Z_1}{Z_2} = \frac{\dfrac{a^4}{12} \Big/ \dfrac{a}{2}}{\dfrac{a^4}{12} \Big/ \dfrac{a}{\sqrt{2}}} = \sqrt{2}$$

The ratio is $(M.R)_1 : (M.R)_2 = \sqrt{2} : 1$

Q-9: ***A water main of 1200 mm internal diameter and 12 mm thick is running full. If the bending stress is not to exceed 56 MPa, find the longest span on which the pipe may be freely supported. Steel and water weight 76.8 kN/m³ and 10 kN/m³ respectively.***

[12 Marks, ESE-2019]

Sol:



Let the unsupported length $= l$m

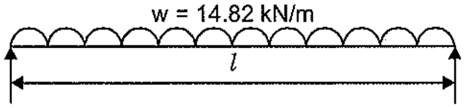
and the water main is simply supported

$$\therefore \quad \text{The total load} = \left[\frac{\pi}{4}\times(1.2)^2\times l\times 10\right]+\left[\frac{\pi}{4}(1.224^2-1.2^2)\times l\times 76.8\right]$$

$$= (11.31l+3.51l)\text{kN} = 14.82l\text{kN}$$

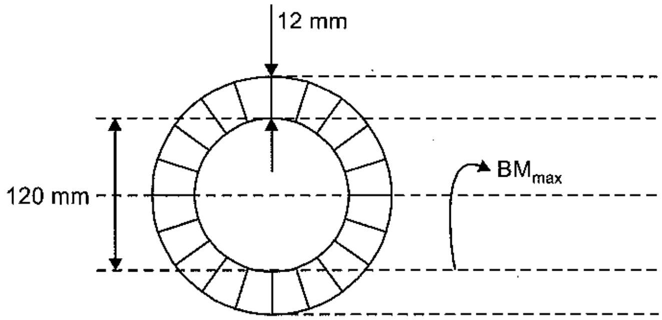
$$\therefore \quad \text{Uniformly distributed (w)} = \left[\frac{14.82l}{l}\right]\text{kN/m} = 14.82\text{kN/m}$$

and it can be represented as



$$\text{The maximum BM occurs at midspan} = \left(\frac{wl^2}{8}\right) = \left(\frac{14.82l^2}{8}\right)\text{kNm}$$

The mid-span section can be represented as



The maximum bending stress occurs at the outer surface of the water main and the stress value is obtained from the flexural formula.

$$\sigma_{max} = \frac{(BM)_{max}\times\left(\frac{\text{Outer diamter}}{2}\right)}{I}$$

$$I = \text{M.O.I. of cross-section}$$

$$= \frac{\pi}{64}\left[(1224)^4-(1200)^4\right]\text{mm}^4$$

$$= 8.39\times10^9\ \text{mm}^4$$

$$\text{and outer dia.} = (1200+2\times12) = 1224\text{mm}$$

$$\therefore \quad \sigma_{max} = \frac{\left(\frac{14.82 \times l^2}{8}\right) \times 10^6 \text{ Nmm} \times \left(\frac{1224}{2}\right) \text{mm}}{8.39 \times 10^9 \text{ mm}^4}$$

$$= 135.129 \times 10^{-3} l^2 \text{ N/mm}^2$$

As per question,

Allowable bending stress = 56 MPa

$$\therefore \quad (135.129 l^2 \times 10^{-3} \text{ MPa}) \leq 56 \text{ MPa}$$

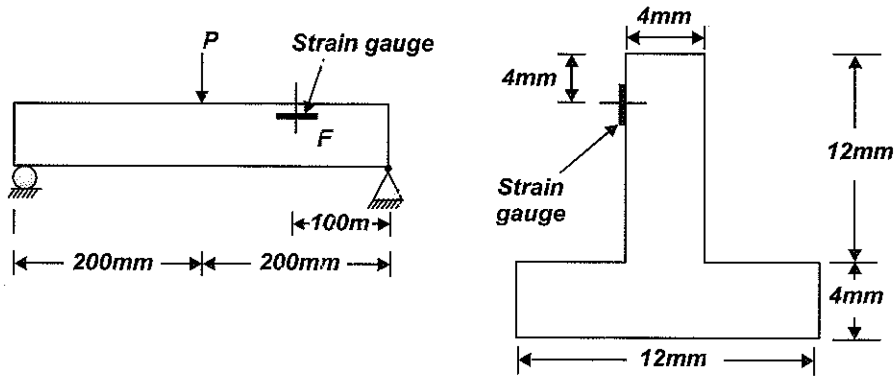
$$\Rightarrow \quad l^2 \leq \left(\frac{56}{135.129 \times 10^{-3}}\right)$$

$$\Rightarrow \quad l \leq \sqrt{\left(\frac{56}{135.129}\right) \times \frac{1}{10^{-3}}} \text{ m}$$

$$\Rightarrow \quad l \leq 20.357 \text{ m}$$

Therefore the longest span on which pipe may freely supported = 20.357 m

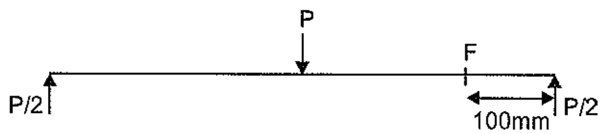
Q-10:



A small T-section is used in inverted positions as a beam and is shown in figure over a span of 400 mm. If due to the application of forces shown, the longitudinal strain gauge at F registers a compressive strain of 1500 microstraints, determine the magnitude of P. Take E = 200 GPa.

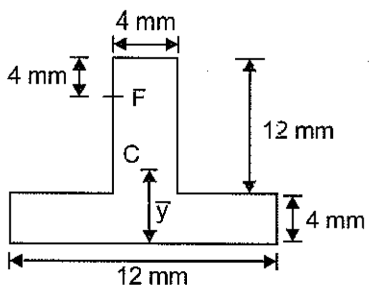
[10 Marks, ESE-2019]

Sol:



B.M at the location of F:

$$M = \frac{P}{2} \times 100 \text{ mm} = 50 \text{ P N-mm} \quad \text{...(i)}$$



C → Centroid of given section

$$\bar{y} = \frac{12\times4\times2+12\times4\times(6+4)}{12\times4+12\times4} = 6\text{ mm}$$

MOI about centroid (C) →

$$I_C = \frac{12\times4^3}{12}+12\times4\times4^2+\frac{4\times12^3}{12} + 4 \times 12 \times 4^2$$

$$I_C = 2176\text{ mm}^4$$

Given strain at F:

$$(\varepsilon_F) = 1500 \times 10^{-6}$$

$$\text{Stress at } F(\sigma_F) = E.\varepsilon_F = 200\times10^3\times1500\times10^{-6}$$

$$= 300\text{ MPa}$$

$$\sigma_F = 300 = \frac{M.y}{I}$$

$$\Rightarrow \quad 300 = \frac{M.(6)}{2176}$$

$$\Rightarrow \quad M = 108800\text{ N-mm}$$

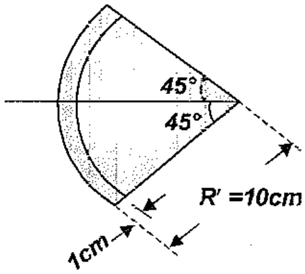
From (i) $\quad 50P = 108800$

$$\Rightarrow \quad P = 2176\text{ N} = 2.176\text{ kN}$$

CHAPTER 7

SHEAR STRESS IN BEAMS

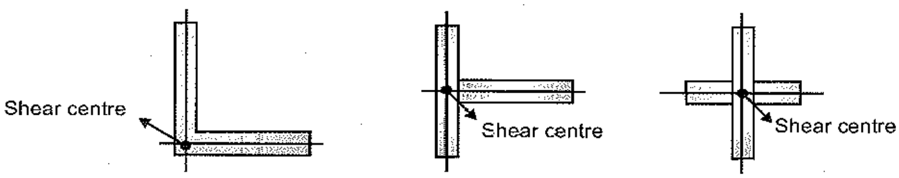
Q-1: ***What do you understand by SHEAR CENTRE? Determine the location of shear centre of an arc section shown in the figure.***

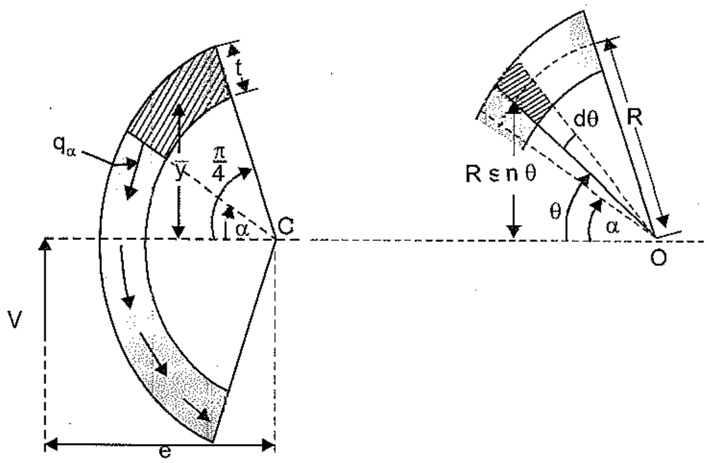


[15 Marks, ESE-1996]

Sol:

- Shear centre is the property of the section.
- The point through which loading should pass such that there is only bending and no twisting is called shear centre.
- It is that point through which resultant shear stress passes.
- It is also known as flexural centre.
- If there are two symmetrical axes, then the shear centre will lie on the point of intersection of both the axes.
- Shear centre of the sections consisting of two intersecting narrow rectangles always lies in the junction of rectangles





Determination of q_α (Shear flow)

$$q_\alpha = \frac{VA\bar{y}}{I}; \quad I = \text{M.I of the full arc section}$$

and, $A\bar{y} = \int y\,dA$ [For solid hatched part of 2nd figure as shown above]

$$A\bar{y} = \int y\,dA = \int_{\alpha}^{\pi/4} (R\sin\theta).(Rd\theta.t) = R^2t\,[\cos\alpha - \cos\pi/4]$$

$$I = \int y^2 dA = 2\int_0^{\pi/4} R^2\sin^2\theta\times Rd\theta\times t = 2R^3t\int_0^{\pi/4}\sin^2\theta\,d\theta$$

$$= 2R^3t\int_0^{\pi/4}\left[\frac{(1-\cos 2\theta)d\theta}{2}\right] = R^3t\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\pi/4} = R^3t\left[\frac{\pi}{4}-\frac{1}{2}\right]$$

$\therefore$
$$q_\alpha = \frac{V\times R^2t\left[\cos\alpha-\cos\frac{\pi}{4}\right]}{R^3t\left[\frac{\pi}{4}-\frac{1}{2}\right]} = \frac{V}{R}\frac{\left[\cos\alpha-\cos\frac{\pi}{4}\right]}{\left(\frac{\pi}{4}-\frac{1}{2}\right)}$$

For no twisting

$$\Sigma M_0 = 0$$

$$Ve = \int_{-\pi/4}^{\pi/4}\left(\frac{q_\alpha}{t}\times Rd\alpha\times t\right)\times R$$

$\Rightarrow$
$$Ve = \int_{-\pi/4}^{\pi/4}\frac{V}{R}\frac{\left(\cos\alpha-\cos\frac{\pi}{4}\right)}{\left(\frac{\pi}{4}-\frac{1}{2}\right)}\times\frac{1}{t}\times R\times d\alpha\times R\times t$$

$\Rightarrow$
$$e = \frac{R}{\left(\frac{\pi}{4}-\frac{1}{2}\right)}\int_{-\pi/4}^{\pi/4}\left(\cos\alpha-\cos\frac{\pi}{4}\right)d\alpha$$

$$e = \frac{R}{\left(\frac{\pi}{4}-\frac{1}{2}\right)}\left[\sin\alpha-\alpha\cos\frac{\pi}{4}\right]_{-\pi/4}^{\pi/4}$$

$$e = \frac{R}{\left(\frac{\pi}{4}-\frac{1}{2}\right)}\left[\sin\frac{\pi}{4}-\frac{\pi}{4}\cos\frac{\pi}{4}-\sin\left(-\frac{\pi}{4}\right)+\left(-\frac{\pi}{4}\right)\cos\frac{\pi}{4}\right]$$

$$e = \frac{2R}{\left(\frac{\pi}{4}-\frac{1}{2}\right)}\left[\frac{1}{\sqrt{2}}-\frac{\pi}{4}\times\frac{1}{\sqrt{2}}\right] \qquad [\because \sin(-\theta) = -\sin\theta]$$

Here, $R = 10 - 0.5 = 9.5$ cm $\qquad \left[\because R = R' - \frac{t}{2}\right]$

$$e = \frac{2\times 9.5}{0.2854}[0.707-0.555]$$

$\therefore$ $e = 10.13$ cm **Ans.**

Q-2: ***A simply supported rectangular beam of length L carries a udl over its entire length. Determine the critical length at which the shearing stress 'τ' and flexural stress 'σ' reach their allowable values simultaneously. The breadth of the beam section. is 'b' and the depth is 'd'.***

[4 Marks, ESE-2013]

Sol: For rectangular section max shear stress $= \frac{3}{2} \times \tau_{av}$

$$\Rightarrow \quad t_{max} = \frac{3}{2} \times \frac{V}{bd} = \frac{3V}{2bd}$$

$$\text{Max bending stress} = \sigma_{max} = \frac{M \times \frac{d}{2}}{\frac{bd^3}{L2}} = \frac{6M}{bd^2}$$

For max shear stress and max flexural stress to reach this max permissible value simultaneously

We have to satisfy $\quad M = \sigma_{max} \frac{bd^2}{6}$ and $V = \frac{2}{3} \tau_{max} \times bd$

Let at a distance x from the support,

$$V = \frac{wL}{2} - wx$$

$$M = \frac{wL}{2} x - \frac{wx^2}{2}$$

$$\Rightarrow \quad \frac{M}{V} = \frac{(Lx - x^2)/2}{\frac{(L-2x)}{2}} = \frac{Lx - x^2}{L - 2x}$$

$$\Rightarrow \quad \frac{Lx - x^2}{L - 2x} = \frac{\sigma_{max} \frac{bd^2}{6}}{\tau_{max} \times \frac{2}{3} bd}$$

Max value of bending stress will occur at mid span

$$\Rightarrow \quad \frac{wl^2}{8} = \sigma_{max} \times \frac{bd^2}{6}$$

Max value of shear stress will occur at support

$$\Rightarrow \quad \frac{3}{2} \times \frac{wl}{2bd} = \tau_{max}$$

$$\Rightarrow \quad \tau_{max} \times \frac{2}{3}(bd) = \frac{wl}{2}$$

$$\Rightarrow \quad \frac{Lx - x^2}{L - 2x} = \frac{wl^2/8}{\frac{wl}{2}} = \frac{l}{4}$$

$$L^2 - 2Lx = 4Lx - 4x^2$$

$$\Rightarrow \quad 4x^2 - 6Lx + L^2 = 0$$

$$\Rightarrow \qquad x = \frac{6L \pm \sqrt{36L^2 - 4\times4\times L^2}}{8}$$

$$= \frac{6L \pm \sqrt{20\,l^2}}{8} = (6 \pm 2\sqrt{5})\,\frac{l}{8} = 1.309l \;, 0.191l$$

Hence the critical length will be 0.191 *l* from end A.

Q-3: ***A bar with a square section of side 100mm is used as a beam and placed in such a way that the diagonal is horizontal. When the section of the beam is subjected to a shear force of 600 kN, determine the maximum shear stress in the section and its location across its depth.***

[20 Marks, ESE-2017]

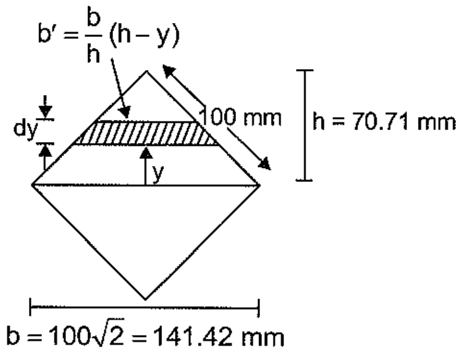
Sol: **Given that:** Square of side = 100 mm

$$V = 600 \text{ KN}$$

So,
$$A\bar{y} = \frac{1}{2}\left(\frac{b}{h}(h-y)^2\right)\left(y + \frac{1}{3}(h-y)\right)$$

$$= \frac{b}{2h}(h-y)^2\,\frac{(2y+h)}{3}$$

$$= \frac{b(h-y)^2(2y+h)}{6h}$$



and
$$I = 2\int_0^h \left(\frac{b}{h}(h-y)dy\right)y^2$$

$$= \frac{2b}{h}\int_0^h (hy^2 - y^3)dy$$

$$I = \frac{bh^3}{6}$$

Alternatively,

MOI of triangle about its base $= \dfrac{bh^3}{12}$

MOI of diamond about its diagonal $= \dfrac{bh^3}{12} \times 2 = \dfrac{bh^3}{6}$

So, $$\tau = \frac{VA\bar{y}}{Ib'} = \frac{V \times (b(h-y)^2(2y+h))}{6h \times \frac{bh^3}{6} \times \frac{b}{h}(h-y)} = \frac{V}{bh^3}(h-y)(2y+h)$$

So, for max. value of τ, $$\frac{d\tau}{dy} = 0$$

$$\Rightarrow \qquad (h-y)(2) + (2y+h)(-1) = 0$$

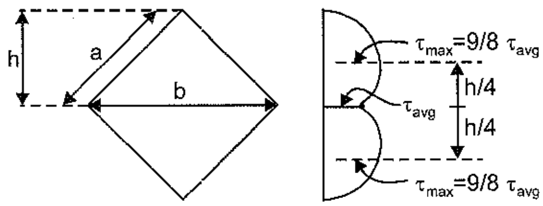
$$\Rightarrow \qquad 2h - 2y - 2y - h = 0$$

$$\Rightarrow \qquad y = \frac{h}{4} = \frac{70.71}{4} = 17.6775 \text{ m}$$

Max. shear stress occurs at a distance of 17.6775 m from the horizontal diagonal.

So,
$$\tau_{max} = \frac{V}{bh^3}\times\left(h-\frac{h}{4}\right)\times\left(\frac{2\times h}{4}+h\right)$$
$$= \frac{V}{bh^3}\times\frac{3h}{4}\times\frac{6h}{4} = \frac{9}{8}\times\frac{V}{bh}$$
$$= \frac{9}{8}\times\frac{600\times1000}{141.42\times70.71}$$
$$= 67.501 \text{ N/mm}^2$$

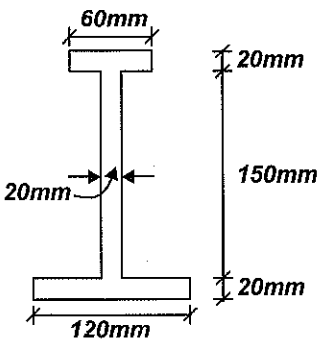
Note: For diamond section shear stress distribution is as shown below.



$$\tau_{avg} = \frac{V}{2\times\left(\frac{1}{2}\times b\times h\right)} = \frac{V}{bh}$$

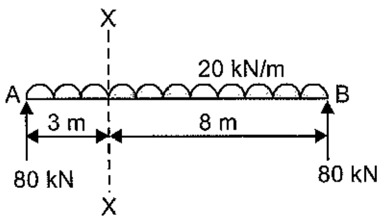
$$\tau_{max} = \frac{9}{8}\times\frac{V}{bh} = \frac{9}{8}\times\frac{600\times10^3}{100\sqrt{2}\times\frac{100}{\sqrt{2}}} = 67.5 \text{ N/mm}^2$$

Q-4: ***A beam of unsymmetrical I section shown in figure is simply supported over a span of 8 m. It carries a uniformly distributed load of 20 kN/m over the entire span. Draw the sketch for shear stress variation across the depth of the cross-section located at a distance of 3 m from left end A.***



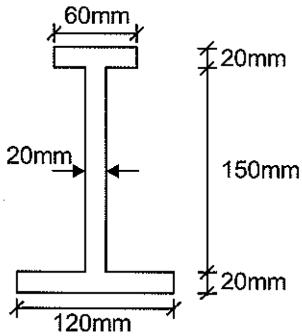
[20 Marks, ESE-2018]

Sol: **Given that:** Unsymmetrical I-section



Shear force at section X-X = 80 – (20 × 3) = (80 – 60) kN

$$\boxed{F = 20 \text{ kN}}$$



Calculation for distance of NA

Distance of NA (centroid) of the section from the top fibre be y_1.

Then, $$y_1 = \frac{(60\times20\times10)+(150\times20)\left(20+\frac{150}{2}\right)+(20\times120\times180)}{[(60\times20)+(150\times20)+(120\times20)]}$$

$$\boxed{y_1 = 110.45\text{ mm}}$$

and, $$I = \frac{1}{12}\times 60\times(20)^3 + 60\times20\times(110.45-10)^2$$
$$+\frac{1}{12}\times20\times150^3 + 20\times150\times(110.45-95)^2$$
$$+\frac{1}{12}\times20^3\times120 + 20\times120\times(110.45-180)^2$$

$$\boxed{I = 30.1786\times10^6\text{ mm}^4}$$

Shear stress at the bottom of the top flange

$$= \frac{F}{bI}\cdot(a\bar{y})$$

$$= \frac{(20\times10^3)\text{ N}}{(60)\times(30.1786\times10^6)}\times(60\times20)\times(110.45-10)$$

$$= 1.33\text{ N/mm}^2$$

Shear stress at the same level, but in web

$$= \frac{(20\times10^3)}{(20)\times(30.1786\times10^6)}\times(60\times20)\times(110.45-10)$$

$$= 3.99\text{ N/mm}^2$$

Shear stress at NA

$$a\bar{y} = a\bar{y}\text{ of top flange} + a\bar{y}\text{ of web above NA}$$

$$= \frac{20\times10^3}{20\times(30.1786\times10^6)}\times202352.025$$

$$= 6.705\text{ N/mm}^2$$

Shear stress at the junction of web and lower flange

Considering the lower side of the section for finding $a\bar{y}$, we get

$$a\bar{y} = (120 \times 20) \times (180 - 110.45)$$
$$= 166920 \text{ mm}^3$$

$\therefore$ Shear stress at the junction of web and lower flange

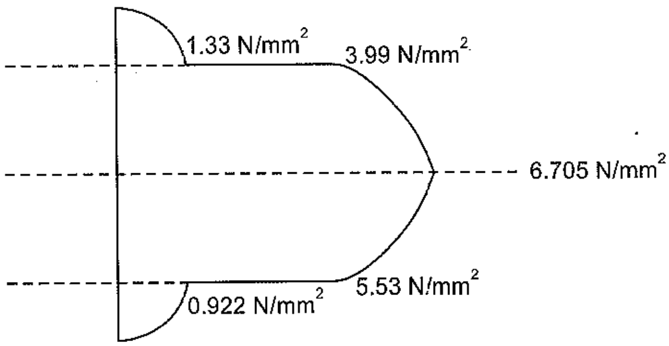
$$= \frac{20 \times 10^3}{20 \times 30.1786 \times 10^6} \times 166920$$
$$= 5.53 \text{ N/mm}^2$$

But at the above level, but in web

$$\text{Shear stress} = \frac{20 \times 10^3}{120 \times 30.1786 \times 10^6} \times 166920 = 0.922 \text{ N/mm}^2$$

and at the extreme fibres, shear stress is zero.

Hence, the variation of shear stress across the depth of the section is as shown below.



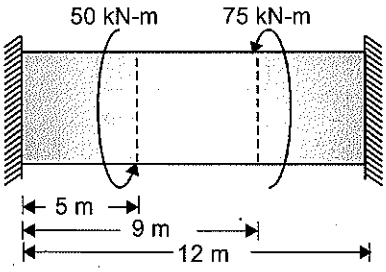
Variation of shear stress across the depth

CHAPTER 8 TORSION OF CIRCULAR SHAFT

Q-1: ***A horizontal shaft 12m in length is fixed at its ends. When viewed from its left end axial couples of 50 kN-m clockwise and 75 kN-m counter clockwise act at 5m and 9m from the left end respectively. Determine the end fixing couples and the position where the shaft suffers no angular twist.***

[10 Marks, ESE-1995]

Sol:

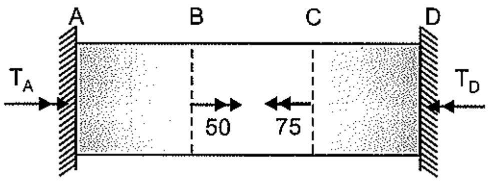


Determine:

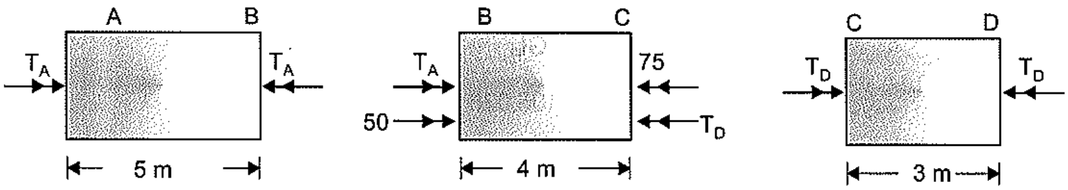
(a) The end fixing moments

(b) Position where the shaft suffers no angular twists.

Representation of problem in simple form:



Splitting the complex structure into three parts and drawing F.B.D. (free body diagram).



Note: Double arrow method is one way of representing torsional moments which follows the right hand thumb rule .

Conceptual background

Sign Scheme: At any section if the vector points towards the section it is taken as +ve otherwise –ve.

Consider Section BC, $50 + T_A = 75 + T_D$

$\therefore$ $\boxed{T_A - T_D = 25}$ ... (i)

Since ends A and D are fixed, So,

$\theta_{AD} = 0$; θ_{AD} = Angle of twist b/w A to D

And we know that, $\theta_{AD} = \theta_{AB} + \theta_{BC} + \theta_{CD}$

So, $\theta_{AB} + \theta_{BC} + \theta_{CD} = 0$

Using $\theta = \frac{TL}{GJ}$ where $J = I_P$ for each section

$$\frac{5T_A}{GI_P} + \frac{4(50+T_A)}{GI_P} + \frac{3T_D}{GI_P} = 0$$

$$\Rightarrow \quad 5T_A + 200 + 4T_A + 3T_D = 0$$

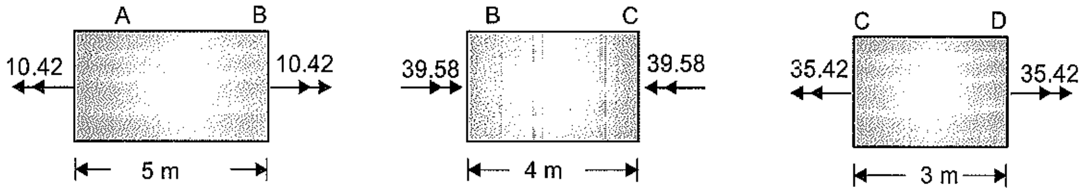
$$\Rightarrow \quad \boxed{9T_A + 3T_D = -200} \quad \text{... (ii)}$$

By solving equation (i) and (ii) we get

$$T_A = -10.42 \text{ kN-m} \quad \text{and } T_D = -35.42 \text{ kN-m}$$

and, $\quad T_B = T_C = 39.58$ kN-m

Pictorial representation



So, $\quad \theta_{AB} = \frac{-10.42 \times 5}{GJ} = \frac{-52.10}{GI_P}, \theta_A = 0$ (–ve) because vector points away from the section

$\therefore \quad \theta_{AB} = \theta_A - \theta_B = -\theta_B = \frac{-52.10}{GI_P}$, [Since A is fixed]

$$\Rightarrow \quad \theta_B = \frac{52.10}{GI_P}$$

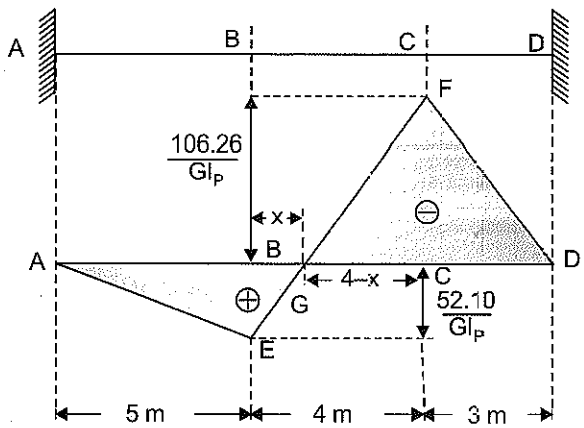
and, $\quad \theta_{CD} = \frac{-35.42 \times 3}{GI_P} = \frac{-106.26}{GI_P}$

Since θ_D is also fixed So, $\quad \theta_C - \theta_D = \frac{-106.26}{GI_P}$

$$\therefore \quad \theta_C = -106.026 / GI_P$$

We know that $\theta = \frac{TL}{GI_P}$ in which T, G, I_P is constant for each section so, θ having linear variation with L.

So, Angle of twist diagram is as shown below



Sign Scheme :
Anticlockwise twist has been taken (+ve), if viewed from D towards A

For finding the location of point G where twist will be zero, we use similar triangle concept,

$$\frac{BG}{GC} = \frac{BE}{FC} \Rightarrow \frac{x}{4-x} = \frac{52.10}{106.26} \Rightarrow 106.26\,x = 208.4 - 52.10\,x$$

$\Rightarrow$ $x = 1.316$ m

So, point G is 5 + 1.316 = 6.316 m away from A.

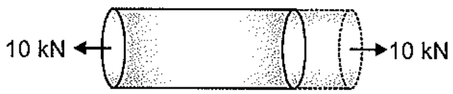
Hence, $T_A = -10.42$, $T_D = -35.42$ **Ans.**

Distance of point where angle of twist is zero from left end = 6.316 m **Ans.**

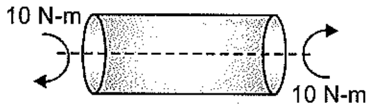
Q-2: ***A steel specimen of 12 mm diameter extends by 6.31 × 10⁻² mm over a gauge length of 150 mm when subjected to an axial load of 10 kN. The same specimen undergoes a twist of 0.5° on a length of 150 mm over a twisting moment of 10 N-m. Using the above data, determine the elastic constants E, μ, G and K.***

[12 Marks, ESE-2001]

Sol:



gauge length = 150 mm
$\Delta l = 6.31 \times 10^{-2}$ mm
dia = 12 mm



gauge length = 150 mm

$$\theta = 0.5° = \left[\frac{0.5}{180} \times \pi\right] \text{radian} = 8.73 \times 10^{-3}$$

dia = 12 mm

Calculate E μ, G, K:

E Calculation:

We know that, $\Delta L = \frac{PL}{AE}$ $\quad \therefore E = \frac{PL}{A \times \Delta L}$

$\therefore$

$$E = \frac{(10\times10^3)\text{N}\times(150)\text{mm}}{\left(\frac{\pi}{4}\times(12)^2\right)\text{mm}^2\times(6.31\times10^{-2})\text{mm}} = 2.10\times10^5\ \text{N/mm}^2$$

$$\boxed{E = 2.1\times10^5\ \text{N/mm}^2}$$

Again from torsion data

$$\theta = \frac{TL}{GJ} \text{ where } J = I_P = \frac{\pi}{32}\times d^4 = \frac{\pi}{32}\times 12^4\ \text{mm}^4 = 2.03575 \times 10^3\ \text{mm}^4$$

G Calculation:

$\therefore$

$$G = \frac{TL}{\theta.J} = \frac{(10\times10^3)\text{N-mm}\times150\text{mm}}{(8.73\times10^{-3})\times(2.03575\times10^3\ \text{mm}^4)} = 8.44 \times 10^4\ \text{N/mm}^2$$

$\therefore$

$$\boxed{G = 8.44\times10^4\ \text{N/mm}^2}$$

μ Calculation:

$$E = 2G(1+\mu) \quad \therefore \mu = \left(\frac{E}{2G}-1\right) = 0.245 \Rightarrow \boxed{\mu = 0.245}$$

K Calculation:

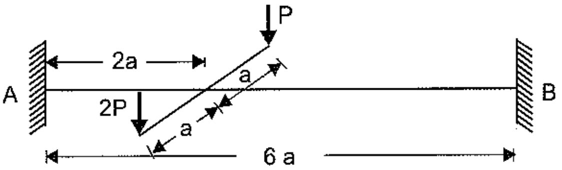
$$E = 3K(1-2\mu) \therefore \frac{E}{3(1-2\mu)} = K$$

$$\boxed{K = 1.373\times10^5\ \text{N/mm}^2}$$

Q-3: ***A horizontal beam of length '6a' has built-in-ends. The beam has two horizontal normally projecting cantilevers of length 'a' at one third points which carry vertical loads of magnitudes 'p' and '2p', one at each end. Draw bending moment, S.F. and torsion diagrams.***

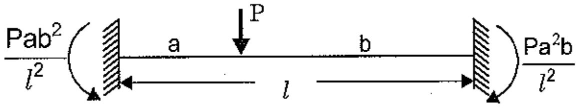
[20 Marks, ESE-2002]

Sol:

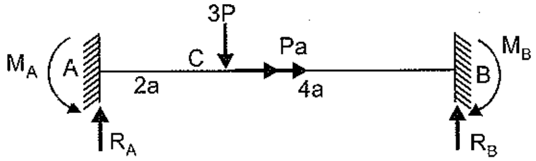


Draw: BMD, shear force and torsion diagrams

Since we know that for built-in ends beam,



Hence for the equivalent loading



3P = Equivalent loading

Pa = Equivalent torsional moment

$$M_A = \frac{3P(2a)(4a)^2}{(6a)^2} = \left(\frac{8Pa}{3}\right) \quad \text{(In this direction as shown above)}$$

$$M_B = \frac{3P(2a)^2(4a)}{(6a)^2} = \left(\frac{4Pa}{3}\right) \quad \text{(In this direction as shown above)}$$

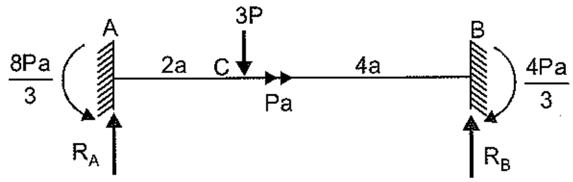
Now calculation of R_A and R_B.

$$\boxed{R_A + R_B = 3P} \quad \text{... (i)}$$

Taking moment equilibrium about any point say A.

$$\Sigma M_A = 0$$

$$\frac{8Pa}{3} + R_B \times 6a = \frac{4Pa}{3} + 3P(2a)$$



$$\Rightarrow \quad R_B \times 6a = \frac{-4Pa}{3} + 6Pa$$

$$\Rightarrow \quad R_B = P - \frac{2}{9}P = \left(\frac{7P}{9}\right)$$

$$\therefore \quad R_A = \frac{20P}{9} \quad \text{... from (i)}$$

Drawing BMD diagram and S.F.D.

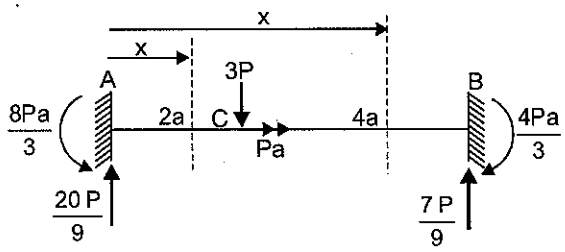
for, part AC,

$$SF = \frac{+20P}{9}$$

$$BM = \frac{+20Px}{9} - \frac{8Pa}{3}$$

$$M|_C = \frac{20P}{9} \times 2a - \frac{8Pa}{3}$$

$$= \frac{40Pa - 24Pa}{9} = \left\{\frac{16Pa}{9}\right\}$$



The position where M = 0 is given by

$$\therefore \quad \frac{20Px}{9} = \frac{8Pa}{3}$$

$$x = \frac{24a}{20} = 1.2a$$

For part (BC)

$$SF = \frac{-7P}{9}$$

$$BM = \frac{-4Pa}{3} + \frac{7P}{9}(6a - x)$$

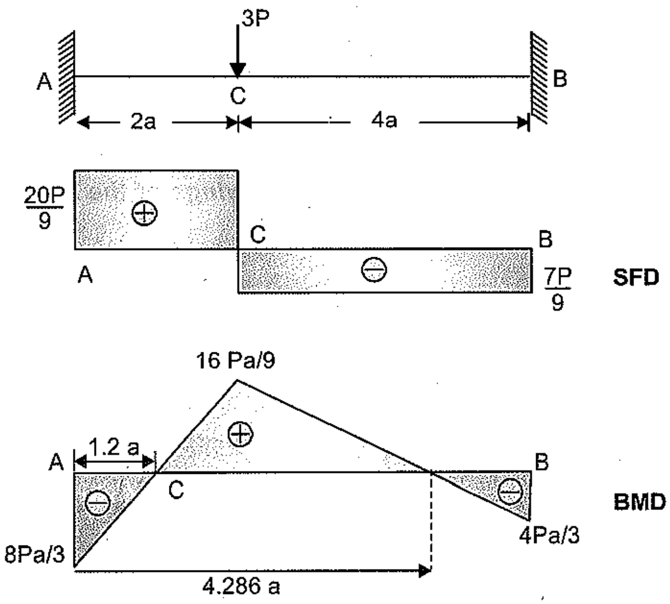
$$\Rightarrow \quad BM = \frac{30\,Pa}{9} - \frac{7Px}{9}$$

at $x = 2a$; $BM = \frac{16\,Pa}{9}$

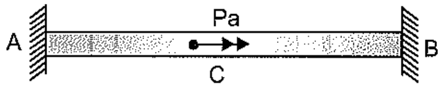
For position where BM = 0,

$$\frac{30\,Pa}{9} = \frac{7Px}{9} \Rightarrow x = \frac{30a}{7} = 4.286\,a$$

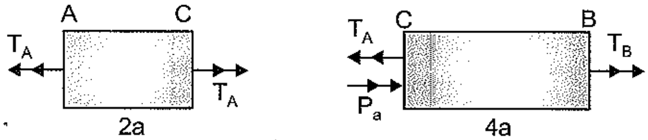
Hence,



Calculation of torsional moments



FBD of torsion



Equilibrium condition

$$\Rightarrow \qquad T_B = T_A - Pa$$

$$\Rightarrow \qquad T_A - T_B = Pa \qquad \text{... (i)}$$

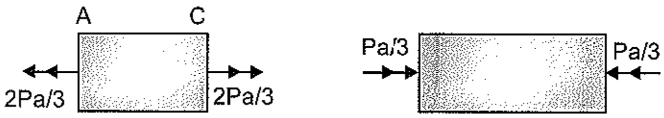
Compatibility condition,

$$\theta_{AC} + \theta_{CB} = 0$$

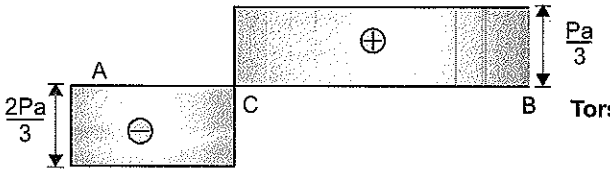
$$\frac{-T_A \times 2a}{GI_P} - \frac{T_B \times 4a}{GI_P} = 0$$

$$\therefore \qquad \boxed{T_A + 2T_B = 0} \qquad \text{... (ii)}$$

By solving these two equation we get, $T_A = \frac{+2Pa}{3}$ $\quad T_B = \frac{-Pa}{3}$



So,

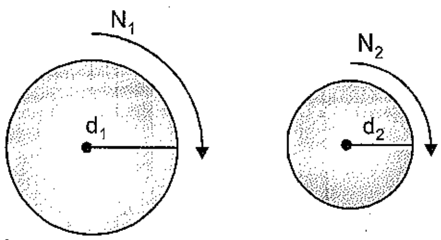


Torsional force diagram

Q-4: Calculate the diameters of two solid shafts to transmit 200 horse power (metric) each without exceeding a shear stress of 700 kg/cm². One of these shafts operates at 20 rpm & the other at 20,000 r.p.m. Comment on the result with reference to savings of the material.

[10 Marks, ESE-2005]

Sol: 1 Metric horse power = 735.5 watt



Power transmitted = (P) = 200 HP = 200 × 0.7355 kW = 147.1 kW = 147.1×10^3 W

$$\text{Permissible shear stress} = \tau_{max} = 700\,kg/cm^2 = \frac{700\times 9.8}{10^{-4}m^2} = 6960\times 10^4\,N/m^2$$

$$\text{Frequency } (N_1) = 20\text{rpm} \Rightarrow \text{Angular velocity } (\omega_1) = \frac{2\pi N_1}{60}$$

and $(N_2) = 20{,}000$ rpm $\Rightarrow$ Angular velocity $(\omega_2) = \dfrac{2\pi N_2}{60}$

For Ist shaft:

$$P = \frac{2\pi N_1 T_1}{60}$$

$$\therefore \quad T_1 = \frac{60P}{2\pi N_1} \Rightarrow \frac{60\times 147.1\times 10^3}{2\times\pi\times 20}$$

$$T_1 = 70235.08\text{ Nm}$$

Suppose the diameter of shaft is d_1

then, $$\frac{\tau_{max}}{R} = \frac{T}{I_P}$$

$$\therefore \quad \frac{2\tau_{max}}{d_1} = \frac{T_1}{\frac{\pi}{32}d_1^4}$$

$$\Rightarrow \quad \frac{2\times\tau_{max}\times\pi}{32\times T_1} = \frac{1}{d_1^3}$$

$$d_1 = \left[\frac{32\times T_1}{2\times\tau_{max}\times\pi}\right]^{1/3} = \left\{\frac{32\times 70235.08}{2\times 6960\times 10^4\times\pi}\right\}^{1/3}$$

$$d_1 = 0.1726\text{ m} = 172.6\text{ mm}$$

Same as above,

$$T_2 = \frac{60P}{2\pi N_2} = \frac{60\times 147.1\times 10^3}{2\times\pi\times 20000} = 70.235\text{Nm}$$

Hence $$d_2 = \left\{\frac{32\times T_2}{2\pi\,\tau_{max}}\right\}^{1/3} = \left\{\frac{32\times 70.235}{2\pi\,\tau_{max}}\right\}^{1/3} = 0.01726\text{ m} = 17.26\text{ mm}$$

We have derived that, $$d = \left\{\frac{32\times T}{2\pi\tau_{max}}\right\}^{1/3}$$

Since $$T = \frac{60P}{2\pi N}$$

$$\Rightarrow \quad d = \left\{\frac{32\times 60\times P}{4\pi^2\times N\times\tau_{max}}\right\}^{1/3}$$

if P and τ_{max} is constant then $$d \propto \frac{1}{N^{1/3}}$$

$$\Rightarrow \quad \frac{\pi}{4}d^2 \propto \frac{1}{N^{2/3}}$$

$\therefore$ (Area of shaft) $\propto \frac{1}{N^{2/3}}$

We know that area of shaft is proportional to the material vol.

Hence, we can conclude that for transmitting same power without exceeding a given shear stress value, the shaft revolving with lower frequency requires more materials than the shaft revolving with higher frequency.

In our case.

$$\frac{A_2}{A_1} = \frac{\frac{\pi}{4}\times d_2^2}{\frac{\pi}{4}\times d_1^2} = \left(\frac{d_2}{d_1}\right)^2 = \left\{\frac{17.26}{172.6}\right\}^2$$

$$\Rightarrow \quad \left(\frac{A_2}{A_1}\right) = \left(\frac{1}{10}\right)^2 \qquad \therefore \quad \frac{A_2}{A_1} = \frac{1}{100}$$

Area required in second case is 100 times lesser than the area required in Ist case.

Q-5: ***The internal diameter of a steel shaft is 70% of external diameter. The shaft is to transmit 3500 kW at 200 rpm. If the maximum allowable stress in the shaft material is 50 MPa, calculate the diameter of the shaft. Find also the maximum twist of the shaft when it is stressed to the maximum permissible value. The length of the shaft is 4 m. Take G = 80 MPa.***

[10 Marks, ESE-2006]

Sol: **Given:** Power transmitted by the shaft = 3500 kW = 3500 × 10^3 Watt

Frequency = 200 rpm

Length of shaft (L) = 4m

$$\therefore \quad \text{Angular velocity} = \frac{2\pi N}{60} = \left[\frac{2\pi\times 200}{60}\right] \text{rad/sec}$$

Permissible stress = 50 N/mm² 50 ×10^6 N/m²

$$\text{Power (P)} = \omega\times T = \frac{2\pi\times 200}{60}\times T$$

$$\therefore \quad T = \frac{60P}{2\pi\times 200} = \frac{60\times 3500\times 10^3}{2\times\pi\times 200} = 167.11 \text{ kN-m}$$

and,

$$\frac{\tau_{max}}{\left(\frac{D_0}{2}\right)} = \frac{T}{J_p}$$

$$\Rightarrow \quad \frac{50}{\frac{D_0}{2}} = \frac{167.11\times 10^6}{\frac{\pi}{32}\times\left(D_0^4 - D_1^4\right)} = \frac{167.11\times 10^6}{\frac{\pi}{32}(1-(0.7)^4)D_0^4}$$

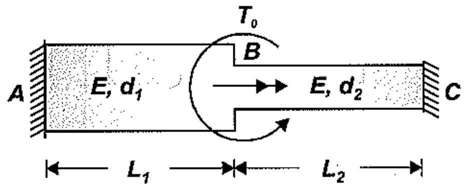
$$\Rightarrow \quad D_0 = \left[\frac{167.11\times 10^6\times 32}{\pi\times(1-0.7)^4\times 100}\right]^{1/3} = 281.89 \text{ mm}$$

$\Rightarrow$ Outer dia = 281.89 mm [Inner dia = 0.7 D_0 (given)]

Inner dia = 197.324 mm

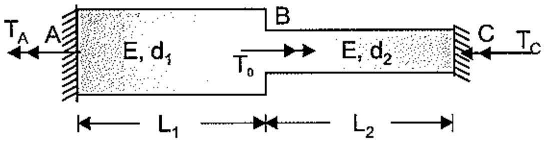
$$\text{Angle of twist } (\theta) = \frac{TL}{GI_P} = \frac{167.11\times 10^6\times 4000}{80\times\frac{\pi}{32}\times\left(281.89^4 - 197.32^4\right)} = 17.737 \text{ rad}$$

Q-6: ***Two circular shaft AB and BC of the same material but different diameter are welded together at point B as shown in the figure. Ends A and C are fixed. An external torque T_o is applied to the shafts at point B. Find the torques exerted on the ends of the shaft at A & C.***



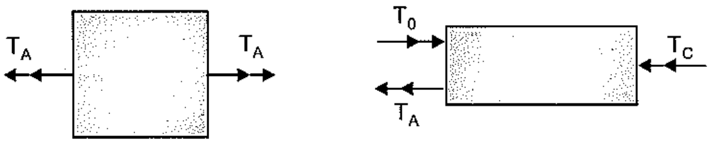
[15 Marks, ESE-2006]

Sol:



Since the material is same so, E, G, μ are same.

F.B.D.



For equilibrium, $$T_O - T_A = T_C$$

$$\Rightarrow \quad T_O = T_A + T_C \quad \text{... (i)}$$

Since the ends are fixed, so $$\theta_{AC} = 0$$

$$\Rightarrow \quad \theta_{AB} + \theta_{BC} = 0$$

$$\Rightarrow \quad \frac{-T_A \times L_1}{G \times \frac{\pi}{32} d_1^4} + \frac{T_C \times L_2}{G \times \frac{\pi}{32} d_2^4} = 0$$

$$\Rightarrow \quad \frac{T_A L_1}{d_1^4} = \frac{T_C L_2}{d_2^4}$$

$$\Rightarrow \quad T_A = T_C\left(\frac{L_2}{L_1}\right) \times \left(\frac{d_1}{d_2}\right)^4 \quad \text{... (ii)}$$

Putting the value of T_A from equation (ii) to equation (i)

$$T_0 = T_C\left(\frac{L_2}{L_1}\right) \times \left(\frac{d_1}{d_2}\right)^4 + T_C$$

$$\Rightarrow \quad T_0 = T_C\left(\frac{L_2 d_1^4 + L_1 d_2^4}{L_1 d_2^4}\right)$$

$$\therefore \quad \boxed{T_C = \frac{T_0 d_2^4 L_1}{L_2 d_1^4 + L_1 d_2^4}}$$

and, $$T_A = \frac{T_0 d_2^4 \times L_1}{L_2 d_1^4 + L_1 d_2^4} \times \left(\frac{L_2}{L_1}\right) \times \left(\frac{d_1}{d_2}\right)^4$$

$$\boxed{T_A = \frac{T_0\, d_1^4\, L_2}{L_2\, d_1^4 + L_1\, d_2^4}}$$

Q-7: ***A solid shaft transmits 250 kW at 100 r.p.m. If the shear stress is not to exceed 75 N/mm², what should be the diameter of the shaft?***

If this shaft is to be replaced by a shallow shaft whose internal diameter shall be 0.6 times the outer diameter, determine the size and percentage saving in weight, maximum stresses being the same.

[10 Marks, ESE-2011]

Sol: Given,
$$\tau_{max} = 75 \text{N/mm}^2$$
$$N = 100 \text{ rpm}$$
$$P = 250 \text{ kW}$$

We know that $$P = T\omega = T \times \frac{2\pi N}{60}$$

$$\Rightarrow \quad T = \frac{60\,P}{2\pi N}$$

Torque, $$T = \frac{60 \times 250}{2\pi \times 100} = 23.87 \text{ KN-m} = 23.87 \times 10^6 \text{ N-mm}$$

Now, $$\frac{\tau}{r} = \frac{T}{I_P} = \frac{G\theta}{l}$$

or $$\frac{\tau_{max}}{\left(\frac{D}{2}\right)} = \frac{T}{I_P} \quad \text{... (i)}$$

$$\Rightarrow \quad \frac{75 \times 2}{D} = \frac{23.87 \times 10^6}{\frac{\pi}{32} \times D^4}$$

$$\boxed{D = 117.468 \text{ mm.}}$$

When this shaft is replaced by a hollow shaft whose,

$$\text{Internal Dia} = D_i$$

Also, $$\text{Outer Dia} = D_0$$

It is given that $$D_i = 0.6\, D_0$$

From (i) $$\frac{\tau_{max}}{\left(\frac{D}{2}\right)} = \frac{T}{I_P}$$

$$\Rightarrow \quad \frac{75}{\left(\frac{D_0}{2}\right)} = \frac{23.87 \times 10^6}{\frac{\pi}{32} \times \left(D_0^4 - D_i^4\right)}$$

$$\Rightarrow \quad \frac{150}{D_0} = \frac{23.87 \times 10^6}{\frac{\pi}{32} \times (D_0^4 - (0.6 D_0)^4)}$$

Solving, we get;
$$D_0 = 123.03 \text{ mm}$$
$$D_i = 73.82 \text{ mm}$$

$$\% \text{ saving in weight} = \frac{\text{Cross-sectional area of solid shaft} - \text{Cross-sectional area of hollow shaft}}{\text{Cross-sectional area of solid shaft}}$$

$$= \frac{\frac{\pi}{4}D^2 - \frac{\pi}{4}\left(D_0^2 - D_i^2\right)}{\frac{\pi}{4}D^2} = \frac{D^2 - \left(D_0^2 - D_i^2\right)}{D^2}$$

$$= \frac{117.468^2 - \left(123.03^2 - 73.82^2\right)}{117.468^2} = 0.2979$$

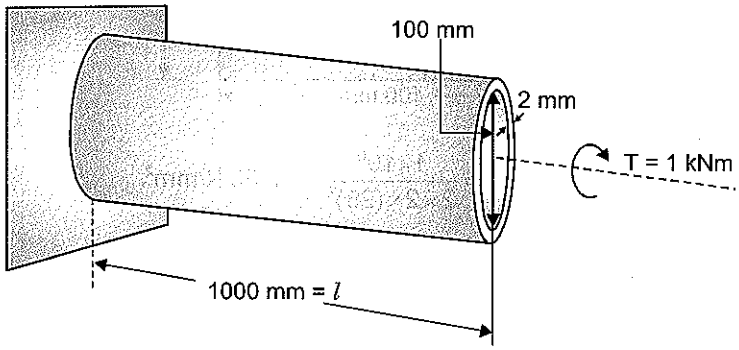
$\Rightarrow$ % saving in material is 29.79%

Q-8: ***A thin walled tube of circular cross-section with outer diameter 100 mm, thickness 2 mm and length 1000 mm is fixed at one end. It is subjected to a twisting moment of 1 kN-m at the free end. Find the shear stress in the wall of the tube and the angle of twist at the free end. E = 2 × 10^5 MPa and Poisson's ratio = 0.25.***

What will be the shear stress in the wall of the tube if the cross-section of the tube is square with outside dimensions 100 mm × 100 mm and wall thickness 2 mm?

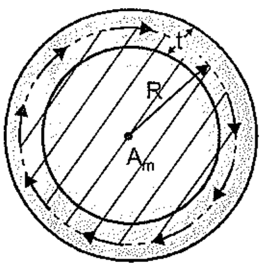
[10 Marks, ESE-2012]

Sol:



By using thin walled tube concept

For thin walled tube, maximum shear stress is taken corresponding to mean radius.



$$J = (2\pi Rt) \times R^2$$

$$\tau = \frac{TR}{J} = \frac{T}{2\pi R^2 t} \qquad [\tau = \text{shear stress in tube}]$$

$$A_m = \pi R^2 \qquad [A_m = \text{area under mean circle}]$$

$$\Rightarrow \quad \tau = \frac{T}{2t\,A_m}$$

$$\Rightarrow \quad \boxed{\tau \times t = \frac{T}{2A_m}}$$

It is important to note that although this formula has been derived by assuming section as circular, but the same is valid for other thin walled sections also.

For thin walled circular tube, $$\tau = \frac{1\times10^6 \text{ Nmm}}{2\times\frac{\pi}{4}(98)^2\times2} = \boxed{33.16 \text{ N/mm}^2}$$

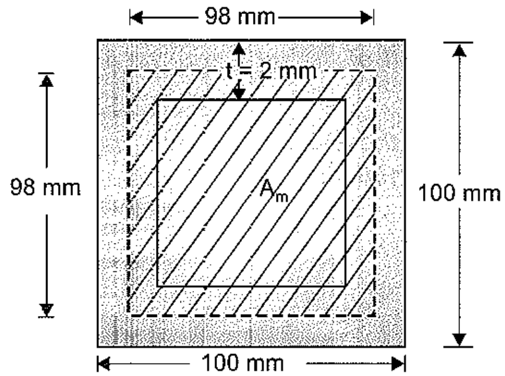
Also, $$\frac{\tau}{R} = \frac{G\phi}{L}$$

$$G = \frac{E}{2(1+\mu)} = \frac{2\times10^5}{2\times1.25} = 8\times10^4 \text{ N/mm}^2$$

$$\Rightarrow \qquad \phi = \frac{\tau\times L}{GR} = \frac{33.16\times1000}{8\times10^4\times49}$$

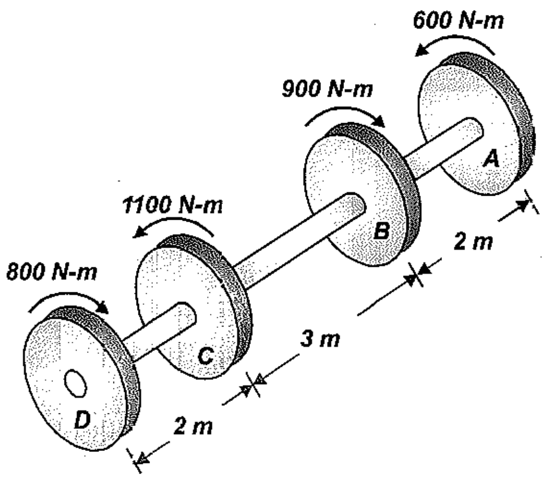
$$\Rightarrow \qquad \boxed{\phi = 8.459\times10^{-3} \text{ rad}}$$

If the tube were square in x-sec. as shown below then



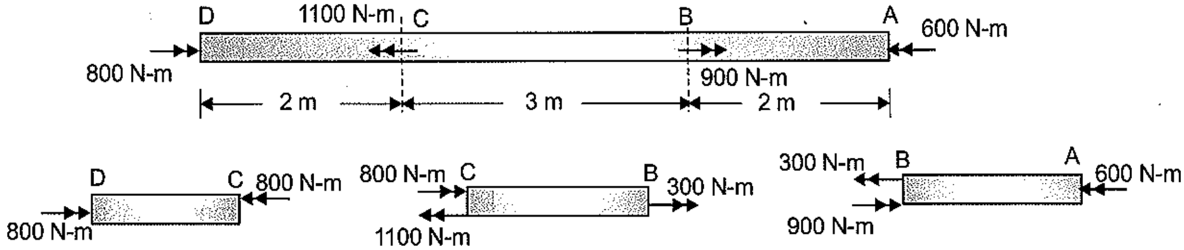
$$\tau = \frac{T}{2A_m t} = \frac{1\times10^6}{2\times2\times(98)^2} = 26.03 \text{ N/mm}^2$$

Q-9: ***An aluminium shaft with a constant diameter of 50 mm is loaded by torques applied to gears attached to it as shown figure below. Using G = 28 GPa, determine the relative angle of twist of geat D relative to gear A.***



[4 Marks, ESE-2013]

Sol:



Here torques are represented by double arrows following right hand thumb rule.

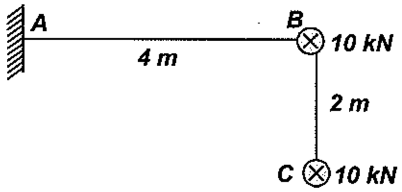
$$\phi_{DA} = \phi_{DC} + \phi_{CB} + \phi_{BA} \qquad [\phi = \text{Angle of twist} = \frac{TL}{GJ}]$$

$$= \frac{800\ \text{Nm} \times 2\ \text{m}}{GJ} + \frac{(-300)\times 3}{GJ} + \frac{600\times 2}{GJ}$$

$$= \frac{1600 - 900 + 1200}{GJ} = \frac{1900\ \text{Nm}^2}{GJ}$$

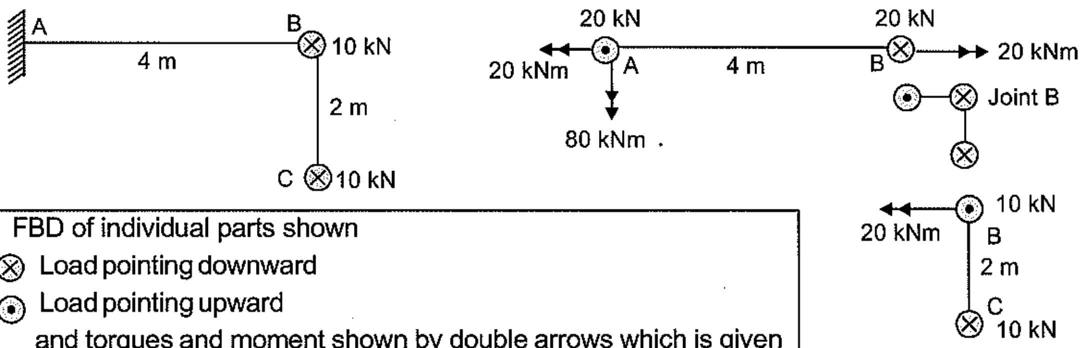
$$= \frac{1900\times 10^6\ \text{Nmm}^2}{28\times 10^3\ \text{N/mm}^2 \times \frac{\pi}{32}(50)^4\ \text{mm}^4} = 0.1106\ \text{rad}$$

Q-10: ***A rigid bent ABC is of uniform C/S and is in a horizontal plane. It is fixed at A and free at C as shown in Fig. It carries two vertical loads 10 kN each at B and C. Draw BMD, SFD and TMD in the bent***



[10 Marks, ESE-2013]

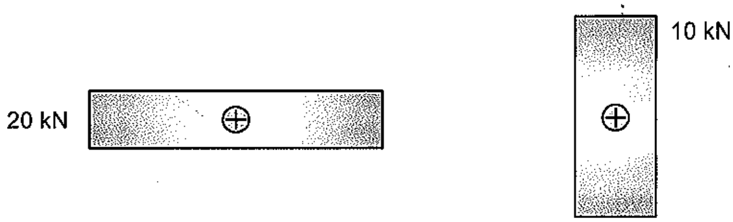
Sol:



FBD of individual parts shown
⊗ Load pointing downward
⊙ Load pointing upward
and torques and moment shown by double arrows which is given by right hand thumb rule

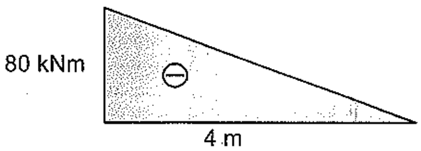
SFD for AB

SFD BC

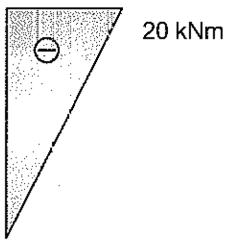


sign to SFD given by taking load pointing out of the plane of paper as (+)ve

BMD for AB



BMD for BC



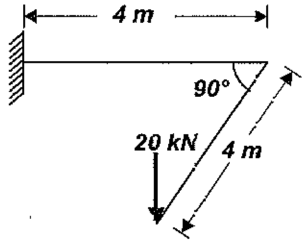
sign to BMD given as hogging moment is taken as (–)ve

TMD for AB

TMD for BC

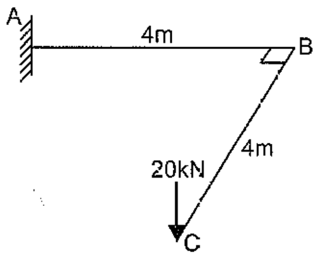
Sign to TMD
If double headed arrow points away from any section it is taking (–)ve

Q-11: ***Draw the bending moment and torsional moment diagram for the beam as shown. The load is perpendicular to the beam in plane.***

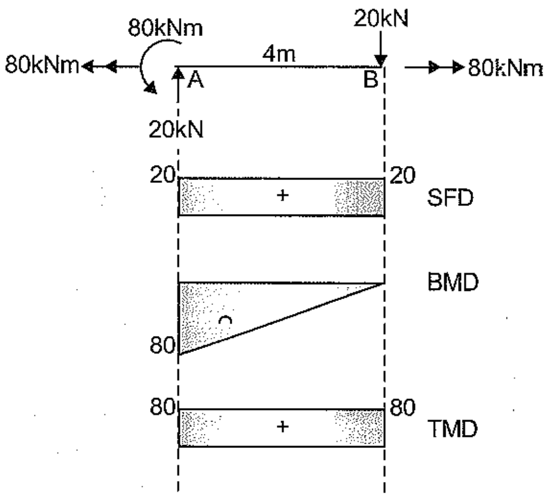


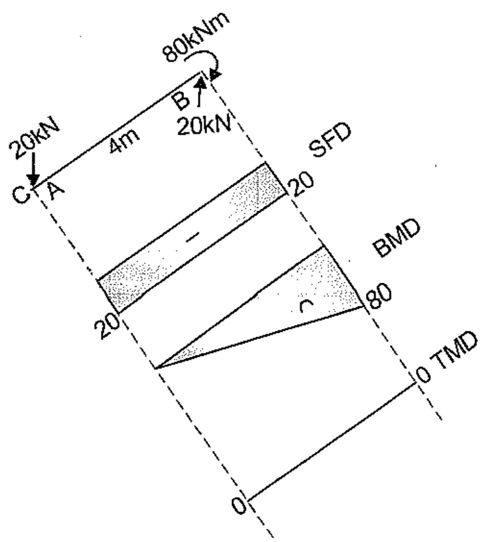
[5 Marks, ESE-2015]

Sol:



Drawing free body diagram of the given structure:





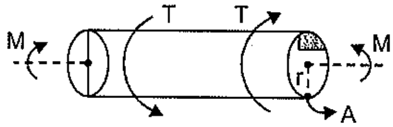
Considering anticlockwise torsion as +ve.

Q-12: ***A cylindrical shaft made of steel for which the tensile yield strength σ_y = 550 MPa is subjected to static loads consisting of a bending moment of 700 kNmm and torsion of 400 kNmm. Determine the diameter of shaft for a factor of safety equal to 1.5 if E = 200 GPa and Poisson's ratio v = 0.25; Adopt maximum shear stress failure theory***

[15 Marks, ESE-2016]

Sol:

$$\sigma_y = 550 \text{ MPa}$$
$$M = 700 \text{ kN-mm} = 7 \times 10^5 \text{ Nmm}$$
$$T = 400 \text{ kN-mm} = 4 \times 10^5 \text{ Nmm}$$
$$FOS = 1.5$$
$$E = 200 \times 10^3 \text{ MPa}$$
$$\mu = 0.25$$

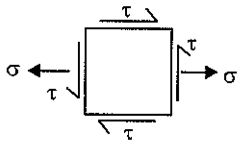


The loading condition on cylindrical shaft is:

Condition of stress for a point of cylindrical shaft at location A:

$$\sigma = \frac{M \times d/2}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32}{\pi d^3} M$$

$$\tau = \frac{T \times d/2}{J} = \frac{T \times d/2}{\frac{\pi}{32} \times d^4} = \frac{16}{\pi d^3} T$$



Maximum shear stress, $\tau_{max} = \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{16}{\pi d^3} M\right)^2 + \left(\frac{16}{\pi d^3} T\right)^2}$

$$\tau_{max} = \frac{16}{\pi d^3} \sqrt{M^2 + T^2}$$

As per maximum shear stress theory,

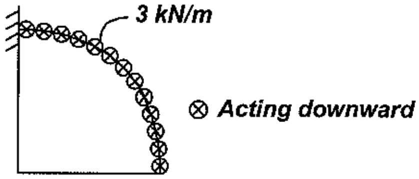
$$\tau_{max} \le \frac{1}{2} \times \left(\frac{\sigma_y}{FOS}\right)$$

$$\Rightarrow \quad \frac{16}{\pi d^3} \times \sqrt{(7 \times 10^5)^2 + (4 \times 10^5)^2} \le \frac{1}{2} \times \left(\frac{550}{1.5}\right)$$

$$\Rightarrow \quad d^3 \ge 22396.768 \text{ mm}^3$$

$$\Rightarrow \quad d \ge 28.19 \text{ mm}$$

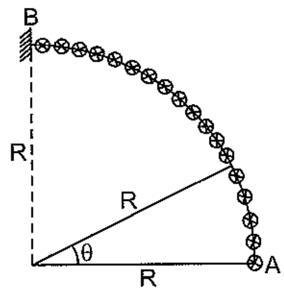
Q-13: ***Compute and draw the vertical shear, bending moment and torque diagrams of the cantilever circular bow girder shown in Figure in the shape of a quadrant of a circle 3m and carrying a uniformly distributed load of 3kN/m (normal load). The girder lies in the horizontal plane.***



[20 Marks, ESE-2017]

Sol: Loading is downward = 3 kN/m (w)

R = 3m



Sign convention:

upward loading (+)ve

(+) M (sagging) (−) M (hogging)

[+] v (shear) [−] v (shear)

[+] (torsion) [−] (torsion)

(a) Shear force at 0 will be $V_\theta = wR\theta \ (0 < \theta < \pi/2)$

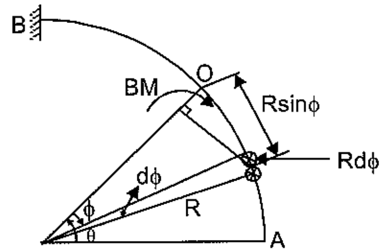
at free end $V_{\theta=0} = 0$

at fixed end $V_{\theta=\pi/2} = wR \times \pi/2 = 3 \times 3 \times \pi/2$

$= \boxed{14.14 \text{ kN}}$

(b) For BM at O, take a small element of length $Rd\phi$ at ϕ angle from O.

Load on element = $wRd\phi$.



BM on O due to small element = $-wRd\phi \times R\sin\phi$

[(–ve) sign shows hogging moment]

BM on O due to load from A to O $(0 < \phi < \theta)$

$$\int dM_\theta = -w\int_0^\theta Rd\phi \times R\sin\phi$$

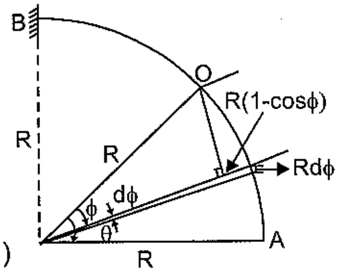
$$M_\theta = -wR^2[-\cos\phi]_0^\theta$$

$$= -wR^2[1-\cos\theta] \qquad \left[0<\theta<\frac{\pi}{2}\right]$$

at free end $M_{\theta=0} = 0$

at fixed end $M_{\theta=\pi/2} = -wR^2 = -3 \times 3 \times 3$

$= \boxed{-27 \text{ kNm}}$



(c) For twisting moment at O

Load on element = $wRd\phi$.

Twisting moment on O due to small element = $wRd\phi \times R(1-\cos\phi)$

Twisting moment on O due to load from A to O $(0 < \phi < \theta)$

$$\int dT_\theta = w\int_0^\theta Rd\phi \times R(1-\cos\phi)$$

$$T_\theta = wR^2[\phi - \sin\phi]_0^\theta$$

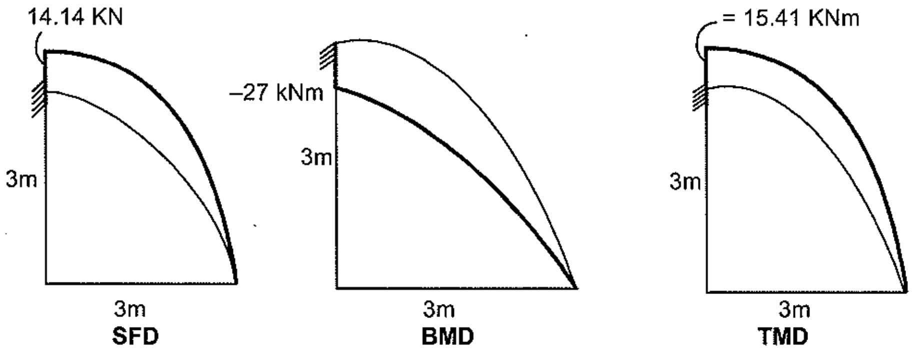
$$= wR^2[\theta - \sin\theta] \qquad \left[0<\theta<\frac{\pi}{2}\right]$$

at free end $T_{\theta=0} = 0$

at fixed end $T_{\theta=\pi/2} = wR^2\left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right)$

$$= 3 \times 3 \times 3 \left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right)$$

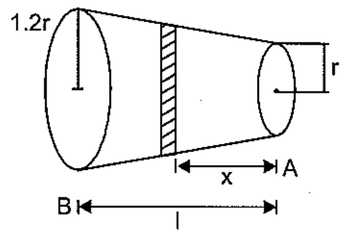
$$= \boxed{15.41 \text{ kNm}}$$



Q-14: ***A solid circular shaft has a uniform taper from one end to the other. The ratio of radius at the larger end to that at the smaller end is 1.2. Determine the error that occurs if the angle of twist for a given length is calculated using the mean radius of the shaft.***

[20 Marks, ESE-2017]

Sol:



Taking elementary section at distance x from end A

$$\int d\theta = \int \frac{T\,dx}{GJ}$$

$$r_x = \left(r + \frac{0.2r}{l}x\right)$$

$$= \frac{r}{l}(l + 0.2x)$$

So, $$J = \frac{\pi}{2}r^4$$

So, $$\theta = \int d\theta = \int_0^l \frac{T\,dx}{G\left(\frac{r}{l}\right)^4 \times \frac{\pi}{2}(l+0.2x)^4}$$

$$= \frac{T}{G\frac{\pi}{2}\left(\frac{r}{l}\right)^4}\left[\frac{(l+0.2x)^{-3}}{-3 \times 0.2}\right]_0^l$$

$$= \frac{T}{G\frac{\pi}{2}\left(\frac{r}{l}\right)^4 l^3}\left[\frac{1-\frac{1}{(1.2)^3}}{0.6}\right]$$

$$= \frac{Tl(0.702)}{G\frac{\pi}{2}\left[r^4\right]}$$

Angle of twist calculation from Average Radius

So, $$r_{avg} = 1.1r$$

$$\theta = \frac{Tl}{G\frac{\pi}{2}(1.1r)^4}$$

$$\theta_{avg} = \frac{(0.683)Tl}{G\frac{\pi}{2}r^4}$$

$$\text{Percentage error} = \frac{0.702-0.682}{0.702}\times 100$$

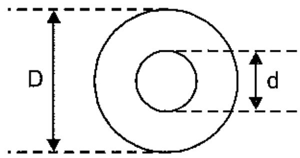
$$= \boxed{2.706\%}$$

Q-15: ***A single solid circular shaft 400 mm diameter running at 200 RPM, is to be replaced by two hollow circular shafts of equal size running at 100 RPM and developing 50% additional power. The internal diameter of hollow shaft may be taken as one-third of their external diameters. If the working stress of the new shaft is 30% greater than that of the former, find the external and internal diameters of the hollow shafts.***

[20 Marks, ESE-2018]

Sol: Given that:

Solid circular shaft of diameter, $(\phi) = 400$ mm



Solid shaft running at 200 rpm and hollow shaft running at 100 rpm

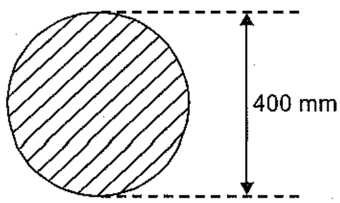
Power development, $[P_2 = 1.5P_1]$

$$\text{Internal diameter of hollow shaft} = \frac{1}{3}\times \text{External diameter}$$

$$\boxed{d = \frac{D}{3}}$$

Working stress of new shaft = 1.3 × former shaft

$$\sigma_2 = 1.3\,\sigma_1$$



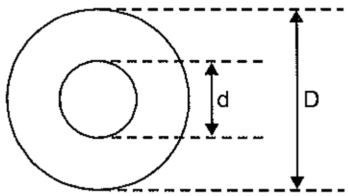
$$P_1 = \frac{2\pi N}{60}\times T$$

and $$\frac{T}{J} = \frac{\sigma_1}{\frac{400}{2}}$$

$$T = \sigma_1 \times \frac{J}{\left(\frac{400}{2}\right)}$$

$$P_1 = \left(\frac{2\pi}{60}\times 200\right)\times\left(\frac{\pi(400)^4}{32}\right)\times\left(\frac{2}{400}\right)\times\sigma_1$$

For hollow shaft



$$P_2 = 2\times\left[\left(\frac{2\pi}{60}\right)\times(100)\times\left(\frac{\pi(D^4-d^4)}{32}\right)\times\left(\frac{2}{D}\right)\right]\times\sigma_2$$

According to the given data,

$$\boxed{P_2 = 1.5P_1}$$

$$\therefore \quad 2\sigma_2\left[\left(\frac{2\pi}{60}\right)\times(100)\times\frac{\pi(D^4-d^4)}{32}\times\left(\frac{2}{D}\right)\right] = 1.5\left[\left(\frac{2\pi}{60}\right)\times 200\times\frac{\pi(400)^4}{32}\times\left(\frac{2}{400}\times\sigma_1\right)\right]$$

$$2\sigma_2\frac{(D^4-d^4)}{D} = 1.5\times 2\times\frac{(400)^4}{400}\sigma_1$$

Also, $\boxed{\sigma_2 = 1.3\sigma_1}$ (given)

and, $\boxed{d = \frac{D}{3}}$ (given)

$$2\times 1.3\times\left[1-\left(\frac{1}{3}\right)^4\right]D^3 = 1.5\times 2\times\frac{(400)^4}{400}$$

External diameter, $\boxed{D = 421.28\text{ mm}}$

Internal diameter, $\boxed{\frac{D}{3} = 140.4\text{ mm}}$

Q-16: ***A solid steel shaft has to transmit 75 kW at 200 rpm. Taking allowable shear stress as 70 N/mm², find suitable diameter for the shaft, if the maximum torque transmitted at each revolution exceeds the mean by 30%.***

[8 Marks, ESE-2019]

Sol: **Given:**

Power transmitted = 75 kW

Speed (N) = 200 rpm

Rotational speed (ω) = $\left(\frac{200\times 2\pi}{60}\right)$ rad/sec = 20.944 rad/sec

Allowable shear stress (τ_{per}) = 70 N/mm²

Also, Maximum Torque transmitted = $1.3\times T_{mean}$

Let, the dia. of shaft = d mm

Also, Power = $T_{mean}\times\omega$

$\Rightarrow$ $75\times 10^3 = T_{mean}\times(20.944)$

$\Rightarrow$ T_{mean} = 3.58 kN-m

$\therefore$ Maximum torque transmitted (T_{max}) = $1.3\times T_{mean}$

$\Rightarrow$ $T_{max} = 1.3\times 3.58$ kN-m

T_{max} = 4.654 kN-m

$$\Rightarrow \qquad \frac{T_{max} \times \left(\frac{d}{2}\right)}{J} \le \tau_{per}$$

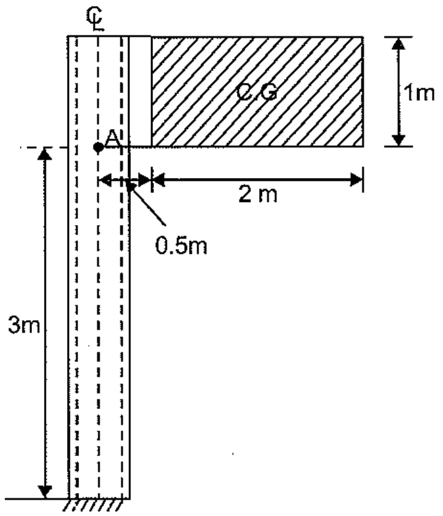
$$\Rightarrow \qquad \frac{T_{max} \times \frac{d}{2}}{\frac{\pi}{32} d^4} \le \tau_{per}$$

$$\frac{16 \times 4.654 \times 10^6}{\pi \times d^3} \le 70$$

$$d \ge 69.7 \text{ mm.}$$

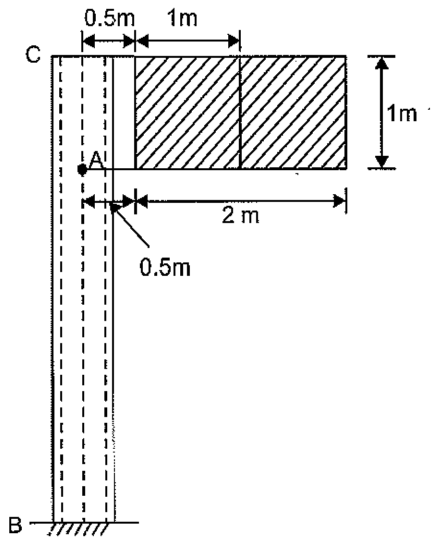
The suitable diameter of shaft would be 69.7 mm.

Q-17: ***A steel tube is to be used as a post for a road sign board as shown in Figure. The maximum wind pressure on the sign board is 1960 N/m². The angle of rotation of the tube at the bottom of the sign board marked as A must not exceed 4° and the maximum shear stress (due to torsion only) must not be greater than 38 MPa. Determine the mean diameter of the tube if the wall thickness is 4.2 mm. Take G = 70 GPa. Assume wind is transmitting only over the sign board portion.***



[20 Marks, ESE-2020]

Sol:



$$\theta = 4° = 4 \times \frac{\pi}{180} = 0.0698 \text{ rad.}$$

Net force (F) due to wind pressure will act at the centroid of sign board which will lead to development of torque in the steel tube.

$$F = \text{Pressure} \times \text{Area}$$

$$F = 1960 \times 2 \times 1 = 3920\text{N}$$

$$\text{So, torque in tube} = F \times R$$

$$= 3920 \times (1+0.5)$$

$$T = 5880 \text{ N-m}$$

$$T = 5880 \text{ N-m}$$

assuming hollow section and mean diameter of tube as d and radius as R.

For hollow circular section

$$J = 2\pi R^3 t$$

Using torsional formula.

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

As per given condition $(\theta \ngtr 4°)$

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\Rightarrow \quad \frac{5880 \times 10^3}{2\pi R^3 \times 4.2} = \frac{0.7 \times 10^5 \times 0.0698}{3000}$$

$$\Rightarrow \quad R = 51.53 \text{ mm}$$

So, $\text{dia of tube} = 2 \times 51.53 = 103.05 \text{ mm}$

For maximum shear stress condition

$$\tau_{max} \ngtr 38$$

$$\frac{TR}{J} = 38$$

$$\frac{5880 \times 10^3 \times R}{2\pi R^3 \times 4.2} = 38$$

$$R = 76.57 \text{ mm}$$

$$\text{So, dia. of tube} = 2 \times 76.57 = 153.14 \text{ mm}$$

$$\text{So, mean dia. of tube} = 153.14 \text{ mm}$$

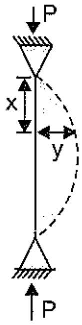
CHAPTER 9

COLUMNS

Q-1: ***Prove that the compressive strength of a compression member is inversely proportional to slenderness ratio. Further prove that Euler's formula is valid for long columns.***

[8 Marks, ESE-2007]

Sol. Consider a pin jointed column at both ends as shown in figure below.



We know that,

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

where, M = –Py

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{-Py}{EI} \Rightarrow \frac{d^2y}{dx^2} + \frac{Py}{EI} = 0$$

The solution of this equation can be written as,

$$y = A \sin mx + B \cos mx, \text{ where } m = \sqrt{\frac{P}{EI}}$$

Applying boundary condition at x = 0, y = 0

$$0 = 0 + B \quad \therefore \; B = 0$$

$$\text{Boundary condition at } x = l \quad \therefore \; y = 0$$

$$\therefore \quad A \sin l = 0$$

'A' can't be zero otherwise for all x we get y = 0.

$$\therefore \quad \sin ml = 0$$

$$\therefore \quad ml = n\pi$$

$$\Rightarrow \quad m^2 l^2 = n^2\pi^2$$

$$\Rightarrow \quad \frac{Pl^2}{EI} = n^2\pi^2$$

$$\Rightarrow \quad P = \frac{n^2\pi^2 EI}{l^2} \text{ where } n = 0, 1, 2, 3, ...$$

$\therefore$ Considering least practical value,

$$P = \frac{\pi^2 EI}{l^2}$$

$$\therefore \quad \text{Buckling stress} = \frac{\pi^2 EI}{l^2 A} = \frac{\pi^2 E2}{(l/r)^2} = \frac{\pi^2 E}{\lambda^2}$$

$\therefore$ Buckling stress $\propto \frac{1}{\lambda^2}$.

In the derviation of Euler's theory, we have assumed a bucked shape in the starting. Hence the Euler's formula is valid only for long column in which failure occurs due to buckling.

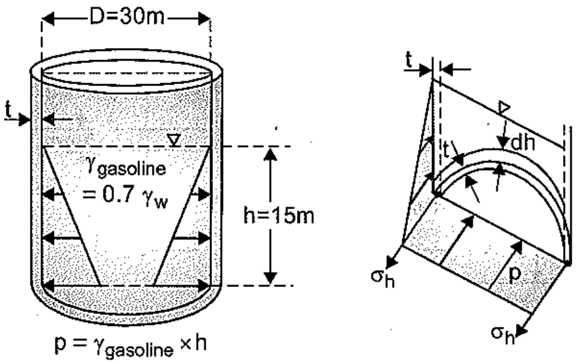
CHAPTER 10

THICK AND THIN CYLINDER SPHERE

Q-1: ***A vertical cylindrical steel storage tank has 30 m diameter and the same is filled upto a depth of 15 m with the gasoline of relative density 0.74. If the yield stress for steel is 250 MPa, find the thickness required for the wall plate. Adopt a factor of safety of 2.5 and neglect localized bending effects, if any***

[10 Marks, ESE-2015]

Sol:



Maximum stress will be at the bottom of the cylinder which will be equal to the hoop stress in the wall of cylinder.

$\therefore$ Pressure at base × projected area = Hoop stress in wall × area of wall × $\frac{1}{FOS}$

$$\Rightarrow \quad (\gamma_{gasoline} \times h) \times (D \times dh) = (\sigma_h) \times 2 \times (t \times dh) \times \frac{1}{FOS}$$

$$\Rightarrow \quad 0.74 \times 9810 \times 15 \times 30 = 250 \times 10^6 \times 2 \times 2\,t \times \frac{1}{2.5}$$

$$\Rightarrow \quad t = 16.33 \text{ mm}$$

CHAPTER 11

MOMENT OF INERTIA

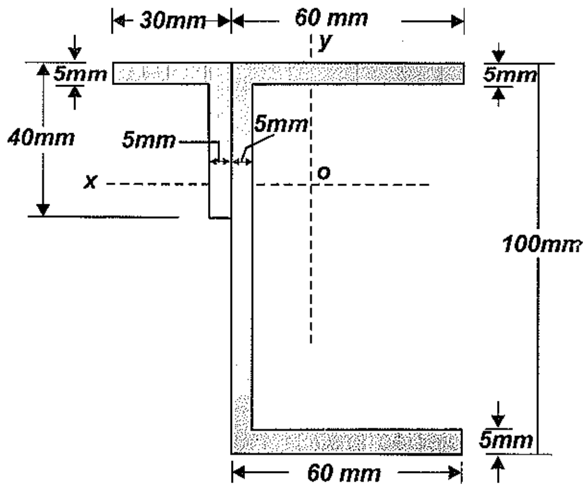
Q-1: ***The combined angle and channel section shown in fig. below forms part of a runway beam. Calculate:***

(a) ***Coordinates of the centroid***

(b) ***Second moment of area about X-X***

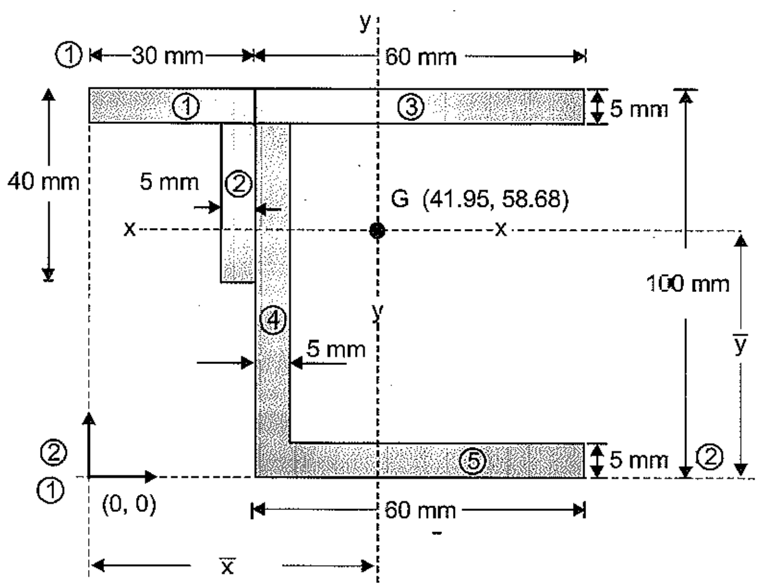
(c) ***Second moment of area about Y-Y***

(d) ***Product of inertia about O***



[10 + 10 + 10 + 10 = 40 Marks, ESE-1999]

Sol:



(a) Determination of centroid $(\bar{x}, \bar{y})$

Component	Area a (mm²)	Centroidal distance x from (1-1) in mm	Centroidal distance y from (2-2) in mm	ax (mm³)	ay (mm³)
(1)	$30\times5=150$	15	$100-5+2.5=97.5$	2250	14625
(2)	$35\times5=175$	$30-5+2.5=27.5$	$60+\frac{35}{2}=77.5$	4812.5	13562.5
(3)	$60\times5=300$	$30+30=60$	97.5	18000	29250
(4)	$(100-2\times5)\times5=450$	$30+2.5=32.5$	50	14,625	22500
(5)	$60\times5=300$	$30+30=60$	2.5	18,000	750
	$\Sigma=1375$			$\Sigma=57687.5$	$=80687.5$

$\therefore$ Centroidal distance along x-direction from 1-1 axis,

$$\bar{x} = \frac{\sum ax}{\sum a} = \frac{57687.5}{1375} = 41.95 \text{ mm}$$

and, centroidal distance along y-direction from 2.2 axis,

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{80687.5}{1375} = 58.68 \text{ mm}$$

(b) & (c) Calculation of 2nd moment of inertia about C.G. (I_{xx}) and I_{xy}):

Sr. No.	Area	x	y	x^2	y	ax^2	ay^2
1.	150	15	97.5	225	9506.25	33750	1425937.5
2.	175	27.5	77.5	756.25	6006.25	132343.75	1051093.75
3.	300	60	97.5	3600	9506.25	1080000	2851875
4.	450	32.5	50	1056.25	2500	475312.5	1125000
5.	300	60	2.5	3600	6.25	1080000	1875
	$\Sigma=1375$					$\Sigma=2801406.25$	$\Sigma=6455781.25$

$$\sum I_{self}\Big|_{\text{along } x\text{-}x \text{ direction}} = \frac{5^3\times30}{12}+\frac{35^3\times5}{12}+\frac{60\times5^3}{12}+\frac{90^3\times5}{12}+\frac{60\times5^3}{12}$$

$$= 323177.0833$$

Hence M.O.I. about the 2-2 axes= $\sum I_{self}+\sum ay^2 = 323177.0833 + 6455781.25$

$$= 6778958.33 \text{ mm}^4$$

and from parallel axes theorem,

$$I_{2\text{-}2} = I_{C.G.} + A_{Total}\,(\bar{y})^2$$

Hence

$$I_{2\text{-}2}\Big|_{\text{along } x} = I_{C.G.}\Big|_{\text{along } x} + A_{total}(\bar{x})^2$$

$$\Rightarrow \quad 6778958.33 = I_{C.G.}\Big|_{\text{along } x} + 1375\times58.68^2$$

$$\Rightarrow \quad I_{C.G.}\Big|_{\text{along } xx} = 2044362.52 \text{ mm}^4 \quad \text{[i.e M.O.I. about x-x axis]}$$

$$\sum I_{self}\Big|_{\text{along yy direction}} = \frac{5\times30^3}{12}+\frac{35\times5^3}{12}+\frac{5\times60^3}{12}+\frac{90\times5^3}{12}+\frac{5\times60^3}{12}$$

$$= 192552.0833 \text{ mm}^4$$

Hence M.O.I. about 1-1 axes = $\sum I_{self} + \sum ay^2$ = 192552.0833 + 2801406.25= 2993958.333 mm^4

from parallel axes theorem,

$$I_{2\text{-}2}\big|_{\text{along y-axes}} = I_{C.G.}\big|_{\text{along y}} + A_{total}(\bar{y})^2$$

$$\Rightarrow \quad 2993958.333 = I_{C.G.}\big|_{\text{along y}} + 1375 \times 41.95^2$$

$$I_{C.G.}\big|_{\text{along yy}} = 574229.895 \text{ mm}^4 \quad \text{[Moment of inertial about y-y axis]}$$

(d) Product of inertia about *xy*,

$$I_{xy} = I_{xyc} + \sum Axy$$

Hence I_{xyc} is zero for each individual area because each individual area have symmetrical centroidal axes.

Calculation of product of inertia about C.G.

Sr. No.	Area	x	y	Axy
1.	150	– 26.95	+ 38.82	– 156929.85
2.	175	– 14.45	+ 18.82	– 47591.075
3.	300	+ 18.05	+ 38.82	+ 210210.3
4.	450	– 9.45	– 8.68	+ 36911.7
5.	300	+ 18.05	– 56.18	– 304214.7
	Σ = 1375			Σ = –261613.625

Here, x and y are distance of c.g. of individual areas (with sign)from the centroid of the section as a whole.

Thus, $I_{xy} = \Sigma A_{xy} = -261613.625$ mm^4

UNIT-2 STRUCTURE ANALYSIS

SYLLABUS

Analysis of determinate and indeterminate structures; Trusses, beams, plane frames; Rolling loads, influence Lines. Unit load method and other methods; Free and Forced vibrations of single degree and multi degree freedom system; Suspended Cables; Concepts and use of Computer Aided Design.

CONTENTS

CHAPTER 1

DETERMINACY INDETERMINACY AND STABILITY OF STRUCTURES

Q-1: ***What do you understand by static indeterminacy and kinematic indeterminacy of example of a fixed end beam.***

[5 Marks, ESE-2014]

Sol: **Static indeterminacy (D_S):**

It is defined as the number of equilibrium conditions, also referred as compatibility conditions, required in excess of static equilibrium equations to fully analyze a statically indeterminate structure.

$$D_S = \begin{bmatrix} \text{No. of unknown forces} \\ \text{in member or at support reactions} \end{bmatrix} - \begin{bmatrix} \text{Equations of static equilibrium} \\ \text{available} \end{bmatrix}$$

$$\underbrace{D_S}_{\text{Total indeterminacy}} = \underbrace{D_{Si}}_{\text{Degree of internal static indeterminacy}} + \underbrace{D_{Se}}_{\text{Degree of external static indeterminacy}}$$

For a 2D-framed structure:

Frames are rigid jointed structures. Restraining of deformation at support or any joint gives rise to three internal reactions. (R_x, R_y, M_z) in each member.

At every joint; three equilibrium equations are available.

$$\Sigma F_x = 0, \ \Sigma F_y = 0, \Sigma M_Z = 0$$

Let

m = No. of members in a 2D framed structure

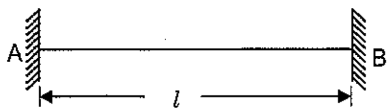
R = Total no of reaction

R_e = Total no of external reaction from support

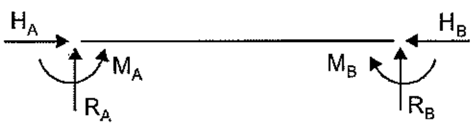
j = Total no. of joints in the structure

Then,
$$R = 3m + Re$$
$$D_S = 3m + R_e - 3j$$

Ex: A Beam fixed at both ends



FBD



Here $R_e = 3 + 3 = 6$

$m = 1$

$j = 2$

$\therefore \quad D_s = 3m + R_e - 3j = 3 \times 1 + 6 - 3 \times 2 = 3$

Kinematic indeterminacy (D_K):

It is defined as the total no. of unknown joint displacements at all joints or the total no. of available degree of freedom at all joints.

$$D_K = \begin{bmatrix} \text{Total possible degree} \\ \text{of freedon} \end{bmatrix} - \begin{bmatrix} \text{No. of available support reactions which} \\ \text{generate as a result of restrained joint displacements} \end{bmatrix}$$

For a 2D- framed structure:

Each joint has three possible degrees of freedom, $(\Delta_x, \Delta_y, \theta_z)$.

For a frame having j no. of joints

Total possible degrees of freedom = 3j

But some displacements will be constrained by the supports provided

Let R_e be the total no. of external support reactions

$$\therefore \quad D_K = 3j - R_e$$

If the members are considered inextensible then there will be no axial deformations and

$$D_K = 3j - R_e - m$$

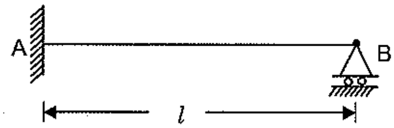
$$m = \text{no. of members}$$

Ex:

$j = 2$

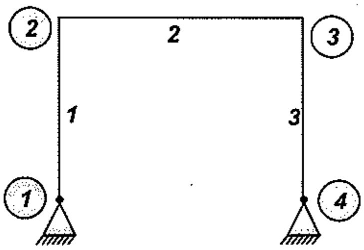
$R_e = 3 + 1 = 4$

$m = 1$



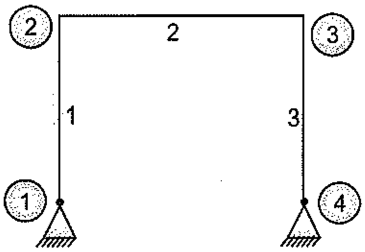
$D_K = 3j - R_e = 3 \times 2 - 4 = 2 \ (\Delta_x, \theta_z \text{ at B})$

Q-2: ***A plane frame is shown in figure. Determine the size of the reduced stiffness matrix with axial deformations and without axial deformations after introducing the boundary conditions. Also, show the active degrees of of freedome in both cases. Node number is shown in circles.***



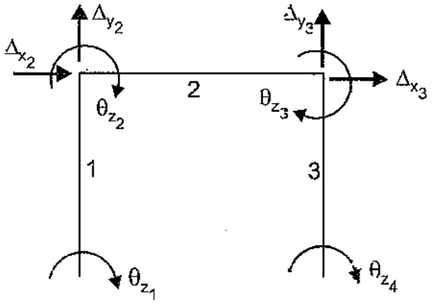
[10 Marks, ESE-2014]

Sol:



Case I: With axial deformations

Size of reduced stiffness = Degree of kinematic indeterminacy of the structure



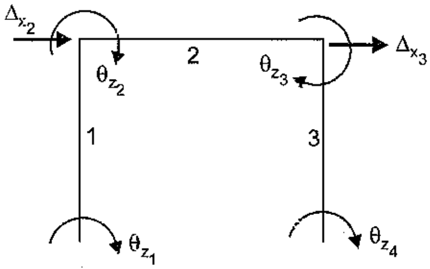
$$D_K = 1 + 3 + 3 + 1 = 8$$

Alternative:

$$D_K = 3j - R_e$$
$$j = \text{Total no. of joints}$$
$$R_e = \text{No. of external support reactions}$$
$$j = 4$$
$$R_e = 4\ (R_{x1}, R_{y1}, R_{x4}, R_{y4})$$
$$D_K = 3 \times 4 - 4 = 8$$

∴ Size of reduced stiffness matrix will be [8 × 8]

Case II: Without axial deformations



Here, $\Delta x_2 = \Delta x_3$ ∵ Size of member 2 is unchanged

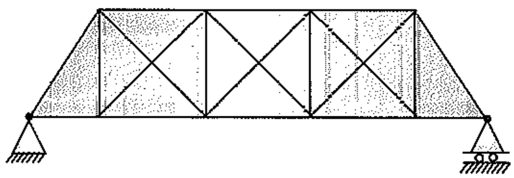
$$D_K = 1 + 1 + 1 + 1 + 1 = 5$$

Alternate method:

$$D_K = 3j - R_e - m$$
$$m = \text{Total no. of member} = 3$$
$$D_K = 3 \times 4 - 4 - 3 = 5$$

Size of reduced stiffness matrix will be [5 × 5]

Q-3: ***Find the total degree of statical indeterminacy (both internal and external) for bridge truss shown in the figure.***



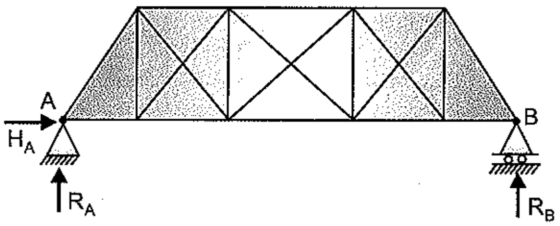
[5 Marks, ESE-2015]

Sol: {Total degree of static indeterminacy} = {Degree of external indeterminacy} + {Degree of internal indeterminacy}

$$D_{s,\,tot} = D_{s,\,e} + D_{s,\,i}$$

$$D_{s,\,e} = \text{(Total no. of support reactions)} - \text{(Total no. of equations of static equilibrium)}$$

$$= 3 - 3 = 0$$



($\because$ Support reactions = H_A, R_A, R_B; equilibrium equations $= \Sigma F_H = 0; \Sigma F_V = 0; \Sigma M = 0$)

$$D_{s,\,tot} = m + r - 2j$$

where, m = No. of members of the truss = 20

r = No. of support reactions = 3

j = No. of joints of the truss = 10

$\Rightarrow$ $$D_{s,\,tot} = 20 + 3 - 2 \times 10 = 3$$

$\Rightarrow$ $$D_{s,\,i} = D_{s,\,tot} - D_{s,\,e} = 3 - 0 = 3$$

$\Rightarrow$ $$D_{s,\,i} = 3$$

$$D_{s,\,e} = 0$$

$$D_{s,\,tot} = 3$$

CHAPTER 2

FORCE METHOD OF ANALYSIS (STATICALLY INDETERMINATE STRUCTURES)

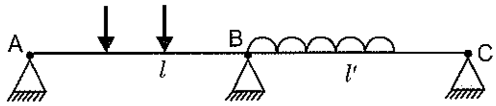
Q-1: ***Employing the equation of 3 moments obtain the central deflection in a simply supported beam of span l loaded with a uniformly distributed load of w/unit length. EI is constant for the beam.***

[15 Marks, ESE-1995]

Sol: ***Conceptual Background:***

Clapeyron's theorem of three moments: re ationship among the bending moments at three consecutive supports of a horizontal beam.

$$M_A . l + 2M_B (l + l') + M_C l' = \frac{6a_1 x_1}{l} + \frac{6a_2 x_2}{l'}$$



a_1 = Area of BM diagram due to vertical loads on AB

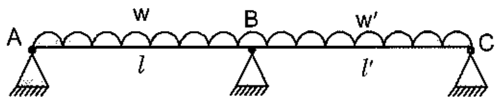
a_2 = Area of BM diagram due to vertical loads on BC

x_1 = Distance from A to c.g for the BMD for AB

x_2 = Distance from C to c.g for the BMD for BC

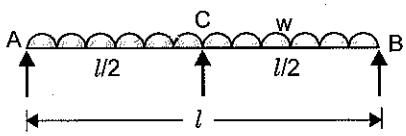
If weight distributed uniformly i.e., udl on both spans, equation can also be written as,

$$M_A l + 2M_B (l + l') + M_C l' = \frac{wl^3}{4} + \frac{w'l'^3}{4}$$



3 Moments equation of clapeyron's theorem is applicable for a continuous span where we get a relationship among the bending moments at three consecutive supports of a horizontal beam.

To a calculate the central deflection, let use assume a support at centre of beam AB. Thus applying 3 moments equation, we get,



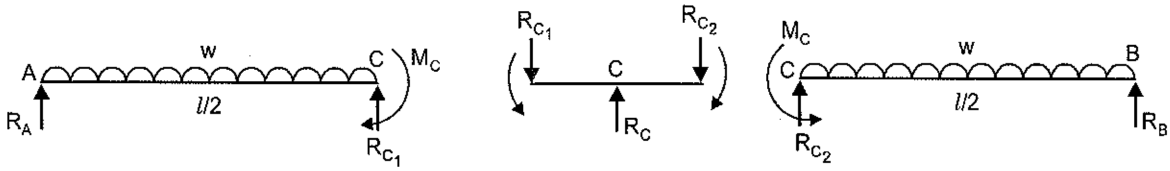
$$M_A \frac{l}{2} + 2M_C \left(\frac{l}{2} + \frac{l}{2}\right) + M_B \times \frac{l}{2} = \frac{w\left(\frac{l}{2}\right)^3}{4} + \frac{w\left(\frac{l}{2}\right)^3}{4}$$

$$\Rightarrow \quad M_A + 4M_C + M_B = \frac{2wl^3}{4 \times 8} \times \frac{2}{l} = \frac{wl^2}{8}$$

As simply supported ends, $\Rightarrow$ $\qquad M_A = M_B = 0$

$$\Rightarrow \quad M_C = \frac{wl^2}{8} \times \frac{1}{4} = \frac{wl^2}{32}$$

Now, to calculate reactions at C, we use FBD of beam



$$R_{C_1} = \frac{wl/2}{2}+\frac{M_C}{l/2} = \frac{wl}{4}+\frac{wl^2}{32\times\frac{l}{2}} \qquad R_{C_2} = \frac{w(l/2)}{2}+\frac{M_C}{l/2}$$

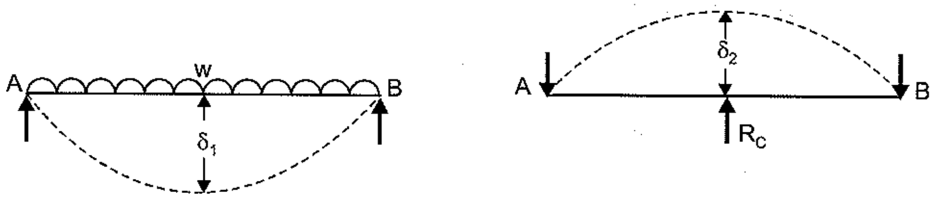
$$= \frac{wl}{4}+\frac{wl}{16} \qquad = \frac{wl}{4}+\frac{wl}{16}$$

$$\Rightarrow \quad R_{C_1} = \frac{5wl}{16} \qquad \Rightarrow \quad R_{C_2} = \frac{5wl}{16}$$

$$\Rightarrow \quad R_C = R_{C_1}+R_{C_2} = \frac{5wl}{16}+\frac{5wl}{16} = \frac{5wl}{8}$$

Now using compatibility condition that the net downward deflection due to udl on span AB is equal to net upward deflection due to R_C acting alone.

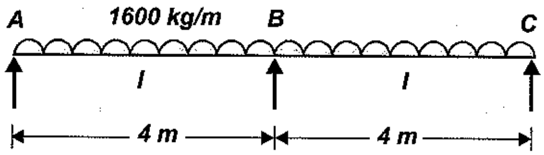
$\Rightarrow$



$$\Rightarrow \quad \delta_1 = \delta_2 = \frac{R_C l^3}{48\,EI} = \frac{5\,wl}{8}\times\frac{l^3}{48EI} = \frac{5wl^4}{384EI}$$

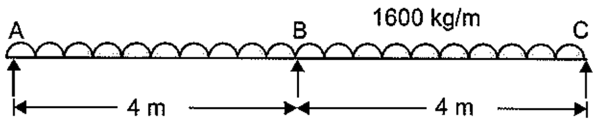
Thus the central deflection in a simply supported beam is $\frac{5\,wl^4}{384\,EI}(\downarrow)$

Q-2: ***A continuous beam ABC is simply supported at A, B and C as shown in fig. The support C is yielding and settle at the rate of 3 mm per 1000 kg. EI = 8 × 10¹⁰ kg cm². Analyse the beam and draw bending moment and shear force diagrams.***



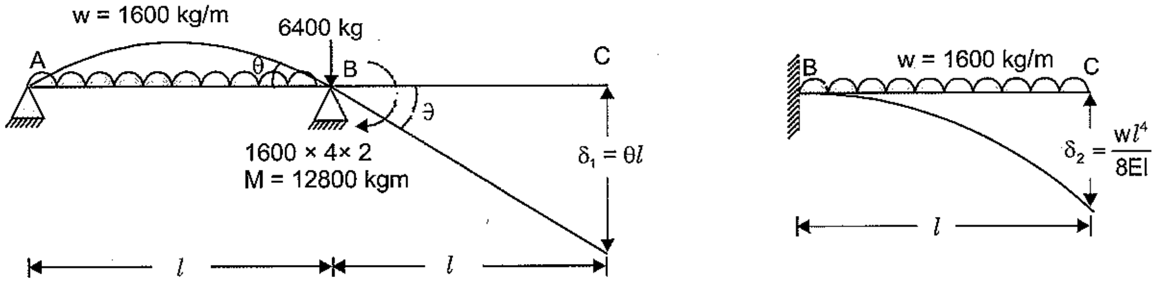
[20 Marks, ESE-1997]

Sol:



Support C yields at 3 mm per 1000 kg. If R_c is the reaction at C in kg, settlement at C will be $\frac{3R_c}{1000}$ mm

Let us solve the problem by consistent deformation method. If we remove the support at C, the total downward deflection at C will be given as $\delta_1+\delta_2$.



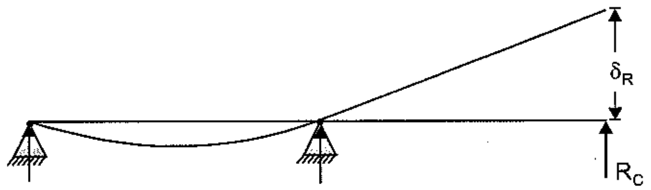
$$\theta = \frac{-wl^3}{24EI} + \frac{Ml}{3EI}$$

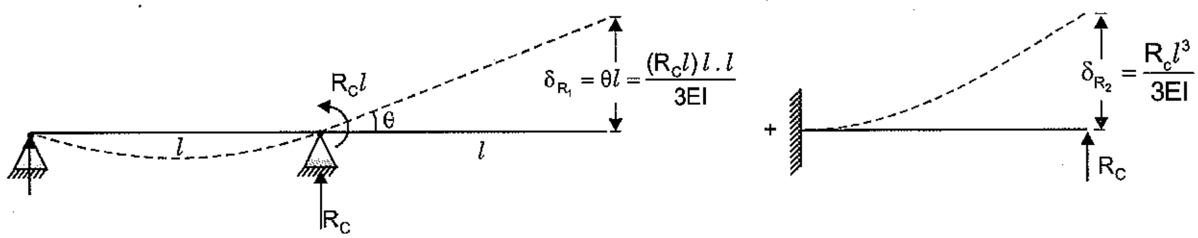
$$\Rightarrow \qquad \delta_1 = \frac{-wl^4}{24EI} + \frac{Ml^2}{3EI}$$

$$\delta_2 = \frac{wl^4}{8EI}$$

$$\Rightarrow \qquad \delta = \delta_1 + \delta_2 = \frac{-wl^4}{24EI} + \frac{Ml^2}{3EI} + \frac{wl^4}{8EI} = \frac{wl^4}{12EI} + \frac{Ml^2}{3EI}$$

Upward deflection due to reaction R_c





$$\delta_R = \delta_{R_1} + \delta_{R_2} = \frac{(R_c l)l^2}{3EI} + \frac{R_c l^3}{3EI} = \frac{2R_c\, l^3}{3EI}$$

Net deflection (downward) of support C = $\frac{3R_c}{1000}$ mm where R_C is in kg

$$\Rightarrow \qquad \frac{wl^4}{12EI} + \frac{Ml^2}{3EI} - \frac{2R_c l^3}{3EI} = \frac{3R_c}{10000} \text{ cm}$$

$$\text{w in kg/cm} = 16 \text{ kg/cm}$$

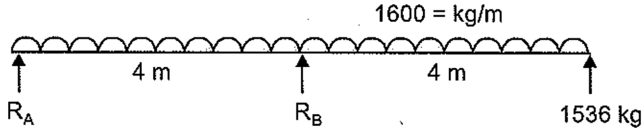
$$\text{l in cm} = 400 \text{ cm}$$

$$\text{EI kg cm}^2 = 8 \times 10^{10} \text{ kg cm}^2$$

$$\text{M} = \text{kg cm}$$

$$\Rightarrow \qquad \frac{16\,(400)^4}{12 \times 8 \times 10^{10}} + \frac{1280000\,(400)^2}{3 \times 8 \times 10^{10}} - \frac{2R_c\,(400)^3}{3 \times 8 \times 10^{10}} = \frac{3R_c}{10000}$$

$$R_c = 2800 \text{ kgm}$$



From $\Sigma F_V = 0$

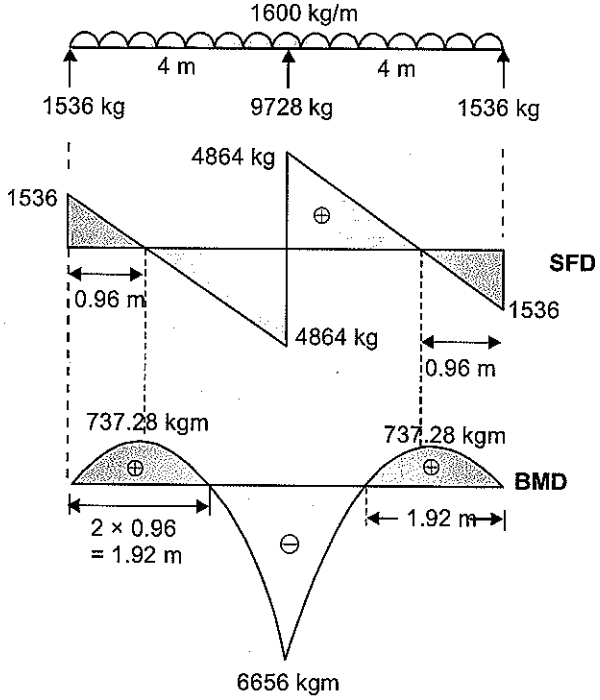
$$R_A + R_B + 1536 = 1600 \times 8 \text{ kg}$$
$$R_A + R_B = 11264 \text{ kg}$$

By taking moment about B

$$R_A \times 4 = 1536 \times 4$$
$$R_A = 1536 \text{ kg}$$

$\Rightarrow$ $$R_B = 11264 - 1536 = 9728 \text{ kg}$$

Hence finally, the reaction are



Q-3: ***Using Betti's theorem, detemine the elongation of a bar 10 mm diameter and 1 m long when it is subjected to a force of 10 kN at diameterically opposite points at mid point of its length. E = 2 × 10⁵ MPa, μ = 0.3***

[8 Marks, ESE-2001]

Sol: As per Betti's theorem, virtual work done by P-force system in going through the deformation caused by Q-force system is equal to the virtual work done by Q-force system in going through the deformation caused by P-force system.

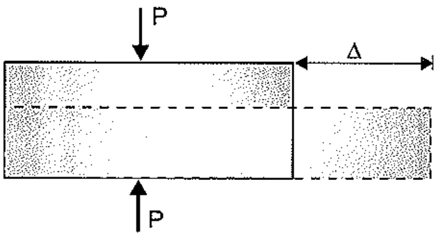


Fig A

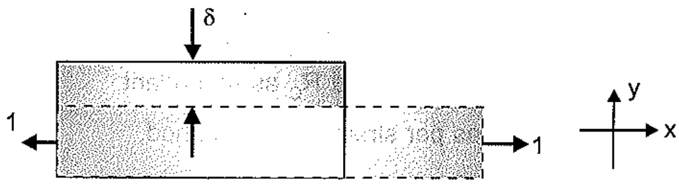


Fig B

$$\Rightarrow \qquad P.\delta = 1.\Delta$$

$$\Rightarrow \qquad \Delta = P.\delta$$

We have to calculate Δ and for this we need to calculate δ.

$$\delta = \varepsilon_y.d \quad \text{[in fig B]}$$

$$\delta = \left[\frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}\right]d$$

$$\delta = 0 - \frac{\mu\sigma_x}{E}d \quad \text{[In Fig B]}$$

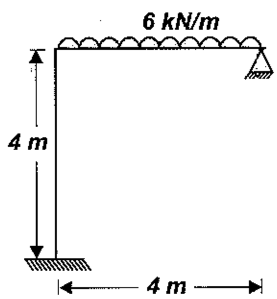
$$= \frac{-\mu d}{E}\left[\frac{1}{A}\right], \quad A = \text{cross sectional area of bar}$$

$$\delta = \frac{-\mu d}{E}\left[\frac{1}{\pi d^2/4}\right]$$

$$\Rightarrow \qquad \Delta = P\delta = \frac{P\mu d \times 4}{\pi d^2 \times E} \quad \text{[magnitude wise]}$$

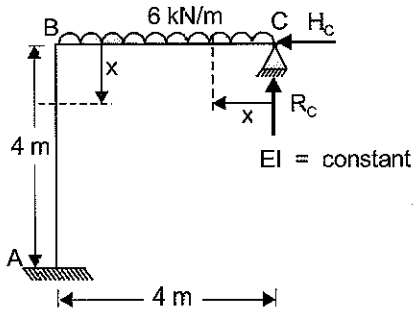
$$\Delta = \frac{4\mu P}{\pi dE} = \frac{4\times 0.3\times 10\times 10^3\,\text{N}}{3.14\times 10\,\text{mm}\times 2\times 10^5\,\text{N/mm}^2} = 1.911 \times 10^{-3}\ \text{mm}$$

Q-4: ***Analyse the portal frame shown by strain energy method and draw the BMD EI is constant.***



[18 Marks, ESE-2001]

Sol:



Degree of static indeterminacy of the above frame = 2

Let us take H_C and R_C as redundant

hence as per strain energy method,

$$\frac{\partial U}{\partial H_C} = 0 \quad \text{and} \quad \frac{\partial U}{\partial R_C} = 0$$

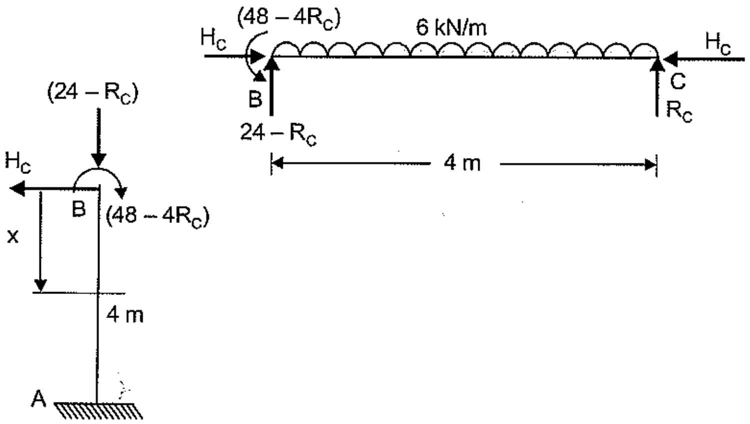
$$U = \int \frac{M^2\, dx}{2EI}$$

$$\Rightarrow \qquad \frac{\partial U}{\partial H_C} = \int \frac{M \cdot \dfrac{\partial M}{\partial H_C} dx}{EI} = 0$$

$$\text{and} \qquad \frac{\partial U}{\partial R_C} = \int \frac{M \dfrac{\partial M}{\partial R_C} \cdot dx}{EI} = 0$$

Let us measure x in CB force C and that in BA from A.

Segment	Moment = M	$\frac{\partial M}{\partial R_C}$	$\frac{\partial M}{\partial H_C}$	Limit of x
CB	$R_C x - \frac{6x^2}{2}$	x	0	0 – 4
BA	$-(48-4R_C)+H_C\,x$	4	x	0 – 4



$$\Rightarrow \quad \int \frac{M \dfrac{\partial M}{\partial H_C} dx}{EI} = \int_0^4 \frac{(R_C\,x - 3x^2)\times 0\, dx}{EI} + \int_0^4 \frac{(H_c\,x - 48 + 4R_C)\,xdx}{EI} = 0$$

$$= \frac{1}{EI}\left[\frac{H_C x^3}{3} - \frac{48x^2}{2} + \frac{4R_C x^2}{2}\right]_0^4 = 0$$

$$\frac{64}{3} H_C + 32 R_C - 384 = 0$$

$$\Rightarrow \quad 64 H_C + 96 R_C - 1152 = 0 \qquad \text{... (i)}$$

$$\text{Also,} \quad \int \frac{M \dfrac{\partial M}{\partial R_C} dx}{EI} = \int_0^4 \frac{(R_C x - 3x^2)xdx}{EI} + \int_0^4 \frac{(H_C x + 4R_c - 48)\times 4dx}{EI}$$

$$\Rightarrow \quad \frac{1}{EI}\left[\frac{R_C x^3}{3} - \frac{3x^4}{4}\right]_0^4 + \frac{1}{EI}\left[\frac{4H_C x^2}{2} + 16R_C x - 48\times 4x\right]_0^4 = 0$$

$$\frac{64\,R_C}{3} - 3\times 64 + 2H_C \times 16 + 16\,R_C \times 4 - 192\times 4 = 0$$

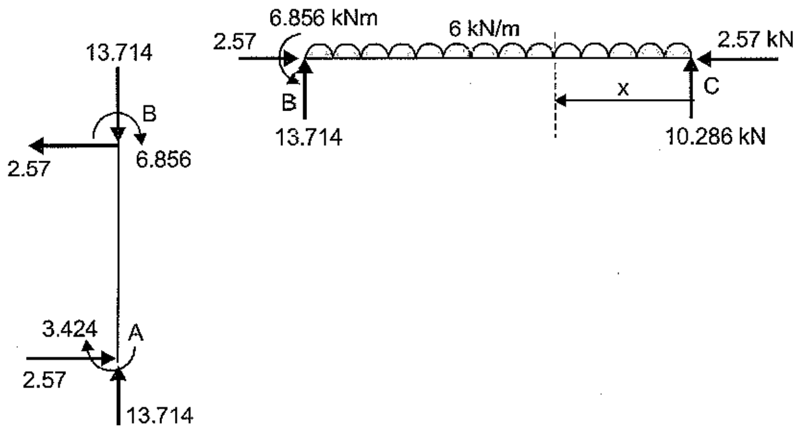
$$\frac{64 R_C}{3} + 64 R_C + 32 H_C - 960 = 0$$

$$256 R_C + 96 H_C - 2880 = 0 \quad \text{... (ii)}$$

From (i) and (ii), $R_C = 10.286$ kN

$H_C = +2.57$ kN

The free body diagram is thus as shown below

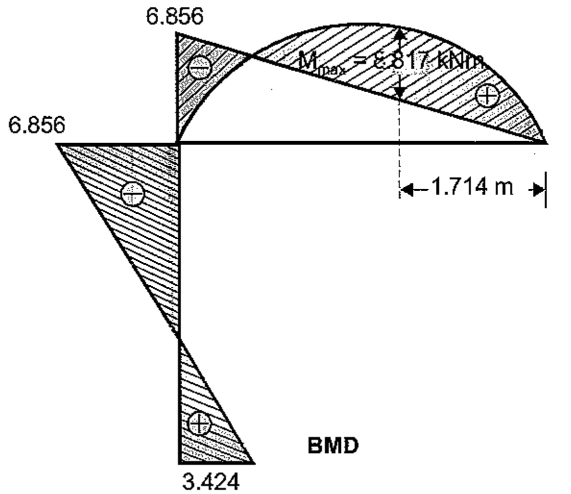


Let SF at a distance x from end C be zero

$\Rightarrow$ $10.286 - 6x = 0$

$\Rightarrow$ $x = 1.714$ m

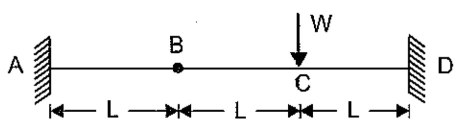
$\Rightarrow$ $M_{max} = 10.286\,(1.714) - \dfrac{6\,(1.714)^2}{2} = 8.817$ kNm



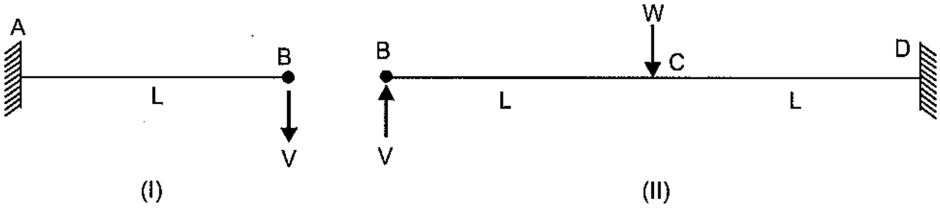
Q-5: ***Two prismatic beams of same area of cross-section AB & BD, have fixed ends at A & D form a hinge at B. Beam BD carries a single vertical load 'W' at mid-point C. AB = BC = CD = L. If A, B, C & D lie in one horizontal line with AD = 3L, analyse the beam and draw bending moment and shear force diagrams.***

[20 Marks, ESE-2002]

Sol:



For vertical loading, Degree of static indeterminacy for the beam is one. Let us take SF at B to be redundant.

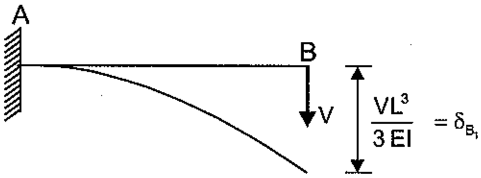


EI = constant

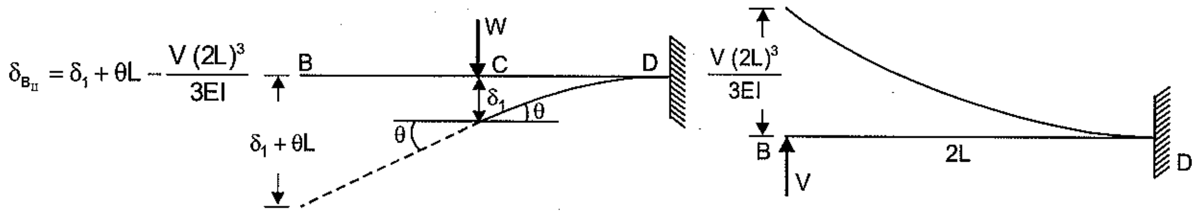
BM at B = 0, because there is internal hinge at B. The value of V can be determined using compatibility equation.

The compatibility equation will be that, downward deflection of B in part I is equal to downward deflection of B in part II

Downward deflection at B (in Part I)



Downward deflection at B (in Part II)



$$\delta_1 = \frac{WL^3}{3EI}$$

$$\theta = \frac{WL^2}{2EI}$$

$$\Rightarrow \qquad \delta_{B_{II}} = \frac{WL^3}{3EI} + \frac{WL^2}{2EI} \times L - \frac{V(2L)^3}{3EI}$$

From compatibility equation

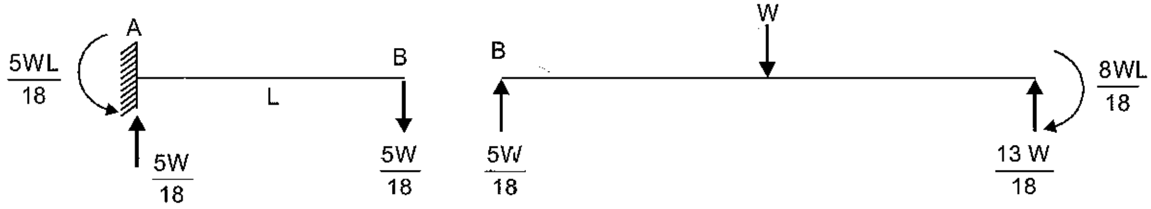
$$\Rightarrow \qquad \frac{VL^3}{3EI} = \frac{WL^3}{3EI} + \frac{WL^3}{2EI} - \frac{8VL^3}{3EI}$$

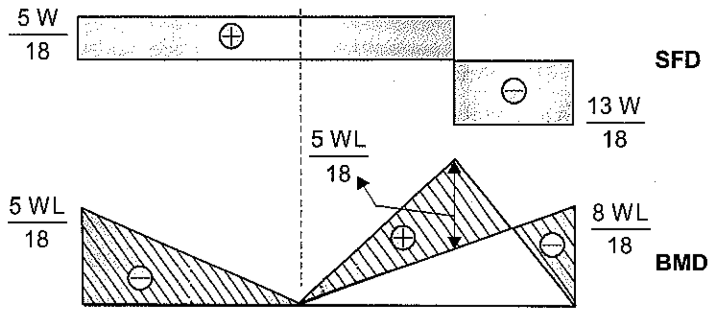
$$\Rightarrow \qquad \frac{9VL^3}{3EI} = \frac{5WL^3}{6EI}$$

$$\Rightarrow \qquad V = \frac{5WL^3}{6EI} + \frac{3EI}{9L^3} = \frac{5W}{18}$$

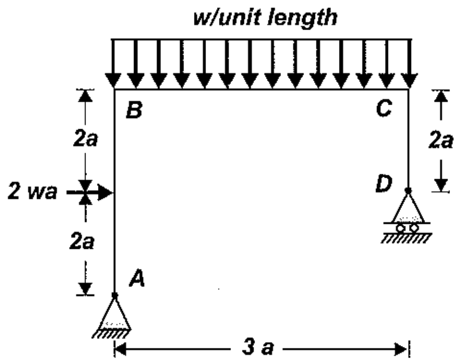
$$\boxed{V = \frac{5W}{18}}$$

Thus the free body diagram is an shown below



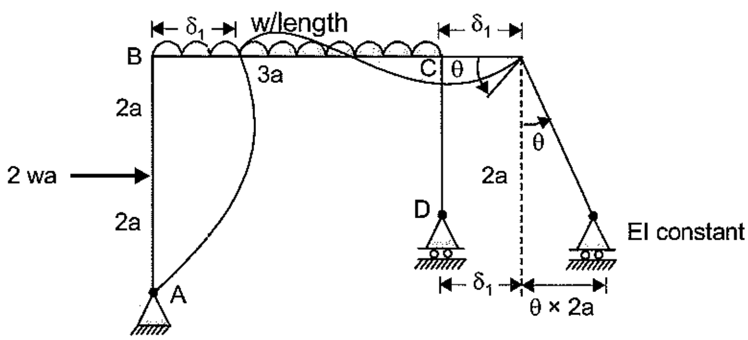


Q-6: ***A rigid frame ABCD of uniform EI shown in the following figure is subjected to horizontal and vertical forces. Determine horizontal displacement at the support D.***



[15 Marks, ESE-2003]

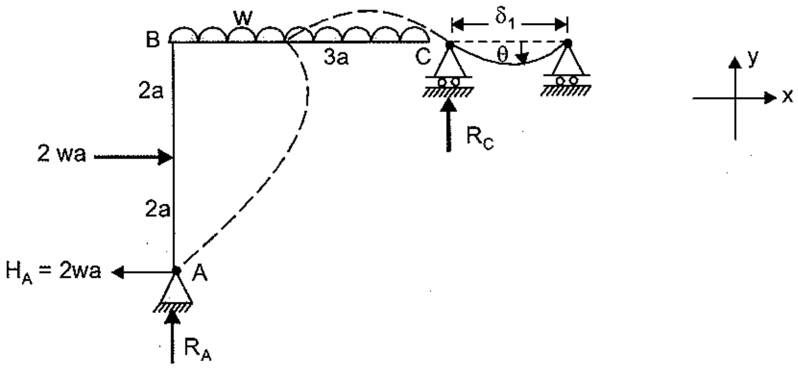
Sol:



As there can be no BM at D and no horizontal reaction D, there well be no bending of member CD . Total horizontal displacement of joint D = $[\delta_1 + \theta \times 2a]$

Thus if we know δ_1 and θ, we can find out horizontal displacement at D.

The displacement δ_1 and θ can be calculated from the following structure



$\Sigma F_y = 0$

$$\Rightarrow \quad R_A + R_C = 3wa \qquad \text{... (i)}$$

$\Sigma M_A = 0 \Rightarrow R_C \times 3a - w \times 3a \times 1.5\,a - 2\,wa\,(2a) = 0$

$$\Rightarrow \quad R_C = 1.5\,wa + \frac{4wa}{3} = \frac{8.5\,wa}{3} \qquad \text{... (ii)}$$

$$R_A = 3wa - \frac{8.5\,wa}{3} = \frac{0.5\,wa}{3}$$

$\Rightarrow \quad \Sigma F_x = 0$

$$\Rightarrow \quad H_A = 2\,wa \qquad \text{... (iii)}$$

Let us use unit load method to calculate δ_1 and θ

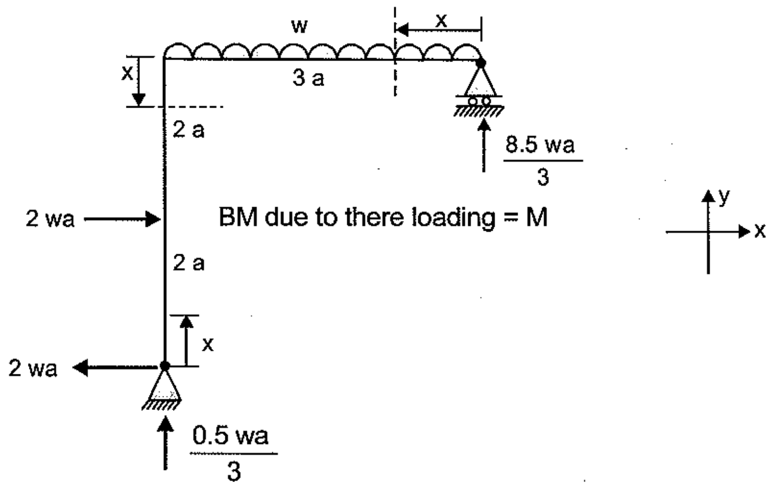
$$\delta_1 = \int \frac{M\,m_1\,dx}{EI}$$

$$\theta = \int \frac{M\,m_\theta . dx}{EI}$$

where M = BM due to external loading

m_1 = BM when external loads are removed and unit load is applied at C to the right

m_θ = BM when external loads are removed and a unit anticlockwise couple is acting at C



From $\Sigma F_y = 0$

$$R_C - R_A = 0 \quad \text{... (i)}$$

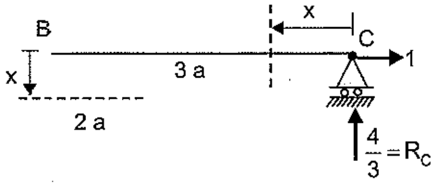
$\Rightarrow \quad \Sigma M_A = 0$

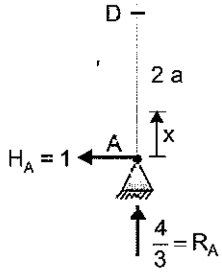
$\Rightarrow \quad R_C \times 3a - (1 \times 4a) = 0$

$\Rightarrow \quad R_C = \dfrac{4}{3}$

$\Rightarrow \quad R_A = \dfrac{4}{3} \quad \text{... (ii)}$

$\Sigma F_x = 0 \Rightarrow H_A = 1 \quad \text{... (iii)}$





BM due this loading = m_1

From $\Sigma F_x = 0$,

$H_A = 0$

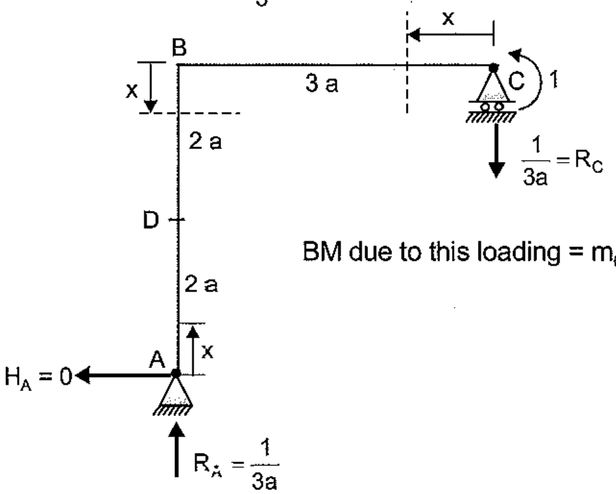
$\Sigma M_A = 0$

$\Rightarrow \quad R_C \times 3a - 1 = 0$

$\Rightarrow \quad R_C = \dfrac{1}{3a}$

From $\Sigma F_y = 0$

$R_A = \dfrac{1}{3a}$



BM due to this loading = m_θ

Segment	M	m_1	m_θ	Range of x
CB	$\dfrac{8.5\,wax}{3} - \dfrac{wx^2}{2}$	$\dfrac{4x}{3}$	$\left(1 - \dfrac{x}{3a}\right)$	0 – 3a
BD	$8.5\,wa^2 - w \times 3a \times 1.5a = 4wa^2$	$4a - x$	0	0 – 2a
AD	$2\,wax$	x	0	0 – 2a

$$\delta_1 = \int \frac{M m_1\, dx}{EI}$$

$$= \int_0^{3a} \frac{\left(\dfrac{8.5\,wa\,x}{3} - \dfrac{wx^2}{2}\right)\dfrac{4x}{3}\,dx}{EI} + \int_0^{2a} \frac{4\,wa^2\,(4a - x)\,dx}{EI} + \int_0^{2a} \frac{2wax^2\,dx}{EI}$$

$$= \frac{8.5 \times 4}{9\,EI}\,\frac{wa\,(3a)^3}{3} - \frac{4w}{6EI}\,\frac{(3a)^4}{4} + \frac{16\,wa^3\,(2a)}{6EI} - \frac{4wa^2}{EI}\,\frac{(2a)^2}{2} + \frac{2wa}{EI}\,\frac{(2a)^3}{3}$$

$$= \frac{34\,wa^4}{EI} - \frac{81\,wa^4}{6EI} + \frac{32\,wa^4}{EI} - \frac{8wa^4}{EI} + \frac{16\,wa^4}{3EI} = \frac{299}{6EI}\,wa^4$$

$$\theta = \int \frac{M m_\theta \, dx}{EI} = \int_0^{3a} \frac{\left(\frac{8.5\,wax}{3} - \frac{wx^2}{2}\right)\left(1 - \frac{x}{3a}\right)dx}{EI} + 0 + 0$$

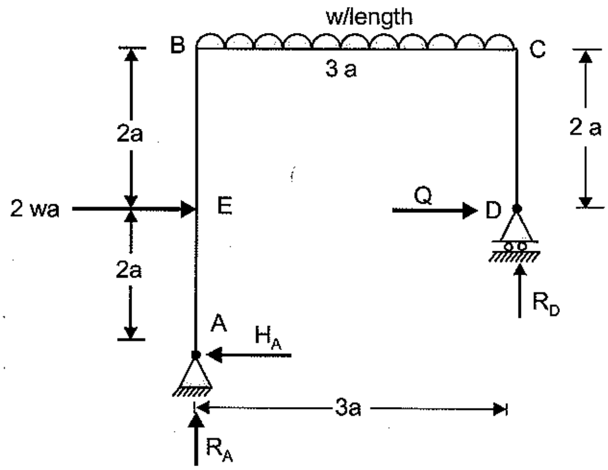
$$= \int_0^{3a} \left(\frac{8.5\,wax}{3EI} - \frac{8.5\,wax^2}{9a\,EI} - \frac{wx^2}{2EI} + \frac{wx^3}{6a\,EI}\right)dx$$

$$= \frac{8.5\,wa}{3EI}\frac{(3a)^2}{2} - \frac{8.5\,wa\,(3a)^3}{27\,a\,EI} - \frac{w\,(3a)^3}{6EI} + \frac{w(3a)^4}{24a\,EI}$$

$$\theta = \frac{25\,wa^3}{8\,EI}$$

$\Rightarrow$ Horizontal deflection at D $= \delta_1 + \theta \times 2a = \left[\frac{299}{6\,EI} + \frac{25}{8EI} \times 2\right] wa^4 = \frac{673\,wa^4}{12\,EI}$ $(\rightarrow)$

Alternatively



Apply a fictitious force Q in the direction along which the deflection is desired and let us suppose the reactions are R_A, H_A and R_D at the point A and D as shown in figure.

Finding reactions in terms of Q and W.

$$\Sigma F_x = 0 \Rightarrow H_A = 2wa + Q$$

$$\Sigma F_y = 0 \Rightarrow R_A + R_D = 3wa$$

$$\Sigma M_A = 0$$

$$R_D \times 3a - Q \times 2a - w \times 3a\frac{3a}{2} - 2wa \times 2a = 0$$

$$\Rightarrow \quad R_D = \frac{2aQ + \frac{9wa^2}{2} + 4wa^2}{3a} = \frac{2aQ + 8.5wa^2}{3a}$$

$$\therefore \quad R_D = \frac{2a}{3a}Q + \frac{17}{6a}wa^2$$

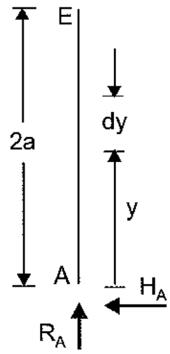
$$\Rightarrow \quad R_D = \frac{2Q}{3} + \frac{17}{6}wa$$

Also,
$$R_A = 3wa - \frac{2Q}{3} - \frac{17}{6}wa = \frac{wa}{6} - \frac{2Q}{3}$$

$$\therefore \quad R_A = \frac{wa}{6} - \frac{2Q}{3}$$

$$\text{Total strain energy (U)} = U_{AB} + U_{BC} + U_{CD} = U_{AE} + U_{EB} + U_{BC} + U_{CD}$$

U_{AE}



$$U_{AE} = \int_0^{2a} \frac{M^2 dy}{2EI}$$

$$U_{AE} = \int_0^{2a} \frac{(H_A y)^2 dy}{2EI}$$

$$\therefore \quad U_{AE} = \int_0^{2a} \frac{(2wa+Q)^2 y^2 dy}{2EI}$$

$$\therefore \quad \frac{\partial U_{AE}}{\partial Q} = \int_0^{2a} \frac{2(2wa+Q)y^2 dy}{2EI} = \frac{1}{EI}\int_0^{2a}(2wa+Q)y^2 dy$$

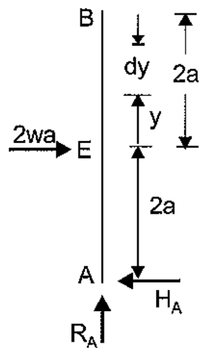
$$= \frac{2wa+Q}{EI}\left[\frac{y^3}{3}\right]_0^{2a}$$

$$= \frac{2wa+Q}{EI} \times \frac{8a^3}{3}$$

$$\therefore \quad \left.\frac{\partial U_{AE}}{\partial Q}\right|_{Q=0} = \frac{2wa}{EI} \times \frac{8a^3}{3} = \frac{16wa^4}{3\,EI}$$

U_{EB}

$$U_{EB} = \int_0^{2a} \frac{M^2 dy}{2EI}$$



$$= \int_0^{2a} \frac{[H_A(2a+y) - 2way]^2 dy}{2EI}$$

$$= \int_0^{2a} \frac{[(2wa+Q)(2a+y) - 2way]^2 dy}{2EI}$$

$$= \int_0^{2a} \frac{\left[4wa^2 + 2way + 2aQ + Qy - 2way\right]^2 dy}{2EI}$$

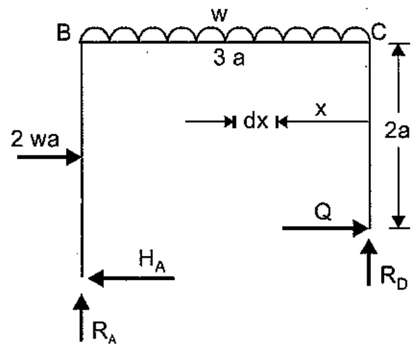
$$= \int_0^{2a} \frac{(4wa^2 + 2aQ + Qy)^2 dy}{2EI}$$

$$\therefore \quad \frac{\partial U_{EB}}{\partial Q} = \int_0^{2a} \frac{2(4wa^2 + 2aQ + Qy)(2a+y)dy}{2EI}$$

$$\frac{\partial U_{EB}}{\partial Q} = \frac{1}{EI}\int_0^{2a}(8wa^3 + 4wa^2 y + 4a^2 Q + 2aQy + 2aQy + Qy^2)dy$$

$$\left.\frac{\partial U_{EB}}{\partial Q}\right|_{Q=0} = \frac{1}{EI}\int_0^{2a}(8wa^3+4wa^2y)dy = \frac{1}{EI}\left[8wa^3y+\frac{4wa^2y^2}{2}\right]_0^{2a}$$

$$\left.\frac{\partial U_{EB}}{\partial Q}\right|_{Q=0} = \frac{24wa^4}{EI}$$



Strain energy stored in BC, $[U_{BC}]$

$$M_x = 2Qa + R_Dx - \frac{wx^2}{2}$$

$$M_x = 2Qa+\left(\frac{2Q}{3}+\frac{17}{6}wa\right)x-\frac{wx^2}{2}$$

$$M_x = 2Qa+\frac{2Qx}{3}+\frac{17}{6}wax-\frac{wx^2}{2}$$

$$\therefore \quad U_{BC} = \int_0^{3a}\frac{\left(2Qa+\frac{2Q}{3}x+\frac{17}{6}wax-\frac{wx^2}{2}\right)^2dx}{2EI}$$

$$\therefore \quad \frac{\partial U_{BC}}{\partial Q} = \int_0^{3a}\frac{2\left(2Qa+\frac{2Q}{3}x+\frac{17}{6}wax-\frac{wx^2}{2}\right)\left(2a+\frac{2}{3}x\right)dx}{2EI}$$

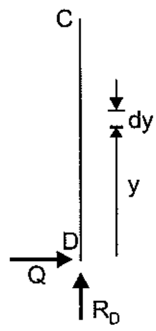
$$\left.\frac{\partial U_{BC}}{\partial Q}\right|_{Q=0} = \int_0^{3a}\frac{2}{2EI}x\left(\frac{17}{6}wax-\frac{wx^2}{2}\right)\left(2a+\frac{2}{3}x\right)dx$$

$$\left.\frac{\partial U_{BC}}{\partial Q}\right|_{Q=0} = \frac{1}{EI}\int_0^{3a}\left(\frac{17}{3}wa^2x+\frac{17}{9}wax^2-wax^2-\frac{w}{3}x^3\right)dx$$

$$\left.\frac{\partial U_{BC}}{\partial Q}\right|_{Q=0} = \frac{1}{EI}\left(\frac{17}{3}wa^2\frac{x^2}{2}+\frac{17}{9}wa\frac{x^3}{3}-\frac{wax^3}{3}-\frac{w}{3}\frac{x^4}{4}\right)_0^{3a}$$

$$= \frac{1}{EI}\left[\frac{17}{3}wa^2\frac{9a^2}{2}+\frac{17}{9}wa\frac{27a^3}{3}-\frac{wa\times 27a^3}{3}-\frac{w}{3}\times\frac{81a^4}{4}\right] = \frac{107}{4}\frac{wa^4}{EI}$$

Strain energy stored in portion CD,



$$U_{CD} = \int_0^{2a}\frac{Q^2y^2dy}{2EI} \qquad [\because M = Qy]$$

$$\frac{\partial U_{CD}}{\partial Q} = \int_0^{2a}\frac{2Qy^2dy}{2EI}$$

$$\left.\frac{\partial U_{CD}}{\partial Q}\right|_{Q=0} = 0$$

$$\therefore \quad U = U_{AE} + U_{EB} + U_{BC} + U_{CD}.$$

$$\Rightarrow \quad \frac{\partial U}{\partial Q} = \frac{\partial U_{AE}}{\partial Q}+\frac{\partial U_{EB}}{\partial a}+\frac{\partial U_{BC}}{\partial Q}+\frac{\partial U_{CD}}{\partial Q}$$

$$\Rightarrow \quad \frac{\partial U}{\partial Q} = \frac{16wa^4}{3EI} + \frac{24wa^4}{EI} + \frac{107}{4} \times \frac{wa^4}{EI} + 0$$

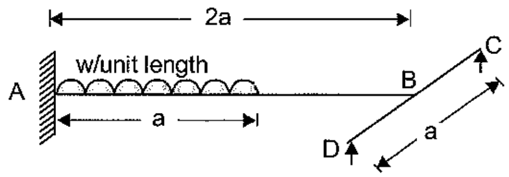
$$\therefore \quad \frac{\partial U}{\partial Q} = \frac{673}{12}\frac{wa^4}{EI} (\rightarrow)$$

Thus, deflection is $\frac{673}{12}\frac{wa^4}{EI}$ towards right at D.

Q-7: ***A cantilever beam AB of length '2a' carries a uniformly distributed load of intensity w over half the span adjoining fixed support. The beam is strengthened by supporting its free end over the mid point of transverse simply supported beam of length 'a'. EI is same for both the beams. Draw bending moment diagram for the cantilever beam.***

[15 Marks, ESE-2003]

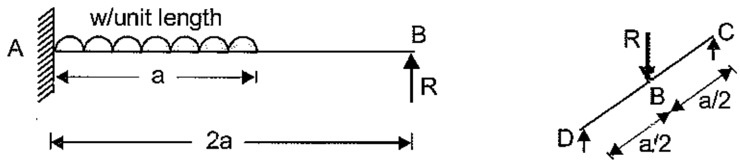
Sol:



As the cantilever beam is supported freely at its free end, there will be one unknown in the analysis of cantilever portion.

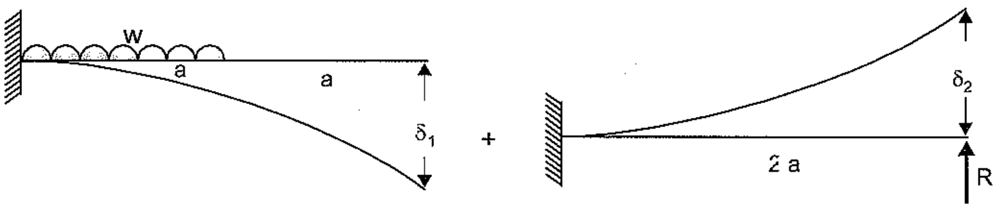
Let the reaction at common point D be 'R'.

Hence the FBD will be as shown below;



To determine R, we will use the compatibility equation. The compatibility equation is that deflection of end B in span AD will be same as deflection of point B in span DC.

Deflection at point B, for the cantilever, (Δ_B)



$$\Delta_B = \left[\frac{wa^4}{8EI} + \frac{wa^3}{6EI} \times a\right] - \frac{R(2a)^3}{3EI} (\downarrow)$$

$$= \frac{wa^4}{8EI} + \frac{wa^4}{6EI} - \frac{8Ra^3}{3EI}$$

$$\Rightarrow \quad \Delta_B = \frac{7}{24}\frac{wa^4}{EI} - \frac{8Ra^3}{3EI}$$

Deflection at B in span DC = Δ_{BSSB}

$$\Delta_{BSSB} = \frac{Ra^3}{48EI}$$

Thus from compatibility conduction

$$\Rightarrow \quad \frac{7}{24}\frac{wa^4}{EI}-\frac{8Ra^3}{3EI} = \frac{Ra^3}{48EI}$$

$$\Rightarrow \quad \frac{7}{24}\frac{wa^4}{EI} = \frac{Ra^3}{48EI}+\frac{8Ra^3}{3EI}$$

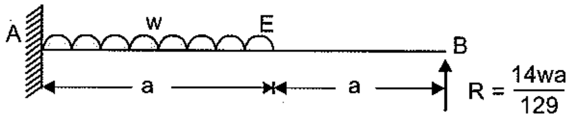
$$\frac{7}{24}\frac{wa^4}{EI} = \frac{Ra^3+128Ra^3}{48EI}=\frac{129Ra^3}{48EI}$$

$$\Rightarrow \quad \frac{7}{24}\frac{wa^4}{EI} = \frac{129Ra^3}{48EI}$$

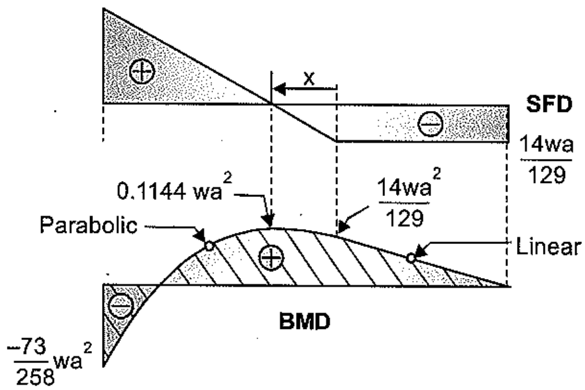
$$\Rightarrow \quad \frac{7\times 48}{24\times 129}wa = R$$

$$\Rightarrow \quad \boxed{R = \frac{14}{129}wa}$$

Hence, F.B.D for cantilever beam,



Bending moment diagram



$SF_x = 0$

$$\Rightarrow \quad -\frac{14wa}{129}+wx = 0$$

$$\Rightarrow \quad x = \frac{14a}{129}$$

$$BM_{max} = \frac{14wa}{129}\times(a+x)-\frac{wx^2}{2}=\frac{14wa}{129}\times\left(a+\frac{14a}{129}\right)-\frac{w}{2}\left(\frac{14a}{129}\right)^2$$

$$= 0.1144\ wa^2 \quad \text{(sagging)}$$

$$BM_A = \frac{14wa}{129}\times 2a-\frac{wa^2}{2}=\frac{28wa^2}{129}-\frac{wa^2}{2}$$

$$= -\frac{73}{258}wa^2 \text{ (hogging)}$$

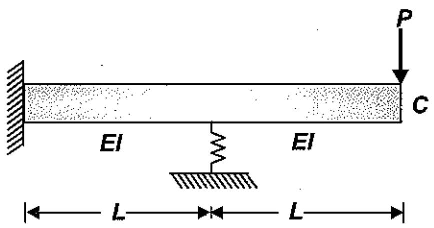
Q-8: ***Compare clearly force and displacement methods of structural analysis.***

[10 Marks, ESE-2006]

Sol:

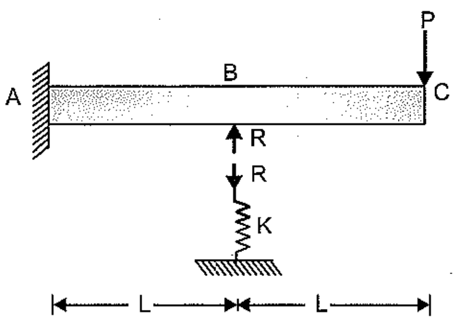
Force Method	Displacement Method
1. Unknowns are member forces or support reactions.	1. Unknowns are member end displaements like θ and δ of the joint.
2. Force-displacement equations are written and solution for unknown forces is obtained with the help of compatibility equation. Finally, member forces are found out using equilibrium equations.	2. Force-displacement equation are written and solution for unknown displacements are obtained using equilibrium equations. Finally, internal forces are found using load displacement relationship.
3. As compatibility equations are required to solve force-displacement relationship, hence this method is also called compatibility method.	3. As equilibrium equations are required to find displacement components, hence it is also called equilibrium method.
4. This method is also called flexibility method because in the compatibility equations, flexibility of member come into the picture.	4. In this method, stiffness of member comes into picture, so it is also said to be stiffness method.
5. Force method is generally used when $D_s < D_k$ where, D_s = Static determinacy D_k = Kinematic determinacy example.  $D_s = 2$, $D_K = 4$	5. Displacement method is generally used when $D_k < D_S$. example  $D_k=2$, $D_s=5$
6. Examples of force method are, • Virtual work method/unit load method • Clapeyron's three moment method • Castigliano's theorem /strain energy method. • Flexibility matrix method • Column analogy method.	6. Example of displacement method are, • Moment distribution method. • Slope deflection method • Kani's method. • Stiffness matrix method.

Q-9: ***Using the force (flexibility/comatibility) method, analyse the structure in the figure.***



[20 Marks, ESE-2006]

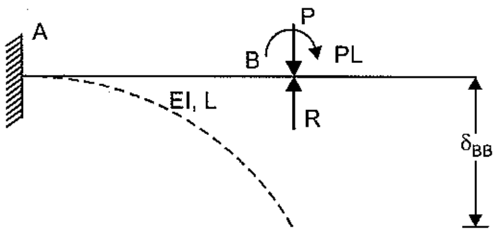
Sol:



Using compatibility condition,

Downward deflection of beam ABC at B = Compression of spring

Downward deflection of beam ABC at B (δ_{BB})



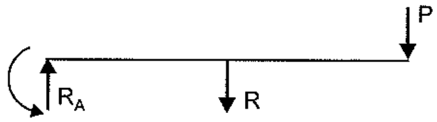
$$\delta_{BB} = \frac{(P-R)L^3}{3EI} + \frac{(PL)L^2}{2\,EI}$$

Compression of spring (δ_{BS})

$$\delta_{BS} = \frac{R}{k}$$

Hence from compatibility condition,

$$\Rightarrow \qquad \frac{(P-R)L^3}{3EI} + \frac{PL^3}{2EI} = \frac{R}{K}$$



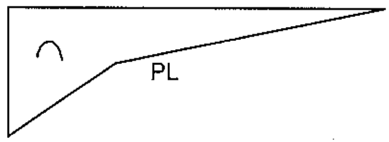
$$\frac{5PL^3}{6EI} = R\left(\frac{1}{K} + \frac{L^3}{3EI}\right)$$

$$\Rightarrow \qquad R = \frac{5PL^3}{6EI\left[\frac{1}{K} + \frac{L^3}{3EI}\right]} = \frac{5PL^3}{\left[\frac{6EI}{K} + 2L^3\right]}$$

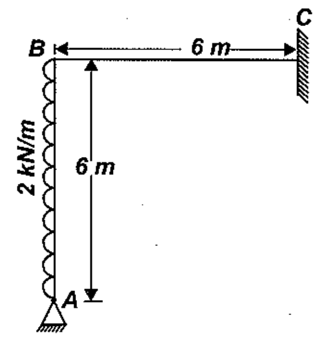
$$-M_A - R \times L + P \times 2L = 0$$

$$\Rightarrow \qquad M_A = \frac{12\,PL\frac{EI}{K} - PL^4}{2L^3 + \frac{6EI}{K}}$$

BMD

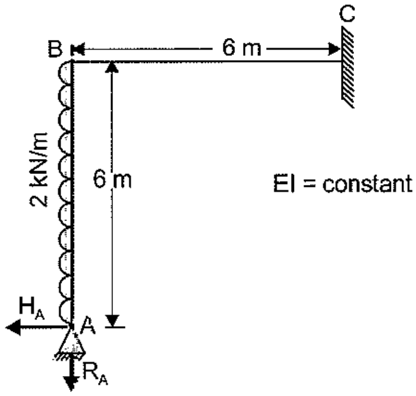


Q-10: ***Analyse the frame shown above by compatibility method. EI is constant. Draw BM and SF diagrams.***



[25 Marks, ESE-2012]

Sol:



Degree of static indeterminacy of the frame is two.

Let us assume reactions at A as redundant (i.e. R_A and H_A as redundant)

Let us remove the redundant reactions and calculate horizontal (Δ_{AH}) $(\rightarrow)$ & vertical $(\Delta_{AV})(\downarrow)$ deflections at A.

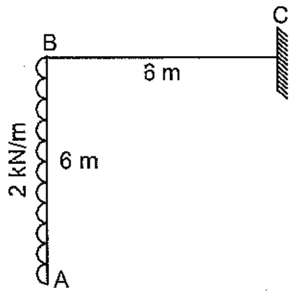
Now remove external loads and apply redundant forces R_A & H_A and find out $\delta_{AH}(\rightarrow)$ & $\delta_{AV}(\downarrow)$ deflection.

From compatibility conditions

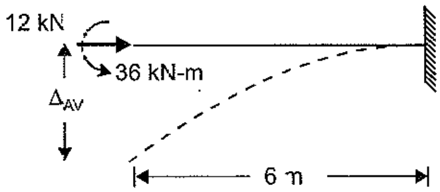
$$\Delta_{AH} + \delta_{AH} = 0$$

and
$$\Delta_{AV} + \delta_{AV} = 0$$

Calculation of Δ_{AH} and Δ_{AV}

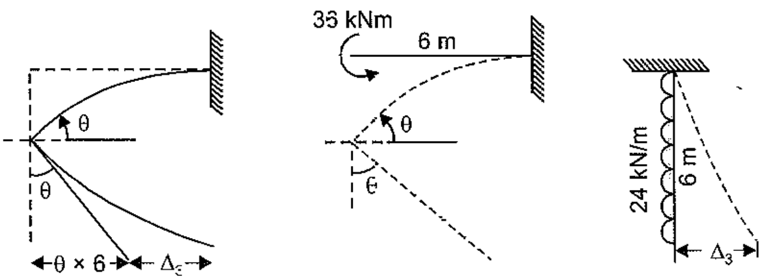


Vertical deflection of A will be equal to vertical deflection of B.



$$\boxed{\Delta_{AV} = \frac{36(6)^2}{2EI} = \frac{648}{EI}\downarrow \quad \text{EI in kNm}^2,\ \Delta_{AV} \text{ in meter}}$$

Horizontal deflection of A is given as below



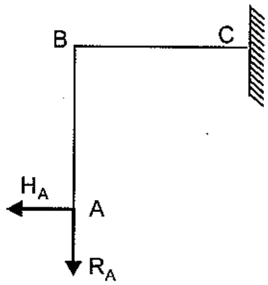
$$\theta = \frac{36\,(6)}{EI} = \frac{216}{EI}$$

$$\Delta_3 = \frac{wl^4}{8EI} = \frac{2\times(6)^4}{8\,EI} = \frac{324}{EI}$$

$$\Rightarrow \quad \Delta_{AH} = \theta\times 6+\Delta_3 = \frac{216\times 6}{EI}+\frac{324}{EI} = \frac{1620}{EI}$$

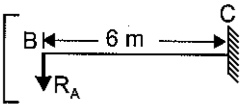
$$\boxed{\Delta_{AH} = \frac{1620}{EI}}$$

Calculation of δ_{AH} and δ_{AV}



Vertically downward deflection due to R_A

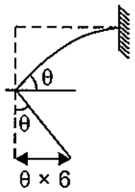
$$= \frac{R_A\,(6)^3}{3EI}(\downarrow)$$



[Note that vertically downward deflection of A due to R_A is equal to vertically ↓ deflection of B]

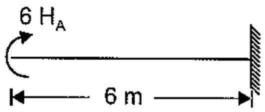
Horizontal deflection due to R_A

$$= \frac{R_A\,(6)^2}{2EI}\times 6\,(\rightarrow)$$



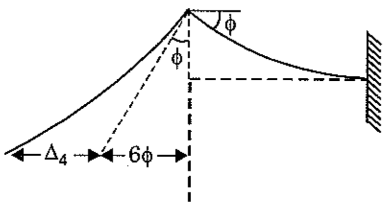
Vertical deflection of A due to H_A

Vertical deflection of A due to H_A is equal to vertical deflection of B



$$= \frac{6H_A\,(6)^2}{2EI}\uparrow$$

Horizontal deflection of A due to H_A



$$= \frac{6H_A\,(6)}{EI}\times 6+\frac{H_A\,(6)^3}{3EI} \qquad \left\{\phi = \frac{6H_A\,(6)}{EI}\right\}$$

$$= \frac{H_A\,(6)^3}{EI}+\frac{H_A\,(6)^3}{3EI} = \frac{4H_A\,(6)^3}{3EI}\;(\leftarrow)$$

Thus,

$$\delta_{AH}(\rightarrow) = \frac{R_A\,(6)^3}{2EI}-\frac{4\,(H_A)\,(6)^3}{3EI}$$

$$\delta_{AV}(\downarrow) = \frac{R_A\,(6)^3}{3EI}-\frac{H_A\,(6)^3}{2EI}$$

Now from compatibility equation

$$\Delta_{AH} + \delta_{AH} = 0$$

and $$\Delta_{AV} + \delta_{AV} = 0$$

$$\Rightarrow \quad \frac{1620}{EI} + \frac{R_A \times 108}{EI} - \frac{288 H_A}{EI} = 0$$

$$\Rightarrow \quad \boxed{108 R_A - 288 H_A + 1620 = 0} \quad \text{... (i)}$$

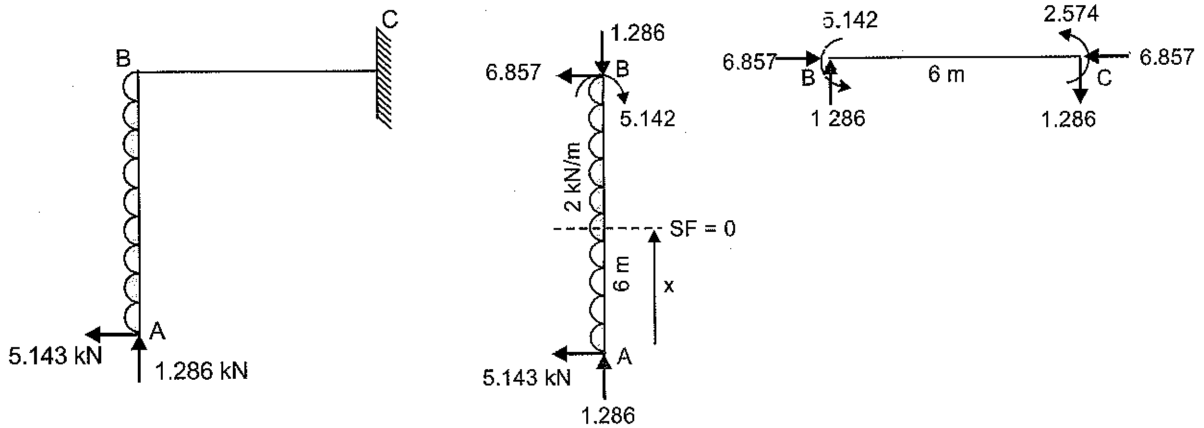
$$\frac{648}{EI} + \frac{72 R_A}{EI} - \frac{108 H_A}{EI} = 0$$

$$\Rightarrow \quad \boxed{72 R_A - 108 H_A + 648 = 0} \quad \text{... (ii)}$$

From (i) and (ii), we get

$$\Rightarrow \quad H_A = 5.143 \text{ kN}$$

$$R_A = -1.286 \text{ kN}$$



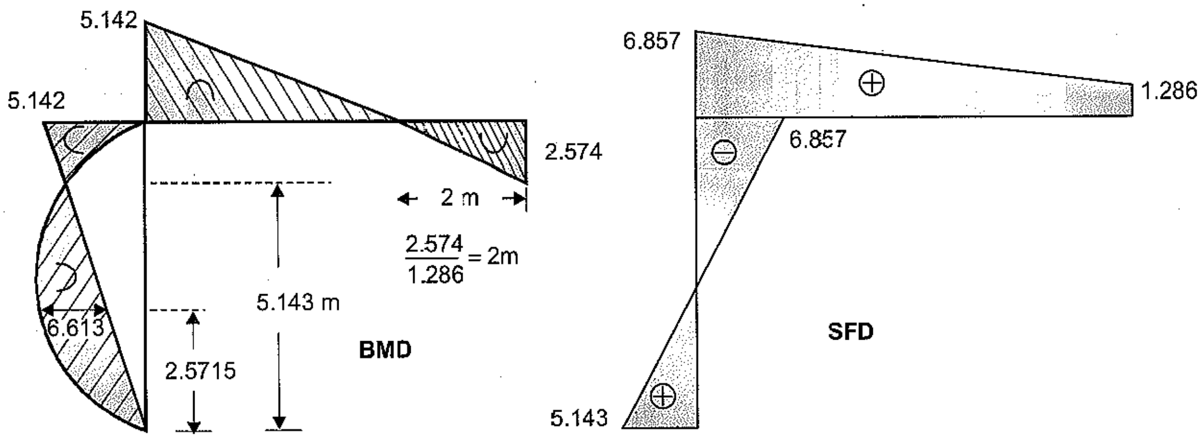
SF is zero at location x

$$5.143 - 2x = 0$$

$$\Rightarrow \quad x = \frac{5.143}{2} = 2.5715 \text{ m}$$

$$BM_x = 5.143 \times 2.5715 - \frac{2(2.5715)^2}{2} = 6.613 \text{ kNm}$$

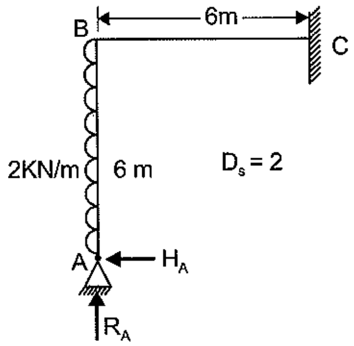
Also, $BM_x = 5.143x - \frac{2(x)^2}{2} \Rightarrow$ BM will be zero at $x = 5.143$ m



Alternative method for calculation of R_A and H_A

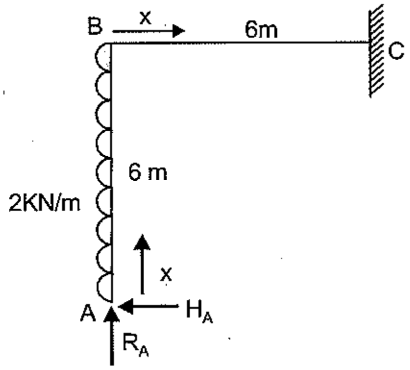
Choose R_A & H_D as Redundant

Apply Castigliano Theorem.



$$\delta_H = \frac{\int M \frac{\partial M}{\partial H_A} dx}{EI} = 0$$

$$\delta_V = \frac{\int M \frac{\partial M}{\partial R_A} dx}{EI} = 0$$



Member	Range	I	M
AB	0 – 6	I	$H_A x - x^2$
BC	0 – 6	I	$R_A x + H_A \times 6 - \frac{2\times 6^2}{2}$ $= R_A x + 6H_A - 36$

$\frac{\partial M}{\partial H_A}$	$\frac{\partial M}{\partial R_A}$
x	0
6	x

$$\delta_H = \int_0^6 \frac{(H_A x - x^2) x\, dx}{EI} + \int_0^6 \frac{(R_A x + 6H_A - 36) 6\, dx}{EI} = 0$$

$$\left[H_A \frac{x^3}{3} - \frac{x^4}{4} + R_A \cdot 6 \cdot \frac{x^2}{2} + 6^2 H_A x - 216x \right]_0^6 = 0$$

$$H_A \left(\frac{6^3}{3} + 6^3 \right) + R_A \frac{6^3}{2} - \frac{6^4}{4} - 216 \times 6 = 0$$

$$288H_A + 108\, R_A - 324 - 1296 = 0$$

$$288\, H_A + 108\, R_A - 1620 = 0$$

$$24\, H_A + 9R_A - 135 = 0$$

$$8H_A + 3R_A - 45 = 0 \quad \text{... (1)}$$

$$\delta_v = \int_0^6 \frac{(R_A x + 6H_A - 36) x\, dx}{EI} = 0$$

$$\left[R_A \frac{x^3}{3} + 6 \cdot H_A \frac{x^2}{2} - 36 \frac{x^2}{2} \right]_0^6 = 0$$

$$R_A \frac{6^3}{3} + H_A \frac{6^3}{2} - 36 \times \frac{6^2}{2} = 0$$

$$108\, H_A + 72\, R_A - 648 = 0$$

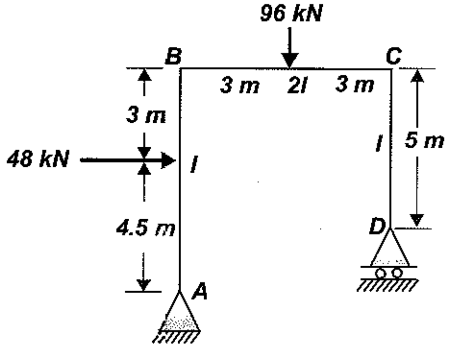
$$9H_A + 6\, R_A - 54 = 0$$

$$3H_A + 2R_A - 18 = 0 \quad \text{... (2)}$$

$$H_A = \frac{36}{7} \quad R_A = \frac{9}{7} = 1.286 \text{ kN}$$

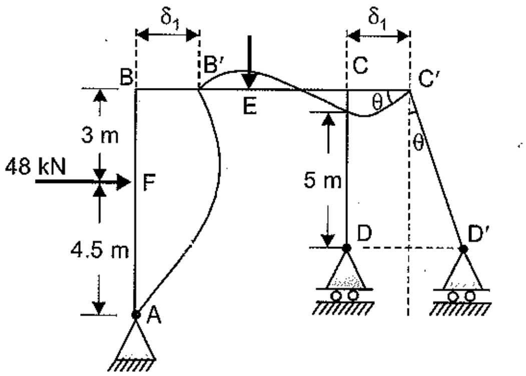
$$\boxed{H_A = 5.143 \text{ kN}} \qquad \boxed{R_A = 1.286 \text{ kN}}$$

Q-11: ***For the rigid frame shown in figure below, find by unit load method, the slopes and deflections at A, C.***

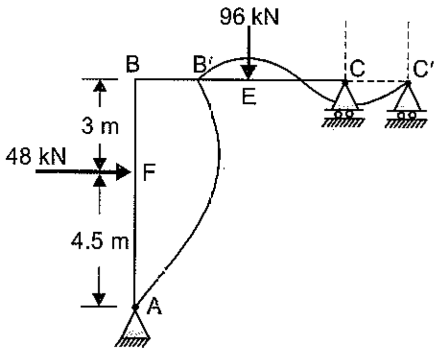


[20 Marks, ESE-2013]

Sol:



Since support D is a roller support, there will be no horizontal reaction at D, hence no BM in the member CD. Thus the structure can be modified as



Thus as support A is a hinged support, deflection at A = 0

Also, from the modified diagram, we can say that vertical deflection at C = 0

Thus we have to calculate slope at A & slope and horizontal deflection at C.

To calculate horizontal deflection & slope at C & slope at A

Using unit load method,

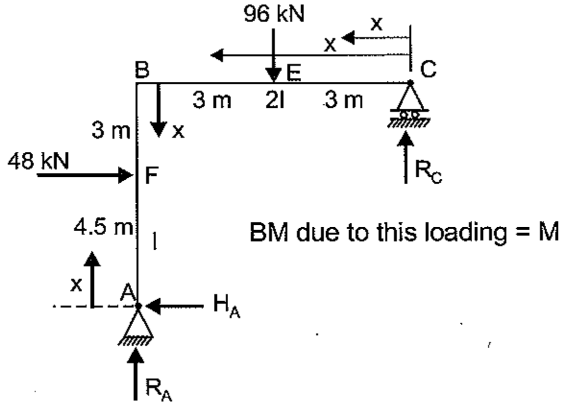
$$\delta_{HA} = \frac{Mm_1 dx}{EI} \; ; \; \theta_c = \frac{Mm_{\theta_c} dx}{EI} \; ; \; \theta_A = \frac{Mm_{\theta_A} dx}{EI}$$

where, M = BM due to external loading

m_1 = BM when external loads are removed and a horizontal unit load is applied at C

m_{θ_c} = BM when external loads are removed & a unit anticlockwise couple is applied at C.

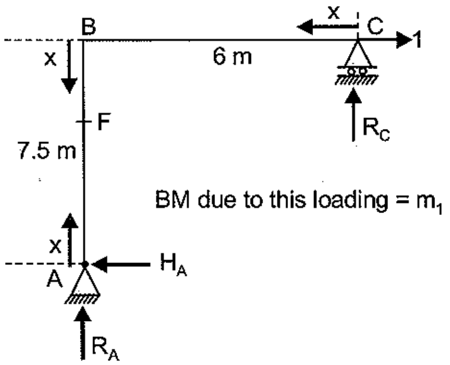
m_{θ_A} = BM when external loads are removed and a unit clockwise couple is applied at A



$H_A = 48$ kN {from $\Sigma F_x = 0$}

$$R_A = \frac{96}{2} - \frac{48 \times 4.5}{6} = 12 \text{ kN}$$

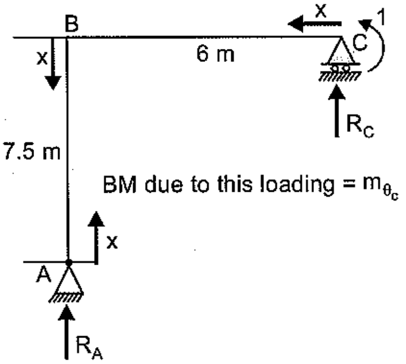
$R_C = 96 - R_A = 96 - 12 = 84$ kN



$H_A = 1$ {from $\Sigma F_x = 0$}

$$R_A = -\frac{1 \times 7.5}{6} = -1.25$$

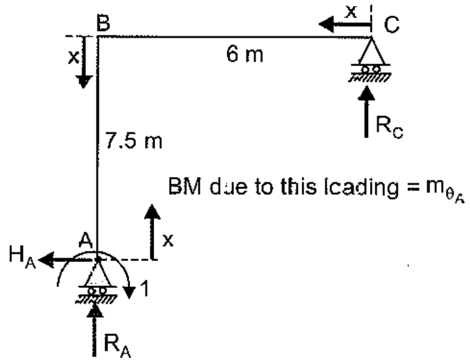
$R_C = -R_A = 1.25$



$H_A = 0$ {from $\Sigma F_x = 0$}

$$R_A = +\frac{1}{6} = 0.167$$

$R_C = -R_A = -0.167$



$$H_A = 0 \qquad \{\text{from } \Sigma F_x = 0\}$$

$$R_A = -\frac{1}{6} = -0.167$$

$$R_C = -R_A = +0.167$$

Segment	I	M	m_1	m_{θ_c}	m_{θ_A}	Range of x
CE	2I	84x	1.25x	$\left(1-\frac{x}{6}\right)$	$\frac{x}{6}$	0–3
EB	2I	84x–96(x–3)				3–6
BF	I	84×6 – 96×3 = 216	1.25×6–x =(7.5–x)	$1-\frac{x}{6}\times 6 = 0$	$\frac{x}{6}\times 6 = 1$	0–3
AF	I	48x	x	0	1	0–4.5

$$\delta_1 = \frac{Mm_1 d_x}{EI} = \int_0^3 \frac{84x(1.25x)dx}{2EI} + \int_3^6 \frac{(288-12x)(1.25x)dx}{2EI} + \int_0^3 \frac{216(7.5-x)dx}{EI} + \int_0^{4.5} \frac{48x^2 dx}{EI}$$

$$= \frac{472.5+1957.5+3888+1458}{EI}$$

$$\boxed{\delta_1 = \frac{7776}{EI}}$$

$$\theta_c = \frac{Mm_{\theta_c}dx}{EI} = \int_0^3 \frac{84x(1-\frac{x}{6})dx}{E(2I)} + \int_3^6 \frac{\{84x-96(x-3)\}\{1-\frac{x}{6}\}dx}{E(2I)} + 0 + 0 = \frac{126+90}{EI}$$

$$\boxed{\theta_c = \frac{216}{EI}}$$

$$\theta_A = \frac{Mm_{\theta_A}dx}{EI} = \int_0^3 \frac{84x\times\frac{x}{6}dx}{E(2I)} + \int_3^6 \frac{[(84x-96(x-3)]}{E(2I)}\times\frac{x}{6}dx + \int_0^3 \frac{216\times 1dx}{EI} + \int_0^{4.5} \frac{48x\times 1dx}{EI}$$

$$= \frac{63+261+648+486}{EI}$$

$$\boxed{\theta_A = \frac{1458}{EI}}$$

Thus the final values are:-

$$\boxed{\theta_A = \frac{1458}{EI}} \; (\circlearrowright)$$

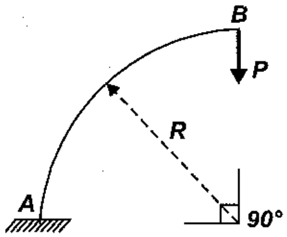
$$\boxed{\delta_A = 0}$$

$$\boxed{\theta_c = \frac{216}{EI}} \; (\circlearrowleft)$$

$$\boxed{\delta_{HC} = \frac{7776}{EI}} \; (\rightarrow)$$

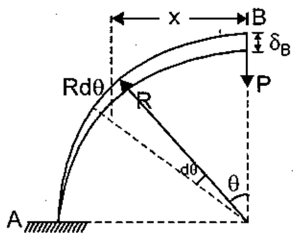
$$\boxed{\delta_{VC} = 0}$$

Q-12: ***Determine the vertical deflection at the free end of a circular cantilever frame shown in figure using the unit load method. Take EI = Constant.***



[5 Marks, ESE-2014]

Sol:



By strain energy method, we know that deflection is given by

$$\delta_B = \int \frac{M.\frac{\partial M}{\partial P}}{EI}.dx \qquad \text{... (i)}$$

Bending moment at any distance 'x' from point B is

$$M = P.x \text{ (consider hogging as +ve)}$$

From figure, we can write

$$x = R\sin\theta$$

$$\therefore \quad M = P.R\sin\theta \qquad \text{... (ii)}$$

Partially differentiating equation (ii), with respect to P, we get

$$\frac{\partial M}{\partial P} = R\sin\theta \qquad \text{... (iii)}$$

We can also write $\quad dx = R.d\theta \qquad$ ... (iv)

Putting values of M, $\frac{\partial M}{\partial P}$ and dx in equation (i), we get

$$\delta_B = \int_0^{\pi/2} \frac{P.R\sin\theta.R\sin\theta.Rd\theta}{EI}$$

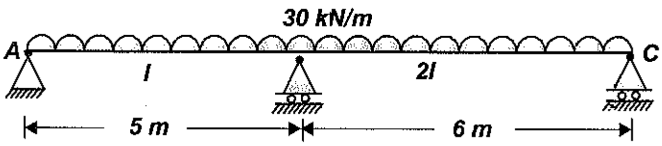
$$\therefore \quad \delta_B = \frac{PR^3}{EI}\int_0^{\pi/2} \sin^2\theta d\theta$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2}\left(\frac{1-\cos 2\theta}{2}\right) = \frac{PR^3}{2EI}\Big|_0^{\pi/2}\left(\theta - \frac{\sin 2\theta}{2}\right)$$

After integrating, we get

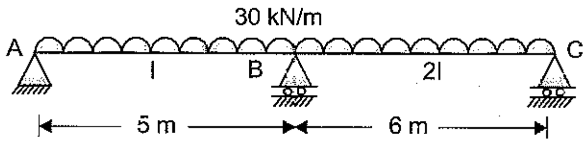
$$\delta_B = \frac{\pi PR^3}{4EI} \downarrow$$

Q-13: ***Analyze the beam shown in figure using the strain energy method and draw bending moment diagram and shear force diagram.***



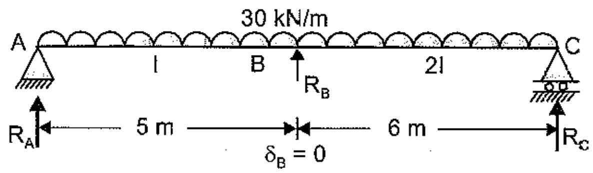
[15 Marks, ESE-2014]

Sol:



It is a statically indeterminate structure with degree of indeterminacy equal to 1.

Let us assume the reaction at support B be R_B. Therefore, we can also draw the above structure as shown below, with the condition that deflection at B should be zero.



Support reaction at A, $$R_A = \frac{30\times 11}{2} - \frac{6R_B}{11}$$

$$= 165 - \frac{6R_B}{11} \quad \text{...(i)}$$

Support reaction at C, $$R_C = \frac{30\times 11}{2} - \frac{5R_B}{11}$$

$$= 165 - \frac{5R_B}{11} \quad \text{...(ii)}$$

Moment in span AB at any distance 'x' measured from A

$$M_{AB} = \left(165 - \frac{6R_B}{11}\right)x - 30.x.\frac{x}{2}$$

$$= \left(165 - \frac{6R_B}{11}\right)x - 15x^2 \quad \text{... (iii)}$$

$$\Rightarrow \quad \frac{\partial M_{AB}}{\partial R_B} = -\frac{6x}{11} \quad \text{...(iv)}$$

Similarly, moment in span CB at any distance 'x' measured from C

$$M_{CB} = \left(165 - \frac{5R_B}{11}\right)x - 30.x.\frac{x}{2}$$

$$= \left(165 - \frac{5R_B}{11}\right)x - 15x^2 \quad \text{...(v)}$$

$$\Rightarrow \quad \frac{\partial M_{CB}}{\partial R_B} = -\frac{5x}{11} \quad \text{... (vi)}$$

$\therefore$ Total strain energy (v) in beam,

$$U = \int_0^5 \frac{M_{AB}^2 dx}{2EI} + \int_0^6 \frac{M_{CB}^2}{2E(2I)} dx$$

$\therefore$ Deflection at, B (δ_B), $$\delta_B = \frac{\partial u}{\partial R_B} = \int_0^5 \frac{2M_{AB}\dfrac{\partial M_{AB}}{\partial R_B}}{2EI} dx + \int_0^6 \frac{2M_{CB}\dfrac{\partial M_{CB}}{\partial R_B}}{4EI} dx$$

Putting value from equation (iii), (iv), (v) and (vi), we get

$$\delta_B = \int_0^5 \frac{\left[\left(165 - \frac{6R_B}{11}\right)x - 15x^2\right].\left(-\frac{6x}{11}\right)dx}{EI}$$

$$+ \int_0^6 \frac{\left[\left(165 - \frac{5R_B}{11}\right)x - 15x^2\right]\left(-\frac{5x}{11}\right)dx}{2EI}$$

Now, as there is a support at B, therefore deflection at B must be zero

i.e., $\delta_B = 0$

$$\Rightarrow \quad \int_0^5 \frac{\left[\left(165 - \frac{6R_B}{11}\right)x - 15x^2\right]\left(-\frac{6x}{11}\right)dx}{EI} + \int_0^6 \frac{\left[\left(165 - \frac{5R_B}{11}\right)x - 15x^2\right]\left(-\frac{5x}{11}\right)dx}{2EI} = 0$$

After solving this equation, we get

$$R_B = 205.025 \text{ kN}$$

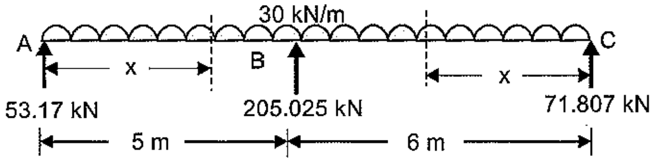
From equation (i), $$R_A = 165 - \frac{6R_B}{11} = 165 - \frac{6 \times 205.025}{11}$$

$$= 53.17 \text{ kN}$$

From equation (ii), $$R_B = 165 - \frac{5R_B}{11} = 165 - \frac{5 \times 205.025}{11}$$

$$= 71.807 \text{ kN}$$

Shear force diagram



Portion AB('x' measured from A)

$$\text{S.F.} = 53.17 - 30x$$

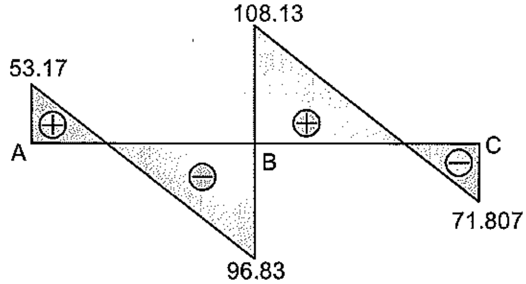
$\therefore$ $$\text{S.F (just left of B)} = 53.17 - 30 \times 5 = -96.83 \text{ kN}$$

Portion CB ('x' measured from C)

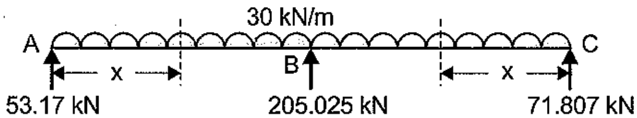
$$\text{SF} = -71.807 + 30x$$

$\therefore$ $$\text{S.F. (just right of B)} = -71.807 + 30 \times 6 = 108.13 \text{ kN}$$

S.F.D. will be



Bending moment diagram



Portion AB('x' measured from A)

$$M = 53.17x - 30.x.\frac{x}{2} = 53.17x - 15x^2$$

$\therefore$ Bending moment at B

$$M_B = 53.17 \times 5 - 15 \times 5^2 = -109.15 \text{ kN-m}$$

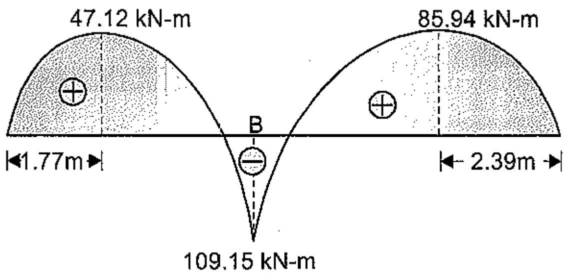
For maximum M, $$\frac{dM}{dx} = 0 \Rightarrow x = 1.77 \text{ m}, M_{max} = 47.12 \text{ kN-m}$$

Portion CB('x' measured from C)

$$M = 71.807x - 15x^2$$

For maximum M, $$\frac{dM}{dx} = 0 \Rightarrow x = 2.39\text{m}, M_{max} = 85.94 \text{ kN-m}$$

BMD will be



Q-14: ***Distinguish between Flexibility method and Stiffness method used for analysis of structures.***

[4 Marks, ESE-2018]

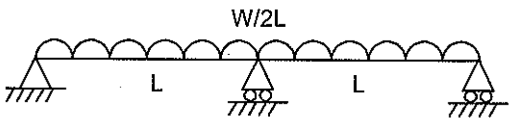
Sol:

	Flexibility Method		**Stiffness Method**
1.	This method is also known as force method of analysis	1.	This method is also known as displacement method of analysis
2.	Redundant forces are the unknowns	2.	Degree of freedoms are the unknowns
3.	Used when degree of static indeterminacy is less than degree of kinematic indeterminacy	3.	used when $D_K < D_S$
4.	Compatability equations are used	4.	Joint equilibrium equations are used
5.	For example,	5.	For example,
	(a) Consistent deformation method		(a) Slope deflection method
	(b) Virtual work method		(b) Moment distribution method
	(c) Flexibility matrix method		(c) Stiffness matrix method
	(d) Energy method		

Q-15: ***A beam of uniform cross-section and of length 2L is simply supported by rigid supports at its ends and by an elastic prop at its centre. If the prop deflects by an amount λ times the load it carries and if the beam carries a total uniformly distributed load of W, find the load carried by the prop if EI is constant throughout the length of beam.***

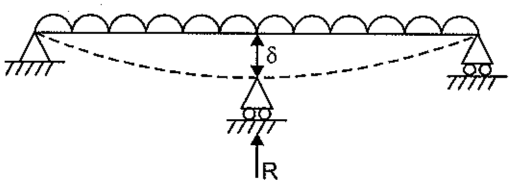
[12 Marks, ESE-2019]

Sol: **Given:**



EI = Constant

Total uniformly distributed load = W



Let the deflection at elastic prop be δ, then according to question,

$$\delta = \lambda.R$$

From compatibility condition:

$$\frac{5.\frac{W}{2L}\times(2L)^4}{384EI} - \lambda.R = \frac{R.(2L)^3}{48EI}$$

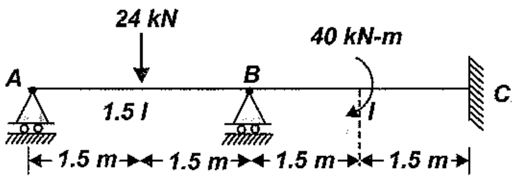
or,
$$\frac{5WL^3}{48EI} = R\left(\frac{L^3}{6EI}+\lambda\right)$$

$$R = \frac{\frac{5WL^3}{48EI}}{\left(\frac{L^3}{6EI}+\lambda\right)} = \frac{5WL^3}{8(L^3+6\lambda EI)}$$

CHAPTER 3

DISPLACEMENT METHOD OF ANALYSIS: (SLOPE DEFLECTION METHOD)

Q-1: ***Analyse the beam loaded as shown in fig. using the slope deflection method. Portion AB has a second moment of area as 1.5 I and BC has this value as I. Draw the bending moment and shear force diagrams.***

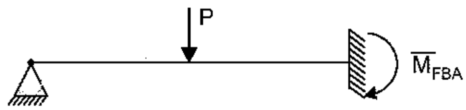


[20 Marks, ESE-1995]

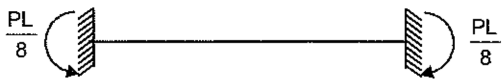
Sol: **Fixed end moments**

$$\overline{M}_{FBA} = +\frac{3PL}{16} = \frac{+3\times24\times3}{16} = +\frac{27}{2} = +13.5 \text{ kN-m}$$

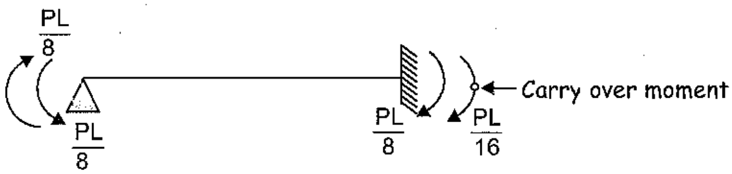
Conceptual Background:

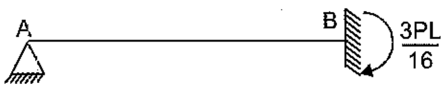


We know that



Hence





Hence, $M_{FBA} = +\frac{3PL}{16}$

$$M_{FBC} = +\frac{M_0}{4} = +\frac{40}{4} = 10 \text{ kNm}$$

$$M_{FCB} = 10 \text{ kNm}$$

Hence, $\overline{M}_{FBA} = +13.5 \text{ kNm}, M_{FBC}$

$= +10 \text{ kNm}, M_{FCB} = +10 \text{ kNm},$

Slope-deflection equations :

$$M_{BA} = \bar{M}_{FBA} + \frac{3EI}{L}\left(\theta_B - \frac{\delta}{l}\right) = 13.5 + \frac{3EI(1.5I)}{3}(\theta_B)$$

$$\Rightarrow \quad M_{BA} = 13.5 + 1.5EI\theta_B \qquad \text{... (1)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\{2\theta_B + \theta_c - 3\delta/l\} = 10 + \frac{2EI}{3}\{2\theta_B\}$$

$$\Rightarrow \quad M_{BC} = 10 + \frac{4EI\theta_B}{3} \qquad \text{... (2)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left\{2\theta_C + \theta_B - \frac{3\delta}{l}\right\} = 10 + \frac{2EI}{3}\{\theta_B\}$$

$$\Rightarrow \quad M_{CB} = 10 + \frac{2EI\theta_B}{3} \qquad \text{... (3)}$$

Joint equilibrium equation

$$M_{BA} + M_{BC} = 0$$

$$13.5 + 1.5EI\theta_B + 10 + \frac{4EI\theta_B}{3} = 0$$

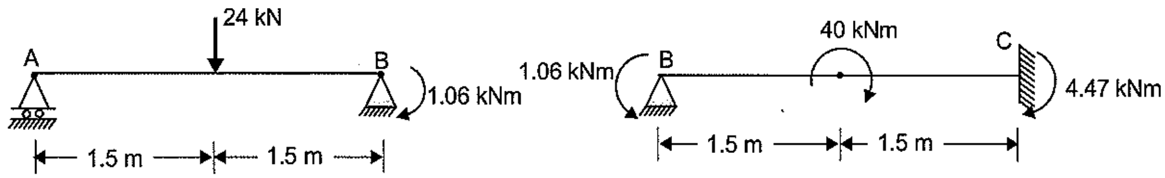
$$\Rightarrow \quad -23.5 = \left(1.5 + \frac{4}{3}\right) EI\theta_B$$

$$\Rightarrow \quad EI\theta_B = -8.294$$

By putting this value in Eqn. (1), Eqn. (2) and Eqn. (3)

$$M_{BA} = 1.06 \text{ kNm}$$
$$M_{BC} = -1.06 \text{ kNm}$$
$$M_{CB} = +4.47 \text{ kNm}$$

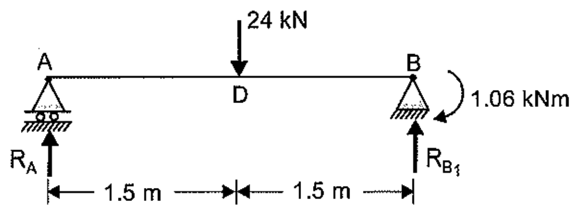
F.B.D



Bending moment diagram and SFD :

For Part AB

$$R_A + R_{B1} = 24$$
$$\Sigma M_B = 0$$
$$R_A \times 3 = 24 \times 1.5 - 1.06$$
$$\Rightarrow \quad R_A = 11.646 \text{ kN}$$
and $$R_{B1} = 12.354 \text{ kN}$$



From A to D $\quad M = R_A \times x = 11.646x$ kNm

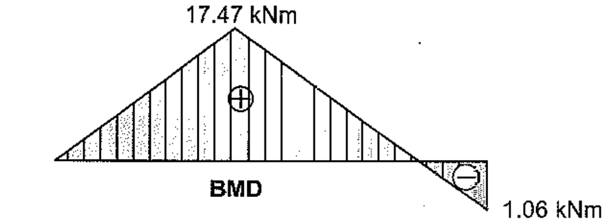
$$M|_{x=1.5m} = (11.646 \times 1.5) = 17.469 \text{ kNm}$$

From B to D (x from right side)

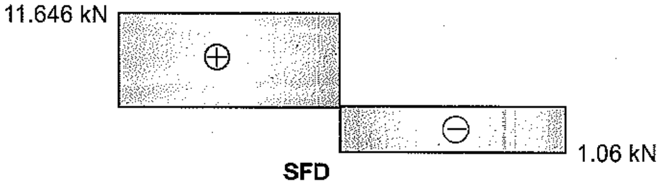
$$M = R_{B_1} \times x - 1.06$$

$$M|_{x=1.5m} = 12.354 \times 15 - 1.06$$
$$= 18.531 - 1.06 = 17.471 \text{ kNm}$$

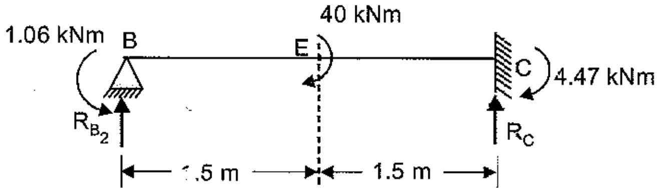
$M|_{x=0}$ = –1.06 kNm. Hence the BMD will be



Shear force diagram will be as shown below



For Part BC



$$\Sigma M_C = 0$$

$$R_{B2} \times 3 - 1.06 + 40 + 4.47 = 0$$

$$\Rightarrow \quad R_{B_2} = -\frac{-40 - 4.47 + 1.06}{3} = -14.47 \text{kN}$$

Also, $$R_{B_2} + R_C = 0$$

$$\therefore \quad R_C = +14.47 \text{ kN}$$

BMD (Between B and E)

$$M = R_{B_2} x - 1.06$$

$$M|_{x=0} = -1.06$$
$$M|_{x=1.5m} = -14.47 \times 1.5 - 1.06 = -22.765 \text{ kNm}$$

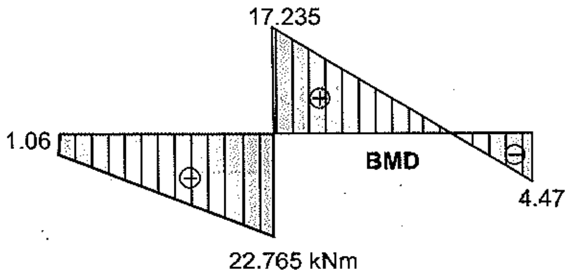
BMD (Between C and E) (x from right side)

$$M = R_C x - 4.47$$

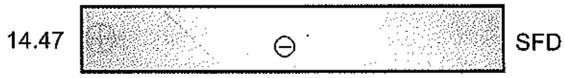
$$M|_{x=0} = -4.47 \text{ kNm}$$

$$M|_{x=1.5\,m} = +14.47 \times 1.5 - 4.47 = 17.235 \text{ kNm}$$

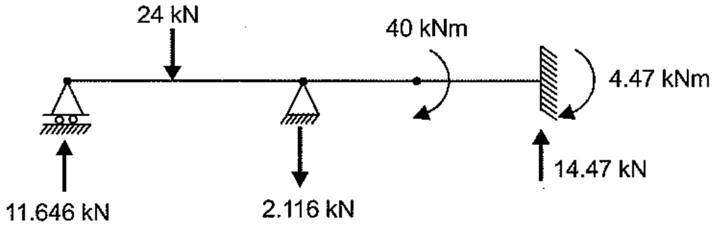
Hence the BMD for part BC can be known as follows



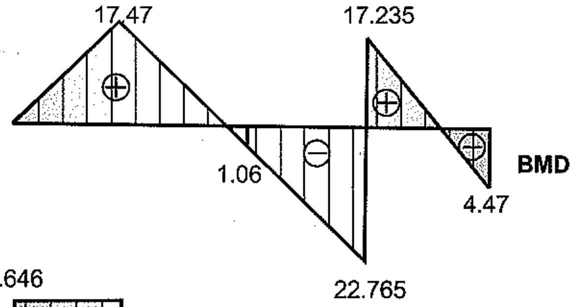
SFD for part BC is shown as below

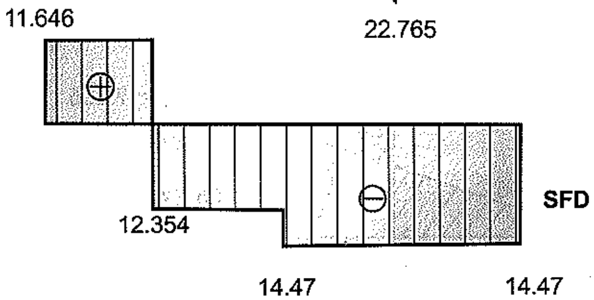


By combining both part,

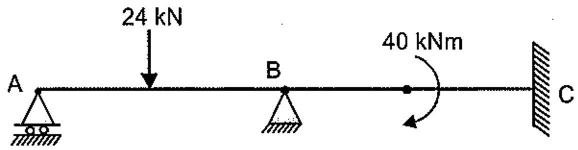


$$\left[\begin{aligned} R_B &= R_{B_1} + R_{B_2} \\ &= 12.354 - 14.47 \\ &= -2.116 \text{ kN} \end{aligned}\right]$$

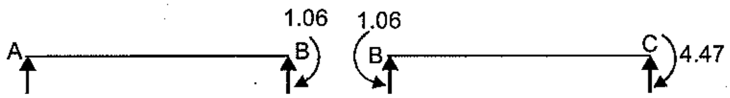


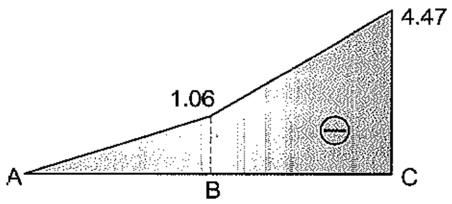


Alternative method for drawing BMD:



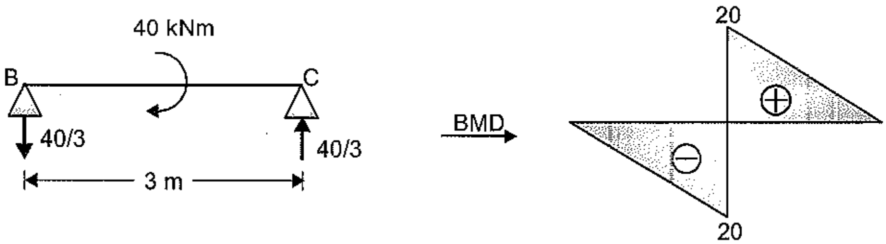
First draw the end moment diagram





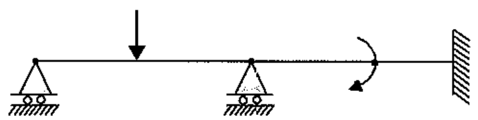
After plotting end moments, Free moment diagram (assuming simply supported condition with loading) is superimposed.

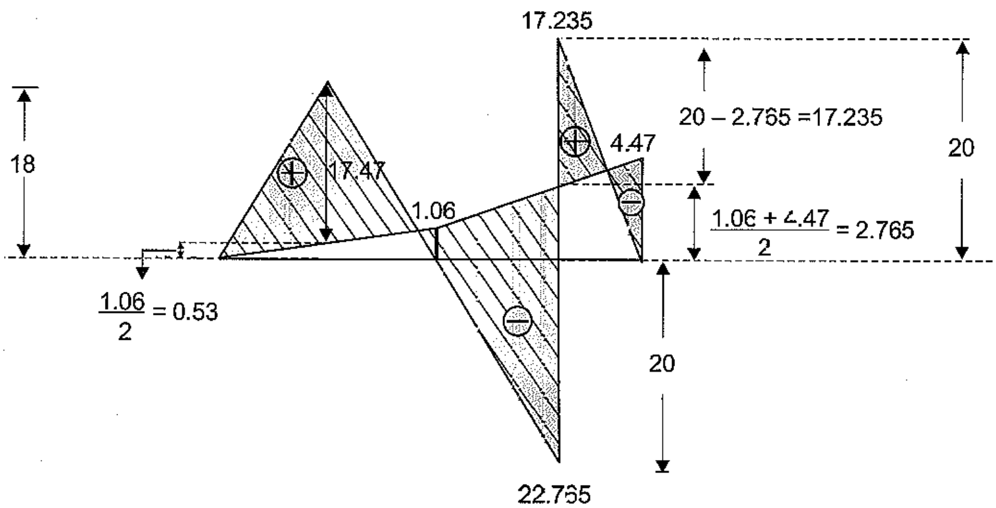
Free moment diagram for AB



Free moment diagram for BC

After superimposition, the BMD will look like as shown below



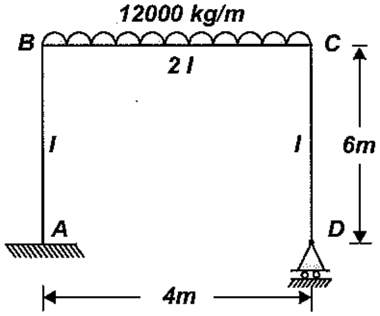


Note:

- The Bending moment diagram from both the method is same only the way of representation is different.
- In solid mechanics generally we deal with deterministic structures. Hence we can directly calculate the unknown reactions and if someone can calculate reactions easily then the 1st method will become easy to daw BMD or SFD.

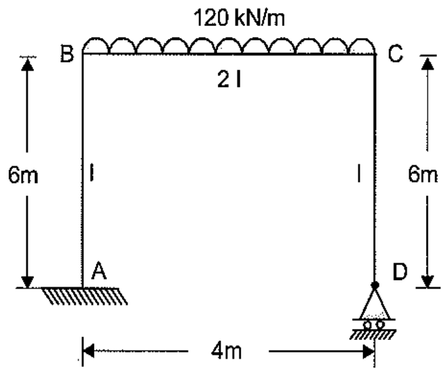
- In structure analysis part, we generally deal with indeterminate structures where we can't find the reactions directly by using equilibrium equations. For that we have to estimate the end moments first. Hence in these cases the fixed end moment method i.e. 2nd one will be easy.
- For shear force, we have to determine reactions as in previous method, and after that we can draw S.F.D. which will be some as in previous method.

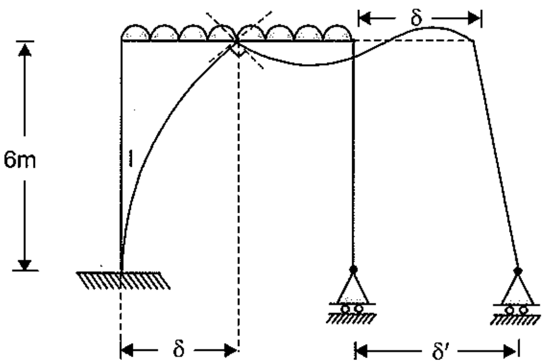
Q-2: ***Analyse the frame shown in fig using slope Deflection Method and draw bending moment and shear force diagrams. End A is fixed, joints B and C are rigid, end D is on a rocker and roller bearing.***



[20 Marks, ESE-1997]

Sol:





Since the supports are unsymmetrical, hence the beam will sway.

Hence we assume the deflected shape of frame as shown in figure above

Fixed end moment calculation

$$M_{FAB} = M_{FBA} = 0 \quad \text{(Because AB part of frame has no force on it)}$$

$$M_{FBC} = \frac{-Wl^2}{12} = \frac{-120\times 4^2}{12} = -160\text{kNm}$$

$$M_{FCB} = +160 \text{ kNm}$$

$$M_{FCD} = M_{FDC} = 0$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\delta}{L}\right]$$

$$M_{AB} = \frac{2EI}{6}\left\{\theta_B - \frac{3\delta}{6}\right\} \quad \text{.... (A)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - \frac{3\delta}{L}\right]$$

$$M_{BA} = \frac{2EI}{6}\left\{2\theta_B - \frac{3\delta}{6}\right\}$$

$$M_{BA} = \frac{2EI\theta_B}{3} - \frac{EI\delta}{6} \quad \text{... (B)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\delta}{L}\right] \qquad [\because \delta = 0]$$

$$= -160 + \frac{2E(2I)}{4}\{2\theta_B + \theta_C\}$$

$$M_{BC} = -160 + 2EI\theta_B + EI\theta_C \quad \text{... (C)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left[2\theta_C + \theta_B - \frac{3\delta}{L}\right]$$

$$= 160 + \frac{2E(2I)}{4}\{2\theta_C + \theta_B\}$$

$$M_{CB} = 160 + 2EI\theta_C + EI\theta_B \quad \text{... (D)}$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left[2\theta_C + \theta_D - \frac{3(\delta - \delta')}{L}\right]$$

$$M_{CD} = \frac{2EI}{6}\left[2\theta_C + \theta_D - \frac{3(\delta - \delta')}{6}\right]$$

$$M_{CD} = \frac{2EI\theta_C}{3} + \frac{EI\theta_D}{3} - \frac{EI(\delta - \delta')}{6} \quad \text{... (E)}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left[2\theta_D + \theta_C - \frac{3(\delta - \delta')}{L}\right]$$

$$M_{DC} = \frac{2EI}{6}\left[2\theta_D + \theta_C - \frac{3(\delta - \delta')}{6}\right]$$

$$M_{DC} = \frac{2EI\theta_D}{3} + \frac{EI\theta_C}{3} - \frac{EI(\delta - \delta')}{6} \quad \text{... (F)}$$

Joint equilibrium equation $\sum M_B = 0$

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow \quad \frac{2EI\theta_B}{3} - \frac{EI\delta}{6} - 160 + 2EI\theta_B + EI\theta_C = 0$$

$$\Rightarrow \quad \frac{8}{3}EI\theta_B + EI\theta_C - \frac{EI\delta}{6} = 160$$

$$\Rightarrow \quad 16EI\theta_B + 6EI\theta_C - EI\delta = 960$$

$$16\theta_B + 6\theta_C - \delta = \frac{960}{EI} \quad \text{.....(i)}$$

Joint equilibrium equation $\sum M_C = 0$

$$M_{CB} + M_{CD} = 0$$

$$160 + 2EI\theta_C + EI\theta_B + \frac{2EI\theta_C}{3} + \frac{EI\theta_C}{3} - \frac{EI(\delta - \delta')}{6} = 0$$

$$\Rightarrow \quad 960+6EI\theta_B+16EI\theta_C+2EI\theta_D-EI(\delta-\delta') = 0$$

$$6\theta_B+16\theta_C+2\theta_D-(\delta-\delta') = -\frac{960}{EI} \quad .....(ii)$$

Joint equilibrium equation M_{DC} = 0

$$\frac{2}{3}EI\theta_D+\frac{EI\theta_C}{3}-\frac{EI(\delta-\delta')}{6} = 0$$

$$4\theta_D+2\theta_C-(\delta-\delta') = 0 \quad .... (iii)$$

Shear equation

Since the point D is roller supported,

Hence, $H_D = 0$

and according to shear equation, $H_A = 0$

$$H_D = \frac{M_{CD}+M_{DC}}{L} = 0$$

$$\Rightarrow \quad \frac{2}{3}EI\theta_C+\frac{EI\theta_D}{3}-\frac{EI(\delta-\delta')}{6}+\frac{2}{3}EI\theta_D+\frac{EI\theta_C}{3}-\frac{EI(\delta-\delta')}{6} = 0$$

$$\Rightarrow \quad EI\theta_C+EI\theta_D-\frac{EI(\delta-\delta')}{3} = 0$$

$$\Rightarrow \quad 3\theta_C+3\theta_D-\delta+\delta' = 0 \quad ... (iv)$$

$$H_A = \frac{M_{AB}+M_{BA}}{L} = 0$$

$$\Rightarrow \quad \frac{EI\theta_B}{3}-\frac{EI\delta}{6}+\frac{2EI\theta_B}{3}-\frac{EI\delta}{6} = 0$$

$$\Rightarrow \quad EI\theta_B-\frac{EI\delta}{3} = 0$$

$$\Rightarrow \quad 3\theta_B = \delta \quad ... (v)$$

Substituting the value of δ from equation (v) in equation (iv)

$$3\theta_C+3\theta_D-3\theta_B+\delta' = 0$$

$$\Rightarrow \quad \delta' = 3\theta_B-3\theta_C-3\theta_D$$

Substituting the value of δ and δ' in equation (i), (ii) and (iii).

$$16\theta_B+6\theta_C-3\theta_B = \frac{960}{EI}$$

$$\Rightarrow \quad 13\theta_B+6\theta_C+0.\theta_D = \frac{960}{EI} \quad ... (\alpha)$$

$$6\theta_B+16\theta_C+2\theta_D-3\theta_B+3\theta_B-3\theta_C-3\theta_D = \frac{960}{EI}$$

$$\Rightarrow \quad 6\theta_B+13\theta_C-\theta_D = -\frac{960}{EI} \quad ... (\beta)$$

$$2\theta_C+4\theta_D-3\theta_B+3\theta_B-3\theta_C-3\theta_D = 0$$

$$\Rightarrow \quad 0.\theta_B-\theta_C+\theta_D = 0 \quad ... (\gamma)$$

By solving three equation α, β and γ we get

$$\theta_B = \frac{144}{EI}, \theta_C = \frac{-152}{EI} \text{ and } \theta_D = \frac{-152}{EI}$$

$$\therefore \qquad \delta = 3\theta_B = \frac{432}{EI}$$

$$\delta' = 3(\theta_B - \theta_C - \theta_D) = 3\frac{(144+152\times 2)}{EI} = \frac{1344}{EI}$$

Substituting these value in equation (A) to (F) we get the end moments.

$$M_{AB} = \frac{EI\theta_B}{3} - \frac{EI\delta}{6} = \frac{144}{3} - \frac{432}{6} = -24$$

$$M_{BA} = \frac{2EI\theta_B}{3} - \frac{EI\delta}{6} = \frac{2\times 144}{3} - \frac{432}{6} = 24$$

$$M_{BC} = -160 + 2EI\theta_B + EI\theta_C = -160 + 2\times 144 + (-152) = -24$$

$$M_{CB} = 160 + 2EI\theta_C + EI\theta_B = 160 + 144 - 152\times 2 = 0$$

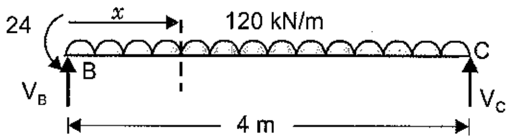
$$M_{CD} = \frac{2}{3}EI\theta_C + \frac{EI\theta_D}{3} - \frac{EI(\delta-\delta')}{6}$$

$$= \frac{2}{3}\times(-152) + \frac{-152}{3} - \frac{432}{6} + \frac{1344}{6} = 0$$

and $\qquad M_{DC} = 0$

Calculation of vertical reaction V_A and V_D

Consider, BC



$$\Sigma M_c = 0$$

$$\Rightarrow \qquad V_B \times 4 = 24 + 120 \times 4 \times 2$$

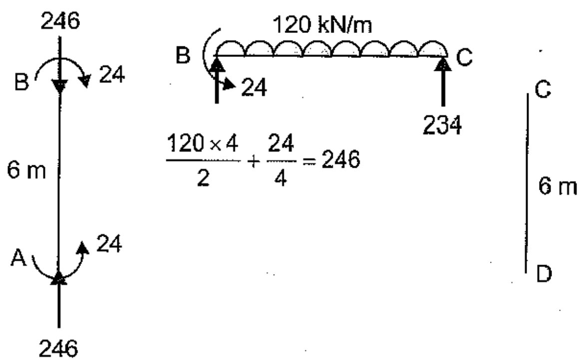
$$\Rightarrow \qquad V_B = 6 + 240 = 246 \text{ kN}(\uparrow)$$

and $\qquad V_C = 120 \times 4 - 246 = 234 \text{ kN}(\uparrow)$

Hence reaction at A, $\qquad V_A = 246 \text{ kN}(\uparrow)$

and reaction at D, $\qquad V_D = 234 \text{ kN}(\uparrow)$

The freebody diagram will look like as shown below:



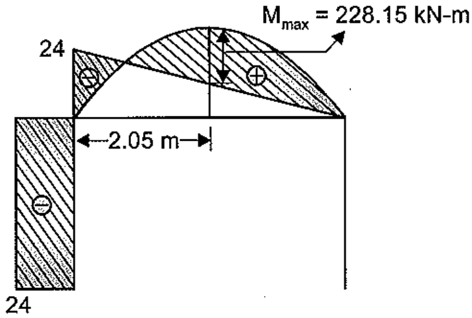
Max BM will occur in BC where SF = 0

$$\Rightarrow \qquad 246 - 120\,x = 0$$

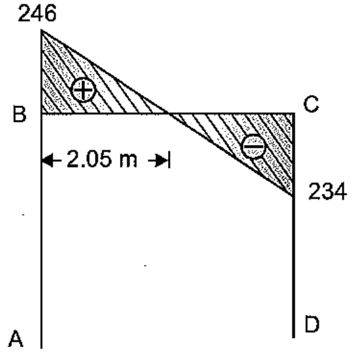
$$\Rightarrow \qquad x = 2.05 \text{ m}$$

$$M_{max} = -24 + 246 \times 2.05 - \frac{120\,(2.05)^2}{2} = 228.15 \text{ kNm}$$

Hence the B.M. diagram is as follows:

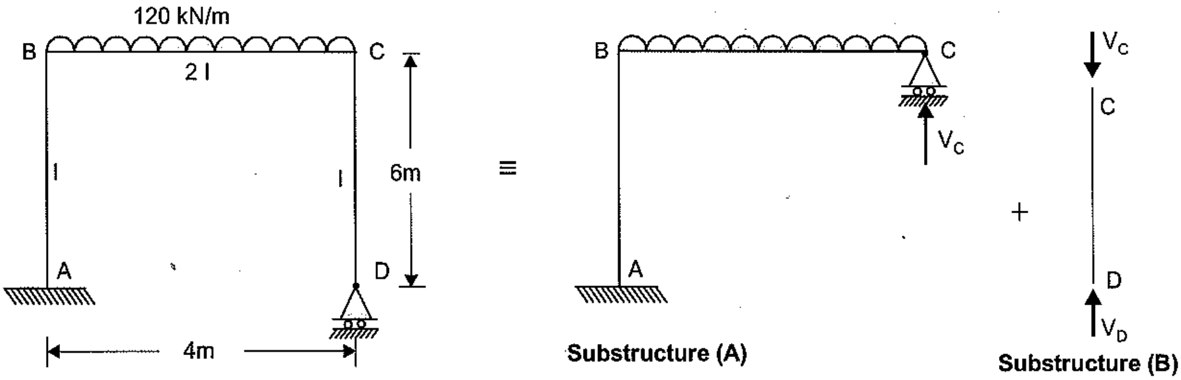


SFD

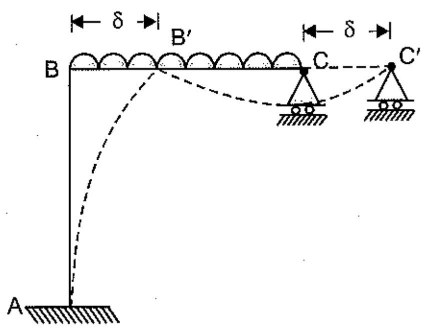


Alternative Method

For analysis purpose, the structure can be divided into two parts i.e., substructure A & substructure B.



Analysis of substructure A



Fixed end moments

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = -160\text{kN-m}$$

$$M_{FCB} = +160\text{ KN-m}$$

$$M_{AB} = \frac{EI\theta_B}{3} - \frac{EI\delta}{6} \quad \text{... (A)}$$

$$M_{BA} = \frac{2}{3}EI\theta_B - \frac{EI\delta}{6} \quad \text{... (B)}$$

$$M_{BC} = \left(M_{FBC} - \frac{M_{FCB}}{2}\right) + \frac{3EI}{L}\left(\theta_B - \frac{\delta}{L}\right) = -160 - \frac{160}{2} + \frac{3E(2I)}{4}(\theta_B)$$

$$= -160 - 80 + \frac{3EI}{2}(\theta_B)$$

$$M_{BC} = -240 + \frac{3EI\theta_B}{2} \quad \text{... (C)}$$

Equillibrium equation at joint B is given by

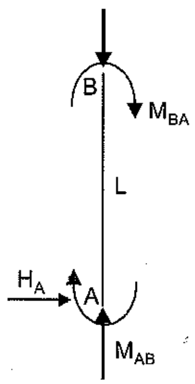
$$M_{BA} + M_{BC} = 0$$

$$\frac{2}{3}EI\theta_B - \frac{EI\delta}{6} + \left(-240 + \frac{3EI\theta_B}{2}\right) = 0$$

$$\Rightarrow \quad \frac{13}{6}EI\theta_B - 240 - \frac{EI\delta}{6} = 0$$

$$\Rightarrow \quad 13\theta_B - \delta = \frac{1440}{EI} \quad \text{... }(\alpha)$$

Shear equation:



$$H_A = 0$$

$$\frac{M_{AB} + M_{BA}}{L} = 0$$

$$\Rightarrow \quad \frac{EI\theta_B}{3} - \frac{EI\delta}{6} + \frac{2}{3}EI\theta_B - \frac{EI\delta}{6} = 0$$

$$\Rightarrow \quad EI\theta_B = \frac{EI\delta}{3}$$

$$\therefore \quad \delta = 3\theta_B \quad \text{... }(\beta)$$

Hence by solving above two equations α and β;

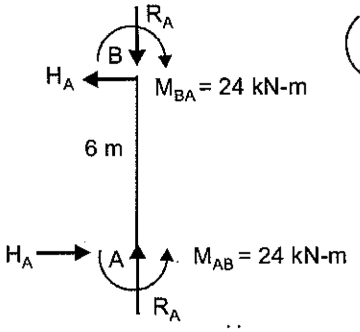
$$\theta_B = \frac{144}{EI} \text{ and } \delta = \frac{432}{EI}$$

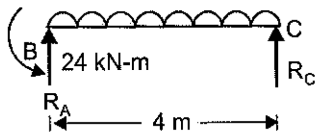
By putting these value in slope deflection equations A, B and C we have;

$$M_{BA} = \frac{EI\theta_B}{3} - \frac{EI\delta}{6} = \frac{144}{3} - \frac{432}{6} = -24\text{ kN-m}$$

$$M_{BA} = \frac{2}{3}EI\theta_B - \frac{EI\delta}{6} = \frac{288}{3} - \frac{432}{6} = 24\text{ kN-m}$$

$$M_{BC} = -24\text{ kN-m}$$





Finding reaction at C,

$$\Sigma M_B = 0$$

$$\Rightarrow \quad R_C \times 4 + 24 - 120 \times \frac{4^2}{2} = 0$$

$$\Rightarrow \quad R_C \times 4 = 960 - 24$$

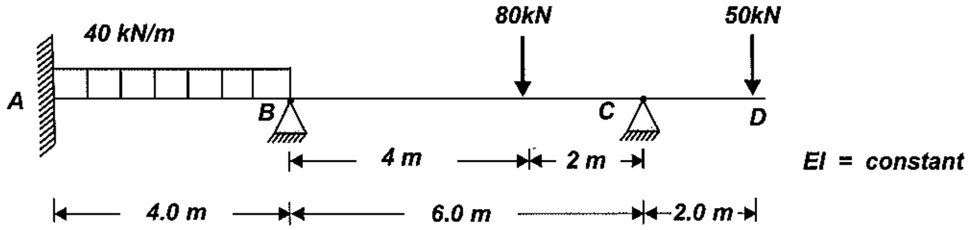
$$\Rightarrow \quad R_C = 234\,\text{kN}\,(\uparrow)$$

Hence $\quad R_A = 120 \times 4 - 234 = 480 - 234 = 246$ kN ($\uparrow$)

Thus BMD and SFD will be drawn as done before.

The substructure B, carries only axial force. Hence BMD and SFD for substructure B will not exist.

