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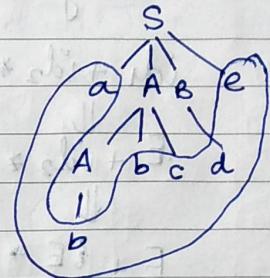
Bottom up Parsing

$L \rightarrow$ left to right scan

$R \rightarrow$ rightmost derivation in reverse order.

$w \rightarrow a b b c d e$

First check right-hand side of the production



Rightmost derivation →

$$S \xrightarrow{rm} aABC \xrightarrow{rm} aAde \xrightarrow{rm} aAbcde \xrightarrow{rm} abbcede$$

* Handle :

A Handle of a string is a substring that matches the right side of a production and who's reduction to the nonterminal on the left side of the production that represents one step along the reverse of a rightmost derivation

$$S \rightarrow aA^{\dagger}B^{\dagger}e$$

$$A \rightarrow Abc \mid b$$

$$B \rightarrow d$$

a b b c d e

aA b c d e

$$A \rightarrow b$$

- this b is a handle.

a A b c d

$A \rightarrow (Abc)$ → this is the handle

a Ade

$B \rightarrow @ \rightarrow$ this substring is the handle.

Date _____ / _____ / _____

$$S \rightarrow aABe$$

↓
this substring is the handle.

- * Consider the following grammar:

$$E \rightarrow E + E$$

$$E \rightarrow E \times E$$

$$E \rightarrow (E)$$

$$E \rightarrow id$$

String is $id_1 + id_2 * id_3$

$$id_1 + id_2 * id_3$$

$$\downarrow \\ E + id_2 * id_3$$

$$\downarrow \\ E + E * id_3$$

$$\downarrow \\ E + E$$

$$\downarrow \\ E$$

$$E \xrightarrow{?m} E + E \xrightarrow{?m} E + E * E$$

$$\Rightarrow$$

$$id_1 + id_2 * id_3$$

$$E \rightarrow id_1$$

$$E \rightarrow id_2$$

$$E \rightarrow id_3$$

$$E \rightarrow (E \times E)$$

$$E \rightarrow (E + E)$$

$\rightarrow id_1$ is a handle

$\rightarrow id_2$ is a handle

$\rightarrow id_3$ —

handle

handle

Right Derivation Form

Handle Reducing Production

$$id_1 + id_2 * id_3$$

$$id_1$$

$$E \rightarrow id_1$$

$$E + id_2 * id_3$$

$$id_2$$

$$E \rightarrow id_2$$

$$E + E * id_3$$

$$id_3$$

$$E \rightarrow id_3$$

$$E + E * E$$

$$E \cdot E^* E$$

$$E \rightarrow E^* E$$

$$E + E$$

$$E + E$$

$$E \rightarrow E + E$$

$$E$$

$$\$ -$$

$$\$ -$$

① Expression \rightarrow expression + term

Expression \rightarrow expression * term

expression \rightarrow term

$$\begin{array}{l} A \rightarrow \beta \quad A' \\ \text{expr}^* \rightarrow \alpha \text{ term expr}' \\ \text{expr}' \rightarrow \underbrace{+ \text{termexpr}'}_{\alpha} \mid \underbrace{- \text{termexpr}'}_{\alpha} \mid \underbrace{\text{termexpr}'}_{A'} \mid \epsilon \end{array}$$

$$S \rightarrow A^*CB \mid CbB' \mid Da$$

$$A \rightarrow da \mid BC$$

$$B \rightarrow g \mid \epsilon$$

$$C \rightarrow h \mid \epsilon$$

$$f_i(S) = \{d, h, b, a, g, \epsilon\} \quad f_0(S) \rightarrow \{\$\}$$

$$f_i(A) = \{d, g, h, \epsilon\} \quad f_0(A) \rightarrow \{h, g, \$\}$$

$$f_i(B) = \{g, \epsilon\} \quad f_0(B) \rightarrow \{\$, a, g, h\}$$

$$f_i(C) = \{h, \epsilon\} \quad f_0(C) \rightarrow \{b, g, \$, h\}$$

$$\text{FOLLOW}(B) = \text{FOLLOW}(S) \cup \text{FIRST}(a) \cup \text{FIRST}(c) \cup \text{FOLLOW}(A) = \{\$, a, g, h\}$$

$$\text{FOLLOW}(c) = \text{FIRST}(b) \cup \text{FOLLOW}(A) \cup \text{FIRST}(B) \cup \text{FOLLOW}(S)$$

* Stack Implementation of Shift-Reduce Parsing (Bottom-up)

Initial Config	\$	w \$
Stack		
Input		:

The primary operation of the parser are shift & reduce, there are actually 4 possible actions a shift-reduce parser can make; shift, reduce, accept, error.

