

Numbers Relations and functions

Q. $1397 \times 1397 = (1400 - 3)^2$
 $= 1400^2 + 9 - (2 \times 1400 \times 3)$
 $= 1951609$

Q. which of the following are divisible by 132?
 264, 462, 396, 792, 968, 2178, 5184, 6336
 $\Rightarrow 132 \Rightarrow 11, 4, 3$
 no. divisible by 11, 4, 3 are divisible by 132.

Q. $(935421 \times 625) = ?$
 $\rightarrow 935421 \times 625 = 935421 \times (5)^4 = 935421 \times \left(\frac{10}{2}\right)^4$
 $= 935421 \times \frac{100000}{16}$
 $= 584638125$

Q. Largest 4 digit no. divisible by 88?
 \rightarrow Largest 4 digit no. = 9999
 divide it by 88 remainder is obtained and subtracted from 9999 which gives no. divisible by 88.

$$\begin{array}{r} 88) \overline{9999}(132 \\ \underline{-88} \\ 119 \\ \underline{-88} \\ 319 \\ \underline{-264} \\ 55 \end{array} \quad 9999 - 55 = 9944$$

Q. Unit digit at $(6374)^{1793} \times (625)^{317} \times (341)^{491} = ?$

Unit digit of $(6374)^{1793} \Rightarrow (4)^{1793} \Rightarrow 4$
 Unit digit of $(625)^{317} \Rightarrow (5)^{317} \Rightarrow 5$
 Unit digit of $(341)^{491} \Rightarrow (1)^{491} \Rightarrow 1$
 Unit digit = $4 \times 5 \times 1 = 0$

Q. $5358 \times (51) = 5358 \times (50+1)$
 $= 267900 + 5358$
 $= 273258$

Q. Smallest 3 digit prime no?

→ 100 is not prime

$\sqrt{101} < 11$ hence we will check for no. upto 10
prime no. upto 11 are 2, 3, 5, 7, 11 these no.
don't divide 101 hence 101 is prime.

Q. Sum of first 45 natural no.

$$\text{Sum of } n \text{ no.} = \frac{n(n+1)}{2} = \frac{45(45+1)}{2} = \frac{45 \times 46}{2}$$

$$= 45 \times 23$$

$$= 1035$$

Q. $\frac{753 \times 753 + 247 \times 247 - 753 \times 247}{753 \times 753 \times 753 + 247 \times 247 \times 247} = ?$

$$\frac{a^2 + b^2 - ab}{a^3 + b^3} = \frac{1}{(a+b)} = \frac{1}{753+247} = \frac{1}{1000}$$

Q. difference of proper fraction and reciprocal
is $9/20$ what is no.

$$\rightarrow x - \frac{1}{x} = \frac{9}{20}$$

$$\boxed{x = \frac{4}{5}}$$

Q. on dividing by 56 we get 29 remainder the divisor
by 8 we get?

$$\rightarrow \frac{29}{8} = 5$$

$$\boxed{5}$$

Q. If n is natural no. $(6n^2+6n)$ is always divisible
by

$$\rightarrow (6n^2+6n) = 6n(n+1)$$

it always divisible by 6 and 12 both

Q. $107 \times 107 + 93 \times 93 = ?$

$$\rightarrow = (107)^2 + (93)^2$$

$$= (100+7)^2 + (100-7)^2$$

$$= 2[100^2 + 7^2]$$

$$= 20098$$

$$--- (2[a^2+b^2] = (a+b)^2 + (a-b)^2)$$

Q. How many 3 digit no. divisible by 6?

→ consider A.P. where

$$a = 102$$

$$d = 6$$

$$l = 996$$

$$a + (n-1)d = l$$

$$102 + (n-1)6 = 996$$

$$\boxed{n = 150}$$

Q. $(963+476)^2 + (963+476)^2 = ?$

$$(963 \times 963 + 476 \times 476)$$

$$\rightarrow 2(a^2+b^2) = (a+b)^2 + (a-b)^2$$

$$\therefore 2$$

Q. which no. obtain by interchanging positions and original two digit no has difference of 36

what is diff. of digits?

$$\rightarrow (10x+y) - (10y+x) = 36$$

$$\boxed{x-y = 4}$$

Q. two digit no. such that product of digit is 18 and if no. reverse and 12 added the no. digits reversed

$$xy = 18$$

$$10x+y + 12 = 10y+x$$

$$\therefore 10x+y = 24$$

$$\boxed{\text{no.} = 24}$$

Q. sum of square of three no. 138 and product of two at time is 131 what is sum of no?

$$a^2 + b^2 + c^2 + 2(ab+bc+ca) = (a+b+c)^2$$

$$\therefore 138 + 2(131) \quad \sqrt{400}$$

$$= (a+b+c)^2$$

Q. If unit digit exceed tenth digit by 2, product given no and sum of digit equal to 144 no.

$\rightarrow x+2$ is unit

$10x$ is tenth digit

$$\therefore (10x + (x+2)) \times (x+x+2) = 144$$

$$\therefore (11x+2)(2x+2) = 144$$

$$22x^2 + 26x - 140 = 0$$

$$x = 2$$

[no. is 24]