Q-3: ***Analyse the continuous beam shown in figure by the slope deflection method. Draw the shearing force and bending moment diagrams. Sketch also the deflected shape.***



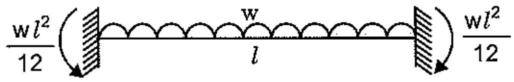
[20 Marks, ESE-1998]

Sol: **Fixed end moments**

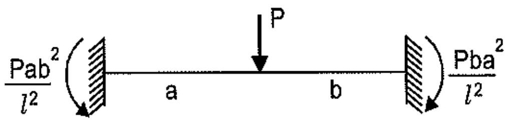
$$M_{FAB} = \frac{-wl^2}{12} = -\frac{-40 \times 4^2}{12} = -\frac{-40 \times 16}{12}$$

$$= -53.33 \text{ kNm}$$

$$M_{FBA} = +53.33 \text{ kNm}$$



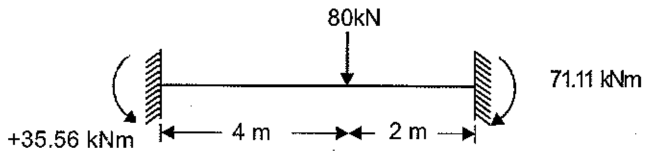
$$M_{FBC} = \frac{-80 \times 4 \times 2^2}{36} = -35.56 \text{ kN/m}$$

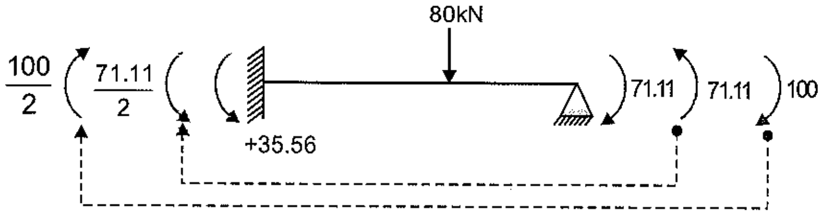


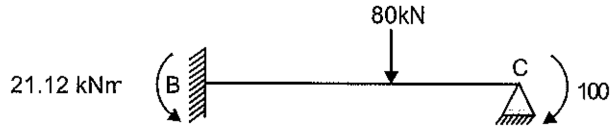
$$M_{FCB} = \frac{80 \times 2 \times 4^2}{36} = 71.11 \text{ kN/m}$$

Determination of $\overline{M}_{FBC}$ to apply the formula $M_{BC} = \overline{M}_{FBC} + \frac{3EI}{L}\left(\theta_B - \frac{\delta}{L}\right)$

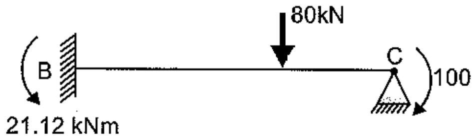
Since the beam is having an overhang, the final moment at C will be 50 × 2 = 100 kNm (clockwise)







Thus the fixed end moment to be used in equation $M_{BC} = \bar{M}_{FBC} + \frac{3EI}{l}\left(\theta_B \frac{\delta}{l}\right)$ is $\overline{M_{FBC}}$ = –21.12 kNm



$\bar{M}_{FBC} = -21.12 \text{kN-m}$

Slope – deflection equations:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$M_{AB} = -53.33 + \frac{2EI\theta_B}{4} \quad \text{... (i)}$$

$$M_{BA} = 53.33 + \frac{2EI}{4}(2\theta_B) = 53.33 + \frac{4EI\theta_B}{4}$$

$$M_{BA} = 53.33 + EI\theta_B \quad \text{... (ii)}$$

and
$$M_{BC} = \overline{M_{FBC}} + \frac{3EI}{l}\left\{\theta_B - \frac{\delta}{l}\right\}$$

⇒
$$M_{BC} = -21.12 + \frac{3EI}{6}(\theta_B) \quad \text{... (iii)}$$

Joint equilibrium,

At B,
$$M_{BA} + M_{BC} = 0$$

$$53.33 + EI\theta_B - 21.12 + \frac{EI\theta_B}{2} = 0$$

$$\frac{3EI\theta_B}{2} = -32.21$$

⇒
$$\theta_B = \left(-\frac{21.47}{EI}\right)$$

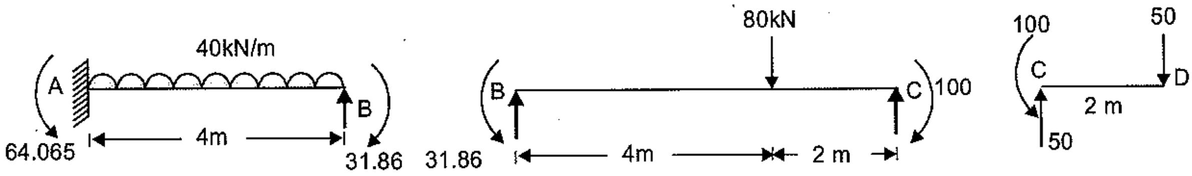
Substituting the value of θ_B in the slope deflection equations (i), (ii) and (iii) we have

$$M_{AB} = -53.33 + \frac{2EI}{4} \times \left(-\frac{21.47}{EI}\right) = -64.065 \text{ kNm}$$

$$M_{BA} = 53.33 + EI \times \left(-\frac{21.47}{EI}\right) = 31.86 \text{ kNm}$$

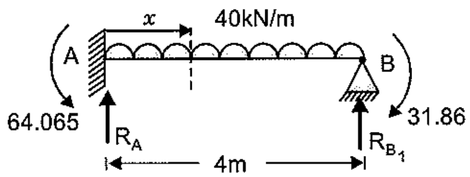
$$M_{BC} = -31.86 \text{ kNm}$$

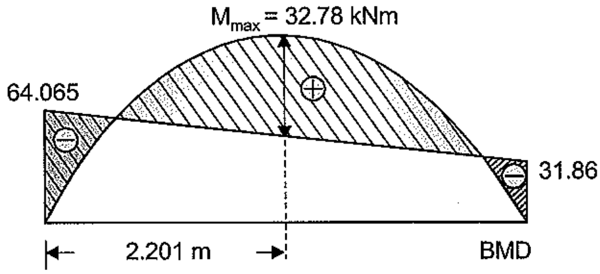
The free body diagram for the beam is as shown below



Bending moment diagram for different parts

Part AB





$$\Sigma M_B = 0 \Rightarrow R_A \times 4 - 64.065 - \frac{40 \times 4^2}{2} + 31.86 = 0$$

$\Rightarrow$ $R_A = 88.05$ kN

Also, $R_{B_1} + R_A = 40 \times 4$

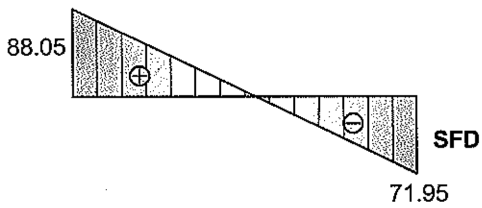
$\Rightarrow$ $R_{B_1} = 71.95$ kN

Let SF be zero at a distance x from A

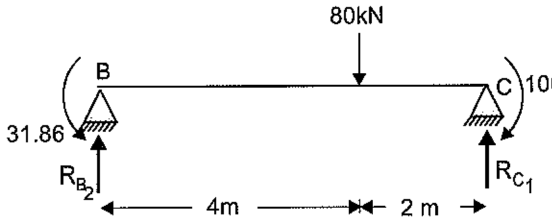
$\Rightarrow$ $88.05 - 40x = 0$

$\Rightarrow$ $x = 2.201$ m

Thus, $M_{max} = 88.05\,(2.201) - 64.065 - \frac{-40\,(2.201)^2}{2} = 32.78$ kNm



Part BC



$$\Sigma M_C = 0$$

$$R_{B_2} \times 6 - 31.86 - 80 \times 2 + 100 = 0$$

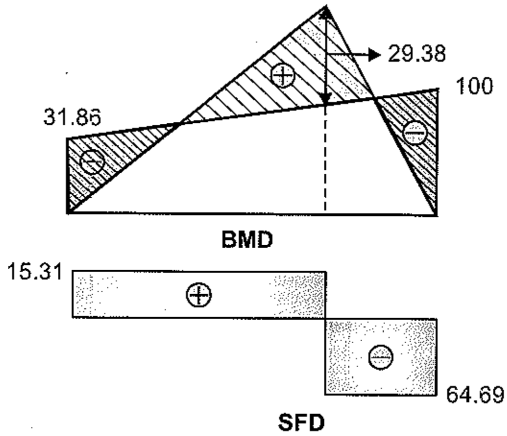
$\Rightarrow$ $R_{B_2} = 15.31$ kN

$$R_{B_2} + R_{C_1} = 80$$

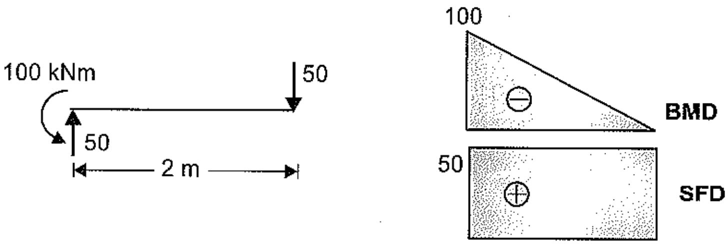
$\Rightarrow$ $R_{C_1} = 80 - 15.31 = 64.69$ kN

BM under the load

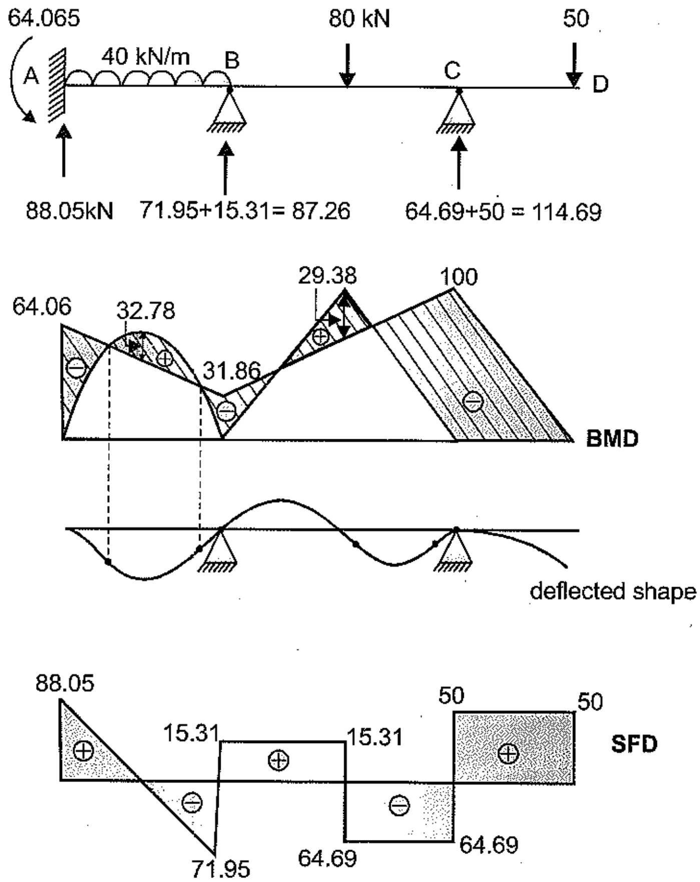
$$M_{max} = R_{B_2} \times 4 - 31.86 = 15.31 \times 4 - 31.86 = 29.38 \text{ kNm}$$



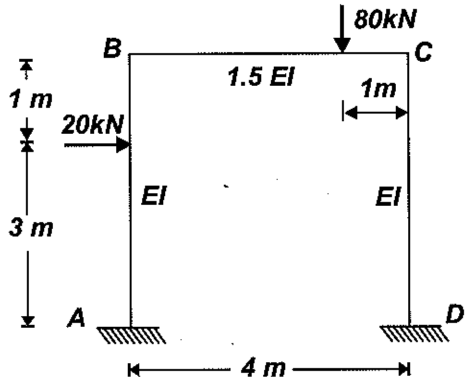
Part CD



BMD, SFD and deflected shape of whole structure:

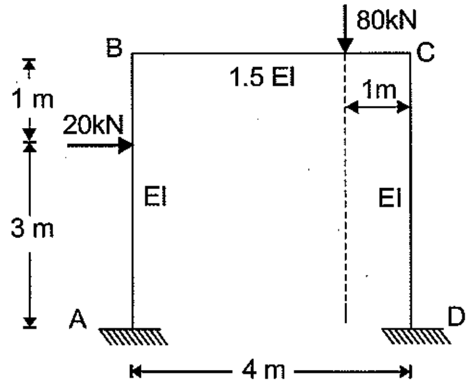


Q-4: ***Analyse the rigid jointed frame shown in the figure using SLOPE DEFLECTION method. Find all the reaction components at the supports 'A' and 'D'. Draw B.M. and S.F. diagrams for all the members.***



[30 Marks, ESE-2005]

Sol:



The unknown joint displacements are θ_B, θ_C and $\delta_B = \delta_C = \delta$

Fixed end moments

$$M_{FAB} = \frac{-20\times 3\times 1^2}{4^2} = -3.75\,\text{kN-m}$$

$$M_{FBA} = \frac{20\times 1\times 3^2}{4^2} = 11.25\,\text{kN-m}$$

$$M_{FBC} = \frac{-80\times 3\times 1^2}{4^2} = -15\,\text{kN-m}$$

$$M_{FCB} = \frac{80\times 1\times 3^2}{4^2} = 45\,\text{kN-m}$$

$$M_{FCD} = 0 = M_{FDC}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$$M_{AB} = -3.75 + \frac{2EI}{4}\left(\theta_B - \frac{3\delta}{4}\right)$$

$$\boxed{M_{AB} = -3.75 + \frac{EI\theta_B}{2} - \frac{3EI\delta}{8}} \quad \text{... (i)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

$$M_{BA} = 11.25 + \frac{2EI}{4}\left(2\theta_B - \frac{3\delta}{4}\right)$$

$$\boxed{M_{BA} = 11.25 + EI\theta_B - \frac{3EI\delta}{8}} \quad \text{... (ii)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right)$$

$$M_{BC} = -15 + \frac{2E(1.5I)}{4}(2\theta_B + \theta_C)$$

$$\boxed{M_{BC} = -15 + \frac{3EI\theta_B}{2} + \frac{3EI\theta_C}{4}} \quad \text{... (iii)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$$

$$M_{CB} = 45 + \frac{2E(1.5I)}{4}(2\theta_C + \theta_B)$$

$$\boxed{M_{CB} = 45 + \frac{3EI\theta_C}{2} + \frac{3EI\theta_B}{4}} \quad \text{... (iv)}$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\delta}{4}\right)$$

$$M_{CD} = \frac{2EI}{4}\left(2\theta_C - \frac{3\delta}{4}\right)$$

$$\boxed{M_{CD} = EI\theta_C - \frac{3EI\delta}{8}} \quad \text{... (v)}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\delta}{L}\right)$$

$\therefore$
$$M_{DC} = \frac{2EI}{4}\left(\theta_C - \frac{3\delta}{4}\right)$$

$$\boxed{M_{DC} = \frac{EI\theta_C}{2} - \frac{3}{8}EI\delta} \quad \text{.... (vi)}$$

Joint equilibrium equation:

$$M_{BA} + M_{BC} = 0$$

$\Rightarrow$
$$11.25 + EI\theta_B - \frac{3EI\delta}{8} + 1.5EI\theta_B + 0.75EI\theta_C - 15 = 0$$

$\Rightarrow$
$$\boxed{0.75EI\theta_C + 2.5EI\theta_B - \frac{3EI\delta}{8} = 3.75} \quad \text{... (A)}$$

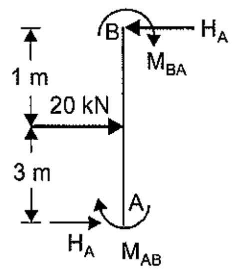
Joint equilibrium equation

$$M_{CB} + M_{CD} = 0$$

$$45 + 1.5EI\theta_C + 0.75EI\theta_B + EI\theta_C - \frac{3}{8}EI\delta = 0$$

$$\boxed{2.5EI\theta_C + 0.75EI\theta_B - \frac{3EI\delta}{8} = -45} \quad \text{... (B)}$$

For horizontal equilibrium, (or, shear equation)



$$H_A + H_D + 20 = 0$$

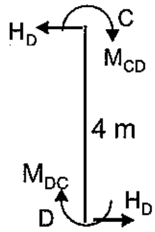
Finding (H_A):

$$\Sigma M_B = 0$$

$$\Rightarrow \quad M_{BA} + M_{AB} = 20 \times 1 + H_A \times 4$$

$$\Rightarrow \quad H_A = \frac{M_{AB} + M_{BA} - 20}{4}$$

Finding (H_D):



$$\Sigma M_C = 0$$

$$H_D \times 4 = M_{DC} + M_{CD}$$

$$\Rightarrow \quad \boxed{H_D = \frac{M_{DC} + M_{CD}}{4}}$$

Shear equations :

$$H_A + H_D + 20 = 0$$

$$\therefore \quad \frac{M_{AB} + M_{BA} - 20}{4} + \frac{M_{DC} + M_{CD}}{4} + 20 = 0$$

$$\Rightarrow \quad M_{AB} + M_{BA} + M_{DC} + M_{CD} + 60 = 0$$

$$\Rightarrow \quad -3.75 + \frac{EI\theta_B}{2} - \frac{3EI\delta}{8} + 11.25 + EI\theta_B - \frac{3EI\delta}{8} + EI\theta_C - \frac{3EI\delta}{8} + 0.5EI\theta_C - \frac{3EI\delta}{8} + 60 = 0$$

$$\Rightarrow \quad 1.5EI\theta_B + 1.5EI\theta_C - \frac{12EI\delta}{8} = -67.5$$

$$\boxed{EI\theta_B + EI\theta_C - EI\delta = -45} \quad \text{... (C)}$$

Solving (A, B, and C) equation we get.

$$EI\theta_B = 12.43$$
$$EI\theta_C = -15.43$$
$$EI\delta = 42$$

Moments calculation

$$M_{AB} = -3.75 + \frac{12.43}{2} - \frac{3}{8} \times 42 = -13.285 \text{ kN-m}$$

$$M_{BA} = 11.25 + 12.43 - \frac{3}{8} \times 42 = 7.93 \text{ kN-m}$$

$$M_{BC} = -15 + \frac{3}{2} \times 12.43 + \frac{3}{4} \times (-15.43) = -7.93 \text{ kN-m}$$

$$M_{CB} = 45 + 0.75 \times 12.43 - 1.5 \times 15.43 = 31.18 \text{ kN-m}$$

$$M_{CD} = -15.43 - 0.375 \times 42 = -31.18 \text{ kN-m}$$

$$M_{DC} = -0.5 \times 15.43 - 0.375 \times 42 = -23.465 \text{ kN-m}$$

Reaction calculation

$$H_A = \frac{M_{AB} + M_{BA} - 20}{4} = -6.34 \text{ kN}$$

$$\therefore \quad H_A = 6.34 \text{ kN } (\leftarrow)$$

$$H_D = \frac{M_{DC} + M_{CD}}{4} = -13.66 \text{ kN}$$

$$\therefore \quad H_D = 13.66 \text{ kN } (\leftarrow)$$

From $\Sigma M_D = 0$

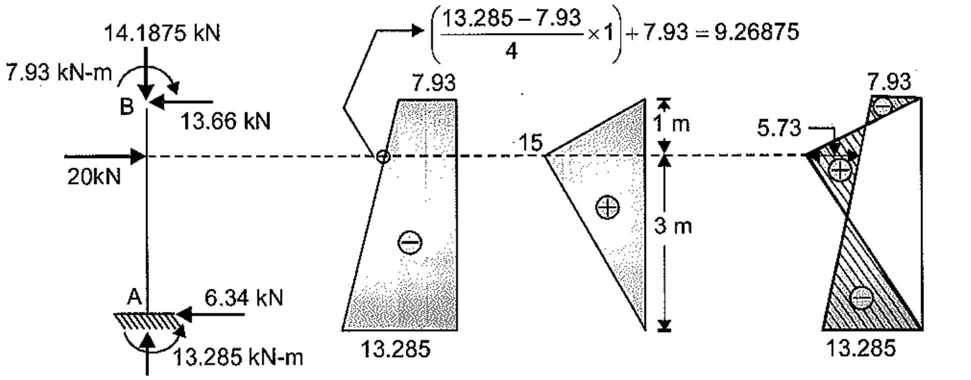
$\therefore \quad R_A \times 4 + 20 \times 3 - 80 \times 1 + M_{AB} + M_{DC} = 0$

$\Rightarrow \quad R_A \times 4 + 60 - 80 + (-13.285) + (-23.465) = 0$

$\therefore \quad R_A = 14.1875 \text{ kN } (\uparrow)$

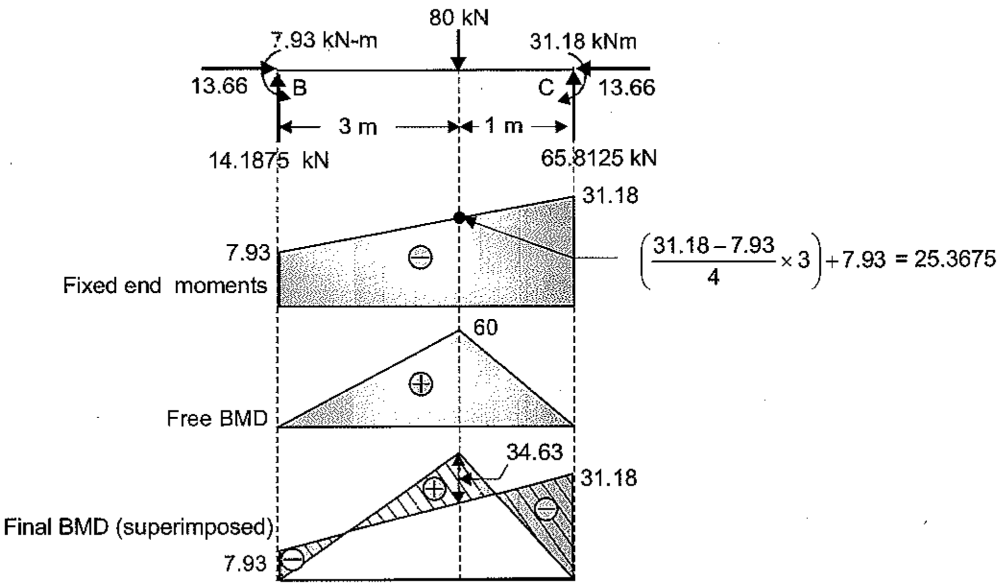
$\Rightarrow \quad R_D = 80 - 14.1875 = 65.8125 \text{ kN } (\uparrow)$

SFD and BMD

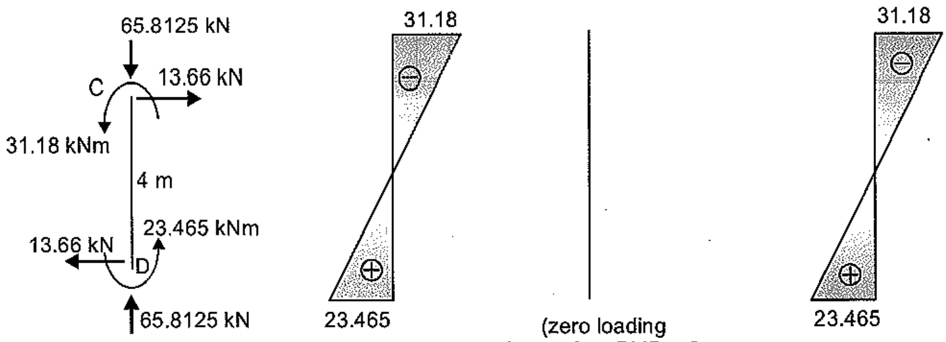


(Fixed end moment diagram) Free moment diagram Resultant BMD for AB

FBD for part BC

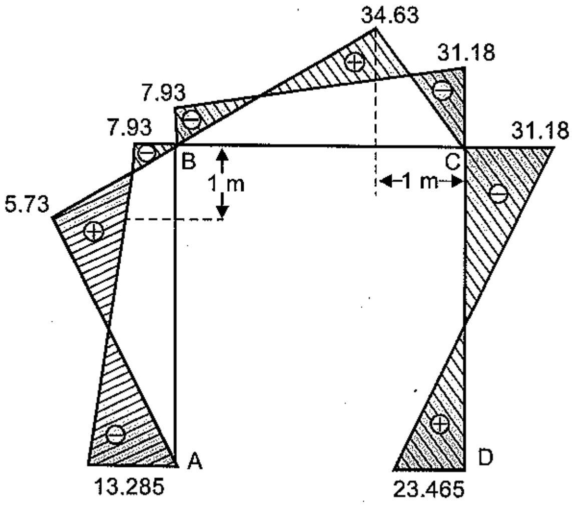


For Part CD

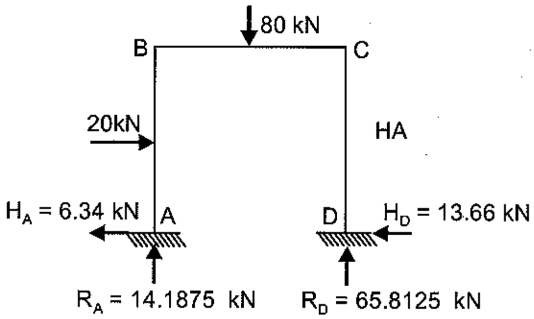


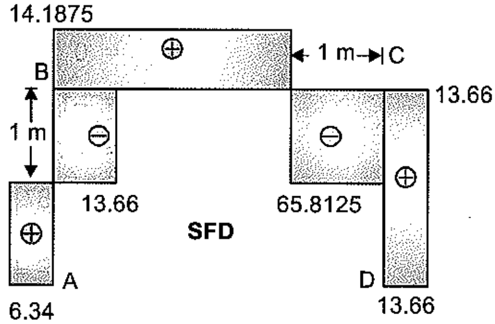
(Fixed end moment) (Final B.M)

Hence final BMD



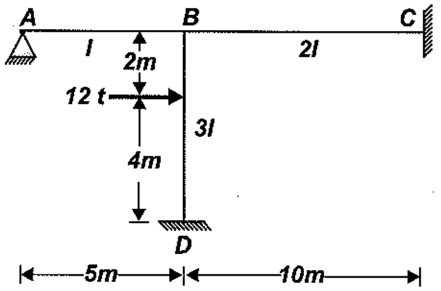
BMD





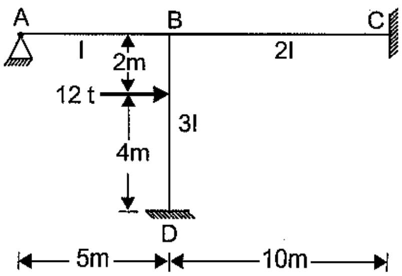
SFD SFD

Q-5: ***Employing slope-deflection method, evaluate BM at salient points of the frame shown below. Draw BM diagram indicating BM values on it.***



[18 Marks, ESE-2010]

Sol:



The frame is a restrained frame hence it is not going to sway. Only degree of freedom in this case is θ_B

$$M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = 0$$

$$M_{FBD} = \frac{12 \times 2 \times 16}{36} = 10.67 \text{ t-m}$$

$$M_{FDB} = \frac{-12 \times 4 \times 4}{36} = -5.33 \text{ t-m}$$

$$M_{BA} = M'_{FBA} + \frac{3EI}{5}(\theta_B) = \frac{3EI\,\theta_B}{5}$$

$$M_{BC} = 0 + \frac{2E\,(2I)}{10}(2\theta_B) = \frac{8EI\theta_B}{10}$$

$$M_{BD} = 10.67 + \frac{2E\,(3I)}{6}(2\theta_B) = 10.67 + 2EI\theta_B$$

From equilibrium equation $M_{BA} + M_{BC} + M_{BD} = 0$

$$\Rightarrow \quad 0.6\,EI\theta_B + 0.8\,EI\theta_B + 10.67 + 2EI\theta_B = 0$$

$$EI\theta_B = \frac{-10.67}{3.4} = -3.138$$

$$M_{AB} = 0$$

$$\Rightarrow \quad M_{BA} = -1.884 \text{ t-m}$$

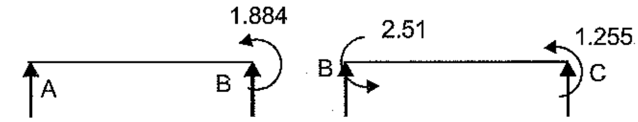
$$M_{BC} = -2.51 \text{ t-m}$$

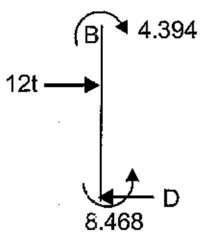
$$M_{BD} = 10.67 - 2 \times 3.138 = 4.394 \text{ t-m}$$

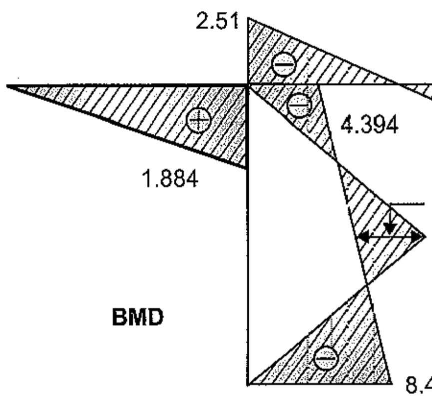
$$\Rightarrow \quad M_{CB} = \frac{M_{BC}}{2} = \frac{-2.51}{2} = -1.255 \text{ t-m}$$

$$M_{DB} = M_{FDB} + \frac{M_{BD} - M_{FBD}}{2}$$

$$= -5.33 + \frac{4.394 - 10.67}{2} = -8.468 \text{ t-m}$$







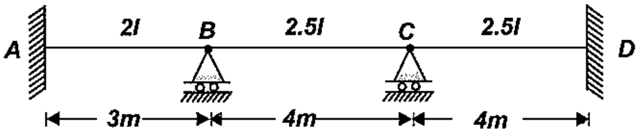
$$10.248 = -\left[4.394 + \frac{8.468 - 4.394}{6} \times 2\right] + \frac{12 \times 2 \times 4}{6}$$

$$= -5.752 + 16 = 10.248 \text{ t-m}$$

BMD

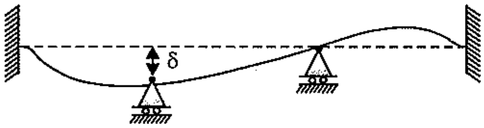
Q-6: ***Analyse the continuous beam shown in the figure below by slope deflection method. Support B settles down by 5 mm.***

$$E = 2 \times 10^5\ N/mm^2,\ I = 36 \times 10^6\ mm^4$$



[10 Marks, ESE-2011]

Sol:



Fixed End moments:

$$M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = M_{FCD} = M_{FDC} = 0$$

Slope deflection equation:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$$\Rightarrow \quad M_{AB} = \frac{2E(2I)}{3}\left\{\theta_B - \frac{3 \times 0.005}{3}\right\}$$

$$\Rightarrow \quad M_{AB} = \frac{4EI}{3}(\theta_B - 0.005) \qquad \text{... (i)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

$$\Rightarrow \quad M_{BA} = \frac{4EI}{3}(2\theta_B - 0.005) \qquad \text{... (ii)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{4}\left[2\theta_B + \theta_C - \frac{3\delta}{L}\right]$$

$$\Rightarrow \quad M_{BC} = \frac{2EI(2.5I)}{4}\left(2\theta_B + \theta_C + \frac{3 \times 0.005}{4}\right)$$

$$M_{BC} = \frac{5EI}{4}\left(2\theta_B + \theta_C + \frac{0.015}{4}\right) \qquad \text{... (iii) } (\delta \text{ for BC is } (-)\text{ve})$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$$

$$\Rightarrow \quad M_{CB} = \frac{2E(2.5I)}{4}\left(2\theta_C + \theta_B + \frac{0.015}{4}\right)$$

$$M_{CB} = \frac{5EI}{4}\left(2\theta_C + \theta_B + \frac{0.015}{4}\right) \qquad \text{... (iv)}$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\delta}{L}\right)$$

$$\Rightarrow \quad M_{CD} = \frac{2E(2.5I)}{4}(2\theta_C)$$

$$\Rightarrow \quad M_{CD} = \frac{10EI\theta_C}{4} \quad \text{... (v)}$$

$$M_{DC} = \frac{5EI\theta_C}{4} \quad \text{... (vi)}$$

Joint Equilibrium

$$M_{BA} + M_{BC} = 0$$

$$\frac{4}{3}EI(2\theta_B - 0.005) + \frac{5EI}{4}\left(2\theta_B + \theta_C + \frac{0.015}{4}\right) = 0$$

$$\Rightarrow \quad \frac{8}{3}\theta_B - 0.0067 + 2.5\theta_B + 1.25\theta_C + 0.0046875 = 0$$

$$\Rightarrow \quad 5.167\theta_B + 1.25\theta_C = 0.0019791 \quad \text{... (A)}$$

$$M_{CB} + M_{CD} = 0$$

$$\frac{5}{4}EI\left(2\theta_C + \theta_B + \frac{0.015}{4}\right) + \frac{10EI\theta_C}{4} = 0$$

$$\Rightarrow \quad \frac{10}{4}\theta_C + \frac{5}{4}\theta_B + 0.0046875 + \frac{10}{4}\theta_C = 0$$

$$\Rightarrow \quad 5\theta_C + 1.25\theta_B = -0.0046875$$

$$\Rightarrow \quad 1.25\theta_B + 5\theta_C = -0.0046875 \quad \text{... (B)}$$

Hence, by solving above two equations we get,

$$\theta_B = 6.49 \times 10^{-4}$$

$$\theta_C = -1.099 \times 10^{-3}$$

By putting these value in slope deflection equation:

$$EI = 2 \times 10^5 \times 36 \times 10^6 \text{ Nmm}^2 = 72 \times 10^5 \text{ Nm}^2$$

$$M_{AB} = \frac{4EI}{3}\{\theta_B - 0.005\} = \frac{4 \times 72 \times 10^5}{3}(6.49 \times 10^{-4} - 0.005)$$

$$= -41.77 \text{ kNm}$$

$$\boxed{M_{AB} = -41.77 \text{ kN-m}}$$

$$M_{BA} = \frac{4EI}{3}(2\theta_B - 0.005) = \frac{4 \times 72 \times 10^5}{3}(2 \times 6.49 \times 10^{-4} - 0.005)$$

$$\boxed{M_{BA} = -35.54 \text{ kN-m}}$$

$$M_{BC} = \frac{5EI}{4}\left(2\theta_B + \theta_C + \frac{0.015}{4}\right)$$

$$= \frac{5 \times 72 \times 10^5}{4} \times \left(2 \times 6.49 \times 10^{-4} + (-1.099 \times 10^{-3}) + \frac{0.015}{4}\right)$$

$$\Rightarrow \quad \boxed{M_{BC} = 35.54 \text{ kNm}}$$

$$M_{CB} = \frac{5EI}{4}\left(2\theta_C + \theta_B + \frac{0.015}{4}\right)$$

$$= \frac{5\times72\times10^5}{4}\times\left(2\times(-1.099\times10^{-3})+6.49\times10^{-4}+\frac{0.015}{4}\right)$$

$$\boxed{M_{CB} = 19.81\text{ kN-m}}$$

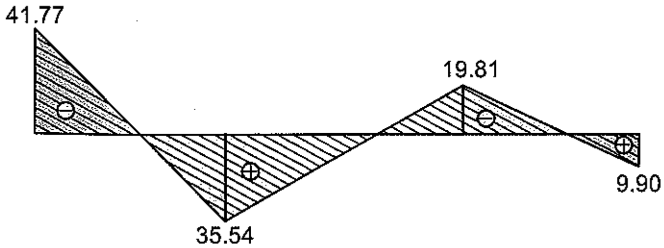
$\Rightarrow$ $$\boxed{M_{CD} = -19.81\text{ kN-m}}$$

$$M_{DC} = \frac{5EI}{4}\times\theta_C = \frac{5\times72\times10^5}{4}\times(-1.099\times10^{-3}) = -9.90\text{ kN-m}$$

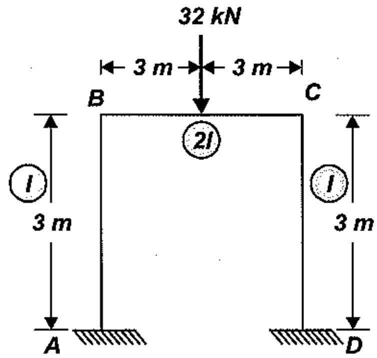
$\Rightarrow$ $$\boxed{M_{DC} = -9.90\text{ kNm}}$$

Hence the free body diagram will look like as shown below

Hence the BMD will be



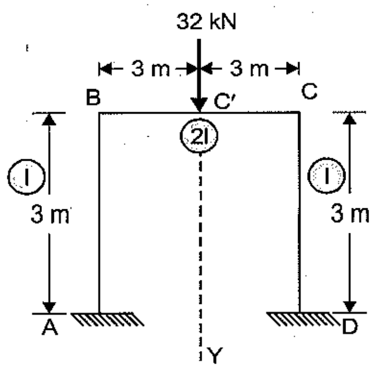
Q-7:



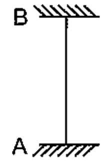
In the rigid frame shown in figure A central concentrated load is acting on "BC". Find slopes θ_A, θ_B, θ_C, θ_D

[10 Marks, ESE-2015]

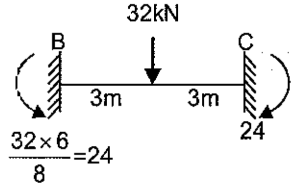
Sol:



FEMs:



$$M_{FBA} = M_{FAB} = 0$$



$$M_{FBC} = -24 \text{ kNm}$$

$$M_{FCB} = 24 \text{ kNm}$$

Equilibrium equation: $\quad M_{BA} + M_{BC} = 0$ ...(i)

Using slope - deflection method:

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$= 0 + \frac{2EI}{3}(2\theta_B) = \frac{4EI\theta_B}{3} \qquad (\theta_A = 0, \delta = 0)$$

$$M_{BC} = -24 + \frac{2E(2I)}{6}\left(\theta_B + \theta_C - \frac{3\delta}{l}\right) \qquad (\delta = 0)$$

Due to symmetry $\quad \theta_B = -\theta_C$

$\therefore$

$$M_{BC} = -24 + \frac{2EI}{3}\theta_B$$

From (i),

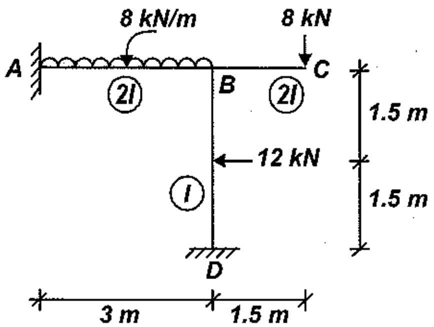
$$\frac{4EI\theta_B}{3} - 24 + \frac{2EI\vartheta_3}{3} = 0$$

$\Rightarrow$ $$\theta_B = \frac{12}{EI}(\text{clock wise})$$

$\Rightarrow$ $$\theta_C = -\frac{12}{EI}(\text{anti-clockwise})$$

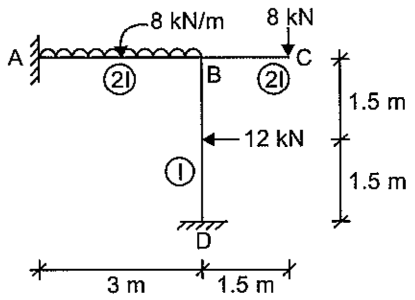
and, since supports A & D are fixed, $\theta_A = \theta_D = 0$

Q-8: ***Analyse the frame shown in figure by slope deflection method and draw the BMD and SFD.***



[20 Marks, ESE-2018]

Sol: **Given that:**



To draw: SFD and BMD by slope deflection method

Unknown displacement in this case is θ_B for which equilibrium equation required is

$$M_{BA} + M_{BC} + M_{BD} = 0$$

Calculation for fixed end moments :

$$M_{FAB} = \frac{-WL^2}{12} = \frac{-8(3)^2}{12} = -6 \text{ kNm}$$

$$M_{FBA} = \frac{+WL^2}{12} = \frac{+8(3)^2}{12} = +6 \text{ kNm}$$

$$M_{FBD} = \frac{-WL}{8} = \frac{-12\times 3}{8} = -4.5 \text{ kNm}$$

$$M_{FDB} = \frac{+WL}{8} = \frac{12\times 3}{8} = 4.5 \text{ kNm}$$

$$M_{BC} = -(8 \times 1.5) = -12 \text{ kNm}$$

Slope deflection equation:

For member AB

$$M_{AB} = M_{FAB} + \frac{2E(2I)}{3}(2\theta_A + \theta_B)$$

$$M_{AB} = -6 + \frac{4EI}{3}(0 + \theta_B) \qquad [\theta_A = 0]$$

$$\boxed{M_{AB} = -6 + \frac{4EI\theta_B}{3}}$$

and, $$M_{BA} = 6 + \frac{2E(2I)}{3}(2\theta_B + \theta_A),\ [\theta_A = 0]$$

$$\boxed{M_{BA} = 6 + \frac{8EI\theta_B}{3}}$$

For member BD

$$M_{BD} = M_{FBD} + \frac{2EI}{3}(2\theta_B + \theta_D)$$

$$\boxed{M_{BD} = -4.5 + \frac{4EI\theta_B}{3}} \qquad [\theta_D = 0]$$

$$M_{DB} = 4.5 + \frac{2EI}{3}(2\theta_D + \theta_B)$$

$$\boxed{M_{DB} = 4.5 + \frac{2EI\theta_B}{3}}$$

Joint equilibrium condition at 'B'

$$M_{BA} + M_{BD} + M_{BC} = 0$$

On substituting the respective values, we get

$$\Rightarrow \quad 6 + \frac{8EI\theta_B}{3} - 4.5 + \frac{4EI\theta_B}{3} - 12 = 0$$

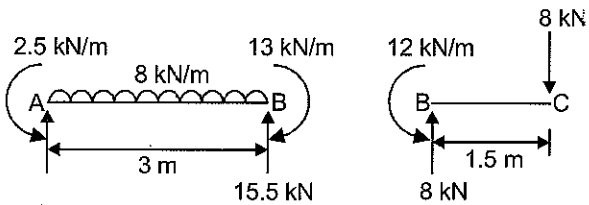
$$\Rightarrow \quad -10.5 + 4EI\theta_B = 0$$

$$\boxed{EI\theta_B = 2.625}$$

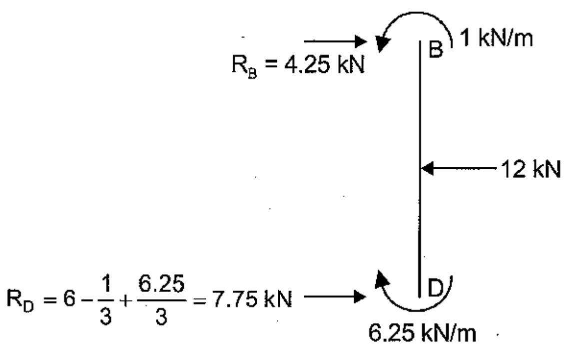
By putting the values of $EI\theta_B$ in slope deflection equations, we have

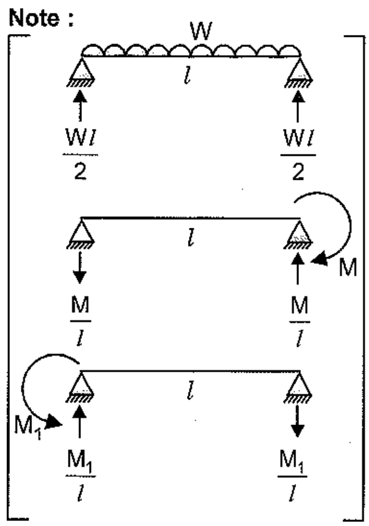
$$M_{AB} = -2.5 \text{ kNm}$$
$$M_{BA} = 13 \text{ kNm}$$
$$M_{BD} = -1 \text{ kNm}$$
$$M_{DB} = 6.25 \text{ kNm}$$
$$M_{BC} = -12 \text{ kNm}$$

Free body diagram:



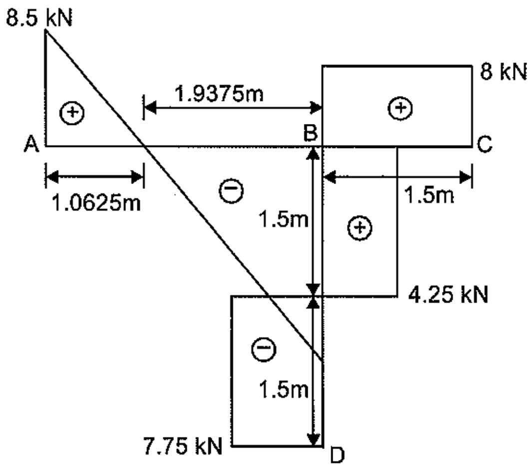
$$R_A = \frac{24}{2} - \frac{13}{3} + \frac{2.5}{3} = 8.5 \text{ kN}$$



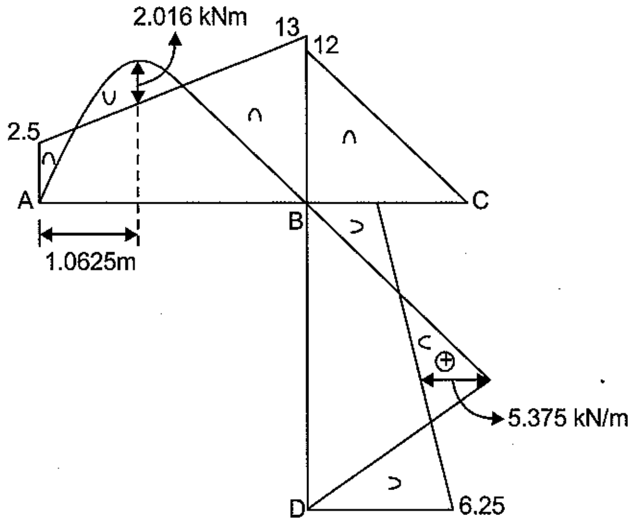


$$R_D = 6 - \frac{1}{3} + \frac{6.25}{3} = 7.75 \text{ kN}$$

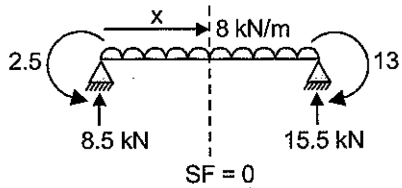
SFD:



BMD:



Max sagging BM in the span

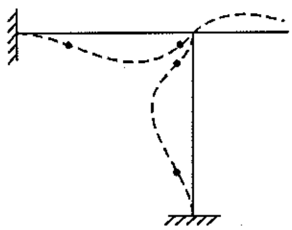


$$\Rightarrow \quad 8.5 - 8x = 0$$

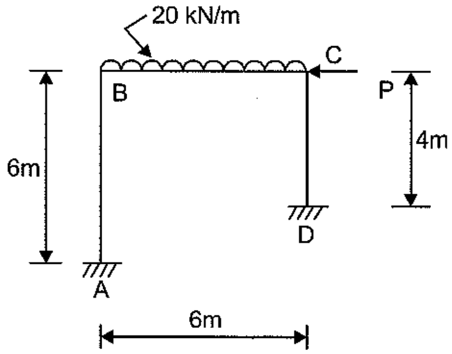
$$x = \frac{8.5}{8} = 1.0625$$

$$M_x = -2.5 + 8.5x - \frac{8x^2}{2} = -2.5 + 9.031 - 4.516 = 2.016 \text{ kN-m}$$

Deflected shape:

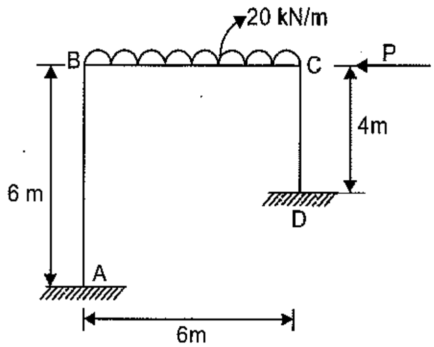


Q-9: ***Frame ABCD shown in Figure, is acted upon by a udl of intensity 20 kN/m on the horizontal span. What should be value of horizontal force 'P' applied at C, that will prevent sway of the frame? Draw BMD.***



[20 Marks, ESE-2020]

Sol:



Using Slope Deflection Equation ;

FEM

$$M_{F_{AB}} = M_{F_{BA}} = 0$$

$$M_{F_{BC}} = \frac{-20 \times 6^2}{12} = -60 \text{ kN-m}$$

$$M_{F_{CB}} = \frac{+20 \times 6^2}{12} = 60 \text{ kN-m}$$

$$M_{F_{CD}} = M_{F_{DC}} = 0$$

Frame will sway so, for prevention of sway we applied force P at C.

For AB

$$M_{AB} = M_{F_{AB}} + \frac{2EI}{6} \times \left(2\theta_A + \theta_B - \frac{3\Delta}{6}\right)$$

$$\theta_A = 0, \; \Delta = 0$$

$$M_{AB} = \frac{2EI}{6} \times \theta_B$$

$$M_{BA} = M_{FBA} - \frac{2EI}{6}\left(2\theta_B + \theta_A - \frac{3\Delta}{6}\right)$$

$$M_{BA} = \frac{2EI}{6} \times 2\theta_B = \frac{4EI\theta_B}{6}$$

For member BC

$$M_{BC} = M_{FBC} - \frac{2EI}{6} \times (2\theta_B + \theta_C)$$

$$= -60 + \frac{2EI}{6} \times 2\theta_B + \frac{2EI}{6}\theta_C$$

$$= -60+\frac{4EI\theta_B}{6}+\frac{2EI\theta_C}{6}$$

$$M_{CB} = M_{FCB}+\frac{2EI}{6}\times(2\theta_C+\theta_B)$$

$$= 60+\frac{4EI\theta_C}{6}+\frac{2EI\theta_B}{6}$$

For member CD :

$$M_{CD} = M_{FCD}+\frac{2EI}{4}\left(2\theta_C+\theta_D-\frac{3\Delta}{4}\right)$$

$$\Delta = 0,\ \theta_D = 0$$

$$= \frac{4EI\theta_C}{4}$$

$$M_{DC} = M_{F_{DC}}+\frac{2EI}{4}\left(2\theta_D+\theta_C-\frac{3\Delta}{4}\right)$$

$$= 0+\frac{2EI\theta_C}{4}$$

Equilibrium at joint B

$$M_{BA} + M_{BC} = 0$$

$$\frac{4EI\theta_B}{6}-60+\frac{4EI\theta_B}{6}+\frac{2EI\theta_C}{6} = 60$$

$$\frac{8}{6}EI\theta_B+\frac{2}{6}EI\theta_c = 60 \quad \text{... (i)}$$

Equilibrium at joint C :

$$M_{CB}+M_{CD} = 0$$

$$60+\frac{4EI\theta_C}{6}+\frac{2EI\theta_B}{6}+\frac{4EI\theta_C}{4} = 0$$

$$\frac{5}{3}EI\theta_C+\frac{EI\theta_B}{3} = -60 \quad \text{....(ii)}$$

Solving eqn. (i) and (ii), we get

$$EI\theta_B = 56.842$$

$$EI\theta_C = -47.368$$

Put value of $EI\theta_B$ and $EI\theta_C$ in slope deflection equation for each member.

$$M_{AB} = \frac{2}{6}\times 56.842 = 18.947 \text{ kN-m}$$

$$M_{BA} = \frac{4}{6}\times 56.842 = 37.894 \text{ kN-m}$$

$$M_{BC} = -60+\frac{4}{6}\times 56.842+\frac{2}{6}\times(-47.368)$$

$$= -37.894 \text{ kN-m}$$

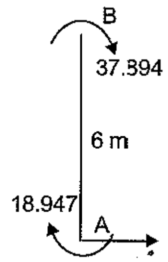
$$M_{CB} = 60+\frac{4}{6}\times(-47.368)+\frac{2}{6}\times 56.842$$

$$= 47.368 \text{ kN-m}$$

$$M_{CD} = \frac{4}{4}\times(-47.368) = -47.368 \text{ kN-m}$$

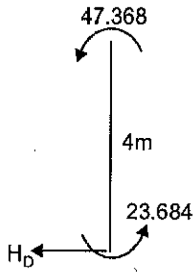
$$M_{DC} = \frac{2}{4}\times(-47.368) = -23.684 \text{ kN-m}$$

For AB



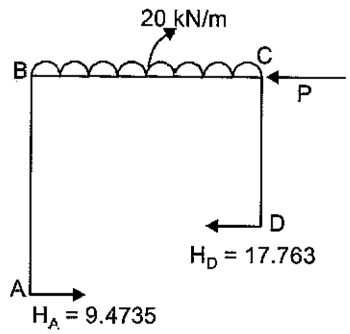
$$H_A = \frac{37.894 + 18.947}{6} = 9.4735 \text{ kN}$$

For CD



$$H_D = \frac{47.368 + 23.684}{4} = 17.763 \text{ kN}$$

Shear Equation for Frame :



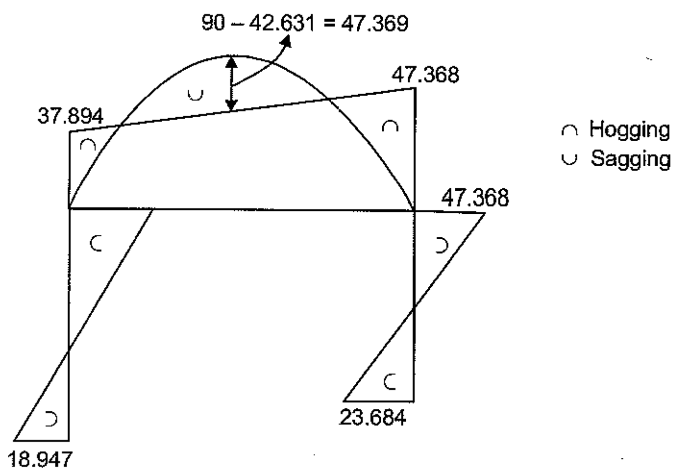
$$H_A = H_D + P$$

$$9.4735 - 17.763 = P$$

$$= -8.2894 \text{ kN}$$

$$P = 8.2895\,(\rightarrow)$$

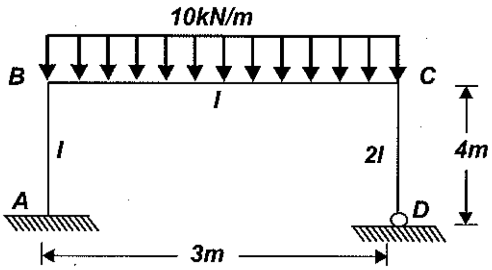
BMD (kN-m)



CHAPTER 4

DISPLACEMENT METHOD OF ANALYSIS: (MOMENT DISTRIBUTION METHOD)

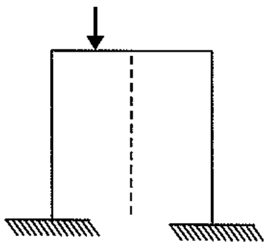
Q-1: ***Analyse the frame loaded as shown in fig and draw the bending moment diagram. The frame is fixed at A and hinged at D. The relative second moment of the areas are also indicated in the figure.***



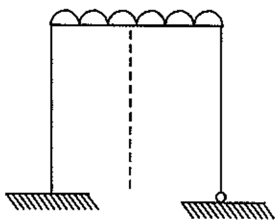
[20 Marks, ESE-1995]

Sol: ***Conceptual Background:***

This is an Unrestrained frame with side sways. Side sway occurs if there is a lack of symmetry either with respect to loading or with respect to geometry. For example, following frames will have side sway.



(Symmetrical frame with Unsymmetrical loading)



(Unsymmetric frame with symmetrical loading)

Moment distribution for frames side with sways:

In this case analysis is performed in two steps

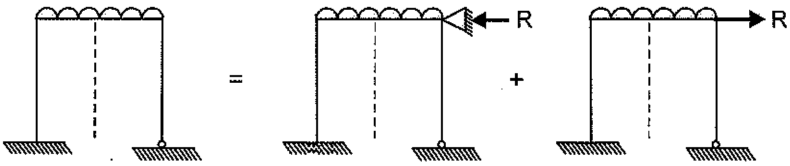
Step (a): The frame is restrained by applying some external loading (restraining force) and analysis is performed to find out end moments. It is called nonsway analysis.

It is important to note that the restraining force should be applied at the joint such that no fixed end moments generate because of restraining force.

Step (b): All external loadings are removed and sway force of magnitude equal to R (restrained force) and opposite to the direction of restraining force is applied and analysis is performed. This is called sway analysis.

Finally, end moments are found using principle of superposition

(Final end moments at joint) = (End moments due to nonsway analysis) + (End moments due to sway analysis)



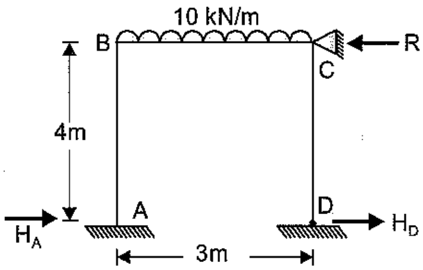
(Nonsway Analysis) (Sway Analysis)

Distribution factors

Joint	Member	Relative stiffness	Total relative stiffness	Distribution factor
B	BA	I/4	7I/12	3/7
	BC	I/3		4/7
C	CB	I/3	17I/24	8/17
	CD	$\frac{3}{4}\times\frac{2I}{4}=\frac{3I}{8}$		9/17

Nonsway Analysis

Objective: To find restraining force R.



Fixed end moment

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = \frac{-10\times 3^2}{12} = -7.50 \text{ kNm}$$

$$M_{FCB} = 7.50 \text{ kNm}$$

$$M_{FCD} = M_{FDC} = 0$$

A		B		C	D	
	3/7	4/7	8/17	9/17		
0	0	–7.50	7.50	0	0	FEM
	+3.21	+4.29	–3.53	–3.9		Bal
+1.605		–1.765	+2.145			C/O
	+0.756	+1.009	–1.009	–1.136		Bal
+0.378		– 0.505	0.505			C/O
	+0.216	+0.289	– 0.238	–0.267		Bal
+0.108		–0.119	+0.145			C/O
	+0.051	+ 0.068	–0.068	–0.077		Bal
+0.026		– 0.034	+0.034			C/O
	+0.015	+0.019	– 0.016	–0.018		Bal
2.117	4.248	– 4.248	5.468	–5.468	0	

Hence, M_{AB} = 2.117, M_{BA} = 4.248, M_{BC} = – 4.248, M_{CB} = 5.468, M_{CD} = – 5.468, M_{DC} = 0

Conceptual Background:

Sign scheme adopted for moment distribution,

+ve B.M = clockwise+ve slope = clockwise rotation

–ve B.M = anticlockwise–ve slope = anticlockwise rotation

Note that we can avoid calculation of moments on legs AB and CD while doing moment distribution. Moments of these legs can finally be obtained using equilibrium equation once M_{BC} and M_{CB} have been calculated. For example, the analysis can be done in the central part follows:

A		B	C		D	
		$\frac{4}{7}$	$\frac{8}{17}$			
0	0	–7.5	7.5	0	0	FEM
		$7.5\times\frac{4}{7}$	$-7.5\times\frac{8}{17}$			Bal
		$\frac{-7.5\times 4}{17}$	$7.5\times\frac{2}{7}$			C/O
		$\frac{7.5\times 4}{17}\times\frac{4}{7}$	$-7.5\times\frac{2}{7}\times\frac{8}{17}$			Bal
						C/O
		–4.248	+5.468			

Now,
$$M_{BA} + M_{BC} = 0 \Rightarrow M_{BA} = -M_{BC} = 4.248$$
$$M_{CB} + M_{CD} = 0 \Rightarrow M_{CD} = -M_{CB} = -5.468$$

Also, from (i) and (ii) ;

$$M_{AB} = M_{FAB} + \frac{M_{BA} - M_{FBA}}{2}$$

$$\left[\begin{array}{l} M_{AB} = M_{FAB} + \frac{2EI}{l}(\theta_B) \quad ...(i) \\ M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B) \quad ...(ii) \\ \text{Since } \delta = 0 \text{ and end A is fixed} \end{array}\right]$$

$$M_{AB} = 0 + \frac{4.248 - 0}{2}$$

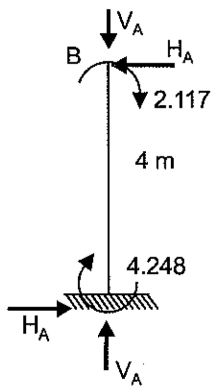
$$M_{AB} = 2124 \approx 2.117$$

$$M_{DC} = 0 \quad [\because \text{ hinge}]$$

Free body diagram

For part AB

Our aim is to calculate H_A

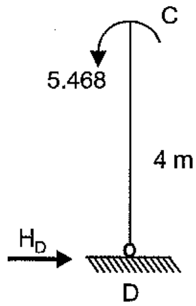


$$\Sigma M_B = 0$$

$$\Rightarrow \quad H_A \times 4 = 2.117 + 4.248$$

$$\Rightarrow \quad H_A = 1.59125 \text{ kN}$$

For part CD:



$$\Sigma M_C = 0$$

$$\Rightarrow \quad H_D \times 4 = -5.468$$

$$\Rightarrow \quad H_D = -1.367 \text{ kN}$$

$$\text{Restraining force, } R = H_A + H_D \quad [\text{from } \Sigma F_H = 0]$$

$$R = H_A + H_D$$

$$\Rightarrow \quad R = 1.59125 - 1.367$$

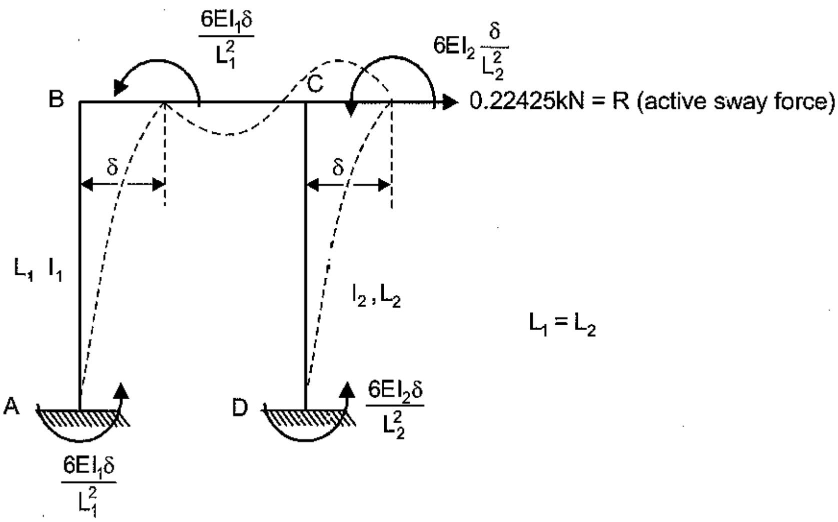
$$\Rightarrow \quad R = 0.22425 \text{ kN}$$

Conceptual Background:

Sway Analysis

- Remove all external loading
- Apply arbitrary sway force (S) opposite to the restraining force i.e., rightward in this case.
- In sway analysis, fixed end moment are not known in advance hence arbitrary values are assumed for fixed end moments, like 100. 10 etc.
- Joint moment determined
- Arbitrary sway force determined
- Correction factor determination; correction factor = $\dfrac{\text{Actual sway force (R)}}{\text{Arbitrary sway force (S)}}$
- Actual sway moment calculation = end moments x (correction factor)
- Total end moment = (Actual sway moment) + (Nonsway moment)

Sway analysis



if we suppose, $\quad \dfrac{6EI_1\delta}{L_1^2} = 10 \quad$ [Here only magnitude considered]

then, $\quad M_{FAB} = M_{FBA} = -10 \quad$ [(–ve) sign taken because of anticlockwise moment]

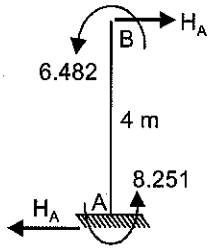
Hence, $\quad \dfrac{6E(2I)\delta}{L^2} = 20$

$$M_{FCD} = M_{FDC} = -20 \qquad [\because l_2 = 2l]$$

So,

A	B	B	C	C	D	
	3/7	4/7	8/17	9/17		
–10	–10	0	0	–20	–20	FEM → considering all joints to be fixed
				+10	+20	In actual case, joint D is hinged hence BM at D is permanently released
–10	–10	0		–10	0	New FEM
	4.286	5.714	4.706	5.294		Bal
2.143		2.353	2.857			C/O
	–1.008	–1.345	–1.344	–1.513		Bal
–0.504		–0.672	–0.673			C/O
	0.288	0.384	0.317	0.356		Bal
0.144		0.159	0.192			C/O
	–0.068	–0.091	–0.091	–0.101		Bal
–0.034		–0.046	–0.046			C/O
	0.02	0.026	0.022	0.024		Bal

Calculation of H_A:



$$M_{AB} = -8.251,$$

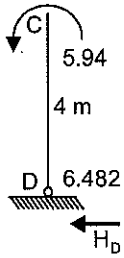
$$M_{BA} = -6.482$$

$$\Sigma M_B = 0$$

$$H_A \times 4 = 8.251 + 6.482$$

$$\Rightarrow \qquad H_A = 3.68325 \text{ kN}$$

Calculation of H_D

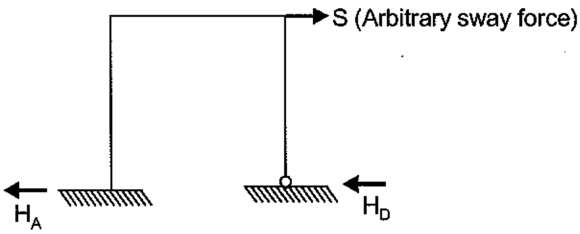


$$M_{CD} = -5.94$$

$$\Sigma M_C = 0$$

$$H_D \times 4 = 5.94$$

$$\Rightarrow \qquad H_D = 1.485 \text{ kN}$$



Hence, $\quad$ Arbitrary sway force $= (S) = 3.68325 + 1.485 = 5.16825$ kN

$$\text{Correction factor} = \frac{R}{S} = \frac{0.22425}{5.16825} = 0.04338$$

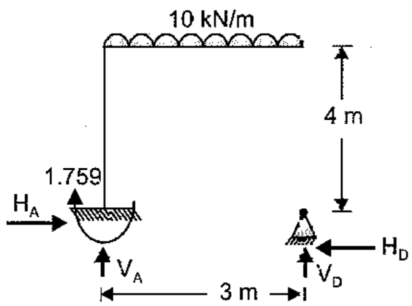
$\therefore$ Actual end moments (for sway) are:

$$M_{AB} = -8.251 \times 0.04338 = -0.358 \text{ kNm}$$
$$M_{BA} = -6.482 \times 0.04338 = -0.281 \text{ kNm}$$
$$M_{BC} = 6.482 \times 0.04338 = +0.281 \text{ kNm}$$
$$M_{CB} = 5.94 \times 0.04338 = +0.2576 \text{ kNm}$$
$$M_{CD} = -5.94 \times 0.04338 = -0.2576 \text{ kNm}$$
$$M_{DC} = 0 \times 0.4338 = 0$$

$\Rightarrow$ Total end moments :

Member	(Non sway end moments)	(Actual sway end Moment)	Total end Moment
AB	2.117	– 0.358	1.759
BA	4.248	– 0.281	3.967
BC	– 4.248	+ 0.281	– 3.967
CB	5.468	+ 0.2576	5.7256
CD	– 5.468	– 0.2576	– 5.7256
DC	0	0	0

Determining the actual reactions:



$$\Sigma F_x = 0$$
$$V_A + V_D = 30$$

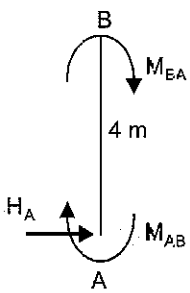
and $$\Sigma M_D = 0$$

$\Rightarrow$ $$1.759 + V_A \times 3 = 10 \times 3 \times 3/2$$

$\Rightarrow$ $$V_A = 15 - \frac{1.759}{3} = 14.4137 \text{ kN} \cong 14.42 \text{ kN}$$

$\Rightarrow$ $$V_D = 15.58 \text{ kN}$$

Part AB

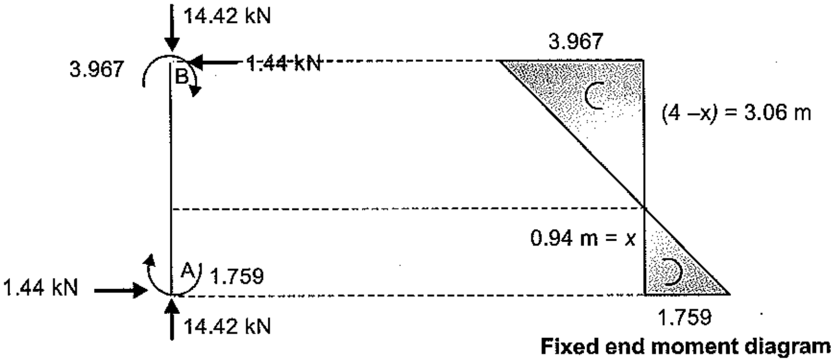


and, $$H_A = \frac{M_{AB} + M_{BA}}{4} = \frac{1.759 + 3.967}{4} = 1.44 \text{ kN}$$

As $$H_A - H_D = 0 \;; \Rightarrow H_D = 1.44 \text{ kN}$$

Drawing Bending Moment Diagrams

For part AB,



Fixed end moment diagram

Finding x,

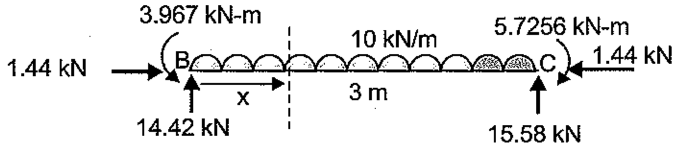
$$\frac{1.759}{x} = \frac{3.967}{4-x} \Rightarrow \frac{4-x}{x} = \frac{3.967}{1.759}$$

$\Rightarrow$ $$\frac{4}{x} = 1+\frac{3.967}{1.759}$$

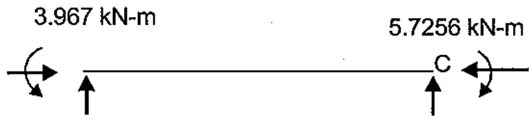
$\Rightarrow$ $$x = 0.94 \text{ m and } 4 - x = 3.06 \text{ m}$$

Since there is no loading in span AB, hence free moment diagram will not exist

For part BC,

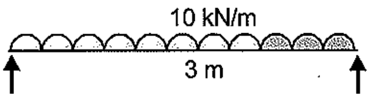


For

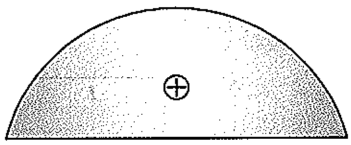


(End Moment Diagram)

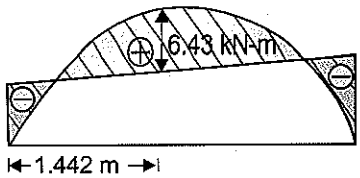
For



Free moment diagram



On superimposition, we get;



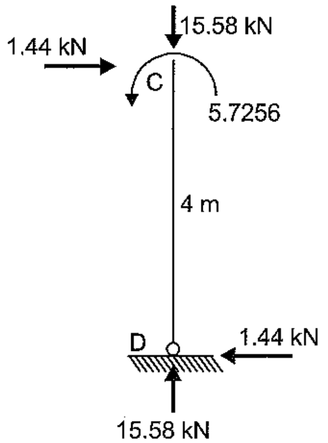
Max BM in BC will occur at the location of zero SF

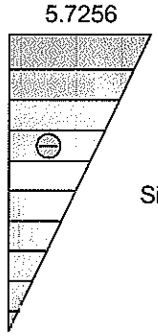
Let at x distance from B, SF = 0

$\Rightarrow \quad 14.42 - 10x = 0 \Rightarrow x = 1.442$ m

$$\Rightarrow \quad BM_{max} = 14.42 \times 1.442 \; \frac{-10\,(1.442)^2}{2} - 3.967 = 6.43 \text{ kNm}$$

For part CD,



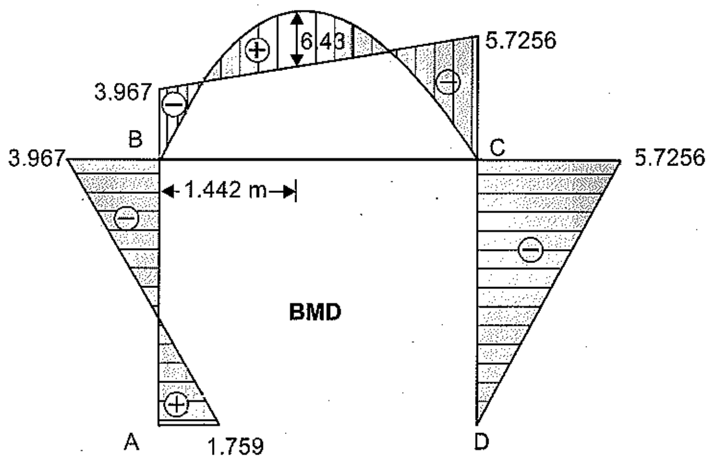


Sign to BMD given by looking from left

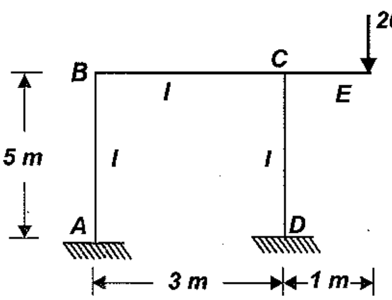
(End Moment Diagram)

Since there is no loading on CD, hence free moment diagram will not exit

By combining all the parts together, we get



Q-2: ***Analyse the rigid jointed frame in the figure and draw bending moment and shear force diagrams. Assume constant EI for all the members.***



[20 Marks, ESE-1996]

Sol: **Distribution factor calculation:**

Joint	Member	Relative stiffness	Total Rel. stiffness	D.F.
B	BA	I/5 = 3I/15	8I/15	3/8
	BC	I/3 = 5I/15		5/8
C	CB	I/3 = 5I/15	8I/15	5/8
	CD	I/5 = 3I/15		3/8

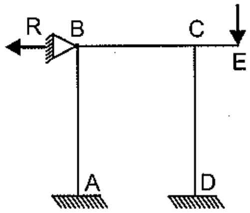
Non sway analysis:

Fixed ends moments

$M_{FAB} = 0$ $\quad\quad M_{FBA} = 0$

$M_{FBC} = 0$ $\quad\quad M_{FCB} = 0$

$M_{FCD} = 0$ $\quad\quad M_{FDC} = 0$



Conceptual Background:

AB and CD have no loading. In the analysis of this type of forces the vertical members BA and CD can be completely left out while calculating internal moments using moment distribution method.

By analysis, M_{BC} and M_{CB} can be calculated and form equillibrium equation $M_{BA} + M_{BC} = 0$; M_{BA} can be calculated.

and, then $$M_{AB} = \frac{M_{BA}}{2}$$

[$\because$ $M_{FAB} = M_{FBA} = 0$ and joint A being fixed and there is no sway as because it is non-sway analysis]

and, $$M_{CB} + M_{CD} + M_{CE} = 0 \quad \text{[Joint equilibrium equation]}$$

$$\Rightarrow \quad M_{CB} + M_{CD} - 20 = 0$$

$$\Rightarrow \quad M_{CD} = 20 - M_{CB}$$

and, $$M_{DC} = \frac{M_{CD}}{2} \quad \text{[Same reason as for } M_{AB} = \frac{M_{BA}}{2}\text{]}$$

Hence with the help of M_{BC} and M_{CB} we can calculate M_{AB} and M_{BA}. M_{CD} and M_{DC}

A	B		C		E
	3/8	5/8	5/8	0	
0	0	0	0	−20	
			+12.5		
		+6.25			
		−3.90			
			−1.95		
			+1.22		
		+0.61			
		−0.38			
			−0.19		
			+0.12		
		0.06			
		−0.0375			
		+2.6025	+11.7		

$$M_{BC} = +2.6025 \text{ t-m}$$

$$\Rightarrow \quad M_{BA} = -2.6025 \text{ t-m}$$

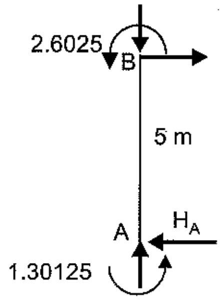
$$\therefore \quad M_{AB} = \frac{M_{BA}}{2} = -1.30125 \text{ t-m}$$

$$M_{CB} = 11.7 \text{ t-m}$$

$$\therefore \quad M_{CD} = 20 - 11.7 = 8.3 \text{ t-m}$$

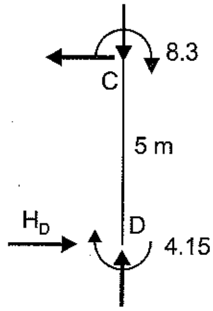
and $$M_{DC} = \frac{M_{CD}}{2} = \frac{8.3}{2} = 4.15 \text{ t-m}$$

Horizontal Reactions at A and D



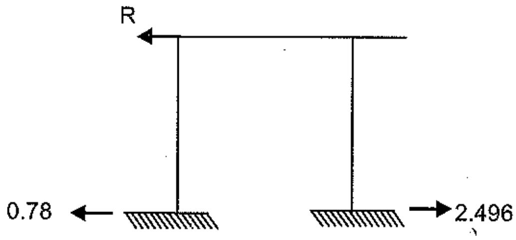
From $\Sigma M_B = 0$

$$H_A = \frac{2.6025 + 1.30125}{5} = 0.78t(\leftarrow)$$



From $\Sigma M_C = 0$

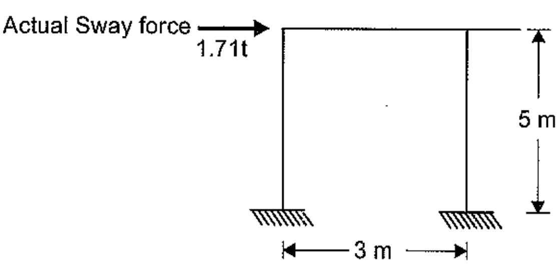
$$H_D = \frac{8.3 + 4.15}{5} = 2.496(\rightarrow)$$

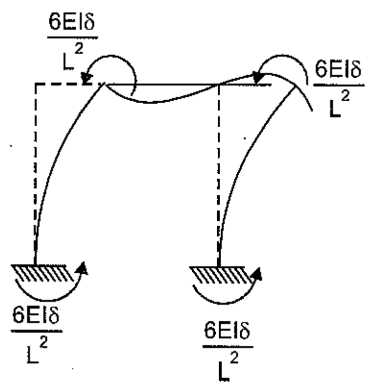


$$\text{Hence the restraining force} = (2.49 - 0.73) \leftarrow$$

$$= (1.71t) \leftarrow$$

Sway Analysis:





$$M_{FAB} = \frac{-6EI\delta}{L^2} = \frac{-6EI\delta}{25} = M_{FBA}$$

$$M_{FBC} = M_{FCB} = 0$$

$$M_{FCD} = M_{FDC} = \frac{-6EI\delta}{25}$$

Thus we can assume arbitrary fixed end moments as,

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = -10$$

and $$M_{FBC} = M_{FCB} = 0$$

A		B		C	D
	3/8	5/8	5/8	3/8	
–10	–10	0 +6.25	0 +6.25	–10	–10
		+3.13 –1.96	+3.13 –1.96		
		–0.98 +0.61	–0.98 +0.61		
		0.31 –0.19	0.31 –0.19		
		– 0.1 0.06	–0.1 0.06		
		0.03 –0.02	0.03 –0.02		
		+7.14	+7.14		

$$M_{BC} = 7.14$$

$$M_{BA} + M_{BC} = 0$$

$\Rightarrow$ $$M_{BA} = -7.14 \text{ t-m}$$

Also, $$M_{AB} = M_{FAB} + \frac{M_{BA} - M_{FBA}}{2}$$

In this case although there is sway but it is assumed to be a known sway

$$M_{AB} = F_{FAB} + \frac{2EI}{l}\left(\theta_B - \frac{3\delta}{l}\right)$$

$$M_{AB} = M_{FAB} - \frac{-6EI\delta}{l^2} + \frac{2EI\theta_B}{l}$$

$$M_{AB} = \underbrace{\left(M_{FAB} - \frac{6EI\delta}{l^2}\right)}_{M_{FAB}'} + \frac{2EI\theta_B}{l}$$

$M_{FAB'}$ can be treated as new fixed end moment.

Thus the concept discussed in question no. (2) can be used.

$$M_{AB} = M_{FAB'} + \frac{M_{BA} - M_{FBA'}}{2} = -10 + \frac{-7.14-(-10)}{2} = -8.57 \text{ t-m}$$

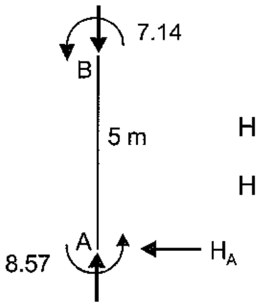
$$M_{BC} = 7.14 \text{ t-m}$$

$$M_{CB} = 7.14 \text{ t-m}$$

$$M_{CB} + M_{CD} = 0 \ \Rightarrow \ M_{CD} = -7.14 \text{ t-m}$$

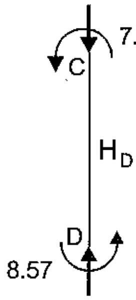
$$M_{DC} = M_{FDC'} + \frac{M_{CD} - M_{FCD'}}{2} = -10 + \frac{-7.14-(-10)}{2} = -8.57 \text{ t-m}$$

Horizontal reaction H_A and H_D

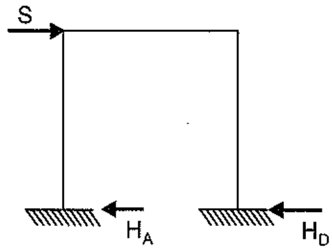


$$H_A = \frac{8.57+7.14}{5}$$

$$H_A = 3.14t\ (\leftarrow)$$



$$H_D = 3.14(t)\ (\leftarrow)$$



$\Rightarrow$ $$S = H_A + H_D$$

Hence, sway force for these reaction = 3.14 + 3.14 = 6.28 $(\rightarrow)$

$$\text{Correction factor} = \frac{R}{S} = \left(\frac{1.71}{6.28}\right) = 0.273$$

$\therefore$ Actual end moments are, (for sway)

$$M_{AB} = -8.57 \times 0.273 = -2.33$$

$$M_{BA} = -7.14 \times 0.273 = -1.95$$

$$M_{BC} = 7.14 \times 0.273 = 1.95$$

$$M_{CB} = 7.14 \times 0.273 = 1.95$$

$$M_{CD} = -7.14 \times 0.273 = -1.95$$

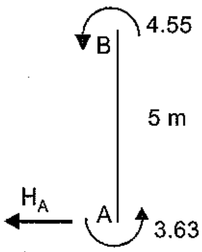
$$M_{DC} = -8.57 \times 0.273 = -2.33$$

$\therefore$ Total End Moments:

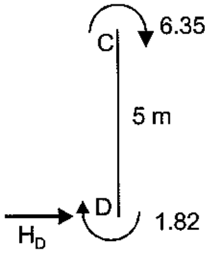
Member	Non sway end moment (t-m)	Actual sway end moment (t-m)	Total end moment (t-m)
AB	−1.30	−2.33	−3.63
BA	+2.60	−1.95	−4.55
BC	+2.60	1.95	+4.55
CB	11.7	1.95	+13.65
CD	8.3	−1.95	+ 6.35
DC	4.15	−2.33	+ 1.82

Calculation of horizontal reactions

From $\Sigma M_B = 0$ $$H_A = \frac{3.63+4.55}{5} = 1.635t(\leftarrow)$$



From $\Sigma M_C = 0$ $\qquad H_D = \dfrac{6.35 + 1.82}{5} = 1.635t\,(\rightarrow)$

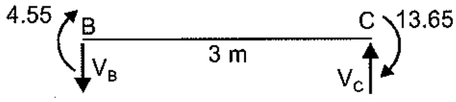


Calculation of vertical reactions

Consider part BC

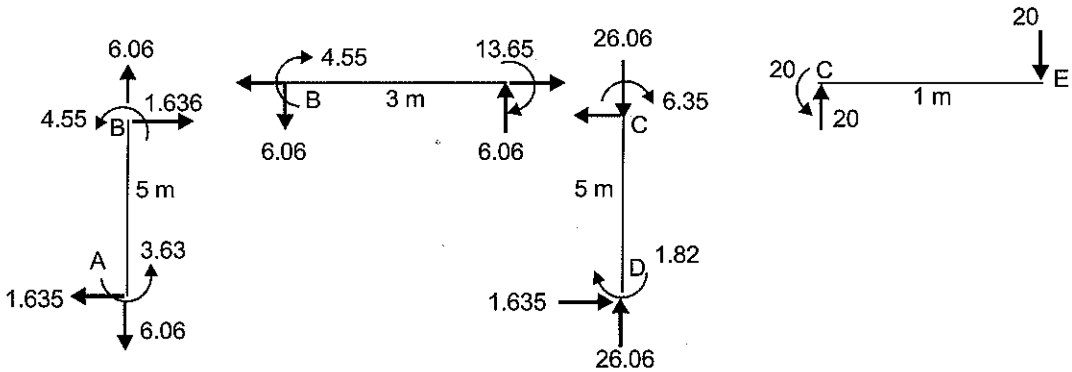
From $\Sigma M_C = 0$

$$V_B = \frac{4.55 + 13.65}{3} = 6.06t(\downarrow)$$



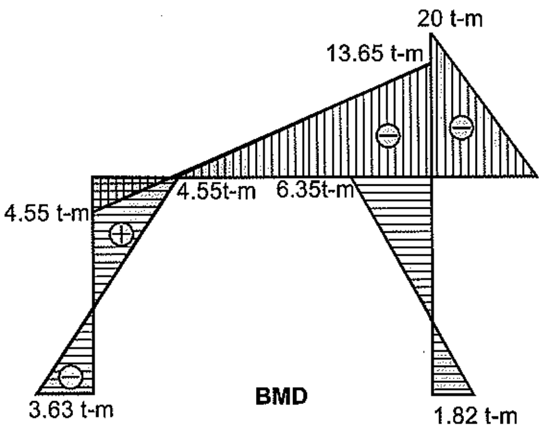
$$V_C = 6.06\text{ t}\,(\uparrow)$$

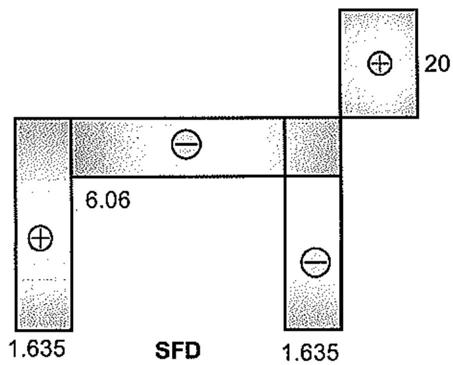
Hence reaction at A (V_A) = 6.06 t $(\downarrow)$ and reaction at D (V_D) = 20 + 6.06 = 26.06 t $(\uparrow)$



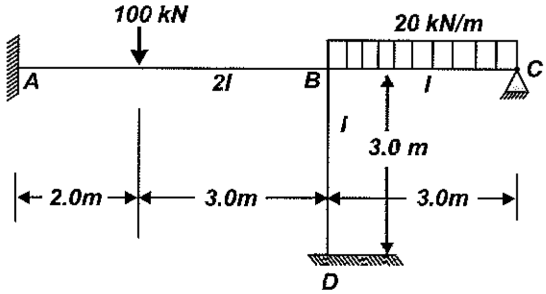
Hence final free body diagram is as shown above

BMD and SFD





Q-3: ***Analyse the frame shown in figure by moment distribution method. Draw the bending moment diagram. The second moment of the area are indicated in the figure.***



[20 Marks, ESE-1998]

Sol: **Distribution factor calculation:**

Joint	Member	Relative stiffness	Total relative stiffness	Distribution factor
	BA	$\frac{2I}{5}=\frac{24I}{60}$		0.41
B	BC	$\frac{3}{4}\times\frac{I}{3}=\frac{I}{4}=\frac{15I}{60}$	$\frac{59I}{60}$	0.25
	BD	$\frac{I}{3}=\frac{20I}{60}$		0.34

Fixed End Moments:

$$M_{FAB} = \frac{-100\times 2\times 3^2}{5^2} = -72\text{KN}-\text{m}$$

$$M_{FBA} = \frac{+100\times 3\times 2^2}{5^2} = +48\text{KN}-\text{m}$$

$$M_{FBC} = \frac{-20\times 3^2}{12} = -15\text{KN}-\text{m}$$

$$M_{FCB} = +15\text{ KN-m}$$

$$M_{FBD} = M_{FDB} = 0$$

While performing moment distribution, leg BD can be left out and finally once BA and BC becomes known, M_{BD} can be obtained from equilibrium equation

$$M_{BA} + M_{BC} + M_{BD} = 0$$

Also, $$M_{DB} = \frac{M_{BD}}{2} \quad \left[\begin{array}{l}\because\ M_{DB} = M_{FDB} + \frac{2EI}{l}\left(2\theta_D - \theta_B - \frac{3\delta}{l}\right) \text{ and } M_{BD} = M_{FBD} + \frac{2EI}{l}\left(2\theta_B + \theta_D - \frac{3\delta}{l}\right)\\ \text{Fixed moments} = 0,\text{ end D is fixed and sway} = 0\end{array}\right]$$

A		B	C	
	0.41	0.25		
–72	48	–15	15	
		–7.5	–15	
–72	48	–22.5	0	New FEM
	–10.455	– 6.375		Bal
–5.23			0	
–77.23	+37.545	–28.875	0	

$$M_{AB} = -77.23 \text{ KN-m}$$
$$M_{BA} = 37.545 \text{ KN-m}$$
$$M_{BC} = -28.875 \text{ KN-m}$$

Applying joint equilibrium equation, at point B,

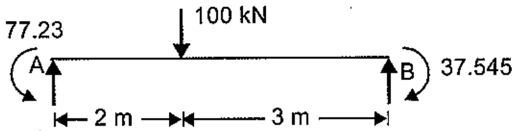
$$M_{BA} + M_{BC} + M_{BD} = 0$$

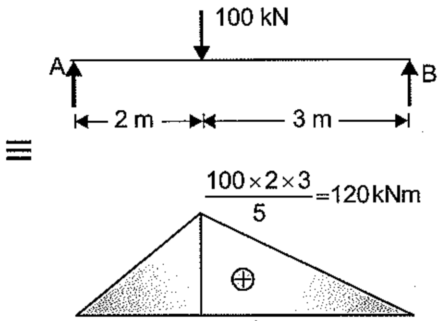
$$\Rightarrow \quad M_{BD} = 28.875 - 37.545 = -8.67 \text{ KN-m}$$

$$M_{DB} = \frac{M_{BD}}{2} = \frac{-8.67}{2} = -4.335 \text{ kN-m}$$

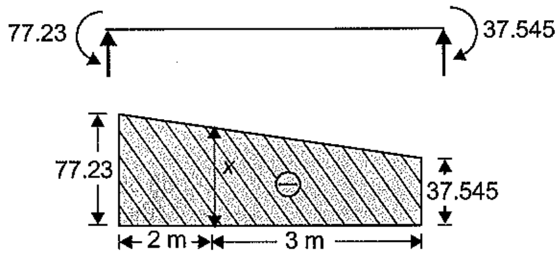
Bending Moment Diagram:

For Span AB





+

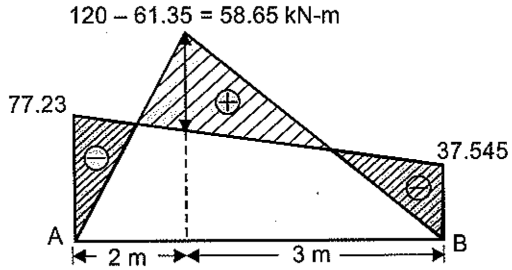


Free BMD

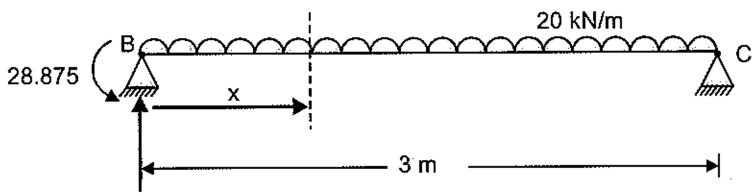
$$x = 37.545 + \frac{77.23 - 37.545}{5} \times 3 = 61.356 \text{ kNm}$$

End moment Diagram

On super imposition we get



For Span BC



$$\frac{20 \times 3}{2} + \frac{28.875}{3} = 39.625 \text{ kN}$$

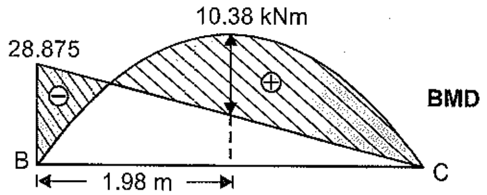
Let SF at a distance x from B

$$\Rightarrow \quad 39.625 - 20x = 0$$

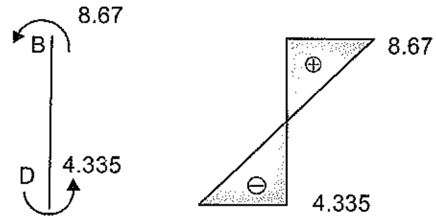
$$x = 1.98 \text{ m}$$

$$M_{max} = 39.625 \times 1.98 - 28.8775 - 20 \times \frac{1.98^2}{2} = 10.38 \text{ kNm}$$

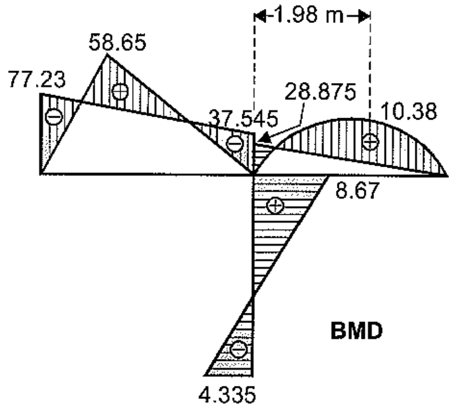
Hence the BMD for part BC will be



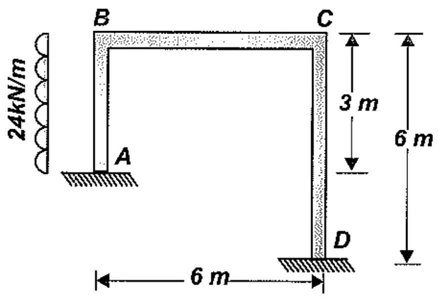
For span BD



Hence by combining the diagram the resultant diagram will look like as shown below



Q-4: ***Moment-Distribution: The rigid - joined portal frame shown in fig. is fully fixed at point A and D and is to be analyzed by the moment distribution method, for the load system shown.***



It can be assumed that EI = 1 for all members.

(a) Estimate the horizontal deflection at B.

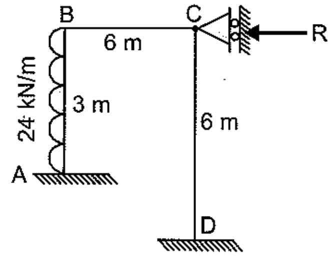
(b) Draw the bending moment diagram throughout the structure.

[10 + 30 = 40 Marks, ESE-1999]

Sol: **Distribution Factor Determination**

Joint	Member	Relative stiffness	Total relative stiffness	Distribution factor
B	BA	$\frac{I}{3}=\frac{2I}{6}$	$\frac{3I}{6}$	$\frac{2}{3}$
	BC	$\frac{I}{6}$		$\frac{1}{3}$
C	CB	$\frac{I}{6}$	$\frac{2I}{6}$	$\frac{1}{2}$
	CD	$\frac{I}{6}$		$\frac{1}{2}$

Nonsway Analysis



Fixed End Moments

$$M_{FAB} = \frac{-24\times 3^2}{12} = -18\,\text{KN-m}$$

$$M_{FBA} = +18 \text{ kN-m}$$

$$M_{FBC} = M_{FCB} = M_{FCD} = M_{FDC} = 0$$

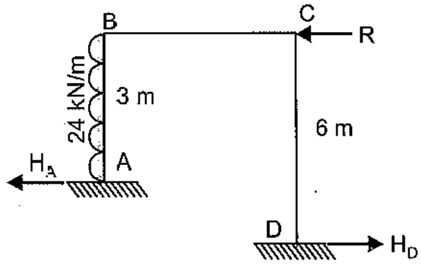
A	B		C		D
	2/3	1/3	1/2	1/2	
–18	18	0	0	0	0
	–12	–6			
–6			–3		
			+1.5	+1.5	
		0.75			0.75
	–0.50	–0.25			
–0.25			–0.125		
			+0.06	+0.06	
		0.03			0.03
	–0.02	–0.01			
–0.01			– 0.005		
			– 0.0025	– 0.0025	
–24.26	+5.48	–5.48	–1.57	+1.57	+0.78

M_{AB} = –24.26 KN-m $\quad$ M_{CB} = –1.57 KN-m

M_{BA} = +5.48 KN-m $\quad$ M_{CD} = +1.57 KN-m

M_{BC} = –5.48 KN-m $\quad$ M_{DC} = 0.78 KN-m

Calculation of actual sway force (R)



$$\Sigma F_H = 0$$

$$\Rightarrow \quad H_A - H_D + R - 24\times 3 = 0$$

$$\Rightarrow \quad R = 72 + H_D - H_A$$

$$H_A = \frac{24.26 - 5.48 + 24\times 3\times 1.5}{3} = 42.26(\leftarrow) \qquad H_D = \frac{0.78 + 1.57}{6} = 0.39\text{kN}(\rightarrow)$$

$\therefore$ Restraining force (R) = 24 × 3 + 0.39 – 42.26 = 30.13 KN $(\leftarrow)$

So, Sway force (R) = 30.13 KN $(\rightarrow)$

Sway Analysis :

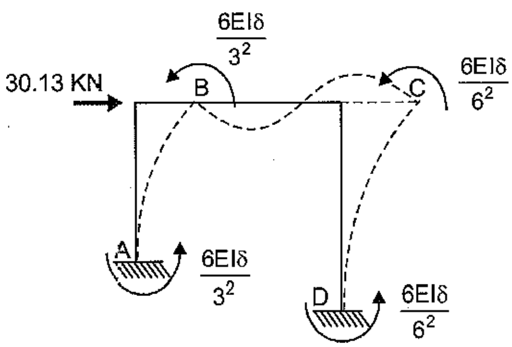
$$M_{FAB} = M_{FBA}$$

$$= \frac{-6EI\delta}{3^2} = \frac{-6EI\delta}{9}$$

$$M_{FBC} = M_{FCB} = 0$$

$$M_{FCD} = M_{FDC}$$

$$= \frac{-6EI\delta}{6^2} = \frac{-6EI\delta}{36}$$



Assume $\quad M_{FAB} = M_{FBA} = -40$ KN-m

$M_{FBC} = M_{FCD} = 0$

$M_{FCD} = M_{FDC} = -10$ KN-m

A		B		C	D
	2/3	1/3	1/2	1/2	
–40	–40	0	0	–10	–10
		13.33	5		
		2.5	6.67		
		–0.83	–3.33		
		–1.67	–0.46		
		0.56	0.23		
		0.11	0.28		
		–0.04	–0.14		
		–0.07	–0.02		
		+0.02	+0.01		
		+13.91	8.24		

$$M_{BC} = 13.91 \text{ KN-m}$$

$$M_{CB} = 8.24 \text{ kNm}$$

From equilibrium eq. $$M_{BA} + M_{BC} = 0$$

$$M_{BA} = -13.91 \text{ kNm}$$

From equilibrium eq $$M_{CB} + M_{CD} = 0$$

$$M_{CD} = -8.24 \text{ kNm}$$

$$M_{AB} = M_{FAB} + \frac{M_{BA} - M_{FBA}}{2} = -40 + \frac{(-13.91)-(-40)}{2}$$

$$= -26.96 \text{kNm}$$

$$M_{DC} = M_{FDC} + \frac{M_{CD} - M_{FCD}}{2} = -10 + \frac{-8.24-(-10)}{2}$$

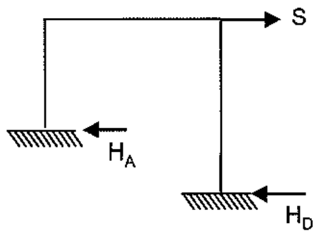
$$= -9.12 \text{ kNm}$$

$M_{AB} = -26.96$ KN-m $\qquad M_{CB} = 8.24$ KN-m

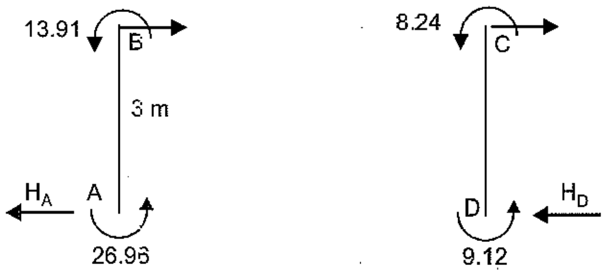
$M_{BA} = -13.91$ KN-m $\qquad M_{CD} = -8.24$ KN-m

$M_{BC} = 13.91$ KN-m $\qquad M_{DC} = -9.12$ KN-m

Calculation of Arbitrary sway froce (S)



$$S = H_A + H_D$$



$$H_A = \frac{13.91 + 26.96}{3} = 13.62\text{KN}(\leftarrow) \qquad H_D = \frac{8.24 + 9.12}{6} = 2.89(\leftarrow)$$

Hence, arbitrary sway force (S) = 13.62 + 2.89 = 16.51 KN

$\therefore$ Correction factor $= \frac{R}{S} = \frac{30.13}{16.51} = 1.825$

Hence actual sway moments are as follows:

$$M_{AB} = -26.96 \times 1.825 = -49.20$$
$$M_{BA} = -13.91 \times 1.825 = -25.38$$
$$M_{BC} = 13.91 \times 1.825 = 25.38$$
$$M_{CB} = 8.24 \times 1.825 = 15.04$$
$$M_{CD} = -8.24 \times 1.825 = -15.04$$
$$M_{DC} = -9.12 \times 1.825 = -16.65$$

Member	Nonsway end moment	Actual sway end moment	Total sway end moment
AB	–24.26	–49.20	–73.46
BA	5.48	–25.38	–19.90
BC	–5.48	25.38	+19.90
CB	–1.57	15.04	13.47
CD	1.57	–15.04	–13.47
DC	0.78	–16.65	–15.87

We have assumed in the sway analysis that the joints B and C sway by δ such that

$$\frac{-6\,EI\delta}{9} = -40$$

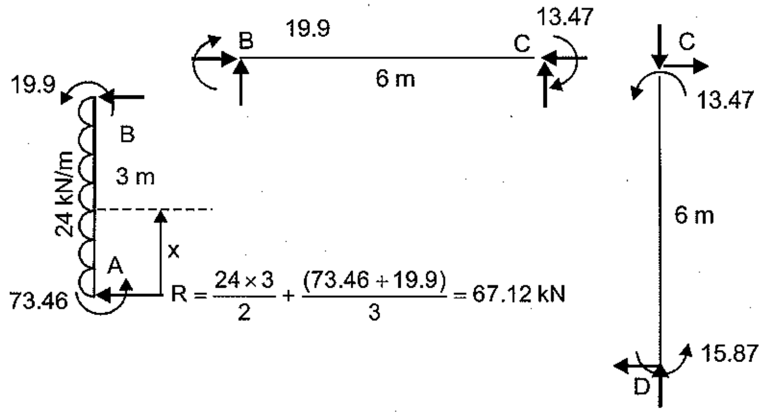
$$\delta = \frac{40 \times 9}{6\,EI}$$

This is arbitrary sway. The actual amount of sway is obtained by multiplying the arbitrary sway by correction factor

$$\delta_{actual} = \frac{40 \times 9}{6EI} \times 1.825 = \frac{109.5}{EI}$$

In this relation if we put EI in kNm^2, we get δ in m

The free body diagram is as given below.



For AB

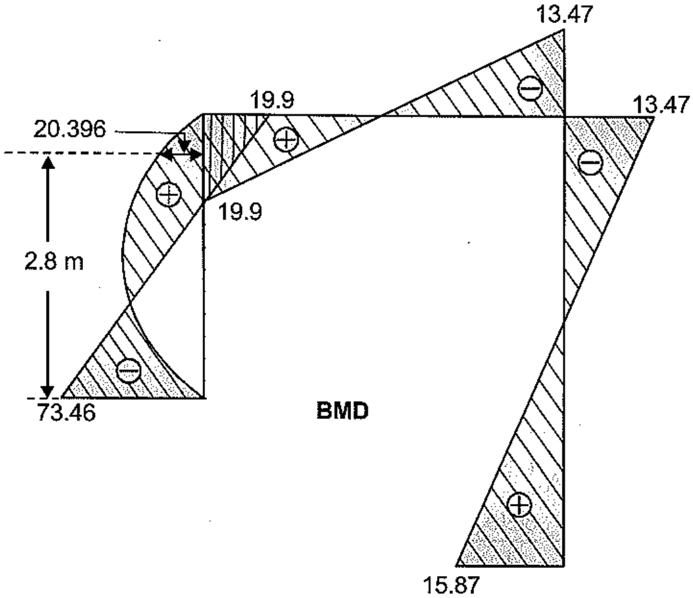
Let SF at distance x = 0

$\Rightarrow$ $$67.12 - 24x = 0$$

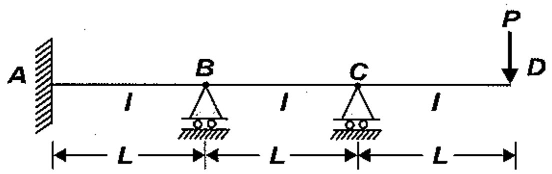
$\Rightarrow$ $$x = 2.8 \text{ m}$$

$$M_{max} = -73.46 + 67.12(2.8) - \frac{24(2.8)^2}{2} = 20.396 \text{ kNm}$$

Hence the BMD is as follows



Q-5: ***The continous beam ABCD shown in the figure is fixed at the end A, simply supported at B and C, and free at the end D. The beam carries a concentrated load P at the free end. Analyse the beam by using***



(i) slope-deflection method, and (ii) moment distribution method.

Also draw bending moment and shear force diagrams.

[20 Marks, ESE-2000]

Sol: **Slope deflection method:**

$$M_{FAB} = M_{FBA} = 0 \qquad \text{[Because no load present]}$$

$$\bar{M}_{FBC} = M_{FCB} = 0$$

M_{FBC} determination to apply in equation

$$M_{BC} = \bar{M}_{FBC} + \frac{3EI}{L}\left(\theta_B - \frac{\delta}{L}\right)$$

Hence $$\bar{M}_{FBC} = \frac{PL}{2}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right) \qquad [\because M_{FAB} = 0, \theta_A = 0, \delta = 0]$$

$$M_{AB} = \frac{2EI\theta_B}{L} \qquad \text{... (i)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right) \qquad [\because M_{FBA} = 0, \theta_A = 0, \delta = 0]$$

$$M_{BA} = \frac{4EI\theta_B}{L} \qquad \text{... (ii)}$$

$$M_{BC} = \bar{M}_{FBC} + \frac{3EI}{L}\left(\theta_B - \frac{\delta}{L}\right)$$

$$M_{BC} = \frac{PL}{2} + \frac{3EI\theta_B}{L} \qquad \text{... (iii)}$$

Equilibrium equation at joint B,

$$M_{BA} + M_{BC} = 0$$

$$\frac{4EI\theta_B}{L} + \frac{PL}{2} + \frac{3EI\theta_B}{L} = 0$$

$$\Rightarrow \qquad \frac{7EI\theta_B}{L} = \frac{-PL}{2}$$

$$\Rightarrow \qquad \theta_B = \frac{-PL^2}{14EI}$$

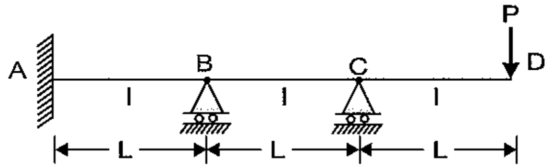
Substituting the value of θ_B in equation (i), (ii), and (iii), we have;

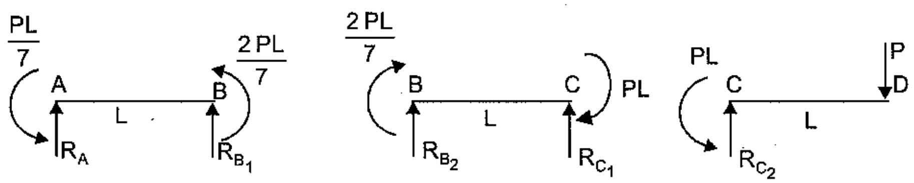
$$M_{AB} = \frac{2EI\theta_B}{L} = \frac{2EI}{L} \times \frac{-PL^2}{14EI} = \frac{-PL}{7}$$

$$M_{BA} = \frac{4EI\theta_B}{L} = \frac{-2PL}{7}$$

$$M_{BC} = \frac{2PL}{7}$$

Hence the free body diagram will look like as shown below





Consider part AB

$$R_A + R_{B_1} = 0 \qquad \text{... (A)}$$

$$\Sigma M_B = 0 \Rightarrow R_A \times L = \frac{3PL}{7}$$

$$\therefore \quad R_A = \frac{3P}{7} \text{ and, } R_{B_1} = -\frac{3P}{7} \text{ [From eqn. (A)]}$$

Consider part BC

$$R_{B_2} + R_{C_1} = 0 \qquad \text{... (B)}$$

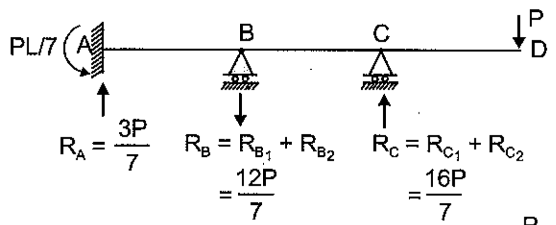
$$\Rightarrow \quad \Sigma M_C = 0$$

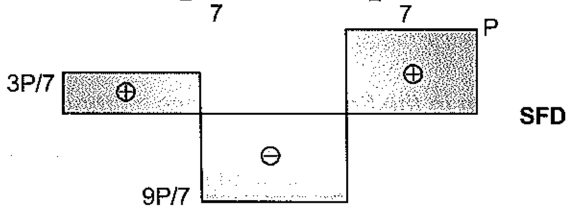
$$\Rightarrow \quad R_{B_2} \times L = -\frac{2PL}{7} - PL \Rightarrow R_{B_2} = \frac{-9P}{7}$$

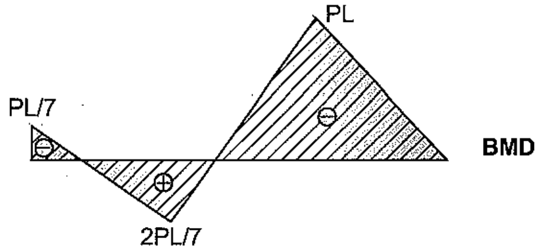
$$\Rightarrow \quad R_{C_1} = \frac{+9P}{7} \text{ [From eqn. (B)]}$$

Consider part CD

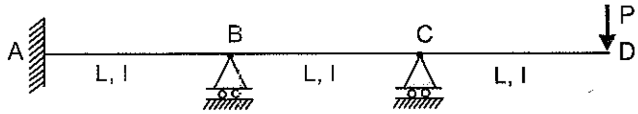
$$R_{C_2} = P \qquad \text{... (C)}$$







Solution by moment distribution method



Distribution Factor

Joint	Members	Relative Stifness	Total stiffness	Distribution factor
B	BA	$\frac{I}{L}$	$\frac{7I}{L}$	$\frac{4}{7}$
	BC	$\frac{3}{4}\frac{I}{L}$		$\frac{3}{7}$

Fixed end moments are all zero

A	B		C	D	
	$\frac{4}{7}$	$\frac{3}{7}$			
0	0	0	0	$-PL$	
		$\frac{PL}{2}$	PL		
0	0	$\frac{PL}{2}$	PL		New FEM
	$\frac{-4PL}{14}$	$\frac{-3PL}{14}$			Bal
$\frac{-2PL}{14}$			0		
$\frac{-2PL}{14}$	$\frac{-4PL}{14}$	$\frac{4PL}{14}$	PL	$-PL$	

$\Rightarrow$

$$M_{AB} = \frac{-PL}{7}$$

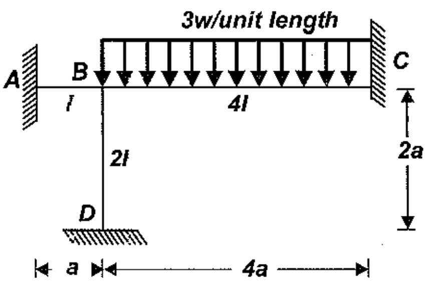
$$M_{BA} = \frac{-2PL}{7}$$

$$M_{BC} = \frac{2PL}{L}$$

$$M_{CB} = PL$$

$$M_{CD} = -PL$$

Q-6: ***Analyse the structure shown in the figure by stiffness method. Draw bending moment diagram and deflected shape.***



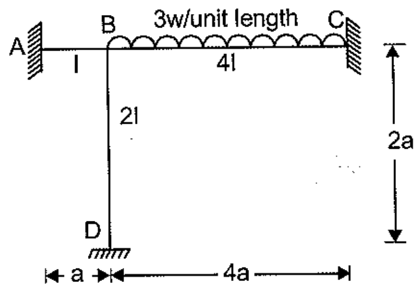
[10 Marks, ESE-2000]

Sol: The various stiffness methods are:

(1) Slope deflection method

(2) Moment distribution method

(3) Stiffness matrix method, etc.



Let us adopt moment distribution method

Fixed end Moments :

$$M_{FAB} = 0 \quad M_{FBA} = 0$$

$$M_{FBC} = -\frac{(3w)\times(4a)^2}{12} = -4wa^2$$

$$M_{FCB} = 4wa^2$$

$$M_{FBD} = M_{FDB} = 0$$

Distribution Factor

Joint	Members	Relative Stifness	Total stiffness	Distribution factor
B	BA	$\frac{I}{a}$	$\frac{3I}{a}$	$\frac{1}{3}$
	BC	$\frac{4I}{4a}$		$\frac{1}{3}$
	BD	$\frac{2I}{2a}$		$\frac{1}{3}$

Let us leave leg BD from the analysis. The moment in this leg will be calculated after M_{BA} and M_{BC} has been determined.

A	B	B	C	
	$\frac{1}{3}$	$\frac{1}{3}$		
0	0	$-4wa^2$	$4\ wa^2$	FEM
	$\frac{4wa^2}{3}$	$\frac{4wa^2}{3}$		
$\frac{2wa^2}{3}$			$\frac{2wa^2}{3}$	C/0
$\frac{2wa^2}{3}$	$\frac{4\ wa^2}{3}$	$\frac{-8wa^2}{3}$	$\frac{14wa^2}{3}$	

$$M_{AB} = \frac{2wa^2}{3}$$

$$M_{BA} = \frac{4wa^2}{3}$$

$$M_{BC} = \frac{-8wa^2}{3}$$

$$M_{CB} = \frac{14\ wa^2}{3}$$

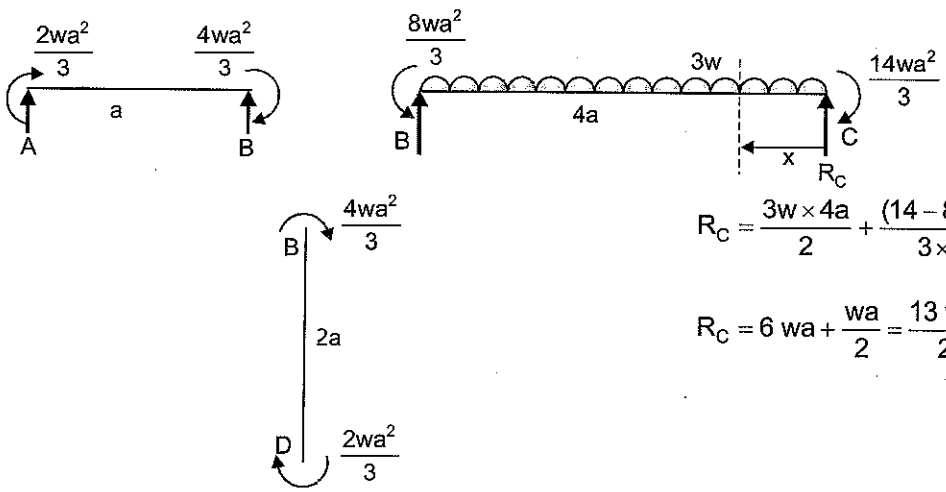
From equilibrium equation,

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$\Rightarrow \quad M_{BD} = -(M_{BA} + M_{BC}) = \frac{4wa^2}{3}$$

$$\Rightarrow \quad M_{DB} = \frac{M_{BD}}{2} = \frac{2wa^2}{3}$$

The free body diagram is as shown below.



$$R_C = \frac{3w \times 4a}{2} + \frac{(14-8)\,wa^2}{3 \times 4a}$$

$$R_C = 6\,wa + \frac{wa}{2} = \frac{13\,wa}{2}$$

Let SF in span BC = 0 at a distance of x from C

$$\Rightarrow \qquad \frac{13\,wa}{2} - 3wx = 0$$

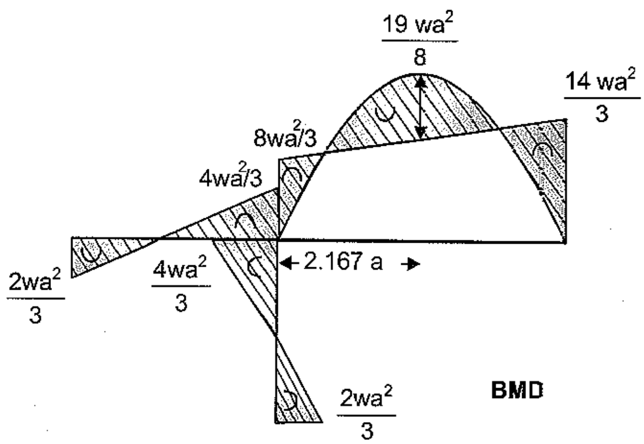
$$x = \frac{13a}{6}$$

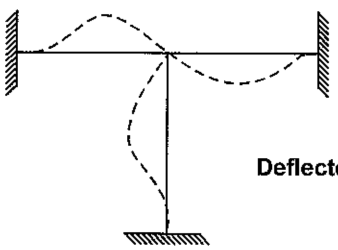
$$\Rightarrow \qquad M_{max} = \frac{13wa}{2} \times \left(\frac{13a}{6}\right) - \frac{14wa^2}{3} - \frac{3w\left(\frac{13\,a}{6}\right)^2}{2}$$

$$= \frac{169\,wa^2}{12} - \frac{56\,wa^2}{12} - \frac{169\,wa^2}{24}$$

$$M_{max} = \frac{19\,wa^2}{8}$$

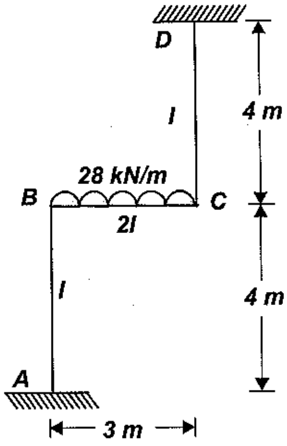
Hence the BMD will be





Deflected shape

Q-7: ***Analyse the rigid frame shown in figure below by moment distribution method.***

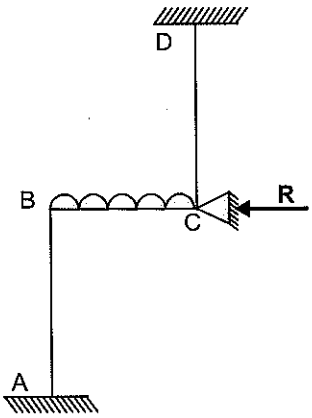


[40 Marks, ESE-2001]

Sol: **Distribution factor:**

Joint	Members	Relative Stifness	Total stiffness	Distribution factor
B	BA	$\frac{I}{4}=\frac{3I}{12}$	$\frac{11I}{12}$	$\frac{3}{11}$
	BC	$\frac{2I}{3}=\frac{8I}{12}$		$\frac{8}{11}$
C	CB	$\frac{2I}{3}=\frac{8I}{12}$	$\frac{11I}{12}$	$\frac{8}{11}$
	CD	$\frac{I}{4}=\frac{3I}{12}$		$\frac{3}{11}$

Nonsway Analysis:



Fixed End Moment,

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = \frac{-28\times 3^2}{12} = -21 \text{ kN-m}$$

$$M_{FCB} = +21 \text{ kN-m}$$

$$M_{FCD} = M_{FDC} = 0$$

A	B		C		D	
	3/11	8/11	8/11	3/11		
0	0	–21	21	0	0	FEM
		15.27	–15.27			Bal
		–7.63	+7.63			C/O
		+5.55	–5.55			Bal
		–2.78	+2.78			C/O
		2.02	–2.02			Bal
		–1.01	+1.01			C/O
		0.74	–0.74			Bal
		–0.37	0.37			C/O
		0.27	–0.27			Bal
		–0.14	0.14			C/O
		0.1	–0.1			Bal
		–8.98	8.98			

$$M_{BC} = -8.98$$
$$M_{CB} = 8.98$$

From equilibrium equation, $V_{BA} + M_{BC} = 0$

$$M_{BA} = +8.98 \text{ kNm}$$

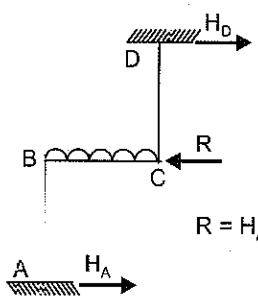
Also as there is no sway and end A is fixed,

$$M_{AB} = \frac{M_{BA}}{2} = 4.49 \text{ kNm}$$

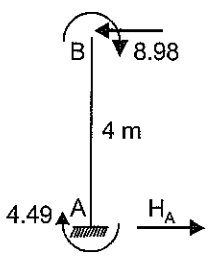
Similarly $M_{CB} + M_{CD} = 0 \Rightarrow M_{CD} = -8.98$ kNm $\left\{\because M_{DC} = \frac{M_{CD}}{2} \text{ as no sway \& end D fixed}\right\}$

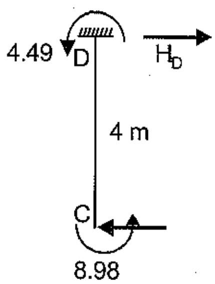
$\Rightarrow M_{DC} = -4.49$ kNm

Calculation of Redundant force (R)



$$R = H_A + H_D$$



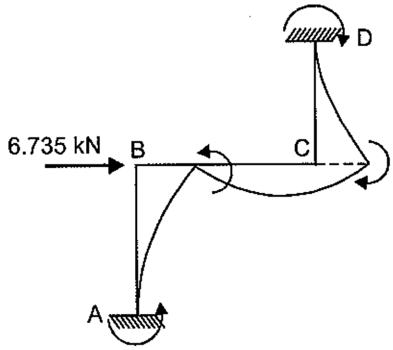


$$H_A = \frac{8.98+4.49}{4} = 3.3675 \text{ kN}(\rightarrow) \qquad H_D = \frac{4.49+8.98}{4} = 3.3675\text{kN}(\rightarrow)$$

$\Rightarrow$ Redundant force (R) = 6.735 kN($\leftarrow$)

and Hence, Sway force = 6.735 kN($\rightarrow$)

Sway analysis



$$M_{FAB} = M_{FBA} = \frac{-6EI\delta}{16}$$

$$M_{FBC} = M_{FCB} = 0$$

$$M_{FCD} = M_{FDC} = \frac{6EI\delta}{16}$$

Hence By Assuming the arbitrary fixed end moments as

$$M_{FAB} = M_{FBA} = -11$$

$$M_{FBC} = M_{FCB} = 0$$

$$M_{FCD} = M_{FDC} = +11$$

A		B	C		D	
	3/11	8/11	8/11	3/11		
–11	–11	0	0	11	11	FEM
		8	–8			Bal
		– 4	4			C/O
		2.91	–2.91			Bal
		– 1.455	1.455			C/O
		1.06	–1.06			Bal
		– 0.53	0.53			C/O
		0.385	–0.385			Bal
		– 0.193	0.193			C/O
		0.14	– 0.14			Bal
		– 0.07	0.07			C/O
		0.05	– 0.05			Bal
		6.297	– 6.297			

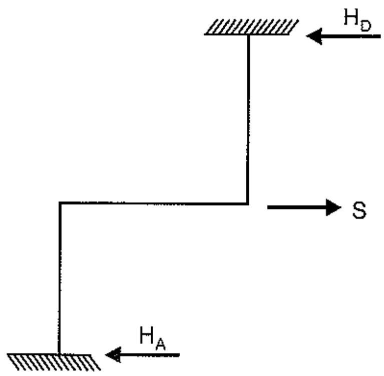
$$M_{BC} = 6.297$$

$$M_{BA} = -6.297 \text{ (Since, } M_{BA} + M_{BC} = 0)$$

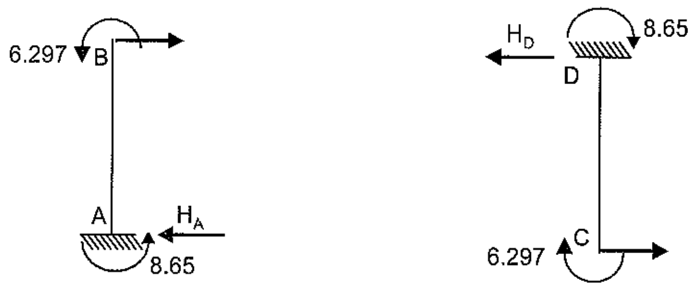
$$M_{AB} = M_{FAB} + \frac{M_{BA} - M_{FBA}}{2} = -11 + \frac{-6.297-(-11)}{2} = -8.65 \text{ kN-m}$$

Similarly, $$M_{CD} = 6.297, M_{DC} = 11 + \frac{6.297-11}{2} = 8.65 \text{ kNm}$$

Calculation of arbitrary sway force (S)



$$\Rightarrow S = H_A + H_D$$



$$H_A = \frac{6.297 + 8.65}{4} = 3.737 \text{ kN} \qquad H_D = \frac{8.65 + 6.297}{4} = 3.737 \text{ kN}$$

$\Rightarrow$ Arbitrary sway force, $S = H_A + H_D$

$\therefore$ Hence, $S = 7.474$ kN

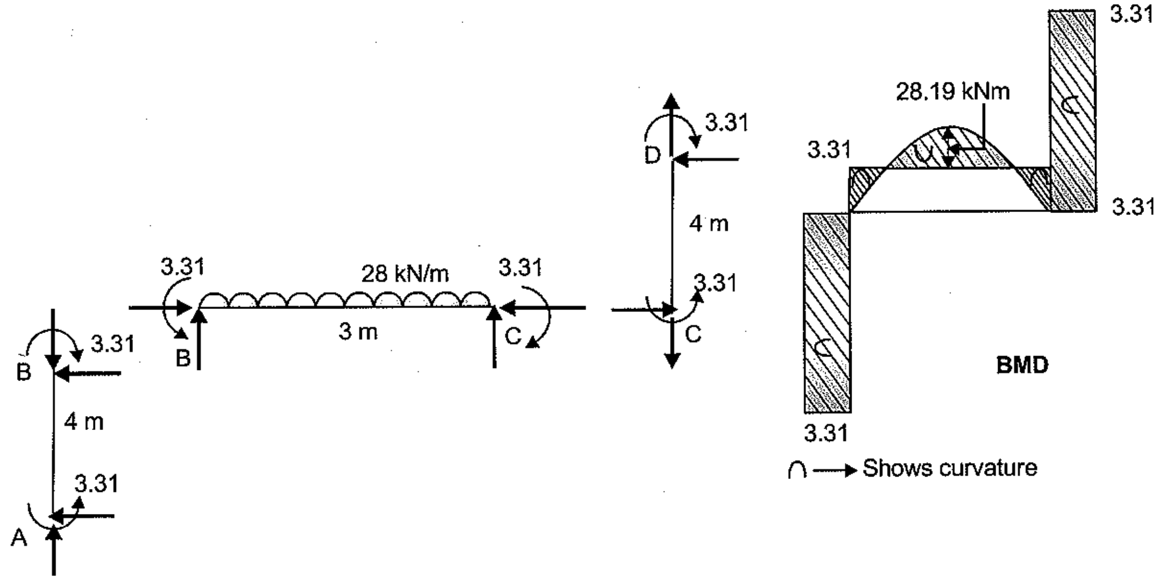
$\therefore$ Correction factor $= \frac{R}{S} = \left(\frac{6.735}{7.474}\right) = 0.9011$

Actual Sway Moments

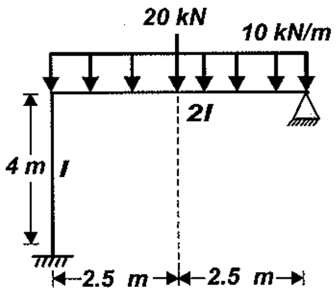
$$M_{AB} = -8.65 \times 0.9011 = -7.80 \text{ kN-m}$$
$$M_{BA} = -6.297 \times 0.9011 = -5.67 \text{ kN-m}$$
$$M_{BC} = 6.297 = 0.9011 = +5.67 \text{ kN-m}$$
$$M_{CB} = -6.297 \times 0.9011 = -5.67 \text{ kN-m}$$
$$M_{CD} = 6.297 \times 0.9011 = +5.67 \text{ kN-m}$$
$$M_{DC} = 8.65 \times 0.9011 = +7.80 \text{ kN-m}$$

Members	Non Sway Moment	Actual Sway moment	Total Moment
AB	4.49	–7.80	–3.31
BA	8.98	–5.67	3.31
BC	– 8.98	5.67	–3.31
CB	8.98	–5.67	3.31
CD	– 8.98	5.67	–3.31
DC	–4.49	7.80	3.31

Thus the free body diagram will be as shown below ;

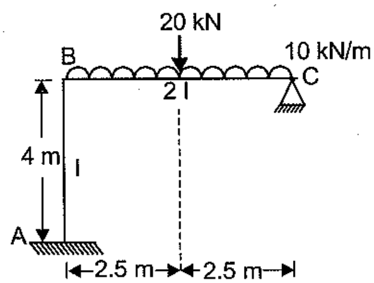


Q-8: ***Use stiffness method to analyse the rigid frame shown in the figure below. Draw neatly the bending moment diagram.***



[20 Marks, ESE-2003]

Sol:



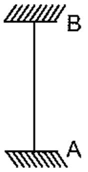
Let us use moment distribution method as it is also a stiffness method of analysis.

This is a case of no-sway (the frame being restrained frame)

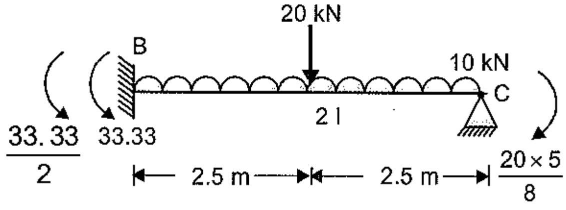
Distribution factor calculation

Joint	Members	Relative Stifness	Total stiffness	Distribution factor
B	BA	$\frac{I}{4}$	$\frac{11I}{20}$	$\frac{5}{11}$
	BC	$\frac{3}{4}\times\frac{2I}{5}$		$\frac{6}{11}$

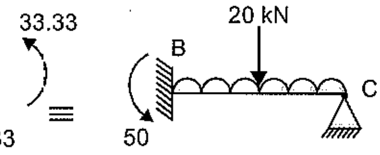
Fixed End moment



$$M_{FAB} = M_{FAB} = 0$$



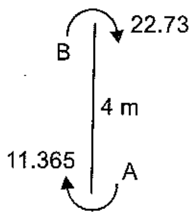
$$\frac{20\times 5}{8} + \frac{10\,(5)^2}{12} = 33.33$$

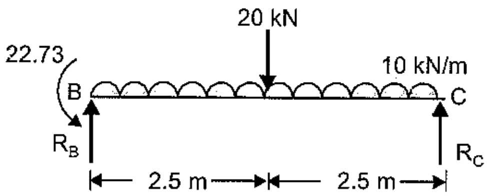


$$\overline{M}_{FBC} = -50\ \text{kNm}$$

A		B		C	
	$\frac{5}{11}$	$\frac{6}{11}$			
0	0	−50		0	FEM
	22.73	27.27			Bal
11.365				0	
11.365	22.73	−22.73		0	

The free body diagram will look like as shown below;





$$R_B + R_C = 5 \times 10 + 20 = 70\ \text{kN}$$

$$\Sigma M_C = 0$$

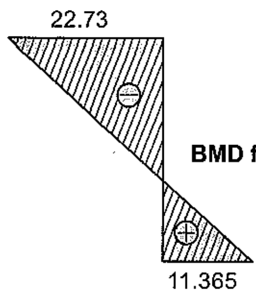
$$\Rightarrow \quad R_B \times 5 - 22.73 - 20\times 2.5 - 10\times 5\times 2.5 = 0$$

$$\Rightarrow \quad R_B = 39.546\ \text{kN}$$

$$R_C = 30.454\ \text{kN}$$

BMD for AB

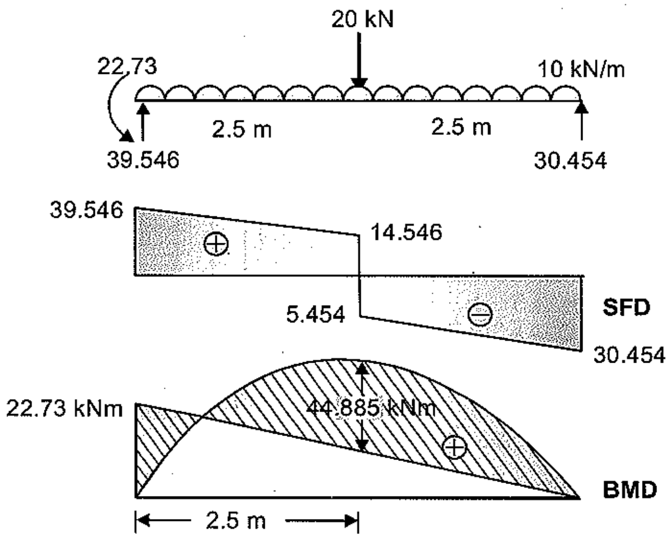
End moment diagram will be the resultant BMD for part AB because there is no load on span AB and hence, free BMD does not exist.



BMD for AB

BMD for part BC

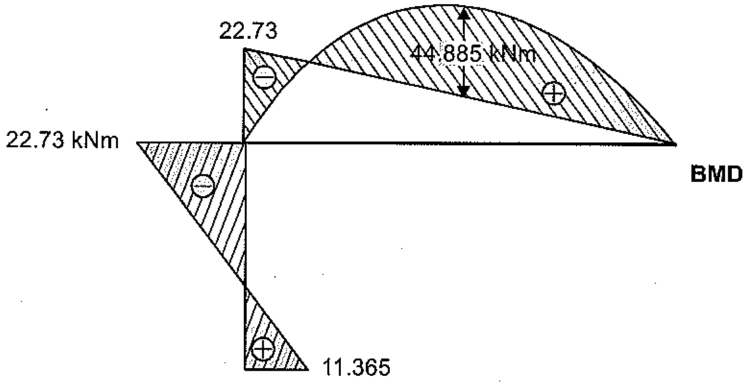
As there is point load and udl on the span, the location of max BM could be either below the point load or at the point of zero shear. Hence let us Ist of all draw the SFD.



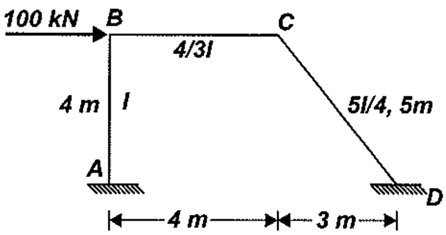
Looking at the SFD, we conclude that max BM will occur only below the point load because there is no point of zero shear in this case:

$$\text{Max BM below point load} = -22.73 + 39.546 \times 2.5 - 2.5 \times 10 \times 1.25$$
$$= 44.885 \text{ kNm}$$

Hence, BMD of the whole frame is as shown



Q-9: ***Analyse the frame ABCD shown in figure. using moment distribution method. The properties of the sections geometry of the frame and the loading are given in figure. Draw the B.M. and axial force diagrams.***

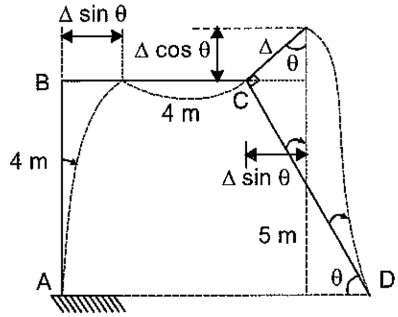


[40 Marks, ESE-2004]

Sol: As the loading is acting only at joint hence FEM = 0.

$\Rightarrow$ Only sway analysis needs to be performed.

Joint	Member	Relative Stiffness	DF
B	BA	I/4	3/7
	BC	$\frac{4}{3}\frac{I}{4}=\frac{I}{3}$	4/7
C	CB	$\frac{4}{3}\frac{I}{4}=\frac{I}{3}$	4/7
	CD	$\frac{5}{4}\times\frac{I}{5}=\frac{I}{4}$	3/7



$$M_{FAB} = \frac{-6EI\Delta\sin\theta}{l^2} = \frac{-6EI\Delta\times\frac{4}{5}}{16} = \mathbf{-0.3EI\Delta}$$

$$M_{FBA} = -0.3EI\Delta$$

$$M_{FBC} = \frac{6E\left(\frac{4}{3}I\right)\Delta\cos\theta}{16} = \frac{\frac{6\times4}{3}\times EI\Delta\times\frac{3}{5}}{16} = 0.3EI\Delta$$

$$M_{FCB} = 0.3EI\Delta$$

$$M_{FCD} = \frac{-6E\left(\frac{5}{4}I\right)\Delta}{25} = -0.3EI\Delta$$

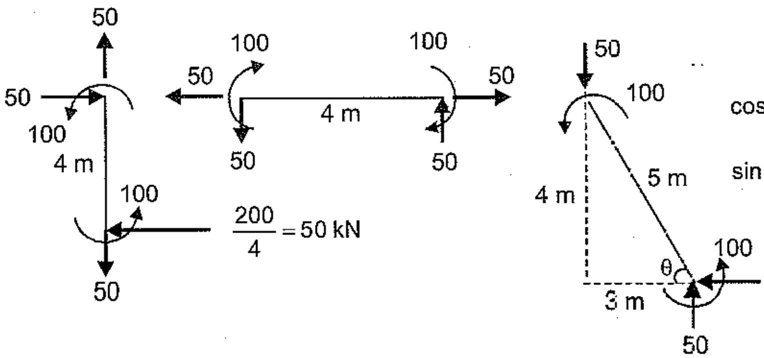
$$M_{FDC} = -0.3EI\Delta$$

Take an arbitrary value of 0.3 EIΔ as 100

	A	B		C		D
DF		3/7	4/7	4/7	3/7	
FEM	–100	–100	100	100	–100	–100

Note that joints are balanced hence no further distribution needs to be carried out.

The free body diagram is as shown below.



$$\frac{200}{4} = 50\text{ kN}$$

$$\cos\theta = \frac{3}{5} \qquad \sin\theta = \frac{4}{5}$$

$$\frac{200}{5\sin\theta} + \frac{50\times3}{4} = \frac{200}{5\times\frac{4}{5}} + \frac{150}{4}$$

$$= \frac{350}{4} = 87.5\text{ kN}$$

Total sway force corresponding to assumed deflection

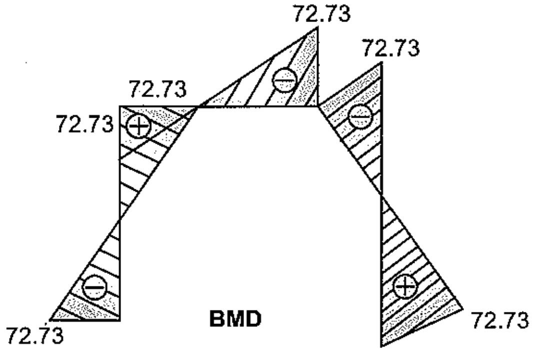
$$= 50 + 37.5 = 137.5\text{kN} \rightarrow$$

$$\Rightarrow \qquad \text{Correction factor} = \frac{100}{137.5} = 0.7273$$

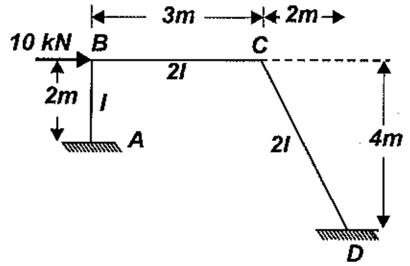
Hence corrected moments are

A	B		C		D
−72.73	−72.73	72.73	72.73	−72.73	−72.73

The BMD is as shown below.

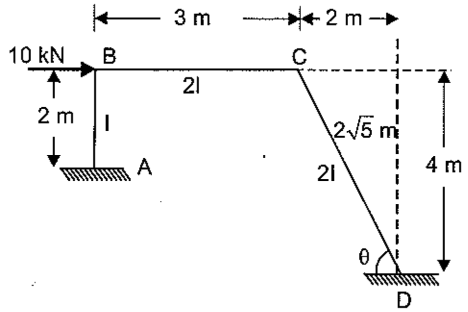


Q-10: ***Analyse the frame ABCD shown in the figure using moment distribution method. The geometry of the frame, loading and properties are shown in the figure. Also draw the Bending Moment diagram.***



[20 Marks, ESE-2008]

Sol:



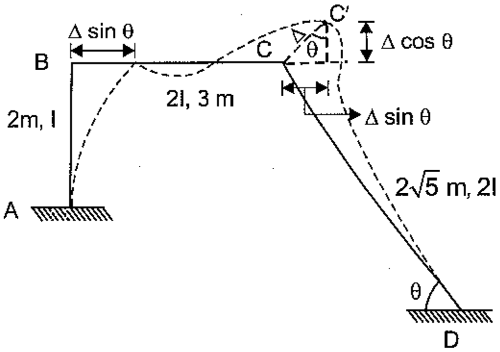
$$\tan\theta = \frac{4}{2} = 2 \qquad \sin\theta = \frac{2}{\sqrt{5}} \qquad \cos\theta = \frac{1}{\sqrt{5}}$$

Distribution factor:

Joint	Members	Stiffness	Total stiffness	Distribution factor
B	BA	$\frac{4EI}{2}$	$\frac{14\,EI}{3}$	$\frac{3}{7}$
	BC	$\frac{4EI\,(2I)}{3}$		$\frac{4}{7}$
C	CB	$\frac{4E\,(2I)}{3}$	4.555 EI	$0.5985 \simeq 0.6$
	CD	$\frac{4EI\,(2I)}{2\sqrt{5}}$		$0.4015 \simeq 0.4$

We need to perform only sway analysis as non sway analysis is not required

Sway analysis



Fixed end moments

$$M_{FAB} = \frac{-6EI\,\Delta \sin\theta}{4} = -\frac{6\times 2}{\sqrt{5}\times 4}EI\Delta = -1.35\,EI\Delta$$

$$M_{FBA} = -1.35\,EI\Delta$$

$$M_{FBC} = +\frac{6E\,(2I)\,\Delta\cos\theta}{9} = \frac{12\cos\theta}{9}EI\Delta = 0.6\,EI\Delta$$

$$M_{FCB} = 0.6\,EI\Delta$$

$$M_{FCD} = \frac{-6EI\,(2I)\Delta}{(2\sqrt{5})^2} = -0.6\,EI\Delta$$

$$M_{FDC} = -0.6\,EI\Delta$$

	A		B		C	D	
		3/7	4/7	0.6	0.4		
	– 1.35 EIΔ	– 1.35 EIΔ	0.6 EIΔ	0.6 EIΔ	–0.6 EIΔ	0.6 EIΔ	FEM
Assumed FEM	– 45	– 45	20 14.29	20	– 20	– 20	FEM Bal
				7.15 – 4.29			C/O Bal
			– 2.15 1.23				C/O Bal
				0.615 – 0.37			C/O Bal
			– 0.185 0.106				C/O Bal
				0.053 – 0.032			C/O Bal
			33.29	23.126			

From Joint equilibrium equation

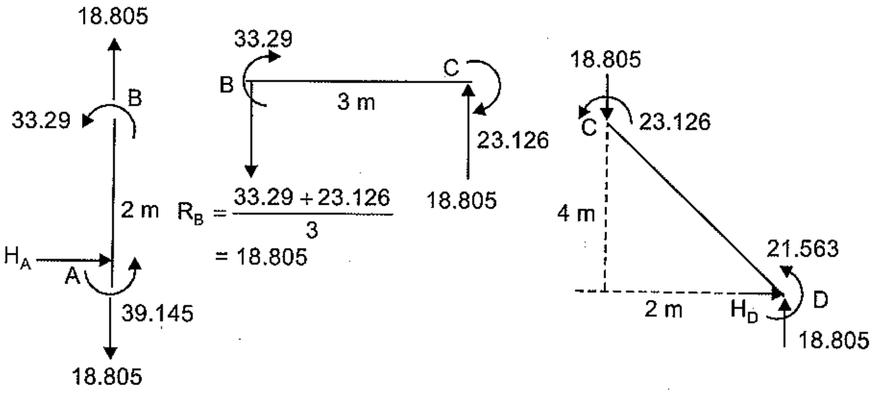
$$M_{BC} + M_{BA} = 0$$

$$\Rightarrow \quad \boxed{M_{BA} = -33.29}$$

Similarly $$\boxed{M_{CD} = -23.126}$$

$$M_{AB} = M_{FAB} + \frac{M_{BA} - M_{FBA}}{2} = -45 + \frac{(-33.29 - (-45))}{2} = -39.145 \text{ kNm}$$

$$M_{DC} = M_{FDC} + \frac{M_{CD} - M_{FCD}}{2} = -20 + \frac{(-23.126) - (-20)}{2} = -21.563 \text{ kNm}$$



$$H_A = \frac{M_{AB} + M_{BA}}{2} = \frac{-39.145 - 33.29}{2} = -36.2175$$

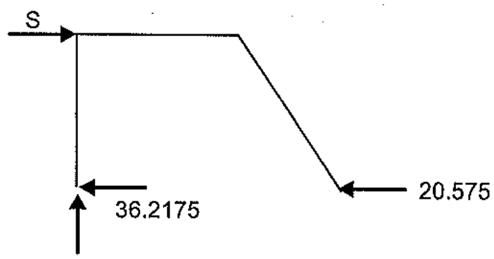
$$\Sigma M_C = 0$$

$$H_D \times 4 + 18.805 \times 2 + 21.563 + 23.126 = 0$$

$$H_D = -20.575 \text{ kN}$$

$$\Rightarrow \quad S = 36.2175 + 20.575$$

$$= 56.7925 \text{ kN } (\rightarrow)$$



Actual sway force = 10 kN

$$\Rightarrow \quad \text{Correction factor} = \frac{10}{56.7925} = 0.176$$

Corrected sway moments are

$$M_{AB} = -39.145 \times 0.176 = -6.89 \text{ kNm}$$

$$M_{BA} = -33.29 \times 0.176 = -5.86 \text{ kNm}$$

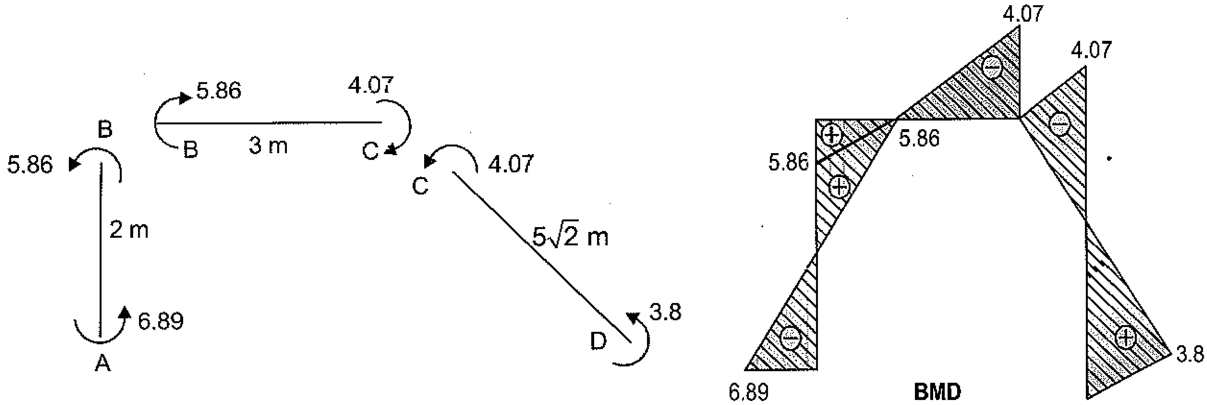
$$M_{BC} = 5.86 \text{ kNm}$$

$$M_{CB} = 23.126 \times 0.176 = 4.07 \text{ kNm}$$

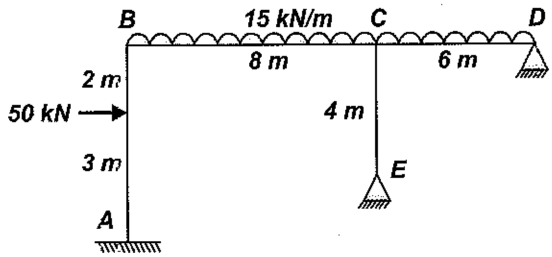
$$M_{CD} = -4.07 \text{ kNm}$$

$$M_{DC} = -21.563 \times 0.176 = -3.8 \text{ kNm}$$

Free body diagram showing end moment



Q-11: ***Analyse the rigid frame shown in the figure below by moment distribution method taking flexural rigidity EI to be uniform for all members.***



[20 Marks, ESE-2012]

Sol: This problem is a problem of no-sway.

As discussed in the previous problem, the member *BA* and *CE* can be completely left out and finally they are calculated using joint equilibrium equations

$$M_{BA} + M_{BC} = 0$$

and
$$M_{CB} + M_{CE} + M_{CD} = 0$$

Once M_{CE} is known, M_{EC} will be calculated. In this case $M_{EC} = 0$ as end E is hinged.

Once M_{BA} is known M_{AB} can be calculated.

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(\theta_B)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B)$$

Thus,
$$M_{AB} = M_{FAB} + \frac{M_{BA} - M_{FBA}}{2}$$

Now, M_{BA}, M_{FAB}, M_{FBA} being known, M_{A3} can be calculated.

Joint	Member	Relative stiffness	DF
B	BA	$\frac{4EI}{5} = 0.8\,EI$	0.615
	BC	$\frac{4EI}{4} = 0.5\,EI$	0.385
C	CB	$\frac{4EI}{8} = 0.5\,EI$	0.286
	CD	$\frac{3EI}{6} = 0.5\,EI$	0.286
	CE	$\frac{3EI}{4} = 0.75\,EI$	0.428

$$M_{FAB} = -\frac{50\times 3(2)^2}{25} = -24 \text{ kNm}$$

$$M_{FBA} = +\frac{50\times 2(3)^2}{25} = 36 \text{ kNm}$$

$$M_{FBC} = -\frac{15(8)^2}{12} = -80 \text{ kNm}$$

$$M_{FCB} = 80 \text{ kNm}$$

$$M_{FCD} = \frac{15(6)^2}{12} = -45 \text{ kNm}$$

$$M_{FDC} = 45 \text{ kNm}$$

Joint	A		B		C		
Member	AB		BC	CB	CD	DC	
			0.385	0.286	0.286		
	–24	36	–80	80	–45 –22.5	45 –45	FEM
1.	–24	36	–80	80	–67.5	0	Final FEM
2.			16.94	–3.575	–3.575		
3.			–1.788	8.47			
4.			+0.688	–2.42	–2.42		
5.			–1.21	0.344			
6.			0.466	–0.098	–0.098		
7.			–0.049	0.233			
8.			0.02	–0.067	–0.067		
Σ1–8			–64.93	82.89	–73.66		

$$M_{BC} = -64.93 \text{ kNm}$$

$$M_{CB} = 82.89 \text{ kNm}$$

$$M_{CD} = -73.66 \text{ kNm}$$

$$M_{BA} + M_{BC} = 0$$

$$M_{BA} = -M_{BC} = 64.93$$

$$M_{CB} + M_{CD} + M_{CE} = 0$$

$$\Rightarrow \quad M_{CE} = -(82.89 - 73.66) = -9.23 \text{ kNm}$$

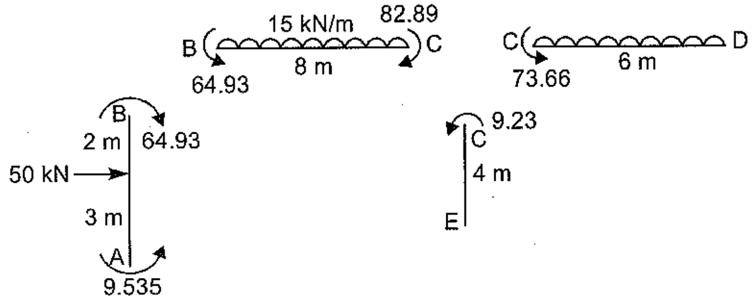
$$M_{AB} = M_{FAB} + \frac{M_{BA} - M_{FBA}}{2} = -24 + \frac{64.93-36}{2}$$

$$M_{AB} = -9.535 \text{ kNm}$$

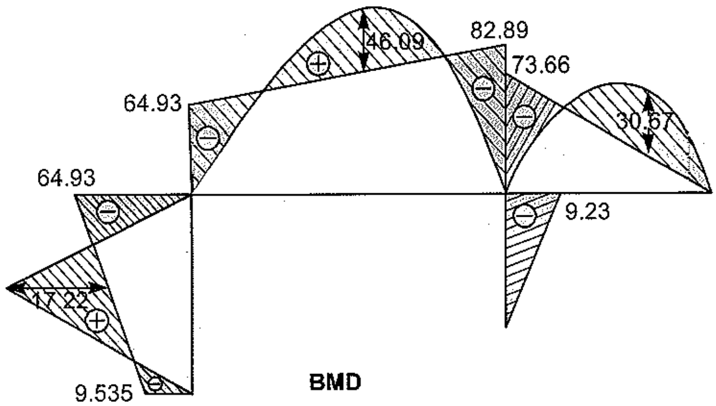
$$M_{EC} = 0$$

$$M_{DC} = 0$$

The free body diagram is as shown below.

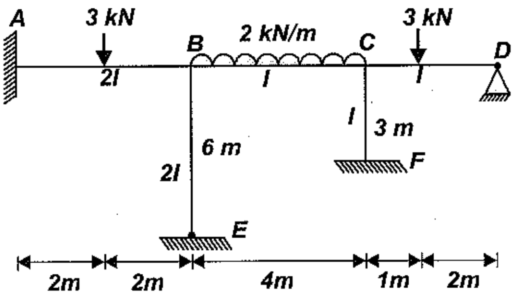


Based on the free body diagram BMD will be as shown below



BMD

Q-12: ***Find the end moments of the frame shown below use moment distribution method. Also draw BMD***



[20 Marks, ESE-2013]

Sol: **Distribution factor:**

Joint	Members	Relative Stifness	Total stiffness	Distribution factor
B	BA	$\frac{2I}{4}$	$\frac{4I}{4}$	$\frac{1}{2}$
	BE	$\frac{3}{4}\times\frac{2I}{6}$		$\frac{1}{4}$
	BC	$\frac{I}{4}$		$\frac{1}{4}$
C	CB	$\frac{I}{4}$	$\frac{10I}{12}$	0.3
	CF	$\frac{I}{3}$		0.4
	CD	$\frac{3}{4}\times\frac{I}{3}$		0.3

Fixed end Moments

$$M_{FAB} = \frac{-3\times 4}{8} = -1.5$$

$$M_{FBA} = 1.5$$

$$M_{FBE} = M_{FEB} = 0$$

$$M_{FBC} = \frac{-2\times 16}{12} = -2.67$$

$$M_{FCB} = 2.67$$

$$M_{FCD} = \frac{-3\times 1\times 4}{9} = -1.33$$

$$M_{FDC} = \frac{3\times 2\times 1}{9} = +0.67$$

For easy analysis, we will leave leg BE and CF and once M_{BA} and M_{BC} is known, M_{BE} can be calculated from equilibrium eq $M_{BA} + M_{BC} + M_{BE} = 0$

$$M_{EB} = 0 \qquad [\because \text{ Hinged end}]$$

Similarly from $M_{CB} + M_{CD} + M_{CF} = 0$, M_{CF} can be calculated and $M_{FC} = \dfrac{M_{CF}}{2}$

A		B		C	D	
	$\frac{1}{2}$	$\frac{1}{4}$	0.3	0.3		
–1.5	1.5	–2.67	2.67	–1.33	0.67	FEM
				–0.33	–0.67	
–1.5	1.5	–2.67	2.67	–1.66	0	New FEM
	0.585	0.2925	–0.30	–0.30		Bal
0.2925		–0.15	0.146		0	C/O
	0.075	0.0375	–0.0438	–0.0438		Bal
0.0375		–0.0219	0.01875		0	C/O
– 1.17	2.16	– 2.51	2.49	– 2.00	0	

$$\boxed{M_{AB} = -1.17\text{ kNm}}$$

$$\boxed{M_{BA} = 2.16\text{ kNm}}$$

$$\boxed{M_{BC} = -2.51\text{ kNm}}$$

$$M_{BA} + M_{BC} + M_{BE} = 0$$

$$\Rightarrow \quad \boxed{M_{BE} = 0.35\text{ kNm}}$$

$$\boxed{M_{EB} = 0}$$

$$\boxed{M_{CB} = 2.49\text{ kNm}}$$

$$\boxed{M_{CD} = -2.00\text{ kNm}}$$

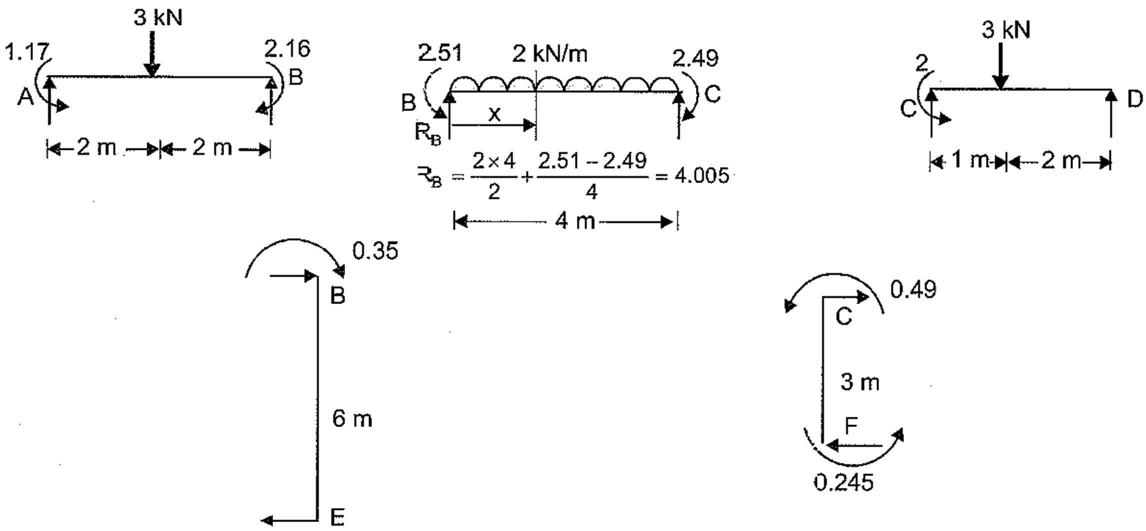
$$M_{CB} + M_{CD} + M_{CF} = 0$$

$$\Rightarrow \quad \boxed{M_{CF} = -0.49\text{ kNm}}$$

$$\boxed{M_{FC} = \frac{M_{CF}}{2} = -0.245\text{ kNm}}$$

$$\boxed{M_{DC} = 0}$$

The free body diagram will look like as shown below

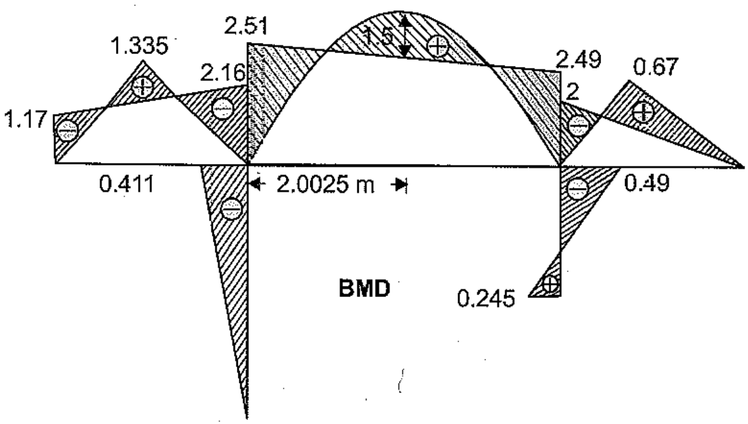


Let BM in B be max at a distance x from end B

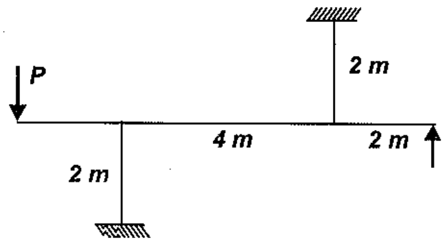
$\Rightarrow \quad (SF)_x = 0 \qquad \Rightarrow \quad 4.005 - 2x = 0$

$\Rightarrow \quad x = 2.0025 \text{ m}$

$$M_{max} = -2.51 + 4.005\,x - \frac{2x^2}{2} = 1.5 \text{ kNm}$$

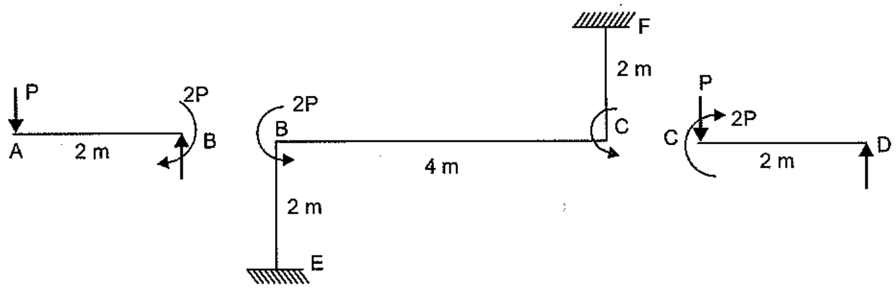


Q-13: ***Sketch the elastic curve for the frame as shown below.***



[4 Marks, ESE-2013]

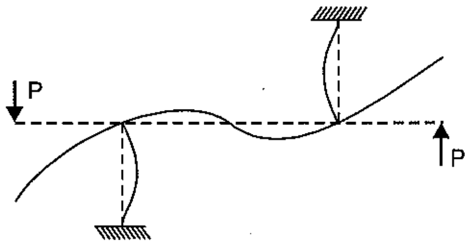
Sol: FBD of structure is



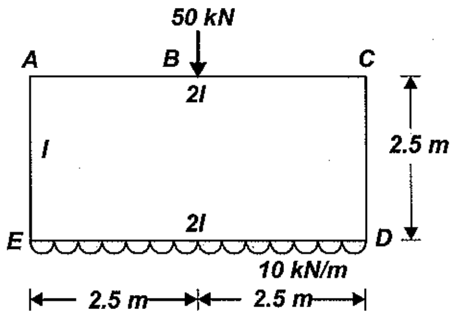
If we cut a section from middle of the frame EBCF, the left side sways to the left and right side sways to the right and the magnitude of moment being same, we can conclude that the frame will not sway.

Alternatively, the frame is anti-symmetric with anti-symmetric loading, thus making the frame behave as a symmetical condition. Thus there is no sway.

Thus the resultant deflected shape is as shown below:

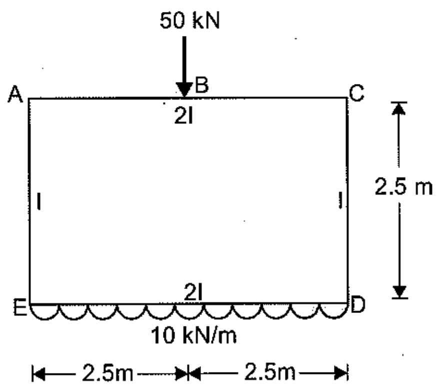


Q-14: ***Analyze the plane box frame shown in figure the moment distribution method and making use of symmetry. Also, draw bending moment diagram.***

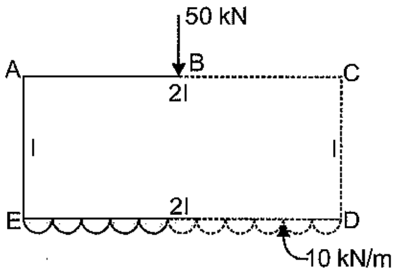


[15 Marks, ESE-2014]

Sol:



The structure is symmetrical, hence symmetrical beam approach will be used. We will consider only half of the structure i.e.,



Joint	Member	Stiffness	D.F.
A	AC	$2E \times \frac{2I}{5}$	0.33
	AE	$4E \times \frac{I}{2.5}$	0.67
E	EA	$\frac{4EI}{2.5}$	0.67
	ED	$2E \times \frac{2I}{5}$	0.33

Fixed end moments

$$M_{FAC} = -\frac{PL}{8} = -\frac{50 \times 5}{8} = -31.25$$

$$M_{FAE} = 0$$

$$M_{FEA} = 0$$

$$M_{FED} = \frac{WL^2}{12} = \frac{10 \times 5^2}{12} = 20.83$$

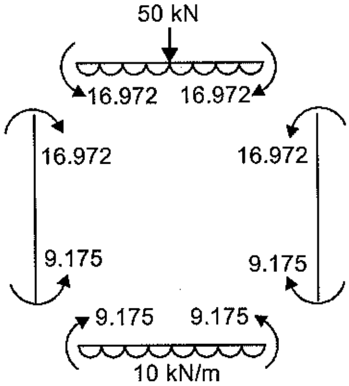
Joint / Member	A: AC	A: AE	E: EA	E: ED
D.F.	0.33	0.67	0.67	0.33
FEM	–31.250	0	0	20.830
Bal	10.420	20.830	–13.890	–6.940
CO	0	–6.945	10.415	0
Bal	2.315	4.630	–6.943	–3.472
CO	0	–3.472	2.315	0
Bal	1.157	2.315	–1.543	–0.772
CO	0	–0.772	1.157	0
Bal	0.257	0.515	–0.772	–0.385
CO	0	–0.386	0.257	0
Bal	0.129	0.257	–0.171	–0.086
Sum	–16.972	16.972	–9.175	9.175

$\Rightarrow$ $$M_{AC} = -16.972$$

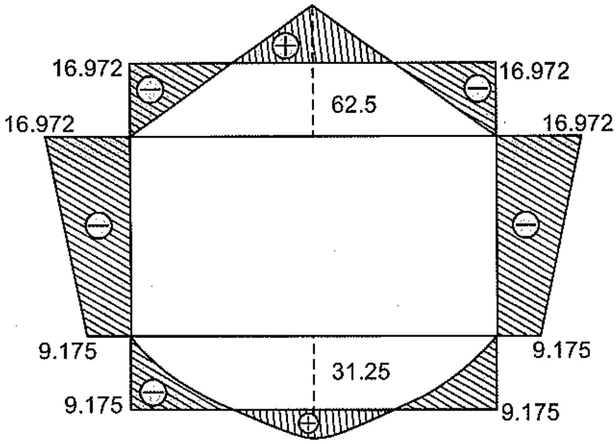
$\therefore$ From symmetry $M_{CA} = 16.972$

Similarly other end moment can be determined.

The free body diagram is as shown

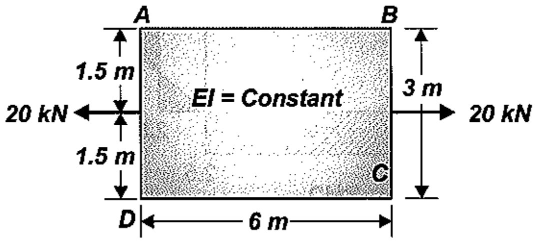


BMD will be (considering sagging as positive and hogging as negative) $\left[M = \frac{PL}{4} = \frac{50 \times 5}{4} = 62.5\right]$



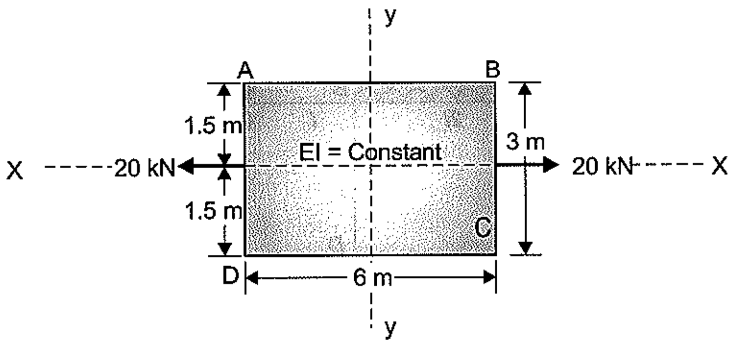
$$\left[M = \frac{\omega L^2}{8} = \frac{10 \times 25}{8} = 31.25\right]$$

Q-15: ***Analyse the box-frame shown in the figure below by Moment Distribution Method. Draw Bending Moment diagram with relevant values.***



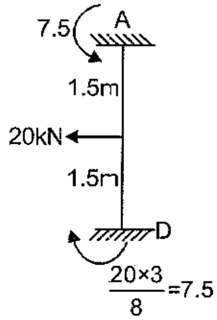
[20 Marks, ESE-2015]

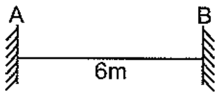
Sol:



Since there is symmetry about X - X & Y - Y axis, we will analyze only joint A of the box frame.

FEMs:





M_{FAD} = – 7.5 kNm; M_{FAB} = 0

Distribution Factor

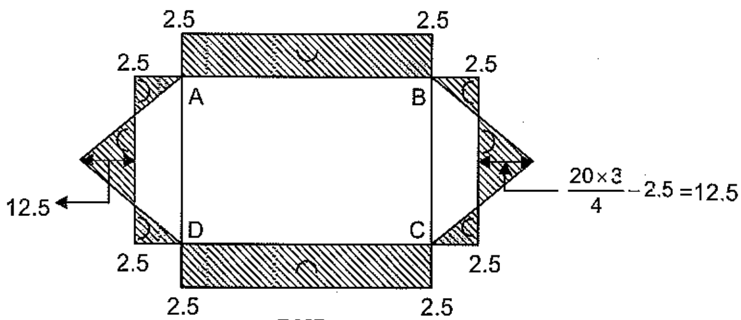
Joint	Member	Member stiffness	Joint stiffness	D.F.
A	AD	$\frac{2EI}{3}$	EI	2/3
	AB	$\frac{2EI}{6}=\frac{EI}{3}$		1/3

D	A (AD)	A (AB)	B	
	2/3	1/3		
	–7.5	0		FEM
	5	2.5		
	–2.5	2.5		

$\Rightarrow$ M_{AB} = 2.5 kNm

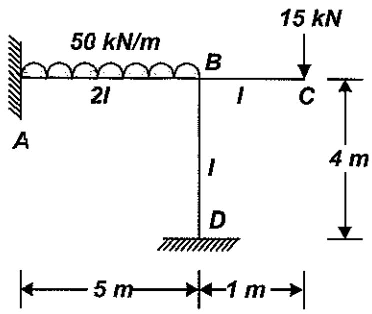
M_{AD} = – 2.5 kNm

Similarly, M_{BA} = – 2.5 kNm; M_{BC} = 2.5 kNm



BMD

Q-16: ***Analyze the frame shown in figure by the moment distribution method and draw bending moment diagram.***



[15 Marks, ESE-2016]

Sol: Fixed end moments

$$M^F_{AB} = \frac{-50(5)^2}{12} = -104.167 \text{ kNm}$$

$$M^F_{BA} = \frac{50(5)^2}{12} = 104.167 \text{ kNm}$$

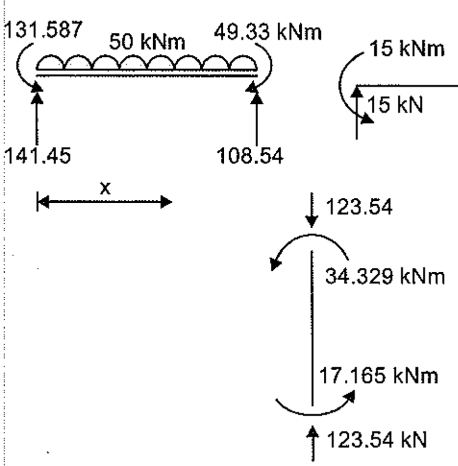
Distribution and carry-over factors

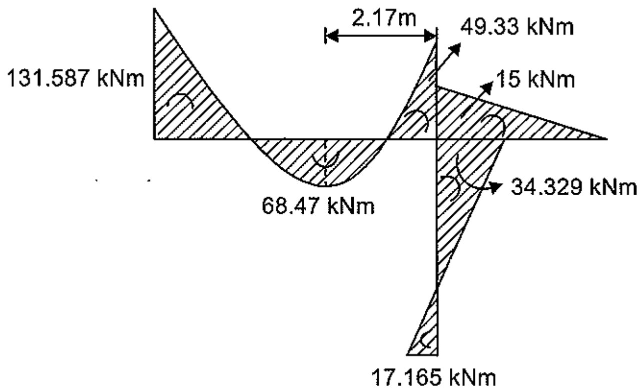
$$K_{BA} : K_{BD} = \frac{4(2EI)}{5} : \frac{4EI}{4} = \frac{8}{5} : 1$$

$$d_{BA} = 0.615$$

$$d_{BD} = 0.385$$

	AB	BA	BC	BD	DB
DF		0.615		0.385	
FEM Bal.	–104.167	104.167 –54.837	–15	0 –34.329	0
CO	–27.42				–17.165
Final Moments	–131.587	49.33	–15	–34.329	–17.165

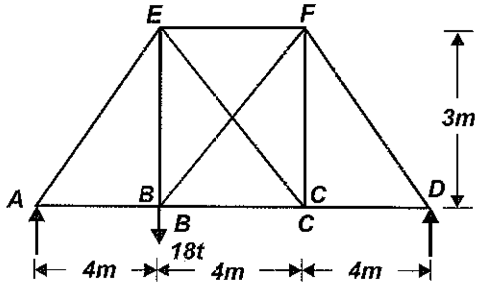




CHAPTER 5

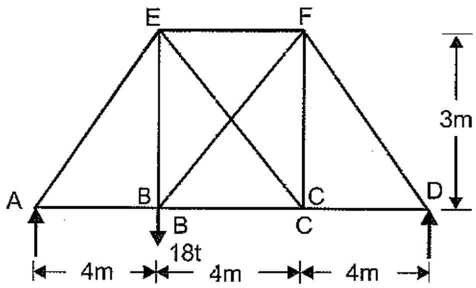
TRUSSES

Q-1: ***All the members of the steel truss shown in the fig may be assumed pin-jointed. Calculate force in all the members. Area of cross-section of all the members is same and equal to 30 sq cm.***



[20 Marks, ESE-1996]

Sol:



Degree of statical indeterminacy of given truss = $D_S = m + r_e - 2j$

here $m = 10$, $r_e = 3$ and $j = 6$

$\therefore$ $$D_S = 10 + 3 - 12 = 1$$

Degree of external indeterminacy = $r_e - 3 = 3 - 3 = 0$

Hence the truss has 1 degree of internal indeterminacy.

We can take either any member force or any support reaction as redundant provided that removal of the member or the support reaction does not make the structure unstable. Let us take force in member BF as redundant

As per unit load method (Maxwell method)

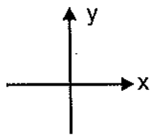
$$\text{Redundant force, } R = \frac{-\sum_{i=1}^{n-1} \dfrac{P_i u_i l_i}{A_i E_i}}{\sum_{i=1}^{n} \dfrac{u_i^2 l_i}{A_i E_i}}$$

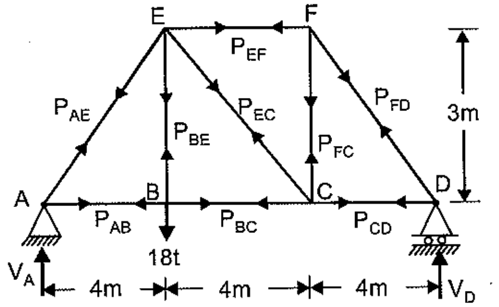
where P_i = Member forces in i^{th} member due to external load after removal of redundant member.

u_i = Member force in i^{th} member due to unit load applied at joint B and F along member BF

l_i, A_i, E_i are length, area and modulus of elasticity of i^{th} member.

Member forces due to external load after removal of redundant member (P_i)





Calculation of reactions

From, $\Sigma F_Y = 0$

$\Rightarrow$ $$V_A + V_D - 18 = 0 \quad \text{... (i)}$$

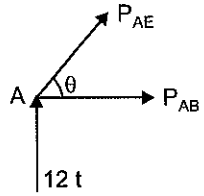
and, from $\Sigma M_D = 0$

$\Rightarrow$ $$V_A \times 12 - 18 \times 8 = 0$$

$\Rightarrow$ $$V_A = 12t$$

$\Rightarrow$ $$V_D = 18 - 12 = 6t$$

Consider Joint A



$$\tan\theta = \frac{3}{4}$$

$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \frac{4}{5}$$

$\Rightarrow$

(Assumed direction of Forces)

$$\Sigma F_Y = 0$$

$\Rightarrow$ $$P_{AE}\sin\theta = -12$$

$\Rightarrow$ $$P_{AE} \times 3/5 = -12$$

$\Rightarrow$ $$\boxed{P_{AE} = -20\,t \quad \text{[Compression]}}$$

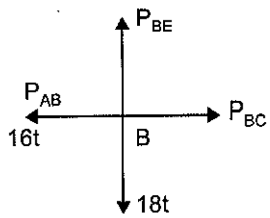
$$\Sigma F_x = 0$$

$\Rightarrow$ $$P_{AB} + P_{AE}\cos\theta = 0$$

$\Rightarrow$ $$P_{AB} = 20\cos\theta = 20 \times 4/5$$

$$\boxed{P_{AB} = 16t \ \text{(tension)}}$$

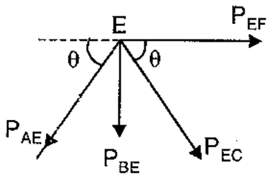
Joint B: From equilibrium of forces



$$P_{BC} = 16t \ \text{(tension)}$$

$$P_{BE} = 18t \ \text{(tension)}$$

Joint E:



$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad P_{EC}\sin\theta + P_{BE} + P_{AE}\sin\theta = 0$$

$$\Rightarrow \quad P_{EC}\sin\theta = -P_{BE} - P_{AE}\sin\theta$$

$$= -18-(-20)\times\frac{3}{5} = -6$$

$$\Rightarrow \quad \boxed{P_{EC}\ \frac{-6}{\sin\theta} = -\frac{6}{3/5} = -10\text{ t}}$$

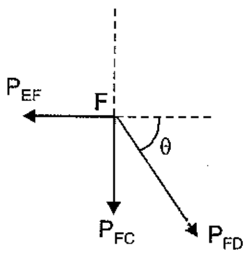
$$\Sigma F_x = 0$$

$$\Rightarrow \quad P_{EF} + P_{EC}\cos\theta - P_{AE}\cos\theta = 0$$

$$\Rightarrow \quad P_{EF} - 10\times\frac{4}{5} + 20\times\frac{4}{5} = 0$$

$$\Rightarrow \quad \boxed{P_{EF} = -8\text{t (compression)}}$$

Joint F:



$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$$\Sigma F_x = 0$$

$$\Rightarrow \quad P_{FD}\cos\theta - P_{EF} = 0$$

$$\boxed{P_{FD} = -8\times\frac{5}{4} = -10\text{t (compression)}}$$

$$\Sigma F_y = 0$$

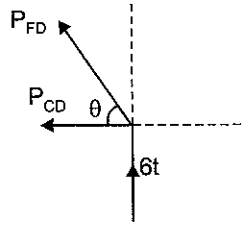
$$P_{FD}\sin\theta + P_{FC} = 0$$

$$\Rightarrow \quad P_{FD}\times\frac{3}{5} + P_{FC} = 0$$

$$\Rightarrow \quad -10\times\frac{3}{5} + P_{FC} = 0$$

$$\Rightarrow \quad \boxed{P_{FC} = +6\text{t (tension)}}$$

Joint D:



$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$$\Sigma F_x = 0$$

$$P_{FD}\cos\theta + P_{CD} = 0$$

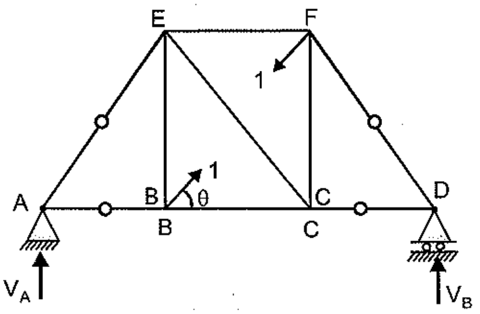
$$P_{CD} = -(-10)\times\frac{4}{5} = 8t \text{ (tension)}$$

Here p-force system is given as,

$$P_{AB} = 16t$$
$$P_{AE} = -20t$$
$$P_{BC} = 16t$$
$$P_{BE} = 18t$$
$$P_{EC} = -10t$$
$$P_{EF} = -8t$$
$$P_{FC} = 6t$$
$$P_{FD} = -10t$$
$$P_{CD} = 8t$$

Calculation of u_i

It can be calculated by removing external loads and applying a pair of unit load at B and F along the member BF. (as shown below)

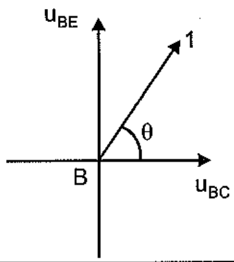


Since the net force applied on truss is zero and the moment produced by applied forces is also zero and truss is externally determinate, reactions produced will be zero

So, $V_A = 0$ and $V_B = 0$

Which implies that the member forces in AB, AE, FD and CD will be zero. [Because two non-concurrent forces are existing at joint and no external load acts at joints]

Joint B:



$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$$\Sigma F_x = 0$$

$$u_{BC} + 1 \times \cos\theta = 0$$

$\Rightarrow$ $$\boxed{u_{BC} = -\frac{4}{5} = -0.8 \text{ (compression)}}$$

$$\Sigma F_y = 0$$

$$u_{BE} = -1 \times \sin\theta$$

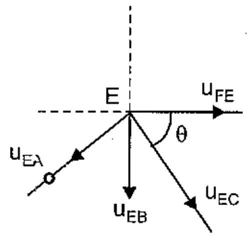
$\Rightarrow$ $$\boxed{u_{BE} = -0.6 \text{ (compression)}}$$

Since joint B is similar to joint F,

Hence $\boxed{u_{FC} = u_{BE} = -0.6}$

and $\boxed{u_{FE} = u_{BC} = -0.8}$

At joint E:



$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{4}$

$\cos\theta = \frac{4}{5}$

$$\Sigma F_x = 0$$

$\Rightarrow$ $$u_{EC}\cos\theta + u_{FE} = 0$$

$\Rightarrow$ $$u_{EC} \times \frac{4}{5} - 0.8 = 0.8$$

$\Rightarrow$ $$\boxed{u_{EC} = 1 \text{ (tension)}}$$

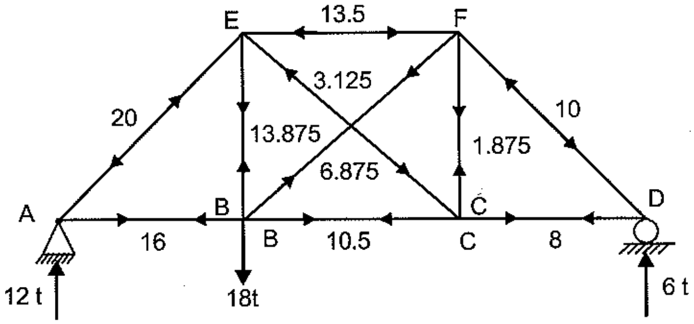
Member	P	u	L	PuL	u^2L	F = P + uR
AB	16	0	4	0	0	16 t
AE	–20	0	5	0	0	–20 t
BC	16	–0.8	4	–51.2	2.56	10.5 t
BE	18	–0.6	3	–32.4	1.08	13.875t
EC	–10	1	5	–50	5	–3.125t
EF	–8	–0.8	4	+25.6	2.56	–13.5t
FC	6	–0.6	3	–10.8	1.08	1.875t
FD	–10	0	5	0	0	–10t
CD	8	0	4	0	0	8t
BF	0	1	5	0	5	6.875t
				$\Sigma PuL = -118.8$	$\Sigma u^2L = 17.28$	

Value of AE is same for all members {given}

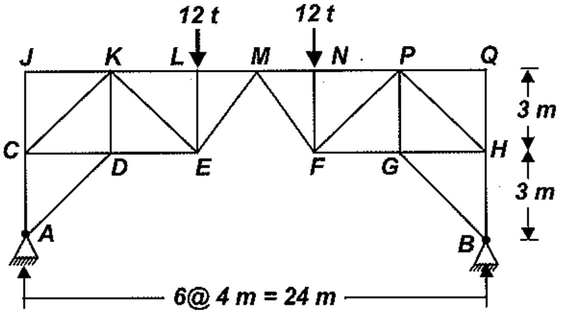
$$\Rightarrow \qquad R = \frac{-\sum PuL}{\sum u^2L} = \frac{-(-118.8)}{17.28} = 6.875$$

Value of R is (+)ve . This implies that force in member BF is of same nature as that of the applied unit loads at B and F i.e., tension.

Hence the force in each member has been tabulated as $F_i = P_i + u_iR$

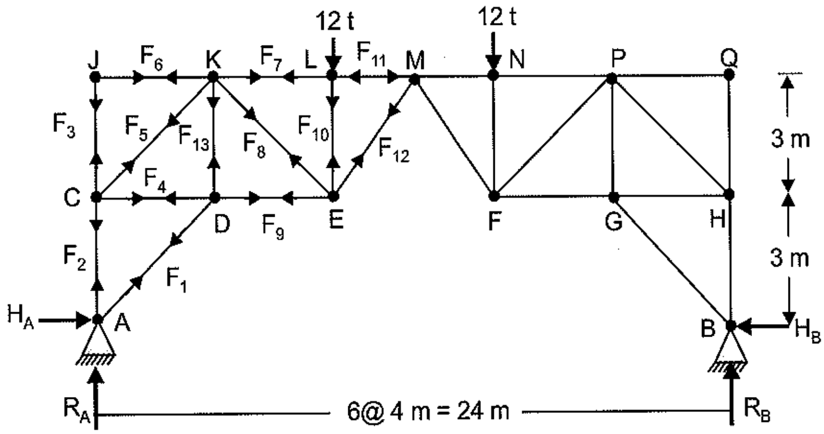


Q-2: ***Assuming all members of the truss shown in figure to be pin-jointed, calculate forces in them.***



[15 Marks, ESE-1997]

Sol:



Due to symmetry in loading

$$R_A = R_B = 12 \text{ t}$$

Moment of forces about hinge M will be zero for forces on either side of M.

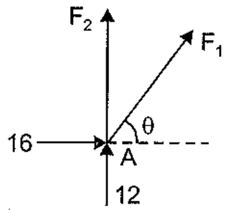
$$\Rightarrow \qquad R_A \times 12 - H_A \times 6 - 12 \times 4 = 0$$

$$12 \times 12 - H_A \times 6 - 12 \times 4 = 0$$

$$H_A = \frac{12\times12-12\times4}{6} = \frac{12\times8}{6} = 16t$$

$$H_A = H_B \quad \text{[From the equilibrium of horizontal forces]}$$

Analysis of joint A



$$\tan\theta = 3/4$$

$$\sin\theta = 3/5$$

$$\cos\theta = 4/5$$

$$F_1\cos\theta + 16 = 0$$

$$F_1 = \frac{-16\times5}{4} = -20\text{ t} \Rightarrow \boxed{F_1 = -20t}$$

$$F_1\sin\theta + F_2 + 12 = 0$$

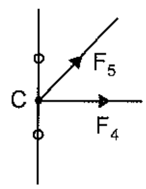
$$-20\times\frac{3}{5}+F_2+12 = 0 \Rightarrow \boxed{F_2 = 0}$$

Member JK and JC are zero force members because two non-concurrent members meet at joint 'J' and no external force is acting at J

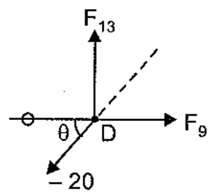
$$\Rightarrow \boxed{F_6 = 0}$$

$$\boxed{F_3 = 0}$$

once $F_3 = 0$ and $F_2 = 0$, F_4 and F_5 will also be zero force member because F_4 and F_5 are two non-concurrent members meeting at joint C and no external force act at C.



At joint D



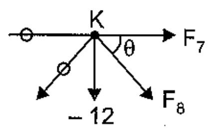
$$F_{13} + 20\sin\theta = 0$$

$$\Rightarrow \boxed{F_{13} = -20\times\frac{3}{5} = -12\text{ t}}$$

$$F_9 + 20\cos\theta = 0$$

$$\boxed{F_9 = -20\times\frac{4}{5} = -16t}$$

At Joint K

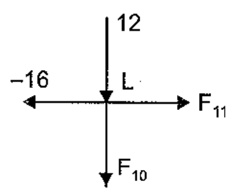


$$F_8\sin\theta - 12 = 0$$

$$\boxed{F_8 = 12\times\frac{5}{3} = 20\text{ t}}$$

$$F_8\cos\theta + F_7 = 0 \Rightarrow \boxed{F_7 = -20\times\frac{4}{5} = -16\text{ t}}$$

At Joint L



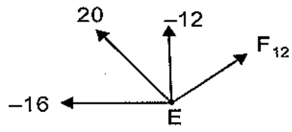
$$12 + F_{10} = 0$$

$$\Rightarrow \boxed{F_{10} = -12\text{ t}}$$

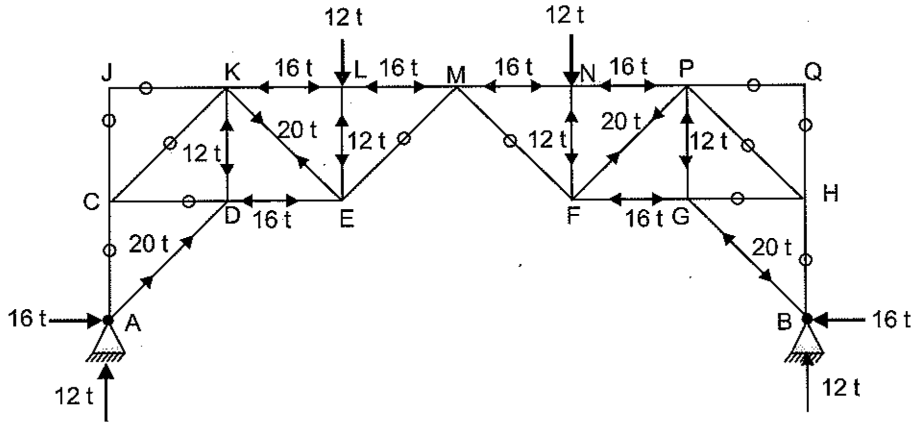
$$\boxed{F_{11} = -16\text{ t}}$$

At joint E

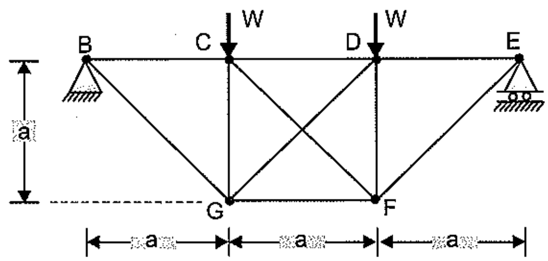
Resultant of –12 t & –16 t force will balance 20 t load hence force in F_{12} = 0.



As there is symmetry about M, forces in corresponding members will be same. Hence the forces in various members are as shown below.



Q-3: ***Figure below shows a plane pin-jointed truss BCDEFG of the given dimensions and carrying the loading shown. All members have the same axial stiffness.***



(a) Calculate the axial force in member FG.

(b) Evaluate the force in the same member if it is fabricated 0.1% too short, and the structure carries no external load.

[30 + 10 = 40 Marks, ESE-1999]

Sol: (a) Degree of indeterminacy of given truss is,

$$D_s = m + r - 2j$$

Here, m = 10, r = 3 and j = 6

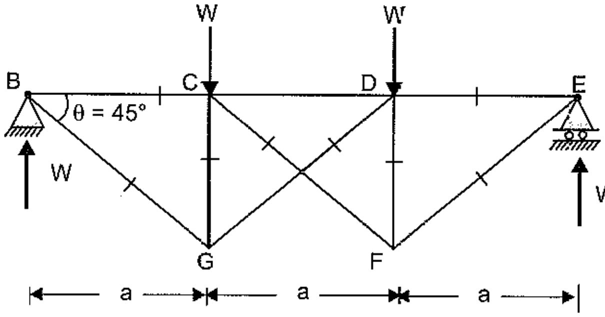
$$\therefore \quad D_s = 10 + 3 - 12 = 1$$

Degree of external indeterminacy = r – 3 = 0

Hence degree of internal indeterminacy = 1.

$\therefore$ Truss is internally indeterminate with 1 degree.

Any of the member or support reactions can be taken as unknown provided that removal of it does not make the structure unstable. Let us take member force in GF as redundant. We could have chosen any other member as redundant but removal of GF keeps the truss symmetrical. Thus calculation will become easier.

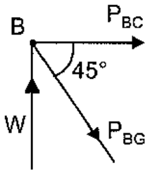


Due to symmetry in the loading and geometry, we can directly write the reactions at B and E as;

$V_B = V_E = W$, horizontal reaction at E = 0 and hence horizontal reactions at B = 0

Calculation of member forces due to external loading when redundant member is removed (P)

Joint B



$$\Sigma F_y = 0$$

$$\frac{P_{BG}}{\sqrt{2}} = W$$

$$\therefore \quad \boxed{P_{BG} = W\sqrt{2} \text{ (tension)}}$$

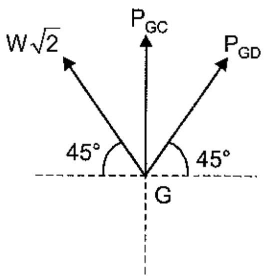
$$\Sigma F_x = 0$$

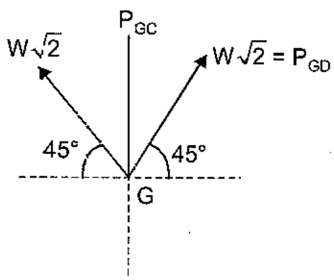
$$P_{BG}\cos 45° + P_{BC} = 0$$

$$\frac{P_{BG}}{\sqrt{2}} = -P_{BC}$$

$$\boxed{P_{BC} = -W \text{ (compresssion)}}$$

Joint G





$$\Sigma F_x = 0$$

$$\Rightarrow \quad \frac{P_{GD}}{\sqrt{2}} = \frac{W\sqrt{2}}{\sqrt{2}}$$

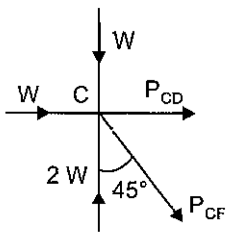
$$\boxed{P_{GD} = W\sqrt{2} \text{ (tension)}}$$

$$\Sigma F_y = 0$$

$$P_{GC} + \frac{P_{GD}}{\sqrt{2}} + \frac{W\sqrt{2}}{\sqrt{2}} = 0$$

$$\Rightarrow \quad \boxed{P_{GC} = -2W \text{ (compression)}}$$

Joint C



$$\Sigma F_y = 0 \qquad\qquad \Sigma F_x = 0$$

$$P_{CF} \times \sin 45° + W - 2W = 0 \qquad P_{CF} \times \frac{1}{\sqrt{2}} + P_{CD} + W = 0$$

$$\Rightarrow \quad P_{CF} = W \times \sqrt{2} \qquad \Rightarrow \quad W + P_{CD} + W = 0$$

$$\Rightarrow \quad \boxed{P_{CF} = \sqrt{2}\,W \text{ (tension)}} \qquad \Rightarrow \quad \boxed{P_{CD} = -2W \text{ (compression)}}$$

Since the truss is symmetric forces in the members are as follows:

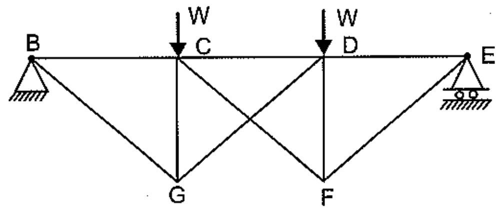
$$P_{BG} = P_{EF} = \sqrt{2}\,W$$

$$P_{BC} = P_{ED} = -W$$

$$P_{CG} = P_{FD} = -2W$$

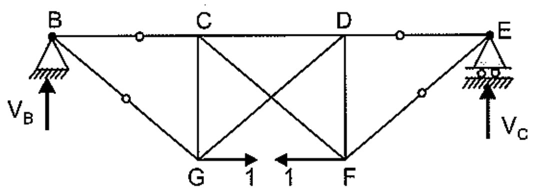
$$P_{CF} = P_{DG} = \sqrt{2}\,W$$

$$P_{CD} = -2W$$



Calculation of member forces due to unit load (u)

Remove all external loads and apply unit force on joints G and F along the direction of GF.

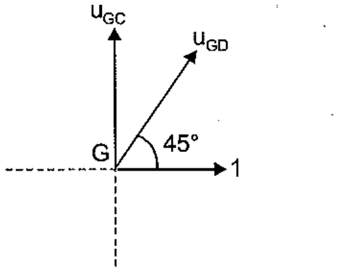


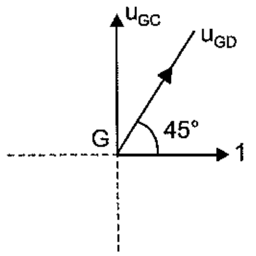
In externally determinate structures equal, opposite and colinear forces will not produce any support reactions.

$$\Rightarrow \qquad V_B = V_E = 0$$

Since V_B and V_E are equal to zero. Hence force in member BG, BC, DE and EF = 0

Joint G



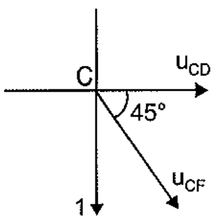


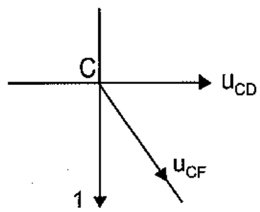
$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$

$$u_{GD} \cos 45° + 1 = 0 \qquad \frac{u_{GD}}{\sqrt{2}} + u_{GC} = 0$$

$$\Rightarrow \quad \boxed{u_{GD} = -\sqrt{2}} \qquad \Rightarrow \quad \boxed{u_{GC} = +1}$$

Joint C





$$\Sigma F_y = 0 \qquad \Sigma F_x = 0$$

$$\Rightarrow \quad \frac{u_{CF}}{\sqrt{2}} = -1 \qquad u_{CD} = -\frac{u_{CF}}{\sqrt{2}}$$

$$\Rightarrow \quad \boxed{u_{CF} = -\sqrt{2}} \qquad \Rightarrow \boxed{u_{CD} = 1}$$

Similarly, $\boxed{u_{DF} = u_{CG} = 1}$

$$\boxed{U_{GD} = u_{CF} = -\sqrt{2}}$$

Member	P	u	L	PuL	u^2L
BC	$-W$	0	a	0	0
BG	$\sqrt{2}W$	0	a	0	0
CD	$-2W$	1	a	$-2Wa$	a
CF	$\sqrt{2}W$	$-\sqrt{2}$	$\sqrt{2}a$	$-2\sqrt{2}Wa$	$2\sqrt{2}a$
GC	$-2W$	1	a	$-2Wa$	a
ED	$-W$	0	a	0	0
EF	$\sqrt{2}W$	0	a	0	0
FD	$-2W$	1	a	$-2Wa$	a
GF	0	1	a	0	a
GD	$\sqrt{2}W$	$-\sqrt{2}$	$\sqrt{2}a$	$-2\sqrt{2}Wa$	$2\sqrt{a}$

$$\Sigma PuL = -\left(6Wa + 4\sqrt{2}Wa\right)$$

$$\Sigma u^2L = 4a + 4\sqrt{2}a$$

Since, $$R = -\frac{\Sigma PuL / AE}{\Sigma u^2L / AE}$$

As AE is same for all members ;

$$\therefore \quad R = -\frac{\Sigma PuL}{\Sigma u^2L} = \frac{6Wa + 4\sqrt{2}Wa}{4a + 4\sqrt{2}a} = \left(\frac{6+4\sqrt{2}}{4+4\sqrt{2}}\right)W = 1.207\ W$$

Hence force in GF member $= P + uR$

$\Rightarrow \quad = 0 + 1 \times 1.207\ W = 1.207\ W$ (tension)

(b) When the member GF is fabricated (0.001 a) too short and no external load is acting, force in member GF is equal to R, where;

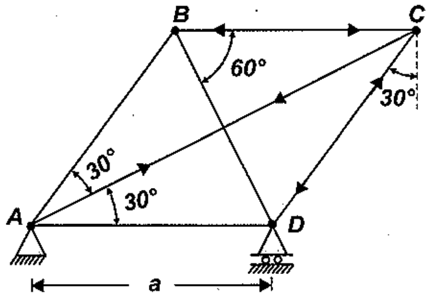
$$R = \frac{-\Sigma u(\lambda)}{\frac{\Sigma u^2L}{AE}} = \frac{-1(-0.001a)}{\frac{4a + 4\sqrt{2}a}{AE}}$$

$$R = \frac{a}{\frac{[4000(1+\sqrt{2})a]}{AE}} = \frac{AE}{400(1+\sqrt{2})}$$

$$\Rightarrow \quad \boxed{F_{GF} = 0 + 1 \times R = R = \frac{AE}{400(1+\sqrt{2})}}$$

(+)ve value of R means, redundant is in the direction of applied unit load ie tensile.

Q-4: ***When fabricating the pin-jointed plane frame shown in the above figure, it was found that the member AC is fabricated Δ too long. Determine the forces in the members after assembly. All members have same axial stiffness (AE).***



[20 Marks, ESE-2000]

Sol: Degree of indeterminacy is given by

$$D_S = m + r - 2j$$

Here, m = 6, r = 3, j = 4

$$\therefore \quad D_S = 1$$

Structurally the frame is internally indeterminate by 1 degree. We can take any of the member or support reaction as redundant provided that removal of that member of support reaction does not make the structure unstable.

Let us take force in member DC as redundant

$$\Rightarrow \quad R = \frac{-\sum u_i (\lambda_i)}{\sum \frac{u_i^2 (l_i)}{A_i E_i}}$$

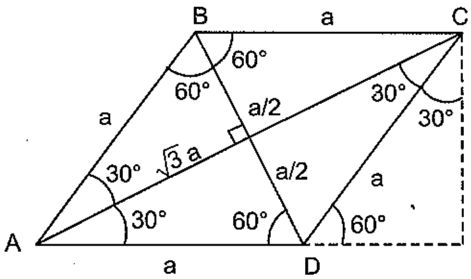
u_i = member force in i^{th} member

Calculation of length of members

$$AC = 2\sqrt{a^2 - \left(\frac{a}{2}\right)^2}$$

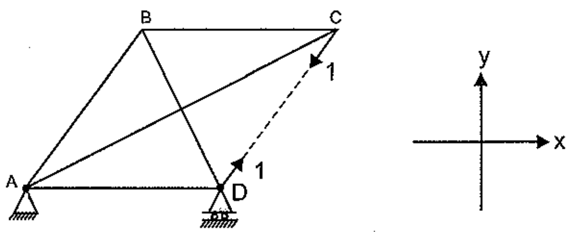
$$= 2a\sqrt{1 - \frac{1}{4}}$$

$$= \frac{2\sqrt{3}\,a}{2} = \sqrt{3}\,a$$

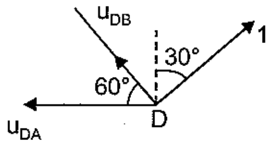


AE = constant for all members

Since the structure is externally determinate, application of equal, opposite and collinear forces will not cause any support reactions



Joint D



$$\Sigma F_x = 0$$

$\Rightarrow$ $$1 \sin 30° - u_{DB} \cos 60° - u_{DA} = 0$$

$$u_{DA} + \frac{u_{DB}}{2} + \frac{1}{2} = 0 \qquad \text{...(i)}$$

$$\Sigma F_y = 0$$

$\Rightarrow$ $$u_{DB} \sin 60° + 1 \cos 30° = 0$$

$\Rightarrow$ $$u_{DB} = \frac{-\cos 30°}{\sin 60°} = -1$$

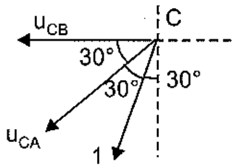
$$\boxed{u_{DB} = -1} \text{ (compressive)}$$

From (i)

$\Rightarrow$ $$u_{DA} = +\frac{1}{2} + \frac{1}{2} = +1$$

$$\boxed{u_{DA} = +1} \text{ (tension)}$$

At joint C



$$\Sigma F_x = 0$$

$\Rightarrow$ $$u_{FCB} + u_{CA} \cos 30° + 1 \cos 60° = 0$$

$\Rightarrow$ $$u_{CB} + \frac{u_{CA}\sqrt{3}}{2} + \frac{1}{2} = 0 \qquad \text{...(ii)}$$

$$\Sigma F_y = 0$$

$$u_{CA} \cos 60° + 1 \cos 30° = 0$$

$\Rightarrow$ $$u_{CA} = \frac{-\cos 30°}{\cos 60°} = \frac{-\sqrt{3}}{2 \times \frac{1}{2}} = -\sqrt{3}$$

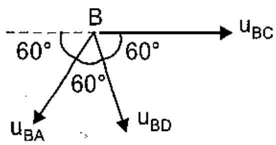
$$\boxed{u_{CA} = -\sqrt{3}} \text{ (Compression)}$$

$\Rightarrow$ From (ii)

$$u_{CB} = \frac{-1}{2} + \frac{\sqrt{3} \times \sqrt{3}}{2}$$

$$\boxed{u_{CB} = 1} \text{ (tension)}$$

At joint B



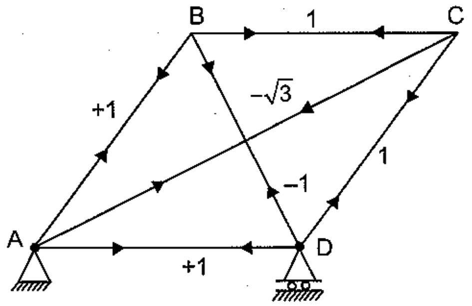
$$\Sigma F_x = 0$$

$\Rightarrow$ $$u_{BC} + u_{BD} \cos 60° - u_{BA} \cos 60° = 0$$

$\Rightarrow$ $$u_{BA} = \frac{u_{BC} + u_{BD} \cos 60°}{\cos 60°}$$

$$= \frac{+\left(1 - 1 \times \frac{1}{2}\right)}{\frac{1}{2}} = +1$$

$$\boxed{u_{BA} = +1} \text{ (Tension)}$$



$$\Rightarrow \qquad R = \frac{-(-\sqrt{3})\,\Delta}{\frac{1}{AE}[(1\times a)+(1\times a)+(1\times a)+(1\times a)+(1\times a)+3\sqrt{3}a]}$$

$$= \frac{\sqrt{3}\,\Delta AE}{(5+3\sqrt{3})\,a}$$

Actual force in members

$$F_i = u_i R + 0$$

Tension (+)ve, compression (–)ve

$$\Rightarrow \qquad F_{AB} = \frac{+1\times\sqrt{3}}{5+3\sqrt{3}}\frac{AE\Delta}{a}$$

$$F_{BC} = \frac{1+\sqrt{3}}{5+3\sqrt{3}}\frac{AE\Delta}{a}$$

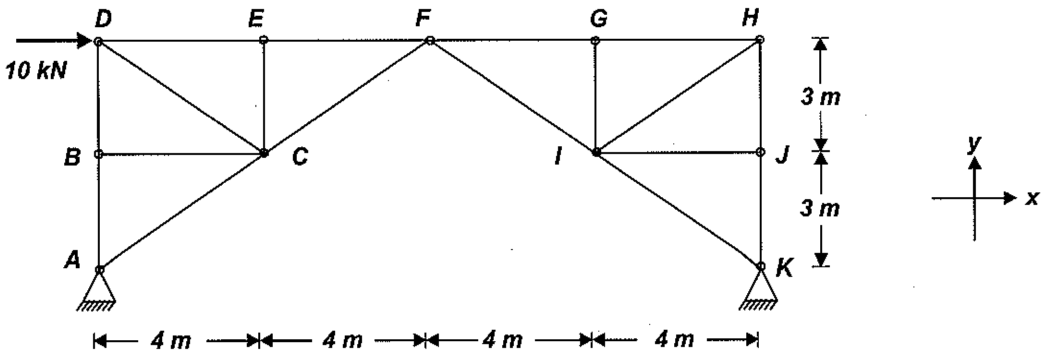
$$F_{CD} = \frac{\sqrt{3}}{5+3\sqrt{3}}\frac{AE\Delta}{a}$$

$$F_{DA} = \frac{+\sqrt{3}}{5+3\sqrt{3}}\frac{AE\Delta}{a}$$

$$F_{AC} = \frac{-\sqrt{3}\times\sqrt{3}}{5+3\sqrt{3}}\frac{AE\Delta}{a} = \frac{-3}{5+3\sqrt{3}}\frac{AE\Delta}{a}$$

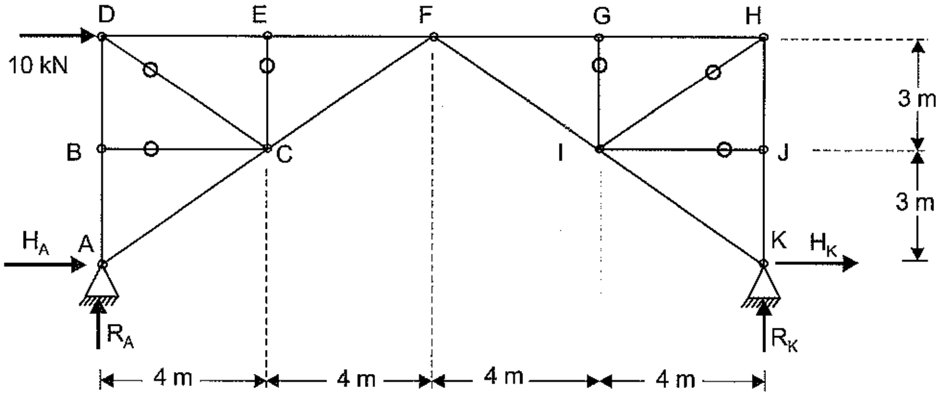
$$F_{BD} = \frac{-\sqrt{3}}{5+3\sqrt{3}}\frac{AE\Delta}{a}$$

Q-5: ***All members of the truss shown in the figure are pin-jointed. Calculate the reactions and forces in all members.***



[20 Marks, ESE-2002]

Sol:



We know that, for plane trusses, statical indeterminacy = m + r – 2j

i.e., $D_s = m + r - 2j$

Here m = 18 r = 4 j = 11

m = Number of members

r = Number of external reaction

j = Number of joints

$\therefore$ $D_S = 18 + 4 - 2 \times 11 = 0$, i.e structure is statically determinate.

From, $\Sigma F_y = 0 \Rightarrow R_A + R_K = 0 \Rightarrow R_A = -R_K$

$\Sigma M_A = 0 \Rightarrow R_K \times 16 = 10 \times 6 \Rightarrow R_K = 3.75\text{ kN}(\uparrow)$

$\therefore \Rightarrow$ $R_A = -3.75\text{kN}$ i.e. $3.75\text{kN}(\downarrow)$

Also, from, $\Sigma F_x = 0 \Rightarrow H_A + H_K + 10 = 0$

Point F can be treated as an internal hinge at which BM is zero

$$R_A \times 8 - H_A \times 6 = 0 \Rightarrow H_A = \frac{-8 \times 3.75}{6} = -5\text{ kN i.e., } 5\text{kN}(\leftarrow)$$

$\therefore$ $H_K = -5\text{ kN i.e., } 5\text{kN}(\leftarrow)$

Analysis of member forces

Member force $F_{EC} = 0$ because there is nothing to balance the member force in EC at joint E.

Similarly, $F_{BC} = 0$

and since, $F_{CB} = 0$ and $F_{EC} = 0$, $F_{DC} = 0$

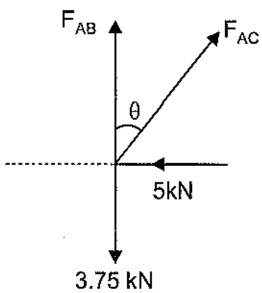
on the same logic $F_{GI} = 0$

$F_{JI} = 0$

$F_{IH} = 0$

Joint A

From $\Sigma F_x = 0$, we have



$\tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$$F_{AC}\sin\theta = 5$$

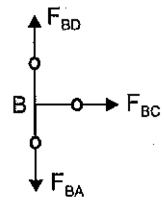
$$\Rightarrow \quad F_{AC} = \frac{5 \times 5}{4} = \frac{25}{4} = 6.25\text{ kN}$$

From $\Sigma F_y = 0$

$$F_{AB} + F_{AC}\cos\theta = 3.75$$

$$\Rightarrow \quad F_{AB} + 6.25 \times \frac{3}{5} = 3.75$$

$$F_{AB} = 0$$



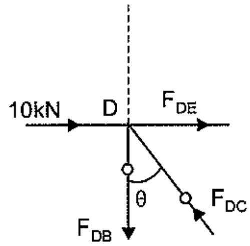
At joint B

Since we get $\quad F_{BA} = 0$

$\Rightarrow \quad F_{BD} = 0$

Also $\quad F_{BC} = 0$

At joint D

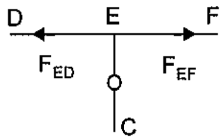


$$\Sigma F_x = 0$$

$\Rightarrow \quad F_{DE} + 10 = 0 \qquad [\because F_{DC} = 0 \ \& \ F_{DB} = 0]$

$\Rightarrow \quad F_{DE} = -10 \text{ kN}$

At joint E

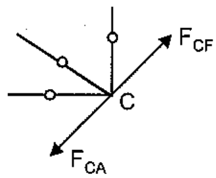


From, $\quad \Sigma F_x = 0$

$$F_{DE} = F_{EF} = -10 \text{ kN}$$

At joint C

$$F_{CA} = F_{CF} = 6.25 \text{ kN}$$

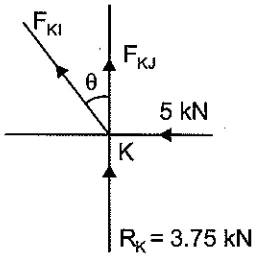


At joint K

From, $\quad \Sigma F_x = 0$

$$F_{KI}\sin\theta + 5 = 0$$

$\therefore \quad F_{KI} = \dfrac{-5}{\sin\theta} = \dfrac{-5\times 5}{4} = -6.25 \text{ kN}$



$$\tan\theta = \frac{4}{3}$$

$$\sin\theta = \frac{4}{5}$$

$$\cos\theta = \frac{3}{5}$$

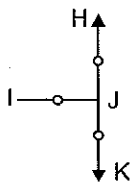
From $\quad \Sigma F_y = 0$

$\therefore \quad F_{KJ} + F_{KI}\cos\theta + 3.75 + 0 = 0$

$\Rightarrow \quad F_{KJ} - 6.25 \times \dfrac{3}{5} = -3.75$

$\Rightarrow \quad F_K = 0$

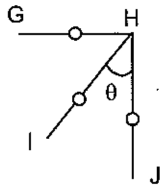
At joint J



$$F_{IJ} = 0$$

$$F_{JH} = F_{JK} = 0$$

At joint H

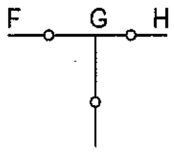


Since $\quad F_{HJ} = 0$

$$F_{HI} = 0$$

$\Rightarrow \quad F_{GH} = 0$

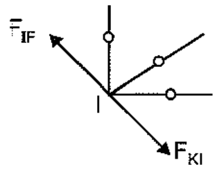
At joint G



$$F_{GF} = 0$$

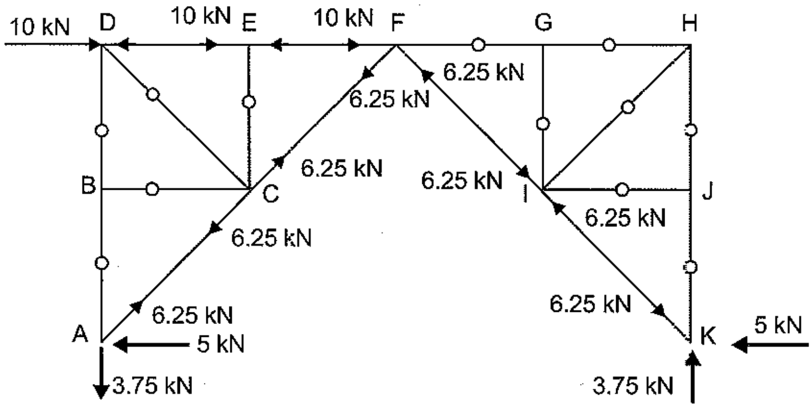
$$F_{GI} = 0$$

At joint I

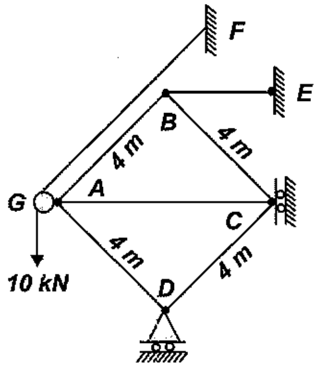


$\Rightarrow$ $$F_{IF} = F_{KI} = -6.25 \text{ kN}$$

Hence, forces in various members are as shown below.



Q-6: ***The square truss ABCD shown below carries a load of 10 kN attached to a string GF and passing over a friction-less pulley at A. GF and AB are parallel. The truss is supported on rollers at C & D and by a link BE at B. Find the forces in the members of the truss.***



[15 Marks, ESE-2003]

Sol:

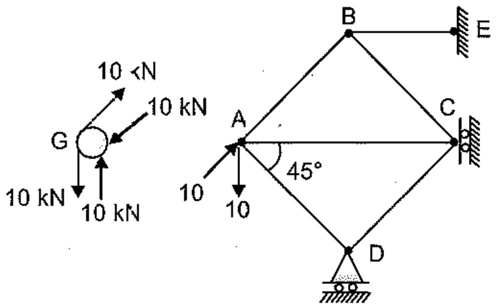


Fig. (i)

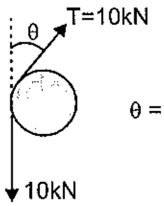
The degree of indeterminacy of the square truss,

$$D_s = m + r - 2j$$

Here m = 6, r = 4 and j = 5

$\therefore$ $$D_s = 6 + 4 - 10 = 0$$

Thus, the given truss is statically determinate.



To solve the truss problem we start from the point where loads are known, thus we start from point A of truss.

Since the pulley is frictionless, hence the tension 'T' generated in GF will be equal to 10 kN as shown in figure,

θ = 45° [since GF || AB]

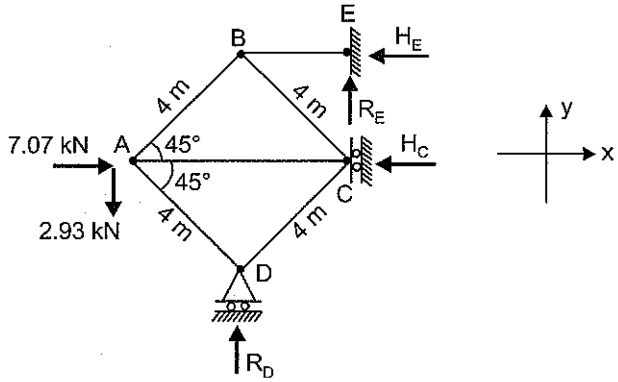
Hence, forces on the truss are as shown above in Fig. (i)

Hence the net force applied on the truss at point A is,

$$F_V|_{\text{at point A}} = 10 - 10\cos 45° = 10 - \frac{10}{\sqrt{2}} = 2.93 \text{ kN}$$

$$F_H|_{\text{at point A}} = 10\sin 45° = \frac{10}{\sqrt{2}} = 7.07 \text{ kN}$$

Now our question simplifies to the figure shown below:



Step-II: Finding reactions

For link BE, $\Sigma M_B = 0$ (from right side)

$\Rightarrow \quad R_E \times BE = 0$

$\Rightarrow \quad R_E \times \frac{4}{\sqrt{2}} = 0$

$\Rightarrow \quad R_E = 0$

For whole structure, $\Sigma F_y = 0$

$\Rightarrow \quad R_E + R_D = 2.93$

$\Rightarrow \quad R_D = 2.93\text{kN}(\uparrow)$

$\Sigma M_B = 0$ (from left side)

$$7.07 \times \frac{4}{\sqrt{2}} + 2.93 \times \frac{4}{\sqrt{2}} - H_C \times \frac{4}{\sqrt{2}} + R_C \times 0 = 0$$

$\Rightarrow \quad H_C = 10\text{kN}(\leftarrow)$

For whole structure, $\Sigma F_x = 0$

$H_E + H_C = 7.07$

$\Rightarrow \quad H_E = 7.07 - 10 = -2.93$ kN

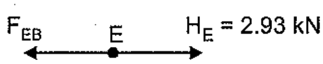
$H_E = 2.93\text{kN}(\rightarrow)$

Step-III: After finding out all reaction, we will proceed joint wise to determine member forces

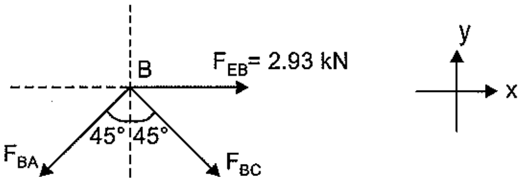
Joint E:

From $\quad \Sigma_{FX} = 0;$

$$\boxed{F_{EB} = 2.93 \text{ kN} \quad \text{(tension)}}$$



Joint B:



$$\Sigma F_x = 0$$

$$F_{BC}\sin 45° + F_{EB} - F_{BA}\sin 45° = 0$$

$$\Rightarrow \frac{F_{BC}}{\sqrt{2}} - \frac{F_{BA}}{\sqrt{2}} = -2.93$$

$$\Rightarrow F_{BA} - F_{BC} = 2.93\sqrt{2} \quad \text{... (i)}$$

$$\Sigma F_y = 0$$

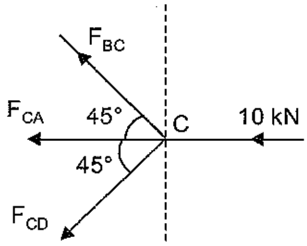
$$F_{BA}\cos 45° + F_{BC}\cos 45° = 0$$

$$\Rightarrow F_{BA} = -F_{BC}$$

$$\Rightarrow F_{BA} + F_{BC} = 0 \quad \text{... (ii)}$$

Solving these two equations, F_{BA} = 2.07 kN (tension) F_{BC} = – 2.07 kN (compression)

Joint C:



$$\Sigma F_y = 0$$

$$\Rightarrow -F_{CD}\sin 45° + F_{BC}\sin 45° = 0$$

$$F_{CD} = +F_{BC}$$

$$\therefore F_{CD} = -2.07 \text{ kN [compression]}$$

$$\Sigma F_x = 0$$

$$\Rightarrow -F_{BC}\cos 45° - F_{CD}\cos 45° - F_{CA} - 10 = 0$$

$$\Rightarrow +2.07 \times \frac{1}{\sqrt{2}} - \left(-2.07 \times \frac{1}{\sqrt{2}}\right) - 10 = F_{CA}$$

$$\Rightarrow F_{CA} = -7.07 \text{ kN [compression]}$$

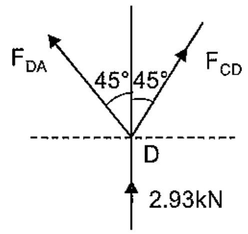
Joint D:

$$\Sigma F_x = 0$$

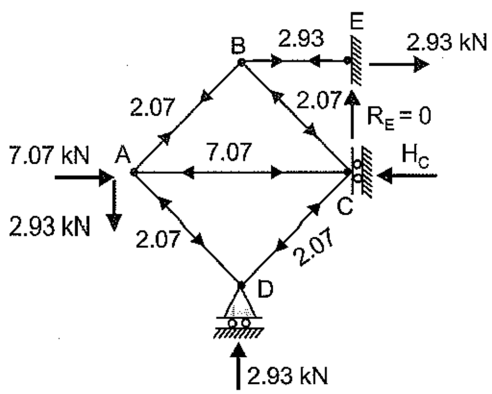
$$-F_{DA}\sin 45° + F_{CD}\sin 45° = 0$$

$$\Rightarrow -\frac{F_{DA}}{\sqrt{2}} + \frac{F_{CD}}{\sqrt{2}} = 0$$

$$\Rightarrow F_{DA} = -2.07 \text{ kN (compression)}$$

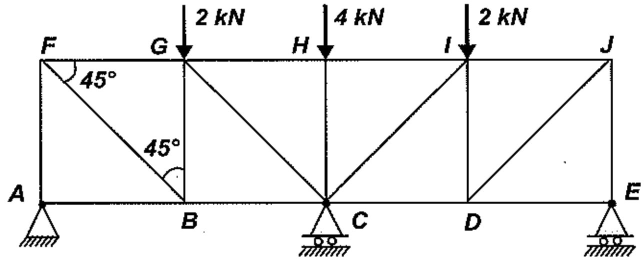


Hence now we can show each member force,



(All forces are in kN)

Q-7: ***Find the forces in the members of the truss shown in figure. The truss is hinged at A and simply supported at C and E. It carries 2 kN, 4 kN and 2 kN at nodes G, H and I respectively. The diagonals makes 45° with the horizontal and vertical members.***



[30 Marks, ESE-2004]

Sol: Degree of indeterminacy of given truss,

$D_S = m + r - 2j;\quad m = 17;\quad r = 4;\quad j = 10$

$D_s = 17 + 4 - 10 \times 2;\quad D_s = 1$

External indeterminacy in plane truss is given by,

$$D_{se} = r - 3 = 4 - 3 = 1$$

$\Rightarrow \therefore$ (Internal indeterminacy) $= D_s - D_{se} = 0$

Conclusion: Truss is internally determinate. Thus we should take support reaction as redundant.

Let us take support reaction at C as redundant.

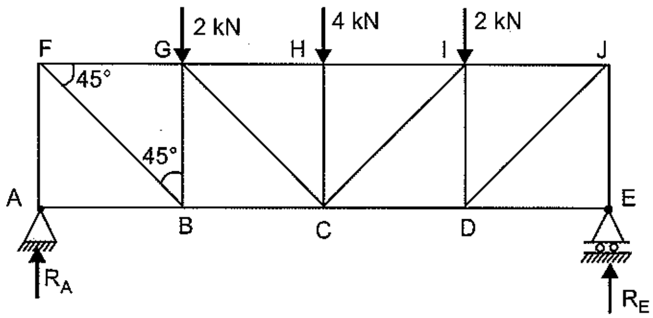
Support reaction C is called R_C;

$$R_C = -\frac{\sum \frac{pul}{AE}}{\sum \frac{u^2 l}{AE}}$$

where, P = Member forces due to external loading when support reaction at C has been removed

u = Member forces due to unit load applied in the direction of reaction R_C when external load has been removed

Calculation of P



Due to symmetry of loading

$$R_A = R_E = \frac{2+4+2}{4} = 4\text{ kN}$$

Joint A:

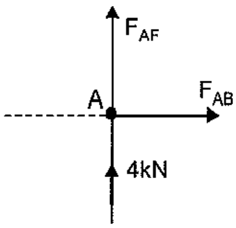
$\Sigma F_x = 0$

$\Rightarrow \quad F_{AB} = 0$

$\Sigma F_y = 0$

$F_{AF} + 4 = 0$

$\Rightarrow \quad F_{AF} = -4\text{kN}$ (compression)



Joint F:

$$\Sigma F_y = 0$$

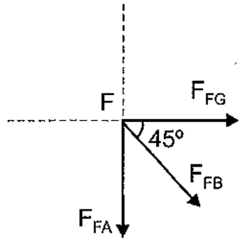
$$F_{FB} \times \frac{1}{\sqrt{2}} - 4 \text{ kN} = 0$$

$$\Rightarrow \quad F_{FB} = 4\sqrt{2}(\text{tension})$$

$$\Sigma F_x = 0$$

$$F_{FB} \cos 45° + F_{FG} = 0$$

$$\Rightarrow \quad F_{FG} = -4\sqrt{2} \times \frac{1}{\sqrt{2}} = -4\text{kN}(\text{Comp.})$$



Joint B:

$$\Sigma F_x = 0$$

$$F_{BC} = F_{FB} \sin 45°$$

$$F_{BC} = 4\sqrt{2} \times \frac{1}{\sqrt{2}}$$

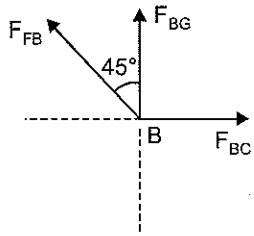
$$= 4\text{kN (tension)}$$

$$\Sigma F_y = 0$$

$$F_{BG} + F_{FB} \cos 45° = 0$$

$$\Rightarrow \quad F_{BG} = -4\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$= -4\text{kN (compression)}$$



Joint G:

$$\Sigma F_y = 0$$

$$\Rightarrow \quad 2 + F_{GC} \cos 45° + F_{GB} = 0$$

$$\Rightarrow \quad 2 + \frac{F_{GC}}{\sqrt{2}} + (-4) = 0$$

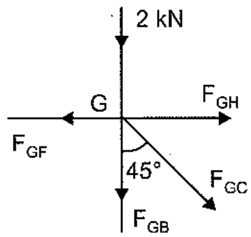
$$\Rightarrow \quad F_{GC} = 2\sqrt{2}(\text{tension})$$

$$\Sigma F_x = 0$$

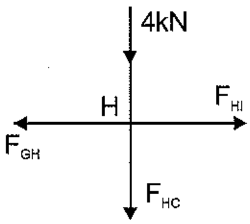
$$F_{GH} + F_{GC} \sin 45° - F_{GF} = 0$$

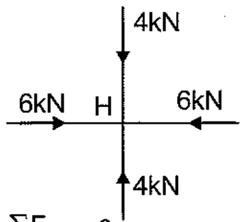
$$\Rightarrow \quad F_{GH} + 2\sqrt{2} \times \frac{1}{\sqrt{2}} - (-4) = 0$$

$$\Rightarrow \quad F_{GH} = -6\text{kN (comp.)}$$



Joint H:





$$\Sigma F_x = 0$$

$$F_{HI} - F_{GH} = 0$$

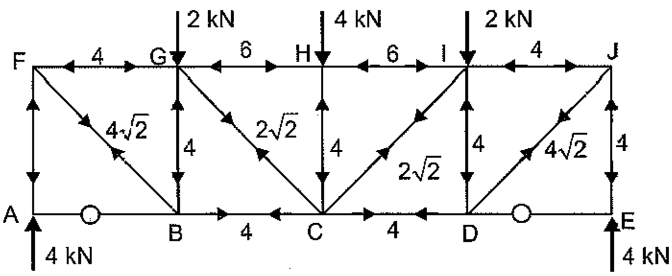
$$\Rightarrow \quad F_{HI} = -6\text{kN (compression)}$$

$$\Sigma F_y = 0$$

$$F_{HC} + 4 = 0$$

$$F_{HC} = -4\text{kN (compression}$$

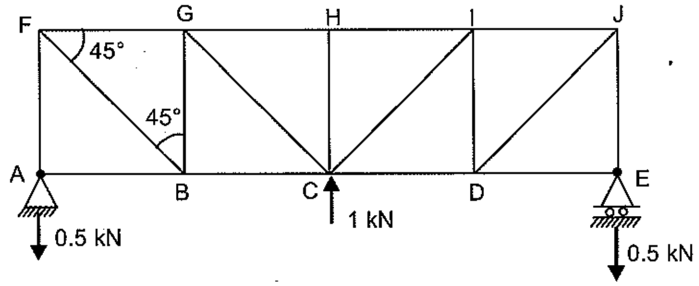
Since the loading is symmetrical, force in the member in other half of truss can be calculated.



(All member forces are in kN)

Calculation of u:

Let the direction of reaction at C be upwards, hence apply unit load at C upwards and find member forces in truss after external loading has been removed.



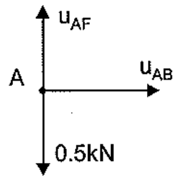
Joint A:

$$\Sigma F_x = 0$$

$$\Rightarrow \quad u_{AB} = 0$$

$$\Sigma F_y = 0$$

$$u_{AF} = 0.5 \text{ kN (tension)}$$



Joint F:

$$\Sigma F_y = 0$$

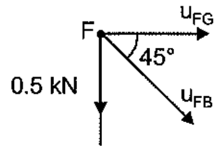
$$0.5 + u_{FB} \times \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow \quad u_{FB} = -0.5\sqrt{2} \text{ (compression)}$$

$$\Sigma F_x = 0$$

$$u_{FG} = -u_{FB} \cos 45°$$

$$= 0.5 \text{ kN (tension)}$$



Joint B:

$$\Sigma F_y = 0$$

$$\Rightarrow \quad u_{BF} \cos 45° + u_{BG} = 0$$

$$\Rightarrow \quad u_{BG} = -(-0.5\sqrt{2}) \times \frac{1}{\sqrt{2}} = 0$$

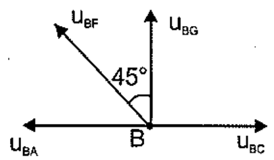
$$\Rightarrow \quad u_{BG} = 0.5 \text{ kN (tension)}$$

$$\Sigma F_x = 0$$

$$u_{BC} - u_{BA} - u_{BF} \sin 45° = 0$$

$$\Rightarrow \quad u_{BC} = -0.5\sqrt{2} \times \frac{1}{\sqrt{2}} + 0$$

$$\Rightarrow \quad u_{BC} = -0.5 \text{ kN (compression)}$$



Joint G:

$$\Sigma F_y = 0$$

$$\Rightarrow \quad u_{GB} + u_{GC} \cos 45° = 0$$

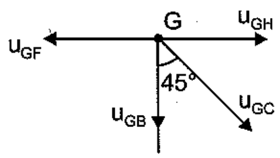
$$\Rightarrow \quad u_{GC} = -0.5\sqrt{2} \text{ kN (compression)}$$

$$\Sigma F_x = 0$$

$$\Rightarrow \quad u_{GC} \sin 45° - u_{GF} + u_{GH} = 0$$

$$\Rightarrow \quad u_{GH} = 0.5 - (-0.5\sqrt{2}) \times \frac{1}{\sqrt{2}}$$

$$= 1 \text{ kN (tension)}$$



Considering Joint H:

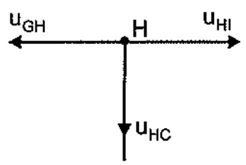
$$\Sigma F_x = 0$$

$$u_{HI} - u_{GH} = 0$$

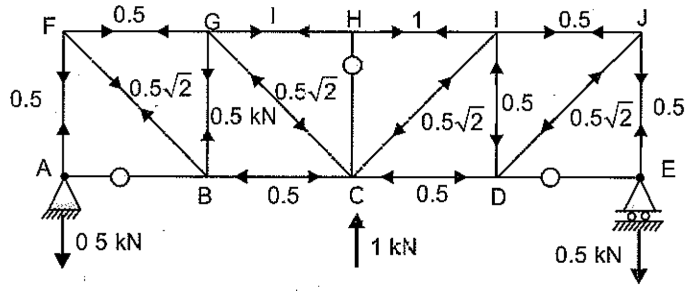
$$\Rightarrow \quad u_{HI} = 1 \text{ kN (tension)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad u_{HC} = 0$$



Due to symmetry of loading and truss, forces in other members can be calculated as shown below.



Considering the length of vertical and horizontal member as 'a'.

Member	P	u	L	PuL	u^2L	$P + uR_C = F$
AB	0	0	a	0	0	0
AF	–4	0.5	a	–2a	0.25a	–0.71
FG	–4	0.5	a	–2a	0.25a	–0.71
FB	$4\sqrt{2}$	$-0.5\sqrt{2}$	$\sqrt{2}\,a$	$-4\sqrt{2}\,a$	$0.5\sqrt{2}\,a$	1
BG	–4	0.5	a	–2a	0.25a	–0.71
BC	4	–0.5	a	–2a	0.25 a	0.71
GH	– 6	1	a	– 6a	a	0.58
GC	$2\sqrt{2}$	$-0.5\sqrt{2}$	$\sqrt{2}\,a$	$-2\sqrt{2}\,a$	$0.5\sqrt{2}\,a$	– 1.82
ED	0	0	a	0	0	0
EJ	–4	0.5	a	–2a	0.25a	–0.71
JI	– 4	0.5	a	– 2a	0.25a	–0.71
JD	$4\sqrt{2}$	$-0.5\sqrt{2}$	$\sqrt{2}\,a$	$-4\sqrt{2}\,a$	$0.5\sqrt{2}\,a$	1
DI	– 4	0.5	a	– 2a	0.25a	–0.71
DC	4	– 0.5	a	– 2a	0.25a	0.71
HI	–6	1	a	– 6a	a	0.58
IC	$2\sqrt{2}$	$-0.5\sqrt{2}$	$\sqrt{2}\,a$	$-2\sqrt{2}\,a$	$0.5\sqrt{2}\,a$	–1.82
HC	– 4	0	a	0	0	–4

$\therefore$
$$\Sigma PuL = -(28a + 12\sqrt{2}\,a)$$

and
$$\Sigma u^2L = 4a + 2\sqrt{2}\,a$$

$\therefore$
$$R_C = -\frac{\sum \frac{PuL}{AE}}{\sum \frac{u^2L}{AE}}$$

Assuming truss members to have same axial stiffness,

$$R_C = \frac{-\Sigma PuL}{\Sigma u^2L}$$

$$\therefore \qquad R_C = \frac{-\left\{-(28a+12\sqrt{2}\,a)\right\}}{4a+2\sqrt{2}\,a}$$

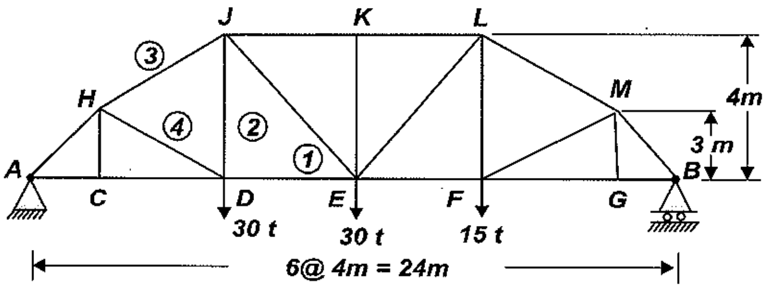
$$R_C = \frac{28+12\sqrt{2}}{4+2\sqrt{2}}$$

$$R_C = 6.586 \text{ kN}$$

(+) ve value of R_C means reaction is in the direction of applied unit load.

Hence, Forces in each members = $P + uR_C$ has been listed in above table.

Q-8: ***Calculate the magnitude and nature of forces in members marked 1, 2, 3 and 4. Assume all members to be pinjointed***



[20 Marks, ESE-2005]

Sol: Degree of indeterminacy (D_s)

$$D_s = m + r - 2j$$

Here, $m = 21$

$r = 3$

$j = 12$

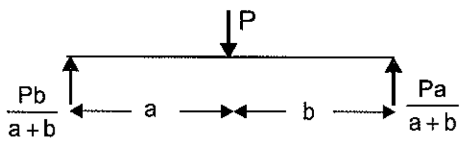
$$\therefore \qquad D_s = 21 + 3 - 2 \times 12 = 0$$

Hence the given truss is statically determinate.

Reaction calculation

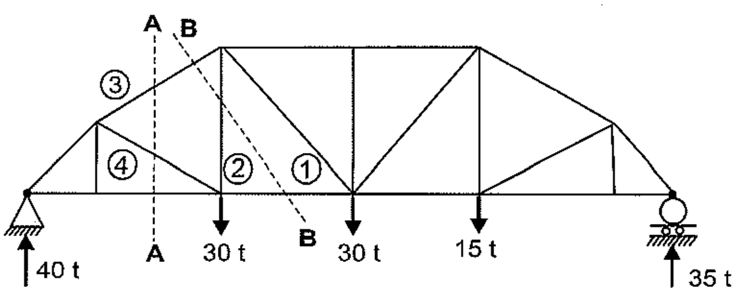
$$R_A = \frac{30\times 4}{6}+\frac{30\times 3}{6}+\frac{15\times 2}{6}$$

$$= 20 + 15 + 5 = 40 \text{ kN}$$

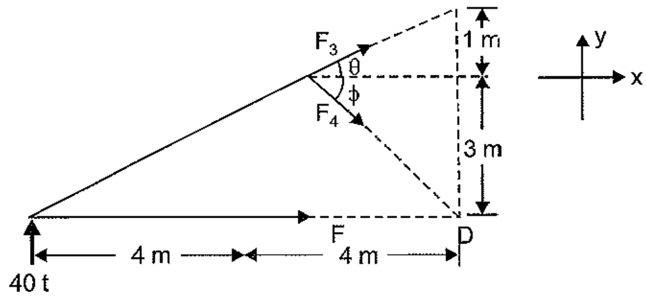


$$\Rightarrow \qquad R_B = 30 + 30 + 15 - 40$$

$$R_B = 35 \text{ kN}$$



From Section an A – A



$\tan\theta = \frac{1}{4}$	$\tan\phi = \frac{3}{4}$
$\sin\theta = \frac{1}{\sqrt{17}}$	$\sin\phi = \frac{3}{5}$
$\cos\theta = \frac{4}{\sqrt{17}}$	$\cos\phi = \frac{4}{5}$

From $\Sigma F_y = 0$

$$F_3 \sin\theta - F_4 \sin\phi + 40 = 0$$

$$\Rightarrow \quad \frac{F_3}{\sqrt{17}} - \frac{F_4 \times 3}{5} + 40 = 0 \qquad \text{... (i)}$$

From $\Sigma M_D = 0$

$$F_3 \cos\theta \times 3 + F_3 \sin\theta \times 4 + 40 \times 8 = 0 \qquad \text{... (ii)}$$

$$\frac{12\,F_3}{\sqrt{17}} + \frac{4F_3}{\sqrt{17}} + 320 = 0$$

$$F_3 = \frac{-320 \times \sqrt{17}}{16} = -20\sqrt{17} = -82.46\text{ t}$$

$$\boxed{F_3 = -82.46\text{ t (compression)}}$$

From (i)
$$F_4 = \left(\frac{F_3}{\sqrt{17}} + 40\right) \times \frac{5}{3} = (-20 + 40) \times \frac{5}{3} = 33.33\text{ t}$$

$$\boxed{F_4 = 33.33\text{t (tension)}}$$

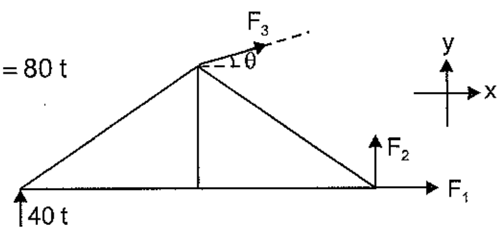
From section BB

From $\Sigma F_x = 0$

$$F_3 \cos\theta + F_1 = 0$$

$$\Rightarrow \quad F_1 = -F_3 \cos\theta = +20\sqrt{17} \times \frac{4}{\sqrt{17}} = 80\text{ t}$$

$$\boxed{F_1 = 80\text{ t}}$$



From $\Sigma F_y = 0$

$$F_3 \sin\theta + F_2 + 40 = 0$$

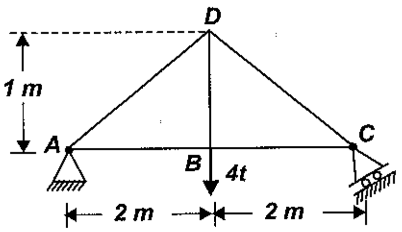
$$F_2 = -F_3 \sin\theta - 40 = +20\sqrt{17} \times \frac{1}{\sqrt{17}} - 40 = 20 - 40$$

$$\boxed{F_2 = -20\text{ t}}$$

Thus,
$F_1 = 80$ t (tension)
$F_2 = 20$ t (compression)
$F_3 = 82.46$ t (compression)
$F_4 = 33.33$ t (tension)

Q-9: ***A small truss as shown in the figure has all the pinjointed members having 20 sq. cm. cross-sectional area. Plane of roller is inclined and parallel to member AD. Determine the movement of joint 'C' on application of a vertical load of 4t at joint 'B'.***

$$E = 2 \times 10^6 \text{ kg/cm}^2$$



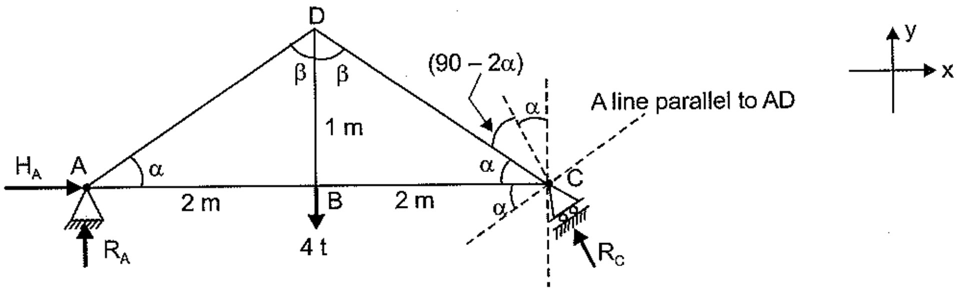
[20 Marks, ESE-2005]

Sol: Static indeterminacy of the given truss,

$$D_s = m + r - 2j$$

Here,
$$m = 5$$
$$r = 3$$
and
$$j = 4$$
$\therefore$
$$D_s = 5 + 3 - 8 = 0$$

Hence structure is statically determinate,



$$\tan\alpha = \frac{1}{2} \quad \Big| \quad \tan\beta = \frac{2}{1}$$
$$\sin\alpha = \frac{1}{\sqrt{5}} \quad \Big| \quad \sin\beta = \frac{2}{\sqrt{5}}$$
$$\cos\alpha = \frac{2}{\sqrt{5}} \quad \Big| \quad \cos\beta = \frac{1}{\sqrt{5}}$$

Calculation of reaction

From
$$\Sigma F_x = 0$$
$$H_A - R_C \cos(90 - \alpha) = 0$$
$$H_A - R_C \sin\alpha = 0 \quad \text{... (i)}$$

From
$$\Sigma F_y = 0$$
$$R_A + R_C \cos\alpha - 4 = 0 \quad \text{... (ii)}$$

From
$$\Sigma M_C = 0$$
$$R_A \times 4 - 4 \times 2 = 0 \quad \text{... (iii)}$$
$$\boxed{R_A = 2t}$$

$$\Rightarrow \qquad R_c = \frac{(4-2)}{\cos\alpha} = \frac{(4-2)\sqrt{5}}{2} = \sqrt{5}$$

$$\boxed{R_C = \sqrt{5}t}$$

$$\Rightarrow \qquad H_A = R_C \sin\alpha = \sqrt{5}\times\frac{1}{\sqrt{5}} = 1$$

$$\boxed{H_A = 1t}$$

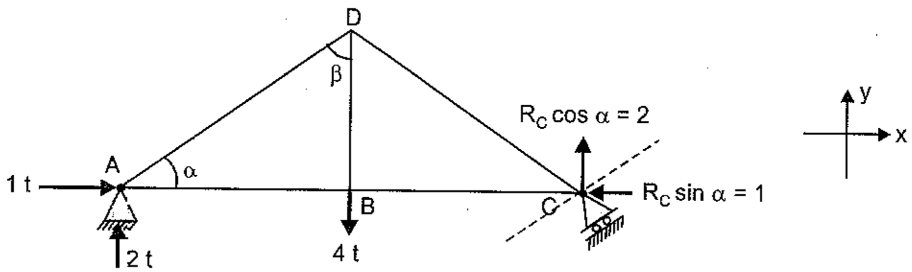
Deflection of Joint C will be along line parallel to AD

$$\Delta = \sum\frac{PuL}{AE}$$

P = Member forces due to external load

u = Member forces due to unit load applied along line parallel to AD after removal of external load

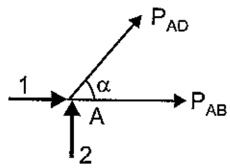
Calculation of P



From $\qquad \Sigma F_x = 0$

$$P_{AB} + 1 + P_{AD}\cos\alpha = 0 \qquad \text{... (i)}$$

From $\qquad \Sigma F_y = 0$



$$P_{AD}\sin\alpha + 2 = 0$$

$$\Rightarrow \qquad \boxed{P_{AD} = \frac{-2}{\sin\alpha} = -2\sqrt{5}t} \qquad \text{... (ii)}$$

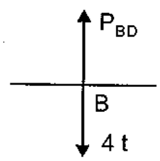
$$\Rightarrow \qquad P_{AB} = -1 - P_{AD}\cos\alpha$$

$$= -1-\left(-2\sqrt{5}\right)\times\frac{2}{\sqrt{5}}$$

$$\boxed{P_{AB} = 3t}$$

Joint B

$$\Sigma F_y = 0$$

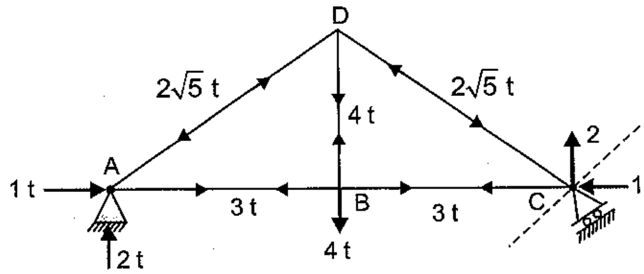


$$\Rightarrow \qquad \boxed{P_{BD} = 4t}$$

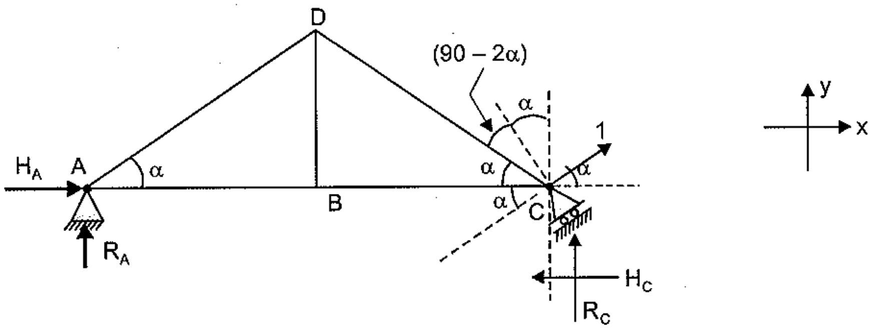
From symmetry of loading,

$$P_{CD} = P_{AD} = -2\sqrt{5}t$$

$$P_{CB} = P_{AB} = 3t$$



Calculation of u



From $\Sigma F_x = 0$

$$H_A - H_C + 1 \times \cos\alpha = 0 \quad \text{...(i)}$$

From $\Sigma F_y = 0$

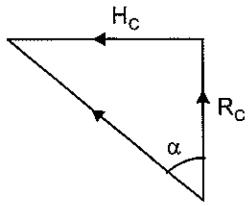
$$R_A + R_C + 1\sin\alpha = 0 \quad \text{...(ii)}$$

From $\Sigma M_C = 0$

$$\boxed{R_A = 0} \quad \text{...(iii)}$$

⇒ From (ii) $R_C = -\sin\alpha$

$$\boxed{R_C = -\frac{1}{\sqrt{5}}}$$



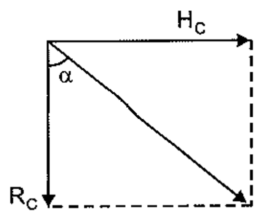
⇒ $$\tan\alpha = \frac{H_C}{R_C}$$

⇒ $$H_C = R_C \tan\alpha = -\frac{1}{\sqrt{5}} \times \frac{1}{2}$$

$$\boxed{H_C = -\frac{0.5}{\sqrt{5}}}$$

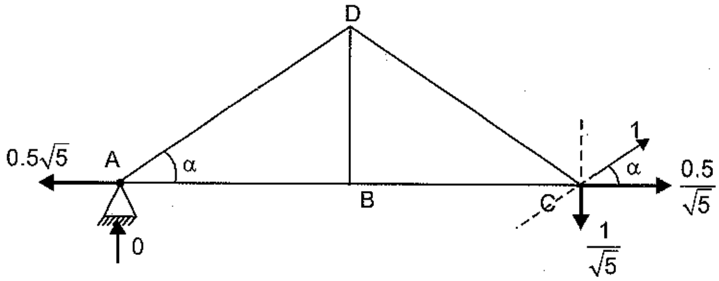
⇒ H_C will be in the direction opposite to that shown above, hence the actual direction is as shown

From (i),

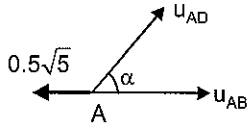


⇒ $$H_A = H_C - \cos\alpha = \frac{-0.5}{\sqrt{5}} - \frac{2}{\sqrt{5}} = -\frac{2.5}{\sqrt{5}}$$

$$\boxed{H_A = -0.5\sqrt{5}}$$



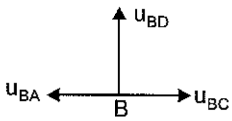
From Joint A



$$u_{AD} = 0$$

$$u_{AB} = 0.5\sqrt{5} \text{ (tension)}$$

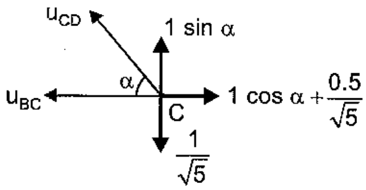
From Joint B



$$u_{BC} = 0.5\sqrt{5} \text{ (tension)}$$

$$u_{BD} = 0$$

From Joint C

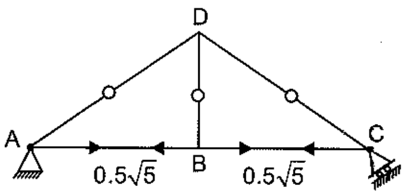


$$\Sigma F_y = 0$$

$$1.\sin\alpha + u_{CD}\sin\alpha - \frac{1}{\sqrt{5}} = 0$$

$$\Rightarrow \qquad u_{CD} = \frac{\frac{1}{\sqrt{5}} - \sin\alpha}{\sin\alpha} = 0$$

Hence u-forces in members are as shown below.