① In a shift action, the ~~parser~~ next symbol is shifted onto the top of the stack.

② In a reduce action, the parser knows the right end of the handle each at the ~~point~~ top of the stack.

Date _____ / _____ / _____

- ③ It must then locate the left end of the handle within the stack and reside with what non-terminal to replace the handle.
- ④ In an accept action the parser announces successful completion of parsing.
- ⑤ In an error action, the parser discovers that a syntax error, and calls an ^{own} recovery routine.

	<u>Stack</u>	<u>Input</u>	<u>Action</u>
1	\$	id ₁ +id ₂ * id ₃ \$	Shift
2	\$ id ₁	+ id ₂ * id ₃ \$	Reduce by E → id
3	\$ E	+ id ₂ * id ₃ \$	Shift
4	\$ E +	id ₂ * id ₃ \$	Shift
5	\$ E + id ₂	* id ₃ \$	Reduce by E → id
6	\$ E + E	* id ₃ \$	Reduce by E → E+E
7	\$ E	* id ₃ \$	Shift
8	\$ E *	id ₃ \$	Red Shift
9	\$ E * id ₃	\$	Reduce E → E id
10	\$ E * E	\$	Reduce E → E * E
11	\$ E	\$	Accept

Operator Precedence Parsing

Operator Grammar: A grammar have the operator property that no production on the right side has two adjacent non-terminal.

Eg.: $E \rightarrow E + E \mid E - E \mid E \times E \mid E/E \mid E \times E \mid (E) \mid id$.

not an eg: $\frac{EAE \mid (E) \mid id}{A \rightarrow + \mid - \mid \times \mid / \mid \mid \uparrow}$

In precedence parsing we define three disjoint precedence relation $, <, =, >$ between pair of terminals. This precedence relationship guide the selection of handle and have the following meaning. E.g. $a < b$

$\times F \rightarrow c$
 $\times F \rightarrow A A$

Relation	Meaning
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$a < b$ "a" yields precedence to "b"

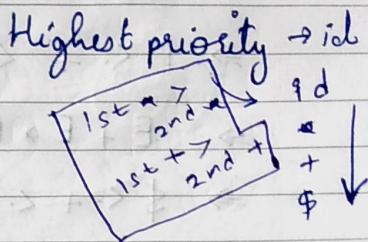
$a = b$ "a" has same precedence over "b"

$a > b$ "a" takes precedence over "b"

Operator precedence Relation matrix:

$$\begin{aligned} E &\rightarrow E + E \quad E * E \mid id \\ W &= id_1 + id_2 * id_3 \end{aligned}$$

	id	+	*	\$
id	:	>	>	>
+	<	>	<	>
*	<	>	>	>
\$	<	<	<	.



Q. When we are given a string, insert \$ at both ends of the string.

$$W = \$ id + id * id \$$$

$$W = \$ < id > + < id > * < id > \$$$

Then find the handle using the steps:

Step 1: Scan the string from the left end until the first $>$ is encountered.

Step 2: Then scan backward (to the left) over any $=$ until a $<$ is encountered.

Step 3: The handle contains everything to the left to the first $>$ and to the right of the $<$ encountered in Step 2, including any intervening surrounding non-terminal.