Q. 476 **0 divisible by 3 & 11 ** no are respectively

$$\rightarrow (4+6+b)-(7+a+0) = 0 \text{ or } 11$$

$$b-a = -3 \text{ or } 8$$

[From options 8 & 5 are no.]

$$Q. 11^2 + 12^2 + 13^2 \dots 20^2 = ?$$

$$\rightarrow (1^2 + 2^2 \dots 20^2) - (1^2 + 2^2 + \dots 10^2)$$

$$\left[\frac{n(n+1)(2n+1)}{6} \right]_{1-20} - \left[\frac{n(n+1)(2n+1)}{6} \right]_{1-10}$$

$$\frac{20(21)(41)}{6} - \frac{10(11)(21)}{6}$$

$$= 2485$$

Q. $(49^{15}-1)$ divisible by ?

$\rightarrow (x^n-1)$ divisible by $(x+1)$ when n is even.

\therefore to make n even

$$= (7^{2 \times 15} - 1)$$

$$= (7^{30} - 1) \text{ where } n = 30$$

\therefore it is divisible by $(x+1)$ i.e. 8.

$$Q. \left[-\frac{1}{n} \right] + \left[1 - \frac{2}{n} \right] + \left[1 - \frac{3}{n} \right] + \left[1 - \frac{4}{n} \right] \dots \text{ up to } n$$

$$= (1 + 1 + \dots n) + \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots \frac{n}{n} \right)$$

$$= \frac{n(n+1)}{2} + n = n - \frac{n(n+1)}{2 \times n} = n - \frac{n+1}{2}$$

$$= \frac{(n+1)}{2}$$

- Q. On dividing 2272 & 875 by 3 digit no. we get same remainder find sum of digits
 → When we subtract two no. who gives same remainders we get no. which is exactly divisible.

$$\therefore 2272 - 875 = 1397$$

$$= 11 \times 127$$

$$\boxed{\text{Ans} \Rightarrow 1+2+7 = 10}$$

- Q. A boy multiply 987 by a no. get 559981 as ans if both g in ans are wrong what is the ans?
 → from option look one by one -

$$987 = 3 \times 7 \times 47$$

Ans should be divisible by 3 & 7

$$\boxed{\therefore 555681}$$

- Q. Nearest no. divisible by 88 to 9217 is?

$$\overline{88)9217(104}$$

$$65$$

now when 65 subtracted from 9217 no is divisible by 88 but not nearest next no. to 9217 is nearest.

$$\therefore 9217 + (88-65)$$

$$\boxed{\therefore = 9240}$$

Q. $\frac{(489+375)^2 - (489-375)^2}{(489 \times 375)} = ?$

$$\frac{(a^2+b^2+2ab) - (a^2+b^2-2ab)}{ab} = \frac{4ab}{ab} = 4$$

Q. $(3^{25}+3^{26}+3^{27}+3^{28}) = ?$

→ From option choose

$$3^{25}(1+3^2+3^3+3) = 3^{25} \times 40 = 3^{25} \times 30 \times 4 \text{ divisible by } 30 \text{ as co-factor of } 30 \text{ are } 10 \times 3$$

as 30 is factor of given no., no. is divisible by 3.

- Q. When no. is divided by 6 remainder is 3 then what is remainder is square no. divided by 6?
- Trick - As no is not mentioned and no condition is mentioned for that we can take randomly any no.

Let no. be 21 which leaves 3 as remainder

$$\therefore 21^2 = 441 \Rightarrow 6) \overline{441}$$

$$\therefore \boxed{\text{Remainder} = 3}$$

- Q. When 17^{200} is divisible by 18 remainder?
- $(17^{200} - 1^{200})$ is divisible by $(17-1)$
- $\therefore \boxed{\text{Remainder is } 1}$

- Q. difference of two consecutive even integers squares is divisible by?

→ Let no. $(2n)$ as it is even and second is $(2n+2)$ then

$$\begin{aligned}(2n+2)^2 - (2n)^2 &= 4n^2 + 4 + 8n - 4n^2 \\ &= 8n+4 \\ &= 4(2n+1)\end{aligned}$$

As it is multiple of 4 divisible by 4.

Trick - as it is generalised Q. we can consider any no. according to given cond of even and consecutive.

