KNOWLEDGE REPRESENTATION

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For the purpose of solving complex problems c\encountered in AI, we need both a large amount of knowledge and some mechanism for manipulating that knowledge to create solutions to new problems. A variety of ways of representing knowledge (facts) have been exploited in AI programs. In all variety of knowledge representations , we deal with two kinds of entities.

A. Facts: Truths in some relevant world. These are the things we want to represent.

B. Representations of facts in some chosen formalism . these are things we will

actually be able to manipulate.

One way to think of structuring these entities is at two levels : (a) the knowledge level, at which facts are described, and (b) the symbol level, at which representations of objects at the knowledge level are defined in terms of symbols that can be manipulated by programs.

The facts and representations are linked with two-way mappings. This link is called representation mappings. The forward representation mapping maps from facts to representations. The backward representation mapping goes the other way, from representations to facts.

One common representation is natural language (particularly English) sentences. Regardless of the representation for facts we use in a program , we may also need to be concerned with an English representation of those facts in order to facilitate getting information into and out of the system. We need mapping functions from English sentences to the representation we actually use and from it back to sentences.

Representations and Mappings

- In order to solve complex problems encountered in artificial intelligence, one needs both a large amount of knowledge and some mechanism for manipulating that knowledge to create solutions.
- Knowledge and Representation are two distinct entities. They play central but distinguishable roles in the intelligent system.
- Knowledge is a description of the world. It determines a system’s competence by what it knows.
- Moreover, Representation is the way knowledge is encoded. It defines a system’s performance in doing something.
- Different types of knowledge require different kinds of representation.

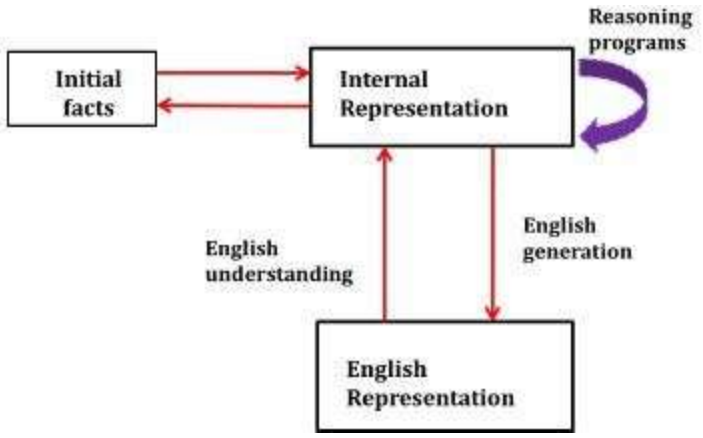


Fig: Mapping between Facts and Representations

The Knowledge Representation models/mechanisms are often based on:

- Logic
- Rules
- Frames
- Semantic Net

Knowledge is categorized into two major types:

1. Tacit corresponds to "informal" or "implicit"
    - Exists within a human being;
    - It is embodied.
    - Difficult to articulate formally.
    - Difficult to communicate or share.
    - Moreover, Hard to steal or copy.
    - Drawn from experience, action, subjective insight
2. Explicit formal type of knowledge, Explicit
    - Explicit knowledge
    - Exists outside a human being;
    - It is embedded.
    - Can be articulated formally.
    - Also, Can be shared, copied, processed and stored.
    - So, Easy to steal or copy
    - Drawn from the artifact of some type as a principle, procedure, process, concepts.

A variety of ways of representing knowledge have been exploited in AI programs.

There are two different kinds of entities, we are dealing with.

1. Facts: Truth in some relevant world. Things we want to represent.
2. Also, Representation of facts in some chosen formalism. Things we will actually be able to manipulate.

These entities structured at two levels:

1. The knowledge level, at which facts described.
2. Moreover, The symbol level, at which representation of objects defined in terms of symbols that can manipulate by programs

Framework of Knowledge Representation

- The computer requires a well-defined problem description to process and provide a well-defined acceptable solution.

- Moreover, To collect fragments of knowledge we need first to formulate a description in our spoken language and then represent it in formal language so that computer can understand.
- Also, The computer can then use an algorithm to compute an answer.