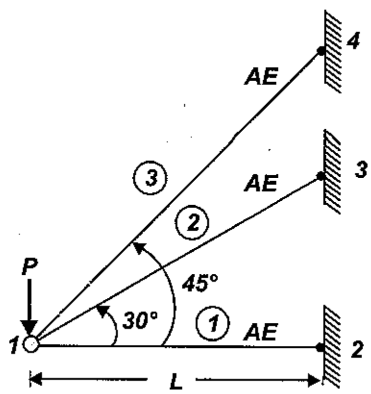
$$\Delta_D = \frac{\Sigma PuL}{AE} = \left[\frac{3\times 0.5\sqrt{5}}{AE}\times 2\right]\times 2$$

$$= \frac{3\times 4\times 0.5\sqrt{5}}{20\text{ cm}^2\times 2\times 10^6\text{ kg/cm}^2\times 10^{-3}}\text{ m}$$

$$= 3.354\times 10^{-4}\text{ m}$$

$$= 0.3354\text{ mm}$$

Q-10: ***For a three bar truss shown in the figure, compute the vertical displacement of node 1 by the displacement (stiffness/equilibrium) method.***

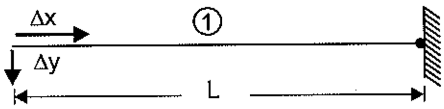


[20 Marks, ESE-2006]

Sol: **Forces induced in each member due to deflection of point 1;**

For member (1),

Axial compression of member 1 = Δ_x



$\therefore$ Axial force (compression) = $\frac{AE}{L}\Delta_x$

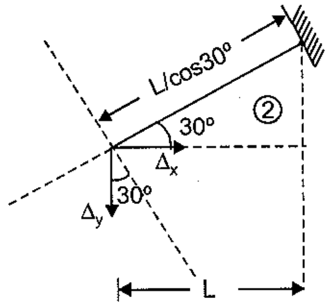
$\therefore$ P_{12} (compression) = $\frac{AE}{L}\Delta_x$

For member (2),

Component of deflections along the axes of member,

$$\Delta_{axial}\big|_{member\ 2} = \Delta_x \cos 30^\circ - \Delta_y \cos 60^\circ$$

$$\Delta_{axial}\big|_{compression} = \frac{\sqrt{3}}{2}\Delta_x - \frac{\Delta_y}{2}$$



Compressive force induced in member (2),

$$P_{13} = \frac{AE\cos\theta}{L}\left\{\frac{\sqrt{3}}{2}\Delta_x - \frac{\Delta_y}{2}\right\}$$

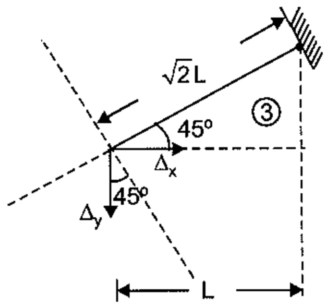
$$\Rightarrow \quad P_{13} = \frac{\sqrt{3}\,AE}{2L}\left\{\frac{\sqrt{3}}{2}\Delta_x - \frac{\Delta_y}{2}\right\}$$

For member (3),

Component of deflections along the axes of member 3,

$$\Delta_{axial}\big|_{member\ 3} = \frac{\Delta_x}{\sqrt{2}} - \frac{\Delta_y}{\sqrt{2}}$$

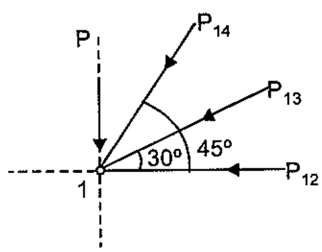
$$\Delta_{axial}\big|_{compression} = \frac{(\Delta_x - \Delta_y)}{\sqrt{2}}$$



Compressive force, $$P_{14} = \frac{AE}{\sqrt{2}L}\left[\frac{\Delta_x - \Delta_y}{\sqrt{2}}\right]$$

$$P_{14} = \frac{AE}{2L}\left[\Delta_x - \Delta_y\right]$$

Equilibrium equation at joint 1:



$$\Sigma F_x = 0 \Rightarrow P_{12} + P_{13}\cos 30^\circ + P_{14}\cos 45^\circ = 0$$

$$\Rightarrow \quad \frac{AE}{L}\Delta_x + \frac{\sqrt{3}\,AE}{2L}\left\{\frac{\sqrt{3}}{2}\Delta_x - \frac{\Delta_y}{2}\right\}\times\frac{\sqrt{3}}{2} + \frac{AE}{2L}[\Delta_x - \Delta_y]\times\frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow \quad \frac{AE}{L}\Delta_x + \left(\frac{3}{4}\frac{AE}{L}\Delta_x - \frac{\sqrt{3}}{4L}AE\,\Delta_y\right)\times\frac{\sqrt{3}}{2} + \frac{AE\Delta_x}{2\sqrt{2}L} - \frac{AE\,\Delta_y}{2\sqrt{2}L} = 0$$

$$\Rightarrow \quad \Delta_x + \frac{3\sqrt{3}}{8}\Delta_x + \frac{1}{2\sqrt{2}}\Delta_x - \frac{3}{8}\Delta_y - \frac{1}{2\sqrt{2}}\Delta_y = 0$$

$$\Rightarrow \quad 2.003\,\Delta_x - 0.7286\,\Delta_y = 0 \quad \text{... (i)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad -P = P_{14}\sin 45^\circ + P_{13}\sin 30^\circ$$

$$\Rightarrow \quad -P = \frac{AE}{2L}[\Delta_x - \Delta_y]\times\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\frac{AE}{L}\left\{\frac{\sqrt{3}}{2}\Delta_x - \frac{\Delta_y}{2}\right\}\times\frac{1}{2}$$

$$\Rightarrow \quad -\frac{PL}{AE} = \frac{\Delta_x}{2\sqrt{2}} + \frac{3}{8}\Delta_x - \frac{\Delta_y}{2\sqrt{2}} - \frac{\sqrt{3}}{8}\Delta_y$$

$$\Rightarrow \quad -\frac{PL}{AE} = 0.7286\,\Delta_x - 0.57\,\Delta_y$$

$$\Rightarrow \quad -0.7286\Delta_x + 0.57\Delta_y = +\frac{PL}{AE} \quad \text{... (ii)}$$

By solving equation (i) and (ii) we get

$$\Delta_x = \frac{1.1927\,PL}{AE}$$

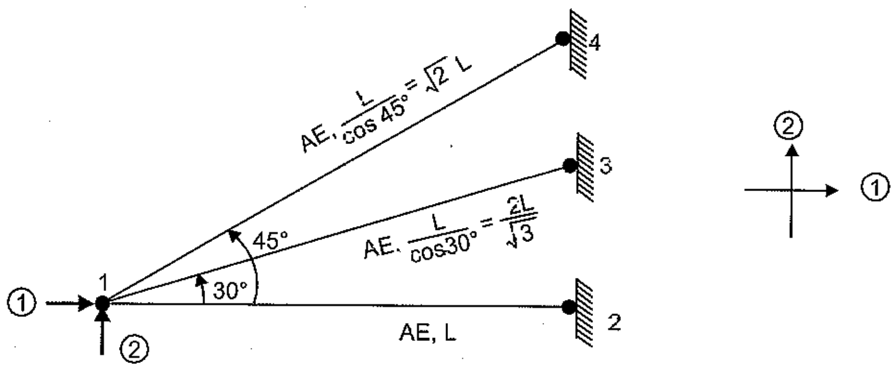
$$\Delta_y = \frac{3.279\,PL}{AE}$$

$$\Rightarrow \quad \text{Downward deflection of joint 1} = \frac{3.279\,PL}{AE}$$

Alternatively

The degree of freedom in the truss is two hence the stiffness matrix will be 2 × 2

Let us choose the co-ordinate system as (1) and (2) as shown below.



Members	$\frac{AE}{L}$	θ	$\frac{AE}{L}\cos^2\theta$	$\frac{AE}{L}\sin\theta\cos\theta$	$\frac{AE}{L}\sin^2\theta$
1 – 2	$\frac{AE}{L}$	0	$\frac{AE}{L}$	0	0
1 – 3	$\frac{\sqrt{3}AE}{2L}$	30°	$\frac{3\sqrt{3}AE}{8L}$	$\frac{3AE}{8L}$	$\frac{\sqrt{3}AE}{8L}$
1 – 4	$\frac{AE}{\sqrt{2}L}$	45°	$\frac{AE}{2\sqrt{2}L}$	$\frac{AE}{2\sqrt{2}L}$	$\frac{AE}{2\sqrt{2}L}$

$$K_{11} = \sum\frac{AE}{L}\cos^2\theta = 2.003\frac{AE}{L}$$

$$K_{12} = K_{21} = \sum \frac{AE}{L}\sin\theta\cos\theta = 0.7286\frac{AE}{L}$$

$$K_{22} = \sum \frac{AE}{L}\sin^2\theta = 0.57\frac{AE}{L}$$

Hence the stiffness eq is

$$\begin{bmatrix} 2.003\frac{AE}{L} & 0.7286\frac{AE}{L} \\ 0.7286\frac{AE}{L} & 0.57\frac{AE}{L} \end{bmatrix}\begin{bmatrix}\Delta_1 \\ \Delta_2\end{bmatrix} = \begin{bmatrix}0 \\ -P\end{bmatrix}$$

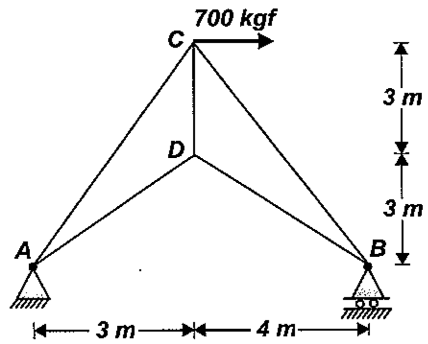
$$\Rightarrow \quad \begin{bmatrix}\Delta_1 \\ \Delta_2\end{bmatrix} = \begin{bmatrix} 2.003\frac{AE}{L} & 0.7286\frac{AE}{L} \\ 0.7286\frac{AE}{L} & 0.57\frac{AE}{L} \end{bmatrix}^{-1}\begin{bmatrix}0 \\ -P\end{bmatrix}$$

$\Rightarrow$ Solving we get, $\Delta_1 = \dfrac{1.192\,PL}{AE}$; $\Delta_2 = \dfrac{-3.279PL}{AE}$

$\Rightarrow$ Vertical deflection in the co-ordinate direction is $\dfrac{-3.279\,PL}{AE}$

$\Rightarrow$ Vertical deflection of joint 1 in ($\downarrow$) = $\dfrac{3.279\,PL}{AE}$

Q-11: ***Calculate forces in all the pin-jointed members of the truss as shown in the figure.***



Note that the truss is not symmetrical. Show the magnitude and nature of forces on the truss diagram.

[20 Marks, ESE-2007]

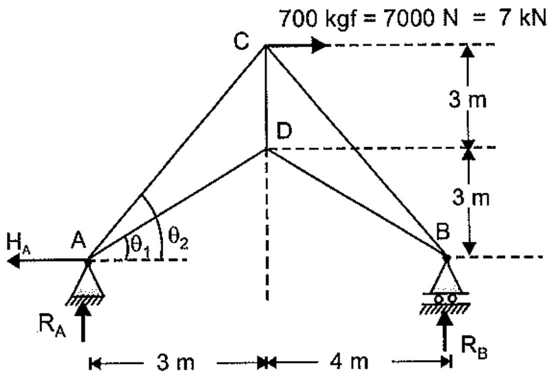
Sol: Degree of statical indeterminacy,

$$D_S = m + r - 2j$$

Hence, $m = 5,\ r = 3,\ j = 4$

$\therefore$ $D_s = 5 + 3 - 8 = 0$

Hence the given truss is statically determinate and unsymmetrical.



Reactions:

$$\Sigma F_y = 0 \Rightarrow R_A + R_B = 0$$

$$\Rightarrow \quad [R_A = -R_B]$$

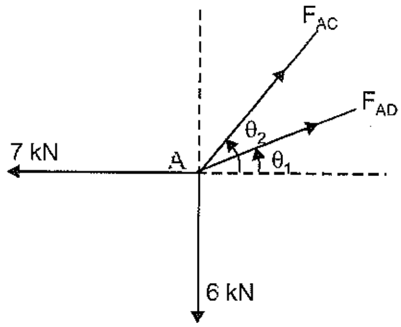
$$\Sigma M_A = 0 \Rightarrow R_B \times 7 - 7 \times 6 = 0$$

$$\Rightarrow \quad R_B = 6 \text{ kN } (\uparrow)$$

$$\therefore \quad R_A = -6 \text{ kN} \Rightarrow R_A = 6 \text{ kN } (\downarrow)$$

$$\Sigma F_x = 0 \Rightarrow H_A = 7 \text{ kN } (\leftarrow)$$

Joint A:



$$\tan\theta_1 = \frac{3}{3}$$

$$\tan\theta_2 = 2$$

$$\Rightarrow \sin\theta_1 = \frac{1}{\sqrt{2}}$$

$$\cos\theta_1 = \frac{1}{\sqrt{2}}$$

$$\sin\theta_2 = \frac{2}{\sqrt{5}}$$

$$\cos\theta_2 = \frac{1}{\sqrt{5}}$$

$$\Sigma F_x = 0$$

$$F_{AD}\cos\theta_1 + F_{AC}\cos\theta_2 = 7$$

$$F_{AD} \times \frac{1}{\sqrt{2}} + F_{AC} \times \frac{1}{\sqrt{5}} = 7$$

$$\Rightarrow \frac{F_{AD}}{\sqrt{2}} + \frac{F_{AC}}{\sqrt{5}} = 7 \quad \text{... (i)}$$

$$\Sigma F_y = 0$$

$$F_{AD}\sin\theta_1 + F_{AC}\sin\theta_2 - 6 = 0$$

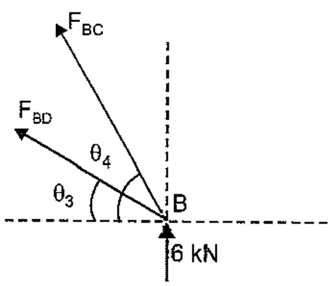
$$\Rightarrow \frac{F_{AD}}{\sqrt{2}} + \frac{2F_{AC}}{\sqrt{5}} = 6 \quad \text{... (ii)}$$

By solving these two equations, we get;

$$F_{AD} = 11.314 \text{ kN (tensile)}$$

$$F_{AC} = -2.236 \text{ kN (compressive)}$$

Joint B:



$$\tan\theta_3 = \frac{3}{4}$$

$$\tan\theta_4 = \frac{6}{4} = \frac{3}{2}$$

$$\Rightarrow \sin\theta_3 = \frac{3}{5}$$

$$\cos\theta_3 = \frac{4}{5}$$

$$\sin\theta_4 = \frac{3}{\sqrt{13}}$$

$$\cos\theta_4 = \frac{2}{\sqrt{13}}$$

$$\Sigma F_x = 0$$

$$\Rightarrow \quad F_{BC}\cos\theta_4 + F_{BD}\cos\theta_3 = 0$$

$$\Rightarrow \quad \frac{2F_{BC}}{\sqrt{13}} + \frac{4F_{BD}}{5} = 0$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad F_{BC}\sin\theta_4 + F_{BD}\sin\theta_3 + 6 = 0$$

$$\Rightarrow \quad \frac{3F_{BC}}{\sqrt{13}} + \frac{3F_{BD}}{5} = -6$$

By solving these two equations we get,

$$F_{BC} = -14.42 \text{ kN (compressive)}$$

$$F_{BD} = 10 \text{ kN (tensile)}$$

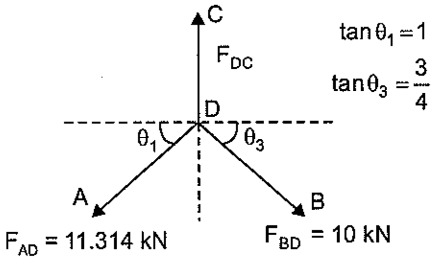
Joint D:

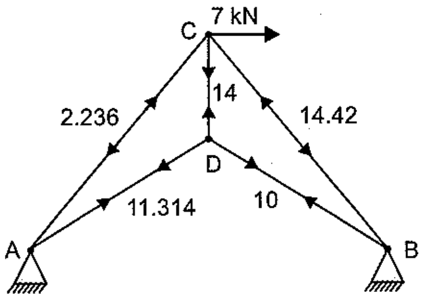
$$\Sigma F_y = 0$$

$$F_{AD}\sin\theta_1 + F_{BD}\sin\theta_3 = F_{DC}$$

$$\Rightarrow \quad 11.314 \times \frac{1}{\sqrt{2}} + 10 \times \frac{3}{5} = F_{DC}$$

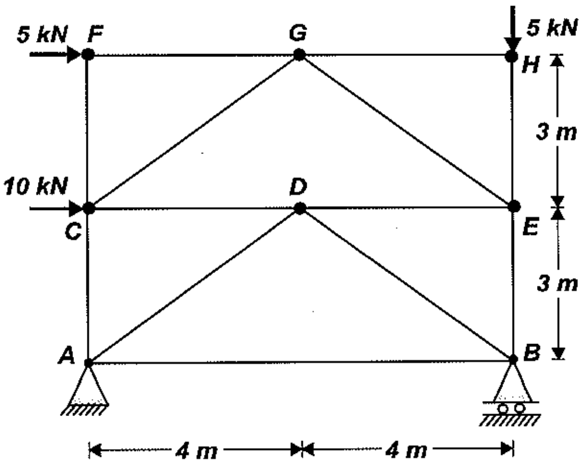
$$\Rightarrow \quad F_{DC} = 14 \text{ (tensile)}$$





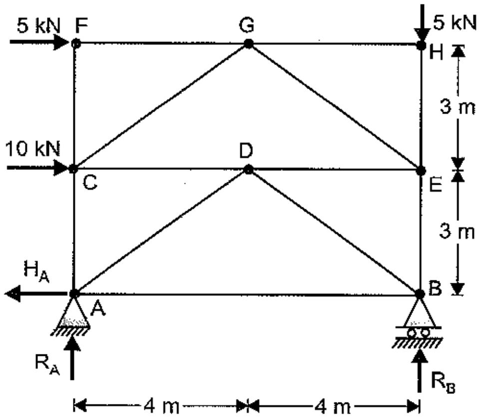
(All forces are in kN)

Q-12: ***Find the magnitude and the nature of the forces in the members of the truss shown in the figure. Find also the reaction components at the supports. Tabulate the results.***



[25 Marks, ESE-2008]

Sol:



Degree of static indeterminacy for given plane truss,

$$D_S = m + r - 2j$$

where,

$$m = 13$$
$$r = 3$$
$$j = 8$$

$\therefore$

$$D_S = 13 + 3 - 16 = 0$$

Hence the given truss is statically determinate.

Reaction Determination

$$\Sigma F_x = 0 \Rightarrow H_A = 10 + 5 = 15 \text{ kN}(\leftarrow)$$

$$\Sigma F_y = 0 \Rightarrow R_A + R_B = 5$$

$$\Rightarrow \Sigma M_B = 0 \Rightarrow R_A \times 8 + 10 \times 3 + 5 \times 6 = 0$$

$$\Rightarrow 8R_A = -60$$

$$\Rightarrow R_A = -7.5 \text{ kN}$$

$$\Rightarrow R_A = 7.5 \text{ kN } (\downarrow)$$

$$\therefore R_B = 5 - (-7.5) = 12.5 \text{ kN } (\uparrow)$$

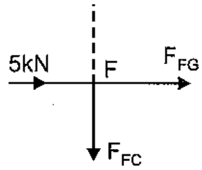
Joint F:

$$\Sigma F_y = 0$$

$$\Rightarrow F_{FC} = 0$$

$$\Sigma F_x = 0$$

$$F_{FG} = -5\text{kN}$$



Joint H:

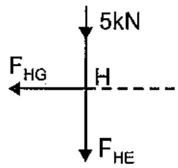
$$\Sigma F_x = 0$$

$$\Rightarrow F_{HG} = 0$$

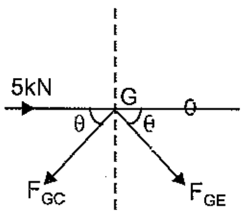
$$\Sigma F_y = 0$$

$$F_{HE} + 5 = 0$$

$$\Rightarrow F_{HE} = -5 \text{ kN}$$



Joint G:



$$\tan\theta = \frac{3}{4}$$

$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \frac{4}{5}$$

$$\Sigma F_x = 0$$

$$F_{GC}\cos\theta - F_{GE}\cos\theta = 5$$

$$\Rightarrow \quad F_{GC} - F_{GE} = \frac{5}{\cos\theta}$$

$$\Rightarrow \quad F_{GC} - F_{GE} = \frac{25}{4}$$

$$\Sigma F_y = 0$$

$$F_{GC}\sin\theta = -F_{GE}\sin\theta$$

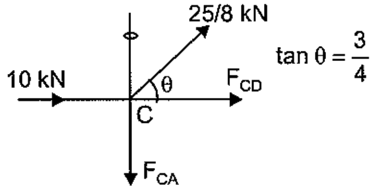
$$\Rightarrow \quad F_{GC} + F_{GE} = 0$$

By solving above two equations, we get;

$$F_{GC} = 25/8 \text{ kN}$$

$$F_{GE} = -25/8 \text{ kN}$$

Joint C:



$$\Sigma F_x = 0$$

$$F_{CD} + \frac{25}{8}\cos\theta + 10 = 0$$

$$F_{CD} = -\frac{25}{8}\times\frac{4}{5} - 10$$

$$F_{CD} = -2.5 - 10$$

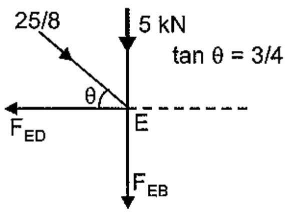
$$F_{CD} = -12.5$$

$$\Sigma F_y = 0$$

$$F_{CA} = \frac{25}{8}\sin\theta = \frac{25}{8}\times\frac{3}{5} = \frac{15}{8}$$

$$F_{CA} = 1.875 \text{ kN}$$

Joint E:



$$\Sigma F_x = 0$$

$$\Rightarrow \quad F_{ED} = \frac{25}{8}\cos\theta = \frac{25}{8}\times\frac{4}{5}$$

$$\Rightarrow \quad F_{ED} = 2.5 \text{ kN}$$

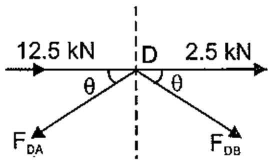
$$\Sigma F_y = 0$$

$$\Rightarrow \quad F_{EB} + \frac{25}{8}\sin\theta + 5 = 0$$

$$\Rightarrow \quad F_{EB} = -5 - \frac{25}{8}\times\frac{3}{5}$$

$$\Rightarrow \quad F_{EB} = -6.875 \text{ kN}$$

Joint D:



$$\tan\theta = \frac{3}{4}$$

$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \frac{4}{5}$$

$$\Sigma F_y = 0$$

$$F_{DA}\times\sin\theta + F_{DB}\sin\theta = 0$$

$$\Rightarrow \quad F_{DA} = -F_{DB}$$

$$F_{DA} + F_{DB} = 0$$

$$\Sigma F_x = 0$$

$$F_{DA}\cos\theta - 12.5 - F_{DB}\cos\theta - 2.5 = 0$$

$$\Rightarrow \quad F_{DA} - F_{DB} = \frac{15}{\cos\theta}$$

$$\Rightarrow \quad F_{DA} - F_{DB} = \frac{15\times 5}{4}$$

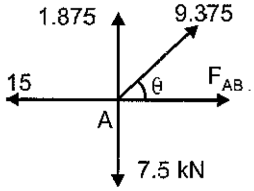
$$\Rightarrow \quad F_{DA} - F_{DB} = 18.75$$

$\therefore$ By solving;

$$F_{DA} = 9.375 \text{ kN}$$

$$F_{DB} = -9.375 \text{ kN}$$

Joint A:



$$\tan\theta = \frac{3}{4}$$

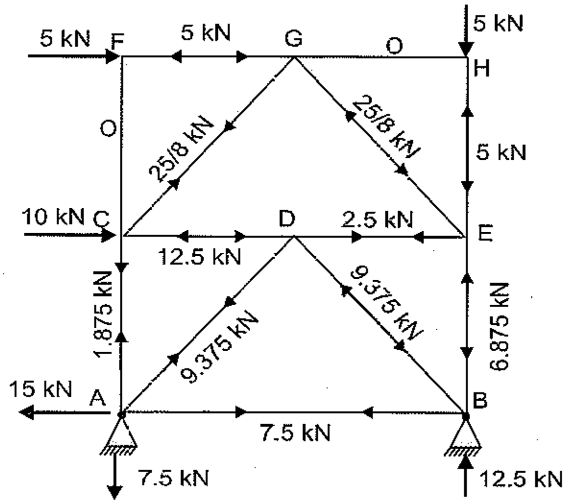
$$\Sigma F_x = 0$$

$$\Rightarrow \quad F_{AB} + 9.375\cos\theta = 15$$

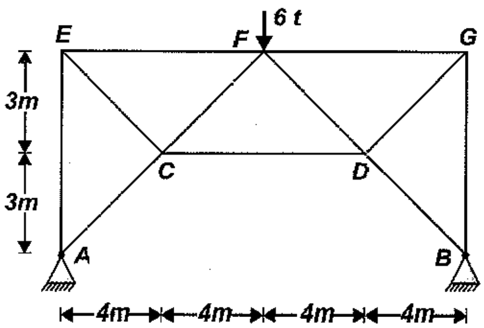
$$\Rightarrow \quad F_{AB} = 15 - 9.375 \times \frac{4}{5}$$

$$\Rightarrow \quad F_{AB} = 7.5 \text{ kN}$$

Hence the member forces and reaction at the support is as shown in figure below:



Q-13: ***All the members of steel truss shown in the figure are pin-jointed and have same area of cross-section.***



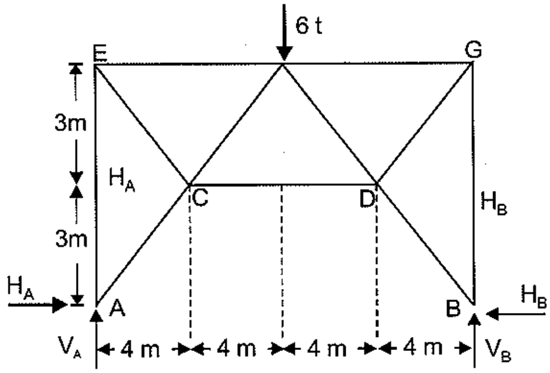
Calculate the vertical and horizontal components of the reactions at A and B.

[20 Marks, ESE-2009]

Sol: Degree of statical indeterminacy = $(D_S) = m + r - 2j$

Here, $m = 11$; $r = 4$; $j = 7$

$\therefore \quad D_S = 11 + 4 - 14 = 1$



Degree of external indeterminacy $= r_e - 3 = 4 - 3 = 1$

Degree of internal indeterminacy $= D_S - D_e = 0$

Hence the given truss is externally indeterminate by one degree.

We can analyze the truss by taking any member force or support reaction as redundant provided that removal of that member or support reaction does not make the structure unstable.

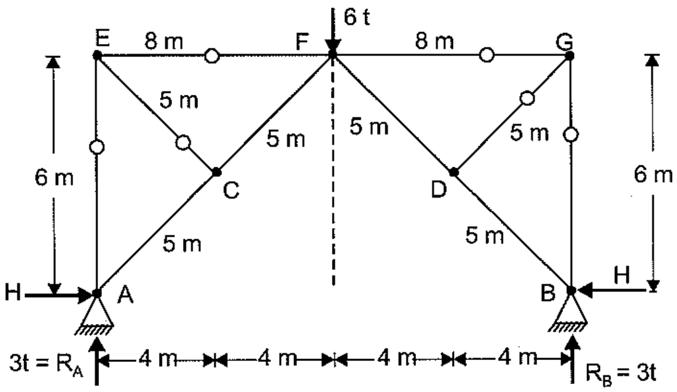
Let use take force in member CD to be the redundant force (R)

$$\Rightarrow \qquad R = \frac{-\sum \frac{P_i u_i l_i}{A_i E_i}}{\sum \frac{u_i^2 l_i}{A_i E_i}}$$

P_i = Member forces in i[th] member due to external loads when member CD has been removed

u_i = Member force in i[th] member due to equal and opposite unit loads applied at C and D

Calculation of (P_i)



From symmetry of truss and loading, $R_A = R_B = \frac{6}{2} = 3t$

From BM at F = 0,

$$3 \times 8 - H \times 6 = 0 \Rightarrow \boxed{H = 4t}$$

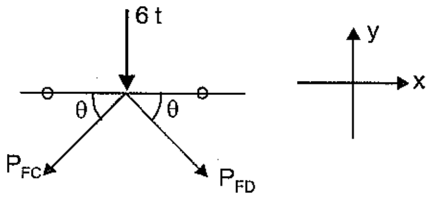
Forces in member CE = 0 because at joint C three members are meeting and two of them are concurrent and there is no external force acting.

Hence, $\qquad P_{CE} = 0$

On similar lines, $\qquad P_{DG} = 0$

$$\Rightarrow \quad P_{EA} = 0$$
$$P_{EF} = 0$$
$$P_{GF} = 0$$
$$P_{GB} = 0$$

At Joint F



$\tan\theta = \dfrac{3}{4}$

$\sin\theta = \dfrac{3}{5}$

$\cos\theta = \dfrac{4}{5}$

From $\quad \Sigma F_x = 0$

$$\Rightarrow \quad (P_{FD} - P_{FC})\cos\theta = 0$$

$$\Rightarrow \quad P_{FD} = P_{FC} \qquad \text{... (i)}$$

From $\quad \Sigma F_y = 0$

$$(P_{FD} + P_{FC})\sin\theta + 6 = 0$$

$$\Rightarrow \quad P_{FD} = P_{FC} = \frac{-3}{\sin\theta} = -5t$$

$$\boxed{P_{FD} = P_{FC} = -5t}$$

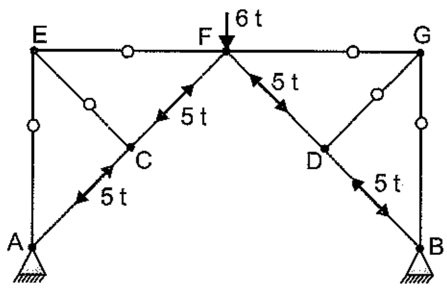
At Joint C

$$\boxed{P_{FC} = P_{CA} = -5t}$$

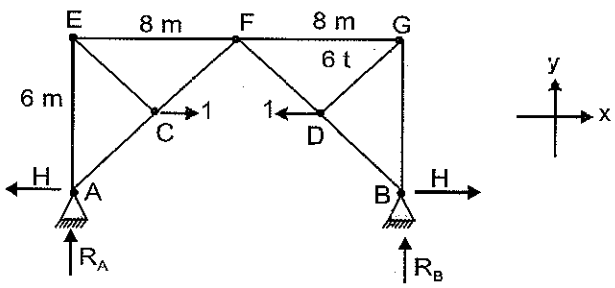
At Joint D

$$\boxed{P_{FD} = P_{DB} = -5t}$$

$\Rightarrow$ Hence P_i forces are as shown



Calculation of (u_i)



From $\quad \Sigma F_y = 0;\ R_A + R_B = 0$

$$\Sigma M_B = 0;\ R_A \times 16 + H \times 0 + 1 \times 3 - 1 \times 3 = 0$$

$$\Rightarrow \quad R_A = 0$$

$$\Rightarrow \quad R_B = 0$$

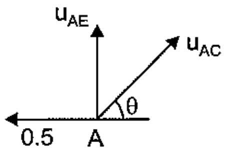
From BM at F to be zero,

$$R_A \times 8 + H \times 6 - 1 \times 3 = 0$$

$$\Rightarrow \quad 0 \times 8 + 6H - 3 = 0$$

$$\Rightarrow \quad \boxed{H = 0.5}$$

At Joint A



$$\tan\theta = \frac{3}{4}$$

$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \frac{4}{5}$$

From $\quad \Sigma F_x = 0$

$$u_{AC}\cos\theta - 0.5 = 0$$

$$\Rightarrow \quad u_{AC} = \frac{0.5 \times 5}{4} = \frac{2.5}{4} = 0.625$$

$$\boxed{u_{AC} = 0.625}$$

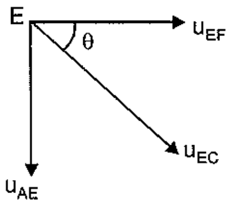
$$\Sigma F_y = 0,$$

$$u_{AC}\sin\theta + u_{AE} = 0$$

$$\Rightarrow \quad u_{AE} = -0.625 \times \frac{3}{5}$$

$$\Rightarrow \quad \boxed{u_{AE} = -0.375}$$

At joint E



$$\tan\theta = \frac{3}{4}$$

$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \frac{4}{5}$$

From $\quad \Sigma F_y = 0$

$$u_{EC}\sin\theta + u_{AE} = 0$$

$$\Rightarrow \quad \boxed{u_{EC} = \frac{0.375 \times 5}{3} = 0.625}$$

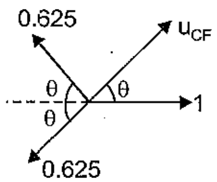
From $\quad \Sigma F_x = 0$

$$u_{EF} + u_{EC}\cos\theta = 0$$

$$u_{EF} + 0.625 \times \frac{4}{5} = 0$$

$$\Rightarrow \quad \boxed{u_{EF} = -0.5}$$

At joint C



$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

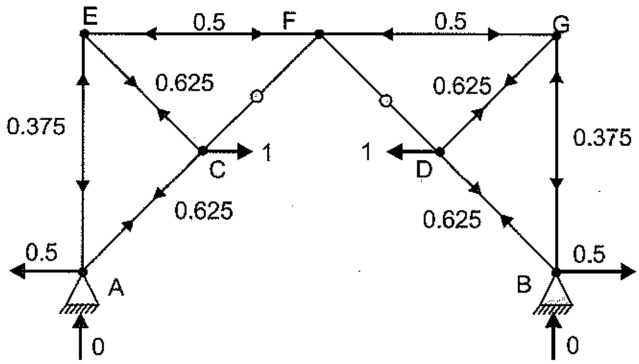
$\cos\theta = \frac{4}{5}$

From $\Sigma F_x = 0$

$$u_{CF}\cos\theta - 2 \times 0.625\cos\theta + 1 = 0$$

$\Rightarrow$ $\boxed{u_{CF} = 0}$

Similarly, on the basis of symmetry, forces in other members can be found out



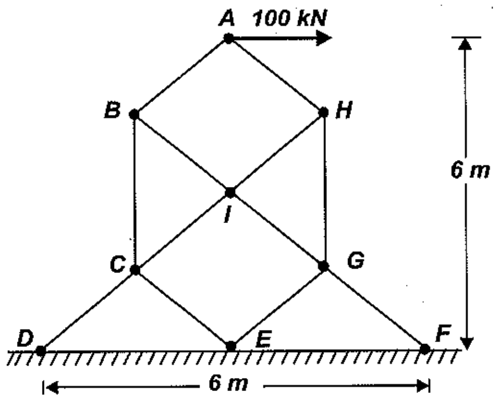
Member	P_i (t)	u_i	l_i (m)	$P_i u_i l_i$	$u_i^2 l_i$
AE	0	–0.375	6	0	0.84375
AC	–5	0.625	5	–15.625	1.953125
EF	0	– 0.5	8	0	2
CF	–5	0	5	0	0
EC	0	0.625	5	0	1.953125
BG	0	–0.375	6	0	0.84375
BD	–5	0.625	5	–15.625	1.953125
GF	0	–0.5	8	0	2
DF	–5	0	5	0	0
DG	0	0.625	5	0	1.953215
CD	0	1	8	0	8
				Σ = –31.25	**Σ = 21.5**

$$\Rightarrow \quad R = \frac{-\sum \frac{PuL}{AE}}{-\sum \frac{u^2L}{AE}} = \frac{31.25}{21.5} = 1.4535\text{ t}$$

We know that final force = $P_i + u_iR$, Hence

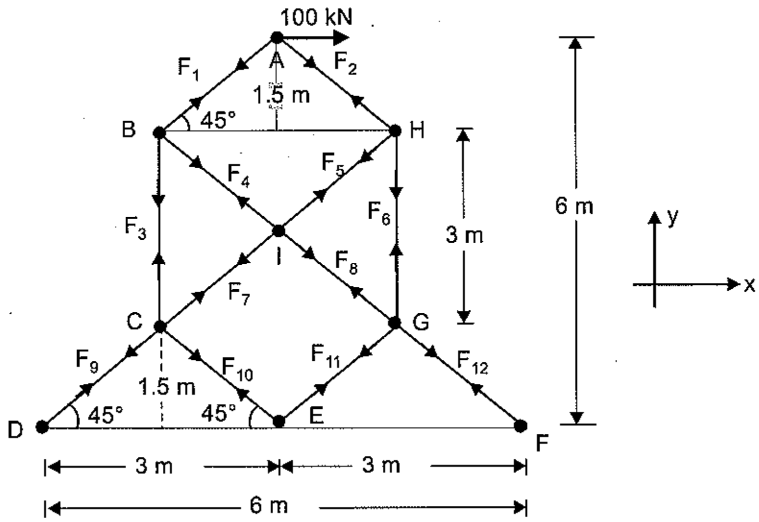
$$\Rightarrow \quad \boxed{\begin{aligned} R_A &= 3 + 0 \times 1.4535 = 3t \\ R_B &= 3 + 0 \times 1.4535 = 3t \\ H &= 4 - 0.5 \times 1.4535 = 3.27325\text{ t} \end{aligned}}$$

Q-14: ***Calculate the bar forces in all the members of the plane truss shown below which is used as a tower. All inclined members have a slope of 45° to the horizontal plane. Also, find reactions at supports D, E and F.***



[20 Marks, ESE-2010]

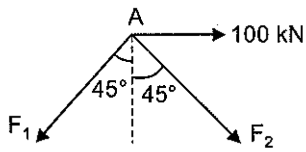
Sol:



Joint A

$$\Rightarrow \qquad \Sigma Fy = 0$$

$$(F_1 + F_2)\cos 45° = 0 \Rightarrow F_1 = F_2$$



$$\Sigma F_x = 0 \qquad F_2 \sin 45° - F_1 \sin 45° + 100 = 0$$

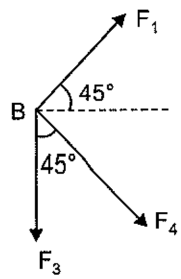
$$(F_2 - F_1)\sin 45° + 100 = 0$$

$$[F_2 - (-F_2)]\sin 45° + 100 = 0$$

$$\frac{2F_2}{\sqrt{2}} = -100$$

$$\boxed{F_2 = -50\sqrt{2}\text{ kN}}$$

$$\boxed{F_1 = +50\sqrt{2}\text{ kN}}$$



Joint B

$$\Sigma F_x = 0$$

$$\Rightarrow \qquad (F_4 + F_1)\times\frac{1}{\sqrt{2}} = 0$$

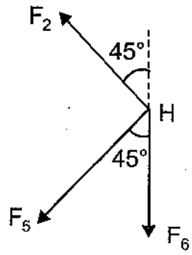
$$\boxed{F_4 = -F_1 = -50\sqrt{2}\text{ kN}}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad F_3 + (F_4 - F_1)\frac{1}{\sqrt{2}} = 0 \Rightarrow F_3 = \left(\frac{F_1 - F_4}{\sqrt{2}}\right) = \frac{50\sqrt{2} - (-50\sqrt{2})}{\sqrt{2}}$$

$$\boxed{F_3 = 100\text{ kN}}$$

At Joint H



$$\Sigma F_x = 0$$

$$\Rightarrow \quad F_2 + F_5 = 0$$

$$F_5 = -F_2 = -(-50\sqrt{2}) = 50\sqrt{2}\text{ kN}$$

$$\boxed{F_5 = 50\sqrt{2}\text{ kN}}$$

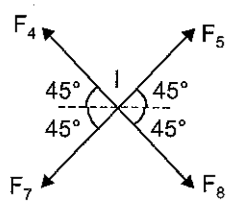
$$\Sigma F_y = 0$$

$$(F_6 + (F_5 - F_2)\frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow \quad F_6 = \frac{-50\sqrt{2} - 50\sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow \quad \boxed{F_6 = -100\text{ kN}}$$

At Joint I



$$\Rightarrow \quad \Sigma F_y = 0$$

$$\Rightarrow \quad (F_7 + F_8 - F_4 - F_5) \times \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow \quad F_7 + F_8 = F_4 + F_5$$

$$F_7 + F_8 = 0 \qquad \text{...(A)}$$

$$\Sigma F_x = 0$$

$$(F_4 - F_5 + F_7 - F_8)\frac{1}{\sqrt{2}} = 0$$

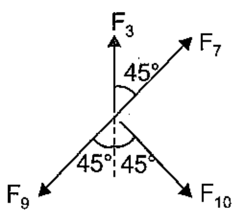
$$F_7 - F_8 = F_5 - F_4 = 50\sqrt{2} - (-50\sqrt{2}) = 100\sqrt{2}$$

$$F_7 - F_8 = 100\sqrt{2} \qquad \text{...(B)}$$

From (A) and (B) $\boxed{F_7 = 50\sqrt{2}}$

$$\boxed{F_8 = -50\sqrt{2}}$$

At joint C



$$\Sigma F_x = 0$$

$$\Rightarrow \quad (F_7 + F_{10} - F_9)\frac{1}{\sqrt{2}} = 0$$

$$F_9 = F_7 + F_{10} \qquad \text{...(C)}$$

$$\Sigma F_y = 0$$

$$F_3 + \frac{1}{\sqrt{2}}\,[F_7 - F_9 - F_{10}] = 0$$

$$F_9 + F_{10} = F_3 \times \sqrt{2} + F_7$$

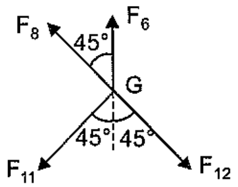
$$F_9 + F_{10} = 100\sqrt{2} + 50\sqrt{2} \qquad \text{...(D)}$$

Also, $$F_9 - F_{10} = 50\sqrt{2} \text{ from equation (C)}$$

$$\boxed{F_{10} = 50\sqrt{2}}$$

$$\boxed{F_{10} = 100\sqrt{2}}$$

At Joint G



$$\Sigma F_x = 0$$

$$\Rightarrow \quad (F_8 + F_{11} - F_{12})\frac{1}{\sqrt{2}} = 0$$

$$F_{11} - F_{12} = -F_8 = 50\sqrt{2}$$

$$F_{11} - F_{12} = 50\sqrt{2} \qquad \text{... (E)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad F_6 + \frac{F_8}{\sqrt{2}} - \frac{F_{11}}{\sqrt{2}} - \frac{F_{12}}{\sqrt{2}} = 0$$

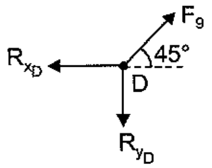
$$F_{11} + F_{12} = \sqrt{2}\,F_6 + F_8 = -100\sqrt{2} - 50\sqrt{2}$$

Also, $$F_{11} - F_{12} = 50\sqrt{2} \text{ from (E)}$$

$$\boxed{F_{11} = -50\sqrt{2}}$$

$$\boxed{F_{12} = -100\sqrt{2}}$$

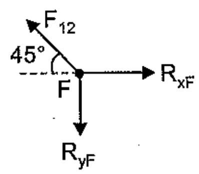
At Joint D



$$R_{y_D} = \frac{F_9}{\sqrt{2}} = 100$$

$$R_{xD} = \frac{F_9}{\sqrt{2}} = 100$$

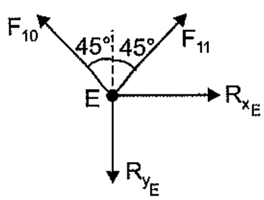
At Joint F



$$R_{xF} = \frac{F_{12}}{\sqrt{2}} = -100$$

$$R_{yF} = \frac{F_{12}}{\sqrt{2}} = -100$$

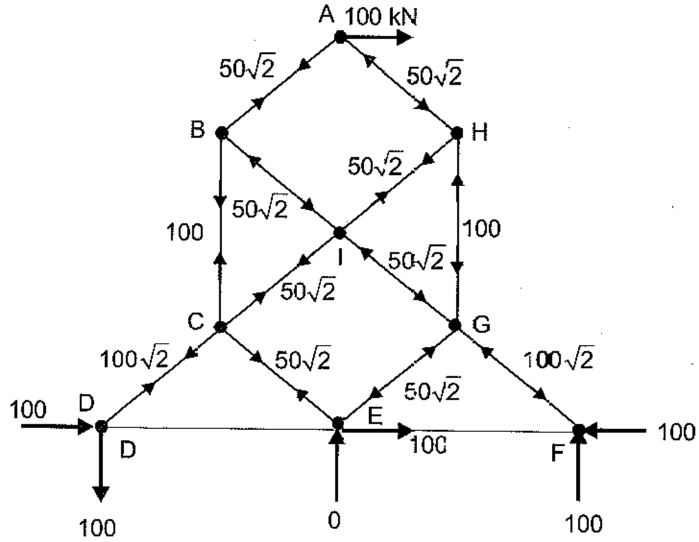
At Joint E



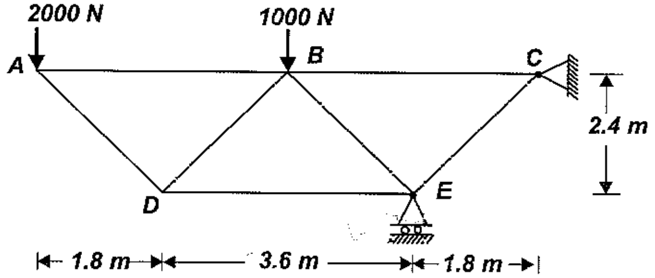
$$\frac{(F_{10} + F_{11})}{\sqrt{2}} = R_{y_E}$$

$$\Rightarrow \quad R_{y_E} = 0$$

$$R_{X_E} = \frac{F_{10} - F_{11}}{\sqrt{2}} = 100$$



Q-15: ***Analyse the truss shown in the figure below and list out the forces in the members.***



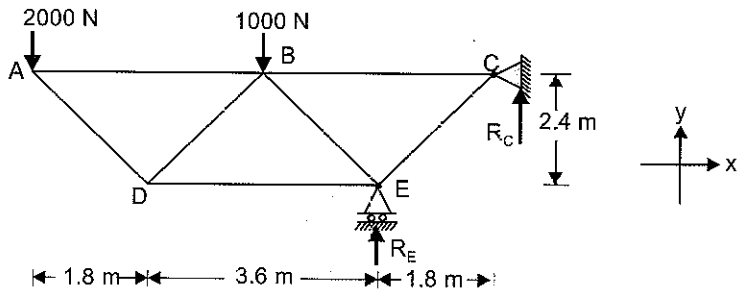
[14 Marks, ESE-2011]

Sol: Degree of statical indeterminacy = $D_S = m + r - 2j$

Here $m = 7$; $r = 3$ and $j = 5$

$\therefore$ $D_S = 7 + 3 - 2 \times 5 = 0$

Hence the given structure is statically determinate



Determination of Reactions:

$$\Sigma F_y = 0$$

$$R_E + R_C = 3000$$

$$\Sigma M_C = 0$$

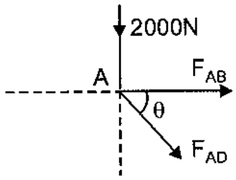
$$R_E \times 1.8 = 1000 \times 3.6 + 2000 \times 7.2$$ [Here perpendicular bisector from B to DE divides DE in equal halves]

$\Rightarrow$ $$R_E = 10{,}000 \text{ N } (\uparrow)$$

$\therefore \quad R_C = 3000 - 10,000 = -7000 \text{ N}$

$\therefore \quad R_C = 7000 \text{ N} (\downarrow)$

At Joint A



$\tan\theta = \frac{2.4}{1.8} = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\Sigma F_y = 0$

$\Rightarrow \quad F_{AD}\sin\theta = -2000$

$\Rightarrow \quad F_{AD} = \frac{-2000\times 5}{4}$

$\boxed{F_{AD} = -2500}$

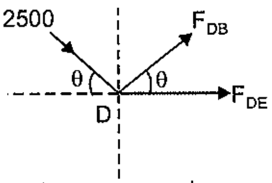
$\Sigma F_x = 0$

$F_{AB} + F_{AD}\cos\theta = 0$

$\Rightarrow \quad F_{AB} = 2500\cos\theta$

$\boxed{F_{AB} = 2500\times\frac{3}{5} = 1500}$

At Joint D:



$\tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\Sigma F_y = 0$

$\Rightarrow \quad F_{DB}\sin\theta = 2500\sin\theta$

$\Rightarrow \quad F_{DB}\times\frac{4}{5} = 2500\times\frac{4}{5}$

$\Rightarrow \quad \boxed{F_{DB} = 2500}$

$\Sigma F_x = 0$

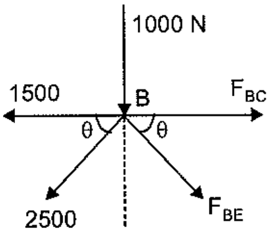
$\Rightarrow \quad F_{DB}\cos\theta + F_{DE} = -2500 \times \cos\theta$

$\Rightarrow \quad 2500\times\frac{3}{5} + F_{DE} = -2500\times\frac{3}{5}$

$\Rightarrow \quad F_{DE} = -1500 - 1500$

$\Rightarrow \quad \boxed{F_{DE} = -3000\text{N}}$

At Joint B:



$\tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\Sigma F_y = 0$

$\Rightarrow \quad F_{BE}\sin\theta + 2500\sin\theta + 1000 = 0$

$\Rightarrow \quad F_{BE}\times\frac{4}{5} + 2500\times\frac{4}{5} + 1000 = 0$

$\Rightarrow \quad F_{BE}\times\frac{4}{5} = -3000$

$\Rightarrow \quad \boxed{F_{BE} = -3750\text{N}}$

$\Sigma F_x = 0$

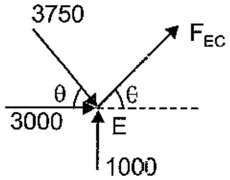
$F_{BC} + F_{BE}\cos\theta - 2500\cos\theta - 1500 = 0$

$\Rightarrow \quad F_{BC} - 3750\times\frac{3}{5} - 2500\times\frac{3}{5} - 1500 = 0$

$\Rightarrow \quad F_{BC} - 2250 - 1500 - 1500 = 0$

$\Rightarrow \quad \boxed{F_{BC} = 5250 \text{ N}}$

At joint E



$$\tan\theta = \frac{2.4}{1.8} = \frac{4}{3}$$

$$\Rightarrow \quad \sin\theta = \frac{4}{5}$$

$$\cos\theta = \frac{3}{5}$$

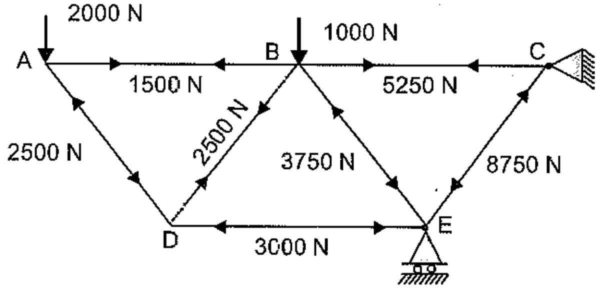
From $\qquad \Sigma F_x = 0$

$$F_{EC}\cos\theta + 3000 + 3750\cos\theta = 0$$

$$\Rightarrow \qquad F_{EC} = -\frac{3000 + 3750\times 3/5}{3/5} = -8750\text{ N}$$

$$\Rightarrow \qquad \boxed{F_{EC} = -8750\text{N}}$$

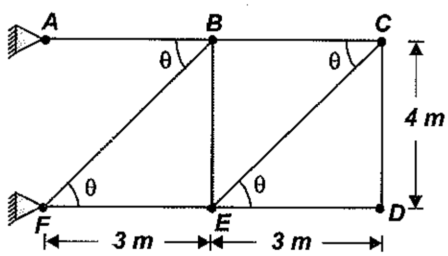
Hence the member forces is as shown in figure.



Q-16: ***Three members of the truss shown in the figure below have been cut either too long (+ ve) or too short (– ve) as per details given below. Calculate the vertical displacement of joint D due to discrepancies in the length of these members by unit load method.***

$$\delta_{AB} = -3\text{ mm};\ \delta_{BC} = 2\text{ mm};\ \delta_{EF} = 4\text{ mm}$$

All members have the same area of cross-section.



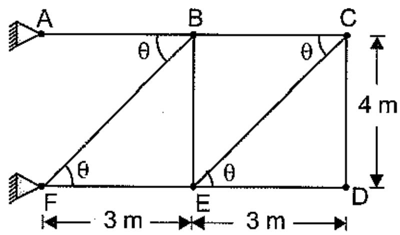
[10 Marks, ESE-2011]

Sol:

$$\tan\theta = \frac{4}{3}$$

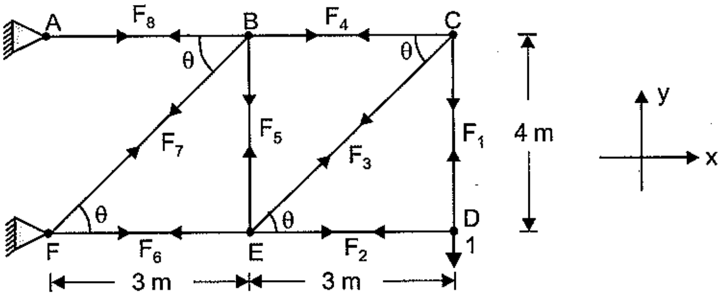
$$\sin\theta = \frac{4}{5}$$

$$\cos\theta = \frac{3}{5}$$



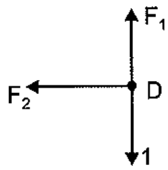
To calculate the vertical displacement at D, we will apply unit load at D in vertically downward direction and analyze the truss to obtain the member forces. The truss is a determinate truss because $m + r - 2j = 0$

$$m = 8 ; r = 4 ; j = 6$$



At joint D

$$\Sigma F_x = 0$$

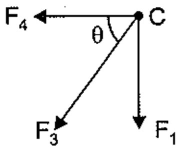


$$\Rightarrow \quad \boxed{F_2 = 0}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad F_1 - 1 = 0 \Rightarrow \boxed{F_1 = 1}$$

At Joint C



$\tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$$\Sigma F_x = 0$$

$$\Rightarrow \quad F_4 + F_3 \cos\theta = 0$$

$$\Rightarrow \quad F_4 + \frac{3F_3}{5} = 0 \quad \text{...(i)}$$

$$\Rightarrow \quad \Sigma F_y = 0$$

$$\Rightarrow \quad F_1 + F_3 \sin\theta = 0$$

$$F_3 = -\frac{F_1}{\sin\theta} = \frac{-1}{4/5} = \frac{-5}{4}$$

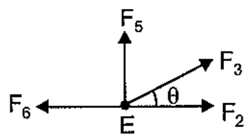
$$\boxed{F_3 = -\frac{5}{4}}$$

From (i)

$$F_4 = \frac{-3F_3}{5} = \frac{-3}{5}\left(-\frac{5}{4}\right) = \frac{+3}{4}$$

$$\boxed{F_4 = \frac{+3}{4}}$$

At joint E



$\tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$$\Sigma F_x = 0$$

$$\Rightarrow \quad F_3 \cos\theta + F_2 - F_6 = 0$$

$$\Rightarrow \quad F_6 = 0 + \left(\frac{-5}{4}\right)\cos\theta$$

$$= \frac{-5}{4} \times \frac{3}{5} = \frac{-3}{4}$$

$$\boxed{F_6 = \frac{-3}{4}}$$

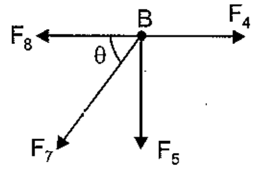
$$\Sigma F_y = 0$$

$$\Rightarrow \quad F_5 + F_3 \sin\theta = 0$$

$$F_5 = -F_3 \sin\theta = -\left(-\frac{5}{4}\right) \times \frac{4}{5} = 1$$

$$\boxed{F_5 = 1}$$

At Joint B



$\tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$$\Sigma F_x = 0$$

$$\Rightarrow \quad F_8 + F_7 \cos\theta - F_4 = 0$$

$$\Rightarrow \quad F_8 + F_7 \times \frac{3}{5} = \frac{+3}{4} \quad \ldots\text{(ii)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad F_5 + F_7 \sin\theta = 0$$

$$F_7 = \frac{-F_5}{\sin\theta} = \frac{-1}{4/5} = \frac{-5}{4}$$

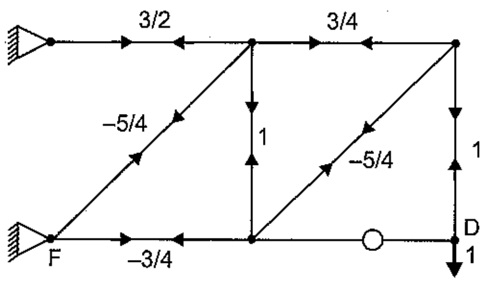
$$\boxed{F_7 = -\frac{5}{4}}$$

From (ii)

$$F_8 = \frac{-3}{5}F_7 + \frac{3}{4} = \frac{-3}{5}\left(\frac{-5}{4}\right) + \frac{3}{4} = \frac{3}{4} + \frac{3}{4} = \frac{3}{2}$$

$$\Rightarrow \quad \boxed{F_8 = \frac{3}{2}}$$

Hence the forces in the members of truss are as shown below



Deflection of Joint D

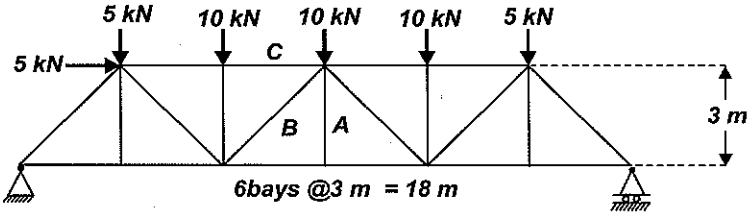
$$\Delta_{Dv} = \Sigma udl = [F_8(-3) + F_4(+2) + F_6(+4)]$$

$$= \left[\frac{6}{4}(-3) + \frac{3}{4}(+2) + \left(\frac{-3}{4}\right)(+4)\right] = \frac{-24}{4} + 3 = -6 \text{ mm}$$

$$\Delta_{Dv} = -6 \text{ mm}$$

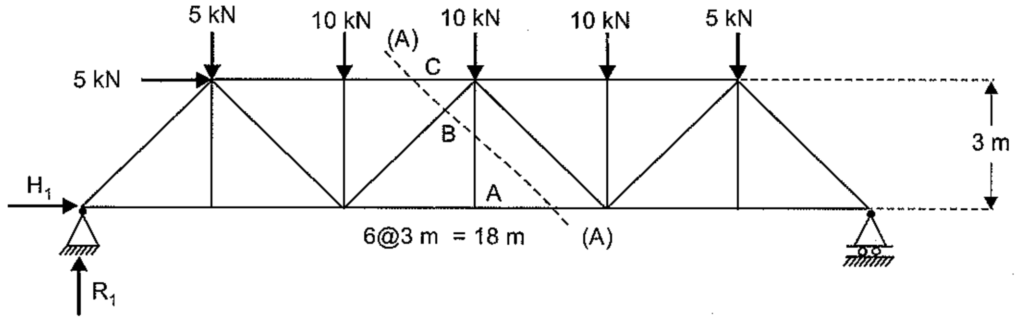
$\Rightarrow$ Deflection of Joint D is opposite to the direction of applied unit load i.e., upwards by 6 mm

Q-17: ***Find the member forces in the member marked A, B, and C for the truss as shown.***



[10 Marks, ESE-2013]

Sol:



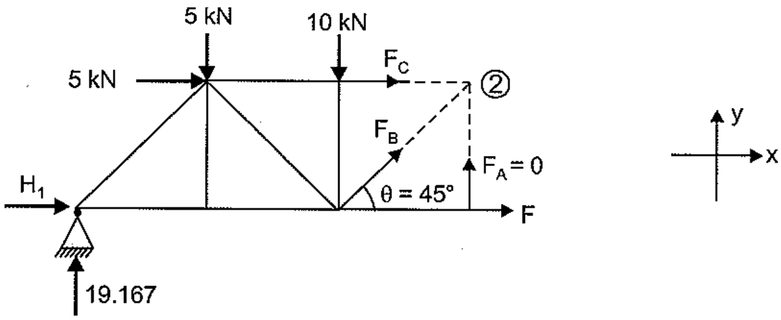
Force in member A = 0 [Because there is no force to balance the force in A]

$$\Rightarrow \quad \boxed{F_A = 0}$$

Reaction at left support is obtained by taking moment of all forces about the right support

$$\Rightarrow \quad R_1 \times 18 + 5\times 3 - 5 \times 15 - 10 \times 12 - 10\times 9 - 10 \times 6 - 5 \times 3 = 0$$

$$\Rightarrow \quad R_1 = 19.167 \text{ kN}$$



From $\Sigma F_y = 0$

$$F_B \sin 45° + 19.167 - 5 - 10 = 0$$

$$\Rightarrow \quad F_B = -4.167 \times \sqrt{2} = -5.893 \text{ kN}$$

$$\boxed{F_B = -5.893 \text{ kN}}$$

Taking moment of all forces about ②

$$-5\times 6 - 10\times 3 + 19.167\times 9 - F\times 3 = 0$$

$$\Rightarrow \quad F = 37.501 \text{ kN}$$

From $\Sigma F_x = 0$

$$F_C + F_B\cos 45° + F + 5 = 0$$

$$\Rightarrow \quad F_C = -5-37.501-(-5.893)\times\frac{1}{\sqrt{2}}$$

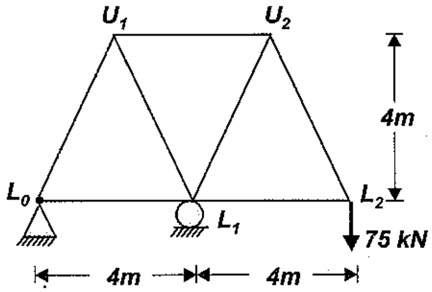
$$= -38.334 \text{ kN}$$

$$\Rightarrow \quad F_A = 0$$

$$F_B = 5.896 \text{ kN (compression)}$$

$$F_C = 38.334 \text{ kN (compression)}$$

Q-18: ***Determine the vertical deflection of joint L_2 of the truss shown in Figure 5. The area of cross-section of the members in 20 cm² each. Take E = 200 GPa.***



[15 Marks, ESE-2014]

Sol: We will use unit load method to determine the deflection of given truss. Basic formula of unit load method is

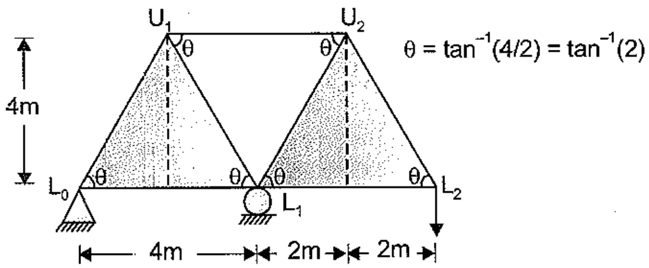
$$\Delta = \Sigma\mu_i(\Delta L)_i$$

in which, μ = Axial force due to unit load

ΔL = Change in length of member by any reason

In this, $\Delta L = \dfrac{PL}{AE}$ where P = axial force due to given load

μ-force system

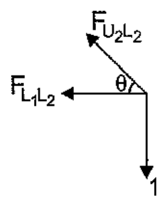


It can easily be seen that this is a statically determinate truss. To obtain deflection of L_2, we apply unit load at L_2 and find axial forces in each member.

Joint L_2

For joint equilibrium

$$\Sigma F_x = 0; \quad \Sigma F_y = 0$$



$$\Rightarrow \quad F_{U_2L_2}\cos\theta + F_{L_1L_2} = 0 \qquad \text{...(i)}$$

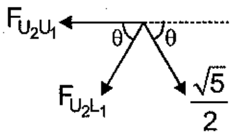
$$F_{U_2L_2}\sin\theta - 1 = 0 \qquad \text{... (ii)}$$

Solving (i) and (ii) and putting $\theta = \tan^{-1}(2)$, we get

$$F_{U_2L_2} = \frac{\sqrt{5}}{2}$$

$$F_{L_1L_2} = -\frac{1}{2} \quad \text{[–ve sign indicate compression)}$$

Joint U_2



$$F_{U_2L_1}\sin\theta + \frac{\sqrt{5}}{2}\sin\theta = 0 \qquad \text{...(iii)}$$

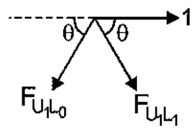
$$F_{U_2U_1} + F_{U_2L_1}\cos\theta - \frac{\sqrt{5}}{2}\cos\theta = 0 \qquad \text{...(iv)}$$

Solving (iii) and (iv), we get

$$F_{U_2U_1} = 0$$

$$F_{U_2L_1} = -\frac{\sqrt{5}}{2} \text{ (–ve sign indicate compression)}$$

Joint U_1



$$F_{U_1L_0}\sin\theta + F_{U_1L_1}\sin\theta = 0 \qquad \text{...(v)}$$

$$F_{U_1L_0}\cos\theta - 1 - F_{U_1L_1}\cos\theta = 0 \qquad \text{...(vi)}$$

Solving (v) and (vi), we get

$$F_{U_1L_0} = \frac{\sqrt{5}}{2}$$

$$F_{U_1L_1} = -\frac{\sqrt{5}}{2} \text{ (–ve sign indicate compression)}$$

Similarly by writing equation for support L_1, we will find $F_{L_1L_0} = -\frac{1}{2}$

P-force system:

We have already find member force due to unit load at L_2. Now, to find member force when 75 kN load acts at L_2, we simply multiply member forces due to unit load (at L_2) by 75.

P-force = (μ-force) × 75

Member	μ	P(kN)	L(m)	$\mu\frac{PL}{AE}$
U_1L_0	$\frac{\sqrt{5}}{2}$	$75\times\frac{\sqrt{5}}{2}$	$\sqrt{20}$	$\frac{419.26}{AE}$
U_1U_2	1	75×1	4	$\frac{300}{AE}$
U_2L_2	$\frac{\sqrt{5}}{2}$	$75\times\frac{\sqrt{5}}{2}$	$\sqrt{20}$	$\frac{419.26}{AE}$
L_2L_1	$-\frac{1}{2}$	$75\times\left(-\frac{1}{2}\right)$	4	$\frac{75}{AE}$
L_1L_0	$-\frac{1}{2}$	$75\times\left(-\frac{1}{2}\right)$	4	$\frac{75}{AE}$
U_1L_1	$-\frac{\sqrt{5}}{2}$	$75\times\left(-\frac{\sqrt{5}}{2}\right)$	$\sqrt{20}$	$\frac{419.26}{AE}$
U_2L_1	$-\frac{\sqrt{5}}{2}$	$75\times\left(-\frac{\sqrt{5}}{2}\right)$	$\sqrt{20}$	$\frac{419.26}{AE}$
				$\frac{\Sigma\mu.PL}{AE} = \frac{2127.05}{AE}$

$$\therefore \quad \Delta_{L2} = \frac{2127.05}{AE}$$

Now we put P-forces in kN and L in meter

$$A = 20 \text{ cm}^2$$

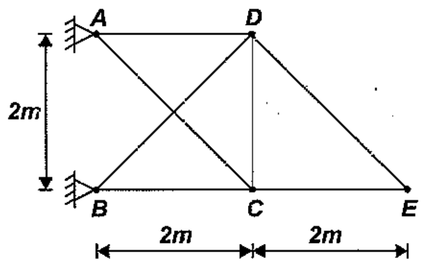
$$E = 200 \times 10^3 \text{ N/mm}^2$$

We should convert all units in 'N' and 'mm'

$$\therefore \quad \Delta_{L_2} = \frac{2127.05 \times 10^3 \times 10^3}{20 \times 10^2 \times 200 \times 10^3} \text{mm}$$

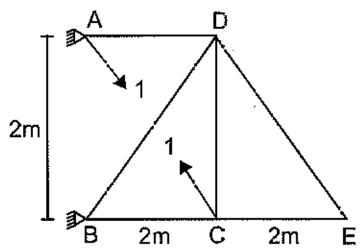
$$\Delta_{L_2} = 5.32\text{mm}(\downarrow)$$

Q-19: ***Determine the forces in all the members of the truss as shown in Figure due to a fall of temperature of the member BD by 30°C. Cross-sectional area of all the members = 1500mm². $\alpha = 0.00001/°C$.***



[20 Marks, ESE-2017]

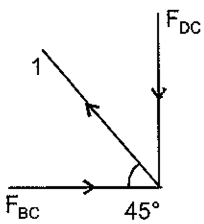
Sol:



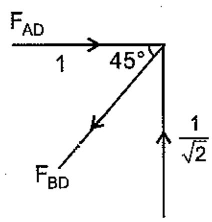
Structure has one degree of static indeterminacy. So removing AC and applying the unit load in the direction of member, AC.

$F_{DE} = F_{CE} = 0$ (since no external load on joint E)

at Joint C



at Joint D



So,

$$F_{BC} = \frac{1}{\sqrt{2}} \text{ (c)} \qquad F_{AD} = \frac{1}{\sqrt{2}} \text{(c)}$$

$$F_{DC} = \frac{1}{\sqrt{2}} \text{(c)} \qquad F_{BD} = 1 \text{ (T)}$$

$$l_i \alpha_i (\Delta T_i) = 2\sqrt{2} \times 1 \times 10^{-5} \times (-30)$$

$$= -8.485 \times 10^{-4}$$

Sign Convention: Tension = (+)ve, Comp = (–)ve

If redundant force comes out to be (+)ve it is in the direction of applied unit load otherwise opposite to it.

(ΔT_i) is (+)ve if temp increases and (–)ve if temp falls.

Member	Length (m)	u_i	$u_i \ell_i \alpha_i (\Delta T_i)$ (m)	$u_i^2 \ell$ (m)
AD	2	$-\frac{1}{\sqrt{2}}$	0	1
AC	$2\sqrt{2}$	1	0	$2\sqrt{2}$
BD	$2\sqrt{2}$	1	-8.485×10^{-4}	$2\sqrt{2}$
BC	2	$-\frac{1}{\sqrt{2}}$	0	1
CD	2	$-\frac{1}{\sqrt{2}}$	0	1
DE	$2\sqrt{2}$	0	0	0
CE	2	0	0	0

$$R = \frac{-\sum u_i \left(l_i \alpha_i (\Delta T_i)\right)}{\dfrac{\sum u_i^2 l_i}{A_i E_i}}$$

Since A_i E_i is constant for all members

$$R = \frac{-AE \sum u_i \left(l_i \alpha_i (\Delta T_i)\right)}{\sum u_i^2 l_i} \text{ kNm/m}$$

$$\sum u_i \left(l_i \alpha_i (\Delta T_i)\right) = -8.485\times10^{-4} \text{ m}$$

$$\sum u_i^2 l_i = 3 + 4\sqrt{2} \text{ m}$$

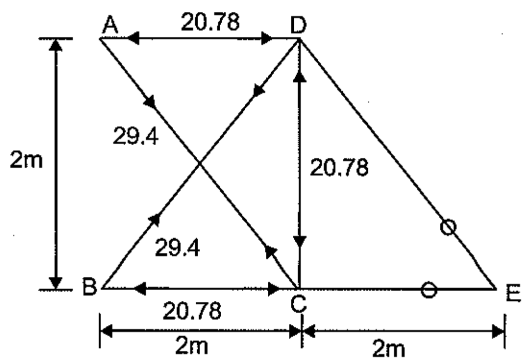
$$\Rightarrow \quad R = \left(-\frac{8.485\times10^{-4}}{3+4\sqrt{2}}\right)\left(1500\times2\times10^5\times10^{-3}\right)\text{kN}$$

(Assuming E = 2 × 10^5 Mpa)

So, force in the member of the truss.

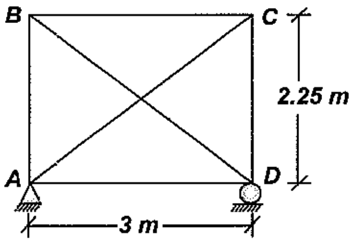
$$F_i = u_i \times R$$

Member	$F_i = u_i R$ (kN)
AD	–20.78
AC	29.4
BD	29.4
BC	–20.78
CD	–20.78
DE	0
CE	0



Shown in last column of the table drawn above.

Q-20: ***The frame shown in figure is fabricated using members with cross-sectional areas as given below:***



Diagonal members = 2000 sq. mm

Other members = 1000 sq. mm

E for the material of members = 2 × 10⁵ N/sq. mm

Member AC was fitted last and the length of the member was 1 mm short. Determine the forces developed in the members when AC was pulled and fitted in position.

[20 Marks, ESE-2018]

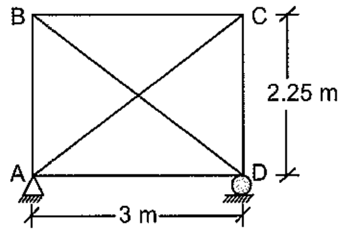
Sol: Cross-sectional area of diagonal members = 2000 mm^2

Cross-sectional area of other members = 1000 mm^2

E (for the material of member) = 2×10^5 N/mm^2

AC is short in length by 1 mm

To calculate: Forces developed



Total degree of redudancy of the frame = m + r – 2j

$$D_S = 6 + 3 - (2 \times 4)$$

$$\boxed{D_S = 1}$$

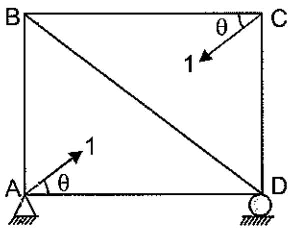
Degree of external redundancy $(\Delta_e) = r - 3 = 3 - 3 = 0$

$\therefore$ Degree of internal redundancy $(\Delta_i) = D_S - D_e = 1$

Note: Normally when there is internal determinacy, it is recommended that the redundant should be taken as the member in which there is lack of fit. But this is not necessary. Any other member can be taken as redundant member.

Let us choose AC as redundant.

Remove member AC and apply equal and opposite load at joint A and C.

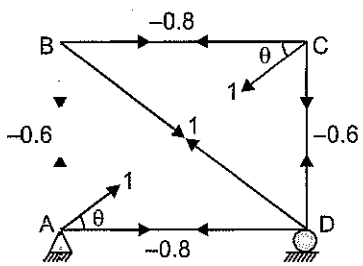


$$\sin\theta = \frac{2.25}{3.75} = 0.6$$

$$\cos\theta = \frac{3}{3.75} = 0.8$$

Note : Reactions will be zero in this case as equal and opposite collinear loads are acting and external redundancy is zero.

Calculation for the redundant 'R'



Member	u	l(m)	u^2l	λ (m)	$u\lambda$ (m)
AB	–0.6	2.25	0.81	0	0
BC	–0.8	3	1.92	0	0
CD	–0.6	2.25	0.81	0	0
DA	–0.8	3	1.92	0	0
AC	1	3.75	3.75	-1×10^{-3}m	-10^{-3}
BD	1	3.75	3.75	0	0
Σ			12.96		-10^{-3}

Then,
$$\Sigma u\lambda = -10^{-3}\text{ m}$$
$$R = \frac{-\Sigma u\lambda}{\Sigma\dfrac{u^2l}{AE}}$$

Calculation for $\Sigma\dfrac{u^2l}{AE}$

$$= \frac{1}{2\times10^5\ (\text{N/mm}^2)}\left[\frac{(0.81+1.92+0.81+1.92)}{1000}+\frac{(3.75+3.75)}{2000}\right]$$
$$= 4.605\times10^{-8}\text{ m/N}$$

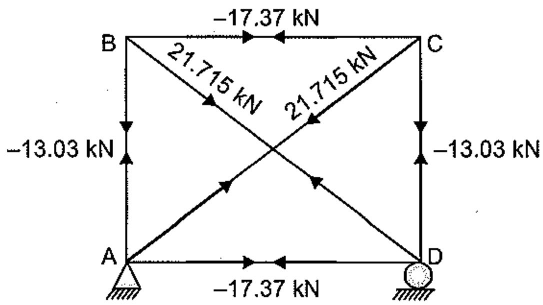
Now,
$$R = \frac{-(-10^{-3})\text{ m}}{4.605\times10^{-8}\text{ m/N}}$$
$$\boxed{R = 21.715\text{ kN}}$$

R is positive (+ve), this implies that the direction of redundant is in the direction of applied load i.e. tensile.

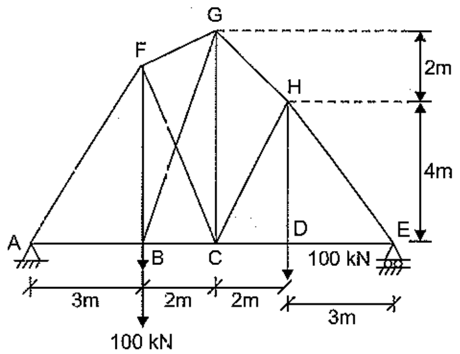
Calculation for member forces

Member	uR
AB	$-0.6\times21.715=-13.03$ kN
BC	$-0.8\times21.715=-17.37$ kN
CD	$-0.6\times21.715=-13.03$ kN
DA	$-0.8\times21.715=-17.37$ kN
AC	$1\times21.715=21.715$ kN
BD	$1\times21.715=21.715$ kN

Member forces are as shown follows:



Q-21: ***Determine the forces in the members of the truss shown in figure. All members have same axial rigidity.***

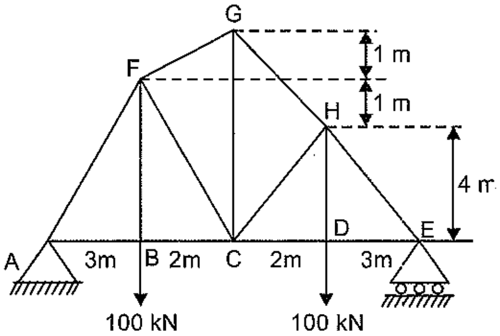


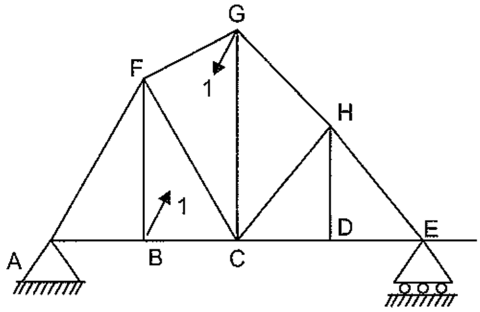
[20 Marks, ESE-2020]

Sol: **Assumption :** BF = 5 m

Truss is indeterminate with 1-degree.

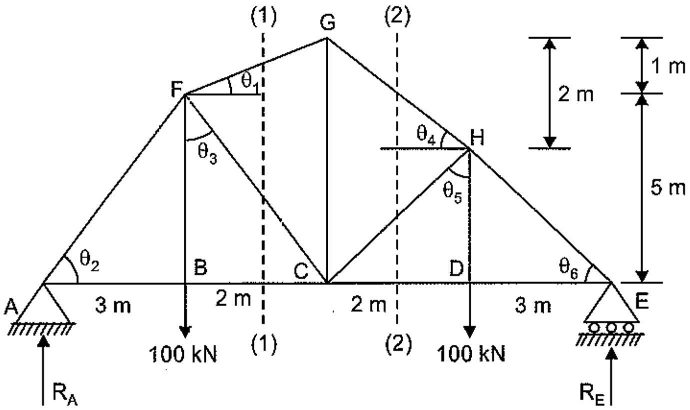
Take member force BG as redundant "R".





$$R = -\frac{\frac{\sum P_i u_i l_i}{AE}}{\frac{\sum u_i^2 l}{AE}}$$

P_i Calculation



$\Sigma M_E) = 0$

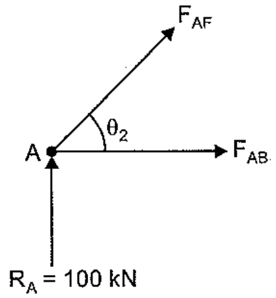
$$R_A \times 10 - 100 \times 7 - 100 \times 3 = 0$$
$$R_A = 100 \text{ kN}$$

$\Sigma F_y \uparrow = 0$

$$R_A + R_E = 200 \text{ kN}$$

$$R_E = 100 \text{ kN}$$

Take joint A,



$$\tan\theta_2 = \frac{5}{3}$$

$$\theta_2 = 59.036°$$

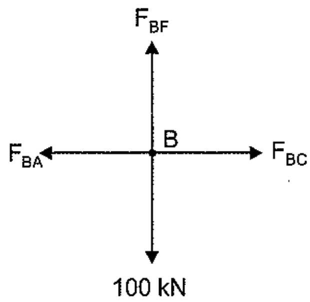
$$F_{AF} = -\frac{100}{\sin\theta_2} = \frac{-100}{\sin 59.036} = -116.619\text{kN}$$

$$F_{AB} = 100\cot\theta_2 = 100\cot 59.036 = 60\text{kN}$$

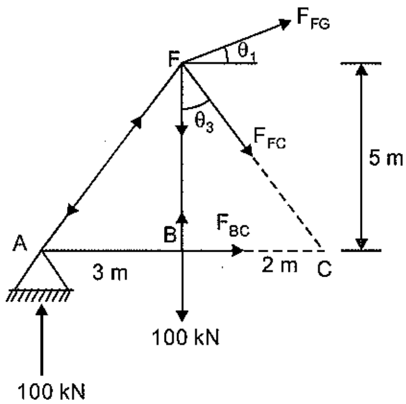
Take Joint B:

$\Sigma F_x = 0 \Rightarrow F_{BC} = F_{BA} = 60\text{kN}$

$\Sigma F_y = 0 \Rightarrow F_{BF} = 100\text{kN}$



Take section (1) - (1)



$$\tan\theta_1 = \frac{1}{2}$$

$$\theta_1 = 26.565°$$

$$\tan\theta_3 = \frac{2}{5}$$

$$\theta_3 = 21.801°$$

$\Sigma M_C = 0$

$$100\times 5 - 100\times 2 + F_{FG}\sin\theta_1\times 2 + F_{FG}\cos\theta_1\times 5 = 0$$

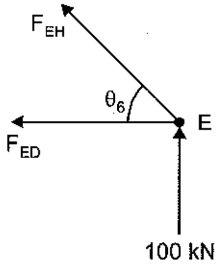
$$-\frac{300}{[2\sin 26.565° + 5\cos 26.565°]} = F_{FG}$$

$\Sigma F_y \uparrow = 0$

$$-F_{FC}\cos\theta_3 + F_{FG}\sin\theta_1 = 0$$

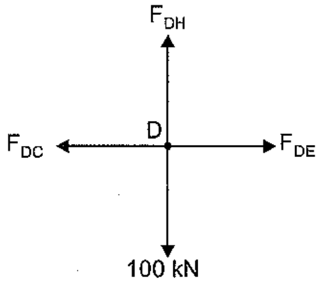
$$F_{FC} = -55.901\times\frac{\sin 26.565°}{\cos 21.801°} = -26.925\text{kN}$$

Take joint E



$$\tan\theta_6 = \frac{4}{3}$$
$$\theta_6 = 53.13^\circ$$
$$F_{EH} = -\frac{100}{\sin 53.13^\circ} = -125\text{ kN}$$
$$F_{ED} = 125\cos 53.13^\circ = 75\text{ kN}$$

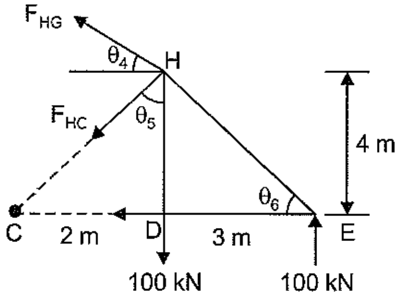
Take joint D



$\Sigma F_y \uparrow = 0 \quad F_{DH} = 100\text{ kN}$

$\Sigma F_x = 0 \quad F_{DC} = F_{DE} = 75\text{ kN}$

Take section (2) - (2)



$$\tan\theta_4 = \frac{2}{2}$$
$$\theta_4 = 45^\circ$$
$$\tan\theta_5 = \frac{2}{4}$$
$$\theta_5 = 26.565^\circ$$

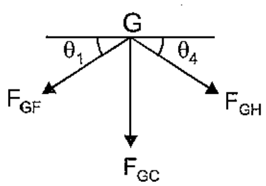
$\Sigma M_C = 0$

$$-100\times 5 + 100\times 2 - F_{HG}\cos\theta_4\times 4 - F_{HG}\sin\theta_4\times 2 = 0$$
$$300 + F_{HG}\times\cos 45^\circ\times 4 - F_{HG}\sin 45^\circ\times 2 = 0$$
$$F_{HG} = -\frac{300}{[4\cos 45^\circ + 2\sin 45^\circ]} = -70.7106\text{ kN}$$

$\Sigma F_y \uparrow = 0$

$$F_{HG}\sin\theta_4 - F_{HC}\cos\theta_5 = 0$$
$$-70.7106\times\frac{\sin 45^\circ}{\cos 26.565^\circ} = F_{HC}$$
$$F_{HC} = -55.901\text{ kN}$$

Take Joint G



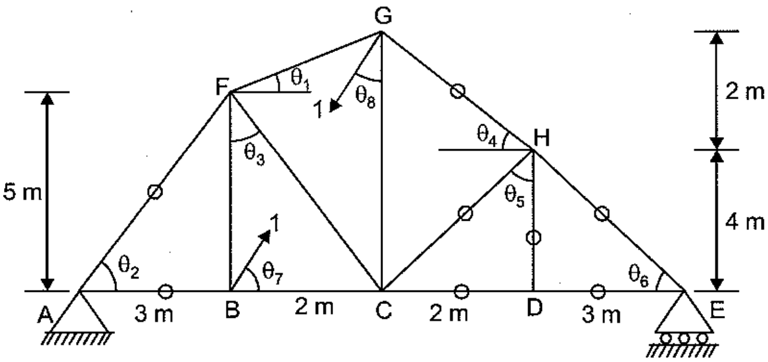
$\Sigma F_y = 0$

$$F_{GF} \sin\theta_1 + F_{GH} \sin\theta_4 + F_{GC} = 0$$

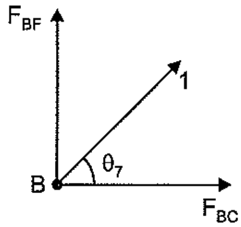
$$F_{GC} - 55.901 \times \sin 26.565° - 70.7106 \times \sin 45° = 0$$

$$F_{GC} = 75 \text{ kN}$$

Calculation :



Take Joint B :



$$\tan\theta_7 = \frac{6}{2}$$

$$\theta_7 = 71.565°$$

$\Sigma F_y = 0$

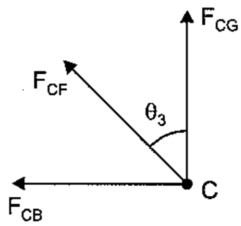
$$F_{BF} + 1\sin\theta_7 = 0$$

$$F_{BF} = -\sin 71.565 = -0.9486$$

$\Sigma F_x = 0$

$$F_{BC} = -\cos\theta_7 = -\cos 71.565 = -0.3162$$

Take Joint C :



$$\tan\theta_3 = \frac{2}{5}$$

$$\theta_3 = 21.801°$$

$\Sigma F_x = 0$

$$F_{CF} \sin 21.801° + F_{CB} = 0$$

$$F_{CF} = -\frac{F_{CB}}{\sin 21.801°}$$

$$F_{CF} = -\frac{(-0.3162)}{\sin 21.801} = 0.8154$$

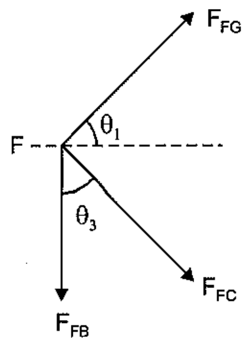
$\Sigma F_y = 0$

$$F_{CF} \cos\theta_3 + F_{CG} = 0$$

$$F_{CF} = -0.854 \cos 21.801°$$

$$F_{CG} = -0.791$$

Take joint F :



$$\tan\theta_1 = \frac{1}{2}$$

$$\theta_1 = 26.565°$$

$\Sigma F_x = 0$

$$F_{FG}\cos\theta_1 + F_{FC}\sin\theta_3 = 0$$

$$F_{FG} = -\frac{0.8514 \times \sin 21.801°}{\cos 26.565°}$$
$$= -0.3535$$

Member	Force F_i (KN)	u_i	L(m)	PuL	u^2L
AB	60	0	3	0	0
AF	–116.619	0	5.831	0	0
BF	100	–0.9486	5	–474.3	4.499
BC	60	–0.3162	2	–37.944	0.200
FC	–26.925	0.8514	5.385	–123.445	3.903
FG	–55.901	–0.3535	2.236	44.1856	0.279
GC	75	–0.791	6	–355.95	3.754
GH	–70.711	0	2.828	0	0
CH	–55.901	0	4.472	0	0
CD	75	0	2	0	0
HD	100	0	4	0	0
HE	–125	0	5	0	0
DE	75	0	3	0	0
BG	0	1	6.325	0	6.325

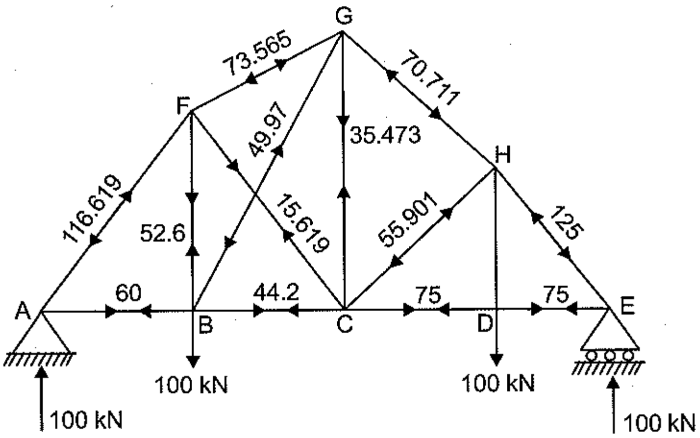
$$\Sigma PuL = -947.45$$
$$\Sigma u^2L = 18.96$$

So,
$$R = \frac{\Sigma PuL}{\Sigma u^2L}$$
$$R = -\left(-\frac{947.45}{18.96}\right)$$
$$R = 49.97 \text{ kN}$$

So, force in member using F = P + KR

$$F_{AF} = P + KR = -116.619 + (49.97 \times 0) = -116.619 \text{kN}$$
$$F_{AB} = 60 + 0 = 60 \text{ kN}$$
$$F_{BF} = 100 + (49.97 \times -0.9486) = 52.6 \text{kN}$$
$$F_{BC} = 60 + (49.97 \times -0.3162) = 44.2 \text{ kN}$$
$$F_{FC} = -26.925 + (49.97 \times 0.8514) = 15.619 \text{kN}$$
$$F_{FG} = -55.901 + (49.97 \times -0.3535) = -73.565 \text{ kN}$$
$$F_{GC} = 75 + (49.97 \times -0.791) = 35.473 \text{kN}$$
$$F_{GH} = -70.711 + 0 = -70.711 \text{kN}$$
$$F_{CH} = -55.901 \text{ kN}$$
$$F_{CD} = 75 \text{ kN}$$
$$F_{HD} = 100 \text{ kN}$$

$$F_{HE} = -125 \text{ kN}$$
$$F_{DE} = 75 \text{ kN}$$
$$F_{BG} = -49.97 \text{ kN}$$

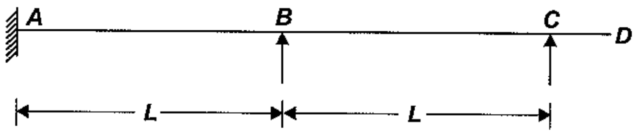


(Member forces shown in figure is in kN)

CHAPTER 6

INFLUENCE LINE DIAGRAM

Q-1: ***What is Muller-Breslau's principle? Draw qualitatively the influence lines for the beam shown in the figure.***



(i) ***Vertical reactions at A, B and C***

(ii) ***Moment at A***

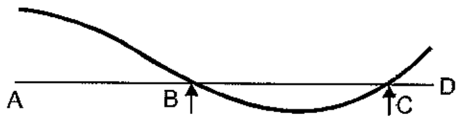
(iii) ***Shear force at X, a section between A and B.***

[15 Marks, ESE-1997]

Sol: **Muller-Breslau Principle**

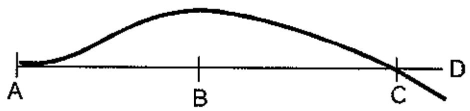
Influence line for any stress function may be obtained by removing the restraint offered by that function and introducing a directly related generalized unit displacement at the location and in the direction of the function.

(i) (a)



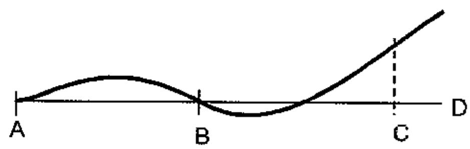
ILD for R_A

(b)



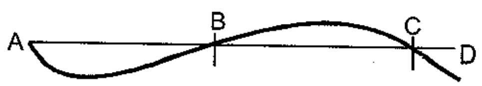
ILD for R_B

(c)



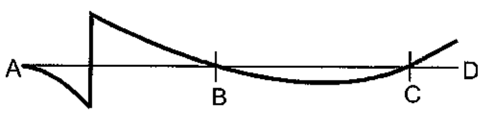
ILD for R_C

(ii)



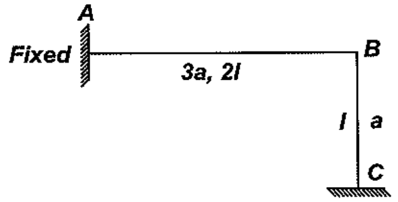
ILD for M_A

(iii)



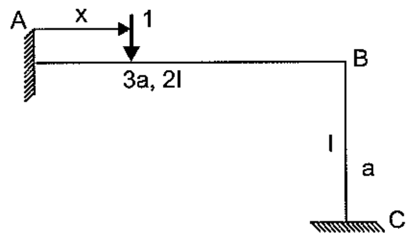
ILD for shear at x–x,
where x–x section is b/w A and B

Q-2: ***State Muller-Breslau principle. Derive the equation for influence line for moment M_B at the joint B of the rigid frame ABC shown in the above figure, when a unit load travels over the beam AB.***



[20 Marks, ESE-2000]

Sol: **Muller-Breslau principle**



Influence line for any stress function may be obtained by removing the restraint offered by that function and introducing a directly related generalized unit displacement at that location in the direction of the stress function.

To derive the equation of influence line for moment M_B at joint B, we will use moment distribution method

Let unit load be placed at a distance x from end A

Fixed end moments

$$M_{FAB} = -\frac{1\times x\,(3a-x)^2}{9a^2}$$

$$M_{FBA} = \frac{1\times(3a-x)\,x^2}{9a^2}$$

$$M_{FBC} = M_{FCB} = 0$$

Distribution factor

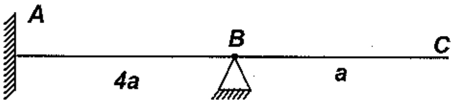
	Member	Stiffness	Total stiffness = Joint stiffness	DF
Joint B	BA	$\frac{4E\,(2I)}{3a}$	$\frac{20\,EI}{3a}$	$\frac{8}{20}$
	BC	$\frac{4EI}{a}$		$\frac{12}{20}$

	A	B (BA)	B (BC)	C	
		$\frac{8}{20}$	$\frac{12}{20}$		
	$\frac{-x\,(3a-x)^2}{9a^2}$	$\frac{(3a-x)x^2}{9a^2}$	0	0	FEM
		$\frac{-(3a-x)\,x^2\times 8}{9a^2\times 20}$	$\frac{-(3a-x)x^2\times 12}{20\times 9a^2}$		Bal
	$\frac{-(3a-x)x^2\times 4}{180a^2}$			$\frac{-(3a-x)x^2\times 6}{180a^2}$	C/O
		$\frac{12x^2\,(3a-x)}{180\,a^2}$	$\frac{-12x^2\,(3a-x)}{180\,a^2}$		

We are only interested in M_{BA} or M_{BC} hence we have calculated the values of M_{BA} and M_{BC} only

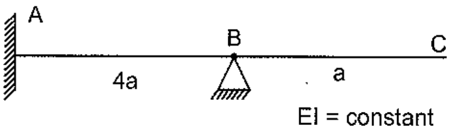
$$\Rightarrow \quad \boxed{\text{BM at B} = \frac{x^2(3a-x)}{15a^2}}$$

Q-3: ***State Muller-Breslau principle. Derive the equation for influence line for the reaction R_B for the beam shown in the figure. EI is constant throughout.***



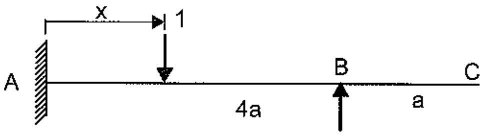
[12 Marks, ESE-2003]

Sol: **Muller-Breslau principle:** Influence line for any stress function may be obtained by removing the restraint offered by that function and introducing a directly related generalized unit displacement at that location in the direction of the stress function.



We have to find out equation of influence line for reaction at B

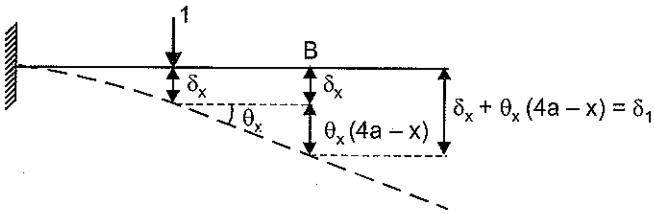
(1) When unit load is in span AB as shown below



We will use consistent deformation method to find R_B

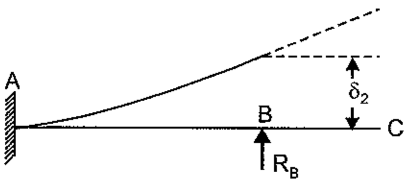
$$\delta_x = \frac{1\times x^3}{3EI}$$

$$\theta_x = \frac{1.x^2}{2EI}$$



$$\delta_1 = \frac{x^3}{3EI} + \frac{x^2(4a-x)}{2EI}$$

$$\delta_2 = \frac{R_B(4a)^3}{3EI}$$



From consistent deformation method

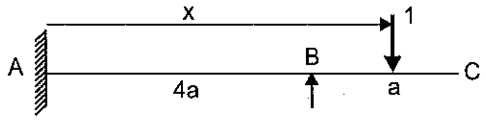
$$\delta_1 = \delta_2$$

$$\Rightarrow \quad \frac{x^3}{3EI} + \frac{x^2(4a-x)}{2EI} = \frac{64R_B a^3}{3EI}$$

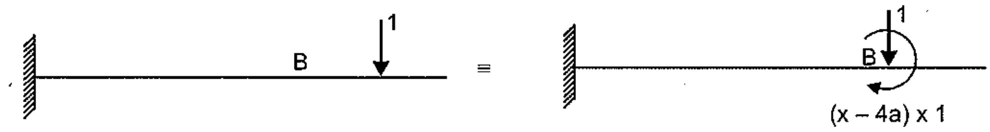
$$R_B = \frac{x^3}{64a^3} + \frac{x^2(4a-x)\times 3}{2\times 64a^3} = \frac{x^3}{64a^3} + \frac{12x^2a}{128a^3} - \frac{3x^3}{128a^3}$$

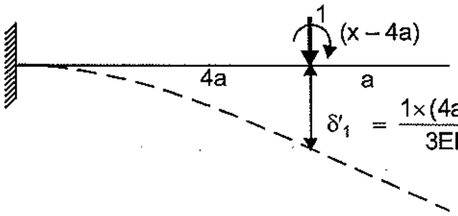
$$= \frac{12x^2a}{128a^3} - \frac{x^3}{128a^3} = \boxed{\frac{x^2(12a-x)}{128a^3} = R_B}$$

(2) When unit load is in B to C



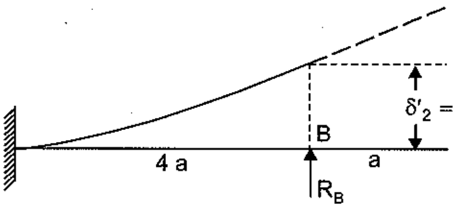
Downward deflection at B due to the above load when support at B is removed is given by





$$\delta'_1 = \frac{1 \times (4a)^3}{3EI} + \frac{(x-4a)(4a)^2}{2EI}$$

Upward deflection at B due to force R_B at B



$$\delta'_2 = \frac{R_B (4a)^3}{3EI}$$

From consistent deformation method

$$\delta'_1 = \delta'_2$$

$$\Rightarrow \quad \frac{(4a)^3}{3EI} + \frac{(x-4a)(4a)^2}{2EI} = \frac{R_B (4a)^3}{3EI}$$

$$\Rightarrow \quad R_B = 1 + \frac{(x-4a)\times 3}{4a \times 2} = 1 + \frac{3x}{8a} - \frac{12a}{8a} = 1 - 1.5 + \frac{3x}{8a}$$

$$\boxed{R_B = \frac{3x}{8a} - 0.5}$$

Thus the equation of influence line for reaction at B is

$$R_B = \frac{x^2 (12a - x)}{128 a^3} \quad ; \quad 0 \le x \le 4a$$

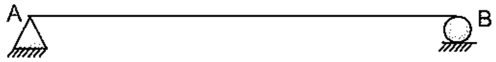
$$R_B = \left(\frac{3x}{8a} - 0.5\right) \quad ; \quad 4a < x \le 5a$$

Q-4: ***State, explain and prove 'Muller-Breslau' influence line theorem for a statically determinate beam. State whether the theorem is also applicable to statically indeterminate beams.***

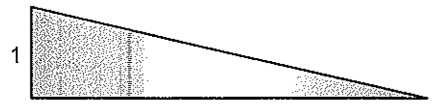
[10 Marks, ESE-2005]

Sol: Muller-Breslau principles states that the influence line for any stress function (reaction, bending moment and shear force) in a structure is represented by its deflected shape obtained by removing the restraint offered by that stress function and introducing a directly related generalized unit displacement in the direction of stress function.

Suppose, we consider a statically determinate beam AB as shown below and it is required to find

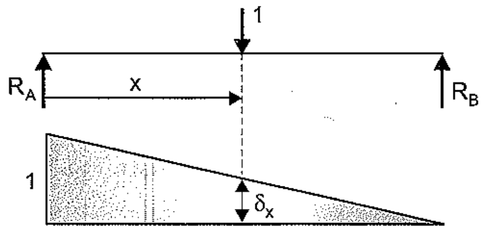


the influence line for support reactior at A. Now, using Muller-Breslau principle, it is seen that the influence line for support reaction at A will be deflected shape of the beam when restraint offered by reaction is removed and a unit displacement in the direction of reaction is applied as shown below.



Influence line for R_A

To prove this, consider a load at any point 'C' at a distance x from A. Then from the principle of virtual work,



$$R_A \times 1 - 1 \times \delta_x = 0$$

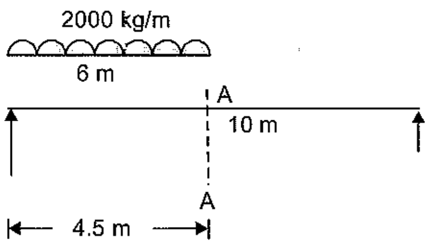
or
$$R_A = \delta_x$$

Muller Breslau principle is applicable statically indeterminate beams also. For statically determinate beams, the influence lines are straight lines while for indeterminate beams, it is curved.

Q-5: ***A uniformly distributed load of 2000 kg/m, 6 m long crosses over a girder simply supported at ends over a span of 10 m from left to right. Calculate maximum bending moment in the girder at a point 4.5 m from left hand end using influence lines.***

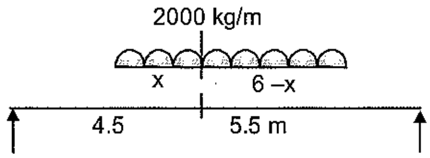
[20 Marks, ESE-2009]

Sol:



For max BM at A – A, udl should be placed in such a way that section divides the load in same ratio as it divides the span

$$\frac{x}{6-x} = \frac{4.5}{5.5}$$



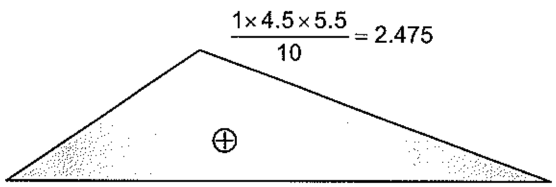
$$\Rightarrow \quad 5.5x = 27 - 4.5x$$

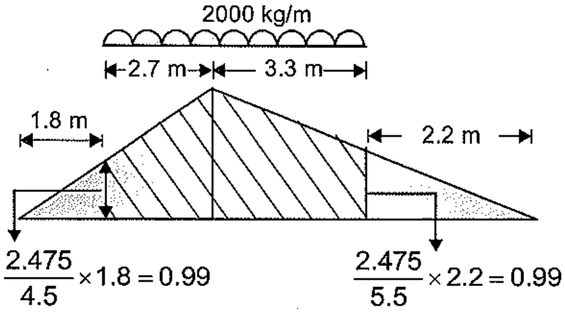
$$10x = 27$$

$$x = 2.7 \text{ m}$$

$$6 - x = 3.3 \text{ m}$$

ILD for BM at section A – A is given as below





$$\text{BM at section A – A} = (\text{Area under udl}) \times (\text{udl})$$

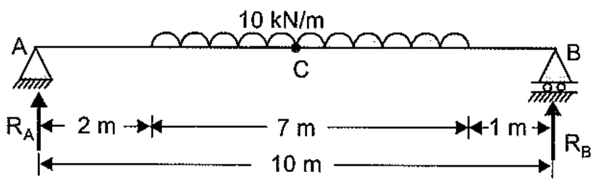
$$= \left[\frac{1}{2}(0.99+2.475)\times2.7+\frac{2.475+0.99}{2}\times3.3\right]\times2000\text{ kgm}$$

$$= \frac{(0.99+2.475)\times6}{2}\times2000 = 20790\text{ kgm}$$

Q-6: ***A simply supported beam has a span of 10 m. A 7m long udl of 10 kN/m intensity crosses the beam from left to right. When the head of the load is 1m from the right support, find the support reactions, BM and SF at the mid-span using the influence line diagram.***

[10 Marks, ESE-2014]

Sol:



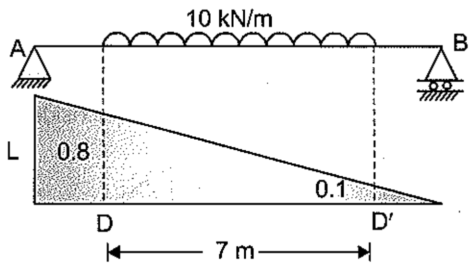
Let R_A = Reaction at A

R_B = Reaction at C

V_C = Shear force at mid-point

M_C = Bending moment at mid-point

ILD for support reaction at A will be

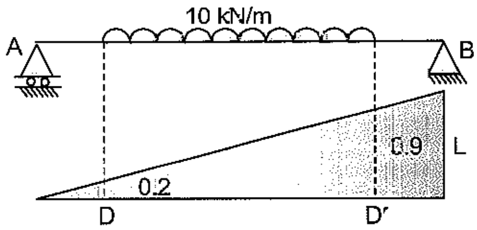


Reaction at A,

$$R_A = (\text{Area of ILD between point D to D}') \times (\text{Loading})$$

$$= \left(\frac{0.8+0.1}{2}\right)\times7\times10\text{ kN} = 31.5\text{ kN}$$

ILD for support reaction at B will be



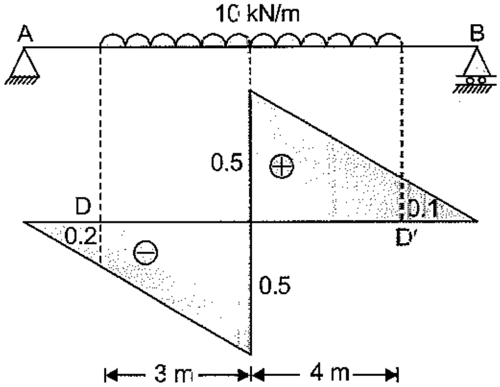
Reaction at B,

$$R_B = (\text{Area of ILD between point D to } D') \times \text{Loading}$$

$$= \left(\frac{0.2+0.9}{2}\right) \times 7 \times 10 \text{ kN}$$

$$= 38.5 \text{ kN}$$

ILD for shear force at mid span will be



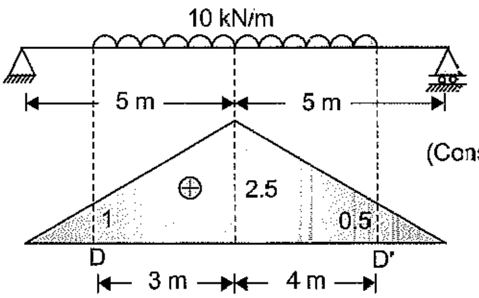
Shear force at mid span,

$$V_c = (\text{Area of ILD between point D to } D') \times \text{Loading}$$

$$V_c = \left(-\left(\frac{0.2+0.5}{2}\right) \times 3 + \left(\frac{0.5+0.1}{2}\right) \times 4\right) \times 10$$

$$= 1.5 \text{ kN}$$

ILD for bending moment at mid-span will be



(Consider sagging as positive)

Bending moment at mid span,

$$M_c = (\text{Area of ILD between point D to } D') \times (\text{Loading})$$

$$\therefore \quad M_c = \left(\left(\frac{1+2.5}{2}\right) \times 3 + \left(\frac{2.5+0.5}{2}\right) \times 4\right) \times 10$$

$$= 112.5 \text{ kN.m (sagging)}$$

Q-7: ***If member U_1 L_1 is fabricated 10mm too short, determine the vertical deflection at L_2.***

[5 Marks, ESE-2014]

Sol: **Given:**

$$\Delta u_1 L_1 = -10\text{mm}$$

From previous question μ-force in member U_1L_1 is $-\frac{\sqrt{5}}{2}$

$$\therefore \quad \Delta L_2' = \left(-\frac{\sqrt{5}}{2}\right)\times(-10) \quad [\because \Delta = \Sigma\mu.\Delta L]$$

$$\therefore \quad \Delta L_2' = 11.18 \text{ mm}$$

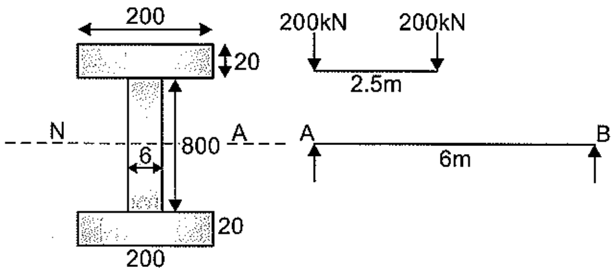
$$\therefore \quad \text{Total deflection at } L_2 = 5.32 + 11.18 = 16.5 \text{ mm} (\downarrow)$$

Q-8: ***Two wheels, placed at a distance of 2.5m apart, with a load of 200 kN on each of them, are moving on a simply supported girder (I-session) of span 6.0 m from left to right. The top and bottom flanges of the I-sections are 200 × 20 mm and the size of web plate is 800 × 6 mm.***

If the allowable bending compressive, bending tensile and average shear stresses are 110 MPa, 165 MPa and 100 MPa respectively, check the adequacy of the section against bending and shear stresses. Self weight of the girder may be neglected.

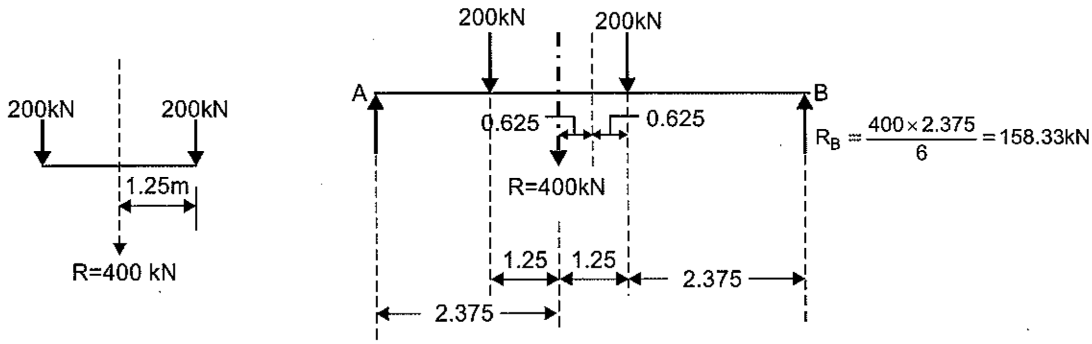
[20 Marks, ESE-2015]

Sol:



To check adequacy of the section against bending, max. bending stress at any section of the beam should be less than the min. permissible bending stress (tensile or compressive).

Now, absolute BM in a beam subjected to moving points loads occurs under the max. load when resultant of the point loads & max. load are equidistant from mid-span.



$$BM_{max} = R_B \times 2.375 = 158.33 \times 2.375 = 376.04 \text{ kNm}$$

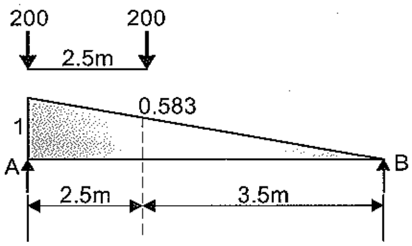
$$\sigma_{max} = \frac{M_{max}y}{I}$$

$$I = \frac{200\times840^3}{12} - \frac{194\times800^3}{12} = 16.01 \times 10^8 \text{ mm}^4$$

$$\Rightarrow \quad \sigma_{max} = \frac{376.04\times10^6\times420}{16.01\times10^8}$$

$$= 98.65 \text{ MPa} < \sigma_{perm,tensile} (= 165 \text{MPa})$$

$$< \sigma_{perm,ccmpressive} (= 110 \text{ MPa})$$



For absolute max. shear force in the beam,

$$V_{max} = \text{Shear force at A}$$

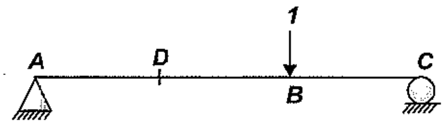
$$= 200 \times 1 + 200 \times 0.583 = 316.66 \text{ kN}$$

$$\tau_{avg} = \frac{V_A}{\text{area}} = \frac{316.66\times10^3}{(2\times200\times20+800\times6)} = 24.74 \text{MPa}$$

$$< \tau_{perm,} \text{shear}(100 \text{MPa})$$

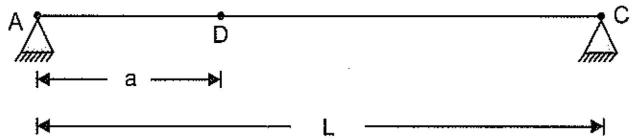
$\Rightarrow$ Section is safe in bending as well as shear

Q-9: ***State the Muller-Breslau principle. Hence draw influence line for moment at D in the beam shown in Figure.***

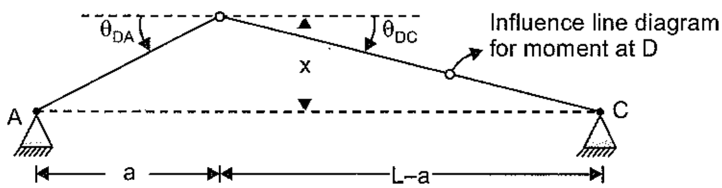


[8 Marks, ESE-2016]

Sol: **Muller Breslau Principle:** Influence line for any stress function in a structure may be obtained by removing the restraint offered by that stress function and introducing a directly related unit displacement at that location in the direction of the restraint offered by that stress function.



For obtaining ILD for BM at D, we will introduce a hinge at D and will give unit rotation at D.



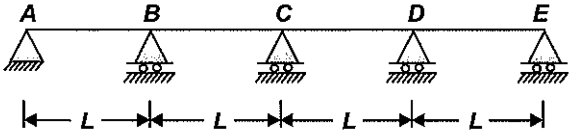
$$\theta_{D_A} + \theta_{D_C} = 1$$

$$\Rightarrow \quad \frac{x}{a} + \frac{x}{L-a} = 1$$

$$xL - xa + xa = a(L-a)$$

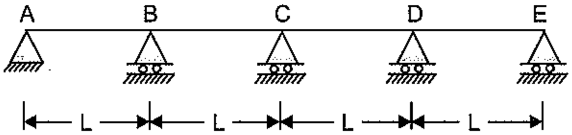
$$\Rightarrow \quad x = \frac{a(L-a)}{L}$$

Q-10: ***A continuous beam ABCDE is subjected to Dead load and live load. Draw the arrangement of Live Load that produce maximum Bending moment at support B and span BC of the beam.***



[8 Marks, ESE-2016]

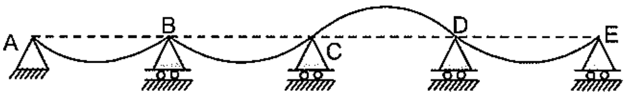
Sol:



Dead load will act throughout the continuous beam over whole span AE.

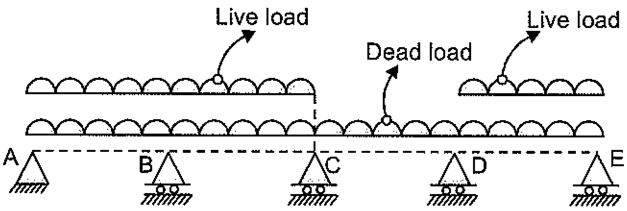
(1) Arrangement of live load to produce maximum BM at support B.

As per Muller Breslau Principle, influence line diagram for bending moment at support B is qualitatively given as:



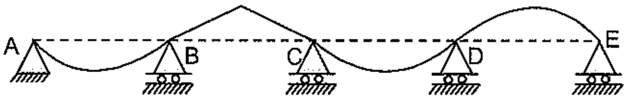
∴ Live load should be place over span AB, BC &DE.

Arrangement of loads:



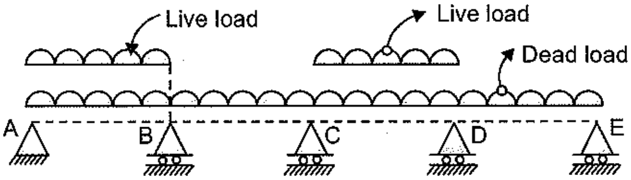
(2) Arrangement of live load to produce maximum BM at span BC.

ILD for BM at span BC as per Muller Breslau principle is qualitatively represented as:

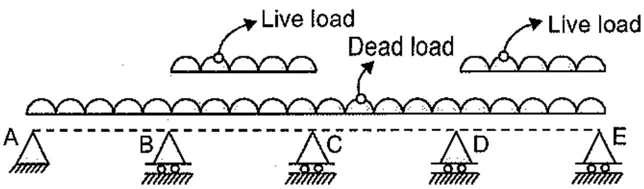


∴ Either span AB and CD to be loaded or span BC and DE to be loaded.

Arrangement of loads:



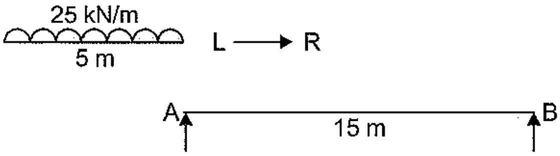
OR



Q-11: ***A beam AB is simply supported over a span of 15 m. An u.d.l. of 25 kN/m intensity and 5 m length moves over the beam from end A and B. Draw the influence line diagram for bending moment and shear force at section C located at 6 m from end A. Hence calculate the maximum bending moment and shear force at section C.***

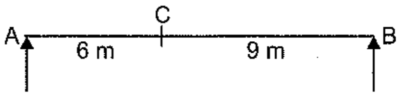
[12 Marks, ESE-2018]

Sol: **Given that:** Simply supported beam (AB) of span 15 m



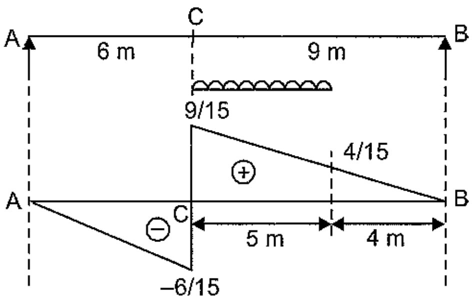
To find out:

(i) ILD for bending moment and shear force at section C



(ii) Maximum bending moment and shear force at section C

Calculation for shear force at section 'C'



ILD for shear force at 'C'

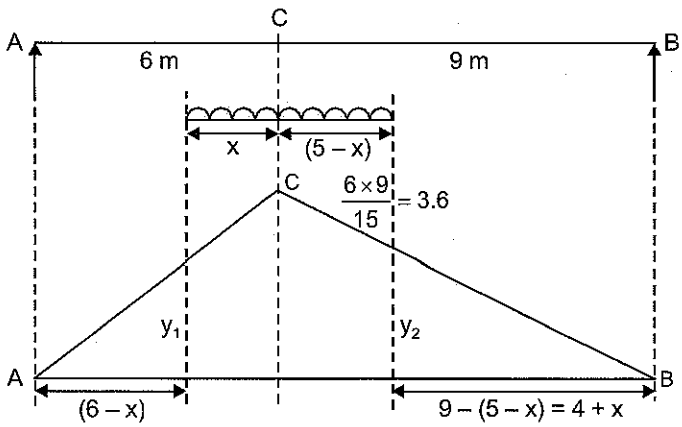
For maximum shear force at 'C'

udl should be placed along span CB such that its rear end just touches the right of section 'C'.

$$(V_{max})_C = 25\times\left[\frac{1}{2}\times 5\times\left(\frac{9+4}{15}\right)\right] = 54.167 \text{ kN}$$

Hence, the maximum shear force at section 'C' is 54.167

Calculation for maximum bending moment at section 'C'



ILD for bending moment at section 'C'

For maximum BM at section 'C', udl should be placed at section 'C' such that average load on span AC would be equal to the average load on span CB.

i.e. $$\frac{Wx}{6} = \frac{W(5-x)}{9}$$

$\Rightarrow$ $$\boxed{x = 2\text{ m}}$$

For similar triangle concept,

$\therefore$ $$\frac{3.6}{6} = \frac{y_1}{(6-x)} = \frac{y_1}{4}$$

$$\boxed{y_1 = 2.4\text{ m}}$$

Also, $$\frac{3.6}{9} = \frac{y_2}{(4+x)} = \frac{y_2}{6}$$

$$\boxed{y_2 = 2.4\text{ m}}$$

Hence, max value of BM at 'C'

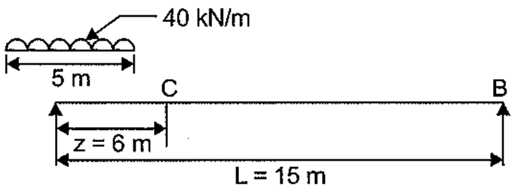
$$= \left[\left\{\frac{1}{2}(2.4+3.6)\times 2\right\} + \left\{\frac{1}{2}\times(2.4+3.6)\right\}\times 3\right]\times 25$$

$$\boxed{(M_{max})_C = 375\text{ kNm}}$$

Q-12: ***A uniformly distributed load of 40 kN/m and 5 m long crosses a simply supported beam of span 15 from left to right. Draw the influence line diagram for shear force and bending moment at a section 6 m from left end. Use these diagrams to get the maximum shear force and bending moment at this section.***

[20 Marks, ESE-2019]

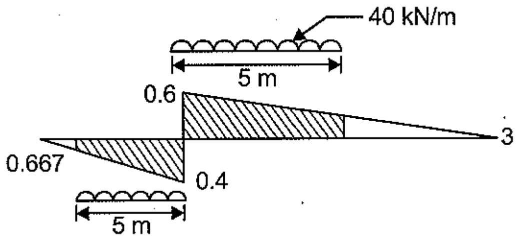
Sol:



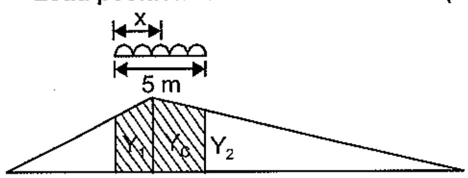
The beam is shown in Figure. For point C, which is at $z = 6$ m from A, *ILD* for shear force F_C and bending moment M_C are to be found.

ILD for F_C: ILD ordinate at just to the left of C is

$$= -\frac{z}{L} = -\frac{6}{15} = -0.4$$



Load position for maximum +ve and (–ve) F_c



Load position for maximum M_c

ILD ordinate at just to right of C

$$= \frac{L-z}{L} = \frac{15-6}{15} = 0.6$$

ILD for F_C is as shown in Figure

At C, negative S.F. is maximum when the head of load touches C. At this time, tail of the udl is at a distance of 1 m from A as shown in Figure. Ordinate under tail end of load is

$$= \frac{1}{6} \times 0.4 = 0.0667$$

$$\therefore \quad \text{Negative maximum } F_C = 40 \times \left[\frac{0.0667+0.4}{2}\right] \times 5 = 46.67 \text{ kN}$$

For maximum positive S.F. at C, tail of the load should be at C as shown in Figure. Ordinate under head to the load

$$= \frac{4}{9} \times 0.6 = 0.267$$

$$\therefore \quad \text{Maximum positive SF} = 40 \times \text{Area of ILD under the load}$$

$$= 40 \times \frac{0.6+0.267}{2} \times 5 = 86.67 \text{ kN}$$

Maximum shear force at C = 86.67 kN

ILD for moment at C is as shown in Figure in which

$$y_C = \frac{z(L-z)}{L} = \frac{6 \times 9}{15} = 3.6$$

For maximum moment, load position should be such that the section divides the load in the same ratio as it divides the span. Referring to Figure.

$$\frac{x}{5-x} = \frac{6}{9}$$

or $$9x = 30 - 6x$$

or $$x = 2 \text{ m}$$

$$y_1 = \left(\frac{6-2}{6}\right) y_C = \frac{4}{6} \times 3.6 = 2.4$$

y_2 will be same as y_1

$$\text{Maximum moment} = w \times \text{Area of ILD for } M_C \text{ under the loaded length}$$

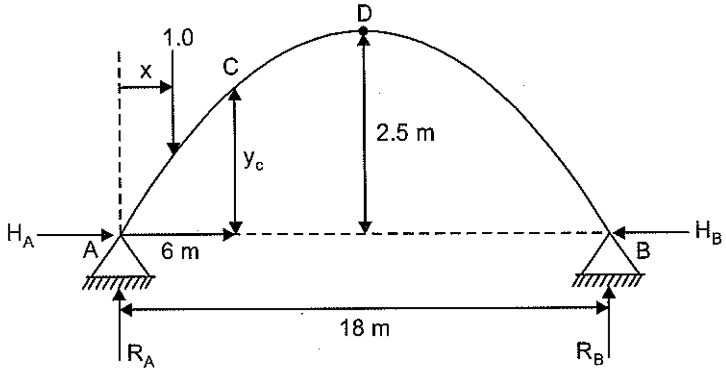
$$= 40\left[\left(\frac{2.4+3.6}{2}\right) \times 2 + \left(\frac{3.6+2.4}{2}\right) \times 3\right]$$

$$= 600 \text{ kNm}$$

Q-13: ***Sketch influence line diagram for the bending moment at a point 'C' located 6 m from one of the supports of a three hinged symmetrical parabolic arch having span of 18 m and central rise 2.5 m. Locate the point from where the moving load changes the sign of bending moment at C.***

[20 Marks, ESE-2020]

Sol: ILD for bending moment at C for three-hinged arch



When unit load is left of C.

$\sum M_B = 0$

$$R_A \times 18 - 1 \times (18 - x) = 0$$

$$R_A = \frac{18-x}{18}$$

$\sum F_y = 0$

$$R_B = \frac{x}{18}$$

$\sum M_D = 0$ right of D.

$$H_B \times 2.5 - R_B \times 9 = 0$$

$$H_B = \frac{x}{18} \times \frac{9}{2.5} = \frac{x}{5}$$

Bending moment at C

$\sum M_C = 0$

$$M_C + R_A \times 6 - H_A \times y_c - 1 \times (6 - x) = 0 \qquad \text{....(1)}$$

Equation of parabolic arch

$$y = \frac{4hx(\ell - x)}{\ell^2}$$

$$= \frac{4 \times 2.5x(18-x)}{18^2} = \frac{5}{162}x(18-x)$$

At x = 6 m

$$y_c = \frac{5}{162} \times 6 \times (18-6) = 2.22\,\text{m}$$

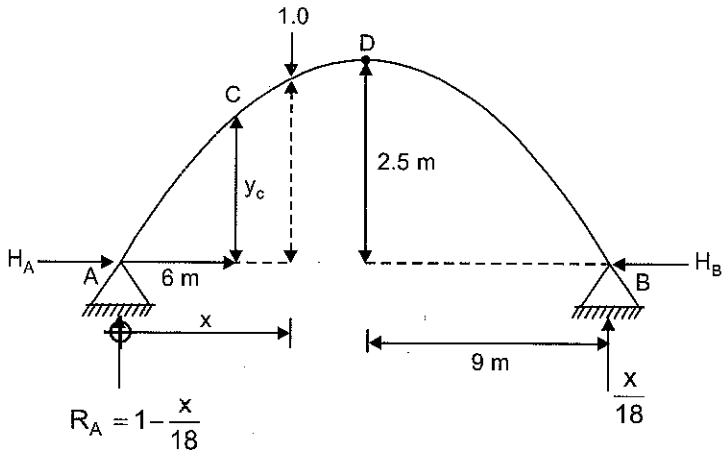
Put value of y_c in equation (1)

$$M_C + \left(1 - \frac{x}{18}\right) \times 6 - \frac{x}{5} \times 2.22 - 1 \times (6-x) = 0$$

$$M_C + 6 - \frac{x}{18} \times 6 - \frac{x}{5} \times 2.22 - 6 + x = 0$$

$$M_C = -\frac{2}{9}x \qquad 0 \le x \le 6\,\text{m}$$

When unit load b/w C & D



$\sum M_D = 0$ right of D.

$$-\frac{x}{18}\times 9 + H_B \times 2.5 = 0$$

$$H_B = H_A = \frac{x}{5}$$

$\sum M_C = 0$

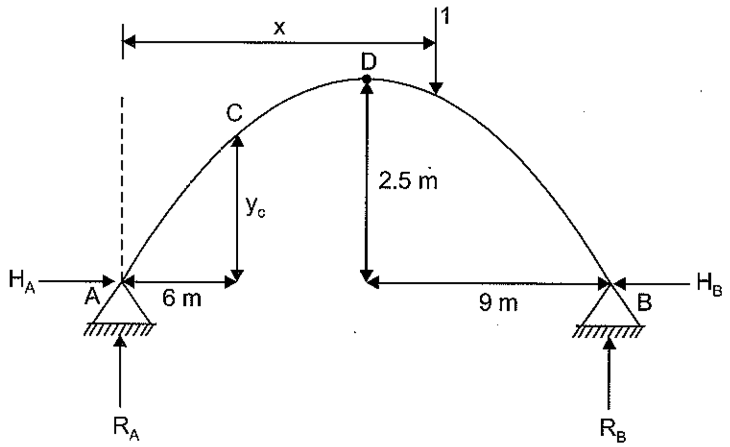
$$M_C + R_A \times 6 - H_A \times y_c = 0$$

$$M_C = \frac{x}{5}\times\frac{20}{9} - \left(1-\frac{x}{18}\right)\times 6$$

$$= \frac{4}{9}x - 6 + \frac{x}{18}\times 6$$

$$= \frac{7}{9}x - 6 \qquad \text{for } 6 \le x \le 9\,\text{m}$$

When unit load b/w D & B



$$R_A = 1-\frac{x}{18}$$

$\sum M_D = 0$ for left of D

$$R_A \times 9 - H_A \times 2.5 = 0$$

$$H_A = \left(1-\frac{x}{18}\right)\times\frac{1}{2.5}\times 9$$

$$= 3.6 - 0.2x$$

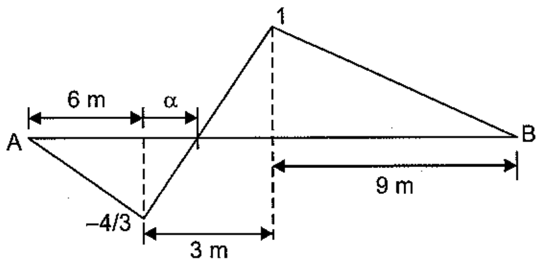
$\sum M_C = 0$ $\qquad M_C + R_A \times 6 - H_A y_c = 0$

$$M_C = (3.6 - 0.2x) \times \frac{20}{9} - \left(1 - \frac{x}{18}\right) \times 6$$

$$= 8 - 6 - x\left[0.2 \times \frac{20}{9} - \frac{6}{18}\right]$$

$$= 2 - \frac{x}{9} \qquad 9 \le x \le 18$$

ILD for M_C



Using similar triangle

$$\frac{\alpha}{\frac{4}{3}} = \frac{3 - \alpha}{1}$$

$$\alpha = 4 - \frac{4}{3}\alpha$$

$$\alpha = 1.71 \text{ m}$$

Location of point from where B.M. changes sign from support A

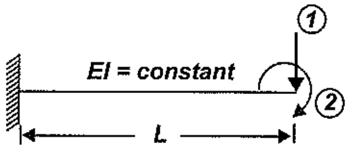
$$= 6 + \alpha$$

$$= 6 + 1.71 = 7.71 \text{ m}$$

CHAPTER 7

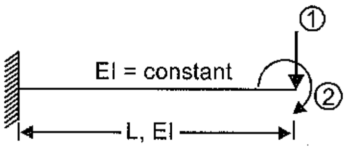
MATRIX METHODS OF ANALYSIS

Q-1: ***Develop the flexibility matrix for the cantilever beam for the degrees of freedom shown below.***



[10 Marks, ESE-2010]

Sol:



As the no. of co-ordinates chosen are 2, the flexibility matrix will be a 2 × 2 matrix. To find out Ist column of flexibility matrix, apply unit load in the direction ① and find out displacements in all co-ordinate directions.

f_{ij} = Displacement in the direction i due to unit load applied in the direction j

= Element of ith row and j^{th} column of flexibility matrix

$$\Rightarrow \quad f_{11} = \frac{1\times L^3}{3EI}$$

$$f_{21} = \frac{1\times L^2}{2EI}$$

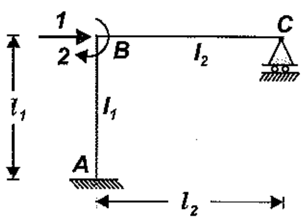
$$f_{12} = \frac{1\times L^2}{2EI}$$

$$f_{22} = \frac{1\times L}{EI}$$

Hence the flexibility matrix is

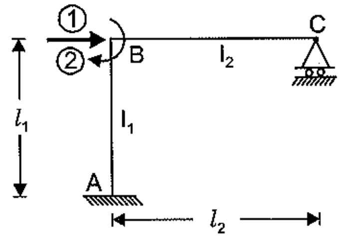
$$[f] = \begin{bmatrix} L^3/3EI & L^2/2EI \\ L^2/2EI & L/EI \end{bmatrix}$$

Q-2: ***Develop the stiffness matrix for the frame shown in the figure below with coordinates 1 and 2 indicated.***



[10 Marks, ESE-2011]

Sol: As the no. of co-ordinate directions are two, the stiffness matrix will be a 2 × 2 matrix.

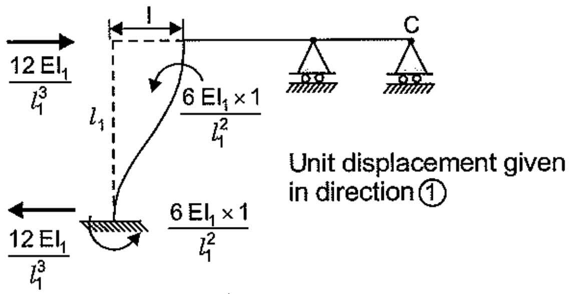


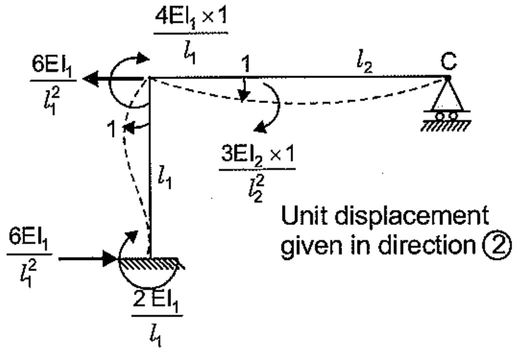
To determine the Ist column of stiffness matrix we give unit displacement in the direction ① without giving displacement in any other co-ordinate direction and find out the forces developed in all co-ordinate directions. The elements obtained are

K_{11} and K_{21}

K_{11} = Force developed in the direction ① when unit displacement was given in direction ① without giving displacement in any other co-ordinate direction.

K_{21} = Force developed in the direction ② when unit displacement was given in direction ① without giving displacement in any other co-ordinate direction.



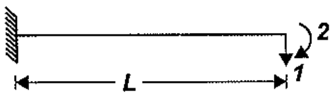


$$\Rightarrow K_{11} = \frac{12EI_1}{l_1^3};\quad K_{21} = \frac{-6EI_1}{l_1^2}$$

$$\Rightarrow K_{12} = -\frac{6EI_1}{l_1^3};\quad K_{22} = \frac{4EI_1}{l_1^2} + \frac{3EI_2}{l_2^2}$$

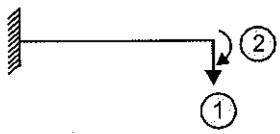
$\Rightarrow$ Stiffness matrix is
$$[K] = \begin{bmatrix} \frac{12EI_1}{l_1^3} & -\frac{6EI_1}{l_1^2} \\ -\frac{6EI_1}{l_1^2} & \frac{4EI_1}{l_1^2} + \frac{3EI_2}{l_2^2} \end{bmatrix}$$

Q-3: ***Derive the flexibility matrix of the plane beam shown in figure with respect to the degress of freedom shown. Take EI = constant.***

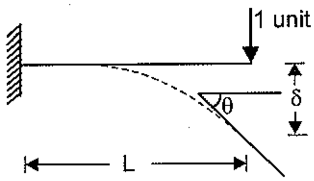


[15 Marks, ESE-2014]

Sol:



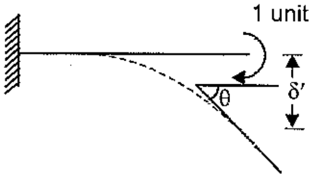
To generate 1st column of flexibility matrix, we apply unit load in the direction of (1) and calculate the deflection it causes in the direction of (1) and (2)



$$\therefore \quad f_{11} = \delta = \frac{PL^3}{3EI} = \frac{L^3}{3EI}$$

$$f_{21} = \theta = \frac{PL^2}{2EI} = \frac{L^2}{2EI} \quad [\because P = 1\,\text{unit}]$$

Similarly, to generate 2nd column of flexibility matrix, we apply unit moment in the direction of (2) and calculate the deflection it causes in the direction of (1) and (2)



$$\therefore \quad f_{12} = \delta' = \frac{ML^2}{2EI} = \frac{L^2}{2EI} \quad [\because M = 1\,\text{unit}]$$

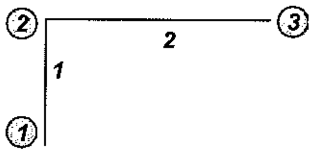
$$f_{22} = \theta' = \frac{ML}{EI} = \frac{L}{EI}$$

Flexibility matrix will be

$$[f] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

$$[f] = \begin{bmatrix} \dfrac{L^3}{3EI} & \dfrac{L^2}{2EI} \\ \dfrac{L^2}{2EI} & \dfrac{L}{EI} \end{bmatrix}$$

Q-4: ***A plane frame is shown in figure. The stiffness matrix of elements 1 and 2 with respect to global axes are also shown. Assemble the global stiffness matrix of the frame. Node number is shown in circles.***



Member 1, $$K_{6\times6} = \begin{bmatrix} K11 & K12 \\ K21 & K22 \end{bmatrix}_{6\times6}$$

Member 2, $$K_{6\times6} = \begin{bmatrix} K'11 & K'12 \\ K'21 & K'22 \end{bmatrix}_{6\times6}$$

where, each of the k_{ij} is a 3 × 3 sub-matrix.

For Member 1,

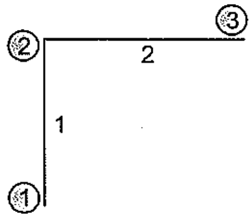
$$\textit{Sub matrix } K_{11} = \begin{bmatrix} k11 & k12 & k13 \\ k21 & k22 & k23 \\ k31 & k32 & k33 \end{bmatrix}$$

For Member 2,

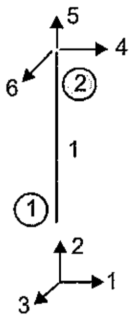
$$\textit{Sub-matrix } k' = \begin{bmatrix} k41 & k42 & k43 \\ k51 & k52 & k53 \\ k61 & k62 & k63 \end{bmatrix}$$

[8 Marks, ESE-2015]

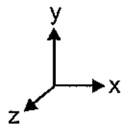
Sol:



For member 1

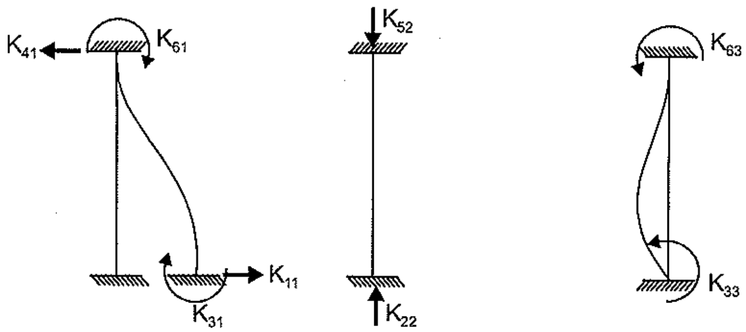


Global coordinates



$$K_{6\times6} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$K_{11} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & k_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \quad K_{21} = \begin{bmatrix} K_{41} & K_{42} & K_{43} \\ K_{51} & K_{52} & K_{53} \\ K_{61} & K_{62} & K_{63} \end{bmatrix}$$



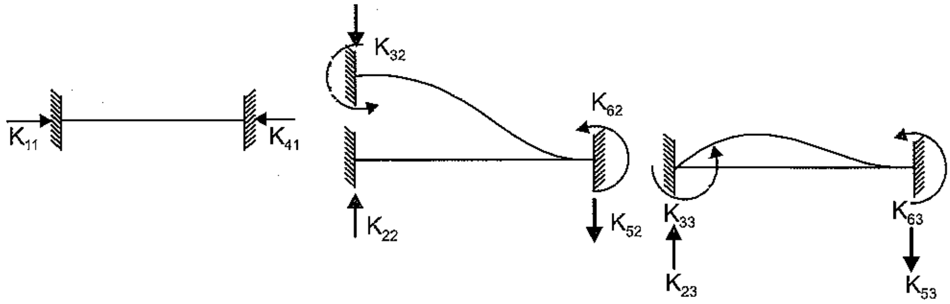
$$K_{21} = \begin{bmatrix} K_{41} & K_{42} & K_{43} \\ K_{51} & K_{52} & K_{53} \\ K_{61} & K_{62} & K_{63} \end{bmatrix} \; K_{22} = \begin{bmatrix} K_{44} & K_{45} & K_{46} \\ K_{54} & K_{55} & K_{56} \\ K_{64} & K_{65} & K_{66} \end{bmatrix}$$

$$K_{66}\text{, member 1} = \left[\begin{array}{ccc|ccc} K_{11} & 0 & -K_{11} & -K_{14} & 0 & -K_{16} \\ 0 & K_{22} & 0 & 0 & -K_{25} & 0 \\ -K_{31} & 0 & K_{33} & K_{34} & 0 & K_{36} \\ \hline -K_{41} & 0 & K_{43} & K_{44} & 0 & K_{46} \\ 0 & -K_{52} & 0 & 0 & K_{55} & 0 \\ -K_{61} & 0 & K_{63} & K_{64} & 0 & K_{66} \end{array}\right]$$

$$K_{63} = \frac{K_{33}}{2} \quad K_{36} = \frac{K_{66}}{2}$$

Similarly For member-2

$$K'_{11} = \begin{bmatrix} K'_{11} & K'_{12} & K'_{13} \\ K'_{21} & K'_{22} & K'_{23} \\ K'_{31} & K'_{32} & K'_{33} \end{bmatrix} \quad K'_{31} = \begin{bmatrix} K'_{41} & K'_{42} & K'_{43} \\ K'_{51} & K'_{52} & K'_{53} \\ K'_{61} & K'_{62} & K'_{63} \end{bmatrix}$$



$$K_{66}\text{, member 2} = \left[\begin{array}{ccc|ccc} K'_{11} & 0 & 0 & -K'_{14} & 0 & 0 \\ 0 & +K'_{22} & K'_{23} & 0 & -K'_{25} & K'_{26} \\ 0 & +K'_{32} & K'_{33} & 0 & -K'_{35} & K'_{36} \\ \hline -K'_{41} & 0 & 0 & K'_{44} & 0 & 0 \\ 0 & -K'_{52} & -K'_{53} & 0 & K'_{55} & -K'_{56} \\ 0 & +K'_{62} & K'_{63} & 0 & -K'_{65} & K'_{16} \end{array}\right]$$

$$K'_{63} = \frac{K'_{33}}{2} \quad K'_{36} = \frac{K'_{66}}{2}$$

Since both matrix of member 1 and 2 are in global coordinate system

$\therefore$ Global matrix of frame will be a 9 × 9 matrix.

$$K_{9,9} = \left[\begin{array}{ccc|ccc|ccc} K_{11} & 0 & -K_{13} & -K_{14} & 0 & -K_{16} & 0 & 0 & 0 \\ 0 & K_{22} & 0 & 0 & -K_{25} & 0 & 0 & 0 & 0 \\ -K_{31} & 0 & K_{33} & K_{34} & 0 & K_{36} & 0 & 0 & 0 \\ \hline -K_{41} & 0 & K_{43} & K_{44}+K'_{11} & 0 & K_{46} & K'_{14} & 0 & 0 \\ 0 & -K_{52} & 0 & 0 & K_{55}+K'_{22} & K'_{23} & 0 & -K'_{25} & K'_{26} \\ -K_{61} & 0 & K_{63} & K_{64} & K_{32} & K_{66}+K'_{33} & 0 & -K'_{35} & K'_{36} \\ \hline 0 & 0 & 0 & -K'_{41} & 0 & 0 & K'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & -K'_{52} & -K'_{53} & 0 & -K'_{55} & -K'_{56} \\ 0 & 0 & 0 & 0 & +-K'_{62} & K'_{63} & 0 & --K'_{65} & K'_{66} \end{array}\right]$$

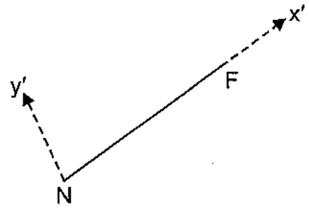
Q-5: ***A member in the plane truss is having the following data : L = length of the member; A = area of cross-section; E = Young's modulus of elasticity and α = angle in the first quadrant from x-axis in anticlockwise direction.***

Derive the element stiffness matrix of the plane truss member in local and global coordinate system.

[20 Marks, ESE-2015]

Sol: A member of truss is as shown below with direction of local coordinates indicated.

N and F represent near end and far end of the member.

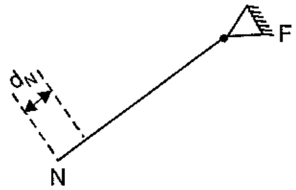


local stiffness matrix for only member of truss will have four elements.

When far end is pinned and unit displacement is given in the coordinate direction at near end.

$$q'_N = \frac{AE}{L} d_N$$

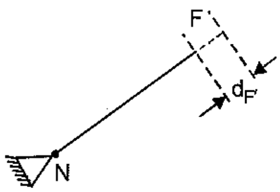
$$q'_N = -\frac{AE}{L} d_N$$



When near end is pinned and unit displacement is given at the far end.

$$q''_N = -\frac{AE}{L} d_F$$

$$q''_F = \frac{AE}{L} d_F$$



hence

$$\begin{bmatrix} q_N \\ q_F \end{bmatrix} = \begin{bmatrix} \frac{AE}{L} & -\frac{AE}{L} \\ -\frac{AE}{L} & \frac{AE}{L} \end{bmatrix} \begin{bmatrix} d_N \\ d_F \end{bmatrix}$$

$$q = k'd$$

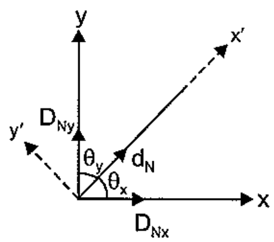
$$k' = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \text{local stiffness matrix}$$

Global Stiffness Matrix

x-y are global coordinates.

D_{Nx} = Displacement at near end in X-global coordinate direction.

D_{Ny} = Displacement at near end in Y-global coordinate direction



If d_N be the displacement in local coordinate direction at near end.

$$d_N = D_{Nx}\cos\theta_x + D_{Ny}\cos\theta_y$$

Similarly, $$d_F = D_{F_x}\cos\theta_x + D_{F_y}\cos\theta_y$$

$$\begin{bmatrix} d_N \\ d_F \end{bmatrix} = \begin{bmatrix} \cos\theta_x & \cos\theta_y & 0 & 0 \\ 0 & 0 & \cos\theta_x & \cos\theta_y \end{bmatrix} \begin{bmatrix} D_{N_x} \\ D_{N_y} \\ D_{F_y} \end{bmatrix}$$

Let, T be the transformation matrix from global coordinates to local coordinates.

$\therefore$ T^T will be the transformation matrix from local coordinates to global coordinates.

$$T^T = \begin{bmatrix} \cos\theta_x & 0 \\ \cos\theta_y & 0 \\ 0 & \cos\theta_x \\ 0 & \cos\theta_3 \end{bmatrix}$$

Force transformation matrix.

$$d = T^T q$$

hence

$$q = k'd$$

$$d = TD$$

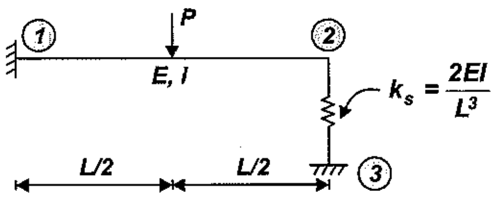
$$q = k'TD$$

$$d = T^T k'TD$$

hence global stiffness matrix $k = T^T k'T$

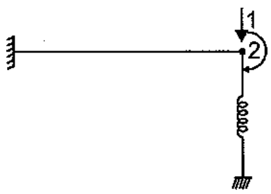
$$k = \frac{AE}{L}\begin{bmatrix} \cos\theta_x & 0 \\ \cos\theta_y & 0 \\ 0 & \cos\theta_x \\ 0 & \cos\theta_y \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} \cos\theta_x & \cos\theta_y & 0 & 0 \\ 0 & 0 & \cos\theta_x & \cos\theta_y \end{bmatrix}$$

Q-6: ***A propped cantilever beam is shown in Figure. Analyze it using the stiffness matrix method and find the reactions in the spring. Neglect axial deformation in the beam. Show the degrees of freedom.***



[12 Marks, ESE-2017]

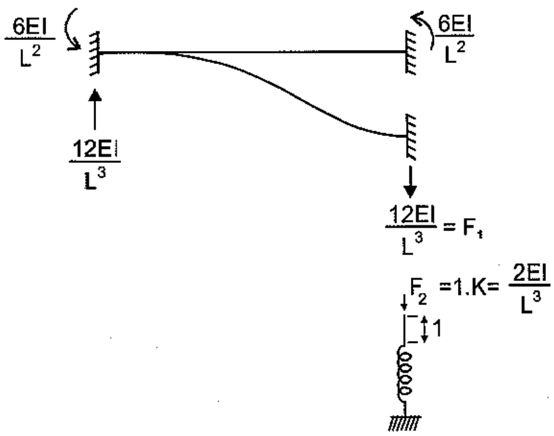
Sol:



Degree of freedom of the structure are Δ_1 & θ_2 ie. displacement in (1) direction & rotation in (2) direction.

Formatiom of Stiffness Matrix : Stiffness matrix of the structure will be a 2×2 matrix.

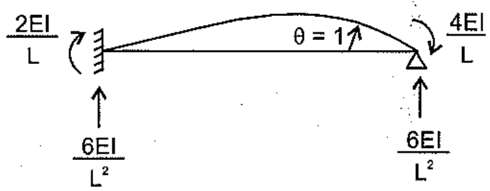
1st column



$$K_{11} = F_1 + F_2 = \frac{12EI}{L^3} + \frac{2EI}{L^3} = \frac{14EI}{L^3}$$

$$K_{21} = -\frac{6EI}{L^2}$$

2nd column

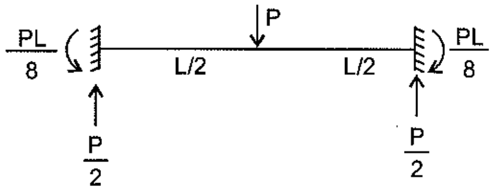


$$K_{12} = \frac{-6EI}{L^2}$$

$$K_{22} = \frac{4EI}{L}$$

So,
$$K = \begin{vmatrix} \frac{14EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{4EI}{L} \end{vmatrix}$$

Fixed End Moment



$$\begin{vmatrix} F_1 \\ F_2 \end{vmatrix} = \begin{bmatrix} -P/2 \\ \frac{PL}{8} \end{bmatrix} + \begin{bmatrix} \frac{14EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} \Delta \\ \theta \end{bmatrix}$$

Where F_1 & F_2 are external forces in co-ordinate direction (1) & (2)

$$\Rightarrow \qquad \begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} -P/2 \\ \frac{PL}{8} \end{vmatrix} + \begin{vmatrix} \frac{14EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} \end{vmatrix} \begin{vmatrix} \Delta \\ \theta \end{vmatrix}$$

Solving
$$\Delta = \frac{PL^3}{16EI}$$

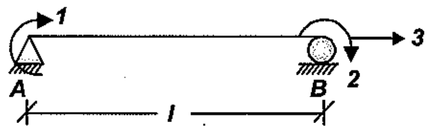
$$\theta = \frac{PL^2}{16EI}$$

So, reaction in the spring is
$$F_s = K\Delta$$

$$F_s = \frac{2EI}{L^3} \times \frac{PL^3}{16EI}$$

$$\boxed{F_S = \frac{P}{8}}$$

Q-7: ***Briefly explain the procedure and then develop the stiffness matrix for the beam element shown in figure with respect to the degrees of freedom 1, 2 and 3. The cross-sectional area A and flexural rigidity EI are constant for the beam.***



[16 Marks, ESE-2018]

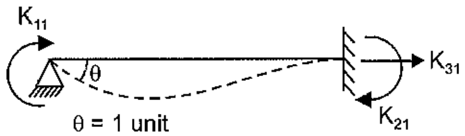
Sol: Procedure to develop stiffness matrix

Step (a): Here, DOF = 3

∴ Stiffness matrix will be 3 × 3 matrix

$$K = \begin{array}{c} \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \\ \begin{matrix} (1) & (2) & (3) \end{matrix} \end{array}$$

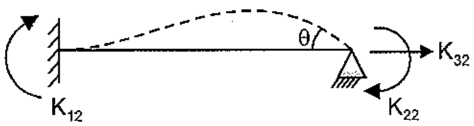
Step (b): To find 1st column i.e. K_{11}, K_{21}, K_{31}, we need to apply unit displacement along DOF = 1 and restrain 2 and 3 DOF. Then we need to find forces developed along 1, 2 and 3.

∴ 

[∵ 2 and 3 restrained]

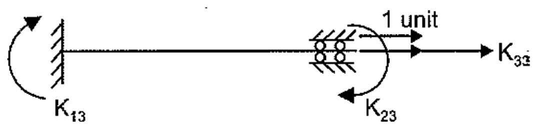
$$\therefore \quad \boxed{K_{11} = \frac{4EI}{L} \qquad K_{21} = \frac{2EI}{L} \qquad K_{31} = 0}$$

Step (c): Now, restrain (1) and (3) and apply unit rotation at (2).

∴ 

$$\boxed{K_{22} = \frac{4EI}{L} \qquad K_{12} = \frac{2EI}{L} \qquad K_{32} = 0}$$

Step (d): Now restrain (1) and (2) and apply unit displacement along (3) to find 3rd column of matrix.

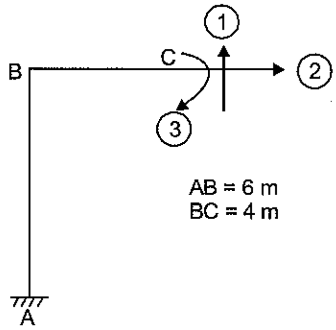


$$K_{13} = 0, \; K_{23} = 0, \; K_{33} = \frac{AE}{L}$$

∴ Matrix K is

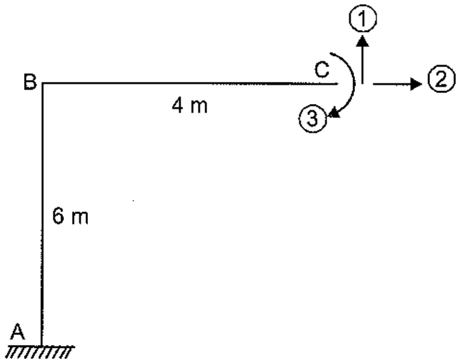
$$K = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & 0 \\ \frac{2EI}{L} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{AE}{L} \end{bmatrix}$$

Q-8: ***Develop the flexibility matrix for the beam shown in Figure, with respect to the generalized coordinates mentioned. EI is constant for all members.***



[20 Marks, ESE-2020]

Sol:

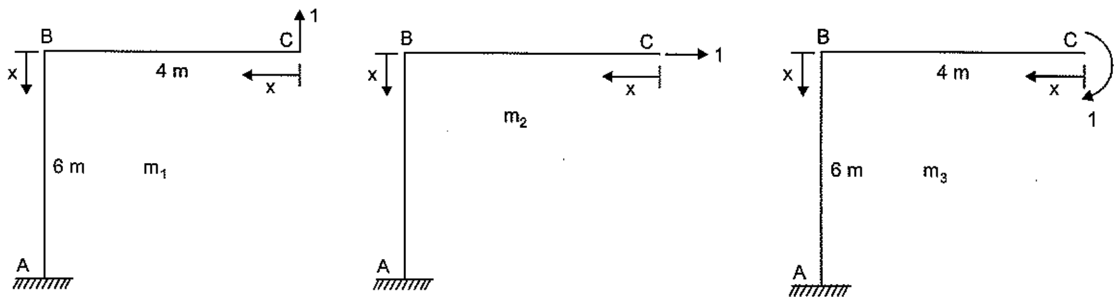


Order of Flexibility matrix is 3.

$$[f] = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

For 1st Column of $[f]_{3\times3}$

To obtain 1st column, apply unit load in direction of coordinate (1) and get displacement in all coordinate.



$$f_{11} = \int \frac{m_1 m_1}{EI}.dx$$

$$f_{21} = \int \frac{m_2 m_1}{EI}.dx$$

$$f_{31} = \int \frac{m_3 m_1}{EI}.dx$$

Member	l	EI	range of x	m_1	m_2	m_3
CB	4m	EI	0 – 4	x	0	–1
BA	6m	EI	0 – 6	4	–x	–1

$$f_{11} = \int_0^4 \frac{x^2.dx}{EI} + \int_0^6 \frac{4^2.dx}{EI} = \frac{117.333}{EI}$$

$$f_{21} = \int_0^4 \frac{0.dx}{EI} + \int_0^6 \frac{(-x)4}{EI}.dx = -\frac{72}{EI}$$

$$f_{31} = \int_0^4 \frac{-x.dx}{EI} + \int_0^6 \frac{-4}{EI}dx = -\frac{32}{EI}$$

2nd Column of matrix :

Apply unit load in direction of coordinate (2) and get displacement in all coordinate.

$$f_{12} = \int \frac{m_1 m_2 dx}{EI}$$

$$f_{22} = \int \frac{m_2 m_2 dx}{EI}$$

$$f_{32} = \int \frac{m_3 m_2 dx}{EI}$$

$$f_{12} = \int_0^6 \frac{-4x\,dx}{EI} = -\frac{72}{EI}$$

$$f_{22} = \int_0^6 \frac{x^2 dx}{EI} = \frac{72}{EI}$$

$$f_{32} = \int_0^6 \frac{(-x)(-1)}{EI}.dx = \frac{18}{EI}$$

3rd Column of Matrix :

Apply unit load in direction of coordinate (3) and get displacement in all coordinate.

$$f_{13} = \int \frac{m_1 m_3}{EI} dx$$

$$f_{23} = \int \frac{m_2 m_3}{EI} dx$$

$$f_{33} = \int \frac{m_3 m_3}{EI} dx$$

$$f_{13} = \int_0^4 \frac{(x)(-1)}{EI} dx + \int_0^6 \frac{(4)(-1)}{EI} dx$$

$$= -\frac{32}{EI}$$

$$f_{23} = \int_0^6 \frac{x}{EI} dx = \frac{18}{EI}$$

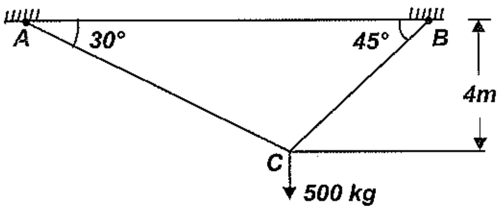
$$f_{33} = \int_0^4 \frac{(-1)^2}{EI} dx + \int_0^6 \frac{(-1)^2}{EI} = \frac{10}{EI}$$

$$[f] = \begin{bmatrix} 117.333 & -72 & -32 \\ -72 & 72 & 18 \\ -32 & 18 & 10 \end{bmatrix}$$

CHAPTER 8

ARCHES AND CABLES

Q-1: ***Two straight wires AC and BC, 8 mm dia meet at joint C as shown in fig. Supports A and B (at the same level) are unyielding. Calculate the horizontal and vertical components of deflection of joint C when a vertical load of 500 kg is applied at C E = 2 × 10⁶ kg/cm². Joints may be assumed as hinged.***

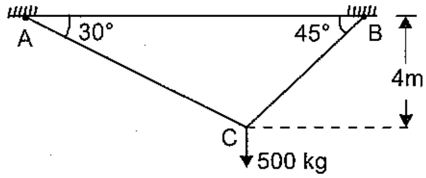


[15 Marks, ESE-1997]

Sol: **Given:**

Diameter of bars = 8 mm = 0.8 cm

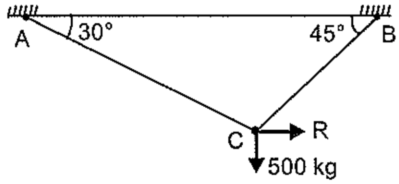
$$E = 2 \times 10^6 \text{ kg/cm}^2$$



$$L_{CA} = \frac{4}{\sin 30^\circ} = 8 \text{ m}$$

$$L_{CB} = \frac{4}{\sin 45^\circ} = 4\sqrt{2}\text{m}$$

To find out the vertical and horizontal deflection component of deflection of joint C, we will use castigliano theorem.

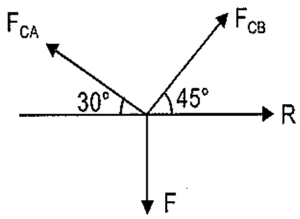


$$\text{Vertical deflection of joint C} = \left.\frac{\partial U}{\partial F}\right|_{\substack{F=500\text{ kg}\\R=0}}$$

$$\text{Horizontal deflection of Joint C} = \left.\frac{\partial U}{\partial R}\right|_{\substack{F=500\text{ kg}\\R=0}}$$

U = Strain energy of system

$$U = U_{CA} + U_{CB}$$



$$U_{CA} = \frac{F_{CA}^2\, l_{CA}}{2AE}$$

$$U_{CB} = \frac{F_{CB}^2\, l_{CB}}{2AE}$$

$$\Sigma F_V = 0$$

$$\Rightarrow \quad F_{CA} \sin 30^\circ + F_{CB} \sin 45^\circ - F = 0 \qquad \ldots(i)$$

$$\Sigma F_H = 0$$

$$\Rightarrow \quad -F_{CA} \cos 30^\circ + F_{CB} \cos 45^\circ + R = 0 \qquad \ldots(ii)$$

From (i) and (ii)

$$F_{CA}\,(\sin 30^\circ + \cos 30) - F - R = 0$$

$$\boxed{F_{CA} = \frac{F+R}{\sin 30^\circ + \cos 30^\circ} = 0.732\,(F+R)}$$

$$F_{CB} = \frac{F - F_{CA}\sin 30^\circ}{\sin 45^\circ} = -0.518\,(F+R) + 1.414\,F$$

$$\boxed{F_{CB} = 0.896\,F - 0.518\,R}$$

$$\Rightarrow \quad U = \frac{F_{CA}^2\, l_{CA}}{2AE} + \frac{F_{CB}^2\, l_{CB}}{2AE}$$

$$AE = \frac{\pi}{4}(0.8)^2\ \text{cm}^2 \times 2\times 10^6\ \text{kg/cm}^2 = 100.48 \times 10^4\ \text{kg}$$

$$\Rightarrow \quad U = \frac{F_{CA}^2\ \text{kg}^2 \times 800\ \text{cm}}{2\times 100.48\times 10^4\ \text{kg}} + \frac{F_{CB}^2\text{kg}^2 \times 565.685\ \text{cm}}{2\times 100.48\times 10^4\ \text{kg}}$$

$$U = \frac{F_{CA}^2}{2\times 1256}\ \text{kgcm} + \frac{F_{CB}^2}{2\times 1776.253}\ \text{kg cm}$$

$$\frac{\partial U}{\partial F} = \left[\frac{2F_{CA}\cdot \frac{\partial F_{CA}}{\partial F}}{2\times 1256} + 2F_{CB}\cdot\frac{\frac{\partial F_{CB}}{\partial F}}{2\times 1776.253}\right]\text{cm}$$

$$\frac{\partial U}{\partial R} = \left[\frac{2F_{CA}\cdot \frac{\partial F_{CA}}{\partial R}}{2\times 1256} + \frac{2F_{CB}\,\frac{\partial F_{CB}}{\partial R}}{2\times 1776.253}\right]\text{cm}$$

$$\Delta_{V_C} = \left.\frac{\partial U}{\partial F}\right|_{\substack{F=500\ \text{kg}\\ R=0}} = \frac{2\times 0.732\,(500)\times 0.732}{2\times 1256}\ \text{cm} + \frac{2\times(0.896\times 500)\times(0.896)}{2\times 1776.253}$$

$$= \frac{0.427\ \text{cm} \times 0.452\,\text{cm}}{2}$$

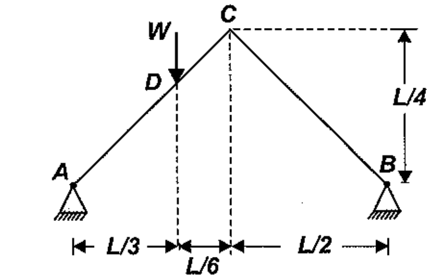
$$\Delta V_C = \frac{0.879\ \text{cm}}{2} = \frac{8.79\ \text{mm}}{2} = 4.395\ \text{mm}\downarrow$$

$$\Delta_{HC} = \left.\frac{\partial U}{\partial R}\right|_{\substack{F=500\ \text{kg}\\ R=0}} = \frac{2\times 0.732\ (500)\times 0.732}{2\times 1256} + \frac{2\times 0.896\times 500\times(-0.518)}{2\times 17776.253}$$

$$= \frac{0.427 - 0.260}{2} = 0.0835\ \text{cm} = 0.835\ \text{mm}\ \ (\rightarrow)$$

(+) ve value of deflections mean that deflections are in the direction of force F and R i.e., downward and rightward.

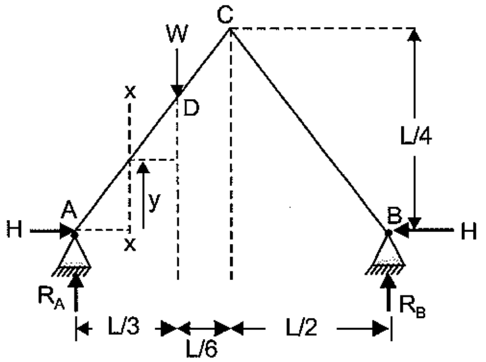
Q-2: ***A symmetrical triangular shaped arch ACB is hinged at supports A and B. The two members of the arch are of the same cross-section. A vertical load 'W' is placed as shown in the figure. Joint C is rigid.***



Determine the components of reactions and draw bending moment diagram indicating maximum +ve and –ve magnitudes.

[20 Marks, ESE-2007]

Sol: **Reaction:**



Reaction Calculation:

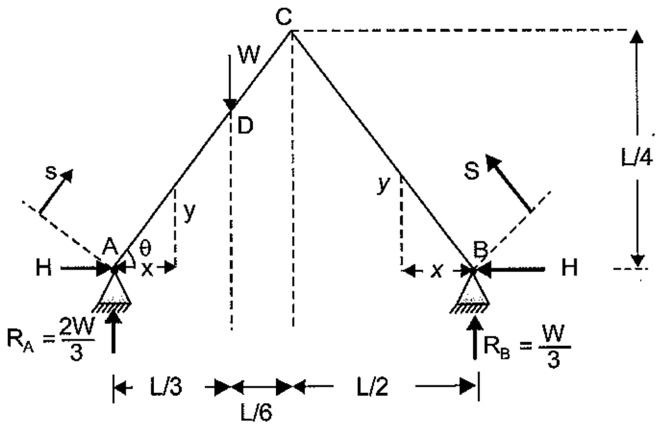
$$\Sigma F_y = 0$$

$$\Rightarrow \quad R_A + R_B = W$$

$$\Sigma M_A = 0$$

$$\Rightarrow \quad R_B \times L = \frac{WL}{3} \Rightarrow R_B = \frac{W}{3} (\uparrow)$$

$$\therefore \quad R_A = \frac{2W}{3} (\uparrow)$$



$$\tan\theta = \frac{1}{2}$$

$$\sin\theta = \frac{1}{\sqrt{5}}$$

$$\cos\theta = \frac{2}{\sqrt{5}}$$

$$\frac{y}{x} = \frac{1}{2}$$

$$\Rightarrow y = \frac{x}{2}$$

From strain energy concept, $\frac{\partial U}{\partial H} = 0$

$$\Rightarrow \int \frac{\frac{M\partial M}{\partial H} ds}{EI} = 0$$

$$\Rightarrow \int M\frac{\partial M}{\partial H}.\, ds = 0$$

[EI = constant in the whole span]

$(ds)\cos\theta = dx$

$$ds = \frac{dx}{\cos\theta} = \frac{\sqrt{5}\,dx}{2}$$

Segment	M	$\frac{\partial M}{\partial H}$	Range of x
AD	$\frac{2W}{3}x - Hy = \frac{2Wx}{3} - \frac{Hx}{2}$	$-\frac{x}{2}$	$0-\frac{L}{3}$
DC	$\frac{2W}{3}x - Hy - W\left(x-\frac{L}{3}\right) = \frac{2Wx}{3} - \frac{Hx}{2} - Wx + \frac{WL}{3}$	$-\frac{x}{2}$	$\frac{L}{3}$ to $\frac{L}{2}$
BC	$\frac{W}{3}x - Hy = \frac{Wx}{3} - \frac{Hx}{2}$	$-\frac{x}{2}$	$0-\frac{L}{2}$

$$\int M.\frac{\partial M}{\partial H} ds = 0$$

$$\Rightarrow \int M\frac{\partial M}{\partial H} \times \frac{dx}{\cos\theta} = 0$$

$$\Rightarrow \int M.\frac{\partial M}{\partial H}.dx = 0$$

$$\Rightarrow \int_0^{L/3}\left(\frac{2Wx}{3}-\frac{Hx}{2}\right)\left(\frac{-x}{2}\right)dx + \int_{L/3}^{L/2}\left(-\frac{Wx}{3}+\frac{WL}{3}-\frac{Hx}{2}\right)\left(\frac{-x}{2}\right)dx + \int_0^{L/2}\left(\frac{Wx}{3}-\frac{Hx}{2}\right)\left(\frac{-x}{2}\right)dx = 0$$

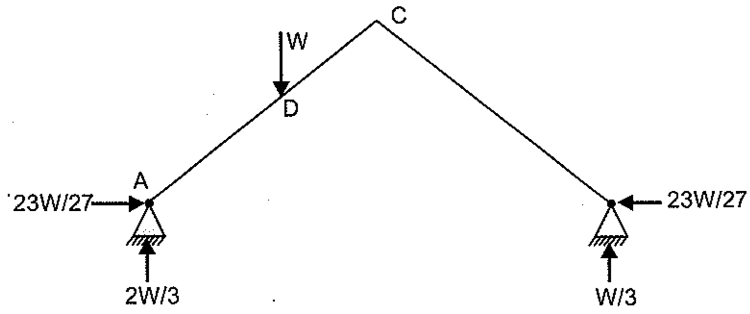
$$\Rightarrow \frac{-2W}{6}\frac{(L/3)^3}{3} + \frac{H}{4}\frac{(L/3)^3}{3} + \frac{W}{6}\left[\frac{(L/2)^3}{3} - \frac{(L/3)^3}{3}\right] - \frac{WL}{6}\left[\frac{(L/2)^2}{2} - \frac{(L/3)^2}{2}\right]$$

$$+\frac{H}{4}\left[\frac{(L/2)^3}{3} - \frac{(L/3)^3}{3}\right] - \frac{W}{6}\frac{(L/2)^3}{3} + \frac{H}{4}\frac{(L/2)^3}{3} = 0$$

$$\Rightarrow WL^3\left[-\frac{2}{18\times27}+\frac{1}{18\times8}-\frac{1}{18\times27}-\frac{1}{12\times4}+\frac{1}{12\times9}-\frac{1}{18\times8}\right]+HL^3\left[\frac{1}{12\times27}+\frac{1}{12\times8}-\frac{1}{12\times27}+\frac{1}{12\times8}\right]=0$$

$$\frac{-23WL^3}{1296}+\frac{HL^3}{48}=0 \quad \Rightarrow H = \frac{23\,WL^3}{27L^3} = \frac{23W}{27}$$

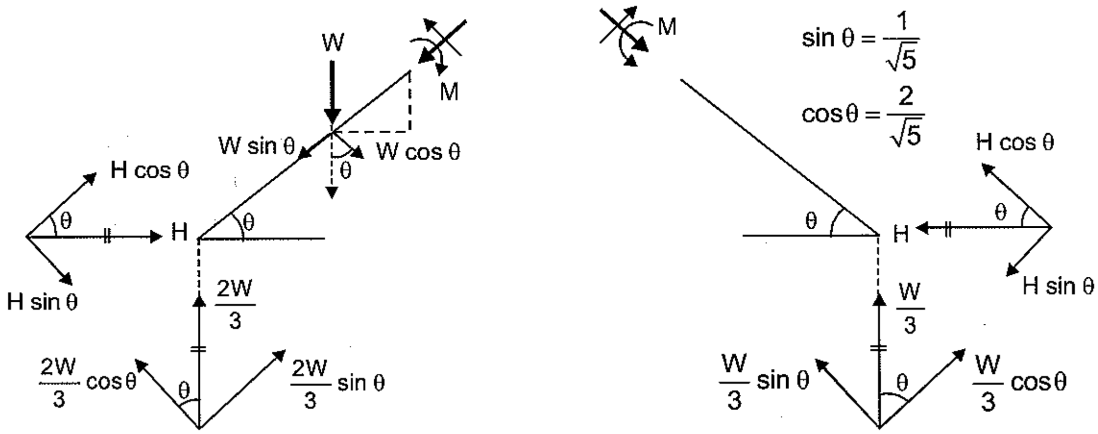
Thus the reactions are as shown below



$$\tan\theta = \frac{L/4}{L/2} = \frac{1}{2}$$

$$\sin\theta = \frac{1}{\sqrt{5}}$$

$$\cos\theta = \frac{2}{\sqrt{5}}$$



$$\frac{2W}{3}\sin\theta = \frac{2W}{3}\times\frac{1}{\sqrt{5}} = 0.298\text{ W}$$

$$H\cos\theta = \frac{23W}{27}\times\frac{2}{\sqrt{5}} = 0.762\text{ W}$$

$$\frac{2W}{3}\cos\theta = \frac{2W}{3}\times\frac{2}{\sqrt{5}} = 0.596\text{ W}$$

$$H\sin\theta = \frac{23W}{27}\times\frac{1}{\sqrt{5}} = 0.381\text{ W}$$

$$\frac{W}{3}\sin\theta = 0.149\text{ W}$$

$$\frac{W}{3}\cos\theta = 0.298\text{ W}$$

$$\text{Length of AC} = \sqrt{\frac{L^2}{16}+\frac{L^2}{4}} = 0.559\,L$$

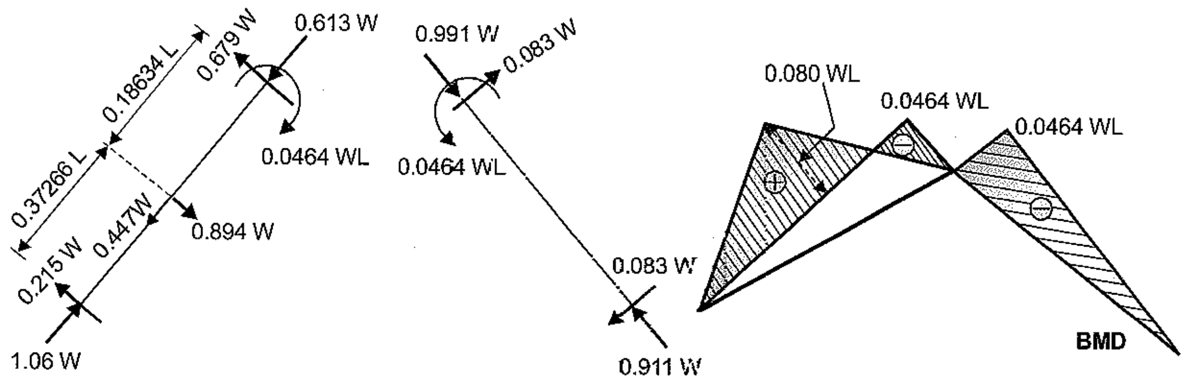
$\Rightarrow$ $$\text{Length of AD} = \sqrt{\left(\frac{L}{6}\right)^2+\left(\frac{L}{12}\right)^2} = 0.18634\,L$$

$\Rightarrow$ From $\Sigma M_C = 0$,

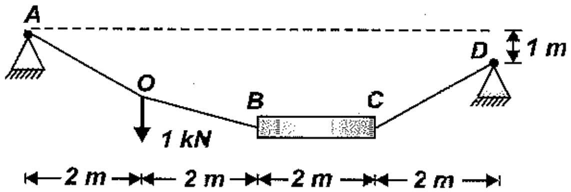
$$\left(\frac{2W\cos\theta}{3} - H\sin\theta\right)\times 0.559\,L - W\cos\theta\times 0.18634\,L + M = 0$$

$\Rightarrow \quad M = 0.0464\ WL$

Hence the FBD will look like as follows:

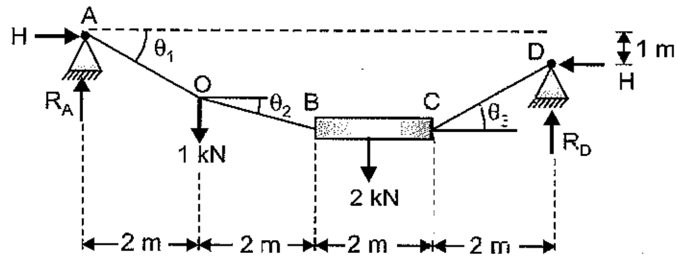


Q-3: ***A rigid uniform straight rod BC of 2 kN weight and 2 m length is held in horizontal position by two inextensible flexible strings AB and CD as shown in the figure. Another 1 kN weight is also suspended on the string AB at O. Calculate the horizontal and vertical reaction components at the supports A and D, and the sag at O.***



[10 Marks, ESE-2008]

Sol: Calculation of reactions:



$$\Sigma F_y = 0 \Rightarrow R_A + R_D = 3\ kN$$

$$\Sigma M_D = 0 \Rightarrow R_A \times 8 + H \times 1 - 1 \times 6 - 2 \times 3 = 0 \Rightarrow R_A = \frac{12-H}{8}$$

BM at every point in the string is zero

$\Rightarrow$ From left side, BM at O = 0

$$\Rightarrow \quad R_A \times 2 + H \times 2 \tan\theta_1 = 0$$

$$\Rightarrow \quad \left(\frac{12-H}{8}\right)\times 2 + 2H\tan\theta_1 = 0$$

$$\Rightarrow \quad \tan\theta_1 = -\left[\frac{12-H}{8H}\right] \quad \text{...(i)}$$

From right side, BM at C = 0

$$\Rightarrow \quad R_D \times 2 + H \times 2\tan\theta_3 = 0$$

$$\Rightarrow \quad 2\times(3-R_A)+2H\tan\theta_3 = 0$$

$$\Rightarrow \quad 2\times\left(3-\frac{12-H}{8}\right)+2H\tan\theta_3 = 0$$

$$\Rightarrow \quad 2\times\left(\frac{12+H}{8}\right)+2H\tan\theta_3 = 0$$

$$\Rightarrow \quad \tan\theta_3 = -\left(\frac{12+H}{8H}\right) \qquad \text{...(ii)}$$

Also, from left side, BM at B = 0

$$R_A\times4+H\times(2\tan\theta_1+2\tan\theta_2)-1\times2 = 0$$

$$\Rightarrow \quad \left(\frac{12-H}{8}\right)\times4+2H(\tan\theta_1+\tan\theta_2) = 2$$

$$\Rightarrow \quad \tan\theta_1+\tan\theta_2 = \frac{2-\left(\frac{12-H}{8}\right)\times4}{2H} = \frac{H-8}{4H}$$

$$\Rightarrow \quad \tan\theta_2 = \frac{H-8}{4H}+\frac{12-H}{8H} = \frac{2H-16+12-H}{8H} \qquad \text{...(iii)}$$

$$\Rightarrow \quad \tan\theta_2 = \frac{H-4}{8H}$$

From the geometry,

$$2\tan\theta_1+2\tan\theta_2-2\tan\theta_3 = 1$$

$$\Rightarrow \quad 2\left[\frac{-(12-H)}{8H}+\frac{H-4}{8H}+\frac{12+H}{8H}\right] = 1$$

$$-12+H+H-4+12+H = 4H$$

$$3H-4 = 4H$$

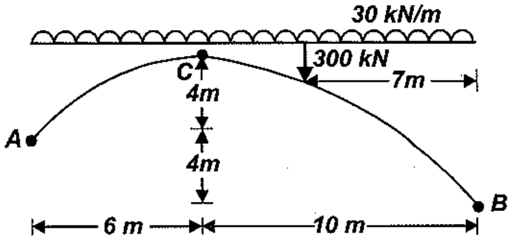
$$\Rightarrow \quad \boxed{H=-4\,\text{kN}}$$

$$\Rightarrow \quad \boxed{R_A=\frac{12-H}{8}=\frac{12+4}{8}=2\,\text{kN}}$$

$$\Rightarrow \quad \boxed{R_D = 3-2=1\,\text{kN}}$$

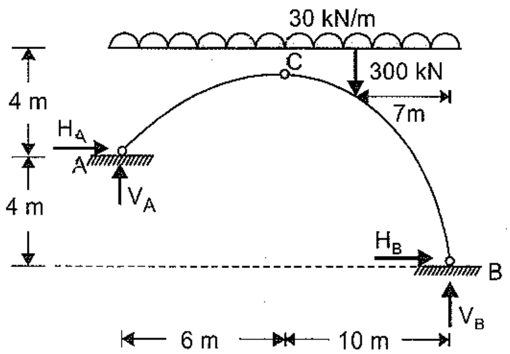
The sag at O = $2\tan\theta_1 = 2\left[-\frac{(12-H)}{8H}\right]=-2\left[\frac{12+4}{8\times(-4)}\right]=1\,\text{m}$

Q-4: ***A3-hinged parabolic arch of 16m span has its abutments A and B at a depth of 4m and 8m respectively below the crown C. It is loaded as shown in figure. Determine the horizontal thrust and the vertical reactions at the supports.***



[15 Marks, ESE-2014]

Sol:



$$V_A = \text{Vertical reaction at A}$$
$$V_B = \text{Vertical reaction at B}$$
$$H_A = \text{Horizontal reaction at A}$$
$$H_B = \text{Horizontal reaction at B}$$

For equilibrium

$$\Sigma F_x = 0; \ \Sigma F_y = 0$$
$$\Sigma F_x = 0$$
$$\therefore \quad H_A + H_B = 0 \quad \ldots(i)$$
$$\Sigma F_y = 0$$
$$\therefore \quad V_A + V_B = 30 \times 16 + 300$$
$$\Rightarrow \quad V_A + V_B = 780 \quad \ldots(ii)$$

Moment about any point should be zero. Let us choose point B and equate moment about point B equal to zero.

$$\therefore \quad H_A \times 4 + V_A \times 16 - 30 \times 16 \times 8 - 300 \times 7 = 0$$
$$\therefore \quad 16V_A + 4H_A = 5940 \quad \ldots(iii)$$

As point C is hinged. Moment at point C from both sides should be zero. Writing total moment from left side of C and equate it to zero we get

$$H_A \times 4 + 30 \times 6 \times 3 - V_A \times 6 = 0$$
$$\Rightarrow \quad 6V_A - 4H_A = 540 \quad \ldots(iv)$$

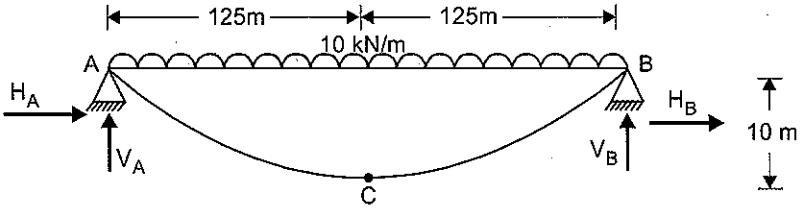
Solving equation (i), (ii), (iii) & (iv), we get

$$V_A = 294.54 \text{ kN}$$
$$V_B = 485.46 \text{ kN}$$
$$H_A = 306.82 \text{ kN}$$
$$H_B = -306.82 \text{ kN}$$

Q-5: ***A cable suspends across a gap of 250 m and carries a udl of 10 kN/m horizontal. Calculate the maximum tension if the maximum sag is 1/25. Also compute sag at 50m from one end.***

[10 Marks, ESE-2014]

Sol: As it is not mentioned clearly whether both support are at same level or not. We will assume that both supports are at same level



Maximum sag will be at mid span, which is equal to $\frac{L}{25} = \frac{250}{25} = 10\text{m}$

V_A = Vertical reaction at support A

V_B = Vertical reaction at support B

H_A = Horizontal reaction at support A

H_B = Horizontal reaction at support B

Now,

$$\Sigma F_x = 0$$

$$\Rightarrow \quad H_A + H_B = 0 \quad \text{...(i)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad V_A + V_B = 10 \times 250$$

$$V_A + V_B = 2500 \quad \text{...(ii)}$$

Equating moment about B to zero, we get

$$V_A \times 250 - 10 \times 250 \times 125 = 0$$

$$\Rightarrow \quad V_A = 1250 \text{ kN}$$

Putting value of V_A in equation (ii), we get

$$1250 + V_B = 2500$$

$$\Rightarrow \quad V_B = 1250 \text{ kN}$$

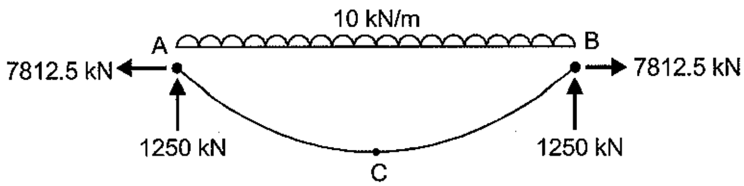
As cable is not able to transfer moment, bending moment at every point in a cable must be zero. Therefore, bending moment at C is also zero.

$$\Rightarrow \quad (B.M)_c = 0$$

$$\therefore \quad H_A \times 10 + V_A \times 125 - 10 \times 125 \times \frac{125}{2} = 0$$

$$\therefore \quad H_A = \frac{10 \times 125 \times \frac{125}{2} - 1250 \times 125}{10}$$

$$\Rightarrow \quad H_A = -7812.5 \text{ kN}$$



Maximum tension in cable is at A or B, therefore

$$T_{max} = \sqrt{H_A^2 + V_A^2}$$

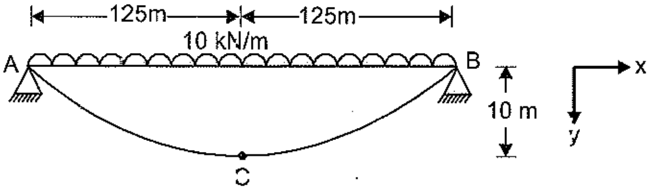
$$= \sqrt{(7812.5)^2 + (1250)^2}$$

$$T_{max} = 7911.87 \text{ kN}$$

Let the equation of cable profile be

$$y = ax^2 + bx + c$$

with A as origin and axes as shown below



Boundary conditions are

(i) at $x = 0$; $y = 0$

$\Rightarrow$ $y = ax^2 + bx + c$

$0 = a.(0)^2 + b.(0) + c$

$\therefore$ $c = 0$

(ii) at $x = 250$; $y = 0$

$y = ax^2 + bx$

$\Rightarrow$ $0 = a.(250)^2 + b.(250)$ $[\because c = 0]$ ...(iii)

(iii) at $x = 125$; $y = 10$

$y = ax^2 + bx$

$\Rightarrow$ $10 = a.(125)^2 + b.(125)$ ...(iv)

Solving equation (iii) and (iv), we get

$a = -6.4 \times 10^{-4}$

$b = 0.16$

$\therefore$ Cable profile is

$y = (-6.4 \times 10^{-4})x^2 + 0.16x$ ...(v)

To find sag at 50m from one end, we have to put $x = 50$ in equation (v)

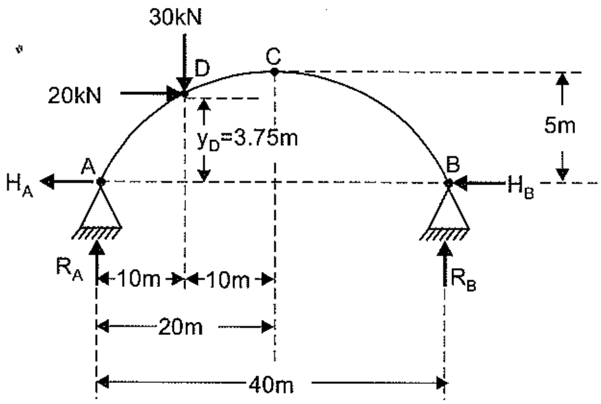
$\therefore$ $y = (-6.4 \times 10^{-4}).50^2 + 0.16 \times 50 = 6.4\text{m}$

$\therefore$ Sag at 50m from one end will be 6.4m.

Q-6: ***A symmetrical three hinged parabolic arch has 40 m span and 5 m rise. A vertical downward load of 30 kN and a horizontal load of 20 kN (acting in the right hand side direction) act at one quarter span from left hand support. Determine reactions at the supports.***

[5 Marks, ESE-2015]

Sol:



Let equation of parabola be $x^2 = -ky$ (taking C as centre i.e. $C \equiv (0, 0)$)

$$\therefore \quad B \equiv (20, 5)$$

$$\Rightarrow \quad 400 = -k(-5) \Rightarrow k = 80$$

$$\Rightarrow \quad x^2 = -80y$$

For pt. D,

$$x = -10$$

$$\Rightarrow \quad (-10)^2 = -80y$$

$$y = 1.25 \text{ m}$$

$$y_D = 5 - 1.25 = 3.75 \text{ m}$$

Taking moment about C from left

$$\Sigma M_C) = 0$$

$$\Rightarrow \quad R_A \times 20 + H_A \times 5 - 30 \times 10 - 20 \times 1.25 = 0$$

$$\Rightarrow \quad 4R_A + H_A = 65.0 \quad \text{...(i)}$$

$$\Sigma M_C) = 0 \quad \text{(from right)}$$

$$-R_B \times 20 + H_B \times 5 = 0$$

$$\Rightarrow \quad H_B = 4R_B \quad \text{...(ii)}$$

$$\Sigma F_H = 0$$

$$\Rightarrow \quad H_A + H_B = 20$$

$$\Rightarrow \quad (65 - 4R_A) + 4R_B = 20$$

$$\Rightarrow \quad 4R_A - 4R_B = 45 \quad \text{...(iii)}$$

$$\Sigma F_V = 0$$

$$\Rightarrow \quad R_A + R_B = 30$$

$$\Rightarrow \quad 4R_A + 4R_B = 120 \quad \text{...(iv)}$$

Solving (iii) and (iv), we get

$$\Rightarrow \quad R_A = 20.625 \text{ kN}$$

$$\Rightarrow \quad R_B = 9.375 \text{ kN}$$

$$\Rightarrow \quad H_B = 37.5 \text{ kN}$$

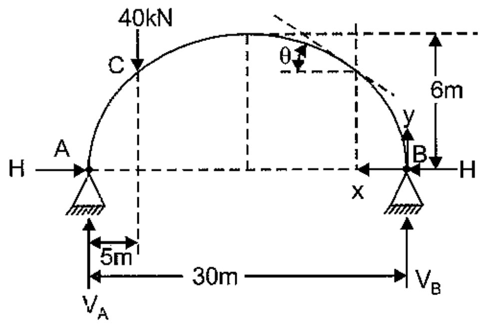
$$H_A = -17.5 \text{ kN}$$

Q-7: ***A two hinged symmetrical parabolic arch of span 30 m and central rise 6 m carries a point load of 40 kN at a distance of 5 m from the left support. find the horizontal thrust at each support. Also find the maximum bending moment.***

[10 Marks, ESE-2016]

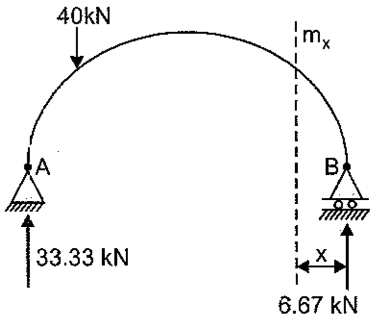
Sol: Degree of static indeterminacy $D_s = 1$

Taking horizontal reaction at B (H) as redundant.



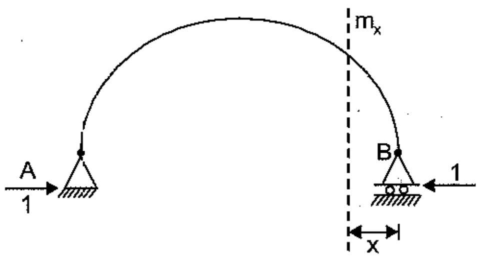
Forces in structure without redundants:

Moment at distance x from B = M_x



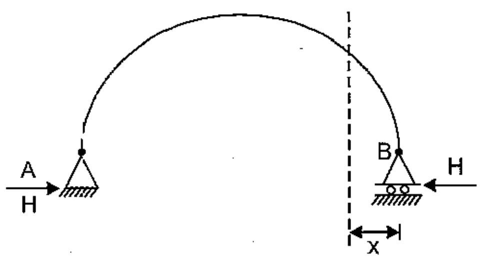
Applying unit load at B in direction of redundant.

Moment at distance x from B = m_x.



Applying redundant reaction (H)

Moment at distance x from B = $H \times m_x$



Let equation of parabolic arch be $y = ax(30-x)$; $y = 6$ at $x = 15$

$$\Rightarrow \qquad a = \frac{6}{15\times 15}$$

$$\therefore \qquad y = \frac{2}{75}x(30-x)$$

$$\tan\theta = \frac{dy}{dx} = \frac{2}{75}(30-2x)$$

$$\sec^2\theta = 1+\tan^2\theta = 1+\left(\frac{2}{75}\right)^2(30-2x)^2$$

$$\cos^2\theta = \frac{5625}{5625+4(30-2x)^2}$$

$$\sec\theta = \frac{\sqrt{5625+4(30-2x)^2}}{75}$$

From principle of virtual work,

Total internal virtual work = Total external virtual work.

$$\Rightarrow \qquad \int_0^l m_x \times \frac{M_x ds}{EI} + \int_0^l \frac{m_x \times (H\times m_x)ds}{EI} = 1 \times 0$$

$$\Rightarrow \qquad H = \frac{-\int_0^l m_x \frac{M_x ds}{EI}}{\int_0^l \frac{m_x^2 ds}{EI}} = \frac{-A/EI}{B/EI}$$

$$m_x = -1\times y = -1\times\frac{2}{75}x\times(30-x)$$

$$M_x = 6.67x \text{ for } 0 \le x \le 25$$

$$= 33.33\ (30-x) \text{ for } 25 < x \le 30$$

$$A = \int_0^{30} m_x M_x \times dx \sec\theta$$

$$A = \int_0^{25}\left[\frac{-2}{75}\times x\times(30-x)\right][6.67x]\frac{\sqrt{5625+4(30-2x)^2}}{75}dx$$

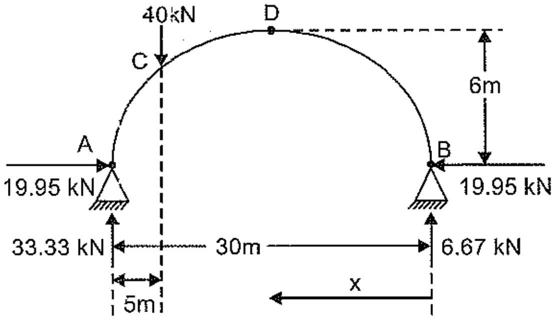
$$+\int_{25}^{30}\left[\frac{-2}{75}\times x\times(30-x)\right][33.33(30-x)]\times\frac{\sqrt{5625+4(30-2x)^2}}{75}dx$$

$$A = -10856.81 - 1135.46 = -11992.27$$

$$B = \int_0^{30}\left[\frac{-2}{75}\times x\times(30-x)\right]^2\times\frac{\sqrt{5625+4(30-2x)^2}}{75}dx$$

$$B = 601.10$$

$$\Rightarrow \qquad H = \frac{-(-11992.27)}{601.10} = 19.95\text{kN} = H$$



Bending moment at distance x from B,

$$M_x = 6.67x - 19.95 \times y = 6.67x - 19.95 \times \frac{2}{75} 5x \times (30 - x) \qquad [\text{For } x \le 25]$$

$$M_x = 6.67x - 19.95 \times \frac{2}{75} \times x \times (30 - x) - 40(x - 25) \qquad [\text{For } 25 \le x \le 30]$$

For maximum M_x, $\frac{dM_x}{dx} = 0$

(a) For $x \le 25$

$$6.67 - 19.95 \times \frac{2}{75} \times (30 - 2x) = 0$$

$\Rightarrow$ x = 8.73 m

$$M_{x\,max} = 6.67 \times 8.73 - 19.95 \times \frac{2}{75} \times 8.73 \times (30 - 8.73) = 58.23 - 98.79 = -40.56 \text{ kNm}$$

(b) For $25 < x \le 30$

$$6.67 - 19.95 \times \frac{2}{75} \times (30 - 2x) - 40 = 0$$

$\Rightarrow$ x = 46.33 m > 30 m (not OK)

(c) For x = 25m

$$M_c = 33.33 \times 5 - 19.95 \times \frac{2}{75} \times 5 \times (30 - 5)$$

M_c = 166.65 – 66.5

M_c = 100.15 kNm

$\therefore$ Horizontal thrust = 19.95 kN

Maximum BM = 100.15 kNm

Q-8: ***A suspension cable of 160 m span and 16 m central dip carries a load of 1/2 kN per linear horizontal meter. Calculate the maximum and minimum tension in the cable. Also find the horizontal and vertical forces in each pier under the following alternate conditions.***

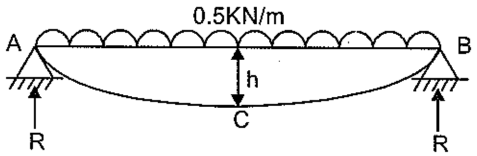
(i) If the cable passes over the frictionless pulley on the top of the piers.

(ii) If the cable is firmly clamped to saddles carried on frictionless roller on the top of the piers.

In each case the backstay is inclined at 30° to the horizontal.

[20 Marks, ESE-2019]

Sol:



$$\text{Reaction (R)} = \frac{0.5\times160}{2} = 40\text{kN}$$

Using, $$T_{min} = \frac{wL^2}{8h} = \frac{0.5\times160^2}{8\times16} = 100\text{kN}$$ [Minimum tension occur at mid span]

$$\text{Maximum tension } (T_{max}) = \sqrt{100^2+40^2} = 107.703\text{kN}$$ [Maximum tension occur at supports]

Assuming, shape to be parabola.

Taking origin at top of pier

Then, $$y = \frac{4h}{L^2}x(L-x)$$

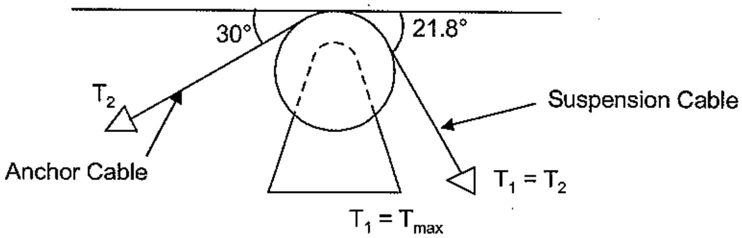
$$\text{Slope of cable} = \frac{dy}{dx} = \frac{4.h}{L^2}(L-2x)$$

$$\text{Slope of cable at support } \frac{dy}{dx} = \frac{4.h}{L} \text{ at } x = 0$$

$$\tan\theta = \frac{4\times16}{160} = 0.4$$

$\therefore$ $$\theta = 21.80°$$

Case-a: Cable passes over frictionless pulley on the top of piers (tension in cable on two side will be same).



$$\text{Vertical pressure on pier (V)} = T_{max}(\sin 21.8° + \sin 30°)$$

$$= 107.703(\sin 21.8° + \sin 30°)$$

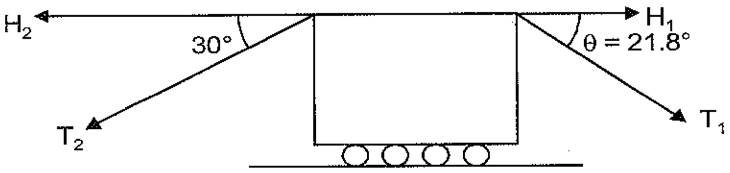
$$= 93.85 \text{ kN}$$

$$\text{Horizontal thrust on pier (H)} = T_{max}(\cos 21.8° - \cos 30°)$$

$$= 107.703(\cos 21.8° - \cos 30°)$$

$$= 6.73 \text{ KN}$$

Case-b: When cable is firmly clamped to saddles carried on frictionless roller on the top of pier. (horizontal force will be same)



$$T_1 \neq T_2$$

Here, $$H_1 = H_2$$

or, $$T_1 \cos 21.8° = T_2 \cos 30°$$

$$T_2 = \frac{T_1 \cos 21.8°}{\cos 30°} = \frac{107.703 \times \cos 21.8°}{\cos 30°}$$

$$= 115.47 \text{ kN}$$

$$H_1 = H_2 = T_2 \cos 30° = 100 \text{ kN}$$

$$\text{Vertical pressure on pier} = T_1 \sin 21.8° + T_2 \sin 30°$$

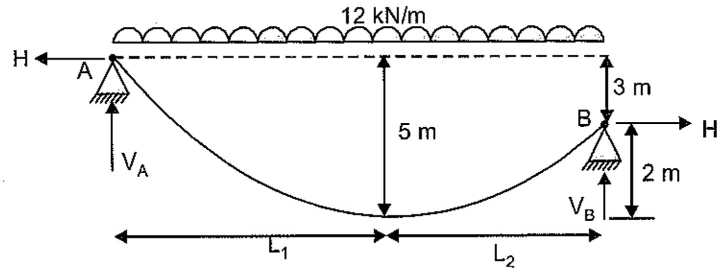
$$= 107.703 \times \sin 21.8° + 115.47 \times \sin 30°$$

$$= 97.732 \text{ kN}$$

Q-9: ***A cable of uniform cross-section hangs between two points A and B, which are 150 m apart. The end 'A' of the cable is 3 m above the other end of the cable. The sag of the cable measured from 'B' is 2 m. If the cable carries a UDL of 12 kN/m, determine the maximum tension in the cable. Also find the horizontal pull.***

[12 Marks, ESE-2020]

Sol: Given data



Using $$L_1 = L\left(\frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right) = 150\left(\frac{\sqrt{5}}{\sqrt{5}+\sqrt{2}}\right)$$

$$L_1 = 91.886 \text{ m}$$

$$L_2 = 150 - 91.886 = 58.114 \text{ m}$$

Again using $$H = \frac{w_0 \ell_1^2}{2h_1} = \frac{w_0 \ell_2^2}{2h_2}$$

$$= \frac{12 \times 91.886^2}{2 \times 5} = 10131.67 \text{ kN}$$

For vertical reaction at A.

$$\Sigma M_B = 0$$

$$\Rightarrow \qquad V_A \times 150 - H \times 3 = 12 \times \frac{150^2}{2}$$

$$\Rightarrow \qquad V_A \times 150 - 10131.67 \times 3 = 135000$$

$$\boxed{V_A = 1102.62 \text{ kN}}$$

and

$$V_B = 12 \times 150 - 1102.63$$

$$\boxed{V_B = 697.36 \text{ kN}}$$

$$\text{Maximum tension} = \sqrt{V_A^2 + H^2}$$

$$= \sqrt{1102.63^2 + 10131.67^2}$$

$$\boxed{T_{max} = 10191.49 \text{ kN}}$$

So,

$$\text{maximum tension} = 10191.49 \text{ kN}$$

$$\text{Horizontal pull} = 10131.67 \text{ kN}$$

UNIT-3

STRUCTURAL DYNAMICS

SYLLABUS

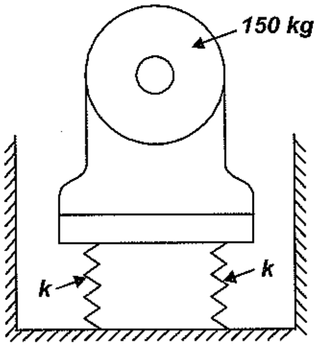
Free and Forced vibrations of single degree and multi degree freedom system.

CONTENTS

CHAPTER 1

STRUCTURAL DYNAMICS

Q-1: ***A motor of 150 kg mass is supported by four springs as shown in figure. Each of the spring has a stiffness of 120 kN/m. The unbalance of the rotor is equivalent to a mass of 40 g located at 150 mm from the axis of rotation. The motor is constrained to move vertically.***



k = 120 kN/m

Find:

(i) ***The speed of motor at which resonance will occur***

(ii) ***The amplitude of vibration of the motor when the speed is 1000 rev./m.***

[12 Marks, ESE-2018]

Sol: Given that:

Mass of motor (m) = 150 kg

Stiffness of each spring (K) = 120 kN/m

Mass of rotor = 40 gm

Distance of rotor from the axis of rotation (r) = 150 mm

To find out:

(i) Speed of motor at which resonance will occur.

(ii) Amplitude of vibration of the motor when the speed is 1000 rev/m

The given problem is the case of undamped vibration.

(i) Calculation for the speed of motor at which resonance will occur i.e. resonance speed

The resonance speed is equal to the natural circular frequency (ω_n) of the free vibration of the motor.

Then, Mass of motor (m) = 150 kg (given)

$$\text{Equivalent spring stiffness } (K_{eq}) = 4 \times K$$
$$= 4 \times 120 \text{ kN/m}$$
$$= 480 \text{ kN/m}$$

$\therefore$ Natural circular frequency or resonance speed

$$(\omega_n) = \sqrt{\frac{K_{eq}}{m}} = \sqrt{\frac{480\times10^3}{150}}$$

$$\omega_n = 56.57 \text{ rad/s} = 540.20 \text{ rpm}$$

Hence, the speed of motor at which resonance will occur is 56.57 rad/s or 540.20 rpm.

(ii) Calculation for amplitude of vibration at 1000 rev/min

$$\text{Angular velocity of motor } (\omega) = 1000 \text{ rev/min}$$

$$= \frac{1000\times2\pi}{60}\text{ rad/sec} = 104.72 \text{ rad/sec}$$

$$\text{Mass Causing Force } (m) = 0.04 \text{ kg}$$

The magnitude of the centrifugal force due to the unbalance of the rotar is

$$P_o = ma_n = mr\omega^2$$

$$= (0.04 \text{ kg})\,(15 \text{ cm})\times10^{-2}\,(104.72 \text{ rad/s})^2$$

$$\boxed{P_o = 65.79 \text{ N}}$$

The static deflection that would be caused by a constant load P_o is

$$\frac{P_o}{K} = \frac{65.79\text{N}}{(480\times10^3)\text{N/m}} = 0.137\times10^{-3}\text{ m}$$

The forced circular frequency (ω) of the motor is the angular velocity of the motor,

$$\boxed{\omega_f = \omega = 104.72 \text{ rad/sec}}$$

Amplitude is given by:

$$\boxed{u_o = \frac{(P_o/K)}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}}$$

$$= \frac{0.137\times10^{-3}}{1-\left(\frac{104.72}{56.57}\right)^2}$$

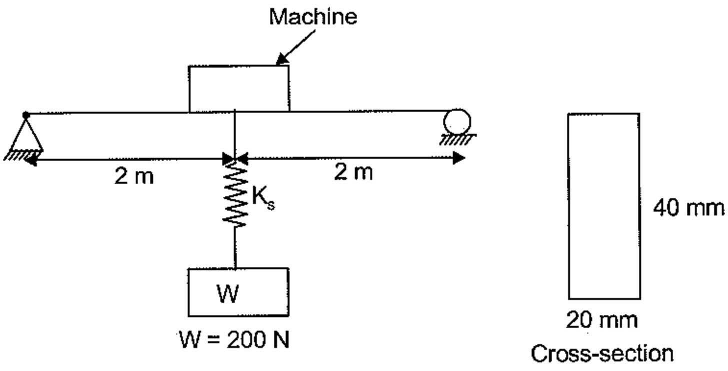
$$\boxed{u_o = -0.0565\times10^{-3}\text{ m}}$$

Note: Since, $\omega_f > \omega_n$, the vibration is 180° out of phase with the centrifugal force due to the unbalance of the rotar.

Q-2: ***A machine is mounted at the centre of a simple supported beam that can exert a harmonic load F(t) = 20 sin (0.12 t) kN in vertical direction. The length of beam is 4 m and its cross-section is uniform throughout. Cross-section of beam : width 20 mm and depth 40 mm. A weight W = 200 N is suspended from the centre of the beam by a spring of spring constant K_s = 40 N/m. Determine the natural frequency of the weight W. Neglect mass of the beam and weight of machine. E = 2×10^5 MPa.***

[12 Marks, ESE-2020]

Sol: Given,



$$\text{Spring constant } (K_S) = 40 \text{ N/m}$$

$$E = 2 \times 10^5 \text{ MPa.}$$

$$F(t) = 20 \sin (0.12t)$$

$$\text{Stiffness of beam is given as } (K_b) = \frac{48\,EI}{L^3}$$

Hence,
$$I = \frac{1}{12} \times 20 \times 40^3 = 106666.67 \text{ mm}^4$$

$\therefore$
$$K_b = \frac{48 \times 2 \times 10^5 \times 106666.67}{(4 \times 10^3)^3}$$

$$= 16 \text{ N/mm}$$

$$= 16000 \text{ N/m}$$

The two springs beam and spring are in series

Equivalent spring stiffness

$$\frac{1}{K_e} = \frac{1}{k_b} + \frac{1}{K_s}$$

$$\frac{1}{K_e} = \frac{1}{16000} + \frac{1}{40}$$

$\therefore$
$$K_{eq} = 39.9 \text{ N/m}$$

$$\text{Natural frequency } (\omega_n) = \sqrt{\frac{k_{eq}}{m}}$$

$$= \sqrt{\frac{39.9}{\frac{200}{9.81}}}$$

$$= 1.398 \text{ rad/s}$$

Note: If there is the case of forced vibration, then frequency is equal to that of the force itself, i.e., 0.12 rad/sec.

UNIT-4

DESIGN OF STEEL STRUCTURE

SYLLABUS

Principles of Working Stress methods, Design of tension and compression members. Design of beams and beam column connections, built-up sections. Girders, Industrial roofs, Principles of Ultimate load design.

CONTENTS

CHAPTER 1

GENERAL DESIGN REQUIREMENTS

Q-1: ***What are the various limit states of design for a steel structure as per IS : 800-2007?***

[5 Marks, ESE-2014]

Sol: In limit state design, we prefer to use the term 'limit states', rather than 'failure'. Thus a limit state is a state of impending failure, beyond which a structure ceases to perform its intended function satisfactorily.

Various limit states considered by 15:800-2007 are as follows:

(i) Ultimate limit state

- Strength (including yielding, buckling and transformation into a mechanism)
- Stability against overturning and sway
- Failure due to excessive deformation of rupture
- Fracture due to fatigue
- Brittle fracture

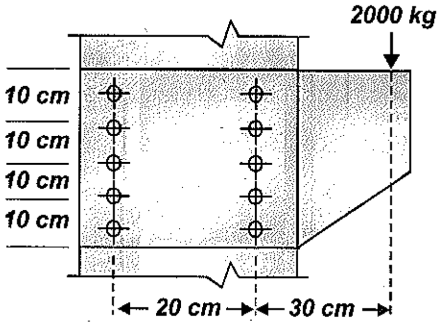
(ii) Serviceability limit state

- Deformation and deflection
- Vibration (e.g., wind-induced oscillations, floor vibration, etc.)
- Repairable damage due to fatigue (cracking)
- Corrosion and durability
- Fire

CHAPTER 2

CONNECTIONS

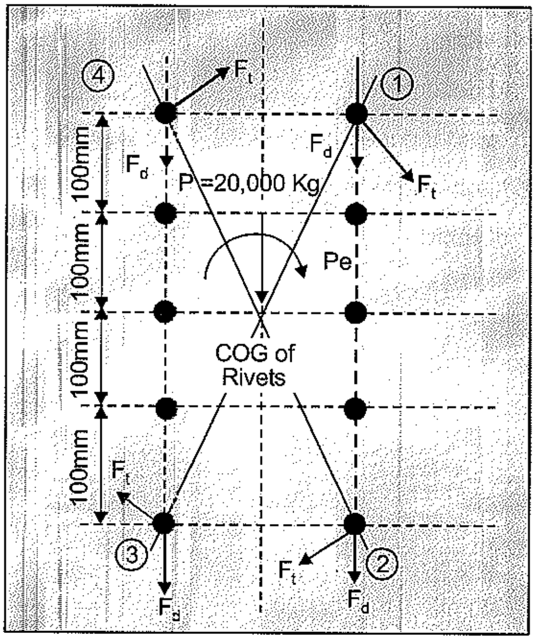
Q-1: ***A load of 20,000 kg is carried by a plate bracket rivetted to a column as shown in the fig Calculate the maximum force taken up by any rivet.***



[10 Marks, ESE-1996]

Sol: Assuming the diameter of all the rivets to be same.

Drawing diagram:



- Every bolt will experience direct shear Force (F_d) and torsional shear Force (F_t).
- On every rivet, the direction of F_d will be downward while the direction of F_t will be perpendicular to radial distance from the C.O.G as shown above.
- If all the rivets are of same diameter then most critical rivet is the one which is farthest from the C.G.
- If there are more than one rivets equally distant from C.O.G, then the rivet in which the angle between F_d and F_t is lesser will be critical.

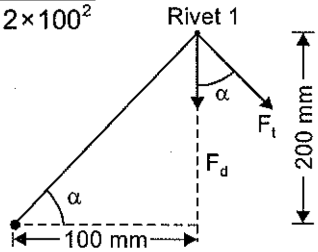
- Hence among the 1, 2, 3 and 4, the maximum Force will act on (1) and (2).

$$F_d = \frac{P}{n} = \frac{20000}{10} = 2000 \text{ kg}$$

where n = number of rivets = 10

and
$$F_t = \frac{Per}{\Sigma r^2} = \frac{20000 \times (100+300) \times \sqrt{200^2+100^2}}{4(200^2+100^2)+4(100^2+100^2)+2\times 100^2}$$

$$= 5962.848 \text{ Kg}$$



$$\tan\alpha = \frac{200}{100}$$

$$\therefore \quad \cos\alpha = \frac{100}{\sqrt{200^2+100^2}} = 0.447$$

$\therefore$ Resultant force on (1) or (2) rivet $= \sqrt{F_d^2 + F_t^2 + 2F_d f_t \cos\alpha}$

$$= \sqrt{2000^2 + 5962.85^2 + 2\times 2000 \times 5962.85 \times 0.447}$$

$$= 7086.406 \text{ Kg}$$

Q-2: ***Design a riveted splice for a tie of a steel bridge, 20 cm wide, 20 mm thick, carrying an axial tensile force of 50,000 kg. Use 12 mm thick cover plates, 22 mm dia rivets. Permissible stresses:***

Tension in plates = 1500 kg/cm²

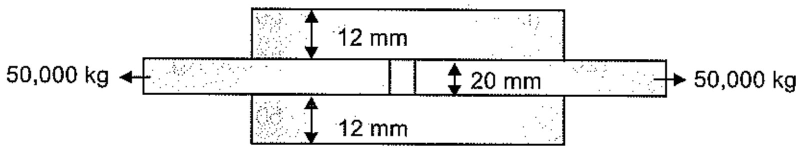
Shear in rivets = 1000 kg/cm²

Bearing in rivets = 3000 kg/cm²

Give a neat sketch of the arrangement.

[15 Marks, ESE-1997]

Sol: Designing the splice as a double cover butt joint as it will give maximum efficiency.



Thickness of cover plate = 12 mm

Thickness of main plate = 20 mm

width of cover plate = 200 mm

Nominal dia of rivets = 22 mm

$\therefore$ Gross dia = 22 + 1.5 = 23.5 m = 2.35 cm

Calculation of number of rivets required,

Shear strength of rivet in double shear $= \left(\frac{\pi}{4} \times d_h^2\right) \times 2 \times \sigma_S = (0.7854 \times (2.35)^2 \times 2 \times 1000)$

$$= 8674.723 \text{ Kg}$$

Bearing strength of rivet $= d_h \times t \times \sigma_{br}$

where, t = {min (combined thickness of two cover plate, main plate thickness)}

t = {min (12 + 12, 20)}

t = 20 mm = 2 cm

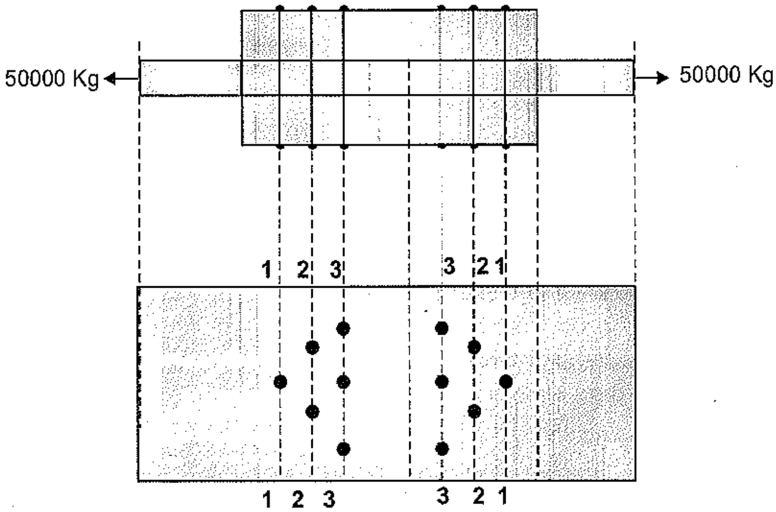
∴ Bearing strength = (2.35 × 2 × 3000) N = 14100 Kg

∴ R_v (rivet value) = min {shear strength, bearing strength}

= 8674.723 Kg

∴ Number of rivet required = $\frac{50000}{8674.723} = 5.76 \cong 6$ rivets

6 numbers of rivets can be arranged in various patterns. However, diamond pattern is generally most economical.



Check for safety of joint in tearing:

For main plate:

At sec (1)-(1)

$$(B - d_h) \times t \times \sigma_{at} \geq 50000 \text{ Kg}$$

$$(20 - 2.35) \times 2 \times 1500 \geq 50000 \text{ Kg}$$

⇒ $52950 \geq 50000$ Kg (OK)

Hence sec (1)-(1) for main plate is safe in tearing.

At sec (2)-(2)

$$R_v + (B - 2d_h)t + \sigma_{at} \geq 50000 \text{ Kg}$$

$$8674.723 + (20 - 2 \times 2.35) \times 2 \times 1500 \geq 50000 \text{ Kg} \quad \text{(OK)}$$

⇒ $54574.723 \geq 50000$ Kg

Hence section (2)-(2) for main plate is safe in tearing.

At sec (3)-(3)

$$(B - 3d_h)\, t\, \sigma_{at} + 3R_v \geq 50000 \text{ Kg}$$

$$(20 - 3\times2.35) \times 2\times1500 + 3\times8674.723 \geq 50000 \text{ Kg}$$

$$64874.169 \geq 50000 \text{ Kg} \quad \text{(OK)}$$

Section (3)-(3) for main plate is safe in tearing.

For cover plate: For cover plate the most critical section is 3-3

Assuming width of cover plate to be same as that for main plate.

i.e., $(B - 3d_h) \times t \times \sigma_{at}$ should be > 50000 Kg

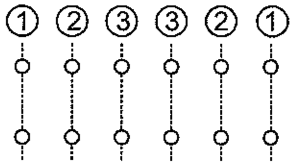
$\Rightarrow$ $(20 - 3 \times 2.35) \times 2.4 \times 1500 \geq 50000$ Kg

$\Rightarrow$ $46620 \text{ Kg} \leq 50000 \text{ Kg}$

Hence cover plate is not safe.

Note that as section (3)-(3) is the 1st line of rivet encountered for cover plate, we do not need to add rivet values for tearing strength of cover plate.

Selecting chain rivet



Section (1 – 1) of main is plate checked for the safety

$(B - 2d_h) \times t \times \sigma_{at}$ should be ≥ 50000 Kg

$\Rightarrow$ $(20 - 2 \times 2.35) \times 20 \times 1500 \geq 50000$

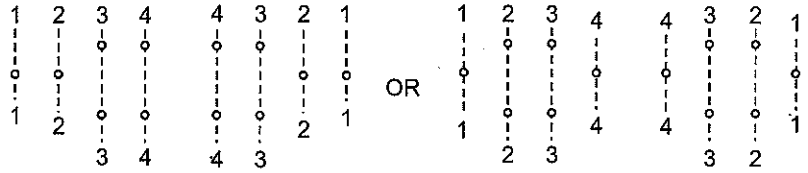
$\Rightarrow$ $45900 \geq 50000$

$\Rightarrow$ Chain riveting is not safe for main plate.

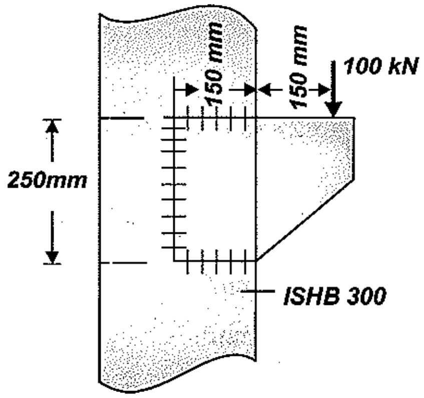
Conclusion: At section 1 –1 we can apply one rivet

and At section 3 – 3 we can apply two rivets

So, the two possible design is as follows:

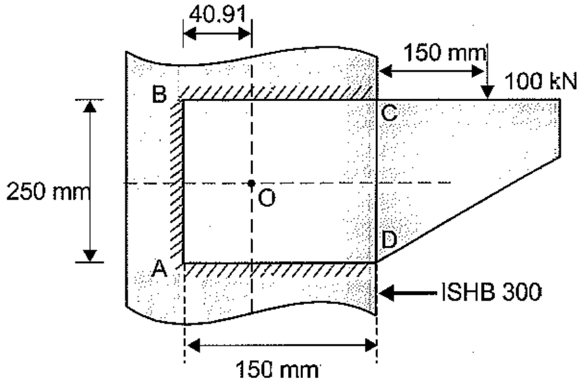


Q-3: ***A welded bracket connects a plate to the column flange shown in figure. Determine the size of the weld if the allowable stress in the weld is 110 N/mm²***



[12 Marks, ESE-1998]

Sol:



Allowable stress in the weld is 110 N/mm^2

Let the size of weld = S

$\therefore$ Effective throat thickness = 0.7S = t.

$$\text{Shear stress due to direct loading } (f_d) = \frac{\text{Direct load}}{\text{Area}} = \frac{100\times10^3}{(2\times150+250)\times t}$$

$$\therefore \quad f_d = \left\{\frac{100\times1000}{550\times t}\right\}$$

and $$f_d = \frac{181.82}{t}\ \text{N/mm}^2$$

shear stress due to torsional moment (f_t),

$$f_T = \frac{(Pe).r}{J}$$

Step 1: Locate the centroid: (consider A as origin)

$$\bar{x} = \frac{250\times t\times0+(150\times t\times75)\times2}{250t+(150t)\times2} = 40.91\text{mm}$$

From symmetry, $\bar{y}$ will be midway between AD & BC

Step 2: Determination of e = 300 − 40.91 = 259.09 mm

Step 3: Determination of r_{max} = i.e, OC or OD in our case,

$$OC = \sqrt{125^2+(150-40.91)^2} = 165.91\,\text{mm}$$

Step 4: Calculation of J,

$$J = I_{xx}+I_{yy}$$

$$I_{xx} = t\times\frac{250^3}{12}+2\times150\times t\times125^2 = 5t(5989583.33)\text{mm}^4$$

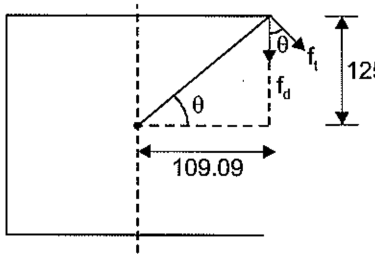
$$I_{yy} = 250\times t\times40.91^2+\frac{2\times t\times150^3}{12}+2\times150\times t\times(75-40.91)^2$$

$$= t(1329545.455)\text{mm}^4$$

$$\therefore \quad J = t(7319128.785)\text{mm}^4.$$

$$\therefore \quad f_t = \frac{P\times e\times r}{J} = \frac{100\times259.09\times165.91\times10^3}{(7319128.785)t} = \frac{587.30}{t}\text{N/mm}^2$$

For finding the resultant force we have to find the angle between f_T and f_d.



$$\tan\theta = \frac{125}{109.09}$$

$$\Rightarrow \quad \cos\theta = 0.6575$$

$$\therefore \quad f_R = \sqrt{f_d^2 + f_T^2 + 2f_d.f_T \cos\theta}$$

$$= \frac{1}{t}\sqrt{(181.82)^2 + (587.3)^2 + 2(181.82)(587.3)\times 0.6575}$$

$$= \frac{720}{t} \text{N/mm}^2$$

For section to be safe

$$\Rightarrow \quad \frac{720}{t} \leq 110$$

$$\Rightarrow \quad t \geq 6.545$$

$$\Rightarrow \quad 0.7s \geq 6.545$$

$$s \geq 9.35 \text{ mm}$$

Let us adult size of weld as 10 mm.

Q-4: ***In an industrial shed an edge support consisting of 2L – 110 × 110 mm is to be connected to a 16 mm gusset plate for a tensile load of 650 kN. Design the moment free welded connection. The distances of centroid of angle from the backside of legs are $C_x = C_y = 30.9$ mm. The strength of weld per mm thickness per mm length is 76 MPa.***

[20 Marks, ESE-2003]

Sol: Total force to be carried = 650 kN. Each angle will carry 325 kN force.

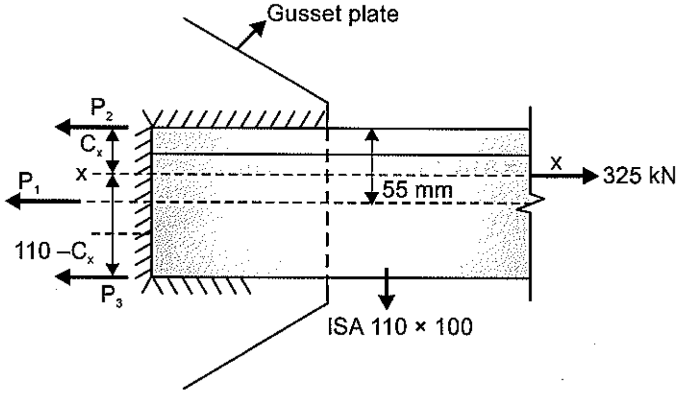


Fig (A)

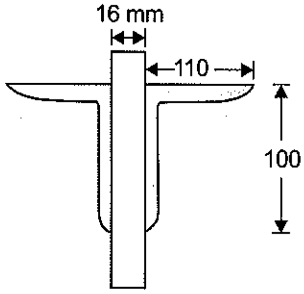


Fig (B)

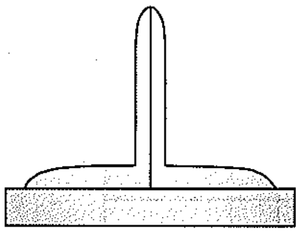


Fig (C)

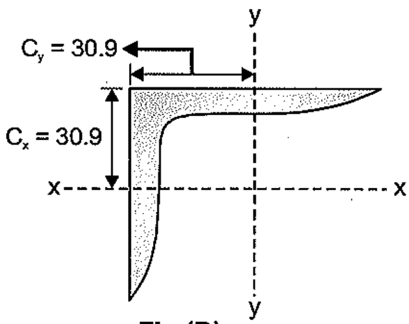


Fig (D)

Strength of weld per mm thickness and per mm length = 76 N/mm^2

The two angles attached to the gusset plate in two ways as shown in figure (B) and (C). To create moment free connection, the figure B is the correct one.