\therefore Let 4 & 6 then

$$36 - 16 = 28 \text{ divisible by } 4 \text{ & } 7$$

Let 2 & 4 then

$$16 - 4 = 12 \text{ divisible by } 4 \text{ & } 3$$

$\therefore \boxed{4}$

Q. Which of following is prime?

119, 187, 247, 551 or none of these.

$$\sqrt{551} > 22$$

We find prime no. up to ~~22~~ 22, and check for divisibility
no. 2, 3, 5, 7, 11, 13, 17, 19, 23

∴ Ans none of these]

Q. Sum of even no. from 1 to 31

$$a = 2, d = 2, l = 30$$

$$S = \frac{n(a+l)}{2} = \frac{15(2+30)}{2} = 240$$

$$a + d(n-1) = l$$

$$2 + 2(n-1) = 30$$

$$n = 15$$

$$= 240$$

Q. How many no. from 6+12+18+... gives 1800

$$a = 6, d = 6, l = ?, n = ?$$

$$a + (n-1)d = l \Rightarrow 6 + (n-1)6 = l \Rightarrow 6n = l$$

$$S = \frac{n(a+l)}{2} \Rightarrow 1800 = \frac{n(6+l)}{2} \Rightarrow 6n + ln = 3600$$

$$6n + 6n^2 = 3600$$

$$n = 24$$

Q. Which of following is prime number?

161, 221, 373, 437

$$\sqrt{437} < 21$$

Prime no. up to 21 = 2, 3, 5, 7, 11, 13, 17, 19

Check prime no. divide them or not.

∴ 373 prime

Q. How many terms are there in $3, 6, 12, 24, \dots, 384$?
 → GP.

$$a = 3, r = 2, x_n = 384, n?$$

$$\text{Given } x_n = ar^{n-1}$$

$$384 = 3 \times 2^{n-1}$$

$$\boxed{n = 8}$$

Q. $2^2 + 2^3 + 2^4 + 2^5 + \dots + 2^9 = ?$

$$a = 2, r = 2, n = 9$$

$$S = a \left(\frac{r^n - 1}{r - 1} \right) = 2 \left(\frac{2^9 - 1}{2 - 1} \right) = 2 \times (512 - 1) = 1022$$

Q. If x & y are integers such that $(3x+7y)$ is multiple of 11 then which of following divisible by 11?

→ randomly took $x=5, y=1$

: $(3 \times 5 + 7 \times 1)$ divisible by 11 -

: $(4 \times 5 + 6 \times 1)$ not divisible

: $(5 \times 5 + 6 \times 1)$ not divisible

: $(9 \times 5 + 4 \times 6)$ not divisible

: $(4 \times 5 - 9 \times 6)$ divisible .

Q. $2^2 + 4^2 + 6^2 + \dots + 20^2 = ?$

$$= 2^2 + (2 \times 2)^2 + (2 \times 3)^2 + (2 \times 4)^2 + \dots + (10 \times 2)^2$$

$$= 2^2 [1^2 + 2^2 + 3^2 + \dots + 10^2]$$

$$= 2^2 \times \frac{n(n+1)(2n+1)}{6}$$

$$= 4 \times \frac{10(10+1)(20+1)}{6}$$

$$= \frac{2 \times 10 \times 21}{3} = 140 \times 10$$

$$= 1504$$

Q. $217 \times 217 + 183 \times 183 = ?$

$$\begin{aligned} &= (200+17)^2 + (200-17)^2 = (a+b)^2 + (a-b)^2 \\ &= 2(a^2 + b^2) \\ &= 2(200^2 + 17^2) \\ &= 2 \times 40000 + 289 \\ &= 80289 \end{aligned}$$

Q. Unit digit of $(784 \times 618 \times 917 \times 463)$ is -

$$\begin{aligned} &\rightarrow = 4 \times 8 \times 7 \times 3 \\ &= 672 \\ &= 2 \end{aligned}$$

Q. $(4^{61} + 4^{62} + 4^{63} + 4^{64}) = ?$

$$\begin{aligned} &= 4^{61}(1 + 4 + 4^2 + 4^3) \\ &= 4^{61} \times 85 \\ &= 4^{60} \times 4 \times 85 \\ &= 4^{60} \times 340 \quad \text{divisible by 10} \end{aligned}$$

as if one factor is there that factor divide no.

Q. $106 \times 106 - 94 \times 94 = ?$

$$\begin{aligned} &= 106^2 - 94^2 \\ &= (106+94)(106-94) \\ &= (200 \times 12) \\ &= 2400 \end{aligned}$$

Q. Number when divided successively by 4 & 5 we get remainders 1 & 4 respectively. If divided by 5 & 4 what will be remainder?

Q $8796 \times 273 + 8796 \times 17 = ?$
→ $= 8796 (273 + 17)$
= $8796 (300)$
= 2638800