So, This process illustrated as,

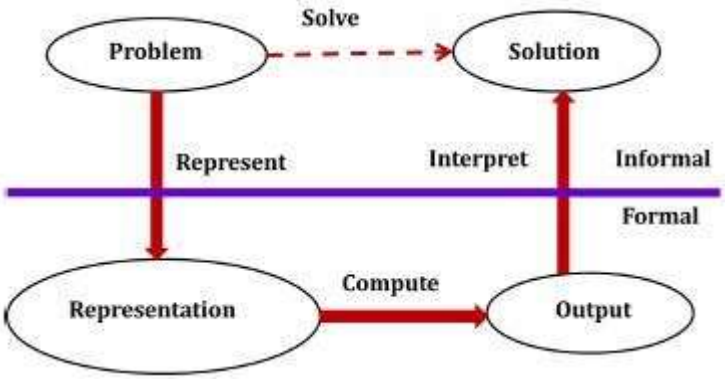


Fig: Knowledge Representation Framework

The steps are:

- The informal formalism of the problem takes place first.
- It then represented formally and the computer produces an output.
- This output can then represented in an informally described solution that user understands or checks for consistency.

The Problem solving requires,

- Formal knowledge representation, and
- Moreover, Conversion of informal knowledge to a formal knowledge that is the conversion of implicit knowledge to explicit knowledge.

Mapping between Facts and Representation

- Knowledge is a collection of facts from some domain.
- Also, We need a representation of "facts" that can manipulate by a program.
- Moreover, Normal English is insufficient, too hard currently for a computer program to draw inferences in natural languages.
- Thus some symbolic representation is necessary.

A good knowledge representation enables fast and accurate access to knowledge and understanding of the content.

A knowledge representation system should have following properties.

1. Representational Adequacy
    - The ability to represent all kinds of knowledge that are needed in that domain.
2. Inferential Adequacy
    - Also, The ability to manipulate the representational structures to derive new structures corresponding to new knowledge inferred from old.
3. Inferential Efficiency
    - The ability to incorporate additional information into the knowledge structure that can be used to focus the attention of the inference mechanisms in the most promising direction.
4. Acquisitional Efficiency
    - Moreover, The ability to acquire new knowledge using automatic methods wherever possible rather than reliance on human intervention.

Knowledge Representation Schemes

Relational Knowledge

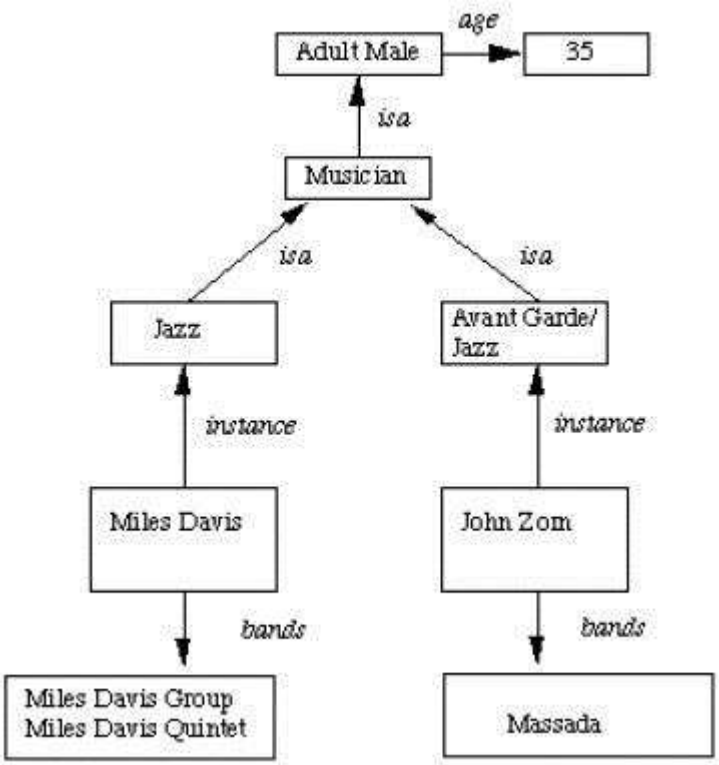
- The simplest way to represent declarative facts is a set of relations of the same sort used in the database system.
- Provides a framework to compare two objects based on equivalent attributes. o Any instance in which two different objects are compared is a relational type of knowledge.
- The table below shows a simple way to store facts.
    - Also, The facts about a set of objects are put systematically in columns.
    - This representation provides little opportunity for inference.

