Propositional Logic– Solution

1) Translate the following Propositional Logic to English sentences.
Let:

- E=Liron is eating
- H=Liron is hungry

(a) $E \Rightarrow \neg H$
Answer: If Liron is eating, then Liron is not hungry

(b) $E \wedge \neg H$
Answer: Liron is eating and not hungry

(c) $\neg(H \Rightarrow \neg E)$
Answer: Liron is hungry and eating

2) Translate the following English sentences to Propositional Logic.
Propositions: (R)aining, Liron is (S)ick, Liron is (H)ungry, Liron is (HA)appy, Liron owns a (C)at, Liron owns a (D)og

(a) It is raining if and only if Liron is sick
Answer: $R \Leftrightarrow S$

(b) If Liron is sick then it is raining, and vice versa
Answer: $(S \Rightarrow R) \wedge (R \Rightarrow S)$ (which is equivalent to $R \Leftrightarrow S$)

(c) It is raining is equivalent to Liron is sick
Answer: $R \Leftrightarrow S$

(d) Liron is hungry but happy
Answer: $H \wedge HA$

(e) Liron either owns a cat or a dog
Answer: $(C \wedge \neg D) \vee (\neg C \wedge D)$

3) Which of the following propositions are tautologies? Which are contradictions? Why?

(a) Three is a prime number.
Answer: neither a tautology nor a contradiction

(b) It is raining or it is not raining.
Answer: tautology

(c) It is raining (P) and it is not raining ($\neg P$).
Answer: contradiction

Example reasoning:
All rows in the truth table evaluate to false.

P	$P \wedge \neg P$
t	f
f	f

4) Which of the following propositions are tautologies? Why?

(a) P
Answer: not a tautology

(b) $P \Rightarrow P$
Answer: tautology

(c) $(P \Rightarrow P) \Rightarrow P$
Answer: not a tautology
Example reasoning:
Not all rows in the truth table evaluate to true.

P	P ⇒ P	(P ⇒ P) ⇒ P
t	t	t
f	t	f

(d) $P \Rightarrow (P \Rightarrow P)$
Answer: tautology

5) Which of the two following propositions are equivalent in the sense that one can always be substituted for the other one in any proposition without changing its truth value? Why?

(a) first proposition:$P \Rightarrow Q$ second proposition: $\neg P \vee Q$
Answer: yes
Example reasoning:
All rows in the truth table evaluate to the same truth value.

P	Q	P ⇒ Q	¬ P ∨ Q
t	t	t	t
t	f	f	f
f	t	t	t
f	f	t	t

(b) first proposition: $\neg P$ second proposition: $P \Rightarrow False$
Answer: yes

(c) first proposition: $\neg P$ second proposition: $False \Rightarrow P$
Answer: no

(d) first proposition: $\neg P$ second proposition: $\neg P \vee Q$
Answer: no

6) Is it possible that

(a) $(KB \models S)$ and $(\neg KB \models S)$
Answer: Yes. For example, if $S \equiv TRUE$, then any interpretation that satisfies KB or $\neg KB$ also satisfies S.