Assuming the thickness of angle 8 mm, the minimum size of weld = 5 mm because thickness of thicker plate 16 mm, which lies in the range of 10 – 20

Providing welded connection as shown in figure A, in which force resisted by side fillet needs are P_2 and P_3 and that resisted by end fillet weld equal to P_1.

$$P_1 \div P_2 + P_3 = 325$$

$$\Rightarrow \qquad \frac{\left[110 \times \frac{5}{\sqrt{2}} \times 76\right]}{1000} + P_2 + P_3 = 325$$

$$29.557 + P_2 + P_3 = 325$$

$$\Rightarrow \qquad \boxed{P_2 + P_3 = 295.443 \text{ kN}} \qquad \text{...(i)}$$

Taking moment about X-X axes

$$P_1 \times (55 - C_x) + P_3(110 - C_x) - P_2(C_{xx}) = 0$$

$$\Rightarrow \qquad 29.557\ (55 - C_x) + P_3\ (110 - C_x) - P_2\ (C_x) = 0$$

$$\Rightarrow \qquad 712.324 + 79.1\ P_3 - 30.9\ P_2 = 0$$

$$\therefore \qquad \boxed{30.9\,P_2 - 79.1\,P_3 = 712.324} \qquad \text{...(ii)}$$

By solving equation (1) and (2), we get

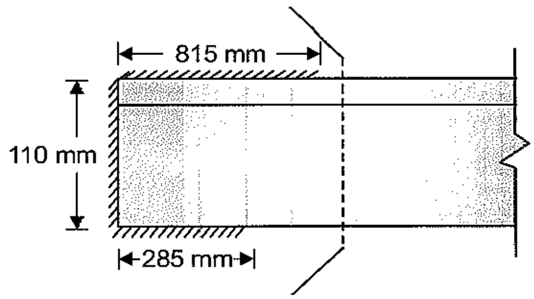
$$P_2 = 218.926 \text{ kN}$$

$$P_3 = 76.517 \text{ kN}$$

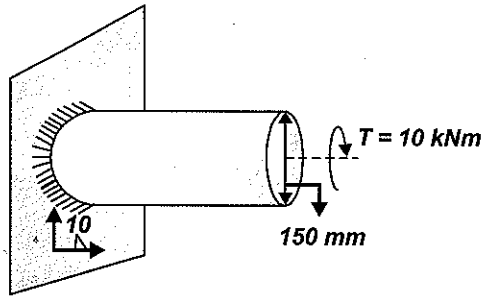
Hence length required, $\quad l_2 = \dfrac{P_2 \times 1000}{76 \times \frac{5}{\sqrt{2}}} = 814.758 \text{ mm} = 815 \text{ mm}$

[Thickness of weld is taken as throat thickness = $5/\sqrt{2}$]

and $\quad l_3 = \dfrac{P_3 \times 1000}{76 \times \frac{5}{\sqrt{2}}} = \dfrac{76.517 \times 1000}{76 \times \frac{5}{\sqrt{2}}} = 284.767 \text{ mm} = 285 \text{ mm}$

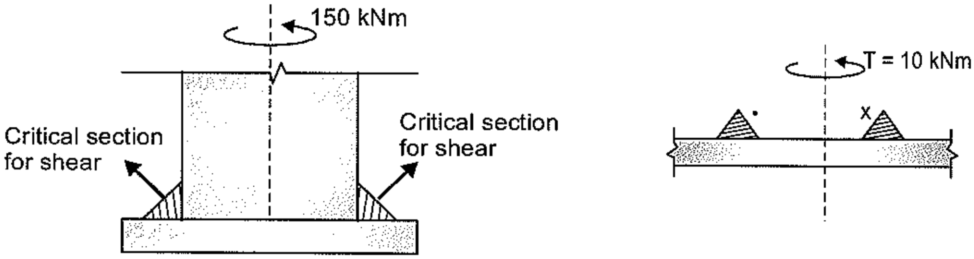


Q-5: ***A circular shaft of 150 mm is welded to a rigid plate by an external all round fillet weld of size 10 mm. If a torque of 10 kNm is applied to the shaft, find the max stress in the weld.***



[10 Marks, ESE-2004]

Sol: We have to calculate max stress in weld.



Polar moment of inertia for resisting section (exact calculation)

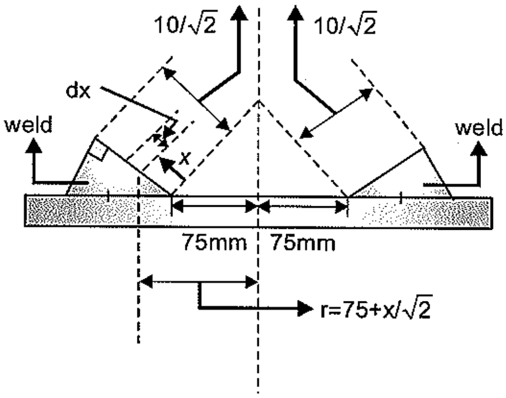
$$I_P = \int 2\pi r\, dx \times r^2$$

$$= 2\pi \int_0^t \left(75 + \frac{x}{\sqrt{2}}\right)^3 dx$$

$$= \left. \frac{2\pi\left(75 + \frac{x}{\sqrt{2}}\right)^4}{4} \times \sqrt{2} \right|_0^t$$

$$= \frac{2\pi \times \sqrt{2}}{4}\left[\left(75 + \frac{10}{2}\right)^4 - (75)^4\right]$$

$$= 20.7025 \times 10^6 \text{ mm}^4$$



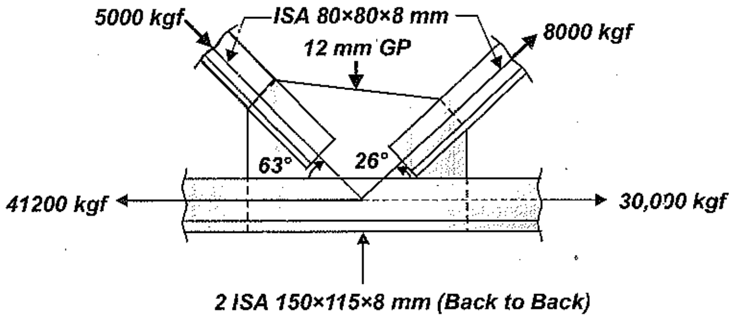
$$\text{Max shear stress} = \frac{Tr}{I_P} = \frac{10 \times 10^6 \text{ Nmm} \times \left(75 + \frac{10}{\sqrt{2} \times \sqrt{2}}\right)}{20.7025 \times 10^6} = 38.64 \text{ N/mm}^2$$

Alternate approach

Max shear stress in weld can also be approximately calculated by treating the weld as a line area.

$$\tau_{max} = \frac{Tr}{J} = \frac{10 \times 10^6 \times \frac{150}{2}}{\pi \times 150 \times t \times (75)^2} = 40.02 \text{ N/mm}^2$$

Q-6: ***Determine the number and pattern of 20 mm diameter power driven shop rivets for the truss connection as shown in the figure below. Sketch the arrangement of rivets.***

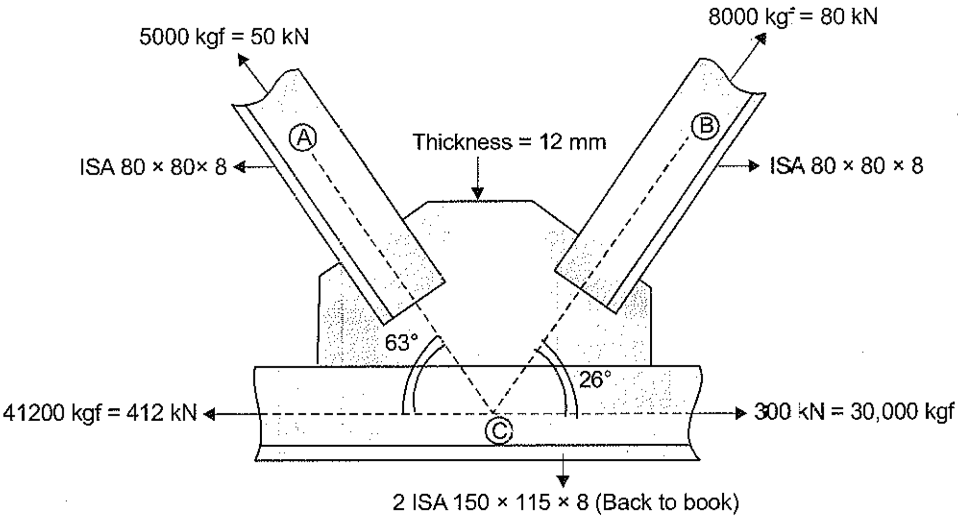


Assume the stresses as follows:

$$\left.\begin{array}{l}\textit{Shear} \ \text{®}\ 1025 \text{ kgf/cm}^2 \\ \textit{Bearing} \ \text{®}\ 2360 \text{ kgf/cm}^2\end{array}\right\}(\textit{Power driven shop rivets})$$

[12 Marks, ESE-2007]

Sol:



$$\sigma_s^{rivet} = 102.5 \text{ N/mm}^2$$

$$\sigma_{br}^{rivet} = 236 \text{ N/mm}^2$$

Nominal diameter of rivet = 20 mm

Gross diameter of rivet = 21.5 mm

$\therefore$ Shearing strength of one rivet in single shear $= \frac{\pi}{4} \times 21.5^2 \times \frac{102.5}{1000} = 37.21$ kN

and bearing strength of one rivet $= \frac{236 \times 8 \times 21.5}{1000} = 40.59$ kN

Hence rivet value of rivets used for section A and B is = 37.21 kN

Hencenumber of rivets required for section A $= \frac{50}{37.21} = 1.34 \cong 2$ rivets

and number of rivets required for section B $= \frac{80}{37.21} = 2.149 \cong 3$ rivets

For section C, rivet value will be changed because the rivets are in double shear and the thickness of thinner member will be 12 mm,

$$\text{Shear strength of one rivet in double shear} = \frac{\pi}{4} \times 21.5^2 \times 2 \times \frac{102.5}{1000} = 74.42 \text{ kN}$$

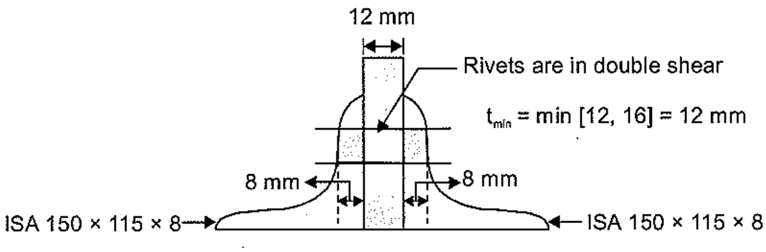
$$\text{Bearing strength of one rivet} = \frac{236 \times 21.5 \times 12}{1000} = 60.89 \text{ kN}$$

Hence rivet value of rivet attached in section (C) = 60.89 kN

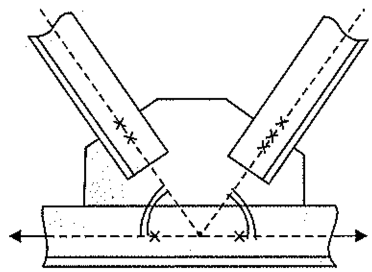
$$\therefore \quad \text{Number of rivet required} = \frac{412 - 300}{60.89} = 1.83 \text{ @ } 2$$

$$\text{Pitch of rivet} = 2.5 \times d_m = 2.5 \times 20 = 50 \text{ mm}$$

$$\text{Edge distance} = 2 \times d_n = 2 \times 21.5 = 43 \text{ mm}$$



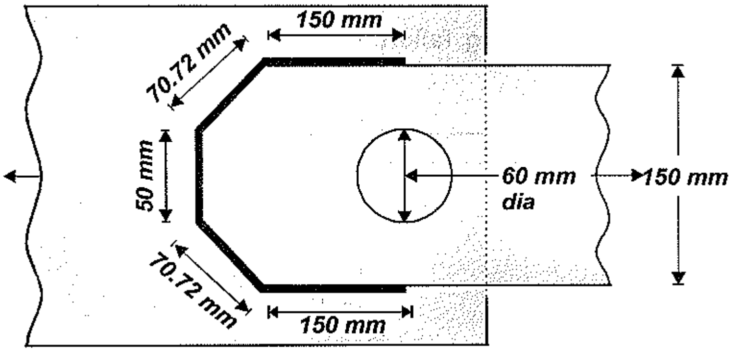
Section C sectional view



Pattern of rivets

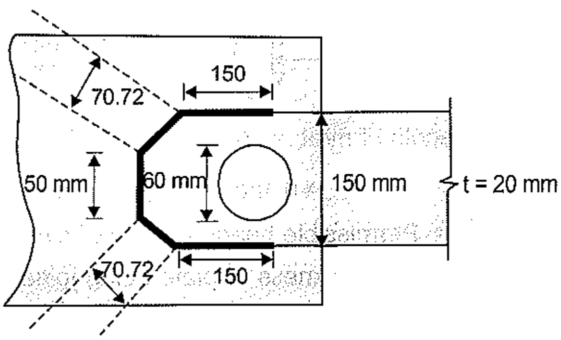
As the size of angle is 150 × 115 mm hence if longer leg is connected leg, two rows of rivets can be attached on the longer leg (length being larger than 125 mm).

Q-7: ***A plate of 150 mm width and 20 mm thick is welded to another plate by fillet weld as shown in the figure. The size of the weld is 12 mm throughout. Compute the average shear stress produced in the weld for the full strength of the plate if the allowable stress is 150 N/mm².***



[10 Marks, ESE-2008]

Sol:



There is a hole in the plate, the hole dia is 60 mm. Hence strength of plate will be reduced due to the presence of hole.

$$\text{Allowable tensile stress} = 150 \text{ N/mm}^2 = \sigma_{at}$$

The full strength of plate is given by

$$P = (\text{Net Area of cross-section of plate}) \times (\text{Permissible stress in tension})$$

$$= \frac{(150-60)\times 20\times 150}{1000} = 270 \text{ kN}$$

This force of 270 kN will be resisted by the fillet weld at the edges.

Let the average shear stress produced in the weld be f_s, then

$$P = l_{eff} \times \frac{S}{\sqrt{2}} \times f_s \qquad [S = \text{Size of weld}]$$

$$\Rightarrow \quad [(\text{Total length of fillets weld at edge}) \times \frac{S}{\sqrt{2}} \times f_s] = P$$

$$\Rightarrow \quad P = [(2\times 150 + 2\times 70.72 + 50)] \times \frac{12}{\sqrt{2}} \times f_s$$

$$\Rightarrow \quad 270\times 10^3 = 491.44 \times \frac{12}{\sqrt{2}} \times f_s$$

$$\Rightarrow \quad f_s = 64.748 \text{ N/mm}^2$$

Q-8: ***Discuss the failure of rivets.***

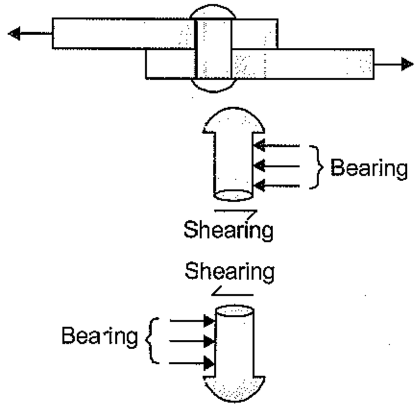
[3 Marks, ESE-2010]

Sol: There are three types of failure can take place in rivet, if applied stresses exceeds the permissible stresses.

(a) Bearing failure (b) Shearing failure (c) Axial failure

(a) Bearing failure and shearing failure

Single Shear Case:



$$\text{Bearing strength of rivet} = \sigma_{br} \times t_{min} \times d$$

If $F_{applied} > \sigma_{br} \times t_{min} \times d$ the bearing failure takes place

when σ_{br} = Permissible bearing stress of rivet

t_{min} = Min thickness of plate being joined.

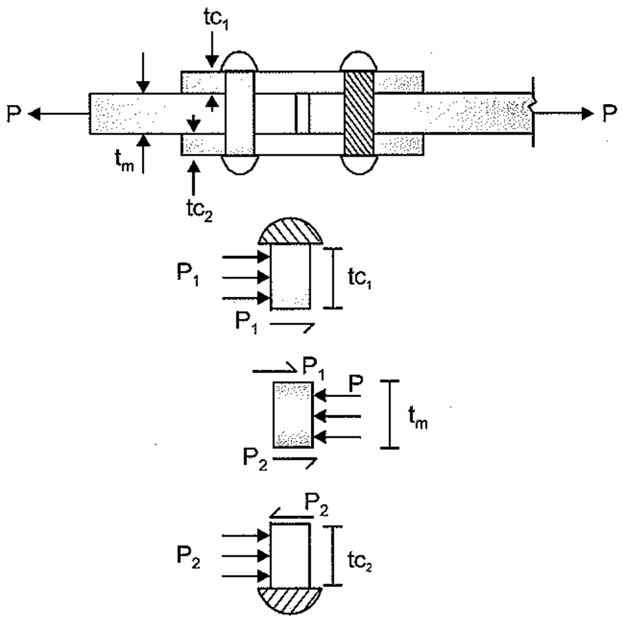
d = Diameter of hole,

$$\text{Shear strength of rivet} = \frac{\pi}{4} \times d^2 \times \sigma_s$$

d = Dia of hole or gross dia. of rivet.

If $F_{applied} > \frac{\pi}{4} d^2 \sigma_s$, shearing failure of rivets occur.

Double Shear Case:



In this case if $P > \sigma_{br} \times t \times d$, Bearing failure of Rivets occur.

Where, $t = \min\{(tc_1 + tc_2), t_m\}$

If $P > \frac{\pi d^2}{4} \times \sigma_s \times 2$, Shearing failure of Rivets occur.

(b) Axial Tension Case:

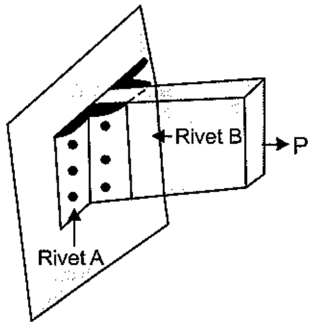
Failure of Rivet A occurs due to axial tension.

If $P > \sigma_{at} \times \frac{\pi d^2}{4}$, Axial tensile failure of rivets occur

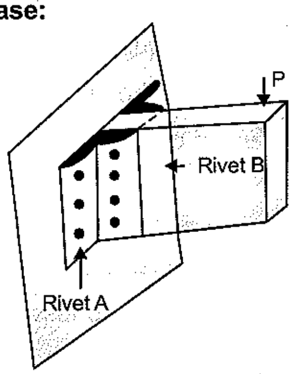
where, σ_{at} = Permissible axial tensile stress.

d = Dia. of hole or gross dia of Rivet.

Failure of Rivet B occure due to shearing or Bearing.



(c) Axial Tension & Shear Case:



Failure of Rivet A occurs due to combination of axial tension & shearing a bearing.

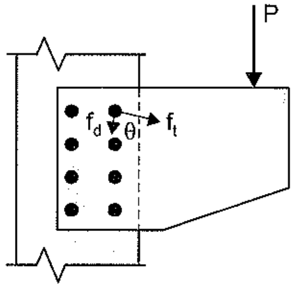
where as failure of Rivet B occur due to shearing or bearing.

For no Failure of Rivet $A = \dfrac{f_{at}cal}{\sigma_{at}} + \dfrac{f_{scal}}{\sigma_s} \not> 1.4$ (In working stress method)

where, f_{atcal} = Calculated axial tensile stress in Rivet A

f_{scal} = Calculated shear stress in Rivet A σ_{at} & σ_s are the permissible axial tensile stress & shear stress respectively.

(d) Direct & Torsional Shear Case



In this case failure of Rivets occur due to resultant shear stress

$$F_r = \sqrt{F_d^2 + F_t^2 + 2f_dF_t\cos\theta}$$

If F_r > Rivet value, failure of Rivets occur.

F_d = Direct shear force in Rivet

F_t = Torsional shear force in Rivet.

Q-9: ***Explain the importance of welded connection in building connection.***

[5 Marks, ESE-2010]

Sol:

- Welded connection leads to saving in expenditure on material & labour.
- It leads to Rapid construction, thus saving in time of Project..
- Welding permits architects & structural engineer complete freedom of design.
- The compactness & rigidity of welded joints permits design assumption to be realised more accurately.
- Welded joints are better for fatigue, impact & vibration as compared to bolted joint.
- It leads to rigid connection which in turn leads to reduced beam depth & weight
- Rigidity of joint, also leads to reduced deflections.

Q-10: ***List out five merits and three demerits of welded joints.***

[4 Marks, ESE-2011]

Sol: **Advantages of welded joints**

(i) In case of welding no holes are required, hence the structural member are more effective in taking loads.

(ii) overall weight of structural steel required is reduced by use of welded joints.

(iii) Since less labour and materials are required for this, so it become economical.

(iv) The speed of fabrication is higher with the welding process

(v) Any shape of joint can be made and complete rigid joint can be provide with the welding process leading to lesser deflection and smaller cross-sectional requirement

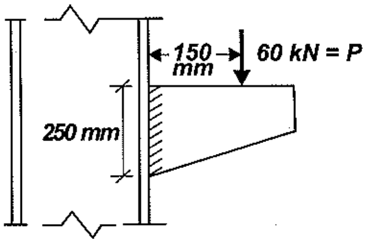
Disadvantages of welded joint

(i) Due to uneven heating and cooling undesirable internal stresses and warping produced.

(ii) Welded joints may be more brittle leading to less fatigue strength

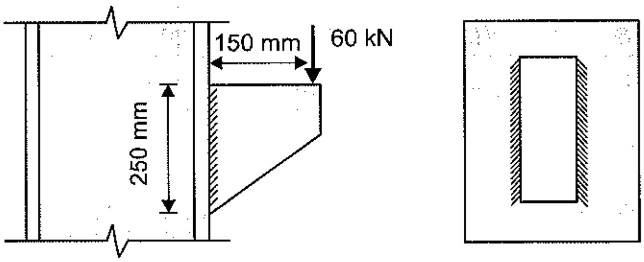
(iii) Defects like internal air pocket, slag inclusion and incomplete penetration are difficult to detect.

Q-11: ***Determine the size of the fillet weld required to join a bracket plate with the flange of a column section shown in the figure below permissible stress in weld = 108 MPa.***



[10 Marks, ESE-2011]

Sol:



Let the thickness of weld throat = t mm

$$\text{Vertical shear stress } (f_v) = \frac{P \times 1000}{250 \times t \times 2} = \frac{60 \times 1000}{500 \times t} = \frac{120}{t} \text{N/mm}^2$$

Horizontal shear stress due to bending at extreme fibre (f_h)

$$f_h = \frac{(P \times e) \times d/2}{I_{xx}} = \frac{P \times e \times \frac{d}{2}}{2 \times t \times \frac{d^3}{12}} = \frac{6Pe}{2td^2}$$

$$\therefore \quad f_h = \frac{3 \times 60 \times 1000 \times 150}{t \times 250^2} = \frac{432}{t} \text{N/mm}^2$$

$$\therefore \quad \text{Resultant stress} = \sqrt{fv^2 + fh^2} = \sqrt{\frac{120^2}{t^2} + \frac{432^2}{t^2}} = \frac{448.36}{t} \text{N/mm}^2$$

For safe connection, $f_R \leq 108$ N/mm²

$$\frac{448.36}{t} \leq 108$$

$$\Rightarrow \quad t \geq \frac{448.36}{108}$$

$$\Rightarrow \quad t \geq 4.15 \text{ mm}$$

$$\Rightarrow \quad \text{Size of weld} \geq 4.15\sqrt{2} = 5.87 \text{ mm} \cong 6 \text{ mm (say)}$$

Q-12: ***Design an unequal angle section to serve as a tie member of 1.6 m length in a roof truss. It has to carry an axial load of 118 kN. The gusset plate is connected to the longer leg of the angle. Also design the fillet weld.***

Permissible stress in weld = 108 N/mm².

Permissible stress in axial tension = 150 N/mm².

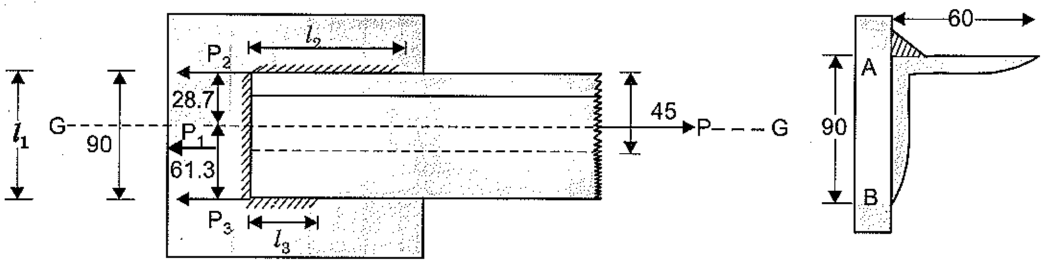
Thickness (t), sectional area (A), weight /m run (w), distance of centres of gravity along x-axis and y-axis (c_x and c_y), and maximum and minimum radius of gyration r_{max} and r_{min} are given for some angles for design.

Angle	t	A	w	c_x	c_y	r_{max}	r_{min}
	6	746	5.9	26.4	11.6	26.9	10.7
80 × 50	8	978	7.7	27.3	12.4	26.6	10.6
	10	1202	9.4	28.1	13.2	26.3	10.6
	6	865	6.8	28.7	13.9	30.7	12.8
90 × 60	8	1137	8.9	29.6	14.8	30.4	12.7
	10	1401	11.0	30.4	15.5	30.1	12.7

[12 Marks, ESE-2011]

Sol: Basis of design will be to chose a section which will lead to most economical condition in terms of wt. of section.

- The net Area required = $\dfrac{118 \times 1000}{150} = 786.67\ mm^2$
- This net area is greater then the gross area of ISA 80 × 50 × 6. Hence we should choose next heavier section. Thus let us choose the section ISA 90 × 60 × 6 which is having gross cross-sectional area of 865 mm^2 with the longer leg welded to the gusset plate.



- Since the outstanding leg is not attached, hence shear lag take place. Thus

∴ $$A_{net} = A_1 + KA_2$$

where $$K = \frac{3A_1}{3A_1 + A_2}$$

and $$A_1 = \left(90 - \frac{6}{2}\right) \times 6 = 522\ mm^2$$

$$A_2 = \left(60 - \frac{6}{2}\right) \times 6 = 342\ mm^2$$

∴ $$K = \frac{3 \times 522}{3 \times 522 + 342} = 0.82$$

Hence, $$A_{net} = 522 + 0.82 \times 342 = 802.44 > 786.67\ mm^2$$

the section ISA 90×60×6 can be used.

⇒ Check for slenderness ratio,

$$\lambda = \frac{l_{eff}}{r_{min}}$$

where l_{eff} = 1600 mm, r_{min} = 12.8 mm

$$\therefore \quad \lambda = 125 \not> 350$$

Hence slenderness ratio is also within permissible limits

Therefore we can use ISA 90 × 60 × 6

It is clear from the figure that at point A, we have a square edge and at point B there is round edge.

And we know that, for square edge,

$$\text{Maximum size} = \{(\text{Thickness of thinner plate being joined}) - 1.5 \text{ mm}\}$$

$$= 6 - 1.5 = 4.5 \text{ mm}$$

and for round edge

$$\text{Maximum size} = \frac{3}{4}\text{th of (Nominal thickness of round edge)}$$

$$= \frac{3}{4} \times 6 = \frac{9}{2} = 4.5 \text{ mm}$$

Hence adopting size of weld = 4.5 mm

and permissible stress in weld = 108 N/mm²

$$\therefore \quad \text{Effective length of weld required } (l_{eff}) = \left[\frac{118 \times 1000}{0.7 \times 4.5 \times 108}\right] = 346.86 \text{ mm}$$

Consider the figure A,

$$l_1 + l_2 + l_3 = 346.85$$

and

$$l_1 = 90 \text{ mm}$$

$$\therefore \quad \boxed{l_2 + l_3 = 256.86 \text{ mm}} \quad \text{...(i)}$$

$$P_1 = \frac{0.7 \times 4.5 \times 90 \times 108}{1000} = 30.618 \text{ kN}$$

∴ Taking moment about G-G axes,

$$P_2 \times 28.7 - P_1 \times (45 - 28.7) - P_3(61.3) = 0$$

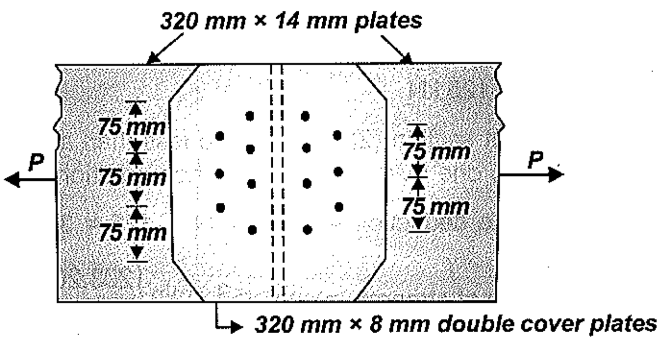
$$\Rightarrow \quad \frac{0.7 \times 4.5 \times 108}{1000} \times 28.7 \times l_2 - \frac{0.7 \times 4.5 \times 108}{1000} \times 61.3 \times l_3 = 30.618 \times (45 - 28.7)$$

$$\Rightarrow \quad 9.764\, l_2 - 20.854\, l_3 = 499.07 \quad \text{...(ii)}$$

By solving the two equations we get,

$$l_1 = 90 \text{ mm}, \; l_2 = 191.24 \text{ mm}, \; l_3 = 65.62$$

Q-13:



A double cover butt joint is provided with the following details:

Size of plates to be spliced 320 mm × 14 mm

Size of cover plates 320 mm × 8 mm

No. of 20 mm dia rivets provided = 7 (as shown in the above figure)

Allowable stress in tension = 125 MPa

Allowable stress in shear = 80 Mpa

Allowable stress in bearing = 250 MPa

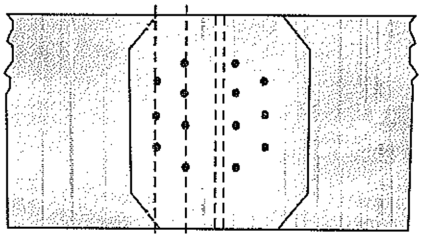
(i) Determine the strength of the connection.

(ii) Find the force on the extreme rivet when the connection is subjected to a pull of 280 kN with an eccentricity of 20 mm.

(iii) Find the limiting value of eccentricity if force on any rivet is not to exceed its strength.

[25 Marks, ESE-2012]

Sol: (i)



Strength of connection is the minimum of shearing, bearing and tearing strength

Shearing strength of connection

$$= 7 \times 2 \times \frac{\pi}{4}(d')^2 \times f_s$$

$$d' = 21.5 \text{ mm}$$

$$f_s = 80 \text{ MPa}$$

[f_s = permissible shearing stress
d' = die of hole
Rivets are in double shear]

$\Rightarrow$ Shearing strength of Joint $= 7 \times 2 \times \frac{\pi}{4}(21.5)^2 \times 80 = 4.6410 \text{ kN}$

Bearing strength of Joint

$$= 7 \times d' \times t \times f_{br}$$

$$= 7 \times 21.5 \times 14 \times 250 \text{ N}$$

$$= 526.75 \text{ kN}$$

[F_{br} = permissible bearing stress = 250 MPa
d' = dia of hole
t = min of combined thickness of two cover plates on the thickness of main plate whichever is smaller]

Tearing strength of main plate at sec (1) – (1)

$$= (320 - 3 \times 21.5) \times 14 \times 125 \text{ N}$$

$$= 447.125 \text{ kN}$$

Tearing strength of main plate at section (2) – (2) will be less than at (1) – (1) only if loss of strength of plate due to deduction due to one rivet hole is more than addition of strength due to $3R_v$

Loss due to deduction $= 21.5 \times 14 \times 125 = 37.625$ kN

Gain of strength $= 3R_v = 3 \times \min\left\{\frac{203.205}{7}, \frac{526.75}{7}\right\} = 87.087$ kN

$\Rightarrow$ Gain of strength > loss due to deduction of hole

hence tensile strength of main plate at section (2) – (2) will be more than that at (1) – (1)

Tearing strength of cover plates at (2) – (2)

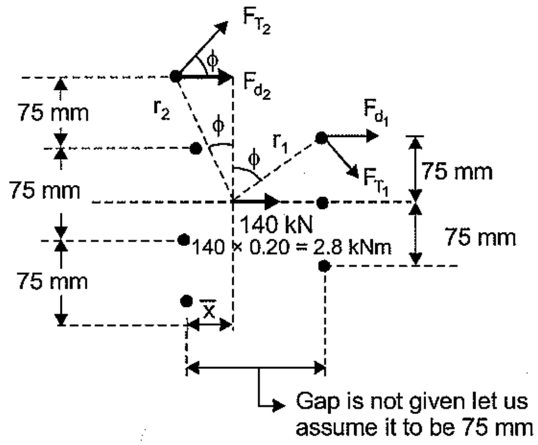
$$= (320 - 4 \times 21.5) \times (8 + 8) \times 125 = 468 \text{ kN}$$

Thus, $\boxed{\text{Strength of joint} = 406.410 \text{ kN}}$

(ii) When the connection is subjected to a pull and 280 kN at an eccentricity of 20 mm

The connection transfers half of the load through one cover plate.

Hence force on Rivet at one section is due combined action of 140 kN load at an eccentricity of 20 mm



$$\bar{x} = \frac{4A \times 0 + 3A \times 75}{7A} = \frac{3 \times 75}{7} = 32.143 \text{ mm}$$

$$r_1 = \sqrt{(75 - 32.143)^2 + (75)^2} = 86.381 \text{ mm}$$

$$r_2 = \sqrt{(32.143)^2 + (112.5)^2} = 117 \text{ mm}$$

$$\tan\theta = \frac{75 - 32.143}{75} \Rightarrow \theta = 29.7448°$$

$$\tan\theta = \frac{32.143}{112.5} \Rightarrow \phi = 15.945°$$

$$\Sigma r^2 = 2\,(86.381)^2 + (75 - 32.143)^2 = 49016.92 \text{ mm}^2$$
$$+ 2\,(117)^2 + 2\,((32.143)^2 + (37.5)^5)$$

$$F_{d_1} = \frac{140}{7} = 20 \text{ kN}$$

$$F_{d_2} = 20 \text{ kN}$$

$$F_{T_1} = \frac{2.8 \times 0.086381}{49016.92 \times 10^{-6}} = 4.93 \text{ kN}$$

$$F_{T_2} = \frac{2.8 \times 0.177}{49016.92 \times 10^{-6}} = 10.11 \text{ kN}$$

$$F_{r_1} = \sqrt{F_{D_1}^2 + F_{T_1}^2 + 2F_{D_1} \cdot F_{T_1} \cos\theta} = 24.4 \text{ kN}$$

$$F_{r_2} = \sqrt{F_{D_2}^2 + F_{T_2}^2 + 2F_2 \cdot F_{T_2} \cos\phi} = 29.85 \text{ kN}$$

$\Rightarrow$ $\boxed{\text{Max force on extreme rivet} = 29.85 \text{ kN}}$

(iii) $\therefore$ Strength of rivet in shearing on one face $= \frac{\pi}{4}(21.5)^2 \times 80 = 29.03$ kN

Strength in bearing on cover plate $= 21.5 \times 8 \times 250 = 43$ kN

$\Rightarrow$ Strength of rivet on one section $= 29.03$ kN

Let the eccentricity be e

$$\Rightarrow \quad F_{d_2} = 20 \text{ kN}$$

$$F_{T_2} = \frac{140 \times 10^3 \times e \times 117}{49016.92} \text{ N} = 334.17 \text{ e N} = 0.3342 \text{ e kN}$$

$$\Rightarrow \quad F_{r_2} = \sqrt{(20)^2 + (0.3342\,e)^2 + 2 \times 20 \times 0.3342\,e \cos 15.945°}$$

For critical eccentricity, $F_{r_2} = 29.03$ kN

$$Fr_2^2 = (29.03)^2$$

$$\Rightarrow \quad (20)^2 + (0.3342\,e)^2 + 12.8537\,e = (29.03)^2$$

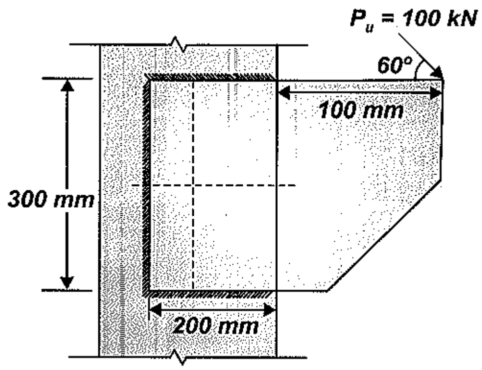
$$\Rightarrow \quad 0.11169\,e^2 + 12.8537e - 442.74 = 0$$

$$e = \frac{-12.8537 \pm \sqrt{(12.8537)^2 + 4 \times 442.74 \times 0.11169}}{2 \times 0.11169}$$

$$= 27.752 \text{ mm}$$

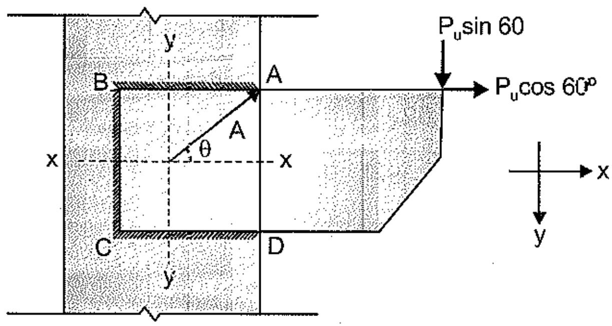
$\Rightarrow$ Limiting eccentricity is 27.752 mm

Q-14: ***A plate is connected to the flange of an ISMB as shown. The factored load is 100 kN. Find the size of the weld. Assume shop weld and ultimate strength of weld as 410 MPa.***



[20 Marks, ESE-2013]

Sol: The loading P_u can be resolved as shown below.



Most critical location for the weld will be point A where angle between torsional shear stress and direct shear stress is minimum and torsional shear stress has maximum value.

Thus, we will calculate resultant shear stress at A and by limiting it to be less than the max possible shear stress, value of size of weld can be calculated.

Let us calculate the C.G. of weld ($\bar{x}$)

$$\bar{x} = \frac{2(200\times t)\times 100+0}{700t} = \frac{400}{7} = 57.143\,\text{mm} \qquad \text{[t = Throat thickness of weld]}$$

$$r_A = \sqrt{(200-57.143)^2+(150)^2} = 207.143 \text{ mm}$$

$$\theta = \tan^{-1}\left(\frac{150}{200-57.143}\right) = 46.397^\circ$$

f_{dx} = Direct shear stress in x-direction

$$= \frac{P_u\cos 60^\circ}{700t} \qquad \text{[t = Throat thickness in mm]}$$

$$= \frac{100\cos 60^\circ\times 10^3\,\text{N}}{700t\,\text{mm}^2} = \frac{500}{7t}\text{N/mm}^2 = \frac{71.429}{t}\text{N/mm}^2$$

f_{dy} = Direct shear stress in y-direction

$$= \frac{P_u\sin 60^\circ}{700t} = \frac{123.718}{t}\ \text{N/mm}^2$$

$$f_t = \text{Torsional shear stress} = \frac{Tr_A}{j}$$

$T = P_u \sin 60^\circ \times (100 + 200 - 57.1432) + P_u \cos 60^\circ \times 150$

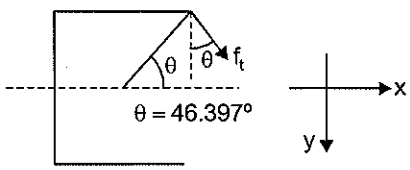
$= 28532.033$ kNmm $= 28.532$ kN/m

$J = I_{xx} + I_{yy}$

$$= \frac{t(300)^3}{12} + 2(200\times t)(150)^2 + 300\times t(57.143)^2$$

$$+2\times\frac{t(200)^3}{12} + 2\times 200\times t(100-57.143)^2$$

$= 14297619.05t$ mm^4



$$f_t = \frac{28.532\times 10^6\,\text{Nmm}\times 207.143\,\text{mm}}{14297619.05t} = \frac{413.909}{t}$$

$$f_{tx} = f_t\sin\theta = \frac{413.909}{t}\times\sin 46.397 = \frac{299.726}{t}\text{N/mm}^2$$

$$f_{ty} = \frac{413.909}{t}\times\cos\theta = \frac{413.909}{t}\times\cos 46.397^\circ = \frac{285.455}{t}$$

$$f_x = f_{dx} + f_{tx} = \frac{71.429+299.726}{t} = \frac{371.155}{t}\text{N/mm}^2$$

$$f_y = f_{dy} + f_{ty} = \frac{123.718+285.455}{t} = \frac{409.173}{t}\text{N/mm}^2$$

$$\text{Resultant shear stress} = \sqrt{f_x^2 + f_y^2} = \frac{552.43}{t} \text{N/mm}^2$$

$$\text{For safety of connection resultant shear stress} < \frac{f_u}{\sqrt{3} \times \gamma_{mw}}$$

$$\Rightarrow \quad \frac{552.43}{t} < \frac{410}{\sqrt{3} \times 1.25} \quad [\text{for shop weld } \gamma_{mw} = 1.25]$$

$$t > \frac{552.43 \times \sqrt{3} \times 1.25}{410}$$

$$t > 2.917 \text{ mm}$$

$$\Rightarrow \quad 0.7\,S > 2.917 \text{ mm}$$

$$S > 4.167 \text{ mm}$$

Adopt size of weld as 5 mm.

Q-15: ***State the assumptions made for designing riveted connections in steel.***

[5 Marks, ESE-2014]

Sol: Assumptions made for designing riveted connections in steel are as follows:

(i) The tensile stress is uniformly distributed on the portions of the plate between the rivets.

(ii) The friction between the plates is neglected

(iii) The shearing stress is uniformly distributed on the cross-section of the rivets.

(iv) The rivets fill the holes completely.

(v) The rivets in a group share the direct load equally.

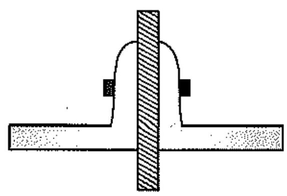
(vi) Bending stress in rivets is neglected.

(vii) Bearing stress distribution is uniformly and the contact area is d × t where d is the diameter and t is the thickness of the plate.

Q-16: ***21SA75 × 75 × 8 carry a load of 150 kN and are placed back to back through a 6mm gusset plate. The permissible shear stress is 100 MPa and bearing stress is 300 MPa. Design the riverted connection and show the arrangement with a neat sketch.***

[10 Marks, ESE-2014]

Sol: Given that 2 ISA 75 × 75 × 8 are placed back to back through a 6 mm gusset plate. Therefore, structure should be like:



Therefore, rivets are in double shear.

Diameter of rivet:

By Unwins's formula

$$d = 6.04\sqrt{t_{min}}$$

$$t_{min} = 6 \text{ mm}$$

$$\therefore \quad d = 6.04 \times \sqrt{6} = 14.79 \text{ mm}$$

Let us use 16 mm dia rivets

$$\therefore \quad \text{Gross dia of rivet, } \phi_g = 16 + 1.5 = 17.5 \text{ mm}$$

Rivet value:

Rivet value is minimum of the bearing strength and shearing strength of the rivet.

$$\text{Shearing strength of rivet} = 2\times\frac{\pi}{4}\times\phi_g^2\times f_s \qquad [\because \text{ Rivets are in double shear}]$$

where f_s = Permissible shear stress in rivet

$$\therefore \quad \text{Shearing strength} = 2\times\frac{\pi}{4}\times 17.5^2\times 100\times 10^{-3}\text{ kN} \qquad [\because f_s = 100\text{ MPa}]$$

$$= 48.105\text{ kN} \qquad \ldots(i)$$

$$\text{Bearing strength of rivet} = \phi_g\times t_{min}\times f_b$$

where f_b = Permissible bearing stress in rivet

$$t_{min} = \min(8 + 8, 6)$$

$$\therefore \quad t_{min} = 6\text{ mm}$$

$$\Rightarrow \quad \text{Bearing strength} = 17.5\times 6\times 300\times 10^{-3}\text{ kN} \qquad [\because f_b = 300\text{ MPa}]$$

$$\therefore \quad = 31.5\text{ kN} \qquad \ldots(ii)$$

$$\therefore \quad \text{Rivet value} = \min.(48.105, 31.5) = 31.5\text{ kN}$$

$$\text{No. of rivets required} = \frac{\text{Total load}}{\text{Rivet value}} = \frac{150}{31.5}$$

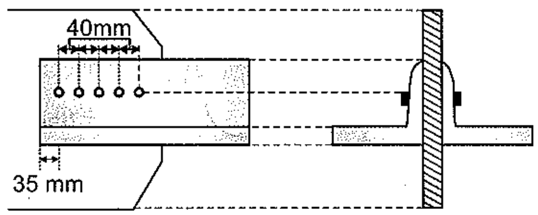
$$= 4.76 \approx 5\text{ rivets}$$

$$\text{Pitch of rivets} = 2.5\times\phi_g = 2.5\times 16 = 40\text{ mm}$$

Let us provide pitch, $p = 40$ mm

$$\text{Minimum edge distance} = 2.0\times\phi_g = 2.0\times 17.5 = 35\text{ mm}$$

Arrangement



Q-17: ***What are High Strength Grip Bolts? State their advantages.***

[5 Marks, ESE-2015]

Sol: **High Strength Friction Grip Bolts**

- High strength friction grip bolts are made of high strength steel and their surface is kept unfinished, i.e., as rolled and rough.
- High initial tension is developed in such bolts in the initial stage of tightening, and this tension clamps the joining plates between the bolt head and the nut.
- The tightening of the bolt to a very high tension, reaching their proof load, is done through calibrated torque wrenches.
- This high pre-compression causes clamping action due to which the load is transmitted from one plate to the other by friction, with negligible slip.

Advantages of High Strength bolts

- It gives rigid joint as there is no slip between plates at working loads.
- It gives high static strength due to high frictional resistance.
- Smaller load is transmitted at net section of plates.
- There are no shearing or bearing stresses in the bolts.
- It has high fatigue strength.
- As the bolts are in tension upto proof load they do not permit loosening of the nut and the washer.

Q-18: ***The vertical member of triangular pratt truss is composed of 2 Nos. ISA 75 × 75 × 6 (connected back to back on each side of the gusset of 10 mm thickness). The factored forces in the member are:***

107 kN (Compression)

79 kN (Tension)

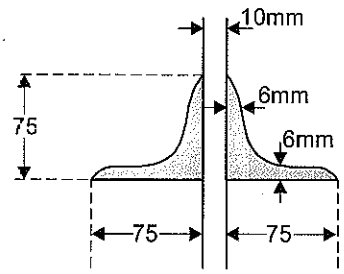
Design the fillet weld connection. Assume f_u = 410 MPa and $\gamma_{mw} = 1.25$. Welding shall done along the length of the member.

[10 Marks, ESE-2015]

Sol: Design of fillet weld is independent of the nature of force i.e. compress ve or tensile. Therefore maximum of the two forces will be considered for the design of fillet weld.

$$\therefore \quad F_u = 107 \text{ kN}$$

$$\text{Force taken by each angle member} = \frac{107}{2} = 53.5 \text{kN}$$



For square edge,

Max. size of weld

$$= \{(\text{Thickness of thinner plate being joined}) - 1.5 \text{ mm}]$$

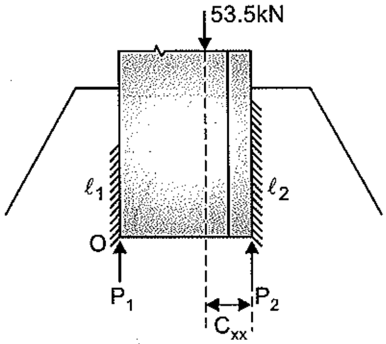
$$= 6 - 1.5 = 4.5 \text{ mm}$$

For round edge, $\quad \text{Max. size} = \frac{3}{4} \times \text{Nominal thickness of round edge}$

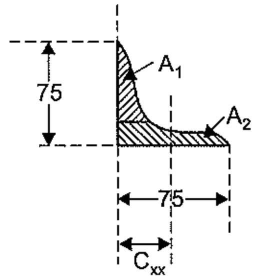
$$= \frac{3}{4} \times 6 = 4.5 \text{mm}$$

$\Rightarrow \quad$ Adopt size of weld = 4.5 mm

Welding is to be done along the length of the member



Since the value of C_{xx} is not given, we will calculate it as follows:



$$C_{xx} = \frac{A_1\bar{X}_1 + A_2\bar{X}_2}{A_1 + A_2} = \frac{[(75-6)\times 6\times 3]+[75\times 6\times 37.5]}{(75-6)\times 6+75\times 6} = 20.97\,\text{m}$$

Now, taking moment about pt O,

$$P_2 \times 75 - 53.5\,(75 - 20.97) = 0$$

$\Rightarrow$ $P_2 = 38.54$ kN

and, $P_1 + P_2 = 53.5$

$\Rightarrow$ $P_1 = 14.96$ kN

Now, $$P_1 = \frac{f_u}{\sqrt{3}\,\gamma_{mw}} \times \ell_1 \times (0.7s)$$

$\Rightarrow$ $$14.96 \times 10^3 = \frac{410}{\sqrt{3}\times 1.25} \times \ell_1 \times 0.7 \times 4.5$$

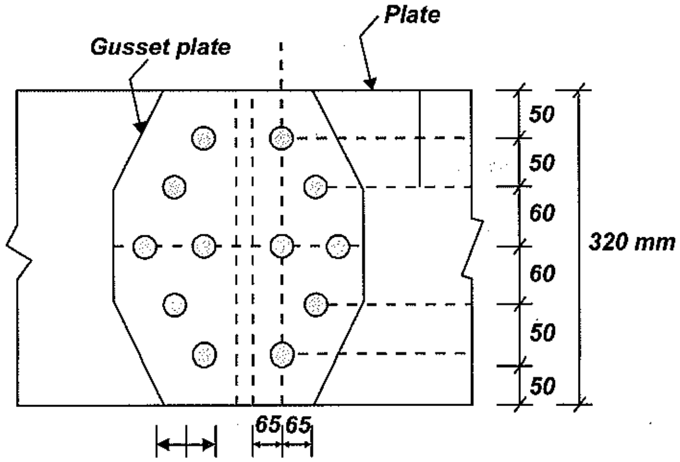
$\Rightarrow$ $l_1 = 25.08\,\text{mm} \simeq 25.5\,\text{mm}$

Similarly, $$38.54 \times 10^3 = \frac{410}{\sqrt{3}\times 1.25} \times \ell_2 \times 0.7 \times 4.5$$

$\Rightarrow$ $l_2 = 64.61$ mm $\simeq 65$mm

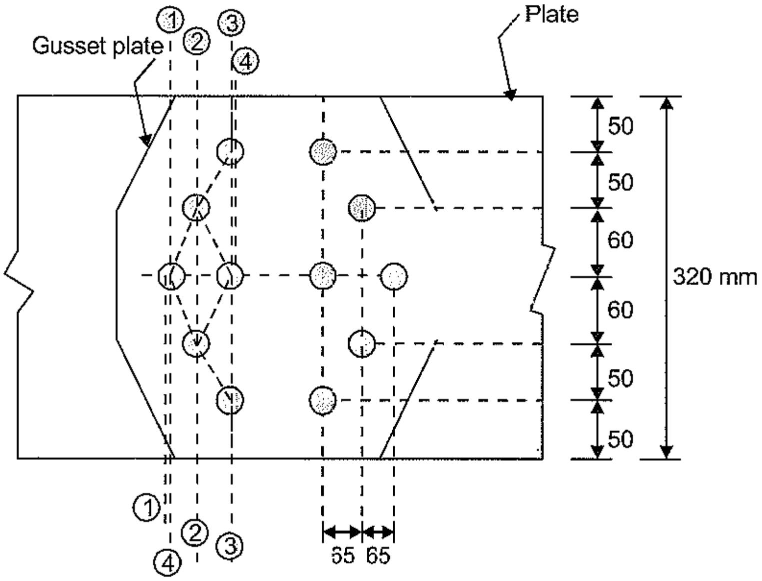
$$\left.\begin{matrix} l_1 = 25.5\text{ mm} \\ l_2 = 65\text{mm} \end{matrix}\right\}\text{Ans.}$$

Q-19: ***Two steel plates in axial tension are to be connected by bolts as shown in figure Determine its tensile strength. Bolt dia = 25 mm, Hole dia = 28 mm, Plate thickness = 16 mm each, f_u = 440 MPa, f_y = 290 MPa.***



[20 Marks, ESE-2016]

Sol:



$d = 25$ mm; $d_0 = 28$ mm; $t = 16$ mm; $f_u = 440$ MPa; $f_y = 290$ MPa;

$P = 65$ mm; $e = 50$ mm; $g_1 = 50$ mm; $g_2 = 60$ mm

Tensile strength of section will be minimum of:

(i) 6 × bolt value

(ii) Strength of plate in gross section yielding

(iii) Strength of plate in net section rupture.

(i)
$$\text{Bolt value} = \text{min of} \begin{cases} \text{Shear strength of bolt, } V_{ds} \\ \text{bearing strength of bolt, } V_{dpb} \end{cases}$$

$$V_{dsb} = \frac{f_u}{\sqrt{3}\,\gamma_{m1}} \times (A_{nb} \times n_b + A_{sb} \times n_s)$$

$$\gamma_{m1} = 1.25$$

$$A_{nb} = \text{Area of bolt at thread} = 0.78 \times \frac{\pi}{4} \times d^2$$

$$A_{sb} = \text{Area of bolt at shaft} = \frac{\pi}{4} \times d^2$$

n_n, n_s = no. of shear plane through thread & shaft, respectively

Note: Taking the joint as double cover bolt joint.

Assuming $n_s = 1$ and $n_n = 1$

$$\therefore \quad V_{dsb} = \frac{440}{\sqrt{3} \times 1.25}\left(1 \times 0.78 \times \frac{\pi}{4} \times 25^2 + 1 \times \frac{\pi}{4} \times 25^2\right) \text{N} = 177.57 \text{ kN}$$

$$V_{bpb} = 2.5 \frac{f_u}{\gamma_{m1}} k_b dt$$

$$K_b = \text{min of} \left[\frac{e}{3d_0}, \frac{P}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1.0\right]$$

$$= \text{min of} \left[\frac{50}{3 \times 28}, \frac{65}{3 \times 28} - 0.25, \frac{440}{440}, 1.0\right]$$

$$= \text{min of } [0.60, 0.52, 1.0, 1.0] = 0.52$$

$$V_{dpb} = 2.5 \times \frac{440}{1.25} 0.52 \times 25 \times (16) N = 183.04 \text{ kN}$$

$\therefore$ Bolt value = 177.57 kN = V_{db}

6 × bolt value = 6 × 177.57 = 1065.42 kN

(ii) Strength of plate in gross section yielding.

$$f_{yg} = \frac{f_y}{\gamma_{m0}} \times A_g = \frac{290}{1.1} \times 320 \times 16 N$$

$$f_{yg} = 1349.82 \text{ kN}$$

(iii) Strength of plate in net section rupture, f_{un}

(a) Strength of main plate at section (1) - (1)

$$= 0.9 \frac{f_u}{y_{m1}} A_{net}$$

$$= 0.9 \times \frac{440}{1.25} \times [(320 - 28) \times 16] N = 1480.09 \text{ kN}$$

(b) Strength of main plate for section (2) - (2)

$$= 0.9 \times \frac{440}{1.25} \times [(320 - 2 \times 28) \times 16] + V_{db}$$

$$= 1338.16 + 177.57 = 1515.73 \text{ kN}$$

$$> 1480.09 \text{ kN}$$

Since, strength of main plate at (2)–(2) > at (1)-(1)

hence, strength of main plate at (3)–(3) will be > at (2)-(2)

(c) Strength of gusset plate at section (3)-(3)

$$= 0.9 \times \frac{440}{1.25} \times [(320 - 3 \times 28) \times 16 \times 10^{-3}] + 3 \times 177.57$$

$$= 1728.95 \text{ kN}$$

(d) Strength of main plate at section (4)-(4)

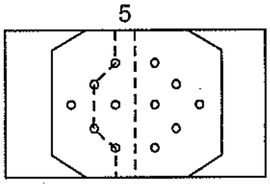
$$= 0.9 \times \frac{440}{1.25} \times \left[\left(320 - 3 \times 28 + \frac{65^2}{4 \times 50} + \frac{65^2}{4 \times 60}\right) \times 16\right] N$$

$$= 1392.547 \text{ kN}$$

(e) Strength of main plate for section 5-5

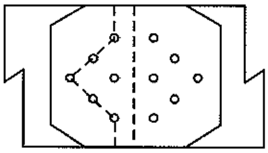
$$= 0.9 \times \frac{440}{1.25}\left[320 - 4 \times 28 + \left(\frac{65^2}{4 \times 50}\right) \times 2\right] \times 16 \times 10^{-3} + 1 \times 177.57$$

$$= 1446 \text{ kN}$$



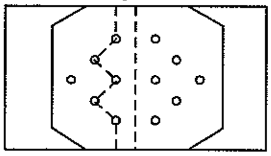
(f) Strength of main plate for section 6-6

$$= \frac{0.9\times440}{1.25}\left[320-5\times28+2\left(\frac{65^2}{4\times50}+\frac{65^2}{4\times60}\right)\right]\times16\times10^{-3} = 1305 \text{ kN}$$



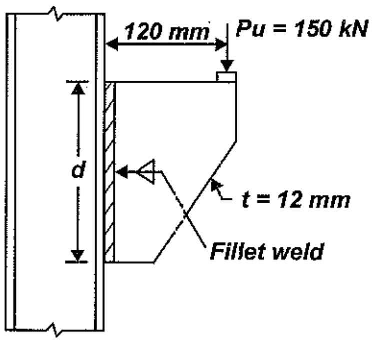
(g) Strength of main plate for section 7-7

$$= \frac{0.9\times440}{1.25}\left[320-5\times28+2\left(\frac{65^2}{4\times50}+\frac{65^2}{4\times60}\right)\right]\times16\times10^{-3}+177.57 = 1482.57 \text{ kN}$$



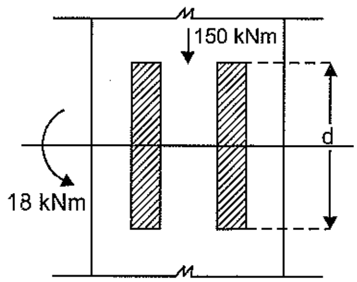
Strength of joint = 1065.42 kN (Governed by strength of bolts)

Q-20: ***A welded bracket connection is shown in figure. It supports a factored load of 150 N at a distance of 120 mm from the face of the column. Design the fillet weld on two sides. Grade of steel = Fe 410, f_y = 250 MPa.***



[10 Marks, ESE-2016]

Sol:



$$P = 150 \text{ kN}$$
$$M = 18 \text{ kNm}$$
$$f_u = 410 \text{ MPa}$$
$$f_y = 250 \text{ MPa}$$
$$t = 12 \text{ mm}$$

Min. size of weld for t = 12 m is 5 mm

Max. size of weld = 12 – 1.5 = 10.5 mm

Adopt weld size, s = 10 mm

$$\text{Shear stress, } q = \frac{150\times10^3}{d\times2\times0.7\times10} \text{N/mm}^2$$

$$q = \frac{10714.286}{d} N/mm^2$$

$$\text{Normal stress, } f = \frac{M \times y}{I}$$

$$f = \frac{18 \times 10^6 \times d/2}{(2 \times 0.7 \times 10) \times \frac{d^3}{12}} N/mm^2 = \frac{7714.285714 \times 10^3}{d^2} N/mm^2$$

For safety, $$\sqrt{f^2 + 3q^2} < \frac{f_u}{\sqrt{3}\gamma_{m_1}}$$

$$\Rightarrow \left(\frac{7714.285714 \times 10^3}{d^2}\right)^2 + 3x\left(\frac{10714.286}{d}\right)^2 \le \left(\frac{410}{\sqrt{3} \times 1.25}\right)^2$$

$$\frac{59510.204 \times 10^9}{d^4} + \frac{344.3878 \times 10^6}{d^2} \le 35861.333$$

$$\Rightarrow 35861.333d^4 - 344.3878 \times 10^6 d^2 - 59510.204 \times 10^9 \ge 0$$

$$\Rightarrow d^2 > 45820.06$$

$$\Rightarrow d > 214.06 \text{ mm}$$

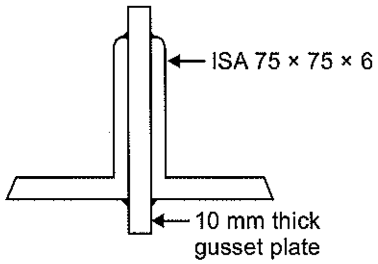
Hence adopt 10 mm size filled weld on both sides of length, d = 215mm.

Q-21: ***A rafter member of a truss consists of double angle ISA, 75×75×6 welded on the opposite sides of a 10mm gusset plate. Design the fillet weld (shop) for the member. The factored compressive force due to DL + LL = 240 kN and the factored tensile force due to DL + WL = 200 kN. Assume E 250 grade steel $C_{XX} = C_{YY}$ = 20.6mm, $\gamma_{mw} = 1.25$.***

[12 Marks, ESE-2017]

Sol: Given that factored compressive force = 240 KN

Factored tensile load = 200 KN

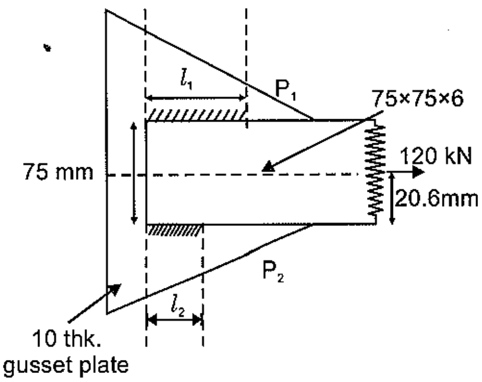


The worst condition is that when the compressive force acting alone.

So $$\text{force in one angle} = \frac{240}{2} = 120 \text{ KN}$$

So $$P_2 = 120 \times \frac{(75 - 20.6)}{75} = 87.04 \text{ KN}$$

$$P_1 = 120 - 87.04 = 32.96 \text{ KN}$$



So Maximum size of weld available

$$= 3/4 \times 6 = 4.5 \text{ mm}$$

Minimum size of weld = 3 mm (for plate thickness ≤ 10mm)

So Let us adopt the weld size = S = 4.5 mm

So Throat thickness = $f_t = 4.5 \times 0.7$ (k = 0.7) = 3.15 mm

Strength of weld per mm length

$$= \left(\frac{410}{\sqrt{3} \times 1.25} \times 3.15 \times 1\right) \quad (\gamma_{mw} = 1.25, \text{ shop weld})$$

$$= 596.52 \text{ N/mm}$$

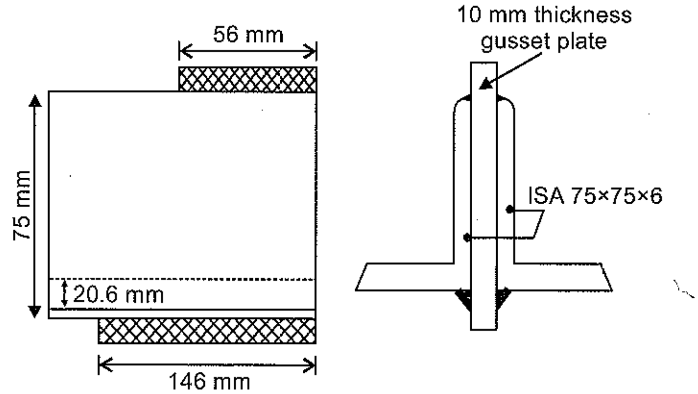
So, length of the weld for P_2

$$= \frac{87.04 \times 1000}{596.52} = 145.91 \text{ mm} \approx 146 \text{ mm}$$

Length of the weld for P_1

$$= \frac{32.96 \times 1000}{596.52} = 55.25 \text{ mm} \approx 56 \text{ mm}$$

So, adopting 146 mm weld length for P_1 and 56 mm for P_2



Q-22: ***A bracket plate is connected to a flange of ISMB 500 as shown in figure. Find the safe load P carried by the joint. M 16 bolts of grade 4.6 are provided at a pitch of 50 mm and end distance 30 mm.***

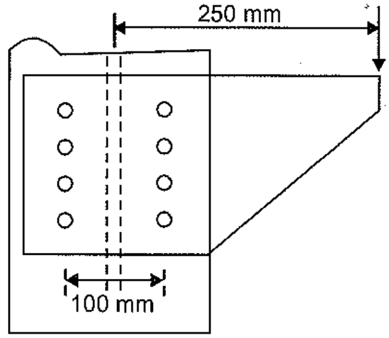
Given:

Thickness of bracket plate = 10 mm

Width of flange b_f = 180 mm

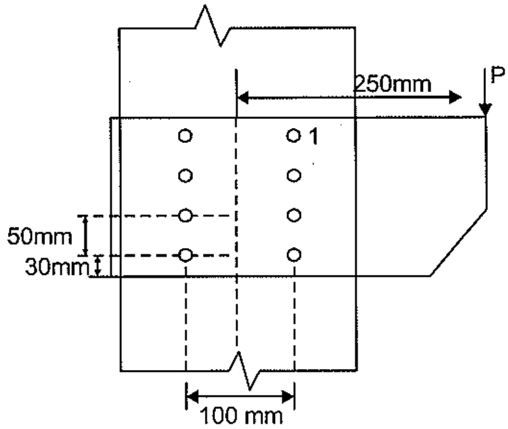
Thickness of flange t_f = 17.2 mm

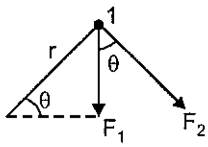
Steel grade of bracket plate Fe 410



[20 Marks, ESE-2020]

Sol:





Given that ISMB 500,

M16 bolts, Grade of bolt = 4.6

Pitch, p = 50 mm and end distance, e = 30 mm

For Fe 410, f_u = 410 MPa, f_y = 250 MPa.

For bolt grade 4.6, f_{ub} = 400 MPa.

Diameter of bolt, d = 16 mm.

Diameter of hole, $d_o = d + 2$

$d_o = 16 + 2 = 18$ mm

Net area of bolt = $0.78\times\frac{\pi}{4}\times d^2 = 0.78\times\frac{\pi}{4}\times 16^2 = 156.83\text{ mm}^2 \simeq 157\text{ mm}^2$

Critical bolt should be number 1

Strength of bolt in single shear:

$$V_{dsb} = A_{nb}\times\frac{f_{ub}}{\sqrt{3}\,\gamma_{mb}} = 157\times\frac{400}{\sqrt{3}\times 1.25} = 29\times 10^3\text{ N}$$

$$= 29\text{ kN}$$

Strength of bolt in bearing:

$$V_{dpb} = 2.5\,k_b d t_{min}\frac{f_u}{\gamma_m}$$

$$\text{Diameter of hole} = d + 2 = 16 + 2 = 18\text{ mm}$$

$$k_b = \text{minimum of}\left[\frac{e}{3d_o}, \frac{p}{3d_o} - 0.25, \frac{f_{ub}}{f_u}, 1.0\right]$$

$$= \text{Minimum of}\left\{\frac{30}{3\times 18}, \frac{50}{3\times 18} - 0.25, \frac{400}{410}, 1.0\right\}$$

$$= \text{minimum}\,[0.56,\ 0.67,\ 0.976, 1]$$

$$\boxed{k_b = 0.56}$$

$$V_{dpb} = 2.5\times 0.56\times 16\times 10\times\frac{410}{1.25} = 73.472\times 10^3\text{ N}$$

$$= 73.47\text{ kN}$$

Hence strength of bolt = 29 kN

Let Q be the factored load

$$\text{Safe load (P)} = \frac{Q}{1.5}$$

$$\text{The number of bolt} = 8$$

$$\text{The direct force} = \frac{Q}{8} = F_1$$

The force in the bolt due to torque $(F_2) = \frac{Qer}{\Sigma r^2}$

$$\text{radius (r)} = \sqrt{(75)^2 + (50)^2} = 90.14 \text{ mm}$$

$$\Sigma r^2 = 4 \times \left(\sqrt{75^2 + 50^2}\right)^2 + 4 \times \left(\sqrt{25^2 + 50^2}\right)^2$$

$$= 45000 \text{ mm}^2$$

$$F_2 = \frac{Q \times 250 \times 90.14}{45000} = 0.5Q$$

$$\tan\theta = \frac{75}{50} \Rightarrow \theta = 56.3°$$

The resultant force (F_R)

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{\left(\frac{Q}{8}\right)^2 + (0.5Q)^2 + 2 \times \frac{Q}{8} \times 0.5Q \times \cos 56.31}$$

$$= 0.579Q$$

$$F_R \leq \text{Strength of bolt}$$

$$0.579Q \leq 29 \text{ kN}$$

$$Q \leq 50.08 \text{ kN}$$

$$\text{Safe load} = \frac{Q}{1.5} = \frac{50.08}{1.5} = 33.39 \text{ kN}$$

Q-23: ***A tie member of a truss consisting of an angle section ISA 65 × 65 × 6 is welded to a gusset plate. Design a fillet weld to transmit a load equal to E 250 (Fe 410). Also sketch the welded length.***

$$A = 744 \text{ mm}^2$$

$$C_z = 18.1 \text{ mm}$$

Thickness of gusset plate is 10 mm.

[12 Marks, ESE-2020]

Sol: Given that:

For Fe 410 grade steel

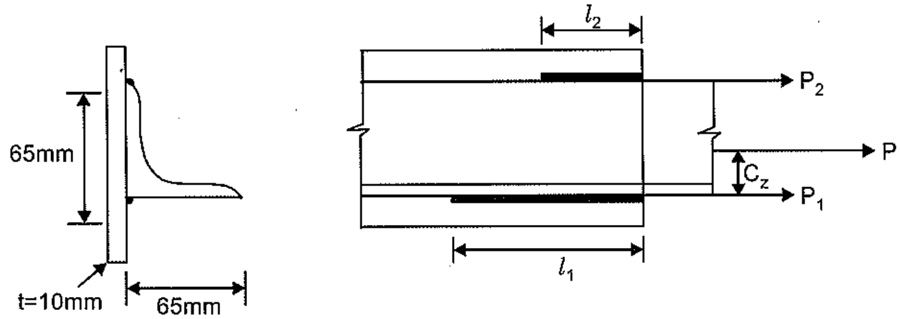
$f_u = 410$ MPa, $f_y = 250$ MPa

Angle section ISA 65 × 65 × 6

$$A = 744 \text{ mm}^2$$

$$C_z = 18.1 \text{ mm}$$

thickness of gusset plate = 10 mm



Tensile strength of member

When member governed yielding

$$T_{dg} = \frac{A_g f_y}{\gamma_{m0}} = \frac{744 \times 250}{1.1} = 169.1 \times 10^3 N$$

$$= 169.1 \text{ kN}$$

Therefore weld will be designed for 169.1 kN

Consider moment about P_2

$$P_1 \times 65 = P \times (65 - 18.1)$$

$$P_1 = \frac{169.1 \times 46.9}{65} = 122.01 \text{ kN}$$

$$P_2 = 169.1 - 122.01 = 47.09 \text{ kN}$$

Minimum size of weld = 3mm

Maximum size of weld = $\frac{3}{4} \times 6 = 4.5$ mm (for round edges)

Let us assume size of weld = 4mm

Effective throat thickness of the weld (t_t) = KS = 0.7 × 4 = 2.8 mm

The design strength of weld (P_{dw}) = $\ell_w \times t_t \times \frac{f_u}{\sqrt{3}\,\gamma_{mw}}$

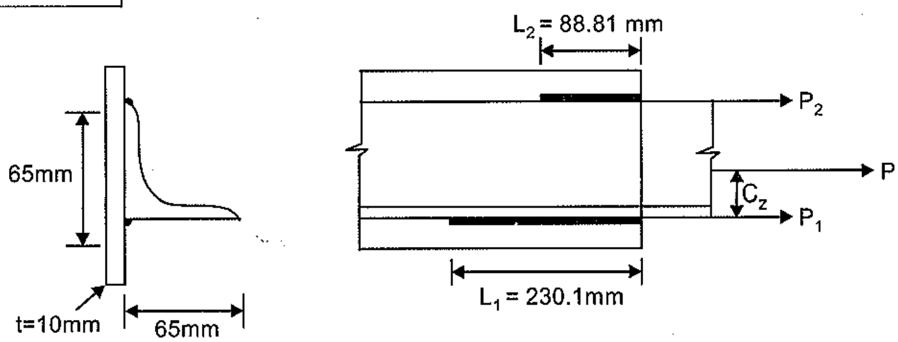
γ_{mw} for shop weld = 1.25

$$P_{dw_1} = P_1 = 122.01 \times 10^3 = \frac{\ell_1 \times 2.8 \times 410}{\sqrt{3} \times 1.25}$$

$\Rightarrow \boxed{\ell_1 = 230.1\text{mm}}$

$$P_2 = 47.09 \times 10^3 = \frac{\ell_2 \times 2.8 \times 410}{\sqrt{3} \times 1.25}$$

$\Rightarrow \boxed{\ell_2 = 88.81\text{mm}}$



CHAPTER 3

DESIGN OF TENSION MEMBERS

Q-1: ***A single ISA 100 × 75 × 10 is used as a tension member with the longer leg connected to a 10 mm thick gusset plate. The connection is made wtih the help of a lug angle Design the connection and sketch the rivet details. Use 20 mm dia rivets. σ_{at} = 150 MPa. Permissible shear and bearing stress in rivets is 100 MPa and 300 MPa respectively. Sections available for lug angle are***

ISA 60 × 60 × 8 - 896 mm²

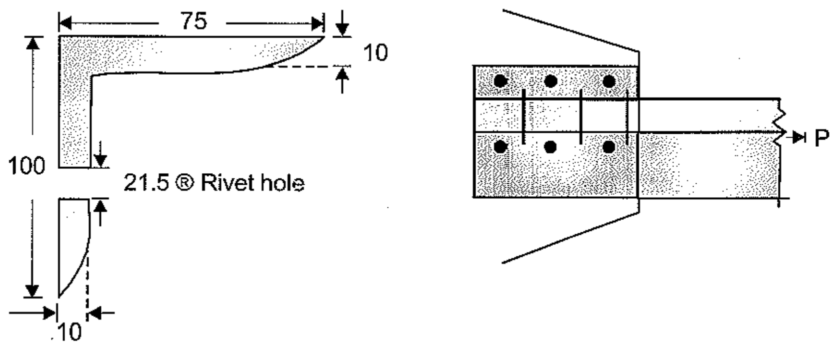
ISA 60 × 60 × 10 - 1100 mm²

ISA 70 × 70 × 8 - 1200 mm²

[15 Marks, ESE-1995]

Sol: In this problem force for which design is to be done is not given hence, we will design it for maximum force that main member can resist.

The main member will be reduced by minimum of one rivet hole, as shown in figure below:



$$A_1 = \left(100 - \frac{10}{2} - 21.5\right) \times 10 = 735 \text{ mm}^2$$

$$A_2 = \left(75 - \frac{10}{2}\right) \times 10 = 700 \text{ mm}^2$$

$$\therefore \quad F_{design} = \frac{150 \times (735 + 700)}{1000} = 215.25 \text{ kN}$$

$$\therefore \quad \text{Force in outstanding leg of main angle} = \frac{F_{design} \times A_2(\text{without hole})}{A_1 (\text{without hole}) + A_2 (\text{without hole})}$$

$$= \frac{215.25 \times (75 - 5) \times 10}{(100 + 75 - 10) \times 10} = 91.318 \text{ kN}$$

[This is an approximately method to calculate the force in outstanding leg]

Lug angle and its connection with the gusset plate is designed for force 1.2 times the force in outstanding leg.

Hence design force for lug angle = 1.2 × 91.318 = 109. 58 kN

Rivets connecting to lug angle with the outstanding leg of main member will be designed for force

$$= 1.4 \times 91.318 = 127.84 \text{ kN}$$

Section of lug angle required

$$A_{\text{net required}} = \frac{109.58 \times 1000}{150} = 730.53 \text{ mm}^2$$

Net area provided by ISA 60 × 60 × 8 is taken as gross area minus deduction due to one hole. This is because at one section lug angle will be reduced by one rivet hole.

$$\text{Net area Provided} = 896 - 21.5 \times 8 = 724 \text{ mm}^2 < 730.53 \text{ mm2 (ie } A_{\text{net req.}})$$

Hence this angle is not safe for design.

Gross area provided by ISA 60 × 60 × 10 = 1100mm^2

Assuming one hole reduction, net area provided = 1100 – 21.5 × 10 = 885 mm^2 > 730.53 mm^2

Hence we adopt Lug angle as ISA 60 × 60 × 10

Designing of rivet

Rivet value calculation

$$\text{Shearing strength in single shear} = \frac{\pi}{4} \times 21.5^2 \times 100 = 36.305 \text{ kN}$$

$$\text{Bearing strength} = d' \times t \times \sigma_{br} = 21.5 \times 10 \times 300 = 64.5 \text{ kN}$$

$\Rightarrow$ Rivet value (Smaller of shearing & bearing strength of Rivet) = 36.305 kN

This rivet value can be used for all connections because everywhere these is single shear and governing thickness for bearing strength calculation is also 10 mm everywhere.

(1) Number of rivets used for connecting the lug angle to the main plate

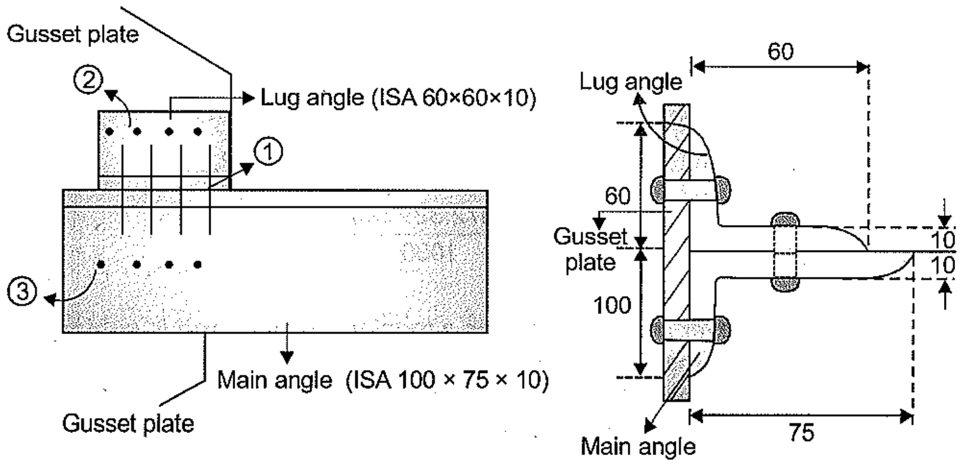
$$= \frac{127.84}{36.305} = 3.52 \cong \text{Adopt 4}$$

(2) Number of rivets used for connecting the Lug angle to the gusset plate

$$= \frac{109.50}{36.305} = 3.018 \cong \text{Adopt (4)}$$

(3) Number of rivets used for connecting the main plate to the gusset plate

$$= \frac{215.25 - 91.318}{36.305} = 3.41 \cong \text{adopt (4)}$$



It is important to note while drawing the diagram that the rivet connecting outstanding leg of main member with the lug angle (i.e., rivet 1) should start in advance of all other rivets. This will ensure that forces are transferred to the lug angles effectively.

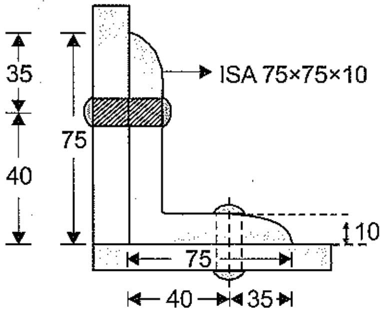
Q-2: ***An angle section ISA 75 × 75 × 10 mm is used as a tension member and is connected to the gusset plates at the joint with 16 mm diameter rivets in single line on both the legs each at a pitch of 50 mm. The rivets on one leg are staggered by 25 mm relative to the rivets on another leg. The rivet line is at a distance of 35 mm centrally from the toe edge of the each leg.***

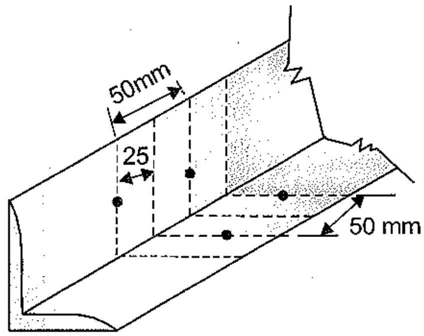
Find the net area of the tension member after making the sketch of it at the joint.

Find the maximum tensile force in the member if the permissible tension stress σ_{st} = 150 MPa

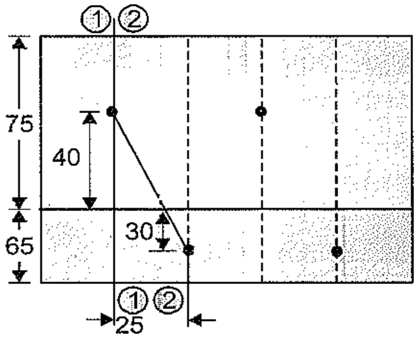
[6 + 4 Marks, ESE-2004]

Sol:





As both legs are connected with the gusset plate, hence the angle will behave as a plate. The angle can be opened up into a plate as shown below:



As the dia of Rivet is 16 mm hence dia of hole = 17.5 mm

$$A_{net} \text{ for section } 1-1 = (140 - 17.5) \times 10 = 1225 \text{ mm}^2$$

$$A_{net} \text{ for section } 2-2 = (140 - 2\times 17.5)\times 10 + \frac{25^2}{4\times 70}\times 10 = 1072.32 \text{ mm}^2$$

Hence minimum of the two area will be the net area

$\Rightarrow$ $$A_{net} = 1072.32 \text{ mm}^2$$

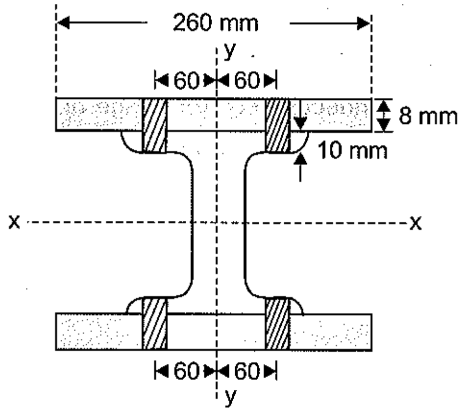
(ii) The maximum tensile force carried by the tension member (P_{max}),

$$P_{max} = \sigma_{at} \times \text{Area} = \frac{1072.32 \times 150}{1000} = 160.85 \text{ kN}$$

Q-3: ***An I-section with two cover plates is used as a tension member and is subjected to a load of 120 t over a length of 16 m. If the allowable tensile stress is 1500 kg/cm², is the member design safe? If so, find the factor of safety. ISWB 300@ 48.1 kg/m, A = 61.33 cm², I_{xx} = 9821.6 cm⁴, I_{yy} = 990.1 cm⁴. There are two rows of fasteners in each flange, thus making at least four holes in one section. The fastener diameter is 22 mm, cover plate size 260 mm × 8 mm, flange thickness of I-section = 10 mm. The centres of fastener is located at 60 mm from centre line of the I-section.***

[10 Marks, ESE-2006]

Sol: According to given data the cross-section is as shown below



Given:

Load, $P = 120t = 1200$ kN

Permissible tensile stress, $\sigma_{at} = 1500 \text{ kg/cm}^2 = 150 \text{ MPa} = 150 \text{ N/mm}^2$

$\therefore$ Net area required $= \dfrac{P}{\sigma_{at}} = \dfrac{1200\times10^3}{150} = 8000 \text{ mm}^2$

Net area of I-section provided = Gross area – Area of fastners holes

$= 6133 - 4 \times (23.5) \times 10 = 5193 \text{ mm}^2$

Net area provided by plates $= [260 \times 8 - 2 \times 23.5 \times 8] \times 2 = 3408 \text{ mm}^2$

Total net area $= 5193 + 3408 = 8601 \text{ mm}^2 > 8000 \text{ mm}^2$

So the tension member is safe under that-loading.

Max. Actual load can be applied $= (A_{net})\text{provided} \times \sigma_{sat} = \dfrac{8601\times150}{1000} = 1290.15 \text{ kN}$

Factor of safety $= \dfrac{\text{Actual load}}{\text{Load Applied}} = \dfrac{1290.15}{1200} = 1.075$

Q-4: ***Explain how the net effective section is calculated for angles and T-sections in tension as per IS 800.***

[10 Marks, ESE-2008]

Sol: An angle is usually connected to a gusset plate by one leg and tee is connected through its flange only. This causes an eccentric loading on the member which results in a non-uniform stress distribution on the section. The connected leg will be more highly stressed than the outstanding leg. To account for this non-uniform stress distribution, we can assume the net effective area to be smaller than the actual net area. Indian Standard Code IS : 800 - 1982 recommends the following net effective areas for angles and tees.

Case-1: In case of single angles in tension, connected by one leg only,

$$\text{Net effective area} = A_1 + k_1A_2 \quad \text{...(i)}$$

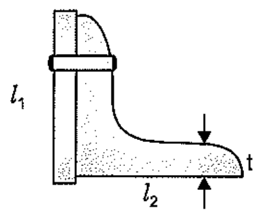
where A_1 = Net sectional area of the connected leg

A_2 = Area of the outstanding leg.

$$k_1 = \frac{3A_1}{3A_1 + A_2} \quad \text{...(ii)}$$

$$A_1 = \left(l_1 - \frac{t}{2} - d_n\right)t$$

$$A_2 = \left(l_2 - \frac{t}{2}\right) \times t$$



d_n = Dia. of hole due to rivet.

Case 2: In the case of a pair of angles back to back (or single tee) in tension connected by only one leg of each angle (or by the flange of a tee) to the same side of the gusset,

$$\text{Net effective area} = A_1 + k_2A_2 \quad \text{...(iii)}$$

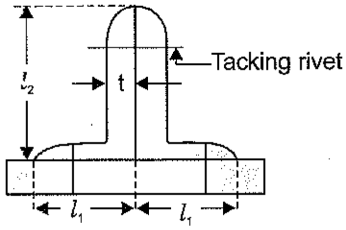
where, A_1 = Net area of the connected legs (or flange of the tee);

A_2 = Area of outstanding leg (or web of the tee)

$$k_2 = \frac{5A_1}{5A_1 + A_2} \quad \text{...(iv)}$$

$$A_1 = 2\left(l_1 - \frac{t}{2} - d_n\right)t$$

$$A_2 = 2\left(l_2 - \frac{t}{2}\right) \times t$$

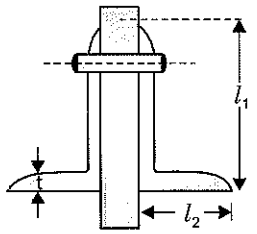


In order that the two angles act as one member, it is necessary to connect them together along their length by tacking rivets. The maximum spacing of tacking rivets shall be 1.0 metre. If tacking rivets or welds are not used, both the angles bend separately and their net effective area should be computed according to Eq. (i)

Case 3: For double angles or tees, carrying direct tension, placed back to back and connected to both sides of the gusset or to both sides of a part of a rolled section, the area to be taken in computing the mean tensile strength shall be the full gross area less the deduction for holes, provided the members are connected together along their length by tacking rivets, the spacing of which shall not exceed 1 m.

$\therefore$ Net effective area = Gross area of section – Area of holes

If the tacking rivet are not used, each angle or tee will behave separately. Then the effective section of an angle will be computed by Eq. (i) and the effective section of a tee by eq. (iii).



$$A_{net} = 2(l_1 + l_2 - t) \times t - 2d_nt$$

Q-5: ***Design a single equal-angle tension member. It is 4 m long and subjected to a tensile load of 25 t. It is connected to a gusseted plate through one leg only. Assume rivet diameter of 18 mm.***

σ_t = 1500 kg/cm²

Check for slenderness ratio. Given, r_{min} = 1.94 cm

[15 Marks, ESE-2010]

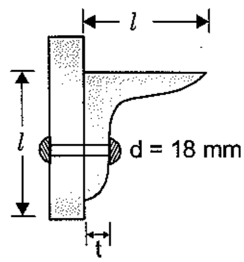
Sol:

$$d = 18 \text{ mm}$$

$$d' = 18 + 1.5 = 19.5 \text{ mm}$$

$$\sigma_t = 1500 \text{ kg/cm}^2$$

$$r_{min} = 1.94 \text{ cm}$$



$$\therefore \quad \text{Slenderness ratio} = \frac{l_{eff}}{r_{min}} = \left\{\frac{400}{1.94}\right\} = 206.185 < 350$$

Hence for slenderness ratio, the tension member is safe.

$$\text{Area required} = \frac{P}{\sigma_{at}} = \frac{25 \times 1000}{1500} = 16.67 \text{ cm}^2 = 1667 \text{ mm}^2$$

From figure, Area of connected leg $(A_1) = \left(l - \frac{t}{2} - d\right) \times t$

Assume t = 8 mm

$$A_1 = (l - 4 - 19.5) \times 8 = (8\ell - 188)$$

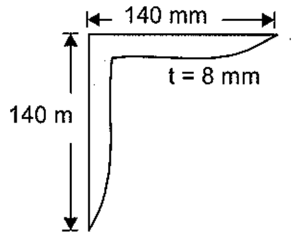
and $A_2 = (l - 4) \times 8 = (8\ell - 32)$

$$\therefore \quad K = \frac{3A_1}{3A_1 + A_2} = \frac{3(8\ell - 188)}{3(8\ell - 188) + (8\ell - 32)} = \frac{24\ell - 564}{24\ell - 564 + 8\ell - 32}$$

$$\therefore \quad K = \frac{24l - 564}{32l - 596}$$

$$\therefore \quad A_{net} = (A_1 + kA_2) = (8l - 188) + \left(\frac{24l - 564}{32l - 596}\right) \times (8\ell - 32)$$

$$= \frac{(8l - 188)(32l - 596) + (24l - 564)(8l - 32)}{(32l - 596)}$$



$$448\ell^2 - 16064\ell + 130096 = 1667(32\ell - 596)$$

$$\Rightarrow \quad 448\ell^2 - 69408\ell + 1123628 = 0$$

$$\ell = 136.56, 18.36$$

Hence take $\ell = 140$

Hence the angle is 140 × 140 × 8 mm.

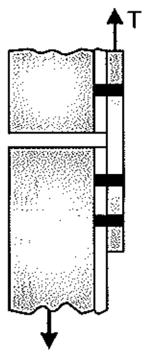
Q-6: ***What is the principle of design of a splice in a steel member subjected to an axial tensile force? Explain with the help of neat sketch.***

[5 Marks, ESE-2014]

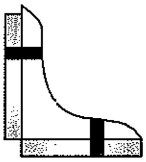
Sol: When the required length of a tension member is less than available length or when two lengths of a tension member have different cross-section tension splices are provided to join the two lengths of the member.

Principle of design of a tension splice:

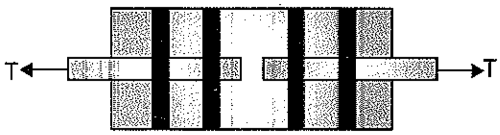
- The effect of eccentricity is neglected, as far as possible it should be avoided. Figure below shows an angle section spliced on one leg of the only. Such an arrangement causes eccentricity and introduces bending moments.



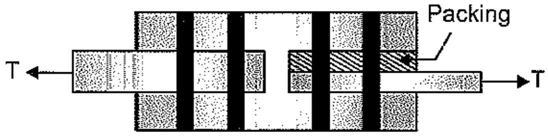
- To overcome this both the legs of the angle should be spliced as shown below :



- The splice cover plates or angles and its connections should be designed to develop the net tensile strength of the main member.
- The forces in the main member are transferred to the cover plate angle sections through the bolt/ welding and carried through these covers across the joint and is transferred to the other portion of the section through the fastener.

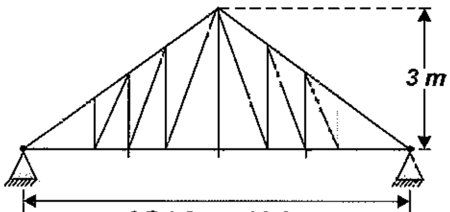


Splicing when both tension members are of same cross-section



Splicing when tension members are of different cross-section.

Q-7:



The bottom chord of the truss is composed of 2 Nos. ISA 60 × 60 × 6 with gusset thickness of 10 mm (back to back and both sides of the gusset). Calculate the factored load carrying capacity of the member under compression only.

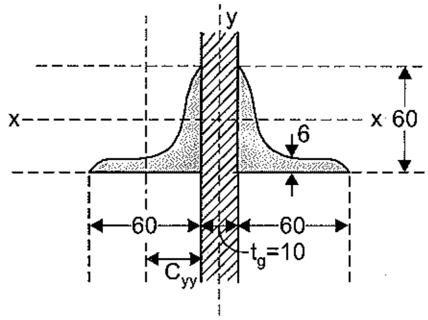
(ISA 60 × 60 × 6 : A = 684 mm², $I_{XX} = I_{YY}$ = 226000 mm⁴, $C_{XX} = C_{YY}$ = 16.9 mm)

KL/r	80	90	100	110	120	130	140	150
f_{cd} (MPa)	118	105	92.6	82.1	73	65.2	58.4	52.6

Longitudinal ties are provided at alternate node of the bottom chord.

[15 Marks, ESE-2015]

Sol: The bottom chord of a truss is under tension, but due to reversal of loads, it can be under compression also.



∴ Effective length of bottom chord member, l_{eff} = 1.8m

Slenderness ratio, $$\lambda = \frac{l_{eff}}{r_{min}}$$

$$r_{min} = \sqrt{\frac{I_{min}}{A}}$$

$$I_{xx} = 2I_{xx,one}$$

$$I_{yy} = 2I_{yy,one} + 2A\left(C_{yy} + \frac{t_g}{2}\right)^2$$

$\Rightarrow$ $$I_{xx} < I_{yy}$$

$\Rightarrow$ $$I_{min} = I_{xx}$$

$\Rightarrow$ $$r_{min} = \sqrt{\frac{2I_{xx,one}}{2A}} = \sqrt{\frac{I_{xx,one}}{A}}$$

$$= \sqrt{\frac{226000}{684}} = 18.177$$

$\Rightarrow$ $$\tau = \frac{1800}{18.177} = 99.025$$

From the given table, $$f_{cd} = 92.6 + \frac{105-92.6}{100-90}(100-99.025)$$

$$= 93.81 \text{ N/mm}^2$$

$\Rightarrow$ $$\text{Factored load} = f_{cd} \times 2A = 93.81 \times 2 \times 684 = 128.33 \text{ kN}$$

Q-8: ***In a roof truss, a diagonal consists of an ISA 60 mm × 60 mm × 8 mm (ISA 6060 @ 0.07 kN/m) and it is connected to gusset plate by one leg only by 18 mm diameter rivets in one chain line along the length of the member. Determine tensile strength of the member, if yield stress for steel is 250 MPa.***

[12 Marks, ESE-2019]

Sol: We will follow the recommendation of IS 800 : 1984 since the necessary data for analysis is not given like the location of connection of bolt, f_u.

$$\text{Diameter of rivet hole} = 18 + 1.5 = 19.5 \text{ mm}$$

$$\text{Net effective area provided. } A_{eff} = A_1 + KA_2$$

where, A_1 = Net sectional area of the connected leg

A_2 = Area of the outstanding leg.

If angle connected by one leg only

$$K = \frac{3A_1}{3A_1 + A_2}$$

Here, $$A_1 = \left(60 - \frac{8}{2} - 19.5\right) \times 8 = 292 \text{ mm}^2$$

$$A_2 = \left(60 - \frac{8}{2}\right) \times 8 = 448 \text{ mm}^2$$

$$K = \frac{3 \times 292}{3 \times 292 + 448} = 0.6616$$

$$A_{eff} = 292 + 0.6616 \times 448 = 588.41 \text{ mm}^2$$

$$\text{Allowable tensile stress, } (f_a) = 0.6 \times f_y = 0.6 \times 250 = 150 \text{ N/mm}^2$$

So, $$\text{Tensile strength} = f_a \times A_{eff} = 150 \times 588.41$$

$$= 88.261 \text{ kN}$$

CHAPTER 4 DESIGN OF COMPRESSION MEMBERS

Q-1: ***A built up column of 6 metre length comprises of 4 ISA 100 100 10, adequately laced, placed with their corners coincident with the corners of a square of 400 mm side. The column is hinged at the base and continuous at the top. Compute the maximum load that the column can support. Design a slab base for the column if the safe bearing pressure on concrete is 3 MPa and the safe bearing capacity of soil is 200 kN/m².***

L / r	***10***	***20***	***30***	***40***	***50***	***60***
σ_{ac}(MPa)	***150***	***148***	***145***	***139***	***132***	***122***

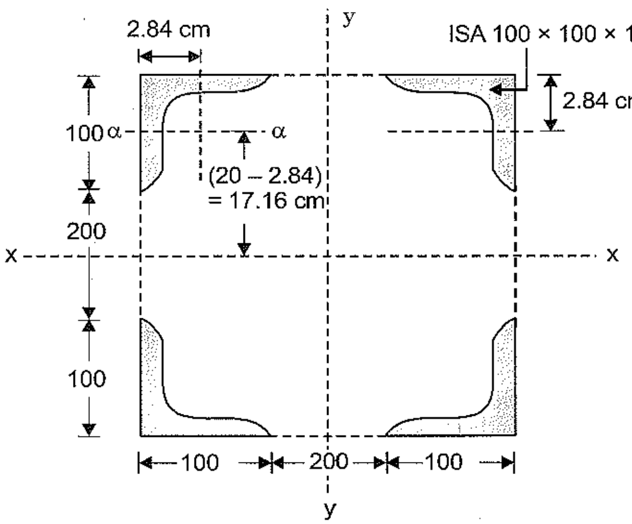
σ_{bs} = 185 MPa;

Properties of ISA 100 100 10:

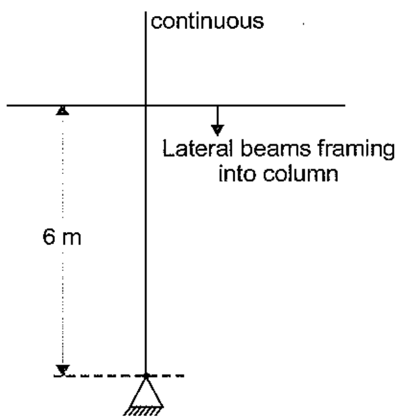
Area = 19.03 cm²; $I_x = I_y$ = 177 cm⁴; $C'_x = C'_y$ = 2.84 cm

[15 Marks, ESE-1995]

Sol:

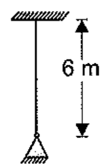


x-Section of column



To calculate the max load that the column can support, we assume the continuous end to be fixed due to heavy beam and column framing into each other at the continuous end.

Thus effective length of column is corresponding to the column support condition shown below.



$l_{eff} = 0.8l$ (as per is code recommendation)

Axial load carrying capacity = σ_{ac}. A

Area = 4 × 19.03 cm² = 76.12 cm²

σ_{ac} is a function of slenderness ratio (λ)

$$\text{Slenderness ratio} = \frac{\ell_{eff}}{r_{min}}$$

where, r_{min} = Minimum radius of gyration

Minimum radius of gyration is about xx or yy axis because xx and yy axis are the axis of symmetry and hence are the principal axes. As the x–x and y – y axis are the axis of symmetry, $r_{xx} = r_{yy}$ $r_{min} = r_{max}$.

$$r_{xx} = \sqrt{\frac{I_{xx}}{A}}$$

$$I_{xx} = 4 \times [I_{xx} + A_{one}\,(17.16)^2]$$

$$= 4\,[177 + 19.03\,(17.16)^2]\ \text{cm}^4$$

$$I_{xx} = 23122.72\ \text{cm}^4$$

$$r_{xx} = \sqrt{\frac{23122\cdot 72}{4 \times 19.03}}\ \text{cm} = 17.429\ \text{cm}$$

$$\Rightarrow \qquad \lambda = \frac{l_{eff}}{r_{min}} = \frac{l_{eff}}{r_{xx}} = \frac{0.8 \times 600}{17.429} = 27.54$$

$$\Rightarrow \qquad \sigma_{ac} = 148 - \frac{3 \times 7.54}{10} = 145.738\ \text{N/mm}^2$$

$$\Rightarrow \qquad \text{Max load carrying capacity} = \sigma_{ac} \times A = 145.738 \times 4 \times 19.03 \times 10^2\ \text{N}$$

$$= 1109.357\ \text{kN}$$

Design of slab base

As the loading is axial, slab base will also be symmetrical

$$\text{Area of base plate required} = \frac{\text{Load}}{\text{Bearing capacity of concrete}}$$

Let us increase the load by 10% to account for the D.L of the structure

$$\Rightarrow \qquad \text{Area required} = \frac{1.1 \times 1109.357\ \text{kN}}{3 \times 10^6\ \text{N/m}^2} = \frac{1.1 \times 1109.357 \times 10^3\ \text{N}}{3 \times 10^6\ \text{N/m}^2}$$

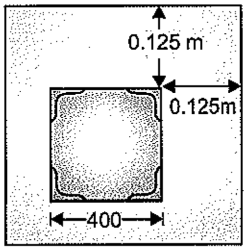
$$\Rightarrow \qquad B \times B = (0.638)^2\ \text{m}$$

$$\Rightarrow \qquad B = 0.638\ \text{m}$$

$$\text{Adopt} \quad B = 0.65\ \text{m}$$

To calculate the thickness of base plate we use the formula

$$t = \sqrt{\frac{3W}{\sigma_{bs}}\left(a^2 - \frac{b^2}{4}\right)}$$



$$\text{where, } w = \text{base pressure} = \frac{1.1 \times 1109.357 \times 10^3\ \text{N}}{(0.65 \times 10^3)^2\ \text{mm}^2} = 2.89\ \text{N/mm}^2$$

$$\sigma_{bs} = 185\ \text{N/mm}^2$$

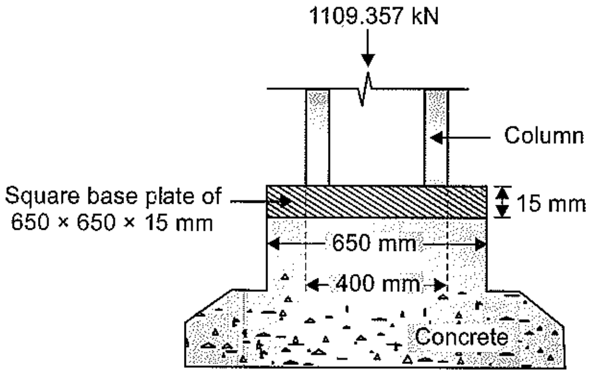
$$a = \text{Larger overhang} = 0.125\ \text{m}$$

$$b = \text{Smaller overhang} = 0.125\ \text{m}$$

$$\Rightarrow \qquad t = \sqrt{\frac{2.89}{185}(0.125)^2 \times \frac{3}{4}}$$

$$t = 0.01353 \text{ m} = 13.53 \text{ mm}$$

let us adopt t = 15 mm



Q-2: ***A column of length 9.4 m is effectively held in position at both ends but not restrained in direction. It is to be constructed by using four equal angle sections ISA 100 × 100 × 10 mm thickness, properties of which are : A = 19.03 cm²; $C_{xx} = C_{yy}$ = 2.84 cm; $I_{xx} = I_{yy}$ = 177 cm⁴; $r_{xx} = r_{yy}$ = 3.05 cm; r_{uu} = 3.85 cm and r_{vv} = 1.94 cm.***

Determine a suitable arrangement to carry an axial load of 75000 kg. Permissible stresses in compression may be interpolated from the following table.

Slenderness ratio (l/r)	***60***	***70***	***80***	***90***	***100***
Permissible stress in kg/sq. cm	***1220***	***1120***	***1010***	***900***	***800***

Specify the spacing of lacings. According to Code the maximum spacing of lacing bars shall be such that the minimum slenderness ratio (l/r) of the components of the member between consecutive connections is not greater than 50 or 0.7 times the most unfavorable slenderness ratio of the member as a whole, whichever is less, where l is the distance between the centres of connection of lattice bars to each component.

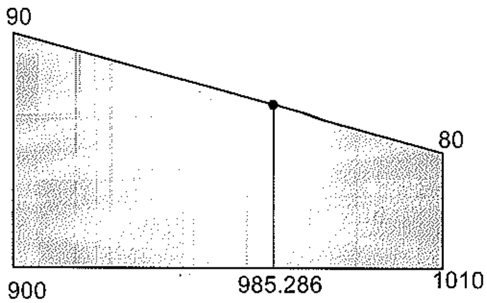
[15 Marks, ESE-1996]

Sol: Given loading for which we have to design the compression in member is (P) = 75000 kg

Area of section Provided = 4 × 19.03 × 100 = 7612 mm² = 76.12 cm2

$$\therefore \qquad \sigma_{ac} = \frac{P}{\text{Area}} = \frac{75000}{76.12}\text{kg} = 985.286 \text{ kg/cm}^2$$

Hence slenderness ratio (l) can be determined by inter polation as follows



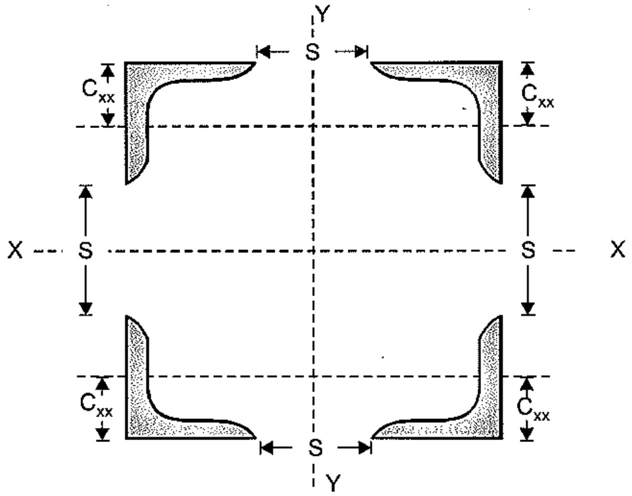
$$\lambda = 80 + \frac{90-80}{1010-900} \times (1010 - 985.286) = 82.247$$

Effective length of column = 9.4 m

$$\therefore \quad r_{min} = \frac{l_{eff}}{\lambda} = \frac{9.4 \times 1000}{82.247} = 114.29 \text{ mm}$$

$$\therefore \quad I_{min} = Ar^2_{min} = 4 \times 1903 \times 114.29^2 = 99.4295 \times 10^6 \text{ mm}^4$$

Let the four angles are arranged in such a way that the spacing (s) between them is equal on all the four faces of the column as shown below



For most efficient coloumn section,

$$I_{xx} = I_{yy} = I_{min}$$

$$\Rightarrow \quad I_{xx} = 4\left[177 \times 10^4 + 1903 \times \left(100 + \frac{S}{2} - C_{xx}\right)^2\right]$$

$$\Rightarrow \quad 99.4295 \times 10^6 = 4\left[177 \times 10^4 + 1903\left(100 - 28.4 + \frac{S}{2}\right)^2\right]$$

$$\Rightarrow \quad 99.4295 \times 10^6 = 7080000 + 7612\,(71.6 + S/2)^2$$

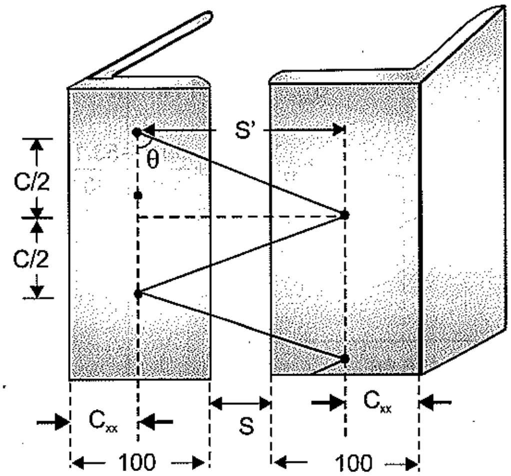
$$\therefore \quad S = 77.09 \text{ mm}$$

$$\therefore \quad \text{Adopting spacing} = 80 \text{ mm}$$

$$\therefore \quad S = 80 \text{ mm}$$

Design of lacing

Assuming the rivets are connected at the C.G of angles



Assuming $\theta = 45°$

And as, $S' = 2(100 + S/2 - C_{xx}) = 2(100 + 40 - 28.4) = 232.2$ mm

$\therefore \quad \tan\theta = \dfrac{S'}{C/2} \Rightarrow C = S' \times 2 = 2 \times 223.2 = 446.4$ mm

$\Rightarrow \quad \dfrac{C}{r_{min} \text{ of single section}} = \dfrac{446.4}{r_{vv}} = \dfrac{446.44}{19.4} = 23.01 \not> 50$ (OK)

and $\quad 0.7\,\lambda_{whole} = 0.7 \times 82.247 = 57.573$

$\Rightarrow \quad 23.01 \not> 57.513$ (OK)

Hence single lacing is sufficient at an angle with vertical equal to 45° and at a spacing of 445 mm. Although the problem is complete at this stage, for illustration purpose, Let us design the lacing.

Assuming diameter of rivet to be used = 20 mm

Hence, Width of the lacing (b) = 60 mm

and Effective length (L) $= \dfrac{S'}{\sin\theta} = \dfrac{S'}{\frac{1}{\sqrt{2}}} = \sqrt{2}S' = \sqrt{2} \times 223.2 = 315.60$ mm

Hence Thickness of lacing bar (t) $= \dfrac{L}{40} = 7.89$ mm

Thus adopt thickness of bar = 8 mm

Check for slenderness ratio of lacing

Slenderness ratio of lacing $(\lambda) = \dfrac{\text{leff}}{\left\{\frac{t}{\sqrt{12}}\right\}} = \dfrac{315.60}{\left(\frac{8}{\sqrt{12}}\right)} = 36.66 \not> 145$ (Hence OK)

calculation of Force for which lacing is to be designed

$$F = \frac{V}{2\sin\theta} = \frac{2.5P}{200\sin\theta} = \frac{2.5 \times P \times \sqrt{2}}{200} = \frac{2.5 \times 75000 \times \sqrt{2}}{200} = 1325.8 \text{ kg}$$

$= 13.258$ kN (assume $g = 10$ m/sec^2)

Check that lacing can sustained the compressive force F or not:

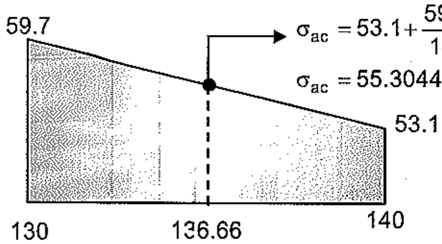
$[(\sigma_{ac})_{\text{of lacing}} \times (A_{gross})_{lacing}]$ should be greater than F

$\lambda_{lacing} = 136.66$

Hence by interpolation (assuming that for $\lambda = 130$, $\sigma_{ac} = 59.7$ N/mm^2 and for

$\lambda = 140$,

$\sigma_{ac} = 53.1$ N/mm^2



$\sigma_{ac} = 53.1 + \dfrac{59.7 - 53.1}{140 - 130}(140 - 136.66)$

$\sigma_{ac} = 55.3044$

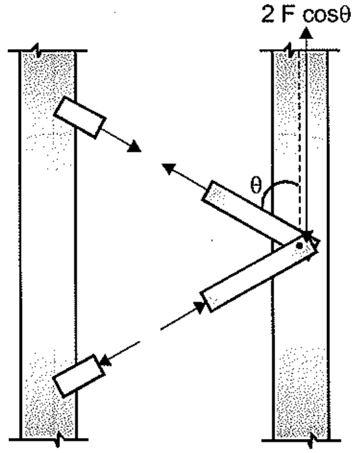
$\therefore \quad \dfrac{(55.3044) \times 60 \times 8}{1000} = 26.546 \text{ kN} > 13.26 \text{ kN}$ (Hence OK)

Check that lacing is safe for tension or not

$$(B-d)\times t\times\sigma_{at} > F$$

$$\Rightarrow \quad \frac{(60-21.5)\times 8\times 0.6\times 250}{1000} = 46.2 > 13.26 \text{ kN (Hence OK)}$$

Connection design



$$\text{number of rivets} = \frac{2F\cos\theta}{R_v}$$

Detemination of Rivet value (R_V)

$$\text{Shearing strength} = \frac{\pi}{4}\times 21.5^2\times 100 = 36.30 \text{ kN}$$

$$\text{Bearing strength} = \sigma_{br}\times t\times d_h = 300\times 8\times 21.5 = 51.6 \text{ kN}$$

$$\therefore \quad R_V = 36.30 \text{ kV.}$$

$$\therefore \quad \text{Number of rivet} = \frac{2\times 13.26\times 1}{36.30\times\sqrt{2}} = 0.516$$

Hence one rivet is sufficient.

Q-3: ***A continuous principal rafter of a truss is 3 m long between intermediate connections. It comprises of two ISA 90 × 90 × 8 connected on both sides of 16 mm thick gusset plate. The two angle sections are tack riveted using 16 mm thick washers along the length at 30 cm interval. Assume effective length between taking rivets = 30 cm. Assuming effective length equal to 0.85 times the distance between intersections, calculate the maximum force that can be taken up by the rafter.***

Allowable stress (f) in compression may be taken from the following table, 'λ' being slenderness ratio:

λ	50	70	90	110	130	150	–
f	*1320*	*1120*	*900*	*720*	*570*	*450*	kg/cm^2

Properties of ISA 90 × 90 × 8 are:

$A = 13.79$ cm^2, $C_{xx} = C_{yy} = 2.51$ cm,

$r_{xx} = r_{yy} = 2.75$ cm, $r_{uu} = 3.47$ cm, $r_{vv} = 1.75$ cm

[10 Marks, ESE-1997]

Sol:

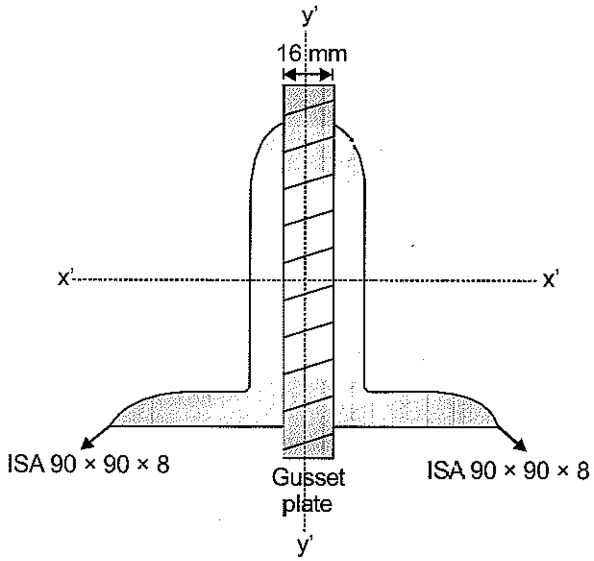


Figure (i)

Properties of ISA 90 × 90 × 8,

$$A = 13.79 \text{ cm}^2$$
$$C_{xx} = C_{yy} = 2.51 \text{ cm}$$
$$r_{xx} = r_{yy} = 2.75 \text{ cm}$$
$$r_{uu} = 3.47 \text{ cm}$$
$$r_{vv} = 1.75 \text{ cm}$$

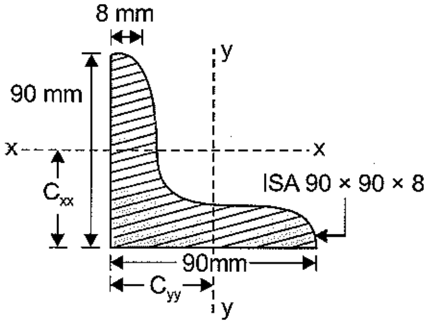


Figure (2)

⇒ Since the member shown above (fig- i) has y-y axis as a axis of symmetry. Hence minimum I of the combined section will be either I'_{xx} or I'_{yy}.

⇒ We can clearly observe that I'_{xx} of combination will become $2I_{xx}$ but the I'_{yy} becomes greater than $2I_{yy}$.

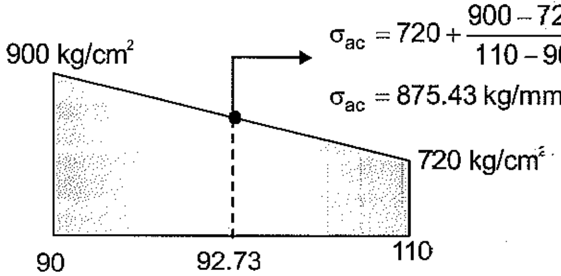
where I_{xx} and I_{yy} = M.O.I of single section along respective axis and I'_{xx} and I'_{yy} are M.O.I. of the combined section about respective axis.

⇒ Hence $(I_{min})_{combination} = 2\,(I_{xx})$single section.

$$\therefore \quad (r_{min})_{combination} = \sqrt{\frac{2I_{xx}}{2A}} = \sqrt{\frac{I_{xx}}{A}} = r_{xx} = 2.75 \text{ cm}$$

$$\Rightarrow \quad \lambda = \frac{l_{eff}}{r_{min}} = \frac{300 \times 0.85}{2.75} = 92.73$$

⇒ By using method of interpolation we can get σ_{ac} at $\lambda = 92.73$



$$\sigma_{ac} = 720 + \frac{900-720}{110-90}(110-92.73)$$
$$\sigma_{ac} = 875.43 \text{ kg/mm}^2$$

$$\lambda = 90 \qquad \sigma_{ac} = 900 \text{ Kg/cm}^2$$

$$\lambda = 110 \qquad \sigma_{ac} = 720 \text{ Kg/cm}^2$$

Hence, at $\lambda = 92.73$, $\sigma_{ac} = 875.43$ Kg/cm^2.

⇒ The maximum force that can be taken by the column is

$$F = \sigma_{ac} \times \text{Area} = 875.43 \times (13.79 \times 2) = 24144.36 \text{ Kg}$$

Check for local buckling: We know that for two sections placed back to back.

$$\lambda \text{ of individual section betwen tacking rivet} = \left[\frac{\ell_{eff}}{(r_{min})\text{ single section}}\right] \ngtr 40/ \ngtr 0.6\lambda \text{ of whole section.}$$

$$\Rightarrow \qquad \frac{30}{r_{vv}} \ngtr 40 \ngtr 0.6 \times 92.73$$

$$\Rightarrow \qquad \frac{30}{1.75} = 17.14 \ngtr 40 \ngtr 55.64$$

Thus section is safe in local buckling.

Q-4: ***How does the design of a tension member differs from that of a compression member?***

[6 Marks, ESE-1999]

Sol: Comparison between tension and compression members is given below illustrates the difference in design of tension & compromise member.

Tension Member	Compression Member
1. Net area is effective	1. Gross-area is effective
2. There is no stability problem	2. There is stability problem because the member may fail by buckling before achieving the full strength
3. Permissible stress is constant	3. Permissible stress depends upon the slenderness ratio which in turn depends upon length and cross-section of the member
4. As permissible stress is fixed, the require area can be determined directly A_{net} required = F/σ_{ac}	4. In this case to find out the area required, the formula that can be applied is A_{net} required = F/σ_{ac} for which we need to know σ_{ac} but σ_{ac} itself depends upon area. Hence we can't have a direct solution. We adopt trial and error solution in this case.

Q-5: ***A built up column consists of two ISMC 400 @ 494 N/m and two plates 500 mm × 10 mm. The clear distance between back to back of channels is 200 mm. One plate is connected to each flange side. Determine the safe load carrying capacity of the section if the effective length of the column is 5.0 m. Use the following data :***

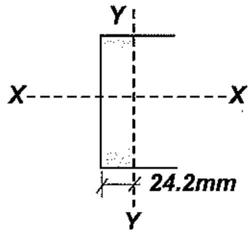
Slenderness Ratio	***30***	***50***	***60***	***70***	***80***	***–***
Permissible Stress	***145***	***132***	***122***	***112***	***101***	***N/m^2***

Determine the safeload carrying capacity of the section if the effective length of the column is 6.0 m.

Properties of ISMC 400 are:

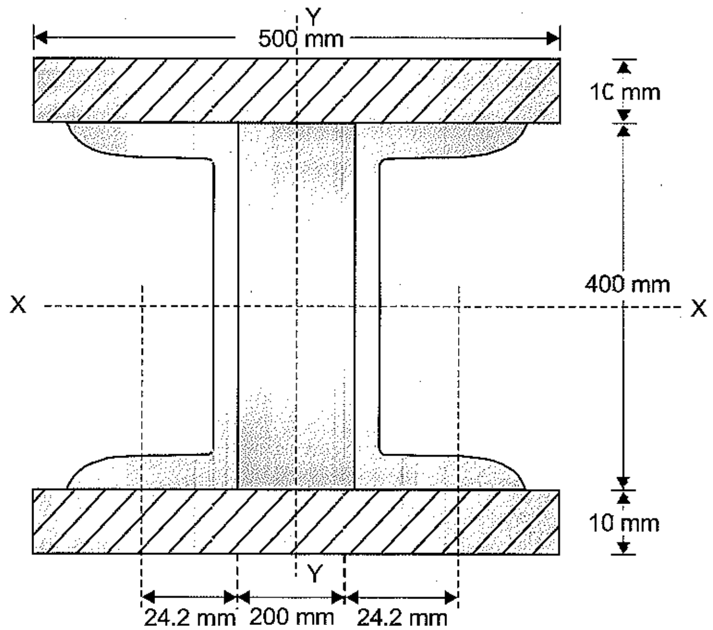
$$\textit{Area of section} = 6.293 \times 10^3 \text{ mm}^2$$
$$I_{xx} = 1,50,828 \times 10^3 \text{ mm}^4$$
$$I_{yy} = 5,048 \times 10^3 \text{ mm}^4,$$

Distance of C.G from the back of the channel = 24.2 mm.



[12 Marks, ESE-1999]

Sol: The cross-section of the column can be shown in figure below,



- The Safe axial load carrying capacity can be determined as,

$$P = \sigma_{ac} \times A_{gross}$$

where P = Load carrying capacity of column

σ_{ac} = Permissible axial compresive stress in column section

A_{gross} = Gross Area of column section.

- Area of column section

$$= 2 \times [\text{Area of channel} + \text{Area of plate}]$$
$$= 2 \times [6293 + 500 \times 10] = 22586 \text{ mm}^2$$

- σ_{ac} depends upon slenderness ratio which in turn depends r_{min}. Hence the problem reduces to findings out r_{min} of the section. When the section has axis of symmetry then it is well known that the minimum M.O.I will be either about the axis of symmetry or, perpendicular to the axis of symmetry. Hence in this case we will have to calculate I_{xx} and I_{yy} for obtaining the minimum M.O.I.

$$I_{xx} = (2 \times 150828 \times 10^3) + \left[\frac{500 \times 10^3}{12} + 500 \times 10(200+5)^2\right] \times 2$$

$$I_{xx} = 721989333.3 \text{ mm}^4 = 721.99 \times 10^6 \text{ mm}^4$$

$$I_{yy} = [2 \times 5048 \times 10^3 + 2 \times 6293 \times 124.2^2] + \frac{10 \times 500^2}{12} \times 2$$

$$I_{yy} = 412576438.4\ mm^4 = 412.576 \times 10^6\ mm^4$$

$$\therefore \quad I_{yy} < I_{xx}$$

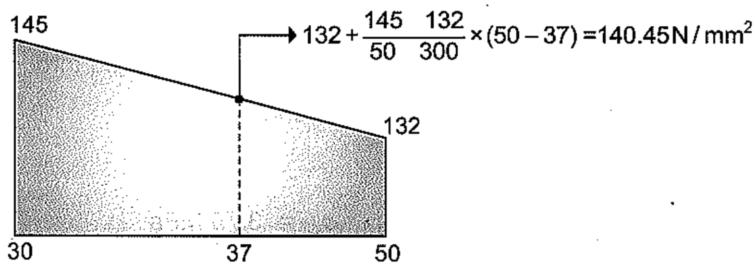
$$r_{min} < \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{412.576 \times 10^6}{(2 \times 6293 + 2 \times 500 \times 10)}} = 135.155\ mm$$

$$\therefore \quad l_{eff} = 5m = 5000\ mm\ \text{(given)}$$

$$\therefore \quad \lambda\ \text{(slenderness ratio)} = \frac{l_{eff}}{r_{min}} = \frac{5000}{135.155} = 36.99 \cong 37 < 180\ \text{(Hence OK)}$$

From the given relationship between λ and σ_{ac} we can interpolate the value of σ_{ac} when λ = 37

λ	σ_{ac} (N/mm^2)
30	145
50	132
37	?



$$\therefore \quad P = \sigma_{ac} \times A = \frac{[140.45 \times 22586]}{1000} = 3172.2\ kN$$

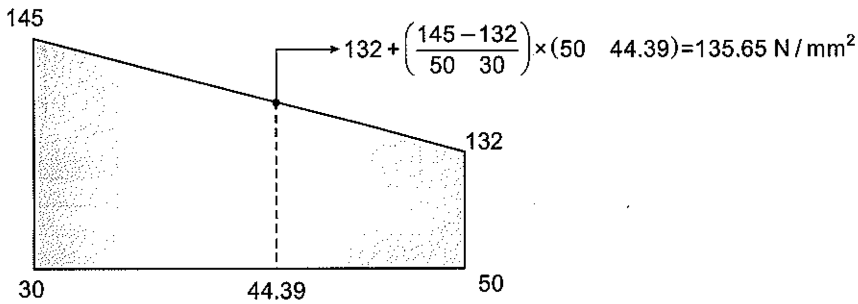
Thus safe load carrying capacity = 3172.2 kN

when the effective length (l_{eff}) = 6 m

$$\lambda\ \text{(slenderness ratio)} = \frac{6000}{135.15} = 44.39$$

By interpolation,

λ	σ_{ac} (N/mm^2)
30	145
50	132
44.33	–

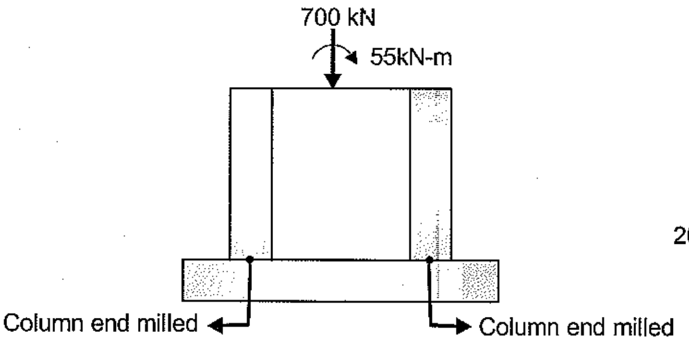


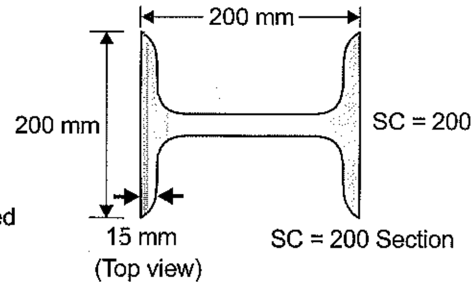
$$\Rightarrow \quad P = \sigma_{ac} \times A = \frac{135.65 \times 22586}{1000} = 3063.71\ kN \quad \text{Ans.}$$

Q-6: ***A column of section SC 200 is to support an axial compressive load of 700 kN and a moment of 55 kNm. Design column slab base, assuming bearing column end to be milled. The allowable bearing stess for concrete in footing is 3.75 MPa, yield stress F_y of steel is 250 MPa. Also design the welded connection between column and the base slab. T for the column flange is 15 mm. Draw a sketch giving the structural details. Take strength of 1 mm fillet weld/mm. Length as 76.0 N; F_{bt} = 0.75 F_y, m= 0.25)***

[15 Marks, ESE-2000]

Sol:





Since column ends are milled so, the axial force is transferred through bearing only.

Thus Connection will be design for moment only.

Given data,

$$\sigma_c = 3.75 \text{ MPa}$$

$$f_y = 250 \text{ MPa}$$

$$\text{Strength of 1 mm fillet weld per mm length} = 76 \text{ N}$$

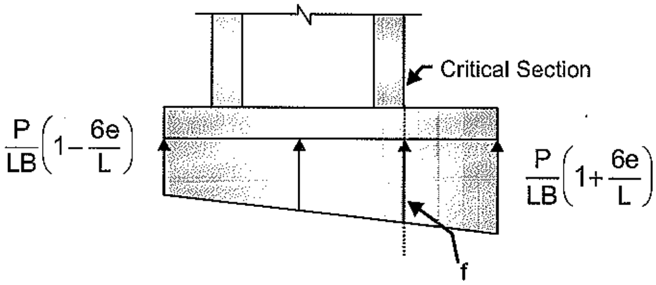
$$f_{bt} = 0.75\, f_y$$

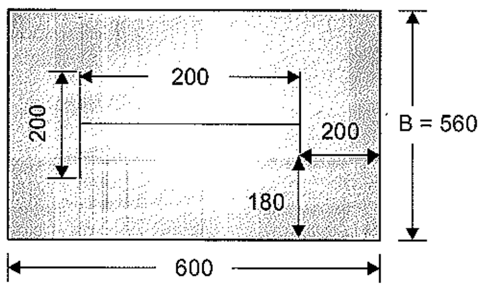
$$\mu = 0.25$$

Eccentricity = $\frac{M}{P} = \frac{55}{700} = 0.0785$ m and we know that if $e < \frac{L}{6}$, there will be compression throughout the base.

For no tension $L \geq 6e \Rightarrow 6 \times 0.0785 = 0.471\text{m}$

Hence adopt $L = 600$ mm.





For safety of concrete

$$\frac{P}{LB}\left(1+\frac{6e}{L}\right) \leq \sigma_c$$

$$\Rightarrow \qquad B \geq \frac{P}{L\sigma_c}\left(1+\frac{6e}{L}\right)$$

$$\Rightarrow \qquad B \geq \frac{700\times1000}{600\times3.75}\left(1+\frac{6\times78.5}{600}\right)$$

$$\therefore \qquad B \geq 555.33$$

adopt B = 560 mm

$$\text{Compressive stress at one end} = \frac{P}{LB}\left(1-\frac{6e}{L}\right) = \frac{700\times1000}{560\times600}\left(1-\frac{6\times78.5}{600}\right)$$

$$= 0.448\ \text{N/mm}^2 > 0$$

$$\text{Compressive stress at other end} = \frac{P}{LB}\left(1+\frac{6e}{L}\right) = \frac{700\times1000}{560\times600}\left(1+\frac{6\times78.5}{500}\right)$$

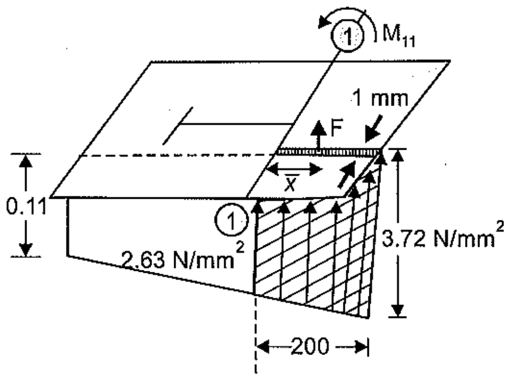
$$= 3.72\ \text{N/mm}^2 < 3.75\text{MPa}.$$

$\therefore$ Compressive stress at critical section can be found by interpolating the value due to linear variation.

$$\text{At critical section (f)} = 0.448+\frac{3.72-0.448}{600}\times400 = 2.63\ \text{N/mm}^2$$

BM about (1)-(1) axis

$$M_{1-1} = F\bar{x}$$



where, $F = \frac{2.63+3.72}{2}\times200\times1 = 635\ \text{N}$

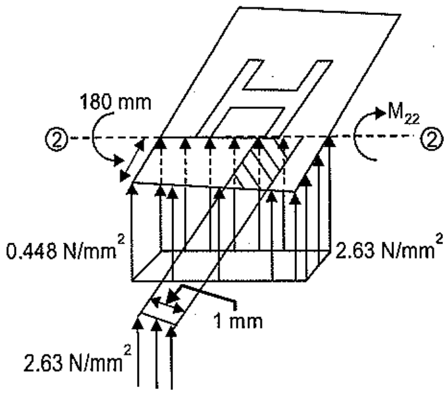
and $\bar{x} = \left(\frac{2a+b}{a+b}\right)\times\frac{h}{3}$ [c.g. distance of stress diagramm from (1) - (1)]

$$= \left(\frac{2\times3.720+2.63}{3.72+2.63}\right)\times\frac{200}{3} = 105.72\ \text{mm}$$

Hence $M_{11} = F\bar{x} = 67.132\times10^3$ N-mm

$$M_{22} = \frac{wl^2}{2} = \frac{2.63\times(180)^2}{2} = 42.606\times10^3\ \text{N-mm}$$

$$f_b = f_{11}-\mu f_{22} = \frac{M_{11}-\mu M_{22}}{1\times\left(\frac{t^2}{6}\right)}$$



We know that for safety,

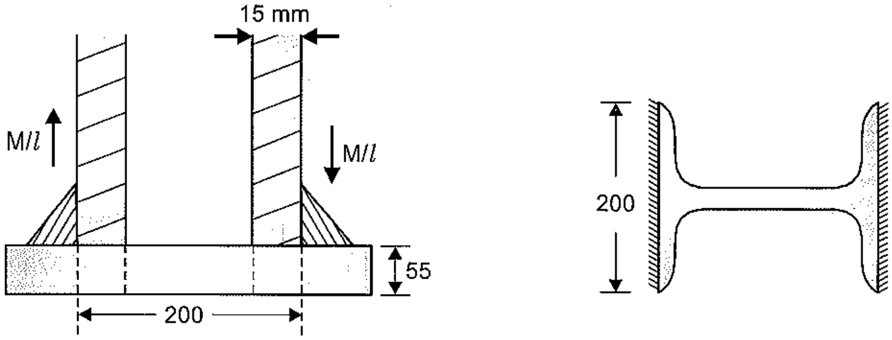
$$f_b \leq 0.75\ f_y$$

$$\Rightarrow \qquad \frac{67.132\times1000-0.25\times42.606\times1000}{t^2/6} \leq 0.75\times250$$

$$\Rightarrow \qquad t \geq 42.51 \text{ adopt } 43\ \text{mm}$$

Hence, adopt base plate as, length = 600 mm ; width = 560 mm ; thickness = 43 mm

Weld design



As the column ends are milled, the direct load will be transferred through direct bearing, hence the fillet weld will be designed to resist force due to bending only.

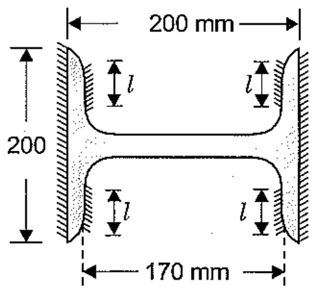
$$\text{Force to be resisted by weld} = \frac{M}{l} = \frac{M}{200} = \frac{55 \text{ kN-m}}{0.2 \text{ m}} = 275 \text{ kN}$$

$$\text{Max size of weld} = (15 - 1.5) \text{ mm} = 13.5 \text{ mm}$$

Hence by adopting size of weld as 13 mm, length of weld required (l) is given by

$$l = \frac{275 \times 1000}{76 \times 13} = 278.34 \text{ mm} \cong 280 \text{ mm}$$

Since the length available is only 200 mm on one side of flange hence we have to use otherside of flange also. The moment arm for the outer side weld will be different from the inner side weld.



$$\text{The net moment resisted by weld} = F_1 \times 200 + F_2 \times 170 \geq 55 \text{ kNm}$$

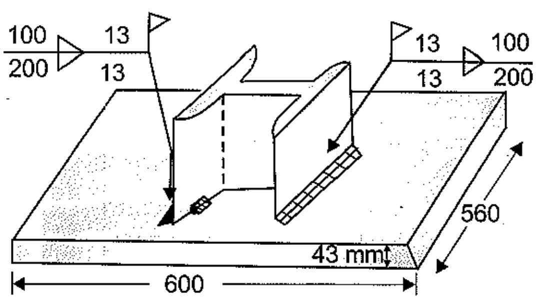
$$= (200 \times 13 \times 76) \times 200 + (2\ell \times 13 \times 76) \times 170 > 55 \times 10^6 \text{ Nmm}$$

$$l = 46.082 \text{ mm}$$

Adopt length as 50 mm.

The connections details areas shown below.

So,



Q-7: ***Design a suitable built up section using two channels for a steel column to carry an axial load of 1200 kN. The effective length of column is 5.8 m. Provide suitable single lacing with 45° angles of inclination. Adopt 22 mm diameter rivets. For Channel ISMC 350***

$$\text{Area of cross-section} = 5366 \text{ sq. mm}$$

$$\text{Width of flange} = 100 \text{ mm}$$

$$I_x = 100.08 \times 10^6 \text{ mm}^4$$

$$I_y = 4.306 \times 10^6 \text{ mm}^4$$

$$C_{yy} = 24.4 \text{ mm}$$

Permissible stress in axial compression for steel with $\sigma_y = 260\,N/sq.\ mm$:

1/r	Permissible Stress N/sq. mm
40	145
50	136
60	126
100	82
110	73
120	64
130	57
140	51
150	46

[28 Marks, ESE-2001]

Sol: Given loading ⇒

$$P = 1200 \text{ kN}$$

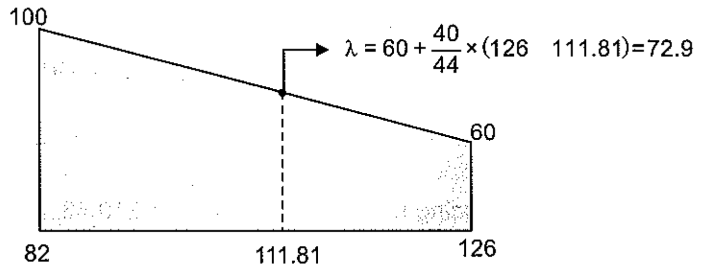
$$\text{Area of section} = 2 \times 5366 = 10732 \text{ mm}^2$$

$$\therefore \quad \sigma_{ac} = \frac{1200 \times 10^3}{10732} = 111.81\,\text{N/mm}^2$$

We can interpolate the value of λ with the help of given chart.

$$\sigma_{ac} = 82 \text{ N/mm}^2 \quad \lambda = 100$$

$$\sigma_{ac} = 126 \text{ N/mm}^2 \ \lambda =;\ 60$$



Hence

$$\lambda = 72.9 \text{ for } \sigma_{ac} = 111.81\,\text{N/mm}^2$$

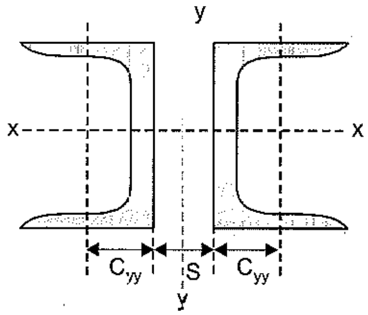
and

$$\lambda = \frac{l_{eff}}{r_{min}}$$

$$\Rightarrow \quad r_{min} = \frac{l_{eff}}{\lambda} = \frac{5800}{72.9} = 79.56 \text{ mm}$$

$$\therefore \quad I_{min} = Ar_{min}^2 = 10732 \times 79.56^2 = 67.9 \times 10^6 \text{mm}^4$$

Let the spacing between the two channel section is 'S'.



For most efficient use of materials $(I_{xx})_{combination} = (I_{yy})_{combination}$.

But $(I_{xx})_{Combination} = 2(I_{xx})_{single} = 2 \times 100.08 \times 10^6 = 200.08 \times 10^6$

But, $I_{min} = 67.9 \times 10^6\ mm^4$.

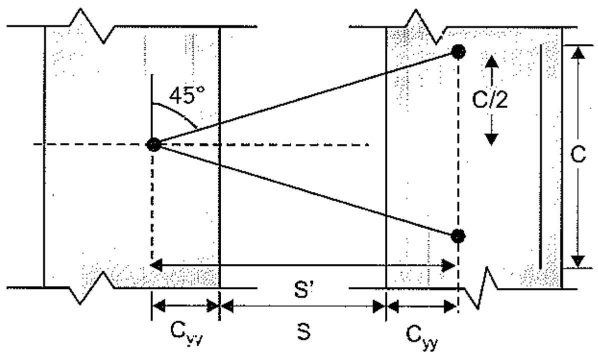
Hence $(I_{yy})_{combination}$

$$\Rightarrow \quad 2 \times 4.306 \times 10^6 + 2 \times 5366 \times \left(24.4 + \frac{S}{2}\right)^2 = 67.9 \times 10^6$$

$\Rightarrow \quad S = 99.85$ mm

Hence take S = 100 mm

Design of lacing: Let us adopt single lacing system



$$S' = S + 2C_{yy} = 100 + 2 \times 24.4 = 148.8 \text{ mm}$$

Let us adopt angle of lacing with vertical as 45°

$$\therefore \quad \tan 45° = \frac{S'}{(C/2)} \Rightarrow C = 2S' = 297.6 \text{ mm}$$

As per question we have to use 22 mm ϕ rivets, which will give the minimum width of lacing bar = 65 mm

$$\text{Length of lacing (L)} = \frac{C/2}{\sin 45°} = \frac{C}{\sqrt{2}} = 210.43 \text{ mm}$$

$$\therefore \quad \text{Thickness of lacing bar (t)} = \frac{L}{40} = \frac{210.43}{40} = 5.26 \text{ mm}$$

Hence adopt thickness of lacing = 8 mm

Check for local bulkling of lacing,

$$\lambda_{lacing} = \frac{l_{eff}}{r_{min}} = \frac{l_{eff}}{\frac{t}{\sqrt{12}}} = \frac{210.43}{\left(\frac{8}{\sqrt{12}}\right)} = 91.12 < 145 \text{ (O.K)}$$

Check for tension

Force carried by lacing bar $F = \dfrac{V}{2\sin\theta} = \dfrac{2.5\times1200\times1}{2\times\frac{1}{\sqrt{2}}\times100} = 21.21\,\text{kN}$ [V = 2.5% of axial load]

$$(B-d)\times t\times\sigma_{at} \text{ should be} > F$$

$$\Rightarrow \quad (65-23.5)\times8\times\frac{150}{1000} > 21.21\,\text{kN}$$

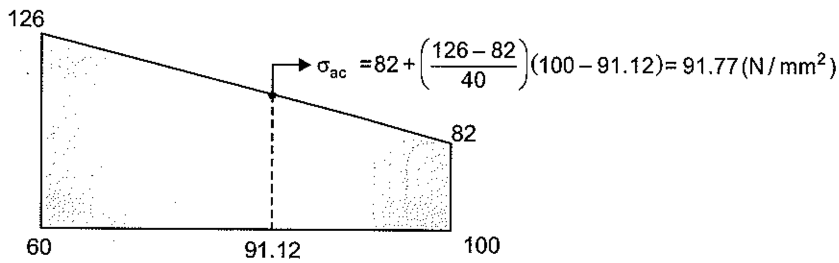
49.8 kN > 21.21 kN Hence OK

Check for compression

$$\sigma_{ac}\times[A_{gross}]_{lacing} > F$$

$$\lambda_{lacing} = \frac{l_{eff}}{\frac{t}{\sqrt{12}}} = 91.12$$

Hence interpolating the value of σ_{ac} for $\lambda = 91.12$



and $\quad [A_{gross}]_{lacing} = 65\times8 = 520\ \text{mm}^2$

$\therefore \quad \sigma_{ac}\times A = 91.77\times520 = 47720.4$ N

$= 47.72$ kN > F (21.21 kN)

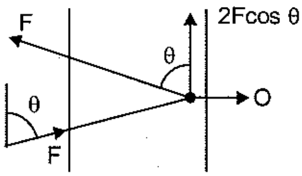
Design of Connection:

Rivet value: (Assuming power driven shop rivet)

$$\text{Shearing strength} = \frac{\pi}{4}\times d^2\times\sigma_S = \frac{\pi}{4}\times23.5^2\times\frac{100}{1000} = 43.37\ \text{kN}$$

$$\text{Bearing strength} = d\times t\times\sigma_{br} = 23.5\times8\times\frac{300}{1000} = 56.4\ \text{kN}$$

Hence $\quad R_V = 43.37$ kN



$$\therefore \quad \text{Number of Rivet} = \frac{2F\cos\theta}{R_y} = \frac{2\times21.21\times1}{43.37\times\sqrt{2}} = 0.69 \cong 1\ \text{rivet}$$

[Since $\theta < 60°$ we will design for $2F\cos\theta$]

Q-8: ***A stanchion of effective length l of 6 m consists of a built up twin box section using an ISMB 250 with two plates of 260 mm × 10 mm size welded each to the tips of the two flanges of ISMB 250 on both the sides of 250 mm depth with 4 mm size fillet weld continuous throughout the height. The geometrical properties of ISMB 250 are***

$h = 250$ mm; $b = 125$ mm; $A = 47.55$ cm^2

$I_{xx} = 5131.6$ cm^4; $I_{yy} = 334.5$ cm^4

Determine the maximum permissible compressive load including self weight for the stanchion if the permissible compressive stress σ_{ac} given by the following formula

$$\sigma_{ac} = \frac{0.60 \times f_{cc} \times f_y}{\left[(f_{cc})^{1.4} + (f_y)^{1.4}\right]^{0.7}}$$

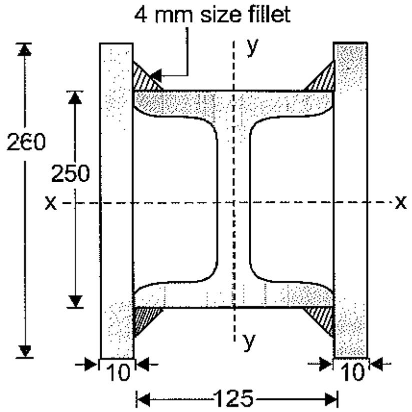
where yield stress, $f_y = 250$ MPa, $f_{cc} = \pi^2 E/\lambda^2$

$l = l/r$, r = radius of gyration

$E = 2.0 \times 10^5$ N/mm^2.

[10 Marks, ESE-2004]

Sol: The diagram of stanchion can be drawn



PLAN

Moment of inertia about x-x axes,

$$I_{xx} = 5131.6 \times 10^4 \text{mm}^4 + \frac{2 \times (260^3 \times 10)}{12} \text{mm}^4$$

$$= 80609333.33 \text{ mm}^4$$

Moment of inertia about y-y axes

$$I_{yy} = 2 \times \left[\frac{260 \times 10^3}{12} + 260 \times 10 \times \left(\frac{125}{2} + 5\right)^2\right] + 334.5 \times 10^4 \text{mm}^4$$

$$= 27080833.33 \text{ mm}^4$$

⇒ I_{yy} is minimum. As x–x & y–y axis are the axis of symmetry of the section, hence I_{xx} & I_{yy} principle axis and as I_{yy} is smaller, it is the minor principal moment of inertia ie. I_{min}.

Area of section = 2 × 260 × 10 + 47.55 × 100 = 9955 mm^2

$$\therefore \quad r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{27080833.33}{9955}} = 52.16 \text{ mm}$$

$$\therefore \quad \lambda = \text{(Slenderness ratio)} = \frac{l_{eff}}{r_{min}} = \frac{6000}{51.16} = 115.04$$

$$\therefore \quad f_{cc} = \frac{\pi^2 E}{\lambda^2} = \frac{\pi^2 \times 2 \times 10^5}{115.04^2} = 149.16\ \text{N/mm}^2$$

and $f_y = 250\ \text{N/mm}^2$

$$\therefore \quad \sigma_{ac} = \frac{0.60 \times f_{cc} \times f_y}{\left[f_{cc}^{1.4} + f_y^{1.4}\right]^{0.7}} = \frac{0.60 \times 149.16 \times 250}{[149.16^{1.4} + 250^{1.4}]^{0.7}}$$

$$= 75.77\ \text{N/mm}^2$$

$$\therefore \quad \text{Maximum permissible load} = \sigma_{ac} \times A_{gross}$$

$$= \frac{75.77 \times 9955}{1000} = 754.29\ \text{kN}$$

Q-9: ***What is the basis for the safety of compression member? Why do we say that the design of compression member is not a direct method?***

[5 Marks, ESE-2004]

Sol: The Basis of safety of compression member is (1) safety against crushing (2) safety agianst buckling.

Short column generally fails by crushing of material and long column fails by buckling of member.

We know that axial load carrying capacity of the section is given by P = σ_{ac} . A_{gross}.

$$A_{gross} = \frac{P}{\sigma_{ac}}$$

Thus area of section required depends or σ_{ac} . But σ_{ac} itself depends on area and distribution of area. Thus we cannot have a direct solution (ie we cannot find out $A_{grooss\ required}$ directly).

Q-10: ***Check the top chord member of a bridge truss for its safety. It is 6 m long and is subjected to an axial compressive force of 600 kN (60 t). The cross-section details of the top chord members are as follows:***

2 ISMC 175, cover plate of 24 cm × 0.8 cm at the top of the channel sections. The channels are facing back to back and spacing of the channels is 9 cm.

Area of the entire cross- section = 67.96 cm²

Neutral Axis depth from the top = 6.96 cm

I_{xx} = *1223 cm⁴,*

I_{yy} = *121 cm⁴ – with reference moment of inetria about their own axis (ISMC 175),*

C_{yy} = *2.20 cm.*

Check the safety of the section and design single lacing system and sketch the Elevation.

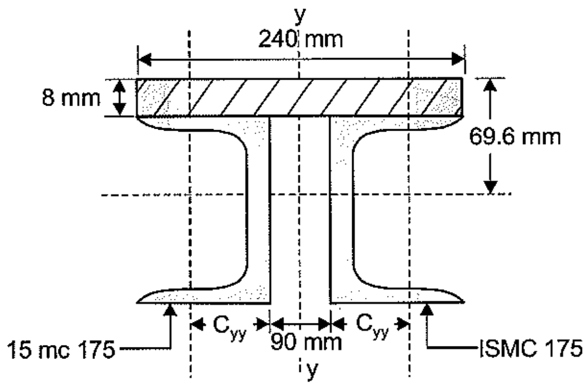
ALLOWABLE STRESS IN AXIAL COMPRESSION

Steel Conforming to		Steel Conforming to IS : 961		
l/r	IS : 226 IS : 2062 & S.T. 44.0 of IS : 1977	HTW Grade thickness not exceeding 32 mm	HT Grade thickness exceeding 45 mm HTW Grade thickness exceeding 32 mm	HT Grade thickness not exceeding 45 mm
	kg/cm²	kg/cm²	kg/cm²	kg/cm²
0	1,250	1,719	1,640	1,719
10	1,246	1,713	1,635	1,713
20	1,239	1,698	1,621	1,698
30	1,224	1,670	1,596	1,670
40	1,203	1,626	1,556	1,626
50	1,172	1,563	1,499	1,563
60	1,130	1,475	1,422	1,475
70	1,075	1,363	1,320	1,363
80	1,007	1,230	1,199	1,230
90	928	1,089	1,067	1,089
100	840	953	938	953
110	753	830	821	830
120	671	724	717	724
130	597	634	629	634
140	531	557	554	557
150	474	493	490	49 3
160	423	438	436	438
170	377	387	386	387
180	336	344	343	344
190	300	307	206	307
200	270	276	274	275
210	243	244	246	247
220	219	223	222	223
230	199	202	202	202
240	181	183	183	183
250	166	167	167	167
300	109	110	110	110
350	76	76	76	76

***Note:** Intermediate values of obtained by linear interpolation.*

[15 Marks, ESE-2005]

Sol: According to the question the intermediate values may be obtained by linear interpolation. given cross-section is, as shown below :



Determination of I_{min}

Since y-y axis is the symmetrical axes for the section, hence I_{min} will be either I_{xx} or I_{yy}

$$\text{Area of one channel} = \frac{67.96 - 24 \times 0.8}{2} = 24.38 \ \text{cm}^2$$

$$I_{xx} = \frac{240 \times 8^3}{12} + 240 \times 8 \times (69.6 - 4)^2 + 2 \times \left[1223 \times 10^4 + \frac{(67.96 \times 100 - 240 \times 8)}{2}\left[\frac{175}{2} - (69.6 - 8)\right]^2\right]$$

$$I_{xx} = 36 \times 10^6 \ \text{mm}^4$$

$$I_{yy} = \frac{8(240)^3}{12} + 2[121 \times 10^4 + 24.38 \times 10^2 \times (45 + 22)^2]$$

$$= 3.52 \times 10^6 \ \text{mm}^4$$

Hence, $$I_{min} = I_{yy} = 33.52 \times 10^6 \ \text{mm}^4$$

$\therefore$ $$r_{min} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{33.52 \times 10^6}{6796}} = 70.23 \text{mm}$$

$\therefore$ $$\lambda(\text{slenderness ratio}) = \frac{l_{eff}}{\gamma_{min}} = \frac{6000}{70.23} = 85.43$$

The value of σ_{ac} can be interpolated for λ = 85.43 (Adopting steel grade other than HT grade))

$$\sigma_{ac} = 928 + \frac{1007 - 928}{10} \times (90 - 85.43) = 964.1 \ \text{kg/cm}^2$$

$\therefore$ Maximum load that can taken by the top chord member is

$$P = \sigma_{ac} \times A' = 964.1 \times 67.96$$

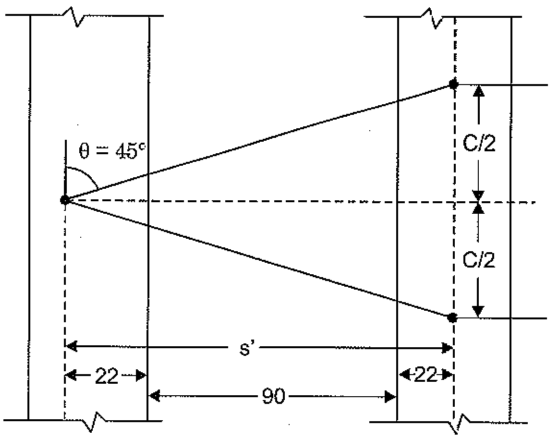
$$= 65520.44 \ \text{kg} = 655.20 \ \text{kN}. \qquad (\text{Assuming } g = 10 \ \text{m/s}^2)$$

Since 655.20 > 600 kN.

Hence, top chord member is safe.

Design of lacing

Assuming θ = 45° and the rivets are connected to the C.G of the channel section



$$S' = 90 + 22 + 22 = 134 \text{ mm}$$

$$\therefore \quad \frac{C}{2} = S' \tan 45° = 134 \text{ mm}$$

$$\therefore \quad C = 268 \text{ mm}$$

Adopting 20 mm ϕ rivets the minimum width of the lacing bar = 60 mm

and,
$$L = \frac{S'}{\sin 45°} = \sqrt{2} \times S' = 189.505 \text{ mm}$$

Hence
$$t = \frac{L}{40} = \frac{189.505}{40} = 4.7376 \text{ mm}$$

$$\boxed{\text{Adopt } t = 8 \text{ mm}}$$

Check for local buckling of lacing bar

$$\lambda = \frac{l_{eff}}{\gamma_{min}} = \frac{l_{eff}}{\frac{t}{\sqrt{12}}} = \frac{189.505\sqrt{12}}{8} = 82.05 < 145 \text{ (Hence OK)}$$

Check for tensile force

$$F = \frac{V}{2\sin\theta} = \frac{2.5 \times 600 \times \sqrt{2}}{100 \times 2 \times 1} = 10.6 \text{ kN}$$

$$\text{Tensile strength of lacing} = (B - d) \times t \times \sigma_{at} = \frac{(60 - 21.5)8 \times 150}{1000} = 46.2. \text{ kN}$$

Since tensile strength (46.2 kN) > 10.61 kN (Hence OK)

Check for compressive force

$$F = 10.61 \text{ kN}$$

$$\text{Compressive strength} = (\sigma_{ac})_{lacing} \times A_{lacing}$$

Since
$$\lambda_{lacing} = 82.05, \text{ Hence } (\sigma_{ac})_{lacing} \text{ can be interpolated}$$

$$(\sigma_{ac})_{lacing} = 928 + \frac{(1007 - 928)}{(90 - 80)} \times (90 - 82.05) = 990.805 \text{ Kg/cm}^2$$

$$\therefore \quad \text{Compressive strength} = 990.805 \times 6 \times 0.8 \text{ kg} = 4755.864 \text{ kg}$$

$$= 47.56 \text{ kN (assuming } g = \frac{10\text{m}}{\text{s}^2}\text{)}$$

Since 47.56 kN > 10.61 kN Hence Ok

Rivets design

Assuming power driven shop rivet

$$\text{Shearing strength} = \frac{\pi}{4}\times 21.5^2\times\frac{100}{1000} = 36.3 \text{ kN}$$

$$\text{Bearing strength} = 21.5 \times 8 \times \frac{300}{1000} = 51.6 \text{ kN}$$

$$\therefore \quad R_V = 36.3 \text{ kN}$$

$$\therefore \quad \text{Number of rivets} = \frac{2F\cos\theta}{36.3} = \frac{2\times 10.61\times 1}{36.3\times\sqrt{2}} = 0.41$$

Hence provide one rivet'

Q-11: ***An unequal angle section 200 mm × 150 mm × 15 mm is to be used in a truss as a strut of length 4.5 m. The cross-sectional properties of the section are as follows :***

$$\textit{Area of cross-section} = 5025 \text{ mm}^2$$

$$I_{XX} = 2\times 10^7 \text{ mm}^4;\quad I_{YY} = 9.7\times 10^6 \text{ mm}^4;$$

$$I_{XY} = -8.3\times 10^6 \text{ mm}^4$$

Using the table of permissible compressive stresses given below, determine the safe load on the member.

Slenderness Ratio	100	110	120	130	140	150	160	170	180
Permissible compressive stress MPa	80	72	64	57	51	45	41	37	33

[20 Marks, ESE-2009]

Sol: The given angle section is unequal, its mean that the structure doesn't have axis of symmetry, The principal moment of inertia can be calculated as,

$$I_{uu}/I_{vv} = \frac{I_{xx}+I_{yy}}{2} \pm \sqrt{\frac{(I_{xx}-I_{yy})^2}{2} + I_{xy}^2}$$

$$= \frac{2\times 10^7 + 0.97\times 10^7}{2} \pm \sqrt{\frac{(2\times 10^7 - 0.97\times 10^7)^2}{2} + (-0.83\times 10^7)^2}$$

$$= 1.485\times 10^7 \pm 0.977\times 10^7$$

$$\therefore \quad I_{uu} = (1.487 + 0.977)\times 10^7 = 2.462\times 10^7 \text{ mm}^4$$

$$I_{vv} = (1.485 - 0.977)\times 10^7 = 0.508\times 10^7 \text{ mm}^4.$$

$$\therefore \quad I_{min} = I_{vv} = 0.508\times 10^7 \text{ mm}^4$$

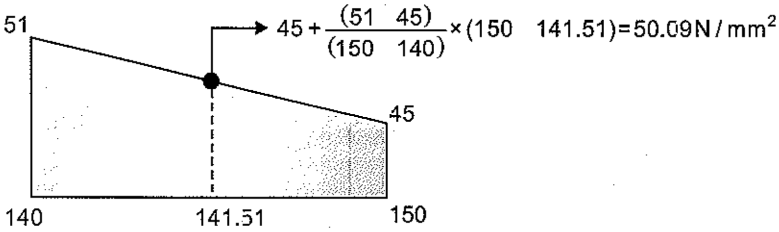
$\therefore$ Minimum radius of gyration,

$$r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{0.508\times 10^7}{5025}} = 31.8 \text{ mm}$$

$$\text{Effective length of strut} = 4.5 \text{ m} = 4500 \text{ mm (Assumed)}$$

$$\therefore \quad \lambda(\text{Slenderness ratio}) = \frac{l_{eff}}{r_{min}} = \frac{4500}{31.8} = 141.51$$

With the help of given table and using interpolation technique,



$$\lambda = 140 \qquad \sigma_{ac} = 51\,\text{N/mm}^2$$

$$\lambda = 150 \qquad \sigma_{ac} = 45\,\text{N/mm}$$

$\therefore$ $$\lambda = 141.51 \qquad \sigma_{ac} = 50.094\ \text{N/mm}^2$$

Hence, $$\text{Safe load} = \frac{50.094 \times 5025}{1000} = 251.72\,\text{kN}$$

Q-12: *Explain why the design of compression member is done by indirect method.*

[5 Marks, ESE-2010]

Sol: In tension member,, stress (σ_{at}) is constant, independent of the sectional area. Hence, for a given load 'P' we can determine the required area (area) direcly as the expressed below,

$$\boxed{A_{req} = \frac{P}{\sigma_{at}}}$$

Whereas in compression member, permissible stress σ_{ac} depends upon slenderness ration (λ) and so on the area itself. Hence, we have to assume area first and then we check whether the applied force is less than the permissible force or not

$$P < \sigma_{at} \times \text{Area}_{\text{ assumed}}$$

Hence, in case of compression memer, we use indirect way to design.

Q-13: *Rolled steel section ISWB 300 is used as a column of height 6 m, fixed at base and pinned at top. Find the permissible compressive load on the column using the table of permissible compressive stresses as given in the table below:*

Cross-section properties of ISWB 300 section are as follows:

Area of cross-section = 6133 mm²

Flange width = 200 mm

Flange thickness = 10 mm

Web thickness = 7.4 mm

$$I_{xx} = 98.216 \times 10^6\ \text{mm}^4$$

$$I_{yy} = 9.9 \times 10^6\ \text{mm}^4$$

λ	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
σ_{ac}	132	122	112	101	90	80	72	64	57	51	45	41	37	33	30	28

λ = ***Slenderness ratio***

σ_{ac} = ***Allowable compressive stress in MPa***

Use linear interpolation for intermediate values of λ.

[15 Marks, ESE-2013]

Sol: The load carrying capacity of the coloumn is given by

$$P = \sigma_{ac} \times A$$

Since σ_{ac} depends upon λ. Hence first we will calculate λ.

$$r_{min} = \sqrt{\frac{I_{min}}{A}}$$

$$I_{min} = I_{yy} = 9.9 \times 10^6 \text{ mm}^4$$

$$\therefore \quad r_{min} = \sqrt{\frac{9.9 \times 10^6}{6133}} = 40.177 \text{ mm}$$

$$\therefore \quad \lambda(\text{Slenderness ratio}) = \frac{l_{eff}}{r_{min}} = \frac{6000 \times 0.8}{40.177} = 119.47$$

[For a coloumn fixed at base and pinned at top, $l_{eff} = 0.8l$]

$\therefore$ By interpolation, σ_{ac} value at $\lambda = 119.47$

$$\sigma_{ac} = 64 + \frac{72-64}{10} \times (120 - 119.47) = 64.424 \text{ N/mm}^2$$

$$\therefore \quad P = \sigma_{ac} \times A = \frac{64.42 \times 6133}{10^3} = 395.11 \text{kN Ans.}$$

Q-14: ***A steel column consisting of ISMB 400 has one end retrained against translation and rotation, while the other end is restrained against translation only. Its unsupported length is 5m. Determine its axial load carrying capacity at service loads using the limit state design of IS : 800:2007.***

Design compressive strength $f_{cd} = \dfrac{f_y/\gamma_{mo}}{\phi + [\phi^2 - \lambda^2]^{0.5}}$

$$\phi = 0.5(1 + \alpha(\lambda - 0.2) + \lambda^2)$$

$$\lambda = \sqrt{\frac{f_y}{f_{cc}}}$$

Euler buckling stress f_{cc}

$$f_{cc} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2}$$

and α = ***0.34 for buckling class b.***

f_y = ***300 MPa,*** γ_{m0} = ***1.10,*** γ_f = ***1.5,***

E = ***200 GPa***

Minimum radius of gyration for ISMB 400 = 28.2 mm, area of cross-section = 7846 m².

[10 Marks, ESE-2014]

Sol: For a column having one end restrained against translation and rotation and other end restrained against translation only

$$L_{eff} = 0.8\ L = 0.8 \times 5 = 4 \text{ m}$$

Given: Euler Buckling stress (f_{cc}) $= \dfrac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2} = \dfrac{\pi^2 \times 2 \times 10^5}{\left(\dfrac{4 \times 10^3}{28.2}\right)^2}$

$$f_{cc} = 98.10 \text{ N/mm}^2$$

Now, $$\lambda = \sqrt{\frac{f_y}{f_{cc}}} = \sqrt{\frac{300}{98.10}} = 1.7487$$

$\therefore$ $$\phi = 0.5\,(1 + \alpha\,(\lambda - 0.2) + \lambda^2))$$

Given $$\alpha = 0.34 \text{ for buckling class b}$$

$\therefore$ $$\phi = 0.5\,(1 + 0.34\,(1.7487 - 0.2) + (1.7487)^2)$$
$$= 2.2922$$

Now design compressive strength

$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + (\phi^2 - \lambda^2)^{0.5}}$$

Given

$$\gamma_{m_0} = 1.10$$

$$f_{cd} = \frac{\dfrac{300}{1.10}}{2.2922 + [(2.2922)^2 - (1.7487)^2]^{0.5}}$$
$$= 72.26 \text{ N/mm}^2$$

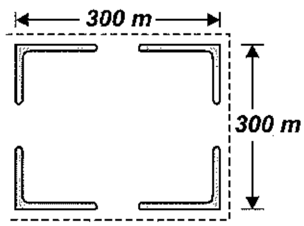
Load carrying capacity of column

$$P = f_{cd} \times A = 72.26 \times 7846 = 566.95 \text{ kN}$$

Load carrying capacity at service load

$$P_S = \frac{P}{\gamma_f} = \frac{566.95}{1.5} = 377.96 \text{ kN}$$

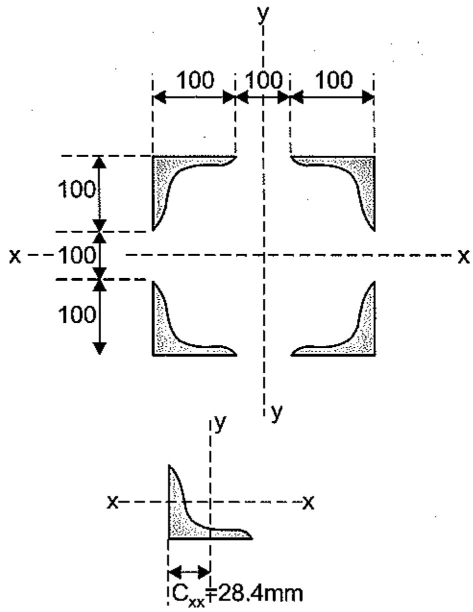
Q-15: ***A column is made of 4 Nos. ISA 100 × 100 × 10 as shown. The effective length of column is 6 m. The lacing of column consists of 60 mm × 10 mm flat bars arranged in a single laced system by bolts and inclined at an angle of 45°. Determine the factored load on the column and check the local buckling of column angles. [ISA 100 × 100 × 10 : A = 1903 mm², $I_{XX} = I_{YY} = 177 \times 10^4$ mm⁴, $C_{XX} = C_{YY} = 28.4$ mm]. Assume gauge length = 55 mm and $r_{VV} = 19.4$ mm.***



KL/r	30	40	50	60	70	80	90
f_{cd} (MPa)	204	185	167	150	133	118	105

[20 Marks, ESE-2015]

Sol:



$$I_{xx} = 4I_{x,x\,one} + 4A\left(\frac{S}{2} + 100 - C_{xx}\right)^2$$

$$= 4 \times 177 \times 10^4 + 4 \times 1903\left(\frac{100}{2} + 100 - 28.4\right)^2$$

$$= 708 \times 10^4 + 11255 \times 10^4 = 11963 \times 10^4 \text{ mm}^4$$

$$r_{min} = \sqrt{\frac{I_{min}}{A}}$$

Due to symmetry, $I_{xx} = I_{yy} = I_{min}$

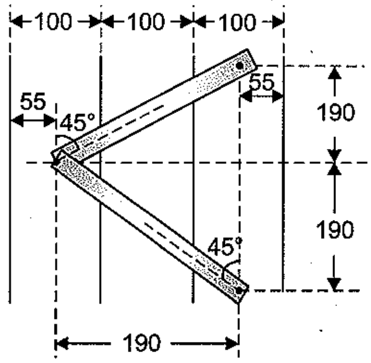
$\therefore$ $r_{min} = \sqrt{\frac{11963 \times 10^4}{4 \times 1903}} = 125.36$ mm

Slenderness ratio, $\lambda = \frac{l_{eff}}{r_{min}} = \frac{6000}{125.36} = 47.86$

From the given table, $f_{cd} = 167 + \left(\frac{185 - 167}{50 - 40}\right)(50 - 47.86) = 170.852$ MPa

$\therefore$ Factored load on column = f_{cd} × (4 A) = 170.852 × 4 × 1903 = 1300.525 kN

Now,



$\Rightarrow$ C = 190 + 190 = 380 mm

r_{vv} = 19.4 mm

For angle sections, check for local buckling:

$$\lambda = \frac{C}{r_{vv}} = \frac{380}{19.4} = 19.587 \not> 50 \quad \text{(OK)}$$

$$0.7 \times \lambda_{\text{whole section}} = 0.7 \times 47.86 = 33.5 \text{ (OK)}$$

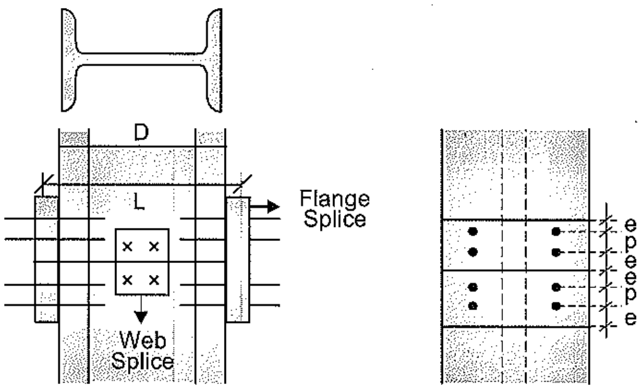
$$19.587 \not> 33.5 \text{ (OK)}$$

$\Rightarrow$ The column angles are safe against local buckling.

Q-16: ***What is the principle of design of splice in a steel member subjected to an axial force ? Explain with the help of neat sketches.***

[5 Marks, ESE-2015]

Sol: Column splices are used to connect two columns when single column length is not available.



Design of Flange Splice:

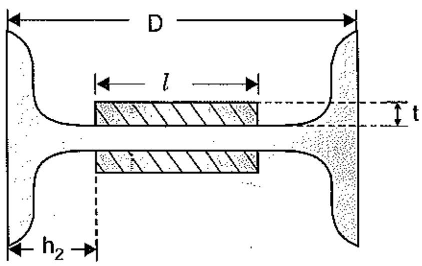
Case I: When the column ends are milled i.e. machine cut and top-storey column flange is resting entirely on bottom storey column flange, in this case 100% of axial load is assumed to be transferred through direct bearing and purpose of the flange splice is only to hold the members together. If moment is acting, in this case each flange splice plate will be designed for $\frac{P}{4} + \frac{M}{L}$.

If moment is not acting we will design it for only $\left(\frac{P}{4}\right)$.

Under this assumption that 50% of load is transferred through flange splices

Case II : If column ends are not milled, (i.e. they are ordinary cut) even in complete bearing of top storey column flange over bottom storey column flange, axial forces will not be transferred through direct bearing and flange splices are designed for 100% of axial loading. Hence design force would be $\left(\frac{P}{2} + \frac{M}{L}\right)$ for each flange splice.

Design of Web Splice:



Web splice is designed to resist shear in the column

$$\text{Shear area} = l \times t \times 2$$

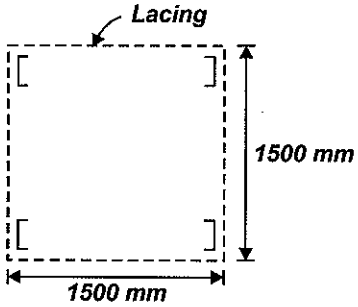
where, $l < D - 2h_2$

Max shear force that can be resisted by web splice = $(l \times t \times 2) \times 0.4\, f_y$

where, 0.4 f_y = permissible stress in shear.

Here, we assume splices to be short columns of zero slenderness ratio.

Q-17: ***A 6m long build-up column carries an axial compressive force and consists of 4nos. ISMC 350 sections as shown in Figure. A lacing is provided in each of the four planes. Explain the principle of design of lacing with the help of a neat sketch.***



[12 Marks, ESE-2017]

Sol: Principles of design of lacing are:

1. The laced column should be strong enough to carry the axial compressive loads.
2. The radius of gyration of whole column perpendicular to the plane of lacing should be more than that parallel to plane of lacing i.e, $r_{yy} > r_{xx}$.
3. Dimensions of lacing should not be varied through the column.
4. Single lacing on opposite faces should be mirror image and on the adjacent faces, should be staggered.
5. Angle of inclination of the lacing bar should lie within 40° to 70° from longitudinal axis.
6. Slenderness ratio of individual section of column between successive lacing point should not be more than 40 or 0.7 × slenderness ratio of whole section

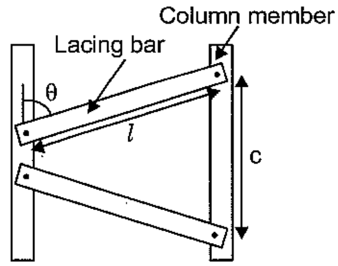
   i.e., $$\frac{C}{r_{min}} \not> 40$$
   $$\not> 0.7\, \lambda_{whole}$$
7. Slenderness ratio of lacing should not be more than 145.
8. Effective length of single lacing is length of lacing between internal rivets or bolts.

   For double lacing, effective length = 0.7 × length stated above.
9. Thickness of lacing bar for single lacing is taken (length of lacing)/40, for double lacing, (length of lacing)/60.
10. Lacing are designed to carry shear force equal to 2.5% of axial compressive load on structure.
11. Lacings are checked for tension and compression for force equal to $\dfrac{V}{N\sin\theta}$

    where $V = \dfrac{2.5\, P_{axial}}{100}$, N = No. of Planes of lacing

12. For connection

if bolting is done at one point then



Lacing arrangement

$$\text{Force on bolt} = \left(\frac{V}{N}\cot\theta\right)$$

if bolt is done at two point then

$$\text{Force on the bolt} = \left(\frac{V}{N}\operatorname{cosec}\theta\right)$$

Bolt is design according to this force.

Q-18: ***A laced column consisting of two ISMC 300 channels placed back-to-back is subjected to factored axial load of 1100 kN. The 10 m long column is restrained in translation but not in rotation at ends. Single lacing at 45° is provided, and connected to flanges by bolts. Verify the capacity of the selected section and determine their spacing. Also determine the size of 50 mm wide lacing rods considering only compressive force in them. Take f_y = 250 MPa and gauge length of lacing rods = 50 mm. The properties of ISMC 300 are as follows :***

A = 4630 mm², B = 90 mm, t = 7.8 mm, T = 13.6 mm, α = 96°, C_Y = 23.5 mm,

$I_{xx} = 6.42 \times 10^7$ mm⁴, $I_{yy} = 3.13 \times 10^6$ mm⁴, r_x = 118.0 mm, r_y = 26.0 mm

Do not design the connection of lacing rod to the channel member.

Table 9(c) of IS : 800 is enclosed for reference.

[20 Marks, ESE-2018]

Sol: Given:

$$\text{Factored load, } P_u = 1100 \text{ kN}$$

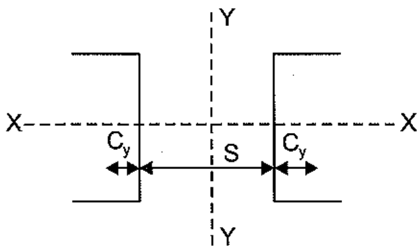
$$\text{Unsupported length, } L = 10 \text{ m}$$

Since the column is restrained in position but not in direction at both ends.

Hence, $\qquad$ Effective length, $l = 1.0\ L = 10$ m

Calculation for spacing of channels :

2 ISMC 300 channels are spaced that the least radius of gyration of built up sections becomes as large as possible. To achieve this, channels are such spaced.



$$I_{YY} \geq I_{ZZ}$$

$$\therefore \qquad 2\left[I_y + A\left(\frac{S}{2}+C_y\right)^2\right] \geq 2I_x$$

$$3.13\times10^6 + 4630\left(\frac{S}{2}-23.5\right)^2 \geq 6.42\times10^7$$

$$S > 182.69 \text{ mm}$$

Provide 2 ISMC 300 channels at a spacing of 200 mm.

Check for compressive strength:

$$\text{Minimum radius of gyration, } r_z = r_{min} = r_x = 118 \text{ mm} \qquad \text{(Given)}$$

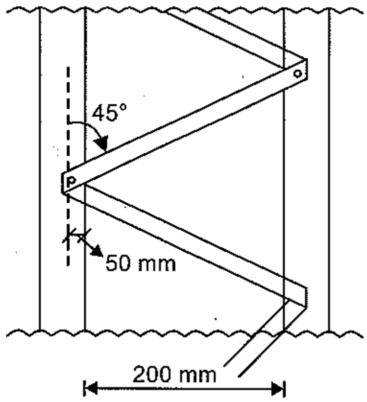
$$\text{Effective slenderness ratio, } \lambda_e = 1.05\frac{l}{r_z} = \frac{1.05\times10000}{118} = 89$$

From the table 9(c)

$$\text{Design compressive strength, } f_{cd} = 136 + \frac{136-121}{80-90}(89-80) = 122.5 \text{ N/mm}^2$$

$$\text{Design compressive strength, } P_d = f_{cd} \cdot A$$
$$= 122.5 \times (2 \times 4630)$$
$$= 1134350 \text{ N}$$
$$= 1134.35 \text{ kN} > 1100 \text{ kN} \quad \text{(OK)}$$

Design of size of lacing rod



$$\text{Maximum shear } V_t = 2.5\% \text{ of factored load}$$
$$= \frac{2.5}{100} \times 1100 = 27.5 \text{ kN}$$

$$\text{Compressive force in lacing bars} = \frac{V_t}{N}\operatorname{cosec}\theta = \frac{27.5}{2}\operatorname{cosec}45° = 19.45 \text{ kN}$$

$$\text{Length of flat lacing bar rod} = (200 + 50 + 50)\operatorname{cosec}45° = 424.3 \text{ mm}$$

$$\text{Effective length of flat lacing rod} = 424.3$$

$$\text{Minimum thickness of lacing rod} = \frac{1}{40} \times 424.3 = 10.6 \text{ mm}$$

Provide a flat section 50 ISF 12 mm

$$\text{Minimum radius of gyration, } r = \frac{t}{\sqrt{12}} = \frac{12}{\sqrt{12}} = 3.46 \text{ mm}$$

$$\text{Slenderness ratio, } \lambda = \frac{l}{r} = \frac{424.3}{3.46} = 122.63 < 145$$

From table 9(c)

$$\text{Design compressive stress, } f_{cd} = 83.7 + \frac{83.7-74.3}{120-130}(122.63-120)$$
$$= 81.23 \text{ MPa}$$

$$\text{Design compressive strength, } P_d = f_{cd} \cdot A$$
$$= 81.23 \times (50 \times 12)$$
$$= 48736.7 \text{ N}$$
$$= 43.74 \text{ kN} > 19.45 \text{ kN}$$

Hence lacing rod is safe against compressive force.

Q-19: ***Check the adequacy of a HB 450 @ 0.872 kN/m rolled steel beam section for a column to carry an axial load of 1100 kN. The column is 4 m long and restrained in position but not in direction at both ends. Allowable axial stress in compression is 105 MPa. The sectional properties of the given section are as follows:***

$$A = 11114\ mm^2,\quad r_{xx} = 187.8\ mm,\quad r_{yy} = 51.8\ mm.$$

[12 Marks, ESE-2019]

Sol: **Given data:**

Axial load, $P = 1100$ kN

Unsupported length = 4 m

Allowable axial stress incompression = 105 MPa

For HB450@ 0.872 kN/m

$$A = 11114\ mm^2$$

$$r_{xx} = 187.8\ mm$$

$$r_{yy} = 51.8\ mm$$

$$\text{Effective length} = kL = 1 \times 4 = 4\ m$$

(K = 1 for restrained in position but not in direction at both end)

$$\text{Slenderness ratio} = \frac{L_{eff}}{r_{min}} = \frac{4000}{51.8} = 77.22 < 250\text{. So, OK.}$$

For $$\frac{KL}{r} = 77.22 \text{ and } f_y = 250$$

$$\sigma_{ac} = 105\ MPa$$

So, $$\text{Load carrying capacity} = 105 \times 11114 = 1166.97\ kN > 1100\ kN$$

So, section is safe.

Q-19: ***A rafter member of a roof truss carries 40 kN compressive load (DL + LL) and 67 kN tensile load (DL + WL). The effective nodal length of the member is 2.1 m. A circular tube section of nominal bore diameter of 50 mm is used. Check the adequacy of the section. Grade of steel = E 250, Young's Modulus E = 200 GPa.***

Given:

Sectional properties of the section $A = 523\ mm^2$, $r = 20.3$ mm.

Outside diameter = 60.3 mm.

Stress reduction factor $\chi = \dfrac{1}{\phi + \sqrt{\phi^2 - \lambda^2}}$

$$\phi = 0.5[1 + \alpha(\lambda - 2) + \lambda^2]$$

$$\lambda = \sqrt{\frac{f_y}{f_{cc}}} \qquad f_{cc} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

Buckling class	***a***	***b***	***c***	***d***
α	***0.21***	***0.34***	***0.49***	***0.76***

[20 Marks, ESE-2020]

Sol. **Given:**

$$\text{Compressive load} = 40 \text{ kN}$$
$$\text{Tensile load} = 67 \text{ kN}$$
$$E = 200 \text{ GPa} = 2 \times 10^5 \text{ MPa}$$

Note: • In the question it is given that $\phi = 0.5[1+\alpha(\lambda-2)+\lambda^2]$

• But **as per IS 800-2007, clause 7.1.2.1**

$$\boxed{\phi = 0.5\left[1+\alpha(\lambda-0.2)+\lambda^2\right]}$$

So, we would solve according to value given in IS code.

$$\text{Factored compressive load} = 1.5 \times 40 = 60 \text{ kN}$$

Check the adequacy of section in compression

$$\chi = \frac{1}{\phi+\sqrt{\phi^2-\lambda^2}}$$

Here,

$$\phi = 0.5[1+\alpha(\lambda-0.2)+\lambda^2]$$

$$f_{cc} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2 \times 200 \times 10^3}{\left(\frac{2.1\times10^3}{20.3}\right)^2}$$

$$= 184.451 \text{ MPa}$$

So,

$$\lambda = \sqrt{\frac{f_y}{f_{cc}}} = \sqrt{\frac{250}{184.45}} = 1.164 \qquad \begin{Bmatrix}\text{for E250} \\ f_y = 250\end{Bmatrix}$$

$$\phi = 0.5\,[1 + 0.21\,(1.164 - 0.2) + 1.164^2]$$

$$\phi = 1.279$$

Assuming the section to be hot rolled

So, tube section comes under buckling class 'a' so $\alpha = 0.21$

Here,

$$\chi = \frac{1}{\phi+\sqrt{\phi^2-\lambda^2}}$$

$$= \frac{1}{1.279+\sqrt{1.279^2-1.164^2}}$$

$$= 0.553$$

$$f_{cd} = \chi\frac{f_y}{\gamma_{mo}}$$

$$= 0.553\times\frac{250}{1.1} = 125.68 \text{ MPa}$$

$$\text{Design compressive strength} = 125.68 \times 523$$
$$= 65730.64 \text{ N}$$
$$= 65.63 \text{ kN}$$

Compressive load < Design Strength

So, Rafter will be safe in compression

Check the adequacy in Tension

There is no information of connection

So check in gross-section yielding

$$\text{Design strength in gross-section yielding} = \frac{f_y \times A_{gross}}{1.1}$$

$$= \frac{250 \times 523}{1.1} = 118863.6 \text{ N} = 118.8 \text{ kN}$$

$$\text{Factored Tensile load} = 67 \times 1.5 = 100.5 \text{ kN}$$

Design Tensile strength > Factored Tensile load so safe in tension

Thus, the given rafter is safe in both tension and compression.

CHAPTER 5

DESIGN OF BEAMS

Q-1: ***A simply supported beam of 6.75 m span is to support a uniformly distributed load of 40 kN/m (inclusive of self weight). Design the beam making use of MB 300 or MB 400 sections which are available in the stores. If required, flange plates may be used. The beam may be assumed to be laterally supported. For the given sections:***

$$F_{bc} = F_{bt} = 165 \text{ MPa}; F_s = 100 \text{ MPa};$$
$$E = 2 \times 10^5 \text{ MPa}$$

For MB 300:
$$I_x = 89.90 \times 10^6 \text{ mm}^4;$$
$$Z_x = 59.9 \times 10^4 \text{ mm}^3;$$
$$B = 140 \text{ mm}, t = 7.7 \text{ mm}$$

For MB 400:
$$I_x = 205 \times 10^6 \text{ mm}^4;$$
$$Z_x = 102.0 \times 10^4 \text{ mm}^3;$$
$$B = 140 \text{ mm}; t = 8.9 \text{ mm}$$

[15 Marks, ESE-2000]

Sol: **Calculation of Z required**

$$Z_{required} = \frac{M_{max}}{\sigma_{bc} \text{ or } \sigma_{bt}}$$

$$M_{max} = \frac{40 \times 6.75^2}{8} = 227.81 \text{ kN-m}$$

$$\therefore \quad Z_{required} = \frac{227.81 \times 10^6}{165} = 138.07 \times 10^4 \text{ mm}^4$$

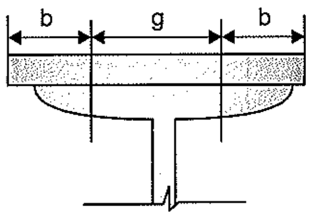
As there is no section providing this much of Z, we should use built up section. Hence we will provide section larger of the two rolled section IS MB 400 and flange plates, to provide for the necessary Z.

Approximate calculation of area required,

$$A_P = \frac{Z_{required} - Z_{rolled}}{D} = \frac{138.07 \times 10^4 - 102 \times 10^4}{400} = 901.75 \text{ mm}^4$$

Let us choose area of plate on each flange as 1800 mm^2 (The area has been arbitrarily chosen as the 100 %. more than req.)

In this question 'g' is not given, hence for checking of local buckling we will assume g = 80 mm from steel table.(Note that in exam, we will not be able to check for local buckling)



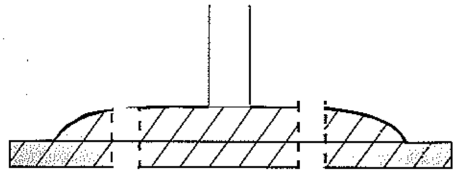
$$g = 80 \text{ mm}$$
$$\text{Thickness} = 8 \text{ mm}$$
$$\text{Length} = 225 \text{ mm}$$
$$\therefore \quad b = \frac{225-80}{2} = 72.5 \text{ mm}$$

and $$\frac{b}{t} = \frac{72.5}{8} = 9.06 < 16 \text{ (Hence OK)}$$

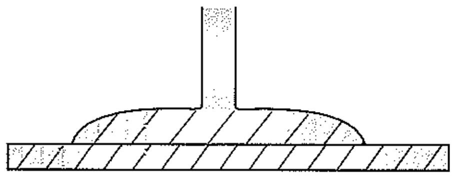
Check for bending

$$\frac{M_{max} \cdot y_c^{max}}{I_{gross}^{xx}} < \sigma_{bc} \text{ (for bending compression)}$$

and $$\frac{M_{max} \cdot y_t^{max}}{I_{gross}^{xx}} \times \frac{\text{gross area of tension flange}}{\text{Net area of tension}} < \sigma_{bt} \text{ (for bending tension)}$$



Net area of tension flange



Gross area of tension flange will be summation of flange plate area and area of flange of rolled section.

As in our problem thickness of flange of rolled section is not given, the ratio of gross-area to net area will be taken, approximately corresponding to the flange plate only.

$$\therefore \quad \frac{\text{Gross area}}{\text{Net area}} = \frac{225 \times 8}{(225 - 21.5 \times 2) \times 8} = 1.236$$

$$\therefore \quad \frac{M_{max} \cdot y_c^{max}}{I_{gross}^{xx}} = \frac{227.81 \times 10^6 \times 208}{205 \times 10^6 + 2 \times \left[\frac{225 \times 8^3}{12} + (225 \times 8) \times (204)^2\right]}$$

$$= 133.54 \text{ N/mm}^2 < 165 \text{ MPa (safe in bending compression)}$$

In tension, $(133.54) \times 1.236 = 165 \text{ N/mm}^2$ Hence Ok.

Check for shear

$$V_{max} = \frac{Wl}{2} = \frac{40 \times 6.75}{2} = 135\text{kN}$$

$$\text{Average shear stress} = \frac{V_{max}}{(D + 2t)t_w} = \frac{135 \times 10^3 \text{N}}{(400 + 2 \times 8)\, 8.9}$$

$$= 36.46 < 100 \text{ (i.e. permissible shear stress) (Hence OK)}$$

Check for deflection

$$\delta_{max} = \frac{5}{384} \times \frac{wl^4}{EI}$$

$$= \frac{\frac{5}{384} \times (40 \text{ N/mm}) \times (6750)^4 \text{ mm}^4}{2 \times 10^5 \times \left[205 \times 10^6 + 2 \times \left[\frac{225 \times 8^3}{12} + 225 \times 8 \times 204^2\right]\right]}$$

$$= 15.235 \text{ mm}$$

and $$\frac{\text{span}}{325} = \frac{6750}{325} = 20.75 \text{ mm} = \text{Permissible deflection.}$$

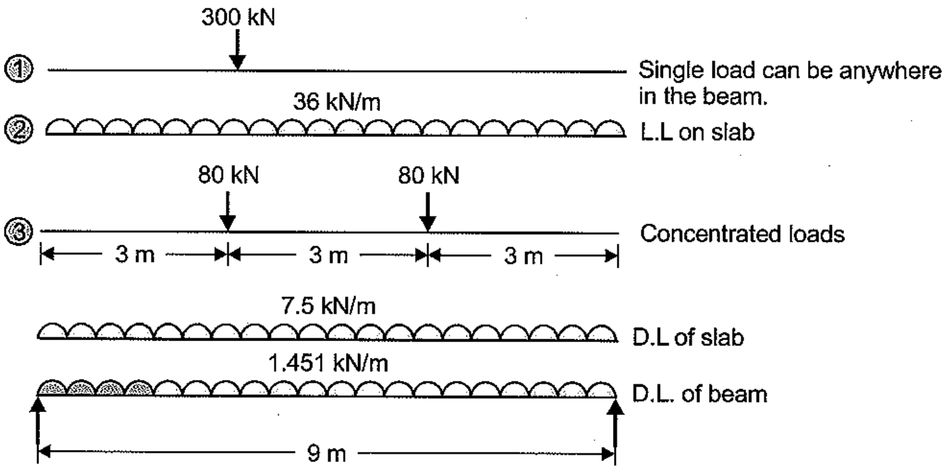
Hence $$\delta_{max} < \frac{\text{span}}{325} \text{ Hence OK. (Safe in deflection)}$$

Q-2: ***A 9 m long simply supported beam supports a 2 m wide and 15 cm thick reinforced concrete slab; in addition, it supports two 80 kN concentrated loads at its one-third span points. The live load on the slab is 1800 kg/m² (18 kN/m²). The Beam is also expected to sustain a single load of 300 kN (30,000 kg) anywhere in the span during the period of maintenance. Given f_b = 1650 kg/cm²; f_s = 945 kg/cm², weight of concrete = 2500 kg/m³, check the cross-sectional dimension of the Beam for its safety.***

The cross-sectional dimensions of the Beam is as follows: ISWB = 600; W = 145.1 kg/m, b_f = 250 mm, t_f = 23.6 mm, t_w = 11.8 mm, D = 600 mm; Plates of dimension 250 mm × 10 mm are provided at the top flange and bottom flange respectively. I_{xx} of ISWB 600 is equal to 115626.6 cm⁴. The Beam is assumed to be restrained laterally.

[20 Marks, ESE-2005]

Sol: Various loads are applied on the beam are as below:



$$\text{D.L of beam} = 145.1 \text{ kg/m} = 1451 \text{ N/m} = 1.451 \text{ kN/m}$$

$$\text{D.L of slab} = \frac{2\times0.15\times9\times25}{9} \text{ kN/m}$$

[Unit weight of concrete = 2500kg/m³ = 25 kN/m³ by taking g = 10 m/s²]

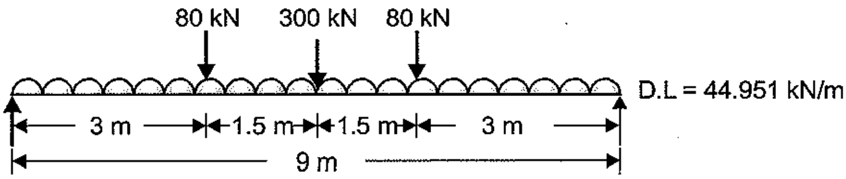
$$= 7.5 \text{ kN/m}$$

$$\text{L.L on slab} = \frac{18\times2\times9}{9} \text{ kN/m} = 36 \text{ kN/m}$$

$$\text{Total udl} = 44.951 \text{ kN/m}$$

For max BM 300 kN load should be at mid span of the beam and for max SF at the section, 300 kN load should be near the support.

Case I:



Max BM will be at mid span,

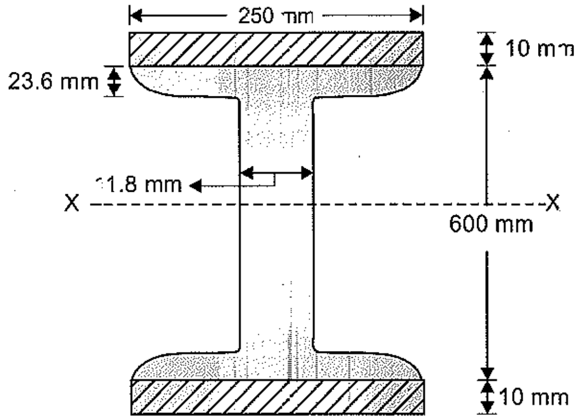
$$\text{Reaction} = 432.2795 \text{ kN}$$

$$M_{max} = 432.2795 \times 4.5 - 80 \times 1.5 - \frac{44.951(9)^2}{8}$$

$$= 1370.129 \text{ kN-m}$$

$$Z_{req} = \frac{M_{max}}{\sigma_{bc} \text{ or } \sigma_{bt}} \quad \text{[As the beam is laterally restrained, } \sigma_{bc} = \sigma_{bt} = f_b\text{]}$$

$$\Rightarrow \quad Z_{req} = \frac{M_{max}}{f_b} = \frac{1370.129 \times 10^6 \text{ N/mm}}{165 \text{ N/mm}^2} = 8.304 \times 10^6 \text{ mm}^3$$



$$I_{xx} = I_{xx} \text{ of ISWB } 600 + 2\left[\frac{250(10)^3}{12} + 250 \times 10 \times (305)^2\right]$$

$$= 1621.43 \times 10^6 \text{ mm}^4$$

$$Z_{xx} = \frac{1621.43 \times 10^6}{310} = 5.23 \times 10^6 \text{ mm}^3$$

$\Rightarrow$ Z_{req} is greater than Z-provided

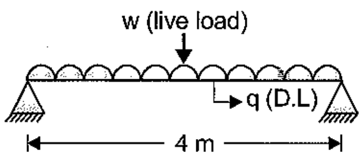
Hence the section is not safe.

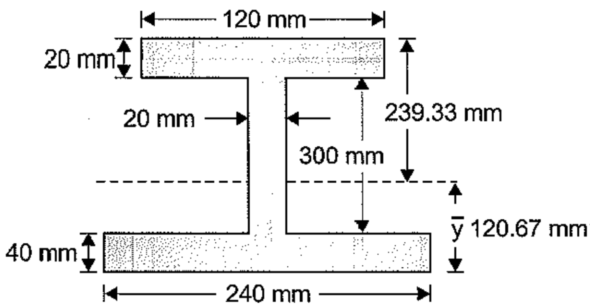
As the beam is not safe in bending we do not need to check for safety in shear and other things.

Q-3: ***The tension flange of a cast iron girder is 240 mm wide and 40 mm thick. The compression flange is 120 mm wide and 20 mm thick and its web is 300 mm × 20 mm. Find the load per metre run which may be carried over a span of 4 m by a beam simply supported at its ends, if the maximum permissible stresses are 90 N/mm² in compression and 30 N/mm² in tension.***

[10 Marks, ESE-2006]

Sol:





Determining the neutral axis depth

Neutral axis will be at the cg. of the section as long as stress is proportional to strain.

$$\bar{y} = \frac{120 \times 20 \times 350 + 300 \times 20 \times 190 + 240 \times 40 \times 20}{120 \times 20 + 300 \times 20 + 240 \times 40}$$

$$= 120.67 \text{ mm}$$

Dead load per meter run for I-sec. (Assuming unit weight of cast iron = 78.5 kN/m^3)

$$q = 78.5\,[0.12 \times 0.020 + 0.3 \times 0.020 + 0.24 \times 0.04]$$

$$= 1.413 \text{ kN/m,}$$

Assume the applied uniformly distributed load = w kN/m & DL = q kN/m

$$\therefore \quad \text{B.M maximum} = \frac{(w + 1.413) \times 4^2}{8} = (2w + 2.826) \text{ kNm.}$$

$$I_{xx} = \left(\frac{120 \times 20^3}{12} + 120 \times 20 \times 229.33^2\right) + \left(\frac{20 \times 300^3}{12} + 20 \times 300 \times 69.33^2\right)$$

$$+ \left(\frac{240 \times 40^3}{12} + 240 \times 40 \times 100.67^2\right)$$

Calculation of Ixx, $I_{xx} = 298.712 \times 10^6 \text{ mm}^4$

and $z_t = \dfrac{298.712 \times 10^6}{239.33} = 1.248 \times 10^6 \text{ mm}^4$

$$z_b = \frac{298.712 \times 10^6}{120.67} = 2.475 \times 10^6 \text{ mm}^3$$

Assuming compresion flange to fail 1st: $\dfrac{BM}{z_t} \le \sigma_{ac}$

$$\frac{(2w + 2.826 \times 10^6)}{1.248 \times 10^6} \le 90$$

$$\Rightarrow \quad w \le 54.75 \text{ kN/m}$$

Assuming tension flange to fail first: $\dfrac{BM}{z_b} \le 30$

$$\frac{(2w + 2.826) \times 10^6}{2.475 \times 10^6} \le 30$$

$$\Rightarrow \quad w \le 35.71 \text{ kN/m}$$

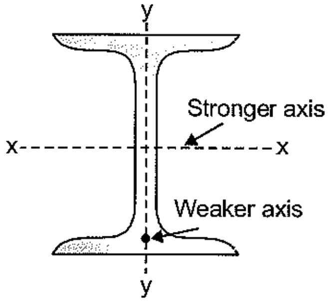
Smaller of the two load ie 35.71 kN/m is the max load per meter length that can be applied on the beam.

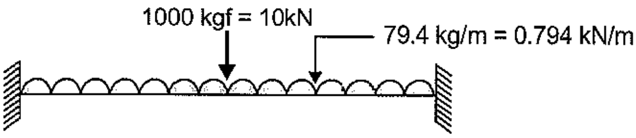
Q-4: ***A steel beam ISWB 450 @ 79.4 kgf/m is fully restrained at its ends and is subjected to a concentrated load of 1000 kgf at its mid point in the direction of its weaker axis. If the effective span of the beam is 8 m and the maximum allowable bending stress is 1650 kgf/cm^2, find the maximum additional concentrated load it can support at its mid point in the direction of strong axis.***

$$Z_{xx} = 1558.1 \text{ cm}^3; \quad Z_{yy} = 170.7 \text{ cm}^3.$$

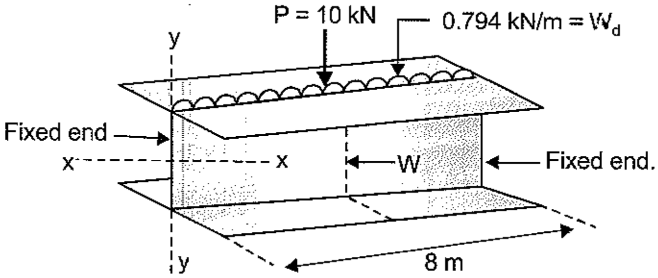
[20 Marks, ESE-2007]

Sol:



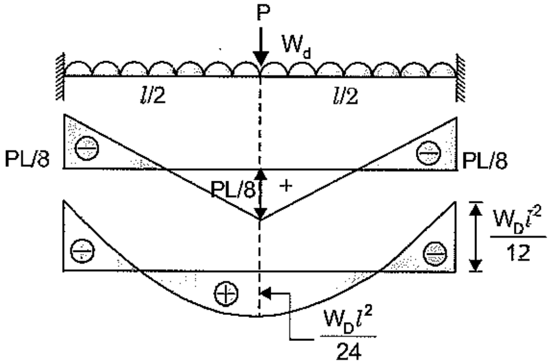


Combined loading situation



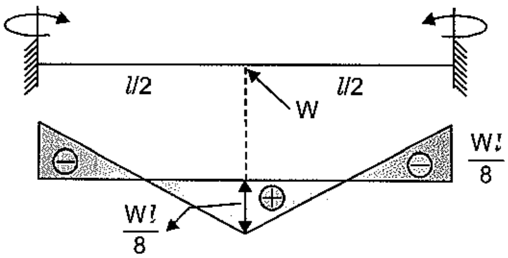
W = Load along strong axis

BMD for vertical loading



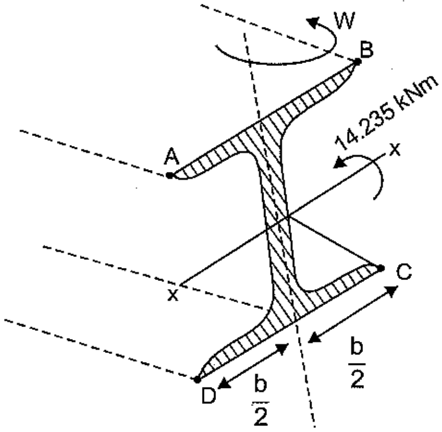
Value of BM in case of vertical loading occurs at the end section. Hence the critical section will be end section.

BMD for horizontal loading



$$\text{Max BM at mid span} = \frac{Wl}{8} = \frac{W \times 8}{8} = W \text{ kNm.}$$

At end section the BM, are as shown below:



Max Bending stress will occur at corner B.

$\Rightarrow$ Max hogging BM at end span $= \dfrac{Pl}{8} + \dfrac{w_D l^2}{12} = \dfrac{10(8)}{8} + \dfrac{0.794(8)}{12} = 14.235$ kNm.

$$\sigma_b = \frac{M_x y}{I_{xx}} + \frac{M_y x}{I_{yy}} = \frac{14.235 \times 10^6}{1558.11 \times 10^3} + \frac{W \times 10^6}{170.7 \times 10^3}$$

For safety $\sigma_b \leq$ Permissible bending stress

$$\sigma_b \leq 165 \text{ N/mm}^2.$$

$$\Rightarrow \quad \frac{14.235 \times 10^6}{1558.11 \times 10^3} + \frac{W \times 10^6}{170.7 \times 10^2} \leq 165$$

$$\Rightarrow \quad W \leq 26.605$$

$\Rightarrow$ Max. load that the beam can support in the direction of stronger axis = 26.605 kN

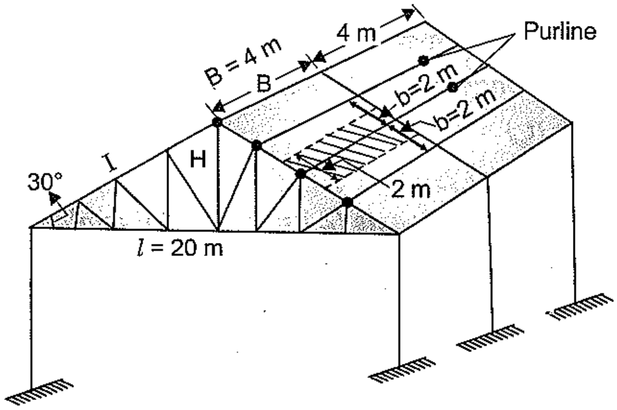
Q-5: ***The roof of an industrial workshop consists of a pitched roof for an effective span of 20 m. The spacing of the trusses is 4 m centre to centre. The spacing of the purlins is 2 m centre to centre. The pitch of the roof is 30°. The weight of the roofing material is 16.2 kg/m² and the nominal wind pressure is 1200 N/m². Check whether the I-section purlin of ISLB 150 is safe for the roof. Permissible bending stress is 1650 kg/cm². The sectional details of ISLB 150 are as follows :***

Self-weight : 14.2 kg/m, Depth of section : 150 mm, Width of flanges : 80 mm, Thickness of flange: 6.8 mm, Thickness of web : 4.8 mm, I_{xx} = 688.2 × 10⁴ mm⁴, I_{yy} = 55.2 × 10⁴ mm⁴.

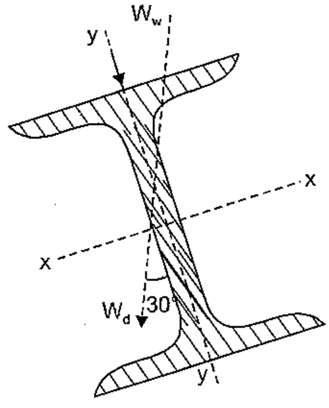
Take the combination of (dead load + wind load) for the design of purlin.

[20 Marks, ESE-2008]

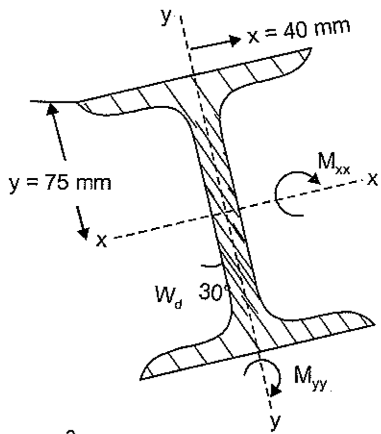
Sol:



Pitch of the truss is expressed as $\frac{H}{l}$. However as in this problem, pitch is given in degrees, we will take slope = 30° load on purlin can be taken as dead load and wind load. Dead load is due to dead weight of purlin + dead load due to roofing material. The dead load will act vertically downwards. The wind load however will be perpendicular to roof



$$W_d = \text{Weight of purlin + D.L on purlin}$$
$$= [14.2 \text{ kg/m} \times 4] + [16.2 \times 4 \times 2] \text{ kg}$$
$$= 186.4 \text{ kg} = 1864 \text{ N (taking } g = 10 \text{ m/s}^2)$$
$$W_w = 1200 \text{ N/m}^2 \times 2 \times 4\text{m}^2 = 960 \text{ N}$$
$$M_{xx} = M_{uu} = \text{Moment about major axis (max value)}$$
$$= (W_w + W_d \cos 30°) \times \frac{4}{10}$$
$$= (1864 \cos 30° + 960) \times \frac{4}{10} = 4485.709 \text{ Nm}$$
$$M_{yy} = M_{vv} = \text{Moment about minor axis (max value)}$$
$$M_{vv} = M_{yy} = w_d \sin 30° \times \frac{B}{10}$$
$$= 1864 \times \sin 30° \frac{4}{10} = 372.8 \text{ Nm}$$



$$\text{Max bending stress} = \frac{M_{xx}y}{I_{xx}} + \frac{M_{yy}X}{I_{yy}} = \frac{4485.706 \times 10^3 \times 75}{688.2 \times 10^4} + \frac{372.8 \times 10^3 \times 40}{55.2 \times 10^4} = 75.9 \text{ N/mm}^2$$

Max permissible bending stress = 165 N/mm^2

Thus section ISLB 150 is safe.

Q-6: ***A beam MB 600 @ 123 kgf is supported over an effective span of 9 m. Two floor joists transmit floor loads at a distance of 1.5 m on either side of the midspan. Determine the safe load which the two floor joists can transmit on the beam if the beam is effectively restrained laterally by the floor joists.***

Properties of MB 600 @ 123 kgf are:

Overall depth (D) = 600 mm

Width of flange (b) = 210 mm

Depth of web (d_1) = 558.4 mm

Mean thickness of flange (t_f) = 20.8 mm

Thickness of web (t_w) = 12 mm

Radius of gyration about yy-axis (r_y) = 41.2 mm

Modulus of section about xx-axis (Z_x) = 3060 × 10^3 mm^3

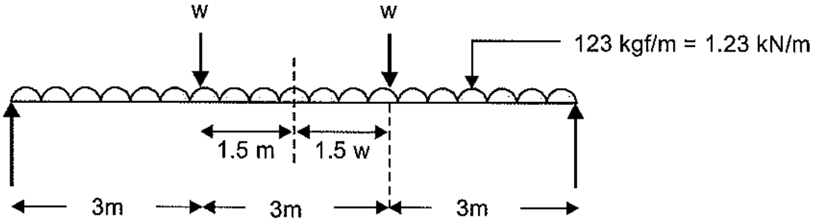
Modulus of section about yy axis (Z_y) = 252 × 10^3 mm^3

Maximum permissible bending stress σ_{bc}(N/mm^2) for t_f / t_w < 2 and d_1 / t_w < 85 is given in the table below as a function of slenderness ratio (L / r) and D/t_f:

l/r \ D/t_f	20	25	30	35	40
60	151	150	149	149	149
65	148	147	146	146	145
70	146	144	143	142	142
75	143	141	140	139	138
80	140	138	136	135	134

[12 Marks, ESE-2011]

Sol: Let W be the max safe load that can be Transmitted on beam.



For the given section,

$$\frac{t_f}{tw} = \frac{20.8}{12} = 1.73 < 2$$

and

$$\frac{d_1}{tw} = \frac{558.4}{12} = 46.53 < 85$$

Hence we can use the data given in the table for determining the maximum permissible stress.

$$\frac{D}{t_f} = \frac{600}{20.8} = 28.85$$

$$\frac{l}{r} = \frac{3000}{41.2} = 72.82$$

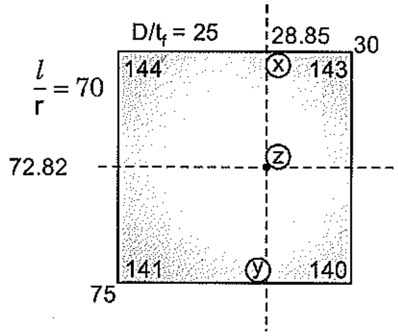
To calculate maximum permissible stress we can use interpolation technique as shown below,

$$x = 144 - \frac{1 \times 3.85}{5} = 143.23$$

$$y = 141 - \frac{1}{5} \times 3.85 = 140.23$$

$$Z = 143.23 - \frac{143.23 - 140.23}{5} \times 2.82$$

$$Z = 141.538 \text{ N/mm}^2$$



$\Rightarrow$ $\sigma_{cbc} = \sigma_{bt} = 141.538$ N/mm2 (Beam is laterals restrained)

$\therefore$ Moment of resistance of beam

$$MR = (\text{Permissible stress}) \times Z_x$$
$$= 141.538 \times 3060 \times 1000 \text{ N-mm}$$
$$= 433.107 \text{ kN-m}$$

The bending moment that has to be carried by the beam,

$$B.M\Big|_{\text{at centre}} = \frac{1.23 \times 9^2}{8} + w \times 4.5 - w \times 1.5 = 12.45 + 3w$$

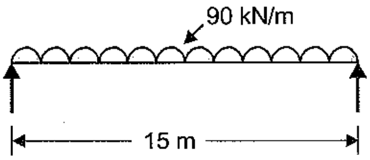
$\therefore$ $3w + 12.45 = 433.107$

$\Rightarrow$ $W = 140.22$ kNm

Q-7: ***Design a suitable section for steel plate beam girder carrying a uniformly distributed load of 60 kN/m (including self weight) over an effective span of 15 m. Check for shear and deflection also. Use limit state method of design. Use ISWB 600 @ 145.1 kgf/m with I_{xx} = 106198.5 cm^4 and I_{yy} = 4700.5 cm^4.***

[20 Marks, ESE-2013]

Sol: Uniformly distributed load inclusive of self wt. = 60 kN/m



$$\text{Factored udl} = 1.5 \times 60 = 90 \text{ kN/m}$$

$$\text{Max. Factored BM} = M = \frac{90(15)^2}{8} = 2531.25 \text{ kN/m}$$

$$\text{Max. Factored } V = \frac{90 \times 15}{2} = 675 \text{ kN}$$

$$Z_p \text{ (Plastic modulus) required for trial section} = \frac{M}{f_y / \gamma_{mo}} = \frac{2531.25 \times 10^6 \text{ Nmm}}{\frac{250}{1.1} \text{N/mm}^2} = 11137.5 \text{ cm}^3$$

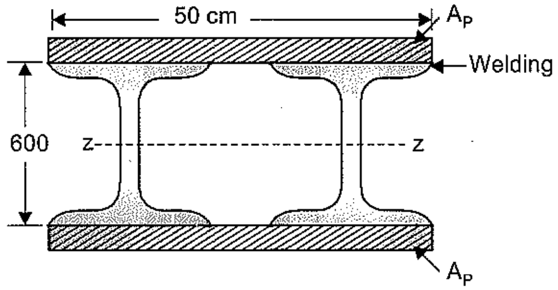
Assume laterally restrained beam condition.

As the Z_P value for the steel section is not given, let us assume the shape factor to be 1.1

$$Z_e \text{ for ISWB 600} = \frac{106198.5}{30} = 3539.95 \text{ cm}^3$$

$$\text{Approx } Z_p = 3539.95 \times 1.1 = 3893.945 \text{ cm}^3$$

Let us use two ISWB 600 section along with plate.



$$\text{Area of plate required } A_P = \frac{Z_{p\,req} - Z_{p\,rolled}}{h}$$

$$= \frac{11137.5 - 2 \times 3893.945}{60} = 55.826 \text{ cm}^2$$

Let us adopt 20 mm thickness plate

$\Rightarrow$ Width of plate required $= \dfrac{55.826}{2.0} = 27.913 \text{ cm}$

Adopt width of plate as 50 cm

Hence plate dimension is 50 cm × 20 mm

As the width (b_f), (t_w), (t_f) etc are not given, Classification of the section is not possible.

Let us assume the beam section to be **plastic section.**

Check for Shear

$$V_d = \left[\frac{f_y}{\sqrt{3} \times \gamma_{mo}} \times h t_w\right] \times 2 \qquad \text{[Shear area = web area]}$$

$$= \frac{250}{\sqrt{3} \times 1.1} \times 600 \times 11 \times 2 \qquad [t_w \text{ has not been given, we have assumed it to be 11 mm}]$$

$$= 1732.05 \text{ kN}$$

$$V < V_d \quad \Rightarrow \quad \text{Beam is safe in shear.}$$

$$0.6\,V_d = 1732.05 \times 0.6 = 1039.23 \text{ kN}$$

$$V = 675 \text{ kN}$$

$$\Rightarrow \quad V < 0.6\,V_d \quad \Rightarrow \quad \text{It is a low shear case.}$$

Note:

$$\frac{d}{t_w} = \frac{600-(t_f+R)}{t_w},$$

Neglecting t_f + R [t_f = thickness of flange R = radius of fillet]

$$\frac{d}{t_w} = \frac{600}{11} = 54.54 < 67\varepsilon \qquad \left[\varepsilon = \sqrt{\frac{250}{f_y}} = 1\right]$$

⇒ Check for web crippling not required

Check for Bending

$$M_d = \beta_b \,.\, Z_p.\frac{f_y}{\gamma_{mo}}$$

$$< 1.2\frac{Z_e f_y}{\gamma_{mo}}$$

$$\beta_b = 1 \text{ (because plastic section has been assumed)}$$

$$Z_p \text{ of the built up section} = Z_{p\,rolled} + A_p.h = 2 \times 3893.945 + 50 \times 2.0 \times 62$$

$$= 13987.89 \text{ cm}^3$$

$$Z_e \text{ for the burst up section} = \frac{I_{zz}}{32\text{ mm}}$$

$$= \frac{2\times 106198.5+2\times\left[\frac{50(2)^3}{12}+50\times 2(31)^2\right]}{32} = \frac{404663.67}{32}$$

$$= 12645.74 \text{ cm}^3$$

$$\beta_b.Z_p.\frac{f_y}{\gamma_{mo}} = \left[1\times 13987.89\times 10^3\text{mm}^3\times\frac{250}{1.1}\text{N/mm}^2\right]\times 10^{-6}\text{ kNm}$$

$$= 3179.065 \text{ kNm}$$

$$1.2\times Z_e\frac{f_y}{\gamma_{mo}} = 1.2\times 12645.74\times 10^3\times\frac{250}{1.1}\times 10^{-6}\text{kNm} = 3448.84 \text{ kN}$$

$$\Rightarrow \quad M_d = 3179.06 \text{ kNm}$$

$$M = 2531.25 \text{ kNm}$$

Since $M < M_d$

⇒ Section is safe in bending.

Check for deflection

Moment of inertia about bending axis I = 404663.67 cm^4

$$\text{Max. deflection} = \frac{5}{384}.\frac{wl^4}{EI} \qquad \text{[W = Service load]}$$

$$= \frac{5}{384} \times \frac{60\frac{N}{mm} \times (15000)^4 mm^4}{2 \times 10^5 \times \frac{N}{mm^2} \times 404663.67 \times 10^4 mm^4}$$

$$= 48.87 \text{ mm}$$

$$\text{Permissible Deflection} = \frac{\text{span}}{300} = \frac{15000}{300} = 50 \text{mm}$$

$\Rightarrow$ Max. deflection < Permissible deflection.

$\Rightarrow$ Section is safe in deflection.

Q-8: ***An ISMB 400 beam is spliced at a section carrying factored bending moment of 120 kNm and factored shear force of 80 kN. The splice is to be designed so that the flange splice will carry the bending moment and the web splice will carry the shear force. Field welding with 8 mm fillet will be used. Determine the size of 100 mm wide flange plate using the following data:***

$$t_f = 16 \text{ mm}; \ t_w = 8.9 \text{ mm and } b_f = 140 \text{ mm}$$

[12 Marks, ESE-2018]

Sol: **Given:**

Factored bending moment, M = 120 kN-m

Factored shear force, V = 80 kN

Width of flange plate, b = 100 mm

t_f = 16 mm, t_w = 8.9 mm, b_f = 140 mm

Size of weld, S = 8 mm

Force in the flange due to bending moment,

$$F = \frac{M}{h - t_f} = \frac{120 \times 10^3}{400 - 16} = 312.5 \text{ kN}$$

Let the thickness of splice plate be t mm.

$$\text{Capacity of splice plate} = \frac{A_g f_y}{\gamma_{mo}}$$

$$312.5 = \frac{100t \times 250 \times 10^{-3}}{1.1}$$

$$t = 13.75 \text{ mm}$$

Providing a flange plate of 14 mm thickness

Design of weld:

$$\text{Effective throat thickness, } t_t = ks = 0.7 \times 8 = 5.6 \text{ mm}$$

Effective length of fillet weld required (lw)

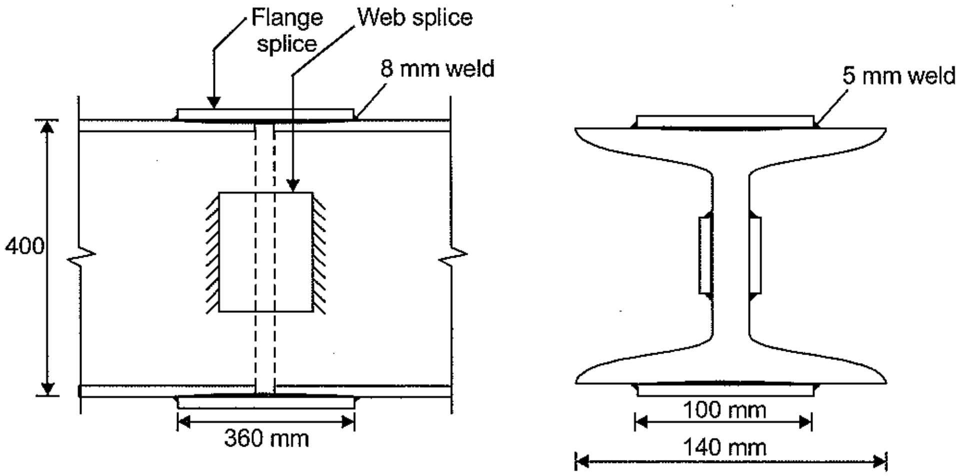
$$F = \frac{f_y t_t l_w}{\sqrt{3}\gamma_{mw}}$$

$$312.5 \times 10^3 = \frac{410 \times 5.6 l_w}{\sqrt{3} \times 1.5}$$

$$l_w = 353.6 \text{ mm}$$

Adopt a flange plate of length 360 mm.

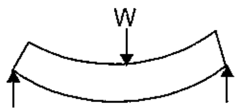
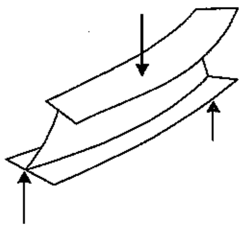
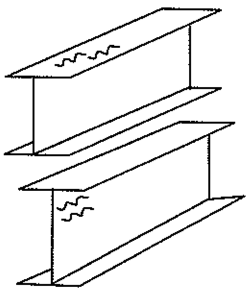
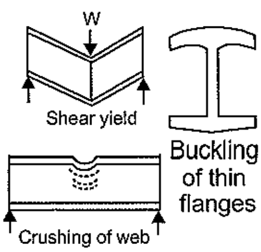
Provide two flange splice plate of size 360 × 100 × 14 mm



Q-9: ***What are the various modes of failure for a steel beam?***

[6 Marks, ESE-2019]

Sol:

Failure mode	Description	Illustration	Comments
Excessive bending	Beam is adequately braced in the lateral plane. Plate components are not too Components are not too thin (compact section) Beam fails due to excessive deformation in the plane of loading.		This is the mode of failure if all other modes of failure are prevented
Lateral torsional buckling	In this case failure occurs due to lateral deflection as well as twist. The load at which this failure occurs depends upon the proportion of the beam, manner of loading and support conditions.		By providing suitable lateral bracing this can be prevented.
Local buckling	Flange may buckle due to compression. Web may buckle due to shear or due to combined effect of shear and bending or due to direct vertical compression under a concentrated load		This failure is unlikely for hot rolled sections since their proportioning are made suitably. However in plate girders web stiffening may be necessary. Bearing stiffeners are provided at supports and under point loads.
Local Failure	This may occur due to the following: (i) Web may yield due to shear (ii) Web may suffer local crushing (iii) Local failure around any openings if present in the web		These are likely in short span or deep beam. Suitable web stiffening should be done. Regions surrounding web holes may be strengthened by local reinforcement

Q-10: ***Find the web buckling and web crippling strength of a beam (ISLB 350) simply supported at both ends. Assume the stiff bearing length 100 mm and grade of steel E 250.***

Section properties of ISLB 350:

$$t_w = 7.4 \text{ mm}$$
$$t_f = 11.4 \text{ mm}$$
$$R = 16 \text{ mm}$$

R = Radius of root.

Given: Design compressive stress f_{cd}, N/mm²

KL / r	*f_{cd}*
90	*121*
100	*107*
110	*94.6*

[12 Marks, ESE-2020]

Sol: Given that

ISLB 350, simply supported at both ends.

Stiff bearing length = 100 mm

Grade of steel = E 250

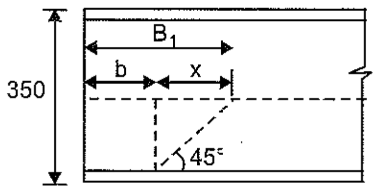
Properties of ISLB 350

t_w = 7.4 mm

t_f = 11.4 mm

R = 16 mm

For web buckling strength



$$x = 350 \times \frac{1}{2} = 175 \text{ mm}$$

Stiff bearing length = 100 mm

Area of bearing (A_b) = $B_1 t_w$ = (b + x) t_w = (100 + 175) × 7.4 = 2035 mm²

Effective length of web = 0.7d = 0.7 [350 – 2 (11.4 + 16)] = 206.64 mm

$$I_{eff} \text{ of web} = \frac{bt_w^3}{12} = \frac{100 \times 7.4^3}{12} = 3376.86 \text{mm}^4$$

$$A_{eff} \text{ of web} = bt_w = 100 \times 7.4 = 740 \text{ mm}^2$$

$$\text{Radius of gyration (r)} = \sqrt{\frac{I_{eff}}{A_{eff}}} = \sqrt{\frac{3376.86}{740}} = 2.136 \text{ mm}$$

$$\text{Slenderness ratio } (\lambda) = \frac{KL}{r} = \frac{206.64}{2.136} = 96.75$$

From table

λ	f_{cd}
90	121
96.75	x
100	107

$$\frac{90-96.75}{96.75-100} = \frac{121-x}{x-107}$$
$$2.1\,x - 224.7 = 121 - x$$
$$\boxed{f_{cd} = x = 111.52 \text{ N/mm}^2}$$

Capacity of web section = $A_b f_{cd}$ = 2035 × 111.52 = 226.94 × 10^3 N = 226.94 kN

For Web cripplign strength:

$$f_w = (b+n_1)\frac{t_w f_y}{\gamma_{m0}}$$
$$n_1 = 2.5 \times (11.4 + 16) = 68.5 \text{ mm}$$
$$F_w = (100+68.5)\times 7.4\times\frac{250}{1.1}$$
$$\boxed{F_w = 283.39\times 10^3 \text{N}}$$
$$\boxed{F_w = 283.39 \text{kN}}$$

CHAPTER 6

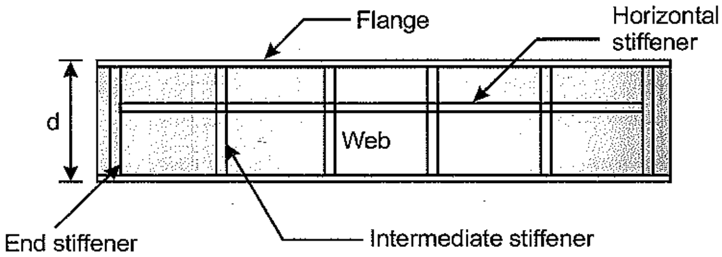
DESIGN OF PLATE GIRDERS

Q-1: ***Show various components of a steel plate grider through neat sketches. Explain design criteria of each and the corresponding limit state.***

[10 Marks, ESE-2016]

Sol: The following are the elements of a typical plate girder:

1. Web
2. Flanges
3. Stiffeners



Web of required depth and thickness are provided to:

(a) Keep flange plates at required distances

(b) Resist the shear in the plate girder

$$V_d = \frac{A_w f_y}{\gamma_{mo}\sqrt{3}}$$

Flanges of required width and thickness are provided to resist bending moment acting on the beam by developing compressive force in one flange and tensile force in another flange.

Assuming moment is resisted by flanges only, and using material partial safety factor for a plastic section,

$$\frac{A_f \times f_y \times d}{1.1} = M$$

Hence area of flange Af may be found. Select $9.4 \in < t_f < 13.6\ b_f \in$ so that bending strength can be found by the formula for semi compact section as per the clause 8.2.1.2 in IS 800.

Stiffeners are provided to safeguard the web against local buckling failure. The stiffeners provided may be classified as :

(a) Transverse (vertical) stiffeners and

(b) Longitudinal (horizontal) stiffeners

Transverse stiffeners are of two types :

(i) Bearing/end stiffeners

(ii) Intermediate stiffeners

End bearing stiffeners are provided to transfer the load from beam to the support. At the end certain portion of web of beam acts as a compression member and hence there is possibility of crushing of web. Hence web needs stiffeners to transfer the load to the support. If concentrated loads are acting on the plate girder (may be due to cross beam) intermediate stiffeners are required.

To resist average shear stress, the thickness of web required is quite less. But use of thin webs may result in to buckling due to shear. Hence when thin webs are used, intermediate transverse stiffeners are provided to improve buckling strength of web.

Many times longitudinal (horizontal) stiffeners are provided to increase the buckling strength of the web. If only one longitudinal stiffners is provided, it will be at a depth of 0.2 d from the compression flange where 'd' is the depth of the web. If another longitudinal stiffener is to be provided it will be at mid depth of the web.

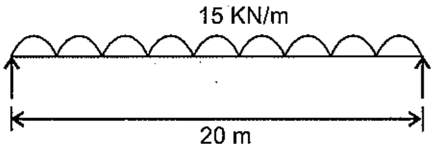
Q-2: ***A welded plate girder of span (simply supported) 20m is subjected to a uniformly distributed load of 100 kN/m including its self-weight. The cross-section of the girder consists of top and bottom flanges (500mm×50mm) and web plate (1000mm×15mm). The section is safe against flexure and shear. Design the connection between the flange and web only. Use $\gamma_{mw} = 1.25$, and E 250 grade of steel. Use the limit state method of design.***

[20 Marks, ESE-2017]

Sol: **Given:**

Service load = 100 KN/m²

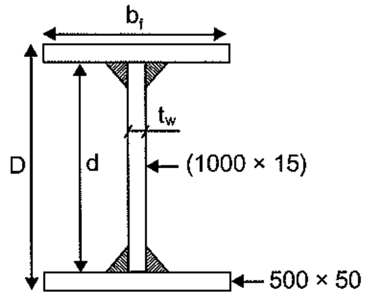
Factored load = 150 KN/m²



So, Maximum shear $= \dfrac{w_u L}{2} = \dfrac{150 \times 20}{2}$ kN

$= 150 \times 10$

$= 1500$ KN



There will be two weld length along the span of each flange to web connection.

So,
$$q_w = \frac{VA_f\bar{y}}{2I}$$

$$I = \frac{b_f D^3}{12} - \frac{(b_f - t_w)d^3}{12}$$

where D = (1000 + 50 + 50) = 1100 mm

$$I = \frac{500 \times 1100^3}{12} - \frac{(500-15)}{12} \times 1000^3 = 1.5042 \times 10^{10}\ \text{mm}^4$$

So,
$$q_w = \frac{1500 \times 1000 \times 500 \times 50 \times \left(500 + \frac{50}{2}\right)}{2 \times 1.5042 \times 10^{10}}$$

$= 654.42$ N/mm

So, let the throat thickness $= t_t$

So strength of weld per meter length

$$= \frac{410}{\sqrt{3} \times 1.25} \times t_t \qquad (\gamma_{mw} = 1.25, \text{ given})$$

So, $$\frac{410}{\sqrt{3} \times 1.25} \times t_t = 654.42$$

$\Rightarrow$ $$t_t = 3.45 \text{ mm}$$

So $$\text{size of weld} = \frac{3.45}{0.7} \quad (k = 0.7)$$

$$= 4.93 \text{ mm}$$

So, adopting weld size = 5 mm

But minimum size of weld is based on thickness of thicker plate i.e. 50 mm,

$\therefore$ Use weld size of 8 mm for first run and 10mm subsequently.

Provide the size of weld define above at top and bottom both side between web and flange plate.

CHAPTER 7

DESIGN OF GANTRY GIRDERS

Q-1: ***Determine the ultimate bending moments and forces due to vertical and horizontal loads that act on a simply-supported gantry girder. Use the following data :***

1. ***Simply supported span = 6 m***
2. ***Distance between crane wheels = 3.6 m***
3. ***Self-weight of girder = 1.5 kN/m***
4. ***Maximum crane wheel load (static) = 220 kN***
5. ***Weight of crab/trolley = 60 kN***
6. ***Maximum hook load = 200 kN***

Take impact factor of 25% and assume double flanged wheels e = 0.15 m while the girder depth, D = 0.60 m.

[20 Marks, ESE-2018]

Sol: **Step 1:** Moments and forces due to self weight

$$\text{Factored self weight, } W_d = 1.5 \times 1.5 = 2.25 \text{ kN/m}$$

Ultimate bending moment at mid span,

$$M_1 = \frac{W_d l^2}{8} = \frac{2.25 \times 6^2}{8} = 10.125 \text{ kN-m}$$

Ultimate reaction at support,

$$V_1 = \frac{W_d . l}{2} = \frac{2.25 \times 6}{2} = 6.75 \text{ kN}$$

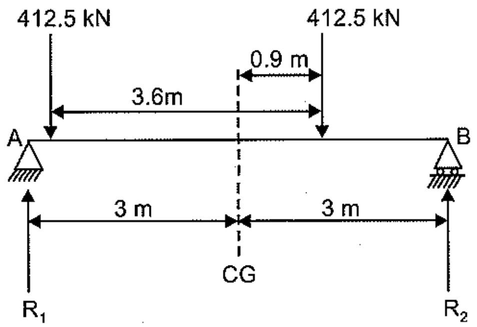
Step 2: Moments and forces due to vertical wheel load,

Factored wheel load (including impact factor of 25%)

$$W_e = 1.5 \times 1.25 \times 220 = 412.5 \text{ kN}$$

Calculation for ultimate BM:

Case 1: Maximum bending moment under wheel load occurs when the two wheels are in such a position that the centre of gravity of wheel loads and one of the wheel load are equidistant from the centre of gantry girder.



$$\Sigma M_A = 0$$

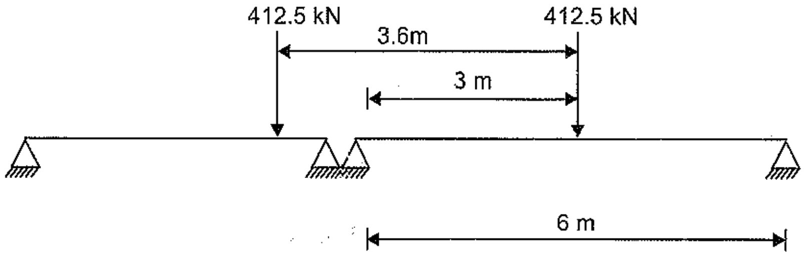
$$412.5 \times (3 + 0.9 - 3.6) + 412.5 \times (3 + 0.9) = R_2$$

$$R_2 = 288.75 \text{ kN}$$

Ultimate moment under wheel load,

$$M_2 = R_2 \times (3 - 0.9) = 288.75 \times 2.1 = 606.4 \text{ kN-m}$$

Case 2: When one of the wheel load is on mid span while the other on other span



Ultimate moment on mid span,

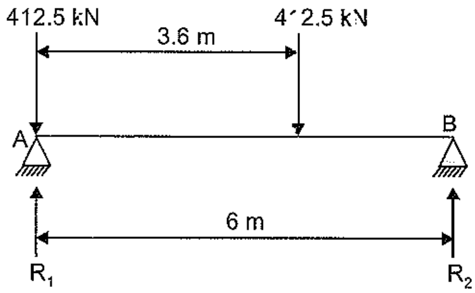
$$M_2 = \frac{W_e.L}{4} = \frac{412.5 \times 6}{4}$$

$$= 618.75 \text{ kN-m}$$

Thus maximum ultimate moment, $M_2 = 618.75$ kN-m

Calculation of ultimate reaction:

The maximum ultimate reaction occurs when one of the wheels is at the support.



$$\Sigma M_B = 0$$

$$412.5 \times (6 - 3.6) + 412.5 \times 6 = R_1 \times 6$$

$$R_1 = 577.5 \text{ kN}$$

Ultimate reaction, $V_2 = R_1 = 577.5$ kN

Step 3: Moment and forces due to horizontal wheel load

Assuming 10% of the weight of the crab and the weight lifted on the crane acts traverse to rail due to impact.

$$\text{Factored horizontal surge load} = 1.5 \times 0.1\ (200 + 60) = 39 \text{ kN}$$

This is divided among the 4 wheels (since the wheel are double flanged)

$$\text{Factored horizontal wheel load, } W_h = \frac{39}{4} = 9.75 \text{ kN}$$

Using calculation similar to those for vertical moments and forces.