Date _____

S

L to R
 $\$ < \cdot id > + < \cdot id > * < \cdot id > \$$

↑
Stop
↔ R to L

→ handle is the id

→ Reduce the first id using non-terminal

 $E + id * id$ reduce $E \rightarrow id$
 $\rightarrow \$ < \cdot + < \cdot id > * < \cdot id > \$$
→ handle = $* id$
 $\Rightarrow E + E * id$ Reduce $E \rightarrow id$
 $\Rightarrow \$ < \cdot + < \cdot * < \cdot id > \$$
 $\Rightarrow E + E * E$ Reduce $E \rightarrow id$
 $\Rightarrow \$ < \cdot + < \cdot * > \$$ handle

 $\Rightarrow E + E$ Reduce by $E \rightarrow E * E$
 $\Rightarrow \$ < \cdot + > \$$
 $\Rightarrow E$ Reduce by $E \rightarrow E + E$
 $\Rightarrow \$$ precedence of $f(3) > g(id)$ 5/09/22

$f(a)$	id	$+$	$*$	$()$	$g(b)$	$\$$
5	3					
4	2					

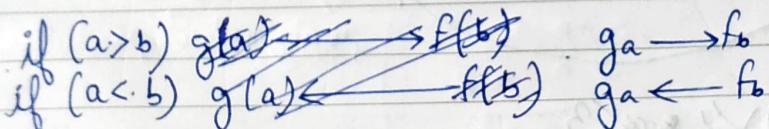
 $a \geq b \quad f(a) > g(b)$
 $a = b \quad f(a) = g(b)$
 $a < b \quad f(a) < g(b)$

construct precedence function

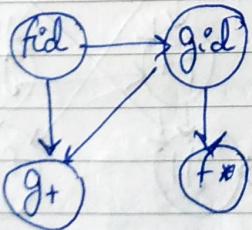
Step

Input \rightarrow operator
precedence
matrix

- * Step 1: Create symbol $f(a)$ and $g(a)$ for each 'a' ie a terminal or end marker.
- * Step 2: Partition the created symbol as many groups as possible in such a way that if $a \leq b$ then $f(a)$ and $g(b)$ are in the same group.
- * Step 3: Create a directed graph, whose nodes are the groups found in Step 2, for any a and b , if $a \leq b$ place an edge from the group of $g(b)$ to the group of $f(a)$. If $a > b$ place an edge from the group of $f(a)$ to that of $g(b)$.



- * Step 4: If the graph constructed in Step 3 has a cycle then no precedence function exist. If there are no cycle, let $f(a)$ be the length of the longest path beginning at the group of $f(a)$ and let $g(a)$ be the length of the longest path from the group g_a .

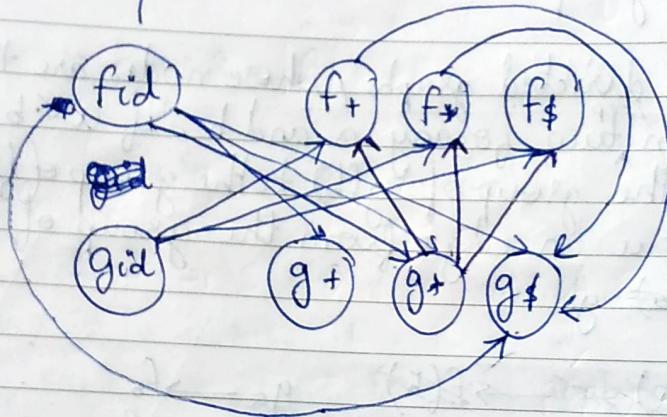


$$\begin{aligned}
 f(id) &= 2 \\
 g(id) &= 1 \\
 g(+) &= 0 \\
 f(*) &= 0
 \end{aligned}$$

Q) operator precedence function for the precedence matrix:

	id	+	*	\$	
id	.	>	>	>	(f)
+	<	>	<	>	
*	<	>	>	>	
\$	<	<	<	.	

