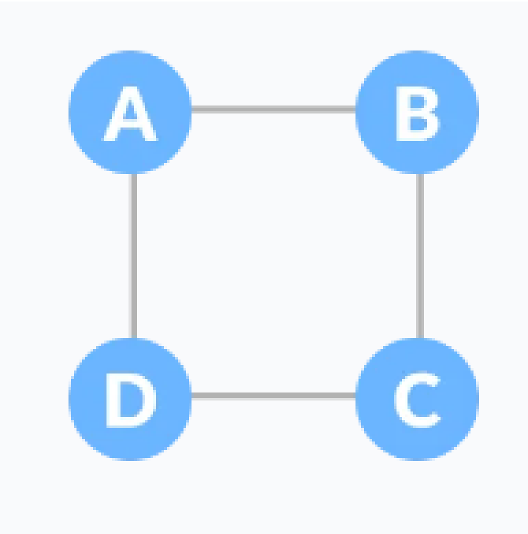
Spanning Tree

Spanning tree

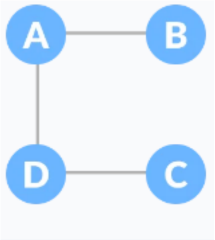
A spanning tree is a sub-graph of an undirected connected graph, which includes all the vertices of the graph with a minimum possible number of edges.

The edges may or may not have weights assigned to them.

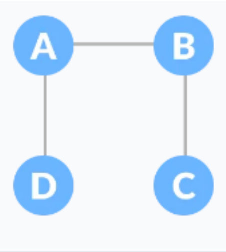
Example of a Spanning Tree



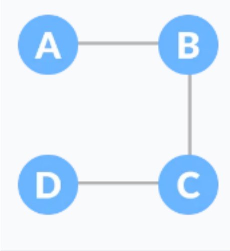
Normal graph



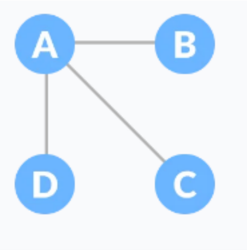
A spanning tree



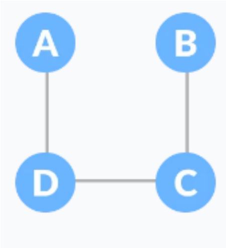
A spanning tree



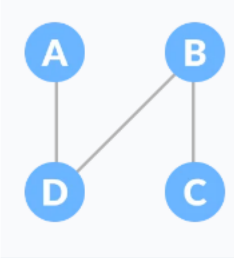
A spanning tree



A spanning tree



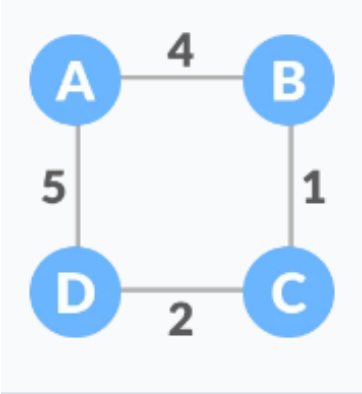
A spanning tree



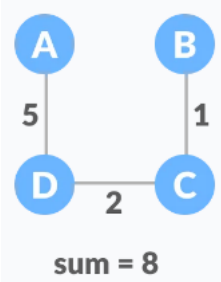
A spanning tree

Minimum Spanning Tree

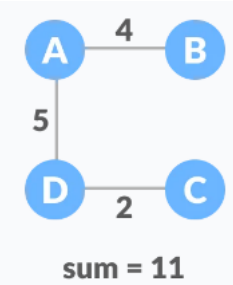
- A minimum spanning tree is a spanning tree in which the sum of the weight of the edges is as minimum as possible.