Ultimate horizontal BM (Case 2)

$$= W_h \times \frac{L}{4} = 9.75 \times \frac{6}{4} = 14.625 \text{ kN-m}$$

Step 4: Calculation for bending moment and forces due to drag force

Eccentricity, $e = 0.15$ m (given)

Depth of the girder, $D = 0.6$ m

$$\text{Reaction, } V_3 = \frac{W_g}{L}\left(\frac{D}{2}+e\right)$$

Factored, $W_g = 1.5 \times 0.05 \times 220 \times 1.25$ (impact is also considered)

$= 20.625$ kN

$$V_3 = \frac{20.625}{6}(0.3+0.15) = 1.55 \text{ kN}$$

Ultimate bending moment due to drag force

$$M_3 = V_3\left(3-\frac{3.6}{4}\right)$$

$$= 1.55(3 - 0.9) = 3.255 \text{ kN-m}$$

Hence, the vertical ultimate bending moment for design

$$= M_1 + M_2 + M_3$$

$$= 10.125 + 618.75 + 3.255$$

$$= 632.13 \text{ kN-m}$$

The ultimate reaction for design (vertical)

$$= V_1 + V_2 + V_3$$

$$= 6.75 + 577.5 + 1.55$$

$$= 585.8 \text{ kN}$$

Ultimate bending moment (horizontal) = 14.33 kN-m

Q-2: ***In an industrial shed, it is proposed to provide a hot rolled section ISMB 500 to carry a two-wheeled system crab on it. The crab can move over the flange of the beam from one end to another end and each wheel of the crab is capable to carry a maximum vertical load of 60 kN (including self weight of wheel).***

The centre to centre distance between supporting ends of the beam is 6 m and the end of the beams are restrained against torsion.

The space between two wheel = 2.4 m. Take impact factor for vertical load as 25%.

Verify the capability of the beam to carry the bending moment developed due to vertical load only. Assume the section is plastic. Grade of steel E 250.

Given data:

Properties of ISMB 500:

$b_f = 180$ mm, $t_f = 17.2$ mm, $t_w = 10.2$ mm

$r_z = 202.1$ mm, $r_y = 35.2$ mm

$Z_{ez} = 1808.7\ cm^3$, $Z_{pz} = 2074.67\ cm^3$

Critical stress, $f_{cr,\,b}$ (MPa)

KL/r	h/t_f = 25	h/t_f = 30
170	136.7	121.3
180	127.1	112.2

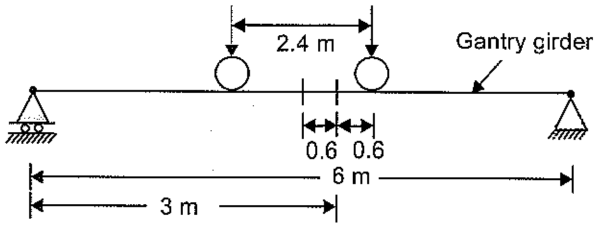
Design bending compressive stress to lateral buckling f_{bd}, for $f_y = 250$ MPa.

$f_{cr,b}$	f_{bd} (MPa)
150	106.8
100	77.3

[20 Marks, ESE-2020]

Sol: Maximum vertical static wheel load (w_u) = 60 kN.

$$\text{Factored wheel load with impact} = 1.5 \times 1.25 \times 60 = 112.5 \text{ kN}$$



Design bending moment and shear force:

The absolute maximum bending occurs under the wheel closest to mid span of gantry girder when the quarter point of the wheel base coinsides with the mid span of gantry girder as shown in above figure:

$$M_{umax} = \frac{w_u}{8L_g} \times [2L_g - C]^2$$

$$= \frac{112.5}{8 \times 6} \times [2 \times 6 - 2.4]^2 = 216 \text{ kN-m}$$

Factored moment carrying capacity of beam $(M_u)_{design}$

$$\frac{kL}{r_{min}} = \frac{6000}{35.2} = 170.45$$

$$\frac{h}{t_f} = \frac{456.6}{17.2} = 27.07$$

$\frac{kL}{r_{min}}$	h/t_f		
	25	27.07	30
170	136.7		121.3
170 45	136.27	129.9	120.89
180	127.1		112.2

$$f_{cr,b} = 129.9 \text{ MPa}$$

$f_{cr,b}$	f_{bd}
150	106.8
129.9	x
100	77.3

$$\frac{106.8 - x}{x - 77.3} = \frac{150 - 129.9}{129.9 - 100}$$

$$106.8 - x = 0.67x - 51.79$$

$$f_{bd}(x) = 94.96 \text{ N/mm}^2$$

Factored moment carrying capacity

$$= \beta_b \cdot Z_{pz} \cdot f_{bd}$$

$$= 1 \times 2074.67 \times 10^3 \times 94.96$$

$$= 197.01 \times 10^6 \text{ N-mm}$$

$$M_{u,design} = 197.01 \text{ kN-m}$$

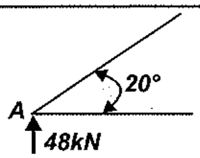
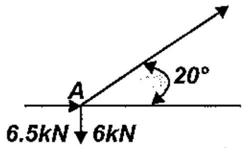
$M_{u,max} > M_{u,design}$, the beam is not safe against bending.

CHAPTER 8

INDUSTRIAL ROOFS

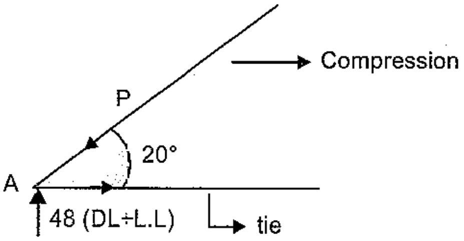
Q-1: ***The joint A of a roof truss is shown below, where the main rafter and the main tie intersect. It is a hinged joint. It is subjected to end reactions as shown below : (1) Dead load + Live load and (2) Dead load + Wind load. Effective length of the main rafter is 2.83 m. Check the assumed section of the main rafter for its safety against compression.***

Properties of the section assumed—double -angle back -to-back, angle being 80 mm × 50 mm × 10 mm having an area of 12.02 cm² each, r_{xx} of the double-angle section = 2.49 cm.

 *(DL + LL)* *l/r*	 *(DL + WL)* *f_c (kg/cm²)*
0	1250
10	1246
20	1239
30	1224
40	1203
50	1172
60	1130
70	1075
80	1007
90	928
100	840
110	753
120	671
130	597
140	531
150	474
160	423
170	377
180	336
190	300
200	270
210	243
220	219
230	199
240	181
250	166
300	109
350	76

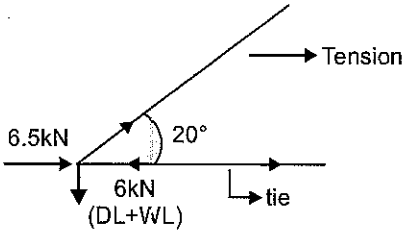
[12 Marks, ESE-2010]

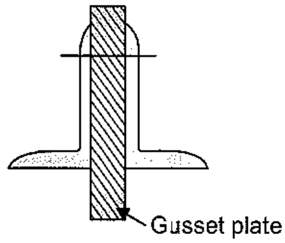
Sol:



$$P \sin 20° = 48$$

$$\Rightarrow \qquad P = \frac{48}{\sin 20} = 140.34 \text{ kN}$$





In the load combination DL + LL, only the rafter is in compression hence, for this case only, we will check the safety of main rafter in compression.

$$r_{xx} \text{ of combination} = 2.49 \text{ cm}$$

Let us assume that r_{xx} the r_{min} value

$$l_{eff} = 2.83 \text{ m} = 283 \text{ cm}$$

$$\text{Slencerness ratio, } \lambda = \frac{l_{eff}}{r_{xx}} = \frac{283}{2.49} = 113.65$$

$\therefore$ σ_{ac} can be determined by interpolation

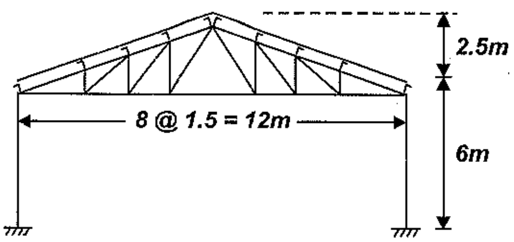
$$\sigma_{ac} = 671 + \frac{753-671}{120-110} \times (120-113.65) = 723.07 \text{ kg/cm}^2$$

$\therefore$ Force that can be resisted $= 723.07 \times 12.02 \times 2 = 17382.60$ kg

$$173.82 \text{ kN} > 140.34 \text{ kN}$$

Hence the section is safe

Q-2: ***A factory shed is to be constructed using truss having a span of 12m. The spacing of the truss is 4m. Galvanised iron sheets are to be placed over purlins located at the nodes as shown in Figure. Determine the design load at the nodes due to live load and wind load for an intermediate truss. Assume design wind pressure = 1500 N/m². C_{pe} = –0.7 (for wind angle 90°), C_{pi} = ±0.2. Loads are to be shown in the diagram separately.***



[20 Marks, ESE-2017]

Sol: **Given:**

$$\text{Spacing of truss} = 4 \text{ m}$$

$$\text{Span of truss} = 12 \text{ m}$$

$$\text{Design wind presure (pd)} = 1500 \text{ kN/m}^2$$

$$C_{Pe} = -0.7,\ Cpi = 0.2$$

$$\therefore \quad \text{Length of rafter} = \sqrt{2.5^2 + \left(\frac{12}{2}\right)^2} = 6.5 \text{ m}$$

$$\text{Distance between panel points} = \frac{6.5}{4} = 1.625 \text{ m}$$

$$\text{Inclination of roof} = \theta = \tan^{-1}\left(\frac{2.5}{6}\right) = 22.62°$$

Load on each intermediate panel due to live load:

For roof with no access, live load up to 10° slope = 750 N/m^2

Let us assume that no access is provided on roof. The live load is reduced by 20 N/m^2 for each one degree above 10° slope.

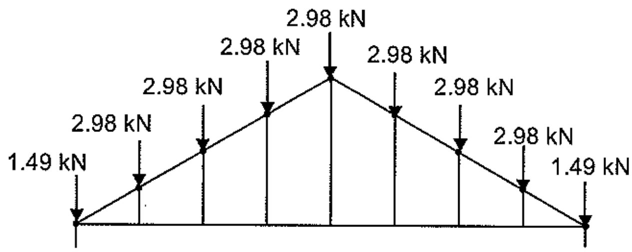
$$\therefore \quad \text{Live load} = 750 - 20 \times (22.62 - 10)$$

$$= 497 \text{ N/mm}^2 \text{ or } 0.497 \text{ kN/m}^2$$

The load on each intermediate panel

$$= \frac{0.497 \times 12 \times 4}{8} = 2.98 \text{ kN}$$

and $$\text{Load on each end panel point} = \frac{2.98}{2} = 1.49 \text{ kN}$$



Live load on panel points

Wind load:

$$\text{Design wind pressure (pd)} = 1.5 \text{ kN/m}^2$$

$$\text{Length of rafter} = 6.5 \text{ m}$$

$$\text{Spacing of truss} = 4 \text{ m}$$

For critical combination of wind load,

$$\text{Net wind pressure coefficient} = -0.7 - 0.2 = -0.9 \qquad [\text{Max (Cpe} - \text{Cpi)}]$$

Wind load acting normal to the individual structural member or cladding unit

$$= (Cpe - Cpi)\, A \times pd. \text{ (IS: 875 part D)}$$

where, Cpe = External pressure coefficient

Cpi = Internal pressure coefficient

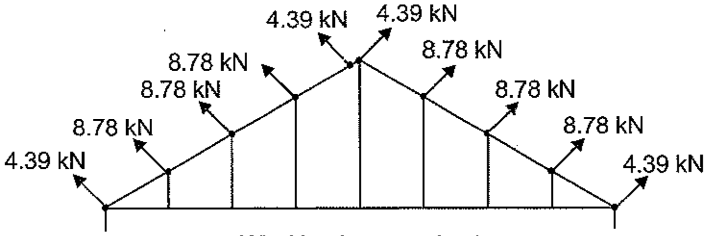
Pd = Design wind pressure

A = Surface area of structural member or cladding unit.

$\therefore$ Wind load on intermediate panel point $= -0.9 \times 4 \times \dfrac{6.5}{4} \times 1.5 = -8.78$ kN

Wind load on end panel points $= \dfrac{-8.78}{2} = -4.39$ kN

(–)ve wind load indicates force acting away from structural element (suction)



Wind load on panel points

Q-3: ***A pitched roof to be provided for a workshop of effective span 18 m. The trusses are spaced at 4 m centre to centre and purlins at 1.6 m centre to centre. The pitch of the roof is 28°, weight of the roofing material is 0.162 kN/m, normal wind pressure is 1.2 kN/m² and permissible bending stress is 165 MPa. check the suitability of ISLB 12575 @ 0.119 kN/m section for purlins, if I_{XX} = 406.8 cm⁴ and I_{YY} = 43.4 cm⁴ for given section.***

[14 Marks, ESE-2019]

Sol: **Given data:**

Effective span = 18 m

Truss spacing = 4 m centre to centre

Purlins spacing = 1.6 m centre to centre

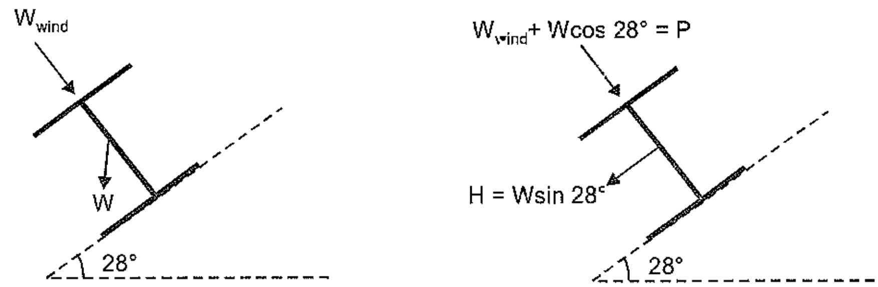
Pitch of the roof = 28°

Weight of the roofing material = 0.162 kN/m

Normal wind pressure = 1.2 kN/m²

Permissible bending stress = 165 MPa

For ISLB 125,75@ 0.119 kN/m



$$I_{xx} = 406.8 \text{ cm}^4$$

$$I_{yy} = 43.4 \text{ cm}^4$$

Dead load of purlins = 119 N/m

Total dead load = 162 + 119 = 281 N/m

The vertical component of dead load parallel to the roof. H = 281 sin 28°

$$H = 131.92 \text{ N/m}$$

$$\text{Wind load} = 1.6 \times 1.2 \times 1000 \text{ N/m}$$

$$= 1920 \text{ N/m}$$

Component of dead load normal to roof

$$= 281 \cos 28°$$

$$= 248.11 \text{ N/m}$$

$$\text{Total load normal to roof} = 1920 + 248.11 = 2168.11 \text{ N/m}$$

$$\text{Moment on purlins } M_{uu} = \frac{PL}{10} = \frac{(2168.11 \times 4) \times 4}{10}$$

$$= 3468.96 \text{ N-m}$$

Similarly,

$$M_{vv} = \frac{HL}{10} = \frac{131.92 \times 4 \times 4}{10}$$

$$M_{vv} = 211.072 \text{ N-m}$$

$$\sigma_{max} = \frac{M_x y_{max}}{I_{xx}} + \frac{M_y x_{max}}{I_{yy}}$$

$$= \frac{3468.96 \times 10^3 \times \frac{125}{2}}{406.8 \times 10^4} + \frac{211.072 \times 10^3 \times \frac{75}{2}}{43.4 \times 10^4}$$

$$= 53.296 + 18.238 = 71.534 \text{ N/mm}^2$$

$$\text{Permissible bending stress} = 165 \text{ MPa} \times 1.33 = 219.45 \text{ MPa}$$

$\Rightarrow$ $\sigma_{max} < \sigma_{per}$ Safe

CHAPTER 9

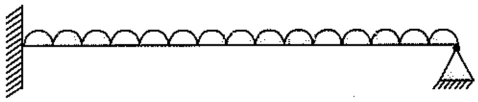
PLASTIC ANALYSIS

Q-1: ***A propped cantilever of span L is loaded by a uniformly distributed load of intensity w/unit length. Obtain the value of w at collapse if fully plastic moment of the section is M_P.***

[10 Marks, ESE-1995]

Sol: ***This problem can be solved by various methods. As this is a basic question, we will solve it by various methods, just for illustration.***

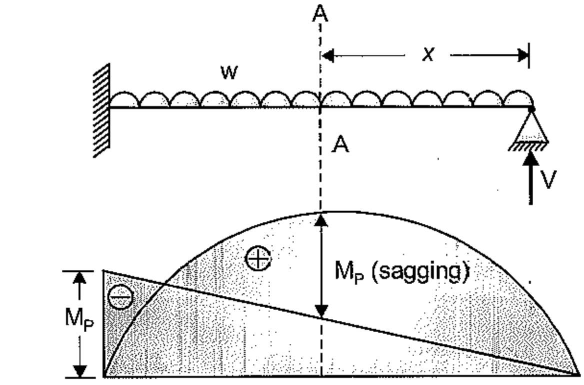
Method-I:



Degree of redundancy = 1.

So, number of hinge required for complete collapse = 1 + 1 = 2

Probable locations of hinges are at fixed support and location where shear force is zero.



(hogging)

Since the beam is prismatic we assume that at a point x distance away from propped end, shear force is zero and plastic hinge is formed.

Let the reaction at the propped end is V.

Hence moment at section A – A,

$$M_P = Vx - \frac{wx^2}{2}$$

and Shear force = 0

$\therefore$ $V = wx$

$\therefore$ $$M_P = wx^2 - \frac{wx^2}{2} = \frac{wx^2}{2} \quad \text{...(i)}$$

and at fixed support,

$$VL - \frac{wL^2}{2} = -M_P$$

$$\Rightarrow \quad wxL - \frac{wL^2}{2} = -\frac{wx^2}{2}$$

$$\Rightarrow \quad x^2 - L^2 + 2xL = 0$$

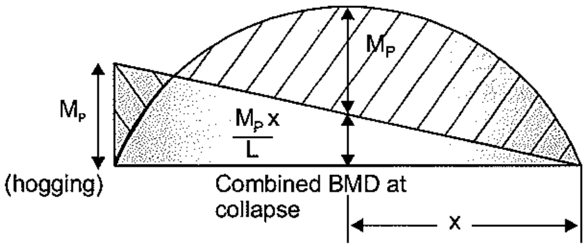
$$\therefore \quad x = 0.414\,L$$

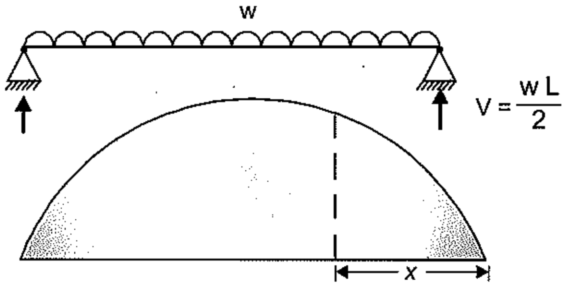
$$\therefore \quad W_u = \frac{2M_p}{x^2} \qquad \text{...[From (1)]}$$

$$\Rightarrow \quad W_u = \frac{2 \times M_P}{(0.414L)^2} = \frac{11.66\,M_P}{L^2}$$

$$\Rightarrow \quad \text{Collapse Load} = \frac{11.66\,M_P}{L^2}$$

Method II:





Ordinate of free BMD at a distance x from propped end

$$\Rightarrow \quad M_P + \frac{M_P x}{L} = Vx - \frac{wx^2}{2}$$

$$\Rightarrow \quad M_P + \frac{M_P x}{L} = \frac{wL}{2} \times x - \frac{wx^2}{2}$$

$$\Rightarrow \quad M_P = \frac{L(xL - x^2)}{2(L + x)} \times w$$

For 'w' to be minimum $\quad W = \frac{2M_p(L + x)}{L(xL - x^2)}$

$$\frac{dw}{dx} = 0$$

$$\Rightarrow \quad (xL - x^2) - (L + x)(L - 2x) = 0$$

$$xL - x^2 - L^2 + 2xL - xL + 2x^2 = 0$$

$$\Rightarrow \quad x^2 - L^2 + 2xL = 0$$

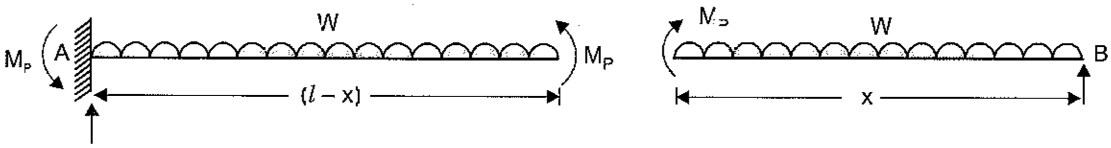
$$\Rightarrow \quad x = 0.414\,L$$

$$\therefore \quad M_P = \frac{L(0.414L^2 - (0.414L)^2)}{2(L + 0.414L)} \times w_u$$

$$\Rightarrow \quad \boxed{\frac{11.66\,M_P}{L^2} = w_u}$$

Method III:

Let us assume that there is max BM at a distance x from propped end. Hence Shear force at this location will be zero. Plastic hinge will form at this location.



At hinge location we can seperate this beam as shown above.

$$\Sigma M_A = 0$$

$$\Rightarrow \quad M_P + M_P = \frac{w(l-x)^2}{2}$$

$$\Rightarrow \quad 2M_P = \frac{w(l-x)^2}{2} \quad \ldots(i)$$

and $$\Sigma M_B = 0$$

$$\therefore \quad M_P = \frac{wx^2}{2} \quad \ldots(ii)$$

From equation, (i) & (ii)

$$\frac{2M_P}{M_P} = \frac{(l-x)^2}{x^2}$$

$$\Rightarrow \quad 2x^2 = l^2 + x^2 - 2lx$$

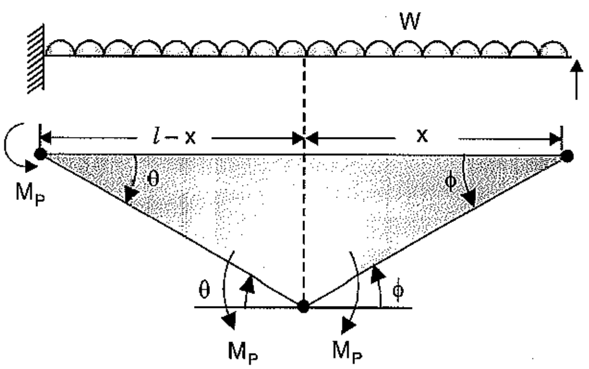
$$\Rightarrow \quad -l^2 + x^2 + 2lx = 0$$

$$\Rightarrow \quad x = 0.414\, l$$

Hence, $$\Rightarrow M_P = \frac{w \times (0.414l)^2}{2}$$

$$\therefore \quad w_u = \frac{11.66\, M_P}{l^2}$$

Method IV:



$$(l-x)\theta = x\phi$$

and, $$-M_P\theta - M_P\theta - M_P\phi + w \times \frac{1}{2} \times (l-x)\theta \times l = 0$$

[Work done due to udl = w × Area under deflected portion]

$$\Rightarrow \quad -2M_P\theta - \frac{M_P(l-x)\theta}{x} + \frac{w(l-x)l\theta}{2} = 0$$

$$\Rightarrow \quad M_P\left[2 + \frac{l-x}{x}\right] = \frac{w(l-x)l}{2}$$

$$\Rightarrow \quad M_P = \frac{w(l-x)lx}{2(l+x)}$$

$$\therefore \quad w = \frac{2M_P(l+x)}{l\,x(l-x)}$$

For w to be minimum,

$$\therefore \quad \frac{dw}{dx} = 0$$

$$\Rightarrow \quad x(l-x) - [l-2x](l+x) = 0$$

$$\Rightarrow \quad lx - x^2 - \{l^2 + xl - 2xl - 2x^2\} = 0$$

$$\Rightarrow \quad lx - x^2 - l^2 - xl + 2xl + 2x^2 = 0$$

$$\Rightarrow \quad x^2 - l^2 + 2xl = 0$$

$$\Rightarrow \quad x = 0.414\,l$$

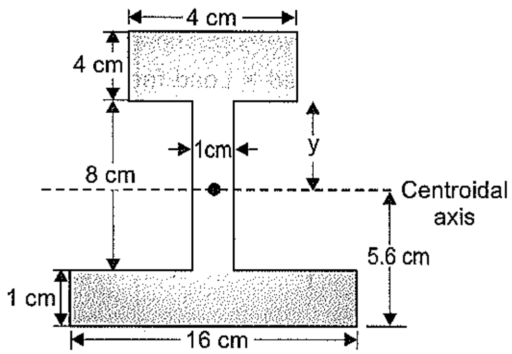
$$\therefore \quad w_u = \frac{2M_P(l+0.414l)}{l \times 0.414l \times (l-0.414l)}$$

$$\Rightarrow \quad w_u = \frac{11.66\,M_P}{L^2}$$

Q-2: ***Define 'Shape Factor'. Determine the shape factor of rail section which is symmetrical about vertical axis, top flange being 4 cm × 4 cm, bottom flange - 16 cm × 1 cm and web 1 cm × 8 cm. Area of section = 40 cm². Centroid of the section lies at 5.6 cm from bottom.***

[15 Marks, ESE-1996]

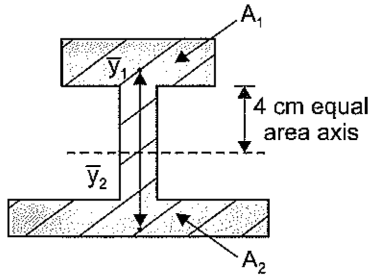
Sol:



- Shape factor is defined as the ratio of plastic moment and the yield moment of the section or, as the ratio of plastic section modulus to elastic section modulus.
- It is depend upon shape of section.
- It signifies the reserve strength available in the section from yielding to fully plastic state.

$$\text{S (shape factor)} = \frac{M_P}{M_y} = \frac{f_y Z_P}{f_y Z} = \frac{Z_P}{Z}$$

Neutral axes determination



For plastic state, Equal area axis will be the neutral axis

Hence, $(4 \times 4) + (1 \times y) = 16 \times 1 + 1 \times (8 - y)$

$\Rightarrow \quad 16 + y = 16 + 8 - y$

$\Rightarrow \quad 2y = 8 \quad \therefore \quad y = 4\text{cm}$

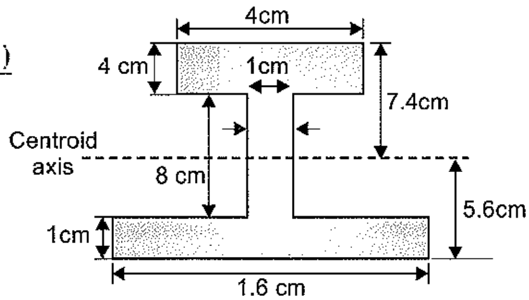
$\therefore \quad Z_p = A_1\bar{y}_1 + A_2\bar{y}_2$

$$A_1\bar{Y}_1 = 4 \times 4 \times (4 + 2) + 4 \times 1 \times 2 = 104 \text{ cm}^3$$

$$A_2\bar{Y}_2 = 16 \times 1 \times 4.5 - 4 \times 1 \times 2 = 80 \text{ cm}^3$$

$\Rightarrow \quad Z_p = 184 \text{ cm}^3$

For yielding, centroial axis will be the neutral axis,



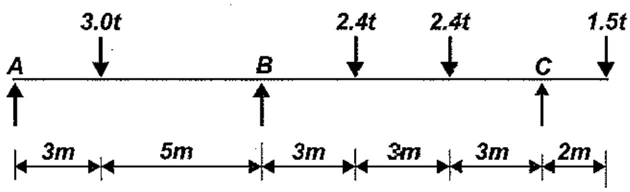
$$I_{c.g.} = \frac{4(7.4)^3}{3} - \frac{3\times(7.4-4)^3}{3} + \frac{16(5.6)^3}{3} - \frac{(16-1)(5.6-1)^3)}{3}$$

$$= 950.933 \text{ cm}^4$$

$$Z = \frac{950.933}{7.4}\text{cm}^4 = 128.5045\text{cm}^3$$

Hence, shape factor (S) $= \frac{Z_P}{Z} = \frac{184}{128.50} = 1.432$

Q-3: ***A steel beam simply supported at A, B and C, is required to carry the loads as shown in fig.***

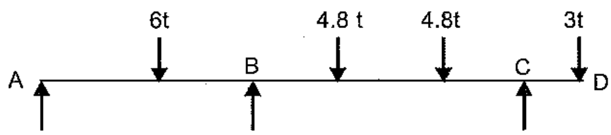


Assuming load factor of 2.0, yield stress for steel = 2500 Kg/cm² shape factor of section = 1.12, determine the minimum value of section modulus of the beam section. 1 t = 1000 kg.

[15 Marks, ESE-1997]

Sol: We know that, collapse load = Working load × Load factor

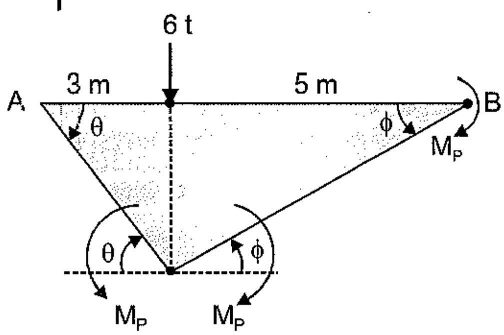
Hence, all the working loads can be converted to ultimate loads as shown below.



Degree of redundancy = 1

$\therefore$ Number of hinges required = 2

(i) Considering Part AB (Mechanism I)



$$-M_P\theta - M_P\phi - M_P\phi + 6\times(3\theta) = 0$$

$$\Rightarrow \quad -M_P - 2M_P\frac{\phi}{\theta} + 18 = 0$$

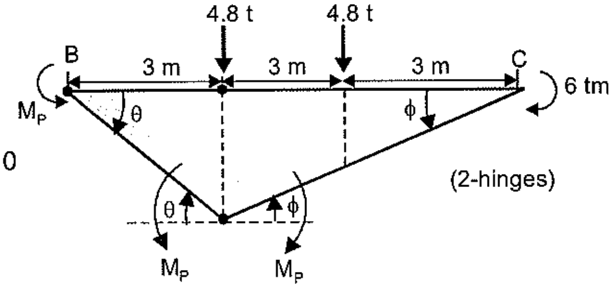
$$3\theta = 5\phi$$

$$\Rightarrow \quad -M_P - 2M_P\frac{3}{5} + 18 = 0$$

$$\Rightarrow \quad \frac{-11M_P}{5} + 18 = 0$$

$$\Rightarrow \quad M_P = \frac{90}{11}\text{tm} = 8.182\text{ tm}$$

(ii) Considering part BC (Mechanism II)



There are two possible cases in BC

$$3\theta = 6\phi$$

$$-2M_P\theta - M_P\phi + (4.8\times6\phi + 4.8\times3\phi) - 6\phi = 0$$

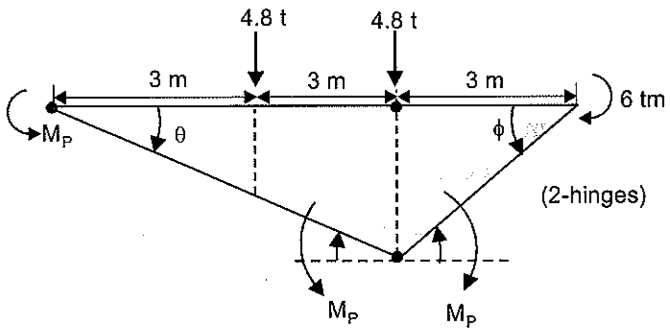
$$\Rightarrow \quad 2M_P\times\theta + M_P\phi = 4.8\times9\phi - 6\phi$$

$$\Rightarrow \quad 4M_P\phi + M_P\phi = 4.8\times9\phi - 6\phi$$

$$\Rightarrow \quad M_P = \left(\frac{4.8\times9-6}{5}\right)\text{tm}$$

$$\Rightarrow \quad M_P = 7.44\text{ tm}$$

(iii) (Mechanism III)



$$6\theta = 3\phi$$

$$-M_P\theta - M_P\theta - M_P\phi + 4.8\times(6\theta + 3\theta) - 6\phi = 0$$

$$\Rightarrow \quad \boxed{\phi = 2\theta}$$

$$-M_P\theta - M_P\theta - 2M_P\theta + 4.8\times(6\theta+3\theta) - 6\phi = 0$$

$$\Rightarrow \quad -2M_P\theta - 2M_P\theta + 4.8\times9\theta - 6\times2\theta = 0$$

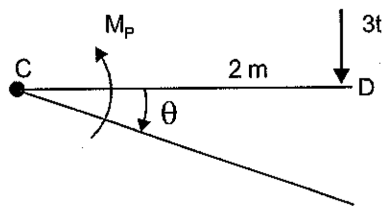
$$\Rightarrow \quad 4M_P\theta = 4.8\times9\theta - 12\theta$$

$$\Rightarrow \quad M_P = 7.8\text{ tm}$$

(iv) Considering part CD

$$-M_P\theta + 3\times2\theta = 0$$

$$M_P = 6\text{tm}$$



Section chosen should be corresponding to largest value of M_P

$$\therefore \quad M_P = 8.18 \text{ tm}$$

But,
$$fy \times Z_P = M_P$$

$$\therefore \quad 2500 \times Z_P = 8.18 \times 100 \times 1000 \text{ Kg cm}$$

$$\therefore \quad Z_P = 327.2 \text{ cm}^3$$

$$\text{Shape factor} = \frac{Z_P}{Z}$$

$$\therefore \quad Z = \frac{Z_P}{S} = \frac{327.2}{1.12} = 292.14 \text{ cm}^3 \text{ Ans.}$$

Q-4: ***Explain the upper bound and lower bound theorems as applied to plastic analysis taking an example of a fixed beam under uniformly distributed load.***

[10 Marks, ESE-1998]

Sol: In plastic analysis, following condition must be satisfied

(i) **Equilibrium condition** $(\Sigma F = 0, \Sigma M = 0)$

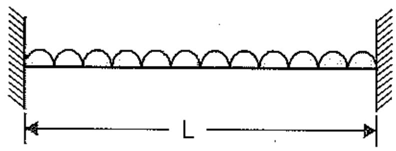
(ii) **Mechanism condition:** At collapse sufficient number of plastic hinge must be developed so as to transform a part or whole of the structure into a mechanism leading to collapse

(iii) **Yield condition:** At collapse, bending moment at any section must not exceed the fully plastic moment capacity of the section

There are two method of analysis by plastic theory:

(A) Upper bound theorem

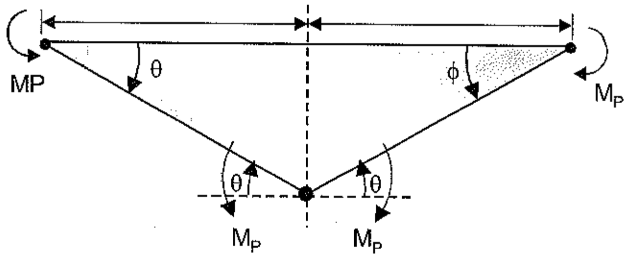
- It is also called kinematic theory.
- This theory satisfies equilibrium and mechanism condition
- The theorem can be stated as of all the possible mechanisms obtained by assuming various position of plastic hinge, the correct mechanism is the one for which loading is minimum and that load is always greater than or equal to the actual collapse load.



Degree of redundancy = 2

$\therefore$ Number of hinge required for complete collapse = 2 + 1 = 3

Probable position of hinges are two nos. at the supports and 1 at the middle point of the span (where shear force = 0).



$$-M_P\theta \times 4 + \text{ work done by udl} = 0$$

Since work done by udl $= w \times$ Area of deflected shape

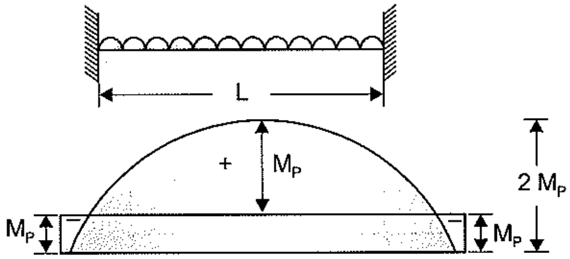
$$= w \times \frac{1}{2} \times L \times \left(\frac{L}{2}\theta\right) = \frac{wL^2\theta}{4}$$

$\therefore$
$$-M_P \times \theta \times 4 + \frac{wL^2}{4}\theta = 0$$

$\Rightarrow$
$$\boxed{\frac{16\,M_P}{L^2} = w}$$

(B) Lower bound theorem

- It is also called static method
- This theorem satisfies equilibrium and yield condition
- According to this theorem, "A load, determined on the basis of any collapse BMD in which BM at any section is less than plastic moment, will be less than or equal to actual collapse load.



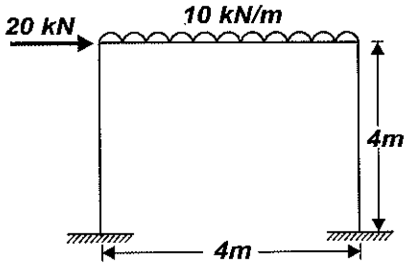
Collapse BMD

$\therefore$
$$2M_P = \frac{wl^2}{8} \quad \text{[Free BMD ordinate]}$$

$\Rightarrow$
$$M_P = \frac{wl^2}{16}$$

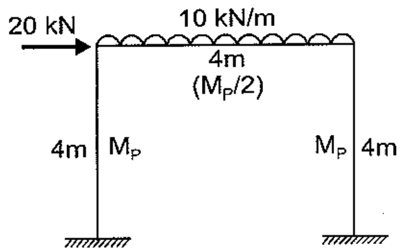
$$W = \frac{16M_P}{\ell^2}$$

Q-5: ***Determine the value of fully plastic moment M_p for the frame shown in fig. Plastic moment capacity of beam = 1/2. Plastic moment capacity of column.***



[12 Marks, ESE-2001]

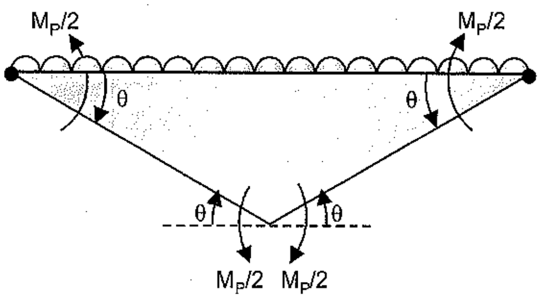
Sol:



Degree of indeterminancy = 6 − 3 = 3

⇒ No. of plastic hinge req. for complete collapse = 4

For beam mechanism



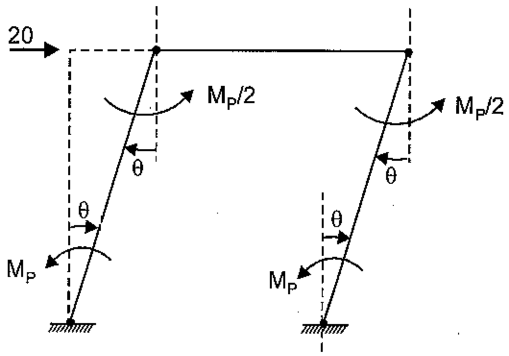
By applying principal of virtual work,

$$-4\left(\frac{M_P}{2}\right)\theta + w\left(\frac{1}{2}\times 4\times 2\theta\right) = 0$$

$$\Rightarrow \quad M_P = 2\,w = 2\times 10 \text{ kNm}$$

$$\Rightarrow \quad M_P = 20 \text{ kN-m}$$

For Sway mechanism



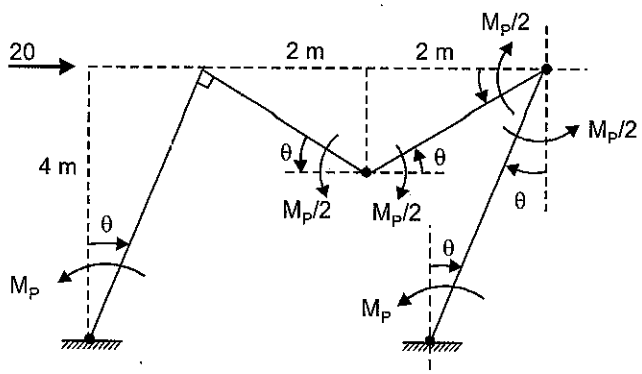
By applying principal of virtual work,

$$-M_P\theta - M_P\theta - 2\times\frac{M_P}{2}\times\theta + 20\times 4\theta = 0$$

$$\Rightarrow \quad -3M_P\theta + 80\theta = 0$$

$$\therefore \quad M_P = \frac{80}{3} = 26.67 \text{ kN-m}$$

Combined Mechanism



By applying principal of virtual work

$$-M_P\theta - \frac{M_P\theta}{2} - \frac{M_P\theta}{2} - \frac{M_P\theta}{2} - \frac{M_P\theta}{2} - M_P\theta + 20\times 4\theta + 10\times\frac{1}{2}\times 4\times 2\theta = 0$$

$$\Rightarrow \quad -4M_P\theta + 80\theta + 40\theta = 0$$

$$\Rightarrow \quad 4\,M_P = 120$$

$$\Rightarrow \quad M_P = 30 \text{ kN-m}$$

Thus the maximum value of M_P = 30 kN-m Ans.

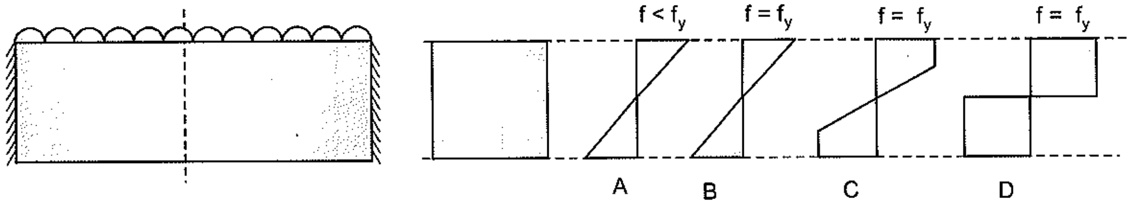
Note that max. value of M_P is chosen so that the given loading leads to just collapse. If we chosen smaller value of M_P, a load less than the given load may also lead to collapse.

Q-6: ***Discuss the concept of plastic hinge. Obtain the shape factor for a circular section.***

[15 Marks, ESE-2002]

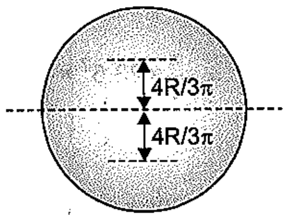
Sol:

- A plastic hinge can be defined as a yielded zone due to flexure in structure in which infinite rotation can take place at a constant resisting moment M_P of the section.



- In the fixed beam shown above when we increase loading gradually, the four condition of bending stress at middle point of span shows the various states of stress viz. yield condition and plastic condition.
- Once the section reaches to stress condition D, the section has lost its capacity to resist any further moment. Hence for moment greater than M_P the section will behave as a hinge. This hinge is called plastic hinge.
- Sometimes, plastic hinge can be thought of as a rusted hinge in which rotation doesn't occur significantly upto a certain moment and beyond that large unrestrained deformation *i.e.*, rotation, occurs.

Shape factor for circular section



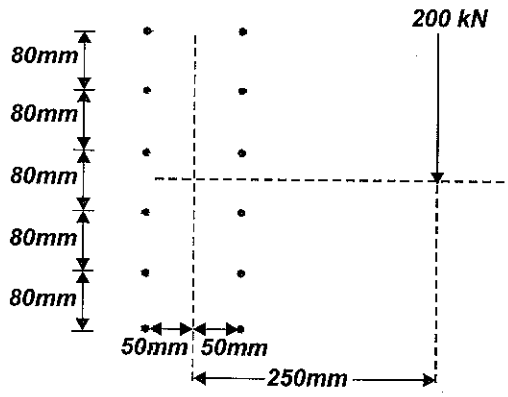
$$\frac{4R}{3\pi} = \frac{2D}{3\pi}$$

$$Z_P = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = \frac{\pi D^2}{8}\left(\frac{2D}{3\pi} + \frac{2D}{3\pi}\right) = \frac{\pi D^2}{8}\left(\frac{4D}{3\pi}\right) = \left(\frac{D^3}{6}\right)$$

and $$Z = \frac{\pi D^3}{32}$$

$$\therefore \quad \text{Shape factor} = \frac{Z_P}{Z} = \frac{D^3/6}{\frac{\pi D^3}{32}} = \frac{32}{6\pi} = 1.698$$

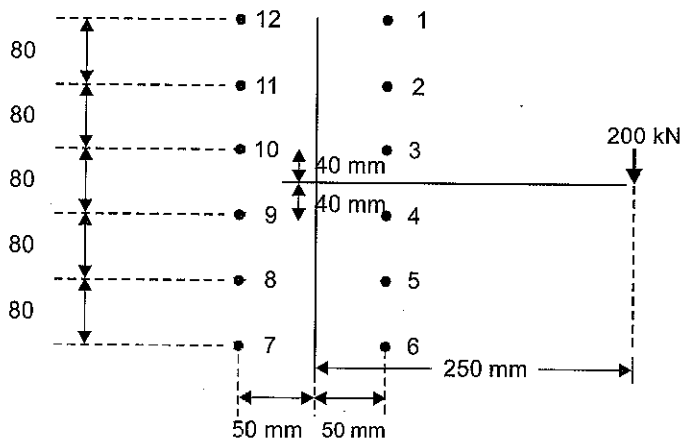
Q-7:



The above figure shows a steel bracket connection using 22 mm dia (12 No.) rivets. The connection transmits an eccentric load of 200 kN, at a distance of 250 mm from the centre of group of rivets. Find the maximum force in the rivets. Check whether the rivets are safe if the total thickness of plates connecting the rivets is 20 mm.

[25 Marks, ESE-2002]

Sol:



Nominal diameter = 22 mm

∴ Gross diameter (dn) = 22 + 1.5 = 23.5 mm

Calculation of Rivet value

$$\text{Shearing strength in one way shear} = \frac{\pi}{4} \times 23.5^2 \times \frac{100}{1000} = 43.37 \text{ kN}$$

[Assuming power driven shop rivet]

$$\text{Bearing strength of one rivet} = 300 \times 23.5 \times \frac{20}{1000} = 141 \text{ kN}$$

[Assuming governing thickness to be 20 mm]

Hence, the rivet value, $R_V = 43.37$ kN.

Force calculation

Force in each rivet due to direct shear,

$$F_d = \frac{200}{12} = 16.67 \text{ kN.}$$

Radial distance calculation

$$r_1 = r_6 = r_7 = r_{12} = \sqrt{200^2 + 50^2} = 206.155\text{ mm}$$

$$r_2 = r_5 = r_8 = r_{11} = \sqrt{120^2 + 50^2} = 130\text{ mm}$$

$$r_3 = r_4 = r_9 = r_{10} = \sqrt{40^2 + 50^2} = 64.03\text{ mm}$$

$$\therefore \quad \Sigma r_i^2 = 4 \times [206.155^2 + 130^2 + 64.03^2] = 253998.9\text{ mm}^2$$

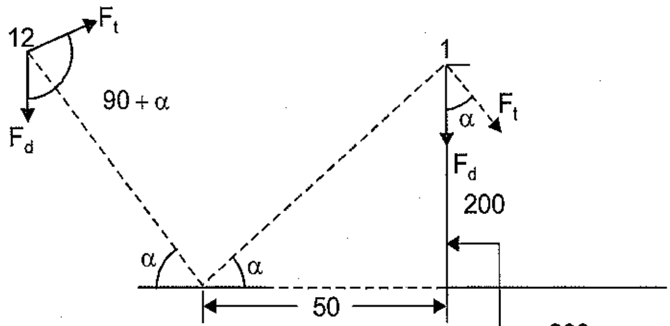
Torsional shear force,

$$F_t = \frac{(Pe)r_i}{\Sigma r_1^2}$$

It has been asked in the problem to calculate max. force in Rivets. The most critical rivets are rivet (1) & (6) because these rivets are at farthest distance from the c.g. of Rivet group and also angle between F_d & F_t is smallest.

$$F_{t1} = F_{t6} = F_{t7} = F_{t12} = \frac{200 \times 10^3 \times 250 \times 206.155}{253998.9 \times 1000} = 40.58\text{ kN}$$

Consider rivet 1



$$\tan\alpha = \frac{200}{50}$$

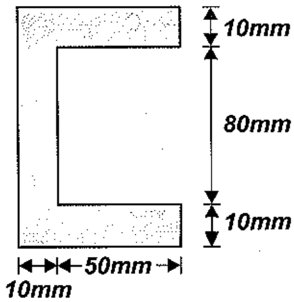
$$\Rightarrow \cos\alpha = 0.243$$

$$\therefore \quad FR|_{\text{for rivet 1}} = \sqrt{F_t^2 + F_d^2 + 2F_tF_d\cos\alpha} = \sqrt{40.58^2 + 16.67^2 + 2 \times 40.58 \times 16.67 \times 0.243} = 47.47\text{ kN}$$

$\Rightarrow$ Max Force in Rivet is 47.47 kN > R_v

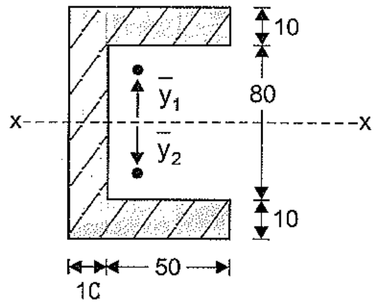
As the force is greater than rivet value, hence the Rivets are not safe.

Q-8: ***Determine shape factor for the cross-section given in the figure below:***



[8 Marks, ESE-2003]

Sol:



Since x-x axis is the axis of symmetry, hence it is both equal area axis and centroidal axis. If $\bar{y}_1$, $\bar{y}_2$ are the distance of C.G. of area above & below. equal area axis then,

$$\bar{y}_2 = \bar{y}_1 = \frac{10\times60\times45+40\times10\times20}{1000} = 35\,\text{mm}$$

$$\therefore \quad Z_P = \frac{A}{2}(\bar{y}_1+\bar{y}_2) = 1000\times(35\times2) = 7\times10^4\ \text{mm}^3$$

In yield condition, neutral axes is the centroides axes and due the symmetry equal area axes and the centroidal axes will be the same.

Hence,
$$I = \left(\frac{60\times(50)^3}{3} - \frac{50\times(40)^3}{3}\right)\times 2$$

$$= 2866666.67\ \text{mm}^4$$

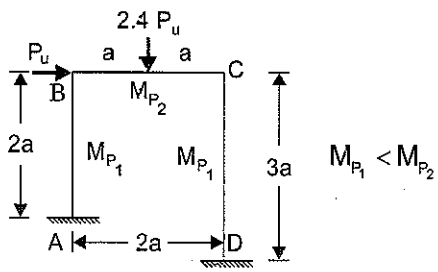
$$\therefore \quad Z = \frac{I}{y_{max}} = \frac{2866666.67}{50} = 57333.33\ \text{mm}^3$$

$$\therefore \quad \text{S (Shape factor)} = \frac{Z_P}{Z} = \frac{7\times10^4}{57333.33} = 1.221\ \text{Ans.}$$

Q-9: **A Portal frame ABCD fixed *at the base carries an ultimate concentrated load of 2.4P_u at the centre of beam BC and ultimate horizontal load of P_u acting at B from left to right. The lengths of columns AB & CD are 2a and 3a respectively and the beam is 2a long. Each column has fully plastic moment of M_{P_1} while fully plastic moment of beam is M_{P_2}. Assuming $M_{P_1} < M_{P_2}$, determine M_{P_1} and M_{P_2} for the condition that collapse may just occur whatever may be the direction of horizontal force.***

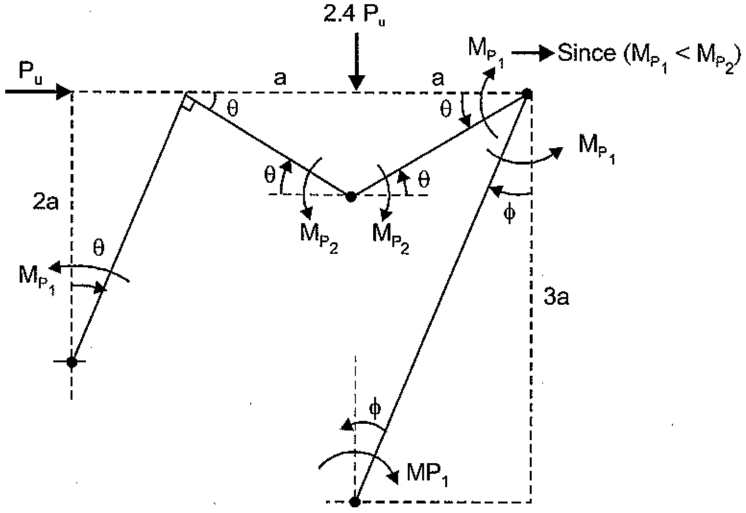
[20 Marks, ESE-2003]

Sol:



- The portal frame according to given loadings is shown above
- We will consider combined mechanism of failure for the two directions of horizontal force, there by giving two equation in Mp_1 & Mp_2. By solving these two equation the values of Mp_1 & Mp_2 can be obtained.

When the Horizontal sway force acts from left to right



$$2a\theta = 3a\phi$$

$\therefore$
$$\phi = \frac{2}{3}\theta$$

Applying principal of virtual work, we get

$$P_u \times 2a\theta + 2.4P_u \times a\theta - M_{P1}\theta - 2M_{P2}\theta - M_{P1}\theta - M_{P1}\phi - M_{P_1}\phi = 0$$

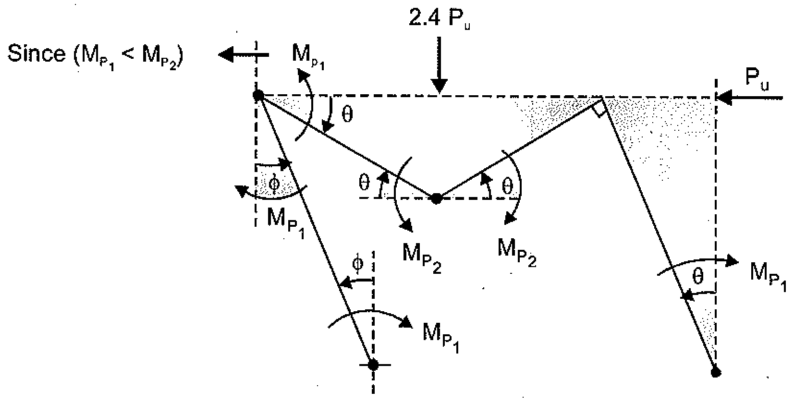
$$\Rightarrow \quad 4.4\,P_u a\theta - 2M_{P_1}\theta - 2M_{P_2}\theta - 2M_{P_1} \times \frac{2}{3}\theta = 0$$

$$\Rightarrow \quad 4.4\,P_u a\theta - 2M_{P_1}\theta - \frac{4M_{P_1}}{3}\theta - 2M_{P_2}\theta = 0$$

$$\Rightarrow \quad 10M_{P_1} + 6M_{P_2} = 4.4 \times 3\,P_u a$$

$$\Rightarrow \quad \boxed{5\,M_{P_1} + 3M_{P_2} = 6.6\,P_u a} \qquad \text{...(i)}$$

When the horizontal sway force acts from right to left



$$2a\phi = 3a\theta$$

$\therefore$
$$\phi = \frac{3}{2}\theta$$

By applying principal of virtual work, we get.

$$P_u \times 3a\theta + 2.4\,P_u \times a\theta - M_{P_1}\theta - 2M_{P_2}\theta - M_{P_1}\theta - 2Mp_1\phi = 0$$

$$\Rightarrow \quad 5.4\,P_u a\theta - 2M_{P_1}\theta - 2M_{P_2}\theta - 2M_{P_1} \times \frac{3\theta}{2} = 0$$

$$\Rightarrow \quad 5.4\,P_u a - 5M_{P_1} - 2M_{P_2} = 0$$

$$\Rightarrow \quad \boxed{5M_{P_1} + 2M_{P_2} = 5.4\,P_u a} \qquad \text{...(ii)}$$

$\therefore$ By solving above two equation; we get

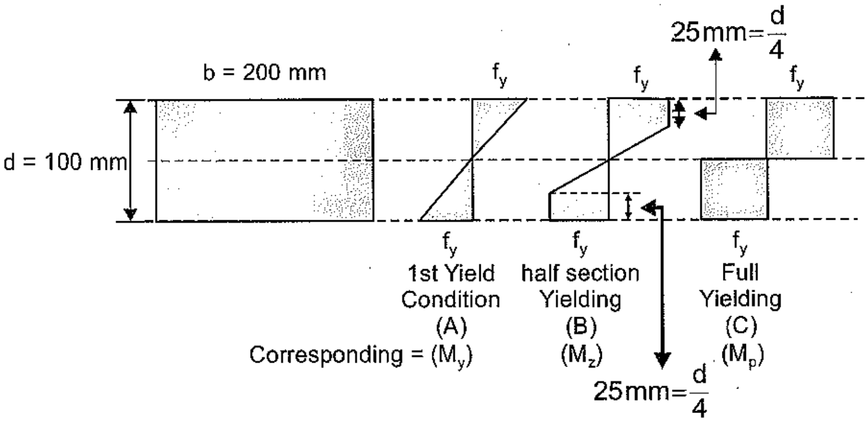
$$\boxed{M_{P_1} = 0.6\,P_u a} \quad \text{and} \quad \boxed{M_{P_2} = 1.2 P_u a} \text{ (Ans.)}$$

Q-10: ***For a mild steel beam of rectangular cross-section with b = 200 mm and d = 100 mm, yield stress f_y = 250 MPa, modulus of elasticity. E = 2.0 × 10^5 MPa, determine the value of bending moment M_y at initial yield of the extreme fibre and also the value of fully plastic moment M_P.***

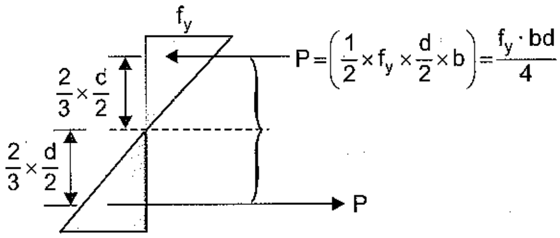
Determine also the curvature k_y at initial yield and also the curvature k_z when half the cross-section has yielded alongwith the corresponding value of the bending moment M_z.

[10 Marks, ESE-2004]

Sol:



Bending moment corresponding to 1st yield (M_y)



$$\therefore \quad \text{Lever arm} = \frac{2d}{3}$$

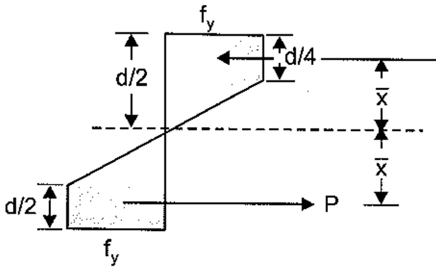
$$\Rightarrow \quad M_y = P \times \frac{2d}{3} = fy.\frac{bd}{4} \times \frac{2d}{3}$$

$$\Rightarrow \quad M_y = \frac{250 \times 200 \times (100)^2 \times 2 \times 100}{6}$$

$$= 83.33 \times 10^6 \text{ Nm-m}$$

$$\boxed{M_y = 83.33\,\text{kNm}}$$

BM corresponding to the case when 1/2 of the section hay Yielded (M_z)



$$P = \left(f_y \cdot \frac{d}{4} \times b\right) + \frac{1}{2}(f_y) \times \frac{d}{4} \times b = \frac{3}{2} \cdot \frac{f_y bd}{4} = \frac{3 f_y bd}{8}$$

$$\text{Lever arm} = 2\bar{x}$$

$$\bar{x} = \frac{\frac{f_y bd}{8} \times \frac{2}{3} \times \frac{d}{4} + \frac{f_y bd}{4} \times \left(\frac{d}{4} + \frac{d}{8}\right)}{\frac{3 f_y bd}{8}}$$

$$\bar{x} = \frac{\frac{2d}{12} + 2 \times \left(\frac{3d}{8}\right)}{3} = \frac{\frac{d}{6} + \frac{3d}{4}}{3}$$

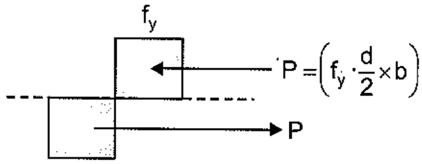
$$\bar{x} = \frac{22d}{72}$$

$$\Rightarrow \quad \text{Lever arm} = 2\bar{x} = \frac{22d}{36} = \frac{11d}{18}$$

$$\Rightarrow \quad M_z = \frac{3 f_y bd}{8} \times \frac{11d}{18} = \frac{11 f_y bd^2}{48}$$

$$\Rightarrow \quad \boxed{M_z = \frac{11 f_y bd^2}{48}} = \frac{11 \times 250 \times 200 \times (100)^2}{48} = 114.583 \text{ kNm.}$$

BM corresponding to fully plastic statye (M_P)



$$M_P = \frac{f_y bd^2}{4} = \frac{250 \times 200 \times 100}{4}$$

$$= 125 \text{ kNm}$$

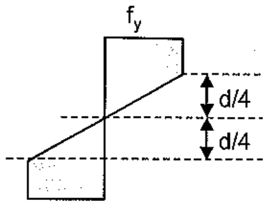
Curvature at initial yield (k_y)

$$\frac{M_y}{I} = \frac{fy}{d/2} = (E.K_y)$$

$$\Rightarrow \quad K_y = \frac{fy}{d/2 \times E} = \frac{250 \times 2}{100 \times 2 \times 10^5} = 2.5 \times 10^{-5} \text{ Per mm}$$

Curvature when 1/2 of the section has Yielded (k_z)

We know that



$$\text{Strain} = \in = \frac{y}{R} = k_z y$$

for

$$Y = \frac{d}{4}$$

$$\in = \frac{fy}{E}$$

$$\Rightarrow \qquad \frac{f_y}{E} = K_z \times \frac{d}{4}$$

$$\Rightarrow \qquad K_z = \frac{f_y}{E\frac{d}{4}} = \frac{4f_y}{Ed} = \frac{4\times250}{2\times10^5\times100} = 5\times10^{-5} \text{ per mm.}$$

Q-11: ***A mild steel T section has the following cross-sectional dimensions:***

Total depth = 200 mm

Width of flange = 120 mm

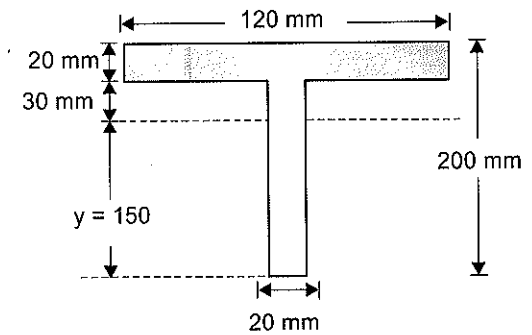
Thickness of flange = 20 mm

Thickness of web = 20 mm

If the yield stress σ_y = 250 MPa determine the plastic moment capacity of the section. Also calculate the shape factor for the section.

[20 Marks, ESE-2009]

Sol: For fully plastic condition:



Total area of cross section = 120 × 20 + 180 × 20 = 300 × 20 = 6000 mm^2

For fully plastic condition neutral axis is the equal area axis.

$$\Rightarrow \qquad 20\,y = \frac{6000}{2}$$

$$y = 150 \text{ mm}$$

For upper part, distance of CG of area above equal area axis = $\bar{y}_1$

$$\bar{y}_1 = \frac{120\times20\times40+30\times20\times15}{3000} = 35 \text{ mm}$$

For lower part, distance of CG of area below equal area axis $\bar{y}_2$

$$\bar{y}_2 = 75 \text{ mm}$$

$$\therefore \qquad M_P = \frac{A}{2}(\bar{y}_1+\bar{y}_2)\times f_y = \left[\frac{6000}{2}(35+75)\times250\times10^{-6}\right]\text{kN-m}$$

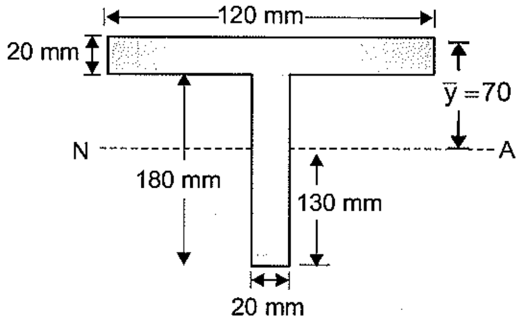
$$\mathbf{M_P = 82.5 \text{ kN-m}}$$

For initial yield

Neutral axis is the centroidal axis

Let us assume the depth of neutral axes from the top of the flange is $\bar{y}$.

$$\therefore \quad \bar{y} = \frac{120\times20\times10+20\times180\times110}{6000} = 70\ \text{mm}$$



Moment of inertia of the section about Neutral axis

$$I_{NA} = \frac{120\times20^3}{12}+120\times20\times60^2+\frac{20\times180^3}{12}+20\times180\times40^2 = 2.42\times10^7$$

$$\therefore \quad Z = \frac{1.8427\times10^7}{110} = 1.8615\times10^5$$

$$\therefore \quad M_y = f_y\times Z = 250\times1.8615\times10^5 = 4.653\times10^7$$

$$\therefore \quad M_y = 46.53\ \text{kN-m}$$

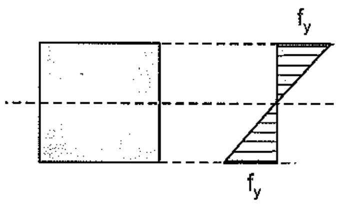
Hence Shape factor (S) $= \dfrac{M_P}{M_y} = \dfrac{82.5}{46.53} = 1.77$

Q-12: ***(i)*** ***Distinguish between yield moment and plastic moment capacity of a section.***

(ii) ***Determine the shape factor from first principles for a triangular section of base width b and height h.***

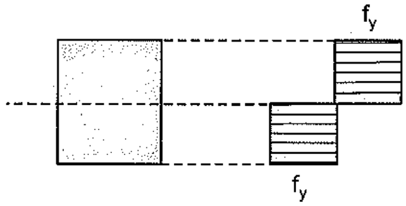
[2 + 12 Marks, ESE-2011]

Sol: (i) Moment of resistance of a section corresponding to just yielding of extreme fibre of the section is called yield moment capacity of the section.



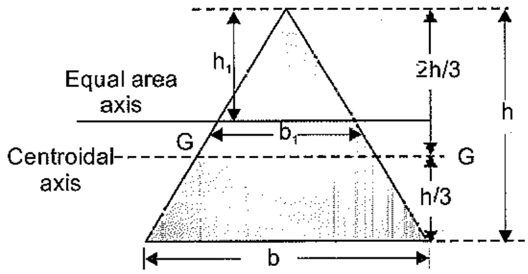
Moment capacity corresponding to this stress distribution is called yield moment capacity (M_y)

Moment of resistance of the section corresponding to complete yielding of the section is called plastic moment capacity of the section (M_p)



Moment capacity corresponding to this stress condition is called M_p.

(ii)



Since $I_{GG} = \frac{bh^3}{36}$

and $y_{max} = \frac{2h}{3}$

$\therefore$ $z_y = \frac{\frac{bh^3}{36}}{\frac{2h}{3}} = \frac{bh^2}{24}$

For plastic analysis, Neutral axes is equal area axes.

$$\frac{1}{2} \times b_1 h_1 = \frac{1}{2} \times (b + b_1) \times (h - h_1)$$

and $\frac{b_1}{h_1} = \frac{b}{h}$ $\quad \therefore \quad b_1 = \frac{bh_1}{h}$ (From similar Triangle)

$\Rightarrow$ $\frac{1}{2} \times \frac{bh_1}{h} \times h_1 = \frac{1}{2}\left(b + \frac{bh_1}{h}\right)(h - h_1)$

$\Rightarrow$ $h_1^2 = (h + h_1)(h - h_1)$

$\Rightarrow$ $h_1^2 = (h^2 - h_1^2)$

$\Rightarrow$ $2h_1^2 = bh^2$

$\Rightarrow$ $h_1 = \frac{h}{\sqrt{2}}$

and $b_1 = \frac{b}{\sqrt{2}}$

Centroid of upper part from neutral axes $(\bar{y}_1)$

$$\bar{y}_1 = \frac{h_1}{3} = \frac{h}{3\sqrt{2}}$$

Centroidal distance of bottom part from neutral axes $(\bar{y}_2)$

$$\bar{y}_2 = \frac{b_1 + 2b}{(b_1 + b)} \times \left(\frac{h - h_1}{3}\right) = \left\{\frac{\frac{b}{\sqrt{2}} + 2b}{\frac{b}{\sqrt{2}} + b}\right\} \times \left\{\frac{h - \frac{h}{\sqrt{2}}}{3}\right\}$$

$$= \left(\frac{1 + 2\sqrt{2}}{\sqrt{2} + 1}\right) \times \left(\frac{\sqrt{2} - 1}{3\sqrt{2}}\right) \times h = 0.155\ h$$

$\Rightarrow$ $Z_P = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$

$$Z_P = \frac{bh}{4}\left(\frac{h}{3\sqrt{2}} + 0.155h\right) = 0.09763\ bh^3$$

$$\Rightarrow \quad \text{Shape Factor} = \frac{M_p}{M_y} = \frac{Z_p}{Z} = 2.343$$

Q-13: ***ISMB 450 is used as a propped cantilever beam of span 12 m. Assuming s_y = 250 MPa determine the factored uniformly distributed load q_u the beam can carry including self weight, if the load is to the applied over the entire span.***

The properties of ISMB 450 are as follows:

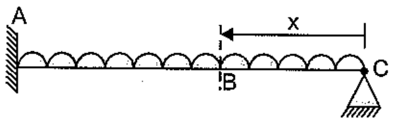
Weight/metre	:	***72.4 kg***
Area of cross-section	:	***9227 mm²***
Width of flange	:	***150 mm***
Thickness of flange	:	***17.4 mm***

$$I_{xx} = 3.039 \times 10^8\ mm^4$$

$$I_{yy} = 8.34 \times 10^6\ mm^4$$

[20 Marks, ESE-2012]

Sol:



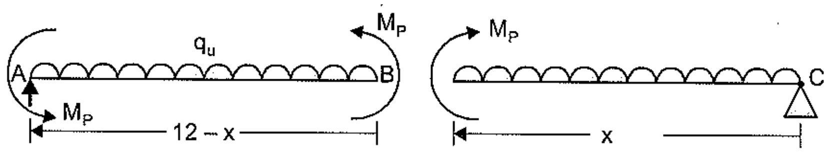
As the thickness of web is not given and also it is not mentioned as to by what approach we should solve the problem, we will use the plastic analysis approach to find out the factored load q_u.

Assume shape factor = 1.1

$$M_y = \text{Yield moment} = f_y.Z_e = \frac{f_y I_{xx}}{d/2} = \frac{2 \times 250 \times 3.039 \times 10^8}{450} = 3.37 \times 10^8 \text{ Nmm}$$

$$M_P = 3.37 \times 10^8 \times 1.1 = 3.71 \times 10^8 \text{ Nmm} \qquad \text{[Assuming shape factor = 1.1]}$$

Let plastic hinges formed for collapse be one at support A and other in the span at B.



$$2M_P = \frac{q_u(12-x)^2}{2} \qquad\qquad M_P = \frac{q_u x^2}{2}$$

$$\Rightarrow \quad = \frac{q_u(12-x)^2}{2} = q_u x^2$$

$$\Rightarrow \quad (12 - x)^2 = 2x^2$$

$$144 + x^2 - 24x = 2x^2$$

$$\Rightarrow \quad x^2 + 24x - 144 = 0$$

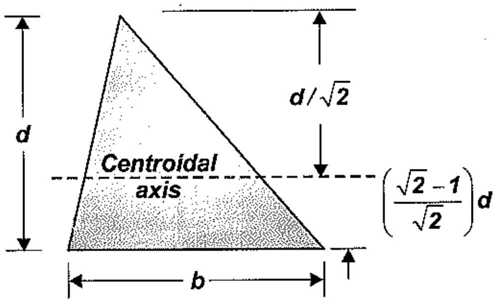
$$\Rightarrow \quad x = 4.97 \text{ m}$$

As, $$M_P = \frac{q_u x^2}{2}$$

$$\Rightarrow \qquad 3.71 \times 10^8 = \frac{q_u(4.97\times10^3)^2}{2}$$

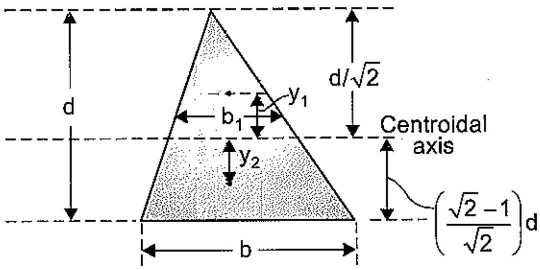
$$q_u = 30.074 \text{ N/mm} = 30.074 \text{ kN/m}$$

Q-14: ***Find the shape factor for a triangular section as shown in the figure.***



[10 Marks, ESE-2015]

Sol:



$$\frac{b_1}{b} = \frac{d/\sqrt{2}}{d}$$

$$\Rightarrow \qquad b_1 = \frac{b}{\sqrt{2}}$$

Shape factor,
$$S = \frac{Z_P}{Z_e} = \frac{\frac{A}{2}(\bar{y}_1 + \bar{y}_2)}{\frac{I}{y}}$$

$$A = \frac{1}{2}bd$$

$$\bar{y}_1 = \frac{1}{3}\frac{d}{\sqrt{2}} = 0.2357\,d$$

$$\bar{y}_2 = \frac{2b+b_1}{b+b_1}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\frac{d}{3} = \frac{2b+b/\sqrt{2}}{b+b/\sqrt{2}}\,\frac{\sqrt{2}-1}{3\sqrt{2}}d = 0.1548\,d$$

$$Z_P = \frac{bd}{4}(0.2357\,d + 0.1548\,d) = 0.0976\ bd^2$$

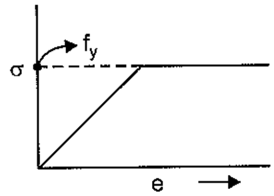
$$Z_e = \frac{I}{y} = \frac{bd^3}{36}\times\frac{3}{2d} = \frac{bd^2}{24}$$

$$S = \frac{0.0976\,bd^2}{bd^2/24} = 2.34$$

Q-15: ***What are the assumptions for plastic analysis of steel sections?***

[5 Marks, ESE-2016]

Sol: Following are the assumptions for plastic analysis of steel sections:



(i) Steel posses ductility so that it can be subjected to deformations in plastic region.

(ii) Strain distribution is linear.

(iii) Strain hardening neglected and stress distribution is given as above.

(iv) Steel behaves same in tension and compression i.e. $\sigma - \varepsilon$ relationship is same in tension and in compression.

(v) Joint is sufficiently strong so as to transfer the plastic moment by forming plastic hinge.

Q-16: ***Show that in a rectangular section subjected to axial force N and bending moment M,***

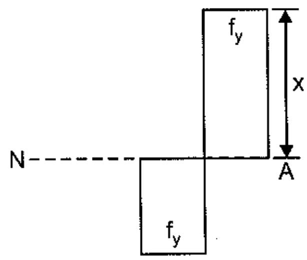
$$\frac{M}{M_p} + \left(\frac{N}{N_p}\right)^2 = 1$$

where, ***N_p = Axial yield capacity***

M_p = Plastic moment capacity

Sol: Axial compression = N

Sagging moment = M



$$C - T = N \quad \text{(Axial load applied)}$$

$$f_y\, bx - f_y\, b\,(d-x) = N$$

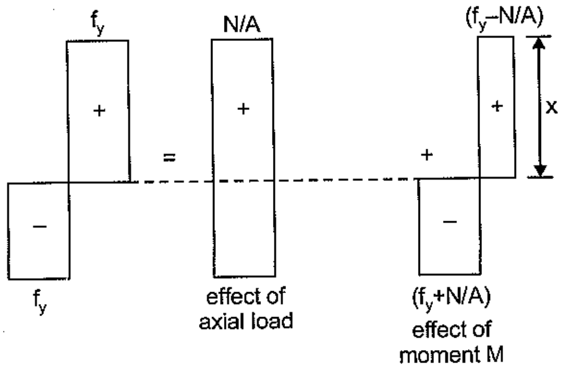
$$f_y\, b\,[x - d + x] = N$$

$$f_y bd\left(\frac{2x}{d} - 1\right) = N$$

$$\frac{2x}{d} - 1 = \frac{N}{f_y bd} \qquad f_y bd = N_p$$

Axial loading capacity = N_p

$$x = \frac{d}{2}\left[\frac{N}{N_p} + 1\right] \qquad \ldots\text{(i)}$$



$$M = (f_y - N/A)\frac{x^2 b}{2} + (f_y + N/A)\frac{(d-x)^2 b}{2}$$

$$= \frac{f_y b}{2}\left[x^2 + (d-x)^2\right] - \frac{Nb}{2A}\left(+x^2 - (d-x)^2\right)$$

$$= f_y \frac{b}{2}(x^2 + d^2 + x^2 - 2dx) - \frac{Nb}{2A}(+x^2 - x^2 - d^2 + 2dx)$$

$$= \frac{f_y b}{2}(2x^2 + d^2 - 2dx) - \frac{Nb}{2A}(2dx - d^2)$$

$$= \frac{f_y b}{2}\left[2\left[\frac{d}{2}\left(\frac{N}{N_p}+1\right)\right]^2 + d^2 - 2d\times\frac{d}{2}\left(\frac{N}{N_p}+1\right)\right] - \frac{Nb}{2A}\left[2d\times\frac{d}{2}\left(\frac{N}{N_p}+1\right) - d^2\right]$$

$$= \frac{f_y bd^2}{4}\left[\left(\frac{N}{N_p}+1\right)^2 + 2 - 2\left(\frac{N}{N_p}+1\right)\right] - \frac{Nbd^2}{2A}\left(\frac{N}{N_p}+1-1\right)$$

$$= \frac{f_y bd^2}{4}\left[\left(\frac{N}{N_p}\right)^2 + 2\left(\frac{N}{N_p}\right) + 1 + 2 - 2\left(\frac{N}{N_p}\right) - 2\right] - \frac{Nbd^2}{2A}\frac{N}{N_p}$$

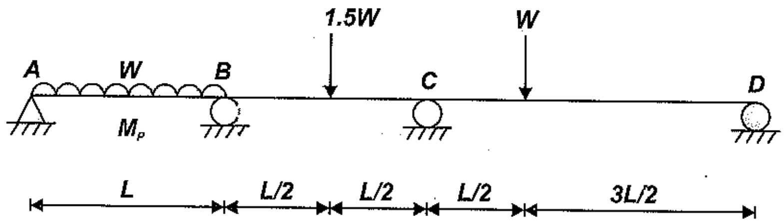
$$= M_p\left[\left(\frac{N}{N_p}\right)^2 + 1\right] - \frac{2N^2}{N_p}\frac{bd^2}{4bd}\times\frac{f_y}{f_y}$$

$$M = M_p\left(\left(\frac{N}{N_p}\right)^2 + 1\right) - 2\left(\frac{N}{N_p}\right)^2 M_p$$

$$\frac{M}{M_p} = \left(\frac{N}{N_p}\right)^2 + 1 - 2\left(\frac{N}{N_p}\right)^2$$

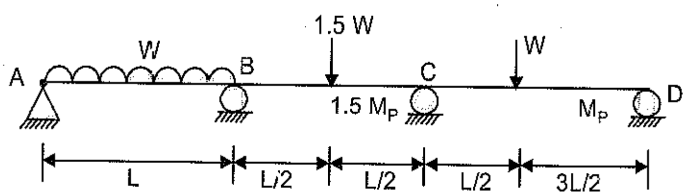
$$\frac{M}{M_p} + \left(\frac{N}{N_p}\right)^2 = 1$$

Q-17: ***Determine the collapse load in a three-span continuous beam shown in Figure. It has a moment capacity of M_P over the exterior spans AB and CD, and 1.5 M_P over the interior span BC.***



[20 Marks, ESE-2017]

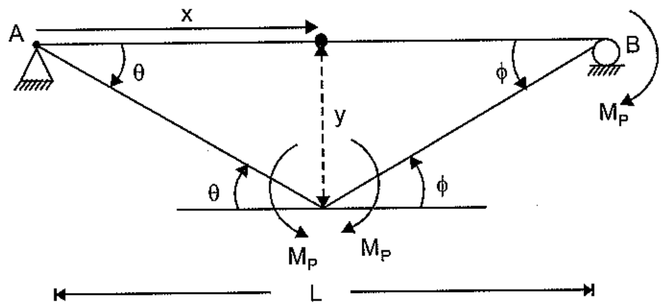
Sol:



Static indeterminacy = 2

No. of hinges for a mechanism formation = 3

Span AB



Let hinge form at distance x from end A

$$y = x\theta = (L-x)\phi \Rightarrow \theta = \left(\frac{L-x}{x}\right)\phi$$

$$\text{External work done} = \frac{1}{2}\times L\times y\times\frac{W}{L} \text{ (udL × Area)}$$

$$\text{Internal work done} = M_p\theta + M_p\phi + M_p\phi$$

$\because$ External work done = internal work done

$$\Rightarrow \quad \frac{1}{2}\times L\times(L-x)\phi\times\frac{W}{L} = M_P\left(\frac{L-x}{x}\right)\phi + 2M_P\phi$$

or
$$W = \frac{M_P\left(\frac{L-x}{x}+2\right)\times 2}{L-x}$$

or
$$W = \frac{M_P(L+x)}{(L-x)x}$$

For w to be minimum

$$\frac{dW}{dx} = 0 \Rightarrow 2M_P\left[\frac{(L-x)x-(L+x)\{L-2x\}}{(L-x)^2x^2}\right] = 0$$

i.e., $Lx - x^2 - L^2 + 2Lx - Lx + 2x^2 = 0$

$\Rightarrow \quad x^2 + 2Lx - L^2 = 0$

$$\Rightarrow \quad x = \frac{-2L\pm\sqrt{4L^2+4L^2}}{2} = \frac{-2L\pm 2\sqrt{2}L}{2} = -L\pm\sqrt{2}L = -L+\sqrt{2L} = 0.414\,L$$

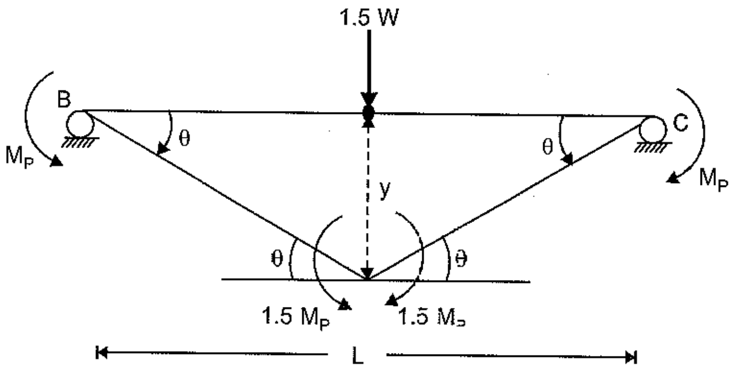
Substituting $x = 0.414\,L$ in equation (1)

$$W = \frac{2M_P(L+0.414L)}{(L-0.414L)\times 0.414L} = \frac{11.656\,M_P}{L}$$

Note: For propped cantiliver beam loaded under udL, plastic hinge forms at 0.414L from propped end and

$M_p = \frac{11.656\,M_p}{L}$.

Span BC



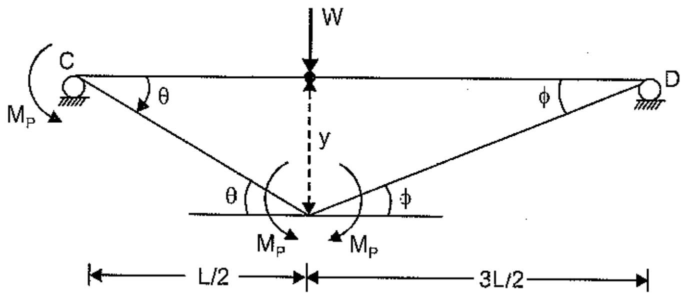
$$\text{Internal virtual work} = \text{External virtual work}$$

$$\Rightarrow \quad M_P\theta + 1.5\,M_P\theta + 1.5\,M_P\theta + M_P\theta = 1.5\,W \times \frac{L}{2}\theta$$

$$\Rightarrow \quad 5M_P\theta = \frac{1.5\,WL}{2}\theta$$

or
$$w = \frac{10}{1.5}\frac{M_P}{L} = 6.67\frac{M_P}{L}$$

Span CD



$$y = \frac{L}{2}\theta = \frac{3L}{2}\phi$$

$$\Rightarrow \quad \theta = 3\phi$$

$$\text{Internal virtual work} = \text{external virtual work}$$

$$\Rightarrow \quad M_P\theta + M_P\theta + M_P\phi = W \times \frac{3L}{2}\phi$$

or
$$2M_P \times 3\phi + M_P\phi = \frac{3WL}{2}\phi$$

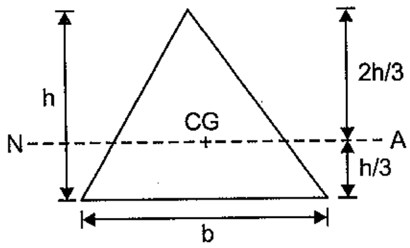
or
$$W = \frac{14}{3} \times \frac{M_P}{L}$$

$$\therefore \quad \text{Collapse load} = \min\left(\frac{11.656\,M_P}{L}, \frac{6.67\,M_P}{L}, \frac{14}{3}\frac{M_P}{L}\right) = \frac{14}{3}\frac{M_P}{L}$$

Q-18: ***Find the shape factor of a triangular section of base b and height h for bending about an axis parallel to the base.***

[12 Marks, ESE-2018]

Sol: Calculation for the shape factor of a triangular section:

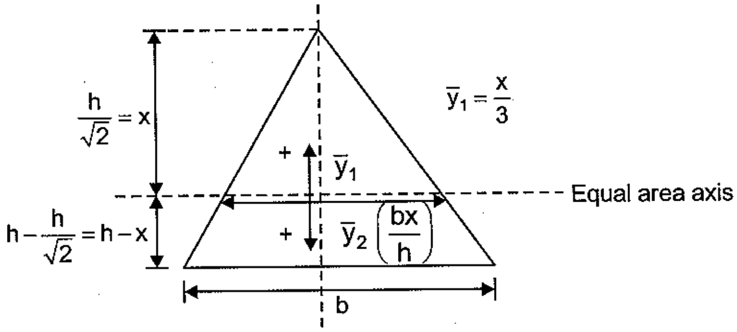


$$I_{NA} = \frac{bh^3}{36}$$

$$y_{max} = \frac{2h}{3}$$

$$\text{Section modulus (x)} = \frac{I_{NA}}{y_{max}} = \frac{bh^3}{36}\times\frac{3}{2h}$$

$$\boxed{z = \frac{bh^2}{24}}$$



Calculation for 'x'

$$\therefore \quad \frac{1}{2}\cdot x\cdot\frac{bx}{h} = \frac{1}{2}\times\frac{bh}{2}$$

$$\boxed{x = \frac{h}{\sqrt{2}}}$$

Then,
$$\bar{y}_1 = \frac{x}{3} = \frac{h}{3\sqrt{2}} = 0.235\,h$$

and,
$$\bar{y}_2 = \frac{2b+\left(\frac{bx}{h}\right)}{b+\left(\frac{bx}{h}\right)}\times\frac{\left(h-\frac{h}{\sqrt{2}}\right)}{3}$$

$$= \frac{\left(2+\frac{1}{\sqrt{2}}\right)}{\left(1+\frac{1}{\sqrt{2}}\right)}\times\frac{\left(1-\frac{1}{\sqrt{2}}\right)}{3}\times h$$

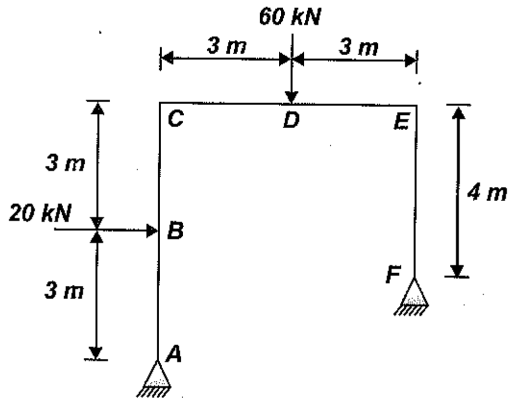
$$\boxed{\bar{y}_2 = 0.155h}$$

$$\text{Plastic modulus } (z_p) = \frac{A}{2}(\bar{y}_1+\bar{y}_2) = \frac{bh}{4}(0.235h+0.155h) = 0.0975\ bh^2$$

Hence,
$$\text{Shape factor (SF)} = \frac{z_p}{z} = \frac{0.0975bh^2}{bh^2}\times 24$$

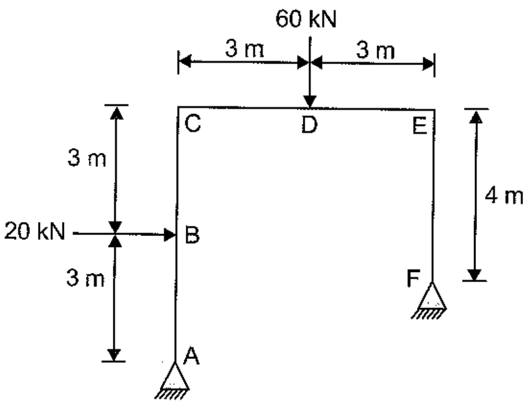
$$\boxed{SF = 2.34}$$

Q-19: ***Find the collapse load for a frame of uniform cross-section shown in figure under the applied forces. Also determine the minimum section in steel required to resist the applied forces.***



[20 Marks, ESE-2018]

Sol: Given that:



To find out:

(i) Collapse load

(ii) Minimum section in steel required to resist the applied force

The possible locations of plastic hinges are B, C, D and E.

Number of possible hinges, N = 4

Degree of redudancy, r = 4 – 3 = 1

Number of possible independent mechanisms = n = N – r = 4 – 1 = 3

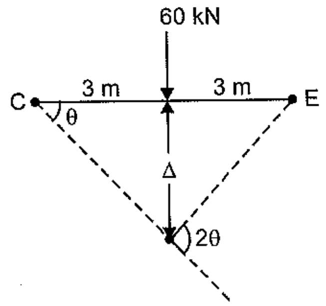
The three independent mechanisms are:

1. Beam mechanism, span CE
2. Beam mechanism, span AC
3. Sway mechanism

Two possible combinations of the independent mechanisms can be made to obtain combined mechanism.

4. Combined mechanism (sway and span CE)
5. Combined mechanism (Sway and span AC)

1. **Beam mechanism (span CE)**

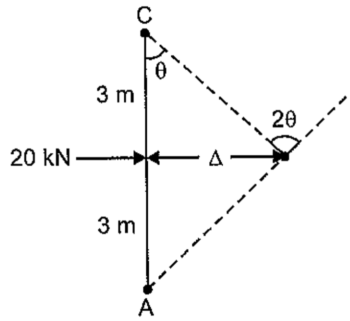


$$\text{External work done} = \text{Load} \times \text{Deflection}$$
$$= 60 \times 3\theta$$
$$= 180\theta$$
$$\text{Internal work done} = M_P\theta + M_P(2\theta) + M_P\theta$$
$$= 4M_P\theta$$

By the principal of virtual work,

$$\text{Internal work done} = \text{External work done}$$
$$4M_P\theta = 180\theta$$
$$\boxed{M_P = 45\text{ kNm}}$$

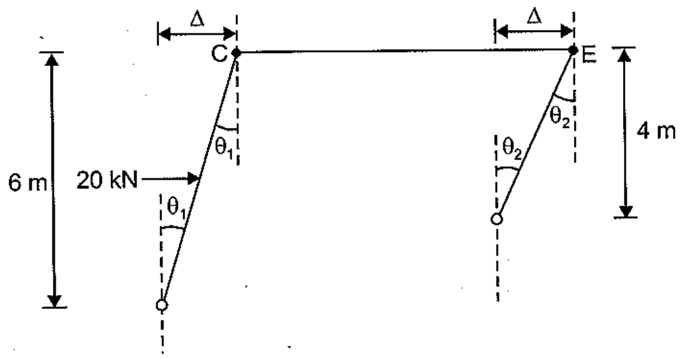
2. **Beam mechanism (span AC)**



By the principle of virtual work,

$$\text{Internal work done} = \text{External work done}$$
$$3M_P\theta = (20) \times 3\theta$$
$$\boxed{M_P = 20\text{ kNm}}$$

3. **Sway mechanism**



To have the same displacements at the joints C and E.

$$\Delta = 6\theta_1 = 4\theta_2$$

$\therefore$ $$\boxed{\theta_2 = 1.5\theta_1}$$

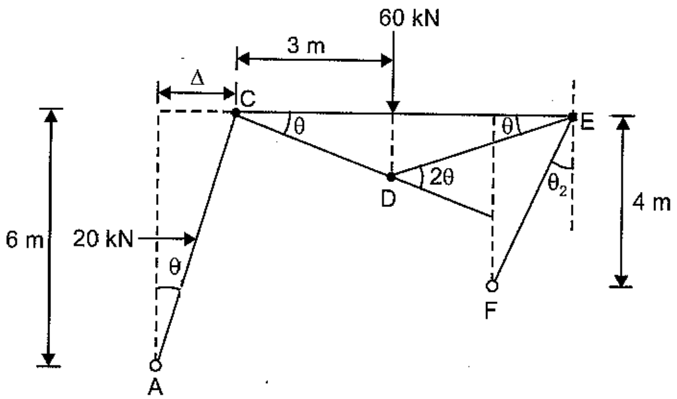
By the principle of virutal work,

$$\text{External work done} = \text{Internal work done}$$
$$(20 \times 3\theta_1) = M_P\theta_1 + M_P\theta_2$$
$$= M_P\theta_1 + 1.5M_P\theta_1$$

$\Rightarrow$ $$60\theta_1 = 2.5M_P\theta$$

$\therefore$ $$\boxed{M_P = 24\text{ kNm}}$$

4. **Combined mechanism (sway and span CE)**



By the principle of virtual work,

$$\text{External work done} = \text{Internal work done}$$

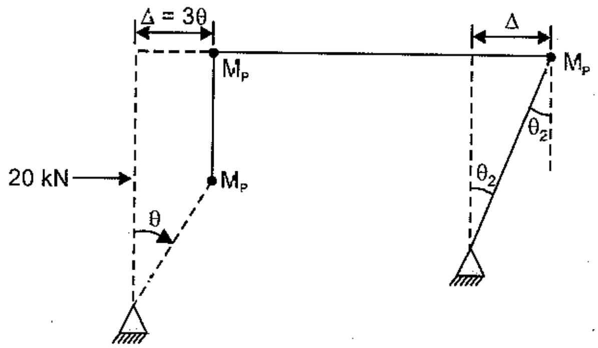
$$[(20\times 3\theta)+(60\times 3\theta)] = M_P(2\theta)+M_P(\theta+\theta_2)$$

$$= M_P(2\theta)+M_P(\theta+1.5\theta) = 4.5M_P\theta$$

$$\Rightarrow \quad 240\theta = 4.5M_P\theta$$

$$\boxed{M_P = 53.33\text{ kNm}}$$

5. **Combined mechanism (sway and span AC)**



By the principle of virtual work,

$$\text{External work done} = \text{Internal work done}$$

$$[(20\times 3\theta)] = M_P(\theta)+M_P\theta_2$$

$$= 2.5M_P\theta$$

$$\boxed{M_P = 24\text{ kN-m}}$$

The plastic moment for the frame will be the maximum of above obtained value.

Hence, the plastic moment (M_P) for the section is 53.33 kNm

For steel, $f_y = 250$ MPa

$$\therefore \quad \text{Plastic modulus required, } Z_{pz} = \frac{M_P}{f_y} = \frac{53.33\times 10^6}{250} = 21.33\times 10^4 \text{ mm}^3$$

UNIT-5

RCC AND PRESTRESSED CONCRETE

SYLLABUS

Limit state design for bending, shear, axial compression and combined forces; Design of beams, Slabs, Lintels, Foundations, Retaining walls, Tanks, Staircases; Principles of pre-stressed concrete design including materials and methods; Earthquake resistant design of structures; Design of Masonry Structure.

CONTENTS

CHAPTER 1

INTRODUCTION

Q-1: ***As part of routine quality control, a sample of three cubes is taken from a site using M30 concrete. The cubes are appropriately cured and tested for their compressive strength. The values obtained for the three cubes are: 31 MPa, 28 MPa and 28 MPa.***

Based on the provision of IS 456-2000, answer the following:

(i) Does the above constitute a valied set of results from the point of view of acceptance? Justify your answer.

(ii) Is the concrete represented by the results acceptable? Clearly indicate if any additional information may be needed before final decision on acceptance is made.

[10 Marks, ESE-2013]

Sol: **As per IS456:2000 (with Amendment no. 3)**

(a) The result of the sample shall be the average of the strength of three specimens. The individual variation should not be more than + 15% of the average. If more, the test results of the sample are invalid

$$\text{Av Strength} = \frac{31+28+28}{3} = 29 \text{ MPa}$$

$$+ 15\% \text{ of aV.} = + 29 \times 0.15 = 4.35 \text{ MPa}$$

$$\Rightarrow \quad \text{Min. acceptable strength} = 29 - 4.35 = 24.65 \text{ MPa}$$

$$\Rightarrow \quad \text{Max. acceptable strength} = 29 - 4.35 = 33.35 \text{ MPa}$$

As all the test results are with in the acceptable range of 24.65 MPa to 33.35 MPa, thus acceptable.

(b) Also for acceptance

$$\left.\begin{aligned}\text{Mean strength of four sample tests} &\geq f_{ck} + 0.825\sigma \\ &\geq f_{ck} + 3\text{N/mm}^2\end{aligned}\right\}\text{Whichever is greater}$$

Note: One sample consists of 3-specimen

Individual test result $\leq f_{ck} - 3$

Normally if σ is not given for M30, we adopt $\sigma = 5$ N/mm^2

Let us check the individual test result

$$f_{ck} - 3 = 30 - 3 - 27\text{N/mm}^2$$

As none of the specimen has strength less than 27 N/mm^2. Hence, result acceptable from individual test result point of view.

However from mean of strength point of view.

(Although code requires av. of 4-samples, but we have only one sample, for which 3-specimen are given. However for illustration we check the mean of three specimen only)

$$\left.\begin{array}{l}\text{Mean} \geq 30 + 0.825 \times 5 \\ \quad\quad\ \geq 30 + 3\end{array}\right\} \text{whichever greater}$$

i.e.

$$\left.\begin{array}{l}\text{Mean} \geq 34.125 \text{ MPa} \\ \quad\quad\ \geq 33 \text{ MPa}\end{array}\right\} \text{whichever greater}$$

i.e. Mean ≥ 34 MPa (Adjusted to nearest 0.5 N/mm²)

However the mean is 29 MPa

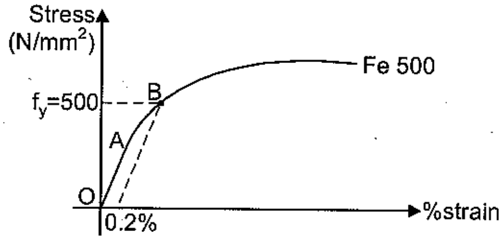
Hence the result is not acceptable finally.

Additional criteria given in code is from the point of view of flexural strength.

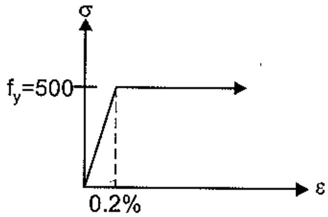
Q-2: ***Discuss the stress-strain curve of steel of grade Fe 500 and its application under Plastic Analysis.***

[5 + 5 Marks, ESE-2015]

Sol: The stress-strain curve for steel grade Fe 500 is given as:



- As there is no clearly visible yield point, 0.2% proof stress is taken as characteristic strength, f_y (= 500 N/mm²).
- Strain-hardening takes place after point B.
- In plastic analysis, the effect of strain hardening is neglected & large deformation under constant stress (= f_y) is assumed as shown below:



Q-3: ***The data of material for making a cement concrete mix may be assumed as follows:***

(i) Water-cement ratio by mass = 0.46

(ii) Entrained air = 2%

(iii)

S.No.	***Material***	***Specific gravity***	***Bulk density (kg/m³)***	***Proportion in mix by dry volume***
1.	***Cement***	***3.15***	***1500***	***1 part***
2.	***Fine aggregate***	***2.61***	***1620***	***1.359 part***
3.	***20 mm nominal size crushed coarse aggregate***	***2.70***	***1530***	***2.79 part***

Determine the following for cement concrete:

(1) ***Absolute volume of fully compacted fresh concrete (ignoring air content) produced by one bag of cement of 50 kg.***

(2) ***Cement content per cubic metre of concrete.***

(3) ***Quantity of materials to make one cubic metre of concrete.***

[10 + 5 + 5 Marks, ESE-2017]

Sol: Given proportion is by dry volume

Changing these proportions from dry volume to dry weight.

$$\Rightarrow \quad \text{Cement : FA : CA} \equiv 1 \times 1500 : 1.359 \times 1620 : 2.79 \times 1530$$

$$\equiv 1 : 1.46772 : 2.8458 \text{ (by weight)}$$

$$\text{Weight of water} = 0.46 \times \text{weight of cement}$$

$$\because \quad \text{Weight of cement} = 50 \text{ kg (given)}$$

$$\Rightarrow \quad \text{Weight of water} = 0.46 \times 50 = 23 \text{ kg}$$

$$\therefore \quad \text{Volume of water} = \frac{23}{1000} = 0.023 \text{ m}^3$$

(1) Calculation of total volume of concrete ignoring air content

$$V = V_w + V_C + V_{FA} + V_{CA}$$

$$= 0.023 + \frac{50}{3.15 \times 10^3} + \frac{1.46772 \times 50}{2.61 \times 10^3} + \frac{2.8458 \times 50}{2.7 \times 10^3}$$

$$= 0.120 \text{ m}^3$$

Abs. volume of fully compacted fresh concrete (ignoring air content) produced by one bag of cement of 50 kg = 0.120 m^3

(2) Let cement content be x kg per m^3 of concrete

$$\Rightarrow \quad V_W + V_C + V_{CA} + V_{FA} + V_{air} = 1 \text{ m}^3$$

$$\text{or} \quad x\left[\frac{0.46}{1000} + \frac{1}{3.15 \times 10^3} + \frac{2.8458}{2.7 \times 10^3} + \frac{1.46772}{2.61 \times 10^3}\right] + 0.02 = 1$$

$$\Rightarrow \quad x = 409.39 \text{ kg}$$

$\therefore$ Mass of cement per m^3 of concrete is 409.39 kg

(3) For one cubic metre of concrete

Let weight of cement be x kg.

$\Rightarrow$ From relations calculated above

x = 409.39 kg = Cement

FA = 1.46772 × 409.39 = 600.87 kg

CA = 2.8458 × 409.39 = 1165.04 kg

Water = 0.46 × 409.39 = 188.319 kg = 188.32 litre

Q-4: ***Explain briefly with an example of the Acceptance Criteria for Concrete as per IS 456-2000.***

[8 Marks, ESE-2019]

Sol: For concrete, we have two acceptance criteria.

(1) Cl. 16.2: Flexural Strength

When both the following conditions are met, the concrete complies with the specified flexural strength.

(a) The mean strength determined from any group of four consecutive test results exceeds the specified characteristic strength by at least 0.3 N/mm^2.

(b) The strength determined from any test result is not less than the specified characteristic strength less 0.3 N/mm^2.

(2) Characteristics compressive strength compliance requirement (Clauses 16.1 and 16.3)

Specified grade	**Mean of the group of 4 Non-overlapping consecutive Test results in N/mm^2**	**Individual Test results in N/mm^2**
(1)	**(2)**	**(3)**
M 15 and above	$\geq f_{ck}$ + 0.825 × established standard deviation (rounded off to nearest 0.5 N/mm^2) or f_{ck} + 3 N/mm^2, whichever is greater	$\geq f_{ck}$–3 N/mm^2

CHAPTER 2

LIMIT STATE METHOD (PART-I)

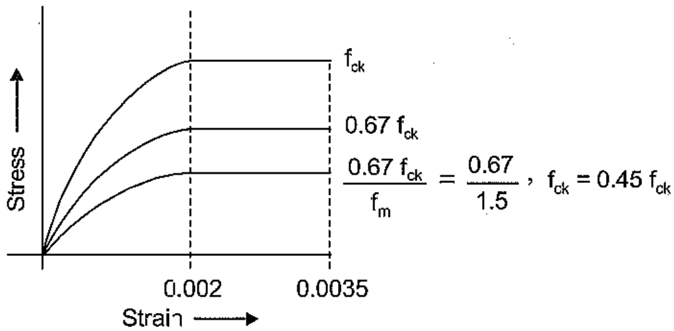
Q-1: ***What are the various assumptions on which the design for the limit state of collapse in flexure is based.***

[10 Marks, ESE-1995]

Sol: The design for the limit state of collapse in flexure as per IS : 456-2000 is based on the following assumptions:

(a) Plane sections normal to the axis remain plane after bending.

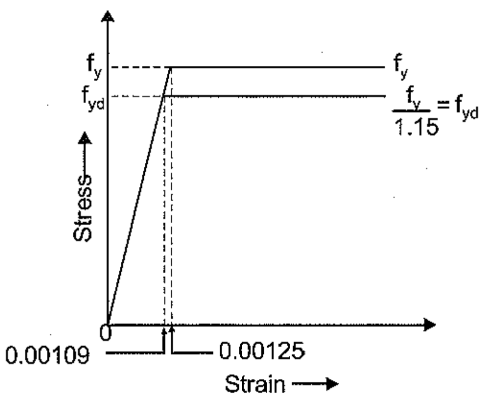
(b) The relationship between compressive stress and strain in concrete can be taken as shown below. However, the relationship between stress & strain can be taken as rectangular, parabolic, trapezoidal or any other shape if the strength calculation based on above is in substantial agreement with the results of test.



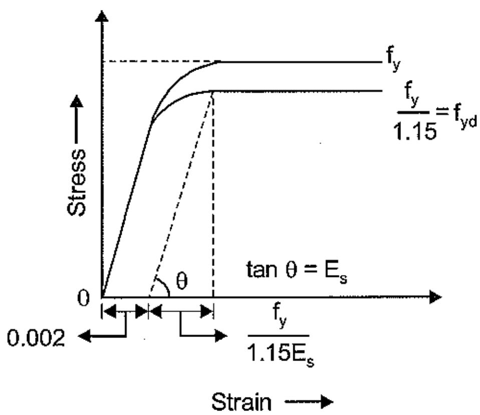
(c) The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in Bending.

(d) Tensile strength of concrete is assumed to be negligible.

(e) For design purpose partial factor of safety for steel = 1.15 and the stress in steel are derived from stress strain curve as shown below:



Stress - strain curve for bars with definite yield point



Stress - Strain curve for cold worked deformed bars

(f) The maximum strain in the tension reinforcement in the section at failure shall not be less than $\dfrac{f_y}{1.15E_s}+0.0020.$

Q-2: ***What are the three assumptions made for design of reinforced concrete section for limit state of collapse in flexure that lead to the limiting value of depth of neutral axis? Calculate the limiting values of neutral axis in terms of effective depth for two grades of steel having characteristic strength f_y = 250 & 415 N/mm².***

[10 Marks, ESE-1996]

Sol: The three assumptions which is necessary for deriving the limiting depth of neutral axis are :

(i) Plane sections normal to the axis remain plane after bending.

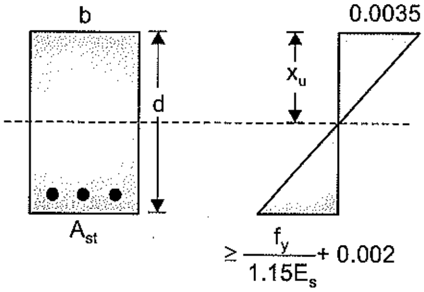
(ii) Maximum strain in concrete at outer most compression fibre is taken as 0.0035 in bending.

(iii) Maximum strain in the steel at failure shall not be less than $\left\{\dfrac{fy}{1.15Es}+0.002\right\}$

where f_y = Characteristic strength of steel

E_s = Modulus of elasticity of steel.

Derivation of limiting depth



Cross section **Strain diagram**

From the recommendation that, $\varepsilon_{st} \geq \dfrac{0.87\,f_y}{E_s}+0.0020$

and strain in concrete at collapse, should be max of 0.0035,

$$\frac{0.0035\,(d-x_u)}{x_u} \geq \frac{0.87\,f_y}{E_s}+0.0020$$

or,

$$\left(\frac{d}{x_u}-1\right) \geq \frac{\dfrac{0.87\,f_y}{E_s}+0.0020}{0.0035}$$

$$\frac{d}{x_u} \geq \frac{\dfrac{0.87f_y}{E_s}+0.0055}{0.0035}$$

$$\frac{x_u}{d} \leq \frac{0.0035}{\dfrac{0.87\,f_y}{E_s}+0.0055}$$

Thus limiting value of Neutral axis depth is given by

$$\boxed{\left(\frac{x_u}{d}\right)_{lim} = \frac{0.0035}{0.0055+\dfrac{0.87\,f_y}{E_s}}}$$

Value of $x_{u\,lim}/d$ for various grades of steel.

	Fe 250	Fe 415
$\dfrac{x_{u,lim}}{d}$	0.5313	0.4791
As per IS code	0.53	0.48

Q-3: ***Prove that the limiting moment of resistance 'M_u' of singly (under) reinforced rectangular concrete section, using stress block parameters of code IS : 456 - 1978, is given by***

$$M_u = 0.87\, f_y \left(\frac{P_t}{100}\right)\left[1 - 1.005\frac{f_y}{f_{ck}}\left(\frac{P_t}{100}\right)\right] bd^2$$

where b and d are width and effective depth of the beam, f_y and f_{ck} are characteristic strength of steel and concrete.

$$P_t = \frac{A_{st} \times 100}{bd}$$

$$A_{st} = \text{Area of steel.}$$

State the assumptions made.

Additional data required:

$$\text{Area of stress block} = 0.36\, f_{ck}\, x_u$$

$$\text{Depth of compressive force from the extreme fibre in compression} = 0.416\, x_u$$

$$\text{Total tension} = 0.87\, f_y\, A_{st}$$

[15 Marks, ESE-1997]

Sol: We know that,

$$\text{M.O.R.} = \text{Tensile force} \times \text{Lever arm} = [0.87\, f_y \times A_{st}] \times (d - 0.416\, x_u)$$

$$= 0.87\, f_y \times \frac{A_{st}}{bd} \times (d - 0.416x_u) \times bd$$

$$= 0.87\, f_y \times \frac{P_t}{100} \times (d - 0.416x_u) \times bd$$

$$= 0.87\, f_y \times \frac{P_t}{100} d(d - 0.416x_u) \times bd$$

$$\text{M.O.R.} = 0.87\, f_y \times \frac{P_t}{100} \times bd^2 \left(1 - 0.42\frac{x_u}{d}\right) \quad ...(i)$$

For Beam to be in equillibrium, C = T

$$\Rightarrow \quad 0.36\, f_{ck}\, ɔ \times x_u = 0.87\, f_y\, A_{st}$$

$$\Rightarrow \quad x_u = \frac{087 f_y \times A_{st}}{0.36 f_{ck} \times b} \quad ...(ii)$$

Putting value of x_u from Eqn. (II) in Eqn. (I) we have

$$\text{M.O.R.} = 0.87\, f_y \times \frac{P_t}{100} \times bd^2 \left(1 - 0.416 \times \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}\right)$$

$$= 0.87\, f_y \times \frac{P_t}{100} \times bd^2 \left(1 - 1.005\frac{f_y \times A_{st}}{f_{ck} \times bd}\right)$$

$$\Rightarrow \quad \text{M.O.R} = 0.87 \times \frac{P_t}{100} \times bd^2 \times \left[1 - 1.005\frac{f_y}{fck} \times \frac{P_t}{100}\right]$$

Q-4: ***Enumerate the situations in which doubly reinforced concrete beams become necessary. What is the role of compression steel?***

[10 Marks, ESE-1998]

Sol: Doubly reinforced concrete beams become necessary in following conditions:

- When the size of the beam is restricted and applied moment is more then the moment of resistanceof balanced section, we would design the section as doubly reinforced section. Compression steelprovided in doubly reinforced section is useful even otherwise because
- It permits smaller size beam
- Reduces long term deflection due to shrinkage & creep and increases ductility of beam
- Can be used as anchor bar for positioning shear reinforcements.
- As compression steel increases ductility, they are provided (even if not required for strength) in seismic zone to withstand repeated reversals produced.

Role of Compression steel

Owing to the continued compressive stress, the concrete undergoes creep strains, which leads to increase in the strain in concrete with time. The steel reinforcement in the compressioon zone is already under compressive stress and the above mentioned creep strains in concrete, along with the creep strain in compressive steel increases the compressive strains in steel many folds.

Hence, as compared to surrounding concrete, the total compressive strain in compresive steel is much greater as a result of the flexure.

So, we see how the compressive steel has taken all the addittional compressive stresses which would have been beyound the permissible compressive stress for concrete, hence making if safe against failure in flexure.

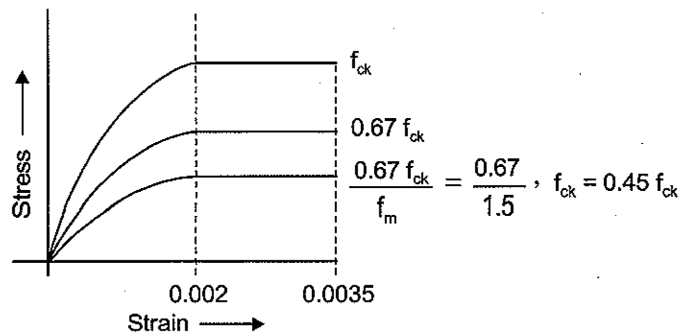
Q-5: ***State the assumptions made in the design for the limit state of collapse in flexure. Are these assumptions justified? Derive the stress block parameters for a rectangular cross-section.***

[15 Marks, ESE-1998]

Sol: The design for the limit state of collapse in flexure as per IS : 456-2000 is based on the following assumptions:

(a) Plane sections normal to the axis remain plane after bending.

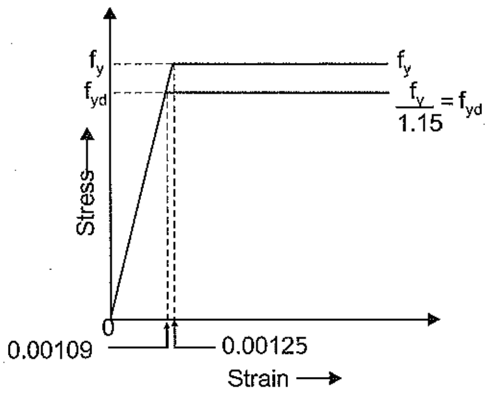
(b) The relationship between compressive stress and strain in concrete can be taken as shown below. However, the relationship between stress & strain can be taken as rectangular, parabolic, trapezoidal or any other shape if the strength calculation based on above is in substantial agreement with the results of test.



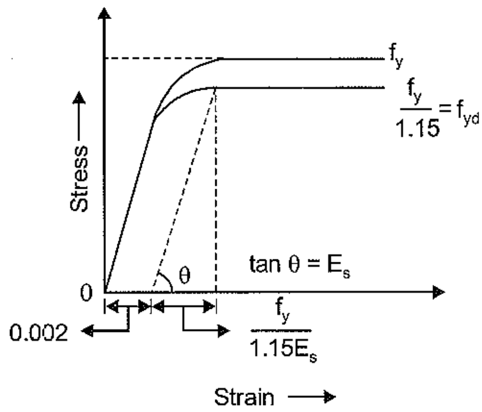
(c) The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in Bending.

(d) Tensile strength of concrete is assumed to be negligible.

(e) For design purpose partial factor of safety for steel = 1.15 and the stress in steel are derived from stress strain curve as shown below:



Stress - strain curve for bars with definite yield point



Stress - Strain curve for cold worked deformed bars

(f) The maximum strain in the tension reinforcement in the section at failure shall not be less than $\frac{f_y}{1.15E_s} + 0.0020$.

Justification of those assumptions

Assumptions, made in limit state of collapse for flexure leads to more effective and economic design.

(a) First assumption plane section normal to the axis remain plane after bending gives a definite linear strain diagram. But due to shear present along with bending, this assumption may not always be correct. however neglecting the shear effect will not lead to significant error. Thus the assumption is justified.

(b) The parabolic and linear combination of stress-strain curve is more safe. The reduction of compressive strength of concrete by 33% and a partial factor of safety = 1.5 increases the reliability of the corresponding RCC structure. This assumpt on is not theoretically 100% correct but as it generally satisfies the experimental observation of strength of beam hence the assumption is justified.

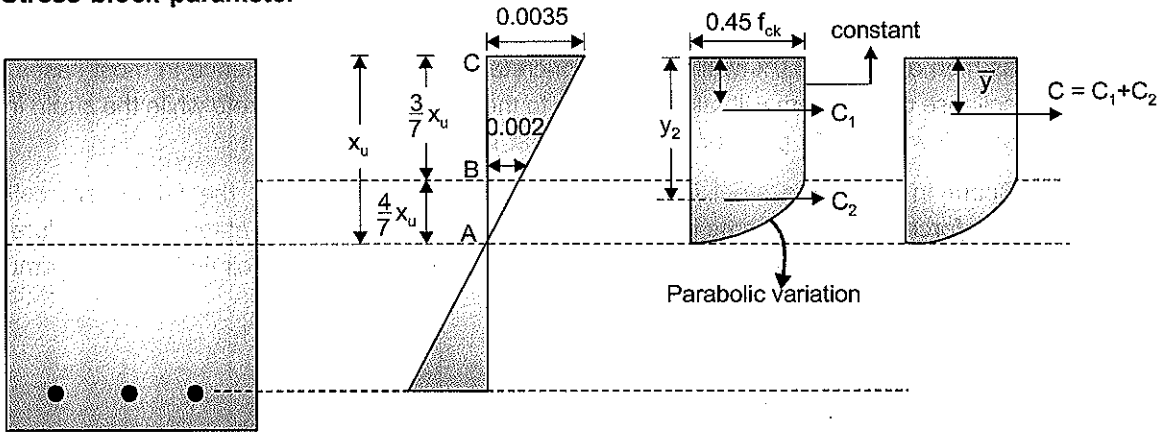
(c) Concrete has very low ductility and so it gets crushed in compression in very low strain, so the value is limited to 0.0035. Thus the assumption is justified.

(d) As the concrete under flexure is mostly cracked, this assumption in justified. Although there can be a small region of uncracked concrete closer to neutral axis. But as this uncracked area is closer to N.A, the lever arm for this area will be very small leading to negligible contribution to MOR, thus the assumption is justified.

(e) Partial factor of safety applied on steel is less because the quality control in production of steel is more certain.

(f) If strain is greater than $\frac{f_y}{0.15E_s} + 0.0020$, it ensures that at ultimate state stress reached is f_{yd} (as evident from the stress strain curve) Thus assumption ensures that yielding has occurred in steel at ultimate state i.e., full strength of steel has been utilized. The assumption is justified because it will ensure the full utilization of costlier material i.e., steel.

Stress block parameter



$$\sigma_\beta = \frac{P}{A} - \frac{Pe'}{Z}$$

By similar triangle, $\frac{0.0035}{x_u} = \frac{0.002}{AB} \Rightarrow AB = \frac{0.002}{0.0035}x_u = \frac{4}{7}x_u$

$\therefore$ $BC = 3/7\ x_u$

For rectangular portion

$$\text{Compressive force } (C_1) = B \times 0\ 45\ f_{ck} \times \frac{3}{7}x_u = 0.1927\ f_{ck}\ Bx_u \text{ and}$$

$$y_1 = \frac{\frac{3}{7}x_u}{2} = \frac{3}{14}x_u$$

For parabolic region,

$$\text{Compressive force } (C_2) = B \times \frac{2}{3} \times 0.45 f_{ck} \times \frac{4}{7}x_4 = 0.1714 f_{ck} B x_u$$

and $$y_2 = \frac{3}{7}x_u + \frac{3}{8} \times \frac{4}{7}x_u = 0.64260x_4$$

$$\text{Total compressive force} = c_1 + c_2 = 0.1927\ f_{ck} \times B \times x_u + 0.1714\ f_{ck} \times B \times x_u$$

$$C = C_1 + C_2 = 0.36\ f_{ck} \times B \times x_u$$

and $\bar{y}$ calculation

$$\bar{y} = \frac{C_1y_1 + C_2y_2}{C_1 + C_2}$$

$$\bar{y} = \frac{(0.1927 f_{ck} \times B \times x_u) \times (3/14\, x_u) + (0.1714 f_{ck} \times B \times x_u) \times 0.64260 x_u}{0.1927 f_{ck} \times B \times x_u + 0.1714 \times B \times x_u}$$

$$\bar{y} = \left\{\frac{0.04129 + 0.11014}{03641}\right\} x_u = 0.416 x_u$$

Q-6: ***Explain 'under-reinforced', 'balanced' and 'over-reinforced' sections in the ultimate load theory.***

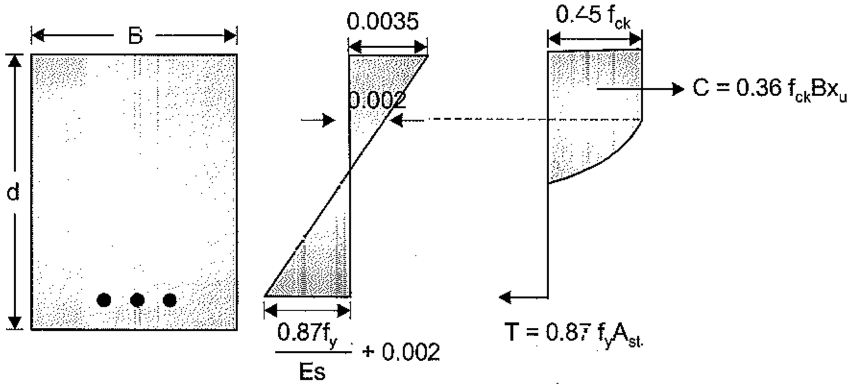
[10 Marks, ESE-1999]

Sol: There are two aspects to describe under reinforced, balanced and over reinforced section.

(a) **Theoretical aspect:** As the bending moment is gradually increasing, the failure of beam will take place only when either steel or concrete reaches its ultimate strain. The ultimate strain in steel is as high as 2.0 to 3.0% while that in concrete is only 0.4 to 0.5. Hence in most cases, the actual collapse of the beam will take place due to the breakdown of concrete while the strain in steel may be below the yield point or above it at that instant.

Beam in which the steel is strained beyond the yield point at the failure are called "Under Reinforced beam". Beams in which steel is still within the elastic stage at their failure are called "over Reinforced section". When concrete and steel both reaches their limiting strain simultaneously, section is called balanced.

(b) **Mathematical aspect:**



Actual depth of neutral axis

$$C = T$$

$$\Rightarrow \quad 0.36\, f_{ck} \times B \times x_u = 0.87 fy\, A_{st}$$

$$\Rightarrow \quad \boxed{\frac{x_u}{d} = \frac{0.87\, f_y\, Ast}{0.36\, f_{ck}\, Bd}}$$

Also limitting value of Neutral axes, is given by

$$\boxed{\frac{x_{u_{lim}}}{d} = \frac{700}{1100 + 0.87 f_y}}$$

(i) When $\frac{x_u}{d} < \frac{x_{u_{lim}}}{d} \rightarrow$ section is under reinforced, steel reaches the strain causing f_{yd} prior to 0.0035 strain in concrete.

(ii) When $\frac{x_u}{d} > \frac{x_{u_{lim}}}{d} \rightarrow$ section is over reinforced $\rightarrow$ Crushing of concrete takes place while strain in steel remains below the the strain corresponding to f_{yd}

(iii) When $\frac{x_u}{d} = \frac{x_{u_{lim}}}{d} \rightarrow$ Balance section$\rightarrow$steel reaches its yielding strain at the same instant when ultimate strain reached in concrete.

Q-7: ***Calculate the ultimate moment of resistance of an R.C.C. rectangular beam with the following data:***

Breadth of beam = 230 mm

Overall depth of beam = 550 mm

Tension steel consists of 4 nos. of 20 mm diameter bars of grade Fe 415

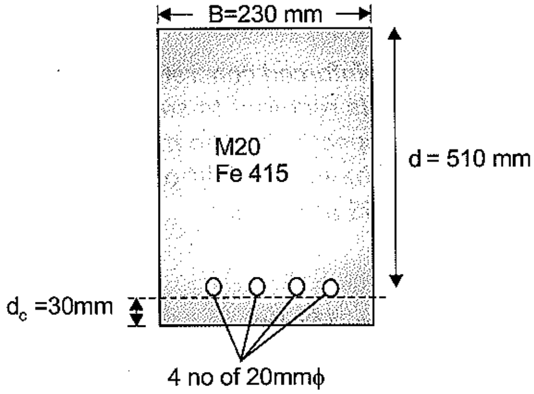
Clear cover 30 mm

Concrete grade - M 20

Hence determine the intensity of safe superimposed load (excluding self-wt) this beam can carry on a simply supported span of 5m.

[25 Marks, ESE-1999]

Sol: (i)



Determine M.O.R. = ?

Step 1: $x_{u_{lim}}$ Calculation

$$x_{u_{lim}} = 0.48 \times d = 0.48 \times 510 = 244.8 \text{ mm}$$

Step 2: x_u calculation (C = T)

$$0.36\, f_{ck} \times B \times x_u = 0.87 f_y \times A_{st}$$

$$\Rightarrow \qquad x_u = \left\{\frac{0.87 \times 415 \times \frac{\pi}{4} \times 20^2 \times 4}{0.36 \times 20 \times 230}\right\} = 273.97\text{mm}$$

Since $x_u > x_{u_{lim}}$

Hence secton is over reinforced.

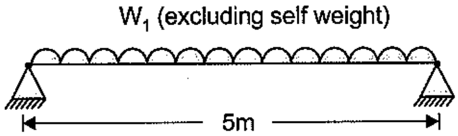
Step 3: Moment of resistance calculation

$$\text{M.O.R.} = 0.36\, f_{ck} \times B \times x_{u_{lim}} (d - 0.42\, x_{u_{lim}})$$

$$= 0.36 \times 20 \times 230 \times 244.8\, (510 - 0.42 \times 244.8)$$

$$\text{M.O.R.} = 165.06 \text{ kNm}$$

(ii)



$$\text{Dead load} = 25 \times 0.230 \times 0.550 = 3.1625 \text{ kN/m}$$

$$\text{Factored dead load} = 3.1625 \times 1.5 = 4.74 \text{ kN/m}$$

Let w_1 is the imposed load per unit run on the beam.

hence,
$$M|_{max} = \frac{(w_1 + 4.74) \times (l_{eff})^2}{8} \text{kN-m}$$

$$MOR = 165.06 \text{ kN-m}$$

$$\therefore \quad \frac{(w_1 + 4.74)(l_{eff})^2}{8} = 165.06$$

$$w_1 = 48.08 \text{ kN/m}$$

$$\Rightarrow \quad \text{Hence safe load} = \frac{48.08}{1.5} = 32.05 \text{ kN/m}$$

Q-8: ***Use limit state method to design a reinforced concrete rectangular beam having an effective simply supported span of 6 m. The beam is required to support live service and superimposed (dead) loads of 14 kN/m and 9.5 kN/m, respectively. The materials to be used are M 20 grade concrete and HYSD steel of grade Fe 415. The unit weight of concrete is 25 kN/m³. Adopt d/b ratio as 2. For the given materials $p_{t.lim}$ = 0.955 per cent.***

[15 Marks, ESE-2000]

Sol: **Given data:**

Live load = 14kN/m

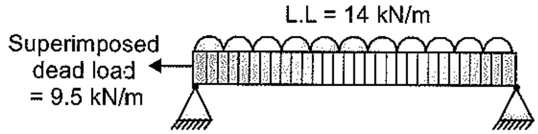
Superimposed dead load = 9.5 kN/m

Effective span = (l) = 6m

f_{ck} = 20 N/mm²

f_y = 415 N/mm²

$\frac{d}{b} = 2$



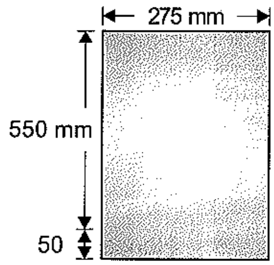
Step 1: Assume overall debth of beam = $\frac{1}{10}$th of effective span

$$\Rightarrow \quad D = \frac{6000}{10} = 600\text{mm}$$

Assume cover of 50 mm.

$$\therefore \quad d = 600 - 50 = 550 \text{ mm}$$

$$\therefore \quad b = \frac{d}{2} = \frac{550}{2} = 275\text{mm.}$$



Step 2: Load calculation

Live load = 14 kN/m

Superimposed dead load = 9.5 kN/m

Self weight of the beam = 0.275 × 0.6 × 1 × 25 = 4.125 kN/m

∴ Total load (W) = 27.625 kN/m

Factored load (W_u) = 1.5 W = 41.4375 kN/m

Step 3: Maximum bending moment (M_{max}) $= \frac{W_u l^2}{8} = \frac{41.4375 \times 6^2}{8}$ = 186.46875 kN-m

$$M_{u\,lim} = 0.87\, f_y\, A_{st\,lim}\left(d - \frac{f_y A_{st\,lim}}{f_{ck}\, b}\right)$$

$$= 0.87\, f_y \left(\frac{A_{st\,lim}}{bd}\right)\left(1 - \frac{f_y}{f_{ck}} \cdot \frac{A_{st\,lim}}{bd}\right) bd^2$$

$$= 0.87\ f_y\left(\frac{0.955}{100}\right)\left(1-\frac{f_y}{f_{ck}}\times\frac{0.955}{100}\right)\times bd^2 \quad (\text{Since } p_{t\,lim} = 0.955\%)$$

$$= 0.87 \times 415\left(\frac{0.955}{100}\right)\left(1-\frac{415}{20}\times\frac{0.955}{100}\right)\times 275\times(550)^2 \text{ N-mm}$$

$$= 229.993 \text{ kNm}$$

Since, $M_{u\,max} < M_{u\,lim}$

$\Rightarrow$ Singly reinforced section will be sufficient.

$$M_u = 0.87\ f_y\ A_{st}\left(d-\frac{f_y A_{st}}{f_{ck}\, b}\right)$$

$$186.46875 \times 10^6 = 0.87 \times 415\ A_{st}\left(550-\frac{415 A_{st}}{20\times 275}\right)$$

$$186.46875 \times 10^6 = 198577.5\ A_{st} - 27.243\ A_{st}^2$$

$$27.243\ A_{st}^2 - 198577.5\ A_{st} + 186.46875 \times 10^6 = 0$$

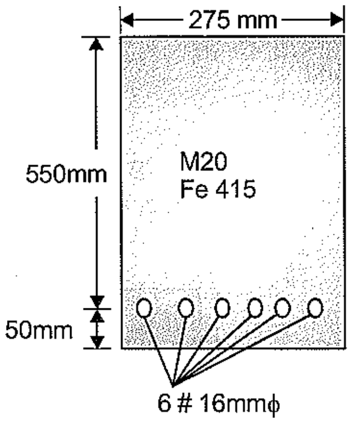
$$A_{st} = \frac{198577.5 \pm \sqrt{(198577.5)^2 - 4\times 27.243\times 186.46875\times 10^6}}{2\times 27.243}$$

$\Rightarrow$ $A_{st} = 1107.205,\ 6181.914$

Adopt $A_{st} = 1107.205\ mm^2$

Adopting 16 mm ϕ bars

Adopt 6 nos. of 16 mmϕ bars.



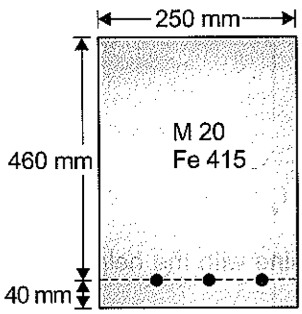
Q-9: ***The size of a R.C. beam is restricted to 250 mm × 500 mm. It carries a superimposed load of 25 kN/m over a span of 6 m. Determine the reinforcements for the beam by L.S.D. method. M_{20} concrete and Fe 415 steel are used. Effective cover to steel = 40 mm.***

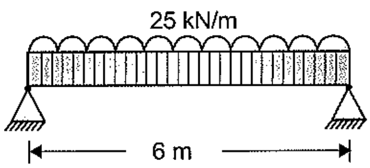
Salient points on design stress-strain curve for steel

Stress	*Strain*
0.8 f_{yd}	*0.00144*
0.85 f_{yd}	*0.00163*
0.90 f_{yd}	*0.00192*
0.95 f_{yd}	*0.00241*
0.975 f_{yd}	*0.00276*
1.0 f_{yd}	*0.00380*

[20 Marks, ESE-2001]

Sol:





Stress	$0.8f_{yd}$	$0.85f_{yd}$	$0.900f_{yd}$	$0.95f_{yd}$	$0.975f_{yd}$	f_{yd}
Strain	0.00144	0.00163	0.00192	0.00241	0.00276	0.00380

$$\text{Self weight ca culation} = 0.25 \times 0.5 \times 25 = 3.125 \text{ kN/m}$$

$$\text{Superimposed load} = 25 \text{ kN/m}$$

$$\text{Total load} = 28.125 \text{ kN/m}$$

$$\text{B.M.} = \frac{wl^2}{8} = \frac{28.125 \times 36}{8} = 126.56 \text{ kN-m}$$

$$\text{Factored B.M.} = (1.5 \times 126.56) = 189.84 \text{ kNm}$$

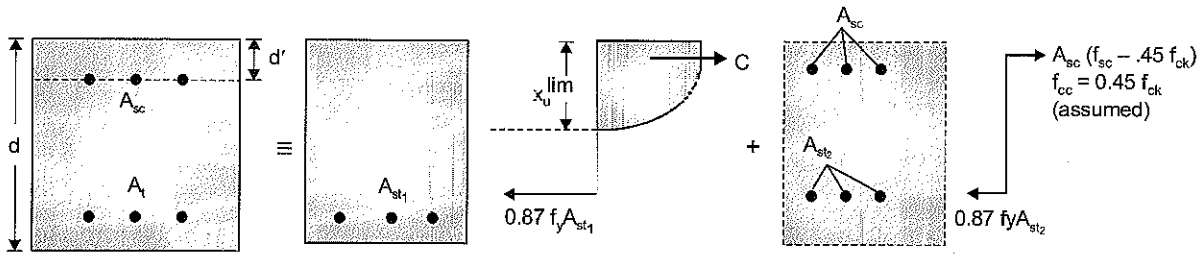
$\therefore$ M.O.R (balanced) for singly reinforced section = $0.36 \times f_{ck} \times B \times x_{u_{lim}} \times (d - 0.42\ x_{u_{lim}})$

$$[x_{u_{lim}} = 0.48 \times 460 = 220.8 \text{ mm}]$$

$$= 0.36 \times 20 \times 250 \times 220.8 \times (460 - 0.42 \times 220.8)$$

$$= 145.96 \text{ kN-m}$$

Since (B.M. due to load) > (M.O.R. of balaneced singly reinforced section) and the size of the beam is fixed, we will go for the doubly reinforcement beam.



Step 1: A_{st_1} calculation

$$\text{Factored B.M} = 189.84 \text{ kN-m} = M \text{ (say)}$$

Now, M_1 of balanced section $= 145.96$ kN-m

$$\Rightarrow \quad A_{st_1} = \frac{145.96 \times 10^6}{0.87 \times 415 \times (460 - 0.42 \times 220.8)}$$

$$\Rightarrow \quad A_{st_1} = 1100.74 \text{ mm}^2.$$

Step 2: A_{st_2} calculation: unbalanced moment $(M_2) = M - M_1 = 189.84 - 145.96 = 43.88$ kN-m

$$\therefore \quad 0.87\ f_y A_{st} \times (d - d') = 43.88 \times 10^6 \ |\text{Assuming } d' = 40\text{mm}$$

$$\therefore \quad A_{st_2} = 289.36 \text{ mm}^2$$

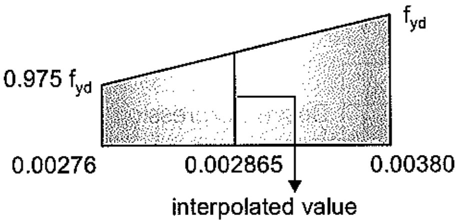
Step 3: A_{sc} calculation

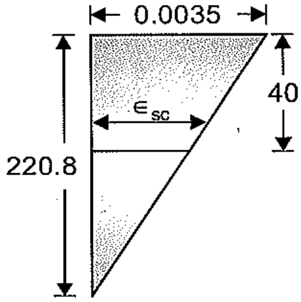
For the Asc calculation C = T (Auxillary section)

$$A_{sc}(f_{sc} - 0.45 f_{ck}) = 0.87 \times f_y \times A_{st_2}$$

for A_{sc} calculation we require f_{sc}. Which can be determine with the help of ϵ_{sc}.

$$\in_{sc} = \frac{0.0035(220.8-40)}{220.8} = 0.0028659 \text{ mm} \qquad [\in > 0.002 \Rightarrow f_{ck} = 0.45 f_{ck}]$$





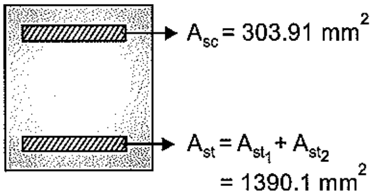
$$= 0.975 f_{yd} + (1-0.975) f_{yd} \times \frac{(0.0028659-0.00276)}{(0.00380-0.00276)}$$

$$= 0.977 f_{yd}$$

$$\therefore \qquad f_{sc} = \frac{0.977\times 415}{1.15} = 352.76 \text{ N/mm}^2$$

$$\therefore \qquad A_{sc} = \frac{0.87\times 415\times 289.36}{(352.76-0.45\times 20)} = 303.91 \text{ mm}^2$$

Pictorial view



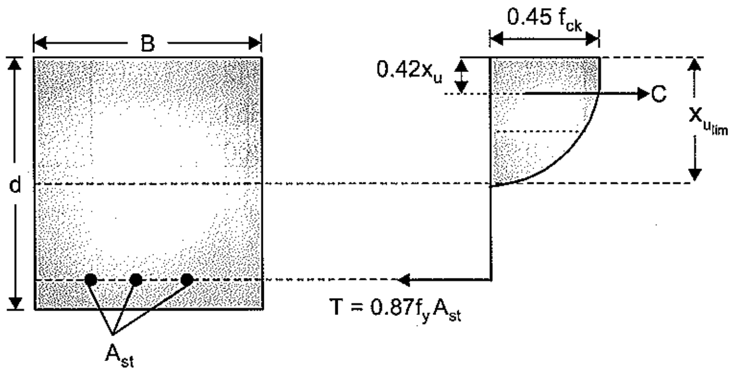
$$A_{st_1} + A_{st_2} = 1100.74 + 289.36$$

$$A_{st} = 1390.1 \text{ mm}^2.$$

Q-10: ***Derive the expression for limiting percentage of tensile reinforcement of a flexual R.C. member.***

[5 Marks, ESE-2002]

Sol:



$\Rightarrow$ For balanced section we calculate limitting % of tensile reinforcement.

$\Rightarrow$ For balanced section C = T,

$$0.36 \times f_{ck} \times B \times x_{u_{lim}} = 0.87 \ f_y \ Ast$$

$$\therefore \qquad x_{u_{lim}} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \times B}$$

$$\Rightarrow \qquad \frac{x_{u_{lim}}}{d} = \frac{0.87 f_y}{0.36 f_{ck}} \times \frac{A_{st}}{Bd}$$

$$\therefore \quad \frac{A_{st}}{Bd} = \frac{0.36 f_{ck}}{0.87 f_y} \times \frac{x_{u_{lim}}}{d}$$

$$\therefore \quad \frac{Ast}{Bd} \times 100 = \frac{36}{0.87} \times \frac{f_{ck}}{f_y} \times \frac{x_{u_{lim}}}{d}$$

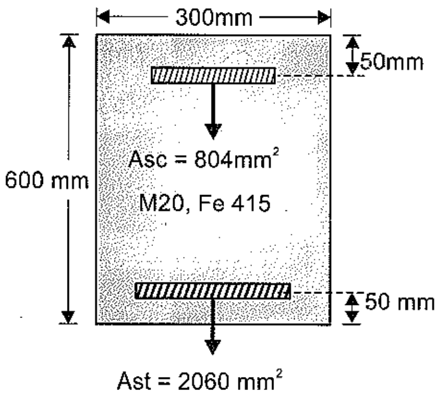
$$\therefore \quad \boxed{\%P_t = 41.38 \times \frac{f_{ck}}{f_y} \times \frac{x_u^{lim}}{d}} \text{ Desired expression}$$

Q-11: ***Determine ultimate moment capacity of a doubly reinforced beam with b = 300 mm, D = 600 mm, A_{st} = 2060 mm², A_{sc} = 804 mm² and effective cover of 50 mm for both tension and compression steels. The materials used are : M 20 concrete and HYSD steel of grade Fe 415. The salient points on stress-strain curve are:***

Strain	*Stress, MPa*
0.00144	*288*
0.00163	*306*
0.00192	*324*
0.00241	*342*
0.00276	*351*
0.00380	360

[15 Marks, ESE-2003]

Sol:



Stress, MPa	Strain
288	0.00144
306	0.00163
324	0.00192
342	0.00241
351	0.00276
360	0.00380

Given data:

B = 300 mm; D = 600 mm; d_c = 50 mm; d′ = 50 mm

d_{eff} = 600 − 50 = 550 mm; A_{st} = 2060 mm²; A_{sc} = 804 mm²

f_{ck} = 20 N/mm²; f_y = 415 N/mm²

Step 1: Limiting depth, $\frac{x_{u_{lim}}}{d} = 0.48 \Rightarrow x_{u_{lim}} = 0.48 \times 550 = 264\text{mm}$

Step 2: Actual depth of neutral axis, C = T

$\Rightarrow \quad 0.36 \times f_{ck} \times B \times x_u + A_{sc} f_{sc} - A_{sc} \times 0.45 f_{ck} = 0.87 \times f_y \times A_{st}$ [Assuming that $f_{cc} = 0.45 f_{ck}$]

$\Rightarrow \quad 0.36 \times 20 \times 300 \times x_u + 804 \times f_{sc} - 804 \times 0.45 \times 20 = 0.87 \times 415 \times 2060$

$\Rightarrow \quad 2160 x_u + 804 f_{sc} = 750999$

$$\therefore \qquad x_u = \left[\frac{750999 - 804f_{sc}}{2160}\right]$$

$$\frac{d'}{d} = \frac{50}{530} = 0.09$$

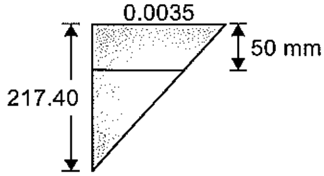
for $\frac{d'}{d} = 0.1$, $f_{sc} = 351.9$ N/mm

start with $f_{sc} = 350$ N/mm^2

Trial-1: Assuming $\quad f_{sc} = 350$ MPa

we get $\quad x_u = \frac{750999 - 804 \times 350}{2160} = 217.40\text{mm}$

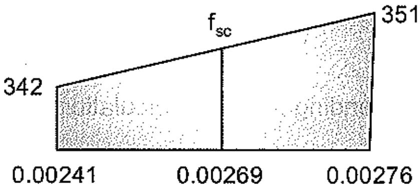
Hence strain at level of A_{sc} is



$$\epsilon_{sc} = \frac{0.0035}{217.40} \times (217.40 - 50)$$

$$\epsilon_{sc} = 0.00269$$

Finding stress fsc for the evaluated E_{sc} with the help of gives chart,



$$f_{sc} = 342 + \frac{(351-342)(0.00269 - 0.00241)}{(0.00276 - 0.00241)}$$

$$= 342 + 7.2 = 349.2 \text{ MPa}$$

Trial-2: Take $f_{sc} = 349.2$ MPa $\quad x_u = \frac{750999 - 804 \times 349.2}{2160}$

$$x_u = 217.70 \text{ mm}$$

$$\text{Value of } \epsilon_{sc} = \frac{0.0035}{217.70} \times (217.70 - 50) = 0.00269\text{mm}$$

$$\Rightarrow \qquad f_{cc} = 0.45 f_{ck} \text{ is valid}$$

$$f_{sc} = 349.2 \text{ MPa}$$

$$\therefore \qquad x_u = 217.70 \text{ mm (Adopted)}$$

$$\therefore \qquad x_u\big|_{actual} < x_u\big|_{limitting}$$

$\therefore$ Section is under reinforced.

Step 3: M.O.R calculation

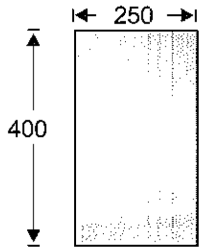
$0.36 f_{ck} \times B \times x_u \times (d - 0.42 x_u) + f_{sc} \times A_{sc}$ (d–dc) – $0.45 f_{ck} A_{sc}$ (d-dc)

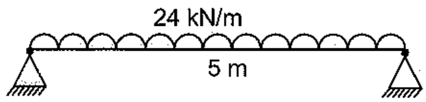
= [0.36 × 20 × 300 × 217.70 × (550 – 0.42 × 217.70) + 349.2 × 804 × (550 – 50) – 0.45 × 20 × 804 × (550 – 50)] = 352.39 kN-m.

Q-12: ***A simply supported beam of 5 m effective span is subjected to 24 kN/m live load : f_{ck} = 20 N/mm², f_y = 415 N/mm². The overall depth of the beam is 400 mm and width is 250 mm. Design the reinforcement of the beam if K = 0.138, J = 0.80.***

[10 Marks, ESE-2006]

Sol:





Given data:

$$f_{ck} = 20\ \text{N/mm}^2; \quad f_y = 415\ \text{N/mm}^2$$

$$k = 0.138; \quad j = 0.80$$

Assume cover of 50mm, then d = 400 – 50 = 350 mm.

Step 1: Calculation of limiting depth,

$$\frac{x_{u_{lim}}}{d} = 0.48$$

$$\Rightarrow \quad \frac{x_{u_{lim}}}{d} = 0.48 \times 350 = 168 \text{ mm.}$$

Step 2: Net applied maximum bending moment calculation

$$\text{Live load} = 24 \text{ kN/m}$$

$$\text{Dead load} = 25 \times 0.25 \times 0.4 \times 1 = 2.5 \text{ kN/m}$$

$$\text{Total load} = 26.5 \text{ kN/m;}$$

$$\text{Factored load} = 26.5 \times 1.5 = 39.75 \text{ kN-m}$$

$$\therefore \quad \text{B.M.}\Big|_{\text{max}}^{\text{at centre}} = \frac{39.75 \times \ell^2}{8} = \frac{39.75 \times 25}{8} = 124.22 \text{ kN/m}$$

Step 3: Calculation of moment of resistance for balanced section

$$M_{u_{lim}} = K f_{ck} bd^2$$

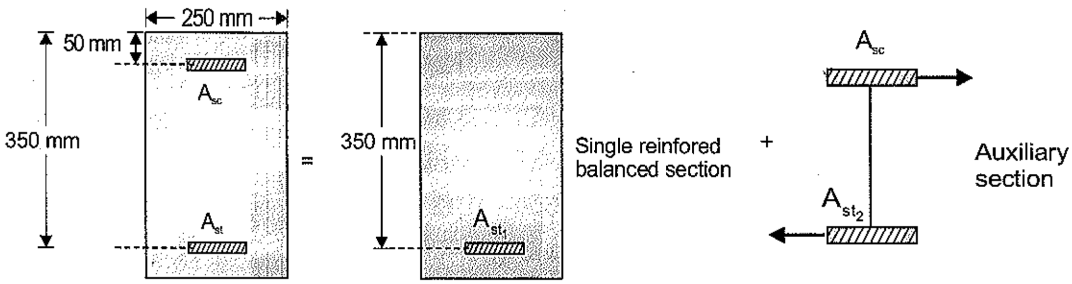
$$= 0.138 f_{ck} bd^2$$

$$= 0.138 \times 20 \times 250 \times (350)^2$$

$$= 84.525 \text{ kNm}$$

Since $M_u > M_{u_{lim}}$ and section dimension are fixed, hence we will have to design doubly reinforced section.

Assuming the cover for compression steel is also 50 mm



Calculation of A_{st_1}

$$M_{u_{lim}} = 84.525 \text{ kN-m}$$

$$\therefore \quad 0.87 \times f_y \times A_{st_1} (Jd) = 84.525 \times 10^6$$

$$\Rightarrow \quad 0.87 \times 415 \times A_{st_1} \times 0.8 \times 350 = 84.525 \times 10^6$$

$$\Rightarrow \quad A_{st_1} = 836.1 \text{ mm}^2$$

Determination of A_{st_2}

$$\text{Unbalanced moment} = 124.22 - 84.50 = 39.72 \text{ kN-m}$$

$$0.87 \times f_y \times A_{st_2} \times (d - d') = 39.72 \times 10^6$$

$$\Rightarrow \quad A_{st_2} = \frac{39.75 \times 10^6}{8.87 \times 415 \times (350 - 50)}$$

$$\Rightarrow \quad A_{st_2} = 366.71 \text{ mm}^2$$

Hence $\quad$ Total $A_{st} = 837.53 + 366.71 = 1204.24$ kN-m

Determination of A_{sc}

$$\frac{d'}{d} = \frac{50}{350} = 0.143$$

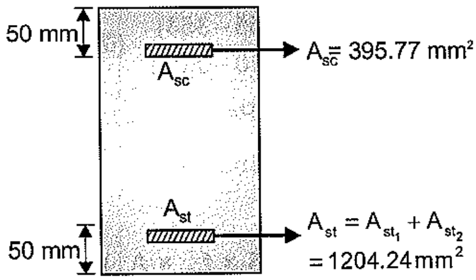
We know that the value of f_{sc} for 415 grade steel is given by

d'/d	f_{sc} N/mm²
0.05	355
0.10	353
0.15	342
0.20	329

$$\Rightarrow \quad f_{sc} = 0.353 - \frac{353 - 342}{(0.15 - 0.1)}(0.043) = 343.54 \text{ N/mm}^2$$

From the relationship $\quad 0.87 f_y A_{st_2} = (f_{sc} - 0.45 f_{ck}) A_{sc}$

$$A_{sc} = \frac{0.87 \times 415 \times 366.71}{(343.54 - 0.45 \times 20)} = 395.77 \text{ mm}^2$$



Q-13: ***An R.C.C. beam is of 230 mm width, 500 mm overall depth. Effective cover to compression steel and tension steel is 40 mm. Compression reinforcement consists of 2 nos. 16 mm diameter bars and tension steel consists of 2 nos. 25 mm diameter bars. This doubly reinforced beam is made of M 20 concrete and Fe 415 steel. Find the moment of resistance of this beam by LIMIT STATE METHOD.***

Codes will not be supplied.

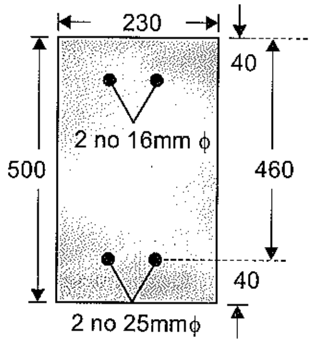
Table : Stress in Compression Reinforcement in M.Pa.

Grade of Steel M.Pa					
	d′/d	**0.05**	**0.10**	**0.15**	**0.20**
Fe415	f_{sc}	**355**	**353**	**342**	**329**

[20 Marks, ESE-2007]

Sol:

d'/d	0.05	0.10	0.15	0.20
f_{sc}	355	353	342	329



Given data: b = 230 mm; D = 500 mm; d′ = 40 mm

$$d = 500 - 40 = 460 \text{ mm}$$

$$f_{ck} = 20 \text{ MPa}$$

$$f_y = 415 \text{ MPa}$$

$$A_{sc} = 2 \times \pi/4 \times 16^2 = 402.12 \text{ mm}^2$$

$$A_{st} = 2 \times \pi/4 \times 25^2 = 981.75 \text{ mm}^2$$

Step 1: Limiting neutral axis depth determination:

$$\frac{x_{u_{lim}}}{d} = 0.48d \Rightarrow x_{u_{lim}} = 0.48 \times 460 = 220.8 \text{ mm}$$

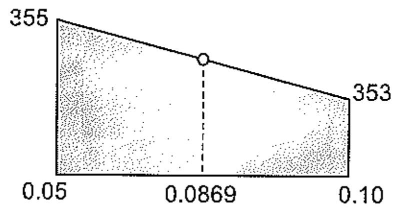
Step 2: Determination of f_{sc}

Since $\frac{d'}{d} = \frac{40}{460} = 0.0869$ which lies between 0.05 and 0.10

$$\frac{d'}{d} = 0.05; \quad f_{sc} = 355 \text{ MPa}$$

$$\frac{d'}{d} = 0.10; \quad f_{sc} = 353 \text{ MPa}$$

$$\frac{d'}{d} = 0.0869; \quad f_{sc} = ?$$



$$f_{sc} = 355 - \frac{(355-353)}{(0.10-0.05)} \times (0.0869 - 0.05)$$

$$= 353.524 \text{ MPa}$$

Step 3: Actual neutral axis depth determination

From, C = T

$$(0.36 f_{ck} \times B \times x_u) + (A_{sc} \times f_{sc} - 0.45 \times f_{ck} \times A_{sc}) = 0.87 \times f_y \times A_{st}$$

$$\Rightarrow 0.36 \times 20 \times 230 \times x_u + 402.12 \times 353.524 - 0.45 \times 20 \times 402.12 = 0.87 \times 415 \times 9814.75$$

$$\Rightarrow 1656 x_u = 215920.8466$$

$$\Rightarrow x_u = 130.39 \text{ mm}$$

Since $x_u\big|_{actual} < x_u\big|_{limiting}$

Hence beam is under reinforced

Step 4: Calculation of moment of resistance

$$0.36\, f_{ck} \times B \times x_u\,(d - 0.42\, x_u) + A_{sc}\,(f_{sc} - 0.45\, f_{ck})\,(d - d')$$

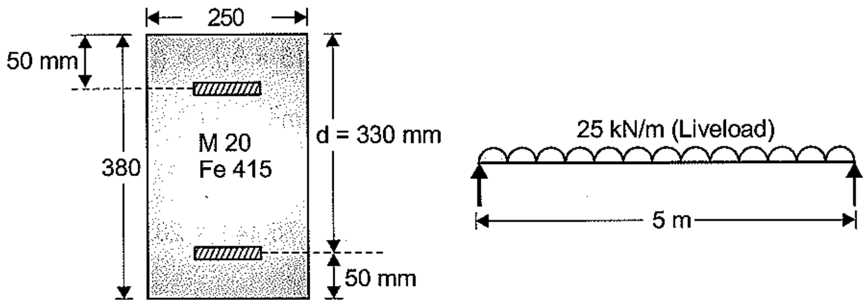
$$\Rightarrow \quad 0.36 \times 20 \times 230 \times 130.39\,(460\text{-}0.42 \times 130.39) + 402.12\,(353.524 - 0.45 \times 20)\,(420)$$

$$= 145.68 \text{ kN-m}$$

Q-14: ***A simply supported beam of 4.5 m effective span is carrying a live load of 25 kN/m. The size of the beam has to be restricted to 250 mm × 380 mm depth. Design the beam for bending using limit state method. The design coefficients are K = 0.138; J = 0.80; K_u = 0.479. Use M20 grade concrete and Fe415 steel.***

[15 Marks, ESE-2008]

Sol:



Step 1: Calculation of limiting neutral axis depth $(x_{u_{lim}})$

Assuming the effective cover for tension and compression steel to be both 50 mm

$$\frac{x_{u_{lim}}}{d} = K_u \quad \Rightarrow \quad x_{u_{lim}} = 0.479 \times [\,380 - 50]\text{ mm}$$

Assuming effective cover 50mm

$$\therefore \quad x_{u_{lim}} = 158.07 \text{ mm}$$

Step 2: Calculation of maximum moment applied.

$$\text{Dead load} = 0.25 \times 0.38 \times 1 \times 25 = 2.375 \text{ kN/m}$$

[Assuming unit wt of concrete = 25 kN/m³]

$$\text{Live load} = 25 \text{ kN/m}$$

$$\therefore \quad \text{Total load} = 27.375 \text{ kN/m}$$

$$\therefore \quad \text{Total factored load} = 27.375 \times 1.5 = 41.0625 \text{ kN/m}$$

$$\therefore \quad \text{B.M}\big|_{max} = \text{B.M}\big|_{\text{atcentreof span}} = \frac{WL^2}{8}$$

$$\therefore \quad \text{B.M}\big|_{max} = \frac{(41.0625)\times 4.5^2}{8} = 103.94 \text{ kN-m}$$

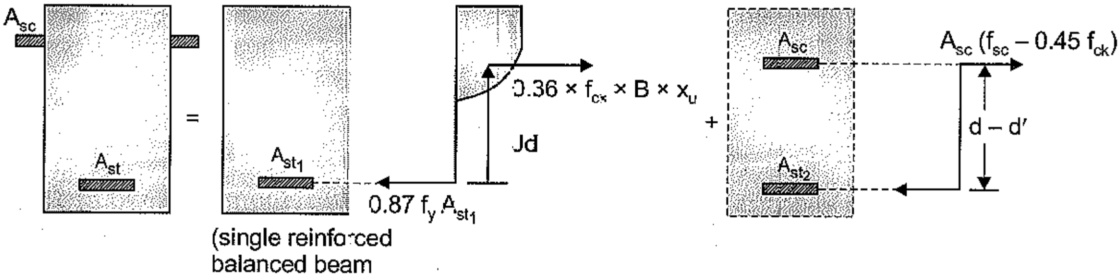
Step 3: Moment of resistance of balanced section

$$\text{M.O.R}\,\big|_{\text{balanced section}} = K\cdot f_{ck}\, bd^2 = 0.138 \times f_{ck}\, bd^2$$

$$= 0.138 \times 20 \times 250\,(330)^2$$

$$= 75.141 \text{ kN/m}$$

Since required moment of resistance is larger than the available moment of resistance of balance section and the size of cross-section of beam is fixed, so we will design it as a doubly reinforced concrete beam.



Step 4: Calculation of A_{st_1} :

$$M.O.R\big|_{\text{balance beam}} = 0.87 \times f_y \times A_{st_1} \times Jd$$

$$75.141 \times 10^6 = 0.87 \times 415 \times A_{st_1} \times (0.8\ d)$$

$$\Rightarrow \quad A_{st_1} = 788.33\ \text{mm}^2$$

Step 5: Calculation of A_{st_2}

$$\text{Unbalance Moment} = 103.94 - 75.141 = 28.8\ \text{kN-m}$$

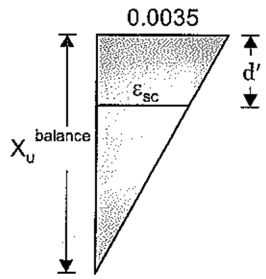
Hence $$0.87 \times f_y \times A_{st_2} \times (d - d') = 28.8 \times 10^6$$

$$\Rightarrow \quad 0.87 \times 415 \times A_{st_2} \times 280 = 28.8 \times 10^6\ [\text{Assumed } d' = 50\ \text{mm}]$$

$$\Rightarrow \quad A_{st_2} = 284.883\ \text{mm}^2$$

$$\therefore \quad A_{st} = A_{st_1} + A_{st_2} = 788.33 + 284.883 = 1073.213\ \text{mm}^2$$

Step-6: Calculation of f_{sc}



$$\frac{d'}{d} = \frac{50}{330} = 0.1515$$

We knows that for Fe 415 steel

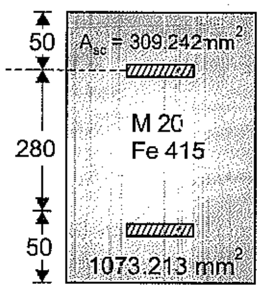
d′/d	f_{sc} N/mm²
0.05	355
0.10	353
0.15	342
0.20	329

$$\Rightarrow f_{sc} = 34.2 - \frac{13}{0.05} \times 0.0015 = 341.161\ \text{N/mm}^2$$

$$A_{sc} = \frac{0.87 f_y\, A_{st_2}}{(f_{sc} - 0.45\, f_{ck})} = \frac{0.87 \times 415 \times 284.883}{(341.61 - 0.45 \times 20)}$$

$$\therefore \quad A_{sc} = 309.242\ \text{mm}^2$$

Hence final reinforced section is,



Q-15: ***A simply supported T-beam of span 9 m in reinforced concrete has the following dimensions:***

Flange width = 2000 mm

Flange thickness = 150 mm

Overall depth = 750 mm

Rib width = 300 mm

The beam is provided with 6 No. 32 mm diameter HYSD bars of grade Fe 500.

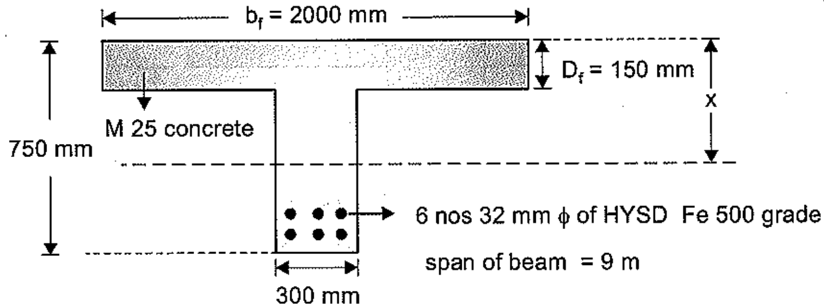
Concrete used is of grade M 25 :

Find the moment of resistance of the beam using limit state method.

Also find the magnitude of two point loads at 3 m distance from the ends.

[12 Marks, ESE-2012]

Sol:



Let us assume the effective cover to be 70 mm

$\Rightarrow$ $$d = 750 - 70 = 680 \text{ mm}$$

$$A_{st} = 6\times\frac{\pi}{4}(32)^2 = 4823.04 \text{ mm}^2$$

Locating Neutral axis

Let us check whether N.A lies in flange or web

If $0.36\, f_{ck}\, b_f\, D_f > 0.87\, f_y\, A_{st}$, N.A lies in flange other wise in web

$$\text{LHS} = 0.36 \times 25 \times 2000 \times 150 \text{ N} = 2700 \text{ kN}$$

$$\text{RHS} = 0.87 \times 500 \times 4823.04 = 2098.02 \text{ kN}$$

$\Rightarrow$ $$\text{LHS} > \text{R.H.S}$$

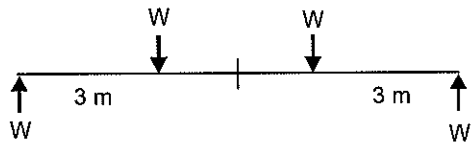
$\Rightarrow$ N.A lies in Flange

Calculation of NA depth

$\Rightarrow$ $$0.36\, f_{ck}\, b_f \cdot x = 0.87\, f_y\, A_{st}$$

$\Rightarrow$ $$x = \frac{0.87\, f_{ck}\, A_{st}}{0.36\, f_{ck}\, b_f} = 116.557 \text{ mm}$$

$\Rightarrow$ $$M_u = 0.36\, f_{ck}\, b_f\, x(d - 0.42\, x) = 1323.951 \text{ kNm}$$



$$\text{Max BM} = W \times 3$$

$$\text{Factored BM} = 1.5 \times W \times 3 = 4.5\, W \text{ kNm}$$ [W is in kN]

$$4.5\, W = M_u = 1323.951 \text{ kNm}$$

$\Rightarrow$ $$W = 294.211 \text{ kN}$$

Q-16: ***Determine the moment of resistance of a Tee-section having the following properties:***

Flange width = 2000 mm

Flange depth = 100 mm

Web width = 250 mm

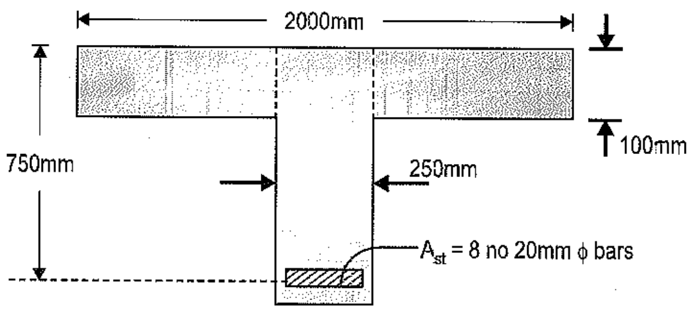
Effective depth = 750 mm

Area of steel = 8 bars of 200 mm ϕ

Material used = M25 grade of concrete and Fe 415 HYSD bars.

[4 Marks, ESE-2013]

Sol:



$$B = 2000 \text{ mm} = B_{eff}$$

(as the span of the beam is not mentioned we can consider it as effective flange width).

$$\text{Assume } x_a < 100,$$

Total tensile force = total compressive force

$$\therefore \quad 0.87 A_{st} f_y = 2000 \times x_a \times 0.36 \times f_{ck}$$

$$\therefore \quad x_a = \frac{0.87 \times \pi/4 \times (20)^2 \times 8 \times 415}{2000 \times 0.36 \times 25} = 50.412 \text{mm}$$

$$x_{u_{lim}} = 0.48d = 360\text{mm}$$

$$\therefore \quad x_a < x_{u_{lim}}, \text{ hence the section is under reinforced.}$$

$$\text{MR of the section} = 0.36 f_{ck} B x_u (d - 0.42 x_u)$$

$$= 0.36 \times 25 \times 2000 \times 50.412 \times (750 - 0.42 \times 50.412)$$

$$= 661349244.7 \text{ N-m}$$

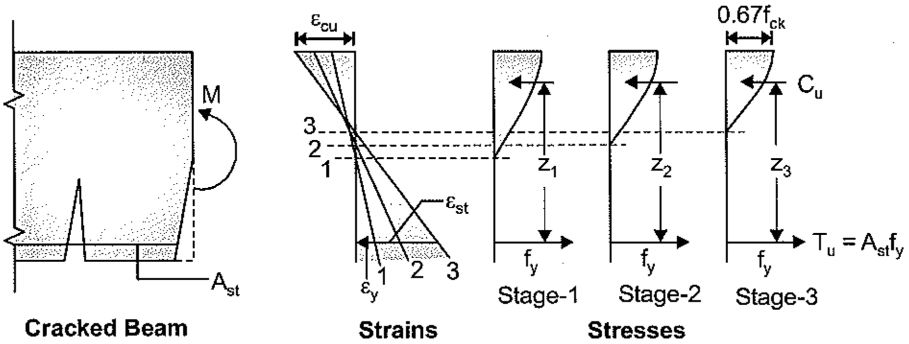
$$\text{MR} = 661.35 \text{ kNm.}$$

Q-17: ***Explain under reinforced and over reinforced failure of a reinforced concrete beam.***

[15 Marks, ESE-2014]

Sol: **Under Reinforced failure:**

- An under-reinforced section is one in which the area of tension steel is such that as the ultimate limit state is approached, the yield strain ε_y is reached in the steel before the ultimate compressive strain is reached in the extreme fibre of concrete.

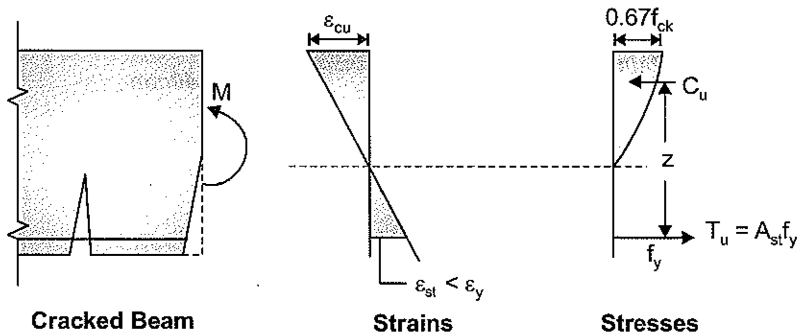


Behaviour of under-reinforced section (tension failure)

- Once steel has yielded it does not take any additional stress. Now if the beam is further loaded then neutral axis and centre of gravity of compressive forces further shifts upward to maintain equilibrium. This result in increase in moment of resistance of beam.
- The process of shift in neutral axis continuous until maximum strain in concrete reaches its ultimate and concrete crushes.
- Such a failure is called under-reinforced failure or tension failure.
- It is ductile in nature.

Over reinforced failure:

- An over-reinforced section is one in which the area of tension steel is such that at the ultimate limit state, the ultimate compressive strain in concrete is reached, however the tensile strain in the reinforcing steel is less than the yield strain ε_y.



Behaviour of over-reinforced section (compression failure)

- The concrete fails in compression before the steel reaches its yield point.
- This type of failure is called 'over-reinforced failure' or 'compression failure'
- It is brittle in nature hence not preferred.

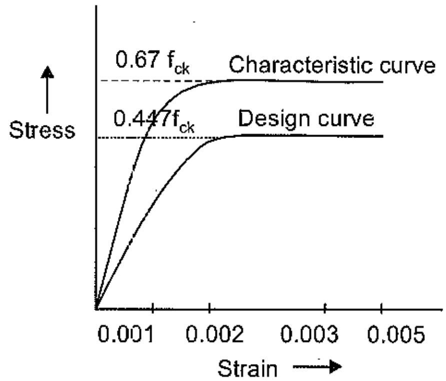
Q-18: ***State of assumption in the limit state of collapse in compression in flexure regarding strain at the highly compressed extreme fibre in concrete. Show with the help of a neat sketch.***

[5 Marks, ESE-2014]

Sol: The assumption in the limit state of collapse in compression in flexure regarding strain at the highly compressed extreme fibre in concrete are:

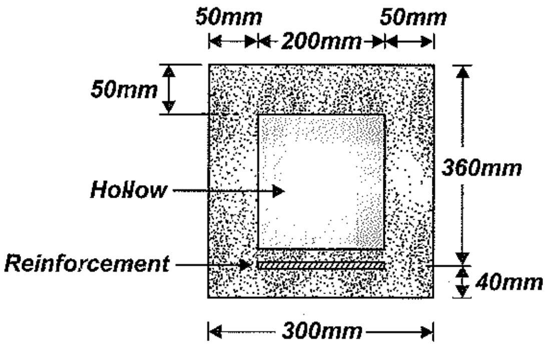
- The maximum compressive strain in concrete (at the outermost fibre) shall be taken as 0.0035. This is so, because regardless of whether the beam is under-reinforced or over-reinforced, collapse invariably occurs by the crushing of concrete.

- The design stress-strain curve of concrete in flexural compression (recommended by the code) is shown below:



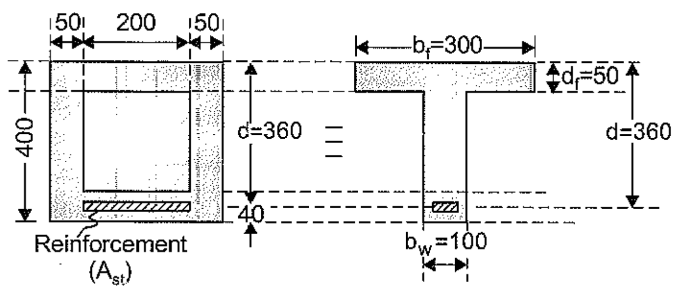
Code also permits the use of any other shape of the stress-strain curve which results in substantial agreement with the result of tests. The partial safety factor of 1.5 is to be applied.

Q-19: ***Determine the reinforcement required to resist a factored bending moment of 40 kN-m acting on a beam of hollow cross section as shown, using M 25 grade of concrete and Fe 415 steel. Effective depth of the beam is 360 mm. Adopt Limit State method of Design.***



[10 Marks, ESE-2015]

Sol:



The hollow concrete block can be assumed to be a T-beam of

$$b_f = 300 \text{ mm}$$
$$D = 400 \text{ mm}$$
$$d = 360 \text{ mm}$$
$$b_w = 100 \text{ mm}$$

Let us suppose, $x_L < d_f$

$\therefore$ At $x_u = d_f$,

$$M_L = 0.36 f_{ck} b_f d_f (d - 0.42 d_f)$$
$$= 0.36 \times 25 \times 300 \times 50 (360 - 0.42 \times 50)$$
$$= 45.765 \text{ kNm} > M_u \text{, applied}$$

$\therefore$ NA lies in flange

$$\therefore \quad M_{u,\ applied} = 0.36\ f_{ck}\ b_f\ x_u\ (d - 0.42\ x_u)$$

$$\therefore \quad 40 \times 10^6 = 0.36 \times 25 \times 300\ x_u\ (360 - 0.42\ x_u)$$

$$\Rightarrow \quad x_u = 43.34 \text{ mm}$$

$$C = T$$

$$0.36\ f_{ck}\ b_f\ x_u = 0.87\ f_y\ A_{st}$$

$$\Rightarrow \quad A_{st} = \frac{0.36 \times 25 \times 300 \times 43.34}{0.87 \times 415} = 324.13 \text{ mm}^2$$

$\Rightarrow$ Provide 3#12ϕbars

$$\Rightarrow \quad A_{st,\ provided} = 339 \text{ mm}^2$$

Check for minimum reinforcement : $\dfrac{0.85}{f_y} = \dfrac{A_{st,min}}{bd}$

$$\Rightarrow \quad A_{st,\ min} = \frac{0.85 \times 300 \times 360}{415} = 221.2\text{mm}^2 < A_{st,provided} \text{ (OK)}$$

Q-20: ***What do you understand by the term limit state?***

[5 Marks, ESE-2016]

Sol: A limit state is a condition beyond which a structural system or a structural component ceases to fulfill its intended function.

There are mainly two types of limit states.

(1) Limit state of collapse (strength limit states): These are the limit state regarding the strength of the material.

Examples are: Limit state of flexure, shear, torsion, bond etc.

(2) Serviceability limit state: These limit states account for appearance, serviceability and performance of structure during its design life.

Examples are: Limit state of deflection, cracking, stability etc.

The acceptable limit for the safety and serviceability requirement of a structure or structural element before failure occurs is called limit state.

Two main limit state are

Limit state of collapse

- To satisfy this limit state, the strength must be adequate to carry the loads.
- Limit state of collapse includes limit state of
  - → Flexure
  - → Compression
  - → Torsion
  - → Shear

Limit state of serviceability

- To satisfy this limit state, deflection, cracking and vibration must not be excessive.
- Excessive deflection can reduce the efficiency of the structure and must be avoided.
- Cracking causes ingress of water.

Q-21: ***Find the moment of resistance of a beam 300 × 600 mm deep if it is reinforced with 3 Nos. of 20 mm dia. bars in compression and tension, each at an effective cover of 40 mm. Use M 20 grade concrete and steel grade Fe415.***

Points on stress-strain curve for Fe 415 steel

Stress level	*Fe 415 grade*	
	Strain	*Stress (N/mm²)*
$0.80\ f_y$	0.00144	288.7
$0.85\ f_y$	0.00163	306.7
$0.90\ f_y$	0.00192	324.8
$0.95\ f_y$	0.00241	342.8
$0.975\ f_y$	0.00276	351.8
$1.00\ f_y$	0.00380	360.9

[12 Marks, ESE-2019]

Sol: **Given:**

$$b = 300 \text{ mm}$$

$$D = 600 \text{ mm}$$

$$A_{sc} = 3\times\frac{\pi}{4}\times 20^2 = 942.48 \text{ mm}^2$$

$$A_{st} = 3\times\frac{\pi}{4}\times 20^2 = 942.48 \text{ mm}^2$$

$$d' = 40 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

Let us assume trial depth of NA = 200 mm.

Strain at the level of compression steel,

$$\varepsilon_{sc} = \frac{0.0035}{200}\times(200-40) = 0.0028$$

and corresponding stress,

$$f_{sc} = 351.8+\frac{360.9-351.8}{0.00380-0.00276}\times(0.00280-0.00276)$$

$$f_{sc} = 352.15 \text{ N/mm}^2$$

New depth of NA

$$0.36 \times 20 \times 300x + (352.15 - 0.45 \times 20) \times 942.48 = 0.87 \times 415 \times 942.48$$

$\Rightarrow$ $$x = 7.81 \text{ mm} \neq x_{assumed}$$

$\therefore$ New trial depth of NA

$$x = \frac{200+7.81}{2} = 103.905 \text{ mm}$$

$$\varepsilon_{sc} = \frac{0.0035}{103.905}\times(103.905-40)$$

$\Rightarrow$ $$\varepsilon_{sc} = 0.0021$$

$$f_{sc} = 324.8 + \frac{342.8 - 324.8}{0.00241 - 0.00192} \times (0.0021 - 0.00192)$$

$$\Rightarrow \quad f_{sc} = 331.4 \text{ N/mm}^2$$

Revised depth of NA

$$0.36 \times 20 \times 300x + (331.4 - 045 \times 20) \times 942.48 = 0.87 \times 415 \times 942.48$$

$$\Rightarrow \quad x = 16.86 \text{ mm} \neq x_{assumed}$$

New depth of NA

$$x = \frac{103.905 + 16.86}{2} = 60.38 \text{ mm}$$

$$\varepsilon_{sc} = \frac{0.0035}{60.38} \times (60.38 - 40) = 0.0012$$

$$f_{sc} = 236.27 \text{ N/mm}^2$$

Depth of NA,

$$0.36 \times 20 \times 300x + (236.27 - 0.45 \times 20) \times 942.48 = 0.87 \times 415 \times 942.48$$

$$x = 58.37 \approx 60.38 \text{ mm}$$

Hence,

$$MOR = 0.36 f_{ck}\, bx\,(d - 0.42x) + (f_{sc} - f_{cc})\, A_{sc}(d - d')$$

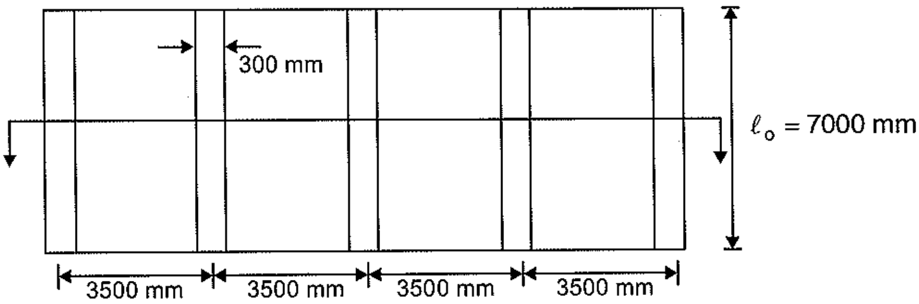
$$= 0.36 \times 20 \times 300 \times 58.37 \times (560 - 0.42 \times 58.37) + (236.27 - 0.45 \times 20) \times 942.48 \times (560 - 40)$$

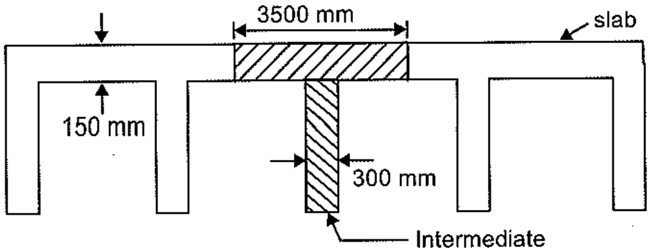
$$\Rightarrow \quad MOR = 178.90 \text{ kN-m}$$

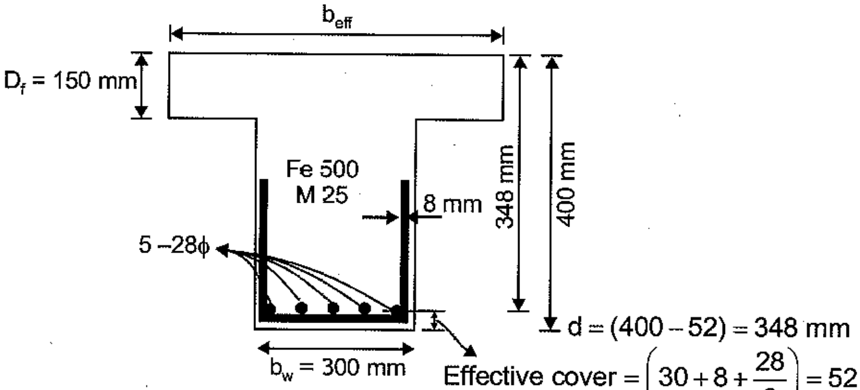
Q-22: ***A floor of an old building consists of 150 mm thick RC slab monolithic with the beam of width 300 mm and total depth 400 mm. The beams are spaced 3.5 c/c and their effective span (simply supported) is 7m. The beams are reinforced with 5 Nos. 28ϕ bars as tension reinforcement. Determine the moment carrying capacity of the beams. Use M 25 and Fe 500. Adopt limit state method of design. Nominal cover = 30 mm. Diameter of the stirrups = 8 mm.***

[12 Marks, ESE-2020]

Sol:







Effective cover $= \left(30+8+\frac{28}{2}\right) = 52$ mm

$$\text{Effective flange width } (b_{eff}) = \text{minimum of } \begin{cases} \frac{\ell_o}{6} + b_w + 6D_f \\ 3500 \end{cases}$$

$$= \text{minimum of } \begin{cases} \frac{7000}{6} + 300 + 6\times150 \\ 3500 \end{cases}$$

$$= 2366.67 \text{ mm}$$

Step (I) To determine x_u:

Assuming x_u to be in the flange and beam is under-reinforced.

$$x_u = \frac{0.87\, f_y A_{st}}{0.36\, f_{ck} b_f}$$

$$= \frac{0.87\times500\times5\times\frac{\pi}{4}\times(28)^2}{0.36\times25\times2366.67}$$

$$= 62.876 \text{ mm} < D_f \text{ (150 mm) OK}$$

So, our assumption of x_u lies in flange is correct.

$x_{u\,lim}$ for the balanced rectangular beam $= 0.46d$ [For Fe500]

$= (0.46 \times 348) = 160.08$ mm

and $x_u < x_{u\,lim} \Rightarrow$ under-reinforced section.

Step (II) To determine M_u (using $b = b_f$ for M_u)

$$M_u = 0.36\, f_{ck} b_f x_u\, (d - 0.42x_u)$$

$$= 0.36 \times 25 \times 2366.67 \times 62.876\, (348 - 0.42 \times 62.876)$$

$$= 430.696 \text{ kNm}$$

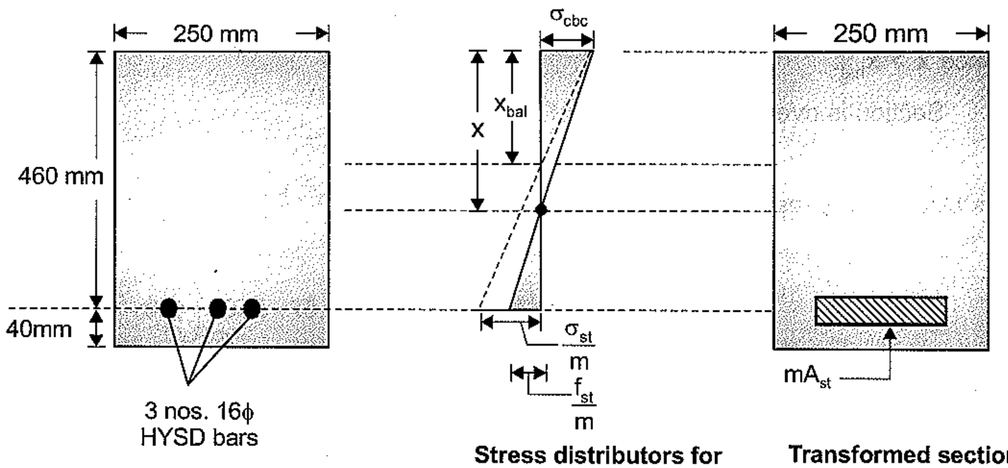
CHAPTER 2

WORKING STRESS (PART-II)

Q-1: ***A rectangular concrete section, 25 cm wide, 50 cm overall deep, is reinforced with three 16 mm dia high yield strength deformed bar at an effective cover of 4 cm from bottom face. If permissible stresses in concrete in bending compression and steel are 50 kg/cm² and 2300 kg/cm² respectively; modular ratio = 19, calculate the moment of resistance of the section using working stress method.***

[15 Marks, ESE-1997]

Sol:



Stress distributors for transformed section

Transformed section

Given data: B = 250 mm; D = 500 mm; d = 460 mm

$$A_{st} = \frac{\pi d^2}{4} \times 3 = \frac{\pi}{4} \times 16^2 \times 3 = 603.18\text{mm}^2$$

According to IS 456 : 2000

σ_{cbc} = 50 kg/cm² = 5N/mm² = Permissible bending compressive stress of concrete

Permissible tensile stress in HYSD bar, σ_{st} = 2300 kg/cm² = 230N/mm²

m = 19 (given)

Step 1: Calculating N.A. for balanced section.

$$X_{bal} = \left(\frac{\sigma_{cbc}}{\sigma_{cbc} + \frac{\sigma_{st}}{m}}\right) d = \left\{\frac{5}{\frac{230}{19} + 5}\right\} 460 = 134.46\text{mm.}$$

Step 2: Actual depth of NA: (x)

$$\frac{Bx^2}{2} = mA_{st}(d - x)$$

Conceptual Background:

Tension = Compression

$$mA_{st} \times \frac{f_{st}}{m} = \frac{1}{2} f_{cb} \times x \times b \quad \ldots(i)$$

From stress diagram

$$\frac{f_{st}/m}{f_{cb}} = \left(\frac{d-x}{x}\right) \quad \ldots(ii)$$

From (i) and (ii)

$$\therefore \quad \frac{1}{2} B x^2 = mA_{st}(d-x)$$

Hence, N.A location is obtained by equating the moment of area of compression side to the tension side.

Concrete reaches maximum permissible stress prior to steel in care of over reinforced section.

$$\Rightarrow \quad \frac{250 \times x^2}{2} = 19 \times 603.18\,(460 - x)$$

$$\Rightarrow \quad 125x^2 + 11460.42x - 5271793.2 = 0$$

$$x = 164.57 \text{ mm}, -256.26 \text{ mm (discarded)}$$

Hence $x > x_{bal}$.

$\Rightarrow$ Section is over reinforced. Thus $f_{cbc} = \sigma_{cbc}$, $f_{st} < \sigma_{st}$

It is clear from stress diagram that M.O.R. $= \left(\frac{1}{2} \times \sigma_{cbc} \times x \times b\right) \times \left(d - \frac{x}{3}\right)$

$$= \frac{1}{2} \times 5 \times 164.57 \times 250 \times \left(460 - \frac{164.57}{3}\right)$$

$$= 41.67 \times 10^6 \text{ N-mm}$$

$$\text{MOR} = 41.67 \text{ kNm}$$

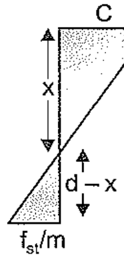
Note: Although in this question actual stress in reinforcement is not required, but we can calculate f_{st} with the help of similar triangle concept.

$$\therefore \quad \frac{\sigma_{cbc}}{x} = \frac{f_{st}}{m(d-x)}$$

$$\Rightarrow \quad f_{st} = \frac{\sigma_{cbc}}{x} m(d-x)$$

$$f_{st} = 19 \times 5\left(\frac{460}{164.57} - 1\right)$$

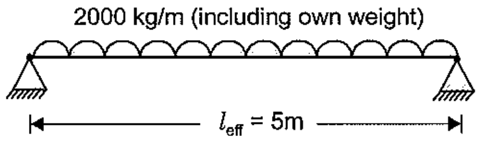
$$f_{st} = 170.54 \text{N/mm}^2$$

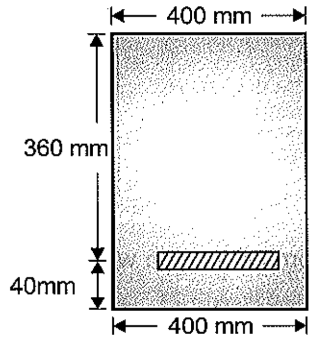


Q-2: ***A rectangular R.C. beam simply supported at ends over an effective span of 5.0 m carries a uniformly distributed load of 2000 kg/m including its own weight. If the permissible stresses in concrete in bending compression and in steel are 70 kg/cm² and 1900 kg/cm² respectively, modular ratio 'm' = 13, design the beam section, for flexure only, by working stress method. The size of the beam is restricted to 40 cm wide × 40 cm overall deep. Assume effective cover = 4.0 cm. Stress in compression reinforcement, if needed may be taken as 1.5 m times the stress in surrounding concrete.***

[15 Marks, ESE-1997]

Sol:





Given data:

$$l_{eff} = 5 \text{ m}$$

$$w = 2000 \text{ kg/m} = 2000 \times 10 \text{ N/m} = 20 \text{kN/m}$$

$$\sigma_{cbc} = 70 \text{ kg/cm}^2 = \frac{70 \times 10}{10^2} \text{N/mm}^2 = 7 \text{N/mm}^2$$

$$\sigma_{st} = 1900 \text{ kg/cm}^2 = 190 \text{ N/mm}^2$$

$$m = 13$$

$$b = 40\text{cm} = 400 \text{ mm}$$

$$D = 40 \text{ cm} = 400 \text{ mm}$$

d = 360 mm (since eff. cover has been assumed to be 4.0 cm)

Step 1: Determination of maximum bending moment.

$$B.M|_{max} = \frac{wl_{eff}^2}{8} = \frac{20 \times 5^2}{8} = 62.5 \text{kN/m}$$

(Since the size of the beam is restricted, we have to check if maximum applied BM is greater than the singly reinforced balanced MOR. If so, singly reinforced section will be sufficient, otherwise we will have to design doubly reinforced section.)

Step 2: Considering balanced section of given size,

$$n_0 = \frac{x_{bal}}{d} = \frac{\sigma_{cbc}}{\sigma_{cbc} + \frac{\sigma_{st}}{m}} = \frac{7}{7 + \frac{190}{13}} = 0.3238$$

and

$$J_0 = 1 - \frac{n_0}{3} = 1 - \frac{0.3238}{3} = 0.8920$$

$$R_{bal} = \frac{1}{2} \times J_0 \times n_0 \times \sigma_{cbc} = \frac{1}{2} \times 0.8920 \times 0.3238 \times 7$$

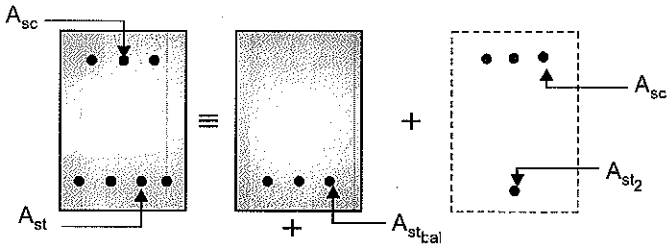
$$= 1.0109 \cong 1.011$$

Hence **$(M.O.R.|_{\text{Balanced section}})$** $= R_{bal} . b.d^2 = 1.011 \times 400 \times 360^2 =$ **$(52.41 \times 10^6 \text{N-mm})$**

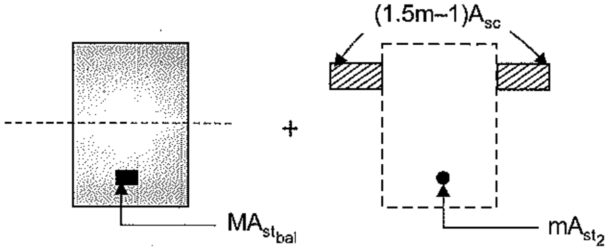
Step 3: Since the size of beam is restricted and $M.O.R|_{applied} > M.O.R.|_{\text{balanced section}}$, doubly reinforced beam is required.

In the design of doubly reinforced section, the overall size is known to us. We have to calculate the area of steel.

$$A_{st} = A_{st_{bal}} + A_{st_2}$$



Transformed section will look like as shown below.



Step 4: Calculating $A_{st_{bal}}$ [Area of steel for singly reinforced balanced section]

$$M_{bal} = \frac{\sigma_{st}}{m} \times mA_{st_{bal}} \times \left(d - \frac{x_{bal}}{3}\right)$$

Since $$x_{bal} = n_0 d$$

$\therefore$ $$A_{st_{bal}} = \frac{52.41 \times 10^6}{190 \times \left\{360 - \frac{0.3238 \times 360}{3}\right\}} = 858.93\text{mm}^2$$

Step 5: Calculating A_{st_2}; [Area of remaining tensile steel in the section with compression reinforcement]

$$A_{st_2} = \frac{M_2}{\sigma_{st}(d-d')} = \frac{B.M|_{max} - M_{bal}}{\sigma_{st}(d-d')} = \frac{(62.5-52.41) \times 10^6}{190 \times (360-40)}$$

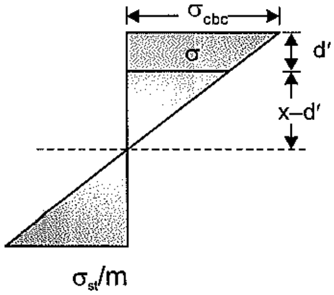
$$= 165.95\text{mm}^2$$

Step 6: A_{sc}, steel as a compression reinforcement

Since $$\text{Compressive force} = \text{Tensile force}$$

$\Rightarrow$ $$(1.5\,m-1)A_{sc}\,\sigma = \sigma_{st} \times A_{st_2}$$

$\therefore$ $$A_{sc} = \frac{\sigma_{st} \times A_{st_2}}{(1.5m-1)\sigma}$$



from simillar traingle concept, $$\frac{\sigma_{cbc}}{x} = \frac{\sigma}{x-d'}$$

$$\frac{\sigma_{cbc}(x-d')}{x} = \sigma$$

$\therefore$ $$\sigma = \frac{7(n_0 d - d')}{n_0 d} = \frac{7(0.3238 \times 360 - 40)}{0.3238 \times 360} = 4.6\text{N/mm}^2$$

$$A_{sc} = \frac{190 \times 165.95}{(1.5 \times 13 - 1)(4.6)} = 370.51\,\text{N/mm}^2$$

Area of compression steel = 370.51mm

Let us adopt 2 no 16ϕ bars

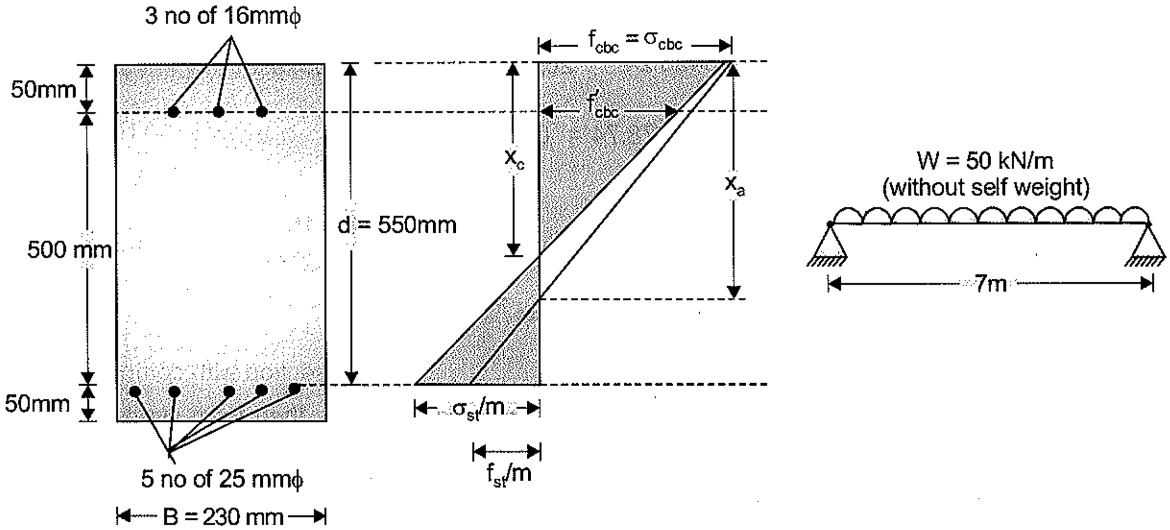
Total area of tensile steel $A_{st} = A_{st_{bal}} + A_{st_2}$ = 858.93 + 165.95 = 1024.88 mm^2.

Let us adopt 4 no 16ϕ & 1 no 20ϕ bars.

Q-3: ***A rectangular beam of overall cross-sectional dimensions 230 mm × 600 mm with 50 mm effective concrete cover is reinforced with 5 numbers of 25 mm diameter bars on the tension side and 3 numbers of 16 mm diameter bars on the compression side. It is carrying an imposed load of 50 kN/m over an effective span of 7.0 m. Check the adequacy of the design. The permissible stresses in concrete and steel are 7 N/mm² and 230 N/mm² respectively ; m = 15 compressive stress in steel bars = 1.5 m times the compressive stress in the surrounding concrete.***

[15 Marks, ESE-1998]

Sol:



Given data: B = 230 mm; d = 550 mm; D = 600 mm

$$A_{sc} = \frac{\pi}{4} \times d^2 \times 3 = \frac{\pi}{4} \times 16^2 \times 3 = 603.1872\text{mm}^2$$

$$A_{st} = \frac{\pi}{4} \times d^2 \times 5 = \frac{\pi}{4} \times 25^2 \times 5 = 2454.3750\text{mm}^2$$

$$\sigma_{cbc} = 7\ \text{N/mm}^2$$

$$\sigma_{st} = 230\ \text{N/mm}^2$$

Calculation of Maximum bending moment (M_{max})

$$M_{max} = \frac{wl_{eff}^2}{8} = \frac{(w_{self\,wt} + W_{imposed\,wt}) \times (l_{eff})^2}{8}$$

$$W_{self} = 0.230 \times .600 \times 25 = 3.45\ \text{kN/m}$$

$$W_{imposed} = 50\ \text{kN/m}.$$

$\therefore$ $(W_{seff} + W_{imposed}) = 53.45$ kN/m

$$\therefore \qquad M_{max} = \frac{(53.45)\times 7^2}{8} = 327.38 \text{ kN-m}$$

For checking the adequacy of structure have to show, that $(M.O.R.)_{beam} > B.M._{max.}$

Calculation of $(M.O.R.)_{beam}$

Step 1: Finding critical depth, (X_c)

$$X_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right)d = \frac{15\times 7\times 550}{230+15\times 7} = 172.39 \text{ mm}$$

Step 2: Calculate actual depth of neutral axes (x_a)

$$\frac{Bx_a^2}{2} + (1.5m-1)A_{sc}\times(x_a - d') = mA_{st}(d-x_a)$$

$$\Rightarrow \quad \frac{230}{2}x_a^2 + (1.5\times 15 - 1)\,603.1872 \times (x_a - 50) = 15 \times 2454.37 \times (550 - x_a)$$

$$\Rightarrow \quad 115x_a^2 + 12968.52.x_a - 648426.24 = 20248552.5 - 36815.55xa$$

$$\Rightarrow \quad 115x_a^2 + 49784.07x_a - 20896978.74 = 0$$

$$x_a = 261.63 \text{ mm}$$

Since x_a (261.63mm) > x_c (172.39)mm,

Hence, the section is over reinforced, $f_{cbc} = \sigma_{cbc} = 7\text{N/mm}^2$

$$f_{st} < \sigma_{st}$$

Step 3: Let f'_{cbc} be the stress at the level of compression steel, then from similar triangle

$$\frac{f'_{cbc}}{x_a - d_c} = \frac{f_{cbc}}{x_a} \quad \Rightarrow \quad f'_{cbc} = \frac{7}{261.63} \times (261.63 - 50) = 5.66 \text{ N/mm}^2$$

Step 4: Moment of resistance calculation,

$$MR = B \times x_a \times \frac{f_{cbc}}{2}\left(d - \frac{x_a}{3}\right) + (1.5m-1)A_{sc}\times f'_{cbc}(d-d')$$

$$= 230 \times 261.63 \times \frac{7}{2}\times\left(550 - \frac{261.63}{3}\right) + (1.5\times 15 - 1) \times 603.18 \times 5.66 \times (550 - 50)$$

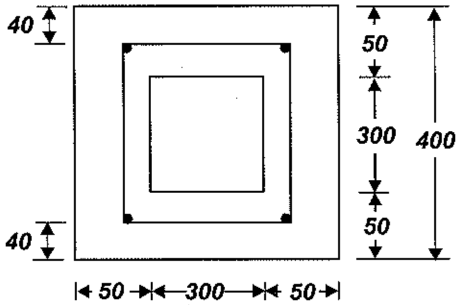
$$= 974691969 + 36700487.1 = 134.17 \text{ kN-m}$$

Since, $\quad M.O.R_{section}$ (134.17 kN-m) < BM_{max} required (327.38 kN-m)

Hence it is unsafe to use this section for given loading.

Q-4: ***Find the moment of resistance of the hollow box section R.C. beam using working stress method. M-20 concrete and Fe-415 steel are used. The walls of the box are 50 mm thick. It is reinforced with four 16 mm bars (one bar at each corner as shown). The effective cover to main steel is 40 mm. The hollow portion of the beam is 300 mm × 300 mm and outer dimensions of beam are 400***

mm × 400 mm. Allowable stress in concrete = 7 MPa, allowable stress in steel = 230 MPa. Modular ratio is 13.33 and 20 in tension and compression zones.



[25 Marks, ESE-2005]

Sol: It is doubly reinforced section,

$$A_{st} = 2 \times \pi/4 \times 16^2 = 402.12 \text{ mm}^2$$
$$A_{sc} = 2 \times \pi/4 \times 16^2 = 402.12 \text{ mm}^2$$
$$\sigma_{cbc} = 7 \text{ N/mm}^2$$
$$\sigma_{st} = 230 \text{ N/mm}^2$$
$$m = 13.33 \rightarrow \text{tension}$$
$$m' = 1.5 \text{ m} = 19.99 \sim 20 \rightarrow \text{compression.}$$

Step 1: Neutral axis depth of balanced section,

$$\frac{x_{bal}}{d} = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}$$

$$\Rightarrow \quad \frac{x_{bal}}{d} = \frac{13.33 \times 7}{230 + 13.33 \times 7} = 0.288$$

$$\Rightarrow \quad x_{bal} = 0.288 \times 360$$
$$= 103.89 \sim 103.9 \text{ mm}$$

Step 2: Actual neutral axis depth x_a.

To check whether the N.A lies in the top 50 mm depth or below it we will take moment of transformed area above 50 mm depth and moment of transformed area below 50 mm depth about an axis at 50 mm depth. If the moment of transformed area above 50 mm depth is greater, neutral axis lies in the top 50 mm depth, otherwise below 50 mm depth.

Moment of transformed area above 50mm depth

$$= 400 \times 50 \times 25 + (1.5 \text{ m} - 1) A_{sc} \times 10$$
$$= 576383.4271 \text{ mm}^3$$

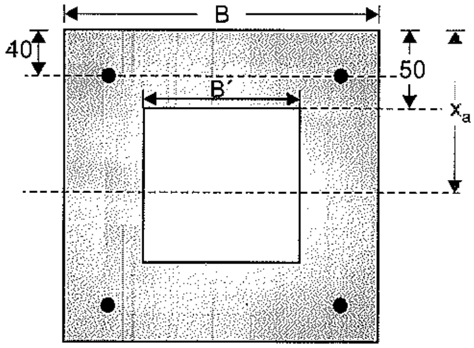
Moment of area below 50mm depth = m A_{st} × (350 – 40) = 1661696.425

Moment of transformed area above depth 50 mm < moment of transformed area below 50 mm depth (w.r.t. 50 mm depth)

Hence,

$\Rightarrow$ x_a > 50mm

$\Rightarrow$ 50 mm < x_a < 350 mm



$$B\times\frac{x_a^2}{2}-B'\times\frac{(x_a-50)^2}{2} + (1.5\ m-1)\ A_{sc}\ (x_a - d_c) = mA_{st}\ (d - x_a)$$

$$\Rightarrow \quad 400 \times \frac{x_a^2}{2} - 300 \times \frac{(x_a-50)^2}{2} + 19 \times 402.12\ (x_a - 40) = 13.33 \times 402.12\ (360 - x_a)$$

$x_a = 81.393$ mm

and $50\text{ mm} < x_a < 350\text{ mm}$

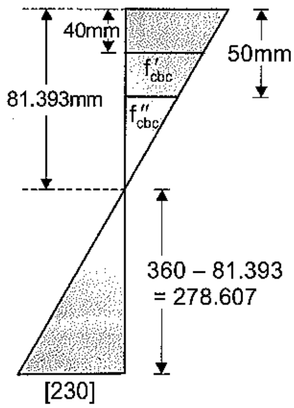
Hence neutral axes is $x_a = 81.393$ mm.

Since $x_a < x_{bal}$. So section is under reinforced

$\therefore \quad f_{st} = \sigma_{st} = 230$ MPa

but $f_{cbc} < \sigma_{cbc}$

Calculation of f_{cbc} and stress in concrete at level of compression steel,



Calculation of actual stress at different levels of concrete

$$\frac{f_{cbc}}{81.393} = \frac{230/m}{278.607} \text{ [from simillar traingle]}$$

$$\Rightarrow \quad f_{cbc} = \frac{230\times 81.393}{13.33\times 278.607} = 5.041 \text{ N/mm}^2$$

Similarly,
$$\frac{f_{cbc}}{81.393} = \frac{f'_{cbc}}{81.393-40}$$

$$\Rightarrow \quad \frac{f_{cbc}}{81.393} = \frac{f'_{cbc}}{41.393}$$

$$\Rightarrow \quad f'_{cbc} = \frac{f_{cbc}\times 41.393}{81.393} = \frac{5.041\times 41.393}{81.393} = 2.563 \text{ N/mm}^2$$

Similarly,
$$\frac{f_{cbc}}{81.393} = \frac{f''_{cbc}}{81.393-50}$$

$$\Rightarrow \quad f''_{cbc} = \frac{5.091\times 31.393}{81.393} = 1.944 \text{ N/mm}^2$$

Actual Stress

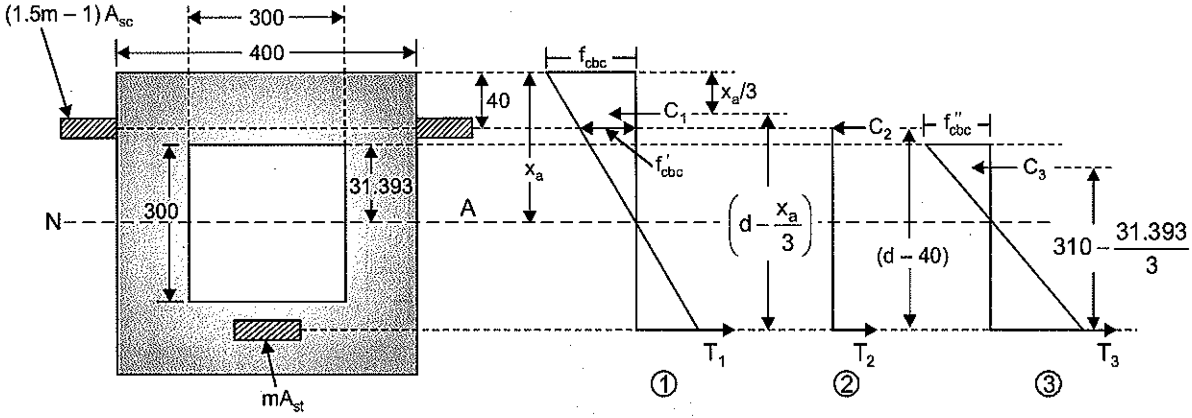
At extreme concrete fibre = $f_{cbc} = 5.041$ N/mm²

At depth 50 mm at concrete = $f''_{cbc} = 1.944$ N/mm²

At the level of compression steel = $f'_{cbc} = 2.563$ N/mm²

At tension steel f_{st} = $\sigma_{st} = 230$ N/mm²

At compression steel f_{sc} = $1.5m \times f'_{cbc} = 51.247$ N/mm²



Moment of resistance = MOR of overall section — MOR of hollow section

= MOR due to (1) and (2) — MOR due to (3)

MOR of overall section

$$MOR_{1\,\&\,2} = 1/2 \times 400 \times x_a \times f_{cbc} \times (d - x_a/3) + (1.5m - 1)\, A_{sc}\; f'_{cbc}(d-d_c)$$

$$= 0.5 \times 400 \times 81.393 \times 5.041 \times \left(360-\frac{81.393}{3}\right) + (1.5 \times 13.33-1) \times 402.12 \times 2.563 \times (360-40) = 33.58 \text{ kNm}$$

MOR of hollow section

$$MOR_3 = 0.5 \times 300 \times 31.393 \times 1.944 \times \left(310-\frac{31.393}{3}\right)$$

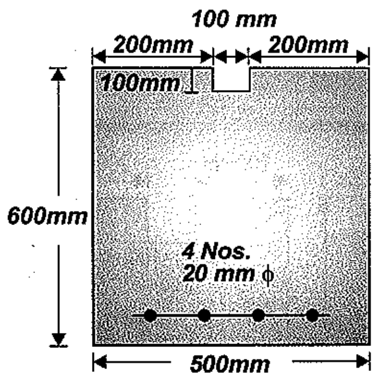
$$MOR_3 = 2.742 \text{ kNm}$$

Moment of resistance of the hallow box section

$$= MOR_{1\,\&\,2} - MOR_3 = 30.838 \text{ kNm}$$

$$MOR = 30.838 \text{ kNm}$$

Q-5: ***A reinforced concrete beam 500 mm wide and 600 mm deep has a 100 mm wide and 100 mm deep groove in the compression side, the reinforcement being on the tension side only, consisting of 4 Nos., 20 mm diameter bar. Determine position of neutral axis and calculate the stresses developed for a bending of 85,000 Nm. Assume m = 15.***

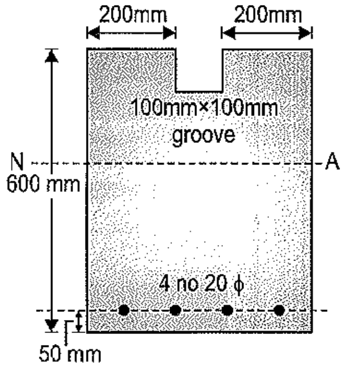


[15 Marks, ESE-2006]

Sol:

Assume cover = 50 mm.

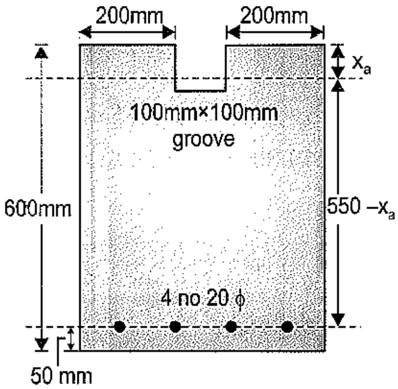
Effective debth = 600 – 50 = 550 mm



$$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$$

Finding actual debth of NA

Case I:



Assuming $x_a < 100$ mm

$$2 \times \left[200 \times x_a \times \frac{x_a}{2}\right] = mA_{st} \times (550 - x_a)$$

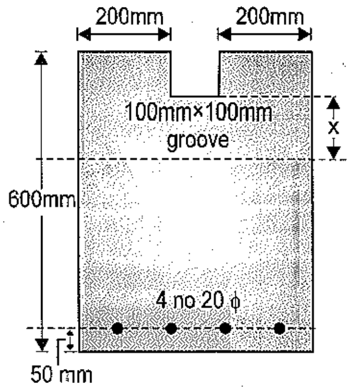
$\Rightarrow$ $$200 \times x_a^2 = 1256.64\,(550 - x_a) \times 15$$

$\Rightarrow$ $$200 \times x_a^2 = 18849.6\, x_a - 10367280 = 0$$

$$x_a = 185.37 > 100 \text{ mm}$$

Hence assumption is not valid.

Case II:



Assuming $x_a \geq 100$ mm

$$2 \times [200 \times 100 \times (50 + x)] + \left[500 \times x \times \frac{x}{2}\right] = m \times Ast \times (550 - 100 - x)$$

$\Rightarrow$ $2000000 + 40000x + 250x^2 = 15 \times 1256.64 \times (450 - x)$

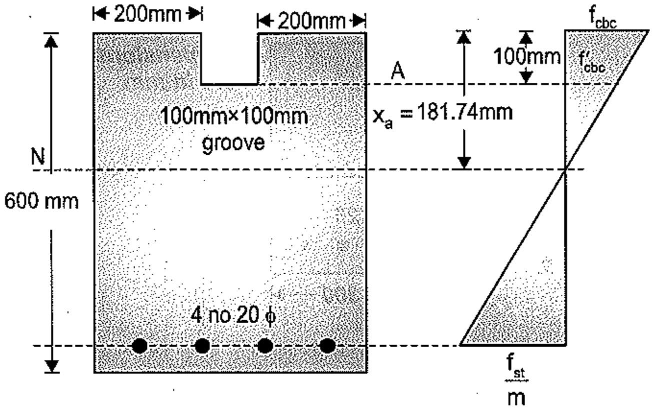
$\Rightarrow$ $250x^2 + 40000x + 2000000 = 8480700 - 18849.6x$

$\Rightarrow$ $250x^2 + 58849.6x - 6480700 = 0$

$\Rightarrow$ $x = 81.74$ mm

$\therefore$ NA depth from top = 100 + 81.74 = 181.74 mm > 100 mm

This is the correct actual debth of NA.

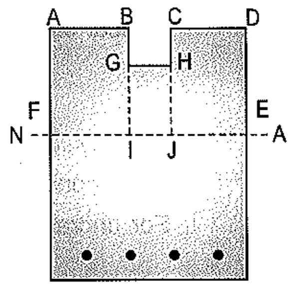


From similar triangle properties we can say

$$\frac{f_{cbc}}{x_a} = \frac{f'_{cbc}}{(x_a - 100)}$$

$$\therefore \quad f'_{cbc} = \frac{181.74 - 100}{181.74} f_{cbc} = 0.45 f_{cbc}$$

From these stresses we can calculate an expression for the corresponding moment of resistance.



$\therefore$ MR of the section = 2 × MR of ABIF + MR of GHJI

$$\therefore \quad MR = 2 \times 0.5 \times f_{cbc} \times 200 \times x_a \times (d - x_a/3)$$
$$+ 0.5 \times f'_{cbc} \times 100 \times (x_a - 100) \times \left(d - \frac{x_a - 100}{3}\right)$$

$$\therefore \quad MR = 2 \times 0.5 \times f_{cbc} \times 200 \times 181.75 \times \left(550 - \frac{181.74}{3}\right)$$
$$+ 0.5 \times 0.45\, f_{cbc} \times 100 \times 81.74 \times \left(450 - \frac{81.74}{3}\right)$$

$$\therefore \quad MR = 17789438.16\, f_{cbc} + 777506.793\, f_{cbc}$$

Applied moment mentioned in question is = 85000 Nm

Equating we get

$$\therefore \quad 85000 \times 10^3 = 177894380.16\, f_{cbc} + 777506.793\, f_{cbc}$$

$$\therefore \quad f_{cbc} = \mathbf{4.578\ N/mm^2}$$

and $$f'_{cbc} = \mathbf{2.06\ N/mm^2}$$

From stress diagram we can say-

$$\frac{f_{cbc}}{x_a} = \frac{f_{st}/m}{d - x_a}$$

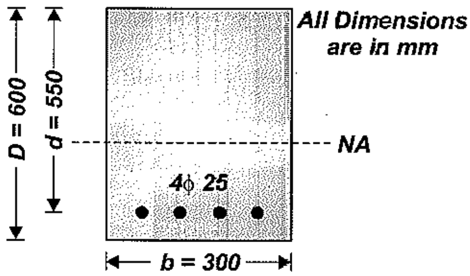
$$\therefore \quad f_{st} = \frac{f_{cbc}}{x_a} \times (d - x_a) \times m$$

$$\therefore \quad \mathbf{f_{st} = 139.146 \ N/mm^2}$$

Stress at concrete top fibre = f_{cbc} = 4.578 N/mm²

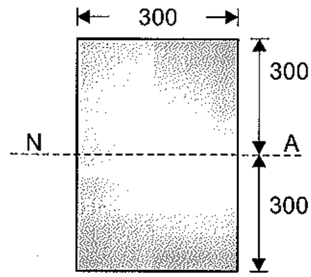
Stress at steel = f_{st} = 139.146 N/mm²

Q-6: ***A reinforced concrete beam of rectangular cross-section (600 mm × 300 mm) is shown in the above figure. Assume M20 grade concrete and Fe 415 grade steel. Permissible compressive stress of*** concrete in bending $\sigma_{cbc} = 7.0$ ***MPa for M 20 concrete. Compute a maximum stresses in concrete and steel when a moment of 50 kN-m is applied to the cross-section.***



[25 Marks, ESE-2009]

Sol: We know that if steel area is neglected, neutral axis will shift up as compared to the original depth of N.A. Hence stress in concrete on tension side will increase above the actual stress. If this increased stress is also not more than the critical stress at which concrete cracks is bending than the actual case in which steel area is accounted for, will also not lead to cracking of section. Thus, we should first check that whether for the given moment, cracking will occur or not.



Steel Area Neglected

Stress at the extreme fibre of concrete

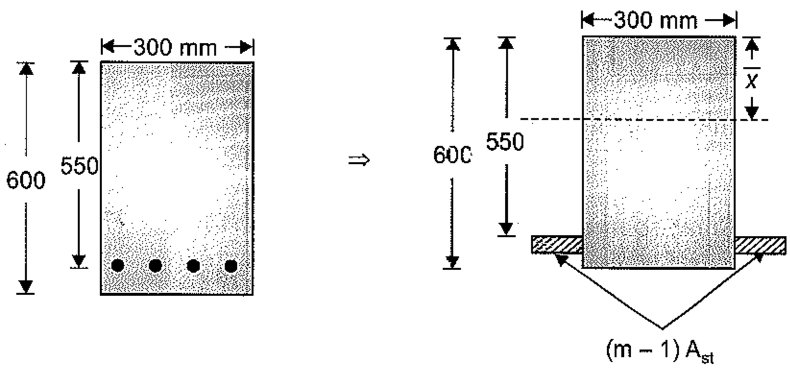
$$= \frac{My}{I} = \frac{50 \times 10^6 \text{ Nmm} \times 300 \text{ mm}}{300 \text{ mm} \frac{(600)^3 \text{ mm}^3}{12}} = 2.78 \text{ N/mm}^2$$

Critical stress at which concrete cracks

$$= f_{cr} = 0.7\sqrt{f_{ck}} = 0.7\sqrt{20} = 3.13 \text{ N/mm}^2$$

⇒ Max tensile stress developed in beam in bending will be less than the critical stress at which concrete cracks. Thus, the beam will be an uncracked beam.

For uncracked beam, the transformed section will be as shown below.



b = 300 mm; d = 550 mm; D = 600 mm

Location of neutral axis

N.A will be at the C.G. of x-section of transformed section.

$$\bar{x} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2}{A_1 + A_2} = \frac{bD\times\frac{D}{2} + ((m-1)A_{st})\times d}{bD + (m-1)A_{st}}$$

$$= \frac{300\times\frac{(600)^2}{2} + \left[\left(\frac{280}{3\times7}\right)-1\right]\times4\times491\times550}{300\times600 + \left(\frac{280}{3\times7}-1\right)\times4\times491}$$

$$= 329.65 \text{ mm}$$

Moment of inertia of the transformed section:

$$I = \frac{bx^3}{3} + \frac{b(D-x)^3}{3} + (m-1)A_{st}(d-x)^2$$

$$= \frac{300(329.65)^3}{3} + \frac{300(600-329.65)^3}{3}$$

$$+ \left(\frac{280}{3\times7}-1\right)\times4\times491\times(550-329.65)^2$$

$$= 6734\times10^6 \text{ mm}^4$$

B.M at which section cracks is given by

$$M_{cr} = \frac{f_{cr}I}{(D-x)} = \frac{0.7\sqrt{20}\times673\times10^6}{(600-329.65)} \text{ Nmm}$$

$$= 77.976 \text{ kN-m} > 50 \text{ kN-m}$$

As the applied BM is less than this, the section will not crack.

Stress in steel in transformed section corresponding to applied BM of 50 kNm is

$$\frac{f_{st}}{m} = \frac{M(d-x)}{I}$$

$$\Rightarrow \quad f_{st} = \frac{mM(d-x)}{I} = \frac{\left(\frac{280}{3\times7}\right)(50\times10^6)(550-329.65)}{6734\times10^6}$$

$$= 21.815 \text{ N/mm}^2$$

$$\text{Stress in concrete} = \frac{Mx}{I} = \frac{50\times10^6\times329.65}{6734\times10^6} = 2.448 \text{ N/mm}^2$$

Q-7: ***Design a T-beam for a commercial complex with reference to the data as stated below by working stress method:***

Clear span of the T-beam = 10 m

Spacings of the T-beam = 2.5 m

Live load = 4 kN/m²

Thickness of the slab = 15 cm

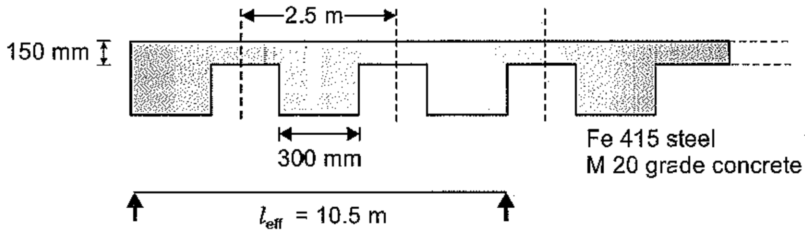
Material = M-20 grade concrete and Fe-415 HYSD bars

Effective span of the beam = 10.5 m

Assume the width of the rib = 300 mm

[15 Marks, ESE-2010]

Sol:



$$\text{Adopt overall depth to be } \frac{l}{12} = \frac{10\,000}{12} = 833.33 \text{ m}$$

Let us adopt overall depth to be 850 mm

Adopting effective cover to be 35 mm, the effective, depth becomes 815 mm

$$\boxed{d = 815 \text{ mm}}$$

$$b_f = b_w + 6D_f + \frac{l_0}{6}$$

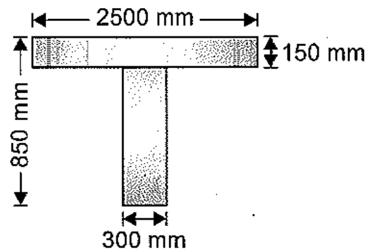
[b_w = Width of web, D_f = Slab thickness, l_0 = Effective span of beam]

$$= 300 + 6\times150 + \frac{10500}{6} = 2950 \text{ mm}$$

$$b_f \not> b_w + \frac{l_1 + l_2}{2} \qquad \text{[i.e., spacing of T-beam]}$$

i.e., $b_w \not> 2.5$ m

hence adopt $\boxed{b_f = 2500 \text{ mm}}$



$$\text{D.L of beam} = \left[2.5\times0.15\times10.5 + \frac{(850-150)\times300\times10.5}{10^6}\right]25 \text{ kN}$$

$$= 153.5625 \text{ kN}$$

$$w_d = \frac{153.5625}{10.5} = 14.625 \text{ kN/m}$$

$$\text{Live load } w_L = \frac{4\times2.5\times10.5}{10.5} = 10 \text{ kN/m}$$

$$\text{Total D.L + L.L} = 24.625 \text{ kN/m}$$

$$\text{Max BM} = \frac{wl^2}{8} = \frac{24.625\,(10.5)^2}{8} = 339.363 \text{ kNm}$$

Max SF at the face of beam = $\frac{24.625 \times 10}{2} = 123.125$ kN m

Approximate area of steel required on tension side in given by

$$A_{st} = \frac{M}{\sigma_{st}\left(d - \frac{D_f}{2}\right)} = \frac{339.363 \times 10^6 \text{ Nmm}}{230\left(815 - \frac{150}{2}\right)\text{N/mm}^2} \text{mm}^2$$

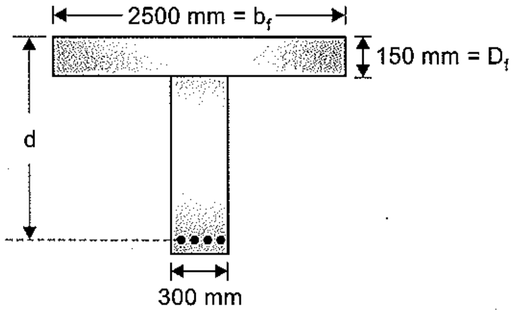
$$= 1903.86 \text{ mm}^2$$

Let us provide 25 mm dia bars

$$\text{No. of bars of 25 mm dia required} = \frac{1903.86}{\frac{\pi}{4}(25)^2} \simeq 4 \text{ bars}$$

Area of steel provided = $1962.5\text{mm}^2 > 1903.86$ mm² OK.

Once the dia of bar and its number chosen, we will provide that and will check for safety in bending



If $\quad \frac{b_f D_f^2}{2} \geq mA_{st}(d - D_f)$, N.A lies in flange otherwise in web

$$\frac{b_f D_f^2}{2} = \frac{2500(150)^2}{2} = 28.125 \times 10^6 \text{ mm}^3$$

$$mA_{st}(d - D_f) = \frac{280 \times 1962.5}{3 \times 7}(815 - 150) \quad \left[\begin{array}{l} m = \frac{280}{3\sigma_{cbc}} \\ \sigma_{cbc} = 7 \text{ N/mm}^2 \text{ for M20} \end{array}\right]$$

$$= 17.4 \times 10^6 \text{ mm}^3$$

$$\Rightarrow \qquad \frac{b_f D_f^2}{2} > mA_{st}(d - D_f)$$

$\Rightarrow$ Neutral axis lies in flange

The neutral axis depth can be calculated as

$$\frac{b_f x^2}{2} = mA_{st}(d - x)$$

$$\frac{2500}{2}x^2 = \left(\frac{280}{3 \times 7}\right) \times 1962.5(815 - x)$$

$$1250x^2 = 21325833.33 - 26166.67x$$

$$1250x^2 + 26166.67x - 21325833.3 = 0$$

$$x = \frac{-26166.67 \pm \sqrt{(26166.67)^2 + 4 \times 1250 \times 21325833.33}}{1250 \times 2}$$

$$x = 120.57 \text{ mm}$$

$$\Rightarrow \quad \frac{x}{d} = \frac{120.57}{815} = 0.148$$

$$n_0 = 0.2887 \text{ For Fe 415 steel}$$

$$\Rightarrow \quad \frac{x}{d} < n_0 \qquad \text{(i.e. critical N.A depth ratio)}$$

⇒ Section is under reinforced

Moment of Resistance of the section

$$MOR = A_{st}\sigma_{st}\left(d - \frac{x}{3}\right) = 1962.5 \times 230\left(815 - \frac{120.57}{3}\right)$$

$$= 349.73 \text{ kNm}$$

$$\text{Max applied BM} = 339.363 \text{ kNm}$$

$$\Rightarrow \quad MOR > M_{max} \text{ applied}$$

⇒ Section in safe in bending.

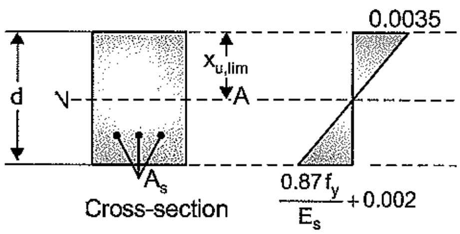
As the data required for shear design in not given we will only design the section for bending.

Q-8: ***"In limit state method of design or working stress method of design, balanced neutral axis depth is not dependent on compressive strength of concrete." Justify the statement with the help of suitable formula.***

[5 Marks, ESE-2015]

Sol: In LSM,

$$\varepsilon_{st} = \frac{0.87 f_y}{E_s} + 0.002 = \frac{0.0035(d - x_{u,lim})}{x_{u,lim}}$$



Strain diagram

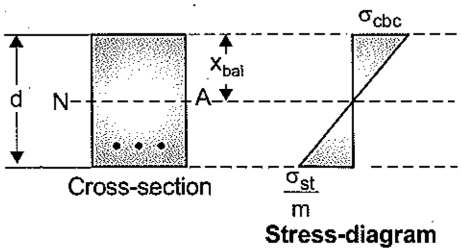
$$\Rightarrow \quad \boxed{x_{u,lim} = \frac{0.0035}{0.0055 + 0.87\frac{f_y}{E_s}} \cdot d}$$

where, $x_{u,lim}$ = balanced neutral axis depth.

⇒ $x_{u,lim}$ is independent of compressive strength of concrete.

In WSM

$$\frac{d - x_{bal}}{x_{bal}} = \frac{\sigma_{st}/m}{\sigma_{cbc}}$$



Also, $$m = \frac{280}{3\sigma_{cbc}}$$

$\Rightarrow$ $$\boxed{x_{bal} = \frac{280}{280 + 3\sigma_{st}} \cdot d}$$

where, x_{bal} = balanced neutral axis depth

$\Rightarrow$ x_{bal} is independent of compressive strength of concrete.

CHAPTER 3 LIMIT STATE OF COLLAPSE IN SHEAR

Q-1: ***A 5 m effective span simply supported beam is subjected to a load of 40 kN/m including its self weight. The size of the beam is 250 mm × 500 mm. The beam is reinforced with $4-20\phi$ at bottom (out of which two bars are (curtailed) and $2-12\phi$ at top. Design the beam against shear force and show the reinforcement details. Use M20 and Fe415. Use limit state method of design.***

% tension steel	0.15	0.25	0.5	0.75	1.0	1.25	1.5	2.0	2.5	3 & more
τ_c (MPa)	0.28	0.36	0.48	0.56	0.62	0.67	0.72	0.79	0.82	0.82

[20 Marks, ESE-2013]

Sol: Design shear force, $V_u = \dfrac{1.5\times40\times5}{2} = 150\text{kN}$

Let us assume the effective cover to be 50 mm

$\Rightarrow \quad d = 500 - 50 = 450$ mm

(i) $\tau_v = \dfrac{V_u}{bd} = \dfrac{150\times10^3}{250\times450} = 1.33\ \text{N/mm}^2$

(ii) $\tau_{c_{max}} = 2.8\ \text{N/mm}^2$

(iii) $\tau_v < \tau_{c_{max}}$

(iv) Percentage of tension steel $(P_t) = \dfrac{2\times\pi/4\times20^2}{bd}\times100 = \dfrac{628.32}{250\times450}\times100 = 0.558\%$

Now we have to calculate τ_c corresponding to above P_t.

P_t	τ_c
0.5	0.48
0.558	?
0.75	0.56

$$\therefore \quad \tau_c = 0.48 + \frac{(0.56-0.48)}{(0.75-0.5)}\times(0.558-0.5)$$

$$\therefore \quad \tau_c = 0.499\ \text{N/mm}^2$$

(v) Now providing 2 legged 8ϕ stirrups - (vertical)

$$(\tau_v - \tau_c)bd = 0.87\ f_y \times A_{sv} \times d/sv$$

$$\therefore \quad S_v = \frac{0.87 f_y A_{sv}.d}{(\tau_v-\tau_c)bd} = \frac{0.87\times415\times2\times\pi/4\times8^2\times450}{(1.33-0.499)\times250\times450}$$

$$= 174.625\ \text{mm}$$

$$\text{Adopt } S_v = 170 \text{ mm}$$

(vi) Provide S_v = Minimum of 170 mm, 0.75 d = 375 mm, 300 mm,

$$\therefore \quad S_v = 170 \text{ mm}$$

(vii) Minimum shear reinforcement requirement

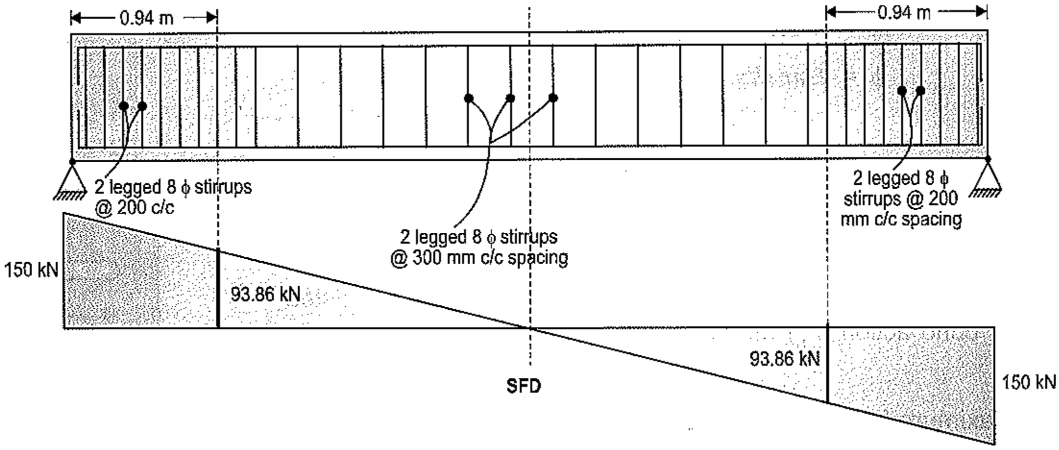
$$\frac{A_{sv}}{b \times S_v} > \frac{0.4}{0.87 f_y}$$

$$\Rightarrow \quad S_v < \frac{0.87 f_y A_{sv}}{0.4 b}$$

$$S_v < 362.78$$

as S_v = 170 mm Hence shear reinforcement is more than the min value.