Player	Height	Weight	Bats - Throws
Aaron	6-0	180	Right - Right
Mays	5-10	170	Right - Right
Ruth	6-2	215	Left - Left
Williams	6-3	205	Left - Right

- Given the facts, it is not possible to answer a simple question such as: "Who is the heaviest player?"
- Also, But if a procedure for finding the heaviest player is provided, then these facts will enable that procedure to compute an answer.
- Moreover, We can ask things like who "bats – left" and "throws – right".

Inheritable Knowledge

- Here the knowledge elements inherit attributes from their parents.
- The knowledge embodied in the design hierarchies found in the functional, physical and process domains.
- Within the hierarchy, elements inherit attributes from their parents, but in many cases, not all attributes of the parent elements prescribed to the child elements.
- Also, The inheritance is a powerful form of inference, but not adequate.
- Moreover, The basic KR (Knowledge Representation) needs to augment with inference mechanism.
- Property inheritance: The objects or elements of specific classes inherit attributes and values from more general classes.
- So, The classes organized in a generalized hierarchy.



- Boxed nodes — objects and values of attributes of objects.
- Arrows — the point from object to its value.
- This structure is known as a slot and filler structure, semantic network or a collection of frames.

The steps to retrieve a value for an attribute of an instance object:

1. Find the object in the knowledge base
2. If there is a value for the attribute report it
3. Otherwise look for a value of an instance, if none fail
4. Also, Go to that node and find a value for the attribute and then report it
5. Otherwise, search through using is until a value is found for the attribute.

Inferential Knowledge

- This knowledge generates new information from the given information.
- This new information does not require further data gathering form source but does require analysis of the given information to generate new knowledge.
- Example: given a set of relations and values, one may infer other values or relations. A predicate logic (a mathematical deduction) used to infer from a set of attributes. Moreover, Inference through predicate logic uses a set of logical operations to relate individual data.
- Represent knowledge as formal logic: All dogs have tails $\forall x: dog(x) \rightarrow hastail(x)$
- Advantages:
    - A set of strict rules.
    - Can use to derive more facts.
    - Also, Truths of new statements can be verified.
    - Guaranteed correctness.
- So, Many inference procedures available to implement standard rules of logic popular in AI systems. e.g Automated theorem proving.

Procedural Knowledge

- A representation in which the control information, to use the knowledge, embedded in the knowledge itself. For example, computer programs, directions, and recipes; these indicate specific use or implementation;
- Moreover, Knowledge encoded in some procedures, small programs that know how to do specific things, how to proceed.
- Advantages:
    - Heuristic or domain-specific knowledge can represent.
    - Moreover, Extended logical inferences, such as default reasoning facilitated.
    - Also, Side effects of actions may model. Some rules may become false in time. Keeping track of this in large systems may be tricky.
- Disadvantages:
    - Completeness — not all cases may represent.
    - Consistency — not all deductions may be correct. e.g If we know that Fred is a bird we might deduce that Fred can fly. Later we might discover that Fred is an emu.
    - Modularity sacrificed. Changes in knowledge base might have far-reaching effects.
    - Cumbersome control information.

USING PREDICATE LOGIC

Representation of Simple Facts in Logic

Propositional logic is useful because it is simple to deal with and a decision procedure for it exists.

Also, In order to draw conclusions, facts are represented in a more convenient way as,

1. Marcus is a man.
    - man(Marcus)
2. Plato is a man.
    - man(Plato)
3. All men are mortal.
    - mortal(men)

But propositional logic fails to capture the relationship between an individual being a man and that individual being a mortal.

- How can these sentences be represented so that we can infer the third sentence from the first two?
- Also, Propositional logic commits only to the existence of facts that may or may not be the case in the world being represented.
- Moreover, It has a simple syntax and simple semantics. It suffices to illustrate the process of inference.
- Propositional logic quickly becomes impractical, even for very small worlds.