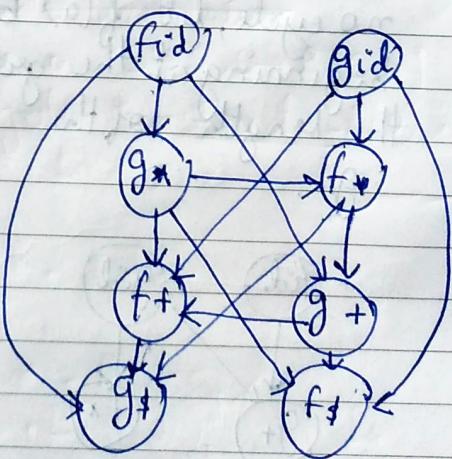
(g)



X

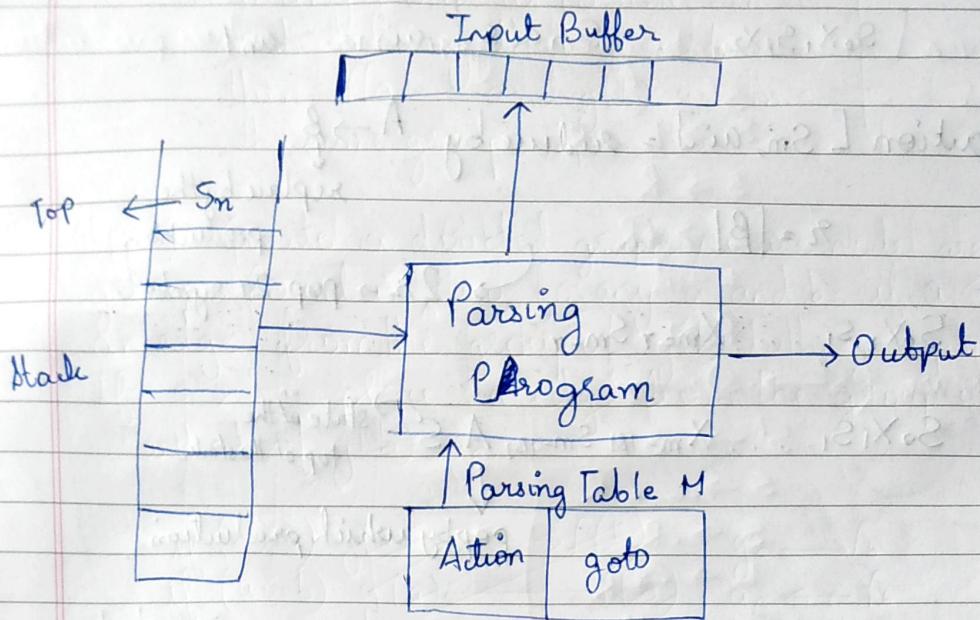
$$\begin{aligned}
 f(id) &= 1 \\
 f(+) &= 2 \\
 f(*) &= 4 \\
 f($) &=
 \end{aligned}$$

	id	+	*	\$
f	5	1	3	0
g	4	2	4	0



LR Parser

Simple Canonical Look Ahead
 SLR CLR LALR



$S_i \rightarrow \text{State}$

$X_i \rightarrow \text{grammar symbol}$

$S_0 X_1 S_1 X_2 S_2 X_3 \dots X_n S_n \rightarrow \text{Stack Configuration}$

Top of the stack is always S_n or a state if not then error

Action \rightarrow terminal / only
 goto \rightarrow non-terminal

$r_1 \rightarrow$ reduce by production no. 1

$s_6 \rightarrow$ shift the handle to 6 (state)



Date / /

($S_0 X_1 S_1 X_2 \dots X_m S_m, a^i a_{i+1} \dots a_n$) depends on this
 Stack Config, Input Config) top of stack
 and input symbol.

Shift \rightarrow action [S_m, a^i] = shift S

$S_0 X_1 S_1 X_2 \dots S_m a^i S, a_{i+1} \dots a_n$

Reduce \rightarrow action [S_m, a^i] = reduce by $A \xrightarrow{r} \beta$

$r = |\beta|$

replace by the production

$\hookrightarrow 2r \rightarrow$ pop 2r symbols

$S_0 X_1 S_1 \dots X_{m-r} S_{m-r}$

$S_0 X_1 S_1 \dots X_{m-1} \beta S_{m-1} S_m, A \xrightarrow{r} \beta$ state at the top of the stack.

\downarrow
pop by which production

13/9/22

SLR parsing table

If G is a grammar, table construction

- 1) Augmented grammar $\Rightarrow \{S' \xrightarrow{} S, S \xrightarrow{} B, C \xrightarrow{} a\}$
- 2) Using closure and goto construct LR(0) item.