For min shear reinforcement however spacing should not be greater then 300 mm



S.F to be resisted by shear stirrups,

$$V = V_u < \tau_c \times bd$$

$$= 150 \times 10^3 - 0.499 \times 250 \times 450$$

$$= 93.86 \text{ kN}$$

Location of min. shear reinforcement

$$\frac{x}{93.86} = \frac{2.5}{150}$$

$$x = 1.56 \text{ m from mid span}$$

CHAPTER 4

BOND AND ANCHORAGE

Q-1: ***Show that development length of a steel bar of dia ϕ embedded in concrete is given by***

$$L_d = \frac{0.87\,\sigma_y \phi}{4\tau_{bd}}$$

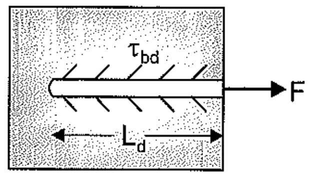
where, τ_{bd} = ***Bond strength of concrete***

τ_y = ***Yield strength of steel***

ϕ = ***Bar dia.***

[4 Marks, ESE-2014]

Sol: L_d = Development length



τ_{bd} = Bond strength of concrete

τ_{bd} acts over the circumference of bar therefore force resisted by bond action (F_b) will be

$$F_b = \pi\phi \times L_c \times \tau_{bd}$$

Maximum force carried by bar (F) will be

$$F = \frac{\pi}{4} \times \phi^2 \times (0.87\sigma_y)$$

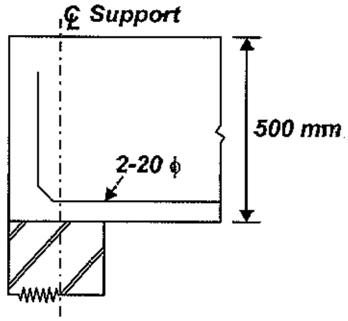
For equilibrium of forces

$$F = F_b$$

$$\therefore \quad \frac{\pi}{4} \times \phi^2 \times 0.87\sigma_y = \pi\phi \times L_d \times \tau_{bd}$$

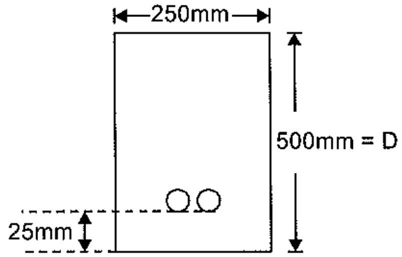
$$\Rightarrow \quad L_d = \frac{0.87\sigma_y\phi}{4\tau_{bd}}$$

Q-2: ***A simply supported beam is 25 × 50 cm deep and has 2-20 mm Fe 415 grade steel bars going into the support shown in Fig. 4. If the shear force at the center of support is 110 kN at service loads, determine the anchorage length. Assume M20 mix. Bond stress for mild steel for M20 concrete is 1.2 MPa in limit state. Take clear cover to steel = 25 mm.***



[20 Marks, ESE-2016]

Sol:



$$A_{st} = 2\times\frac{\pi}{4}\times 20^2 = 200\pi \text{ mm}^2$$

$$V = 1.5\times 110 = 165 \text{ kN}$$

$$\text{Anchorage length} = L_0$$

$$f_{ck} = 20 \text{ MPa}$$

$$\tau_{bd} = 1.6\times 1.2 \text{ MPa}$$

(60% increase for deformed bars)

$$b = 250 \text{ mm}$$

$$d = 500 - 25 - \frac{20}{2} = 465\text{mm}$$

$$\text{Development length, } l_d = \frac{0.87\times f_y\times\phi}{4\tau_{bd}}$$

$$l_d = \frac{0.87\times 415\times 20}{4\times 1.2\times 1.6\times 1.25} = 940.23 \text{ mm}$$

For safety,
$$L_d \le \frac{1.3M_1}{V} + L_0$$

$$M_1 = \text{Moment of resistance at section.}$$

$$M_1 = 0.36\ f_{ck}\ bx_u\ (d - 0.42\ x_u)$$

$$x_u = \text{Depth of N.A. from top of extreme compression fibre}$$

$$x_u = \frac{0.87\times f_y\times A_{st}}{0.36\times f_{ck}\times b} = \frac{0.87\times 415\times 200\times\pi}{0.36\times 20\times 250}$$

$$x_u = 126.03 \text{ mm} < 0.48d\ (0.48\times 465 = 223.2 \text{ mm})$$

$$\Rightarrow \quad M_1 = 0.36\times 20\times 250\times 126.03\times(465 - 0.42\times 126.03) \text{ Nmm}$$

$$M_1 = 93479.14 \text{ kNmm}$$

$$\Rightarrow \quad L_d \le 1.3\frac{M_1}{V} + L_0$$

$$\therefore \quad 940.23 \le \frac{1.3\times 93479.14}{165} + L_0$$

$$L_0 \ge 203.73 \text{ mm}$$

CHAPTER 5 DESIGN OF BEAM AND SLAB

Q-1: ***A rectangular R.C. slab 2 m × 3 m is simply supported along shorter edges such that clear distance between the supporting wall is 2.7 m. The slab is 15 cm thick and reinforced with 16 mm dia mild steel bars spaced at 25 cm centre to centre at effective cover of 25 mm along longer edges and with 10 mm dia bars along shorter edges spaced at 25 cm centre to centre. Concrete used is of M-15 grade for which permissible stresses in bending, shear (nominal) and bond are 50, 3 and 6 kg/ cm² respectively. Permissible tensile stress in mild steel = 1400 kg/cm² Modular ratio = 19. Claculate the maximum safe intensity of load that the slab can carry in addition to its self weight.***

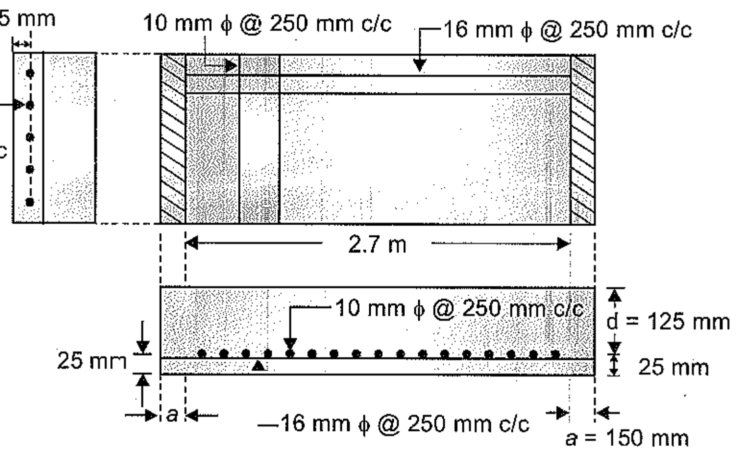
[15 Marks, ESE-1996]

Sol: Effective cover = 25 mm; Width of support = 150 mm; m = 19;

σ_{cbc} = 50 kg/cm² = 5 MPa; σ_{st} = 1400 kg/cm² = 140 MPa

Effective span for slab is smaller of (i) & (ii) as discussed below

(i) Clear span + effective depth = $l_0 + d$ = 2.7 + 0.125 = 2.825 m



(ii) Centre to centre distance between support = $l_0 + a$ [where a = width of support]

= 2.7 + 0.15 = 2.850 m

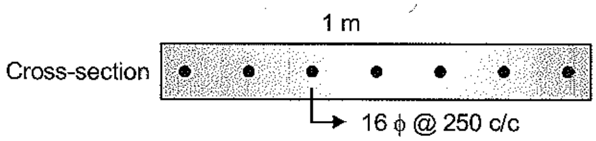
⇒ Effective span, l_{eff} = 2.825 m

Let the total load including self weight of slab that can be carried by slab = w kN/m².

Load calculation from safety against bending

$$\text{Moment bending moment} = \frac{wl_{eff}^2}{8} = \frac{w\times(2.825)^2}{8} = 0.997\ w\ \text{kN-m}$$

$$= 0.997 \times 10^6\ w\ \text{N-mm}$$



$$x_{bal} = \left(\frac{\sigma_{cbc}}{\frac{\sigma_{st}}{m}+\sigma_{cbc}}\right)d = \left[\frac{19\times 5}{140+(19\times 5)}\right]\times 125 = 50.53 \text{ mm}$$

$$A_{st} = \frac{1000}{250}\times\frac{\pi}{4}\times 16^2 = 804 \text{ mm}^2 \text{ (per meter width)}$$

Actual neutral axis depth is given by

$$\frac{bx^2}{2} = mA_{st}(d-x)$$

$$\Rightarrow \quad \frac{1000\times x^2}{2} = 19\times 804\,(125-x)$$

$$\Rightarrow \quad 500x^2 + 15276x - 1909500 = 0$$

$$\Rightarrow \quad x = 48.38 \text{ mm}$$

$\because$ $x < x_{bal}$, $\Rightarrow$ section is under reinforced

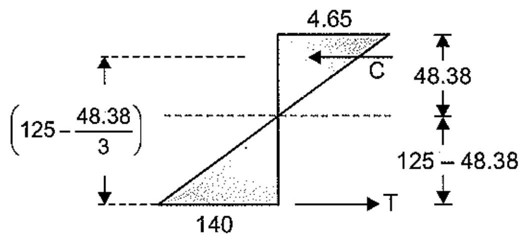
$$\Rightarrow \quad \sigma_{st} = 140 \text{ and } f_{cbc} < \sigma_{cbc}$$

$$\frac{f_{cbc}}{x} = \frac{\sigma_{st}/m}{d-x}$$

$$\therefore \quad \frac{f_{cbc}}{x} = \frac{140}{19}\times\frac{48.38}{(125-48.38)}$$

$$\Rightarrow \quad f_{cbc} = 4.65 \text{ N/mm}^2$$

Now from the stress diagram



$$0.997\times 10^6\, w = b.x\,\frac{f_{cbc}}{2}\left(d-\frac{x}{3}\right)$$

$$= 1000\times 48.38\times\frac{4.65}{2}\left(125-\frac{48.38}{3}\right)$$

$$\Rightarrow \quad \boxed{w = 12.28 \text{kN/m}^2}$$

Load calculation for safety against shear

$$\text{Maximum shear force} = \frac{wl}{2} = \frac{w\times 2.7}{2} = 1.35w \text{ kN}$$

$$\tau_v = \frac{V}{bd} = \frac{1.35\,w\times 10^3}{1000\times 125} = 0.0108\,w$$

$$\text{Permissible stress in shear} = 3 \text{ kg/cm}^2 = 0.3 \text{ MPa}$$

$$\therefore \quad 0.0108\,w = 0.3$$

$$\Rightarrow \quad \boxed{w = 12.78 \text{kN/m}^2}$$

Load calculation from Safety against bond

At simple supports under this condition

$$L_d \le 1.3\frac{M_1}{V} + L_0$$

M_1 = MOR of the section.

V = Shear force at support

L_0 = Anchorage beyond centre of support

Max value of L_0 = d or 12ϕ whichever is greater

$$L_d = \frac{\sigma_s \phi}{4\tau_{bd}}$$

$$\sigma_s = 140 \text{ N/mm}^2$$

$$\phi = 16\text{mm}$$

$$\tau_{bd} = 0.6 \text{ N/mm}^2$$

$$\Rightarrow \quad L_d = \frac{140 \times 16}{4 \times 0.6} = 933.33\text{mm}$$

$$M_1 = bx \cdot \frac{f_{cbc}}{2}(d - x/3)$$

$$= 1000 \times 48.38 \times \frac{4.65}{2}\left(125 - \frac{48.38}{3}\right)\text{Nmm}$$

$$= 12.25 \times 10^6 \text{ Nmm}$$

$$V = \frac{wl_{eff}}{2} = \frac{w \times 2.7}{2} = 1.35w\,\text{kN} = 1350 \text{ wN}$$

Adopting max value of L_0 = $12 \times 16 = 192$mm

$$933.33 \le \frac{1.3 \times 12.25 \times 10^6}{1350\,w} + 192$$

$$\Rightarrow \quad \boxed{w \le 15.91 \text{ kN/m}^2}$$

Thus minimum udl from the above three criteria is

$$w = 12.28 \text{ kN/m}^2$$

$$\text{Self wt of slab} = 25 \times 1 \times 1 \times 0.15 \text{ kN/m}^2 = 3.75 \text{ kN/m}^2$$

$\Rightarrow$ Max safe intensity of load in addit on to self weight = $12.28 - 3.75 = 8.53 \text{ kN/m}^2$

Q-2: ***Design the R.C. floor slab for a room of internal dimensions of 4.0 m × 9.5 m. Assume the slab to be simply supported on 230 mm thick masonry walls. The slab is to support live load of 4.0 kN/m² and surface finish of 1.0 kN/m². Use M-20 grade concrete, HYSD steel of Fe-415 grade. Draw reinforcement details.***

[20 Marks, ESE-2002]

Sol: 1. **As per the vertical deflection criterion, the span to effective depth ratio for spans upto 10 m for a SS slab is given by**

$$\frac{l}{d} = 20$$

$$\Rightarrow \quad d = \frac{l}{20} = \frac{4000}{20} = 200 \text{ mm}$$

2. Effective span in x direction l

(i) Clear span + effective depth = 4.00 + 0.2 = 4.2 m

(ii) Centre to centre distance between supports = 4.0 + 0.23 = 4.23 m

Hence lesser of the above two will be adopted i.e. l_e = 4.2 m = l_{x_0}.

l_{y_0} will be min of the following:

(i) 9.5 + 0.2

(ii) 9.5 + 0.23

$$\frac{l_{y_0}}{l_{x_0}} = \frac{9.7}{4.2} = 2.30 > 2 \text{ Design as a one way slab.}$$

Assuming a nominal cover of 20 mm and 10 mm bar

$$D = 200 + 20 + \frac{10}{2} = 225 \text{ m}$$

3. Bending moment and shear force

(a) Load calculation

Load due to self weight of slab = 0.225 × 1 × 25 = 5.625 kN/m

Superimposed load = 4 × 1 = 4 kN/m

Surface finishes = 1 × 1 = 1 kN/m

Total load = 10.625 kN/m

Total factored load = 15.9375 kN/m

(b) B.M. $= \frac{wl_{eff}^2}{8} = 15.9375 \times \frac{4.2^2}{8} = 35.142$ kN-m

(c) Shear force V $= \frac{wl}{2} = \frac{15.9375 \times 4}{2} = 31.875$ kN

$M_{ulim} = 0.138 f_{ck} bd^2$ for Fe415 grade of steel

$M_{max} \leq M_{ulim}$ for under reinforced design

$$\Rightarrow \quad 31.875 \times 10^6 \text{ Nmm} \leq 0.138 \text{ for } bd^2$$

$$d \geq \sqrt{\frac{31.875 \times 10^6}{0.138 \times 20 \times 1000}}$$

$$d \geq 107.466 \text{ mm}$$

Depth from the strength criteria is quite less as compared to that obtained approx for deflection criteria using basic (l/d) ratio. We know that there would be some modification factor hence l/d ratio increase slightly.

Hence let as adopt overall depth to be 150 mm

$\Rightarrow \quad d = 150 - 20 - \frac{10}{2} = 125 \text{ mm}$

Let us revise the loading on slab for 1 m width of slab

$$\text{Load due to self wt} = 0.15 \times 1 \times 25 = 3.75 \text{ kN/m}$$
$$\text{Super imposed load} = 4 \times 1 = 4 \text{ kN/m}$$
$$\text{Surface finish} = 1 \times 1 = 1 \text{ kN/m}$$
$$\text{Total load} = 8.75 \text{ kN/m}$$
$$\text{Factored load} = 1.5 \times 8.75 = 13.125 \text{ kN/m}$$
$$M_{max} = \frac{13.125(4.2)^2}{8} = 28.94 \text{ kN/m}$$
$$V_{max} = \frac{13.125 \times 4}{2} = 26.25 \text{ kN}$$

4. Area of reinforcement

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$
$$0.36 f_{ck} x_u . b = 0.87 f_y A_{st}$$
$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \text{ put this in above equation}$$
$$M_u = 0.87 f_y A_{st} \left(d - \frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck} b}\right)$$
$$= 0.87 \times 415 A_{st} \left(125 - \frac{0.42 \times 0.87 \times f_y A_{st}}{0.36 \times 20 \times 1000}\right)$$
$$= 361.05 A_{st} (125 - 0.021 A_{st})$$
$$28.94 \times 10^6 = 45131.25 A_{st} - 7.60 A_{st}^2$$
$$\Rightarrow \quad 7.6 A_{st}^2 + 45131.25 A_{st} - 28.94 \times 10^6 = 0$$
$$\Rightarrow \quad A_{st} = 583.84 \text{ mm}^2$$
$$A_{st,min} = 0.12 \frac{bd}{100} = 0.12 \times \frac{1000 \times 150}{100} = 180 \text{ mm}^2$$
$$A_{st} > A_{st,min} \quad \text{O.K.}$$

Use 10 mm dia bar with a spacing = $\dfrac{\frac{\pi}{4} \times 10^2}{583.84} \times 1000 = 134.45$ mm

This spacing $\ngtr$ (i) 3d = 3 × 125 = 375 mm

(ii) 300 mm O.K.

Hence provide 10 mm dia @ 130 mm c/c.

So after curtailment at support of 50% bar, spacing will be less than 300mm c/c and it is safe.

$$\text{Distribution steel} = 0.12 \frac{bD}{100} = 0.12 \times \frac{1000 \times 150}{100} = 180 \text{ mm}^2$$

Use 6 mm dia bar as a distribution steel

$$\text{Spacing} = \frac{\frac{\pi}{4}\times 6^2}{180}\times 1000 = 157 \text{ mm}$$

Spacing $\ngtr$ (i) 5 d = 625 mm

(ii) 450

Provide 6 mm dia bar @ 150 mm c/c

Check for shear

$$\tau_v = \frac{V_u}{bd}$$

$$V_u = 26.25 \text{ kN (for shear force take clear span)}$$

$$\tau_v = \frac{V_u}{bd} = \frac{26.25\times 10^3}{1000\times 125} = 0.21 \text{ N/mm}^2$$

$$P_t = \frac{A_{st}}{bd}\times 100 = \frac{\left(\frac{1000}{260}\right)\times\frac{\pi}{4}\times 10^2}{1000\times 125}\times 100 = 0.242\%$$

(50% bar bent at 0.1 l distance from support, hence, at support spacing will increase to 260 mm c/c)

p_t	τ_c
0.15	0.28
0.25	0.36

for M20 concrete

$$\Rightarrow \quad \tau_c \text{ for } (p_t = 0.242) = 0.28 + \frac{0.08\times(0.242-0.15)}{0.10} = 0.3536$$

overall depth of slab	≤ 150 mm
K	1.30

$$K\tau_c = 1.3 \times 0.3536 = 0.46$$

$$\tau_v < K\tau_c$$

⇒ Slab is safe in shear without shear reinforcement

Check for development length

$$l_d \le 1.3\frac{M_1}{V} + l_0$$

$$l_d = \frac{0.87 f_y \phi}{4\tau_{bd}} = \frac{0.87\times 415\times 10}{4\times 1.6\times 1.2} = 470.117 \text{ mm}$$

τ_{bd} for M_{20} grade for plain bar = 1.2 N/mm²

and for deformed bar it is to be increased by 60%

$$M_1 = 0.87\, f_y A_{st}\,(d - 0.42\, x_u)$$

$$M_1 = 0.87\, f_y A_{st}\left(d - \frac{0.42\times 0.87 f_y A_{st}}{0.36 f_{ck}.b}\right)$$

$$= 0.87\times 415\times\frac{(1000)}{260}\times\frac{\pi\times 10^2}{4}\left(125 - \frac{0.42\times 0.87\times 415\times\left(\frac{1000}{260}\right)\times\frac{\pi}{4}\times 10^2}{0.36\times 20\times 1000}\right)$$

$$M_1 = 12.933 \text{ kNm}$$

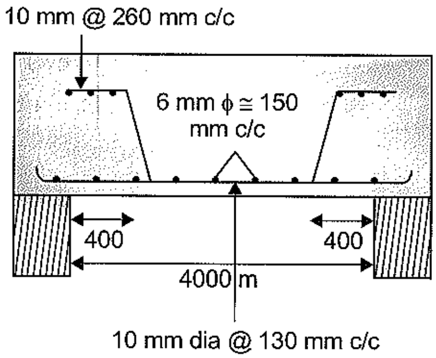
[50% bar bent at 0.1*l* distance from the support hence M_1 at support correspond to 10 mm bar @ 260 c/c]

$$V = 26.25 \text{ kN}$$

$$\frac{M_1}{V} = \frac{12.933 \text{ m}}{26.25} = 492.68 \text{ mm}$$

$$\Rightarrow \quad l_d < 1.3\frac{M_1}{V} + l_0$$

⇒ Safe in development length.



Check for deflection has not been carried out as the necessary data is not available. It can be done if codal data for modification factors are known.

Q-3: ***A simply supported 18 m effective span RCC rectangular beam of 500 mm × 1500 mm (overall depth) section is reinforced throughout with 21 nos. 25 mm diameter bars in three layers of 7 bars each at a clear cover of 37.5 mm on tensile face. The reinforcement on the compression face is 4 – 25 mm + 1 – 20 mm diameter bars in one layer at an effective cover of 50 mm. the clear cover between the different layers on tension face is 25 mm. M 25 grade concrete and Fe 415 grade steel bars are used in the beam throughout. The beam is laterally restrained throughout the span.***

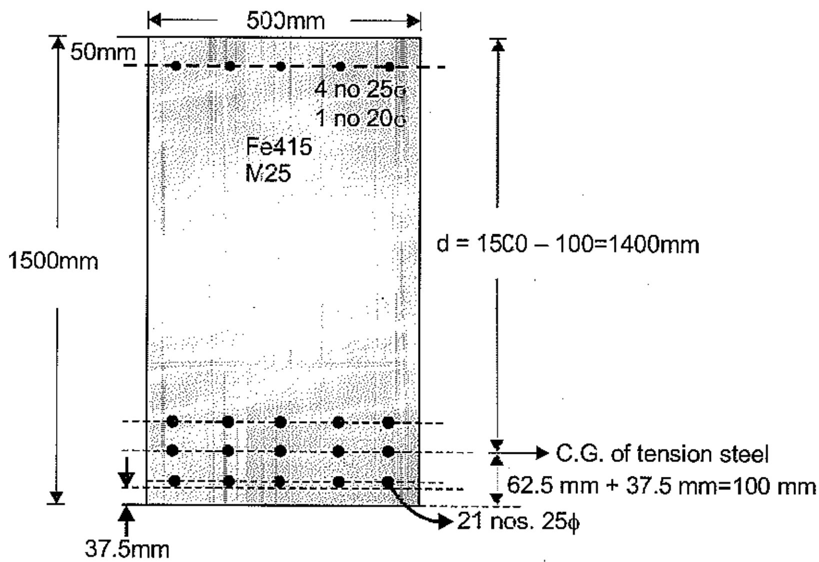
(a) ***Find out the maximum B.M. M_u at limit state of collapse for the beam section assuming the stress in compression reinforcement $\sigma_{sc} = 0.9566\ f_{y_d}$ corresponding to tensile stress f_{y_d} in tensile reinforcement at the limit state.***

(b) ***What shall be the superimposed uniformly distributed load w, the beam can carry at working conditions?***

(c) ***Design the shear reinforcement at support if design shear strength of concrete τ_c is given as follows for different values of p = 100 A_s/bd.***

(%)	*1.25*	*1.5*	*1.75*
τ_c (MPa)	*0.70*	*0.74*	*0.78*

Sol:



Step 1: Determination of limiting neutral axes,

$$x_{u_{lim}} = 0.48d = 0.48 \times 1400 = 672\text{mm}$$

Given data:

$$B = 500 \text{ mm}$$

$$d = 1400 \text{ mm}$$

$$A_{st} = \frac{\pi}{4} \times 25^2 \times 21 = 10308.375\text{mm}^2$$

$$A_{sc} = \frac{\pi}{4} \times 25^2 \times 4 + \frac{\pi}{4} \times 20^2 \times 1 = 2277.66 \text{ mm}^2$$

Stress in tension steel at limit state

$$f_{y_d} = 0.87 f_y$$

$$f_{sc} = 0.9566 f_{y_d} = 0.9566 \times 0.87 \times 415 = 345.38 \text{ N/mm}^2$$

Step 2: Actual neutral axis depth determination:

$$C_1 + C_2 = T$$

$$\Rightarrow \quad 0.36 \times f_{ck} \times B \times x_u + f_{sc} \times A_{sc} - 0.45 \times f_{ck} \times A_{sc} = 0.87 \times f_y \times A_{st}$$

$$\Rightarrow \quad 0.36 \times 25 \times 500 \times x_u + 345.38 \times 2277.66 - 0.45 \times 25 \times 2277.66 = 0.87 \times 415 \times 10308.375$$

$$x_u = 657.96$$

Since $x_u\big|_{actual} < x_u\big|_{Limitting}$

Hence the beam is under reinforced.

(a) M.O.R. calculation

$$\therefore \quad \text{M.O.R.} = 0.36 f_{ck} \times B \times x_u \times (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$$

$$= 0.36 \times 25 \times 500 \times 657.96 \times (1400 - 0.42 \times 657.96)$$

$$+ (345.38 - 0.45 \times 25) \times 2277.66 \times (1400 - 50)$$

$$\Rightarrow \quad \text{M.O.R} = 4354.342 \text{ kN-m}$$

(b) $\text{Mu, lim} = \dfrac{W_u \ell^2}{8}$

$$\therefore \quad \frac{W_u \ell^2}{8} = 4354.342$$

$$\Rightarrow \quad \frac{W_u \times 18^2}{8} = 4354.342 \text{ kN-m}$$

$$\Rightarrow \quad W_u = 107.51 \text{ kN/m}$$

$\therefore$ Load under working condition $= W_{total} = \dfrac{W_u}{1.5} = \dfrac{107.51}{1.5} = 71.676$ kN/m

$\because \quad W_{total} = W_{self\ wt\ of\ beam} + W_{superimposed}$ UDL

$\therefore \quad W_{superim\ posed}$ UDL $= 71.676 - 0.5 \times 1.5 \times 1 \times 25 = 52.926$ kN/m

(c) $W_u = 107.51$ kN/m

$$V_u = \frac{W_u \ell}{2} = \frac{107.51 \times 18}{2} = 967.59 \text{ kN}$$

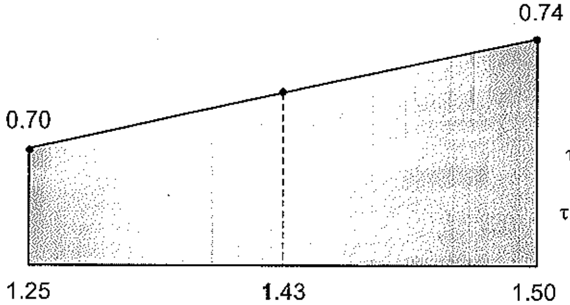
$\therefore$ Nominal shear stress $\tau_V = \dfrac{Vu}{Bd} = \dfrac{967.59 \times 1000}{500 \times 1400} = 1.383 \text{ N/mm}^2$

and $\% p = 100 \times \dfrac{Ast}{Bd} = 100 \times \dfrac{10308.375}{500 \times 1400} = 1.47\%$

For $p = 1.25\%$ we have $\tau_c = 0.70 \text{N/mm}^2$

$p = 1.50\%$ we have $t_c = 0.74 \text{N/mm}^2$

$p = 1.47\%$ $\tau_c = ?$



$$\tau_c = 0.70 + \frac{0.74 - 0.70}{150 - 1.25} \times (1.47 - 1.25)$$

$$\tau_c = 0.7352 \text{ N/mm}^2$$

$\therefore$ $V_c = \tau_c Bd = 0.7352 \times 500 \times 1400 = 514.640$ kN

$\because$ $\tau_v > \tau_c$ Hence shear reinforcement is required

$V_s = V_u - V_c = 967.59 - 514.640 = 452.95$ kN

According to IS 456 : 2000

$$S_V = \frac{0.87 f_y A_{sv} d}{V_s}$$

Adopting 4 legged 8 mm ϕ bars for vertical stirups

$$A_{SV} = 4 \times \pi \times 4^2 = 201.062 \text{mm}^2$$

$$\therefore \quad S_V = \frac{0.87 \times 415 \times 1400 \times 201.062}{452.95} = 224.37 \text{ mm}$$

Providing 210 mm spacing. (at supports)

Check for shear reinforcfement

for vertical strrirups $S_V \not> 0.75\text{ d} = 0.75 \times 1400 = 1050$ mm

$\not> 300$ mm

Hence shear reinforcement of 4 egged 8mm ϕ @ 210mm c/c is OK.

Q-4: ***A reinforced concrete beam, having a simply supported span of 6m, carries a dead load of 15 kN/m (incl. its dead load) and an imposed load of 20 kN/m at service. Design the cross-section of the beam at its mid-span only for flexure and shear at the limit state of collpase. Assume moderate exposure condition and grade of steel as Fe415. Draw a neat sketch showing the reinforcement details.***

[15 Marks, ESE-2014]

Sol:

Span (L) = 6m

Dead load (w_D) = 15 kN/m

Imposed load (w_L) = 20 kN/m

$\therefore$ Total load = 15 + 20 = 35 kN/m

Factored load (w_f) = $1.5 \times 35 = 52.5$ kN/m

We know that, bending moment at mid-span in simply supported beam is $\frac{wL^2}{8}$

$$\therefore \qquad M_u = \frac{w_f L^2}{8}$$

$$M_u = \frac{52.5 \times 6^2}{8} = 236.25 \text{ kN-m} \qquad \ldots(i)$$

We have to design beam for moderate condition therefore grade of concrete should not be less than M25. Let us use M25 and Fe415.

for Fe415, $\quad M_u = 0.138 f_{ck} b d^2$

Assuming, $\quad d = 2b$, we get

$$M_u = 0.138 \times 25 \times b \times (2b)^2$$

$$\Rightarrow \qquad 236.25 \times 10^6 = 0.138 \times 25 \times 4b^3$$

$$\therefore \qquad b^3 = 17.119,565 \Rightarrow b = 257.73 \text{ mm.}$$

Let us adopt $\quad b = 260$ mm

$$\therefore \qquad d = 2 \times 260 = 520 \text{ mm}$$

Overall depth of beam, $\quad D = d + \text{cover}$

Let us provide cover of 30 mm

$$\therefore \qquad D = 520 + 30 = 550 \text{ mm}$$

$\therefore$ Dimensions of beam will be 550 × 260

Area of steel:

$$0.87 F_y A_{st}(d - 0.42 x_u) = M_u$$

$$\Rightarrow \quad 0.37 \times 415 \times A_{st}(520 - 0.42 \times 0.48 \times 520) = 236.25 \times 10^6 \qquad [\because x_u = 0.48d \text{ for Fe415}]$$

$$\therefore \quad A_{st} = 1576 \text{ mm}^2$$

Let us use 16 mm ϕ base

$$\text{Area of 1 bar} = \frac{\pi}{4} \times 16^2 = 64\pi$$

$$\therefore \qquad \text{No. of bars} = \frac{1576}{64\pi} = 7.83 \simeq 8 \text{ nos}$$

Let us provide $8 - 16\,\phi\,\text{Fe415}$ bars.

Design for shear:

Shear force at mid-span is zero therefore only minimum shear reinforcement is provided.

Let us adopt 2-legged 8 mm ϕ stirrups

$$\therefore \qquad A_{sv} = 2 \times \frac{\pi}{4} \times 8^2,$$

where A_{sv} = total cross-sectional area of stirrup legs effective in shear.

$$\Rightarrow \qquad A_{sv} = 32\pi$$

We know, minimum shear reinforcement is

$$\frac{A_{sv}}{b.s_v} \geq \frac{0.4}{0.87 F_y},$$

where s_v = stirrup spacing along the length of member

$$\therefore \qquad S_v \le \frac{32\pi \times 0.87 \times 415}{0.4 \times 260}$$

$$\Rightarrow \qquad S_v \le 349 \text{ mm} \qquad \ldots\text{(ii)}$$

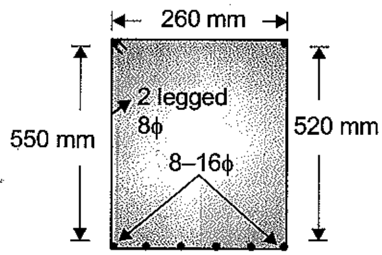
As per 15456 : 2000

$$S_v \le \begin{matrix} 300 \text{ mm} \\ 0.75\text{ d} \end{matrix}$$

$$\therefore \qquad S_v \le 300 \text{ mm} \qquad \ldots\text{(iii)}$$

$$0.75 \times 520 = 390 \text{ mm}$$

From (ii) and (iii), minimum spacing of shear reinforcement is 300 mm. Therefore, provide 2 legged 8 mm ϕ stirrups at C–C spacing of 300 mm



Q-5: ***Design a simply supported roof slab (bending and shear only) for a room 3.5m×8m clear in size if the superimposed dead and live load is 5 kN/m². Use M 25 mix and Fe 415 grade steel. Show reinforcement details. Shear strength of concrete = 0.35 MPa.***

[12 Marks, ESE-2017]

Sol: Given that:

Clear span 3.5 m × 8 m

So using Deflection Condition:

$$\frac{l}{d} = \frac{3500}{23} = 152.17 \text{ m} \qquad \left[\text{Assuming } \frac{l}{d} = 23 \text{ i.e., greater than 20 (taking in effect of } k_1)\right]$$

So Adopting over all depth = 200 mm

So Effective depth = 200 – 20 – 8 = 172 mm

(Assuming 8 mm of bar and 20 mm clear cover)

Load Calculation:

$$\text{D.L.} = 25 \times 0.2 = 5 \text{ KN/m}^2$$

$$\text{LL} = 5 \text{ KN/m}^2 \text{ (given)}$$

So $$\text{Total load} = 5 + 5 = 10 \text{ KN/m}^2$$

So $$\text{Factored load} = 10 \times 1.5 = 15 \text{ KN/m}^2$$

Effective Depth:

$$l_{eff} = l_0 + d = l_0 + w$$

Let the thickness of bearing wall be = 230 mm

So, $(l_{eff})_x$ min of $\begin{pmatrix} 3.5 + 0.172 \\ 3.5 + 0.23 \end{pmatrix}$ m = 3.672 m

$$(l_{eff})_y = \min(8 + 0.172,\ 8 + 0.23) = 8.172 \text{ m}$$

So, $$(l_{effective})_x = 3.672 \text{ m}$$

$$\frac{(l_{eff})_y}{(l_{eff})_x} = \frac{8.172}{3.672} = 2.2254 \Rightarrow \text{one way slab.} \qquad \left(\because \frac{ly}{lx} > 2\right)$$

So, $$\text{B.M.} = \frac{15 \times 3.672^2}{8} \text{ (slab spanning in shorter direction)}$$

$$= 25.28 \text{ KNm/m (per m width)}$$

Check for Depth:

$$M_u = 0.138\, f_{ck} b d^2 \qquad \text{(for Fe 415 steel)}$$

$$\Rightarrow \quad 25.28 \times 10^6 = 0.138 \times 25 \times 1000 \times d^2\ (f_{ck} = 25 \text{ N/mm}^2)$$

$$d = 85.61 \text{ mm} << 172 \text{ mm (o.k)}$$

So, Calculation of Reinforcement:

$$M = 0.87\, f_y A_{st}\left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$$

$$\Rightarrow \quad 25.28 \times 10^6 = 0.87 \times 415 A_{st}\left(172 - \frac{415 \times A_{st}}{25 \times 1000}\right)$$

$$\Rightarrow \quad A_{st} = 424.47 \text{ mm}^2\text{/m}$$

$$(A_{st})_{max} = 0.04 \times bD = 0.04 \times 1000 \times 200 = 8000 \text{ mm}^2 > 424.47 \text{ mm}^2$$

$$\text{min. steel} = 0.12\% \text{ of } bD = \frac{0.12}{100} \times 1000 \times 200 = 240 \text{ mm}^2 < 424.47 \text{ mm}^2 \ \therefore \text{ OK}$$

Using 8 mm ($Ast_{8mm} = 50\text{mm}^2$)

$$\text{Spacing} = \frac{1000 \times 50}{424.47} = 117.79 \text{ mm}$$

So adopting 8 mm of bar @ 100 mm c/c

Here spacing is less than < 300 mm

$$< 3d_1 \text{ i.e, } 3 \times 172 = 516 \text{ mm OK}$$

For Distribution Bar:

$$A_{st} = 0.0012 \times 1000 \times 200 \quad (\text{i.e., } \phi - 12\% \text{ of bD})$$

$$= 240 \text{ mm}^2$$

So $$\text{Spacing} = \frac{1000 \times 50}{240} = 208.33 \text{ mm c/c}$$

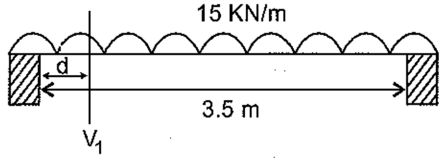
So using 8mm of bar @ 200 mm c/c

Here, spacing < 300 mm. OK

$$5d = 5 \times 172 = 860 \text{ mm (o.k)}$$

Check for Shear:

Given $\tau_c = 0.35$ MPa



(Critical Section)

$$V_1 = 15 \times \frac{(3.5 - 2 \times 0.172)}{2} \quad \text{(Shear at critical section)}$$

$$= 23.67 \text{ kN}$$

So $$\tau_V = \frac{23.67 \times 1000}{1000 \times 127} = 0.14 \text{ N/mm}^2$$

For slab depth D = 200 mm, k = 1.2

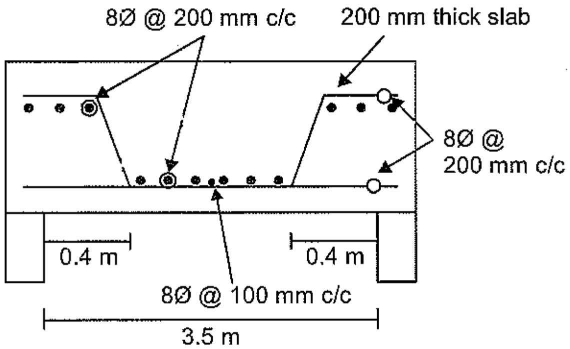
So $$K\tau_C = 1.2 \times 0.35 = 0.42 \text{ N/mm}^2$$

$$\tau_V < K\tau_C \quad \text{(Safe)}$$

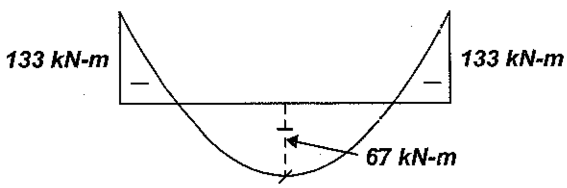
$$\tau_{c\,max} = 3.1 \text{ N/mm}^2 \quad \text{(For M25 grade)}$$

$$\tau_V < 0.5 \times \tau_{c\,max}\ (1.55 \text{ N/mm}^2) \quad \text{Safe}$$

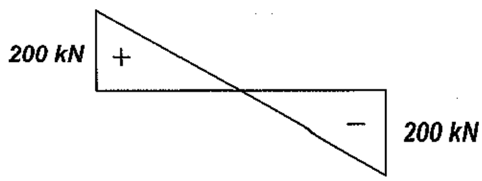
So, no need of shear reinforcement.



Q-6: ***Design the reinforcement at critical section only for a beam for which Bending moment and shear force diagram has been shown in figure.***



Service State Bending Moment Diagram



Service State Shear Force Diagram

Following parameter may be used for design:

1. ***M-20 grade concrete***
2. ***Fe-415 grade steel***
3. ***Nominal concrete cover : 30 mm***

4. *(Depth/Width) ratio of beam : (02)*
5. *Diameter of reinforcing bar : 20 mm for flexure reinforcement and 8 mm for shear reinforcement*
6. *Shear strength of concrete = 0.6 N/mm²*
7. *$(Mu, lim/f_{ck}\, bd^2) = 0.138$*
8. *$(pt, lim\ f_y/f_{ck}) = 19.82$*

[20 Marks, ESE-2018]

Sol: At critical section (at supports)

$$\text{Design bending moment, } M_u = 1.5 \times 133 = 199.5 \text{ kN-m}$$

$$\text{Design shear force, } V_u = 1.5 \times 200 = 300 \text{ kN}$$

Let the beam size be (b × D)

$$\text{Effective depth, d} = D - 30 - 8 - \frac{20}{2} = (D - 48) \text{ mm}$$

Determination of size of beam

$$M_{u,\ lim} \geq M_u$$

$$0.138 f_{ck} bd^2 \geq 199.5 \times 10^6$$

Given: $\frac{D}{b} = 2$

$$\therefore \quad 0.138 \times 20 \times \frac{D}{2} \times (D - 48)^2 \geq 199.5 \times 10^6$$

$$D(D^2 + 2304 - 96D) \geq 144565217.4$$

$$D^3 - 96D^2 + 2304D - 144565217.4 \geq 0$$

$$D \geq 557.31 \text{ mm}$$

Taking, $D = 600$ mm, $b = 300$ mm

$$\text{Effective depth d} = 600 - 48 = 552 \text{ mm}$$

Design of reinforcement for bending moment at support

$$A_{st} = \frac{f_{ck}bd}{2f_y}\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}bd^2}}\right]$$

[f_{ck} = 20 MPa, f_y = 415 N/mm², b = 300 mm, d = 552 mm]

$$A_{st} = \frac{20\times300\times552}{2\times415}\left[1-\sqrt{1-\frac{4.6\times199.5\times10^6}{20\times300\times552^2}}\right]$$

$$= 1174.3 \text{ mm}^2$$

$$\text{No. of bars of 20 mm diameter} = \frac{1174.3}{\frac{\pi}{4}\times20^2} = 3.73 \simeq 4$$

$$A_{st,\ provided} = 4\times\frac{\pi}{4}\times20^2 = 1256.6 \text{ mm}^2$$

$$= \frac{1256.6\times100}{300\times552}\times\frac{415}{20}$$

$$= 15.74 < 19.82 \quad \text{OK}$$

Since the moment is hogging at supports hence reinforcement will be provided at top.

Design of reinforcement for shear force:

$$\text{Nominal shear stress, } \tau_v = \frac{V_u}{bd} = \frac{300\times10^3}{300\times552} = 1.81\,\text{MPa} < \tau_{c,max}\,(2.8\text{MPa})$$

$\Rightarrow$ Safe in diagonal compression failure

$$\text{Shear strength of concrete, } \tau_c = 0.6 \text{ MPa (Given)}$$

Since, $\tau_v > \tau_c$ hence beam is not safe in shear

$$\begin{aligned}\text{Design shear force for stirrups, } V_{us} &= V_u - \tau_c bd \\ &= 300\times10^3 - 0.6\times300\times552 \\ &= 200640 \text{ N}\end{aligned}$$

For 2 legged 8 mm stirrups, $A_{sv} = 2\times\frac{\pi}{4}\times8^2 = 100.53 \text{ mm}^2$

Spacing:

(i)
$$V_{us} = 0.87 f_y A_{sv}\cdot\frac{d}{S_v}$$

$$200640 = 0.87\times415\times100.53\times\frac{552}{S_v}$$

$$S_v = 99.85 \text{ mm}$$

(ii) For minimum shear reinforcement

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87 f_y}$$

$$\frac{100.53}{300S_v} \geq \frac{0.4}{0.87\times415}$$

$$S_v \leq 302.5 \text{ mm}$$

(iii)
$$\begin{aligned}\text{Minimum spacing} &= 0.75\,d \text{ or } 300 \text{ mm whichever lesser} \\ &= 300 \text{ mm}\end{aligned}$$

Hence provide 2 legged 8ϕ stirrups at 90 mm c/c

Design of reinforcement for bending moment at mid-span

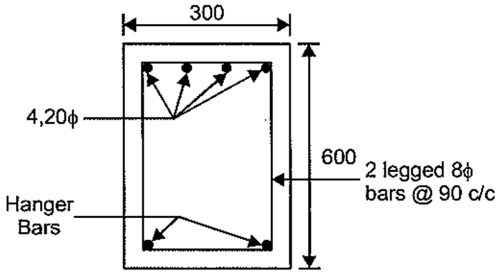
$$M_u = 1.5\times67 = 100.5 \text{ kN-m}$$

$$\therefore \quad A_{st} = \frac{20\times300\times552}{2\times415}\left[1-\sqrt{1-\frac{4.6\times100.5\times10^6}{20\times300\times552^2}}\right]$$

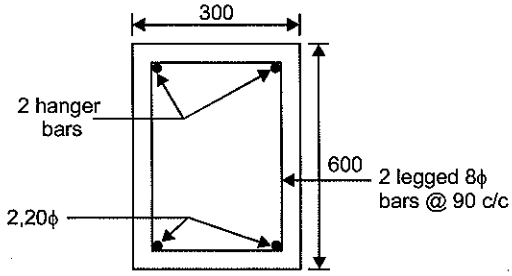
$$= 541.2 \text{ mm}^2$$

$$\text{No. of bars required} = \frac{541.2}{\frac{\pi}{4}\times20^2} = 1.72 \simeq 2$$

This reinforcement will be provided at bottom (sagging moment)

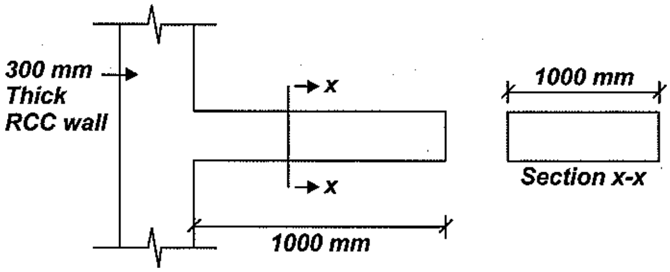


Section near support



Section near mid span

Q-7: ***Design a cantilever slab shown in figure for flexure only.***



Sketch the reinforcement also. Following parameters may be used for design, applying different checks and detailing the reinforcement :

1. ***Span to effective depth : 10 (maximum)***
2. ***Mild exposure condition : Nominal concrete cover 20.0 mm***
3. ***2.0 hours of fire resistance : Nominal concrete cover 25.00 mm***
4. ***Maximum live load : 30 kN/m²***
5. ***Load combination : 1.5 × Dead load + 1.5 × live load***
6. ***Effective length : length to the face of support plus half the effective depth***
7. ***Grade of concrete M-20***
8. ***Grade of reinforcing steel Fe-415***
9. ***Unit weight of RCC : 25 kN/m³***
10. ***Development length in tension : 48 × diameter of reinforcing bar***
11. ***Development length in compression : 37 × diameter of bar***
12. ***Minimum reinforcement : 0.12% of total cross-sectional area***
13. ***Maximum spacing of main reinforcement : 3 × effective depth***
14. ***Maximum spacing of distribution reinforcement : 5 × effective depth***
15. ***Diameter of main reinforcing bar : 10 mm***
16. ***($M_{u,lim}/f_{ck}bd^2$) : 0.138***
17. ***($p_{t,lim}\, f_y/f_{ck}$) = 19.82***

[20 Marks, ESE-2018]

Sol: Given: Span to effective depth = 10 (maximum)

$$\therefore \quad d \geq \frac{l}{10}$$

$$d \geq \frac{1000}{10}$$

$$d \geq 100 \text{ mm}$$

Providing $d = 120$ mm

$$D = d + \text{Nominal cover} + \frac{\phi}{2}$$

Nominal cover = max[20 (given mild exposure), 25 (fire resistance of 2 hours)]

$$D = 120 + 25 + \frac{10}{2}$$

$$D = 150 \text{ mm}$$

$$\text{Dead load, } W_d = 1 \times 0.15 \times 25 \; [\text{Given, } \gamma_{concrete} = 25 \text{ kN/m}^3]$$

$$W_d = 3.75 \text{ kN/m}$$

$$\text{Live load, } W_l = 1 \times 30 = 30 \text{ kN/m}$$

$$\text{Design load, } W_u = 1.5(W_d + W_l)$$

Considering the maximum live load given is service load.

$$W_u = 1.5(3.75 + 30)$$

$$W_u = 50.625 \text{ kN/m}$$

$$\text{Design BM, } M_u = \frac{W_u l_{eff}^2}{2}$$

$$\text{Effective span, } l_{eff} = 1000 + \frac{120}{2}$$

$$\therefore \quad l_{eff} = 1060 \text{ mm} = 1.06 \text{ m}$$

$$M_u = 50.625 \times \frac{1.06^2}{2}$$

$$M_u = 28.44 \text{ kN-m}$$

$$M_{u,\,lim} = 0.138 f_{ck} b d^2 = 0.138 \times 20 \times 1000 \times 120^2$$

$$M_{u,\,lim} = 39.74 \text{ kNm}$$

$$M_u < M_{lim} \quad (\therefore \text{ under reinforced beam})$$

Reinforcement for flexure

$$A_{st,\,required} = \frac{0.5 f_{ck} bd}{f_y} \times \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} bd^2}}\right]$$

$$A_{st,\,required} = 0.5 \times \frac{20}{415} \times 1000 \times 120 \times \left[1 - \sqrt{1 - \frac{4.6 \times 28.44 \times 10^6}{20 \times 1000 \times 120^2}}\right]$$

$$A_{st,\,required} = 755.42 \text{ mm}^2$$

$$\text{Spacing} = \frac{1000}{\frac{755.42}{\frac{\pi}{4} \times 10^2}} = 103.9 \text{ mm} < 3d < 3 \times 120$$

$$= 360 < 300 \text{ mm (Hence OK)}$$

Provide 10 mm dia bars @ 100 mm/cc

$$P_{t,\text{provided}} = \left(\frac{1000\times\frac{\pi}{4}\times 10^2}{100\times 1000\times 120}\right)\times 100 = 0.654$$

$$P_{t,\lim} = \frac{19.82\times f_{ck}}{f_y}$$

$$= \frac{19.82\times 20}{415} = 0.955 > P_{t\text{ provided}} \text{ (Hence OK)}$$

Distribution reinforcement:

For Fe415 $$A_{st\ min} = \frac{0.12\times bD}{100}$$

$$= \frac{0.12\times 1000\times 150}{100} = 180\ mm^2\ (A_{t\ min} = 0.12\%\ BD)$$

Assuming 8 mm dia bar

$$\text{Spacing} = \frac{1000}{\dfrac{180}{\frac{\pi}{4}\times 8^2}} = 279.11\ mm < 5\times 120 = 5d = 600\ mm$$

$$< 300\ mm$$

Provide 8 mm dia bars @ 270 mm c/c for distribution reinforcement.

Check for development length.

$$L_d = 48\phi = 48\times 10 = 480\text{mm}[\text{For tension}]$$

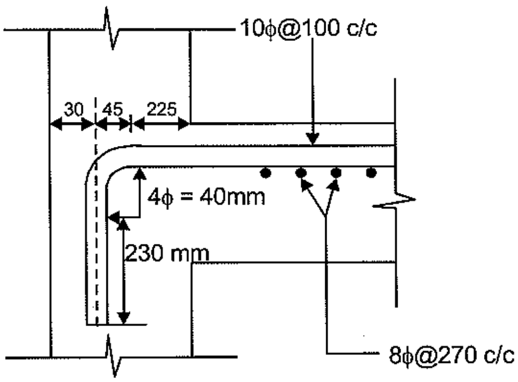
$$L_d = 37\phi = 37\times 10 = 370\text{mm}[\text{For compression}]$$

Taking a clear cover of 25 mm, the length available,

$$= 300 - \left(25 + \frac{\phi}{2}\right)$$

$$= 270\ mm < 480\ mm$$

To provide development length, make a standard 90° bend and provide an extension of 230 mm beyond the end of curve.



∴ Total development length provided on left side of support

$$= 225 + 8\phi + 230 - 4\phi$$

$$= 495\ mm > 480\ mm\ \text{OK}$$

Q-8: ***Design a two way slab for an office room 5.8 m × 4.2 m clear in size if the superimposed load*** is 4 kN/m². ***Use M 25 grade of concrete and steel grade Fe415. The bending moment coefficients for two-way slabs simply supported on four sides is given below :***

l_y/l_x	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
α_x	0.062	0.074	0.084	0.093	0.099	1.104	0.113	0.118
α_y	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029

Assume the edges simply supported and the corners not held down. Assume the shape factor for shear k = 1.3.

Design shear strength of concrete of M 25 grade.

$100\,A_{st}/bd$	τ_c N/mm²
0.25	0.36
0.50	0.49
0.75	0.57
1.00	0.64

[20 Marks, ESE-2019]

Sol: **Given:**

$$l_y = 5.8 \text{ m}$$

$$l_x = 4.2 \text{ m}$$

$$w = 4 \text{ KN/m}^2$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$K = 1.3$$

Assume overall thickness of slab = 180 mm

Effective depth of slab = 160 mm

Effective span will be, $l_x = 4.2 + 0.09 = 4.29$ m

$$l_y = 5.8 + 0.09 = 5.89 \text{ m}$$

$$\frac{l_y}{l_x} = \frac{5.89}{4.29} = 1.373$$

$$\alpha_x = 0.093 + \frac{0.099 - 0.093}{1.4 - 1.3} \times (1.373 - 1.3)$$

$$\alpha_x = 0.0974$$

and,

$$\alpha_y = 0.055 - \frac{0.055 - 0.051}{1.4 - 1.3} \times (1.4 - 1.373)$$

$$\alpha_y = 0.0539$$

Load calculations:

Self weight of slab = $0.18 \times 25 = 4.5 \text{kN/m}^2$

Imposed Load = 4kN/m

$$\text{Factored Total Load} = 1.5\times(4.5+4) = 12.75\ \text{kN/m}^2$$

$$M_x = \alpha_x w l_x^2 = 0.0974\times12.75\times4.29^2 = 22.86\,\text{kN-m}$$

$$M_y = \alpha_y w l_x^2 = 0.0539\times12.75\times4.9^2 = 16.5\,\text{kN-m}$$

Now effective depth of slab (d) is given by

$$d \geq \sqrt{\frac{22.86\times10^6}{0.138\times25\times1000}}$$

$$d \geq 81.4\ \text{mm}$$

Hence, our assumption is correct.

Calculation of steel reinforcements :

Along shorter span, (A_{stx})

$$M_x = 0.87 f_y A_{Stx}\left(d-\frac{f_y A_{Stx}}{f_{Ck} b}\right)$$

$$22.86\times10^6 = 0.87\times415\times A_{Stx}\left(160-\frac{415\times A_{Stx}}{25\times1000}\right)$$

$$\Rightarrow \quad A_{Stx} = 413.46\,\text{mm}^2$$

$$\text{Spacing of 8 mm}\ \phi\ \text{of reinforcements} = \frac{1000 A_\phi}{A_{Stx}}$$

$$= \frac{1000\times\frac{\pi}{4}\times8^2}{413.46} = 121.57\ \text{mm}$$

$$\not> 3d$$

$$\not> 300\ \text{mm}$$

Along the longer span,

$$M_y = 0.87 f_y A_{Sty}\left(d-\frac{f_y A_{Sty}}{f_{Ck}\, b}\right)$$

$$16.5\times10^6 = 0.87\times415\times A_{sty}\left(160-\frac{415\times A_{sty}}{25\times1000}\right)$$

$$\Rightarrow \quad A_{sty} = 294.63\ \text{mm}^2$$

$$\text{Spacing of 8 mm}\ \phi\ \text{reinforcements is} = \frac{1000 A_\phi}{A_{Sty}} = \frac{1000\times\frac{\pi}{4}\times8^2}{294.63}$$

$$\Rightarrow \quad S_V = 170.6\ \text{mm}$$

$$\not> 3d$$

$$\not> 300$$

Hence, provide 8 mm ϕ bars @ 120 mm c/c along shorter direction and provide 8 mm ϕ bars @ 160 mm c/c along the longer direction.

Design For Shear:

Maximum shear Force can be taken as $\frac{\alpha l_x}{2}$ in either direction, where l_x = clear short span.

$$\text{Maximum shear force} = \frac{1}{2} w l_x = \frac{1}{2} \times 12.75 \times 4.29 = 27.35 \text{ kN/m}$$

$$\text{Nominal shear stress, } \tau_V = \frac{27.35 \times 10^3}{1000 \times 160} = 0.171 \text{ N/mm}^2$$

$$\text{Percentage tension steel, } p_t = \frac{100 \times 50 \times \frac{1000}{160}}{1000 \times 160} = 0.195\%$$

We know that for M20 and $p_t \le 0.15\%$, $\tau_c = 0.29 \text{ N/mm}^2$

$$\tau_C = 0.29 + \frac{0.36 - 0.29}{0.25 - 0.15} \times (0.195 - 0.15)$$

$$\Rightarrow \quad \tau_C = 0.3215 \text{ N/mm}^2$$

$$\text{Shear strength in slabs, } \tau_C' = K\tau_C$$

$$= 1.3 \times 0.3215$$

$$= 0.418 \text{ N/mm}^2 > \tau_V$$

Hence, safe in shear.

Q-9: ***A simply supported reinforced concrete beam of size 300 mm × 500 mm is reinforced with 5 Nos. 16 ϕ bars as tension reinforcement. Two bars are curtailed at quarter span from both ends. Find out the load carrying capacity (UDL) of the beam having effective space of 6m. Also design the beam against shear force. Use M 25 and Fe 415. Nominal cover = 30 mm. Use limit state method of design. Show the reinforcement detail (cross-section) also. Use 2 Nos. 12 ϕ bars as hanger bars.***

$\frac{M_u}{bd^2}$	2	2.5	2.75	3	3.25	3.5
p_t	0.51	0.61	0.74	0.83	0.91	–

p_t	0.25	0.5	0.75	1.0	1.25
τ_c, MPa	0.36	0.49	0.57	0.64	0.7

[20 Marks, ESE-2020]

Sol:

$$l = 6 \text{ m}$$

$$b = 300 \text{ mm}$$

$$D = 500 \text{ mm}$$

$$\text{Nominal Cover} = 30 \text{ mm}$$

$$\text{Take effective cover} = 50 \text{ mm} \qquad \left\{30 + \frac{16}{2} + 12\right\}$$

$$d = 500 - 50 = 450 \text{ mm}$$

$$A_{st} = \frac{\pi}{4}\times 16\times 16\times 5 = 1005.30\,\text{mm}^2$$

$$A'_{st} = \text{Area of steel at curtailed point}$$

$$= \frac{\pi}{4}\times 16\times 16\times 3 = 603.185\,\text{mm}^2$$

$$\text{B.M. at mid span} = \frac{w_u l^2}{8}$$

$$\text{B.M. at curtailed section} = \frac{w_u l}{2}\times\frac{l}{d} - \frac{w_u l}{4}\times\frac{l}{8}$$

$$= \frac{3w_u l^2}{32}$$

Depth of N.A.

$$0.36 f_{ck} b.x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87\times f_y\times A_{st}}{0.36\times f_{ck}\times b}$$

$$= \frac{0.87\times 415\times 1005.30}{0.36\times 25\times 300}$$

$$= 134.43 \text{ mm}$$

$$x_{u_{lim}} = 0.40\times d$$

$$= 0.48\times 450 = 216 \text{ mm}$$

$$x_u < x_{u_{lim}}$$

So, under reinforced section.

Now MoR of beam

$$M = 0.36 f_{ck}\times b.x_u(d - 0.42x)$$

$$= 0.36\times 25\times 300\times 134.43(450 - 0.42\times 134.43)$$

$$= 142841834.6$$

$$= 142.84 \text{ kN-m}$$

$$\text{B.M. due to external load} = \text{MoR}$$

$$\frac{w_u l^2}{8} = 142.84$$

$$w_u = 31.74 \text{ kN/m}$$

$$\text{Service Load } w = 21.16 \text{ kN/m}$$

At curtailed section :

Deputy of N.A.

$$0.36 f_{ck} b.x_u = 0.87 f_y A'_{st}$$

$$x = \frac{0.87\times 415\times\frac{\pi}{4}\times 16^2\times 3}{0.36\times 25\times 300}$$

$$= 80.65 \text{ mm}$$

$$x < x_{u_{lim}}$$

$$MOR = 0.36 f_{ck} b.x_u (d - 0.42x_u)$$

$$= 0.36 \times 25 \times 300 \times 80.65(450 - 0.42 \times 80.65)$$

$$= 90.61 \text{ kN-m}$$

$$\frac{3w_u l^2}{32} = 90.61$$

$$w_u = 26.85 \text{ kN/m}$$

$$\text{Service load} = \frac{26.85}{1.5} = 17.90 \text{ kN-m}$$

So, the maximum load carrying capacity = 17.90 kN/m

Design the beam against shear force :

$$\text{Maximum factored shear force} = \frac{w_u l}{2}$$

$$= \frac{17.90 \times 1.5 \times 6}{2}$$

$$= 80.55 \text{ kN}$$

Nominal shear stress

$$\tau_v = \frac{80.55 \times 10^3}{300 \times 450}$$

$$= 0.6 \text{ N/mm}^2$$

For M25 Grade

$$\tau_{cmax} = 3.1$$

$\tau_V < \tau_{Cmax}$ [O.K.]

Now,

$$p_t = \frac{1005.3 \times 100}{300 \times 450} = 0.75\%$$

Now, τ_c from the given table corresponding to p_t.

$$\tau_c = 0.57 \text{ N/mm}^2$$

$\tau_v < \tau_c$ so design for minimum shear reinforcement.

$$0.87 f_y A_{sv} \frac{d}{S_v} = 0.4 \text{ bd}$$

$$\frac{0.87 \times 415 \times \frac{\pi}{4} \times 12^2 \times 2}{S_v} = 0.4 \times 300$$

$$S_v = 680.65 \text{ mm}$$

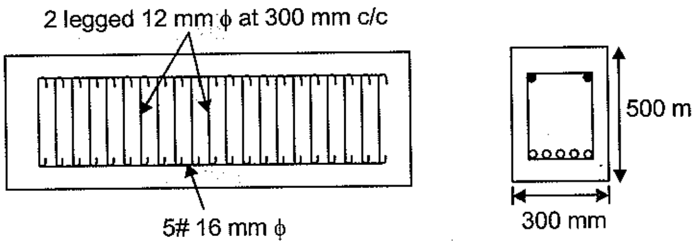
Take

$$S_v = \text{Min}\begin{cases} 0.75d \\ 300 \text{mm} \\ S_v \end{cases}$$

$$= \text{Min}\begin{cases} 337.5\text{m} \\ 300\text{mm} \\ 680.65\text{m} \end{cases}$$

$$S_v = 300 \text{ mm}$$

Provide a 2-legged 12 mm base at spacing 300 mm c/c



CHAPTER 6

LINTEL

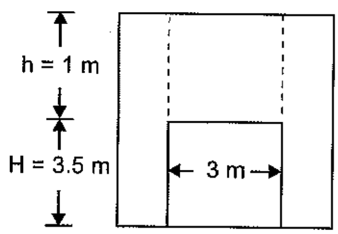
Q-1: ***The opening of a masonry building is 3 m and 3.5 m high. The ceiling of the root is 4.5 m above the floor. The space between top of lintel and bottom of roof is filled with brick masonry. The roof transmits a total load of 25 kN/m run to the lintel. Design the lintel supported on brick walls of width 300 mm. Use M 20 grade concrete and steel grade of Fe 415. Assume the unit weight of the brick masonry is 20 kN/m³ and that of concrete is 25 kN/m³. The design shear strength of concrete is given in table.***

$\frac{100\,A_s}{bd}$	$\frac{\tau_c\ N/mm^2}{M\ 20}$
0.15	0.28
0.25	0.36
0.50	0.48
0.75	0.56
1.0	0.62
1.25	0.67

The design bond stress for MS bars is given by $\tau_{bd} = 1.2\,N/mm^2$ for M 20 grade of concrete.

[20 Marks, ESE-2019]

Sol:



$$l_0 = 3\text{ m}$$

$$h = 1\text{ m}$$

$$t_w = 0.3\text{ m}$$

$$\text{Load from roof to lintel} = 25\text{ kN/m}$$

$$f_{ck} = 20\text{ N/mm}^2$$

$$f_y = 415\text{ N/mm}^2$$

$$\gamma_{wall} = 20\text{ kN/m}^3$$

$$\gamma_c = 25\text{ kN/m}^3$$

$$\tau_{bd} = 1.2\text{ N/mm}^2$$

Let us assume Total depth of lintel beam = 320 mm $\left(\frac{3000}{10} = 300 \text{ mm}\right)$ and d = 290 mm.

Effective span = 3.29 m

Self weight of lintel = 0.3 × 0.32 × 25 = 2.4 kN/m

Weight of brick wall over lintel = 1 × 0.3 × 20

= 6 kN/m

Load from roof to lintel = 25 kN/m

Total weight on the lintel (factored) = 1.5 (2.4 + 6 + 25) = 50.1 kN/m

$$\text{Bending moment} = \frac{50.1 \times 3.29^2}{8} = 67.78 \text{ kN-m}$$

$$\text{Check for the depth, } d \geq \sqrt{\frac{M}{0.138\, f_{ck}\, b}}$$

$$d \geq \sqrt{\frac{67.78\times10^6}{0.138\times20\times300}}$$

$$\geq 286.12 \text{ mm OK}$$

Take $d = 290$ mm

Calculation of A_{st},

$$M_d = 0.87 f_y A_{st}\left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$$

$$67.78 \times 10^6 = 0.87 \times 415\, A_{st}\left(290 - \frac{415\, A_{st}}{20\times300}\right)$$

$$\Rightarrow \quad A_{st} = 799.98 \text{ mm}^2 \text{ and } \frac{A_{st\,min}}{300\times290} = \frac{0.85}{415}$$

$$\Rightarrow \quad A_{st\,min} = 178.2 \text{ mm}^2 < A_{st\,req}.$$

So, provide 4 bars of 16 mm diameter as longitudinal reinforcements

Check for shear

Max. shear force will occur 'd' distance from the face of support.

$$\text{Max. SF} = \frac{50.1\times(3-2\times0.29)}{2} = 60.62 \text{ kN}$$

Nominal shear stress, τ_v

$$\tau_v = \frac{V_u}{bd} = \frac{60.62\times10^3}{300\times290} = 0.697 \text{ N/mm}^2$$

and percentage tensile reinforcement,

$$p_t = \frac{100\times\frac{\pi}{4}\times16^2\times4}{300\times290} = 0.92\%$$

According to Question,

$$\tau_c = 0.56 + \frac{0.62 - 0.56}{1 - 0.75} \times (0.92 - 0.75)$$

$$\Rightarrow \quad \tau_c = 0.601 \text{ N/mm}^2$$

$$\because \quad \tau_v - \tau_c = 0.697 - 0.601 = 0.096 < 0.4 \text{ N/mm}^2$$

Minimum shear reinforcement should be provided.

Let us provide 8 mm diameter 2-legged vertical shear stirrups.

Spacing of shear stirrups, (S_V)

$$0.87 f_y A_{sv} \frac{d}{S_v} \geq 0.4\,bd$$

$$\Rightarrow \quad 0.87 \times 415 \times 2 \times 50 \times \frac{290}{S_v} \geq 0.4 \times 300 \times 290$$

$$\Rightarrow \quad S_v \leq 300.875 \text{ mm}$$

But $\quad S_v \not> 0.75 \times 290 = 217.5$ mm

$$\not> 300 \text{ mm.}$$

Hence, provide 8 mm dia. 2-legged vertical shear stirrups at a distance of 215 mm c/c.

Check for development length

$$L_d = \frac{0.87\, f_y \phi}{4\tau_{bd}}$$

$$L_d = \frac{0.87 \times 415 \times 16}{4 \times 1.6 \times 1.2} = 752.19 \text{ mm}$$

Due to compression confinement,

$$L_d \leq \frac{1.3\, M_1}{V}$$

V = S.F at point of zero B.M $\quad = \dfrac{50.1 \times 3.29}{2} = 82.41 \text{ kN}$

$$L_d \leq \frac{1.3 \times 67.78 \times 10^6}{82.41 \times 10^3}$$

$$L_d \leq 1069.21 \text{ mm} \quad \text{Which is OK}$$

CHAPTER 7

COLUMN

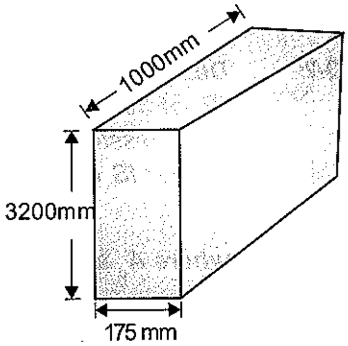
Q-1: ***A reinforced concrete wall of 175 mm thickness and 3.2 m effective height is needed for a compressive load of 1000 kN/m. Design the wall using M 15 grade concrete and mild steel reinforcement.***

[15 Marks, ESE-1995]

Sol: Reinforced concrete wall with axial loading will be designed as a column considering unit length

$$\text{Effective height} = (H) = 3.2 \text{ m} = 3200 \text{ mm}$$

$$\text{Thickness (t)} = 175 \text{ mm}$$



Consider 1 m width i.e., 1000mm width

$$\text{Slenderness ratio} = H/t = \frac{3200}{175} = 18.28 \geq 12$$

Hence the wall should be designed as a long column

We know that for long column,

$$P = C_r(\sigma_{cc} A_c + \sigma_{sc} A_{sc})$$

where σ_{cc} = N/mm² & σ_{sc}= 130 N/mm² [For M15 & Fe 250 i.e. Mild steel]

and $$C_r = \text{Reduction factor} = 1.25 - \frac{L_{eff}}{48t} = 1.25 - \frac{H}{48t}$$

$$= \left\{1.25 - \frac{3200}{48 \times 175}\right\} = 0.869$$

$\therefore$ $$P = 0.869\,(\sigma_{cc} \times (A - A_{sc}) + \sigma_{sc} A_{sc})$$

$\Rightarrow$ $$P = 0.869\,(4 \times (1000 \times 175 - A_{sc}) + 130 \times A_{sc})$$

$\Rightarrow$ $$P = 608300 - 3.476\,A_{sc} + 130 \times A_{sc} \times 0.869$$

$\Rightarrow$ $$P = 1000 \times 10^3 = 608300 + 109.494\,A_{sc}$$

$\Rightarrow$ $$A_{sc} = 3577.4 \text{ mm}^2.$$

Assuming that 16 mm ϕ bars are used and they are used equally on two faces.

$$\text{Hence spacing} = \frac{1000 \times \frac{\pi}{4}(16)^2}{\frac{3577.4}{2}} = 112.407 \text{ mm}.$$

Let us provide 16mmϕ bars @ 110mm c/c equally on two faces.

Spacing $\not> 450$ mm (OK)

Minimum ratio of vertical reinforcement to gross concrete area should be 0.0015

$$\Rightarrow \quad \frac{\dfrac{\frac{\pi}{4}(16)^2 \times 1000}{110} \times 2}{\dfrac{110}{175 \times 1000}} = 0.0209 > 0.0015 \quad \text{(OK)}$$

Horizontal reinforcement provided such that ratio of horizontal reinforcement to gross concrete area ≥ 0.0025 (for mild steel bars)

$$A_u = 0.0025 \times 1000 \times 175 \text{ (for 1m length)}$$
$$= 437.5 \text{ mm}^2$$

Let us provide 10mm bars at a spacing of $\dfrac{1000 \times \frac{\pi}{4}(10)^2}{\dfrac{437.5}{2}}$ = 358.86 mm c/c on two faces

i.e. 10mm bars @ 350mm c/c on two faces

Spacing $\not>$ 3×175 nor greater than 450 mm (OK)

Q-2: ***Design a square section column using M-15 concrete and mild steel bars to carry an axial load (P) of 30,000 kg. Effective length of column (leff) = 4.0 m. Assume permissible stresses in direct compression in M-15 concrete (σ_{cc}) and in mild steel bars (σ_{sc}) as 40 and 1300 kg/cm² respectively. As per IS code.***

$$P = C_R(\sigma_{cc} A_c + \sigma_{sc} A_s)$$

where A_c and A_s are areas of cross-section of concrete and steel respectively,

$$C_r = \text{Reduction Coefficient} = 1.25 - \frac{l_{eff}}{48b} \not> 1.0$$

$$b = \text{lateral dimension of column.}$$

Sketch the arrangement for longitudinal and lateral reinforcement.

[15 Marks, ESE-1996]

Sol: **Given:** l_{eff} = 4m. = 4000 mm

Designing the column, of size = B × B

Assuming area of steel = 0.8% of Ast = $\frac{0.8}{100} \times B^2 = \left\{\frac{8B^2}{1000}\right\}$

(Area of concrete) $A_C = A - A_S = B^2 - \frac{8B^2}{1000} = \frac{992B^2}{1000}$ (Assuming square section)

$\therefore$ σ_{cc} = 40 kg/cm² = 4 N/mm² and σ_{sc} = 130 N/mm²

For short column

$$P = \sigma_{cc} A_c + \sigma_{sc} As$$

$$\Rightarrow \quad 30{,}000 \times 10 = 4 \times 0.992\, b^2 + 130 \times 0.008\, b^2$$

$$\Rightarrow \quad b = 244.75 \text{ mm.}$$

Check: $\frac{\text{Leff}}{\text{B}} = \frac{4000}{244.75} = 16.34 > 12$. Hence column is long column

$$C_R = 1.25 - \frac{Leff}{48B} = 1.25 - \frac{4000}{48B}$$

$$\therefore \quad P = C_R\,(\sigma_{cc}\,Ac + \sigma_{sc}\,As)$$

$$\Rightarrow \quad 30{,}000 \times 10 = \left(1.25 - \frac{4000}{48B}\right)\left(\frac{992B^2}{1000} \times 4 + 130 \times \frac{8B^2}{1000}\right)$$

$$\Rightarrow \quad (30{,}000 \times 10 \times 48B) = (1.25 \times 48B - 4000)(\,0.992B^2 \times 4 + 0.008 \times 130B^2)$$

$$\Rightarrow \quad 14400000B = 5.008B^2\,(1.25 \times 48B - 4000)$$

$$\Rightarrow \quad 300.48B^3 - 20032B^2 - 14400000B = 0$$

$$\Rightarrow \quad B(300.48\,B^2 - 20032B - 14400000) = 0$$

$$B = 254.77 \text{ mm} \cong 260 \text{ mm}$$

Hence, $\quad$ cross – section = 260mm × 260 mm

$$\therefore \quad \text{Min. reinforcement} = \frac{8}{1000}(280)^2 = 540.8\text{mm}^2$$

Provide 16 mm ϕ bar.

$$\Rightarrow \quad \text{No. of bars} = \frac{540.8}{\frac{\pi}{4} \times (16)^2} = 2.69$$

Adopt (4) nos of 16ϕ bars

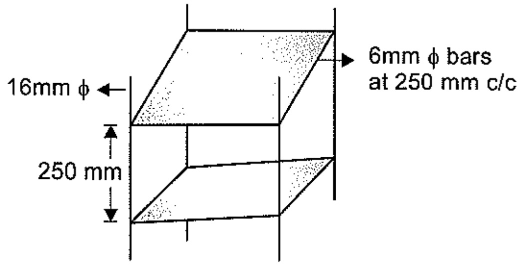
Diameters of lateral ties

Lateral ties should be greater than equal to 1/4th the dia of longest i.e., bar and 6mm whichever is greater

(i) $\frac{1}{4} \times 16 = 4\text{mm}$

(ii) 6 mm

$\Rightarrow$ Provide 6mm ϕ bars as lateral ties.



Max spacing should be smaller of

(i) Least lateral dimension = 260 mm

(ii) 16 × min^m dia of bars = 256 mm

(iii) 300 mm

Hence providing 6mm ϕ bars at 250 mm c/c as lateral ties.

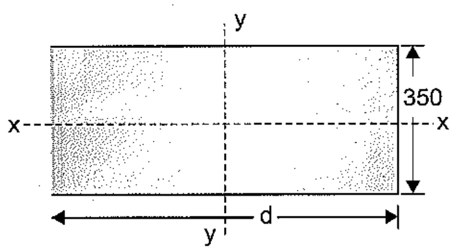
Q-3: ***A reinforced concrete column of effective length of 3.0m is to be designed to support a factored load of 2400 kN. Using limit state method determine the crosssectional dimensions of the column and reinforcement when one side of the column is restricted to 350mm. The concrete used is of grade M20 and reinforcement is of HYSD steel of grade Fe 415.***

[25 Marks, ESE-2000]

Sol: Assume the column is of dimension 350 × d and d > 350.

(1) Slenderness ratio of column = $\frac{l_{eff}}{b} = \frac{3000}{350} = 8.57 < 12$

Hence this is a short column.



As per is code e_{min} is greater of

(i) $\frac{l}{500}+\frac{d}{30}$

(ii) 20 mm

Hence, min eccentricity about x-axis (e_x) is given by

$$\left.\begin{array}{ll}\text{(i)} & \frac{3000}{500}+\frac{350}{30}=17.67 \\ \text{(ii)} & 20\text{ mm}\end{array}\right\} \max = 20\text{ mm}$$

min eccentricity about y-axis (e_y) is given by

$$\left.\begin{array}{ll}\text{(i)} & \left(\frac{3000}{500}+\frac{d}{30}\right) \\ \text{(ii)} & 20\text{ mm}\end{array}\right\} \max$$

Adopting d = 420 mm, $e_{y\,min}$ = 20 mm.

We can apply the short column formula

$P_u = 0.4\, f_{ck} A_c + 0.67\, f_y A_{sc}$ if $e_{x\,min}$ & $e_{y\,min}$ are both less then 0.05 × lateral dimension.

$$\begin{cases} 0.05\times350 = 17.5 \\ 0.05\times420 = 21 \end{cases}$$

$$\Rightarrow \quad \begin{cases} e_{x\min} \not< 0.05\times350 \\ e_{y\min} < 0.05\times420 \end{cases}$$

Hence IS code recommendation for use of short column formula as described above is not valid.

Thus in this case, we have to design the column as uniaxial bending

However, uniaxial bending design requires interaction curves which is not available and stress strain relationship is also not available. Thus neglecting the small variation, we will use the short column formula for design.

$$\text{Factored load} = 2400\text{ kN}$$

$$\Rightarrow \quad 2400 \times 1000 = 0.4 \times f_{ck}\,(A_g - A_{sc}) + 0.67\, f_y\, A_{sc}$$

$$= 0.4 \times 20\,(350 \times 420 - A_{sc}) + 0.67 \times 415\, A_{sc}$$

$$A_{sc} = 4532.493\text{ mm}^2$$

Let us provide 24 mm dia bars

$$\text{No. of bars req} = \frac{453.493}{\frac{\pi}{4}(24)^2} = 10.02\text{nos.}$$

Let us provide 6 nos 24 ϕ and 6 nos. 20 ϕ bars.

$$A_{sc}\text{ provided} = 4596.96\text{ mm}^2$$

$$\%\text{ of steel provided} = 3.127\% < 6\%\text{ ok.}$$

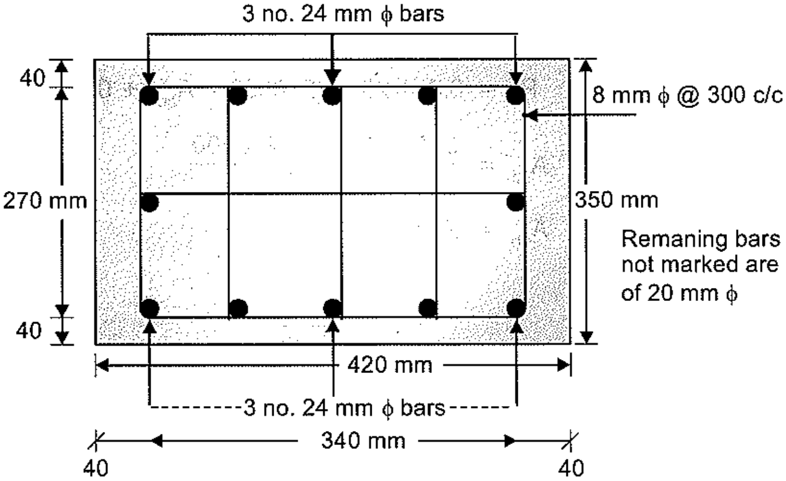
Design of lateral ties

$$\phi_t > \begin{cases} \frac{\phi_{largest}}{4} = \frac{24}{4} = 6\text{mm} \\ 6\text{mm} \end{cases}$$

Let us adopt dia. of ties = 8 mm

$$\text{Spacing } S_t < \begin{cases} D = 350\text{mm(least lateral dimension)} \\ 16 \times \phi_{small} = 16 \times 20 = 320\text{mm} \\ 300 \end{cases}$$

Adopt 8 mm ties at 300 mm c/c spacing.



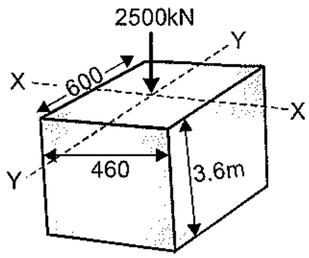
$$48\,\phi_{tie} = 48 \times 8 = 384 \text{ mm}$$

⇒ Longitudinal bars spaced at a distance not exceeding 48 times the dia. of lateral ties are tied in both directions, hence additional longitudinal bars in between these bars need to be tied in one direction only by open ties. Hence ties have been provided based on the above recommendation of IS code.

Q-4: ***A reinforced concrete column of size 460 mm × 600 mm having effective length of 3.6 m is to be designed using limit state method to support an axial service load of 2500 kN. Use M-20 grade concrete and HYSD steel of Fe-415 grade.***

[15 Marks, ESE-2002]

Sol:



Step 1: $\dfrac{\text{Leff}}{\text{Least lateral dimension}} = \dfrac{3600}{460} = 7.83 < 12$

Hence this is short column.

Step 2: $e_{min} = \left.\dfrac{\text{Leff}}{500} + \dfrac{D}{30} \text{ or, } 20\text{mm}\right\}$ whichever is maximum

⇒ e_{min}: $\dfrac{3600}{500} + \dfrac{460}{30} = 22.53\text{mm}$ or, 20 mm

Hence provide ⇒ e_{min} = 22.53 mm for x-axis

Similarly for y-axis $e_{min} = \max\begin{cases} \dfrac{3600}{500} + \dfrac{600}{30} \\ 20 \text{ mm} \end{cases}$

⇒ e_{min} = 27.2 mm

as 2 2.53 < 0.05 × 460 and 27.2 < 0.05 × 600

Hence, we can use, $P_u = 0.4\ f_{ck}\ A_{sc} + 0.67\ f_y Asc$

Step 3: Service load = 2500 kN

Factored load = Pu = 1.5 × 2500 = 3750 kN

Given: $f_{ck} = 20 N/mm^2,\ f_y = 415\ N/mm^2$

$A = 460 \times 600 = 276000\ mm^2$

$\therefore \quad P_u = 0.4 \times 20 \times (276000 - A_{sc}) + 0.67 \times 415 \times A_{sc}$

$$\Rightarrow \quad A_{sc} = \frac{3750 \times 1000 - 2208000}{270.05}$$

$$\therefore \quad A_{sc} = 5710\ mm^2$$

Adopting 36 mm ϕ bars

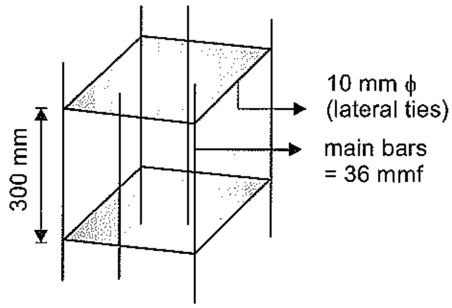
$$\text{Number of bars required} = \frac{5710}{\pi/4 \times 36^2} = 5.6 \cong 6 \text{ bars}$$

Step 4: Diameter of lateral ties:

(a) 1/4 × 36 = 9mm

(b) 6mm

Hence provide 10mm bars as lateral ties



Step 5: Spacing of lateral ties

(a) Least lateral dimension = B = 460 mm

(b) 16 ϕ = 16 × 36 = 576 mm

(c) 300 mm

Minimum of the above three will be the spacing = 300 mm

Q-5: ***Design a circular column with helical reinforcement subjected to a working load of 1500 kN. Diameter of the column is 450 mm. The column has unsupported length of 3.5 m and is effectively held in position at both ends but not restrained against rotation. Use limit state design method. Use M-25 concrete and HYSD Fe-415 steel.***

[20 Marks, ESE-2011]

Sol: **Given:**

$P = 1500$ kN

$\therefore$ Factored load Pu = 1.5 × 1500 = 2250 kN

Circular diameter = 450 mm

$L_{eff} = 3.5$ m

$$\text{Check for slenderness ratio} = \frac{L_{eff}}{D} = \frac{3500}{450} = 7.77 < 12$$

$\therefore$ Column can be designed as short column,

$$\text{Minimum eccentricity} = \frac{L_{eff}}{500} + \frac{D}{30} \text{ or 20 mm whicheve is greater}$$

$$\therefore \quad e_{min} = \frac{3500}{500} + \frac{450}{30} \text{ or } 20 \text{ mm}$$

$$= 7 + 15 \quad \text{or } 20 \text{ mm}$$

$$= 22 \text{ mm} \quad \text{or} \quad 20 \text{ mm}$$

$$\therefore \quad e_{min} = 22 \text{ mm}$$

According to code if $e_{min} < 0.05 \times D = 0.05 \times 450 = 22.5$ mm then only we can apply

$$P_u = (0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}) \times 1.05 \qquad \text{[For spiral column]}$$

Provided that $\dfrac{\text{volume of spiral reinforcement}}{\text{volume of core}} \not< 0.36 \left(\dfrac{Ag}{A_{core}} - 1\right) \dfrac{f_{ck}}{f_y}$

$$\text{Area of concrete } (A_c) = \pi/4 \times 450^2 - A_{sc}$$

where A_{sc} = Area of steel

$$\Rightarrow \quad 2250 \times 10^3 = \{0.4 \times 25 \times [159043.5 - A_{sc}] + 0.67 \times 415 \times A_{sc}\} \times 1.05$$

$$\Rightarrow \quad 2250 \times 10^3 = (1590435 - 10A_{sc} + 278.05A_{sc}) \times 1.05$$

$$\Rightarrow \quad A_{sc} = 2060.89 \simeq 2060.9 \text{ mm}^2$$

$$\text{Checking for } A_{min} = 0.8\% \text{ of gross Area} = \frac{0.8}{100} \times \frac{\pi}{4} \times 450^2 = 1273 \text{mm}^2$$

$$\Rightarrow \quad A_{sc} > A_{min} \text{ O.K.}$$

Main reinforcement area requirement is 2060.9 m^2.

Hence provide 18 mm ϕ bar 4 no. & 20mmϕ bar 4 no.

$$\text{Area provided} = 4\left[\frac{\pi}{4}(18)^2 + \frac{\pi}{4}(20)^2\right] = 2273.36 \text{ mm}^2$$

Check for longitudinal reinforcement

1. $\phi > 12$ mm OK
2. $A_{sc} > A_{scmin}$ OK
3. Spacing between bars along pheriphery $= \dfrac{\pi \times 350}{8} = 137.375 < 300$ mm OK

Design of spiral reinforcement,

Recommendation for the dia of spiral is same as that of lateral ties in column

i.e. $$\phi_{spiral} \not< \frac{1}{4}\phi_{\text{longitudinal bar(largest)}}$$

$$\not< 6 \text{ mm}$$

i.e. in our case $$\phi_{spiral} \not< \frac{20}{5} \text{ mm}$$

$$\not< 6 \text{ mm}$$

Let us adopt the dia of spiral as 6 mm.

Pitch of the spiral:

Where we consider 5% extra strength for the spiral column then we have to satisfy the following recommendations.

(1) $$\frac{\text{Vol. of spiral reinforcement}}{\text{Volume of Core}} \not< 0.36\left(\frac{A_g}{A_{core}}-1\right)\frac{f_{ck}}{f_y}$$

(2) $$\text{Pitch (s)} \not> 75 \text{ mm}$$

$$\not> \frac{1}{6} \times \text{core dia}$$

$$\not< 25 \text{ mm}$$

$$\not< 3 \times \phi_{spiral}$$

Assuming clear cover of 40 mm (d_c)

$$(D_c) \text{ core diameter } = 450 - 40 \times 2 = 370 \text{ mm}$$

$$V_c = \text{Volume of core per unit height of column}$$

$$= \frac{\pi}{4} \times D_c^2 \times 1000 = \frac{\pi}{4} \times 370^2 \times 1000 = 107521260 \text{ mm}^3$$

V_h = Volume of helical reinforcement in unit length

= (Number of turns in unit height) × (Length of one turn) × (c/s area of helical R/F)

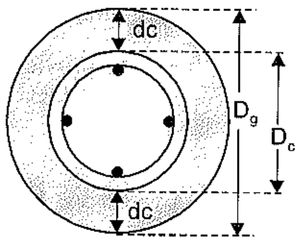
$$\therefore \quad V_h = \frac{1000}{S} \times \pi D_h \times \frac{\pi}{4} \times \phi_h^2$$

where S = Pitch length

$$\therefore \quad V_h = \frac{1000}{S} \times \pi \times (D_c - \phi) \times \frac{\pi}{4} \times \phi^2 = \frac{1000}{S} \times \pi \times (370-6) \times \frac{\pi}{4} \times 6^2$$

$$V_h = \frac{32332899.63}{S}$$

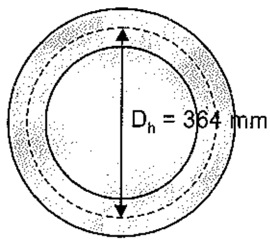
$\therefore$ A/c to code, for helical reinforcement,



$$0.36\frac{f_{ck}}{f_y}\left(\frac{Ag}{Ac}-1\right) \le \frac{V_h}{Vc}$$

$$\Rightarrow \quad 0.36 \times \frac{25}{415}\left[\frac{\frac{\pi}{4}\times 450^2}{\frac{\pi}{4}\times 370^2}-1\right] \le \frac{32332899.63}{S \times 107521260}$$

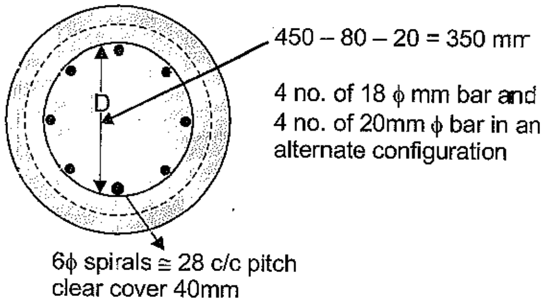
$$\therefore \quad S < 28.93 \text{ mmm}$$



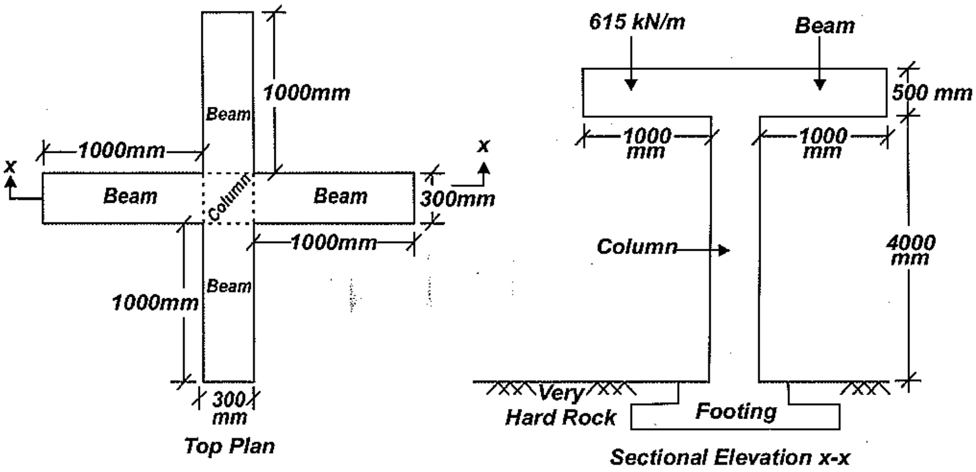
$$\text{As per code } S < \begin{cases} 75 \text{ mm} \\ \frac{DC}{6} = 61.67 \text{ mm} \end{cases}$$

$$\text{and} \quad S > \begin{cases} 25 \text{ mm} \\ 3\phi = 18 \text{ mm} \end{cases}$$

Hence provide 6ϕ spirals at 28 mm pitch.



Q-6:



A square column (500 mm × 500 mm) carries load from two beams, which are mutually perpendicular as shown in figure. Overhang portion of beams carry a total load of 615.0 kN/m (include self-weight). Design the column at top of footing level. Footing is fully embedded in very hard rock. Beams are restrained against rotation at Beam-column junction. The minimum eccentricity is less than 0.05 times the lateral dimension of column. Sketch all details required at column cross-section.

Use:

M-20 grade concrete, Fe-415 grade reinforcing bars,

Appropriate coefficient form 1.0/1.20/1.50/2.0,

Main reinforcing bar : 32 mm diameter ***[20 Marks, ESE-2018]***

Sol: Total load on the column due to overlang portion of beams

$$= 4 \times (615 \text{ kN/m} \times 1 \text{ m})$$

$$= 2460 \text{ kN}$$

Self weight of column needs to be considered as the column is to be designed at the top of footing load.

$\therefore$ Self weight = $(0.5 \times 0.5 \times 4.5) \times 25$ (Assuming unit weight of RCC as 25 kN/m^3)

$$= 28.125 \text{ kN}$$

$\therefore$ Total factored load = $1.5(2460 + 28.125)$

$$P_u = 3732.2 \text{ kN}$$

Check for slenderness ratio:

$\dfrac{l_{ex}}{D} = \dfrac{Kl}{D} = \dfrac{1.2 \times 4}{0.5} = 9.6 < 12$ i.e. short column.

[K = 1.2, for one end fixed and other free against lateral displacement]

Assuming minimum eccentricity is less than 0.05 times of lateral dimension of column

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$3732.2 \times 10^3 = 0.4 \times 20 \times (500^2 - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$3732.2 \times 10^3 = 2000 \times 10^3 - 8A_{sc} + 278.05 A_{sc}$$

$$A_{sc} = 6414.36 \text{ mm}^2$$

No. of reinforcing bar of 32 mm diameter $= \dfrac{6414.36}{\frac{\pi}{4} \times 32^2} = 7.97 \simeq 8$

Hence, provide 8 no. of 32ϕ bars.

Design of lateral ties:

$$\text{Diameter of lateral ties} \not< \frac{1}{4}\text{(main bar diameter) and 6 mm}$$

$$\not< \frac{1}{4} \times 32 = 8 \text{ mm and 6 mm}$$

Use 8 mm lateral ties

$$\text{Spac ng} = \text{minimum}\begin{cases} = \text{least lateral dimension} = 500 \text{ mm} \\ = 16 \times \text{main reinforcing diameter} \\ = 16 \times 32 = 512 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

$\therefore$ Use 8 mm diameter lateral ties at 300 mm c/c

Assuming a clear cover of 40 mm, c/c distance betwen outer bars

$$= 500 - 2 \times \left(40 + 8 + \frac{32}{2}\right)$$

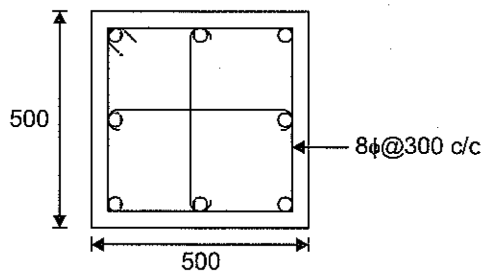
$$= 372 \text{ mm}$$

$$\text{Spacing between bars} = \frac{372}{2} = 186\text{mm} > 75\text{mm}$$

Also, $48\phi_t = 48 \times 8 = 384$ mm

$\Rightarrow$ $372 \text{ mm} < 48\phi_t$

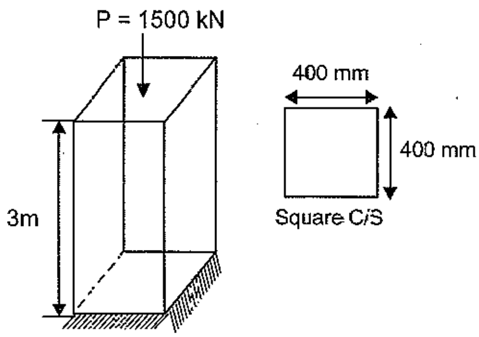
Hence, provide open ties for the intermediate reinforcement as shown.



Q-7: ***Design a square column of height 3 m subjected to an axial load of 1500 kN under dead and live load condition. Use limit state method of design. Assume effective length factor = 1.2. Size of the column is fixed at 400 mm × 400 mm. Show the reinforcement detail (cross-section). Use M25 and Fe500.***

[20 Marks, ESE-2020]

Sol: Given



$$\text{Ultimate Load } (P_u) = 1500 \times 1.5 = 2250 \text{ kN}$$

$$\text{Effective length factor} = 1.2$$

M25 and Fe 500

Using limit state method of design :

Here, effective length (l_{eff}) = 3000 mm × 1.2 = 3600 mm

Checking column is long or short

$$\text{Slenderness ratio } (\lambda) = \frac{l_{eff}}{\text{least lateral dimension}}$$

$$= \frac{3600}{400} = 9 < 12$$

So, column is short.

Checking minimum eccentricity (e_{min}).

$$e_{min} = \max\begin{cases} \dfrac{l_{unsupported}}{500} + \dfrac{D}{30} \\ 20 \text{ mm} \end{cases}$$

$$= \max.\begin{cases} \dfrac{3000}{500} + \dfrac{400}{30} = 19.33 \text{mm} \\ 20 \text{mm} \end{cases}$$

$$= 20 \text{ mm}$$

$e_{min.}$ shall be less than 0.05D = 0.05 × 400 = 20 mm

Calculation of area of steel (A_{sc})

So, $$\boxed{P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}}$$

Here,

$$P_u = 2250 \times 10^3 \text{ N}$$

$$f_{ck} = 25 \text{ MPa}$$

$$f_y = 500 \text{ MPa}$$

$$A_c = \text{Area of concrete} = (A - A_{sc})$$

$$A_{sc} = \text{Area of steel}$$

$$A = \text{Total c/s area} = (400 \times 400) \text{ mm}^2$$

So, $$2250 \times 10^3 = 0.4 \times 25 \times (400 \times 400 - A_{sc}) + 0.67 \times 500 \times A_{sc}$$

or $$2250000 = 1600000 - 10A_{sc} + 335A_{sc}$$

or $$650000 = 325\ A_{sc}$$

$$A_{sc} = 2000 \text{ mm}^2$$

Providing 4 nos. of 25 mm bar at corner and 4 nos. of 16 mm bar in between.

$$\text{Total area of steel provided } (A_{SC}) = 4 \times \frac{\pi}{4} \times 25^2 + 4 \times \frac{\pi}{4} \times 16^2$$

$$= 2767.74 \text{ mm}^2$$

Checking percentage of Steel

$$p' = \frac{A_{SC}}{A} \times 100$$

$$= \frac{2767.74}{400 \times 400} \times 100$$

$$= 1.73\% > 0.8 \text{ (minimum reinforcement)}$$

Design of lateral ties :

$$\text{tie diameter } (\phi_t) \geq \begin{cases} \dfrac{\phi_{\text{longi. max.}}}{4} \\ 6\text{mm} \end{cases}$$

So,

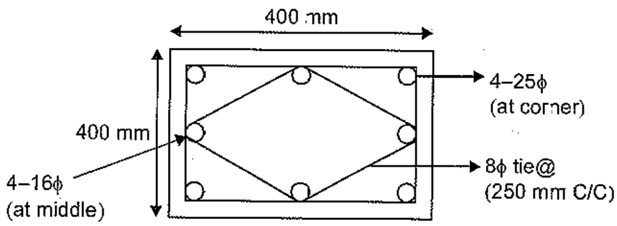
$$\phi \geq \begin{cases} 25/4 = 6.25\text{mm} \\ 6\text{mm} \end{cases}$$

Hence, using 8 mm dia. bars as stirrups.

$$\text{Tie spacing } (s_t) \leq \begin{cases} D \\ 16\ \phi_{\text{longi, min}} \\ 300\text{mm} \end{cases}$$

$$s_t = \begin{cases} 400\text{ mm} \\ 16 \times 16 = 256\text{ mm} \\ 300\text{ mm} \end{cases}$$

Hence, using 8 mm stirrup @ 250 mm C/C..
Providing nominal cover = 40 mm
Finally we get



CHAPTER 8

FOOTING

Q-1: ***Design a sloped square footing for a R.C. column 300 mm × 300 mm size carrying an axial load of 320 kN. The safe bearing capacity of soil is 150 kN/sq. m. M_{20} concrete and Fe 415 steel are used. Use WSD method.***

Allowable shear stress data:

p_t	t_c N/mm
0.25	0.22
0.50	0.30
0.75	0.35
1.0	0.39
1.25	0.42
1.5	0.45

[20 Marks, ESE-2001]

Sol: **Step 1:** Design constants

$$k = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}} = \frac{13 \times 7}{230 + 13 \times 7} = 0.2835$$

$$j = 1 - n/3 = 1 - \frac{0.2835}{3} = 0.9055$$

$$R = \frac{1}{2} \times \sigma_{cbc} \times j \times k = \frac{1}{2} \times 7 \times 0.9055 \times 0.2835 = 0.8984 \cong 0.90$$

Step 2: Design of size

$$\text{Load from column} = 320 \text{ kN}$$

$$\text{Weight of foundation} = \frac{10}{100} \times 320 = 32 \text{ kN}$$

[Assuming 10% of the axial load = Weight of foundation]

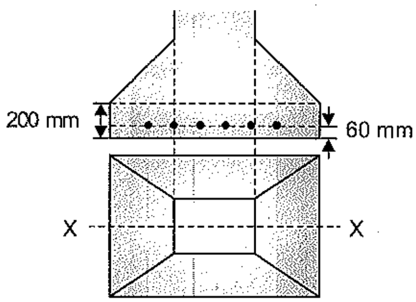
So, $\quad$ Net load = 352kN.

$$\text{Area} = \frac{P_T}{q_0} = \left(\frac{352}{150}\right) m^2 = 2.35 m^2$$

Since the footing is a square footing, therefore $B = \sqrt{A} \Rightarrow B = \sqrt{2.35} = 1.53 \cong 1.55$ m

$\therefore \quad A_0 = 1.55^2 = 2.4025 \text{ m}^2$

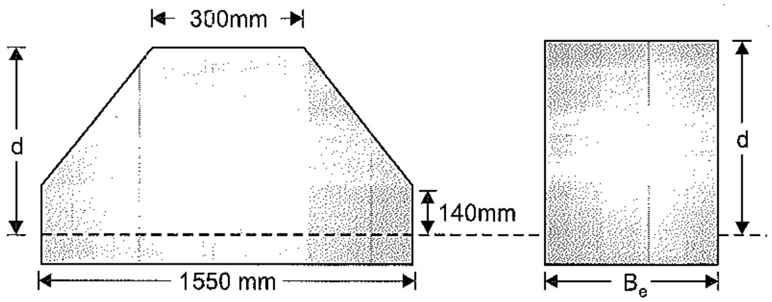
Hence, $\quad$ Net soil pressure becomes $= \dfrac{320}{2.4025} = 133.20 \text{kN/m}^2$



(Top view)

Assuming depth of slab = 200 mm and nominal cover = 60 mm

Step 3: Depth of foundation fo˙ bending moment critical section is at the face of column section



b = 300mm and B = 1550 mm.

$$\text{Width of equivalent section, } B_e = b + \frac{1}{8}(B - b) = 300 + \frac{1}{8}(1550 - 300)$$
$$= 456.25 \text{ mm}$$

[Note : this width is used only for debth calculation]

Step 4: Bending moment,

$$\text{Overhang } (O_x) = \frac{1.55 - 0.3}{2} = 0.625\text{m}$$

$$\text{Bending moment} = \text{W.B.}\frac{O_x^2}{2} = 133.2 \times 1.55 \times \frac{0.625^2}{2} = 40.32 \text{ kN-m}$$

$$\therefore \quad d = \sqrt{\frac{BM}{R.B_e}} = \sqrt{\frac{40.32 \times 10^6}{0.9 \times 456.25}} = 313.40\text{mm}$$

Effective cover = 60 mm

$\therefore$ Total bebth, D = 313.40 + 60 = 373.40 mm

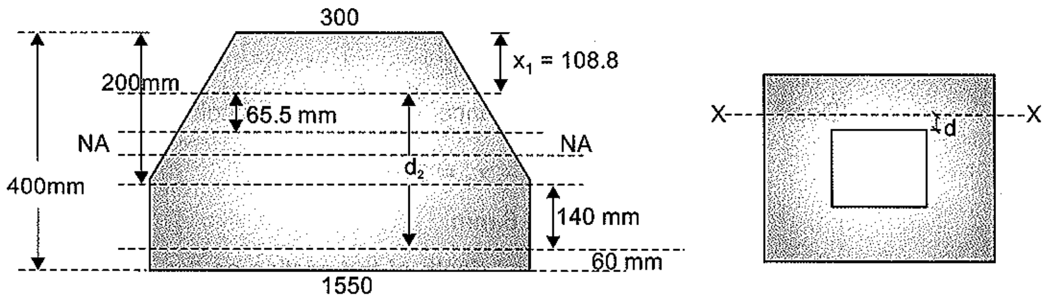
Adopting D = 400 mm

Hence, d = 400 – 60 = 340 mm

Step 5: One way shear

$$\text{Overhang, } O_x = \frac{1.55 - 0.3}{2} - 0.34 = 0.285\text{m} \qquad \text{(at distance d from face)}$$

$$V = W \times B \times Ox$$
$$= 133.20 \times 1.55 \times 0.285$$
$$= 58.84 \text{ kN}$$



$$\text{Shear stress, } \tau_v = \frac{V}{b_2 \times d_2}$$

[b_2 = Top width at N.A. level; d_2 = depth at critical section i.e. at a distance 'd' from face of column]

Total effective depth at a distance 'd' from the face of column = d_2

$$d_2 = 140 + \frac{400-200}{\left(\frac{1550-300}{2}\right)} \times \left[\frac{1550-300}{2} - 340\right]$$

$\Rightarrow$ $$d_2 = 231.2 \text{ mm}$$

Top width at section X-X (i.e. critical section)

$$= 300 + 2 \times 340 = 980 \text{ mm}$$

$$\text{Depth of neutral axis} = nd_2 = 0.2835 \times 231.2 = 65.55 \text{ mm}$$

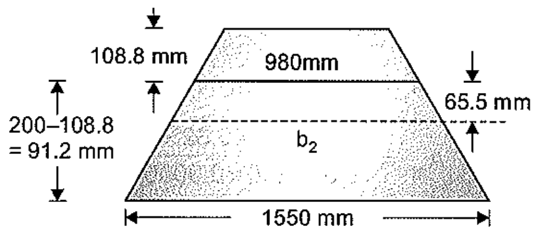
Depth of top at section X-X from the bottom of column

$$= \frac{200 \times 340}{\frac{1550-300}{2}} = 108.8 \text{mm}$$

Now, width of the section of NA = b_2

$\therefore$ $$b_2 = 980 + \left[\frac{1550-980}{91.2}\right] \times 65.5$$

$$b_2 = 1389.4 \text{ mm}$$



$\therefore$ $$\text{Shear stress} = \frac{V}{b_2 d_2} = \frac{58.84 \times 10^3}{1389.4 \times 231.2} = 0.18 \text{ N/mm}^2 < 0.22 \text{ (min } \tau_c)$$

Hence section is safe for shear

Step 6: Check for punching:

$$\text{Net punching force} = P - w\,(a + d)\,(b + d)$$

$$= 320 - 133.2\,(0.3 + 0.34)\,(0.3 + 0.34)$$

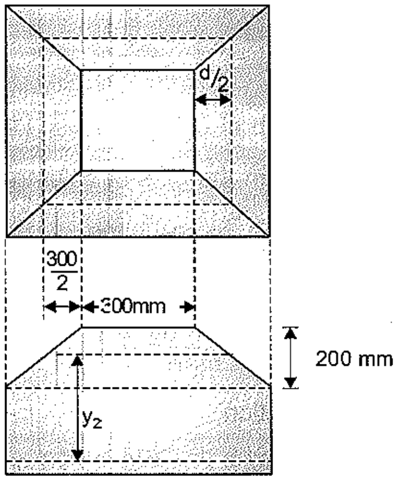
$$P_{net} = 265.44 \text{ kN}$$

$$x_2 = \frac{200}{625} \times 170 = 54.4\text{mm}$$

[Depth of top section at a distance $\frac{d}{2}$ from face of column from the bottom of column]

$$y_2 = 340 - x_2 = 340 - 54.4$$
$$= 285.6 \text{ mm}$$

[Total depth of the section on the section critical for punching.]



$$\text{Punching shear stress} = \frac{\text{Net punching force}}{\text{perimeter} \times \text{debth}} = \frac{265.44 \times 10^3}{2[(a+b)+(b+d)] \times y_2}$$

$$= \frac{265.44 \times 10^3}{2[(0.3+0.34)+(0.3+0.34)] \times 285.6 \times 10^3}$$

$$= 0.36 \text{ N/mm}^2$$

$$\text{Permissible punching stress} = k_c \times \tau_c = 1 \times 0.16\sqrt{f_{ck}} = 1 \times 0.16\sqrt{20}$$
$$= 0.715 \text{ N/mm}^2 > 0.36 \text{ N/mm}^2$$

Hence section is sate in punchig shear.

Step 7: Area of steel calculation

$$M = 40.32 \text{ kN-m}$$

$$A_{st} = \frac{M}{t.jd} = \frac{40.32 \times 10^6}{230 \times 0.9050 \times 340} = 569\text{mm}^2$$

$$\text{Number of 10mm } \phi \text{ bar} = \frac{569}{\frac{\pi}{4} \times 10^2} = 7.24 \cong 8\text{bar}$$

$$\text{Minimum \% of steel} = 0.12\% = \frac{0.12}{100} \times B_e \times d$$

$$= \frac{0.12}{100} \times 456 \times 400$$
$$= 218 \text{ mm}^2 < 569 \text{ mm}^2$$

$\therefore$ Provide 8 no's 10mm ϕ bars in both direction.

Step 8: Check for development length $= \frac{1}{2}(1550 - 300) - 60 = 565$ mm

$$\tau_{bd} \text{ for M20} = 1.2 \text{ N/mm}^2$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$L_d = \frac{\sigma st}{4\tau_{bd}} = \frac{230}{4\times12}\times\phi = 47.9\phi$$

$$= 47.9 \times 10 = 479 \text{ mm} < 565\text{mm hence Ok}$$

Q-2: ***Determine the thickness of the footing slab of uniform thickness for a 400 mm square column transmitting an axial load of 1100 kN. The safe bearing capacity of soil is 150 kN/m². The materials to be used are M 20 concrete and HYSD steel of grade Fe 415. Use limit state design method.***

[15 Marks, ESE-2003]

Sol: **Size of foundation**

While deciding the size footing we work with service load

$\Rightarrow$ Load from column = 1100 kN

Foundation load = 10% of column load

$$= \frac{10}{100}\times1100 = 110 \text{ kN}$$

Hence Total load (P_T) = 1210 kN

$\therefore$ Area of foundation $= \frac{P_T}{q_0} = \frac{1210}{150} = 8.07\text{m}^2$

Adopting square footing, $B = \sqrt{8.07} = 2.84 \simeq 3\text{m}.$

Final area = 9m^2

Net soil pressure $= w = P/A = \frac{1100}{9} = 122.22\text{kN/m}^2$

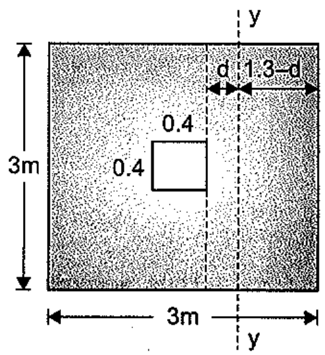
For designing the foundation slab, we will work with factored load.

Factored Net soil pressure = 1.5 × 122.22 = 183.33 kN/m²

Calculation of depth of footing

Depth of footing slab is generally governed by shear. However will also check the depth obtained from shear considerable for safety in bending.

Case I for one way shear



$$V = w \times (1.3 - d) \times 3$$

$$\tau_v = \frac{183.33\times(1.3-d)\times3}{3d}$$

$$\tau_c = ?$$

Assuming τ_c for minimum % age of steel (0.15%)

$\therefore$ $\tau_c = 0.28 \text{ N/mm}^2 = 0.28 \times 10^3 \text{ kN/m}^2$

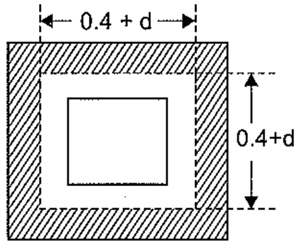
Assuming K = 1

For safety in shear $\tau_v < k.\tau_c$

$$\therefore \quad \frac{183.33\times(1.3-d)\times 3}{3d} < 0.28\times 10^3$$

$$\therefore \quad \boxed{d > 0.514\text{ m}}$$

Case II for two way shear



$$\text{Net punching force} = F_p = w\times(3^2 - (0.4+d)^2]$$

$$\therefore \quad \text{Punching force} = \frac{w(9-(0.4+d)^2)}{4\times(0.4+d)\times d} = \tau_{v_{punching}}$$

$$\therefore \quad \tau_{v_{punching}} = k_s.\tau_c$$

$$\tau_c = 0.25\sqrt{f_{ck}} = 1.118\text{ N/mm}^2$$

$$= 1.118\times 10^3\text{ kN/m}^2$$

$$\Rightarrow \quad k_s = 0.5+\beta \ngtr 1$$

$$\beta = \frac{0.4}{0.4} = 1$$

$$k_s = 1$$

$$\frac{w(9-(0.4+d)^2)}{4\times(0.4+d)\times d} < 1\times 1.118\times 10^3$$

$$\Rightarrow \quad \frac{183.33\{9-(0.4+d)^2\}}{4\times(0.4+d)\times d} < 1\times 1.118\times 10^3$$

$$\Rightarrow \quad \boxed{d > 0.417\text{ m}}$$

Case III for maximum bending moment

$$BM_{max} = \frac{w\ell^2}{2}, \quad l = \frac{(3-0.4)}{2} = 1.3\text{m}$$

[Critical section for bending is at the face of column]

$$w = 183.33\text{ kN/mm for a 1 m strip}$$

$$BM_{max} = 154.914\text{ kNm}$$

$$BM_{max} = M_{u_{lim}}\text{ of the section (Assume)}$$

$$\therefore \quad 154.914\times 10^6 < 0.36\, f_{ck}\times X_{u_{lim}}\times b\times(d-0.42\, X_{u_{lim}})$$

$$\therefore \quad 154.914\times 10^6 < 0.36\times 20\times 0.48\times d\times 1000\,(d-0.42\times 0.48\times d)$$

$$\therefore \quad d > 0.237\text{ m}$$

Thus, according to following three criteria, uniform depth of the footing should be provided = 520 (effective depth) + 40 mm (cover.

$$\Rightarrow \quad \boxed{D = 560\text{ mm}}$$

Q-3: ***Design a constant thickness footing for a reinforced concrete column of 300 mm × 300 mm. The column is carrying an axial working load of 600 kN. The BC of soil is 200 kN/m². Use M-25 concrete and HYSD Fe-415 bars. Use limit state design method.***

$\frac{100A_{st}}{bd}$	0.15	0.25	0.50	0.75	1.0
τ_c (N/mm²)	0.19	0.36	0.49	0.57	0.64

[10 Marks, ESE-2011]

Sol:

$$\text{The BC of the soil} = 200 \text{ kN/m}^2$$

and

$$P = 600 \text{ kN}$$

$$\text{Total load on soil} = \text{Load on column + Self weight of footing}$$

$$= 600 + 600 \times \frac{10}{100} = 660 \text{ kN.}$$

$$\text{Area of footing} = \frac{\text{Total load on soil}}{\text{BC}} = \frac{660}{200} = 3.3\text{m}$$

∴ Size of the footing will be B m × B m.

$$B^2 = 3.3$$

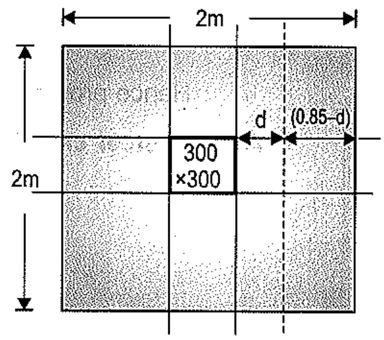
$$\Rightarrow \quad B = 1.817\text{m} \simeq 2 \text{ m.}$$

∴ Provide footing of size 2m × 2m

$$\text{Factored net soil pressure} = 1.5 \times \frac{600}{4} = 225 \text{ kN/m}^2$$

Calculation of depth of footing

(i) For one way shear



$$V = 2 \times (1 - 0.15 - d) \times 225 \text{ kN/m}^2$$

$$\tau_v = \frac{450(0.85-d)}{2d} \text{kN/m}^2$$

Assume 0.25% steel,

$$\tau_c = 0.36 \text{ N/mm}^2 \text{ for M25 grade concrete}$$

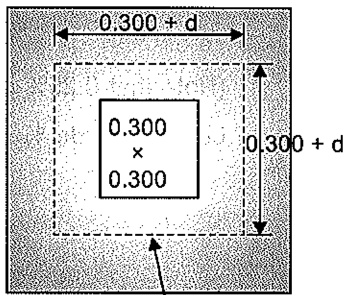
Assuming K = 1, then [for overall depth of slab > 300 mm]

$$\tau_v < k\tau_c$$

$$\frac{450(0.85-d)}{2d} \times 10^{-3} < 0.36$$

$$\therefore \quad \boxed{d > 0.327\text{m}}$$

(ii) For two way shear



$$\text{Net punching force} \quad F_P = \{2 \times 2 - (0.3 + d)^2\} \times 225 \text{ kN}$$

$$F_P = (4 - (0.3 + d)^2) \times 225 \text{ kN}$$

$$\text{Punching shear} = \frac{F_P}{\text{Perimeter} \times d}$$

$$\tau_{v_P} = \frac{\{4-(0.3+d)^2\} \times 225}{4(0.3+d)d} \text{kN/m}^2$$

$\tau_{v_p} < k_s.\tau_{c_p}$ shculd be valid

$$\tau_{c_p} = 0.25\sqrt{f_{ck}} = 1.118 \text{ N/mm}^2$$

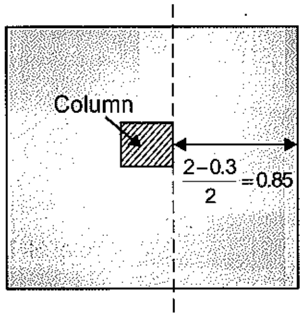
$$k_s = 0.5 + \beta \not> 1 = 0.5 + 1, \not> \left[\beta = \frac{0.3}{0.3} = 1\right]$$

$$\Rightarrow \quad K_s = 1$$

$$\therefore \quad \tau_{v_p} < K_s \tau_{c_p}$$

$$\therefore \quad \frac{\{4-(0.3+d)^2\}\times 225}{4(0.3+d)d}\times 10^{-3} < 1.118 \times 1$$

$$\therefore \quad \boxed{d > 0.303\text{m}}$$



Critical section for bending

(iii) For flexure

$$\text{BM} = \frac{wl^2}{2}\times 2 = \frac{225\times(0.85)^2}{2}\times 2$$

$$= 162.562 \text{ kNm.}$$

For a under reinforced section $M_u \le M_{u_{lim}}$

$$162.562 \times 10^6 \le 0.36\ f_{ck}\ x_{u_{lim}}\ b\ [d - 0.42\ x_{u_{lim}}]$$

$$162.562 \times 10^6 \le 0.36 \times 25 \times 0.48 \times d \times 2000\ [d - 0.42 \times 0.48d]$$

$$d \ge 153.51 \text{ mm}$$

Hence provide a effective depth of footing slab d = 350 mm,

Area of steel calculation

$$M_u = 0.87\ f_y\ A_{st}\left[d - \frac{0.42\times 0.87 f_y \times A_{st}}{0.36 f_{ck} b}\right]$$

$$\therefore \quad 162.562\times 10^6 = 0.87\times 415\times A_{st}\left[350 - \frac{0.42\times 0.87\times 415\times A_{st}}{0.36\times 25\times 2000}\right]$$

$$\therefore \quad 450247.88 = A_{st}\ [350 - 0.010531\ A_{st}]$$

$$\therefore \quad A_{st} = 1340.5\text{mm}$$

$\therefore$ No. of 12ϕ bars are required $= \dfrac{1340.5}{\pi\times 6^2} = 11.85 \simeq 12\text{no.}$

$\therefore$ % of steel provided $= \dfrac{12\times \pi/4\times 12^2}{2000\times 350}\times 100 = 0.194\% < 0.25\%$

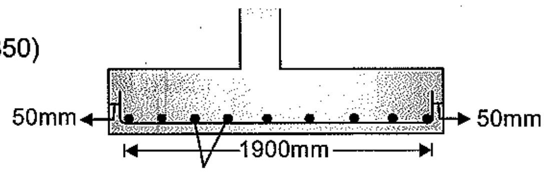
(Note that 0.25% steel has been adopted while checking for one way shear)

Let us provide % of steel = 0.25%

$$A_{st} = \frac{0.25}{100}\ (2000 \times 350)$$

$$\therefore \quad A_{st} = 1750 \text{ mm}^2$$

$\therefore$ 12mmϕ bars provided $= 15.47 \approx 16\text{no.}$



16mm ϕ bar ≅120mm c/c spacing

Thus reinforcement is greater than 0.12% OK

$\therefore$ Spacing will be $= \dfrac{2000 - 50\times2 - (12\times2) - 6\times2}{15} = \dfrac{1864}{15} = 124.26$ mm

Spacing $\not> 3d$ i.e. 3×350 mm

$\not> 300$ mm

Hence spacing is OK.

Check for development length:

Critical section for checking the development length of slab reinforcement is at the face of column development length req. for Fe 415 grade HYSD been and M 25 grade concrete

$$= \frac{0.87 f_y \phi}{4\tau_{bd}}$$

$$= \frac{0.87\times415\phi}{4\times1.6\times1.4} \qquad [\tau_{bd} \text{ for M25 grade concrete} = 1.4 \text{ N/mm}^2]$$

$$= 40.296\ \phi \simeq 41\phi$$

$$= 41 \times 12 = 492 \text{ mm}$$

$$\text{Available length} = \frac{2000-300}{2} = 850 \text{ mm}$$

$\Rightarrow$ 850 > 492 mm KM

As the no. of beam in the column is not given. Hence design of dowel bars is not possible.

Q-4: ***Design a square footing (bending and shear only) for a column load of 1400 kN at service from a 400mm square column containing 8 nos. 20mm bars. The bearing capacity of soil is 100 kN/m² at 1m depth below ground level. The unit weight of the Earth is 20 kN/m³. Use M 25 grade concrete and Fe 415 grade steel, load factor = 1.5. Show reinforcement details. Shear strength of concrete = 0.35 MPa.***

[20 Marks, ESE-2017]

Sol: **Given:**

Column load at service = 1400 kN

Soil bearing capacity = 100 kN/m² = q_g

D_f = 1m

Grade of steel and concrete are Fe 415 and M25 respectively.

Let the side of square footing = B

$$\Rightarrow \quad B \times B = \frac{1400\times1.1}{100} = 15.4 \text{ m}^2$$

(10% increase in load due to self weight of footing)

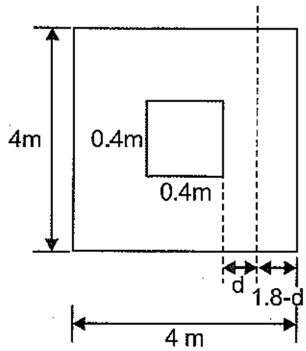
$$\Rightarrow \quad B = \sqrt{15.4} = 3.92 \text{ m}$$

Take B = 4 m

$$\text{Factored load} = w_u = \frac{1400\times1.5}{4^2} = 131.25 \text{ kN/m}^2$$

(To decide about size of footing, we work with service load condition and for design of footing we work with factored load)

One way shear



$$\tau_{v_1} = \frac{V_u}{bd} = \frac{w_u \times 4 \times (1.8-d)}{4d}$$

$$= 131.25\,\frac{(1.8-d)}{d}\ \text{kN/m}^2$$

$\tau_v < K\tau_c$ for safety, assuming K = 1

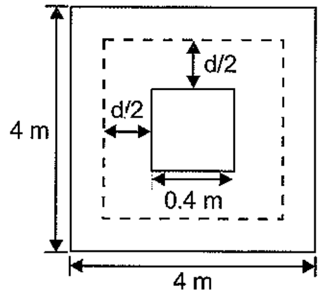
$\tau_c = 0.35 \times 10^3$ kN/m²

$$\Rightarrow \quad \frac{131.25\,(1.8-d)}{d} < 350$$

$\Rightarrow \quad d \geq 0.4909$ m

or $\quad d \geq 491$ mm

Two way shear



Punching shear stress

$$\tau_{v_2} = \frac{[4\times 4-(0.4+d)^2)]\times w_u}{4\,(0.4+d)\times 4}$$

$$= \frac{[16-(0.4+d)^2]\times 131.25}{4\,(0.4+d)\times d}$$

$\tau_{v_1} \not> K_S\tau_{c'}$

$K_S = 0.5+\beta,\ \beta = \frac{4}{4} = 1 \Rightarrow K_s = 1.5 \leq 1$ so $K_s = 1$

$\tau_C{}' = 0.25\sqrt{f_{ck}} = 0.25\sqrt{25} = 1.25\ \text{N/mm}^2$

$= 1250$ kN/m²

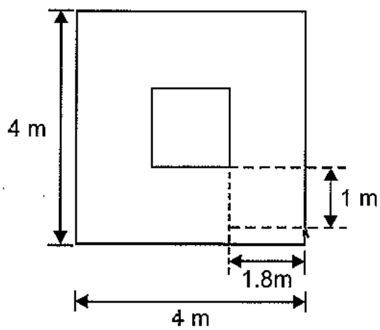
$$\therefore \quad \frac{[16-(0.4+d)^2]131.25}{4(0.4+d)\times d} \leq 1250$$

$d \geq 0.4636$ mm

$d \geq 463.6$ mm

Bending moment criteria

Critical section is at column face



$$M_u = w_u \times 1 \times 1.8 \times \frac{1.8}{2}$$

$$= 131.25 \times \frac{1.8^2}{2}$$

$= 212.625$ kN-m/m

$M_u \leq 0.138\, f_{ck}\, bd^2$ (For Fe415)

$\Rightarrow \quad 212.625 \times 10^6 \leq 0.138 \times 25 \times 1000 \times d^2 \Rightarrow d \geq 248.25$ mm

$\Rightarrow$ Adopt effective depth = 500 mm

Reinforcement calculation

$$M_u = 0.87\ f_y\ Ast\left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$$

$$\Rightarrow \quad 212.625 \times 10^6 = 0.87 \times 415 \times A_{st}\left(500 - \frac{415 \times A_{st}}{25 \times 1000}\right)$$

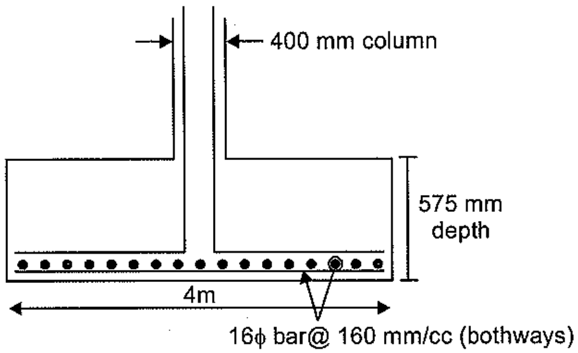
$$\Rightarrow \quad A_{st} = 1227.86\ \text{mm}^2 \text{ for 1 metre width (use } 16\text{mm}\phi\text{bar)}$$

$$= 28892.61\ \text{mm}^2 \text{ (higher value neglected)}$$

$$\text{Spacing} = \frac{1000 \times \frac{\pi}{4} \times 16^2}{1227.86} = 163.75\ \text{mm}$$

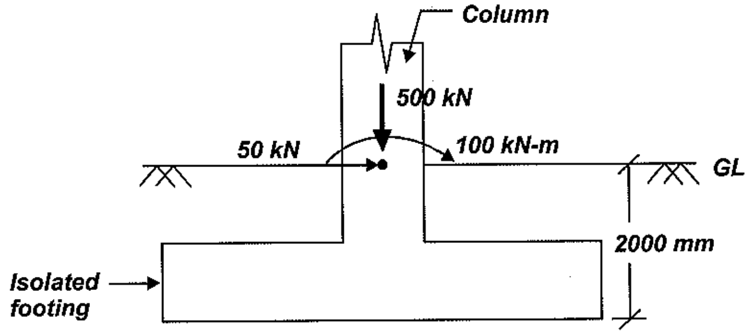
$\therefore$ Provide 16 mmϕ bar @ 160 mm spacing

Total depth of footing = $500 + 50 + \frac{16}{2} + 16 = 574$ mm $\approx$ 575 mm (Taking clear cover = 50 mm and 16ϕ bar)



Reinforcement detail of the footing

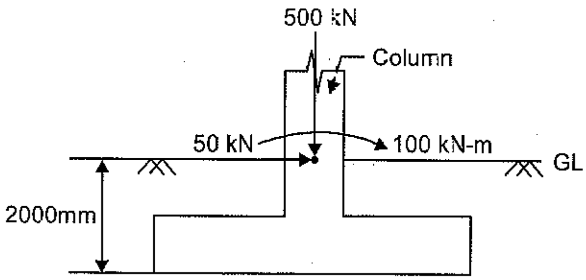
Q-5: ***An isolated footing is transferring load from a column (300 mm × 300 mm) as shown in figure. Arrange the plan dimensions of footing so that there will be uniform soil pressure intensity. Following parameters may be used.***



1. ***Column size : 300 mm × 300 mm***
2. ***Safe Bearing capacity : 100 kN/m²***
3. ***Dead weight of footing and soil weight over it may be taken as 10% of vertical load of column.***

[12 Marks, ESE-2018]

Sol:



Assuming given loads are service loads

Moment at the base of footing due to horizontal force = 50 × 2 = 100 kNm

Total moment at the base of footing = 100 + 100 = 200 kNm

Footing area required = $\frac{500 \times 1.1}{100}$ (Accounting dead weight of footing and soil 10% of vertical load)

$$A = 5.5 \text{ m}^2$$

Footing dimension,

$$B = \sqrt{A} = \sqrt{5.5}$$

$$B = 2.345 \text{ m}$$

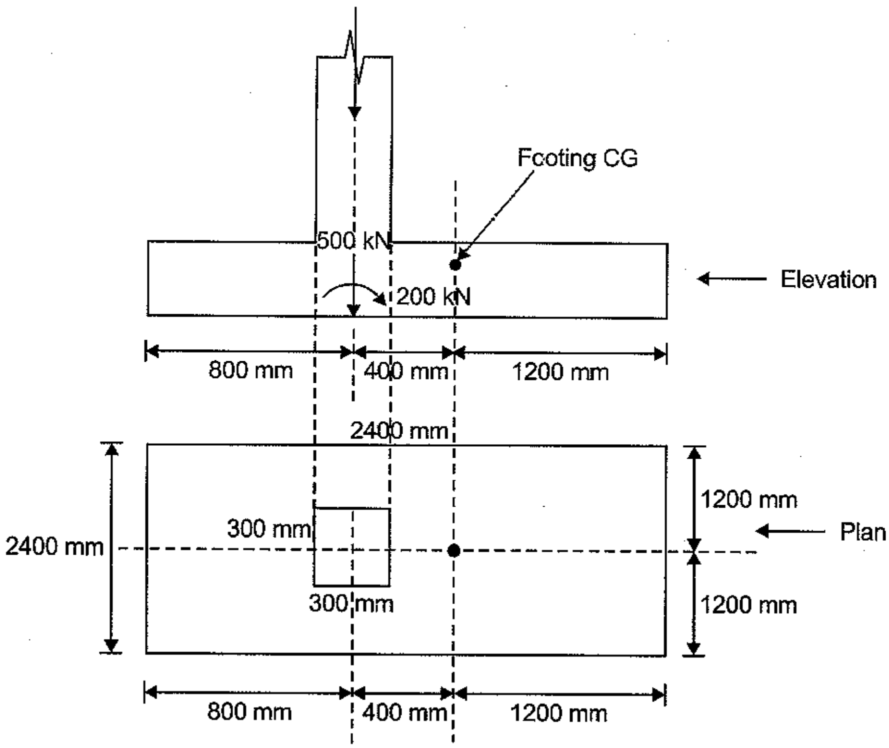
Take B = 2.4 m

Eccentricity required in column and footing centroid so that there is uniform soil pressure intensity.

$$e = \frac{M}{P} = \frac{200}{500} = 0.4 \text{ m} = 400 \text{ mm}$$

(No need to consider dead load of footing and soil, because that is already uniformly distributed)

Arranging plan dimensions:



Q-6: ***A combined footing is to be provided for two columns (size 300 × 300) spaced at 3 m c/c. Axial load on each of the columns is 350 kN. The width of the footing is fixed at 1.4 m. A foundation beam of 400 m × 800 mm is provided along the length. Design the foundation slab using M25 and Fe500. Assume the thickness of the slab varies from 250 mm to 150 mm. Also show the reinforcement detail (in cross-section) of the footing slab. Use limit state method of design. Bearing capacity of the soil is 100 kN/m².***

Given:

$\frac{M_u}{bd^2}$	0.3	0.35	0.4	0.45	0.5
p_t	0.070	0.082	0.094	0.106	0.118

p_t	< 0.15	0.25	0.5	0.75	1.0
τ_c, MPa	0.29	0.36	0.49	0.57	0.64

[20 Marks, ESE-2020]

Sol: **Data given:**

M25 $\Rightarrow f_{ck} = 25$ MPa

Fe 500 $\Rightarrow f_y = 500$ MPa

$f_b = 100$ kN/m² (soil bearing capacity)

Column A: 300 mm × 300 mm

Column B: 300 mm × 300 mm

Centre to centre spacing of column = 3 m

P_A = 350 kN and P_B = 350 kN

Width of footing (B) = 1.4 m

Foundation beam along length = 1.4 m

Foundation beam along = 400 mm × 800 mm

Required: To design foundation slab.

Total load on the two columns = (2 × 350)

= 700 kN

Approximate weight of foundation (10% of column load) = 70 kN

Total load transmitted to the soil = (70 + 700)

= 770 kN

Bearing capacity of soil (f_b) = 100 kN/m²

$$\therefore \quad \text{Area of foundation } (A_f) = \left(\frac{770}{100}\right) \text{m}^2 = 7.7 \text{ m}^2$$

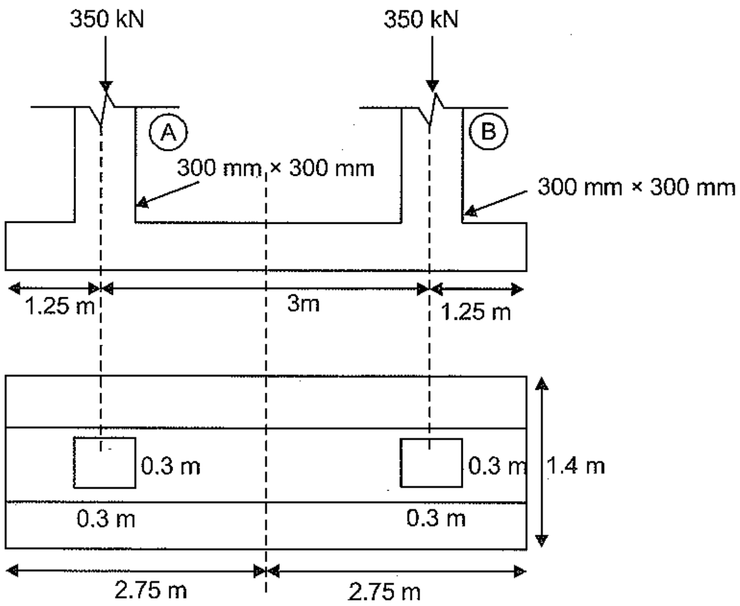
Width of footing (B) = 1.4 m

$$\therefore \quad \text{Length of footing } (L) = \left(\frac{7.7}{1.4}\right) = 5.5 \text{ m}$$

$$\text{Distance of the resultant column load from the axis of column A} = \left(\frac{350 \times 3}{2 \times 350}\right) = 1.5 \text{ m}$$

For the condition of uniform distribution of load to the soil, the line of action of resultant column load must pass through the centroid of the foundation plan. For this condition, cantilever projection on the left side of column A = $\left(\frac{5.5}{2} - 1.5\right)$ = 1.2 m

Similarly, cantilever projection on the right side of the column B = 1.25 m (because symmetric load).



Position of the two columns and the foundation plane.

Net upward pressure intensity on footing = P

$$= \frac{(2\times 350)}{1.4\times 5.5}$$

$$= 90.91 \text{ kN/m}^2$$

Design of footing slab:

Transverse projection of footing slab (a) $= \frac{(1.4-0.3)}{2}$

$= 0.55$ m

(a) Depth from bending compression:

consider a 1m wide strip of footing slab.

$$\text{BM per metre width} = \frac{90.91\times(0.55)^2}{2}$$

$$= 13.75 \text{ kNm}$$

$$\text{Factored moment, } M_u = (1.5 \times 13.75) = 20.625 \text{ kNm}$$

Equating M_{ulim} to M_u,

$$20.625 \times 10^6 = 0.133 \text{ fck bd}^2 = 0.133 \times 25 \times d\ 1000 \times d^2$$

[For Fe 500, $M_{ul\ im}$, = 0.133 fck bd^2]

$$\Rightarrow \quad \boxed{d = 78.76 \text{ mm}}$$

But, the depth based on shear consideration is nearly double than that due to moment considerations. At the face of strap beam.

Hence, adopt effective depth (d_s) = 250 mm

and overall depth (D_s) = 300 mm

and at edge,

adopt effective depth (d_{se}) = 150 mm

and overall depth (D_{se}) = 200 mm

Considering section as under reinforced,

$$M_u = 0.87f_yA_{st}d_s\left[1-\frac{A_{st}f_y}{bd_sf_{ck}}\right]$$

$$\Rightarrow \quad 20.625\times10^6 = 0.87\times500\times A_{st}\times250\left[1-\frac{A_{st}\times500}{10^3\times250\times25}\right]$$

On solving, we get

$$A_{st} = 192.62\ \text{mm}^2$$

But,

$$A_{st,\ min} = (0.12\%\ \text{of bD})$$

$$= \left[\frac{0.12}{100}\times1000\times300\right]$$

$$= 360\ \text{mm}^2$$

Adopt 10 mm diameter bars at spacing of 200 mm centre to centre as main and distribution reinforcement.

$$A_{st}\ \text{provided} = \left[\frac{\pi}{4}\times10^2\times\frac{1000}{200}\right] = 393\,\text{mm}^2$$

Check for Shear Stress :

$$\text{Design shear stress} = V_u = (a-d)\times p\times1.5$$

$$= (0.55-0.25)\times90.91\times1.5$$

$$= 40.91\ \text{kN}$$

$$\tau_V = \left(\frac{V_u}{bd'}\right),$$

where d' is the effective depth at critical section (i.e. at d_s, distance from face of support)

$$= \frac{40.91\times10^3}{10^3\times\left[250-\frac{(250-150)}{550}\times250\right]}$$

$$= 0.2\ \text{N/mm}^2$$

$$P_t = \left(\frac{100A_{st}}{bd'}\right) = \left[\frac{100\times393}{10^3\times\left[250-\frac{(250-150)}{550}\times250\right]}\right]$$

$$= 0.192\%$$

For P_t = 0.192%, $\tau_c = 0.29+\frac{(0.36-0.29)}{(0.25-0.15)}\times(0.192-0.15)$

$$= 0.319\ \text{N/mm}^2$$

$$\Rightarrow \quad K_s\tau_c = (1\times0.319) = 0.319\text{N/mm}^2 > 0.2\text{N/mm}^2$$

Hence, shear stresses are within permissible limits.

Design of Strap Beam :

$$\text{Design ultimate load on beam} = w_u$$

$$= (p \times 1.5 \times 1.4)$$

$$= (90.91 \times 1.5 \times 1.4)$$

$$= 190.911 \text{ kN/m}$$

Neglecting the small cantilever portion of the beam,

$$M_u = \frac{w_u \ell^2}{8} = \frac{190 \times (3)^2}{8}$$

$$= 213.75 \text{ kNm}$$

$$V_u = 0.50.5\, w_u \ell = (0.5 \times 190.911 \times 3)$$

$$= 286.37 \text{ kN}$$

Given, size of strap beam as 400 mm × 800 mm (i.e. b = 400 mm , d_b = 500 mm)

$$M_u = 0.87 \times f_y \times A_{st} \times d_b \left[1 - \frac{A_{st} f_y}{b \times d_b \times f_{ck}}\right]$$

$$\Rightarrow \quad 213.75 \times 10^6 = 0.87 \times 500 \times A_{st} \times 800 \left[1 - \frac{A_{st} \times 500}{400 \times 800 \times 25}\right]$$

On solving, we get

$$\Rightarrow \quad A_{st} = 639.81 \text{ mm}^2$$

$$\frac{A_{st,\,min}}{b \times d_b} = \frac{0.85}{f_y} \Rightarrow A_{st,min} = \left(\frac{400 \times 800 \times 0.85}{500}\right) = 544 m^2 < A_{st}$$

Provide 4 bars of 16 mm dia. (A_{st} provided = 804 mm²)

Design for Shear :

$$\tau_V = \left(\frac{V_u}{b \times d_b}\right) = \left(\frac{286.37 \times 10^3}{400 \times 800}\right) = 0.895 \text{N/mm}^2$$

$$p_t = \left(\frac{100 \times A_{st}}{b \times d_b}\right) = \left(\frac{100 \times 804}{400 \times 800}\right) = 0.25\%$$

$$\text{From table for } p_t = 0.25\%$$

$$\tau_c = 0.36 \text{ N/mm}^2 < \tau_V$$

Hence, shear reinforcements are required to resist the balance shear force computed as

$$V_{us} = \left[286.37 - (0.36 \times 400 \times 800) \times 10^{-3}\right]$$

$$= 171.17 \text{ kN}$$

Using 8 mm diameter 4 legged stirrups, the spacing is

$$S_V = \left(\frac{0.87 \times 415 \times 4 \times 50 \times 800}{171.17 \times 10^3}\right), f_y \not> 415 \text{ MPa}$$

$$= 337.49 \text{ mm}$$

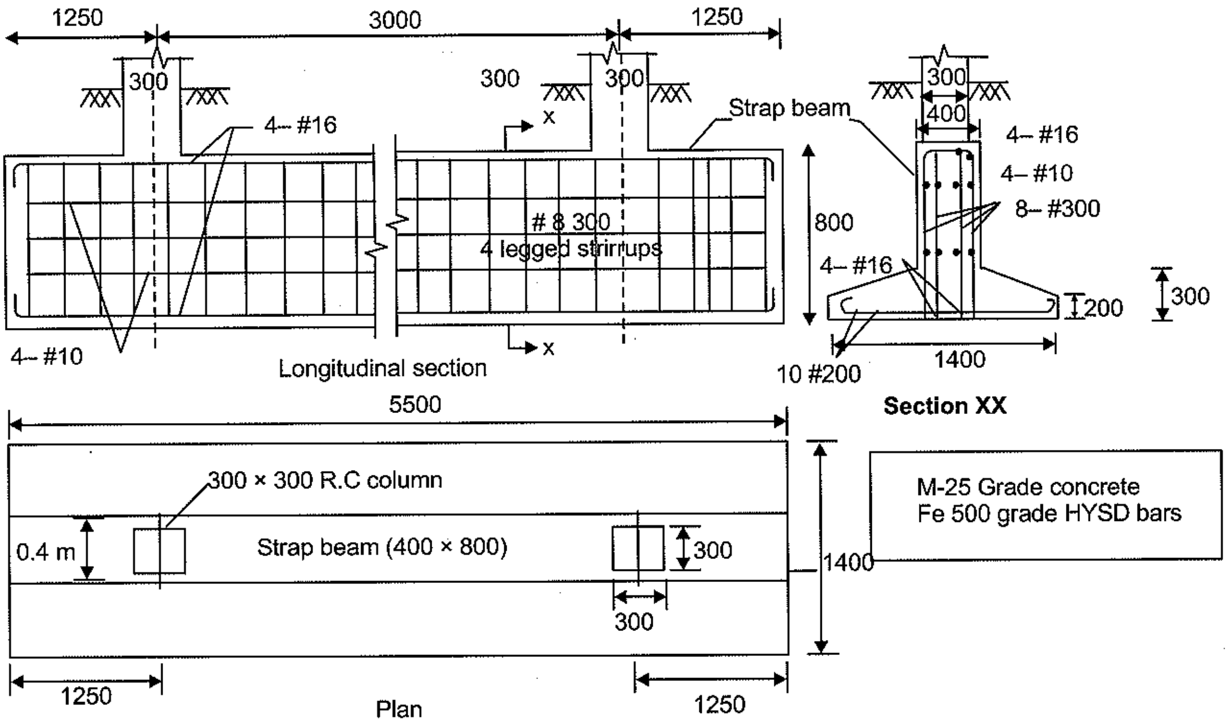
$$\text{Adopt, } S_V = 300 \text{ mm (centre to centre)}$$

$$\text{Check,} \quad S_V \not> 300 \text{ mm}$$

$$\not> 0.75 \times 800 = 600 \text{ mm} \quad \text{OK}$$

Adopt 8 mm diameter 4 legged stirrups at 250 mm centre to centre spacing in strap beam.

Also, side face reinforcement of 0.1% of Web area distributed on both faces as depth of beam is greater than 750 mm.



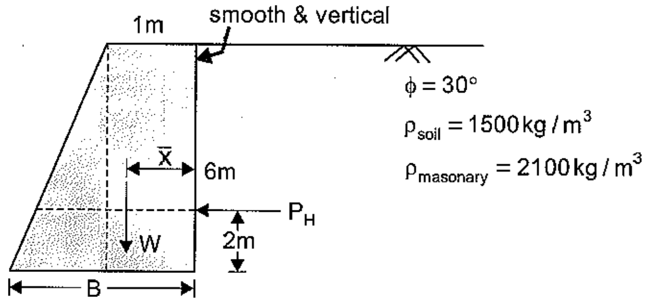
CHAPTER 9

DESIGN OF RETAINING WALLS

Q-1: ***A masonry retaining wall, trapezoidal in section is 1.0 m wide at top. Its earth retaining face is vertical and smooth. The retained earth, having density of 1500 kg/m³ and angle of shearing resistance of 30°, is level with the top of the wall. Height of wall is 6.0 m. Assuming density of masonry as 2100 kg/m³, calculate the minimum bottom width of the wall so that no tension is inducted at the base. Also calculate the maximum base pressure at this width.***

[15 Marks, ESE-1996]

Sol:



We have to calculate minimum bottom width of the wall so that no tension is induced at base. Also we have to calculate max base pressure.

The forces acting on the retaining wall are P_H, W and R where

P_H = Active earth thrust

W = Wt. of wall

R = Reaction from ground

$$P_H = \frac{1}{2}K_A\gamma H^2 = \frac{1}{2}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)\times 1500\times g\times(6)^2$$

$$= \frac{1}{2}\left(\frac{1-\sin 30^\circ}{1+\sin 30^\circ}\right)\times 1500\times 9.81\times 36\frac{N}{m\,width}$$

$$= 88290\,N/m\,width = 88.29\text{ kN/m width}$$

P_H will be acting at 2m from base.

$$W = \frac{B+1}{2}\times 6\times 1\times 2100\times 9.81\text{ N/meter width}$$

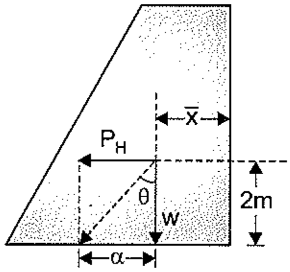
$$= 61.803(B+1)\text{ kN/meter width}$$

$$\bar{x} = \frac{1\times 6\times 0.5 + \frac{1}{2}(B-1)\times 6\times\left[\frac{(B-1)}{3}+1\right]}{\frac{B+1}{2}\times 6}$$

$$= \frac{3 + \frac{3(B-1)(B+2)}{3}}{3(B+1)}$$

$$\bar{x} = \frac{3+(B-1)(B+2)}{3(B+1)}$$

$$\tan\theta = \frac{\alpha}{2} = \frac{P_h}{w}$$



For No. tension; $e \le \frac{B}{6}$

$$\Rightarrow \quad \alpha + \bar{x} - \frac{B}{2} \le \frac{B}{6}$$

$$\alpha + \bar{x} \le \frac{B}{6} + \frac{B}{2}$$

$$\alpha + \bar{x} \le \frac{4B}{6}$$

$$\alpha + \bar{x} \le \frac{2B}{3}$$

$$\frac{2P_H}{w} + \bar{x} \le \frac{2B}{3}$$

$$\frac{2 \times 88.29}{61.803(B+1)} + \frac{3+(B-1)(B+2)}{3(B+1)} \le \frac{2B}{3}$$

$$\frac{20}{7} + \frac{3+(B^2+B-2)}{3} \le \frac{2(B)(B+1)}{3}$$

$$\frac{60}{7} + 3 + B^2 + B - 2 \le 2B^2 + 2B$$

$$B^2 + B - \frac{67}{7} \ge 0$$

$$B > \frac{-1 \pm \sqrt{1^2 + 4 \times \frac{67}{7} \times 1}}{2}$$

$$(B - 2.634)\ (B + 3.634) > 0$$

$$\Rightarrow \quad B \ge 2.634 \text{ m}$$

Adopting $\boxed{B = 2.634 \text{ m}}$

$$W = 61.803\ (3.634) = 224.592 \text{ kN}$$

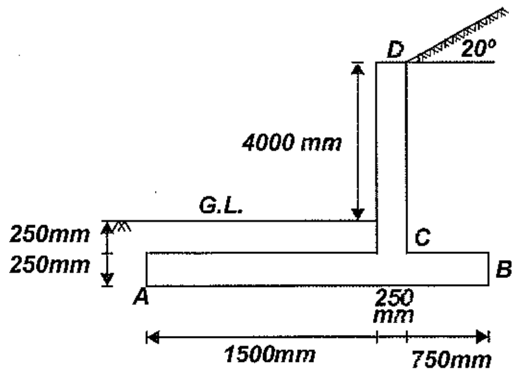
$$\bar{x} = 0.970 \text{ m}$$

$$e = \alpha + \bar{x} - B/2 = \frac{2 \times 88.29}{224.592} + 0.97 - \frac{2.634}{2} = 0.439\text{m}$$

This can also be directly calculated as $e = \frac{B}{6}$

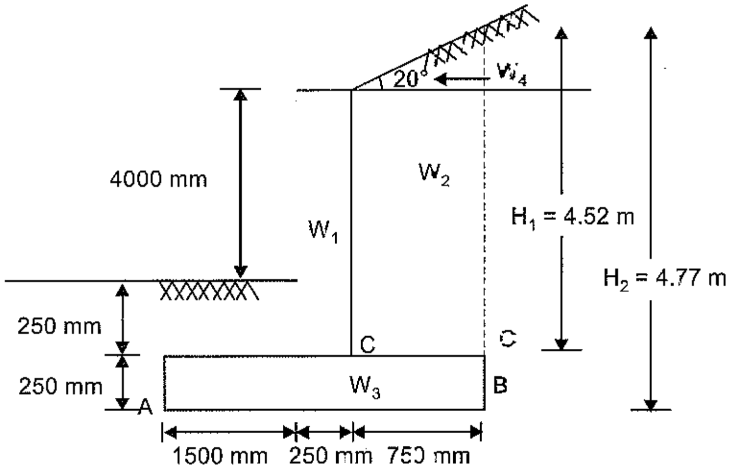
$$\Rightarrow \quad \text{Max compressive stress} = \frac{2W}{B} = \frac{2 \times 224.592}{2.634 \times 1} = 170 \frac{\text{kN}}{\text{m}^2}$$

Q-2: ***Check the safety and stability of the retaining wall shown in Figure. Safe bearing capacity of soil = 80 kN/m², coefficient of friction between soil and concrete = 0.55, angle of repose of earth = 35°, density of earth = 19 kN/m³.***



[20 Marks, ESE-2017]

Sol:



Given: Safe bearing pressure $= 80\ \text{kN/m}^2$

$$\mu = 0.55$$

$$\phi = 35°$$

$$\gamma_{soil} = 19\ \text{kN/m}^3$$

Active earth pressure, $p_a = \frac{1}{2}K_a\gamma H^2$

Active earth pressure coefficient, $K_a = \cos\beta \times \dfrac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$

$$= \cos 20° \times \frac{\cos 20° - \sqrt{\cos^2 20° - \cos^2 35°}}{\cos 20° + \sqrt{\cos^2 20° - \cos^2 35°}} = 0.322$$

$$H_2 = 4 + 0.25 + 0.25 + 0.75 \tan 20° = 4.77\ \text{m}$$

$$P_a = \frac{1}{2} \times 0.322 \times 19 \times 4.77^2 = 69.6\ \text{kN/m}$$

Horizontal component, $P_{aH} = P_a \cos 20° = 65.4\ \text{kN/m}$

Vertical component, $P_{av} = P_a \sin 20° = 23.8\ \text{kN/m}$

S.No	Description	Forces (kN/m)		Lever (m)	Moment about toe	
		Vertical (kN/m)	Horizontal (kN/m)		Clockwise	Counter clockwise
1	$w_1 = 0.25 \times 4.25 \times 25$	26.56		1.625	43.16	
2	$w_2 = 4.25 \times 0.75 \times 19$	60.56		2.125	128.69	
3	$w_3 = 2.5\times 0.25 \times 25$	15.625		1.25	19.53	
4	$w_4 = \frac{1}{2}\times 0.75\times 0.75\tan(20)\times 19$	1.94		2.25	4.365	
5	P_V	23.8		2.5	59.5	
6	P_H		65.4	1.59		103.986
	Total	128.485	65.4		255.245	103.986

$$\text{FOS against overturnning} = \frac{255.245}{103.986} = 2.455 > 2$$

$\therefore$ Safe

$$\text{FOS against sliding} = \frac{\mu\Sigma w}{pH} = \frac{0.55\times 128.485}{65.4} = 1.08 < 1.5$$

Not safe in sliding as per design standards.

$$\bar{x} = \frac{\Sigma M}{\Sigma V} = \frac{255.245 - 103.986}{128.485} = 1.177 \text{ m}$$

$$e = \frac{b}{2} - \bar{x} = \frac{2.5}{2} - 1.177 = 0.073 < \frac{b}{6}$$

$\because$ $$e < \frac{b}{6} \Rightarrow \text{No tension condition.}$$

$$\sigma_{max} = \frac{\Sigma V}{b}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{128.485}{2.5}\left(1 + \frac{6\times 0.073}{2.5}\right)$$

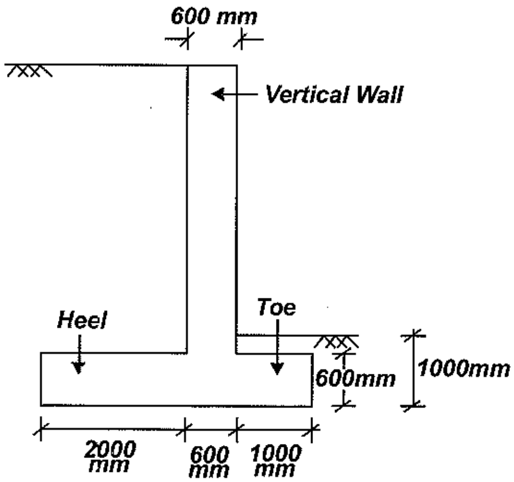
$$= 60.4 \text{ kN/m}^2 < 80 \text{ kN/m}^2 \text{ (Sbc of soil = 80 kN/m}^2\text{)}$$

$$P_{min} = \frac{\Sigma V}{b}\left(1 - \frac{6e}{b}\right) = \frac{128.485}{2.5}\left(1 - \frac{6\times 0.073}{2.5}\right) = 42.4 \text{ kN/m}^2$$

$$\text{Factor of safety against bearing capacity failure} = \frac{80}{60.4} = 1.32$$

Theoretically, the retaining wall is safe, provision of shear key must be provided to increase (FOS) against sliding.

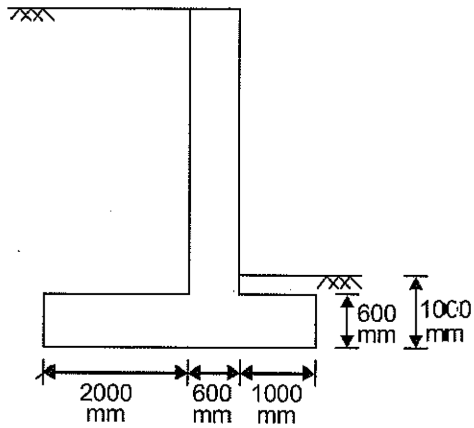
Q-3: ***Design and sketch the reinforcement in vertical wall, Toe slab and Heel slab (for maximum Bending moment and Maximum shear force only) for the cantilever retaining structure shown in figure. Following parameters may be used for design and sketch :***



1. ***Maximum bending moment in vertical wall : 400 kN-m, Toe : 160 kN-m and in Heel : 200 kN-m***
2. ***Maximum shear force in vertical wall : 200 kN, Toe : 120 kN and in Heel : 160 kN***
3. ***Grade of concrete M-20***
4. ***Grade of reinforcing bar Fe-415***
5. ***Nominal concrete cover 20 mm.***
6. ***Development length : 47 × diameter of bar***
7. ***Diameter of main bar in vertical wall : 25 mm, in Heel : 25 mm and in Toe : 25 mm***
8. ***($M_{u, lim}/f_{ck}\, bd^2$) = 0.138***
9. ***($p_{t, lim}\, f_y/f_{ck}$) = 19.82***
10. ***Shear strength of concrete = 0.60 N/mm²***

[20 Marks, ESE-2018]

Sol:



	Vertical wall	Toe slab	Heel slab
BM	400 kN – m	160 kN – m	200 kN – m
SF	200 kN	120 kN	160 kN
Main bar reinforcement dia	25 mm	25 mm	25 mm

$$\text{Factored BM, } M_u = 1.5 \times 400 = 600 \text{ kNm}$$

$$\text{Effective depth, d} = 600 - 20 - \frac{25}{2} = 567.5 \text{ mm}$$

Taking 1000 mm width of stem:

$$M_{u\ lim} = 0.138 f_{ck} bd^2$$
$$= 0.138 \times 20 \times 1000 \times 567.5^2$$
$$= 888.88 \text{ kNm} > M_u$$

So under-reinforced section

$$A_{st} = \frac{0.5 f_{ck} bd}{f_y} \times \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} bd^2}}\right]$$

$$A_{st} = \frac{0.5 \times 20}{415} \times 1000 \times 567.5 \times \left[1 - \sqrt{1 - \frac{4.6 \times 600 \times 10^6}{20 \times 1000 \times 567.5^2}}\right]$$

$$A_{st} = 3337 \text{ mm}^2$$

$$\text{Spacing} = \frac{1000}{\frac{3337}{\frac{\pi}{4} \times 25^2}}$$

$$= 147.02 \text{ mm} \not> 300 \text{ mm}$$

$$\not> 3d = 3 \times 567.5 = 1702.5 \text{ mm}$$

∴ Provide 25 mm dia bars @ 140 mm c/c

$$P_{t\ lim} = 19.82 \times \frac{20}{415} = 0.955$$

$$P_{t,\ provided} = \frac{A_{st\ provided}}{bd} \times 100$$

$$= \frac{1000 \times \frac{\pi}{4} \times 25^2}{140 \times 1000 \times 567.5} \times 100$$

$$= 0.617 < P_{t\ lim} \text{ (Hence OK)}$$

$$A_{st,\ min} = 0.12 \times bD$$

$$= 0.12 \times \frac{1000 \times 600}{100}$$

$$= 720 \text{ mm}^2 < A_{st\ provided}$$

$$\text{Distribution reinforcement} = 0.12\% \ bD = 720 \text{ mm}^2$$

Providing 10 mm dia bars:

$$\text{Spacing} = \frac{1000}{\frac{720}{\frac{\pi}{4} \times 10^2}}$$

$$= 109.03 \text{ mm} \not> 300 \text{ mm}$$

$$\not> 5d = 5 \times 567.5 = 2837.5 \text{ mm}$$

Provide 10 mm dia bars @ 100 mm c/c on outer face of wall in both ways as distribution reinforcement.

Provide temperature reinforcement ϕ 10 mm bars @ 200 mm c/c on inner face.

Note: More distribution reinforcement is required on the outer face as compared to inner face or soil retained face.

$$\text{Shear stress} = \tau_v = \frac{V_u}{bd}$$

$$= \frac{1.5 \times 200 \times 10^3}{1000 \times 567.5} = 0.529 \text{ N/mm}^2$$

$$\tau_v = \tau_c = 0.6 \quad \text{(safe in shear)}$$

So no shear reinforcement required.

Design of reinforcement in toe slab:

$$\text{Design BM, } M_u = 1.5 \times 160 = 240 \text{ kNm}$$

So,

$$A_{st} = \frac{0.5 f_{ck} \times bd}{f_y} \times \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} bd^2}}\right]$$

$$A_{st} = 0.5 \times \frac{20}{415} \times 1000 \times 567.5 \times \left[1 - \sqrt{1 - \frac{4.6 \times 240 \times 10^6}{20 \times 1000 \times 567.5^2}}\right]$$

$$A_{st} = 1226.96 \text{ mm}^2$$

$$\text{Spacing} = \frac{1000}{\dfrac{1227}{\dfrac{\pi}{4} \times 25^2}}$$

$$= 399.86 \text{ mm} \ngtr 300 \text{ mm}$$

$$\ngtr 3d$$

So provide 25 mm dia bars @ 300 mm c/c

Check for shear:

$$\tau_v = \frac{1.5 \times 120 \times 10^3}{1000 \times 567.5} = 0.317 < 0.6 \text{ N/mm}^2 \quad \text{(Hence safe in shear)}$$

Design of reinforcement in heel slab:

$$\text{Design BM, } M_u = 1.5 \times 200 = 300 \text{ kNm}$$

So,

$$A_{st} = 0.5 \times \frac{20}{415} \times 1000 \times 567.5 \times \left[1 - \sqrt{1 - \frac{4.6 \times 300 \times 10^6}{20 \times 1000 \times 567.5^2}}\right]$$

$$A_{st} = 1553.09 \text{ m}^2$$

$$\text{Spacing} = \frac{1000}{\dfrac{1554}{\dfrac{\pi}{4} \times 25^2}}$$

$$= 315.72 \text{ mm} \ngtr 300 \text{ mm}$$

$$\ngtr 3d$$

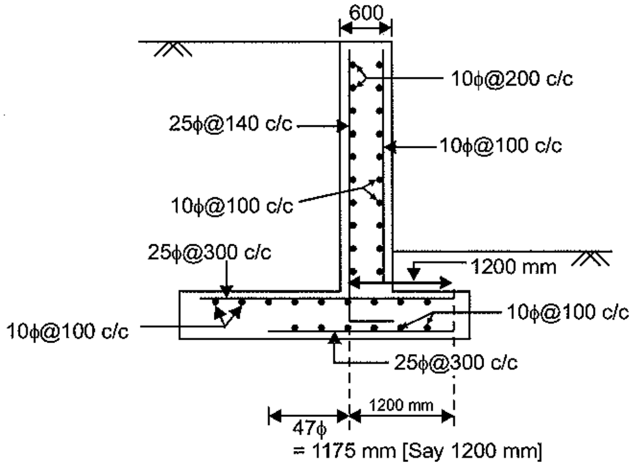
Provide 25 mm dia bars @ 300 mm c/c

Check for shear:

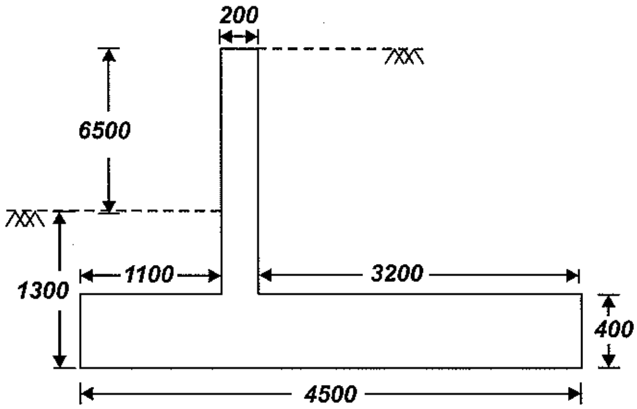
$$\tau_v = \frac{V_u}{bd} = \frac{1.5 \times 160 \times 10^3}{1000 \times 567.5}$$

$$= 0.423 < 0.6 \text{ N/mm}^2 \quad \text{(Given)}$$

Hence, heel is also safe in shear.



Q-4: ***Design the counter forts of a retaining wall to retain earth for a height of 6.5 m above the ground level. The unit weight of soil is 16 kN/m³ and the angle of repose of soil is 30°. the safe bearing capacity of soil is 180 kN/m². Use M 20 grade concrete and steel of grade Fe 415. The cross-section of the retaining wall is given below. The spacing of counterfort is taken as 3.5 m. Assume a cover of 40 mm for counterforts.***



All dimensions are in mm.

Assume the maximum pressure at toe end is 166.05 kN/m² and the minimum pressure at the heel end is 38.92 kN/m². Sketch the reinforcement details.

[20 Marks, ESE-2019]

Sol: **Given:** h = 6.5m, γ = 16 KN/m³, $\phi = 30°$, q = 180 kN/m²

Spacing of counterfort = 3.5m

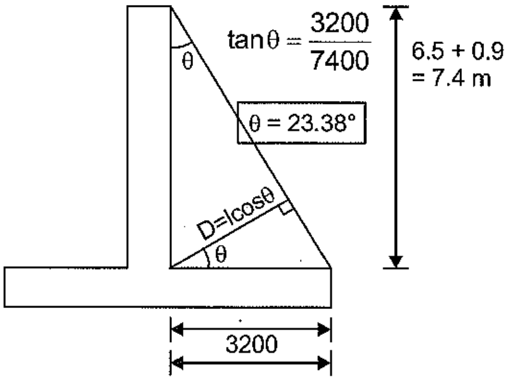
Cover = 40 mm, M20, Fe415.

Assume thickness of counterfort = 400 mm. ($\cong$ 0.05 × h = 0.05 × 7.4)

clear spacing of counterfort = 3.5 m.

Thus each counterfort receives earth pressure from a width of 3.5 + 0.4 = 3.9 m.

$$k_a = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$



$$k_a \gamma H = \frac{1}{3} \times 16 \times 7.4$$
$$= 39.47\ \text{kN/m}^2$$

$$(P)_{\text{at Base}} = k_a \gamma H = \frac{1}{3} \times 16 \times 7.4 = 39.47\ \text{kN/m}^2$$

$$\text{Factored bending moment} = 1.5 \times \frac{1}{2} \times 39.47 \times 7.4 \times \frac{7.4}{3} \times 3.9 = 2107.34\ \text{kN-m}.$$

$$\text{Factored shear force} = 1.5 \times \frac{1}{2} \times 39.47 \times 7.4 \times 3.9 = 219.06 \times 3.9 = 854.33\ \text{kN}$$

From figure $\tan\theta = \frac{3200}{7400} = 0.43,\ \boxed{\theta = 23.38°}$

$$(D)_{\text{base}} = l\cos\theta = 3200 \times \cos 23.38° = 2937.25\ \text{mm}$$

Clear cover 40 mm and 25 mm ϕ bar .

$$d = 2937.25 - 40 - 12.5 = 2884.75 \cong 2885\ \text{mm}.$$

$$A_{st} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-\frac{4.6\,M}{f_{ck}\,bd^2}}\right]bd$$

$$= \frac{20}{2 \times 415}\left[1-\sqrt{1-\frac{4.6 \times 2107.34 \times 10^6}{20 \times 400 \times 2885^2}}\right] \times 400 \times 2885$$

$$= 2104\ \text{mm}^2$$

Provide 25 mm ϕ bar

$$\text{Number of bar} = \frac{2104}{\frac{\pi}{4} \times 25^2} = 4.28 \simeq 5\ \text{bar}$$

Shear force for varying depth:

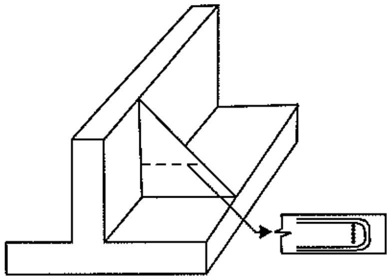
$$(V_u)_{net} = v_u - \frac{M_u}{d} \times \tan\theta$$

$$= 854.52 - \frac{2107.34}{2.885} \tan 23.38$$

$$= 538.78$$

$$(\tau_v)_{net} = \frac{538.78 \times 10^3}{400 \times 2885} = 0.47 \text{ MPa} < (\tau_c)_{max}$$

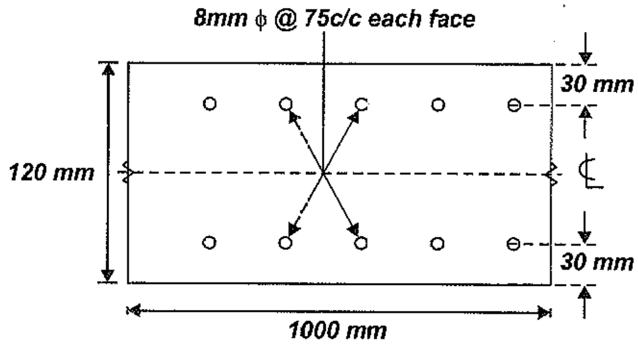
Hence safe in shear.



CHAPTER 10

DESIGN OF WATER TANKS

Q-1: ***The floor of a water tank is subjected to a direct pull of 50 kN/m and bending moment of 0.5 kNm/m in the vertical plane. A section of the floor is shown in Figure. Find the maximum stresses in concrete and steel. Assume M30 concrete. Permissible bending stress in concrete = 10 MPa. Use Fe415.***



[20 Marks, ESE-2017]

Sol: **Given that:**

$$D = 120 \text{ mm}$$

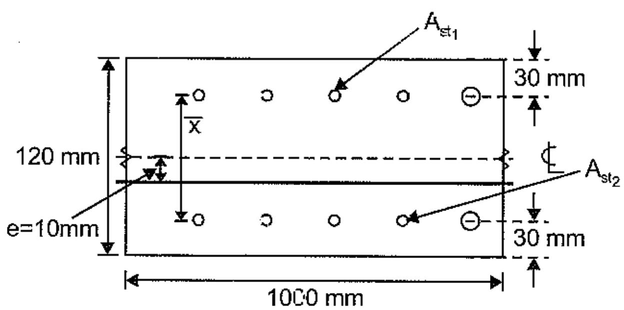
$$A_{St1} = A_{St2} = 8 \text{ mm bar @ 75 mm c/c}$$

So,

$$e = \frac{M}{T} = \frac{0.5\times10^6}{50\times10^3} = 10 \text{ mm}$$

The eccentricity lies within the section and between the two layer of reinforcement.

Taking moment about the bottom steel.



$$\sigma_{st1}\times A_{st1}\times\bar{x} = T\,(30-10)$$

where, $\quad A_{st1}(\text{for 8 mm @ 75 mm c/c }) = 50 \times \dfrac{1000}{75} \text{ mm}^2$

$$\bar{x} = (120-30-30)\text{mm}$$

$$\sigma_{st_1}\times50\times\left(\frac{1000}{75}\right)(120-30-30) = 50{,}000(30-10)$$

$$\Rightarrow \qquad \sigma_{st_1} = \frac{100 \times 10^4}{4 \times 10^4} = 25 \text{ MPa}$$

Also, $\sigma_{st_1} A_{st1} + \sigma_{st_2} A_{st2} = T$

$$\Rightarrow \qquad \sigma_{st_2} = \frac{\left(50{,}000 - 25 \times 50 \times \frac{1000}{75}\right)}{\left(50 \times \frac{1000}{75}\right) \text{mm}^2} \text{ MPa} = 50 \text{ MPa}$$

Modular ratio for M30 concrete

$$m = \frac{280}{3 \times \sigma_{cbc}} = \frac{280}{3 \times 10} = 9$$

Direct tension in concrete

$$\sigma_{ct} = \frac{T}{A_c + (m-1)A_{st}} \qquad (A_{st} = A_{st1} + A_{st2})$$

$$\sigma_{ct} = \frac{50 \times 10^3}{120 \times 1000 + (9-1) \times \left(2 \times 50 \times \frac{1000}{75}\right)}$$

$$= 0.383 \text{ MPa}$$

$$\text{Bending tension in concrete} = \frac{\sigma_{st_2}}{m} = \frac{50}{9} = 5.56 \text{ MPa} > 2 \text{ MPa}$$

So section is cracked section.

Q-2: ***A cylindrical water tank of capacity 500 m³ is resting on ground and have a free-flexible joint at base (vertical wall-base slab connection). Overall height of tank is restricted to 4.3 m (it includes a free board of 0.3 m). Design the vertical cylindrical wall of tank only. Following parameters may be used for design, if required:***

1. $\sigma_{cbc} = 10 \text{ N/mm}^2$
2. $\sigma_{cbt} = 2.0 \text{ N/mm}^2$
3. $\sigma_{ct} = 1.5 \text{ N/mm}^2$
4. $\sigma_{st} = 130 \text{ N/mm}^2$
5. $\gamma_w = 10 \text{ N/mm}^3$
6. ***Main reinforcing bar diameter : 16 mm***
7. $m = 10$

[12 Marks, ESE-2018]

Sol:

$$V = 500 \text{ m}^3$$

$$H = 4 \text{ m}$$

$$V = \frac{\pi}{4} \times D^2 \times 4$$

$$D = 12.62 \text{ m}$$

$$\text{Provide diameter of water tank} = 13 \text{ m}$$

$$\text{Maximum Hoop tension (T)} = \frac{\gamma_w HD}{2} = \frac{10 \times 4 \times 13}{2} \quad \text{[Considering height of water} \cong 4.0\text{m]}$$

$$T = 260 \text{ kN/m}$$

The given value of σ_{bc}, σ_{st} suggests M30, Fe-415 grade of concrete and steel respectively.

Design reinforcement for Hoop tension :

$$A_{st} = \frac{T}{\sigma_{st}} = \frac{260 \times 10^3}{130}$$

$$A_{st} = 2000 \text{ mm}^2$$

For 1000 mm height, spacing of reinforcement

$$= \frac{1000}{\left(\frac{2000}{\frac{\pi}{4} \times 16^2}\right)}$$

$$= 100.48 \text{ mm} \not> 300 \text{ mm}$$

$$\not> \text{Thickness of section}$$

Provide ϕ 16 bars @ 100 mm c/c as Hoop tension reinforcement

Calculation of thickness of wall :

(i) Empirical formula:

$$t = 30H + 50$$

$$t = 30 \times 4 + 50$$

$$t = 170 \text{ mm}$$

(ii) From direct tension in concrete:

$$\sigma_{ct} = \frac{T}{A}$$

$$1.5 = \frac{260 \times 10^3}{1000 \times t + (m-1)A_{st}}$$

$$1.5 = \frac{260 \times 10^3}{1000 \times t + (10-1) \times 2000}$$

$$t = 155.33 \text{ mm}$$

So, provide thickness, t = 170 mm (maximum of above two criteria)

Minimum reinforcement according to IS : 3370 [Part 2 : 2009]

$$A_{st}, \min = \frac{0.24}{100} \times t \times 100$$

$$= \frac{0.24}{100} \times 170 \times 1000$$

$$= 408 \text{ mm}^2 < A_{st,provided}, \text{ OK}$$

As t < 200 mm, hence we can provide steel on one face only.

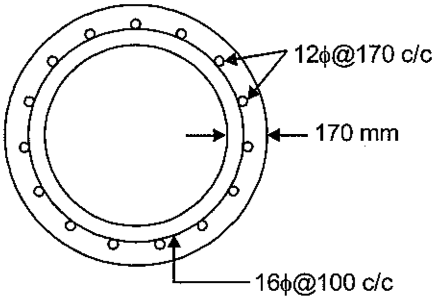
Provide 408 mm² reinforecment is vertical direction using 12 mm ϕ bars,

$$\text{Spacing} = \frac{1000}{\frac{408}{\frac{\pi}{4} \times 12^2}} = 277.2 \text{ mm}$$

But spacing $\ngtr$ 300 mm

$\ngtr$ Thickness of section (170 mm)

So provide 12 mm ϕ bars @ 170 mm c/c.



Cross Section of Tank

Q-3: ***Design the side walls of an underground tank of size 12 m × 3 m × 3 m deep. The angle of repose of soil is 30°. The density of soil is taken as 17 kN/m³. Assume the soil is saturated. Use M 25 grade of concrete and Fe 415 grade of steel. Take Q = 1.156 N/mm² and J = 0.87.***

[20 Marks, ESE-2019]

Sol: **Given that:**

$$Q = 1.156 \text{ N/mm}^2$$

$$j = 0.87$$

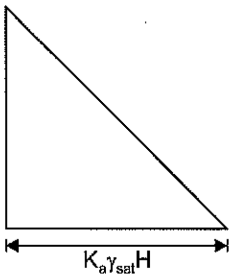
M25, Fe 415

$$\phi = 30°$$

$$\gamma_{sat} = 17 \text{ kN/m}^3$$

$$\text{Size of water tank} = 12 \times 3 \times 3 \text{ m}$$

Hence, $\frac{L}{B} > 2$



(a) For empty water tank

$$K_a = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$

$$(BM) = \frac{1}{2} \times K_a \gamma_{sat} H \times H \times \frac{H}{3}$$

$$= \frac{1}{2} \times \frac{1}{3} \times 17 \times \frac{(3)^3}{3} = 25.5 \text{ kN-m}$$

We know that

$$M = Qbd^2$$

$$d \geq \sqrt{\frac{M}{Qb}}$$

$$\geq \sqrt{\frac{25.5 \times 10^6}{1.156 \times 1000}} = 148.5 \text{ mm}$$

To avoid chance of over-reinforced provide effective depth = 185 mm and cover 45 mm with reinforcement 20 mm ϕ

$$\text{Overall depth} = 185 + \frac{20}{2} + 45 = 240 \text{ mm}$$

Area of steel $\Rightarrow$

$$\sigma_{st}A_{st}jd = M \qquad [\because \sigma_{st} = 130 \text{ MPa for Fe415}$$

$$A_{st} = \frac{25.5\times10^6}{130\times0.87\times185} = 1218.73 \text{ mm}^2$$

Provide 20 mm bar

$$n = \frac{1218.73}{\frac{\pi}{4}\times20^2} \simeq 4 \text{ bar}$$

$$\text{Spacing} = \frac{\frac{\pi}{4}\times20^2\times1000}{1218.73} \simeq 258 \text{ mm}$$

Hence, provide 200 mm c/c spacing (Spacing $\not>$ D or 300 mm)

$$(A_{st})_{min} = \frac{0.35\,b\,D/2}{100} = \frac{0.35\times1000\times\frac{240}{2}}{100} = 420 \text{ mm}^2$$

$$< (A_{st})_{provided.}$$

Direct compression in long wall when tank is empty

$$= \frac{\gamma_{sat}}{2}(H-h)\times B = \frac{17}{2}\times(3-1)\times3 = 51 \text{ kN}$$

which is safe in compression.

(b) Tank fill with water and no earth fill outside.

$$M = \frac{1}{2}\gamma_w\frac{(H)^3}{3} = \frac{1}{2}\times9.81\times\frac{3^3}{3} = 44.145 \text{ kN-m}$$

Hence A_{st} for inner face:

$$\sigma_{st}A_{st}jd = M$$

$$130\times A_{st}\times0.87\times185 = 44.145\times10^6$$

$$A_{st} = 2109.83 \text{ mm}^2$$

Provide 20 mm ϕ bar

$$n = \frac{2109.83}{\frac{\pi}{4}\times20^2} = 7 \text{ bar}$$

$$\text{Spacing} = \frac{\frac{\pi}{4}\times20^2\times1000}{2109.83} = 148.9 \text{ mm}$$

Provide 145 mm c/c.

$$\text{Direct tension in long wall} = \frac{\gamma_w}{2}(H-h)\times B = \frac{9.81}{2}\times2\times3 = 29.43 \text{ kN}$$

$$\text{Tensile stress} = \frac{29.43\times10^3}{240\times1000} = 0.12 \text{ MPa}$$

Hence, safe in tens on.

CHAPTER 11

DESIGN OF STAIRCASES

Q-1: ***What do you understand by a tread riser staircase? List out the steps for design. Draw a sectional elevation of this staircase showing the different reinforcements needed.***

[10 Marks, ESE-1995]

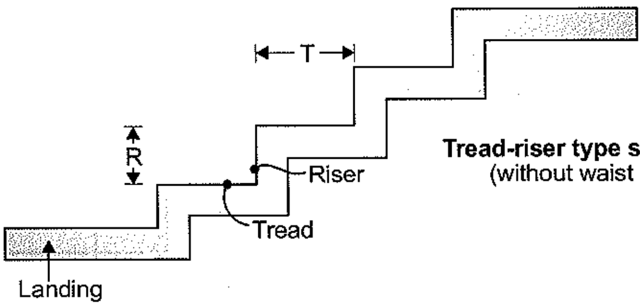
Sol: A staircase consists of a flight of steps, usually with one or more intermediate landing (horizontal slab platform) provided between the floor levels.

A horizontal top portion of a step (where the foot rests) is termed as "tread" and the vertical projection of step (i.e. vertical distance between two neighbouring step) is called "riser".

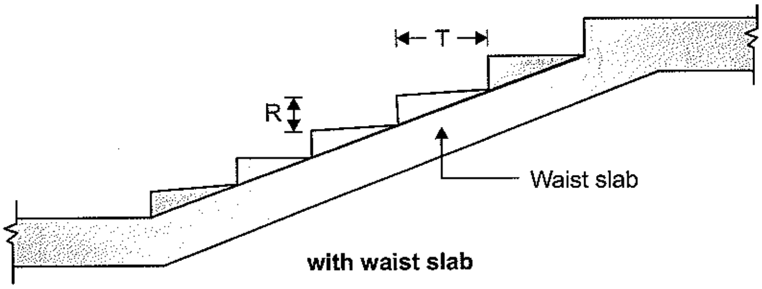
The steps in the flight can be designed in a number of ways:

(a) with waist slab

(b) with tread-riser arrangement (without waist slab)



Tread-riser type stair case
(without waist slab)

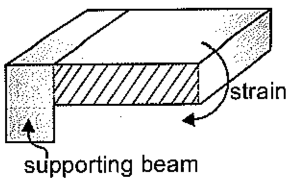


with waist slab

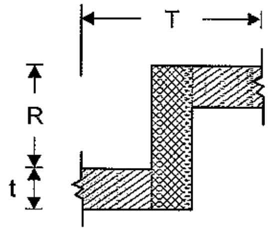
In the tread-riser arrangement, slab is repeatedly folded and behaves eccentrically like a folded plate. For design simplified analysis is done.

Tread-riser unit spanning transversely

Spanning transversely means bending in transverse duration.



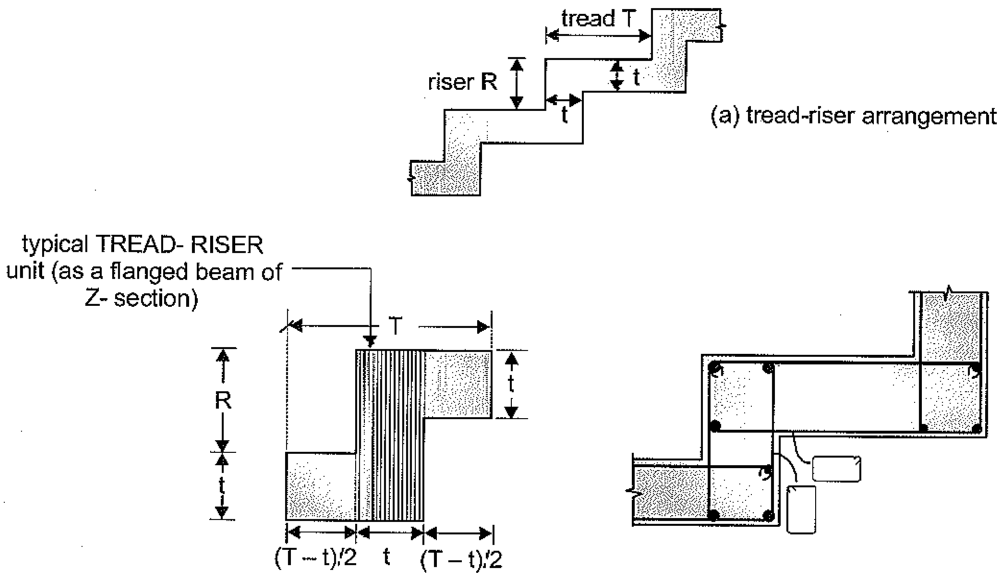
In this case, assumption is made that each tread-riser unit comprising the riser slab and one half of each tread slab on either side can be assumed to behave independently as a beam of Z-section.



Overall depth of beam is (R + t)

In most cases of tread-riser unit spanning transversely, the bending moments are low and hence it generally suffices to provide a nominal slab thickness of t = 100mm. For convenience in calculation, the flange portion of the beam may be ignored and rectangular portion of the riser alone can be considered thus it is found that reinforcement requirement is nominal.

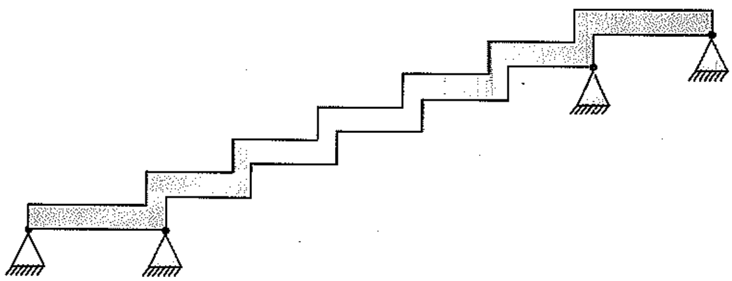
Detailing can be done as follows:



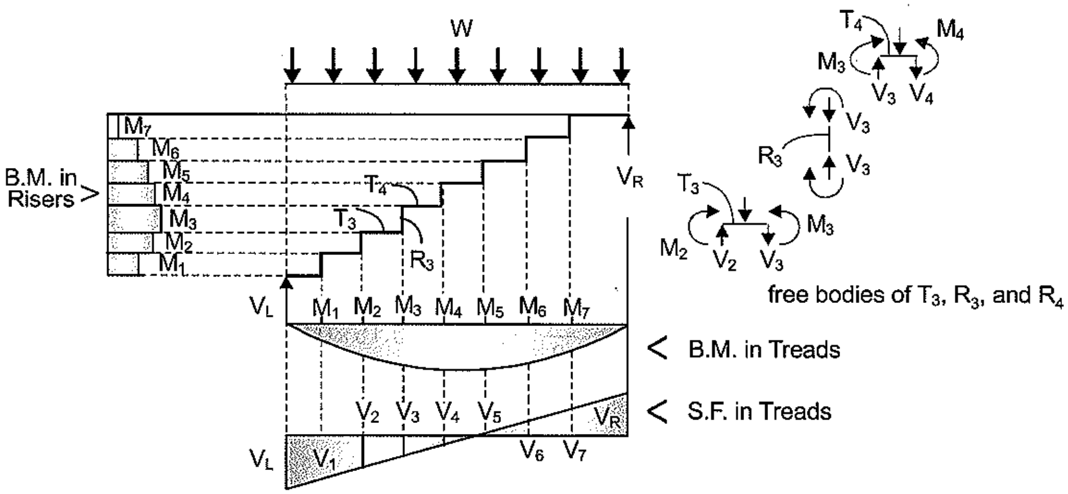
Stairs spanning transversely

For Tread-riser unit spanning longitudinally

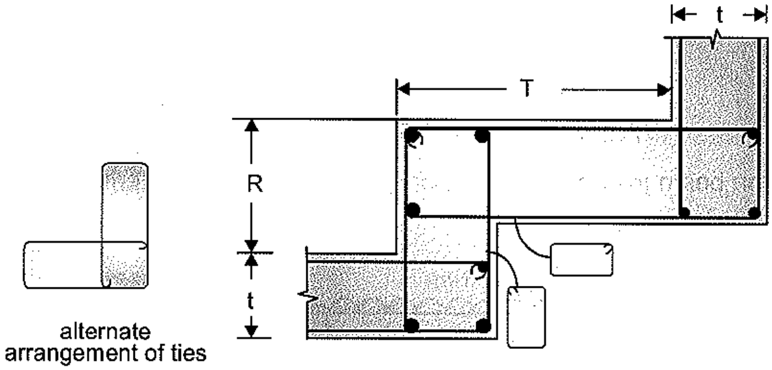
In this case there is no transverse support rather the support is as shown below:



Thus bending occurs in longitudinal direction. BM in tread & riser is as shown below:

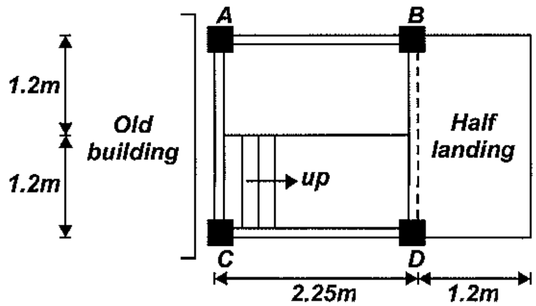


As depicted in free body diagram, each tread slab is subjected to bending moment (which varies slightly along tread) combined with shear force, whereas each riser slab is subjected to BM (which is constant for a given riser) combined with an axial force. It is assumed that connection betweentread and riser is rigid. For all practical purposes, it is sufficient to design both tread slab and riserslab for flexure alone. Slab thickness may be kept same for both tread slab or riser slab and may be taken as $\frac{\text{span}}{25}$ for simply supported stairs and $\frac{\text{span}}{30}$ for continuous stairs. The reinforcement detain is as shown below:



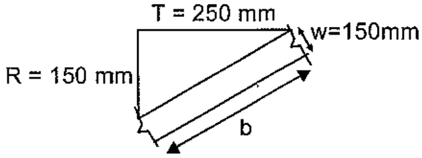
Stairs spanning longitudinally

Q-2: ***A separate two-flight staircase is to be constructed for an old residential house as shown in Figure. The floor-to-floor height of the building is 3m. There are 10 nos. of risers of each 150mm and treads of 250mm. Four columns are located at A, B, C, D. The beams AB, CD and AC are located at floor level and the beam BD is located at half landing level. The thickness of waist slab and the landing is 150mm. Live load = 3kN/m². Half landing slab is cantilevered from the beam BD. Neglect the line load along the edge of the half landing. Draw the bending moment diagram for the flight for design purpose.***



[20 Marks, ESE-2017]

Sol:



Each step is considered equivalent to a horizontal beam of width b = $\sqrt{R^2+T^2}$

$$\therefore \quad b = \sqrt{0.15^2+0.25^2} = 0.2915 \text{ m}$$

$$\text{Dead load of each step per metre} = \frac{RT}{2}\times\gamma_c = \frac{1}{2}\times 0.15\times 0.25\times 25 = 0.469 \text{ kN/m}$$

$$\text{Dead load of waist slab} = \text{Thickness of waist slab} \times b \times \gamma_c$$

$$= 0.15 \times 0.2915 \times 25 = 1.093 \text{ kN/m}$$

$$\text{Total dead load on horizontal area} = \frac{1.093+0.469}{0.25} = 6.256 \text{ kN/m}^2 \quad \text{(Neglecting floor finish loads)}$$

$$(DL + LL) = (6.256 + 3) = 9.256 \text{ kN/m}^2$$

$$\text{Factored load} = 1.5 \times 9.256 = 13.884 \text{ kN/m}^2$$

Load on landing slab:

$$\text{Dead load} = 0.15 \times 25 = 3.75 \text{ kN/m}^2$$

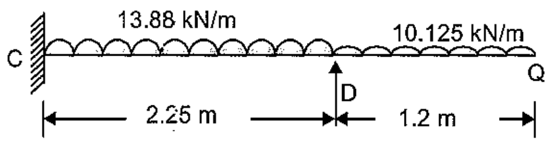
$$\text{Live load} = 3 \text{ kN/m}^2$$

$$(DL + LL) = (3.75 + 3) = 6.75 \text{ kN/m}^2$$

$$\text{Factored load} = 1.5 \times 6.75 = 10.125 \text{ kN/m}^2$$

(Assume column beam joint as fixed support and portion DQ as overhang)

For 1 m width,



Fixed end moment calculation:

$$\bar{M}_{FCD} = -\frac{wL^2}{12} = \frac{13.88\times 2.25^2}{12} = -5.86 \text{ kNm}$$

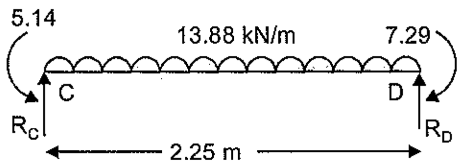
$$\bar{M}_{FDC} = +5.86 \text{ kNm}$$

$$\bar{M}_{FDQ} = -\frac{wL^2}{2} = -\frac{10.125\times 1.2^2}{2} = -7.29 \text{ kNm}$$

Applying moment distribution method

	C	D		Q
		1	0	
FEM	−5.86	+5.86	−7.29	
		+1.43		
C.O	+0.72			
Final	− 5.14	+7.29	−7.29	

To find maximum sagging bending moment



$$\Sigma M_C = 0$$

$$\Rightarrow \quad R_D \times 2.25 - 7.29 - \frac{13.88 \times 2.25^2}{2} + 5.14 = 0$$

$$\Rightarrow \quad R_D = 16.57 \text{ kN}$$

Bending moment is max. at location

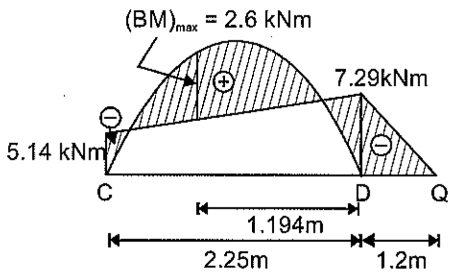
where, S.F = 0

$$\therefore \quad 16.57 - 13.88x = 0 \quad \Rightarrow \quad x = 1.194 \text{ m from D}$$

$$\therefore \quad (BM)_{max} = 16.57 \times 1.194 - 7.29 - \frac{13.88 \times 1.194^2}{2}$$

$$= 2.6 \text{ kNm (sagging)}$$

$$(BM)_{max} = 7.29 \text{ kNm (hogging)}$$



CHAPTER 12

PRESTRESSED CONCRETE

Q-1: ***Explain the essential requirements of steel and concrete for prestressed concrete. What are the advantages of prestressed concrete over reinforced concrete?***

[10 Marks, ESE-1996]

Sol: **Requirements for concrete:**

- A prestressed concrete members is essentially a concrete members only and the tendons are considered as the necessary device to supply the prestress to concrete.
- High strength concrete as well as high tensile steel wires are required to get the maximum advantage of a prestressed concrete member.
- Concrete used for prestressed work should have a min grade of M40 for pre-tensioned system and M30 for post tensioned system.
- The nominal maximum size of coarse aggregate shall be as large as possible subject to the following:
  - (a) In no case greater than one-fourth the minimum thickness of the member, provided that the concrete can be placed without difficulty so as to surround all prestressing tendons and reinforcements and fill the corners of the form.
  - (b) It shall be 5mm less than the spacing between the cables, strands or sheathings where provided.
  - (c) Not more than 40 mm; aggregates having a maximum nominal size of 20mm or smaller are generally considered satisfactory.
- For prestressed concrete construction only 'Design mix concrete' shall be used. The cement content in the mix should preferably not exceed 530 kg/m^3.
- Concrete should meet the acceptance criteria of IS-456.
- Durability requirements are as shown below:

Minimum cement content required in cement concrete to ensure durability under specified conditions of exposure (As per IS 1343 : 1980)

Exposure	Prestressed Concrete	
	Minimum cement content kg / m^3	Maximum water cement ratio
Mild: For example, completely protected against weather, or aggressive conditions, except for a brief period of exposure to normal weather conditions during construction.	300	0.65
Moderate: For example, sheltered from heavy wind driven rain and against freezing, whilst saturated with water, buried concrete in soil and concrete continuously under water	300	0.55
Severe: For example, exposed to sea water, alternate wetting and drying and to freezing whilst wet subject to heavy condensation or corrosive fumes	360	0.45

Note: To minimum cement content is based on 20mm nominal maximum size. For 40mm aggregate, minimum cement content should be reduced by about 10 percent under severe exposure condition only; for 12.5 mm aggregate, the minimum cement content should be increased by about 10 percent under moderate and severe exposure conditions only.

- No hand mixing is permitted.
- Total amount of chloride (as Cl^-) and total amount of soluble sulphate as (SO_3^{2-}) in the concrete at the time of placing should be limited to 0.06% by mass of cement & 4% by mass of concrete respectively.

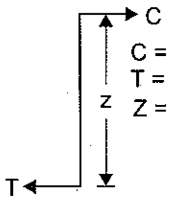
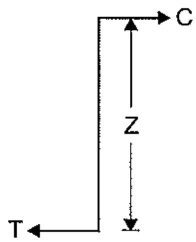
Requirements for steel

- The prestressing steel shall be any one of the following:
  (a) Plain hard-drawn steel wire (b) Cold-drawn indented wire
  (c) High tensile steel bar (d) Uncoated stress relieved strand
- Modulus of elasticity of prestressing steel shall be as follows:

Type of Steel	Modulus of Elasticity, E, kN/mm^2
Plain cold-drawn wires	210
High tensile steel bars rolled or heat treated	200
Strands	195

- The wire, as supplied, shall preferably be self-straightening when uncoiled. If it is not so, the wire may need to be mechanically straightened before use. In this event, care shall be taken to avoid alternation in the properties of the wire during the straightening process and preferably a test shall be made on a sample of the wire after straightening.
- In no case heat shall be applied to facilitate straightening or bending of prestressing steel.

Advantages of prestressed beam over RCC

RCC beam	PSC beam
(i)  C = compressive stress T = tensile stress Z = lever arm (stress diagram) C = Changes according to external load Z = Constant essentially	(i)  (stress diagram) C = Do not change Z = Changes
(ii) Concrete in compression side is alone effective	(ii) Entire section is effective
(iii) x-sectional requirement is more	(iii) x-sectional required is less
(iv) Shear reinforcement is necessary	(iv) Due to curved tendons and pre-comparession a considerable part of shear is resisted.
(v) Cracks occur under working load	(v) Cracks do not occur. If at heavy load the cracks generated, it will close after removal of load.
(vi) Deflection is large as compared to PSC	(vi) Deflection is small as compared to RCC
(vii) In RCC beam there is no way of testing of concrete and steel	(vii) At the time of prestressing we can check both.

Q-2: ***List various methods used for post-tensioning of concrete structures. Describe salient features of Magnel-Blaton system of post-tensioning.***

[10 Marks, ESE-2000]

Sol: The various method used for post-tensioning of concrete structures are:

(1) Aderson (U.S.A.)

(2) Baur-Leon-hardt (Germany)

(3) Billner (U.S.A)

(4) B.B.R.V. (Switzerland)

(5) Dywidag (Germany)

(6) Freyssinet (France)

(7) Gifford-Udall-CCL (Great Bretain)

(8) Leoba (Germany)

(9) Lee-Mccall (Great Britain)

(10) Magnel Blaton (Belgium)

(11) Prescon (U.S.A)

(12) P.S.C. (Monowire) (Great Britain)

(13) Losinger VSL (Switzerland)

(14) Freyssinet (International)

The salient features of Magnel Blaton system of post-tensioning are:

- Tendons used are wires.
- Range of prestressing force is small, medium and large [small up to 130 kN, medium = 130 – 500 kN large = 500 kN – 4500 kN]
- Cable duct provided is rectangular, formed by solid rubber core or by metal sheath around cable.
- The wires are arranged in horizontal rows of four wires spaced by metal grilles at intervals.
- Tensioning is done by hydraulic jack with tensioning of two wires at a time
- Magnel-Blaton post-tensioning system adopts metallic and sandwich plates, flat wedges, and a distribution plate for anchoring the wires. Each sandwich plate can house up to four pairs of wires. The distribution plate may be cast into the member at the desired location.

Q-3: ***What are the various methods of pre-stressing? Also discuss the most widely used methods of pre-tensioning and post-tensioning of concrete elements.***

[10 Marks, ESE-2011]

Sol: The various method of prestressing are:

- Pre bending high strength steel beam and encasing its tensile flange with concrete.
- Mechanical jacking of tendons
- Thermal prestressing by application of electric heat
- Chemical prestressing by means of expansive cement

Pre-tensioning Method

Hoyer system of prestressing

- Hoyer system is generally used for mass production like railway sleepers, poles etc.
- The end abutments are kept sufficient distance apart and several members are cast in a single line.
- The shuttering is provided at the sides and b/w the members.
- This system is also called the long line method.
- The tension is applied by hydraulic jacks or by a movable stressing machine. The wires or strands when tensioned singly or in groups are generally anchored to the abutments by steel wedge. The transfer of prestress to concrete is achieved by large hydraulic or screw jacks by which all the wires are simultaneously released after the concrete attains the requisite compressive strength.

Post-tensioning Method

(1) Aderson (U.S.A.)

(2) Baur-Leon-hardt (Germany)

(3) Billner (U.S.A)

(4) B.B.R.V. (Switzerland)

(5) Dywidag (Germany)

(6) Freyssinet (France)

(7) Gifford-Udall-CCL (Great Bretain)

(8) Leoba (Germany)

(9) Lee-Mccall (Great Britain)

(10) Magnel Blaton (Belgium)

(11) Prescon (U.S.A)

(12) P.S.C. (Monowire) (Great Britain)

(13) Losinger VSL (Switzerland)

(14) Freyssinet (International)

- The freyssinet anchorage system, which is widely used in Europe and India, consists of a cylinder with a conical interior through which the high-tensile wire pass and against the walls of which the wires are wedged by a conical plug lined longitudinally with grooves to house the wires.

  The main advantages of the Freyssinet system is that a large number of wires or strands can be simultaneously tensioned using the double-acting hydraulic jack.

- The Gifford-Udall (C.C.L) system developed in U.K. consist of steel split-cone and cylindrical female-cone anchorages to house the high tensile wires bearing against steel plates.

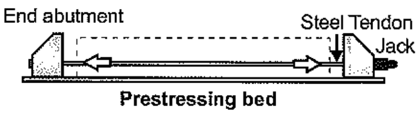
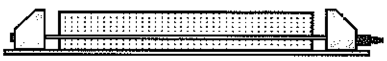
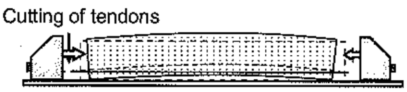
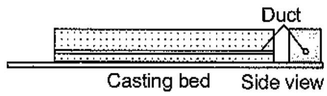
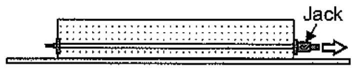
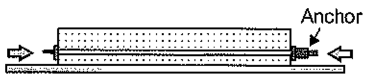
  Each wire is tensioned separately and anchored by forcing a sleeve wedge into a cylindrical grip resting against a bearing plate.

- Magnel-Blaton post-tensioning system adopts metallic and sandwich plates, flat wedges, and a distribution plate for anchoring the wires. Each sandwich plate can house up to four pairs of wires. The distribution plate may be cast into the member at the desired location.

Q-4: ***Distinguish clearly between pretensioned and post-tensioned prestressed concrete bringing out all the operations involved.***

[8 Marks, ESE-2012]

Sol: The difference between pretensioned and post-tensioned prestressed concrete are.

Pre Tensioned	**Post Tensioned**
(1) First, tensioning of wires is done between two abutments and then concrete is casted	(1) First concrete is casted with ducts in it and then wire is placed in the duct and tensioned.
(2) After sufficient strength has developed in concrete, wires are cut off and prestress is induced in the beam due to bond between concrete and wire.	(2) After tensioning of wire, it is fixed by wedges on ends of duct and prestress developed by bearing at the ends.
(3)  **(a) Applying tension to tendons**  **(b) Casting of concrete**  **(c) Transferring of prestress**	(3)  **(a) Casting of Concrete**  **(b) Tensioning of tendons**  **(c) Anchoring the tendon at the stretching end**
(4) The various stages are: (a) Anchoring of tendons against the end abutments (b) Placing of jacks. (c) Applying tension to the tendons (d) Casting of concrete (e) Cutting of tendons	(4) The forces are transmitted to the concrete by (a) Casting of concrete. (b) Placement of tendons. (c) Placement of the anchorage block and jack. (d) Applying tension to the tendons. (e) Seating of the wedges. (f) Cutting of the tendons.
(5) Suitable for precast members produced in bulk	(5) Suitable for heavy cast-in-place members.
(6) Large anchorage devices not required required	(6) Large anchorage devices and grouting equipment
(7) Waiting period in prestressing bed before sufficient strength gained by concrete.	(7) Waiting period in casting bed is less.
(8) Good bond required between concrete wire over transmission length.	(8) Transfer of prestress independent of transmission & length

Q-5: ***List types of losses of pre-stress in pre-tensioning and post-tensioning systems.***

[5 Marks, ESE-2015]

Sol: The different types of losses encountered in pretensioning and post tensioning are as given below.

Pre-tensioning	Post-tensioning
1. Elastic deformation of concrete	1. No loss due to elastic shortening when all bars are simultaneously tensioned. If however, wires are successively tensioned there would be los of prestress due to elastic deformation of concrete
2. Relaxation of steel	2. Relaxation of steel
3. Shrinkage of concrete	3. Shrinkage of steel
4. Creep of concrete	4. Creep of concrete
	5. Frictional losses
	6. Anchorage slip

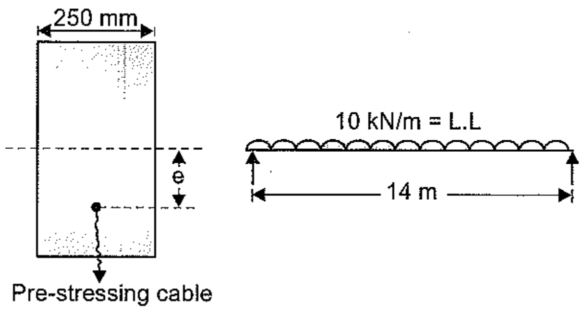
CHAPTER 13

ANALYSIS OF PRESTRESS AND BENDING STRESS

Q-1: ***A post tensioned prestressed concrete beam 250 mm wide has to be designed for a live load of 10 kN/m across a span of 14 metres. The stresses in concrete must not exceed 17 MPa in compression and 1.4 MPa in tension. The loss of prestress may be assumed as 15%. Calculate (i) the minimum possible depth for the beam and (ii) for this depth the minimum prestressing force and the corresponding eccentricity.***

[20 Marks, ESE-1995]

Sol:



Stress in concrete must that exceed 17 MPa in compression and 1.4 MPa in tension

The loss in prestress = 15%

$\Rightarrow$ η = 85% i.e η = 0.85

At transfer

At top $$-\left[\frac{P}{A}-\frac{Pe}{Z}+\frac{M_d}{Z}\right] \leq 1.4 \quad \text{...(i)}$$

At Bottom $$\frac{P}{A}+\frac{Pe}{Z}-\frac{M_d}{Z} \leq 17 \quad \text{...(ii)}$$

At Service Load:

At Bottom

$$-\left[\eta\frac{P}{A}+\frac{\eta Pe}{Z}-\frac{M_d}{Z}-\frac{M_L}{Z}\right] \leq 1.4 \quad \text{...(iii)}$$

At Top

$$\frac{\eta P}{A}-\frac{\eta Pe}{Z}+\frac{M_d}{Z}+\frac{M_L}{Z} \leq 17 \quad \text{...(iv)}$$

From $\eta(\text{i})+(\text{iv})$

$$-\frac{\eta P}{A}+\frac{\eta Pe}{Z}-\frac{r M_d}{Z} \leq 1.4\times\eta$$

$$+$$

$$\frac{\eta P}{A}-\frac{\eta Pe}{Z}+\frac{M_d}{Z}+\frac{M_L}{Z} \leq 17$$

$$\Rightarrow \quad \frac{M_d(1-\eta)+M_L}{Z} \leq (17+1.4\eta)$$

$$Z \geq \frac{M_d(1-\eta)+M_L}{17+1.4\,\eta} \quad ...(A)$$

From η(ii) + (iii)

$$\frac{\eta P}{A}+\frac{\eta Pe}{Z}-\frac{\eta M_d}{Z} \leq 17\,\eta$$

$$+$$

$$-\frac{\eta P}{A}-\frac{\eta Pe}{Z}+\frac{M_d}{Z}+\frac{M_L}{Z} \leq 1.4$$

$$\frac{M_d(1-\eta)+M_L}{Z} \leq (17\eta+1.4)$$

$$Z \geq \frac{M_d(1-\eta)+M_L}{17\eta+1.4} \quad ...(B)$$

Now,

$$W_d = (B\,D\times 24)\text{kN/m} = (0.25\times 24D)\text{kN/m} = 6D \text{ kN/m}$$

$$M_d = \frac{6D(14)^2}{8}$$

$$M_d = 147\ D \text{ kN-m} \qquad \text{(Unit wt. of Concrete = 24 kN/m}^3\text{)}$$

$$M_L = \frac{10\,(14)^2}{8} = 245 \text{ kN-m}$$

From A

$$M_d = 147\ D \text{ kNm, D is in meter}$$

$$M_L = 245 \text{ kNm}$$

$$\left[\frac{M_d(1-\eta)+M_L}{Z}\right]\times\frac{10^3}{10^6} \leq 17 + 1.4\eta$$

$$\Rightarrow \quad Z \geq \frac{[M_d(1-\eta)+M_L]\times 10^3}{17+1.4\eta}$$

$$\Rightarrow \quad \frac{0.25\,D^2}{6} \geq \frac{[147\,D\times 0.15+245]\times 10^{-3}}{17+1.4\times 0.85}$$

$$\Rightarrow \quad 0.7579\,D^2 \geq [22.05\,D+245]\times 10^{-3}$$

$$\Rightarrow \quad 757.9\ D^2 - 22.05\ D - 245 \geq 0$$

$$D = \frac{22.05\pm\sqrt{(22.05)^2+4\times 245\times 757.9}}{2\times 757.9}$$

$$D \geq 0.583 \text{ m}$$

From B

$$\frac{0.25D^2}{6} \geq \frac{(147\,D\times 0.15+245)\times 10^{-3}}{17\times 0.85+1.4}$$

$$660.417\ D^2 - 22.05\ D - 245 \geq 0$$

$$D \geq \frac{22.05\pm\sqrt{(22.05)^2+4\times 245\times 660.417}}{2\times 660.417}$$

$$D \geq 0.626 \text{ m}$$

$$\boxed{\text{Adopt D} = 630 \text{ mm}}$$

Min Prestressing is calculated from

(1) Top Stress at transfer

(2) Bottom stress at Service Condition

$$\Rightarrow \quad -\left[\frac{P}{A}-\frac{Pe}{Z}+\frac{M_d}{Z}\right] \le 1.4 \qquad \ldots(\alpha)$$

$$\Rightarrow \quad -\left[\frac{\eta P}{A}+\frac{\eta Pe}{Z}-\frac{M_d}{Z}+\frac{M_L}{Z}\right] \le 1.4m \qquad \ldots(\beta)$$

$$\Rightarrow \quad -\frac{\eta P}{A}+\frac{\eta Pe}{Z}-\frac{\eta M_d}{Z} \le 1.4\,\eta$$

$$\frac{-2\eta P}{A}+\frac{M_d(1-\eta)}{Z}-\frac{M_L}{Z} \le 1.4\,(1+\eta)$$

$$-\frac{2\eta}{A}P \le 1.4(1+\eta)-\frac{M_d(1-\eta)}{Z}-\frac{M_L}{Z}$$

$$P \ge \left[\frac{M_d(1-\eta)+M_L}{Z}-1.4\,(1+\eta)\right]\frac{A}{2\eta}$$

$$P \ge \left[\frac{[147\times 0.630\times 0.15+245]}{\left(\frac{(0.25)\,(0.630)^2}{6}\right)}\times\frac{1}{10^3}-1.4\times 1.85\right]\times\frac{250\times 630}{2\times 0.85}$$

$$P \ge 1210.416 \text{ kN}$$

From $\alpha\,\&\,\beta$

$$-\frac{\eta P}{A}+\frac{\eta Pe}{Z}-\frac{\eta M_d}{Z} = 1.4\,\eta$$

$$-\frac{\eta P}{A}-\frac{\eta Pe}{Z}+\frac{M_d}{Z}-\frac{V_L}{Z} = +1.4$$

$$+ \quad + \qquad - \qquad + \qquad\quad -$$

$$\frac{2\eta Pe}{Z}-(1+\eta)\frac{M_d}{Z}-\frac{M_L}{Z} = 1.4\,\eta-1.4$$

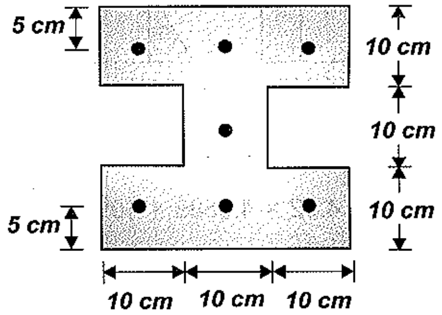
$$\frac{2\eta Pe}{Z} = \frac{(1+\eta)M_d+M_L}{Z}+1.4(\eta-1)$$

$$e = \left[\frac{(1+\eta)M_d+M_L}{Z}-1.4(1-\eta)\right]\frac{Z}{2\eta P}$$

$$e = \left[\frac{[(1.85)\times 147\times 0.630+245]\times 10^6}{\frac{250(630)^2}{6}}-1.4\times 0.15\right]\times\frac{250\times(630)^2}{6\times[2\times 0.85]\times 1210.416\times 10^3}$$

$$e = 200.64 \text{ mm}$$

Q-2: ***An I section in concrete has the following dimensions : Flanges - 30 cm × 10 cm. web - 10 cm × 10 cm. It is pretensioned by 7 steel wires of 8 mm dia, as shown in the figure, to an initial prestress of 10, 000 kg/cm² Modular ratio m = 6. Calculate the stress in concrete and steel immediately on cutting the wires when the member is still supported in the prestressing bed. As summing a loss of prestress of 2000 kg/cm² including loss due to elastic deformation, calculate the maximum moment to permit a maximum compressive stress of 120 kg/cm² and no tension in concrete.***



[15 Marks, ESE-1996]

Sol: **Given data:**

Flanges : 30 cm × 10 cm = 300 mm × 100 mm

Web : 10 cm × 10 cm = 100 mm × 100 mm

m = 6.

$$\text{Area of the section (A)} = 2 \times (300 \times 100) + 100^2 = 70{,}000 \text{ mm}^2$$

$$\text{Moment of Inertia (I)} = \frac{300^4}{12} - \frac{200 \times 100^3}{12}$$

$$= 6.5833 \times 10^8 \text{ mm}^4 \text{ (steel area neglected)}$$

$$\text{Section modulus (Z)} = \frac{6.5833 \times 10^8}{150} = 4.3889 \times 10^6 \text{ mm}^3$$

$$\text{Area of steel wires} = A_{st} = 7 \times \frac{\pi}{4} \times 8^2 = 351.858 \text{ mm}^2$$

$$\text{Initial prestressing force } (P_0) = 1000 \text{ N/mm}^2 \times 351.858 = 351858 \text{ N}$$

$$\left[\because\ 10{,}000 \text{ kg/cm}^2 = \frac{10{,}000 \times 10}{10 \times 10} \text{ N/mm}^2 = 1000 \text{ N/mm}^2\right]$$

Stresses immediately after cutting the wires

$$\text{Stress in concrete} = \frac{P_0}{A} = \frac{351858}{70000} = 5.027 \text{ N/mm}^2$$

$$\text{Stress in steel} = 1000 \text{ N/mm}^2$$

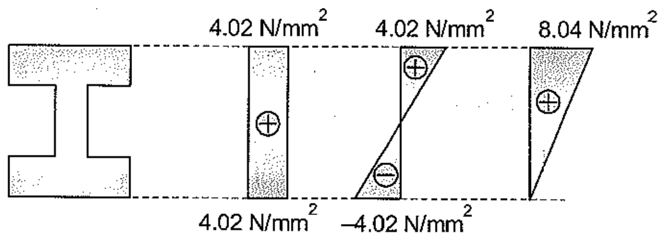
$$\text{The total loss of stress in wires} = 200 \text{N/mm}^2$$

$$\therefore \quad \text{Final stress in steel} = 1000 - 200 = 800 \text{N/mm}^2$$

Final prestressing force (P) = 800 × 351.858 = **281486.4N**

$$\text{Direct stress} = +\ P/A = \frac{281486.4}{70{,}000} = 4.02 \text{N/mm}^2$$

To avoid tensile stress, bending stress in the extreme fibres = ± 4.02N/mm²



Let M = Maximum permissible bending moment

$M = f.Z = 4.02 \times 4.3889 \times 10^6$ N-mm = **17.643 kN-m**

Q-3: Explain the different concepts as may be applied to explain and analyze the basic behaviour of prestressed concrete.

[10 Marks, ESE-1997]

Sol: Different concepts to analyze the basic behaviour of prestress concrete.

- Prestressing is artificially induced compressive stresses in a structure before it is loaded so that any tensile stress, which might be caused by external loads are automatically cancelled & cracks are eliminated.
- To take maximum advantage of prestressed concrete, high strength concrete as well as high strength steel are used in place of ordinary concrete so that prestress losses due to shrinkage, creep and failure of prestressed structure due to bond slip or bearing failure could be avoided.

The various concepts used to analyze the prestressed beam are:

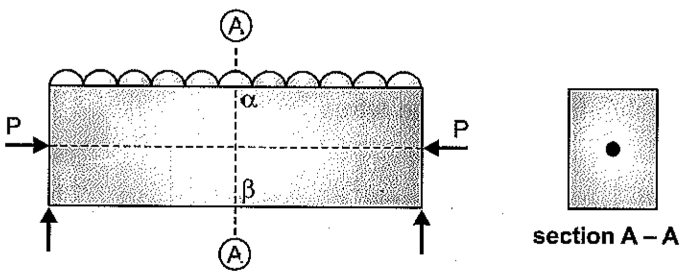
(1) Stress concept

(2) Load balancing concept

(3) C-line, P-line [i.e. thrust line or pressure line concept]

Stress Concept

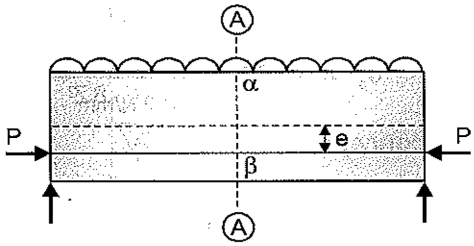
In stress concept, stress at the edges of the section under internal forces in concrete are calculated.



$$\sigma_\alpha = \frac{P}{A} + \frac{M}{Z} \quad \text{[M = BM at section A–A which is sagging]}$$

$$\sigma_\beta = \frac{P}{A} - \frac{M}{Z}$$

However if we put the tendon at some eccentricity as shown below then



$$\sigma_\alpha = \frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z}$$

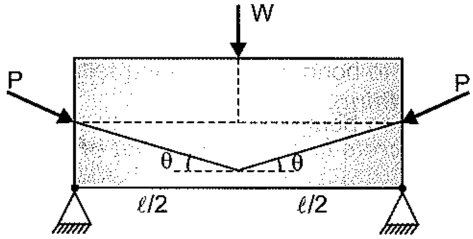
$$\sigma_\beta = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$$

Note that stress caused by external bending moment is counteracted by hogging BM developed due to eccentricity of tendon. Thus by providing tendon at an eccentricity, requirement of prestressing force (P) to counteract the applied moment (i.e. to ensure zero tension or less than acceptable tension) will be reduced significantly.

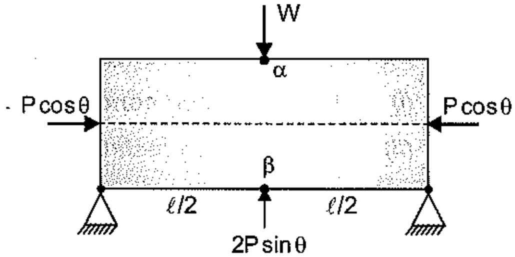
Load balancing Concept

By providing bent tendon, the tendon will exert an upward pressure on the concrete beam and will therefore counteract a part or whole of the external downward loading.

- A parabolic shape of the profile will exert a uniformly distributed upward load on the beam.
- Sharp angles will induce concentrated loads.



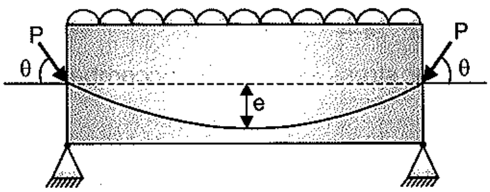
Actual loading



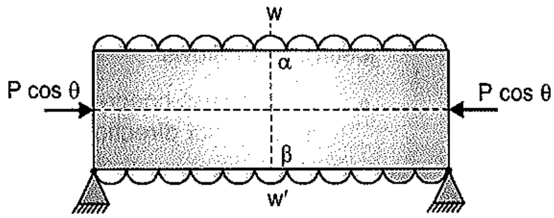
Equivalent Loading

$$\Rightarrow \qquad \sigma_\alpha = \frac{P\cos\theta}{A} + \frac{(w - 2P\sin\theta)l}{4 \times Z}$$

$$\sigma_\beta = \frac{P\cos\theta}{A} - \frac{(w - 2P\sin\theta)l}{4Z}$$



Actual Loading



Equivalent Loading

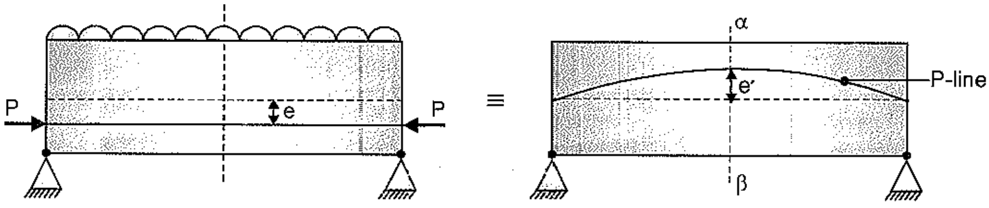
$$w' = \frac{8Pe\cos\theta}{\ell^2}$$

$$\sigma_\alpha = \frac{P\cos\theta}{A} + \frac{\left(w - \frac{8Pe\cos\theta}{l^2}\right)l^2}{8Z}$$

$$\sigma_\beta = \frac{P\cos\theta}{A} - \frac{\left(w - \frac{8Pe\cos\theta}{l^2}\right)l^2}{8Z}$$

Pressure line or thrust line concept

At any section of a prestressed beam, the combined effect of prestressing force and the externally applied load will result in a distribution of concrete stresses that can be resolved into a single force. The locus of the points of application of this resultant force in any structure is termed as the pressure or thrust line.



$$e' = \frac{M}{P} - e$$

$$\sigma_\alpha = \frac{P}{A} + \frac{Pe'}{Z}$$

$$\sigma_\beta = \frac{P}{A} - \frac{Pe'}{Z}$$

Concept of finding out stress using P-line concept is called strength concept or internal restoring couple concept also.

Q-4: ***Comment on the following statements:***

(i) In prestressed concrete, dead loads cost nothing.

(ii) The shearing resistance of prestressed concrete is superior to that of reinforced concrete.

[2 × 4 Marks, ESE-1998]

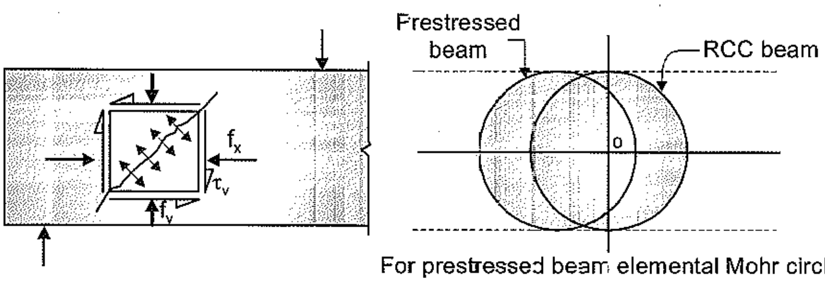
Sol: (i) In prestressed concrete D.L costs nothing because.

- The dead load moments are neutralized by the prestressing force
- Section becomes much more slender, so the dead load reduces a lot.
- Resistance to shear increases due to prestressing leading to saving.
- Reduction of section leads to further saving in construction of other components.

Hence we generally say in prestressed concrete dead load cost nothing

(ii) The shearing resistance of prestress concrete is superior to that of RCC. This statement can be justified as follows

- In prestress concrete member, the shear stress is generally accompanied by a direct stress in the axial direction of the member



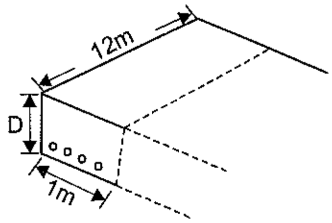
For prestressed beam elemental Mohr circle shifted to the left by reducing the chances of tensile stress.

- In prestress concrete members, the direct stresses f_x & f_y being compressive (Though f_y is variable), the magnitude of the principal tensile stress is considerably reduced, and in some cases even eliminated, so that-under working loads, both major and minor principal stresses are compressive, thereby eliminating the risk of diagonal tension cracks in concrete.
- In case of RCC however, diagonal tensile stress will develope. Thus effect of shear will be significant.

Q-5: ***Design a prestressed concrete slab spanning 12.0 m carrying an imposed load of 20 kN/mm². M_{40} concrete and steel with an ultimate tensile stress of 1600 N/mm² are used. The permissible stresses in concrete are 14 N/mm² in compression and zero in tension. Neglect the loss in prestress. Cables of 12 wires of 5 mm diameter capable of carrying an effective prestress of 225 kN are available. Indicate the zone in which the resultant cable must lie.***

[20 Marks, ESE-1998]

Sol: **Step-1: Considering** 1m wide strip of the slab

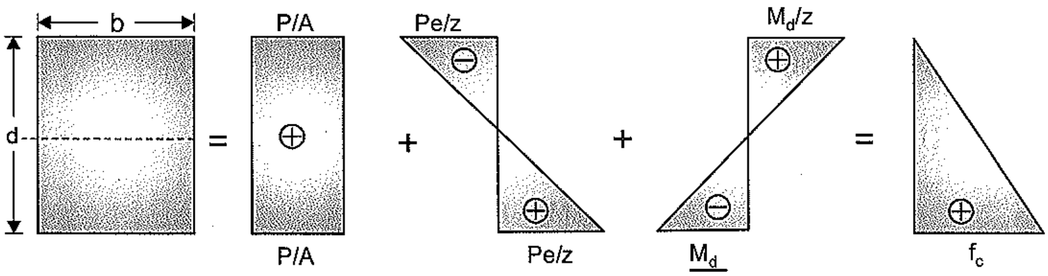


Given live load = 20kN/m²

Moment due to live load, $M_1 = \dfrac{w \times l^2}{8} = \dfrac{20 \times 12^2}{8} = 360\text{kNm}$

Basic theory behind designing

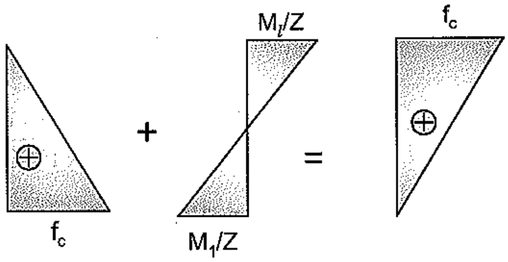
Let the effective total prestress P is at an eccentricity of e. let M_d be the maximum bending moment due to dead load alone. The stress due to direct load, eccentricity of the prestress and due to the dead load moment are shown as below:



$$\frac{P}{A} + \frac{M_d}{Z} - \frac{Pe}{z} = 0 \qquad \text{...(1)}$$

$$\frac{P}{A} + \frac{Pe}{z} - \frac{M_d}{z} = f_c \qquad \text{...(2)}$$

When the live load is applied, let the bending moment due to live load be M_1 such that the resultant stress should be as shown below,



Step-2: To fulfill such condition

$$\frac{M_l}{z} = f_C \qquad \text{... (3)}$$

$$\Rightarrow \qquad \boxed{\frac{M_l}{f_c} = z}$$

Hence $$z_{required} = \frac{360 \times 10^6}{14} = 25714286 \text{mm}^3$$

i.e., $$\frac{BD^2}{6} = 25714286 \Rightarrow D^2 = \left[\frac{25714286 \times 6}{1000}\right]$$

$$\therefore \qquad D = 393 \text{ mm} \cong 400\text{mm}$$

Step-3: Prestressing force (P) = $\frac{f_c}{2} . A = \frac{1000 \times 400 \times 14}{2} = 2800\text{kN}$ [This is obtained by adding eq(1) and (2)].

Step-4: Amount of steel required:

Strength of 1 cable of 12 wires of 5mm ϕ = 225kN (Given)

$$\text{Number of cables} = \frac{2800}{225} = 12.4 \cong 13$$

$$\therefore \qquad A_{st} = 13 \times 12 \times \frac{\pi}{4} \times 5^2 = 3063.05\text{mm}^2$$

Step-5: Bending moment due to dead load (Md)

$$\text{Dead load} = 0.4 \times 1 \times 1 \times 25 = 10\text{kN/m}^2$$

$$\therefore \qquad M_d = \frac{10 \times 12^2}{8} = 180\text{kN-m}$$

Step-6: Eccentricity calculation

From (1) & (2) using the relation $\frac{M_l}{z} = f_C$

$$\frac{P}{A} + \frac{M_d}{z} - \frac{Pe}{z} = 0$$

$$\frac{P}{A} + \frac{Pe}{z} - \frac{M_d}{z} - \frac{M_l}{z} = 0$$

$$2\frac{M_d}{z} - 2\frac{Pe}{z} + \frac{M_l}{z} = 0$$

$$\Rightarrow \qquad 2\frac{Pe}{z} = \frac{2M_d + M_l}{z}$$

$$e = \frac{2M_d + M_l}{2P}$$

$$\therefore \qquad e = \frac{(2 \times 180 + 360) \times 10^6}{2 \times 2800 \times 1000}$$

$$\Rightarrow \qquad e = 128.57 \text{ mm}$$

Step-7: $$\text{Spacing of cables} = \frac{1000}{13} = 76.92 \cong 80\text{mm}$$

Hence, providing 13 cables a 80 mm c/c.

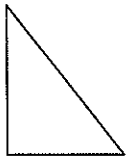
Step-8: Kern distance.

To locate the zone in which cable must lie we have the following condition.

At transfer

$$\frac{P}{A}-\frac{Pe}{z}+\frac{M_d}{z} \geq 0 \qquad \text{...(i)}$$

$$\frac{P}{A}+\frac{Pe}{z}-\frac{M_d}{z} \leq f_c \qquad \text{...(ii)}$$

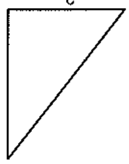


f_c

[f_c = Permissible stress in compression, M_d = dead load moment]

Under service load (when no prestress loss considered)

f_c



$$\frac{P}{A}-\frac{Pe}{z}+\frac{M_d+M_L}{z} < f_c \qquad \text{...(iii)}$$

$$\frac{P}{A}+\frac{Pe}{z}-\frac{M_d+M_L}{z} \geq 0 \qquad \text{...(iv)}$$

Normally out of these condition (ii) & condition (iv) is most critical & govern the limiting zone of cable.

From (ii) $\quad e \leq \dfrac{f_c z}{P}-\dfrac{z}{A}+\dfrac{M_d}{P}$

from (iv) $\quad e \geq -\dfrac{z}{A}+\dfrac{M_d+M_L}{P}$

From (ii) At mid span

$$e \leq \frac{f_c(bd^2/6)}{P}-\frac{bd^2}{6\times bd}+\frac{M_d}{P}$$

$$e \leq \frac{14(1000\times 400^2)}{6\times 2800\times 10^3}-\frac{400}{6}+\frac{180\times 10^6}{2800\times 10^3}$$

$$e \leq 130.952 \text{ mm}$$

At support

$$e \leq \frac{400}{6}$$

$$e \leq 66.67 \text{ mm}$$

From (iv) At mid span

$$e \geq -\frac{bd^2}{6\times bd}+\frac{M_d+M_L}{P}$$

$$e \geq -\frac{400}{6}+\frac{(180+360)\times 10^6}{2800\times 10^3}$$

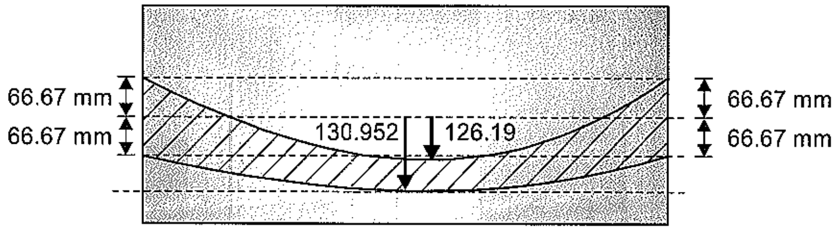
$$e \geq 126.19 \text{ mm}$$

At support

$$e \geq -\frac{400}{6}$$

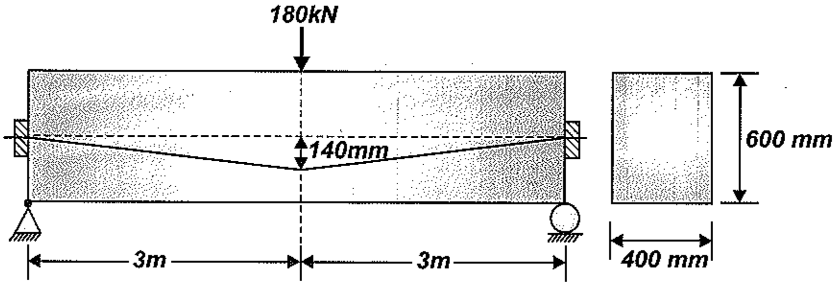
$$e \geq -66.67 \text{ mm}$$

Pictorial representation of the limiting zone of prestressing cable is as shown below.



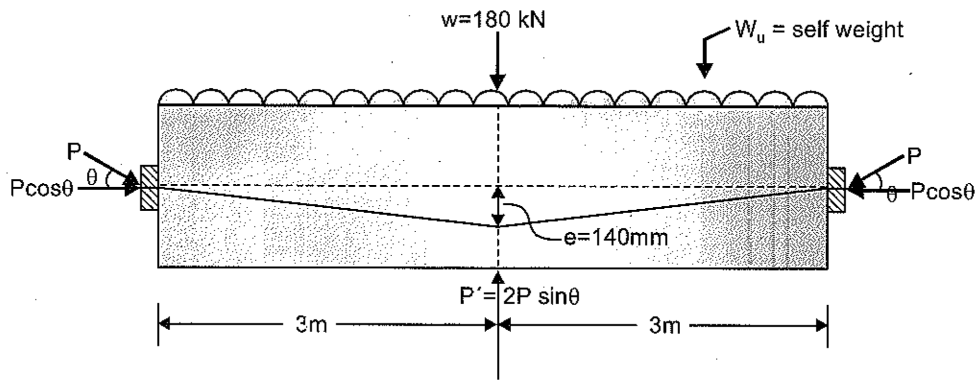
Q-6: ***A prestressed concrete beam 400 mm wide and 600mm deep has a span of 6m. The beam is prestressed with a tendon bent as shown. There is a central concentrated load of 180 kN acting on the beam. Effective prestressing force = 1200 kN.***

Calculate the extreme fibre stresses at mid span taking into account the self weight of the beam also.



[15 Marks, ESE-2005]

Sol: Effective prestressing force P = 1200 kN



θ is very small so $\sin\theta = \theta$ and $\cos\theta = 1$

$\therefore$ Upward reaction on the beam at the centre due to cable profile

$$P' = 2P\theta = 2\times1200\times\frac{0.140}{3} = 112\,\text{kN}.$$

$\therefore$ Self weight acting on the beam = (0.6 × 0.4 × 25) = 6 kN/m

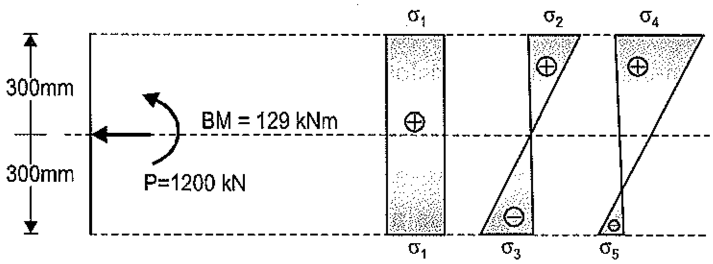
$\therefore$ Effective bending moment at central section–

= Moment due to (conc. load + self wt. – up reaction)

$$\frac{(W-P')\ell}{4} - \frac{w_u\ell^2}{8} = \frac{(180-112)\times6}{4} + \frac{6\times6^2}{8}$$

$$= 102 + 27 = 129 \text{ kNm (sagging)}$$

The section can be represented as



Compressive stress due to direct force $= \dfrac{1200\times1000}{600\times400}$

$$\sigma_1 = 5\text{N/mm}^2$$

Now section modulus of the section $z = \dfrac{bd^2}{6} = \dfrac{400\times600^2}{6} = 24 \times 10^6 \text{ mm}^3$

Compressive stress at extreme top fibre due to BM

$$\sigma_2 = \frac{BM}{z} = 5.375 \text{ N/mm}^2$$

tensile stress at extreme bottom fibre due to BM

$$\sigma_3 = \frac{BM}{z} = 5.375 \text{ N/mm}^2$$

Stress at extreme top fibre of mid span

$$\sigma_4 = \sigma_2 + \sigma_1 = 10.375\text{N/mm}^2$$

Stress at extreme bottom fibre of mid span

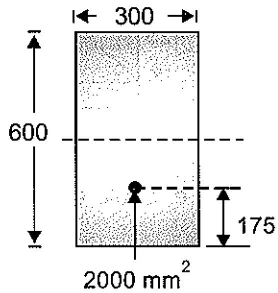
$$\sigma_5 = \sigma_1 + \sigma_3 = -0.375\text{N/mm}^2$$

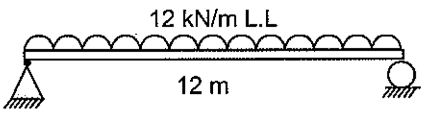
Q-7: ***A prestressed concrete beam with cross-section of 300 × 600, 12m long, carries a live load of 12 kN/m in addition to self weight. It is prestressed with 2000 mm² of high tension steel located at 175 mm from the soffit. The cable profile is straight for full length and stress in it is 800 N/mm². It is bonded with concrete. Determine the location of thrust line in the beam and plot the position at end, and at mid section.***

Given: $\dfrac{E_s}{E_c} = m = 6$.