Predicate logic

First-order Predicate logic (FOPL) models the world in terms of

- Objects, which are things with individual identities
- Properties of objects that distinguish them from other objects
- Relations that hold among sets of objects

- Functions, which are a subset of relations where there is only one "value" for any given "input"

First-order Predicate logic (FOPL) provides

- Constants: a, b, dog33. Name a specific object.
- Variables: X, Y. Refer to an object without naming it.
- Functions: Mapping from objects to objects.
- Terms: Refer to objects
- Atomic Sentences: in(dad-of(X), food6) Can be true or false, Correspond to propositional symbols P, Q.

A well-formed formula (*wff*) is a sentence containing no "free" variables. So, That is, all variables are "bound" by universal or existential quantifiers.

(∀x)P(x, y) has x bound as a universally quantified variable, but y is free.

Quantifiers

Universal quantification

- (∀x)P(x) means that P holds for all values of x in the domain associated with that variable
- E.g., (∀x) dolphin(x) → mammal(x)

Existential quantification

- (∃ x)P(x) means that P holds for some value of x in the domain associated with that variable
- E.g., (∃ x) mammal(x) ∧ lays-eggs(x)

Also, Consider the following example that shows the use of predicate logic as a way of representing knowledge.

1. Marcus was a man.
2. Marcus was a Pompeian.
3. All Pompeians were Romans.
4. Caesar was a ruler.
5. Also, All Pompeians were either loyal to Caesar or hated him.
6. Everyone is loyal to someone.
7. People only try to assassinate rulers they are not loyal to.
8. Marcus tried to assassinate Caesar.

The facts described by these sentences can be represented as a set of well-formed formulas (*wffs*) as follows:

1. Marcus was a man.
    - man(Marcus)
2. Marcus was a Pompeian.
    - Pompeian(Marcus)
3. All Pompeians were Romans.
    - ∀x: Pompeian(x) → Roman(x)
4. Caesar was a ruler.
    - ruler(Caesar)
5. All Pompeians were either loyal to Caesar or hated him.
    - inclusive-or
    - ∀x: Roman(x) → loyalto(x, Caesar) ∨ hate(x, Caesar)
    - exclusive-or
    - ∀x: Roman(x) → (loyalto(x, Caesar) ∧¬ hate(x, Caesar)) ∨
    - (¬loyalto(x, Caesar) ∧ hate(x, Caesar))

6. Everyone is loyal to someone.
    - ∀x: ∃y: loyalto(x, y)
7. People only try to assassinate rulers they are not loyal to.
    - ∀x: ∀y: person(x) ∧ ruler(y) ∧ tryassassinate(x, y)
    - →¬loyalto(x, y)
8. Marcus tried to assassinate Caesar.
    - tryassassinate(Marcus, Caesar)

Now suppose if we want to use these statements to answer the question: ***Was Marcus loyal to Caesar?***

Also, Now let's try to produce a formal proof, reasoning backward from the desired goal: ¬ Ioyalto(Marcus, Caesar)

In order to prove the goal, we need to use the rules of inference to transform it into another goal (or possibly a set of goals) that can, in turn, transformed, and so on, until there are no unsatisfied goals remaining.

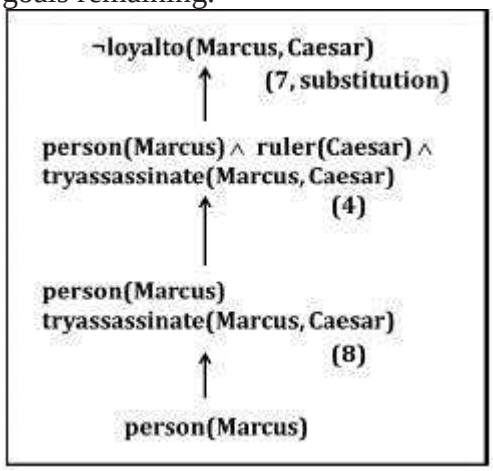


Figure: An attempt to prove ¬loyalto(Marcus, Caesar).