$A \xrightarrow{} \cdot XYZ$

$A \xrightarrow{} X \cdot YZ$

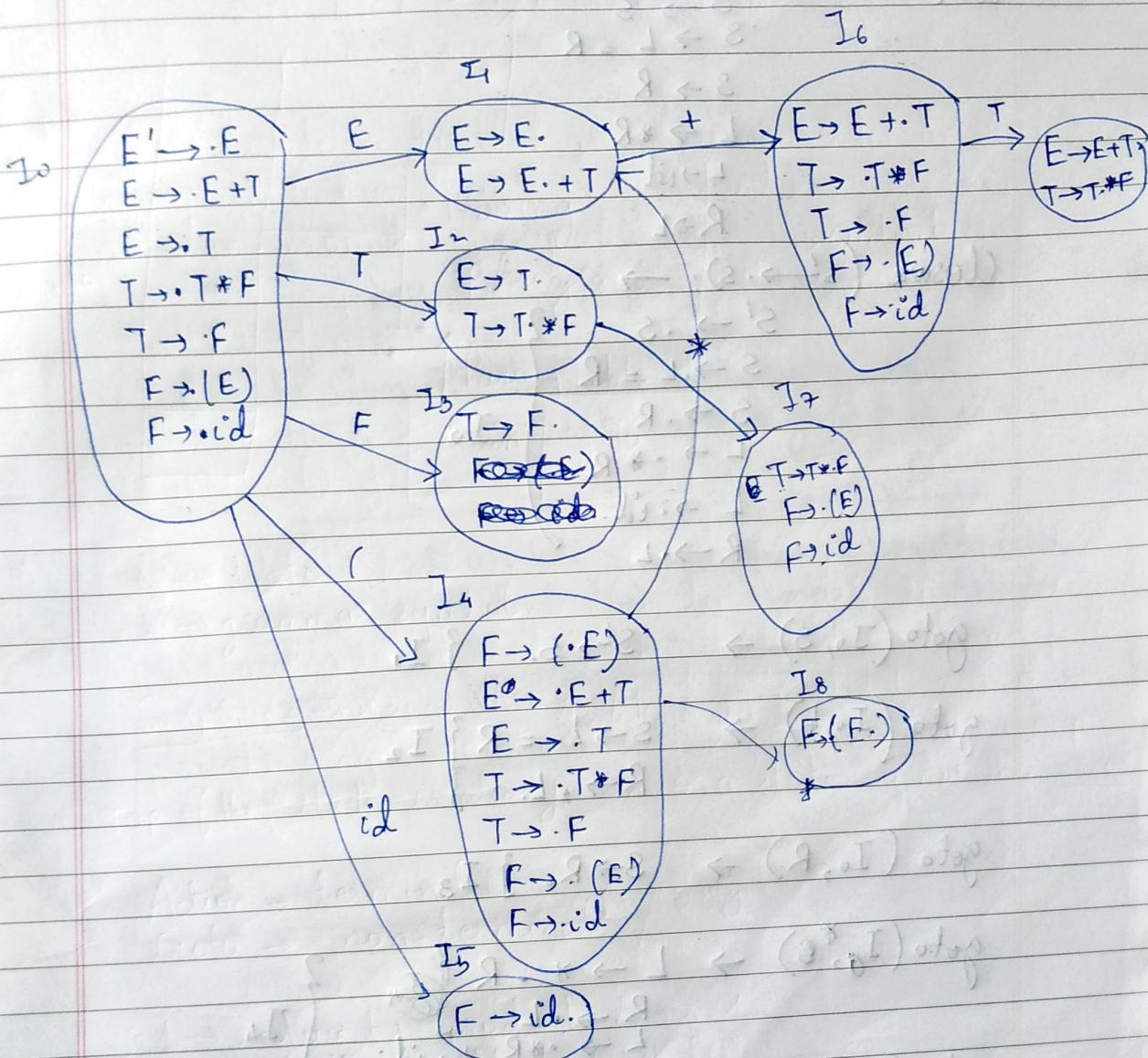
$A \xrightarrow{} XY \cdot Z$

$A \xrightarrow{} XY \cdot Z$

Closure Operation

Augmented
Grammar

$$\left\{ \begin{array}{l} E' \rightarrow E \\ E \rightarrow E + T \\ E \rightarrow T \\ T \rightarrow T * F \\ T \rightarrow F \\ F \rightarrow (E) \\ F \rightarrow id \end{array} \right. \longrightarrow \text{Grammar } G'$$



FOLLOW (F) =

19/09/2022

- ① Every SLR(1) 'G' is unambiguous but there are many
 a Unambiguous grammar that are not SLR(1).