[15 Marks, ESE-2006]

Sol:





When the effect of modular ratio is taken into account i.e., when the deduction due to hole is not made, a transformed area equivalent to (m – 1) A_{st} shall be taken into account.

In these cases the steps in calculation will be as under:

(1) locate the C.G of the transformed section

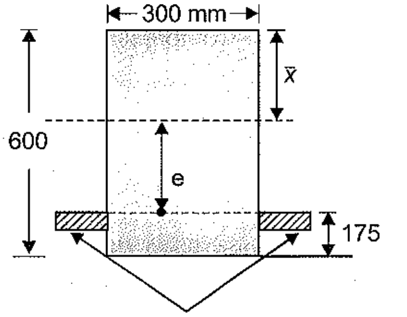
(2) Find out moment of inertia of transformed section about C.G line

(3) Find out section modulus Z_{bottom} and Z_{top}

(4) Find out the eccentricity of prestressing force w.r.t the C.G

(5) Prestressing force will be taken as area of steel tendon × stress in steel.

Further calculation would be the normal calculation



$(m-1)\,A_{st} = (6-1)\times 2000 = 10000\ mm^2$

A_{st} × Stress in tendon = Prestressing force = P

$$\Rightarrow \quad P = 2000 \times 800 = 1600\ kN$$

$$\bar{x} = \frac{300\times 600\times 300 + 10000\times 425}{300\times 600 + 10000}$$

$$\bar{x} = 306.57\ mm$$

$$I_{GG} = \frac{300\times(306.57)^3}{3} + \frac{300\,(293.43)^3}{3} + 10000\,(293.43-175)^2 = 5548 \times 10^6\ mm^4$$

$$Z_{top} = \frac{I_{GG}}{306.57} = 18.1\times 10^6\ mm^3$$

$$Z_{bottom} = \frac{I_{GG}}{293.43} = 18.9\times 10^5\ mm^3$$

$$e = 293.43 - 175 = 118.43\ mm$$

$$D.L = 0.3 \times 0.6 \times 24 = 4.32\ kN/m \text{ (Assuming the unit of concrete to be 24 kN/m}^3\text{)}$$

$$L.L = 12\ kN/m$$

$$\Rightarrow \quad \text{Total udl} = w = 16.32\ kN/m$$

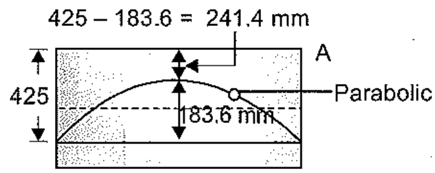
$$M_{mid\ span} = \frac{16.32\,(12)^2}{8} = 293.32\ kN/m$$

To locate the thrust line, we will use the amount of shifting from cable line.

At ends, M = 0

⇒ Amount of shifting will be zero

At mid span, M = 293.76 kNm



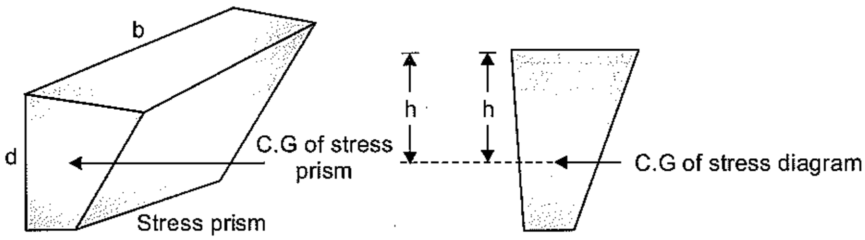
$$\Rightarrow \quad \text{Amount of shifting} = \frac{M}{P} = \frac{293.76\ kNm}{1600\ kNm} = 183.6\ mm$$

Also, as the variation of M is parabolic and cable is straight (i.e. cable eccentricity is constant) hence,

$\left(e' = \frac{M}{P} - e\right)$ i.e., Thrust line eccentricity will be parabolic

Note: If we draw the stress diagram and Find out its C.G, the thrust line at centre will not be 241.4 mm from top.

This is because location of thrust line will be at the C.G of stress prism not at the C.G of stress diagram. Calculation of C.G of stress prism will be difficult in any section in which width is not constant throughout. However if width is constant throughout, C.G of stress prism and that of stress diagram will be at the same vertical elevation.



Hence in case of section where width is not constant throughout, it would be better to compute the location of thrust line using the concept that shift of thrust line from cable line is M/P.

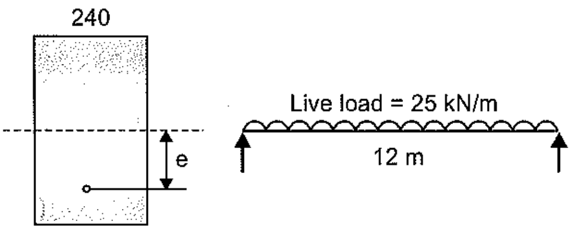
Q-8: ***A post tensioned prestressed beam of rectangular section of 240 mm wide is to be designed for a live load of 25 kN/m uniformly distributed on an effective span of 12m Stress in concrete must not exceed 17 MPa in compression a 1.4 MPa in tension at any time and the loss of prestress should be assumed as 15%.***

(1) Calculate the minimum possible depth of beam.

(2) For the section, designed above, calculate the minimum prestressing force and the corresponding eccentricity.

[20 Marks, ESE-2007]

Sol:



Stress in concrete must not exceed 17 MPa in compression & 1.4 MPa in tension

$$\eta = 0.85$$

At Transfer

At Top

$$-\left[\frac{P}{A}-\frac{Pe}{Z}+\frac{M_d}{Z}\right] \leq 1.4 \qquad \text{...(i)}$$

At Bottom

$$\frac{P}{A}+\frac{Pe}{Z}-\frac{M_d}{Z} \leq 17 \qquad \text{...(ii)}$$

At Service Load:

At Bottom

$$-\left[\eta\frac{P}{A}+\frac{\eta Pe}{Z}-\frac{M_d}{Z}-\frac{M_L}{Z}\right] \leq 1.4 \qquad \text{...(iii)}$$

At Top

$$\frac{\eta P}{A}-\frac{\eta Pe}{Z}+\frac{M_d}{Z}+\frac{M_L}{Z} \leq 17 \qquad \text{...(iv)}$$

From $\eta(i) + (iv)$

$$-\frac{\eta P}{A}+\frac{\eta Pe}{Z}-\frac{\eta M_d}{Z} \le 1.4\times\eta$$

$$+$$

$$\frac{\eta P}{A}-\frac{\eta Pe}{Z}+\frac{M_d}{Z}+\frac{M_L}{Z} \le 17$$

$$\Rightarrow \quad \frac{M_d(1-\eta)+M_L}{Z} \le (17+1.4\eta)$$

$$Z \ge \frac{M_d(1-\eta)+M_L}{17+1.4\eta} \qquad \text{...(A)}$$

From $\eta(ii) + (iii)$

$$\frac{\eta P}{A}+\frac{\eta Pe}{Z}-\frac{\eta M_d}{Z} \le 17\eta$$

$$+$$

$$-\frac{\eta P}{A}-\frac{\eta Pe}{Z}+\frac{M_d}{Z}+\frac{M_L}{Z} \le 1.4$$

$$\frac{M_d(1-\eta)+M_L}{Z} \le (17\eta+1.4)$$

$$Z \ge \frac{M_d(1-\eta)+M_L}{17\eta+1.4} \qquad \text{...(B)}$$

Now,

$$W_d = (B\,D\times 24)\text{ kN/m} = (0.24\times 24D)\text{kN/m} = 5.76D\text{kN/m}$$

$$M_d = \frac{5.76D(12)^2}{8}$$

$$M_d = 103.68\,D$$ (Unit wt. of Concrete = 24 kN/m^3)

$$M_L = \frac{25\,(12)^2}{8} = 450\text{ kN/m}$$

From A

M_d = 103.68 D kNm, D is in meter

M_L = 450 kNm

$$\left[\frac{M_d(1-\eta)+M_L}{Z}\right]\times\frac{10^3}{10^6} \le 17 + 1.4\eta$$

$$\Rightarrow \quad Z \ge \frac{[M_d(1-\eta)+M_L]\times 10^3}{17+1.4\eta}$$

$$\Rightarrow \quad \frac{0.24\,D^2}{6} \ge \frac{[103.68\,D\times 0.15+450]\times 10^{-3}}{17+1.4\times 0.85}$$

$$\Rightarrow \quad 0.7276\,D^2 \ge [15.552\,D+450]\times 10^{-3}$$

$$\Rightarrow \quad 727.6\,D^2 - 15.55\,2\,D - 450 \ge 0$$

$$D = \frac{15.552 \pm \sqrt{(15.552)^2 + 4 \times 450 \times 727.6}}{2 \times 727.6}$$

$$D \geq 0.797 \text{ m}$$

From B

$$\frac{0.24D^2}{6} \geq \frac{(103.68\, D \times 0.15 + 450) \times 10^{-3}}{17 \times 0.85 + 1.4}$$

$$634\, D^2 - 15.552\, D - 450 \geq 0$$

$$D \geq \frac{15.552 \pm \sqrt{(15.552)^2 + 4 \times 450 \times 634}}{2 \times 634}$$

$$D \geq 0.855 \text{ m}$$

$$\text{Adopt } D = 860 \text{ mm}$$

Min Prestressing is calculated from

(1) Top Stress at transfer

(2) Bottom stress at Service Condition

$$\Rightarrow \quad -\left[\frac{P}{A} - \frac{Pe}{Z} + \frac{M_d}{Z}\right] \leq 1.4 \qquad \ldots(\alpha)$$

$$\Rightarrow \quad -\left[\frac{\eta P}{A} + \frac{\eta Pe}{Z} - \frac{M_d}{Z} + \frac{M_L}{Z}\right] \leq 1.4m \qquad \ldots(\beta)$$

$$\Rightarrow \quad -\frac{\eta P}{A} + \frac{\eta Pe}{Z} - \frac{\eta M_d}{Z} \leq 1.4\, \eta$$

$$\frac{-2\eta P}{A} + \frac{M_d(1-\eta)}{Z} + \frac{M_L}{Z} \leq 1.4\,(1+\eta)$$

$$-\frac{2\eta}{A}P \leq 1.4(1+\eta) - \frac{M_d(1-\eta)}{Z} - \frac{M_L}{Z}$$

$$P \geq \left[\frac{M_d(1-\eta) + M_L}{Z} - 1.4\,(1+\eta)\right]\frac{A}{2\eta}$$

$$P \geq \left[\frac{[103.68 \times 0.860 \times 0.15 + 450]}{\left(\frac{(0.24)\,(0.86)^2}{6}\right)} \times \frac{1}{10^3} - 1.4 \times 1.85\right] \times \frac{240 \times 860}{2 \times 0.85}$$

$$P \geq 1587.216 \text{ kN}$$

From $\alpha \,\&\, \beta$

$$-\frac{\eta P}{A} + \frac{\eta Pe}{Z} - \frac{\eta M_d}{Z} = 1.4\, \eta$$

$$-\frac{\eta P}{A} - \frac{\eta Pe}{Z} + \frac{M_d}{Z} - \frac{M_L}{Z} = +1.4$$

$$+ \quad + \quad - \quad +$$

$$\frac{2\eta Pe}{Z} - (1+\eta)\frac{M_d}{Z} - \frac{M_L}{Z} = 1.4\,\eta - 1.4$$

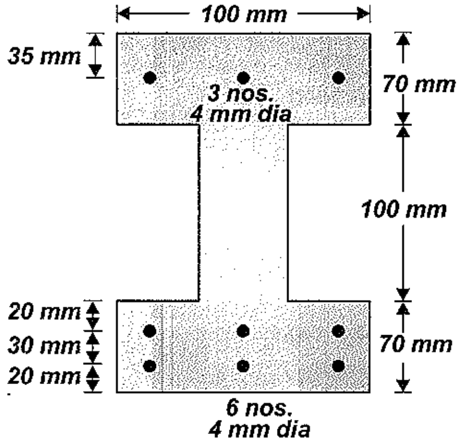
$$\frac{2\eta Pe}{Z} = \frac{(1+\eta)M_d + M_L}{Z} + 1.4(\eta - 1)$$

$$e = \left[\frac{(1+\eta)M_d + M_L}{Z} - 1.4(1-\eta)\right]\frac{Z}{2\eta P}$$

$$e = \left[\frac{(1+\eta)M_d + M_L}{Z} - 1.4(1-\eta)\right]\frac{Z}{2\eta P} \times \frac{240\times(860)^2}{6\times[2\times 0.85]\times 1587.216\times 10^3}$$

$$e = 225.6$$

Q-9: ***The pre-stressed concrete beam shown in the figure, provided with 9 nos. of 4 mm diameter wires as shown is subjected to a pre-stress of 800 N/mm². Determine the sagging moment that can be applied to the section so that the maximum compressive stress in concrete shall not exceed 14 N/mm² and the maximum tensile stress in the concrete shall not exceed 1.4 N/mm². Neglect the losses in pre-stress.***



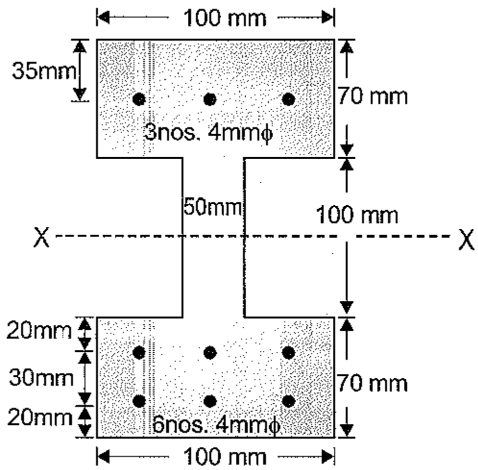
[15 Marks, ESE-2008]

Sol: As the span of the beam section is not given effect of dead load cannot be estimated. Hence we will only account for

(1) effect of axial load P

(2) moment due to eccentricity of axial load (Pe)

(3) externally applied sagging moment.



Neutral axis assumed to be at mid depth (neglecting the area of steel)

$$\text{Area of steel wires in the top flange} = 3 \times \pi/4 \times 4^2 = 37.7 \text{ mm}^2$$

$$\text{Area of steel wires in the bottom flange} = 6 \times \pi/4 \times 4^2 = 75.4 \text{ mm}^2$$

So, $$\text{Total area of steel} = 37.7 + 75.4 = 113.1 \text{ mm}^2$$

$$\therefore \quad \text{Prestressing force (P)} = \frac{(800 \times 113.1)}{1000} = 90.48 \text{ kN}$$

$$\text{Area of beam cross-section} = 100 \times 240 - 100 \times 50 = 19000 \text{ mm}^2$$

Moment of Inertia of beam section along x – x.

$$I = \frac{100 \times 240^3}{12} - \frac{2 \times 25 \times 100^3}{12} = 111033333.3 \text{ mm}^4$$

$$\text{Section modulus } Z = \frac{I}{y_{max}} = \left\{\frac{111033333.3}{120}\right\} = 925277.78 \text{ mm}$$

$$\text{Direct stress } \frac{P}{A} = \frac{90.48 \times 1000}{19000} = 4.762 \text{ N/mm}^2$$

Extreme stresses due to eccentricity in prestressing forces

$$= \mp\left[\frac{75.4 \times 800 \times 85}{925277.8} - \frac{37.7 \times 800 \times 85}{925277.8}\right]$$

$$= \mp 2.77 \text{N/mm}^2$$

Assuming maximum sagging moment M.

Case 1: Limiting the extreme compressive stress at top at 14 N/mm^2

[Normally under service load condition there is compressive stress at top &tensile stress at bottom]

$$\therefore \quad 4.76 - 2.77 + \frac{M}{925277.8} = 14$$

$$\Rightarrow \quad M = 12.01 \text{ kN-m}$$

Case 2: Limiting the extreme tensile stress at bottom to 1.4 N/mm^2

$$4.76 + 2.77 - \frac{M}{925277.8} = -1.4$$

$$\Rightarrow \quad M = 8.26 \text{ kN-m}$$

Hence maximum saging bending moment = 8.26 kN-m

Alternative Solution :

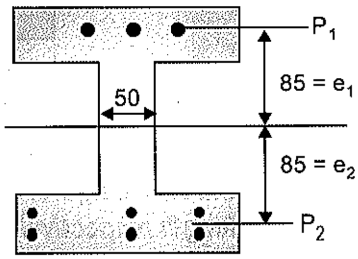
Effect of axial loading

$$P_1 = 3 \times \frac{\pi}{4}(4)^2 \times 800 = 30.16 \text{ kN}$$

$$P_2 = \frac{30.16}{3} \times 6 = 60.32 \text{ kN}$$

$$\text{Total compressive stress} = \frac{P}{A} = \frac{30.16 + 60.32}{100 \times 240 - 50 \times 100}$$

$$= 4.76 \text{ N/mm}^2$$



N.A will be assumed at mid depth

$$\Rightarrow \quad I = \frac{100\times(240)^3}{12} - \frac{50\times100^3}{12}$$

$$I = 111.03 \times 10^6 \text{ mm}^4$$

$$Z = \frac{I}{120} = 925.27\times10^3 \text{ mm}^4$$

Net sagging moment:

$$M + P_1 e_1 - P_2 e_2 = M + 30.16\times85\times10^3 - 60.32\times10^3\times85$$

$$= M - 30.16 \times 10^3 \times 85$$

Stress due to sagging moment:

$$\text{Top} = \frac{M - 30.16\times85\times10^3}{Z}$$

$$\text{Bottom} = -\frac{M - 30.16\times85\times10^3}{Z}$$

$$\text{Total Top stress} = 4.76 + \frac{M - 30.16\times16\times85\times10^3}{925.27\times10^3}$$

$$\text{Total Bottom stress} = 4.76 - \frac{M - 30.16\times85\times10^3}{925.27\times10^3}$$

Note: Normally under service load condition there is lesser stress at the bottom and larger stress at top.

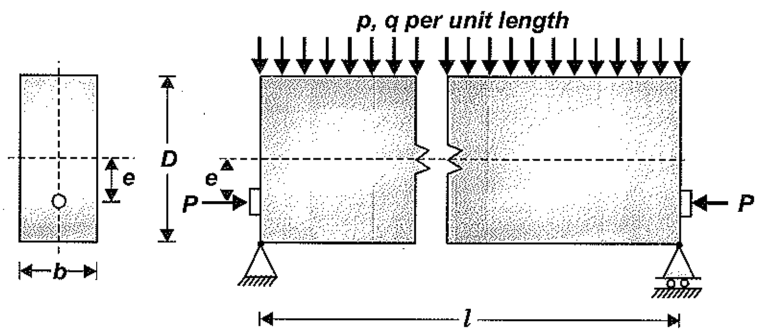
Hence, $$4.76 + \frac{M - 30.16\times85\times10^3\times16}{925.27\times10^3} = 14 \Rightarrow M = 11.11 \text{ kNm}$$

$$4.76 - \frac{M - 30.16\times85\times10^3}{925.27\times10^3} = -1.4 \Rightarrow M = 8.265 \text{ kNm}$$

Adopt max sagging BM = 8.26 kNm
(min of the above two)

Note: If the tension is assumed at the top of the beam and the compression is assumed at the bottom the value of M comes out to be negative i.e., we will have to apply hogging moment for this. Thus not acceptable because in the question it has beam asked to calculate max sagging B.M.

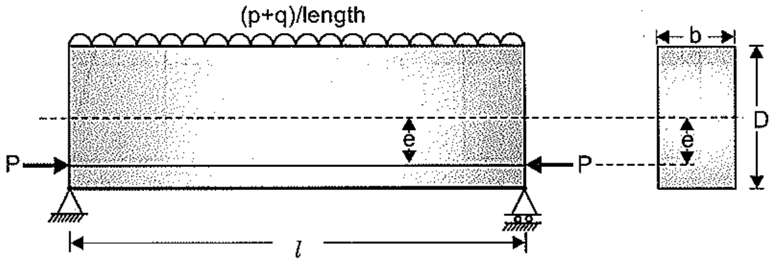
Q-10:



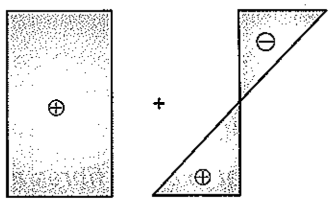
A simply supported high strength concrete beam of rectangular cross-section (b × D), shown in the above figure, supports uniformly distributed dead and live loads of intensities p and q per unit length respectively. The beam has prestressing force P at an eccentricity e. Show clearly and neatly the stress distributions, through the beam depth, due to eccentric prestressing, dead and live loads at a cross-section where maximum stresses occur.

[15 Marks, ESE-2009]

Sol: In this problem we will have to locate the section as which max stress occurs. To locate the section of maximum stress, the constraint that we will apply is that there should not be any tension developed any where in the section.



The load P and its eccentricity e results in constant stress throughout the beam. The resulting stress is max at bottom and minimum at top.



P/A

$$\frac{PeD/2}{\frac{bD^3}{12}} = \frac{6Pe}{bD^2}$$

For No tension without external loading and when D.L is not causing its effect, $\frac{P}{A} \geq \frac{6Pe}{bd^2}$ ...(i)

BM due to dead and live load is always sagging thus leading to tension at bottom and compression at top.

For No tension at base when external moment are also accounted for

$$\frac{P}{bD} + \frac{6Pe}{bD^2} - \frac{6M}{bd^2} > 0$$

$$\frac{P}{bd} + \frac{6Pe}{bD^2} > \frac{6M}{bD^2}$$

$\Rightarrow$ $\frac{12\,Pe}{bD^2} > \frac{6M}{bD^2}$ (taking the min value of load effect, P/bd from (i), $\frac{p}{bd} = \frac{6pe}{bd^2}$)

$\Rightarrow$ $M < 2Pe$

but when $M < 2Pe$

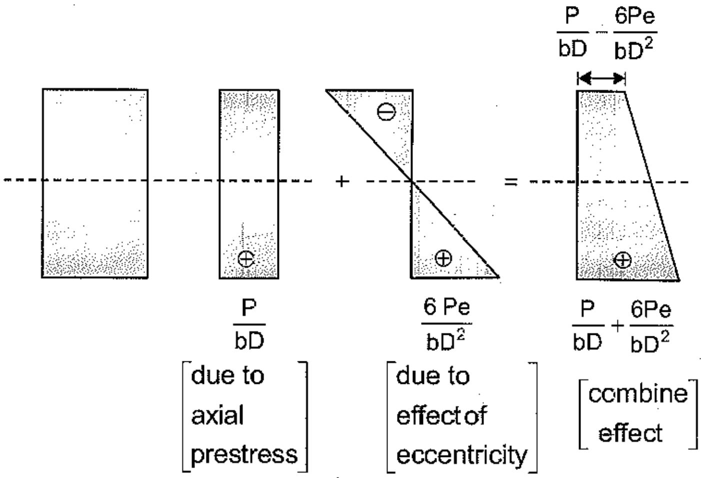
Stress at top = $\frac{P}{bD} - \frac{6Pe}{bD^2} + \frac{6M}{bD^2}$

$\Rightarrow$ Stress at top < $\frac{P}{bD} - \frac{6Pe}{bD^2} + \frac{12\,Pe}{bD^2}$

$\Rightarrow$ Stress at top < $\frac{P}{bD} + \frac{6Pe}{bD^2}$ (i.e. even smaller than the case when external moments were not considered)

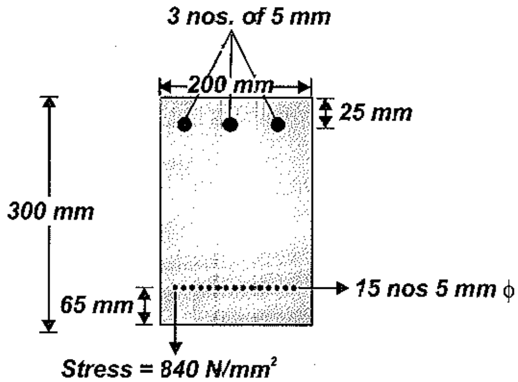
$\Rightarrow$ Thus for max stress, we should consider a section when BM = 0 i.e., at support

Hence stresses at support are



Q-11: ***A rectangular simply supported prestressed concrete beam of cross-sectional area 200 mm × 300 mm is prestressed by 15 nos. 5 mm f bar located at 65 mm from the soffit and 3 nos. of 5 mm f bar at 25 mm from the top. Assuming the effective stress in the steel wire as 840 N/mm²***

(i) Calculate the stress in concrete at extreme fibres at mid span due to prestressing force and also due to its own weight over the span of 6 m



(ii) If a uniformly distributed working load of 6 kN/m imposed on the beam, find the maximum compressive stress in the concrete.

(iii) If the modulus of rapture of concrete is 6.5 N/mm². Estimate the load factor against cracking

[20 Marks, ESE-2012]

Sol: Assuming bars to be unbounded and by neglecting the holes due to steel bars, the neutral axis can be taken at mid depth of beam.

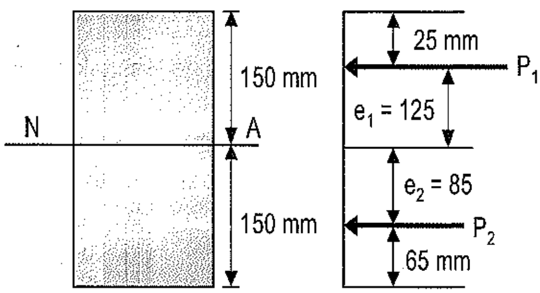
Stresses in concrete will be due to

(1) D.L moment

(2) $(P_1 + P_2)$ compressive

(3) P_1 at some eccentricity e_1.

(4) P_2 at some eccentricity e_2



$$\text{D.L.} = 0.2 \times 0.3 \times \underset{\text{unit wt. of prestressed concrete}}{\underset{\downarrow}{24}} \text{ kN/m} = 1.44 \text{ kN/m}$$

$$\text{B.M due to D.L at mid span} = \frac{Wl^2}{8} = \frac{1.44\,(6)^2}{8} = 6.48 \text{ kNm}$$

Stresses due to dead load

D.L

$$\text{Bending stress at top} = \frac{M_d}{Z_t} = \frac{6.48\times16^6 \text{ Nmm}}{\frac{200\,(300)^2}{6}\text{ mm}^3} = +2.16 \text{ N/mm}^2$$

$$\text{Bending stress at bottom} = \frac{M_d}{Z_b} = -2.16 \text{ N/mm}^2 = -2.16 \text{ N/mm}^2$$

$$\text{Net B.M due to } P_1 \text{ and } P_2 = (P_1 e_1 - P_2\ e_2) \text{ sagging}$$

Stress due to bending caused by P_1 & P_2

$$\text{Top} = \frac{P_1 e_1 - P_2 e_2}{Z}$$

$$\text{Bottom} = \frac{P_1 e_1 - P_2 e_2}{Z}$$

$$P_1 = 3\times\frac{\pi}{4}(5)^2\times840 \text{ N} = 49.480\times10^3 \text{ N}$$

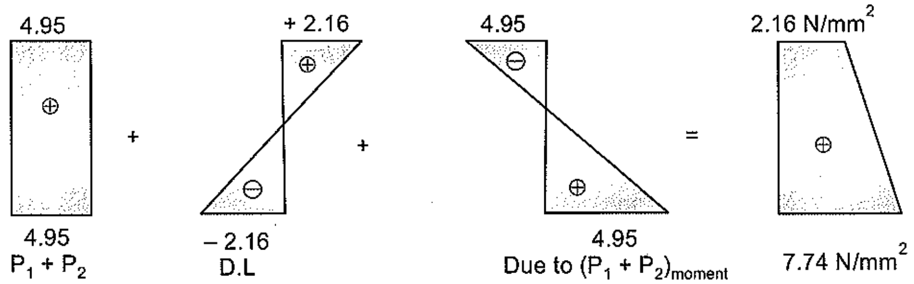
$$P_2 = \frac{49.48}{3}\times15\times10^3 \text{ N} = 247.4\times10^3 \text{ N}$$

$$\text{Top stress} = \frac{49.48\times10^3\times125 - 247.4\times10^3\times85}{\frac{200\,(300)^2}{6}} = -4.95 \text{ N/mm}^2$$

$$\text{Bottom stress} = 4.95 \text{ N/mm}^2$$

Stress due to (P_1 + P_2) will be compressive throughout $= \frac{P_1+P_2}{A} = \frac{(49.48+247.4)\times10^3}{200\times300}\text{ mm}^2 = 4.95$

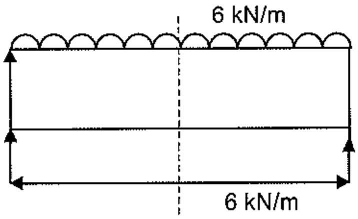
$$(P_1 + P_2)_{stress} = 4.95$$



At mid span **Fig.A**

Part 2: As there is no loss taking place, stress shown in fig. (A) will remain as such only further addition will be due to live load.

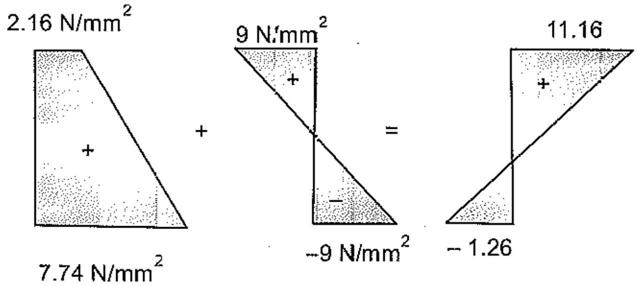
$$\text{Max BM at mid span due to live} = \frac{wl^2}{8} = \frac{6\times6^2}{8} = 27 \text{ kN}$$



Stress

$$\text{Top} = +\frac{27\times10^6}{\frac{200(300)^2}{6}} = 9 \text{ N/mm}^2$$

$$\text{Bottom} = -9 \text{ N/mm}^2$$



Maximum compressive stress in beam with D.L, L.L and prestressing force is equal to 11.16 N/mm²

Part 3: Tensile stress existing is –1.26 N/mm². However, cracking will occur when the tensile stress becomes 6.5 N/mm², (i.e., equal to modulus of rupture of conc.) To create this, additional live load moment required will be

$$= \frac{27}{9} \times (6.5 - 1.26) = 15.72 \text{ kN-m}$$

$\Rightarrow$ Live load moment causing cracking = 27 + 15.72 = 42.72 kNm

$\Rightarrow$ Load factor against cracking w.r.t LL = $\dfrac{W_{\text{live crack}}}{W_{\text{live}}} = \dfrac{M_{\text{live crack}}}{M_{\text{live}}} = \dfrac{42.72}{27}$ = 1.58 w.r.t to live load

Load factor against cracking w.r.t total load = $\dfrac{W_{\text{total crack}}}{W_{\text{total}}} = \dfrac{M_{\text{total crack}}}{M_{\text{total}}} = \dfrac{(M_{\text{dead}} + M_{\text{live}})_{\text{crack}}}{(M_{\text{dead}} + M_{\text{live}})}$

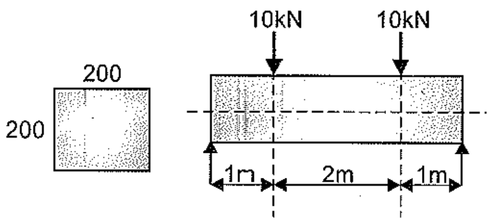
$$= \frac{6.48 + 42.72}{6.48 + 27} = 1.46 \text{ w.r.t total load.}$$

Generally load factor should be calculated with respect to live load.

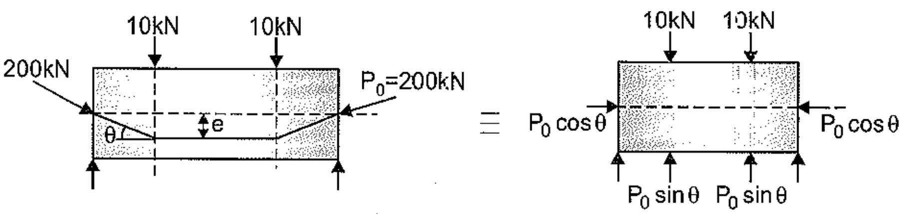
Q-12: ***A PSC simply supported beam of length 4 m, width 200 mm and depth 200 mm is prestressed with a prestressing force of 200 kN. The beam is subjected to two concentrated loads of 10 kN each located 1 m from each support. Neglecting the dead weight of the beam, sketch the cable profile of the tendons for load balancing condition. Determine the maximum stress produced in the concrete.***

[10 Marks, ESE-2015]

Sol:



As per load balancing concept, for concentrated loads cable profile would be as follows:



$$P_0 \sin\theta \simeq P_0 \tan\theta = 10\text{kN}$$

$$\Rightarrow \qquad 200 \times \frac{e}{1000} = 10$$

$$\Rightarrow \qquad e = 50 \text{ mm}$$

$$\cos\theta = 1$$

The effect of 10 kN load will be fully countered by force due to tendons.

$\therefore$ No bending stress will be generated in the beam.

$$\therefore \qquad \text{Max. stress} = \frac{P_0 \cos\theta}{A} = \frac{200 \times 10^3}{200 \times 200} = 5 \text{ N/mm}^2$$

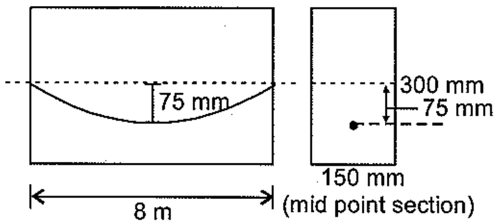
Compressive throughout.

Q-13: ***A prestressed concrete beam of 150mm × 300mm cross-section supports a live load of 5 kN/m over a simple span of 8m. It has a parabolic cable having an eccentricity of 75mm at the mid-span and zero at the ends. Determine the force of prestress if the net resultant stress at the bottom fibre at mid-span is zero under the action of self-weight, live load and prestress force.***

[12 Marks, ESE-2017]

Sol: **Given:**

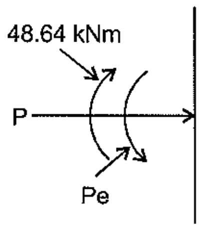
L.L. = 5 KN/m



D.L. = 0.3 × 0.15 × 24 (Unit wt of prestress concrete = 24 kN/m^3)

= 1.08 kN/m

D.L. + L.L. = 1.08 + 5= 6.08 kN/m



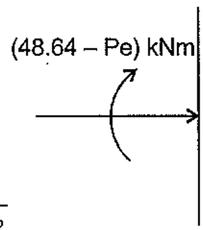
Let the pre-stressing force = P

$$\text{So,} \qquad \text{B.M. due to D.L. and L.L.} = \frac{6.08 \times 8^2}{8} = 48.64 \text{ kNm}$$

$$\text{Section modulus (z)} = \frac{bD^2}{6} = \frac{150 \times 300^2}{6}$$

At Mid Span

Stress at the bottom fiber



$$\sigma_b = \frac{P}{A} + \frac{Pe}{z} - \frac{(M_D + M_L)}{z} = 0$$

$$\sigma_b = \frac{P}{150 \times 300} + \frac{P \times 75}{\frac{1}{6} \times 150 \times 300^2} - \frac{48.64 \times 10^6}{\frac{1}{6} \times 150 \times 300^2}$$

$$\sigma_b = 0$$

$$\Rightarrow \qquad \frac{P}{300 \times 150} = \frac{48.64 \times 10^6 \times 6}{150 \times 300^2} - \frac{Pe \times 6}{300^2 \times 150} \quad (e = 75 \text{ mm})$$

$$\Rightarrow \quad P\left(\frac{1}{300\times150}+\frac{75\times6}{300^2\times150}\right) = \frac{48.64\times6\times10^6}{150\times300^2}$$

So, $P = 389120 \text{ N}$

or $P = 389.12 \text{ kN}$

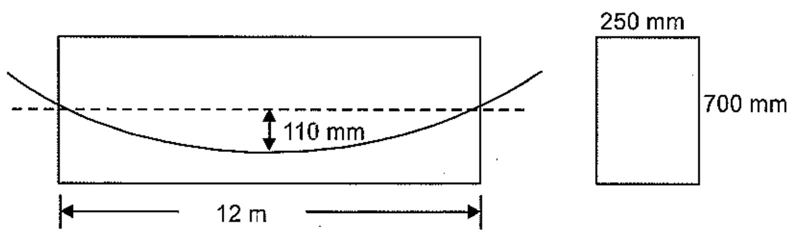
Q-14: ***A prestressed concrete beam supports an imposed load of 6.5 kN/m over an effective span of 12 m. The beam has a rectangular section of width 250 mm and depth of 700 mm. Find the effective prestressing force in the cable if it is parabolic with an centricity of 110 mm at the centre and zero at the ends, for the following conditions :***

(i) If the bending effect of the prestressing force is nullified by the imposed load for the mid-span section (neglecting self weight of the beam).

(ii) If the resultant stress due to self-weight, imposed load and prestressing force is zero at the soffit of the beam for the mid-span section. Assume the density of concrete is 24 kN/m³.

[12 Marks, ESE-2019]

Sol: Given data:



(i) If the bending effect of the prestressing force is nullified by the imposed load for the mid-span section.

Moment due to presstress force = Moment due to imposed load

$$P\times e = \frac{wL^2}{8}$$

$$P = \frac{wL^2}{8e} = \frac{6.5\times12^2}{8\times0.11}$$

$$\boxed{P = 1063.64 \text{ kN}}$$

(ii) Self weight of beam = 24 × 0.25 × 0.7 = 4.2 kN/m

Total udl = 4.2 + 6.5 = 10.7 kN/m

$$\text{Moment due to load} = \frac{wL^2}{8} = \frac{10.7\times12^2}{8} = 192.6 \text{ kN-m}$$

As per given condition

Stress at soffit = 0

$$\frac{P}{A}+\frac{Pe}{z}-\frac{M}{z} = 0$$

$$P\left[\frac{1}{A}+\frac{e}{z}\right] = \frac{M}{z}$$

$$P\left[\frac{1}{250\times700}+\frac{110\times6}{250\times700^2}\right] = \frac{192.6\times10^6\times6}{250\times700^2}$$

$$\boxed{P = 849.7 \text{ kN}}$$

Q-15: ***A simply supported prestressed concrete beam of width 100 m, depth 200 and span 10m, carries a udl of intensity 'w'. If the member is prestressed with a parabolic cable having zero eccentricity at the ends and 60 mm eccentricity at mid, determine the value 'w' for the following conditions, for effective prestressing force of 125 kN.***

(i) Load balancing case

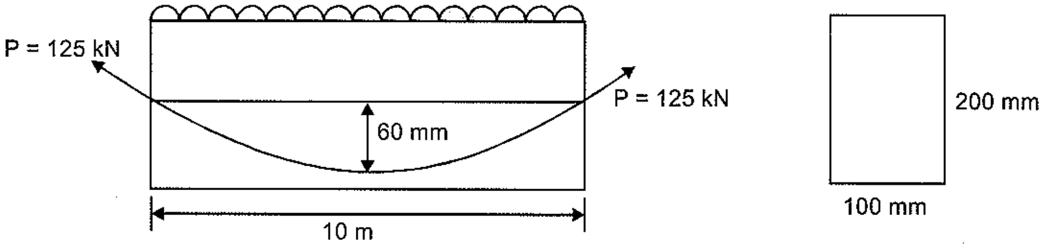
(ii) For no tensile stress condition at mid span

(iii) For cracking condition taking the tensile strength of concrete as 1.5 N/mm²

For all cases neglect the weight of concrete.

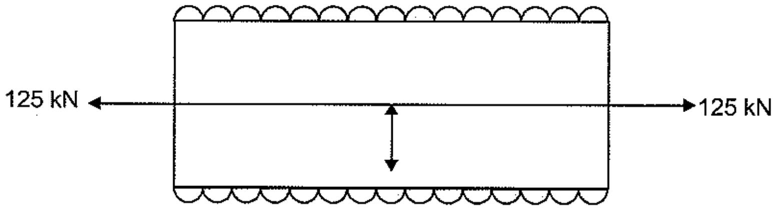
[20 Marks, ESE-2020]

Sol:



(i) Load balancing case:

The upward load w_e due to parabolic profile



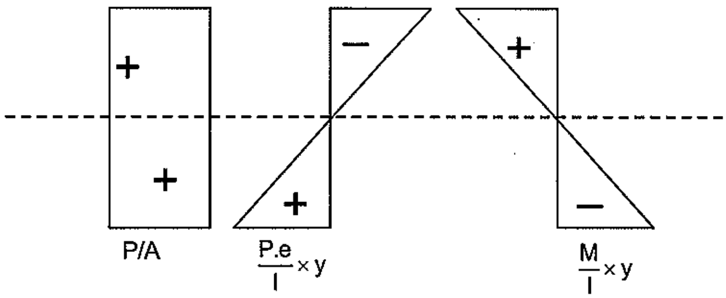
$$w_e = \frac{8Pe}{\ell^2}$$

$$w_e = \frac{8\,P.e}{\ell^2}$$

$$= \frac{8\times125\times60\times10^{-3}}{10\times10}$$

$$w_e = 0.6 \text{ kNm}$$

(ii) For no tensile stress condition of mid span



$$\sigma_{top} = \frac{P}{A} - \frac{Pe}{I}y + \frac{M}{I}\times y$$

$$\sigma_{bottom} = \frac{P}{A} + \frac{Pe}{I}y - \frac{M}{I}\times y$$

For no tension at top

$$\frac{P}{A}-\frac{Pe}{I}y+\frac{M}{I}\times y = 0$$

$$\Rightarrow \quad \frac{125\times10^3}{200\times100}-\frac{125\times10^3\times60}{\left(\frac{100\times200^3}{12}\right)}\times100+\frac{M}{\left(\frac{100\times200^3}{12}\right)}\times100=0$$

$$\Rightarrow \quad 6.25 - 11.25 + M \times 1.5 \times 10^{-6} = 0$$

$$M = 3.33 \times 10^{-6} \text{ N-mm}$$

$$\frac{w\ell^2}{8} = 3.33 \text{ kN-mm}$$

$$w = \frac{8\times3.3}{100} = 0.32 \text{ kN/m}$$

$$\boxed{w = 0.32 \text{ kN/m}}$$

For no tension at bottom

$$\frac{P}{A}+\frac{Pe}{I}\times y-\frac{M}{y}\times y = 0$$

$$6.25 + 11.25 - M \times 1.5 \times 10^{-6} = 0$$

$$M = 11.66 \text{ N-mm}$$

$$\frac{w\ell^2}{8} = 11.66 \text{ kN-m}$$

$$\boxed{w = 0.93 \text{ kN/m}}$$

(iii) For cracking condition tensile stress at the concrete = 1.5 N/mm^2

So, mininimum load for no tension = 0.2664 N/m

$$\sigma_{top} = -1.5$$

$$\frac{P}{A}-\frac{Pe}{I}y+\frac{M}{I}y = -1.5$$

$$6.25 - 11.25 + M \times 1.5 \times 10^{-6} = -1.5$$

$$M = 2.33 \times 10^{6} \text{ N-mm}$$

$$M = 3 \text{ kN-m}$$

$$\frac{w\ell^2}{8} = 3$$

$$\boxed{w = \frac{18.64}{100} = 0.18 \text{ kN/m}}$$

$$\sigma_{bottom} = -1.5$$

$$6.25 + 11.25 - M \times 1.5 \times 10^{-6} = -1.5$$

$$M = 12.67 \text{ kNm}$$

$$\frac{w\ell^2}{8} = 12.67$$

$$w = \frac{12.67\times8}{100} = 1.01 \text{ kN/m}$$

$$\boxed{w = 1.01 \text{ kN/m}}$$

So, minimum load for tensile stress 1.5 N/mm^2 at the concrete = 0.18 N/m

CHAPTER 14

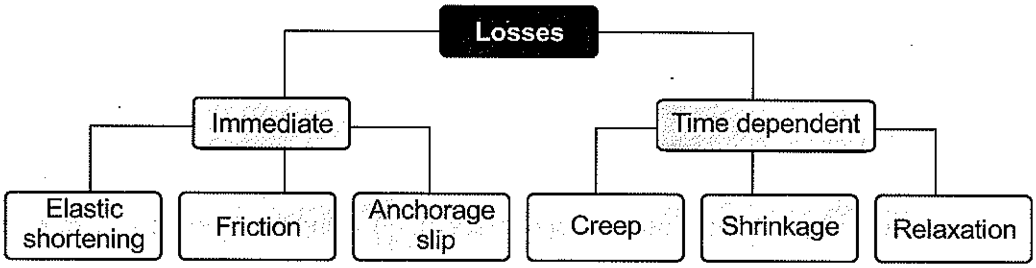
LOSSES IN PRESTRESS

Q-1: *What are the different losses of prestress in pre-tensioned and post-tensioned beams. How are they estimated?*

[10 Marks, ESE-1995, 2004, 2005, 2008]

Sol: **Losses in Prestress**

- In prestressed concrete applications, the most important variable is the prestressing force. In the early days, it was observed that the prestressing force does not stay constant, but reduces with time.
- Even during prestressing of the tendons and the transfer of prestress to the concrete member, there is a drop of the prestressing force from the recorded value in the jack gauge.
- The various reductions of the prestressing force are termed as the losses in prestress.
- The losses are broadly classified into two groups, immediate and time-dependent.
- The immediate losses occur during prestressing of the tendons and the transfer of prestress to the concrete member.
- The time-dependent losses occur during the service life of the prestressed member.
- The various losses in prestress are shown in the following chart.



Various losses in prestress

The different types of losses encountered in pretensioning and post tensioning are as given below.

Pre-tensioning	Post-tensioning
1. Elastic deformation of concrete	1. No loss due to elastic shortening when all bars are simultaneously tensioned. If however, wires are successively tensioned there would be loss of prestress due to elastic deformation of concrete
2. Relaxation of steel	2 Relaxation of steel
3. Shrinkage of concrete	3. Shrinkage of concrete
4. Creep of concrete	4. Creep of concrete
	5. Frictional losses
	6. Anchorage slip

Note: The total losses are around 15 to 20% of the initial prestressing.

- In addition to the above, there may be losses of prestress due to sudden changes in temperature, especially in steam curing of pretensioned units.
- The rise in temperature causes a partial transfer of prestress (due to the elongation of the tendons between adjacent units in the long-line process) which may cause a large amount of creep if the concrete is not properly cured.

Elastic Shortening Loss

Pre-tensioned Members

When the tendons are cut and the prestressing force is transferred to the member, the concrete undergoes immediate shortening due to the prestress. The tendon also shortens by the same amount, which leads to the loss of prestress.

Post-tensioned Members

If there is only one tendon, there is no loss because the applied prestress is recorded after the elastic shortening of the member. For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons.

Elastic Shortening loss Calculation

- The elastic shortening loss is quantified by the drop in prestress (Δf_s) in a tendon due to the change in strain in the tendon ($\Delta\varepsilon_s$).
- It is assumed that the change in strain in the tendon is equal to the strain in concrete (ε_c) at the level of the tendon due to the prestressing force. This assumption is called strain compatibility between concrete and steel.

  Change in strain is

$$\Delta f_s = E_s \Delta\varepsilon_s = E_s \varepsilon_c = E_s\left(\frac{f_c}{E_c}\right)$$

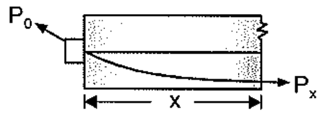
$$\boxed{\Delta f_s = m f_c}; \quad m = \text{Modular ratio}$$

- In case of curved tendon, use average stress ($f_{c_{avg}}$) along the tendons in computing the elastic shortening loss.

Loss of Prestress due to Friction

- The friction generated at the interface of concrete and steel during the stretching of a curved tendon in a post-tensioned member, leads to a drop in the prestress along the member from the stretching end.
- The loss due to friction does not occur in pre-tensioned members because there is no concrete during the stretching of the tendons.
- The friction is generated due to the curvature of the tendon and the vertical component of the prestressing force.
- In addition to friction, the stretching has to overcome the **wobble** of the tendon. The wobble refers to the change in position of the tendon along the duct. The losses due to friction and wobble are grouped together under friction.
- Force in cable at a distance x from jacking end, after frictional loss = Px

$$\boxed{P_x = P_0 e^{-(\mu\alpha + kx)}}$$



where, P_x = Prestressing force at a distance x from jacking end.

P_0 = Prestressing force at jacking end.

k = Coefficient called wobble correction factor.

μ = Coefficient of friction in curve

α = Cumulative angle in radian through which the tangent to the cable profile has turned between any two point under consideration.

- For small values of $\mu\alpha + kx$, the above expression can be simplified by the Taylor series expansion.

$$P_x = P_0(1 - \mu\alpha - kx)$$

$$P_0(\mu\alpha + kx) = P_0 - P_x = \text{loss in prestress}$$

$$\mu\alpha + kx = \frac{P_0 - P_x}{P_0}$$

$$\frac{\text{Loss of prestress}}{\text{Initial stress}} = \mu\alpha + kx$$

$$\boxed{\text{Loss of prestress} = (\mu\alpha + kx)\ \text{Initial stress}}$$

Loss of Prestress Due to Anchorage Slip

- In a post-tensioned member, when the prestress is transferred to the concrete, the wedges slip through a little distance before they get properly seated in the conical space.
- The anchorage block also moves before it settles on the concrete.
- There is loss of prestress due to the consequent reduction in the length of the tendon.

$$\text{This loss due to anchorage slip} = \frac{E_s\Delta}{L} = \left(\frac{\Delta}{L}\right)E_s$$

E_s = Young modulus of steel in N/mm^2

Δ = Anchorage slip in mm

L = Length of cable in mm

Table: Typical values of anchorage slip

Anchorage system	Anchorage slip (Δ)
Freyssinet	
12-5 mm ϕ strands	4 mm
12-8 mm ϕ strands	6 mm
Magnel	8 mm
Dywidag system	1 mm

Loss of Prestress Due to Creep of Concrete

Creep is the property of concrete by which it continues to deform with time under sustained loading.

$$\text{Creep coefficient is defined as f} = \frac{\text{Creep strain}}{\text{Elastic strain}} = \frac{\varepsilon_{cp}}{\varepsilon_c}$$

$$\varepsilon_{cp} = \phi\varepsilon_c \quad \text{... (i)}$$

$$\varepsilon_c = \frac{f_c}{E_c}$$

$$\varepsilon_{cp} = \phi.\frac{f_c}{E_c}$$

$$\text{Loss of stress} = \varepsilon_{cp} \times E_s = \phi.\frac{f_c}{E_c} \times E_s; \; m = \frac{E_s}{E_c}$$

$$\boxed{\text{Loss of stress} = m\phi f_c}$$

m = Modulator ratio

ϕ = Creep coefficient

f_c = Strain in concrete at the level of steel

- Note that elastic shortening loss multiplied by creep co-efficient is equal to loss due to creep.

Age at loading	Creep co-efficient
7 days	2.2
28 days	1.6
14 years	1.1

Loss Due to Shrinkage of Concrete

The shrinkage of concrete in prestressec members results in a shortening of tensioned wires and hence contributes to the loss of stress.

- In the case of pre-tensioned members, generally moist curirg is resorted to in order to prevent shrinkage until the time of transfer.
- Consequently, the total residual shrinkage strain will be larger in pretensioned members after transfer of prestress in comparison with post-tensioned members, where a portion of shrinkage will have already taken place by the time of transfer of stress.
- This aspect has been considered in the recommendations made by the Indian standard code (IS: 1343) for the loss of prestress due to the shrinkage of concrete and is detailed below.

  ε_{cs} = Total residual shrinkage strain having values of 3×10^{-4} for pretensioning and $\left[\frac{2\times10^{-4}}{\log_{10}(t+2)}\right]$ for post-tensioning.

  where, t = age of concrete at transfer in days.
- The value may be increased by 50 per cent in dry atmospheric conditions, subject to a maximum value of 3×10^{-4} units.

  The loss of stress in steel due to the shrinkage of concrete s estimated as,

  Loss of stress = $\varepsilon_{cs} \times E_s$.

  where, E_s = Modulus of elasticity of steel.

Loss of Prestress due to Relaxation of Steel

- Relaxation of steel is defined as the decrease in stress with time under constant strain.
- Due to the relaxation of steel, the prestress in the tendon is reduced with time.
- The relaxation depends on the type of steel, initial prestress (f_{pi}) and the temperature.

- To calculate the drop (or loss) in prestress (Δf_p), the recommendations of **IS:1343 - 1980** can be followed in absence of test data.

Table: Relaxation losses for prestressing steel at 1000 H at 27°C (as per IS 1343 – 1980)

Initial stress (1)	Relaxation loss N/mm^2 (2)
$0.5\ f_p$	0
$0.6\ f_p$	35
$0.7\ f_p$	70
$0.8\ f_p$	90

Note: f_p is the characteristic strength of prestressing steel.

Q-2: ***A simply supported post tensioned concrete beam of span 15 m has a rectangular cross-section 300 mm × 600 mm. The pre-stress at the ends is 1150 kN with zero eccentricity at the supports. The eccentricity at the mid span is 200 mm. The cable profile is parabolic. Assuming friction coefficient k_f = 0.15 per 100 m and μ = 0.35, determine the loss due to friction at the centre of the beam.***

[10 Marks, ESE-2001]

Sol: **Given data:** B = 300 mm; D = 600 mm; Span = 15 m

Initial prestressing force, P_0 = 1150 kN

$$\text{Eccentricity}\begin{bmatrix}\text{at support} = 0\\ \text{at mid span} = 200\,\text{mm}\end{bmatrix}$$

$$k_f = 0.15 \text{ per } 100 \text{ m} = 0.0015 \text{ per meter}$$

$$\mu = 0.35$$

Cable profile is parabolic, the equation of profile is,

$$y = \frac{4ex}{L^2}(L-x)$$

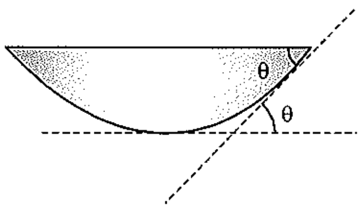
$$\therefore \quad \frac{dy}{dx} = \frac{4e}{L^2}(L-2x)$$

$$\therefore \quad \theta|_{x=0} = \frac{dy}{dx}\Big|_{x=0} = \frac{4e}{L^2}\times L = \frac{4e}{L} = \frac{4\times 200}{15{,}000} = 0.0533 \text{ radian}$$

Since we have to calculate the loss at mid point.

(It is mentioned in question that prestress at both ends is 1150 kN i.e. both the ends are friction loss free and that indicates jacking is done from both the sides.

$$\alpha = \theta = 0.0533 \text{ radian}$$



$$P_x = P_0\ e^{-(k_f x+\mu\alpha)}\left[\text{where } x = \frac{15}{2} = 7.5\text{m}\right]$$

$$= 1150\ e^{-(0.0015\times 7.5 + 0.35\times 0.0533)}$$

$$\boxed{P_x = 1116.12\text{kN}}$$

$\therefore$ Loss of presstress = 1150 – 1116.12 = 33.88 kN

$$\%\text{ loss} = \frac{33.88}{1150}\times 100 = 2.95\%$$

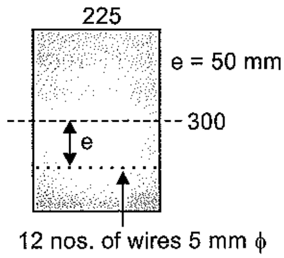
Q-3: ***A pretensioned beam of size 225 × 300 mm deep is prestressed by 12 wires 5 mm diameter initially stressed at 1100 MPa. The centroid of the prestressing wires is located at 100 mm from the bottom. Estimate the loss of prestress due to elastic deformation, creep, shrinkage and relaxation for the following stipulations:***

Grade of concrete	:	*M 40*
Relaxation of steel	:	*5 percent*
E_s	:	*2×10^5 MPa*
Creep coefficient	:	*1.6*
Residual shrinkage strain	:	*3×10^{-4}*

[10 Marks, ESE-2003]

Sol: E_c will be adopted as $5000\sqrt{f_{ck}}$ as it has been revised in IS 456 : 2000, although IS 1343: 1980 says E_C is equal to $5700\sqrt{f_{ck}}$.

(1) Elastic shortening loss = mf_c



$$m = \frac{E_s}{E_c} = \frac{2\times10^5}{5000\sqrt{f_{ck}}} = \frac{2\times10^5}{5000\sqrt{40}} = 6.324$$

$$f_c = \frac{P}{A} + \frac{Pe^2}{I}$$

$$P = 1100\times12\times\frac{\pi}{4}(5)^2 = 259.18\text{ kN}$$

$$f_c \text{ (due to all bars taken together)} = \frac{259.18\times10^3}{225\times300} + \frac{259.18\times10^3\times(50)^2}{225\,(300)^3/12} = 5.12\text{ N/mm}^2$$

Elastic shortening loss = mf_c = 5.12 × 6.324 = 32.38 N/mm² (loss of stress in each bar)

Since elastic shortening loss in a short term oss and creep losses are long term loss, we should use the stress at the level of concrete after elastic shortening loss to calculate creep losses.

Stress in steel after elastic shortening loss = 1100 – 32.38 = 1067.62 kN/mm²

Stress in concrete at the level of steel after elastic shortening loss

$$= \frac{5.12\times1067.62}{1100} = 4.97\text{ N/mm}^2$$

(2) Loss due to creep $\quad m\phi f_c = 6.324 \times 1.6 \times 4.97 = 50.29\text{ N/mm}^2$

(3) Shrinkage loss $\quad 3\times10^{-4}\times 2\times10^5 = 60\text{ N/mm}^2$

(4) Relaxation loss $\quad 1100\times\frac{5}{100} = 55\text{ N/mm}^2$

$$\text{Total loss of stress} = 32.38 + 50.29 + 60 + 55 = 197.67\text{ N/mm}^2$$

$$(\%)\text{ loss} = \frac{197.67}{1100}\times100 = 17.97\%$$

$$\text{Total force loss} = 197.67\times\frac{\pi}{4}(5)^2\times12\text{ N} = 46.551\text{ kN}$$

Q-4: ***Why are there no similar losses in reinforced concrete beams as in case of prestressed concrete beam?***

[4 Marks, ESE-2004]

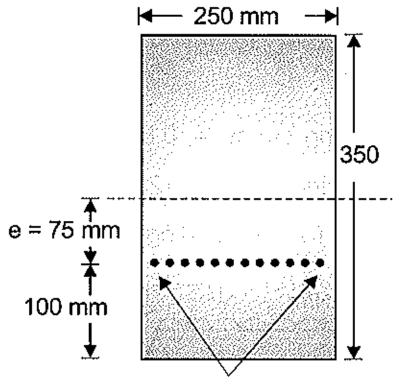
Sol: In prestressed concrete beam compressive stress initially is induced to counteract the tensile stress induced in concrete due to loading. So the loss occurs due to the loss of initial force in steel.

In RCC no initial stress is induced, steel is provided here to compensate the low tensile strength of concrete i.e., to carry tension induced due to the external loading. Hence, as in RCC their is no initial stress induced, so no corresponding losses like pre stress.

Q-5: ***A pretensioned beam of size 250 mm × 350 mm prestressed by 12 wires of 7mm diameter is initially stressed to 1200 N/mm² with the eccentricity located 100 mm from the soffit. Estimate the final loss of prestress due to elastic deformations, creep, shrinkage and relaxation. Given, relaxation*** of steel stress = 90 N/mm², ***E_S = 210 kN/mm², E_C = 35 kN/mm², creep coefficient (f) = 1.6, residual shrinkage strain = 3 × 10^{-4}.***

[20 Marks, ESE-2010]

Sol: Initial prestressing = 1200 N/mm²



(1) Elastic shortening loss = mf_c $\quad m = \frac{E_s}{E_c} = \frac{210}{35} = 6$

f_c = Stress in concrete at the level of prestressing steel = $\frac{P}{A} + \frac{(Pe)e}{I}$

$$P = 1200 \times 12 \times \frac{\pi}{4}(7)^2 = 553.896 \text{ kN}$$

$$\Rightarrow \quad f_c = \frac{553.896 \times 10^3}{250 \times 350} + \frac{553.896 \times 10^3 \times (75)^2}{\frac{250\,(350)^3}{12}}$$

$$= 6.33 + 3.488 = 9.818 \text{ N/mm}^2$$

$\Rightarrow$ Elastic shortening loss = 6 × 9.818 = 58.908 N/mm²

Since elastic shortening loss is a short term loss, we will calculate the prestress after short term loss. This will then be used to calculate long term loss.

Stress after elastic shortening loss = 1200 – 58.908 = 1141.092 N/mm²

$$f_c \text{ after elastic shortening loss} = \frac{9.818 \times 1141.092}{1200} = 9.336 \text{ N/mm}^2$$

(2) Loss due to creep = $mf_c \cdot \phi$ = $6 \times 9.336 \times 1.6$ [ϕ = creep coefficient] = 89.626 N/mm²

(3) Loss due to shrinkage = $\varepsilon_{sc} \cdot E_s = 3 \times 10^{-4} \times 210 \times 10^3$ N/mm² = 63 N/mm²

(4) Loss due to relaxation = 90 N/mm²

Final loss of prestress = 58.908 + 89.626 + 63 + 90 = 301.534 N/mm²

CHAPTER 15

DEFLECTION OF PRESTRESSED BEAM AND MISCELLANEOUS

Q-1: ***A concrete beam simply supported at both ends with a rectangular section 300 mm × 600 mm is prestressed by 2 post tensioned cables of area 500 mm² each. The cables are located at a constant eccentricity of 100mm throughout the beam of span 8m. The cables are stressed to 1600 MPa initially. Calculate the deflection of the beam (maximum) when it carries an imposed load of 20 kN/m allowing 20% loss in prestress. Assume the modulus of elasticity of concrete and steel are 30000 MPa and 2 × 10⁵ MPa respectively. Neglect the effect of shrinkage and creep.***

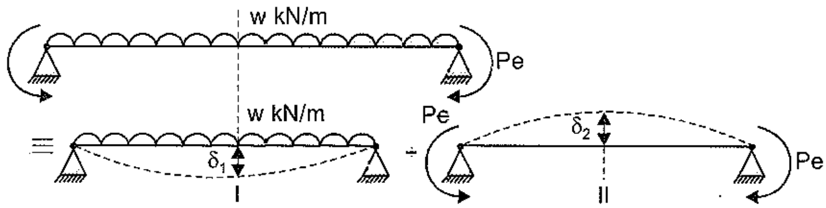
[10 Marks, ESE-2013]

Sol:

$$\text{Final prestressing stress} = 0.8 \times 1600 = 1280 \text{ MPa.}$$

$$\text{Final prestressing force} = 1280 \times 1000 = 1280 \text{ kN.}$$

The beam can be shown as below (on the loading basis).



Max deflection will occur always at centre.

Case I:

$$w = \text{Imposed load} + \text{Dead load}$$

$$= 20 + 0.3 \times 0.6 \times 25$$

$$w = 24.5 \text{ kN/m}$$

$$\delta_I = \frac{5w\ell^4}{384EI} = \frac{5 \times 24.5 \times 8^4 \times 12 \times 10^3}{384 \times 3 \times 10^{10} \times 10^{-12} \times 300 \times 600^3}$$

$$w = 24.5 \text{ kN/m}$$

$$l = 8\text{m}$$

$$I = \frac{300 \times 600^3}{12} \times 10^{-12} \text{ m}^4$$

$$E_c = 3 \times 10^{10} \text{ Pa} = 3 \times 10^6 \text{ N/m}^2$$

$$\delta_I = 8.06 \times 10^{-3} \text{ m}$$

$$\delta_I = 8{,}06 \text{ mm (downwards)}$$

Case II:

Deflection at mid point. $\delta_{II} = \dfrac{ML^2}{8EI}$

$$\delta_{II} = \frac{PeL^2}{8EI} = \frac{1280 \times 10^3 \times 100 \times 10^{-3} \times 8^2 \times 12}{8 \times 3 \times 10^{10} \times 300 \times 600^3 \times 10^{-12}}$$

$$\delta_{II} = 6.32 \times 10^{-3} \text{ m} = 6.32 \text{ mm (upwards)}$$

$\therefore$ Net maximum resultant deflection of the beam at mid span = 8.06 – 6.32 = 1.74 mm (downwards).

CHAPTER 16

EARTHQUAKE RESISTANT DESIGN OF STRUCTURE

Q-1: ***A symmetrical reinforced concrete frame building 25 × 25 m in plane is located in seismic zone IV on hard soil. The height of the building is 30 m. Determine the base shear due to earthquake.***

Given:

$Z = 0.24,\ I = 1.5$

Total dead load = 150000 kN

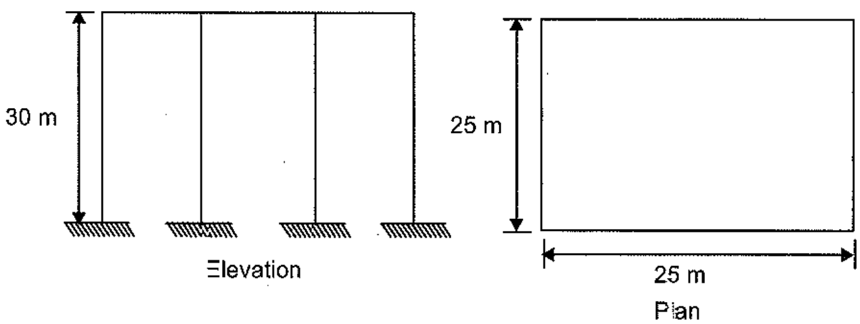
Total live load (effective) = 50000 kN

$$T = 0.09\frac{h}{\sqrt{D}}$$

$$\frac{S_a}{g} = \begin{cases} 1+15T & 0 \le T \le 0.10 \\ 2.5 & 0.10 \le T \le 0.40 \\ 1/T & 0.40 \le T \le 4.0 \end{cases}$$

[15 Marks, ESE-2020]

Sol: We have,



Building is located in zone IV

So, zone factor (Z) = 0.24 (given)

Building is located on hard soil

So, Average response acceleration coefficient $(S_a/g) = \begin{cases} 1+15T & 0 \le T \le 0.10 \\ 2.5 & 0.10 \le T \le 0.40 \\ 1/T & 0.40 \le T \le 4.0 \end{cases}$

The building is reinforced concrete frame building

Zone IV corresponds to severe intensity zone and we usually provide special moment resisting frame.

So, Importance factor (I) = 1.5

Response reduction factor (R) = 5

and width of building (d) = 25 m

Height of building (h) = 25 m

As per clause 7.5.3 of IS 1893: 2002

Base shear due to earthquake is given by

$$\boxed{V_B = A_h.W} \quad \text{... (i)}$$

Here A_h = Design horizontal seismic coefficient

W = total seismic weight of structure

Calculation of design horizontal seismic coefficient (A_h)

As per clause 6.4.2 of IS 1893:2002

$$\boxed{A_h = \frac{ZI}{2R}.\frac{S_a}{g}}$$

Here Z = Zone factor = 0.24

I = Importance factor = 1.5

R = Response reduction factor = 5

$\frac{S_a}{g}$ = Average response acceleration coefficient

Now, **As per clause 7.6.2 of IS 1893:2002**

Fundamental natural period of building (T) = $\frac{0.09\,h}{\sqrt{d}}$

Here, h = height of building (in m) = 30 m

d = Base dimension (in m) = 25 m

$$\therefore \quad T = 0.09 \times \frac{30}{\sqrt{25}}$$

$$= 0.54 \text{ sec}$$

Since, $0.40 < T < 4.0$

So, $$\frac{S_a}{g} = \frac{1}{T} = \frac{1}{0.54} = 1.852$$

Hence, $$A_h = \frac{0.24 \times 1.5}{2 \times 5} \times 1.852$$

$$= 0.0667$$

Calculation of seismic weight (W)

Total seismic weight = Total dead load + effective live load

= 150000 + 50000

= 200000 kN

From equation (i),

Design base shear (V_B) = $A_h.W$

= 0.0667 × 200000

= 13340 kN

UNIT-6

PERT CPM AND CONSTRUCTION EQUIPMENT

SYLLABUS

Construction: *Planning, Equipment, Site investigation and Management including Estimation with latest project management tools and network analysis for different Types of works; Analysis of Rates of various types of works; Tendering Process and Contract Management, Quality Control, Productivity, Operation Cost: Land acquisition; Labour safety and welfare.*

CONTENTS

CHAPTER 1

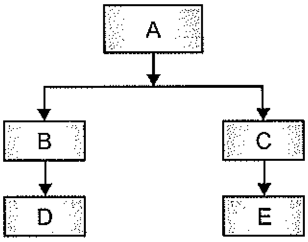
PROJECT MANAGEMENT

Q–1: ***Discuss and differentiate line organisation and staff organisation.***

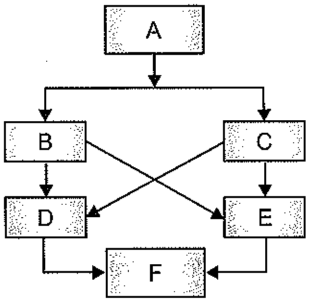
[5 Marks, ESE-2011]

Sol: **Line Organization**

- Oldest type of organization followed.
- Each department is a complete self contained unit, a separate person will look after the activities of the department and he has full control over the department.
- Under this the line organization, authority flows from top to bottom vertically.



Line Organization



Staff Organization

Staff Organization

- Various specialists are selected for various functions performed in an organization.
- Workers receive instruction from various specialists, who are working at a supervision level.

Q–2: ***What do you understand by multiskilling of labour? What are the benefits of multiskilling?***

[8 Marks, ESE-2016]

Sol: **Multiskilling of labour:** Training of single employee in multiple skill-sets which provide for workers to have a range of skills or knowledge for working on several different projects, which may or may not be a part of worker's technical job description. This increases productivity and cuts the bottom line for a company, which does not have to hire additional personnel to do other jobs.

Advantages of multiskilling:

(a) **Flexibility:** Company with multiskilled employees has flexible work force, which provides the employer with the ability to schedule and arrange workers to best suit the needs of the business.

(b) **Decreased labour cost:** A business with a multiskilled labour can operate with a reduce number of employees necessary to conduct business. Workers who are skilled in only one area of business may sit idle while waiting for work to be available.

(c) **Efficiency in planning:** Planning and scheduling workers can make changes to production schedule to meet customer demand without a loss of productivity.

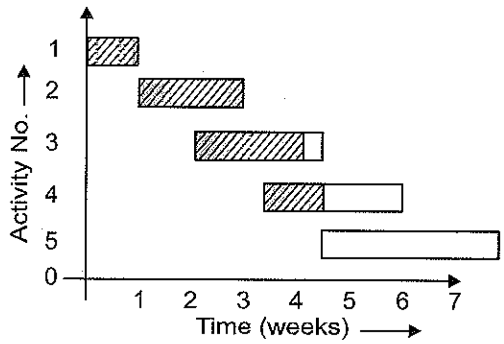
(d) **Employee satisfaction:** Multiskilled workers are not threatened by obsolescence when new technology changes method of production, as workers used to learning new skills consistently can adopt to changes in production.

Q-3: ***Briefly answer the following:***

How can an existing bar chart be modified to depict the project progress made?

[5 Marks, ESE-2017]

Sol: An existing bar chart does not show the progress of the work and hence it cannot be used as controlling device. However and existing bar chart can be modified to depict the progress made. This can be done by showing the progress of each activity, by hatched lines along the corresponding bar of the activity. Generally, hatching is done is half the width of the bar.



Q-4: ***What is a work breakdown structure in Construction Project Management? Define and explain in brief. Further, how Work Breakdown Structure is classified into different levels for making the job convenient? Explain with an example.***

[12 Marks, ESE-2018]

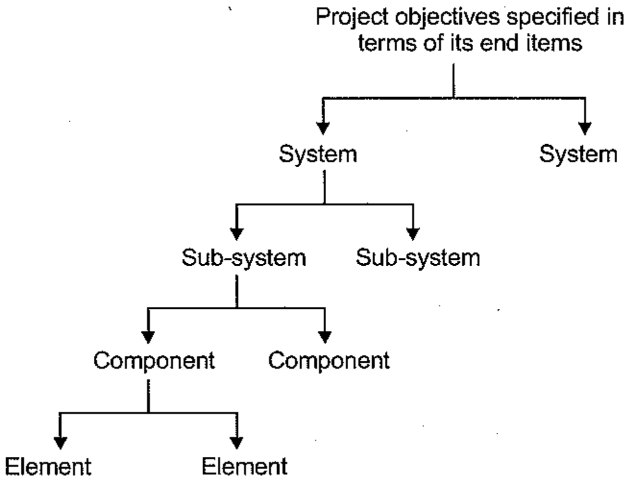
Sol: Work breakdown structure in construction project management

- In any construction project, the various activities that make up the project have to be clearly identified.
- Process of breaking the project into easily identifiable major systems, their sub-systems and discrete activities is called the **work breakdown structure** (WBS).
- In other words, WBS is hierarchical tree structure, obtained through decomposition of the project into phases and tasks.
- Project is broken down into manageable chunks (whole to part)
- It is a framework for planning, scheduling, estimating, budgeting, monitoring and controlling the project.

  For example,

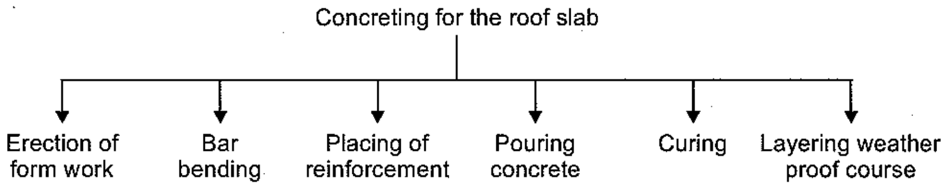
  (i) In a big construction project, the various activities that make up the project have to be clearly identified.

  It is first identified in terms of its end items, then splits into systems, sub-systems, then their components and elements.



(WBS of Big Construction Project)

(ii) The concreting work for the roof slab of a residential building can be split up into various elements as follows:



Benefits of WBS:

1. It forces the team to create detailed steps.
2. It lays the groundwork for schedule and budget
3. It creates accountability
4. It creation breeds commitment

Q–5: ***Explain major activities involved in different stages of planning for a construction project.***

[20 Marks, ESE-2019]

Sol: Planning is the most important technique of management . In a construction project plan includes the estimates, the budget and time schedule and sequences of completion of each part of the project, manpower planning and the plant and equipments.

Following steps are involved in effective planning:

(i) **Crystalizing the opportunity or problem:** The first step in planning would be to find out the problem or identify the opportunity to be seized, this is necessary to be able to formulate practical and realistic objectives.

(ii) **Securing and analyzing necessary information:** Adequate information is required on course of action possible. It is necessary to determine the nature of the information required and where this information will be available. This information must be analysed to establish the relationship and tabulate them for adequate interpretation.

(iii) **Establishing planning premises and constraints:** An analysis of the data so collected will result in the formulation of certain assumptions on the basis of which the plan will be made through a process of forecasting. Constraints such as government control will also exist. Planning will be in the backdrop of such premises and constraints which must be watched to detect changes and their effect on the plans.

(iv) **Ascertaining alternative course of action or plan:** Based on the above analysis, possible alternative course of action will be identified and examined. Generally, every situation will have more than one course of action. Exploitation of the right course will depend to a large content on experience, ingenuity and imagination of the planner.

(v) **Selecting optimum plan:** An evaluation of the above alternate course of action can be carried out either by judgement alone or with the help of quantitative techniques and staff assistant, to best suit the interest of the organisation.

(vi) **Determining derivative plan:** The above selected plan will form the basic plan from which other plans will develop to support it. For example, basic marketing plan may have been evolved which may result in other derivative plans such as the advertising plan.

(vii) **Fixing the timing of introduction:** The question of timing-who will do, what will have to be decided and an appropriate time schedule drawn up with the details of construction work for communication.

(viii) **Arranging future evaluation of effectiveness of the plan:** Since the ultimate aim of the plan to achieve the objective, result or goal, an evaluation at the earliest possible opportunity necessary to evaluate the adequacy of cost and time and determining whether the planned objectives are reached as desired.

Q-6: ***What is work breakdown structure (WBS) with respect to construction planning and management? How is WBS classified into different levels?***

[12 Marks, ESE-2020]

Sol: In any construction project, the various activities that make up the project have to be clearly identified. Work breakdown structure or schedule is a pictorial representation of the entire program. It is a preliminary diagram illustrating, the way in which all the supporting objective goes together and mesh to ensure the attainment of the major objectives. Such a breakdown structure is more essential in complex projects like construction projects.

In work breakdown structure, the top down approach to planning is adopted. It ensures that the total project is fully planned and all derivative plan contributed directly to the desired end objectives. It also aids in the identification of objectives and allows the planner to see the total picture of the project. The development of work breakdown structure begins at the highest level of the program with identification of end items. The major end items are divided into sub component parts and the component part are further divided into their more detailed units. The subdivision of work continues to successively lower levels, until it reaches the level where the end items subdivisions finally become manageable units for planning and control purposes. The end items appearing at this level are than divided into major work package (i.e., engineering manufacturing, testing etc.)

Hierarchies

Very large and complex networks contain more than 200 to 500 work operations. If all the work operations are represented' on a single network diagram, it will become clumsy. To represent these entire operations one can use a family of network or hierarchy of network or hierarchy of increasing detail. There can be several stages in the hierarchy, each to be used by different levels of management i.e. top management, middle and lower management people.

The number of stages in the hierarchy may reflect not only the complexity of project but also the structure of management of organization and system of control and reporting that are in use.

The hierarchy may have generally two or three stages or levels. The number of work operations at each stage of hierarchy will depend on the following factors.

- Purpose of diagram
- Degree of control desired
- Extent of available information
- How the diagram is to be used

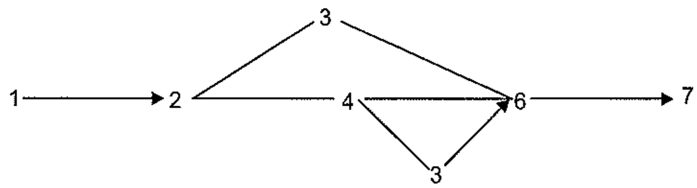
The First Level diagrams

These are used mainly for general information. The purpose is to describe in general terms at top management or the public or client, the overall nature of project and how it is to be accomplished.

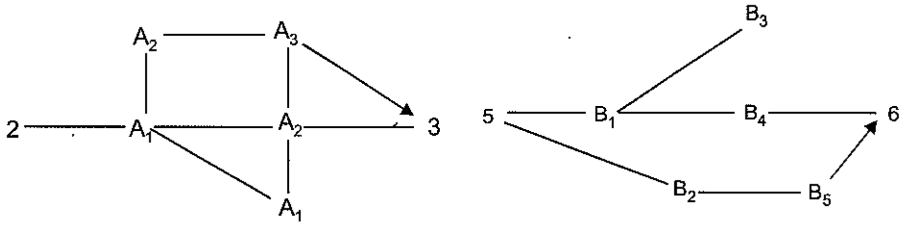
When hierarchy principle is used to split up the large project, the first level network should be proposed by the top administrate in consultation with representatives of various divisions.

The representatives of each division will then prepare more detailed sub networks for middle level and lower level for implementation.

(a) Top level management

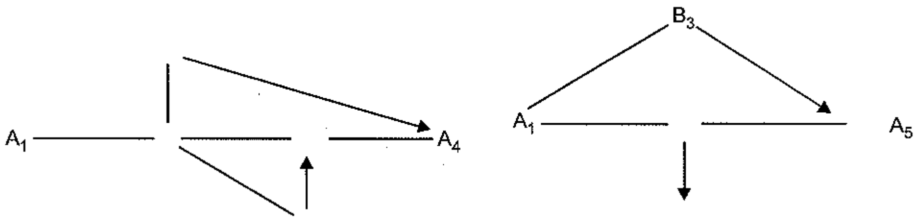


(b) Middle Level Management



Project A Project B

(c) Lower Level Management (Site Control)



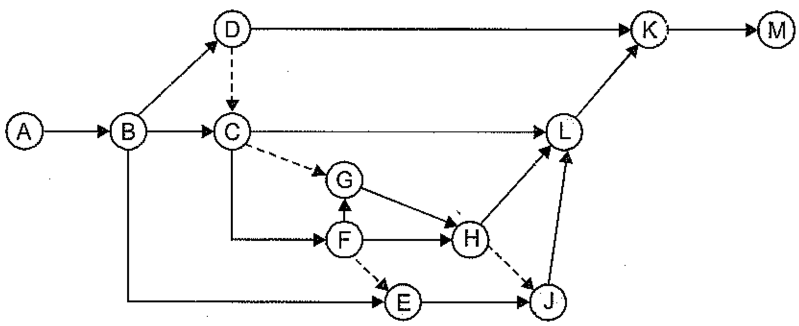
CHAPTER 2

FUNDAMENTALS OF NETWORK

Q–1: ***In a construction project, events have been identified as A, B, C, D, E, F, G, H, J, K, L and M. A is the start event, B occurs after A; C succeeds B and precedes L but restrains the occurrence of G; D occurs after B, before K and restrains C; F succeeds C, precedes H and restrains E; E succeeds B but precedes J; G succeeds F and precedes H; H precedes L and restrains J; L occurs after J but before K; M succeeds K. Draw a PERT network. Number the events according to Fulkerson's rule.***

[15 Marks, ESE-1997]

Sol:



Q–2: ***Explain various steps involved in the development of networks.***

[10 Marks, ESE-2006]

Sol: The CPM/PERT network is an improvement over Gantt's milestone chart in a way that the CPM/PERT network illustrated the interrelationships between and among all the milestones in an entire project. The CPM/PERT network may be developed from Gantt's milestone chart in the following four transitional steps.

First Step : The horizontal bars representing activities or jobs or tasks in a milestone chart are removed and the interrelationships between milestones within each specific activity re represented by arrows as shown in fig. below

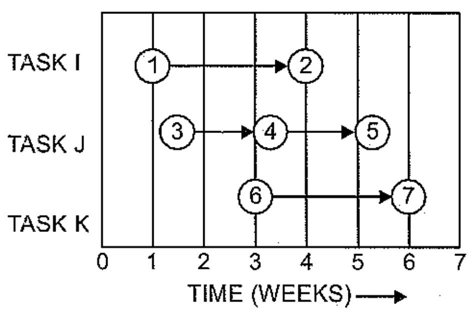
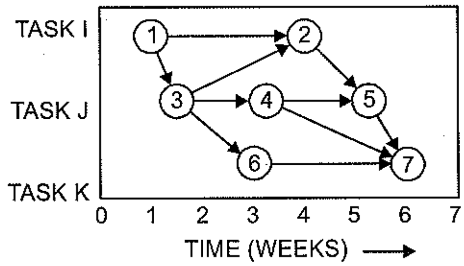


Illustration of first transitional step for the development of CPM/PERT network

Second Step: The milestone of different tasks or activities are interrelated by arrows as shown in fig. This is the major advantage of CPM/PERT network over

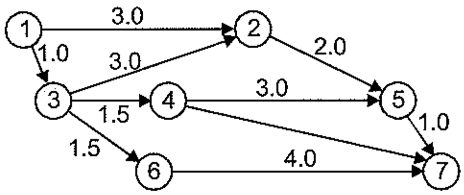


Interrelationship between milestones of different tasks

Gantt's milestone charts. It may be seen from Fig. that milestone 4 can be started soon after the completion of milestone 3 and one need not wait until completion of milestone 2. Similarly milestone 6 can be taken up soon after completion of milestone 3, without waiting for the completion of milestones 4 and 5.

Third Step : Since all the milestones have been interrelated by arrows there is no need to designate each task or activity separately. Therefore the term task or activity, represented along the vertical axis in the Gantt's bar for milestone charts, is omitted in the network (Fig. above).

Fourth Step : The horizontal time scale is also omitted in the network because the times between the milestones are indicated on the arrows between them as shown in fig. The times on the arrows are normally expressed in weeks. For example arrow between milestones 4 and 5 has 3 upon it which indicates that it will take 3 weeks time to each from milestone 4 to milestone 5.



Q–3: ***Draw a network for the following situations:***

(i) ***A is prerequisite on D***

(ii) ***B is prerequisite on D and E***

(iii) ***C is prerequisite on E***

(iv) ***D is prerequisite on F***

(v) ***E is prerequisite on F***

[10 Marks, ESE-2016]

Sol: We have

$\xrightarrow{A} 0 \xrightarrow{D}$ .... (1)

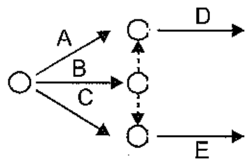
$\xrightarrow{B} 0 \begin{array}{l} \nearrow^{D} \\ \searrow_{E} \end{array}$ .... (2)

$\xrightarrow{C} 0 \xrightarrow{E}$ .... (3)

$\xrightarrow{D} 0 \xrightarrow{F}$ .... (4)

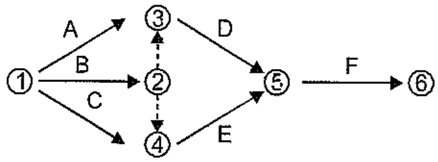
$\xrightarrow{E} 0 \xrightarrow{F}$ .... (5)

Combining (1), (2) and (3)



.... (6)

Combining (4), (5) & (6) to get final network:



Q–4: ***(a) Briefly answer the following:***

(i) Differentiate between the terms 'Activity' and 'Dummy'.

(ii) Differentiate between 'Forward Planning' and 'Backward planning' for network construction.

[20 Marks, ESE-2017]

Sol: (i)

Activity	**Dummy**
1. Activity consumes finite resource and time.	1. Does not consumes any resource and time.
2. Represented by the full line. ($\rightarrow$)	2. Represented by broken line. ($\dashrightarrow$)
3. It has physical significance and must be completed to complete the project.	3. Used only to maintained the logic.

(ii)

Forward Planning	**Backward Planning**
1. Planner starts from initial event and end at last event	1. Planner starts with end event and end at first.
2. Planner asks himself like what comes next, what are dependent values etc.	2. Planner asks himself, if we want to achieve this, event or activities that should have taken place.
3. It is used to estimate the earliest occurrence event.	3. It is used to estimate the latest occurence time of event.
4. Head event earliest time is calculated by the help of tail event.	4. Tail event latest time is calculated by the help of head event.

Q–5: ***Define the terms activity, event and Network.***

[12 Marks, ESE-2019]

Sol: 1. **Activity**

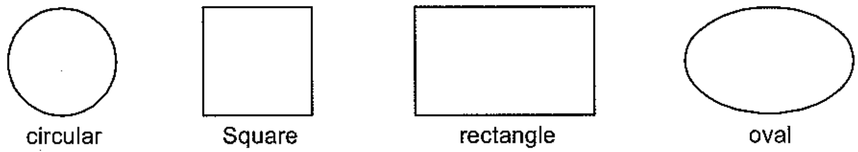
- An activity is a recognizable part of a work project that requires time and resources.
- Activity are denoted by an arrow. The "tail of the arrow" signifies the start of activity and "the arrow head" its termination.

$$\xrightarrow[\text{Duration (T)}]{\text{Description}}$$

- Few example of activities are like excavation, shuttering, bar bending, concreting etc.

2. **Event**

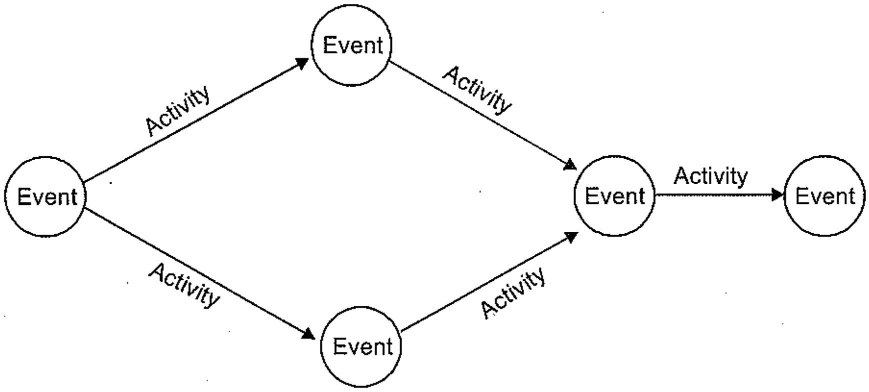
- An event is a instant of time or state at which some specific milestone has been achieved. i.e. completion of preceeding activity (activities) or start of succeeding activity (activities).
- An event does not consume any time or resource.
- Events are represented by nodes. The shape of nodes can be as follows.



- Examples of few events are like, completion of excavation, start of shuttering., completion of Bar-bending and etc.

3. **Network**

- A network is a flow diagram consisting of activities and events, connected logically and sequentially. In the netwrok diagram, an activity is represented by arrows while events are represented, usually, by circles, as shown in Fig.



- Networks are of two types: PERT network and CPM network. PERT network is event-oriented, while CPM network is activity oriented.

CHAPTER 3

PROGRAMME EVALUATION REVIEW TECHNIQUE

Q–1: ***The interdependence of a job consisting of seven activities A to G is given as :***

Activity	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Predecessor activity	–	–	*A*	*B*	*A*	*B*	*C, D*
Succeeding activity	*C, E*	*D, F*	*G*	*G*	–		

The time estimates, in days, for each activity are:

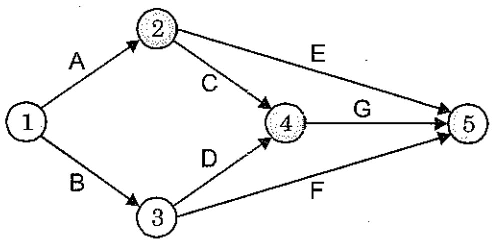
Activity	*Time estimates*
A	*6 – 9 – 18*
B	*5 – 8 – 17*
C	*4 – 7 – 22*
D	*4 – 7 – 16*
E	*4 – 7 – 10*
F	*2 – 5 – 8*
G	*4 – 10 – 22*

Z (+)	*% Probability*
0.8	*78.81*
0.9	*81.59*
1.0	*84.13*
1.1	*86.43*
1.2	*88.49*

Draw the network and determine the probability of completing the job in 35 days.

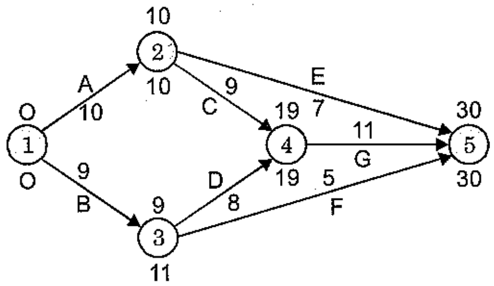
[15 Marks, ESE-1995]

Sol:



Activity	Time estimates (days)	Expected time (days)	Standard deviation (days)
A	6 – 9 – 18	10	2
B	5 – 8 – 17	9	2
C	4 – 7 – 22	9	3
D	4 – 7 – 16	8	2
E	4 – 7 – 10	7	1
F	2 – 5 – 8	5	1
G	4–10–22	11	3

Now the nature diagram



- Expected time fo the completion of project = 30 day and standard deviation along critical path

$$= \sqrt{\sigma_A^r + \sigma_C^r + \sigma_9^r}$$

$$= \sqrt{2^r + 3^r + 3^r} = 4.6904$$

- If the scheduled time = 35 days

then z factor $= \dfrac{35-30}{4.6904} = \dfrac{T_s - T_E}{\sigma} = \dfrac{35-30}{4.6904} = 1.066$

- Corresponding probability of completion

$$= 84.13 + \frac{(86.43-84.13)}{(1.1-1.0)}(1.066-1.0)$$

$$= 85.648\%$$

Q–2: ***Define 'slack' What does negative slack indicate in PERT network analysis?***

[5 Marks, ESE-1996]

Sol:

- Slack or Slack time of any event is defined as the difference between the latest allowable occurrence time and earliest expected occurrence time fo the event.
- It is denoted by S.

$$S = T_L - T_E$$

- Slack is associated with the event.
- It is the excess time available by which occurrence of an event can be delayed without affecting the project completion time.

  Negative Slack (S < 0)

- A negative Slack is obtained when T_L is less than T_E for an event.

$$S(-ve) = T_L - T_E \quad (\because T_L < T_E)$$

- It indicate that:

  (i) This event is behind the schedule by the time period equal to Slack of that event. Any further delay in such events cause more delay in the project hence these events are called super critical events.

  (ii) Resource deployed are not adequate enough.

Q–3: ***A typical serious candidate for services examination take no less than 25 minutes to answer a question and some times as much as 48 minutes, 35 minutes times are most frequent than any other time. If this performance were an activity in a PERT project, estimate***

(i) would be expected time to answer a question, n

(ii) its variance,

(iii) the time to be allocated for answering a question in scheduling the project.

[10 Marks, ESE-2000]

Sol: (i) $t_0 = 25;\ t_m = 35;\ t_p = 48$

$$t_e = \frac{25+4\times35+48}{6} = 35.5 \text{ min}$$

(ii) Variance, $$s^2 = \left(\frac{t_p - t_o}{6}\right)^2 = 14.69 \ \text{min}^2$$

(iii) Time to be allocated for answering a question in scheduling the project will be 35.5 min.

Q–4: ***Define and explain the terms:***

(i) Most likely time,

(ii) Mean time, and

(iii) Expected time as related to various activities of a project.

[15 Marks, ESE-2002]

Sol: **(i) Most likely time:** It is the time required to complete the activity if normal conditions prevails. This timeestimate generally lies between pessimistic and optimistic time estimates.

(ii) Mean time: It is defined as algebraic sum of time durations taken by various jobs divided by number of jobs. It is also called as average time of the distributions.

$$\bar{x} = \frac{\Sigma x}{n}$$

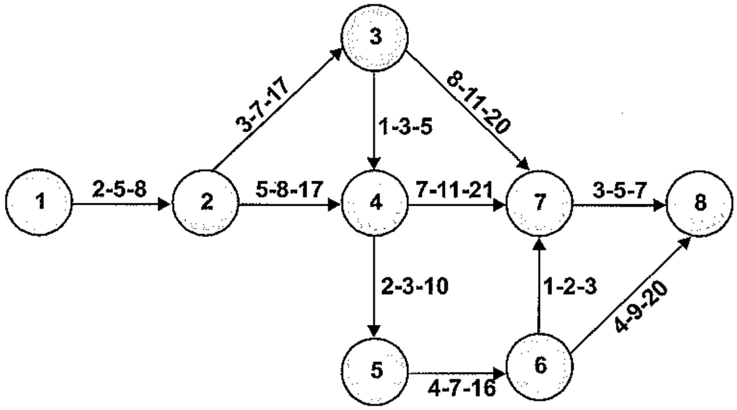
(iii) Expected time

The average or mean time calculated from the probability distribution curve of an activity is expected time, it divide the curve area in two equal halves.

Generally as we don't have the exact curve, we use weighted average to calculate expected time

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Q–5: ***The time estimates in days for optimistic, most likely and pessimistic times of various activities of a project are shown below in the network diagram. Obtain the expected time and the latest allowable time for each event. Draw the critical path and estimate the approximate value of standard deviation for the network.***



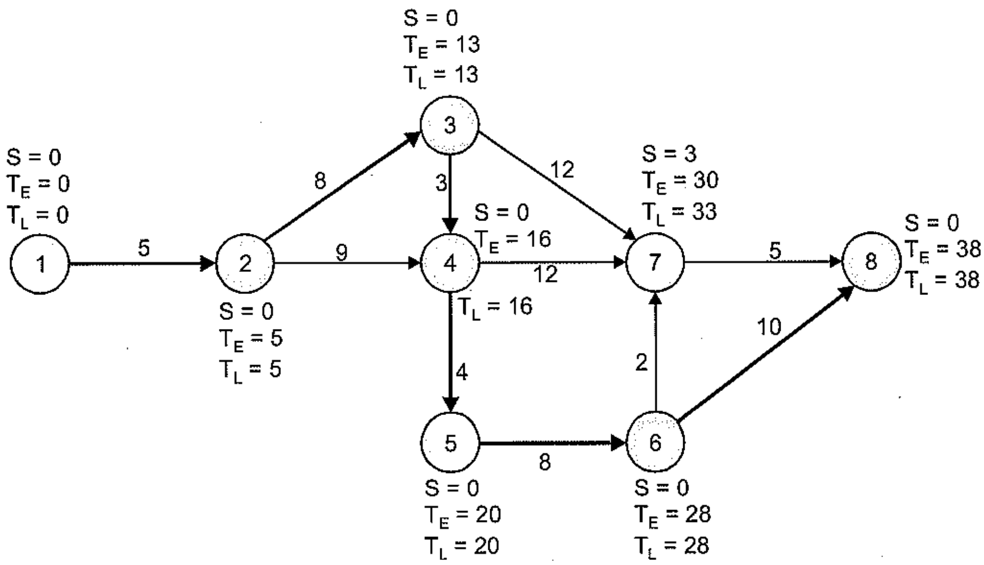
[25 Marks, ESE-2002]

Sol: Calculation of expected time t_e and σ is done in table:

where, $$t_e = \frac{t_o + 4t_m + t_p}{6}; \quad s = \frac{t_p - t_o}{6}$$

Activity	t_o	t_m	t_p	t_e	σ
1-2	2	5	8	5	1
2-3	3	7	17	8	2.33
2-4	5	8	17	9	2
3-4	1	3	5	3	0.67
3-7	8	11	20	12	2
4-7	7	11	21	12	2.33
4-5	2	3	10	4	1.33
5-6	4	7	16	8	2
6-7	1	2	3	2	0.33
6-8	4	9	20	10	2.67
7-8	3	5	7	5	0.67

Calculation of T_E, T_L and Slack is done in Network diagram below:



We can observe that events 1, 2, 3, 4, 5, 6 and 8 are having zero Slack and maximum duration.

$\therefore$ Critical paths is 1 – 2 – 3 – 4 – 5 – 6 – 8

Now we have to calculate standard deviation.

s for 1—2—3—4—5—6—8

$$= \sqrt{1^2 + 2.33^2 + 0.67^2 + 1.33^2 + 2.0^2 + 2.67^2} = 4.45 \text{ days.}$$

Q–6: ***A small project consists of seven activities with the following details:***

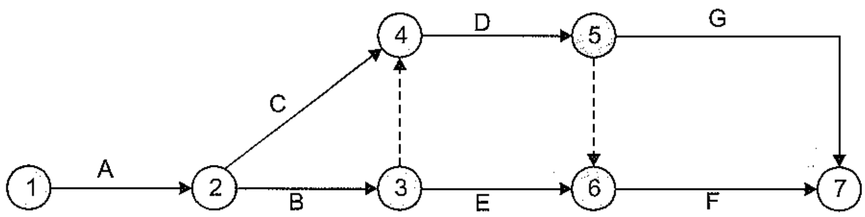
Activity	***Duration, weeks***			
	Most likely	***Optimistic***	***Pessimistic***	***Immediate predecessor***
A	***4***	***2***	***8***	***–***
B	***7***	***3***	***15***	***A***
C	***4***	***4***	***4***	***A***
D	***11***	***5***	***23***	***B, C***
E	***8***	***4***	***16***	***B***
F	***6***	***3***	***15***	***D, E***
G	***5***	***5***	***5***	***D***

(i) ***Draw the network, find critical path, the expected completion time.***

(ii) ***What project duration will have 95% confidence of completion. Take Z = 1.64.***

[20 Marks, ESE-2003]

Sol: Network Diagram

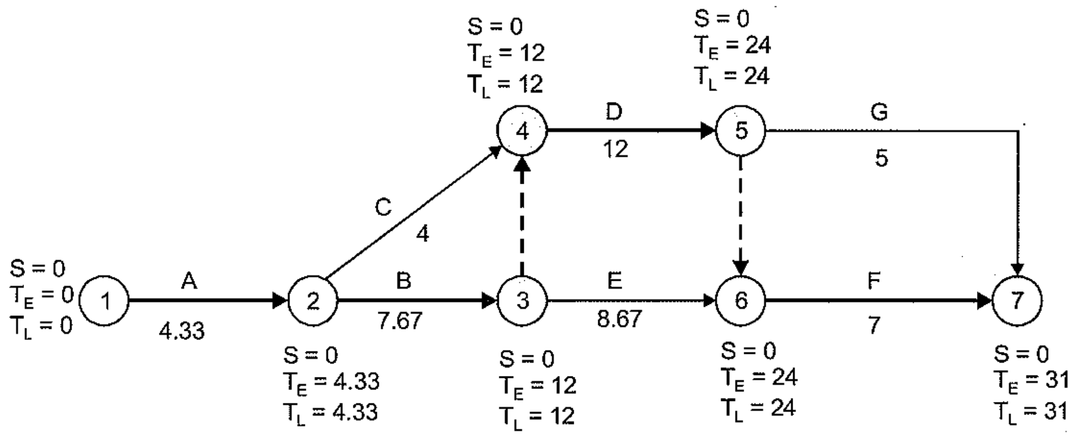


Calculation of t_e and σ of activities is done in table below:

where, $$t_e = \frac{t_o + 4t_m + t_p}{6} \quad ; \quad \sigma = \frac{t_p - t_0}{6}$$

Activity	t_o	t_m	t_p	t_e	σ
A	2	4	8	4.33	1
B	3	7	15	7.67	2
C	4	4	4	4	0
D	5	11	23	12	3
E	4	8	16	8.67	2
F	3	6	15	7	2
G	5	5	5	5	0

Calculation of T_E, T_L and Slack has been done in network diagram below.



As we can observe that events 1, 2, 3, 4, 5, 6, 7 are having Slack 0, hence critical path is 1—2—3—4—5—6—7 or A—B—D—F

(ii) We know that,

$$Z = \frac{T_S - T_E}{\sigma}$$

$$s = \sqrt{1^2 + 2^2 + 3^2 + 2^2} = \sqrt{18} = 4.24$$

$$1.64 = \frac{T_S - 31}{4.24}$$

$$T_S = 1.64 \times 4.24 + 31 = 37.95 \text{ weeks}$$

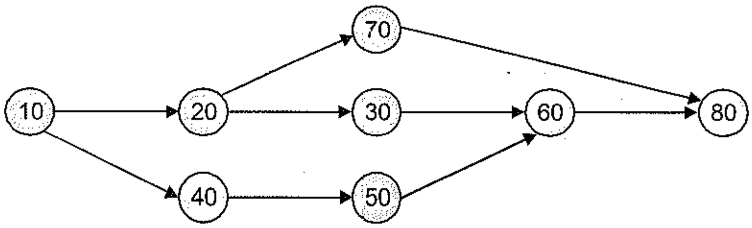
∴ Scheduled completion time for 95% probability is 37.95 weeks.

Q–7: ***With the help of the data given below, draw the network diagram and find the project completion time based on 'Expected Time'.***

Preceding event node number	*Succeeding event node number*	*Optimistic time in weeks*	*Most likely time in weeks*	*Pessimistic time in weeks*
10	20	10	12	20
10	40	5	15	19
20	30	10	15	26
20	70	15	20	25
30	60	5	10	15
40	50	4	8	12
50	60	5	10	15
60	80	2	4	6
70	80	2	4	6

[20 Marks, ESE-2005]

Sol: Network diagram for the given project

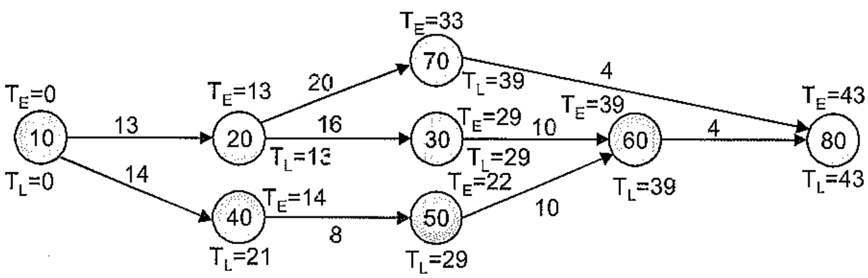


Calculation of expected time is done in table below where

$$t_E = \frac{t_0 + 4t_m + t_p}{6}$$

Activity	Optimistic Time (t_o) (weeks)	Most Likely Time (t_m) (Weeks)	Pessimistic Time (t_p) (Weeks)	Expected Time (t_e) (Weeks)
10 – 20	10	12	20	13
10 – 40	5	15	19	14
20 – 30	10	15	26	16
20 – 70	15	20	25	20
30 – 60	5	10	15	10
40 – 50	4	8	12	8
50 – 60	5	10	15	10
60 – 80	2	4	6	4
70 – 80	3	4	6	4

Calculation of project completion time is done in network diagram below

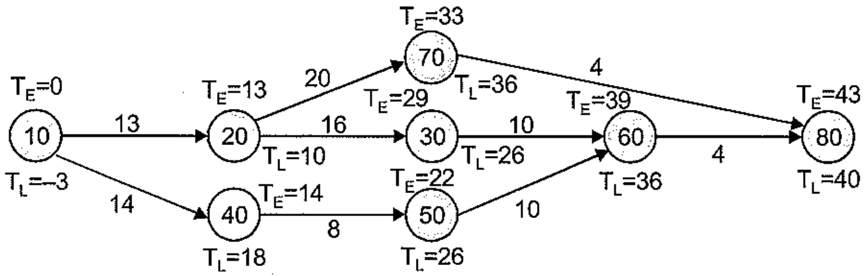


Q–8: ***If the project in previous question is to be completed within 40 weeks, draw a table showing the original early event time, late event time, original slack and revised early event time, revised late event time and revised slack time. Draw the revised network diagram.***

[10 Marks, ESE-2005]

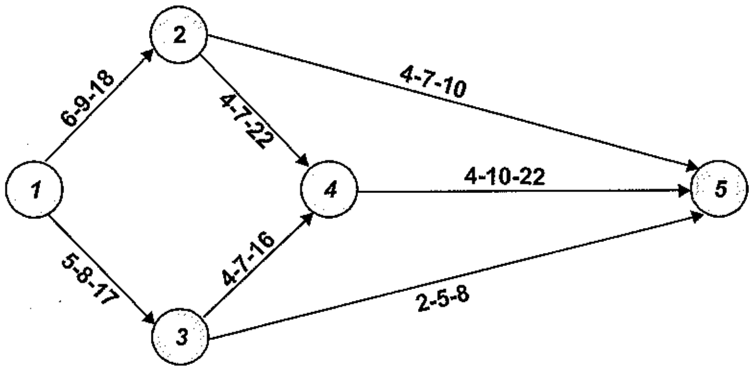
Sol:

Event	Original Early Event Time	Original Late Event Time	Original Stack	Revised Early Event Time	Revised Late Event Time	Revised Slack
10	0	0	0	0	–3	–3
20	13	13	01	13	10	–3
30	29	29	0	29	26	–3
40	14	21	7	14	18	4
50	22	29	7	22	26	4
60	39	39	0	39	36	–3
70	33	39	6	33	36	3
80	43	43	0	43	40	–



Q–9: ***For the network shown, the estimated time in days for each activity is shown in the network diagram. Determine the critical path. Also determine the probability of completing the work in 36 days. The values of Z and corresponding probability are given in the table:***

Z	*1.0*	*1.1*	*1.2*	*1.5*	*2.0*	*3.0*
Probability	*84.13*	*86.43*	*88.9*	*93.92*	*97.92*	*99.87*



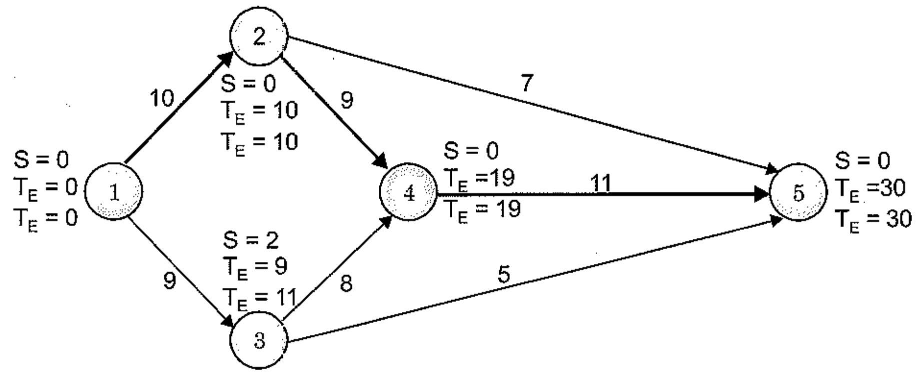
[20 Marks, ESE-2006]

Sol: Calculation of expected time and standard deviation is done in table below:

where, $t_e = \dfrac{t_0 + 4t_m + t_p}{6}$, $\sigma = \dfrac{t_p - t_0}{6}$

Activity	t_o	t_m	t_p	T_e	σ
1 – 2	6	9	18	10	2
1 – 3	5	8	17	9	2
2 – 4	4	7	22	9	3
3 – 4	4	7	16	8	2
2 – 5	4	7	10	7	1
3 – 5	2	5	8	5	1
4 – 5	4	10	22	11	3

Calculation of T_E, T_L and Slack has been done in network diagram below:



∵ Critical path is the path joining events having zero Slack and longest duration.

∴ 1 – 2 –5 and 1 – 2 – 4 – 5 are the path joining zero Slack *i.e.* critical events. But 1 – 2 – 5 is not the longest duration path as its duration is 17 days and 1 – 2 – 4 – 5 is 30 days hence 1 – 2 – 4 – 5 is the critical path.

For calculation of Z we should know σ of project which is of critical path.

∴ Critical path is 1 – 2 – 4 – 5

$$\sigma \text{ of } 1-2-4-5 = \sqrt{\sigma_{1-2}^2 + \sigma_{2-4}^2 + \sigma_{4-5}^2}$$

$$\sigma = \sqrt{2^2+3^2+3^2} = \sqrt{4+9+9} = \sqrt{22} = 4.69 \text{ days}$$

Now we know that,
$$Z = \frac{T_S - T_E}{\sigma}$$

Z = Probability factor

T_S = Scheduled time = 36 days

T_E = Expected completion time = 30 days

σ = Standard deviation of project = 4.69 days

$$Z = \frac{36-30}{4.69} = \frac{6}{4.69} = 1.279$$

By straight line interpolation method, Z = 1.279

$$\Rightarrow \quad p_r\% = 88.49 + \frac{0.079 \times 1.83}{0.1}$$
$$= 88.49 + 1.4457 = 89.9357$$

Q–10: ***Define Optimistic, Pessimistic and Most Likely Time Estimate.***

OR

Enlist and explain Time estimates in PERT.

[5 Marks, ESE-2008]

Sol:

1. Optimistic Time Estimate (t_0)

- It is the minimum time required for an activity if everything goes perfectly well without any problems or adverse conditions developed during the execution of the activity.
- In this time estimate no provisions are made for delays or setbacks and better than normal conditions are assumed to prevail during the execution of the activity.

2. Pessimistic Time Estimate (t_p)

- It is the maximum time required for an activity if everything goes wrong and abnormal situations prevail.
- This time estimate does not include the possible effects of major catastrophes such as flood, earthquakes, fire, labour strikes etc.

3. Most Likely time Estimate (t_l/t_m)

- It is time required to complete the activity if normal conditions prevail.
- This time estimate lies between pessimistic and optimistic time estimates.

Q–11: ***A construction work consists of activities with PERT durations in days as given below:***

Activity	*P*	*Q*	*R*	*S*	*T*	*U*	*W*	*Y*	*Z*
Predecessor	–	*P, T*	*Q*	–	*S*	–	*S*	*S*	*U,W*
t_o	3	4	4	3	8	1	2	4	6
t_m	6	8	5	3	14	4	5	7	15
t_p	9	9	9	3	17	7	14	13	30

Determine:

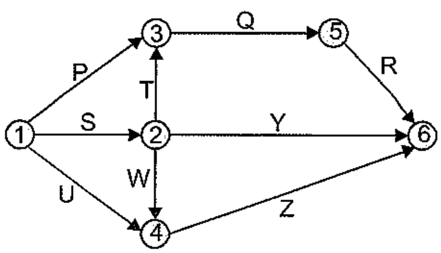
(i) The probability of completing the job in 32 days and

(ii) The completion time with 50% probability.

Z	*Probability*
– 1.5	0.07
– 1.3	0.10
– 1.0	0.16

[10 Marks, ESE-2009]

Sol: **Network Diagram**

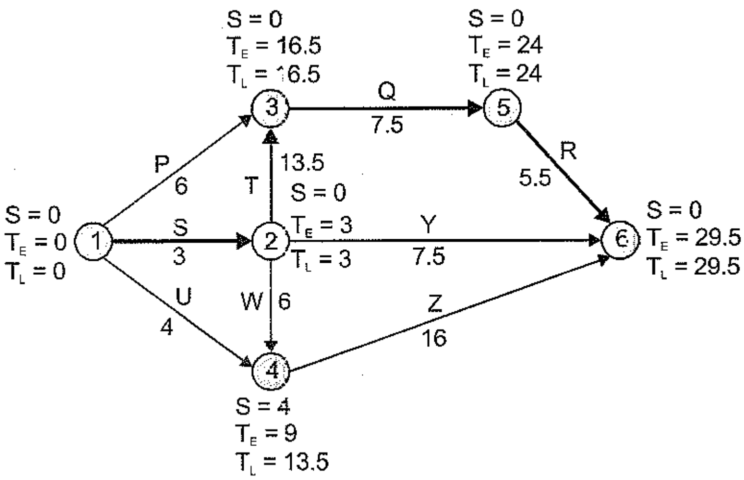


Calculation of T_E, T_L and σ has been done in table below:

where, $t_e = \frac{t_0 + 4t_m + t_p}{6}$, $\sigma = \frac{t_p - t_0}{6}$

Activity	t_o	t_m	t_p	t_e	σ
P	3	6	9	6	1
Q	4	8	9	7.5	0.83
R	4	5	9	5.5	0.83
S	3	3	3	3	0
T	8	14	17	13.5	1.5
U	1	4	7	4	1
W	2	5	14	6	2
Y	4	7	13	7.5	1.5
Z	6	15	30	16	4

Calculation of T_E, T_L and Slack has been done in network diagram below:



As we can observe that Events 1, 2, 3, 5, 6 are having zero Slack.

∴ Critical path is 1–2–3–5–6/S–T–Q–R

Standard deviation along the critical path is,

$$s = \sqrt{0^2 + 1.5^2 + 0.833^2 + 0.833^2} = 1.91 \text{ days}$$

(i) $Z = \frac{T_S - T_E}{\sigma} = \frac{32 - 29.5}{1.91} = 1.31$

∵ if $Z = -1$ then $p = 0.16$ ∴ if $Z = 1$ then $p = 0.84$

∵ if $Z = -1.3$ then $p = 0.10$ ∴ if $Z = 1.3$ then $p = 0.90$

∵ if $Z = -1.5$ then $p = 0.07$ ∴ if $Z = 1.5$ then $p = 0.93$

∴ Probability for $Z = 1.31$

We will do interpolation = $0.90+\left(\frac{0.93-0.90}{1.5-1.3}\right)\times(1.31-1.30) = 0.9015 = 90.15\%$

(ii) For $p = 50 \rightarrow z = 0$

Hence $T_E = T_L = 29.5 = T_S$.

Q–12: ***A contractor intends to bid for a small-size civil engineering project consisting of the following activities and corresponding different time limits. Work out***

(i) Critical path and standard deviation for the whole network

(ii) Z-factor for completing the project in 42 weeks

(iii) Completion time duration for which the contractor should bid considering 93% probability assuming Z-factor as 1.5 and

(iv) TF, FF and IF:

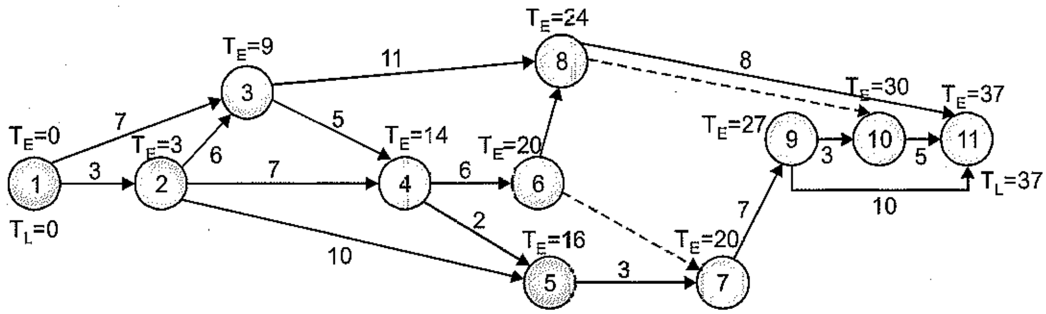
Activity	Optimistic time (in weeks)	Most likely time (in weeks)	Pessimistic time (in weeks)
1 – 2	1	3	5
1 – 3	3	6	15
2 – 3	2	5	14
2 – 4	5	7	9
3 – 4	2	4	12
2 – 5	6	9	18
4 – 5	1	2	3
3 – 8	3	12	15
4 – 6	4	6	8
5 – 7	1	2.5	7
6 – 8	3	4	5
6 – 7	0	0	0
7 – 9	1	8	9
9 – 10	1	3	5
8 – 10	0	0	0
8 – 11	1	9	11
10 – 11	3	4.5	9
9 – 11	4	9	20

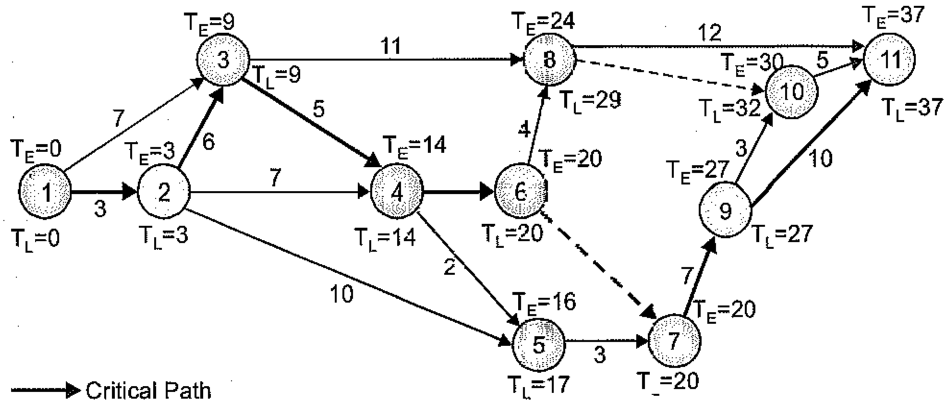
[15 Marks, ESE-2010]

Sol: Calculation of expected time and standard deviation is done in the table below.

Activity	Optimistic Time (t_o) (Weeks)	Most Likely Time (t_m) (Weeks)	Pessimistic Time (t_p) (Weeks)	Expected Time (T_E) $(t_0+4t_m+t_p)/6$	σ $(t_p - t_o)/6$
1 – 2	1	3	5	3	0.67
1 – 3	3	6	15	7	2
2 – 3	2	5	14	6	2
2 – 4	5	7	9	7	0.67
3 – 4	2	4	12	5	1.67
2 – 5	6	9	18	10	2
4 – 5	1	2	3	2	0.33
3 – 8	3	12	15	11	2
4 – 6	4	6	8	6	0.67
5 – 7	1	2.5	7	3	1
6 – 8	3	4	5	4	0.33
6 – 7	0	0	0	0	0
7 – 9	1	8	9	7	1.33
9 – 10	1	3	5	3	0.67
8 – 10	0	0	0	0	0
8 – 11	1	9	11	8	1.67
10 – 11	3	4.5	9	5	1
9 – 11	4	9	20	10	2.67

Network diagram for the given project is





(1)-(2)-(3)-(4)-(6)-(7)-(9)-(11) is the critical path.

$$\sigma \text{ for critical path } = \sqrt{\sigma_{1-2}^2+\sigma_{2-3}^2+\sigma_{3-4}^2+\sigma_{4-6}^2+\sigma_{6-7}^2+\sigma_{7-9}^2+\sigma_{9-11}^2}$$

$$\sigma = \sqrt{(0.67)^2+(2)^2+(1.67)^2+(0.67)^2+0^2+(1.33)^2+(2.67)^2}$$

Standard deviation for project = 4.07 weeks

(ii) Z-factor for completing project in 42 weeks

$$= Z_{42} = \frac{T_S - T_E}{\sigma} = \frac{42-37}{4.07} = 1.2285$$

(iii) Let T_S be the completion duration then

$$Z = \frac{T_S - T_E}{\sigma} \Rightarrow 1.5 = \frac{T_S - 37}{4.07} \Rightarrow T_S = 43.105 \text{ Weeks}$$

Activity	T_E	EST	EFT	LST	LFT	F_T	F_F	F_{ID}
1 – 2	3	0	3	0	3	0	0	0
1 – 3	7	0	7	2	9	2	2	2
2 – 3	6	3	9	3	9	0	0	0
2 – 4	7	3	10	7	14	4	4	4
3 – 4	5	9	14	9	14	0	0	0
2 – 5	10	3	13	7	17	4	3	3
4 – 5	2	14	16	15	17	1	0	0
3 – 8	11	9	20	18	29	9	4	4
4 – 6	6	14	20	14	20	0	0	0
5 – 7	3	16	19	17	20	1	1	0
6 – 8	4	20	24	25	29	5	0	0
6 – 7	0	20	20	20	20	0	0	0
7 – 9	7	20	27	20	27	0	0	0
9 – 10	3	27	30	29	32	2	0	0
8 – 10	0	24	24	32	32	8	6	1
8 – 11	8	24	32	29	37	5	5	0
10 – 11	5	30	35	32	37	2	2	0
9 – 11	10	27	37	27	37	0	0	0

Q–13: ***For an activity of casting a raft foundation of a High rise building, three engineers A, B and C have given the time estimates as follows. State who is more certain about the time of completion of job. Also calculate expected time of completion of each engineer.***

	Times in week		
Engineer	***Optimistic***	***Most likely***	***Pessimistic***
A	***05***	***07***	***09***
B	***04***	***06***	***07***
C	***03***	***05***	***08***

[5 Marks, ESE-2012]

Out of the three engines the one with the least standard deviation will be most certain about the time of completion of job.

Calculation of expected time and standard deviation is done in the table below.

Engineer	Optimistic time (Weeks) (t_o)	Most Likely Time (Weeks) (t_m)	Pessimistic Time (Weeks) (t_p)	Expected Time (Weeks) $\frac{t_o + 4t_m + t_p}{6}$	σ $\frac{t_p - t_o}{6}$
A	5	7	9	7	0.67
B	4	6	7	5.834	0.5
C	3	5	8	5.167	0.834

Now, since engineers B schedule gives the minimum standard deviation, he will be more certain about the time of completion of job.

Q–14: ***PERT calculations indicate that duration of a given project is 72 weeks. With the variance of 15, work out number of weeks within which the project is expected to be completed with probability of 50%, 80% and 98%. Take Z-values of 0.89 and 2.1 for probability of 80% and 98% respectively.***

[5 Marks, ESE-2012]

Sol:

$$\text{Given duration of the project } (T_E) = 72 \text{ weeks}$$

$$\text{Variance} = 15$$

$$\sigma = \sqrt{\text{Variance}} = \sqrt{15} = 3.873 \text{ weeks}$$

Let T_{50}, T_{80} and T_{98} are the number of weeks within which the project is expected to be completed with probability of 50%, 80% and 98% respectively.

Now,
$$Z_{50} = 0$$

Hence,
$$\frac{T_{50} - T_E}{\sigma} = 0 \Rightarrow T_{50} = T_E = 72 \text{ week}$$

$$Z_{80} = 0.89 \text{ (Given)}$$

Hence,
$$\frac{T_{80} - 72}{3.873} = 0.89$$

$\Rightarrow$
$$T_{80} = 0.89 \times 3.873 + 72$$
$$= 75.447 \text{ weeks}$$

$$Z_{98} = 2.1 \text{ (Given)}$$

Hence,
$$\frac{T_{98} - 72}{3.873} = 2.1$$

$\Rightarrow$
$$T_{98} = 2.1 \times 3.873 + 72$$
$$= 80.133 \text{ weeks}$$

Q–15: ***In a small project, three activities, A, B and C, are on the critical path. Their optimistic, most lik and pessimistic time durations (weeks) is given below :***

Activity	*Optimistic duration*	*Most likely duration*	*Pessimistic duration*
A	*6*	*8*	*11*
B	*4*	*6*	*8*
C	*11*	*15*	*18*

Assuming that the non-critical activities are completed within the time :

(i) What is the probability that the proejct can be comnpleted in 29 weeks ?

(ii) What is the probability that the project can be completed in 27 weeks ?

For a probability of 10%, 20% and 30% the Z values may be taken to be –1.26, –0.84 and –0.53, respectively.

[10 Marks, ESE-2013]

Sol:

Activity	t_0	t_m	t_p	t_e	σ
A	6	8	11	8.167	0.833
B	4	6	8	6	0.667
C	11	15	18	14.833	1.167

Expected completion of the project will be in 29 weeks and standard deviation of the project.

$$= \sqrt{0.833^2 + 0.667^2 + 1.167^2} = 1.581$$

(i) The probability that the project can be completed in 29 weeks is 50%.

(ii) $$\text{Z value} = \frac{T_S - T_E}{X} = \frac{27 - 29}{1.581} = -1.265$$

Probability will be <50% as $T_S < T_E$ and given values:

Z	Probability
–1.26	10%
–0.84	20%
–1.265	?

$$\text{Probability} = \left\{10 - \frac{10}{(1.26 - 0.84)} \times (1.265 - 1.26)\right\} = 9.88\%$$

Q–16: ***In PERT network, how is the "expected time" of completion of an activity related to most likely optimistic and the pessimistic times of completion of that activity?***

[4 Marks, ESE-2014]

Sol: For calculation of expected time (t_e) from most likely time (t_m), optimistic time (t_0) and pessimistic time (t_p) we use weighted average method in which weightage of 1 is given to t_0, weightage of 4 is given to t_m and weightage of 1 is given to t_p i.e.,

$$t_e = \frac{t_0 + 4t_m + t_p}{6}$$

17: Define the following terms briefly in the context of construction contracts:
Beta distribution in PERT

[4 Marks, ESE-2004]

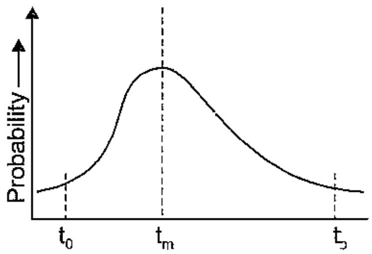
Sol: Beta distribution in PERT:

β – distribution curve is used in PERT network analysis because in this analysis individual activities generally have b distribution which can be left or right skewed. If large number of activities are taken together, the skewedness will die out and for overall project time, the probability distribution function will normal distribution.

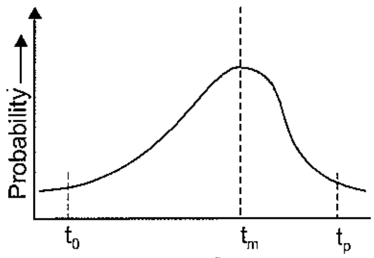
Standard deviation and variance for β – distribution curve can be approximately calculated as below.

β – distribution curve is used in PERT network analysis because in this analysis individual activities generally have b distribution which can be left or right skewed. If large number of activities are taken together, the skewedness will die out and for overall project time, the probability distribution function will normal distribution.

- Standard deviation and variance for β – distribution curve can be approximately calculated as below.
  Standard Deviation, $\sigma = \frac{t_p - t_0}{6}$; Variance, $\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$
- A right skewed β – distribution curve represents pessimistic estimator.
- Where as a left skewed β distribution curve represents optimistic estimator.



Left Skewed β - distribution curve



Right skewed β - distribution curve

Q–18: In a CPM network, the critical path includes five activities. Their durations are tabulated as follows:

Activity	*Duration (Days)*		
	Optimistic Time (T_0)	*Most Likely Time (T_m)*	*Pessimistic Time (T_p)*
A	*2*	*4*	*7*
B	*5*	*8*	*14*
C	*4*	*6*	*8*
D	*2*	*2*	*2*
E	*7*	*10*	*21*

Compute the following values:

(i) The probability that the project will finish by the end of day 32

(ii) The probability that the project will finish by the end of day 34

(iii) The probability that the project will finish before day 30

(iv) ***The probability that the project will finish on the 32^{nd} day***

(v) ***The probability that the project will finish no later than the 35^{th} day***

Z =	***0***	***0.34***	***0.67***	***1.01***
Probability	***50%***	***63.33%***	***74.9%***	***84.4%***

[20 Marks, ESE-2016]

Sol:

$$T_A = \frac{t_0 + 4t_m + t_p}{6} = \frac{2+4\times4+7}{6} = 4.167 \text{ days}$$

$$T_B = \frac{5+4\times8+14}{6} = 8.5 \text{ days}$$

$$T_C = \frac{4+6\times4+8}{6} = 6 \text{ days}$$

$$T_D = \frac{2+2\times4+2}{6} = 2 \text{ days}$$

$$T_E = \frac{7+10\times4+21}{6} = 11.33 \text{ days}$$

Total time for the completion of the project = ΣT_i = 4.167+8.5+6.0+11.33 = 32 days

(i) For $T_S \le 32$ days $\Rightarrow$ p = 50%

(ii) For $T_S \le 34$ day

$$Z = \frac{T_S - T_E}{\sigma}$$

Now
$$\sigma_p = \left(\sum_{i=1}^{n}\left(\frac{t_p - t_0}{6}\right)^2\right)^{1/2}$$

$$= \left(\left(\frac{7-2}{6}\right)^2 + \left(\frac{14-5}{6}\right)^2 + \left(\frac{8-4}{6}\right)^2 + \left(\frac{2-2}{6}\right) + \left(\frac{21-7}{6}\right)^2\right)^{1/2} = 2.97$$

$$Z = \frac{34-32}{2.97} = 0.67$$

For Z = 0.67 $\Rightarrow$ p = 74.9%

(iii) For $T_S \le 30$ days

$$Z = \frac{30-32}{2.97} = -0.67$$

For Z = –0.67 $\Rightarrow$ p = 100 – 74.9 = 25.1%

(iv) For T_S = 32nd day

$$P(T = 32^{nd} \text{ day}) = P(T_S \le 32 \text{ days}) - P(T_S \le 31 \text{ days})$$

For
$$P(T_S \le 31 \text{ days}),\ Z = \frac{31-32}{2.97} = -0.34$$

$$P(T_S \le 31 \text{ days}) = 100 - 63.33 = 36.67\%$$

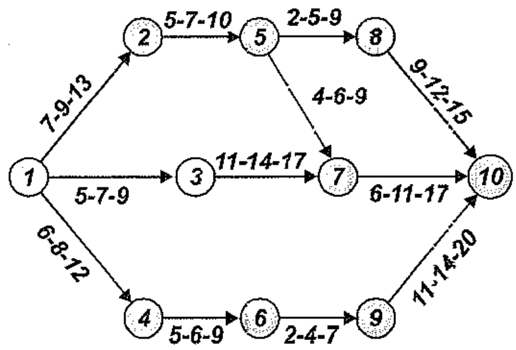
$$P(T = 32^{nd} \text{ day}) = 50 - 36.67 = 13.33\%$$

(v) For $T \le 35$

$$Z = \frac{35-32}{2.97} = 1.01$$

For Z = 1.01 $\Rightarrow$ P = 84.4%

Q–19: ***Calculate the expected time t_E and critical path for the following network shown in Figure. The optimistic, most likely and pessimistic time for each activity is as shown. 1 is the initial event and 10 is the end event.***

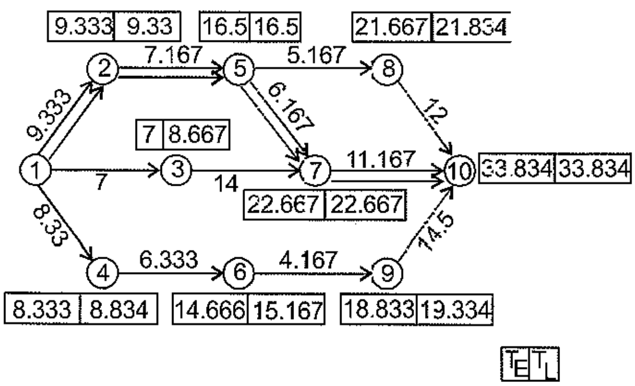


[12 Marks, ESE-2017]

Sol: Calculation of expected time is done in table below where,

$$T_E = \frac{T_0 + 4T_m + T_e}{6}$$

Activity	Optimistic Time (T_0) units	Most Likely Time (T_m)	Pessimistic Time (T_p)	Expected Time (TE)
1 – 2	7	9	13	9.333
1 – 3	5	7	9	7
1 – 4	6	8	12	8.333
2 – 5	5	7	10	7.167
3 – 7	11	14	17	14
4 – 6	5	6	9	6.333
5 – 7	4	6	9	6.167
5 – 8	2	5	9	5.167
6 – 9	2	4	7	4.167
8 – 10	9	12	15	12
7 – 10	6	11	17	11.167
9 – 10	11	14	20	14.5



Expected Time $\qquad T_E$ = 33.834 units

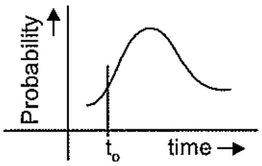
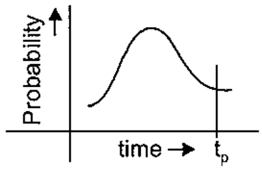
Critical path is 1 – 2 – 5 – 7 – 10

Q–20: ***Briefly answer the following:***

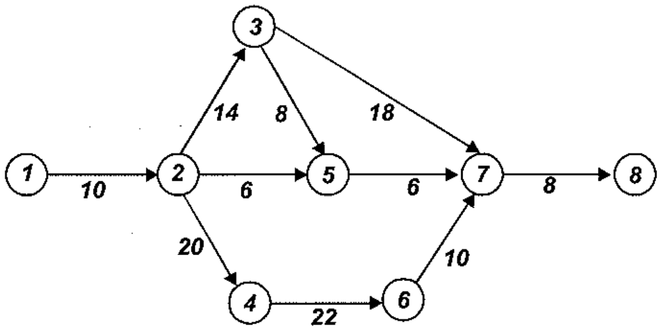
Differentiate between 'Optimistic time estimate' and 'Pessimistic time estimate'.

[5 Marks, ESE-2017]

Sol:

Optimistic time estimate	**Pessimistic time estimate**
1. It is the minimum time in which the activity can be completed.	1. It is the maximum time in which activity can be completed.
2. Most favourable conditions prevail.	2. Most adverse conditions prevail.
 t_o = optimistic time	 t_p = pessimistic time

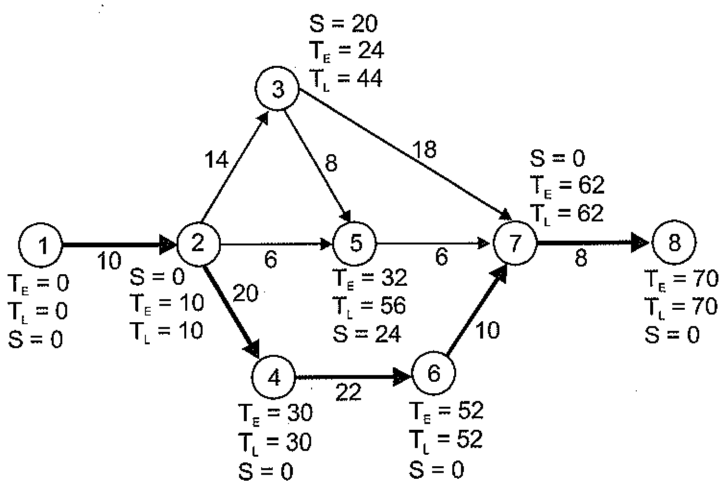
Q–21:



Identify the critical path in the network as shown in figure and determine the project completion time. The duration are in weeks.

[20 Marks, ESE-2019]

Sol:



Event	T_E	T_L	Slack $S = T_L - T_E$	Remark
1	0	0	0	Critical
2	10	10	0	Critical
3	24	44	20	
4	30	30	0	Critical
5	32	56	24	
6	52	52	0	Critical
7	62	62	0	Critical
8	70	70	0	Critical

Critical path having longest path with timewise (having minimum slack)

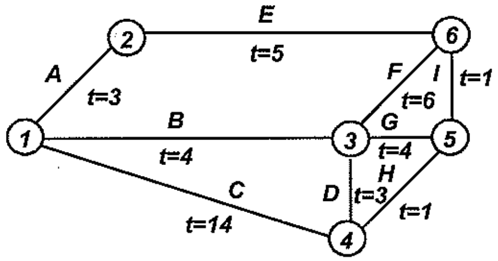
$\therefore$ Critical path is 1 – 2 – 4 – 6 – 7 – 8

Project completion time = 70 weeks

CHAPTER 4

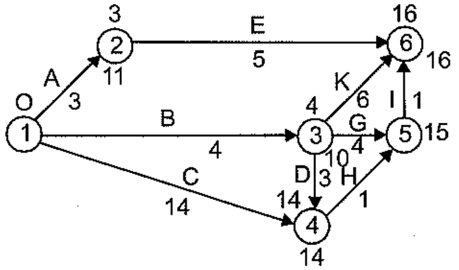
CRITICAL PATH METHOD

Q–1: ***The network shown in the figure below has the estimated duration for each activity marked Determine the total float for each activity and establish the critical path.***



[25 Marks, ESE-1996]

Sol:



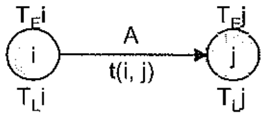
Activity	Est	LFT	T_E	Total float
A	0	11	3	8
B	0	10	4	6
C	0	14	14	0
D	4	14	3	7
E	3	16	5	8
F	4	16	6	6
G	4	15	4	7
H	14	15	1	0
I	15	16	1	0

Q–2: ***What is the different between 'Free Float' and 'Total Float'? Explain the significance of each term.***

[15 Marks, ESE-1999]

Sol: Total float is the maximum time by which an activity can be delayed without affecting the completion time of the project.

On the other hand free float can be said as a part of total float by which an activity can be delayed without affecting the succeeding activity.



Total Float $= \left[T_L j - T_E i - t(i,j)\right]$

Free Float $= \left[T_E j - T_E i - t(i,j)\right]$

Here for activity A -

$T_E i$ = Earliest start time

$T_L i$ = Latest start time

$T_E j$ = Earliest finish time

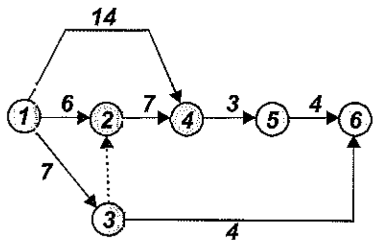
$T_L j$ = Latest finish time

t(i, j) = Activity duration.

- Total float-gives the total available extra time to do an activity but it affects the float of the preceding and succeeding activity.
- Free float gives the total available extra time to do an activity without affecting the succeeding event.

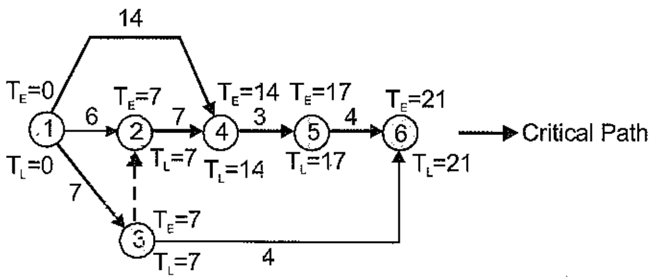
  Hence floats can be efficiently used for resource allocating and strategy making. Thus we can shift our interest from non critical events to critical events.

Q-3: ***Determine the total float, free float and critical path for the installation project represented by the network shown in the figure. The estimated duration in days for the activities are shown in the figure.***



[20 Marks, ESE-2000]

Sol: Given network diagram with estimated time duration for the activities.



Activity	Duration (Days)	Earliest		Latest		Total Float (FT)	Free float (FF)
		Start Time (EST)	Finish Time (EFT)	Start Time (LST)	Finish Time (LFT)		
1–2	6	0	6	1	7	1	1
1–3	7	0	7	0	7	0	0
1–4	14	0	14	0	14	0	0
2–4	7	7	13	7	14	0	0
3–2	0	7	7	7	7	0	0
3–6	4	7	11	17	21	10	10
4–5	3	14	17	14	17	0	0
5–6	4	17	21	17	21	0	0

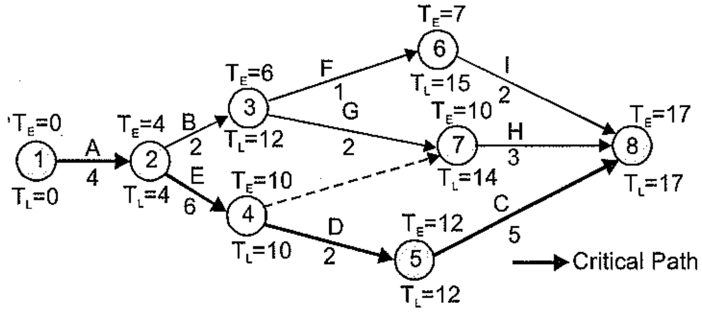
The activities for which total float is zero are the critical activities, and these are 1–3, 1–4, 2–4, 3–2, 4–5 and 5–6. The critical path is therefore along these activities. 1–4–5–6 and 1–3–2–4–5–6 are the critical paths.

Q–4: ***A network is formed by the following activities. The duration of the activities are given below. Draw the network, calculate the project duration and identify the critical path.***

Activity	*Preceded by*	*Duration (Days)*
A	*Starting*	*4*
B	*A*	*2*
C (Terminal)	*D*	*5*
D	*E*	*2*
E	*A*	*6*
F	*B*	*1*
G	*B*	*2*
H (Terminal)	*E, G*	*3*
I (Terminal)	*F*	*2*

[20 Marks, ESE-2001]

Sol: Network for the given activity duration table is as follows:



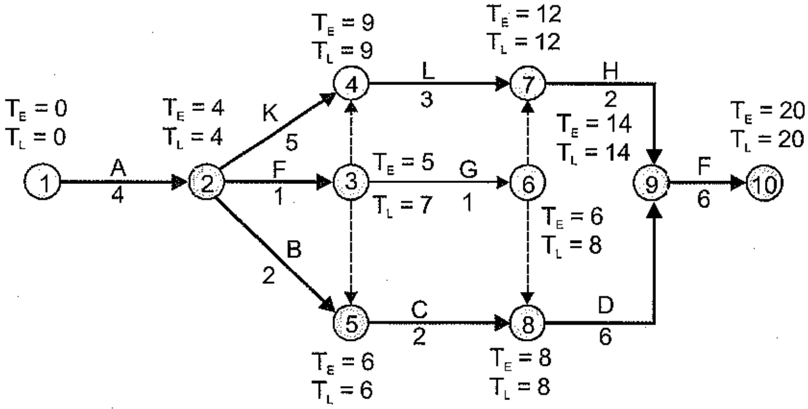
The value of T_E (Earliest Event Time) and T_L (Latest Allowable Event Time) is written in the network diagram shown above critical path is path along which slack of the event is zero. From the above diagram 1–2–4–5–8 or A–E–D–C is the critical path marked with dark line in the diagram above. The project duration is 17 days.

Q–5: ***Draw the network for the following project and indicate the event times and critical path. Also find the project duration and total float for all activities.***

Activity	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*	*L*
Duration (Days)	*4*	*2*	*2*	*6*	*6*	*1*	*1*	*2*	*5*	*3*
Depends Upon	–	*A*	*A, B, F*	*C, G*	*D, G, H*	*A*	*F*	*G, L*	*A*	*F, K*

[20 Marks, ESE-2004]

Sol: **Network Diagram**



Calculation of Float has been done in table below:

where, EST = T_E^i, EFT = $T_E^i + t_e^{ij}$, LST = $T_L^j - t_e^{ij}$, LFT = T_L^j, F_T = LST – EST.

Activity	t^{ij}	EST	EFT	LST	LFT	F_T
A	4	0	4	0	4	0
B	2	4	6	4	6	0
C	2	6	8	6	8	0
D	6	8	14	8	14	0
E	6	14	20	14	20	0
F	1	4	5	6	7	2
G	1	5	6	7	8	2
H	2	12	14	12	14	0
K	5	4	9	4	9	0
L	3	9	12	9	12	0

Total Project duration = 20 days.

Critical path is along A–K–L–H–E and A–B–C–D–E as Total Float of activities is zero along this path.

Q-6: ***Draw a comparison between PERT and CPM.*** ***[10 Marks, ESE-2005]***

Sol:

PERT	CPM
1. Network diagram is event oriented.	1. Network diagram is activity oriented.
2. It uses probabilistic approach and is suitable for research & development and non repetitive project.	2. It uses Deterministic approach and is suitable for repetitive type of project.
3. 3 time estimates are given for completion of an activity.	3. Single time estimate is given for each activity.
4. Follows β- distribution.	4. Follows Normal distribution.
5. Cost of project is directly proportional to time and hence to minimize the project cost the project completion time is minimized.	5. Cost model has to be developed using which min. cost of the project is found.
6. Critical events are identifiec by using the concept of slack.	6. Critical activities are identified by using concept of float.
7. Critical path will be path joining the critical events.	7. Critical path will be the path joining all the critical activities.

Q–7: ***Information on the activities required for a medium size Civil Engineering Project is as follows :***

Name of Activity	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*
Node No.	*1-2*	*1-3*	*1-4*	*2-5*	*3-5*	*3-6*	*3-7*	*4-6*	*5-7*	*6-8*	*7-8*
Duration (in months)	*02*	*07*	*08*	*03*	*06*	*10*	*04*	*06*	*02*	*05*	*06*

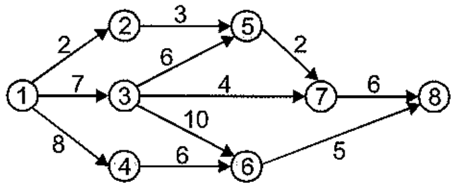
Draw the network and calculate the following :

(i) ***Earliest and latest event time*** ***(ii)*** ***Earliest and latest start time***

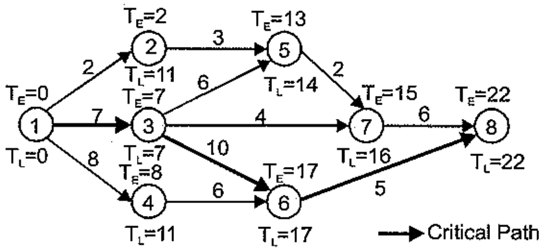
(iii) ***Earliest and latest finish time*** ***(iv)*** ***Critical path***

[20 Marks, ESE-2008]

Sol: Network diagram for the given project is as follows:



Earliest and the latest event time is calculated on the Network diagram below and the critical path is marked on it.



(1)–(3)–(6)–(8) is the critical path. Calculation of Earliest and Latest start and finish time for various activities are done below.

Activity	**Duration**	**Earliest**		**Latest**	
	(Months)	**Start Time**	**Finish Time**	**Start Time**	**Finish Time**
A (1 – 2)	02	0	2	9	11
B (1 – 3)	07	0	07	0	7
C (1 – 4)	08	0	08	03	11
D (1 – 5)	03	2	05	11	14
E (1 – 5)	06	7	13	8	1
F (1 – 6)	10	7	17	7	17
G (1 – 7)	04	7	11	12	16
H (1 – 6)	06	8	14	11	17
I (1 – 7)	02	13	15	14	16
J (1 – 8)	05	17	22	17	22
K (1 – 8)	06	15	21	16	22

Q–8: ***The data for planning a project by network technique is given below:***

Activity	*Duration (in weeks)*	*Activities Immediately*	
		Proceeding	*Following*
A	*4*	–	*B, C*
B	*4*	*A*	*D, F*
C	*6*	*A*	*E, G*
D	*3*	*B*	*E, G*
E	*6*	*C, D*	*H, I*
F	*6*	*B*	*I*
G	*5*	*C, D*	*I*
H	*7*	*E*	–
I	*4*	*E, F, G*	–

For the above data, answer the following:

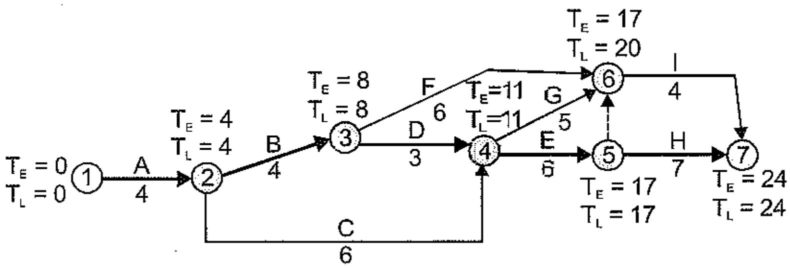
(i) ***Draw the CPM diagram.***

(ii) ***Prepare a CPM schedule and calculate F_T, F_F and F_{IN}.***

(iii) ***State the critical path.***

(iv) ***Indicate the project duration.*** ***[10 Marks, ESE-2011]***

Sol: **Network Diagram**



Calculation of Float has been done in table below:

where, EST = T_E^i, EFT = $T_E^i + t_e^{ij}$, LST = $T_L^j - t_e^{ij}$, LFT = T_L^j, F_T = LST – EST, $F_F = F_T - S_J$, $F_{ID} = F_F - S_i$

Activity	t^{ij}	EST	EFT	LST	LFT	F_T	F_F	F_{ID}
A	4	0	4	0	4	0	0	0
B	4	4	8	4	8	0	0	0
C	6	4	10	5	11	1	1	1
D	3	8	11	8	11	0	0	0
E	6	11	17	11	17	0	0	0
F	6	8	14	14	20	6	3	3
G	5	11	16	15	20	4	1	1
H	7	17	24	17	24	0	0	0
I	4	17	21	20	24	3	3	0

Total Project duration = 24 weeks.

Critical path is along 1–2–3–4–5–7 or A–B–D–E–H as Total Float of activities is zero along this path.

Q-9: ***The data for planning a certain Civil Engineering project by CPM-Network analysis is given below. Draw the network and establish the critical path.***

Also determine the following:

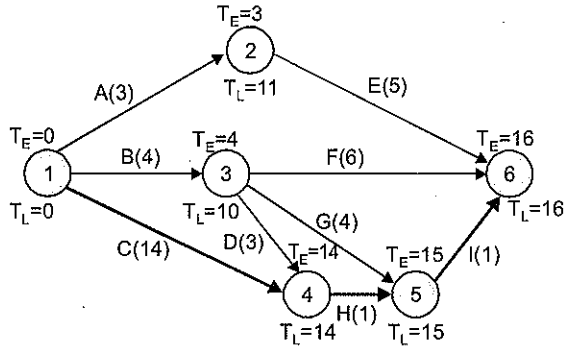
(i) Prepare a CPM schedule and calculate total float, free float and independent float.

(ii) Compute the project duration.

Activity	*Duration in weeks*	*Activity immediately Preceding*	*Activity immediately Following*
A	*03*	—	*E*
B	*04*	—	*D, F, G*
C	*14*	—	*H*
D	*03*	*B*	*H*
E	*05*	*A*	—
F	*06*	*B*	—
G	*04*	*B*	*I*
H	*01*	*C, D*	*I*
I	*01*	*G, H*	—

[10 Marks, ESE-2012]

Sol: Network diagram for the given project is



Values in parentheses shows the duration of the activities.

1–4–5–6 or C – H – I is the critical path.

Activity	Duration (Weeks)	Earliest Start Time	Earliest Finish Time	Latest Start Time	Latest Finish Time	Total Float	Free Float	Independent Float
A	3	0	3	8	11	8	0	0
B	4	0	4	6	10	6	0	0
C	14	0	14	0	14	0	0	0
D	3	4	7	11	14	7	7	1
E	5	3	8	11	16	8	8	0
F	6	4	10	10	16	6	6	0
G	4	4	8	11	15	7	7	1
H	1	14	15	14	15	0	0	0
I	1	15	16	15	16	0	0	0

Duration of the project is 16 weeks.

Q–10: ***(i) Define critical path***

(ii) Explain FLOAT and slack. ***[10 Marks, ESE-2015]***

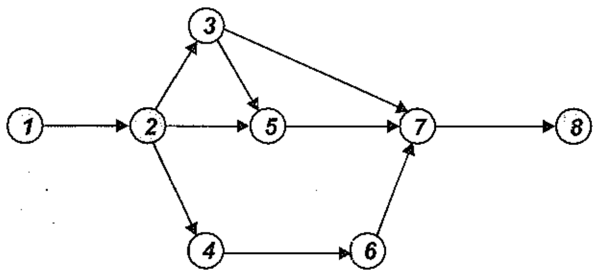
Sol: **(i) Critical Path**

- It is the path in a project network, which commencing from the critical event, connects the events having zero or minimum slack times and terminates at the end event.
- It is the longest path in terms of time or most time consuming path from beginning to the end of the project.
- It is also the shortest path in terms of time in which the project can be completed as early as possible.

(ii) Float and Slack

- Float is the extra available time by which an activity can be delayed without affecting the project completion time.
- Slack is the excess time available by which occurrence of an event can be delayed without affecting the project completion time.
- Float is associated with an activity while slack is associated with an event.

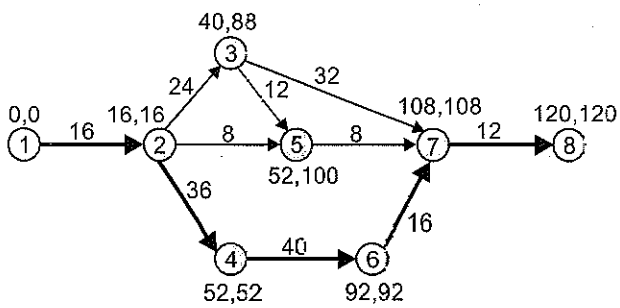
Q–11: ***For the network of a construction project with various activities shown below, determine the project completion time. Determine the total float of each activity. Mention the critical activities.***



Activity	Duration(weeks)
1 - 2	16
2 - 3	24
2 - 4	36
2 - 5	8
3 - 5	12
4 - 6	40
3 - 7	32
5 - 7	8
6 - 7	16
7 - 8	12

[20 Marks, ESE-2015]

Sol:



Critical activity: 1–2–4–6–7–8

Time along critical activity = 120 weeks

Total Float (F_T) = Latest Start Time (LST) – Earliest Start Time (EST)

Activity	Duration	EST	LST	F_T
1 – 2	16	0	0	0
2 – 3	24	16	64	48
2 – 4	36	16	16	0
2 – 5	8	16	92	76
3 – 5	12	40	88	48
4 – 6	40	52	52	0
3 – 7	32	40	76	36
5 – 7	8	52	100	48
6 – 7	16	92	92	0
7 – 8	12	108	108	0

Q–12: ***A construction project has the following characteristics:***

Activity	*Preceding activity*	*Duration (weeks)*
A	*None*	*5*
B	*A*	*2*
C	*A*	*6*
D	*B*	*12*
E	*D*	*10*
F	*D*	*5*
G	*D*	*5*
H	*B*	*9*
I	*C, E*	*1*
J	*G*	*2*
K	*F, I, J*	*3*
L	*K*	*9*
M	*H, G*	*7*
N	*M*	*9*

(i) ***Draw a network for this project.***

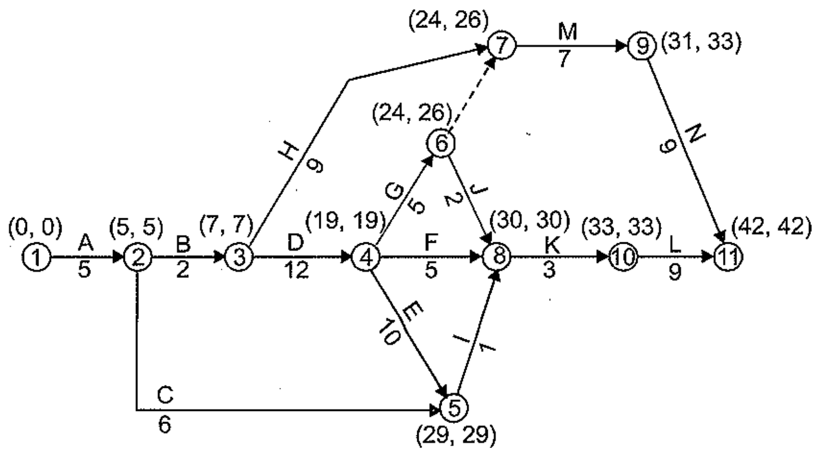
(ii) ***Find various paths and the critical path as well as the project completion time.***

(iii) ***Prepare an activity schedule showing Earliest Start time, Earliest finish time, Latest Start time, Latest Finish time and float for each activity.***

(iv) ***Will the critical path change if activity G takes 10 weeks instead of 5 weeks? If so, what will be the new critical path?***

[12 Marks, ESE-2018]

Sol: (i) According to the given construction project details, the network diagram is



Network diagram

(ii) The various paths and the critical path as well as the project completion time is shown below :

Path	Duration (weeks)
(i) 1–5–3–8–10–11	24
(ii) 1–2–3–7–9–11	32
(iii) 1–2–3–4–6–7–9–11	40
(iv) 1–2–3–4–6–8–10–11	38
(v) 1–2–3–4–8–10–11	36
(vi) 1–2–3–4–5–8–10–11	42

Thus, the critical path is 1 – 2 – 3 – 4 – 5 – 8 – 10 – 11 and the duration of the project is 42 weeks.

(iii) Calculation for the EST, EFT, LST, LFT and total float for each activity is shown below.

Activity	t^{ij}	EST	EFT	LST	LFT	F_T
A	5	0	5	0	5	0
B	2	5	7	5	7	0
C	6	5	11	23	29	18
D	12	7	19	7	19	0
E	10	19	29	19	29	0
F	5	19	24	25	30	6
G	5	19	24	21	26	2
H	9	7	16	17	26	10
I	1	29	30	29	30	0
J	2	24	26	28	30	4
K	3	30	33	30	33	0
L	9	33	42	33	42	0
M	7	24	31	26	33	2
N	9	31	40	33	42	2

where,
$$\text{EST} = T_E^i$$
$$\text{EFT} = T_E^i + t^{ij}$$
$$\text{LST} = T_L^j - t^{ij}$$
$$\text{LFT} = T_L^j$$

and
$$F_T = \text{LST} - \text{EST or LFT} - \text{EFT}$$

(iv) If activity G takes 10 weeks instead of 5 weeks, then the duration of the paths (iii) and (iv) [from table shown in part (ii)] would become 45 and 43 weeks respectively.

In that situation, path 1 – 2 – 3 – 4 – 6 – 7 – 9 – 11 would be critical.

Hence, the new critical path is 1 – 2 – 3 – 4 – 6 – 7 – 9 – 11 and new project completion time is 45 weeks.

Q-13: ***The following table gives the activities in a construction project and other relevant information.***

Activity	***Duration (days)***
1 – 2	***20***
1 – 3	***25***
2 – 3	***10***
2 – 4	***12***
3 – 4	***6***
4 – 5	***10***

(i) ***Draw the network for the project***

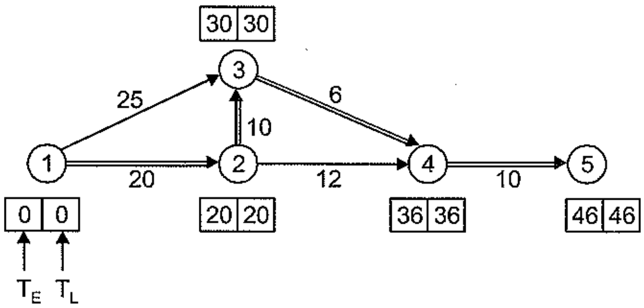
(ii) ***Find the critical path.***

(iii) ***Find free, total and independent floats for each activity.***

[20 Marks, ESE-2020]

Sol:

(i)



(ii) The critical path is the longest duration path (time-wise), i.e.

Time along critical path = 46 days

(iii) Calculation of free, total and independent float :

			Activity times (days)			Floats (days)			
Activity	**t^{ij}**	**EST**	**EFT**	**LST**	**LFT**	**TF**	**FF**	**IF**	**Remark**
1 – 2	20	0	20	0	20	0	0	0	Critical
1 – 3	25	0	25	5	30	5	5	5	
2 – 3	10	20	30	20	30	0	0	0	Critical
2 – 4	12	20	32	24	36	4	4	4	
3 – 4	6	30	36	30	36	0	0	0	Critical
4 – 5	10	36	46	36	46	0	0	0	Critical

Note :

$$\text{EST} = T_E^i$$

$$\text{EFT} = T_E^i + t^{ij}$$

$$\text{LST} = \left(T_L^i - t^{ij}\right)$$

$$\text{LFT} = T_L^i$$

$$\text{Total Float (TF)} = (\text{LST} - \text{EST}) = (\text{LFT} - \text{EFT})$$

$$\text{Free Float (FF)} = (\text{TF} - S_j)$$

$$\text{Independent Float (IF)} = (\text{FF} - S_i)$$

CHAPTER 5

CRASHING

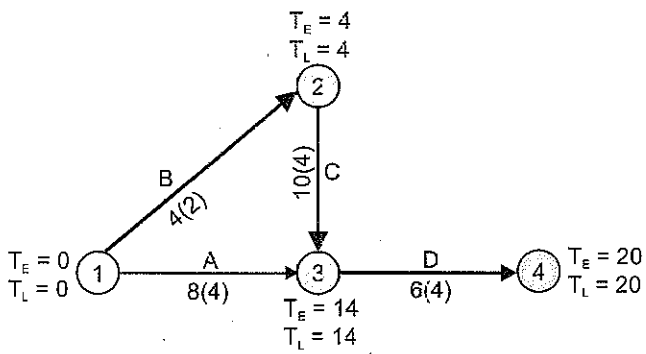
Q–1: ***A, B, C and D are the activities. Their normal and crash durations and associated costs are given in the table below:***

Activity	*Duration in days*		*Direct cost in Rupees*	
	Nomal	*Crash*	*Normal*	*Crash*
A	*8*	*4*	*6000*	*12000*
B	*4*	*2*	*2000*	*14000*
C	*10*	*4*	*4000*	*8000*
D	*6*	*4*	*4000*	*8000*

For the entire project the indirect cost is Rs. 1000 per day. 'A' and 'B' are starting activities; 'C' follows 'B'; 'D' follows 'A' and 'C'; 'D' is finishing activity. Draw CPM Network. Calculate points for PTC graph and plot the same. Determine the optimum cost and optimum duration for the project. PTC is Project-Time-Cost-Trade-off graph.

[25 Marks, ESE-2007]

Sol: **Network Diagram**



Critical path = 1 – 2 – 3 – 4

Cost slope

Activity	Normal		Crash		Cost Slope		
	Time	Cost	Time	Cost	ΔC	Δt	$\Delta C/\Delta t$
A	8	6000	4	12000	6000	4	1500
B	4	2000	2	14000	12000	2	6000
C	10	4000	4	8000	4000	6	666.67
D	6	4000	4	8000	4000	2	2000

Normal Cost at normal project duration

Normal project duration = 20 days

Direct cost = 6000 + 2000 + 4000 + 4000 = 16000

Indirect cost = 20 × 1000 = 20000

Total cost = 16000 + 20000 = 36000

First stage crashing

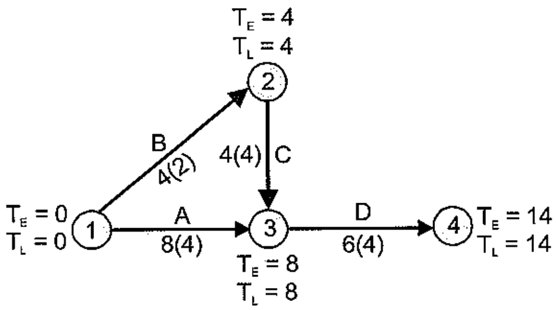
We can observe that among critical activities, activity C has minimum cost slope *i.e.* 666.67 and has crashing potential of 6 days. It can be crashed by 6 day without affecting other parallel activities.

New project duration = 14 days.

Direct cost = 16000 + 6 × 666.67 = 20000

Indirect cost = 14 × 1000 = 14000

Total cost = 20000 + 14000 = 34000



Second stage crashing

We can observe that, now we have 2 critical path A–D and B–C–D. Therefore we have to check various alternatives of combinations of cost slope

(i) C/S of B + C/S of A = 6000 + 1500 = 7500

(ii) C/S of D = 2000

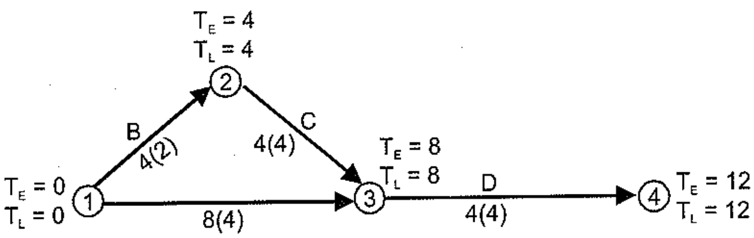
∴ Cost slope of activity D is minimum, therefore it can be crashed for its complete crashing potential that is 2 days.

New project duration = 12 days.

Direct cost = 20000 + 2 × 2000 = 24000

Indirect cost = 12 × 1000 = 12000

Total cost = 24000 + 12000 = 36000



Third Stage Crashing

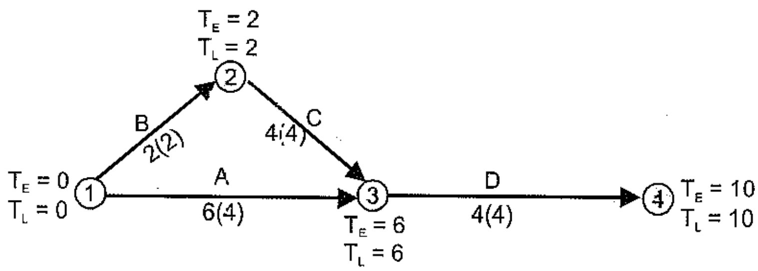
Now we can crash activity A and B simultaneously by 2 days. As these are parallel activity and crashing potential of activity B gets expired.

New project duration = 10 days.

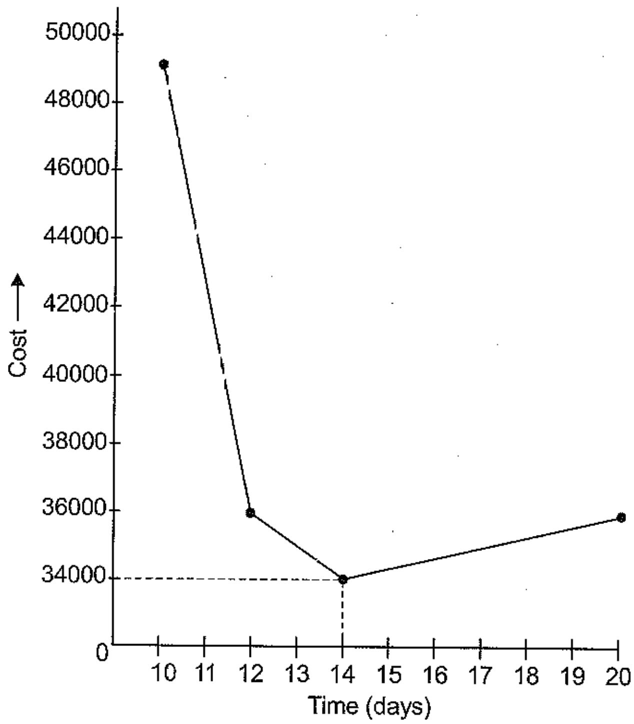
Direct cost = 24000 + 7500 × 2 = 39000

Indirect cost = 10 × 1000 = 10000

Total cost = 39000 + 10000 = 49000



Total Cost Curve



Optimum project duration = 14 days

Optimum project cost = 34000

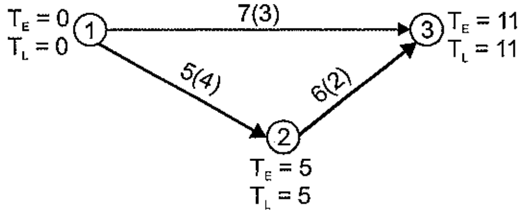
Q–2: ***Calculate optimum cost and optimum duration for jobs of network given in the table below.***

Activity	*Normal Duration Days*	*Normal Cost Rs.*	*Crash Duration days*	*Crash Cost Rs.*
1 – 2	*5*	*4,000*	*4*	*5,000*
1 – 3	*7*	*8,000*	*3*	*10,000*
2 – 3	*6*	*6,000*	*2*	*8,400*

Indirect cost = Rs. 1000 per days. Sketch project time - cost diagram.

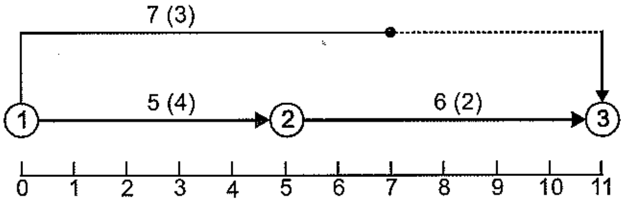
[10 Marks, ESE-2009]

Sol: **Network Diagram**



Critical path is 1 – 2 – 3

Time Scale Version:



Normal duration of project = 11 days.

Cost Slopes

Activity	Normal		Crash		Cost Slope		
	Duration (Weeks)	Cost (Rs.)	Duration (Weeks)	Cost (Rs.)	ΔT	ΔC	$(\Delta C/\Delta T)$
1 – 2	5	4000	4	5000	1	1000	1000
1 – 3	7	8000	3	10,000	4	2000	500
2 – 3	6	6000	2	8,400	4	2400	600

Total Project Cost at Normal Duration

$$\text{Normal Duration} = 11 \text{ days}$$

$$\text{Direct Cost} = 4000 + 8000 + 6000 = 18000$$

$$\text{Indirect Cost} = 11 \times 1000 = 11000$$

$$\text{Total Cost} = 18000 + 11000 = 29000$$

First Stage Crashing

Activity 2 – 3 is critical activity having minimum cost slope among critical activities *i.e.* 600. It has cash potential of 4 days and can be crashed 4 days.

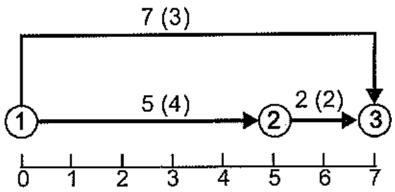
Now, $$\text{new project duration} = 7 \text{ days}$$

$$\text{Direct Cost} = 18000 + 4 \times 600 = 20400$$

$$\text{Indirect Cost} = 7 \times 1000 = 7000$$

$$\text{Total Cost} = 20400 + 7000 = 27400$$

Time Scale Version:



Second Stage Crashing

Now we have two critical path 1 – 2 – 3 and 1 – 3

∴ We have to crash activity 1 – 2 and 1 – 3 simultaneously by 1 day. Note that we can not crash activity 1 – 3 by its full potential because its a parallel activity to activities 1 – 2 and 2 – 3, whose crashing potential is already finished.

Combined cost slope of 1 – 2 & 1 – 3 = 1000 + 500 = 1500

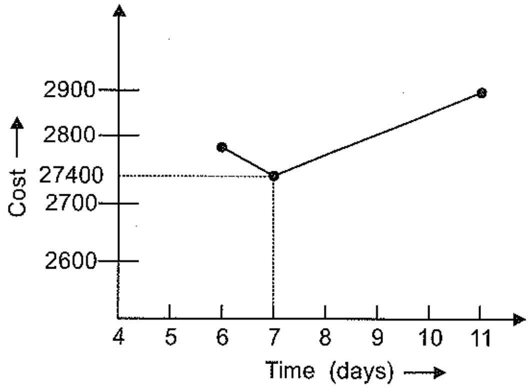
$$\text{New project duration} = 6 \text{ days.}$$

$$\text{Direct Cost} = 20400 + 1500 = 21900$$

$$\text{Indirect Cost} = 6 \times 1000 = 6000$$

$$\text{Total Cost} = 21900 + 6000 = 27900$$

Time V/s Project Cost Curve



$$\text{Optimum project duration} = 7 \text{ days}$$

$$\text{Optimum project cost} = 27400$$

Q-3: ***Determine the minimum cost and optimum, duration for a project network. The duration and cost of each activity of the project is given in the following table. The indirect cost of the project is Rs. 800 per day. Draw the time - scaled version of the network:***

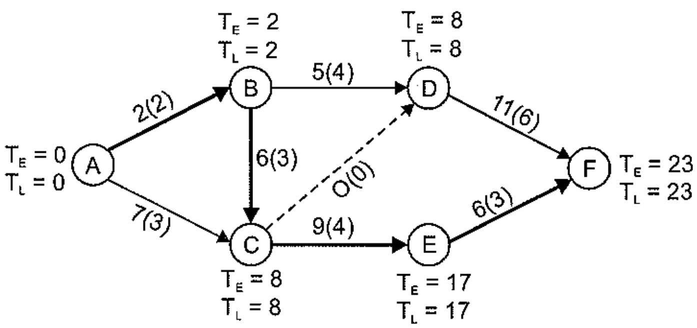
Activity	*Normal duration (in days)*	*Normal cost (in Rs.)*	*Crash duration (in days)*	*Crash cost (in Rs.)*
A – B	*2*	*10,000*	*2*	*10,000*
A – C	*7*	*5,000*	*3*	*9,000*
B – C	*6*	*3,000*	*3*	*4,200*
B – D	*5*	*2,000*	*4*	*2,500*
C – D	*0*	*0*	*0*	*0*
C – E	*9*	*6,000*	*4*	*9,000*
D – F	*11*	*6,000*	*6*	*10,000*
E – F	*6*	*7,000*	*3*	*9,100*

[15 Marks, ESE-2010]

Sol:

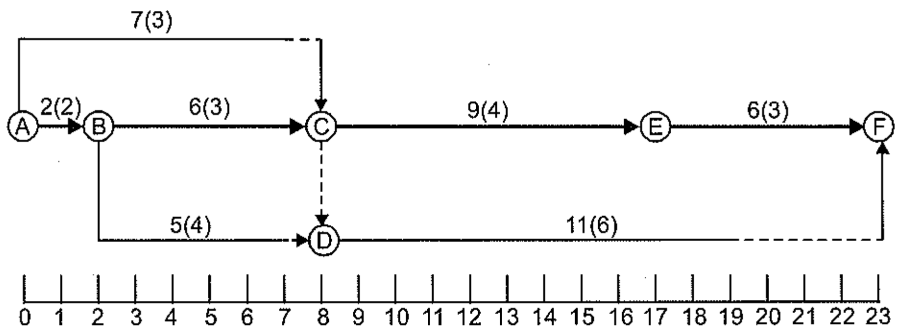
Activity	Normal		Crash		Cost Slope		
	Duration (Days)	Cost (Rs.)	Duration (Days)	Cost (Rs.)	ΔT	ΔC	$(\Delta C/\Delta T)$
A–B	2	10000	2	10000	0	0	0
A–C	7	5000	3	9000	4	4000	1000
B–C	6	3000	3	4200	3	1200	400
B–D	5	2000	4	2500	1	500	500
C–D	0	0	0	0	0	0	0
C–E	9	6000	4	9000	5	3000	600
D–F	11	6000	6	10000	5	4000	800
E–F	6	7000	3	9100	3	2100	700

Network Diagram



Critical path is A-B-C-E-F

Time scale Diagram



Total Project cost at normal duration

Normal Project Duration = 23 days.

Direct cost = 10,000 + 5000 + 3,000 + 2,000 + 0 + 6,000 + 6,000 + 7,000

= 39000

Indirect cost = 23 × 800 = 18400

Total cost = 39000 + 18400 = 57,400

First stage of Crashing

Critical path is A–B–C–E–F & A–B has least C/S but crash duration is zero.

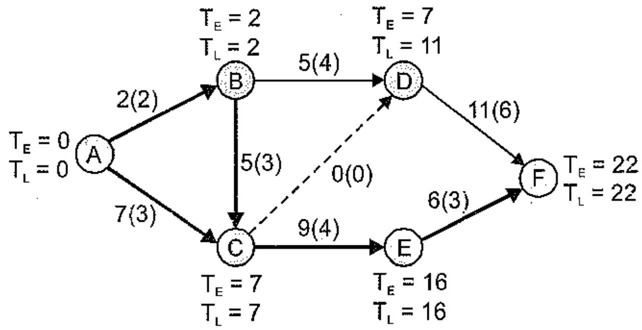
Among B–C, C–E and E–F, B–C has least cost slope and can be crashed by 3 days. But if we crash B–C by 3 days then critical path will get changed to A–C–E–F and will also affect activity B–D. Hence B–C can be crashed by 1 day only.

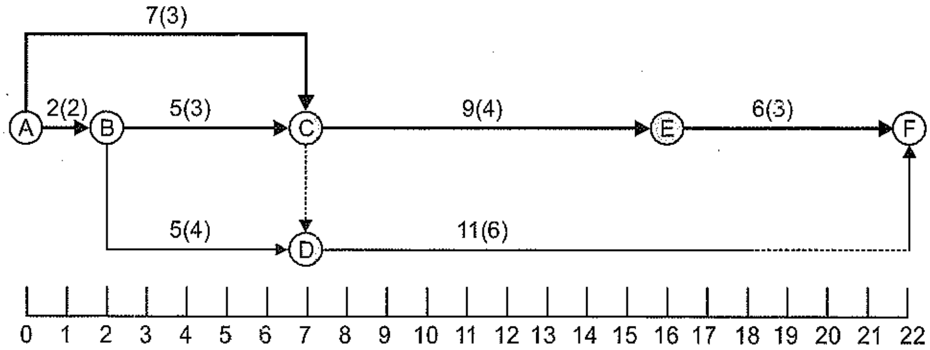
New Project duration = 22 days.

Direct Cost = 39000 + 400 = 39400

Indirect Cost = 22 × 800 = 17600

Total Cost = 39400 + 17600 = 57000





Second stage crashing

Now we have 2 critical path A–C–E–F and A–B–C–E–F. Therefore we will have various combination of crashing alternative we should note that Activity B–D is parallel to Activity A–C and B–C. Therefore it will also come in crashing combination of A–C and B–C.

(i) C/S of BC(+ C/S of AC + C/S of BD = 400 + 1000 + 500 = 1900

(ii) C/S of CE = 600

(iii) C/S of EF = 700

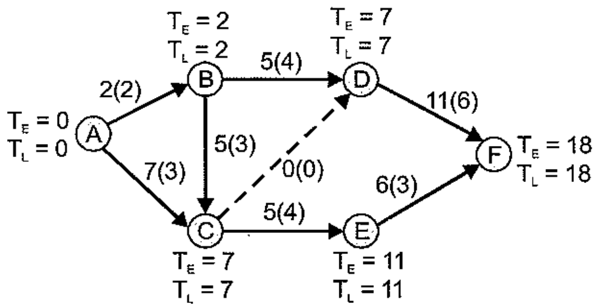
As C/S of CE is least hence we will crash CE by 4 days but it has a crashing potential of 5 days. We are not doing that because than DF which is a parallel activity will get super critical.

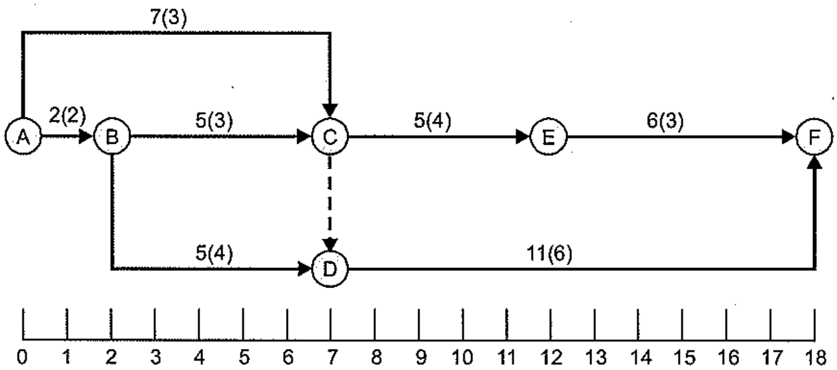
New Project duration = 18 days.

Direct Cost = 39400 + 4 × 600 = 41800

Indirect Cost = 18 × 800 = 14400

Total Cost = 56200





Third Stage Crashing

Now every activity has become critical activity and there are 5 critical path, normally, A–B–C–E–F, A–B–C–D–F, A–B–D–F, A–C–D–F, A–C–E–F

Comparison of various alternatives of crashing for minimum cost slope.

(i) C/S of BC + C/S of AC + C/S of BD = 400 + 1000 + 500 = 1900

(ii) C/S of CE + C/S of DF = 600 + 800 = 1400

(iii) C/S of EF + C/S of DF = 700 + 800 = 1500

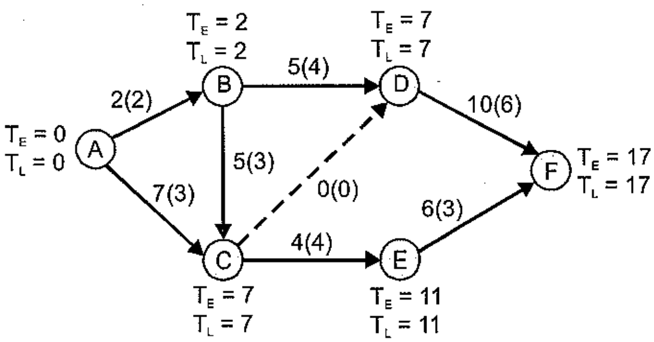
As the cost slope of CE and DF is minimum it can be crashed by 1 day = 1400

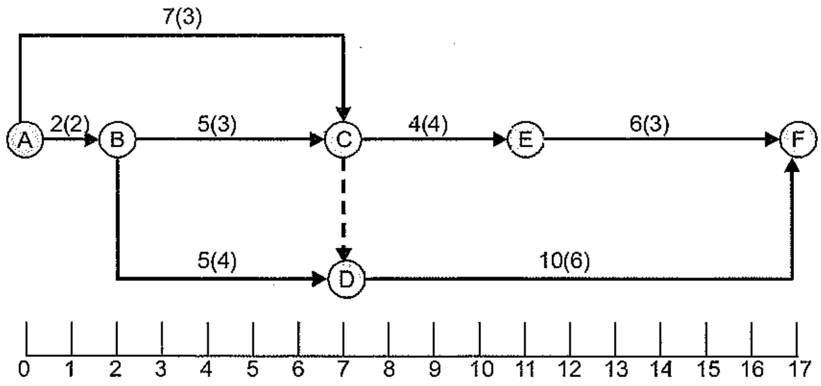
New Project duration = 17 days.

Direct Cost = 41800 + 1400 = 43200

Indirect Cost = 17 × 800 = 13600

Total Cost = 43200 + 13600 = 56800





Stage 4 Crashing

Again, Now comparison of various alternatives of crashing for minimum cost slope.

(i) C/S BC + C/S of AC + C/S of BD = 400 + 1000 + 500 = 1900

(i) C/S EF + C/S of DF = 1500

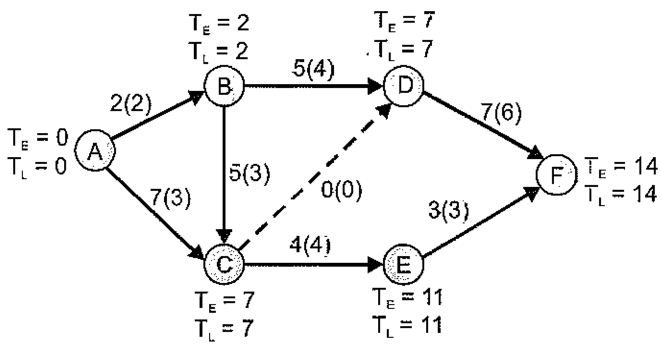
We can observe that C/S of EF and DF is minimum and can be crashed by 3 days.

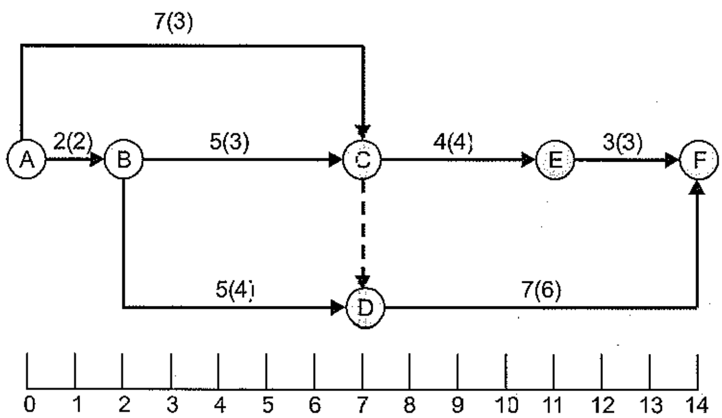
New Project duration = 14 days

Direct Cost = 43200 + 3 × 1500 = 47700

Indirect Cost = 14 × 800 = 11200

Total Cost = 47700 + 11200 = 58900





Stage 5 Crashing

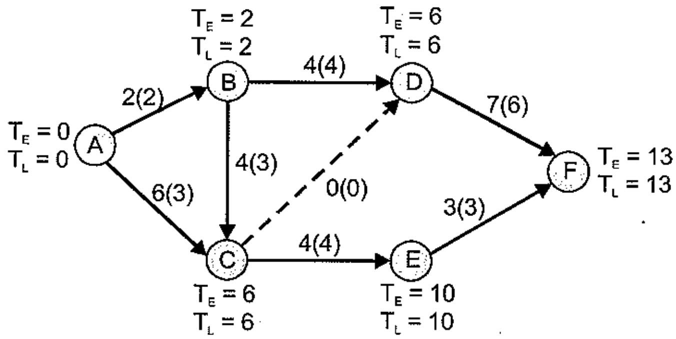
Now we can crash BC, AC and BD simultaneously by 1 day as

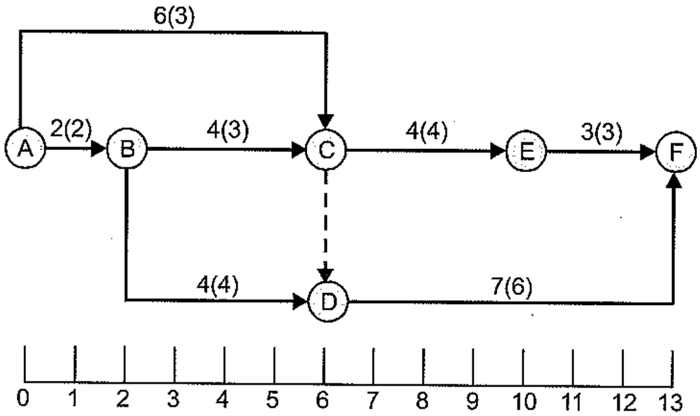
New Project duration = 13 days.

Direct Cost = 47700 + 1 × 1900 = 49600

$$\text{Indirect Cost} = 13 \times 800 = 10400$$

$$\text{Total Cost} = 60000$$





$$\text{Optimum project duration} = 18 \text{ days}$$

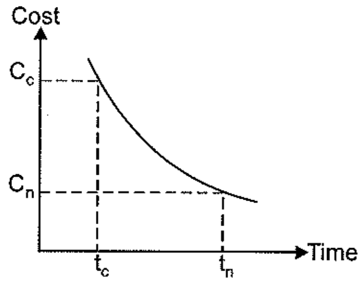
$$\text{Optimum project cost} = 56200$$

Q–4: ***What is direct cost slope?***

[5 Marks, ESE-2015]

Sol: **Direct Cost Slope:**

- $$\text{Direct cost slope} = \frac{C_c - C_n}{t_n - t_c} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$
- It indicates the increase in direct cost when the activity is reduced by one day.
- It is helpful in project cost analysis of direct cost.



Q–5: ***Briefly explain at least five different types of vibrators used in cement concrete making industry.***

[10 Marks, ESE-2019]

Sol: Compaction of concrete by vibration has almost completely revolutionised the concept of concrete technology, making possible the use of low slump stiff mixes for production of high quality concrete with required strength and impermeability.

Internal vibrator: This is also called, "Needle Vibrator", "Immersion vibrator", or "Poker Vibrator". The vibrations are caused by eccentric weights attached to the shaft or the motor or to the rotor of a vibrating element.

Formwork vibrator (external vibrator): Formwork vibrators are used for concreting columns, thin walls or in the casting of precast units. The machine is clamped on to the external wall surface of the formwork. The vibration is given to the formwork so that the concrete in the vicinity of the shutter gets vibrated.

Table Vibrator: This is the special case of formwork vibrator, where the vibrator is clamped to the table or table is mounted on springs which are vibrated transferring the vibration to the table. They are commonly used for vibrating concrete cubes.

Platform Vibrator: Platform vibrator is nothing but a table vibrator, but it is larger in size. This is used in the manufacture of large prefabricated concrete elements such as electric poles, railway sleepers, prefabricated roofing elements etc.

Surface Vibrator: Surface vibrators are sometimes knows as, "Screed Board Vibrators". A small vibrator placed on the screed board gives an effective method of compacting and levelling of thin concrete members, such as floor slabs, roof slabs and road surface.

CHAPTER 6 UPDATING AND RESOURCE ALLOCATION

Q–1: ***Explain Resources Allocation.***

[4 Marks, ESE-2009]

Sol: **Resource Allocation**

Proper allocation and utilization of resources such as manpower money, materials and machinery are essential for completion of project within time and cost. The scheduling and planning should be done considering the distribution of resources throughout the project. Resource allocation should be done in such a way that, there is no variation in resource requirement, requirement of resources never exceeds the supply and no surplus of resources for a single activity.

There are two technique for resource allocation.

(i) **Resource smoothing:** In this procedure total time for the project is not changed but the activities having floats are adjusted to get a uniform demand for resources.

(ii) **Resource levelling:** Overall scheduling here done in such a way that peak demand during project never exceeds the available resources.

Q–2: ***Differentiate resource levelling and resource smoothing.***

[5 Marks, ESE-2011]

Sol:

	Resource Levelling		Resource Smoothing
1.	These techniques are used when sharing of critical resources is required between different activities in a project.	1.	This technique is used to maintain the utilization of project resources at a particular limit.
2.	Resource levelling may cause the critical path to be changed.	2.	It is applied only on activities that have float and thus the critical path is not influenced
3.	Project duration may get changed as it influences the critical path.	3.	Project duration remains unaltered.

Q–3: ***What is Resource Levelling in Construction Project Management and how it is different than Resource Loading?***

[8 Marks, ESE-2018]

Sol: **Resource levelling in construction project management:**

In resources levelling, the activities are rescheduled such that maximum or peak demand of the resource does not exceed the availability of resources.

Underlying assumptions in resource levelling :

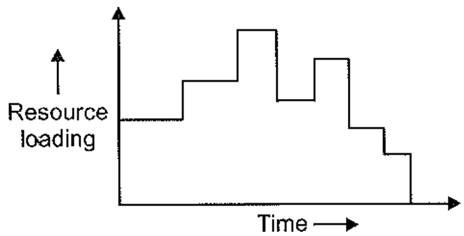
1. Only one type of resource is considered throughout the project.
2. Activities already underway should be completed first.

Important points:

1. Project duration might be changed
2. Resources are limited
3. Critical path may get changed

Resource loading:

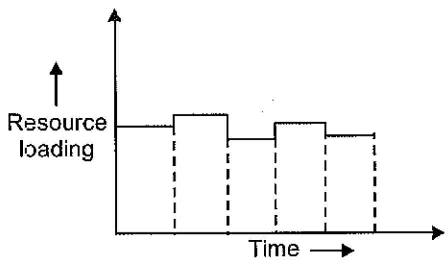
- "Resource loading" refers to the allocation of resources to work elements in a project network.
- A resource loading graph presents a graphical representation of resource allocation over time.
- The graph provides information useful for resource planning and budgeting purposes.



Resource loading graph

Difference of resource levelling from resource loading:

- Resource levelling refers to the process of reducing the period-to-period fluctuations in a resource loading graph.
- If resource fluctuations are beyond acceptable limits, actions can be taken to move activities or resources around in order to level out the resource loading graph.
- It attempts to minimise fluctuations in resource loading by shifting activities within their available slacks.



Resource loading after levelling

- Advantages of resource levelling over resource loading include simplified resource tracking and control, lower cost of resource management, and improved opportunity for learning.

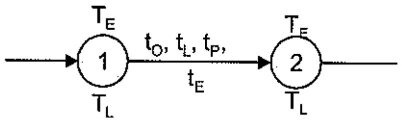
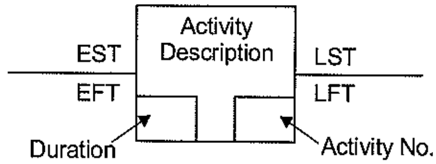
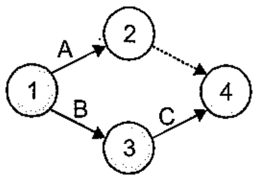
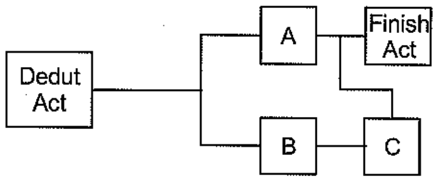
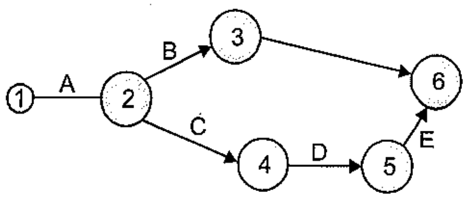
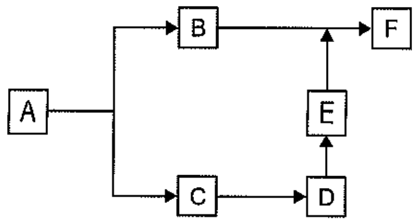
CHAPTER 7

A-O-N DIAGRAM

Q–1: ***With the help of suitable diagrams compare and contrast Activity on the Arrow Networks with Activity on the Node Networks (Precedence Diagrams).***

[10 Marks, ESE-1999]

Sol: Comparison between activity on arrow Networks with activity on the node networks.

AOA Network	AON Network
1. Activity is represented by a arrow and the various calculated duration of the activity is mentioned on it. These arrows connect various nodes representing various events.	1. An activity is represented by a node and various nodes are connected by simple logical relationships, hence no place for events.
2. To evaluate project duration critical path diagram is event dependable. 	2. AON networks are self explanatory as all the activity duration can be shown on the node itself. 
3. To establish logical relationship, we are presence of dummy activities with zero duration sometime. 	3. To establish multiple activities start from or finish to a single pont we introduce debut activity or finish activity of zero duration. 
4. Chronology of activities is less clear.	4. It can show activities which should be allowed to overlap, pre-post operation activities are clearly depicted.
5. For PERT analysis, AOA network is preferable. e.g. 	5. For CPM analysis AON network is preferable e.g. 

Q–2: ***What are the main advantages of Activity on the Node Networks over Activity on the Arrow Networks?***

[15 Marks, ESE-1999]

Sol: Advantages of activity on the Node Networks over Activity on arrow networks:

(i) It is self-sufficient and self-explanatory as all activity times are shown on network itself.

(ii) It eliminates use of dummies.

(iii) It can show activities which should be allowed to overlap each other or must be separated by a time delay.

(iv) Revision and modification can be carried out easily without affecting most of the activities.

(v) Pre and post operation activities are clearly depicted.

Q–3: ***What are the main advantages of Activity on the Node (A-O-N) networks over activity on the Arrow (A-O-A) networks?***

[8 Marks, ESE-2016]

Sol: Followings are the main advantages of activity on the Node (A-O-N) networks over activity on the arrow (A-O-A) networks:

(i) AON networks are more explanatory than AOA networks for the details of the activities. The earliest start time, latest start time, earliest finish time, latest finish time, duration and description of activities are clearly represented in AON networks.

(ii) In AON network the inter dependency between various activities is shown by logical relationship between various activities rather than between events as in case of AON diagram.

Hence, use of dummy activities is eliminated by adopting AON diagram.

(iii) For activity oriented projects like critical path method, AON is more suitable over AOA network.

(iv) The revision and modification in the network diagram is easier in AON network as compared to AOA network.

(v) For in experienced user, AON diagram is easier to understand than AOA diagram.

CHAPTER 8
TENDERING PROCESS AND COST ESTIMATION

Q–1: ***Safety precautions to be taken at a construction site are related to the nature of the construction activity. Discuss this statement with appropriate examples.***

[10 Marks, ESE-2013]

Sol: Safety precautions to be taken at construction site are related to the nature of the construction activity. We can summarize it as below :

General Safety Precautions

(i) Train all personal at worksite safety and operating procedure either on-site or at a training facility.

(ii) In general a safety talk or safety oath at the beginning of work is a little helpful for common workers.

(iii) A thorough walk through of the site is mandatory.

Safety precautions during work at height

(iv) Use harnesses and other safety equipment when performing roof work or working on scaffolds.

(v) Above a threshold height according to the site condition (e.g. 6 feet for general building work), safety harnesses should be compulsory.

(vi) In case of building construction safety net below to working level is mediatory to rest falling materials.

Safety precaution during excavation

(vii) A proper slope or stepping should be maintained to resist landslide.

(viii) Barricading should be done around each excavation site.

Special

(ix) Identify and mark any hazardous material

(x) All equipment should be checked, whether they are working properly or not.

(xi) Personal protective equipment should be provided to all the employees according to work.

Q–2: ***Explain briefly the highlighted terms in the following sentence. (not more than 20 words). The contractor and the client agreed to send the matter for arbitration.***

OR

The contractor was informed that he plea of force majeure for the delay in the completion was not acceptable. ***[5 Marks, ESE-2013]***

Sol: **Arbitration:** Arbitration utilizer a neutral third party to hear a dispute between parties. The hearing is informal and the parties mutually select the arbitrator. The arbitrator is retained to decide how to settle the dispute and the decision is final and binding on the parties. Here the arbitrator, not the parties, who render the terms and conditions of the dispute resolution.

Force Majeure: The term 'force majeure' relates to the law of insurance and is frequently used in construction contracts to protect the parties in the event that a segment of the contract cannot be performed due to causes that are outside the control of the parties, such as natural disasters, that could not be evaded through the exercise of due care.

Q-3: What is the difference between "security deposit" and "mobilization advance" in a construction contract?

[4 Marks, ESE-2014]

Sol: **Security deposit:** Security deposit is the sum of money held in trust with the department. It is generally 10% of the tender amount.

The purpose of security deposit is to guarantee that the contractor will:

- Perform and complete the work according to the contract requirement
- Discharge lawful obligations and satisfy lawful claims against the contractor.
- ensure that sub-contractors discharge their lawful obligations and satisfy lawful claims against them.

Mobilization Advance

For the activation of contractor's physical and manpower resources transfer to construction site until the completion of the project in civil construction project advance is given to contractor known as mobilization advance. It is normally restricted to 10–15% of the contractor value.

The prerequisite for the issue of advance is that the contractor has to provide a guarantee in the shape of bank or insurance equal to the amount being issued to the contractor

Mobilization advance is deducted from the bills of the contractor in equal installments covering the project period, on completion of recovery guarantee provided by the contractor is released.

Q-4: Details of a construction project comprising of three activities are given in the following table:

S.No.	*Activity*	*Unit*	*Estimated quanity*	*Estimated rate per unit*	*Rate of award*
1	A	M^3	5000	1000	850
2	B	MT	4500	40000	4200
3	C	M^2	7000	5000	4800

Based on the information provided in the table, answer the following questions:

(i) What should be the cost of the project for which an "approval" is obtained from the competent authority before proceeding with the advertisement for the job etc.?

(ii) If at a certain point in time, the work done for the actities A, B and C is 2700, 3000 and 4000 in the corresponding units, what is the percentage of the finacial completion of the project?

(iii) Clearly state the assumptions in calculating (ii) above

[8 Marks, ESE-2014]

Sol: (i) Total cost of the project = cost required for the completion of activity (A + B + C)

$$= 5000 \times 850 + 4500 \times 4200 + 7000 \times 4800$$

$$\therefore \quad \text{Total cost of project} = 56{,}750{,}000$$

(ii) At a certain point of time,

$$\text{Total expenditure on the project} = \text{Work done in activities } (A + B + C)$$
$$= 2700 \times 850 + 3000 \times 4200 + 4000 \times 4800$$
$$= 34{,}095{,}000$$

$$\therefore \quad \text{\% of financial completion} = \frac{34{,}095}{56{,}750} \times 100 = 60.079\%$$

(iii) Percentage of completion of project is based on the total expenditure on the project till date i.e., percentage completion $\propto$ expenditure

$$\text{\% of financial completion} = \frac{\text{Expenditure on project till date}}{\text{Total cost of the project}}$$

Q–5: ***Define the following terms briefly in the context of construction contracts:***

(i) EPS contract

(ii) PPP

(iii) Escalation

[10 Marks, ESE-2015]

Sol: **(i) EPC contract:**

- Full form of word EPC is engineering, Procurement and construction.
- EPC contract will carry out the detailed engineering design of the project, procure all the equipments and materials necessary and then construct to deliver a functioning facility or asset to their clients as prescribed by them with in the agreed period of time.

(ii) PPP:

- Full form of word PPP is public private partnership.
- PPP involves a contract between a public sector authority and a private party in which the private party provides a public service or project and assumes substantial financial, technical and operational risk in the project.

(iii) Escalation:

An increase in the cost of performing construction work, resulting from performing the work in a later period of time and at a cost higher than originally anticipated in the bid.

Q-6: ***Explain the different types of contracts adopted in construction.***

[20 Marks, ESE-2019]

Sol. **Types of contract:** Various methods on alternatives that are used in the construction industry either in part or in whole consist of different approaches to contracting for services or production or different set-ups created. This is due to different relationships existing between the owner/ultimate user, or occupant and contractor. Depending upon the size and complex nature of the job, these methods may be modified to suit the requirements. Following are the different types of contracts adopted in construction:

(a) **Lump-sum contract:** This is a traditional method in which a construction project is implemented. The owner, having retained an Architect/Engineer, has a set of definitive documents consisting of design plans and specifications prepared, defining the scope of work required.

The definitiveness of lump-sum contract makes it the established method of contracting in many instances the only method utilised, because of the statutory requirements.

(b) **Cost plus fixed fee contract:** All costs within predetermined yardsticks or in accordance with specific regulations are reimbursed by the owner to the contractor; apart from this contactor is paid a fixed sum which represents profit (his fee).

(c) **Cost plus bid fee contract:** In this method also, the costs of construction are to be reimbursed as incurred within predetermined yardsticks. The fee or profit with one contractor, an owner will issue a request for proposals to a select number of contractors, all of whom he believes are in an equal or at least similar position build the required facility, past experience and performance in the evaluation of the contractors from whom the successful one is selected.

(d) **Guaranteed maximum contract:** This method is often used as either a direct substitute a lump-sum or cost-plus or modification of either contracting system. No owner wishes in a position to issue a blank check for building a new facility; all new buildings, improvements, maintenance or repairs are accomplished within pre-established budgets.

(e) **Negotiated contract (competitive and noncompetitive):** There are many elements common to cost plus method and this traditional method of contracting, but the negotiated contract is often classified separately. The end result of the negotiated contract can be any form of contracting previously described, lump-sum or cost-plus. It is, however, dependent upon plan development status or specific owner or project requirements.

(f) **Unit-price contract:** This type of contract is based on estimated quantities of the items involved in the work. The cost per unit of each item is bid by the contractor and the estimated quantities of these items are given by the owner.

(g) **Design build:** As the term implies, this approach establishes a single administrative, management and professional responsibility for the two separate functions of design and construction. The owner enters into one agreement for both. The method of contracting can be any of the traditional methods or modifications previously described.

(h) **Turn-key contracts:** Turn key construction utilises a single contract for all functions. There is one administrative, management and professional responsibility for design and construction. There is a single party under contract to an owner to fulfil these functions in addition to other functions that may be necessary to implement a project.

CHAPTER 9

ENGINEERING ECONOMY

Q–1: ***Explain the various methods of comparison of civil engineering project alternatives.***

[10 Marks, ESE-2011]

Sol: **Comparison of Alternatives**

- Construction management often involves cost comparisons between alternatives of different engineering efficiency, for example, one with high initial cost and low operation and maintenance cost, compared with a low initial cost but high operating and maintenance cost.
- Using cash-flow diagrams and time value of money, equivalence is studied to identify the better alternative, using a common basis.
- The most common basis of comparison are:
  - (a) The Present worth amount
  - (b) The Annual equivalent amount
  - (c) The Capitalized amount.
  - (d) The rate of Return method.

Present Worth Amount

In this method, present worth of the cash flow in the form of equivalent single sum is calculated using an interest rate, sometimes also known as discounting rate.

This method is based on following condition:

1. Cash flow is known
2. Interest rate is known.

There can be two cases:

Case I: Alternative with equal service life

- In this case alternative with lower present worth is accepted as economical.

Case II - Alternative with unequal service life.

- In this case a common life period is taken which is L.C.M of the given values. Then overall present worth of alternatives are obtained and then compared for the selection of economical alternative.

Annual Equivalent Amount

- This is most widely used for comparison of alternatives because the computations are easy to carry out.
- In this method, cash flow is converted into a series of equal amount by at first calculating the present worth amount and then multiplying it with Capital Recovery factor.
- By observing the annual equivalent amount of each alternative comparison is done.

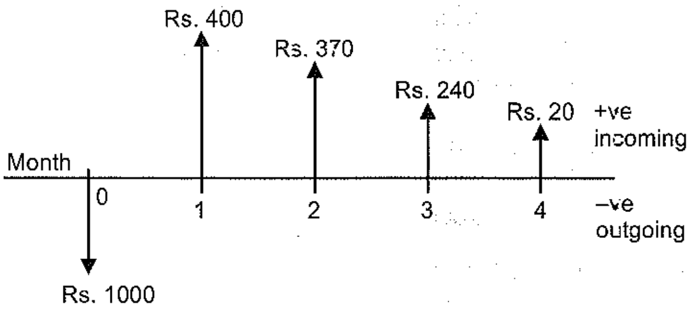
The Capitalized Amount

- This method is very useful to compare long – term projects.
- In this method, the annual equivalent amount obtained is assumed to extend for infinitely long period and then Capitalized amount is obtained as $\frac{'A'}{i}$.

Note: Annual equivalent amount is calculated for replacement and maintenance cost and added to the initial cost of alternative.

Rate of return method

- It represents the interest rate which reduces the present worth amount of a series of receipts and disbursements to zero for each alternative.
- In terms of economics, rate of return is interest earned on the unrecovered balance of an investment such that unrecovered balance is zero at the end of investment life.
- An illustrative computation showing how a part of the cash flow at the end of a year goes towards payment of interest due on the outstanding (unrecovered) balance of investment, and the remainder liquidates the outstanding investment, is shown in Table below.



Cash-flow diagram to illustrate the IRR

At the end of the proposals life (four years), the entire investment has just been recovered, and the applicable rate of interest (10 per cent) is a special and unique rate is called as Rate of return.

End of year t	Cash-flow at EOY,t	Unrecovered balance at the beginning of year t	Interest earned on the unrecovered balance during the year	Unrecovered balance at the beginning of the year (t+1)
0	–1,000	–	–	–1,000
1	400	–1, 000	–100	–700
2	370	–700	–70	–400
3	240	–400	–40	–200
4	220	–200	–20	0

IRR = 10%

Q–2: ***Briefly explain five important factors which should be considered for selection of equipment for a construction project.***

[10 Marks, ESE-2017]

Sol: Various important factors which should be considered for selection of equipment for a construction project are:

(1) **Specific construction operation:**

It is the first factor that must be consider in selection of any equipment. The equipment selected must be capable of doing the work specified. Problems arises in selecting equipment because of variety of machines that can accomplish a task. ex. earth can be excavated by power shovel, hoe, dragline etc.

Trenching can be done by dragline, tractor-scrapers etc. The selected equipment must satisfy constraints imposed by the contract documents and job conditions

(2) **Conditions at job site:** When there is working space limitation, the operating dimension of equipment must be consider in order to ensure that whether there are adequate clearences, reaches and so on.

The available manoeuvring room influences the earth moving operation. If there is limited space, smaller haul units may be used.

(3) **Location of job site:** Location of the job site influences the selection of equipment in several ways. Climatic or logistics conditions may vary at job site.

Weather condition: Temperature, precipitation and wind all affect the performance of construction equipment and ability of the operator to sum his equipment efficiency.

For example. The volumetric and mechanical effieincies of internal combustion engine are affected by temperature when temperature is higher than 100°F, air is less dense than normal and has to be compressed to obtain needed fuel to air-mixture.

(4) **Versatatity and adaptbility of equipment:** This factor must be considered when there are a number of operations requiring similar equipments. If these operations are all for one project, some equipment may be able to work on various operations.

A versatile piece of equipment is one that can be used for several construction operations.

(5) **Economy:** Economy is one of the most important factor in consideration of these equipment. The cost of production, depreciation fuel all combinedly used to predict the furture profit.

(6) **Project timeline considerations:** Project deadlines also affect the selection of equipment. If there is limited time available to complete a project, then companies may prefer highly advanced construction equipment that can reduce a project's completion time significantly.

(7) **Labour considerations:** This also highly affects the selection decision. If there is a shortage of manpower at the jobsites, then the companies may opt for highly automated machines. Further, the selection of construction equipment then the company may or may not opt highly sophisticated equipment.

(8) **Safety considerations:** Any construction site is the locus of multiple high risk activities. There are obvious safety concerns associated with workers operating on the ground, particularly within confined spaces when heavy materials are being moved around. Hence, in such cases, companies may have to select equipment which ensures safety of the workers. Thus, safety considerations also affect the selection of the equipment.

(9) **Replacement Parts:** Prior to purchasing equipment, the buyer should determine where spare parts are obtainable; otherwise the project may be delayed. If parts are not obtainable quickly, it may be wise to purchase other equipment, for which parts are quickly available, even though the latter seems less desirable.

CHAPTER 10

FUNDAMENTAL OF EQUIPMENTS

Q–1: ***Calculate the time required to grade and finish 30 km of road formation of 9.0 m width for two-lane road with motor-grader having width of 3.0 m, using six passes with speed for each of the successive two passes as 5 kmph, 7 kmph and 9 kmph respectively. Assume machine efficiency based on operator skill, machine characteristics and working conditions as 80%.***

[6 Marks, ESE-2012]

Sol:

$$\text{Total area} = 30 \times 9 \times 10^{-3} = 0.27\,\text{km}^2$$

$$\text{Output of grader} = \text{Speed} \times \text{grader width (km}^2\text{ / hour)}$$

Time required in the first two pass

$$t_1 = \frac{\text{Total Area}}{\text{Output}} \times 2$$

$$t_2 = \frac{0.27}{5 \times 3 \times 10^{-3}} \times 2 = 36 \text{ hours}$$

Time required in the consecutive next two pass

$$t_2 = \frac{0.27}{7 \times 3 \times 10^{-3}} \times 2 = 25.71 \text{ hours}$$

Time required in the last two pass

$$t_3 = \frac{0.27}{9 \times 3 \times 10^{-3}} \times 2 = 20 \text{ hours}$$

$\therefore$ Total time required considering 80% efficiency

$$= \frac{1}{0.8}(t_1 + t_2 + t_3) = 102.14 \text{ hours}$$

Q–2: ***The operating time per hour for a bulldozer working in a construction project is 55 minutes to push sandy loam top soil, having swell factor as 1.24, for a hauling distance of 45m. What will be the output of the bulldozer per hour? Assume the forward speed and the reverse speed of the bulldozer as 2.7 kmph and 5.4 kmph respectively. Rated mold board capacity in loose volume may be assumed as 3.6 cum. The gear shifting time of 0.2 minute may be assumed.***

[10 Marks, ESE-2017]

Sol. As given

Operating time of buldozer = 55 minutes

Sandy, loam, swell factor = 1.24

Hauling distance = 45m

Forward speed = 2.7 km/hr

Reverse speed = 5.4 km/hr

Rated mold board capacity in loose volume = 3.6 cum

Gear shifting time = 0.2 minutes

$$\text{Equivalent mold board capacity} = \frac{\text{Rated mold board capacity}}{\text{swell factor}} = \frac{3.6}{1.24} = 2.903 \text{ m}^3$$

Time taken to cover forward hauling distance of 45 m

$$= \frac{45}{2.7 \times \frac{5}{18}} = 60 \text{ sec or } 1 \text{ min}$$

$$\text{Time taken to cover reverse hauling distance of 45 m} = \frac{45}{5.4 \times \frac{5}{18}} \text{ sec} = 30 \text{ sec or } 0.5 \text{ min}$$

Total time taken for one trip = 1 + 0.5 + 0.2 = 1.7 min (Gear shifting speed = 0.2 min)

$$\therefore \quad \text{No. of trips in one hour} = \frac{\text{operating time}}{\text{Time for one trip}} = \frac{55}{1.7} = 32.353$$

∴ Net output of bulldozer per hour

= No. of trips per hour × equivalent mold board capacity.

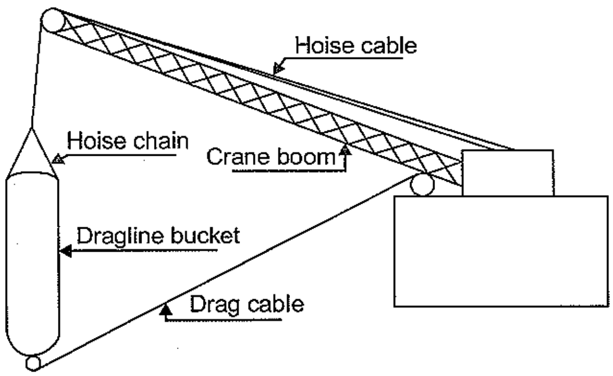
= 32.353 × 2.903 = 93.92 m^3/hr

Q–3: ***Write in brief the principles of Dragline and Clamshell used as excavation equipments, the detail of their components and neat sketches showing their parts. How both the equipments can be compared?***

[20 Marks, ESE-2018]

Sol: **Principles of drag line:**

- As the basic character of the machine is dragging the bucket against the material to be excavated.
- Excavation is started by swinging the empty bucket to the digging position at the same time loosen the drag and the hoist cables.
- Excavation is done by pulling the bucket toward the machine while maintaining tension in the hoist cable.
- When the bucket is filled, the operator takes in the hoist cable while playing out the drag cable.
- Dumping is done by releasing the drag cable.
- Filling the bucket, hoisting, swinging and dumping of the loaded bucket, followed in that order, constitute one cycle.

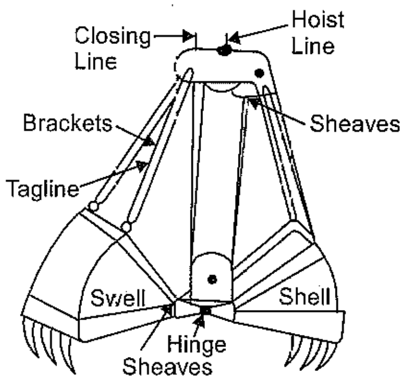


- Size of dragline is expressed by the size of its bucket.

Note: Since, it is difficult to control the accuracy in dumping from a dragline, a larger capacity of haul units is desirable to reduce the spillage.

Principles of Clamshells:

- Clamshell is machine having characteristics of crane and dragline.
- Digging is done like dragline and when bucket is full it works like a crane.
- It has bucket divided into two halves which are hinged at top.
- These are especially suited to vertical lifting of materials from one location to another.
- The limit of vertical movement maybe relatively large when they are used with long crane booms.
- Buckets have teeth, that can be easily removed also. Teeth are used in digging harder types of materials but are not required when a bucket is handling materials.
- The capacity of a clamshell bucket is usually given in m^3.



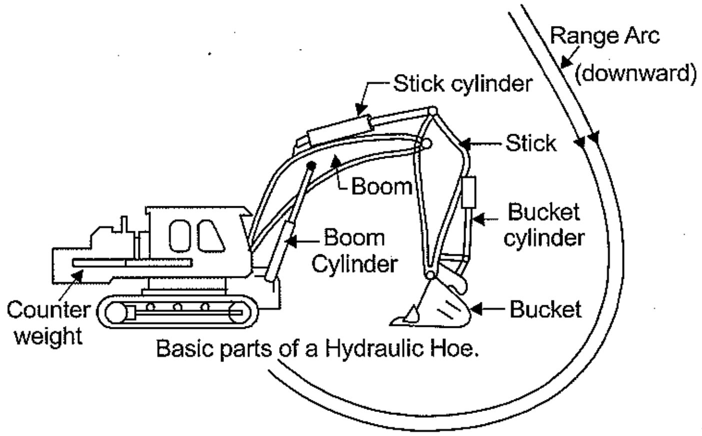
CHAPTER 11

EXCAVATION EQUIPMENT

Q–1: ***List out the basic parts and operations of a Hoe and state its applications :***

[10 Marks, ESE-2001]

Sol: Hoe



Basic parts of a Hydraulic Hoe.

Operation

- Hoes are used primarily to excavate below the natural surface of the ground on which the machine rests. Because of their positive bucket control, they are superior to draglines in overrating on close-range work and loading into haul units.
- Penetration force into the material being excavated is achieved by the stick and bucket cylinder. Maximum crowd force is developed when the stick cylinder operates perpendicular to the stick.
- To break material loose is best at the bottom of the range are because of the geometry of the boom, stick and bucket.

Applications

- Basic application is excavation and loading into haul units.
- By changing attachments it can be used as multi purpose tool platform as rock drillers, earth clearer, grappler, land clearer, impact hammer, demolition jaw, vibratory plate compactor etc.

Q–2: ***List out the basic parts and explain operation of power shovel with a neat sketch.***

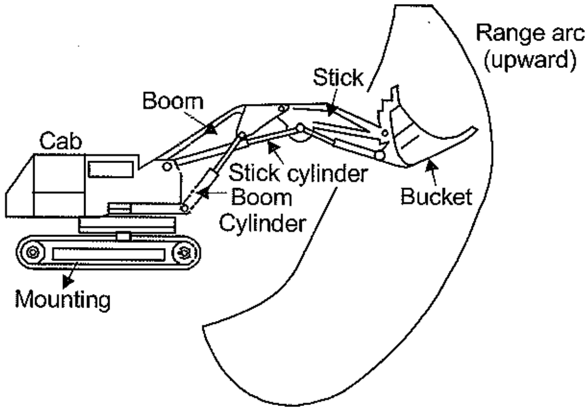
OR

Describe a basic shovel by means of a simple diagram and name the main parts. Also indicate its suitability in field application.

[10 Marks, ESE-2004]

Sol:

- The basic parts of a power shovel include the mounting cab, boom, stick and bucket.
- With a shovel in the correct position, near the face of the material to be excavated, the bucket is lowered to the floor of the pit with the teeth pointing into the face.
- A crowding force is applied by hydraulic pressure to the stick cylinder at the same time the bucket cylinder rotates the bucket through the face.
- Thus, boom cylinder is for lift, stick cylinder is for penetrating and bucket cylinder is to rotate the bucket.



Q–3: ***Briefly explain the different types of bulldozers according to their uses.***

[6 Marks, ESE-2008]

Sol: **1. Position of angles**

(a) Bulldozers- In these blade is set perpendicular to the direction of movement. It pushes the earth forward and dump to some place

(b) Angle Dozers- In these blade is set at an angle with the direction of movement. It pushes the earth forward and to one side.

2. Based on mounting

(a) Wheel mounted

(b) Crawler mounted

Advantages of the crawler-mounted bulldozer

(a) ability to deliver greater tractive effort on soft, loose or muddy soil

(b) ability to travel on muddy surfaces

(c) ability to operate in rock formations, where rubber tyres may get damaged, which may reduce the cost of maintaining haul roads

(d) greater flotation because of lower pressures under the tracks

(e) greater use-versatility on jobs.

Advantages of the wheel-mounted bulldozers:

(a) higher travel speeds on the job or from one job to another

(b) elimination of hauling equipment for transporting the bulldozer to the site

(c) greater output, especially when significant travelling is required

(d) less operator fatigue

(e) ability to travel on bitumen roads without damaging the surface.

3. Based on control- for raising and lowering the blade

(a) Cable controlled

(b) Hydraulically controlled

Advantages of the Cable controlled bulldozers

(a) Simple to install, operate and control

(b) Easy in repairing

(c) Reduction in the danger of damaging a machine

Advantages of the Hydraulically controlled bulldozers

(a) Able produces a high down pressure on blades to force blades into ground

(b) Able to maintain a precise setting of the position of the blade.

Q–4: ***A $\frac{1}{2}$ cubic meter short boom dragline having ideal output of 150 m³/hr. is to be used to excavate hard tough clay the effect of the depth of cut of 5.0 m and angle of swing of 120° shall be 0.89. The operating factor shall be 50 min/hr. Determine the probable output of dragline.***

[4 Marks, ESE-2010]

Sol:

$$\text{Average production} = 0.89 \times 150 = 133.5 \text{ m}^3/\text{hr}$$

$$\text{Probable output} = \frac{50}{60} \times 133.5 = 111.25 \text{ m}^3/\text{hr}$$

Q–5: ***Indicate the performance of power shovel, backhoe dragline and clam-shell for the following conditions in terms of very good, good, fair or poor :***

(i) Excavation in hard soil or rock

(ii) Excavation in wet soil or mud

(iii) Loading efficiency

[6 Marks, ESE-2010]

Sol: **Comparison Between Different Types of Excavating Equipment**

Sl.No.	Items of Comparison	Power Shovel	Back Hoe	Drag line	Clam Shell
1.	Excavation in hard soil or rock	Good	Good	Not Good	Poor
2.	Excavation in wet soil or mud	Poor	Poor	Moderately good	Moderately good
3.	Distance between footing and digging	Small	Small	long	long
4.	Loading Efficiency	Very good	Good	Moderately good	Precise but slow
5.	Footing required	Close to work	Close to pit	Fairly away from pit	Fairly away frompit
6.	Digging level	Digs at or above footing level	Digs below footing level	Digs below footing level	Digs at or below footing level
7.	Cycle time	Short	Sightly more than power shovel	More than the power shovel	More than the other equipment.

Q–6: ***List the different types of equipments used for excavation work.***

[15 Marks, ESE-2011]

Sol. Different Types of Equipments used for excavation work:

(i) Power shovels
(ii) Draglines
(iii) Hoes
(iv) Clamshells
(v) Bulldozers
(vi) Scrapers

Q–7: ***Name various types of Earth Excavating Equipments and give their corresponding digging depth.***

[4 Marks, ESE-2012]

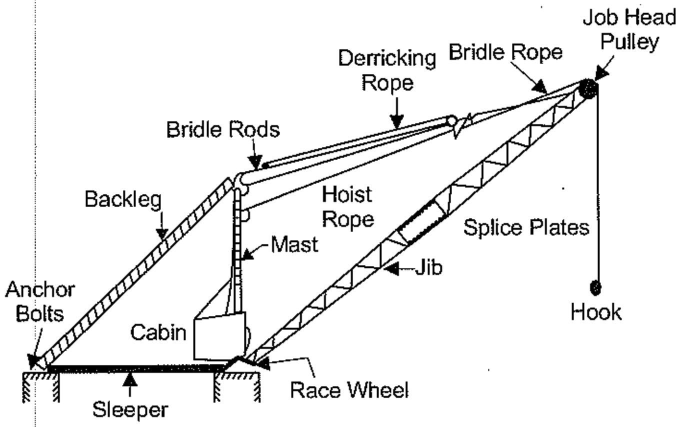
Sol:

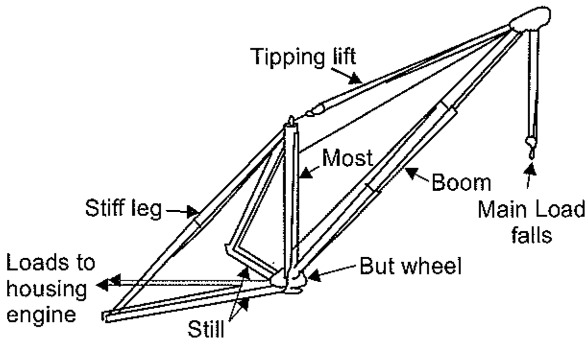
Earth excavating Development	Digging depth in meter
Power shovel	10 – 20 ft.
Hoe	15 – 30 ft.
Dragline	20 – 35 ft.
Clamshell	20 – 35 ft.
Trenchers	Variable depth
Bulldozer	3 – 5 ft.

CHAPTER 12 LOADING AND CONVEYING EQUIPMENT

Q–1: ***Explain the Derrick crane with a neat sketch.*** ***[4 Marks, ESE-2009]***

Sol: Derrick Cranes





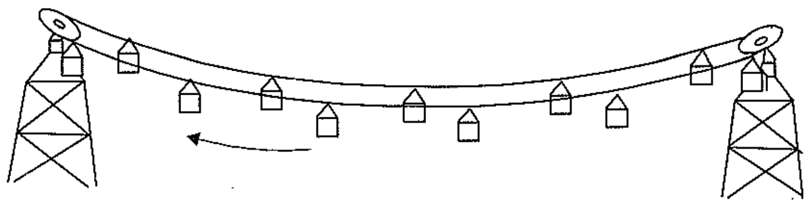
- Derrick crane consist of a mast, a boom and a race wheel on which the boom rotates about a vertical axis.
- These cranes are either electrically operated , diesel operated or diesel- electrically operated.
- Power driven cranes are of mainly two types
  (a) **Guy derrick:** It is used for heavy loads up to 200 tons with 360° rotation.
  (b) **Stiff leg derrick:** It is used for load from 7 to 50 tons with 270°-290° rotation.

Q–2: ***Describe the method of conveying materials with belt conveyors. List the components. State the advantages.***

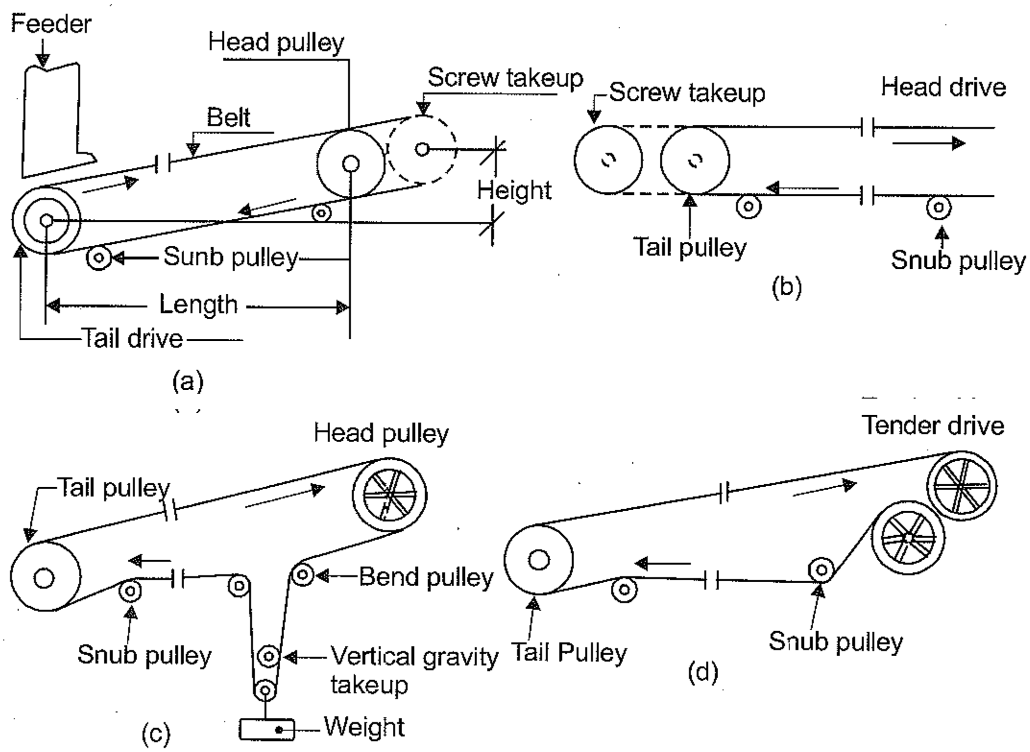
[5 Marks, ESE-2011]

Sol: **Belt Conveyors**

- Conveyor is an equipment which is capable of carrying material in a continuous stream usually having as its special feature - same kind of an endless chain or belt which by its motion constitutes continuous carrying arrangement of the conveyor.
- Conveying may involve a horizontal, inclined or vertical movement. For example aerial transportation through ropeway or tramways.



- When the movement is along horizontal or inclined direction it is called as Conveyor , whereas if it is vertical movement it known as Elevator.
- Belt-conveyor systems are used extensively in construction, where they provide the most economical method of handling and transporting materials like, earth, sand, gravel, crushed stone, concrete, etc.
- Because of the continuous flow of materials at relatively high speeds, belt-conveyors have high capacities.
- The essential parts of a belt-conveyor system include a continuous belt, idlers, a driving unit, driving and tail pulleys, take-up equipment, and a supporting structure.



- A conveyor for transporting materials a short distance may be a portable unit or a fixed installation.
- When a belt-conveyor system is used to transport materials over a considerable distance (up to several kilometers), the system should consist of a number of different flights, as there is a limit to the maximum length of the belt.
- A flight is a complete conveyor unit which discharges its load onto the tail end of the succeeding unit.
- These systems can operate over any terrain provided that slopes do not exceed the limit over which the given material has to be transported.
- Belts are made by joining several layers of woven cotton. These layers are covered with an adhesive which combines them into a unified structure.
- The top and bottom surfaces of a belt are covered with rubber to protect the belt from abrasion and injury from the impact at loading. Various thicknesses of covers may be specified.
- Special types of reinforcement, such as rayon, nylon, and steel cables, are used sometimes to increase the strength of a belt.
- It is necessary to select a belt that is wide enough to transport the material at the required rate.

Power for Belt Conveyor

Power required to drive a loaded belt conveyor is the sum of powers required for the following movement.

(a) Move the load horizontally

(b) Lift the load or to lower the load vertically

(c) Turn all pulleys

(d) Move the empty belt over the idlers

(e) Compensate for driving losses

(f) Operate a tripper, if one is used.

Advantages of belt-conveyor

The principal advantages of a belt conveyer system over other means of haulage are as follows;

(i) Continuous and uniform haulage (ii) Low maintenance cost

(iii) Labour requirement (iv) Ability to cross adverse terrain

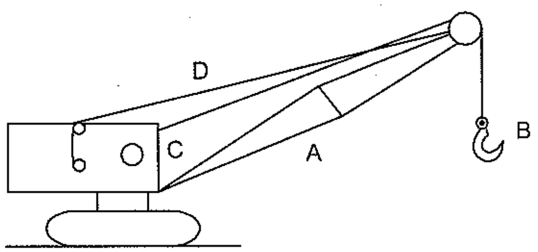
(v) High reliability (vi) Excellent safety.

Q-3: ***What is a crane? How is it used in construction industry? Briefly explain three different types of cranes that are being used in construction works.***

[20 Marks, ESE-2020]

Sol: **Crane**

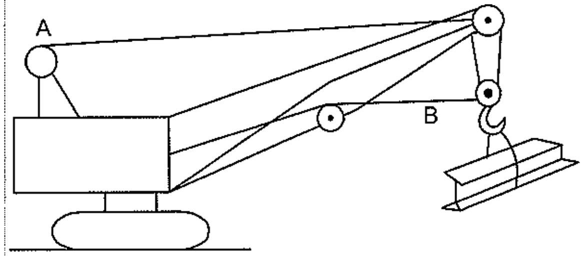
A crane is a machine designed primarily for lifting but adopted for many other uses. It consists basically of a power unit mounted on crawler tracks or wheels, with a boom and control cables for raising & lowering the load and the boom (Fig. i). A gantry is sometimes added (Fig. ii) to provide better boom support. For greater reach, an extension is added to the boom. This is in the form of a boom insert, a section added between the upper and lower ends, or a job, an extension to the end of the boom (Fig. iii)



A Boom C Hoist cable

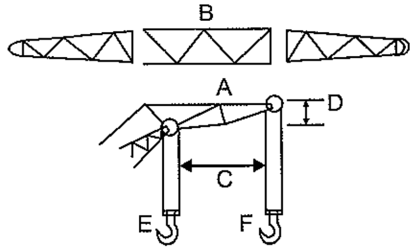
B Load carrier single line D Boom hoist cable for lowering and hoisting the boom

Fig.(i) Crane



A High gantry B Tagline

Fig. (ii) Crane with a high gantry



A Jib D Higher than standard boom

B Boom insert E For heavy loads

Fig. (iii) Crane with a boom insert

Cranes have wide application in construction projects, industries and in shipping etc. These are used for lifting the loads (may be construction materials, loose materials, packages, containers, finished and semifinished products in industries etc.) and placing them at desired place. For this purpose the cranes have three motions in general, namely hoisting. i.e., lifting or lowering, derricking and slewing. The cranes are generally electrically operated, diesel operated or may have diesel-elctric drive.

The cranes are very much useful to pick up a load at one point and be able to deposit it at another point within a restricted area. These are used for loads that are heavy for other types of equipment. Each type of crane is designed and manufactured to work economically in specific site situatons: Modern-day sites often employ more than one type of crane and more than one crane of the same type.

The different types of cranes that are being used in construction works are:

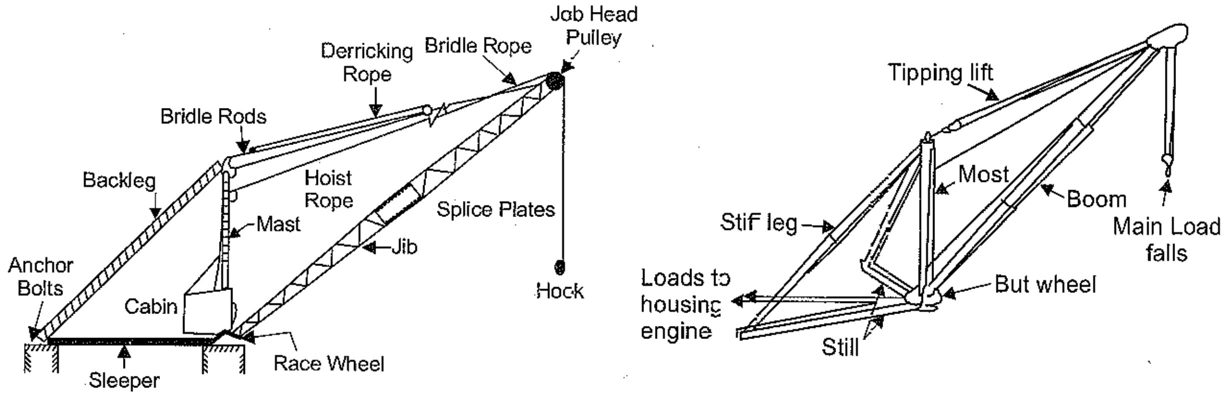
(i) Derrick cranes (Stationary cranes)

(ii) Mobile cranes

(iii) Tower cranes

(i) Derrick Cranes (Stationary Cranes)

- Derrick cranes consist of a mast, a boom, bullwheel on which it rotates about a vertical axis, and supporting members (also known as guys).
- These cranes are very widely used in construction projects, industrial and multistoreyed building consturction, plant erection, loading and unloading of cargoes at ports, in ship building etc. When used with grab, it can handle loose materials like sand, ballast, coal etc.