- The problem is that, although we know that Marcus was a man, we do not have any way to conclude from that that Marcus was a person. Also, We need to add the representation of another fact to our system, namely***: ∀man(x) → person(x)***
- Now we can satisfy the last goal and produce a proof that Marcus was not loyal to Caesar.
- Moreover, From this simple example, we see that three important issues must be addressed in the process of converting English sentences into logical statements and then using those statements to deduce new ones:
    1. Many English sentences are ambiguous (for example, 5, 6, and 7 above). Choosing the correct interpretation may be difficult.
    2. Also, There is often a choice of how to represent the knowledge. Simple representations are desirable, but they may exclude certain kinds of reasoning.
    3. Similalry, Even in very simple situations, a set of sentences is unlikely to contain all the information necessary to reason about the topic at hand. In order to be able to use a set of statements effectively. Moreover, It is usually necessary to have access to another set of statements that represent facts that people consider too obvious to mention.

Representing Instance and ISA Relationships

- Specific attributes **instance** and **isa** play an important role particularly in a useful form of reasoning called property inheritance.
- The predicates instance and isa explicitly captured the relationships they used to express, namely class membership and class inclusion.
- 4.2 shows the first five sentences of the last section represented in logic in three different ways.
- The first part of the figure contains the representations we have already discussed. In these representations, class membership represented with unary predicates (such as Roman), each of which corresponds to a class.
- Asserting that P(x) is true is equivalent to asserting that x is an instance (or element) of P.
- The second part of the figure contains representations that use the ***instance*** predicate explicitly.

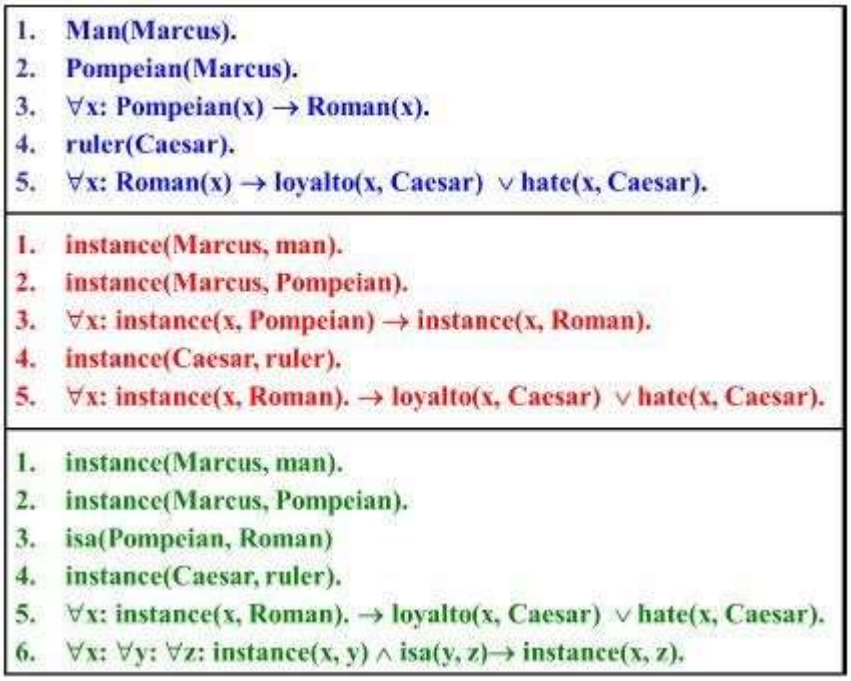


Figure: Three ways of representing class membership: ISA Relationships

- The predicate ***instance*** is a binary one, whose first argument is an object and whose second argument is a class to which the object belongs.
- But these representations do not use an explicit ***isa*** predicate.
- Instead, subclass relationships, such as that between Pompeians and Romans, described as shown in sentence 3.
- The implication rule states that if an object is an instance of the subclass Pompeian then it is an instance of the superclass Roman.
- Note that this rule is equivalent to the standard set-theoretic definition of the subclass-superclass relationship.
- The third part contains representations that use both the ***instance*** and ***isa*** predicates explicitly.
- The use of the ***isa*** predicate simplifies the representation of sentence 3, but it requires that one additional axiom (shown here as number 6) be provided.