Weighted graph



sum = 8



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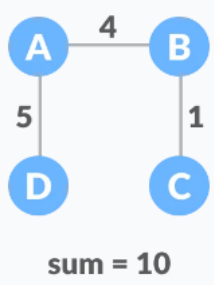
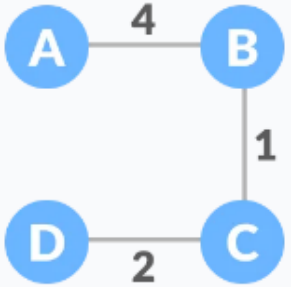


sum = 10



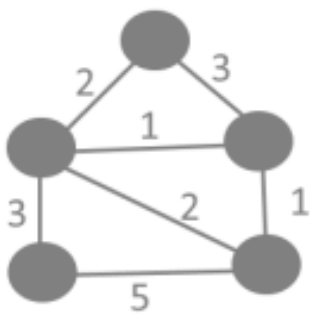
sum = 7

Minimum Spanning Tree

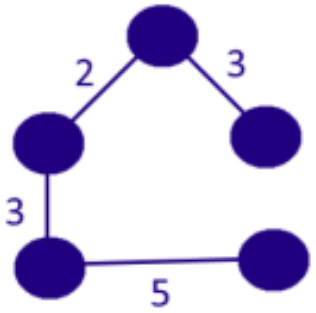
- The minimum spanning tree from a graph is found using the following algorithms:
    - Prim's Algorithm
    - Kruskal's Algorithm

Kruskal's algorithm

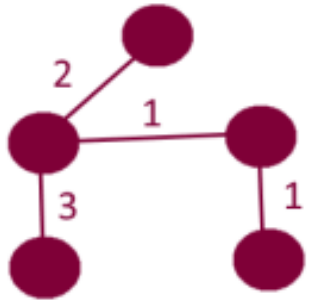
- Kruskal's algorithm is a minimum spanning tree algorithm that takes a graph as input and finds the subset of the edges of that graph which
  - form a tree that includes every vertex
  - has the minimum sum of weights among all the trees that can be formed from the graph