Derrick crane

- Derrick crane consist of a mast, a boom and a race wheel on which the boom rotates about a vertical axis.
- These cranes are either electrically operated , diesel operated or diesel- e ectrically operated.
- Power driven cranes are of mainly two types

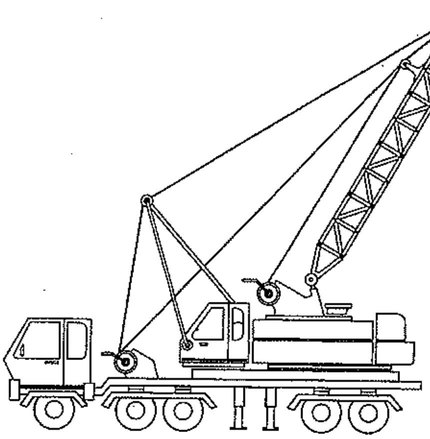
 (a) Guy derrick- It is used for heavy loads up to 200 tons with 360° rotation.

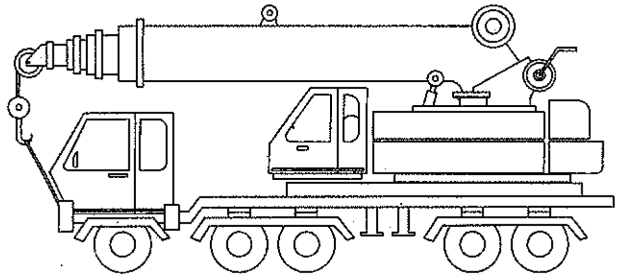
 (b) Stiff leg derrick - It is used for load from 7 to 50 tons with 270°-290° rotation.

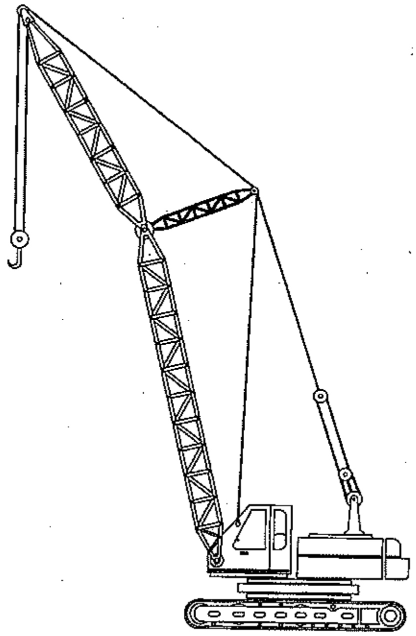
(ii) Mobile Cranes

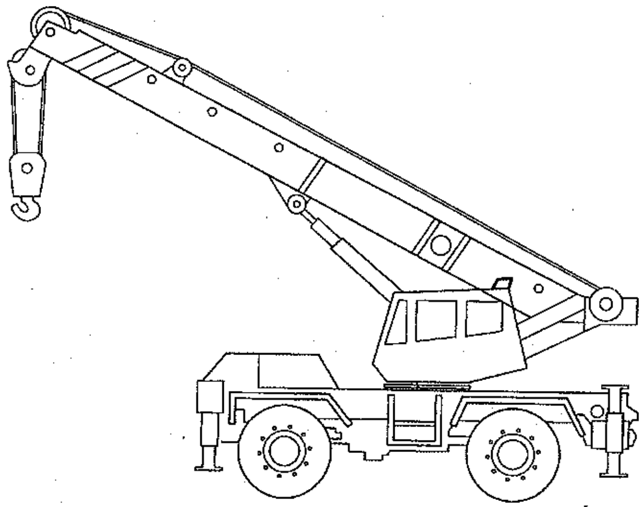
The mobile cranes have four motions viz., hoisting, derricking, slewing and travelling. Thus, it has travelling motino as an extra feature as compared to Derrick cranes.

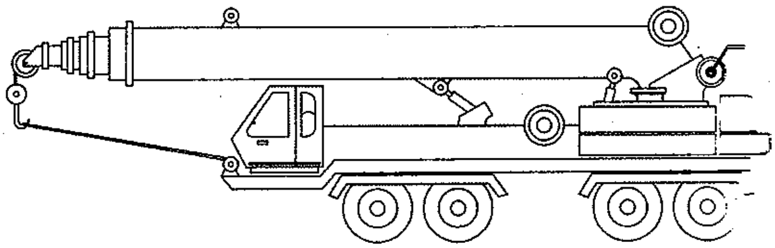
- These cranes are mounted on mobile units-a rubber tyred truck or a crawler.
- Truck cranes have high mobility while the crawler mounted cranes move slowly.
- The crawler's cranes are capable of moving on rough terrain where truck mounted cranes will not travel. Therefore these cranes are, economical where ground conditions are poor, and where long periods of operation within small areas are needed.





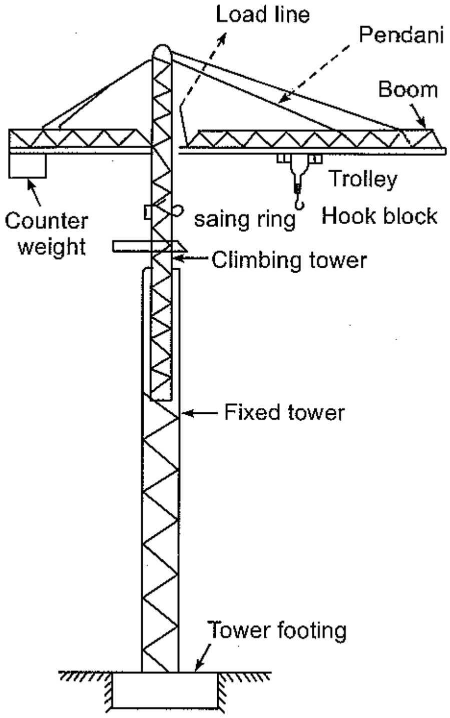


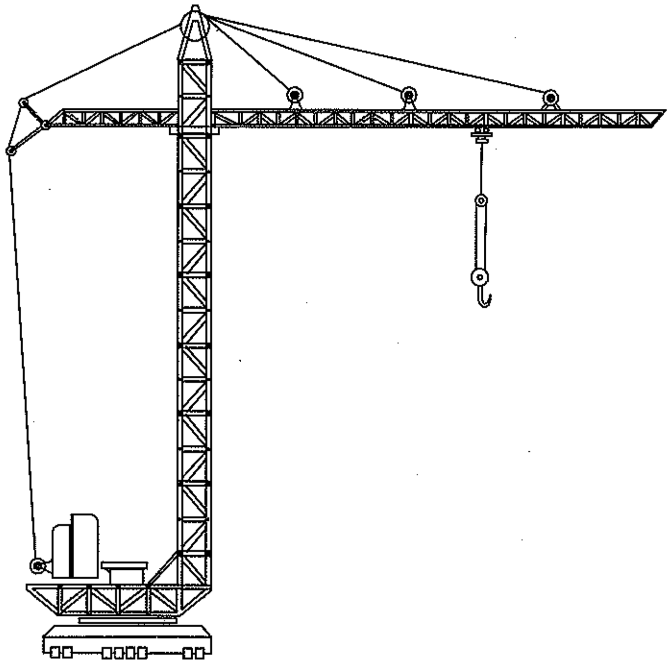




(iii) Tower cranes

- Tower cranes are used in various construction projects of high buildings, bridges, cooling towers, television towers or power plants.
- The construction of the tower crane is such that it can work and then dismentled in restricted space.
- This crane is usually employed in erection of high industrial and residential buildings reaching up to several meters in height.
- It is also usually used in assembling high industrial plants with elements of steel structure.





- Main parts of tower cranes are under carriage, slewing platform, tower with operator's cabin and jibs.

CHAPTER 13

CONCRETING EQUIPMENT

Q–1: ***Describe briefly the working of concrete pumps.***

[10 Marks, ESE-2000]

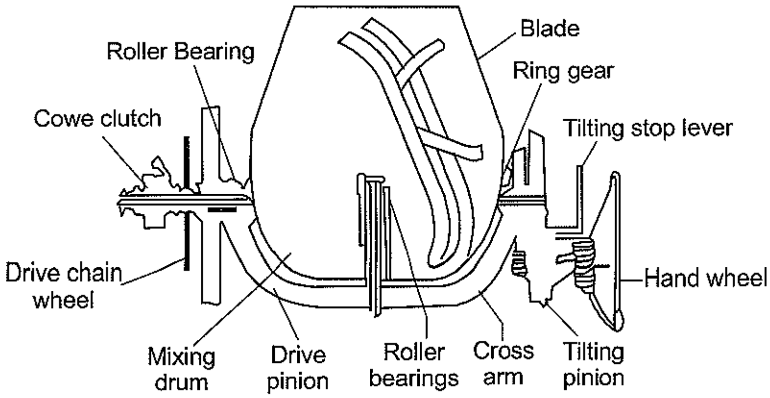
Sol: **Concrete Pumps:**

- In modern projects concrete pumps are proved to be a lot advantageous.
- Projects, having concrete as their primary material to be transported, larger concrete structural elements, sites with accessibility limitations, can be done easily with the help of concrete pumps.
- Three assembles are mainly used now a day.
    (i) **Pump with pipelines:** Pumps and pipeline are separate system. Pipeline is rigid, hence planning is required to lay the pipeline according to the need of concreting.
    (ii) **Pump and boom combination:** Pump and entire pipeline is mounted on the truck. Pipeline is flexible and can be moved with the help of movable boom.
    (iii) **Pump with pipe line and tower mounted boom:** It is combination of rigid pipeline and flexible one generally is used for concreting at tall structures.
- Pump generally of three types:
    (i) Piston pumps
    (ii) Pneumatic pumps
    (iii) Squeeze pressure type.
- The main advantage is large rate of concreting, but that depends on availability of concert and workmanship and criticality of the structure.

Q–2: ***Explain about tilting type of mixers and pan type of mixers used in the concrete mixing.***

[10 Marks, ESE-2006]

Sol: **Tilting mixer**



- The tilting mixer consists of a drum rotating on roller bearing through a ring gear around the periphery.
- Set of blades inside the drum gives continuous agitation of the material while mixing.
- Material for mixing is charged into the drum by a loading hopper made of steel plate. The hopper is operated through the loading lever.
- Mixed concrete is discharged by tilting the drum about the horizontal axis.
- Tilting mixers are useful for large construction works.
- Tilting mixers are easier to clean and can discharge the mix quickly and with minimum segregation.
- It gives better results even with dry concrete.
- It can be used for aggregate size more than 75 mm.

Q–3: ***Enumerate and explain briefly the various types of concrete pumps widely used in the construction industry. Explain briefly the important guidelines to be followed for successful pumping of concrete.***

[8 Marks, ESE-2008]

Sol: Types of concrete pumps:

Today there are three types of pumps being manufactured.

(i) **Piston Pumps:** Most piston pumps currently contain two pistons, with one refracting during the forward stroke of the other to give a more continuous flow of concrete.

(ii) **Pneumatic Pumps :** They normally use a reblending discharge box at the discharge end to bleed off the air and to prevent segregation and spraying.

(iii) **Squeeze Pressure Pumps :** In this case hydraulically powered rollers rotate on the flexible hose within the drum and squeeze the concrete out at the top. The vacuum keeps a steady supply of concrete in the tube from the intake hopper.

Guidelines for Successful Pumping Operation

(i) Use a combined gradation of coarse and fine aggregate that ensures no gaps in sizes that will allow past to be squeezed through the coarser particles under the pressure induced in the line. This is the most often overlooked aspect of good pumping.

(ii) Use a minimum 5-in pipe diameter

(iii) Always lubricate the line with cement paste or mortar before beginning the pumping operation.

(iv) Ensure a steady, uniform supply of concrete with a slump between 60 mm - 130 mm, as it enters the pump.

(v) Always presoak the aggregates before mixing them in the concrete to prevent their soaking up mix water under the imposed pressure.

(vi) Never use aluminium lines. Aluminium particles will be scraped from the inside of the pipe as the concrete moves through and will become part of the concrete. Aluminium and portland cement react, liberating hydrogen gas that can rapture the concrete with disastrous result.

Q–4: ***Enlist major concreting equipments required to carry out following operations : Mixing, transportation delivery and compacting equipment.***

[4 Marks, ESE-2012]

Sol: **Operation Major concreting equipment**

MixingBatching plant, Tilting mixer, Pan type mixer.

Transportation DeliveryTransit mixers, chutes, pans, crane buckets, concrete pumps

Compacting Vibrators and tamping rods.

Q–5: ***Calculate number of transit mixers (TM) required for transporting concrete from central batching plant to site. The cycle time data of a 6 m^3 typical transit mixer is given below:***

Loading time of TM = 6.0 minutes

Travel time of loaded TM to site = 30.0 minutes

Average waiting time at site = 5.0 minutes

Discharge time of concrete at site through concrete pump = 15 minutes

Travel time for return trip = 24 minutes

If the central batching plant having average output of 60 m^3/hr is to run continuously, work out the requirement of no. of concrete pumps and TM.

[6 Marks, ESE-2012]

Sol: Volume of Transit mixer = 6 m^3

Loading time of TM = 6.0 minutes

Travel tie of TM = 30.0 minutes

Average waiting time at site = 5.0 minutes

Discharge time of concrete through pump

= 15 minutes

Travel time for return trip = 24 minutes

Hence, Cycle time for TM = 6 + 30 + 5 + 15 + 24 = 80 minutes

$$\text{Output of TM per hour} = \frac{6}{80/60} = 4.5\ m^3/hr$$

Average output of Batching plant = 60 m^3/hr

Hence, $$\text{No. of TM required} = \frac{60}{4.5} = 13.33 \simeq 14\,\text{Nos.}$$

Moreover a T.M. will reach site every 6 minutes if we provide 13.33 T.M. with plant running continuously, and a pump requires 15 minutes to empty a T.M. hence number of pump is required = 15/6 = 2.5 $\simeq$ 3 No.

Q–6: ***Describe briefly five different types of earthwork equipment (rollers) used for compacting soils.***

[10 Marks, ESE-2014]

Sol: Five different of earthwork equipments used for compacting soils are as fallows:

Sheep's Foot Rollers

- Sheep's foot rollers are suitable for compacting fine grained materials such as clays and mixtures of sand and clay.
- These cannot compact granular soils such as sand and gravel.
- Depth of a layer of soil to be compacted is limited to approximately the length of the feet.
- They are used for manipulation and compaction of plastic clays where stratification must be eliminated, such as clay cores in dams.

Smooth-wheel Rollers

- Smooth-wheel Rollers can be self-propelled or of the towed type with smooth steel roll surfaces.
- These rollers may be classified by type or by weight.
- These rollers are effective in compacting granular soils, such as sand, gravel, and crushed stone and they are also effective in smoothening surfaces of soils that have been compacted by tamping rollers.
- When compacting cohesive soils, these rollers tend to form a crust over the surface, which may prevent adequate compaction in the lower portion of a lift.

Pneumatic-tyred Rollers

- Pneumatic-tyred Rollers are surface rollers, which apply the principle of kneading action to effect compaction below the surface.
- These rollers are used for rolling subgrades, airfield and bases of earthfill dams.
- They can be self-propelled or towed, small-or large-tyred units.
- These rollers rely on dead weight acting or upon pneumatic tyred wheels to produce the compacting effort.

Impact Rollers

- The impact roller is a recent development.
- It consists of a square concrete block with steel covering having rounded corners.
- It relies for its compaction effect on the impacts set up when this specially profiled roller is towed at fairly high speed.
- Impact roller can compact up till a considerable depth with certain soils and to be applicable to a wide range of soil types.

Vibrating Compactors

- Vibratory compactors enhance the performance of static weight rollers by adding dynamic forces, usually achieved by a rotating eccentrically weighed shaft mounted inside the roller.
- Vibrating compactors have shown their abilities to produce excellent densification of soils such as sand, gravel and relatively large stones.
- As these materials are vibrated, the particles shift their position and nestle more closely with adjacent particles to increase the density of the mass.
- Types of Vibrating compactors are:

  (a) Vibrating sheep's foot rollers,

  (b) Vibrating steel-drum rollers,

  (c) Vibrating pneumatic-tyred rollers,

  (d) Vibrating plates or shoes.

CHAPTER 14

COMPACTING EQUIPMENT

Q–1: ***Suggest any one or more of the following compacting equipments, which are most suitable for effective compaction in respect of each one of the gravel, sand silt and clay :***

(i) Sheepfoot Roller ***(ii) Tamping Roller***

(iii) Vibratory Roller ***(iv) Pneumatic Tired Roller***

Also indicate which method of compaction is adopted in these four compacting equipments.

[7 Marks, ESE-2008]

Sol:

Soil Type	**Equipment**
Gravel	Vibratory Roller Pneumatic Tired Roller
Sand	Vibratory Roller Pneumatic Tired Roller
Silt	Tamping Roller Pneumatic Tired Roller Sheepfoot Roller
Clay	Sheepfoot Roller Tamping Roller

Roller Type	**Method of Compaction**
Sheepfoot Roller	Pressure
Tamping Roller	Impact, Pressure
Vibratory Roller	Impact, vibration
Pneumatic Tired Roller	Pressure, Kneading

UNIT-7 BUILDING MATERIAL

SYLLABUS

Stone, Lime, Glass, Plastics, Steel, FRP, Ceramics, Aluminum, Fly Ash, Basic Admixtures, Timber, Bricks and Aggregates; Classification, properties and selection criteria;

Cement: Types, Composition, Properties, Uses, Specifications and various Tests; Lime & Cement Mortars and Concrete: Properties and various Tests; Design of Concrete Mixes: Proportioning of aggregates and methods of mix design.

CONTENTS

CHAPTER 1

CEMENT

Q–1: ***What are initial and final setting times of cement? How are they experimentally determined? Briefly explain the roles of gypsum and calcium chloride in cement.***

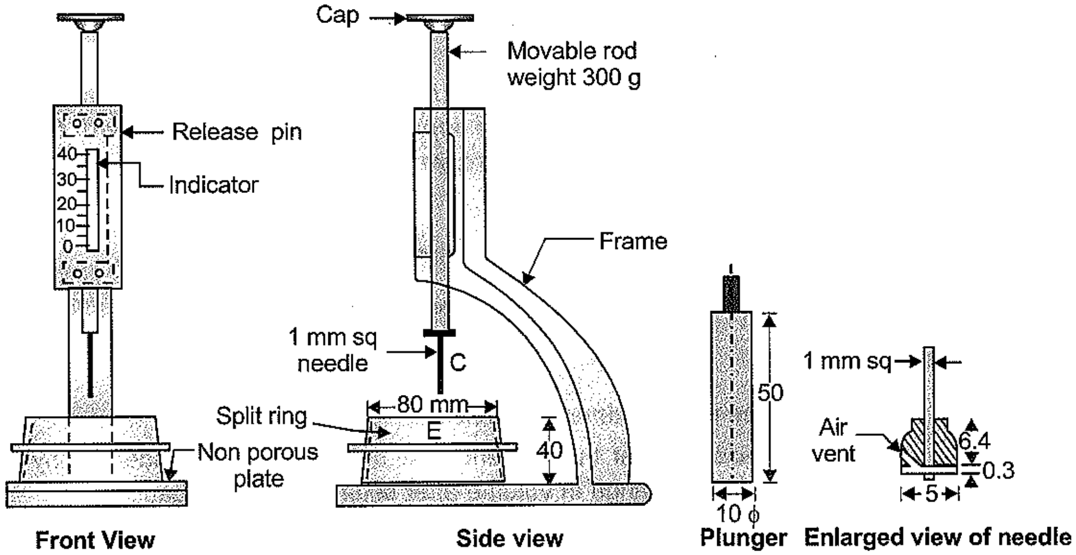
[10 Marks, ESE-1995]

Sol: **A. Initial and Final Setting time of cement**

- On addition of water to cement, the paste thus resulted undergoes stiffening and starts to gain strength and lose consistency simultaneously .
- This stiffening occurs in two stages and these stages are referred as initial and final setting times.
- The time elapsed between the moments the water is added and the paste starts losing its plasticity, is regarded as Initial setting time. As per Vicat's test (Refer Fig. below), it is the time elapsed till the paste stiffens to such an extent that the Vicat's needle cannot go into it within 5 ± .05 mm measured from the bottom of the mould.
- The time elapsed between the moments the water is added and the paste completely loses its plasticity is regarded as Final setting time. As per Vicat's test, it is the time elapsed till the paste attains such firmness that the attachment to the needle fails to leave any mark on it (though the needle will make an impression)

B. Experimental Determination

- The setting times are determined through Vicat's test. The apparatus used is as below.



- A cement paste is prepared by gauging cement with 0.85 times the water required to prepare a paste of standard consistency.

- The stop watch is started at the instant water is added.
- The mould rests on a non-porous plate. It is completely filled with the paste and its surface is leveled smooth with the top of the mould.
- The mould is then placed on Vicat's apparatus and the needle is lowered to contact the test block and is quickly released. The needle penetrates the test block completely at the beginning (40 mm from the top).
- After sometime the needle is unable to pierce the block for more than 33-35 mm from the top as the paste starts to lose its plasticity.
- The stop watch is stopped and the time elapsed gives the initial setting time.
- A circular attachment with needle is provided for determining the final setting time.
- This arrangement is gently lowered onto the surface of the test block and released. If the needle makes an impression (not more than 0.5 mm) but the edge of the circular arrangement fails to do so, the paste is finally set.
- The time elapsed till this moment, as given by stop watch, will the final setting time.
- According to IS code I.S. 12269-1987, minimum initial setting time is 30 minutes and maximum final setting time is 600 minutes

C. Reason of using Gypsum and Calcium Chloride

(i) Gypsum: During grinding of clinkers, 3 to 4 percent of Gypsum is added. It controls the initial setting time of cement, otherwise the cement would set as soon as water is added. Hence, the gypsum acts as a retarding agent, delaying the setting action of cement, making it workable.

(ii) Calcium Chloride: Calcium chloride ($CaCl_2$), has the ability to accelerate cement hydration and reduce set time by as much as two thirds. In addition to the important contributions of cold weather protection and early strength of concrete, it also has following benefits:

- Improves workability-regardless of mixture design, less water is required to produce a given slump when calcium chloride is used.
- Improves strength of air-entrained concrete. Calcium chloride compensates for the reduction in strength with a higher cement factor concrete.
- Reduces bleeding this is due to the early stiffening produced by acceleration and allows earlier final finishing.

Q–2: ***Explain the purpose of conducting SOUNDNESS test of cement. Describe the apparatus and method of test of cement. Describe the apparatus & method of test with the help of neat sketches. What are the permissible limits of observation in the test?***

[5 Marks, ESE-1996]

OR

Describe the procedure to list the soundness of cement. Name the constituents causing soundness.

[5 Marks, ESE-2010]

Sol: A. Significance of soundness of Cement

It is very essential that the cement after setting shall not undergo any appreciable change in volume, because change in volume after setting of cement causes:

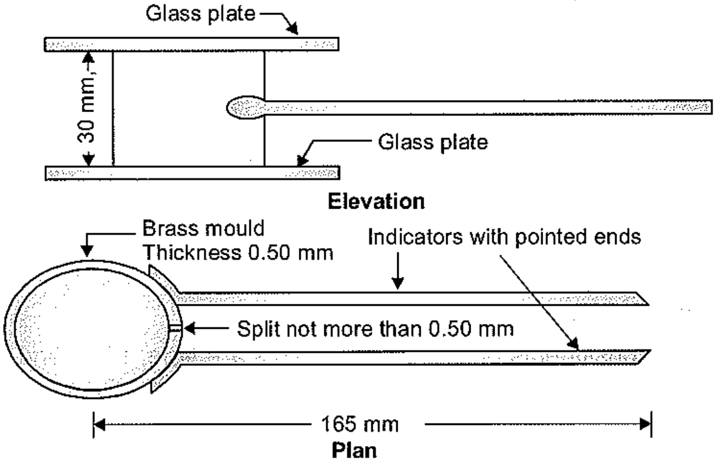
1. Cracks
2. Undue expansion which results in disintegration of concrete
3. Adverse effect on durability

B. Soundness Test of cement

1. Unsoundness in cement does not come to surface for a considerable period of time. Therefore, accelerated tests are required to detect it.
2. It can be tested with Le-Chatelier method or by autoclave method.
3. Le chatelier method is used in case of unsoundness due to free lime only as it does not indicate the presence and after effects of excess of Magnesia.
4. For magnesia content exceeding 3 %, Autoclave test has to be used as it is sensitive to both free lime and magnesia.

Le – Chatelier method

- The sketch of the apparatus used is as shown below:



- It consists of a small split cylinder of spring brass or other suitable metal. It is 30 mm in diameter and 30 mm high.
- On either side of the split are attached two indicator arms 165 mm long with pointed ends.
- Cement is gauged with 0.78 times the water required for standard consistency (0.78 P), in a standard manner and filled into the mould kept on a glass plate.
- The mould is covered on the top with another glass plate. The whole assembly is immersed in water at a temperature of 27°C – 32°C and kept there for 24 hours.
- Measure the distance between the indicator points.
- Submerge the mould again in water. Heat the water and bring to boiling point in about 25-30 minutes and keep it boiling for 3 hours.
- Remove the mould from the water, allow it to cool and measure the distance between the indicator points.
- The difference between these two measurements represents the expansion of cement.

6. **Autoclave method**

- A 25 × 25 × 250 mm specimen is made with neat cement paste
- The moulded specimen is removed from the moist atmosphere after 24 hours and measured for length.
- It is then placed in an autoclave at room temperature, making sure that four sides of the specimen are exposed to saturated steam.
- The temperature is raised at such rate so as to allow the gauge pressure of the steam to rise from 2.1 N/mm^2 in 1 to 1.25 hours from the moment heat is turned on.
- The pressure is maintained for 3 hours
- After the heat supply is turned off, the autoclave is cooled at a rate that pressure is less than 0.1 N/mm^2 after 1 hour.

- The Autoclave is opened and test specimen is placed in water at temperature of 90°C which is gradually brought down to 27±2°C in 15 minutes.
- It is then maintained at this temperature for next 15 minutes and then taken out
- The length of the specimen is measured again.
- The % difference in two lengths gives unsoundness of the cement.

C. The Permissible Limit

S. No.	Type of Cement	Soundness by Le chatelier (Max)	Soundness by Autoclave (Max %)
1.	OPC (33, 43, 53 grade), SRC, PPC,RHC, Slag cement, LHC	10 mm	0.8
2.	2HAC, SAC	5 mm	Not specified
3.	Masonry cement	10 mm	1
4.	IRS-T-40	5 mm	0.8

Q–3: Name the four important constituents of cement and state the role of each in achieving its properties.
[10 Marks, ESE-1998]

Sol: **Chemical Composition of Raw Materials**

- The three basic constituents of hydraulic cements are lime, silica and alumina.
- The relative proportions of these oxide compositions are responsible for influencing the various properties of cement.
- The approximate limits of chemical composition in cement are given below.

Constituents of portland cement (Raw materials)

Ingredient	Function	Composition (%)	Average (%)
Lime (CaO)	• It control strength and soundness. • Its deficiency reduces strength and setting action (time to change of plastic to solid state of paste) and excess of it cause unsoundness (cement to expand and disintegrate).	60 to 65	62
Silica (SiO_2)	• It imparts strength. • Excess of it increase the strength but setting action is prolonged.	17 to 25	22
Alumina (Al_2O_3)	• Responsible for quick setting. • Strength decreases as alumina in excess amount.	3 to 8	5
Calcium sulphate ($CaSO_4$)	• It increase initial setting time (time period during paste remains in plastic state) of cement.	3 to 4	4
Iron oxide (Fe_2O_3)	• Gives colour and helps in fusion of different ingredients. • Excess of it produce a hard clinker which is difficult to grind.	0.5 to 6	3
Magnesia (MgO)	• It imparts colour and hardness (rigidity of paste). • Excess amount makes the cement unsound.	0.5 to 4	2
Sulphur trioxide (SO_3)	• Excess of it makes cement unsound.	1 to 3	1
Alkalies [Soda (Na_2O) and Potash (K_2O)]	• These are residue. • Excess of it cause efflorescence and cracking.	0.5 to 1.3	1

Q–4: ***Describe the hydration of Portland cement and outline the ways in which the Vicat apparatus and the Le Chatelier apparatus can be used to assess the properties of fresh and hand made pastes.***

[15 Marks, ESE-1999]

Sol: **Hydration of Portland cement:**

(i) When water is added to cement, a chemical reaction b/w waters and cement takes place which is known as hydration of cement.

(ii) Heat liberated during this chemical reaction is known as heat of hydration.

(iii) The significant product of hydration is (CaO, SiO_2, H_2O) which is called as Tobermarite gel because of its structural similarity to a naturally occurring mineral Tobermarite and commonly it is referred C–S–H Gel.

Component Elements

O_2	Si	Ca	Al	Fe

↓

Component Oxides

CaO	SiO_2	Al_2O_3	Fe_2O_3

↓

Cement Compounds

C_3S	C_2S	C_3A	C_4AF

↓

Portland Cement

Various Types of Portland Cement

↓

Hydration Products

Gel	$Ca(OH)_2$

- The Vicat apparatus test is to estimate the quantity of mixing water to form a paste of normal consistency.
- Normal consistency is defined as that percentage water requirement of the cement paste, the viscosity of which will be such that the Vicat's plunger penetrates up to a point 5 to 7 mm from the bottom of the Vicat's mould.
- The water requirement for various test of cement depends on the normal consistency of the cement, which itself depends upon the compound composition of the cement.
- The Le-Chatelier apparatus test is to detect the change in volume of cement after setting.
- Soundness of cement is tested by Le-Chatelier method or by autoclave method. For OPL, RHL, LHC and PPC it is limited to 10mm.
- Soundness of cement can be ensured by limiting the quantities of free lime and magnesia which slake slowly causing charge in volume of cement known as unsound).
- It is a very important test to assure the quality of cement since an unsound cement produces cracks, distortion and disintegration ultimately loading to failure.

Q–5: ***Explain how sulphate resisting cement and rapid hardening Portland cement differ from ordinary Portland cement and the specific circumstances in which these cements would be used.***

[15 Marks, ESE-1999]

Sol: **A. Sulphate resisting cement (SRC)**

1. Unlike Ordinary Portland Cement (OPC) which is vulnerable to sulphate attack, SRC is better suited to conditions where sulphate attack is quite likely
2. This is because amount of Tri-calcium aluminate (C_3A) is restricted to lower than 5 % thereby reducing the formation of Ettringite (expansive hydrates). It also has comparatively lower content of C_4AF.
3. Most of the other properties of SRC are similar to that of OPC.
4. The use of SRC is recommended for following applications:
   - Foundations, piles.
   - Basements and underground structures.
   - Sewage and Water treatment plants.
   - Chemical, Fertilizers and Sugar factories.
   - Food processing industries and Petrochemical projects.
   - Coastal works.
   - Also for normal construction works where OPC is used.
   - Construction of builcing along the coastal area within 50 km from sea.

B. Rapid Hardening Cement (RHC)

1. It has higher percentage of tri-calcium silicate (C_3S) than ordinary cement and also has finer grinding.
2. It has similar initial and final setting times as those of OPC but it attains higher strength in early days.
3. Because of higher rate of gaining strength, it also has higher rate of heat development during hydration of cement.
4. It also has better resistance to chemical attack than OPC.
5. Due to its rapid setting, it allows the construction work to be carried out speedily.
6. The use of RHC is recommended for following applications:
   - Concreting in cold weather to avoid vulnerability of concrete to frost damage
   - Road repair works
   - Prefabricated concrete construction
   - Where formwork has to removed early for using elsewhere.

Q–6: ***Name the principal compounds in Portland cement, their relative rates of reaction with water and their approximate proportions.*** ***[5 Marks, ESE-1999]***

OR

Which are the four important compounds formed during the setting action of cement (four principal minerals in ordinary Portland cement)? Mention their relative proportions expressed as percentages and also functions of these compounds.

[5 Marks, ESE-2007]

Sol: The principal compounds in portland cement is also known as Bogue compound.

S.No.	The principal mineral comp. in Portland Cement	Formula	Name	Symbol
1.	Tricalcium silicate	3 $CaO.SiO_2$	Alite	C_3S
2.	Dicalcium silicate	3 CaO SiO_2	Belite	C_2S
3.	Tricalcium aluminate	3 $CaOAl_2O_3$	Celite	C_3A
4.	Tetracalcium aluminate ferrite	4 $CaO.Al_2.Fe_2O_3$	Felite	C_4AF

Properties of portland cement varies significantly with the proportions of the above mentioned compounds, as substantial difference is observed in their individual behaviour.

1. **Tricalcium silicate – (25–50%)– Normally 40%.**
   - It is considered as best cementing material and is well burnt cement.
   - It enables clinker easy to grind, increases resistance to freezing and thawing.
   - It hydrates rapidly generating high heat and develops on early hardness and strength.
   - Raising of C_3S content beyond the specified limits increases heat of hydration and solubility of cement in water.
   - respectively for 7 days strength and hardness.
   - Rate of hydration of C_3S and the character of gel developed are the main cause of the hardness and early strength of cement.
   - Heat of hydration is 500 J/gm.
2. **Dicalcium Silicate (C_2S) (25–40%) (Normally 32%)**
   - It hardness and hardens slowly and takes long time to add to the strength (after a year or more).
   - It imparts resistance to chemical attack.
   - Raising of C_2S content reduces early strength, decreases resistance to freezing and thawing at early ages and decreases heat of hydration.
   - Hydrolysis of C_2S proceeds slowly.
   - At early ages C_2S has little influence on strength and hardness. While after one year, its contribution to the strength and hardness is proportionality almost equal to C_3S.
   - Heat of hydration is 260 J/g.
3. **Tricalcium Aluminate (C_3A) (5-11%) (Normally 10.5%)**
   - It rapidly reacts with water and is reparable for flash set of finely grounded clinker.
   - Rapidity of action regulated by the addition of 2-3% of gypsum at the time of grinding cement.
   - It is responsible for the initial set high heat of hydration and has greater tendency to volume charge causing cracking

- Raising the C_3A content reduces the setting time, weavens resistance to sulphate attack and lower the ultimate strength, heat of hydration and counteraction during air hardening.
- Meat of hydration of 865 J/g.

4. **Tetracalcium Alumino Ferrite (C_4AF 8–14%)**
   - It is responsible for flash set but generates less heat.
   - It has less cementing value.
   - Raising the C_4AF content reduces the strength slightly.
   - Heat of hydration 420 J/g.

Q–7: ***Explain how do portland-pozzolana cement and super sulphate cement differ from ordinary portland cement. Under what specific circumstances these cements would be used.***

[10 Marks, ESE-2000]

Sol: **A. Portland Pozzolana Cement (PPC)**

1. The Portland Pozzolana Cement is a kind of Blended Cement which is produced by either inter-grinding of OPC clinker along with gypsum and pozzolanic materials in certain proportions or grinding the OPC clinker, gypsum and Pozzolanic materials separately and thoroughly blending them in certain proportions.
2. PPC produces less heat of hydration and offers greater resistance to the attack of impurities in water than OPC.
3. PPC is finer than OPC and due to pozzolanic action it improves the pore size distribution and also reduces the micro cracks at the transition zone.
4. PPC is economical than OPC owing to cheaper pozzolanic material as compared to clinkers.
5. Strength similar to those of OPC can be expected only at later stages.
6. If enough moisture is available for continued pozzolanic action, the long term strength of PPC is higher than that of OPC.
7. It is suitable for use in following conditions:
   - For hydraulic structures
   - For marine structures
   - For mass concrete structures like dams, thick foundations etc.
   - For sewage disposal works

B. Super Sulphate cement

1. It is manufactured by grinding together a mixture of 80-85% granulated slag, 10-15% hard burnt gypsum and about 5% Portland cement clinker.
2. The product is ground finer than that of Portland cement.
3. It is more sensitive to deterioration during storage than OPC.
4. It has low heat of hydration and high sulphate resistance.
5. It also has high resistance than Portland cement against attack by sea water.
6. It is suitable for use in following areas:

- Foundation works where chemically aggressive conditions exist
- It is used in marine works
- In fabrication of reinforced concrete pipes which are likely to be buried in sulphate bearing soils.

Q–8: ***List out the products of hydration and their influence on the properties of cement***

[10 Marks, ESE-2001]

Sol:

- The chemical reaction between cement and water is known as hydration of cement.
- The reactions take place between the active compounds of cement (C_4 AF, C_3 A, C_3 S & C_2 S) and water.

  The products generally are:

  (i) C-S-H gel, also known as Tobermorite gel

  (ii) Ca $(OH)_2$

  (iii) Some other minor compounds.

Influences

- The C-S-H gel and $Ca(OH)_2$ Crystallizes in the available free space.
- $Ca(OH)_2$ reacts with water and generate OH^- ions, which are important for the protection of reinforcement in case of RCC.
- Further as the hydration proceeds the two crystal types become more heavily interlocked increasing the strength.
- The main cementing action is generally provided by the gel which occupies two-thirds of the total mass of hydrate.

Q–9: ***(i) Explain the difference between various grades of ordinary Portland Cement.***

OR

(ii) What are the different types of portland cement as per Indian code of practice? Discuss any two.

[10 Marks, ESE–2002, 2011]

Sol: **OPC in India**

1. The cements produced by inter grinding of cement clinker prepared in rotary cement kiln along with 3-5 % gypsum only are called as Ordinary Portland Cement (OPC). Depending upon the strength requirement, OPC is further classified as OPC-33 grade, OPC-43 grade and OPC-53 grade
2. They have 28 days mean compressive strength exceeding 33MPa, 43 MPa and 53 MPa respectively.
3. They are all made from same materials and have same initial and final setting time.
4. Increase in C_3S content and fineness of grinding increases the strength.

A. 33 Grade Grade OPC: The 28 days compressive strength of OPC is 33 MPa.

- Now a days it has been obsoleted, because of lower strength.

B. 43 grade OPC

1. The 43 grade OPC is the most popular general-purpose cement in the country today. The production of 43 grade OPC is nearly 50% of the total production of cement in the country.
2. The 43 grade OPC can be used for following applications:
   - General Civil Engineering construction work.
   - RCC works (preferably where grade of concrete is up to M-30).
   - Precast items such as blocks, tiles, pipes etc.
   - Asbestos products such as sheets and pipes.
   - Non-structural works such as plastering, flooring etc.
3. The compressive strength of cement when tested as per IS code shall be minimum 43 MPa

C. 53 Grade OPC

1. 53 Grade OPC is a higher strength cement to meet the needs of the consumer for higher strength concrete.
2. As per BIS requirements the minimum 28 days compressive strength of 53 Grade OPC should not be less than 53 MPa.
3. For certain specialized works, such as pre-stressed concrete and certain items of precast concrete requiring consistently high strength concrete, the use of 53 grade OPC is found very useful.
4. 53 grade OPC produces higher-grade concrete at very economical cement content. In concrete mix design, for concrete M-20 and above grades, a saving of 8 to 10 % of cement may be achieved with the use of 53 grade OPC.
5. 53 Grade OPC can be used for the following applications.
   - RCC works (Preferably where grade of concrete is M-25 and above).
   - Precast concrete items such as paving blocks, tiles building blocks etc.
   - Pre-stressed concrete components.
   - Runways, concrete Roads, Bridges etc.
   - Runways, concrete Roads, Bridges etc

Q-10: ***Explain pozzolanic action.***

[10 Marks, ESE-2003]

Sol: **Pozzolanic action**

A. Pozzolana is a siliceous or siliceous and aluminous material which as such does not have cementitious properties.

B. It reacts with calcium hydroxide in the presence of water at room temperature through a reaction called pozzolanic reaction to form insoluble calcium silicate hydrate and calcium aluminate hydrate compounds possessing cementitious properties.

C. The reaction can be written as –

Pozzolana + $Ca(OH)_2 + H_2O$ = C-S-H (gel)

D. It is firstly slow and hence heat of hydration and strength development will be accordingly slow.

E. The reduction of $Ca(OH)_2$ improves the durability of cement paste by making the paste dense and impervious.

F. It also reduces the expansion caused by alkali-aggregate reaction in concrete.

Q–11: ***Which are the four important compounds formed during the setting action of cement (four principal minerals in ordinary Portland cement)? Mention their relative proportions expressed as percentages and also functions of these compounds.***

[10 Marks, ESE-2007]

Sol: The lower important compound formed during the setting action of cement are.

1. **Tricalcium aluminate (3CaO, Al_2O_3):** This compound is formed within about 24 hours after addition of water to the cement.
2. **Tetra-calcium alumino- Ferrite (4CaO, Al_2O_3, Fe_2O_3):** This compound is also formed within about 24 hrs. after addition of water to the cement.
3. **Tricalcium silicate (3CaO, SiO_2):** This compound is formed within a week on 50 after addition of water to the cement and it is mainly responsible for importing strength to the cement in early period of setting.
4. **Dicalcium silicate (2CaO, SiO_2):** This compound is formed very slowly and hence it is responsible for giving progressive strength tot he cement.
   - The above four principle minerals in ordinary portland cement are designated in short as C_3A,$C_4AF$$C_3S$,$C_2S$ respectively and their relative proportion expressed or percentage, are as follows:

$$\left.\begin{array}{lll} C_3A & \rightarrow & 4 \text{ to } 14\% \\ C_4AF & \rightarrow & 10 \text{ to } 18\% \end{array}\right\} \text{Celit}$$

$$\begin{array}{lll} C_3S & \rightarrow & 45 \text{ to } 65\% \text{ (alit)} \\ C_2S & \rightarrow & 15 \text{ to } 35\% \text{ (belit)} \end{array}$$

   - When water is added to the cement, the quickest to react with water is C_3A and in order of decreasing rate are C_4AF, C_3S and C_2S.

Q–12: ***List the various laboratory tests for assessing the quality of cement and their importance.***

[10 Marks, ESE-2008]

Sol: The various tests done to determine the quality of cement are as:

A. Fineness test

1. The fineness of cement has an important bearing on the rate of hydration and hence on the rate of gain of strength and also on the rate of evolution of heat. Finer cement offers a greater surface area for hydration and hence faster the development of strength.
2. Increase in fineness of cement is also found to increase the drying shrinkage of concrete.
3. Fineness of cement is tested in two ways :

   (a) **By sieving:** The principle of this is that we determine the proportion of cement whose grain size is larger than specified mesh size.

(b) By determination of specific surface (total surface area of all the particles in one gram of cement) by air-permeability apparatus. Expressed as cm^2/gm. or m^2/kg. Generally Blaine Air permeability apparatus is used.

4. Maximum number of particles in a sample of cement should have a size less than about 100 microns. The smallest particle may have a size of about 1.5 microns. By and large an average size of the cement particles may be taken as about 10 micron.

B. Setting time test

1. The significance of initial and final setting times is in the construction industry. There are various time bound factors involved in cement work such as mixing, transportation, laying, compacting and finishing, which will be facilitated only if cement or concrete is in plastic condition. For this purpose the initial setting time of concrete is determined.
2. Simultaneously, it is also very important that once the concrete is compacted and finished, it attains its firmness as soon as possible to avoid damages from external forces, bringing final setting time into the picture.
3. Vicat's apparatus is used to find these parameters.

C. Compressive Strength test

1. The compressive strength of hardened cement is the most important and most specified of all the properties.
2. Therefor cement is always tested for this strength before employing it for important works.
3. Before starting any project, concrete mix designs are prepared in the lab in accordance with the properties of available materials. For checking the applicability and suitability of these designs, this test is used.
4. It is also employed to check the strength of concrete ready for dispatch from the batching plant.

D. Soundness test

1. It is very essential that the cement after setting shall not undergo any appreciable change in volume, because change in volume after setting of cement causes:
   - Cracks
   - Undue expansion which results in disintegration of concrete
   - Adverse effect on durability
2. It can be tested with Le-Chatelier method or by autoclave method.
3. Le-Chatelier method is used in case of unsoundness due to free lime only as it does not indicate the presence and after effects of excess of Magnesia.
4. For magnesia content exceeding 3 %, Autoclave test has to be used as it is sensitive to both free lime and magnesia.

E. Heat of hydration test

1. It is estimated that exothermic reaction of cement with water generates about 120 calories of heat for 1 gram of cement.
2. A temperature rise of about 50°C is observed in the interior of mass concrete dam. This can cause serious expansion of the body of the dam and subsequent cooling will cause shrinkage which can lead to serious cracking of concrete.

3. So test of heat of hydration is essentially required for low heat cements.
4. This is carried out over a few days by vacuum flask methods, or over a longer period in adiabatic calorimeter.

F. Chemical Composition test

1. Cement mainly consists of lime, silica, alumina and iron oxide.
2. Their relative proportions greatly influence the various properties of cement.
3. So it is of vital importance to carry out chemical composition tests in laboratory.

Q–13: Discuss how consistency of cement is determined.

[5 Marks, ESE-2011]

Sol: A. Consistency

1. The standard consistency of a cement paste is defined as that consistency which will permit the Vicat's plunger to penetrate to a point 5 to 7mm from the bottom of the Vicat's mould.

B. Procedure for consistency

1. The requirements for the test are-
   - Vicat's Apparatus Conforming to IS: 5513-1976.
   - Balance of capacity 1Kg and sensitivity to 1gram.
   - Gauging trowel conforming to IS: 10086-1982.
2. Unless otherwise specified, this test shall be conducted at a temperature 27 ± 20°C and the relative humidity of laboratory should be 65 + 5%.
3. Prepare a paste of weighed quantity of cement (300gms) with weighed quantity of potable or distilled water, taking care that the time of gauging is neither less than 3 minutes nor more than 5 minutes and the gauging is completed before any sign of setting occurs.
4. The gauging is counted from the time of adding water to the dry cement until commencing to fill the mould.
5. Fill the Vicat's mould with this paste resting upon a non-porous plate.
6. Smoothen the surface of the paste, making it level with the top of the mould.
7. Slightly shake the mould to expel the air.
8. In filling the mould, operator's hands and the blade of the gauging trowel shall only be used.
9. Immediately place the test block with the non-porous resting plate, under the rod bearing the plunger.
10. Lower the plunger gently to touch the surface of the test block and quickly release, allowing it to sink into the paste.
11. Record the depth of penetration
12. Prepare trial pastes with varying percentages of water and test as described above until the plunger is 5mm to 7mm from the bottom of the Vicat's mould.

Q–14: Cement is made of solid complexes of oxides of calcium etc. write the names of the four well known complexes along with the notation used to represent these.

[4 Marks, ESE-2013]

Sol: Those four complexes are:

(i) Tricalcium Silicate $3CaO.SiO_2$ (C_3S) (ii) Dicalcium Silicate $2CaO.SiO_2$ (C_2S)

(iii) Tricalcium Aluminate $3CaO.Al_2O_3$ (C_3A) (iv) Tetracalcium Alumina Ferrite $4CaO.Al_2O_3.Fe_2O_3$(C_4AF)

Collectively they are known as Bogue compound.

Q–15: When testing cements as per Indian standard, it may not be possible to decide on their quality only on the basis of the compressive strength results. Why?

[5 Marks, ESE-2013]

Sol:

- When cement is used in a concreting process strength is not the only criteria to decide the quality of cement Quality is a relative term, A cement's superiority depends on the purpose of the concreting.
- There are other parameters like fineness, soundness, heat produced during hydration, initial and final setting time etc which determine the superiority or quality of a cement category.
- e.g. In mass concreting low heat cement is more preferable instead of its lesser strength.
- Soundness of cement is always a considerable parameter

Q–16: What do you understand by the initial and final setting times of cement? What are the typical intial and final setting times of 43 grade OPC cement and Portland pozzolana cement (PPC) as per IS code?

[5 Marks, ESE-2014]

Sol: **Initial and Final Setting time of cement**

- On addition of water to cement, the paste thus resulted undergoes stiffening and starts to gain strength and lose consistency simultaneously .
- This stiffening occurs in two stages and these stages are referred as initial and final setting times.
- The time elapsed between the moments the water is added and the paste starts losing its plasticity, is regarded as Initial setting time. As per Vicat's test (Refer Fig. below), it is the time elapsed till the paste stiffens to such an extent that the Vicat's needle cannot go into it within 5 – 7 mm measured from the bottom of the mould.
- The time elapsed between the moments the water is added and the paste completely loses its plasticity is regarded as Final setting time. As per Vicat's test, it is the time elapsed till the paste attains such firmness that the annular collar (plunger) fails to leave any mark on it (though the needle will make an impression)

		Initial setting time (minutes)	Final setting time (minutes)
(1)	OPC – 43	30	600
(2)	PPC	30	600

Q–17: List out eight chemical ingredients of Portland cement and briefly explain their functions.

[12 Marks, ESE-20018]

Sol: **Chemical ingredients of portland cement:**

- The three basic constituents of hydraulic cements are lime, silica and alumina.

- The relative proportions of these oxide components are responsible for influencing the various properties of cement.
- An increase in lime content beyond a certain value makes it difficult to combine completely with other compounds. Consequently, free lime will result in an unsound cement.
- An increase in silica content at the expense of alumina and oxide makes the cement difficult to fuse and form clinker.

The other chemical ingredients are:

1.	Lime (CaO)	2.	Silica (SiO_2)
3.	Alumina (Al_2O_3)	4.	Iron oxide (Fe_2O_3)
5.	Magnesia (MgO)	6.	Soda (Na_2O)
7.	Potash (K_2O)	8.	Sulphur trioxide (SO_3)

1. Lime (CaO)

- Composition (%) = 60–65%
- Function:
  - Control strength and soundness
  - Its deficiency reduces strength and setting time.
  - Excess of it cause unsoundness

2. Silica (SiO_2)

- Composition (%) = 17–25%
- Function:
  - It gives strength
  - Excess of it causes slow setting

3. Alumina (Al_2O_3)

- Composition (%) = 3–8%
- Function:
  - It is responsible for quick setting
  - If in excess, it lowers the strength

4. Iron oxide (Fe_2O_3)

- Composition (%) = 0.5–6%
- Function:
  - It gives colour
  - It helps in fusion of different ingredients i.e. it acts as a flux

5. Magnesia (MgO)

- Composition (%) = 0.5–4%
- Function:
  - It imparts colour and hardness
  - If in excess, it causes cracks in mortar and concrete and unsoundness

6. **Soda (Na_2O)**
   - Composition (%) = 0.5–1%
7. **Potash (K_2O)**
   - Composition (%) = 0.5–1%
     - Soda and potash are residues, and if in excess cause efflorescence and cracking
8. **Sulphur trioxide (SO_3)**
   - Composition (%) = 1–2%
   - Function:
     - Excess of it makes cement unsound

Q–18: ***What are Bogue's compounds? Briefly mention their functions.***

[8 Marks, ESE-2018]

Sol: **Bogue's compounds:**

When the raw materials of portland cement are put in kiln, then it fuses and following four major compounds are formed and they are known as Bogue compounds.

1. Tricalcium silicate (C_3S)
2. Dicalcium silicate (C_2S)
3. Tricalcium aluminate (C_3A)
4. Tetracalcium alumino ferrite (C_4AF)

Functions of Bogue compounds:

1. **Tricalcium silicate**

Chemical formula : $3CaO \cdot SiO_2$

Name : Alite

Symbol : C_3S

- It enables clinker easy to grind, increases resistance to freezing and thawing.
- It hydrates rapidly generating high heat and develops an early hardness and strength.
- It is considered as the best cementing material and is well burnt cement.
- Hydrolysis of C_3S is mainly responsible for 7 days strength and hardness.
- Rate of hydrolysis of C_3S and the character of gel developed are the main cause of the hardness and early strength of cement paste.

2. **Dicalcium silicate**

Chemical formula : $2CaO \cdot SiO_2$

Name : Belite

Symbol : C_2S

- It imparts resistance to chemical attack.
- It hydrates and hardens slowly and takes long time to add to the strength (after a year or more), i.e. it is responsible for ultimate strength.

- Raising of C_2S content renders clinker harder to grind, reduces early strength, decreases resistance to freezing and thawing at early ages and decreases heat of hydration.
- At early stages, less than a month, C_2S has little influence on strength and hardness, while after one year, it contribution to the strength and hardness is proportionatly almost equal to C_3S.

3. Tricalcium aluminate (C_3A)

Chemical formula : $3CaO \,.\, Al_2O_3$

Name : Celite

Symbol : C_3A

- It is the most responsible for the initial setting, high heat of hydration and has greater tendency to volume changes causing cracking.
- It rapidly reacts with water and is responsible for flash set of finely grounded clinker.
- Rapidity of action is regulated by the addition of 2–3% of gypsum at the time of grinding the cement.
- Raising the C_3A content reduces the setting time, weakens resistance to sulphate attack and lowers the ultimate strength, heat of hydration and contraction during air hardening.

4. Tetracalcium alumino Ferrite

Chemical formula : $4CaO \,.\, Al_2Fe_2O_3$

Name : Felite

Symbol : C_4AF

- It is responsible for flash set but generates less heat.
- It has poorest cementing value.
- Raising the C_4AF content reduces the strength slightly.]

Q–19: ***Explain the products of hydration of C_3S and C_2S (Bogues compounds) giving the relevant equations involving the reactions.***

[4 Marks, ESE-2019]

Sol: Hydration of silicate is given by following equation

- $2C_3S + 6H \longrightarrow C_3S_2H_3 + 3Ca(OH)_2$
- $2C_2S + 4H \longrightarrow C_3S_2H_3 + Ca(OH)_2$
- From these two above equations it can be seen that hydration of C_3S produce lesser calcium silicate hydrates (C–S–H) and more $Ca(OH)_2$ as compared to the hydration of C_2S.
- Since $Ca(OH)_2$ is not a desirable product in the concrete mass because it is soluble in water and gets leached out making the concrete porous, particularly in hydraulic structure, that's why cement with small percentage of C_3S and more C_2S is recommmended for use in hydraulic structure.
- $Ca(OH)_2$ reacts with sulphate present in soil or water to form $CaSO_4$ which further reacts with C_3A and forms calcium sulphoaluminate (ettringite) which causes volume expansion such that expanded volume is approximately 227% of the volume of original aluminates, thus resulting in crack and subsequent disruption.

- The only advantage of $Ca(OH)_2$ is that being alkaline in nature maintain $PH \approx 13$ in concrete which resist the corrosion of reinforcement.

Q–20: ***Write briefly about the following: Role of Flyash as a part replacement of cement.***

[10 Marks ESE–2019]

Sol: Partial replacement of portland cement by pozzolana has to be carefully defined as its specific gravity (or relative density) is much lower than that of cement. Thus replacement by mass results in a considerably greater volume of cementitious material. If equal early strength is required and pozzolana is to be used. e.g because of alkali-aggregate reactivity, then addition of pozzolana rather than partial replacement is necessary.

Q-21: ***Describe how the compounds of clinker affect the properties of cement.***

[12 Marks, ESE-2020]

Sol:

- The various constituents of cement combine in burning and form cement clinker. The compounds formed in the burning process have the properties of setting and hardening in the presence of water. They are known as Bogue compounds after the name of Bogue who identified them.
- The properties of cement varies markedly with the properties of these compounds, reflecting substantial difference between their individual behaviour.
- Bogue's compounds are:

1. Tricalcium Silicate C_3S– (25 – 50%)– Normally 40%

- It is considered as best cementing material and is well burnt cement.
- It enables clinker easy to grind , increases resistance to freezing and thawing.
- It hydrates rapidly generating high heat and develops an early hardness and strength.
- Raising of C_3S content beyond the specified limits increases heat of hydration and solubility of cement in water.
- Hydrolysis of C_3S is mainly responsible for 7 days strength and hardness.
- Rate of hydrolysis of C_3S and the character of gel developed are the main cause of the hardness and early strength of cement paste.
- Heat of hydration is 500 J/gm.

2. Dicalcium Silicate (C_2S) (25 – 40%) (Normally 32%)

- It hydrates and hardens slowly and takes long time to add to the strength (after a year or more) i.e. it is responsible for ultimate strength.
- It imparts resistance to chemical attack.
- Raising of C_2S content renders clinker harder to grind, reduces early strength, decreases resistance to freezing and thawing at early ages and decreases heat of hydration.
- At early ages, less than a month, C_2S has little influence on strength and hardness. While after one year, its contribution to the strength and hardness is proportionately almost equal to C_3S.
- Heat of hydration is 260 J/g

3. Tricalcium Aluminate (C_3A) (5 – 11%) (Normally 10.5%).

- It rapidly reacts with water and is responsible for flash set of finely grounded clinker.
- Rapidity of action is regulated by the addition of 2-3% of gypsum at the time of grinding the cement.
- It is most responsible for the initial setting, high heat of hydration and has greater tendency to volume changes causing cracking.

- Raising the C_3A content reduces the setting time, weakens resistance to sulphate attack and lowers the ultimate strength, heat of hydration and contraction during air hardening.
- Heat of hydration of 865 J/g.

4. Tetracalcium Alumino Ferrite (C_4AF 8 –14%) (Normally 9%)

- It is responsible for flash set but generates less heat.
- It has poorest cementing value
- Raising the C_4AF content reduces the strength slightly.
- Heat of hydration 420 J/g

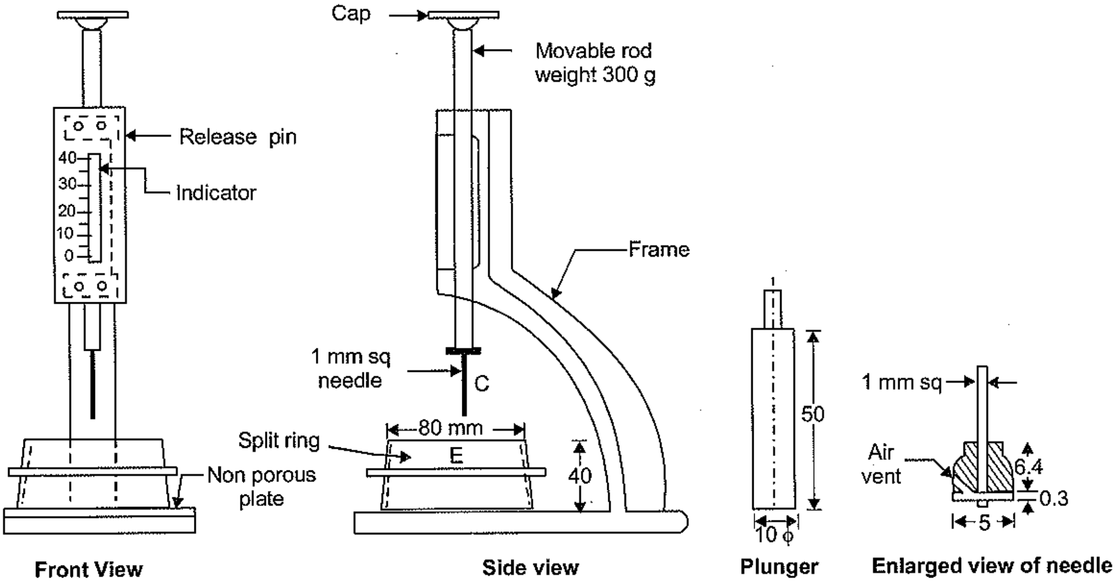
Q-22: ***What do you mean by normal consistency of cement ? What is its significance? How is it tested?***

[8 Marks, ESE-2020]

Sol: **Normal consistency** is defined as that percentage water requirement of the cement paste, the viscosity of which will be such that the Vicat's plunger penetrates up to a point 5 to 7mm from the bottom of the Vicat's mould.

Significance

- The water requirement for various tests of cement depends on the normal consistency of the cement, which itself depends upon the compound composition and fineness of the cement.



(all dimension in mm)

VICATS Apparatus

Attachment	Use
10 mm ϕ Plunger	Consistency test
1 mm² needle	Initial setting time
5 mm ϕ annular collar	Final setting time

The Procedure

1. The requirements for the test are :
   - Vicat's Apparatus Conforming to IS : 5513-1976.
   - Balance of capacity 1 kg and sensitivity to 1 gram.
   - Gauging trowel conforming to IS : 10086-1982.

2. Unless otherwise specified, this test shall be conducted at a temperature 27 ± 20°C and the relative humidity of laboratory should be 65 + 5%.
3. Prepare a paste of weighed quantity of cement (300 gms) with weighed quantity of potable or distilled water, taking care that the time of gauging is neither less than 3 minutes nor more than 5 minutes and the gauging is completed before any sign of setting occurs.
4. The gauging is counted from the time of adding water to the dry cement until commencing to fill the mould.
5. Fill the Vicat's mould with this paste resting upon a non-porous plate.
6. Smoothen the surface of the paste, making it level with the top of the mould.
7. Slightly shake the mould to expel the air.
8. In filling the mould, operator's hands and the blade of the gauging trowel shall only be used.
9. Immediately place the test block with the non-porous resting plate, under the rod bearing plunger (diameter 10 mm and length 50 mm).
10. Lower the plunger gently to touch the surface of the test block and quickly release, allowing it to sink into the paste.
11. Record the depth of penetration.
12. Prepare trial pastes with varying percentages of water and test as described above until the plunger is 5 mm to 7 mm from the bottom of the Vicat's mould.
13. When the reading is 5-7 mm from the bottom of the mould, the amount of water added is considered to be correct percentage of water for normal consistency

CHAPTER 2

CONCRETE

Q–1: ***What are the factors that influence the strength of cement concrete? Briefly discuss the effects of water-cement ratio and workability on the strength of concrete.***

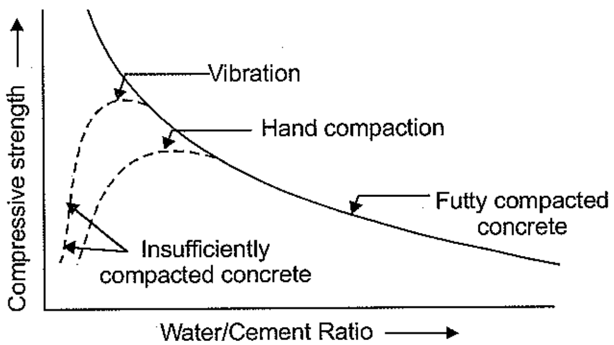
[10 Marks, ESE-1995]

Sol: **A. The factors that influence the strength of concrete are:**

- Water cement ratio, i.e. ratio of cement to mixing water
- Ratio of cement to aggregate
- Grading, surface texture, shape, strength and stiffness of aggregate particles
- Maximum size of aggregate
- Degree of compaction

B. Effect of water- cement ratio on strength of concrete

- According to Abram's law, the strength of workable concrete is only dependent on water-cement ratio.
- The volume of water in fresh concrete is related directly to the volume of empty pore space in hardened concrete. Similarly, the volume of cement in fresh concrete is related directly to the solid volume in hardened concrete. Water-cement ratio is therefore a measure of the void volume relative to the solid volume in hardened cement paste, and its strength goes up as the void volume goes down. So, the lower the water-cement ratio, the lower is the void volume-solid volume ratio, and stronger the hardened concrete.



- In a hardened state concrete, strength is inversely proportional to the water/cement ratio as shown in the graph
- As is indicated from the graph, the strength of concrete does not follow the curve if we decrease the water-cement ratio beyond an optimal range.

- The reason why the compressive strength of concrete does not actually follow a hyperbolic curve at lesser water-cement ratio is when water to cement ratio is low in a fresh mix, then less water is available for the hydration of cement. Hence, some amount of cement paste remains un-hydrated that leads tc internal tension in concrete plus weak bond.
- The strength that might be developed in this situation is now much dependent on the following four factors:

  1. Water to cement ratio
  2. Cement to aggregate ratio
  3. Maximum aggregate size
  4. Physical properties of aggregates.

  Note: The factors (2,3 and 4) are of lesser importance while factor (1) is the major influencing factor.

C. Effect of workability on strength of concrete

- Workability means higher water to cement ratio.
- It is visible from the graph above that compressive strength of concrete also depends on method of compaction. (Vibration method gives higher strength as compared to hand compaction at lower-water cement ratio)
- The objective of compaction is to minimize the air voids in the concrete which will lead to higher strength. Higher the compaction more will be the strength of concrete.
- To get 100% compaction with given efforts, normally a higher water cement ratio than theoretically calculated ratio is required.
- Hence a workable concrete is necessary to get desired level of compaction, hence improve the strength of concrete.

  Note: If the water to cement ratio becomes too high, it will lead to segregation of concrete which is not desirable.

Q–2: ***Using a mix design procedure, mix proportion for the desired grade of concrete have been obtained as 1 : 2.1 : 3.5 (by mass) with water-cement ratio of 0.5 and air content of 3 percent. Calculate the weights of individual ingredients required to make 0.25 m³ concrete. The specific gravities of cement, sand and aggregate wee 3.15, 2.65 and 2.70, respectively.***

[20 Marks, ESE-2000]

Sol: Mix proportion is given as

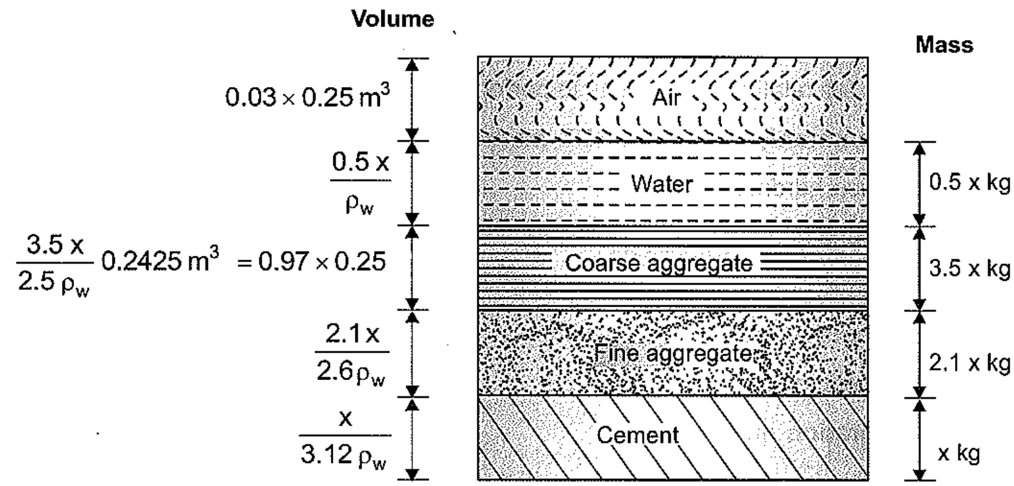
Cement: Fine aggregate: coarse aggregate = 1 : 2.1 : 3.5 (by mass)

$$\text{Water cement ratio} = \frac{\text{weight of water}}{\text{weight of cement}} = 0.5$$

Specific gravities of cement, sand and coarse aggregates are 3.15 : 2.65 : 2.70 respectively

$$\text{Air content} = 3\% = \frac{\text{Volume of air}}{\text{total volume}}$$

Let us use the phase diagram



$$\Rightarrow \quad \frac{x}{3.15\,\rho_w} + \frac{2.1\,x}{2.6\,\rho_w} + \frac{3.5\,x}{2.5\,\rho_w} + \frac{0.5\,x}{\rho_w} = 0.2425\ m^3$$

$$3.025\frac{x}{\rho_w} = 0.2425$$

$$\Rightarrow \quad x = 80.165\ \text{kg}$$

⇒ Mass of cement required for 0.25 m^3 of concrete = 80.165 kg

Mass of F.A required for 0.25 m^3 of concrete = 80.165 × 2.1 = 168.35 kg

Mass of CA required for 0.25 m^3 of concrete = 80.165 × 3.5 = 280.58 kg

Mass of water required for 0.25 m^3 of concrete = 0.5 × 80.165 = 40.08 kg

= 40.08 lit.

Q-3: ***(i)*** ***What is non-destructive testing of concrete? What are its relative merits? Name the methods of non-destructive testing and explain briefly any one method.***

OR

(ii) ***Discuss about non-destructive tests on concrete.***

[? Marks, ESE-1998, 2001]

Sol: (i) **Non-destructive testing of concrete:**

1. Non-destructive testing (NDT) is a wide group of analysis techniques used to evaluate the properties of concrete without causing damage.
2. The strength of concrete is therefore inferred from some other properties values of which for a good concrete sample are already known.
3. So, instead of absolute values, an estimate of its strength, durability and elastic parameters are obtained.
4. Though these tests are easy to perform but their analysis requires special knowledge.

Merits of NDT of concrete

1. They do not cause any damage the structure or specimen and hence save a lot of time and money.

2. We can do these tests to determine strength of existing structures and hence devise repair plan.
3. They are relative y very easy to perform.

Some of the commonly employed NDT methods

1. Surface hardness tests
2. Rebound hammer tests
3. Dynamic or vibration tests
4. Radioactive and nuclear methods
5. Magnetic and electrical methods

Dynamic or vibration method

1. The fundamental principle on which this method is based is velocity of sound through a material
2. The velocity of scund through a solid can be measured by determining the resonant frequency of specimen or by recording the time taken by short pulses of vibration passing through the sample.
3. A mathematical relationship could be established between velocity of sound through specimen and its resonant frequency, and of these two with the modulus of elasticity of the material.

Q-4: ***Calculate the quantities of cement, sand and coarse aggregate required to produce one cubic meter of concrete for mix properties of 1 : 40 : 2.80 (by volume) with water-cement ratio of 0.48 (by mass). Bulk densities of cement, sand and coarse aggregates are 14.7, 16.66 and 18.68 kN/m3, respectively. Percentage of entrained air is 2.0. Specific gravities of cement, sand and coarse aggregate are 3.15, 2.6 and 2.5, respectively***

[10 Marks, ESE-2003]

Sol: Cement : F.A : CA

xm^3 : $1.4x\ m^3$: $2.8\ xm^3s$

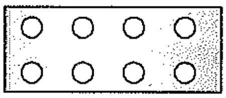
$$\frac{\text{weight of water}}{\text{weight of cement}} = 0.48$$

$$\frac{\text{Weight}}{\text{Volume}} = \text{Absolute density } (e_s) = \frac{W_s}{V_s}$$

$\gamma_{\text{bulk cement}} = 14.7\ kN/m^3$

$\gamma_{\text{bulk FA}} = 16.66\ kN/m^3$

$\gamma_{\text{bulk CA}} = 15.68\ kN/m^3$



$$\frac{\text{Weight}}{\text{Volume}} = \frac{\text{Weight of solid}}{V_s + V_{air}} = \text{Bulk density}$$

$\therefore$ Cement : FA : CA ≡ (14.7 x kN) : (1.4 × 16.66 x kN) : (15.68 × 2.8x)

Weight of water = 0.48 × weight of cement = 0.48 × 14.7 x kN

$$\text{Volume of water} = \frac{0.48 \times 14.7\ x}{\gamma_w}\ m^3 \quad ; \text{Vol. of air} = 0.02\ m^3$$

$$\therefore \quad \frac{14.7x}{3.15\ \gamma_w} + \frac{1.4 \times 16.66x}{2.6\ \gamma_w} + \frac{15.68 \times 2.8\ x}{2.5\ \gamma_w} + \frac{0.48 \times 14.7x}{\gamma_w} + 0.02 = 1m^3$$

$$\therefore \quad x = 0.257\ m^3$$

Now, Weight of cement = 14.7 x = 3.777 kN = 377.7 kg [1 kN = (100 kg)]

$$\text{Weight of FA} = 1.4 \times 16.66\ x = 5.994 \text{ kN} = 599.4 \text{ kg}$$

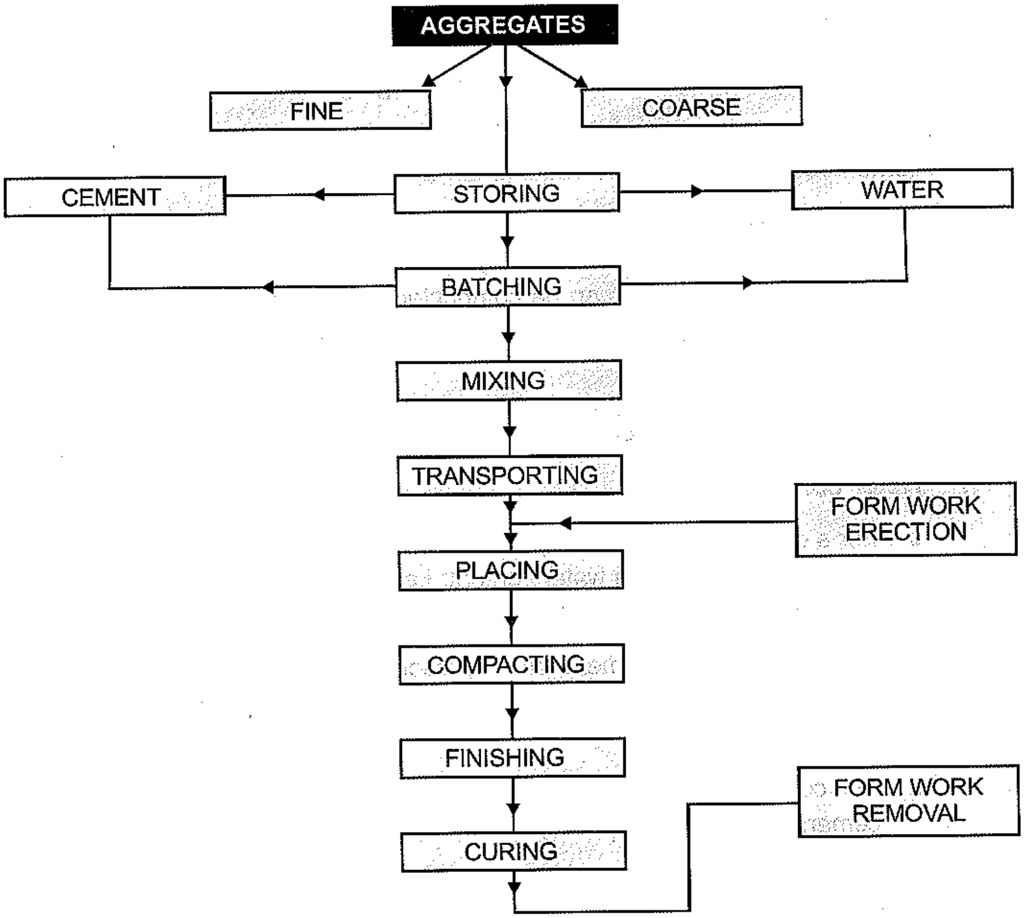
$$\text{Weight of CA} = 15.68 \times 2.8\ x = 11.283 \text{ kN} = 1128.3 \text{ kg}$$

$$\text{Weight of water} = 0.48 \times 14.7\ x = 0.48 \times 377.7 = 181.296 \text{ kg}$$

Q–5: ***Discuss in brief the various operations involved in a concrete construction project.***

[10 Marks, ESE-2004]

Sol:



Generally following operation are performed during concreting:

(i) Storing
- (a) Storing of cement
- (b) Storing of aggregates
- (c) Storing of water.

(ii) Batching
- (a) Batching of cement
- (b) Batching of aggregates
- (c) Batching of water

(iii) Mixing
- (a) Hand mixing
- (b) Machine mixing.

(iv) Handling and Transporting

(v) **Placing**

(vi) **Compacting**

(vii) **Finishing**

(viii) **Curing**

Q-6: ***Define 'workability of concrete'. What are the factors affecting workability of concrete?***

[10 Marks, ESE-2005]

Sol: **A. Workability of Concrete**

1. Workability is referred to as the ease with which a concrete can be transported, placed and consolidated without excessive bleeding or segregation.
2. For maximum strength in concrete, full compaction is required. It means a higher water to cement ratio then theoretical requirements.
3. This is because workability can also be defined as the internal work done in overcoming the frictional forces between concrete ingredients for full compaction. So, water functions as a lubricant so that concrete can be compacted up to maximum possible extent.
4. The optimal workability of concrete will depend upon the given job and hence will vary from situation to situation.

B. Factors affecting workability of concrete

1. Water content

- For a given maximum size of coarse aggregate, the slump or consistency of concrete is a direct function of the water content; i.e., within limits it is independent of other factors such as aggregate grading and cement content.
- At a constant water/cement ratio reduction in the aggregate/cement ratio causes increase in the water content, which consequently results into the increases in consistency of concrete.
- At a constant water content reduction in the aggregate/cement ratio decreases the water/cement ratio and consistency would not be affected

2. Cement content

- In normal concrete, at given water content, a considerable lowering of the cement content tends to produce harsh (i.e., low workable) mixtures with poor finishability.
- Concretes containing a very high proportion of cement or a very fine cement show excellent cohesiveness but tend to be sticky

3. Aggregate characteristics.

- The particle size of coarse aggregate influences the water requirement for a given consistency.
- At same water content very fine sands or angular sands produce lesser workable concrete as compared to a coarser or a well-rounded sand

4. Admixtures

- At given water content, air-entraining admixtures improve the consistency and cohesiveness of the concrete by increasing the volume of paste.

- Pozzolanic admixtures tend to improve the cohesiveness of concrete. Fly ash, when used as a partial replacement for fine aggregate, generally increases the consistency at given water content.
- At a constant water content of a concrete mixture, the addition of a water-reducing admixture will increase the consistency.

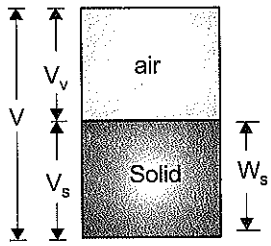
Q–7: ***Estimate the quantities of cement, fine aggregate and coarse aggregate per cubic metre of concrete if the void ratio in cement is 62%, fine aggregate is 41% and coarse aggregate is 45%. The material properties are as follows:***

Mix : 1 : 2 : 4 with a w/c of 0.55, one bag of cement contains 50 kg of cement & its density is 1440 kg/m³. The density of fine aggregate is 1700 kg/m³ & coarse aggregate is 1600 kg/m³ respectively. One bag of cement is equal to 34.7 litres.

[10 Marks, ESE-2006]

Sol: When the mix proportion is given like 1 : 2 : 4, and it is not mentioned whether it is by volume or by weight, we should always take it as by weight like 1 kg cement : 2 kg fine aggregate : 4 kg coarse aggregate

Also, bulk density or simply density of cement means



$$\text{Bulk density or density of cement} = \frac{\text{Mass of cement}}{\text{Vol. of cement}}$$

On the other hand, absolute density or mass density means

$$\text{Absolute density or mass density of cement} = \frac{\text{Mass of cement}}{\text{Vol. of cement solid}}$$

$$\text{Mass density} = \frac{W_s}{V_s}$$

$$\text{Bulk density} = \frac{W_s}{V} = \frac{W_s}{V_s + V_v} = \frac{W_s / V_s}{1+\frac{V_v}{V_s}}$$

$$\boxed{\text{Bulk density} = \frac{\text{Mass density}}{1+e}}$$

where,

$$e = \text{Void ratio}$$

$$e_{\text{cement}} = 0.62$$

$$\text{Bulk density of cement} = 1440 \text{ kg/m}^3$$

$$e_{\text{fine aggregate}} = 0.41$$

$$\text{Bulk density of fine aggregate} = 1700 \text{ kg/m}^3$$

$$e_{\text{coarse aggregate}} = 0.45$$

$$\text{Bulk density of coarse aggregate} = 1600 \text{ kg/m}^3$$

$\Rightarrow$ $$\text{Mass density of cement, } \rho_c = (\text{Bulk density of cement}) \times (1+e_c)$$

$$\rho_c = 1440 \times 1.62 = 2332.8 \text{ kg/m}^3$$

Similarly, $$\rho_{fa} = 1700 \times 1.41 = 2391 \text{ kg/m}^3$$

$$\rho_{ca} = 1600 \times 1.45 = 2320 \text{ kg/m}^3$$

Let the volume of air in 1m^3 of concrete $= 0.02 \text{ m}^3$

Sum of vol. of all ingredients = Vol. of concrete

Let the mass of cement in 1 m^3 of concrete be x kg

$\Rightarrow$ x kg of cement is to be mixed with 2x kg fine aggregate and 4x kg coarse aggregate and as $\frac{W}{C}$ ratio is 0.55, wt. of water is 0.55x.

$$\Rightarrow \quad \frac{x}{2332.8}+\frac{2x}{2397}+\frac{4x}{2320}+\frac{0.55x}{1000}+0.02 = 1$$

$\Rightarrow$ x = 277.06 kg

$\Rightarrow$ Wt. of cement for 1m^3 Concrete = 277.06 kg

$\Rightarrow$ Wt. of F.A for 1m^3 Concrete = 554.12 kg

$\Rightarrow$ Wt. of C.A for 1m^3 Concrete = 1108.24 kg

$\Rightarrow$ Wt. of water for 1m^3 Concrete = 152.383 kg

Q–8: ***Write about various moduli of elasticity of plain cement concrete. Which values are used in design? What are the factors affecting Modulus of Elasticity of concrete?***

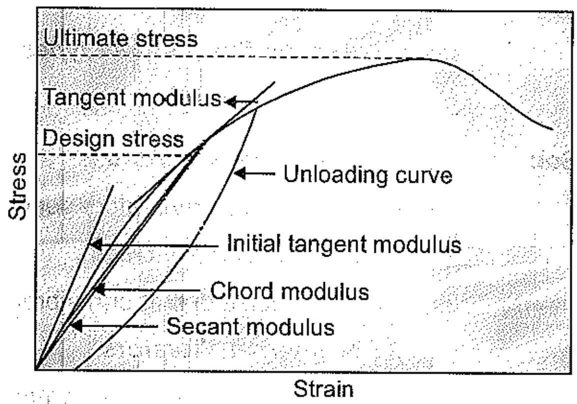
[10 Marks, ESE-2007]

Sol: **A. Moduli of elasticity of plain cement concrete**

1. Concrete is a heterogeneous, multi-phase material, whose rheological behavior is influenced by the elastic properties and morphology of gel materials.
2. As a result, the stress strain curve does not exactly follow the Hooke's law

B. Static modulus of elasticity

1. It is defined as slope of stress-strain curve for concrete under uniaxial tension or compression loading.
2. But since this curve for concrete is not straight at any point, the modulus of elasticity is found out with reference to tangent drawn to the curve at the origin. This is called INITIAL TANGENT MODULUS. But it gives satisfactory results at low stress values only.
3. A tangent drawn from any other point on stress-strain curve will give TANGENT MODULUS. But it also gives satisfactory values in the vicinity of the point considered.
4. The most commonly used modulus is SECANT MODULUS and is given by the slope of the line drawn connecting a specified point on curve to origin of curve.
5. The value of secant modulus decreases with increase in stress, so the stress at which it is found should always be stated.
6. CHORD MODULUS is found out by chord drawn between two specified points on the stress-strain curve.
7. Static modulus of elasticity does not truly represent elastic behavior of concrete due to phenomenon of creep. It will get affected more seriously at higher stresses.



C. Dynamic modulus of elasticity

1. It can be found by subjecting the concrete member to longitudinal vibration at their natural frequency, and is calculated from following relationship

$$E_d = Kn^2L^2\rho$$

where,
E_d = Dynamic modulus of elasticity
K = A constant,
n = Resonant Frequency
L = Length of the specimen
ρ = Density of concrete

D. Factors affecting modulus of elasticity

1. Effects of moisture condition

- Specimens tested in dry condition show about 15% decrease in elastic modulus as compared to the wet specimens. This is explained by the fact that drying produces more micro-cracks in the transition zone, which affects the stress-strain behavior of the concrete.
- This is opposite to its effects on compressive strength. The compressive strength is increased by about 15% when tested dry as compared with the wet specimens.

2. Effects of Aggregate properties

- Porosity of aggregate has the most effect on the elastic modulus of concrete. An aggregate with a low porosity has a high modulus of elasticity. The elastic modulus of concrete is affected by the volume fraction of the aggregate as well as the elastic modulus of the aggregate.

3. Effects of cement matrix

- The lower the porosity of the cement paste, the higher the elastic modulus of the cement paste.
- The higher the elastic modulus of the cement paste, the higher the elastic modulus of the concrete.

4. Effects of transition zone

- The void spaces and the micro-cracks in the transition play a major role in affecting the stress-strain behavior of concrete.
- The transition zone characteristics affect the elastic modulus more than it affects the compressive strength of concrete

Q–9: ***List four different types of vibrators used for compaction of concrete and briefly explain each of them.***

OR

Briefly describe five different types of vibrators used in cement concrete making industry.

[10 Marks, ESE-2007]

Sol: **Types of Concrete Vibrators**

There are four types of vibrators generally used:

1. Needle or internal vibrators
2. Form or external vibrators
3. Vibrating table or platform vibrators
4. Surface vibrators

Needle or internal Vibrators

- This is perhaps the most commonly used vibrator.

- It essentially consists of a steel tube (with one end closed and rounded) having an eccentric vibrating element inside it. This steel tube called poker is connected to an electric motor or a diesel engine through a flexible tube.
- The needle is easily moved from place to place, depending on the consistency of the mix.
- They are available in size varying from 40 to 100 mm diameter. The diameter of the poker is decided from the consideration of the spacing between the reinforcing bars in the form-work.
- The frequency of vibration varies upto 15000 rpm. However a range between 3000 to 6000 rpm is suggested as a desirable minimum with an acceleration of 4g to 10g.
- The normal radius of action of a needle vibrator is 0.50 to 1.0m.
- It would be preferable to immerse the vibrator into concrete at intervals of not more than 600mm or 8 to 10 times the diameter of the poker.
- Period of vibration required may be of the order of 30 seconds to 2 minute. Actual completion of compaction can be judged by the appearance of the surface of the concrete, which should be neither honeycombed nor contain an excess of mortar.
- Vibrator should be immersed through the entire depth of the freshly deposited concrete and into the layer below if this is still plastic or can be brought again to a plastic condition.
- Internal vibrators are comparatively efficient since all the work is done directly on the concrete, unlike other types of vibrators.
- Internal vibration is generally best suited for ordinary construction provided the section is large enough for the vibrator to be manipulated.
- The vibrator should never be used to move concrete laterally, as segregation can easily occur.

Form or External Vibrators

- These vibrators are clamped rigidly to the form work at the predetermined points so that the form and concrete are vibrated.
- The external vibrators are more often used for pre-casting of thin in-situ sections of such shape and thickness as cannot be compacted by internal vibrators.
- They consume more power for a given compaction effect than internal vibrators.
- These vibrators can compact upto 450mm from the face but have to be moved from one place to another as concrete progresses. These vibrators operate at a frequency of 3000 to 9000 rpm at an acceleration of 4g.

Vibrating Tables or Platform Vibrators

- The vibrating table consists of a rigidly built steel platform mounted on flexible springs and is driven by an electric motor.
- The normal frequency of vibration is 4000 rpm at an acceleration of 4g to 7g.
- The vibrating tables are very efficient in compacting stiff and harsh concrete mixes required for manufacture of precast elements in the factories and test specimens in laboratories.

Surface Vibrators or Screeds

- These consist of vibrating-pan or screed vibrators which vibrate the concrete from the surface - usually at the time the concrete is struck off or screeded.
- These are placed directly on the concrete mass.
- These are best suited for compaction of shallow elements and should not be used when the depth of concrete to be vibrated is more than 150 mm.
- Very dry mixes can be most effectively compacted with such vibrators.
- The surface vibrators commonly used are pan vibrators and vibrating screeds.
- The operating frequency is about 4000 rpm at an acceleration of 4g to 9g.
- Surface vibrators are usually used as screeds for slabs or pavements.

Q–10: (i) ***What do you understand by 'workability of concrete'? Describe briefly a test for its in-situ determination. What should be the values of observation from this test for concrete used for different purposes?***

OR

(ii) ***Describe the workability of fresh concrete and its measurement in the light of statement that workability is a composite property and each test measures only a particular aspect of it***

OR

(iii) ***What is meant by the term 'Workability of concrete'? List the methods used for measurement of workability of concrete. Clearly indicate the aspect of workability measured by each method.***

OR

(iv) ***Give a brief description of slump test for measurement of workability of concrete and its merits and demerits. What are the slump values recommended for different types of works?***

[10 Marks, ESE-1996, 2000, 2002, 2008]

Sol: **A. Workability of Concrete**

1. Workability is referred to as the ease with which a concrete can be transported, placed and consolidated without excessive bleeding or segregation.
2. For maximum strength in concrete, full compaction is required. It means a higher water to cement ratio then theoretical requirements.
3. This is because workability can also be defined as the internal work done in overcoming the frictional forces between concrete ingredients for full compaction. So, water functions as a lubricant so that concrete can be compacted up to maximum possible extent.
4. The optimal workability of concrete will depend upon the given job and hence will vary from situation to situation.

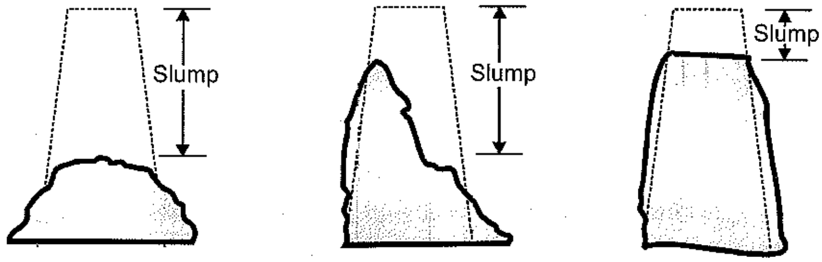
B. Test for in-situ determination

1. Slump test is used for in-situ determination of workability. The procedure is as follows-
   - The apparatus used is a metallic mould in the form of frustum of a cone having internal dimensions as:

     Top diameter – 10 cm
     Bottom diameter – 20 cm
     Height – 30 cm

   - A steel tamping rod of 16 mm dia, 0.6 long with bullet end is used.
   - The mould is placed on a smooth, horizontal, rigid and non-absorbent surface.
   - It is then filled in 4 layers, each approximately ¼ of height of mould.
   - Each layer is tamped 25 times by the tamping rod evenly.
   - After the top layer has been rodded, the concrete is struck off level with a towel and tamping rod.
   - The mould is removed immediately by lifting it vertically carefully.
   - The concrete will then subside and this subsidence is referred as slump of concrete.
   - The difference between the height of mould and the highest point of subsided concrete in mm is taken as slump of concrete.

- The pattern of slump also indicates the characteristic of concrete.
- An even slump is called true slump, and if one half of concrete slides down, it is called shear slump. It may collapse in case of very wet concretes.
- In case of shear slump, the slump value is measured as difference between the height of mould and average value of subsidence.
- Shear slump indicates a non-cohesive concrete and may lead to segregation.



C. Observations of test for concrete used for different purposes-

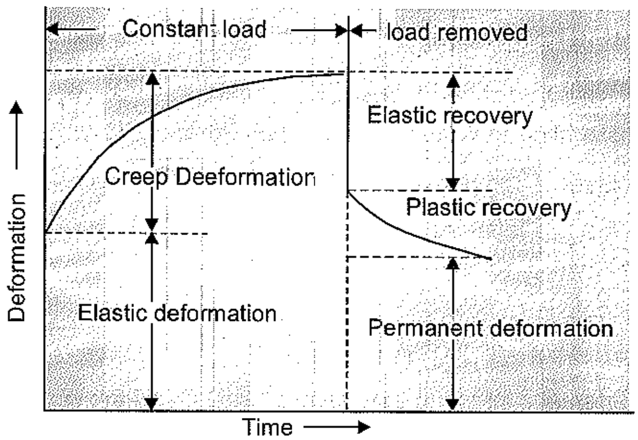
Slump test is suitable for concrete of low to high workability, i.e. in range of 25 to around 150 mm. The details are as in the table:

Degree of Workability	Slump (in mm)	Use of concrete
Low	25-75	1. Roads vibrated by hand operated machines. 2. Mass concrete foundations without vibration or lightly reinforced sections with vibration.
Medium	50-100	1. Manually compacted flat slabs using crushed aggregates 2. Normal reinforced concrete, manually compacted and heavily reinforced sections with vibration
High	100-150	For sections with congested reinforcement

Q–11: ***Draw a sketch showing the typical creep strain time curve under uniaxial compression for concrete.***

[10 Marks, ESE-2009]

Sol: The typical creep strain time curve is as below Draw the graph



Q–12: ***Briefly describe the admixtures generally used in concrete and the properties they impart to the concrete.***

[10 Marks, ESE-2009]

Sol: **A. Admixtures**

They are ingredients other than water, aggregates, hydraulic cement, and fibers that are added to the concrete batch immediately before or during mixing. A proper use of admixtures offers certain beneficial effects to concrete.

B. Generally used admixtures

1. Accelerating admixtures or accelerators

- These admixtures speed up the chemical reaction of the cement and water and so accelerate the rate of setting and/or early gain in strength of concrete.
- They are used where
  - Rapid setting and high early strengths are required (e.g. in shaft sinking)
  - Rapid turnover of moulds or formwork is required
  - Concreting takes place under very cold conditions.
- Among the main types of accelerators are chloride based, non-chloride based and shotcrete accelerators.
- Some examples are $CaCl_2$, NaCl, NaOH& KOH

2. Retarding admixtures or retarders

- These admixtures slow the chemical reaction of the cement and water leading to longer setting times and slower initial strength gain.
- They are used—
  - When placing concrete in hot weather, particularly when the concrete is pumped.
  - To prevent cold joints due to duration of placing.
  - In concrete which has to be transported for a long time.
- The most common retarder is calcium sulphate. Other examples include hydroxylated carboxylic acids, lignins, sugar and some phosphates.

3. Air-entraining admixtures

- An air-entraining agent introduces air in the form of minute bubbles distributed uniformly throughout the cement paste. The main types include salts of wood resins, animal or vegetable fats and oils and sulphonated hydrocarbons
- They are used—
  - Where improved resistance of hardened concrete to damage from freezing and thawing is required.
  - For improved workability, especially in harsh or lean mixes.
  - To reduce bleeding and segregation, especially when a mix lacks fines.

4. **Water reducing admixtures or Plasticizers-**

- When added to a concrete mix, plasticizers (water-reducing agents) are absorbed on the surface of the binder particles, causing them to repel each other and deflocculated. This results in improved workability and provides a more even distribution of the binder particles through the mix.
- There uses are—
  - Plasticizers usually increase the slump of concrete with given water content.
  - Plasticizers can reduce the water requirement of a concrete mix for a given workability, as a rule-of-thumb by about 10%.
  - The addition of a plasticizer makes it possible to achieve a given strength with lower cement content.
  - Plasticizers may improve pump ability.
  - The main types of plasticizers are Lignosulphonic acids and their salts, hydroxylated carboxylic acids and their salts and modifications of both.

Q–13: ***(i) Give a detailed account of the cylinder splitting test of concrete.***

(ii) What are the limitations of the above test in evaluating the real tensile strength of concrete.

[10 Marks, ESE-2009]

Sol: (i) Cylinder splitting test:

1. Due to difficulty in applying uniaxial tension to a concrete specimen, the tensile strength is determined by indirect methods.
2. It is the standard test to determine the tensile strength of concrete in indirect way in accordance with IS: 5816-1970
3. A standard test cylinder of concrete specimen of 300mm X 150mm diameter is placed horizontally between the loading surfaces of compression testing machine.
4. The compression load is applied diametrically and uniformly along the length of cylinder until the failure of the cylinder along vertical diameter.
5. On application of load, a uniform tensile stress acts over two-third of the loaded diameter
6. The magnitude of the tensile stress is obtained by-

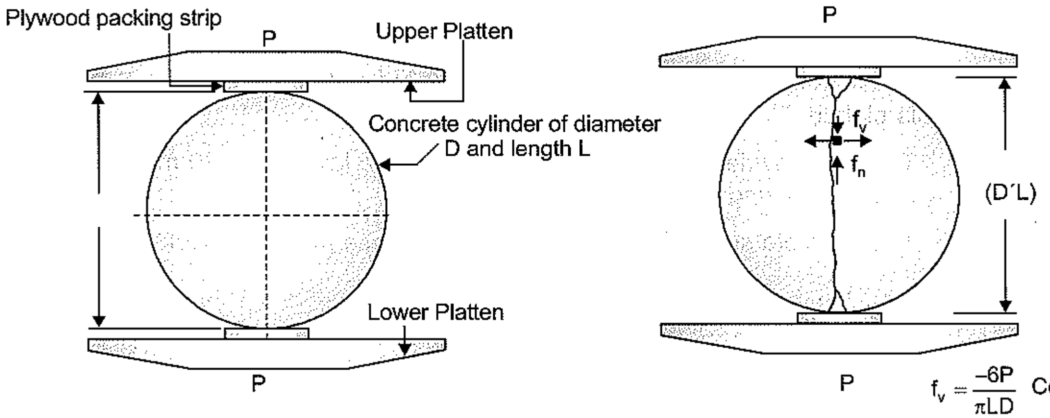
$$\sigma = \frac{2P}{\pi DL}$$

where, P = Applied load

D = Diameter of the cylinder

L = Length of the cylinder

7. The load is applied gradually and increased continuously until failure at a rate within the range of 1.2MPa/min to 2.4MPa/min.
8. Between the loading platens and the specimen cylinder, packing strips of plywood are placed for uniform distribution of load and to avoid high compression stresses near the point of application.



$f_v = \frac{-6P}{\pi LD}$ Compressive

$f_n = \frac{2p}{\pi LD}$ Tensile

Loading Arrangement of Split Tensile Strength Test

(ii) Limitations of the test

1. The test calculates the maximum tensile stress assuming line loads and a uniform distribution of tensile stresses, but concrete has a non-linear stress-strain distribution.
2. The strength of specimen depends upon the diameter of the specimen and hence it is not necessarily a material property, but a reliable value that can be used for comparison and design.

Q–14: ***(i) Describe in brief the rebound hammer method of non destructive testing of concrete.***

OR

(ii) Explain in detail the non-destructive testing of concrete using Rebound hammer.

[10 Marks, ESE-2004, 2010]

Sol: **Rebound Hammer Test**

1. It is done to find out the compressive strength of concrete by using rebound hammer.
2. The principle of the test is that rebound of an elastic mass depends on the hardness of the surface against which it strikes.
3. When the plunger of the rebound hammer is pressed against the surface of the concrete, the spring-controlled mass rebounds and the extent of the rebound depends upon the surface hardness of the concrete.
4. The surface hardness and therefore the rebound are taken to be related to the compressive strength of the concrete.
5. The rebound value is read from a graduated scale and is designated as the rebound number or rebound index. The compressive strength can be read directly from the graph provided on the body of the hammer.
6. The procedure:
   - The rebound hammer should be tested against the test anvil before commencing the test to verify the results.
   - Apply light pressure on the plunger – it will release it from the locked position and allow it to extend to the ready position for the test.
   - Press the plunger against the surface of the concrete, keeping the instrument perpendicular to the test surface. Apply a gradual increase in pressure until the hammer impacts.

- The spring controlled mass when rebounds, it takes with it a rider which slides along a graduated scale. It can be held in position on the scale by depressing the locking button.
- A calibration curve relating to compressive strength of the concrete with the rebound number is plotted.
- The test provides useful information for surface layer up to 30 mm depth and is suitable for concrete having strength of 20-60 MPa.
- The concrete surface must be smooth and loose material should be ground off.

Q–15: ***What is Ferrocement? List the properties of Ferrocement.***

[10 Marks, ESE-2010]

Sol: **A Ferro-cement**

1. The term Ferro-cement implies the combination of ferrous product with cement. Generally this combination is in the form of steel wires meshes embedded in a Portland cement mortar.
2. Ferro-cement reinforcement is assembled into its final desired shape and plastered directly. There is no need for form work. Minimum two layers of reinforcing steel meshes are required.

B. Properties

1. Its strength per unit mass is high.
2. It has the capacity to resist shock load.
3. It can be given attractive finish like that of teak and rose wood.
4. Ferro cement elements can be constructed without using form work.
5. It is impervious

Q–16: ***Explain in detail the ultrasonic pulse velocity method of nondestructive testing of concrete.***

[5 Marks, ESE-2011]

Sol: **Ultrasonic Pulse Velocity Method**

1. The pulse velocity method is a truly nondestructive method, as the technique uses mechanical waves resulting in no damage to the concrete element being tested.
2. A test specimen can be tested again and again at the same location, which is useful for monitoring concrete undergoing internal structural changes over a long period of time.
3. Ultrasonic pulse velocity tests can be carried out on both laboratory-sized test specimens and concrete structures.
4. The basic idea on which the pulse velocity method is established is that the velocity of a pulse of compressional waves through a medium depends on the elastic properties and density of the medium.
5. The transmitting transducer of the pulse velocity instrument transmits a wave into the concrete and the receiving transducer, at a distance L, receives the pulse through the concrete at another point.
6. The pulse velocity instrument display indicates the transit time, t, it takes for the compressional wave pulse to travel through the concrete. The compressional wave pulse velocity V, therefore, is

$$V = L/T$$

7. The factors that affect pulse velocity are-
   - The type and amount of aggregate. In general, the pulse velocity of cement paste is lower than that of aggregate.

- A good contact between transducer and surface has to be maintained otherwise an incorrect pulse velocity reading may result.
- The pulse velocity for saturated concrete is higher than for air-dry concrete.
- Smaller path lengths tend to give more variable and slightly higher pulse velocity because of the inhomogeneous nature of concrete. So, minimum path length of at least 12 inches is recommended.
- The presence of steel reinforcement. The pulse velocity in steel is 1.4 to 1.7 times the pulse velocity in plain concrete. Therefore, pulse velocity readings in the vicinity of reinforcing steel are usually higher than that in plain concrete.
- Whenever possible, test readings should be taken such that the reinforcement is avoided in the wave path. If reinforcements cross the wave path, correction factors should be used.

8. Some major applications-
   - Estimation of Strength of Concrete
   - Establishing Homogeneity of Concrete
   - Determination of Dynamic Modulus of Elasticity

Q–17: ***List the principal constituents of fly ash. Explain its pozzolanic action when used in concrete.***

[10 Marks, ESE-2012]

Sol: The main constituents of fly ash are:

1. Silica 2. Aluminium oxide 3. Ferrous oxide

- Fly ash is a pozzolan. A pozzolan is a siliceous or aluminosiliceous material that, in finely divided form and in the presence of moisture, chemically reacts with the calcium hydroxide released by the hydration of portland cement to form additional calcium silicate hydrate and other cementitious compounds.
- Simply, this reaction can be schematically represented as follows,

$$Ca(OH)_2 + H_4SiO_4 \longrightarrow Ca^2 + H_2SiO_4^{2-} + 2H_2O \longrightarrow CaH_2\ SiO_4 + 2H_2O$$

or, $$CH + SH \longrightarrow C-S-H$$

- The aluminate part will react with calcium hydroxide and water to form calcium aluminate hydrates such as C_4AH_{13}, C_3AH_6 water to form calcium aluminate hydrates such as $C_4\ A\ H_{13}$, $C_3\ AH_6$ or in combination with silica C_2ASH_8 or stratlingite.

Q–18: ***A site is using a concrete where the unit content of water, cement, sand and coarse aggregate is 180 kg/m³, 360 kg/m³, 700 kg/m³ and 1210 kg/m³, respectively. For a portion of the work, the Engineer permits volume batching and rectangular boxes measuring 35 cm by 45 cm have to be fabricated to measure coarse aggregate.***

Assume the following:

(i) The mixer available will mix concrete with one bag of cement (50 kg) at one time.

(ii) 2 (two) boxes of coarse aggregate will be used in a batch (as defined above)

(iii) When filled in a normal manner, the void content in the box is 40%.

(iv) Specific gravity of the coarse aggregate is 2.75 Find the height of the box.

[10 Marks, ESE-2013

Sol: Ratio of water:

Cement : FA : CA d = 180 : 360 : 700 : 1210 = 1 : 2: 3.89 : 6.722.

- At one time cement aceded is 50 kg.
- Coarse aggregate to be added in one batch = $\dfrac{6.722}{168.05\text{kg}}\times 50$ = 168.05 kg
- Actual volume of this coarse aggregate = $\dfrac{168.05\times 10^3}{2.75}$ = 61109.1 cm^3
- If the height of the box is h the volume of that box = V = 45× 35 × h cm^3

 As per the situation in question we can say

$$2 \times 0.6 \times V = 61109.1$$

$$\therefore \qquad h = \frac{61109.1}{2\times 06\times 45\times 35} = 32.33\text{ cm} \simeq 33\text{cm}$$

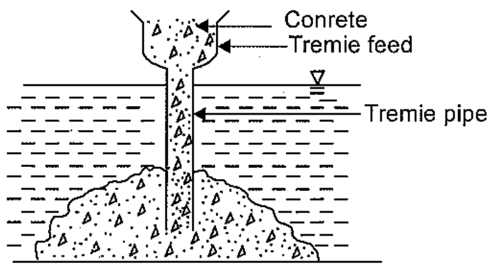
The required height of the bcx is 33 cm

Q–19: ***Explain with appropriate sketches how a 'termie' is used to place concrete underwate.***

[5 Marks, ESE-2013]

Sol: **Tremie Concreting**

- The tremie concrete placement method uses a pipe, through which concrete is placed below water-level. The lower end of the pipe is kept immersed in fresh concrete so that the rising concrete from the bottom displaces the water without washing out the cement content.
- Concrete is often poured through a tremie pipe in order to build caissons, which are the foundations of, among other things, bridges that span bodies of water.



Placing concrete under water

Q–20: ***Corrosion of reinforcement in reinforced concrete construction is matter of serious concern to civil engineers. List and briefly discuss some of the provisions made in codes (such as IS 456-2000) to address this problem during the design and construction of structures likely to be subjected to such deterioration.***

[10 Marks, ESE-2013]

Sol: The major factors influencing the corrosion of steel reinforcement.

(i) The cover to embedded steel

(ii) The cement content and w/c ratio, workmanship.

(i) Cover

- To resist the corrosion of reinforcement steel an optimum depth of cover should be provided.
- If should not be tco thin, otherwise the water will percolate through it.
- it should not be too thick, otherwise on tension side large tension crack in concrete will cause entry of water. codes recommendations on that are:

Exposure	Nominal cover (min)
Mild	20 mm
Moderate	30 mm
Severe	45 mm
Very severe	50 mm
Extreme	75 mm

Structure	Nominal cover (min)
Beams/Slabs	20 mm
Columns	40 mm or size of max ϕ bar
Footing	50 mm

(ii) Cement content and water-cement ratio

- Permeability of concrete depends on the cement content and water cement ratio. Their the corrosion of steel bar also depends on the cement content and water-cement ratio.
- Adequate quantity of cement and low water-cement ratio loads to low permeability and that quantity varies as the exposure condition.

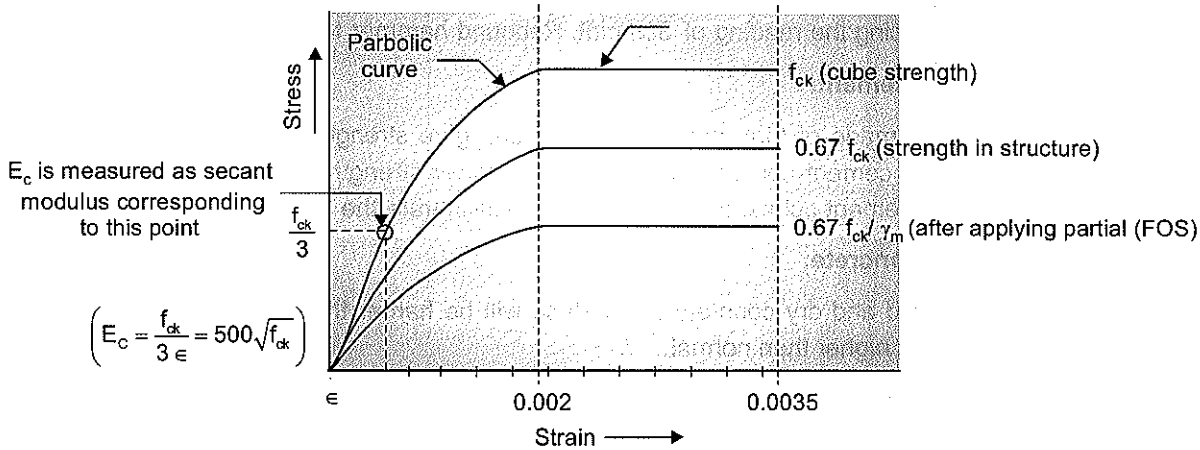
Exposure	Minimum Cement content (Kg/m^3)	Max W/C ratio	min grade of Concrete
Mild	300	0.55	M 20
Moderate	300	0.50	M 25
Severe	320	0.45	M 30
Very severe	340	0.45	M 35
Extreme	360	0.40	M 40

The workmanship is another parameter the compaction should be good to reduce permeability.

Q–21: ***IS 456-2000 suggests use of a certain stress-strain curve of concrete in the absence of actual experimental data. The code also allows use of an expression ($5000\sqrt{f_{ck}}$) to estimate the modulus of elasticity of concrete (E_c). Draw a neat representation of that curve, briefly explain its salient features. The suggested value represents the value of E at which is any. of the points) on the stress-strain curve.***

[5 Marks, ESE-2013]

Sol:



Suggested stress-strain curve for concrete

Q–22: ***Explain the concept of 'Maturity' of concrete.***

[5 Marks, ESE-2013]

Sol:

- The strength of concrete depends upon both the time as well as temperature during the early period of gain in strength.

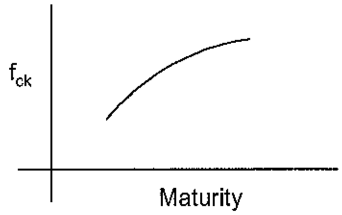
- The maturity of concrete is defined as the summation of product of time and temperature.

  Maturity $= \Sigma(\text{time} \times \text{temperature})$ | unit are °C – hr ; °C days

  e.g. A sample of concrete curved at 18°C for 28 days is taken to be fully matured which is equal to

  M_{28} days = 28 × 24 [18 – (– 11)] = 19488°C hr

  The temperature is reckoned from –11°C as origin since hydration continues to take place upto about this temperature.



Q–23: ***What are some of the steps that can be taken to prevent the formation of 'thermal cracks' in mass concrete?***

[5 Marks, ESE-2013]

Sol:

- In case of mass concreting due to large heat of hydration trapped within concrete, mass tries to expand.
- Hence tensile stress develops outer side and compressive stress develops inner side, and if the value of tensile stress exceeds the limit, then tensile crack generates.
- To reduce this phenomena:

  (i) We can use cement with low heat of hydration.

  (ii) We can add other cementitious material (like fly ash, pozzolana)

  (iii) Site entrapment can be increased to accommodate stress.

  (iv) Externally ice bags can be used to reduce temperature.

Q–24: ***List some of the important factors that are known to affect the readings taken while carrying out non-destructive testing of concrete using Schmidt rebound hammer and explain the effects briefly.***

[5 Marks, ESE-2014]

Sol: Factors influencing the reading of Schmidt. Rebound hammer test

(1) **Type of cement:**

Concrete made of high alumina cement can give strengths upto 100% higher, whereas super sulphated cement concrete can given 50% lower strength compared to calibration obtained on portland cement cubes. It is necessary to recaliberate the hammer for different types of cement.

(2) **Age of concrete:**

- In very old and dry concrete the surface will be harder than the interior giving rebound values somewhat higher than normal.
- New concrete with moist surface generally has a relatively softer surface, resulting in lower rebound value than normal.

(3) **Carbonation of concrete surface:**

Surface carbonation of concrete significantly affect the rebound hammer test result. In old concrete where carbonation layer can be upto 20 mm thick the strength may be overestimated by 50%.

(4) Other factors like smoothness of surface, size and shape of specimen, moisture condition of concrete also affects the readings of Schmidt Rebound Hammer test.

Q–25: ***Explain different types of vibrators and the application of each in construction works.***

[5 Marks, ESE-2015]

Sol: There are four types of vibrators generally used:

1. Needle or internal vibrators
2. Form or external vibrators
3. Vibrating table or platform vibrators
4. Surface vibrators

Needle or internal Vibrators

- It essentially consists of a steel tube (with one end closed and rounded) having an eccentric vibrating element inside it. This steel tube called poker is connected to an electric motor or a diesel engine through a flexible tube.
- The needle is easily moved from place to place, depending on the consistency of the mix.
- They are available in size varying from 40 to 100 mm diameter. The diameter of the poker is decided from the consideration of the spacing between the reinforcing bars in the form-work.
- Internal vibrators are comparatively efficient since all the work is done directly on the concrete, unlike other types of vibrators.
- Internal vibration is generally best suited for ordinary construction provided the section is large enough for the vibrator to be manipulated.

Form or External Vibrators

- These vibrators are clamped rigidly to the form work at the predetermined points so that the form and concrete are vibrated.
- The external vibrators are more often used for pre-casting of thin in-situ sections of such shape and thickness as cannot be compacted by internal vibrators.
- They consume more power for a given compaction effect than internal vibrators.
- These vibrators can compact upto 450mm from the face but have to be moved from one place to another as concrete progresses. These vibrators operate at a frequency of 3000 to 9000 rpm at an acceleration of 4g.

Vibrating Tables or Platform Vibrators

- The vibrating table consists of a rigidly built steel platform mounted on flexible springs and is driven by an electric motor.
- The normal frequency of vibration is 4000 rpm at an acceleration of 4g to 7g.
- The vibrating tables are very efficient in compacting stiff and harsh concrete mixes required for manufacture of precast elements in the factories and test specimens in laboratories.

Surface Vibrators or Screeds

- These consist of vibrating-pan or screed vibrators which vibrate the concrete from the surface - usually at the time the concrete is struck off or screeded.
- These are placed directly on the concrete mass.
- These are best suited for compaction of shallow elements and should not be used when the depth of concrete to be vibrated is more than 150 mm.
- Very dry mixes can be most effectively compacted with such vibrators.
- The surface vibrators commonly used are pan vibrators and vibrating screeds.
- The operating frequency is about 4000 rpm at an acceleration of 4g to 9g.
- Surface vibrators are usually used as screeds for slabs or pavements.

Q–26: ***What is an admixture? List four different mineral and chemical admixtures.***

[10 Marks, ESE-2016]

Sol: Admixtures are chemical compounds other than cement aggregate, water and additives like pozzolana and fibre reinforcement used as an ingredient of concrete or mortar to alter some of its properties.

These are added to concrete to increase workability, reduce water content, accelerate or retardate setting of concrete, increase resistance to chemical attack and/or freezing and thawing, etc.

Types of admixtures:

1. **Chemical admixtures**

(i) **Plasticizer and super plasticizers:** These admixture are water reducers. These chemicals improves workability of the mix so than low water cement ratio can be used to enhance the strength of the mix.

Examples of plasticizers: Lignosulphonic acid & their salts, hydroxy carboxylic acid and their salts.

Examples of super plasticizers: Sulphonated melamine formaldehyde condensates, modified Ligno-sulphonates.

(ii) **Air Entrainers:** These admixtures introduce air bubbles throughout the cement paste. It increase resistance to freezing and thawing and improve workability.

Examples: Wood resins, animal or vegetable fats and oils, aluminium powder etc.

(iii) **Accelerators:** These admixtures speeds up the rate of setting and/or early gain of strength of concrete.

Examples: CaCl2, NaCl, KOH, etc.

(iv) **Retarders:** These admixtures, slow the rate of setting of cement paste and delay the gain of strength of concrete.

Example: calcium sulphate, tartaric acid, etc.

2. **Mineral admixtures**

(i) Fly ash, (ii) Silica fume, (iii) Rice husk ash and (iv) Granulated blast furnance slag

Q–27: ***What do you understand by the term characteristic strength of a material? Please elaborate.***

[8 Marks, ESE-2016]

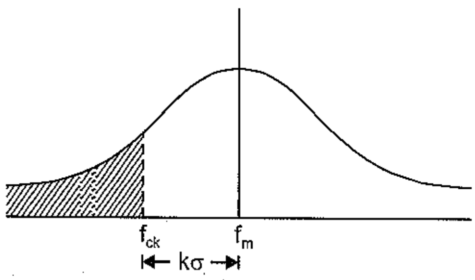
Sol: Characteristic strength of a material is that strength below which not more than 5% of the test results are expected to fall i.e., it is the strength corresponding to 95% confidence limit. Hence there is 95% probability that the strength of material will be greater than or equal to the characteristic strength.

The probability distribution of the strength is given as:

Characteristic strength, $f_{ck} = f_m - k\sigma$

$f_m \rightarrow$ mean strength

$\sigma \rightarrow$ standard deviation



$$\sigma = \sqrt{\frac{\Sigma(f - f_m)^2}{n-1}} \text{ for } n < 30 = \sqrt{\frac{\Sigma(f - f_m)^2}{n}} \text{ for } n \geq 30$$

K = Constant = 1.65 for 95% confidence limit.

Hence, $f_{ck} = f_m - 1.65\sigma$

Q–28: ***(i)*** ***Name the tests to measure workability of fresh concrete.***

(ii) ***What is a super plasticizing admixture?***

(iii) ***What are the three basic qualities of high performance concrete? Discuss the contradictions.***

(iv) ***What is creep of concrete?***

[20 Marks, ESE-2017]

Sol: **(i)** **Test to measure the workability of concrete**

(a) Slump test

(b) Compaction factor test

(c) Consistometer method (Vee Bee method)

(d) Flow test

(ii) **Super Plasticizer**

Super plasticizer is substance that increases the workability at very high value, without loss in the strength of concrete.

When they are used to produce flowing concrete, a rapid loss of workability can be expected and therefore they should be added just prior to placing.

Higher is the molecular mass, higher is the effieciency of super-plasticiser.

But loss of workability is very fast when we mix the plasticizer.

Example: Sulphohated melamine formaldehyde, modified lignosulphate etc.

It is capable of reducing water content by 20 to 40%

(iii) High performance concrete are not special type of concrete but are created by using one ore more cementious material such as fly ash, silica fume or granulated blast furnance slag and usually a super-plasticizer.

Three basic properties of high performance concrete are:

(a) It has very less permeability, toughness and good workability

(b) Energy absorption capacity for earthquake resistant structures

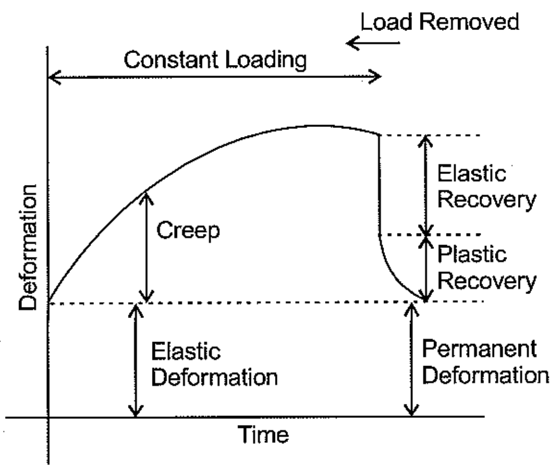
(c) It has high durability and high workability

Contradiction: High performance concrete may not fulfill the criteria of high strength.

(iv) **Creep:** It is continuous deformation with time under constant loading. So, it is basically due to Dead Load and permanent live load.

The long term creep causes change is shape of members due to deformation, which is turn causes stress concentration on the concrete member or high deflections leading to failure in servicibility criteria.

Creep effect can be illustrated by following deformation vs time plot:



Causes of creep are:

(i) High cement content

(ii) Aggregate content is low

(iii) High water cement ratio

(iv) Relative humidity is low

(v) Loading occurs at early age

Q–29: ***What are the various factors that promote the Alkali Aggregate Reaction? How can this be controlled?***

[8 Marks, ESE-2019]

Sol: Alkali aggregate reaction (AAR) is a chemical reaction of alkali in concrete and certain alkaline reactive minerals is aggregate producing a hygroscopic gel which, when moisture present, absorbs water and expand. This gel expansion causes cracking in the concrete.

Factors promoting alkali-aggregate reaction are:

(i) Reactive type aggregate: The petrographic examination of thin rock sections may also immensely help to asses the potential reactivity of the aggregate. This test often requires to be supplemented by other tests.

(ii) High alkali content in cement: The high alkali content in cement is one of the most important factors contributing to the alkali-aggregate reaction.

(iii) Availability of moisture: Progress of chemical reactions involving alkali-aggregate reaction in concrete requires the presence of water. It has been seen in the field and laboratory that lack of water greatly reduces this kind of deterioration.

(iv) Optimum temperature conditions: The ideal temperature for the promotion of alkali-aggregate reaction is in the range of 10 to 38°C. If the temperatures condition is more than or less than the above, it may not provide an ideal situation for the alkali-aggregate reaction.

Q–30: ***The strength of a sample of fully matured concrete is found to be 50 MPa. Find the strength of identical concrete at the age of 7 days when cured at an average temperature of 25°C during day time and 15°C during the night time. Take constants A and B as 32 and 54 respectively. These are the Plowman's Coefficients for Maturity Equation.***

[12 Marks, ESE-2019]

Sol: Maturity of concrete at the age of

$$7 \text{ days} = \Sigma(\text{time} \times \text{temperature})$$

$$M = \Sigma t \times (T - T_c)$$

T_c = Temperature at which maturity of concrete cease (–11°C)

$$= 7 \times 12 \times [25 - (-11] + 7 \times 12 \times [15 - (-11)]$$

$$= 7 \times 12 \times 36 + 7 \times 12 \times 26 = 5208°\text{C hour.}$$

$\therefore$ The percentage strength of concrete at maturity of 5208° C hour $= A + B\log_{10}\dfrac{(5208)}{10^3}$

[$\because$ The value of Plowman's coefficient A and B are 32 and 54 given]

$$= 32 + 54\log_{10}\left(\frac{5208}{1000}\right) = 70.7\%$$

$$\therefore \quad \text{Strength at 7 days} = \left(50 \times \frac{70.7}{100}\right)\text{MPa} = 35.35 \text{ MPa}$$

Q–31: ***Write briefly about the following :***

Role of Flyash as a part replacement of cement.

[10 Marks, ESE-2019]

Sol: **Air entrainers**

- An air-entraining agent introduces air in the form of minute bubbles distributed uniformly throughout the cement paste. The main types include salts of wood resins, animal or vegetable fats and oils and sulphonated hydrocarbons.
- Following are some of the examples of air entraining agents:
  - (i) Natural wood resins and their soaps, of which vinsol resin is the best.
  - (ii) Animal or vegetable fats and oils such as tallow, olive oil and their fatty acids such as stearic acid and oleic acids and their soaps.
  - (iii) Wetting agents such as alkali salts or sulphated and sulphonated organic compounds.
  - (iv) Aluminium powders:
- Air entrainment reduces the strength of concrete and overdosing can cause major loss of strength.
- 1% air may cause a strength loss of 5%.
- The use of ground granulated blast furnace slag (GGBS) and fly ash (FA) tends to reduce the amount of air entrained.

The main consituents of fly ash are:

1. Silca 2. Aluminium oxide 3. Ferrous oxide

Fly ash is a pozzolona. Pozzolana may often be cheaper than the portland cement that they replace but their chef advantage lies in slow hydration and therfore low rate of heat development. hence portaland-pozzolana cement or a partial replacement of portland cement by the pozzolana is used in mass concrete construction.

Q–32: ***Calculate the quantities of ingredients required to produce one cubic meter of structural concrete. The mix is to be used in the proportions of 1 part of cement of 1.42 parts of sand to 2.94 parts of 20 mm nominal size crushed coarse aggregate by dry volumes with a w/c ratio of 0.49 (by mass). Assume the bulk densities of cement, sand and coarse aggregate to be 1500, 1700 and 1600 kg/m³ respectively. The percentage of entrained air is 2.0. Take specific gravity of cement, sand and coarsed aggregate as 3.15, 2.6 and 2.6 respectively.***

[10 Marks, ESE-2019]

Sol: **Given:**

$$\text{Total volume of concrete to be produced} = 1 \text{ m}^3$$

$$C : FA : CA = 1 : 1.42 : 2.94 \text{ (by volume)}$$

$$\text{Water cement ratio} = 0.49 \text{ (by mass)}$$

$$\rho_c = 1500 \text{ kg/m}^3,\ \rho_{FA} = 1700 \text{ kg/m}^3, \rho_{CA} = 1600 \text{ kg/m}^3$$

$$\text{Volume of Air} = 2\%$$

$$G_C = 3.15,\ G_{FA} = 2.6,\ G_{CA} = 2.6$$

Let the volume of cement required to produce 1m³ of concrete (V_c) = x

$\therefore$ Volume of fine aggregates in 1 m³ of concrete ($V_{F.A}$) = 1.41 x and Volume of coarse aggregates in 1 m^3 of concrete (V_{CA}) = 2.94 x

Hence, Mass of cement required (M_C) = 1500 x

Mass of fine aggreate requied (M_{FA}) = 1.42 x × 1700

Mass of coarse aggregate required (M_{CA}) = 2.94x × 1600

Mass of water required (M_w) = 1500 x × 0.49

$\because$ We know that,

$$\frac{M_C}{G_C \rho_W} + \frac{M_{FA}}{G_{FA}\rho_W} + \frac{M_{CA}}{G_{CA}\rho_W} + \frac{M_W}{\rho_W} + V_A = 1$$

$$\frac{1500x}{3.15 \times 1000} + \frac{1.42x \times 1700}{2.6 \times 1000} + \frac{2.94x \times 1600}{2.6 \times 1000} + \frac{1500x \times 0.49}{1000} + 0.02 = 1$$

$$\Rightarrow \boxed{x = 0.248\, m^3}$$

Mass of cement (M_C) = 1500 × 0.248 = 372 kg

Mass of fine aggregate (M_{FA}) = 1.42 × 0.248 × 1700 = 598.67 kg

Mass of coarse aggregate (M_{CA}) = 2.94 × 0.248 × 1600 = 1166.59 kg

Mass of water required (M_W) = 182.28 kg

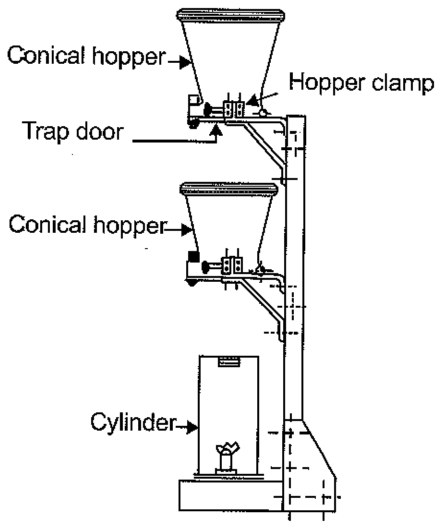
Q-33: ***How is workability of concrete defined as per Indian Standard Specification IS 1199-1959? Briefly explain the method of measurement of workability through compacting factor test.***

[6 Marks, ESE-2020]

Sol: **As per *IS 1199-1959*, workability** is defined as the property of concrete which determines the amount of useful internal work necessary to produce complete compaction.

Compacting Factor test

- This test is more accurate and sensitive than the slump test and it is useful for concrete mixes of medium and low workabilities i.e. compacting factor of 0.9 to 0.8.
- Concrete of very low workability of the order of 0.7 or below, the test is not suitable, because this concrete cannot be fully compacted for comparison in the manner described in the test.
- The test should be made for concrete with maximum size of aggregate $\not>$ 40mm.
- It is primarily designed for laboratory work but can also be used in the field.
- Sample of concrete to be tested is placed gently in the upper hopper, and levelled
- Trap-door is then opened to allow the concrete to fall into the lower hopper.
- Sticked concrete in the upper hopper at sides is gently pushed into lower one.



Compacting Factor Apparatus

- The trap-door of the lower hopper is opened so that the concrete falls in the cylinder.
- The excess of concrete remaining above the level of the top of the cylincer should be cut off and removed.

- Weight of the concrete in the cylinder is then determined to the nearest 10 gm this is known as wt of partially compacted concrete.
- Cylinder is refilled with concrete from the same sample in layers of 50 mm deep, each layer being heavily rammed or preferably vibrated so as to obtain full compaction.
- Top surface of the fully compacted concrete is carefully struck off level with the top of the cylinder.
- The mass of concrete in the cylinder should be measured and it is known as the mass of fully compacted concrete.
- *Compacting factor is defined as ratio of the weight of partially compacted concrete to the weight of fully compacted concrete. i.e.* $\boxed{C.F = \frac{\text{mass of partially compacted concrete}}{\text{mass of fully compacted concrete}}}$
- CF = 0.85 low workability
  CF = 0.92 medium workability
  CF = 0.95 high workability

CHAPTER 3

TIMBER

Q-1: ***What is decay in timber? How is it detected? How can the timber be guarded against decay? Name any two diseases of timber.***

[10 Marks, ESE-1995]

Sol: **A. Decay in Timber**

(i) Timber decay occurs when it loses its value as an engineering material due to deterioration.

(ii) The general causes of decay in timber are the presence of sap, exposure to alternate wet and dry cycles or to moisture accompanied by heat and poor of ventilation.

(iii) Rot in timber is decomposition or putrefaction, generally occasioned by damp, and which proceeds by the emission of gases, chiefly carbonic acid and hydrogen.

(iv) The conditions needed are oxygen, moisture and nutrients, with moisture being the critical component.

(v) If moisture is not present in timber then the fungi will remain dormant, even when oxygen & the nutrients they require are abundant. However, an excess of moisture will prevent the growth of the fungus, but moderate warmth, combined with damp air, accelerates it.

B. Detection of decay

(i) Wood shrinks, darkens and cracks in a 'cuboidal' manner.

(ii) Change in color of wood (brown or white rot).

(iii) Active decay produces a musty, damp odor.

(iv) Appearance of few to numerous small pits.

(v) Soft spots of intense discoloration.

C. Protection of Timber

(i) *Seasoning of Timber*

- It is the process by which moisture content in a freshly cut tree is reduced to a suitable level. By doing so the durability of timber is increased. The various methods of seasoning used may be classified into-
  - ➢ Natural Seasoning
  - ➢ Artificial Seasoning
- Timber needs to be at the same moisture content as it will be in its final use so it will not move or bend once in place, and to have safeguard water in the sap is reduced to deter fungal attack.

(ii) ***Preservation of Timber***

- It refers to the application of treatments (chemicals) to timber to stop the attack of woodworm, fungal decay (wet rot/dry rot) and to protect it from the effects of dampness.
- While chemical treatments add to the cost of the timber, they can significantly increase its lifetime.
- There are three main types of wood preservative: tar oils, waterborne and organic solvent-borne.

D. Diseases of Timber

Some of the diseases are Dry rot, wet rot, sap stain, white rot, brown rot etc.

Q-2: What is meant by the terms : 'seasoning of timber' and 'preservation of timber'? Name the various methods of applying preservatives to timber. Give a brief account of one method.

[10 Marks, ESE-1997]

Sol: A. Seasoning of Timber

(i) It is the process by which moisture content in a freshly cut tree is reduced to a suitable level. By doing so the durability of timber is increased. The various methods of seasoning used may be classified into-

➢ Natural Seasoning

➢ Artificial Seasoning

(ii) Timber needs to be at the same moisture content as it will be in its final use so it will not move or bend once in place, and to have sugars in the sap reduced to deter fungal attack.

B. Preservation of Timber

(i) It refers to the application of treatments (chemicals) to timber to stop the attack of woodworm, fungal decay (wet rot/dry rot) and to protect it from the effects of dampness.

(ii) There are three main types of wood preservative: tar oils, waterborne and organic solvent-borne.

C. Methods of application

The methods of applying preservatives in use include:

- brushing and spreading
- spraying, deluging and fogging
- immersion
- hot and cold steeping in open tanks
- diffusion
- pressure impregnation
- double vacuum

D. Immersion method

(i) Immersion treatment is used with timbers which are to be subjected to a wide variety of hazards.

(ii) The effectiveness of the treatment is related to the time for which the timber is immersed.

(iii) Recommended immersion times ranges from three minutes for many building timbers to several hours for timbers in ground contact or other hazardous situations.

(iv) The clean and dry timbers are totally immersed in a tank of preservative fluid and it is carried out at ambient temperatures, provided these are above freezing.

(v) An exception to total immersion is the butt treatment of fence and other posts, where butt end is immersed to about 300 mm above ground level in preservative for several hours and the remainder by a much shorter immersion or by brush or spray.

(vi) In industrial practice immersion treatments are increasingly carried out in mechanical plant in which several cubic meters of timber are treated at a time and in which the immersion time is fixed by a timing device controlling the immersion mechanism.

Q-3: ***Explain the following: (i) Plywood (ii) Laminated board (iii) Batten board (iv) Fibre board***

[10 Marks, ESE-2001]

Sol: **(i) Plywood**

1. 'PLY' means thin layer. Plywoods are boards which have been prepared from thin layers of wood or veneers.
2. The three or more veneers in odd numbers to get a balanced sheet and reduce warping.
3. Veneers are placed one above the other with the direction of grains of successive layers at right angles to each other.
4. They are held in position by application of a suitable adhesive.
5. Cross-graining reduces the tendency of wood to split when nailed at the edges, it reduces expansion and shrinkage equating to improved dimensional stability, and it makes the strength of the panel consistent across both directions.
6. The outside layers of veneer are called faces, the central layer core and the layers in between the faces and core are called cross bands.
7. They have a wide range of applications like ceilings, furniture, doors, packing cases etc. but are not suitable for applications that involve direct impact.
8. They are available in different forms such as laminated board, batten board, three ply etc.

(ii) Laminated Board

1. It is plywood with a core which consists of strips.
2. The strips do not exceed 7 mm in thickness and are glued together to form a slab. It is in turn glued between outer veneers.
3. The direction of grains of core slab is at right angles to those of adjacent outer veneers.

(iii) Batten Board

1. The core consists of sawn thin wood having thickness of about 20-25 mm.
2. The overall thickness of board is about 50 mm.
3. The direction of grains of core batten is at right angle to those of adjacent outer ply sheets.

(iv) Fiber Board

1. They are rigid boards and are also known as pressed wood or reconstructed wood, and are made from wood fibers.
2. They have a thickness range of 3-12 mm and their weights depend upon the pressure applied during manufacturing.
3. They can be classified as insulating boards, medium hard & super hard boards and laminated boards depending upon their composition and form.
4. They form ideal base for practically all types of decorative finishes such as oil paints.

Q-4: Describe four common defects in the timber.

[10 Marks, ESE-2003]

Sol: Various defects which are likely to occur in timber may be grouped into the following four categories:

A. Defects due to Natural Forces

The following defects are caused by natural forces:

(i) Knots-In the sawn pieces of timber the stump of fallen branches appear as knots. Knots are dark and hard pieces. Grains are distorted in this portion.

(ii) Shakes-The shakes are cracks in the timber which appear due to excessive heat, frost or twisting due to wind during the growth of a tree.

(iii) Wind crack-these are the cracks on the outside of a log due to the shrinkage of the exterior surface

(iv) Upsets-This is due to excessive compression in the tree when it was young. Upset is an injury by crushing. This is also known as rupture.

B. Defects due to Fungi and Insects Attack

(i) Due to fungi attack rotting of wood, takes place. Wood becomes weak and stains appear on it.

(ii) Some of the defects caused by fungi are brown rot, dry rot, blue stain, wet rot, white rot and sap stain.

(iii) Beetles, marine borers and termites (white ants) are the insects which eat wood and weaken the timber.

C. Defects due to Conversion

In the process of converting timber to commercial sizes and shapes the following types of defects are likely to arise:

(i) Chip marks-marks placed by chips on finished surface of timber

(ii) Torn grain-small depression formed on finished surface due to falling of a tool etc.

(iii) Diagonal grain-due to improper sawing

D. Defects due to Seasoning

If seasoning is not uniform, the converted timber may warp and twist in various directions. Some of the defects are:

(i) Twist-Spiral distortion along the length of timber.

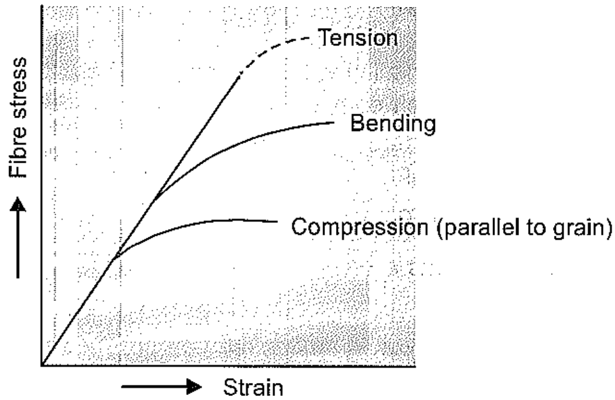
(ii) Cup-A curvature in the transverse direction of the timber

(iii) Bow-A curvature in the direction of its length

Q-5: ***Draw the related stress-strain curves for a timber member under tension, compression and bending parallel to the grains. Discuss the importance of compression test results.***

[10 Marks, ESE-2005]

Sol:



Importance of compression test result

- In compression test two types of strengths are identified along the grain and across the grain.
- Strength parallel to the grain is important for column, props. posts etc, to avoid buckling and failure in bending before crushing.
- Strength of timber across the grain is important in case of beams, sleepers, rollers, wedges bearing blocks etc.

Q-6: ***Explain the methods of improving fire resistance of timber.***

[10 Marks, ESE-2006]

Sol: **A. Fire- resistance of timbers**

(i) Although wood is combustible, timber used in heavy sections attain high degree of fire resistance as it is a bad conductor of heat and hence sufficient heat is required to cause a flame.

(ii) With respect to fire resistance, timber is classified as refractory and non-refractory timber.

(iii) The refractory timber is non-resinous and does not catch fire easily. Some of the examples are Sal, teak etc. However, non-refractory timber is resinous and catches fire readily. Examples include Chir, Deodar etc.

B. Methods to make timber fire resistant-

(i) Application of special chemicals

- A solution of Special chemicals called 'antipyrine' issued to coat the timber to make it more fire resistant.
- When temperature rises, they either melt or give off gases which hinder combustion.
- Such a wood does not inflame even at a higher temperature and merely smolders.
- Antipyrines containing salts of ammonium or boric and phosphoric acids are considered best in making timber fire resistant.

(ii) Sir Abel's Process

- The timber surface is cleaned and is coated with a dilute solution of sodium silicate.
- A cream like paste of slaked fat lime is applied on it.
- Then a concentrated solution of silicate of soda is applied on timber surface.

Q-7: ***Explain the preservative treatment of timber indicating the types, characteristics and methods of applications of preservatives.***

OR

Give a short description of preservation of wood using various wood preservatives.

[10 Marks, ESE-1998, 2008]

Sol: **A. Preservation of timber**

(i) It refers to the application of treatments (chemicals) to timber to stop the attack of woodworm, fungal decay (wet rot/dry rot) and to protect it from the effects of dampness.

(ii) While chemical treatments add to the cost of the timber, they can significantly increase its lifetime.

B. Types of Preservatives

There are three main types of wood preservative-

(i) Tar Oils

- The principal type is creosote which is derived from coal or wood distillation.
- Heavy Creosote Oil is usually applied by vacuum pressure processes; lighter creosotes by dipping, spraying or brushing.
- The main problems are the smell, the risk of tainting, and the emission of toxic smoke if treated timbers catch fire.
- Timber treated with creosote is not suitable for painting, but may be stained after a period of weathering.

(ii) Waterborne Preservatives

- These are usually based on copper, with additions of other pesticides.
- They are applied by high pressure treatment, typically for ground contact uses, e.g. fence posts and decking.
- A different type of waterborne preservative is exemplified by Disodium Octaborate which is applied by a diffusion process to unseasoned wet wood before seasoning.
- These preservatives are not fixed and must be protected both on-site and after installation.
- Also used are micro-emulsion preservatives based on organic biocides, applied by double vacuum treatment, principally for out of ground contact timbers, such as joinery, carcassing etc.

(iii) Organic Solvent Preservatives

- These are based on biocides soluble in hydrocarbon solvents. They have good penetration but also have a strong odor which may taint foodstuffs when fresh.
- They are often flammable and need careful handling until they are dry.
- Treated wood must be allowed to dry before over-painting and this will require from two to seven days with normal ventilation. The solvent will take longer to evaporate in stacked timber.

C. Methods of application

The methods of applying preservatives in use include:

- brushing and spreading
- spraying, deluging and fogging
- immersion
- hot and cold steeping in open tanks
- diffusion
- pressure impregnation
- double vacuum

D. Characteristics of good preservative-

(i) Cheap, durable, easily available and capable of covering a large area with small quantity.

(ii) Free from unpleasant smell

(iii) Strongly toxic to insects, fungi etc.

(iv) Penetrating power into wood fibers should be high (at least for a depth of 6 – 25 mm)

(v) High resistance to the moisture and dampness, and should not be easily washed away by water.

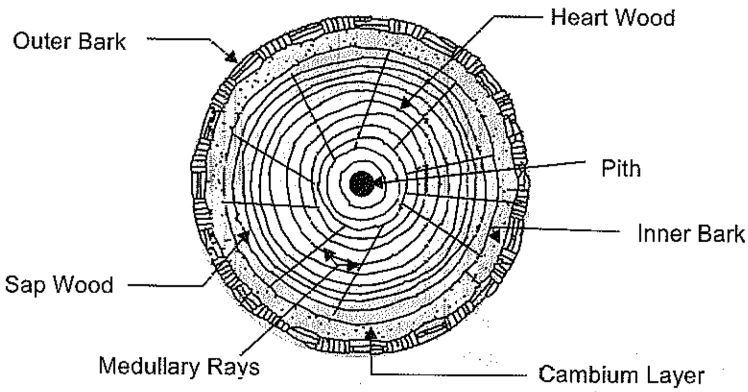
(vi) It should not corrode the metals with which it comes into contact.

Q-8: (i) Draw a neat sketch of macrostructure of exogenous tree.

(ii) Draw neat sketches showing various types of shakes.

[3 + 3 Marks, ESE-2009]

Sol: (i) Macrostructure of exogenous tree



(a) **Pith:** The innermost central portion or core of the tree is called pith or medulla

(b) **Heart wood:** The inner annual ring surrounding the pith is known as heart wood. It imparts rigidity to tree

(c) **Sap wood:** The outer annual rings between heart wood and cambium layer is known as sap wood

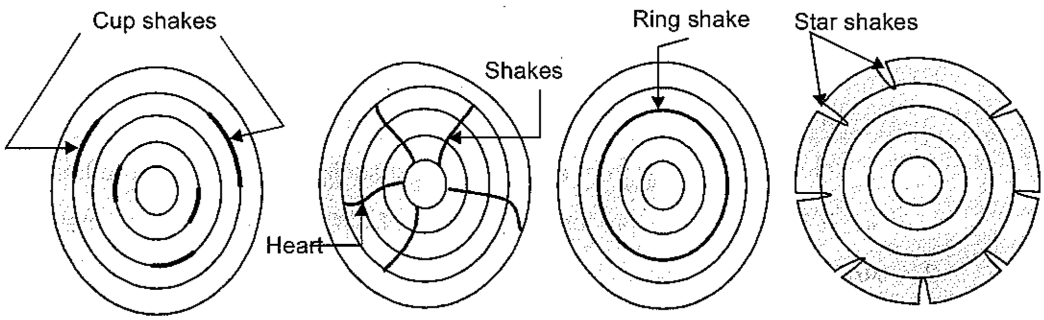
(d) **Cambium layer:** Thin layer of sap between sap wood and inner bark is known as cambium layer

(e) **Inner bark:** The inner skin or layer covering the cambium layer is known as inner bark

(f) **Outer Bark:** The outer skin or cover of the tree is known as outer bark

(g) **Medullary rays:** The thin radial fibers extending from pith to cambium layer are known as medullary rays

(ii) Sketch showing various types of shakes



Cup shakes **Heart Shakes** **Ring shake** **Star shakes**

(a) Cup Shakes- caused by rupture of tissue in a circular direction. It separates one annual ring from the other.

(b) Heart Shakes- occur in center of cross section of tree and they extend from pith to sap wood in direction of medullary rays

(c) Ring Shakes- they occur when cup shakes cover an entire ring.

(d) Star Shakes- they are cracks which extend from bark towards the sap wood.

Q-9: ***(i) What is seasoning of timber and why is it done? Discuss in brief the different methods of seasoning timber.***

OR

(ii) Write briefly about five methods of artificial seasoning of timber.

OR

(iii) Why is seasoning of timber required? List out the methods of seasoning.

[10 Marks, ESE-2004, 2007, 2010]

Sol: **A. Seasoning of Timber:**

1. This is a process by which moisture content in a freshly cut tree is reduced to a suitable level. By doing so the durability of timber is increased.

2. Timber needs to be at the same moisture content as it will be in its final use so it will not move or bend once in place, and to have sugars in the sap reduced to deter fungal attack.

3. Some other objectives of seasoning are:
   - To reduce the weight of timber
   - To make timber easily workable
   - To allow the timber to burn readily if used as fuel

B. The methods of seasoning:

They may be classified into:

1. Natural seasoning

- It is very cheap and simple method in which natural air is used to remove moisture.
- The basic principle is to stack the timber so that plenty of air can circulate around each piece.

- The timber is stacked with wide spaces between each piece horizontally, and with strips of wood between each layer ensuring that there is a vertical separation too.
- Air can then circulate around and through the stack, to slowly remove moisture. In some cases, weights can be placed on top of the stacks to prevent warping of the timber as it dries.
- Over-head cover from effects of direct sunlight and driving weather has to be provided.
- It is a very slow process and drying of different slow process may not be uniform.

2. ***Artificial seasoning***

- In this method timber is seasoned in a chamber with regulated heat, controlled humidity and proper air circulation.
- Therefore, specific conditions for different species can be maintained.

C. Methods for artificial seasoning

1. ***Boiling:*** In this method timber is immersed in water and then water is boiled for 3 to 4 hours. Then it is dried slowly. Instead of boiling water hot steam may be circulated on timber. The process of seasoning is fast, but costly.
2. ***Kiln Seasoning:*** Kiln is an airtight chamber. The process can be summarized as-
   - Timber to be seasoned is placed inside Kiln.
   - Then fully saturated air with a temperature 35°C to 38°C is forced inside it.
   - The heat gradually reaches inside timber. Then relative humidity is gradually reduced and temperature is increased, and maintained till desired degree of moisture content is achieved.
   - The kiln used may be stationary or progressive.
   - In progressive kiln the carriages carrying timber travel from one end of kiln to other end gradually. The hot air is supplied from the discharging end so that temperature increase is gradual from charging end to discharging end. This method is used for seasoning on a larger scale.
3. ***Chemical Seasoning:*** In this method, the timber is immersed in a solution of suitable salt. Then the timber is dried in a kiln. The preliminary treatment by chemical seasoning ensures uniform seasoning of outer and inner parts of timber.
4. ***Electrical Seasoning:*** In this method high frequency alternate electric current is passed through timber. Resistance to electric current is low when moisture content in timber is high. As moisture content reduces the resistance reduces. Measure of resistance can be used to stop seasoning at appropriate level. However it is costly process. This technique has been tried in some plywood industries but not in seasoning of timber on mass scale.

Q-10: Write short notes on: Electrical seasoning of wood

[10 Marks, ESE-2010]

Sol.

Electrical seasoning of wood.

- In this method, use of high frequency alternating current is done.
- Timber when it is green offer less resistance to the flow of electric current and resistance increases heat.

- It is the most rapid method of seasoning but the initial and maintenance cost are such high that it because uneconomical to season timber on commercial base by this method.

Q-11: ***Discuss the factors affecting the strength of timber.***

[5 Marks, ESE-2011]

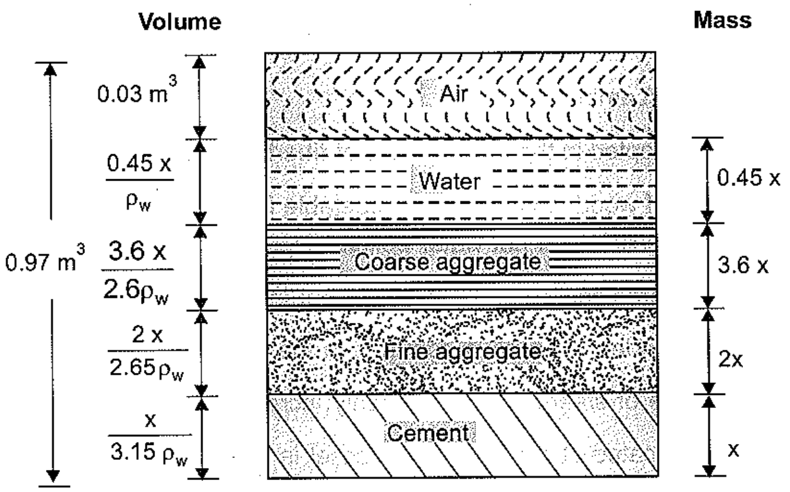
Sol: **Factors affecting strength of timber**

(i) Abnormalities of growth
(ii) Faults in seasoning
(iii) Invasion of insects
(iv) Irregularities of grain
(v) Moisture content
(vi) Presence of knots & shakes etc.
(vii) Way in which a timber piece is cut from the log
(viii) Presence of sap wood
(ix) Upset or rupture due to an injury during growth or bad felling of trees

Q-12: ***For a mix design of proportion 1 : 2 : 3.6 (by mass) with water-cement ratio of 0.45 and air content 3% of the concrete volume, calculate the weight of water, cement, fine aggregate and coarse aggregate to make 1m³ concrete. The specific gravities of cement, F.A and C.A. are 3.15, 2.65, 2.6.***

[10 Marks, ESE-2012]

Sol:



Let the volume of concrete 1 m³

$\Rightarrow$ Vol of air = 0.03 m^3

Let the weight of cement be x kg and unit weight of water be 1000 kg/m³

$\Rightarrow$ Weight of FA = 2x kg

Weight of CA = 3.6x kg

$$\frac{\text{Weight of water}}{\text{Weight of cement}} = \text{w/c ratio} = 0.45$$

$\Rightarrow$ Weight of water = 0.45 x kg

The absolute volume of all the ingredient must add upto 0.97 m^3

$$\Rightarrow \quad \frac{x}{3.15\,\rho_w}+\frac{2x}{2.65\,\rho_w}+\frac{3.6\,x}{2.6\,\rho_w}-\frac{0.45\,x}{\rho_w} = 0.97m^3$$

$$\frac{2.457\,x}{\rho_w} = 0.95$$

$$\Rightarrow \quad x = \frac{0.95\,\rho_w}{2.457}=\frac{0.95\times 1000}{2.457}\text{ kg} = 386.68\text{ kg}$$

$\Rightarrow$ Weight of cement for 1 m^3 of concrete = 386.68 kg

$\Rightarrow$ Weight of FA for 1 m^3 of concrete = 773.366 kg

$\Rightarrow$ Weight of CA for 1 m^3 of concrete = 1392.06 kg

$\Rightarrow$ Weight of water for 1 m^3 of concrete = 174.006 kg = 174.006 litre

Q-13: ***What are the varieties of industrial timber? Indicate the procedure followed for making fibre boards.***

[10 Marks, ESE-2012]

Sol: The timber which is prepared scientifically in a factories termed as the industrial timber and such timber possesses desired shape, appearance, strength etc.

- The line varieties of industrial timber.
  - (i) Veneers
  - (ii) Plywood
  - (iii) Fibreboards
  - (iv) Chipboard/Particle board
  - (v) Block/Batten/Lamin.
- The procedure adopted in the manufacture of fiberboards.
  - (i) The pieces of wood, cone or other vegetable fibres and chippings are collected and they are heated and boiled in a hot water boiler.
  - (ii) The wood fibres separated by heat are put in a vessel.
  - (iii) The steam under pressure is admitted in the vessel
  - (iv) The pressure of steam is then suddenly increased to 7 N/mm^2. (For few seconds only).
  - (v) The valve located at the bottom of vessel is opened and the steam is allowed to exit.
  - (vi) The sudden release of pressure make the wood pieces to explode and in doing so, the natural adhesive contained in the wood fibres is separated out.
  - (vii) The wood fibres are then allowed to flow out.
  - (viii) The fibres are cleaned of all super flows.
  - (ix) Such cleaned fibre are spread an wire seeress in the form of loose sheets or blankets of required thickness.
  - (x) Such loose sheets of wood fibres are prepared b/w steel plates and ultimately, the fibre boards are obtained.

Q-14: ***Describe the defects in timber with the help of neat sketches.***

[5 Marks, ESE-2014]

Sol: Various defects which are likely to occur in timber may be grouped into the following four categories:

A. Defects due to Natural Forces

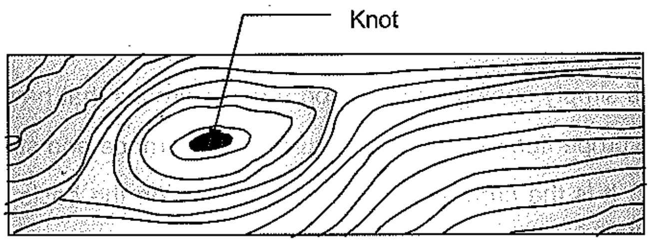
The following defects are caused by natural forces:

(i) Knots- In the sawn pieces of timber the stump of fallen branches appear as knots. Knots are dark and hard pieces. Grains are distorted in this portion.

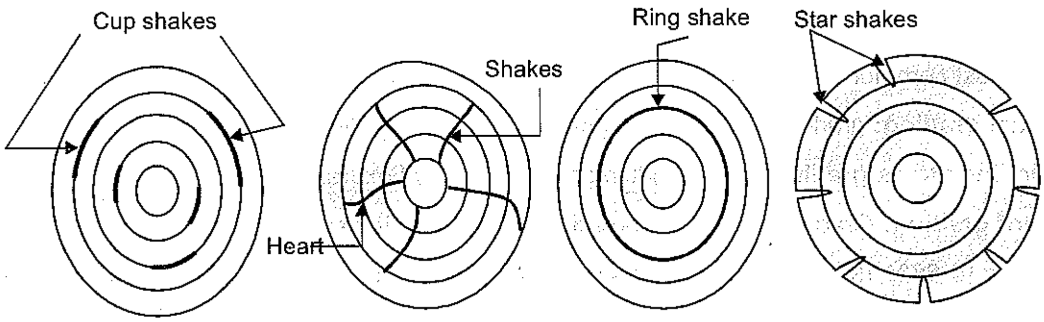
(ii) Shakes- The shakes are cracks in the timber which appear due to excessive heat, frost or twisting due to wind during the growth of a tree.

(iii) Wind crack- these are the cracks on the outside of a log due to the shrinkage of the exterior surface

(iv) Upsets- This is due to excessive compression in the tree when it was young. Upset is an injury by crushing. This is also known as rupture.



Knot



Cup shakes **Heart Shakes** **Ring shake** **Star shakes**

B. Defects due to Fungi and Insects Attack

(i) Due to fungi attack rotting of wood, takes place. Wood becomes weak and stains appear on it.

(ii) Some of the defects caused by fungi are brown rot, dry rot, blue stain, wet rot, white rot and sap stain.

(iii) Beetles, marine borers and termites (white ants) are the insects which eat wood and weaken the timber.

C. Defects due to Conversion

In the process of converting timber to commercial sizes and shapes the following types of defects are likely to arise:

(i) Chip marks- marks placed by chips on finished surface of timber

(ii) Torn grain- small depression formed on finished surface due to falling of a tool etc.

(iii) Diagonal grain- due to improper sawing

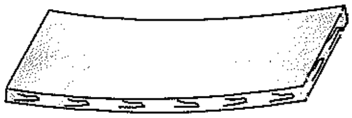
D. Defects due to Seasoning

If seasoning is not uniform, the converted timber may warp and twist in various directions. Some of the defects are:

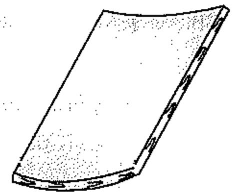
(i) Twist- Spiral distortion along the length of timber.

(ii) Cup- A curvature in the transverse direction of the timber

(iii) Bow- A curvature in the direction of its length



Bow



Cup

Q-15: Elaborate the terms: Grade of timber, location and factor of safety with reference to structural timber.

[10 Marks, ESE-2016]

Sol: **Grading:** Grading is simply sorting a production run into groups that have similar properties.

The grouping of the properties can be any mixture of appearance and structural properties.

Structural grading: It is the process of sorting the timber on the basis of estimates of the structural properties of the timber.

Structural grading uses stress grade which indicate strength and stiffness.

All grading methods uses early measured parameter to correlate with strength and stiffness properties. The most common are.

Visual grading: grade indicator is the presence of visually discernible features.

Machine stress grading: Grade indicator is minor axis flexural stiffeners.

Proof grading: Grade indicator is lower limit of major axis flexural strength.

As per IS 883: 1994, the cut size of structural timber shall be graded into following three grades.

1. Select grade
2. Grade-I
3. Grade-II

Location: Location with reference to structural timber refers to certain locations in the building where permissible stresses (in case of bending, shear and axial loading) shall be established for safety and classification of structural timber.

These locations are as specified below.

Strength
- 1. Parallel to grains
  - (a) Inside location
  - (b) Outside location
  - (c) Wet location
- 2. Perpendicular to grains
  - (a) Inside location
  - (b) Outside location
  - (c) Wet location

Inside location: Position in buildings in which timber remains continuously dry or protected from weather.

Outside location: Position in building in which timbers are occasionally subjected to wetting and drying as in case of open sheds and outdoor exposed structures.

Wet location: Position in buildings in which timbers are almost continuously damp or wet in contact with earth or water such as piles and timber foundation.

Factor of safety: Permissible stresses are obtained by applying appropriate factors to ultimate stress, these factors are called factors of safety.

Q-16: ***What are the various defects in timber? Explain through neat sketches.***

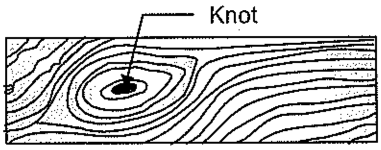
[4 Marks, ESE-2017]

Sol: Defects in timber can be classified as follows:

(a) Defects due to natural forces
(b) Defects due to conversion
(c) Defects due to seasoning
(d) Defects due to fungi.

(a) Various defects due to natural forces are:

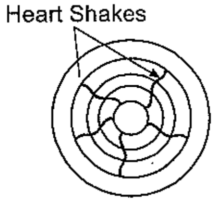
1. **Knots:** Knots are formed in the timber when tree loose its branch



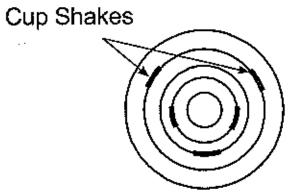
Knot

2. **Shakes:** It is longitudinal cracks in the wood between the annual rings.

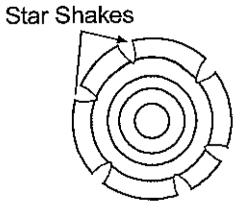
(i) **Heart shakes:** Cracks starts from pith and run towards sapwood. It occurs due to shrinkage of heart wood when tree is overturned



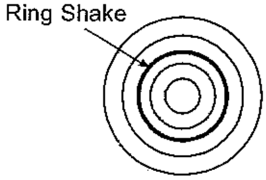
(ii) **Cup shakes:** It appears as curved split which partly or wholly seperates annual rings from one another



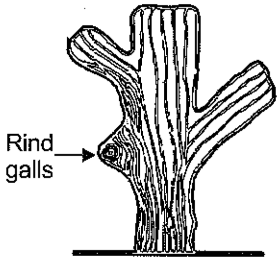
(iii) **Star Shakes :** It is radial split or cracks wide at circumference and diminishing towards the centre.



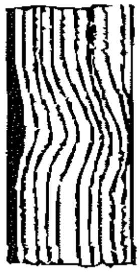
(iv) **Ring shakes:** When cup shakes cover the entire area, they are known as ring shakes



(3) **Rind galls:** The are formed when tree recieves injury in its young age or due to unsuccessful attempts at the formation of branches.



(4) **Upsets (Ruptures):** They indicate the wood fibres which are injured by crushing or compression.



(5) **Twisted fibres:** It is caused by twisting of the young trees constantly in one direction by fast blowing winds.

Twisted

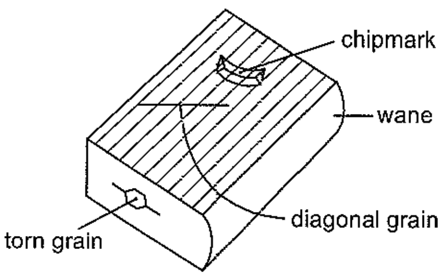
(b) **Various defects due to conversion are:**

(1) **Chip Mark:** Marks placed by chips on the finished surface of timber.

(2) **Diagonal grain:** Formed due to improper sawing

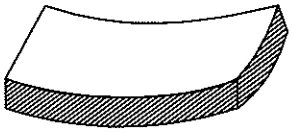
(3) **Torn grain:** Small depression is formed on finished surface of t mber by falling of a tool.

(4) **Wane:** Due to unsound milling pratices. Corner of wood section having some part of bark.

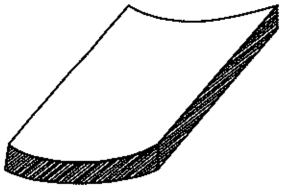


(c) Various defects due to seasoning:

(1) Bow: Timber having curvature along the length and out of plane.

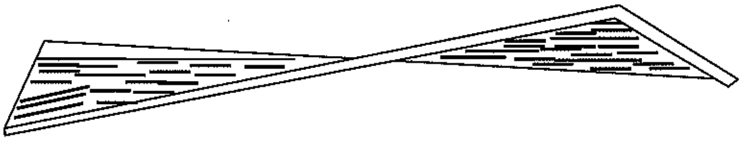


(2) Cup: Timber having curvature along the width and out of plane.



(3) Wrap: Timber having curvature along the length and in the plane

(4) Twist: Curvature both along the length and width.



(d) Various defects due to attack of fungi:

(1) Brown rot: Attack on cellulose from wood and wood attains brown colour

(2) White rot: Attack on lignin of wood and wood attains white mass consisting of cellulose.

(3) Dry rot: Attack on sapwood by fungus due to absence of sunlight, dampness, presence of sap and stagnant air.

(4) Wet rot: Fungi causes chemical decomposition of timber. This is caused by alternate dry and wet conditions.

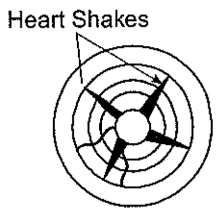
Q-17: **Explain the following defects in timber with neat sketches:**

(A) Shakes *(B) Knots*

[4 Marks, ESE-2019]

Sol: (A) Shakes

- It is longitudinal separations (cracks) in the wood between the annular rings.
- This lengthwise separations reduce the allowable shear strength without much effect on compressive and tensile strength.
- Wood appearance becomes undesirable.



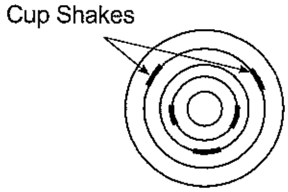
Heart Shakes

(i) ***Heart Shake:***

- It occurs due to shrinkage of heartwood, (interior of a tree) when tree is overmatured.
- Cracks start from pith and run towards sapwood.
- These are wider at centre and diminish outwards.

(ii) Cup ***Shake:***

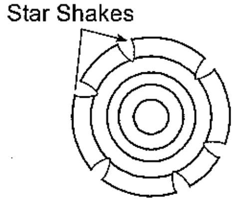
- It appears as curved split which partly or wholly separates annual rings from one another.
- It is caused due to excessive frost action or non-uniform growth.



Cup Shakes

(iii) ***Star Shake:***

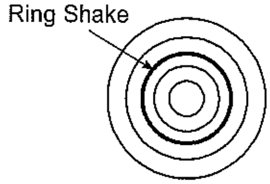
- It is radial splits or cracks wide at circumference (bark) and diminishing towards the centre of the tree.
- It is confined usually to sapwood thus giving star apperance at the end of a piece.
- This may arise from severe frost and fierce heat of sun. Star shakes appear as the wood dries below the fibre saturation point.
- It is fault leading to separation of log into number of pieces when sawn.



Star Shakes

(iv) ***Ring Shake:***

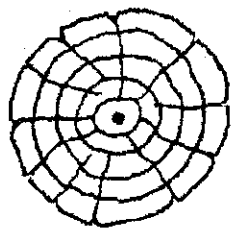
- When cup shakes cover the entire ring, they are known as the ring shakes.



Ring Shake

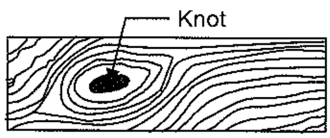
(v) ***Radial Shake:***

- These are similar to star shakes. But they are fine, irregular and numerous.
- This split starts from bark and sapwood and extends to the heartwood and pitch.
- These defect occurs when outer tissues dry at faster rate than inner ones. this defect can also occur during seasoning process due to excessive heart of sun or cold of frost.


Radial shake

(B) ***Knots***

- These are the bases of branches or limbs which are broken or cut, encased by the wood of the free trunk.
- Knots are formed in timber when tree lose its branch. When the tree grows further and puts on more wood, the stumps of these branches which have fallen off are covered and appear as knots in the sawn pieces of timber.
- As continuity of wood fibres is broken by knots they form a source of weakness and reduces the workability of timber.



Knot

Q-18: ***Discuss in brief the methods of preserving timbers by water soluble preservatives.***

[12 Marks, ESE-2020]

Sol: Water-Soluble Preservative

(i) Water soluble (Leachable) Type

Leachable preservatives are those whose amount in the treated material gets gradually diminish owing dissolving property of water. Remedy is to provide water proof paint coating on treated material and properly having maintained. Generally these are odourless having less chances of fire hazard and futher treated material can be varnished or waxed. examples are Boric acid and Borax, sodium fluoride, sodium pentachlorophenate (Na-PCP), Gamma-BHC, Chlorpyrites and Synthetic Pyrethroids.

(ii) Water Soluble (fixed) Type

These preservatives consist of a mixture of various salts, effective against variety of organisms. Fixative salts is usually sodium or potassium dichromate. The chromium fixes the toxic elements such as arsenic, copper and boron in timber so it becomes difficult for them to leach by the action of water. period of drying in fixation process is 2–3 weeks. These should be applied in cold as these will precipitated when heated by specially presence of any reducing substance in timber.

Example are copper-chrome-Arsenic Composition (CCA), Acid cupric-chromate Composition, Copper-Chrome Boron Composition (CCB), Zinc-Meta Arsenite, Borated Copper-Chrome-Arsenic Composition (BCCA), Ammonical Copper Arsenite (ACA).

CHAPTER 4

BRICKS

Q-1: ***How are bricks classified as per IS code? What are the properties associated with tis classification?***

[10 Marks, ESE-1998]

Sol: **A. Classification criteria**

As per IS: 1077-1992 code, bricks are classified on the basis of minimum compressive strength which has been kept at 3.5 MPa.

B. Properties governing quality of bricks:

(i) **Water absorption**- According to IS: 3495-1992, a good brick when immersed in water for about 24 hours, must not absorb water more than 20% by weight up to class 12.5 MPa and 15% by weight for higher classes.

(ii) **Crushing strength**- as per IS: 1077-1992, it is found by using compression testing machine. It must not have a crushing strength less than 3.5N/mm^2.

(iii) **Hardness**- No impression should be left on the brick surface when scratched with a finger nail.

(iv) **Efflorescence**- It is said to be free from soluble salts when a brick immersed in water for 24 hours is allowed to dry in shade, does not show grey or white deposits on its surface.

(v) **Dimensions and tolerance**- as per IS:1077-1992, the standard size of common building bricks is as:

- 19cm X 9cm X 9cm
- 19cm X 9cm X 4cm

(vi) **Soundness**- Two bricks are struck with each other. They should not break and should produce a clear ringing sound.

Q-2: ***Write short notes on the following bricks:***

(i) Table moulded *(ii) Refractory*

(iii) Pressed *(iv) Machine moulded*

[10 Marks, ESE-2001]

Sol: **(i) Table molded bricks**

(a) A table of size 2m X 1m is used by the molder

(b) The clay, mold, water pots, stock board, strikes and pallet boards are kept on this table.

(c) The bricks are molded on it and then sent for drying.

(d) The efficiency of molder decreases gradually because of standing at the same place for a long time

(e) The cost also increases when this method is adopted.

(ii) Refractory

(a) They are also known as fire bricks and are made from fire clay

(b) The process of manufacturing is same as ordinary bricks but the burning and cooling of these bricks is done gradually.

(c) They are white or yellowish white in color and can withstand high temperature without losing their shapes.

(d) They are used for linings of interior surfaces of furnaces, chimneys, kilns, ovens and fireplaces etc.

(e) The compressive strength of these bricks varies from 200-200 Mpa and percentage absorption from 5-10.

(f) There are three types of fire bricks:

- Acidic bricks
- Basic Bricks
- Neutral Bricks

(iii) Pressed bricks

(a) Moist powdered clay is subjected to a great pressure of about 40 kg/cm^2 to make these bricks.

(b) They do not require drying and are burnt directly

(iv) Machine molded

(a) The molding is done with the help of machines

(b) It is economical when bricks have to be manufactured in huge quantities at the same spot in short time

(c) It is also helpful for molding hard and strong clay.

(d) The molding machines are broadly classified into two groups-

- Plastic Clay machines
- Dry Clay machines

Q-3: Explain absorption and saturation factors with regard to bricks.

[10 Marks, ESE-2002]

Sol: Absorption can be broken into two distinct categories:

- Absorption.
- Initial rate of absorption (IRA).

Both are important in selecting the appropriate brick.

A. Absorption of a brick

(i) It is expressed as a percentage, and defined as the ratio of the weight of water that is taken up into its body divided by the dry weight of the unit.

(ii) Water absorption is measured in two ways:

- **Cold water test:** An oven dried specimen of known weight (W_1) is kept immersed in water at room temperature (27 ± 2°C) for 24 hours. The specimen is then removed and is weighed (W_2). The water abscrbed is given by-

$$W_{24} = [(W_2-W_1)/ W_1] \times 100$$

- **Boiling water test:** An oven dried specimen of known weight (W_1) is kept immersed in a tank and water is heated to boiling in one hour and bciled continuously for 5 hours. The water is then allowed to cool to room temperature by natural loss of heat for 16-19 hours. The specimen is again weighed (W_3). The water absorption is given by-

$$W_5 = [(W_3-W_1)/ W_1] \times 100$$

(iii) These two are usec to calculate the saturation coefficient by dividing the 24-hour cold-water absorption (W_{24}) by the five-hour boiling water absorption (W_5). The saturation coefficient is used to help predict durability.

B. Initial rate of absorption

(i) The initial rate of absorption or suction is the rate of how much water a brick draws in during the first minute after contact with water.

(ii) The suction has a direct bearing on the bond between brick and mortar.

(iii) When a brick has high suction, a strong, watertight joint may not be achieved. High suction brick should be wetted for three to 24 hours prior to laying to reduce the suction and allow the brick's surface to dry.

(iv) Very low suction brick should be covered and kept dry on the jobsite.

Q-4: ***List four important tests conducted on the bricks. Describe briefly the method used to determine compressive strength of bricks.***

[10 Marks, ESE-2003]

Sol: **A. Four important tests conducted on bricks:**

(i) Compressive strength (ii) Efflorescence

(iii) Water absorption (iv) Soundness test

B. Compressive strength test

(i) Any unevenness observed on the bed faces of brick is removed by grinding to provide two smooth parallel faces.

(ii) It is then immersec in water at room temperature for 24 hours.

(iii) The frog and all voids in the bed faces of brick are filled flush with cement mortar (1:3)

(iv) It is then stored under the damp jute bags for 24 hours followed by immersion in clean water for 3 days.

(v) The specimen is then placed with flat faces horizontal and mortar filled face facing upwards between plates of the testing machine.

(vi) Load is then applied axially at a uniform rate of 14 MPa per minute till failure occurs and maximum load at failure is noted.

(vii) Compressive strength is given by

$$\text{Compressive strength} = \frac{\text{Maximum load at failure}}{\text{loaded area of brick}}$$

(viii) Average of five results is noted.

Q-5: ***Describe in brief the classification of ordinary bricks according to their qualities into four categories.***

[10 Marks, ESE-2004]

Sol: Ordinary bricks are classified into following four categories as per their qualities:

A. **First class Bricks:** These bricks are of standard shape and size and are table molded. They are burnt in kilns. They fulfill all desirable properties of good bricks i.e. water absorption, compressive strength, hardness etc. They are used for high quality work of permanent nature.

B. **Second Class Bricks:** These bricks are ground molded and burnt in kilns. The edges may not be sharp and uniform. The surface may be somewhat rough. Such bricks are commonly used for the construction of walls which are going to be plastered.

C. **Third Class Bricks:** These bricks are ground molded and burnt in clamps. Their edges are somewhat distorted. They produce dull sound when struck together. They are used for temporary and unimportant structures.

D. **Fourth Class Bricks:** These are the over burnt bricks. They are dark in color. The shape is irregular. They are used as aggregates for concrete in foundations, floors and roads etc. due to the fact that they have compact structures and sometimes are found to be stronger than even first class bricks.

Q-6: ***What are the properties governing the quality of bricks? Discuss the importance of water absorption and strength under compression.***

[10 Marks, ESE-2005]

Sol: The properties governing the quality of bricks.

(i) **Size and shape:** Bricks should have uniform size and plane, rectangular surface with parallel sides and sharp straight edges.

(ii) **Colour:** Bricks should have a uniform deep red or cherry colour as indicative of uniformity in chemical composition and thoroughness in the burning of the brick.

(iii) **Texture and compactness:** Surface should not be too smooth to cause slipping of mortar. The brick should have precompact and uniform texture. A fractured surface should not show fissures, hales grits or lumps of line.

(iv) **Hardness:** Brick should be so hard that when scratched by a finger nail no impression is made.

(v) **Soundness:** When two bricks are struck together, a metallic sound should be produced.

(vi) Water absorption $\leq$ 20 per cent of its dry weight when kept immersed in water for 24 hours.

(vii) Crushing strength $\geq$ 10 N/mm^2.

(viii) Brick earth should be free from stones, grit, organic matter etc.

- The importance of water absorption of bricks depends on their porosity which is due to voids of various size present in the bricks.
- Almost all bricks absorbs water by capillary action.
- But porosity and water absorption does not give proper indication as to whether brick work can keep away the rain water and protect the interior from dampness travelling from outside.
- Permeability measures the travel of water through a brick.
- Percentage of water absorption gives indication of compactness which is obtained from burning.

- The importance of brick strength under compression because bricks are often subjected to large compressive stresses.
- Compressive stresses of brick provides a basics of composition of quality of bricks, but it is of little value in determining the strength of well, or strength of wall mainly depends on the strong of water.
- Common building bricks shall have minimum strength of 35 kg f/cm^2. (3.5 N/mm^2)

Q-7: ***Write about the following tests on clay bricks and mention the desired test limits as per Indian Standards:***

(i) Water absorption ***(ii) Crushing strength***

(iii) Hardness ***(iv) Soundness***

(v) Presence of soluble salts

[10 Marks, ESE-2007]

Sol: Various tests done on clay bricks are as under:

(i) Absorption test: This test is performed as per IS: 3495-1992. It has to be performed in both cold water (for 24 hours) and boiling water (for 5 hours) as water absorption in bricks occurs in pores and sometimes these pores are completely sealed and hence inaccessible to water under ordinary conditions. It must not absorb water more than 20% by weight up to class 12.5 MPa and 15% by weight for higher classes.

(ii) Crushing strength- It is performed as per IS: 1077-1992. It is found by using compression testing machine where it is pressed till it breaks. It must not have a crushing strength less than 3.5 N/mm^2.

(iii) Hardness- It is performed as per IS No impression should be left on the brick surface when scratched with a finger nail.

(iv) Soundness: Two bricks are struck with each other. They should not break and should produce a clear ringing sound.

(v) Presence of soluble salts-The presence of alkalis in brick is not desirable because they form patches of gray powder by absorbing moisture. Hence to determine the presence of alkalis this test is performed by placing the brick specimen in a glass dish containing water to a depth of 25 mm in a well-ventilated room. After all the water is absorbed or evaporated again water is added for a depth of 25 mm. After second evaporation the bricks are observed for white/grey patches. The observation is reported as 'nil', 'slight', 'moderate', 'heavy' or serious to mean:

(a) **Nil:** No patches

(b) **Slight:** 10% of area covered with deposits

(c) **Moderate:** 10 to 50% area covered with deposit but unaccompanied by flaking of the surface.

(d) **Heavy:** More than 50 per cent area covered with deposits but unaccompanied by flaking of the surface.

(e) **Serious:** Heavy deposits of salt accompanied by flaking of the surface.

Q-8: ***What are the principal constituents of brick earth and how do they influence the quality of brick?***

[10 Marks, ESE-2008]

Sol: The principle constituents of brick earth are as under:

A. Silica

(i) The greatest in bulk, is practically sand i.e. in an uncombined state. It is infusible, whether alone or with alumina, without a small quantity of flux in the form of lime or oxide of iron.

(ii) It acts as a preventative to cracking, shrinking, or warping.

(iii) Up to a certain proportion the more silica there is the better the shape and more even the texture of the resulting brick.

(iv) An excess of silica renders a brick too brittle.

(v) It increases the refractoriness of low alumina clays

B. Alumina

(i) It is the principal and most important constituent of good clay, as it absorbs water and imparts the plastic qualities so that the brick can be molded.

(ii) However, it shrinks, cracks, and warps very considerably under the influence of heat, which renders it very hard. So it must not be present in excess quantity than specified. A good brick earth contains about 20-30% alumina.

(iii) Its presence also provides refractory properties to bricks.

C. Lime

(i) It may be called a flux, though its presence in the bulk has a double effect: it both diminishes the contraction in the process of drying the raw material, and it blends the silica and alumina together in the burning.

(ii) The carbonate of lime lowers the fusion point and must be present in very small quantities, comminuted and equally distributed throughout the mass

(iii) If it exists in lumps it will be slaked by moisture, and cause the disintegration of the brick, whether laid or not in the finished work.

D. Oxide of Iron

(i) It is also a flux, and in the presence of nearly equal quantities of silica and alumina.

(ii) It is the coloring matter of most kinds of bricks, the intensity of the color (from a light yellow to a dark red) being in proportion to the quantity of iron oxide present.

(iii) With 8 or 10 per cent of oxide of iron, the color is a dark blue or purple, and the addition of a small proportion of manganese gives almost a black color to the brick; and with lime the two impart a cream color, the one darkening and the other lightening the shade. Magnesia and iron oxide make a yellow brick.

(iv) It also improves impermeability and durability of bricks, besides imparting strength and hardness.

E. Magnesia

(i) It is rarely present in excess of 1% and imparts yellowish color to brick in combination with oxide of iron.

(ii) It slows down the rate of softening of clay during burning and reduces warping.

(iii) Excess of magnesia causes decay of bricks.

Q-9: ***Write briefly on the composition & properties of refractory bricks.***

[6 Marks, ESE-2009]

Sol:

- Refractory bricks are defined as non metallic material suitable for lining of furnaces operated at high temperature.
- There bricks are made from fire clay or refractory clay.

Fire/Refractory clay

- Fire-clay is a term, loosely applied, to include their sedimentary or residual clays which fuse at a very high temperature and which, when so burnt passes great resistance to heat.
- There are pure hydrated silicates of alumina and contain a large proportion of silica 55–75% alumina 20–35%, iron oxide 2–5% with about 1 per cent of lime, magnesia and alkalis.
- If greater is the percentage of alumina, more refractory the clay is found to be.

Properties of refractory clay.

- The colour is whitish yellow or light brown.
- The water absorption of fire-clay bricks varies from 4-10%.
- The minimum average compressive strength of the bricks should be 3.5 N/mm^2.

Q-10: ***Write short notes on:***

(i) Harmful ingredients in brick earth

(ii) Tests on brick to assess its suitability-list the names

[3+3 Marks, ESE-2010]

Sol: (i) **Harmful ingredient in brick earth.**

(a) **Lime:** If lime is present in excess, colour of bricks changes from red to yellow.

- When lime is present in lumps, it absorbs moisture, swells and causes disintegration of the bricks.

(b) **Iron Pyrites:** If iron pyrites are present n brick earth, then bricks tend to oxidize and decompose during burning and may split into pieces.

- Pyrites discolourize the bricks.

(c) Pebbles, Gravels, Grits

- It does not allows clay to be mixed uniformly and thoroughly which will result in weak and porous bricks.

(d) **Alkalies:** Excess of alkalies causes brick to melt and lack their shape.

- There causes efflorescence - when bricks come in contact with moisture, water is absorbed and the alkalies crystallize.

(e) **Organic matter:** During burning of bricks, organic matter gets burnt completely, leaving behind pores and have making bricks porous.

(f) **Water:** A large amount of free water causes shrinkage of bricks on drying, whereas combined water causes shrinkage during burning.

(g) **Sulphur:** It sulphur is present in brick earth and in sufficient time is given (decreasing burning) for oxidation of carbon and sulphur then sulphur will cause the formation of a spongy, swollen structure in the brick over the brick will be discoloured by white blotches

(ii) Test on brick to asses its suitability list the names.

(a) Dimension Test (IS:1077)

(b) Water Absorption test (IS: 3495-Part III)

(c) Compressive strength test (IS:3495-Part I)

(d) Warpage Test (IS: 3495–Part IV)

(e) Efflorescence Test (IS: 3495–Part III)

Q-11: Briefly explain the processes involved in the manufacture of bricks in order.

[5 Marks, ESE-2011]

Sol: The processes involved in manufacturing of bricks are as under:

A. Preparation of clay-

It consists of following steps:

(i) **Unsoiling:** the top layer of soil (about 200 mm in depth) is taken out and thrown away as it contains impurities.

(ii) **Digging:** the clay is then dug out and spread on a levelled ground

(iii) **Cleaning:** clay is cleaned of stones, pebbles, organic matter etc.

(iv) **Weathering:** clay is exposed to atmosphere for softening.

(v) **Blending:** the clay is made lose and any ingredient to be added is spread out at its top and the intimately mixed.

(vi) **Tempering:** water is added to clay and whole mass is kneaded under feet of men or cattle. To obtain a homogeneous mass.

B. Moulding: it is the process of filling the prepared clay into moulds of definite size and pattern. It can be done using hand moulding or machine moulding technique.

C. Drying: Moulded bricks are dried to remove any moisture from the clay as damp bricks are likely to crack if burnt. The bricks are stacked with proper spacing to allow circulation of air and dried till moisture content is reduced to 2% or so.

D. Burning: it imparts hardness and strength to the bricks and makes them dense and durable. It is done either in clamps, which are temporary structures adopted to manufacture bricks on small scale or in Kilns, which are permanent and manufacture bricks on a large scale.

Q-12: Briefly describe the various defects in bricks.

[5 Marks, ESE-2011]

Sol: Defects of Bricks

(i) Over-Burning of Bricks

- If the bricks are over burnt, a soft molten mass is produced and the bricks loose their shape. Such bricks are not used for construction works.

(ii) Under-Burning of Bricks

- When bricks are not burnt properly, the clay is not softened because of insufficient heat and the pores are not closed.
- This results in higher degree of water absorption and less compressive strength
- Such bricks are not recommended for construction works.

(iii) Bloating

- This defect is observed as spongy swollen mass over the surface of burned bricks.
- It is caused due to the presence of excess carbonaceous matter and sulphur in brick-clay.

(iv) Black Core

- When brick-clay contains bituminous matter or carbon and they are not completely removed by oxidation, the brick results in black core mainly because of improper burning.

(v) Efflorescence

- This iscaused because of alkalies present in bricks.
- When bricks come in contact with moisture, water is absorbed and the alkalies crystallize.
- After drying grey or white powder patches appear on the brick surface. This can be reduced by selecting proper clay materials for brick manufacturing, preventing moisture to come in contact with the masonry, by providing waterproof coping and by using water repellent materials in mortar and by providing damp proof course.

(vi) Chuffs

- Deformation of the shape of bricks caused by the rain water falling on hot bricks is known as chuffs.

(vii) Checks or Cracks

- This is because of lumps of lime or excess of water.
- In case of lime, when bricks come in contact with water, the absorbed water reacts with lime nodules causing expansion and a consequent disintegration of bricks, whereas shrinkage and burning cracks result when excess of water is added during brick manufacturing.

(viii) Spots

- If sulphide, is present in the brick clay, it cause dark surface spots on the brick surfaces. Such bricks are not only harmful but also unsuitable for exposed masonry work.

(ix) Blisters

- Broken blisters are generally caused on the surface of sewer pipes and drain tiles due to air imprisoned during their moulding.

(x) Laminations

- It is by the entrapped air in the voids of clay. Laminations produce thin lamina on the brick faces which weather out on exposure.
- Such bricks are weak in structure.

Q-13: (i) Explain various defects in bricks.

OR

(ii) Briefly describe the various defects in bricks.

[10 Marks, ESE-2011]

Sol: The various defects in bricks are as:

A. Efflorescence: If the bricks are manufactured from earth containing excessive soluble salts. These salts dissolved in water (due to rain or due to the entry of moisture) and appear in the form of fine whitish crystals on the exposed brick surface. This is known as efflorescence. The masonry surface will give an ugly appearance.

B. **Over-burning of bricks:** Bricks should be burned at temperatures at which incipient, complete and viscous vitrification occur. However, if the bricks are over-burnt, a soft molten mass is produced and the bricks lose their shape. Such bricks are not used for construction works.

C. **Under-burning of bricks:** When bricks are not burnt to cause complete vitrification, the clay is not softened because of insufficient heat and the pores are not closed. It results in higher degree of water absorption and less compressive strength. Such bricks are not recommended for construction works.

D. **Bloating:** This defect observed as spongy swollen mass over the surface of burned bricks is caused due to the presence of excess carbonaceous matter and sulfur in brick-clay.

E. **Black core:** When brick-clay contains bituminous matter or carbon and they are not completely removed by oxidation, the brick results in black core mainly because of improper burning.

Q-14: ***Discuss the properties imparted to brick-earth by Alumina and silica.***

[10 Marks, ESE-2012]

Sol: **Alumina:** The properties imported to brick earth by its constituents alumina.

- A good brick earth should contain
- Alumina about 20% to 30%.
- It imparts plasticity to the earth so that it can be moulded.
- If alumina is present in excess, with inadequate quantity of sand, the raw bricks. Shrink and warp drying and burning and becomes to hard when burnt.

Silica: A good brick earth should contain about 50% to 60% of silica

- The presence of this constituent prevents cracking, shrinkage and warping of raw bricks.
- It thus imports uniform shape to the bricks.
- The durability of bricks depends on the proper proportion of silica in brick earth.

Q-15: ***Give a detailed procedure for determining the compressive strength of bricks as per I.S. code. Also write about water absorption test. Mention the usual limit.***

[5 Marks, ESE-2015]

Sol: **Compressive strength test**

(i) Any unevenness observed on the bed faces of brick is removed by grinding to provide two smooth parallel faces.

(ii) It is then immersed in water at room temperature for 24 hours.

(iii) The frog and all voids in the bed faces of brick are filled flush with cement mortar (1:3)

(iv) It is then stored under the damp jute bags for 24 hours followed by immersion in clean water for 3 days.

(v) The specimen is then placed with flat faces horizontal and mortar filled face facing upwards between plates of the testing machine.

(vi) Load is then applied axially at a uniform rate of 14 MPa per minute till failure occurs and maximum load at failure is noted.

(vii) Compressive strength is given by

$$\text{Compressive strength} = \frac{\text{Maximum load at failure}}{\text{loaded area of brick}}$$

(viii) Average of five results is noted.

Absorption of a brick

(a) It is expressed as a percentage, and defined as the ratio of the weight of water that is taken up into its body divided by the dry weight of the unit.

(b) Water absorption is measured in two ways:

- **Cold water test:** An oven dried specimen of known weight (W_1) is kept immersed in water at room temperature ($27 \pm 2°C$) for 24 hours. The specimen is then removed and is weighed (W_2). The water abscrbed is given by-

$$W_{24} = [(W_2 - W_1)/ W_1] \times 100$$

- **Boiling water test:** An oven dried specimen of known weight (W_1) is kept immersed in a tank and water is heated to boiling in one hour and boiled continuously for 5 hours. The water is then allowed to cool to room temperature by natural loss of heat for 16-19 hours. The specimen is again weighed (W_3). The water absorption is given by-

$$W_5 = [(W_3 - W_1)/ W_1] \times 100$$

Class A brick should not absorb water greater than 15%

Class B brick should not absorb water greater than 20%

Q-16: ***List the tests for bricks and the corresponding I.S. code.***

[4 Marks, ESE-2017]

Sol: **Test of bricks:**

(a) Dimension Test (IS : 1077)

(b) Water Absorption Test (IS : 3495 (part II))

(c) Compressive Strength Test (IS : 3495 (part I))

(d) Warpage Test (IS : 3495 (part IV))

(e) Efflorescence Test (IS : 3495 (part III))

(f) Soundness Test (IS : 1077)

(g) Hardness Test (IS : 1077)

(h) Structure Test (IS : 1077)

Q-17: ***Explain briefly the various tests conducted on Bricks mentioning the relevant codal provisions.***

[8 Marks, ESE-2019]

Sol: The various tests conducted on Bricks are following:

(i) Compressive Strength test

- Bricks are often subjected to large compressive stresses.
- Compressive stresses of bricks provides a basics of comparison of quality of bricks, but it is of little value in determining the strength of wall, as strength of wall mainly depends on the strength of mortar.
- Any unevenness observed on the bed faces of brick is removed by grinding to provide two smooth parallel faces.

- It is then immersed in water at room temperature for 24 hours.
- The frog and all voids in the bed faces of brick are filled flush with cement mortar (1 cement, 1 clean coarse sand of grade 3mm and down)
- It is then stored under the damp jute bags for 24 hours followed by immersion in clean water for 3 days.
- The specimen is then placed with flat faces horizontal and mortar filled face facing upwards between plates of the testing machine.
- Load is then applied axially at a uniform rate of 14 MPa per minute till failure occurs and maximum load at failure is noted.
- Compressive strength is given by

$$\text{Compressive strength} = \frac{\text{Maximum load at failure}}{\text{loaded area of brick}}$$

- Average of five results is noted.

(ii) Water Absorption Test

- Water absorption of bricks depends on their porosity. Almost all bricks absorbs water by capillary action.
- But porosity and water absorption does not give proper indication as to whether brick work can keep away the rain water and protect the interior from dampness travelling from outside.
- Permeability measures the travel of moisture through a brick.
- Percentage of water absorption gives indication of compactness which is obtained from burning. (Vitrification in real sense).
- Absorption can be broken into two categories:

  (a) Absorption (b) Initial rate of absorption (IRA)

  Both are important in selecting the appropriate brick.

A. Absorption

(i) 24 Hour Immersion Cold water Test

- Dry bricks are kept in oven (110 ± 5°C) till it attains constant mass.
- After cooling the bricks to room temperature its weight is recorded as W_1.
- Now bricks are immersed in water at a temperature of 27° ± 2°C for 24 hours.
- Bricks are then taken out of water and wiped with a damp cloth and weighted as W_2.

$$\boxed{\text{Water absorption in } (W_{24})\,\% = \frac{W_2 - W_1}{W_1} \times 100}$$

(ii) 5 Hours Boiling Water Test

- Weight of the oven dried bricks (W_1) is recorded as above. Then the specimen is immersed in the water and heated to boiling in one hour and boiled continuously for five hours, followed by cooling down to 27° ± 2°C by natural loss of heat within 16–19 hours. Then bricks is taken out of water and wiped with a damp cloth and the weight is recorded as W_3.

$$\text{The water absorption in } \% = \frac{W_3 - W_1}{W_1} \times 100$$

- These two are used to calculate the **saturation coefficient** by dividing the 24-hour cold-water absorption (W_{24}) by the 5-hour boiling water absorption (W_5). The saturation coefficient is used to help predict durability.

B. Initial rate of absorption

(i) The initial rate of absorption or suction is the rate of how much water a brick draws in during the first minute after contact with water.

(ii) The suction has a direct bearing on the bond between brick and mortar.

(iii) When a brick has high suction, a strong, watertight joint may not be achieved. High suction brick should be wetted for three to 24 hours prior to laying to reduce the suction and allow the brick's surface to dry.

(iv) Very low suction brick should be covered and kept dry on the jobsite.

Note:
- Generally bricks are soaked in water before use in masonry work so that they do not absorb water from cement.
- Average water absorption shall not be more than 20% by weight upto class 12.5 and 15% by weight for higher classes.
- For water absorption less than 5% , danger of frost action is negligible.

(iii) Efflorescence Test

- Ends of the brick are kept in a standard porcelain or glass dish (180 mm × 180mm × 40 mm for square shaped) depth containing 25 mm depth of water at room temperature (20°–30°C) till the entire water is absorbed or evaporated. The room should be well ventilated.
- Water is again filled to 25 mm depth in the dish and allowed to be absorbed by the brick or evaporated. Cover the dish with suitable glass cylinder to avoid excessive evaporation.
- Bricks are examined after second evaporation and area of white patches is measured on the brick.
- Presence of efflorescence is classified as:
  1. Nil—Deposit of efflorescence is imperceptible.
  2. Slight—Deposit of efflorescence does not cover more than 10 per cent of the exposed area of the brick.
  3. Moderate—Deposit of efflorescence is more than 10 per cent but less than 50% of the exposed area of the brick.
  4. Heavy—Deposit of efflorescence is more than 50 per cent but the deposits do not powder or flake away the brick surface.
  5. Serious—Deposits are heavy with powdering or flaking the surface.

Note: The specifications limit the efflorescence to be not more than moderate (10–50%) up to class 12.5 and not more than slight (<10%) for higher classes.

CHAPTER 5

BRICK MASONARY

Q-1: ***Enumerate various factors which determine the thickness of brick walls. Give the characteristics of good bricks and name the tests that are carried out to determine them.***

[10 Marks, ESE-1997]

Sol: **A. Factors determining the thickness of brick walls:**

(i) Overall height of brick wall

(ii) Super imposed load per unit length of wall

(iii) Height of the wall between floors

(iv) Damp proofing and expenses

(v) Length of the wall between piers, buttresses, cross-walls

(vi) Strength of brick masonry, which depends upon the quality of bricks, mortar and method of bonding

B. Characteristics of a good brick:

(i) Should be uniform in shape and of standard size

(ii) Should be table molded, well burnt in kilns, copper colored, free from cracks and with sharp and square edges.

(iii) Should give clear metallic ringing sound when struck with another brick.

(iv) Must have bright homogeneous and uniform compact structure, free from voids.

(v) Should not absorb water more than 20% by weight for 1^{st} class bricks and 22% for 2^{nd} class bricks when soaked in cold water for more than 24 hours.

(vi) Should have sufficient hardness such that no impression is left on its surface when it is scratched with finger nail.

(vii) Should not break into pieces when dropped flat on hard ground from a height of about 1 meter.

(viii) Should have low thermal conductivity and should be sound proof.

(ix) When soaked in water for about 24 hours, should not show deposits of white salts when allowed to dry in shade.

(x) Should not have crushing strength below 3.5 N/mm^2.

Tests to determine properties of bricks

(i) **Absorption test-** A weighed dry brick is immersed in water for about 24 hours and then weighed again. It must not absorb water more than 20% by weight.

(ii) **Crushing strength-**It is found by using compression testing machine. It must not have a crushing strength less than 3.5 N/mm^2.

(iii) **Hardness-**No impression should be left on the brick surface when scratched with a finger nail.

(iv) **Presence of soluble salts-**It is said to be free from soluble salts when a brick immersed in water for 24 hours is allowed to dry in shade, does not show grey or white deposits on its surface.

(v) **Structure-** A brick is broken and its structure is examined. It must be homogeneous, compact and free from any defects such as holes, lumps etc.

(vi) **Soundness-**Two bricks are struck with each other. They should not break and should produce a clear ringing sound.

Q-2: ***Describe the various stages in the making of cement mortars. Why are cement mortars not ground like lime mortar?***

[10 Marks, ESE-1997]

Sol: **A. Various stages of making Cement mortar**

(i) Mortars are typically made from a mixture of sand, a binder such as cement or lime, and water with either manual mixing or mechanical mixing

(ii) In manual mixing

- The sand is first sieved, cleaned with water to remove dirt and dust, and then dried.
- This dry sand is then laid uniformly on a watertight platform.
- The requisite quantity of cement is then spread uniformly over it.
- The whole mass is then mixed thoroughly with spades till it becomes uniform in color.
- A depression is then made in the center of the mix and water is added as per required quantity.
- The dry mix from the sides is moved and placed along the edges of the depression till the water is completely absorbed by the mix.
- The wet mix is then worked with spades again to give a uniform consistency to the mortar.

(iii) In mechanical mixing

- A calculated quantity of cement, sand & water are fed into the cylindrical container of the mixer.
- The rotor then rotates and thoroughly mixes the ingredients.

B. **Mortars are typically made from** a mixture of sand, a binder such as cement or lime, and water with either manual mixing or mechanical mixing. Cement mortars employ cement which is already fine. Hence grinding is not required. Lime mortars are made using slaked lime and grinding is done to ensure:

- Crushing of particles of unslaked lime, if any, to ensure slaking.

  Note: Slaking means addition of water to quick lime [CaO] which it absorbs to become slaked lime [$Ca(OH)_2$]. Theoretical amount of water required for lime slaking is about 32% of the weight of CaO.

- Making an intimate mixture of the whole mass so that no two grains of sand are left without an intervening film of binding material.

Q-3: ***What are the properties of a good mortar? Enumerate four main uses of mortar.***

[10 Marks, ESE-2004]

Sol: **1. Properties of good mortar are:**

- It should be capable of developing good adhesion with the building units such as bricks, stones etc.
- The important properties of a good mortar mix are mobility, place-ability and water retention.
- It should be capable of developing good adhesion with the building units such as bricks, stones etc.
- It should be capable of developing the designed stresses.
- It should be capable of resisting penetration of rainwater.
- It should be cheap.
- It should be durable.
- It should be easily workable.
- It should not affect the durability of materials with which it comes into contact.

(ii) The Uses of mortar are:

- To bind the building units such as bricks, stones etc.
- To carry out painting and plaster works on exposed surfaces of masonry.
- To form an even bedding layer for building units.
- To form joints of pipes.
- To improve the appearance of structure.

Q-4: ***Explain the compressive strength test of cement mortar and state its importance.***

[10 Marks, ESE-2005]

Sol: **A. Compressive strength test of cement mortar**

(i) Requirements for the test are 7.06 cm cubes moulds (50cm^2 face area), apparatus for gauging and mixing mortar, vibrator, compression testing machine etc.

(ii) Take 200gm of cement and 600gm of standard sand in a pan.

(iii) Mix the cement and sand in dry condition with a trowel for 1 minutes and then add water.

(iv) The quantity of water shall be (p/4+3)% of combined weight of cement and sand where, p is the % of water required to produce a paste of standard consistency.

(v) Add water and mix it until the mixture is of uniform color. The time of mixing shall not be < 3 minutes & not > 4 minutes.

(vi) Immediately after mixing the mortar, place the mortar in the cube mould and prod with the help of the rod. The mortar shall be prodded 20 times in about 8 sec to ensure elimination of entrained air.

(vii) If vibrator is used, the period of vibration shall be 2 minutes at the specified speed of 12000+- 400 vibrations /minutes.

(viii) Then place the cube moulds in temperature of 27 ± 2°C and 90% relative humidity for 24 hours.

(ix) After 24 hours remove the cubes from the mould and immediately submerge in clean water till testing.

(x) Take out the cubes from water just before testing. Testing should be done on their sides without any packing.

(xi) The rate of loading should be 350 kg/cm^2/minute and uniform.

(xii) Test should be conducted for 3 cubes and the average value should be reported as the test result for both 7day and 28 day compressive strength.

B. Importance

(i) The compressive strength of hardened cement is the most important and most specified of all the properties.

(ii) Therefor cement is always tested for this strength using this method before employing it for important works.

(iii) Before starting any project, concrete mix designs are prepared in the lab in accordance with the properties of available materials. For checking the applicability and suitability of these designs, this test is used.

(iv) It is also employed to check the strength of concrete ready for dispatch from the batching plant.

Q-5: ***Discuss the types of mortar which can be used for the following types of masonry work with suggested proportions:***

(i) Masonry in foundation and plinth ***(ii) Masonry in superstructure***

(iii) Plastering work ***(iv) Pointing***

[5 Marks, ESE-2011]

Sol:

S.No.	Nature of work	Type of Mortar	Proportion
(i)	Masonry in foundation and plinth	Lime mortar	1:2
		Cement mortar	1:6
(ii)	Masonry in super structure	Cement mortar	1:3
		Time mortar	1:1
(iii)	Plastering work	Cement mortar	1:3 to 1:4
		Lime mortar	1:2
(iv)	Pointing work	Cement mortar	1:1 to 1:2

Q-6: ***List the factors that influence the basic permissible compressive stresses for masonry for different mortars. Also elaborate the modification due to each factor.***

[10 Marks, ESE-2016]

Sol: Basic permissible stress in masonry depend upon

(i) slenderness ratio of wall

(ii) eccentricity of load

(iii) area of wall

(iv) shape of the unit

(v) unit strength and masonry strength

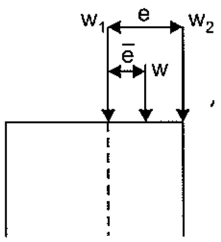
Basic compressive strength = 0.25 × Strength from prism test at 28 days.

Modification due to each factor:

(i) **Slenderness ratio and eccentricity:** Basic permissible strength related to slenderness ratio through. Strength reduction factor and also related to eccentricity.

It depend on mean eccentricity.

$$\bar{e} = \frac{w_2 e}{w_1 + w_2}$$



(ii) **Area of wall:** (Area reduction factor) Probability of failure of small section due to substandard material is more

Thus for A < 0.2 m² K_s is applied.

(iii) **Shape modification factor:** More height to width ratio of the unit means less horizontal joints hence increase strength as brick work. This factor applicable upto 15 MPa unit strength.

$$f_c = f_b \times k_s \times k_a \times k_p$$

f_c = permissible stress

f_b = basic permissible stress

k_s = slenderness ratio and eccentricity modification factor

k_a = area modification factor

k_p = shape modification factor

CHAPTER 6

AGGREGATE

Q-1: ***Explain the phenomenon of BULKING of sand and its significance. How is it determined?***

[10 Marks, ESE-1996]

Sol: **A. Bulking of sand**

(i) The presence of moisture in sand increases the volume of the sand.

(ii) This is due to the fact that moisture causes film of water around sand particles which increases the volume of sand.

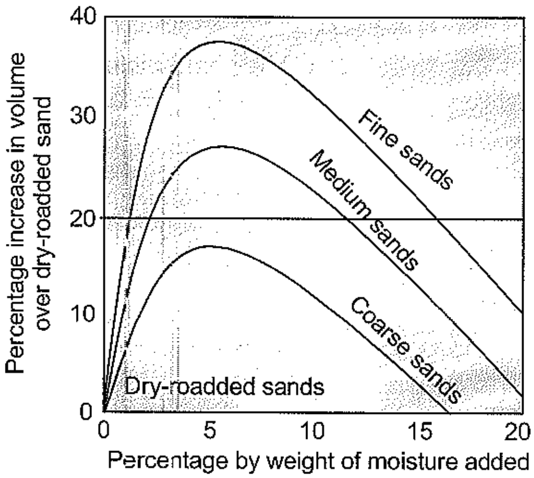
(iii) The moisture exerts surface tension and keeps every particle away from each other denying any point contact between them.

(iv) This phenomenon is called bulking of sand.

(v) The finer the material more will be bulking for given moisture content.

(vi) When moisture content is increased by adding more water, the sand particles pack near each other and the amount of bulking of sand decreases.

(vii) It is evident from the chart above that for a increase in moisture content of about 5-8 %, the increase in volume can be as much as 20-40%



B. Significance of bulking

(i) The bulking of sand affects the volumetric proportioning of sand and hence if suitable allowance is not made for it, it will increase the cost of cement and mortar.

(ii) It will lead to under sanded mixtures which are very difficult for working and laying.

C. Determination of percentage of bulking

(i) A sample of moist sand is taken in a measuring cylinder and its height is noted, say h_1.

(ii) More water is added to the cylinder and is thoroughly stirred by means of rod so as to inundate the sand completely.

(iii) As the volume of flooded sand is same as dry sand, it will offset the bulking effects.

(iv) The level of sanc is again noted and it is say h_2.

(v) The % bulking can be calculated as –

$(h_1 - h_2)/h_2 * 100$

Q-2: ***Explain the following:***

(i) Grading zones of sand

(ii) Grading of coarse aggregate

[10 Marks, ESE-2002]

Sol: **A. Grading zones of sand**

(i) Natural sand are weathered and worn out particles of rocks and are of various grades or size depending on the accounting of wearing

(ii) The sand must be of proper gradation because than it will have less voids and hence the cement required will be less. Such sand will be more economical.

(iii) There is standard specification for Sand. As per IS : 383-1970 It is divided in four gradations known as Zone I, Zone II, Zone III and Zone IV according to percentage passing through 600 μ sieve.

(iv) From grading zone I to IV, the fine aggregates become progressively finer and the ratio of fine to coarse aggregate should be progressively reduced.

(v) An aggregate falls in a particular grading zone if its percentage passing through the 600 μ sieve falls in its range and is not allowed to fall outside the limits of other sieves by more than 5%.

(vi) The permissible limit for crushed stone sand on 150 μ sieve is increased to 20%, but it does not affect 5% allowance permitted to other sieve sizes.

B. Grading of Coarse aggregates

(i) For coarse aggregates, the grading is expressed in terms of percentage by weight retained on or passing through a series of sieves taken in order 80 mm, 40 mm, 20 mm, 10 mm & 4.75 mm.

(ii) A curve called grading curve is made showing cumulative percentages of material passing the sieves on ordinate with the sieve openings to the logarithmic scale represented on the abscissa.

(iii) It indicates whether the grading of given sample is as specified or is too coarse or too fine or deficient in a particular size.

(iv) The main points governing the desired aggregate grading are-

- The surface area of aggregate
- The relative volume occupied by the aggregate
- The workability of the mix
- Tendency to segregate

The smaller the size of aggregate, the greater is the surface area and hence to fill the voids with minimum amount of fine aggregates, the aggregate size should be as large as possible.

Q-3: ***Explain how bulking of fine aggregate takes place and how it is taken care of in the field. Also explain the method of preparation of bulking chart in the laboratory.***

[10 Marks, ESE-2006]

Sol: The bulking of line aggregate takes place due to pressure of moisture cutout, aggregate bulk in volume. The moisture particles form a thin film around the aggregates and exert surface tension. This keeps the particle away from each other and thus aggregate bulk in volume.

Which moisture content is increased by adding more water, the sand particles pack near each other and the amount of bulking of sand is decreased. Thus the dry sand and the sand completely flooded with water have particularly the same volume.

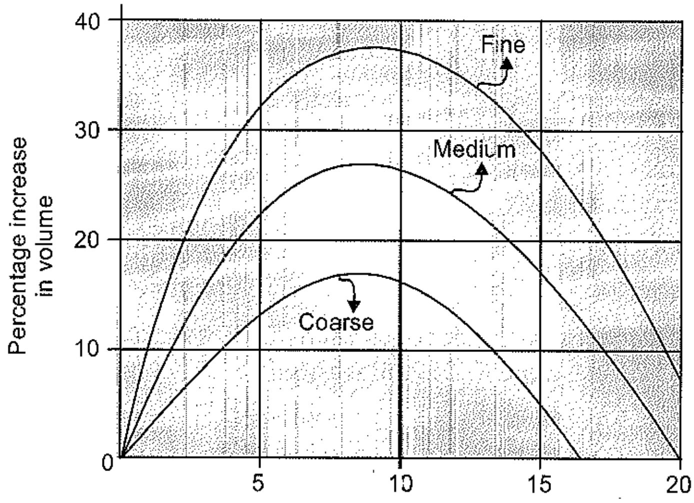


Chart showing bulking of sand

The bulking of line aggregate can be estimates by simple test.

(i) Take a sample of line aggregate is filled into a measuring cylinder in the normal manner.

(ii) The level is noted down say & pour water in the measuring cylinder and completely inundate the sand and shake it.

(iii) Since the volume of the saturated sand is some as that of dry sand, the inundated sand completely

- The level of the sand is again coated down say l_2.
- Then l_1–l_2 shows the bulking of the sample of sand under test.

$$\% \text{ of bulking} = \left(\frac{l_1 - l_2}{l_2}\right) \times 100$$

Q-4: ***What do you understand by bulking of sand? How does it affect quantity of sand of volume batching?***

[5 Marks, ESE-2014]

Sol: A. **Bulking of Sand**

(i) The presence of moisture in sand increases the volume of the sand.

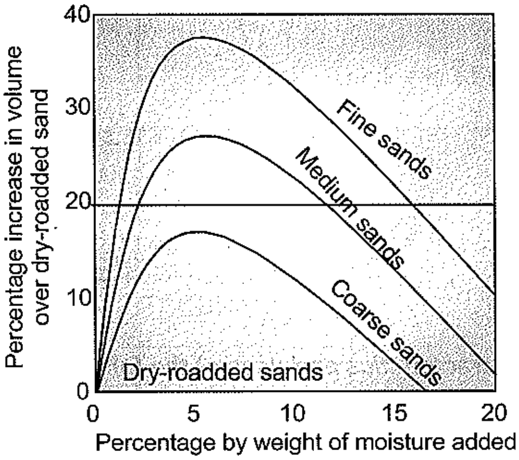
(ii) This is due to the fact that moisture causes film of water around sand particles which increases the volume of sand.

(iii) The moisture exerts surface tension and keeps every particle away from each other denying any point contact between them.

(iv) This phenomenon is called bulking of sand.

(v) The finer the material more will be bulking for given moisture content.

(vi) When moisture content is increased by adding more water, the sand particles pack near each other and the amount of bulking of sand decreases.



(vii) It is evident from the chart above that for a increase in moisture content of about 5-8 %, the increase in volume can be as much as 20-40%

B. Significance of Bulking

(i) The bulking of sand affects the volumetric proportioning of sand and hence if suitable allowance is not made for it, it will increase the cost of cement and mortar.

(ii) It will lead to under sanded mixtures which are very difficult for working and laying.

Q-5: How are aggregates classified based on particle size ? What is bulking of sand ?

[6 Marks, ESE-2020]

Sol: *Classification of Aggregate based on particle size*

(a) Coarse Aggregate

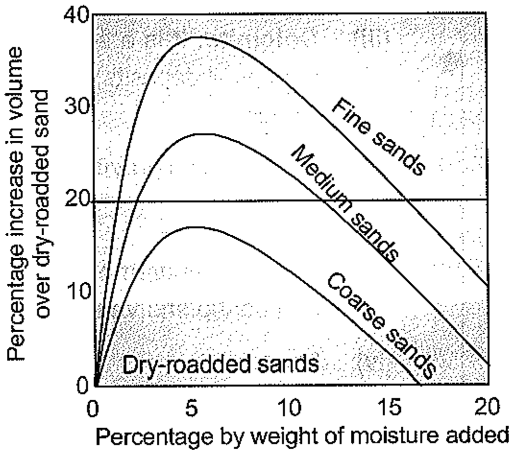
- Aggregates ranging from 80mm – 4.75mm are identified as coarse aggregates.
- These are obtained from natural disintegration or artificial crushing of rocks.

(b) Fine Aggregate

- Aggregate passing through 4.75 mm sieve are defined as fine.
- They may be natural sand—deposited by rivers, crushed stone sand—obtained by crushing stones and crushed gravel sand.
- Smallest size of fine aggregate (sand) is 0.06 mm.
- Depending upon the particle size, fine aggregates are described as fine, medium and coarse sands.

Bulking of Sand

- The presence of moisture in sand increases the volume of the sand. This is due to the fact that moisture causes film of water around sand particles which increases the volume of sand.
- The moisture exerts surface tension and keeps every particle away from each other denying any point contact between them. This phenomenon is called bulking of sand.
- The finer the material more will be bulking for given moisture content.
- When moisture content is increased by adding more water, the sand particles pack near each other and the amount of bulking of sand decreases.



- It is evident from the chart above that for a increase in moisture content of about 5-8 %, the increase in volume can be as much as 20-40%

CHAPTER 7

STONE

Q-1: Give the composition, characteristics and uses of the following stones (i) granite, (ii) quartzite, (iii) dolomite, (iv) murrum and (v) slate.

[10 Marks, ESE-1995]

Sol: **(i) Granite:**

- It is a igneous rock formed by agency of heat, the molten material subsequently become solidified.
- Specific gravity = 2.6 to 2.7
- Crushing or compressive strength = 770 to 1300 kg/cm^2

Uses:

- Granite is a very hard and durable builcing stone suitable for work of importance such as bridge abutments, piers, etc. where weight and durability are essential.
- It is not suitable for curving work.

(ii) Quartzite:

- It is a silicious sand stones under the effect of metamorphic action yield quartzite. It is a very dense and strong stone with stratified structures and crystalline texture, but breaks up into irregular shapes. It is difficult to dress and work
- Specific gravity = 2.65 to 2.95
- Crushing or compressive strength = 550 kg/cm^3

Uses: Quartzite is used for rubble masonry road metalling and also as aggregate for concrete.

(iii) Dolomite: It is a sedimentary rock resulting from the precipitation of salts in drying water basin (chemical deposits) are dolomite usually the rock are well stratified and show well defined bedding planes.

- Specific gravity = 2.85
- Crushing or compressive strength = 720 – 760 kg/cm^2

Uses: It is extremely suitable for ornamental and superior type of building work. It is who suitable for flooring and veneers work.

(iv) Murrum

- It is decomposed laterite rock and acts as fine bondage for roads
- It is used in embankment, filling behind retaining wall, filling below plinth and around foundation

(v) **Slate**

- It is a metamorphic laminated clay rocks with planes of cleavage along which they can be split into very thin slabs.
- Specific gravity = 2.89
- Crushing or compressive strength = 770 to 211 D kg/cm^2

Uses: Slates are used as a roofing and flooring material. Harder varieties of slate are used for dado work (i.e., for lining lower part of walls damp proof material and steps of stairs)

Q-2: ***List the various types of ASHLAR masonry. Briefly describe the construction of walls with any two types.***

[10 Marks, ESE-1996]

Sol: **ASHLAR MASONRY:** This is a costlier, high grade and superior quality of masonry. This built from accurately dressed stones with uniform and very fine joints of about 3 mm thickness. By arranging the stone blocks in various patterns different types of appearances can be obtained. The backing of ashlar masonry walls may be built of ashlar masonry or rubble masonry. The size of stone blocks should be in proportion to wall thickness.

The various types of Ashlar masonry are:

(i) Ashlar fine
(ii) Ashlar rough tooled
(iii) Ashlar rock or quarry faced
(iv) Ashlar chamfered
(v) Ashlar facing
(vi) Ashlar block in course

(i) **Ashlar fine:** At all beds, joints faces the stones should be dressed perfectly so that they confirm to the desired pattern. The size of the stones to be laid in regular causes should not be less than 300 mm in height should not be less than 300 mm in height width of stones should not be less than the height of the courses. Also length of stones should be more than two times the height of the course. Generally face stones are laid as headers and stretchers alternatively the header comes under the middle portion of the stretchers. In order to break the continuity of vertical joints, the stones in the adjacent layers should have a lap of more than half of the height of the course. All the joints either horizontal or vertical should be made of fine mortar with a maximum thickness of 3 mm

(ii) **Ashlar rough tooled (Bastard ashlar):** In this type of masonry, the beds and sides of each stone blocks are finely chisel dressed just in the same manner as for ashlar fine, but the exposed face is dressed by rough tooling. A strip, about 25 mm wide and made by means of a chisel is provided around the perimeter of the rough dressed face of each stone. The rough tooled face when tested with a straight edge 600 mm in length, should not show any point on the surface to vary by more than 3 mm in any direction. This type of masonry is also known as *bastard ashlar*. The size, angle, edges etc. are maintained in order, similar to that for fine dressed ashlar. The thickness of mortar joint should not be more than 6 mm.

Q-3: ***List out at least eight tests required to determine the suitability of stone for engineering use.***

[8 Marks, ESE-2018]

Sol: **Tests to determine the suitability of stone for engineering use are following:**

(i) **Acid Test:** This test is carried out to understand the presence of $CaCO_3$ in building stone.

(ii) **Attrition Test:** This test predicts the rate of wear of stones against the grinding action under traffic. Therefore this test is primarily used for stones to be used in road construction.

(iii) **Crushing Test:** This test is performed to find out the compressive strength of stones under gradually applied load.

(iv) **Freezing and thawing Test:** This test is applicable to the regions where the temperature can go below the freezing point.

(v) **Hardness Test:** This test is used to describe strength of stones in terms of hardness (resistance against abrasion).

(vi) **Impact Test:** Impact test is used to determine the toughness of a stone sample.

(vii) **Microscopic Test:** The stone sample examined under a microscope and studied to predict the quality of stone. The properties that are examined are:

(a) Average grain size

(b) Existence of pores, fissures, veins and shakes

(c) Mineral constituents

(d) Nature of cementing material

(e) Presence of any harmful substance

(f) Texture of stone etc.

(viii) **Smith's Test:** Smith's test is performed to find out the presence of any soluble matter in a stone sample.

(ix) **Water Absorption Test:** This test is done to check how much water the stone may absorb. Aggregates with high water absorption are unsuitable for engineering use.

Q-4: ***Briefly explain the purpose and the procedure for Attrition Test on stone.***

[4 Marks, ESE-2018]

Sol: **Attrition Test:** This test predicts the rate of wear of stones against the grinding action under traffic. Therefore this test is primarily used for stones to be used in road construction.

The test procedure is as follow:

(i) Samples of stones are broken into pieces of about 60 mm size.

(ii) The sample pieces, weighing around 5kg, are put in the two cylinders of Devil's attrition test machine. These cylinders have a diameter of about 20 mm and length 340 mm. Their axes of these cylinders make an angle of 30 degree with the horizontal.

(iii) Cylinders are rotated about the horizontal axis for 5 hours at the rate of 30 rpm.

(iv) The specimen pieces are then taken out from the cylinders and they are passed through a sieve of 1.5 mm mesh.

(v) The quantity of material retained on the sieve is weighed.

(vi) Percentage wear is found to judge the stone.

Percentage wear = loss in weight × 100/initial weight.

CHAPTER 8

LIME

Q-1: ***Describe the various tests performed to assess the suitability of Lime as a cementing material.***

[8 Marks, ESE-2019]

Sol. The tests generally performed on lime can be classified as: laboratory tests and field tests.

Laboratory Tests

IS : 6920- 1973 has specified ten laboratory tests for limes. The most commonly used tests are:

Test 1: Loss on Ignition Test (LOI) The LOI test can be conducted to monitor the relative degree of calcination. It is also used in the testing stage to compare LOI of limestone from different deposits. It should be accompanied by a thorough visual inspection regarding over-burnt and under-burnt stone. Loss on ignition consists of strongly heating a sample of the material at a specified temperature, allowing volatile substances to escape, until its mass ceases to change. The simple test typically consists of placing a few grams of the material in a tare, pre-ignited crucible and determining its mass, placing it in a temperature-controlled furnace for a set time, cooling it in a controlled (e.g., water-free, CO_2 free) atmosphere, and re-determining the mass. The process may be repeated to show that mass-change is complete.

Test 2: Soundness Test The soundness test is a very simple but important test. Its purpose is to determine how effectively the quicklime slakes. Small cores of over-burnt material may be present in the lime hydrate. They will slake very slowly. If a lime containing such cores is used in a plaster, at some future time the core will slake in the wall causing the material around it to pop out, hence the commonly known defect *popping.* To avoid this defect, the lime hydrate supplied must be completely slaked, without any core of over-burnt material. This test is used to control the quality of lime produced during the course of production.

Test 3: Reactivity Assessment of Quicklime The addition of water to quicklime to produce a lime hydrate results in the evolution of heat. Lightly burnt quicklime will evolve heat, i.e., react. at a faster rate than will the hard over-burnt quicklime. This phenomenon is used in this test to monitor the reactivity and hence the degree of burning of the quicklime produced. It is also used in comparing limestones from different deposits for the purpose of selection.

Test 4: Determination of Available Lime by the Rapid Sugar Test This method consists in taking 500 g sieved hydrated lime sample in a flask containing 20 ml distilled water. The corked flask is swirled and heated for two minutes. To this is added 150 ml water and 15 g granulated sugar and flask re-corked and shaken at intervals for five minutes. The solution is allowed to stand for 30 minutes to one hour. The solution in the flask is titrated with the standard HCl solution with two drops phenolphthalein using standard procedure. The reading is noted; l ml of acid solution is equivalent to one per cent available lime expressed CaO.

Field Tests

IS 1624-1974 has specified a number of simple field tests for the limes. They can be readily performed.

Visual Examination Class C lime should be pure white in colour.

Hydrochloric Acid Test The test consists in pouring ½ N hydrochloric acid to lime sample, measuring levelled tablespoonful, taken in a test-tube till effervescence ceases (about 100 ml acid would be required).

The sample is left standing for 24 hours. Bubbling action will indicate presence of lime and volume of insoluble residue the unwanted inert material (adulteration) in the lime. The following observations would helpful:

(i) Formation of good thick gel which does not flow when test-tube is inverted indicates class-A lime.

(ii) Formation of flowing gel indicates class-B lime.

(iii) No gel formation indicates c1ass-C lime.

Ball Test A ball, about the size of an egg, made of the lime sample with just enough water is stored for six hours and then placed in a basin of water. The following inferences can be drawn:

(i) Expansion and disintegration of ball in a few minutes of its placement in water indicates c1ass-C lime.

(ii) Little expansion with a number of cracks in the ball indicates class-B lime.

(iii) No adverse effect indicates that lime belongs to Class-A category.

Impurity Test A known weight of lime sample is mixed with water in a beaker and the solution is decanted. The residue is dried well in hot sun for eight hours and then weighed. If the residue is less than 10 percent then the lime is good; 10 to 20 percent it is fair and above 20 percent it is poor.

Plasticity Test Lime sample is mixed with water to a thick paste and left overnight. It is spread on a blotting paper like butter with a knife to test its plasticity. Good lime is plastic in nature.

Workability Test To judge the workability of lime sample 1 : 3 lime-sand mortar is prepared and thrown on a brick wall by a trowel; if it sticks well, its workability is good.

CHAPTER 9

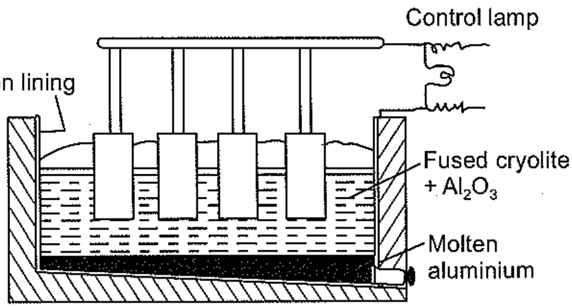
ALUMINIUM

Q-1: ***Briefly explain manufacture of Aluminium and state at least six physical and mechanical properties of Aluminium.***

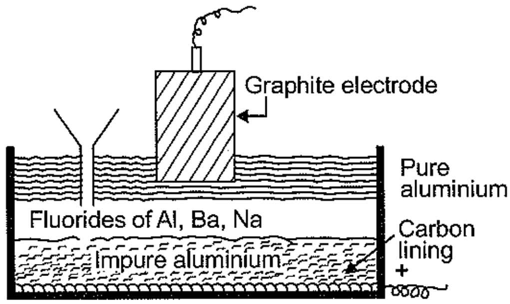
[8 Marks, ESE-2018]

Sol: **Manufacture of Aluminium:**

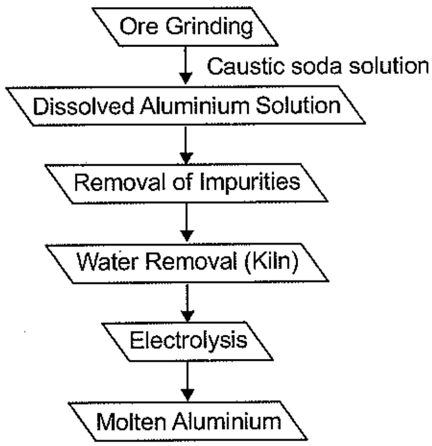
- The **principle constituents** of bauxite ($Al_2O_3 . 2H_2O$) which yield aluminium on a commercial scale are hydrated oxides of aluminium and iron with some silica.
- Some of the aluminium ores are corundum, kaolin or china clay and kryolite.
- The ore is **purified** by **Bayer's process** and is **reduced to aluminium** by **Hall Hiroult's process** in **two stages**.
- In the **first stage**, bauxite is converted into alumina by roasting, grinding, heating (with sodium hydrate) and filtering. Then, it is agitated for several hours to precipitate the hydrate, which is separated, washed and calcined at 1000°C.
- In the next stage, aluminium is extracted by electrolysis of alumina in a molten bath (as shown in figure 1) of crystolite (a fluoride of alumina and sodium).



(a) Extraction of Aluminium by Electrolysis



(b)Hoope's Cell for Refining Aluminium



(c) Flow diagram for Extraction of Aluminium

Physical properties of aluminium:

(i) Aluminium is silver white in colour with a brittle metallic lustre on freshly broken surface.

(ii) It is harder than tin.

(iii) It is very light, soft, strong and durable.

(iv) It can be riveted and welded, but cannot be soldered.

(v) It has high corrosion resistance.

(vi) It provide safety against attack from insects.

(vii) It has good noise control.

(viii) It is easy to transport because of its light weight.

(ix) High reflectivity

(x) Ease in fabrication and assembly

(xi) High scrap value

(xii) Economy in maintenance and aesthetic appearance.

Mechanical properties of aluminium:

(i) It is malleable, less ductile than copper but excels zinc, tin and lead.

(ii) It has low thermal conductivity.

(iii) It can be tempered at 350°C and melting point is 657°C.

(iv) The tensile strength is 117.2 N/mm^2 in the cast ferm and 241.3 N/mm^2 when drawn in wires.

(v) Specific gravity is 2.7 and the modulus of elasticity is 6.89×10^4 N/mm^2.

(vi) Its Poisson's ratio is 0.335

(vii) High – strength to weight ratio

(viii) Capacity to withstand low temperature (cryogenics)

(ix) High conductivity of electricity

Q-2: ***How is the presence of surface oxide film responsible for excellent corrosion resistance of Aluminium?***

[4 Marks, ESE-2019]

Sol: Aluminium has remarkable corrosion resistance due to thin surface layer of aluminium oxide that forms when the the metal is exposed to air, effectively preventing further oxidation. This pheonomen is called passivation. Stronger the aluminium-copper alloy, lesser the corrosion resistance due to galvanic reactions with alloyed copper.

CHAPTER 10

CERAMICS

Q-1: ***Describe the thermal and electrical properties of ceramics.***

[8 Marks, ESE-2019]

Sol: **Thermal properties of ceramic:**

Thermal capacity, conductivity and resistance to shocks need to be considered while using a ceramic.

The specific heat for refractories to be used in regenerator chambers should be more. The heat for refractories to be used in regenerator chamber should be more. The heat in ceramics is conducted by photon conductivity and by the interaction of lattice vibration. The ceramic materials do not have enough free electrons to bring out electronic thermal conductivity. At high temperatures, conduction takes place by transfer of radian energy. The thermal conductivity of refractories should be minimum for lining and maximum for crucibles and retorts.

Electrical Properties of ceramic:

Ceramic materials have no free electrons so they have low electrical conductivity. However, at high temperatures the ionic diffusion is acclerated.

Clay displays a very high dielectric constant – a property of material related to its behvaour when located within an electric field between two electrodes – under static conditons. However, for alternating current, the dielectric constant in clay arises from ion and electron movement.

CHAPTER 11

PLASTICS

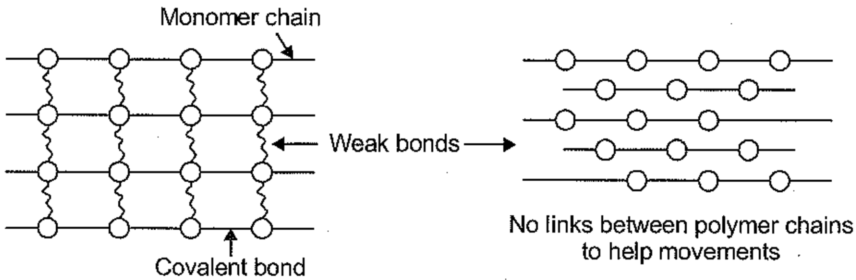
Q-1: ***(i)*** ***Briefly explain Thermoplastic and Thermosetting materials.***

(ii) ***List out six differences between them.***

[4+8 Marks, ESE-2018]

Sol: **(i)** **Thermoplastic materials:**

- Thermoplastic is a class of polymer, which can be easily melted or softened by providing heat in order to recycle the material.
- They have covalent interactions between monomer molecules and secondary weak van der Wall interactions between polymer chains.
- This weak bonds can be broken by heat, and change its molecular structure.



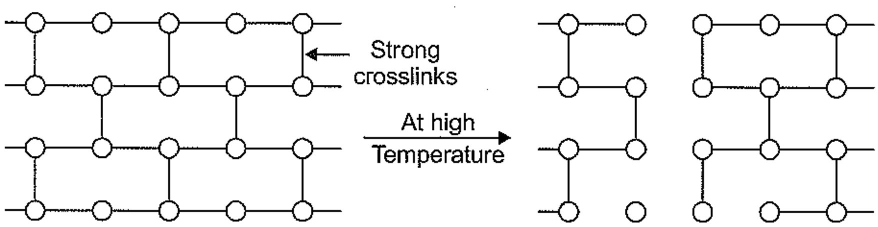
(Thermoplastic polymer) (Thermoplastic softened state)

- This type of polymers can be readily recycled or remoulded, because each time it is reheated, it can be reshaped into a new article.

  Ex. Acrylic, styrene, nylon, poly-vinyl chloride etc.

Thermosetting materials:

- Thermosetting materials are highly cross-linked polymers that have a three-dimensional network of covalently bonded atoms.
- The strong cross-linked structure shows resistance to higher temperatures which provides greater thermal stability than thermoplastic.
- These materials cannot be recycled, remoulded or reformed upon heating.



(Thermosetting plastic) (Thermal degradation)

Ex. Phenolic resins, amino resins, epoxy resins etc.

(ii) Difference between Thermoplastic and Thermosetting:

	Thermoplastic		Thermosetting
1.	These have covalent bonds between monomers and weak van der Waal interactions between monomer chains	1.	They have strong cross-linked and a 3D network of covalently bonded atoms. The stiffness of material increases with the number of cross-links in the structure
2.	It is synthesised by the addition polymerization	2.	It is synthesised by the condensation polymerization
3.	This is lower in molecular weight compared to thermosetting	3.	High in molecular weight
4.	This has a low melting point, low tensile strength, stiffness, brittleness, rigidity and durability	4.	This has high melting point, high tensile strength, stiffness, brittleness, rigidity and durability
5.	It is processed by injection, moulding, extrusion process, blow moulding, thermoforming process, and rotational moulding	5.	It is processed by compression moulding, reaction injection moulding
6.	It is soluble in some organic solvents	6.	Insoluble in organic solvents

Q-2: ***Differentiate in brief between Thermoplastic and Thermosetting plastic are as follows.***

[8 Marks, ESE-2020]

Sol: Difference between Thermoplastic and Thermosetting plastic are as follows:

	Thermoplastic		Thermosetting
1.	These have covalent bonds between monomers and weak vander Waal interactions between monomer chains	1.	They have strong cross-linked and a 3D network of covalently bonded atoms. The stiffness of material increases with the number of cross-links in the structure
2.	It is synthesised by the addition polymerization	2.	It is synthesised by the condensation polymerization
3.	This is lower in molecular weight compared to thermosetting	3.	High in molecular weight
4.	This has a low melting point, low tensile strength, stiffness, brittleness, rigidity and durability	4.	This has high melting point, high tensile strength, stiffness, brittleness, rigidity and durability
5.	It is processed by injection, moulding, extrusion process, blow moulding, thermoforming process, and rotational moulding	5.	It is processed by compression moulding, reaction injection moulding
6.	It is soluble in some organic solvents	6.	Insoluble in organic solvents

CHAPTER 12

FRP

Q-1: ***Discuss about fibre reinforced concrete.***

[10 Marks, ESE-2001]

Sol: **Fiber reinforced concrete**

(i) Fiber reinforced concrete (FRC) can be defined as a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discreet, uniformly distributed suitable fibers.

(ii) Continuous meshes, woven fabrics and long wires and rods are NOT considered to be discreet fibers.

(iii) Concrete made with Portland cement is relatively strong in compression but weak in tension and tends to be brittle.

(iv) In plain concrete, micro-cracks develop due to plastic shrinkage and drying shrinkage even before loading and when loaded they tend to open up and propagate due to stress concentration and additional cracks form in places of minor defects.

(v) Inclusion of small closely spaced and uniformly dispersed fibers act as crack arrester and improve its static and dynamic properties.

(vi) They reduce the permeability of concrete and thus reduce bleeding of water. Some types of fibers produce greater impact, abrasion, and shatter–resistance in concrete

(vii) The use of fibers also alters the behavior of the fiber-matrix composite after it has cracked, thereby improving its toughness.

(viii) For the effective use of fibers in hardened concrete:

- Fibers should be significantly stiffer than the matrix, i.e. have a higher modulus of elasticity than the matrix.
- Fiber content by volume must be adequate.
- There must be a good fiber-matrix bond.
- Fiber length must be sufficient.
- Fibers must have a high aspect ratio, i.e. they must be long relative to their diameter.

Commonly used fibers for reinforcing concrete are steel fibers, glass fibers, synthetic fibers and natural fibers.

Q-2: ***What is CFRP? Indicate its ultimate tensile strength and modulus of elasticity.***

[4 Marks, ESE-2017]

Sol: CFRP stands for carbon fiber reinforced polymer.

It is extremely strong and light fiber reinforced plastic which contains carbon fibers.

CFRP's are composite materials, Its constituents are

(a) **Matrix:** It is usually a polymer resin such as epoxy to bind the reinforcement together.

(b) **Reinforcement:** It is carbon fibers, which provides the strength

The properties of CFRP depends on the layout of carbon fibre relative to the polymer.

CFRP's are used in high rigidity and high-strength to-weight ratio condition and are expensive. They are mainly used in aerospace, automotive, civil engineering etc.

Ultimate tensile strength of CFRP = 2.1 – 5.5 GPa.

Elastic modulus of CFRP = 200 – 800 GPa.

CHAPTER 13

MISCELLANEOUS

Q-1: ***Describe the construction of:***

(i) Concrete flooring

(ii) Terrazzo flooring

(iii) Mosaic flooring

[10 Marks, ESE-1996]

Sol: (i) **Concrete flooring:** This type of flooring is most commonly used these days in residential, commercial, institutional and public building of all types. This flooring is also known as Indian patent stone flooring. The concrete flooring consist of two component.

(a) A base course an subgrade

(b) A wearing course an floor finish

In floor construction, the floor finish over the base course (or subgrade) may be placed in two ways. Firstly, non-monolithically, i.e., topping or floor finish is laid after the base has hardened and secondly, monolithically i.e., topping is laid immediately after laying the base layer while the base is still in plastic state.

The concrete flooring consist of topping of cement concrete 2.5 to 4 cm thick laid an a 10 to 15 cm base of suitable mix of concrete 1 : 3 : 6, depending upon the loading anticipated and type of earth filling on which the base is placed.

(ii) Terrazzo flooring: This is a special type of concrete flooring in which marble chips are used as aggregates, and this concrete on polishing with carbarundum stone presents a smooth surface any desired colour can be obtained by using different colours of cement. The aggregate shades are exposed by grinding the surface. Normally to terrazzo mix having proportion 1:2 to 3 (1 cement : 2 to 3 marble chips). Depending upon the size of marble chips Terrazzo finish is atleast 10 mm thick and comprises a mixture of desired cement (i.e., coloured cement), marble powder and coarse aggregate, such as chippings as marble, quartzite, pearl, glass etc of selected colours and of sizes graded from 2mm to 8 mm sometimes, larger particles up to 10 mm are used.

(iii) Mosaic flooring (or china mosaic tile floors): This flooring which consist of tiles available in variety of pattern and colours, is commonly used in operation theaters, temples etc. For construction of mosaic flooring, first of all, a hard concrete base is laid over concrete base is laid over concrete base, while it is still wet, a 2 cm layer of cement mortar (1 : 2) is evenly laid upon the bed of cement mortar, small pieces of broken tiles are arranged in definite patterns. After this cement or coloured cement is sprinkled at the top and surface is rolled by light stone roller till the even surface is attained. this surface is left for 24 hrs to dry and then it is rubbed with punic stone to get a smooth and polished surface.

NOTE

NOTE