Ans.

Given an unambiguous grammar, whose productions are:

$$\begin{aligned} S' &\rightarrow S \\ S &\rightarrow L = R \\ S &\rightarrow R \\ L &\rightarrow *R \\ L &\rightarrow id \\ R &\rightarrow L \end{aligned}$$

Closure ($S' \xrightarrow{*} S.$) →

$$\left. \begin{aligned} S' &\rightarrow .S \\ S &\rightarrow .L = R \\ S &\rightarrow .R \\ L &\rightarrow .*R \\ L &\rightarrow .id \\ R &\rightarrow L \end{aligned} \right\} I_0$$

goto (I_0, S) → $S \xrightarrow{*} S.$ } I_1

goto (I_0, L) → $\left. \begin{aligned} S &\rightarrow L. = R \\ R &\rightarrow .L \end{aligned} \right\} I_2$

goto (I_0, R) → $S \xrightarrow{*} R.$ } I_3

goto ($I_0, *$) → $\left. \begin{aligned} L &\rightarrow * . R \\ R &\rightarrow . L \\ L &\rightarrow . * R \\ L &\rightarrow . id \end{aligned} \right\} I_4$

goto (I_0, id) → $L \rightarrow id.$ } I_5

goto ($I_2, =$) → $S \xrightarrow{*} L = . R$ } I_6
 $\left. \begin{aligned} R &\rightarrow . L \\ L &\rightarrow . * R \\ L &\rightarrow . id \end{aligned} \right\}$

Date _____ / _____ / _____

$$\begin{aligned} \text{goto } (I_4, R) &\rightarrow L \rightarrow *R. \} I_7 \\ \text{goto } (I_4, L) &\rightarrow R \rightarrow L. \} I_8 \\ \text{goto } (I_4, *) &\rightarrow L \rightarrow *.*R \} I_4 \\ \text{goto } (I_4, id) &\rightarrow I_5 \end{aligned}$$

$$\text{goto } (I_6, R) \rightarrow S \rightarrow L = R. \} I_9$$

~~$\text{goto } (I_6, \dots)$~~ $\text{goto } (I_6, *) \Rightarrow I_4$

$$\text{goto } (I_6, id) \Rightarrow I_5$$

FOLLOW(S) \cup FOLLOW(L)

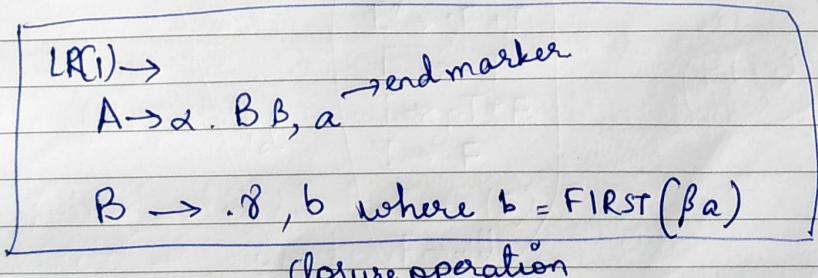
$$\text{FOLLOW}(R) = \{ \$, = \}$$

$$\text{FOLLOW}(L) = \{ =, \$ \} \quad r5$$

$$\text{goto } (I_6, =) = I_6$$

So for the item in column $=$, we see that both S and $r5$ is present. So this grammar is not SLR(1).

* In canonical LR we add a terminal or endmarker at the end of each production to remove the disadvt. obtained in SLR, so it is known as LR(1).



Goto Operation:

$$I_i : A \rightarrow \alpha . X \beta, a$$

$$\text{goto } (I_i, x)$$

$$I_j : A \rightarrow \alpha . X . \beta, a$$