Graph



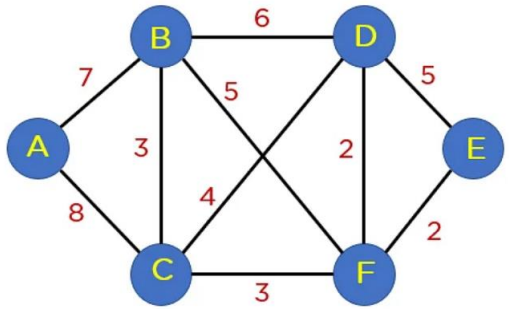
Spanning Tree
Cost = 13



Minimum Spanning Tree, Cost = 7

Kruskal's algorithm

- We start from the edges with the lowest weight and keep adding edges until we reach our goal.
- The steps for implementing Kruskal's algorithm are as follows:
  - Sort all the edges from low weight to high
  - Take the edge with the lowest weight and add it to the spanning tree. If adding the edge created a cycle, then reject this edge.
  - Keep adding edges until we reach all vertices.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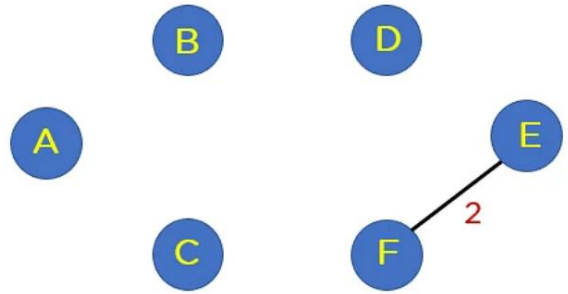
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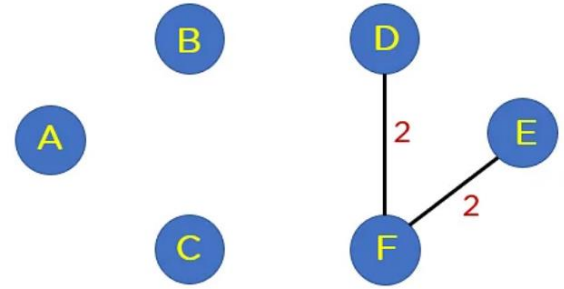
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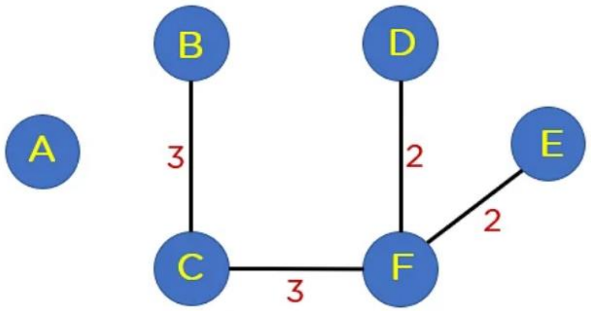
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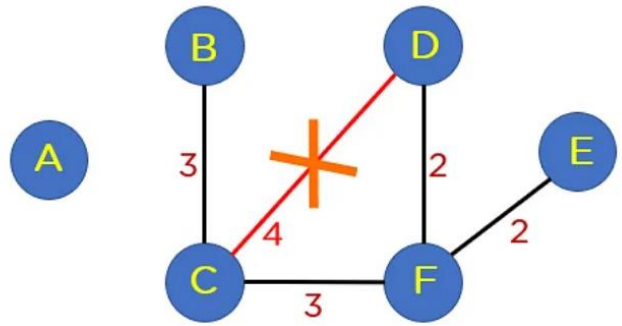
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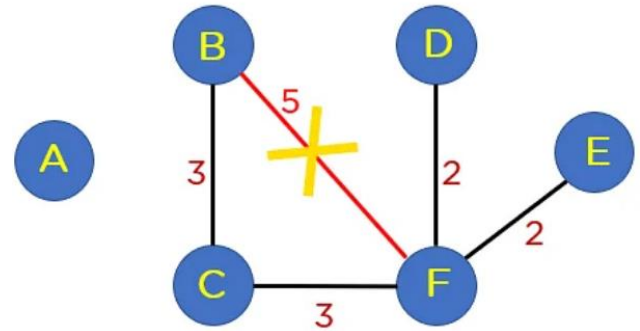
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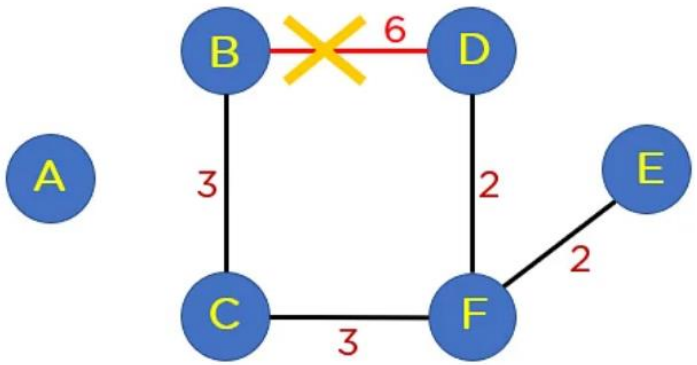
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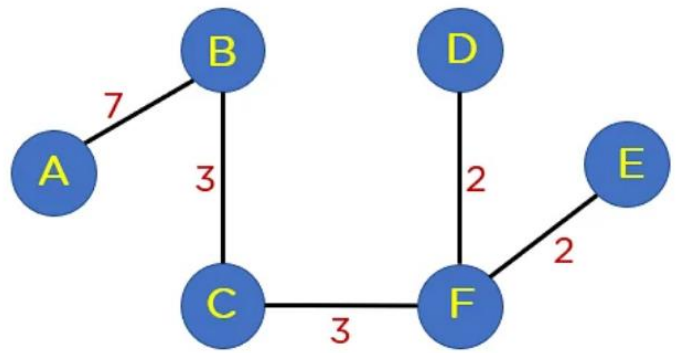
A
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C
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7
3
2
2
3

Prim's algorithm

- We start from one vertex and keep adding edges with the lowest weight until we reach our goal.
- The steps for implementing Prim's algorithm are as follows:
  - Initialize the minimum spanning tree with a vertex chosen at random.
  - Find all the edges that connect the tree to new vertices, find the minimum and add it to the tree
  - Keep repeating step 2 until we get a minimum spanning tree