Computable Functions and Predicates

- To express simple facts, such as the following greater-than and less-than relationships: gt(1,O) It(0,1) gt(2,1) It(1,2) gt(3,2) It(2,3)
- It is often also useful to have computable functions as well as computable predicates. Thus we might want to be able to evaluate the truth of gt(2 + 3,1)
- To do so requires that we first compute the value of the plus function given the arguments 2 and 3, and then send the arguments 5 and 1 to gt.

Consider the following set of facts, again involving Marcus:

1) Marcus was a man.
   man(Marcus)
2) Marcus was a Pompeian.
   Pompeian(Marcus)
3) Marcus was born in 40 A.D.
   born(Marcus, 40)
4) All men are mortal.
   x: man(x) → mortal(x)
5) All Pompeians died when the volcano erupted in 79 A.D.
   erupted(volcano, 79) ∧ ∀ x : [Pompeian(x) → died(x, 79)]
6) No mortal lives longer than 150 years.
   x: t1: At2: *mortal(x) born(x, t1) gt(t2 – t1,150) → died(x, t2)*
7) It is now 1991.
   now = 1991

So, Above example shows how these ideas of computable functions and predicates can be useful. It also makes use of the notion of equality and allows equal objects to be substituted for each other whenever it appears helpful to do so during a proof.

- So, Now suppose we want to answer the question "Is Marcus alive?"
- The statements suggested here, there may be two ways of deducing an answer.
- Either we can show that Marcus is dead because he was killed by the volcano or we can show that he must be dead because he would otherwise be more than 150 years old, which we know is not possible.
- Also, As soon as we attempt to follow either of those paths rigorously, however, we discover, just as we did in the last example, that we need some additional knowledge. For example, our statements talk about dying, but they say nothing that relates to being alive, which is what the question is asking.

So we add the following facts:

8) Alive means not dead.
   x: t: [alive(x, t) → ¬ dead(x, t)] [¬ dead(x, t) → alive(x, t)]
9) If someone dies, then he is dead at all later times.
   x: t1: At2*: died(x, t1) gt(t2, t1) → dead(x, t2)*

So, Now let's attempt to answer the question "Is Marcus alive?" by proving: ¬ ***alive(Marcus, now)***

Resolution

Propositional Resolution

1. Convert all the propositions of F to clause form.
2. Negate P and convert the result to clause form. Add it to the set of clauses obtained in step 1.
3. Repeat until either a contradiction is found or no progress can be made:
    1. Select two clauses. Call these the parent clauses.
    2. Resolve them together. The resulting clause, called the resolvent, will be the disjunction of all of the literals of both of the parent clauses with the following exception: If there are any pairs of literals L and ¬ *L* such that one of the parent clauses contains *L* and the other contains ¬*L,* then select one such pair and eliminate both *L* and ¬ *L* from the resolvent.
    3. If the resolvent is the empty clause, then a contradiction has been found. If it is not, then add it to the set of classes available to the procedure.

The Unification Algorithm

- In propositional logic, it is easy to determine that two literals cannot both be true at the same time.
- Simply look for L and ¬L in predicate logic, this matching process is more complicated since the arguments of the predicates must be considered.
- For example, man(John) and ¬man(John) is a contradiction, while the man(John) and ¬man(Spot) is not.
- Thus, in order to determine contradictions, we need a matching procedure that compares two literals and discovers whether there exists a set of substitutions that makes them identical.
- There is a straightforward recursive procedure, called the unification algorithm, that does it.

Algorithm: Unify(L1, L2)

1. If L1 or L2 are both variables or constants, then:
    1. If L1 and L2 are identical, then return NIL.
    2. Else if L1 is a variable, then if L1 occurs in L2 then return {FAIL}, else return (L2/L1).
    3. Also, Else if L2 is a variable, then if L2 occurs in L1 then return {FAIL}, else return (L1/L2). d. Else return {FAIL}.
2. If the initial predicate symbols in L1 and L2 are not identical, then return {FAIL}.
3. If LI and L2 have a different number of arguments, then return {FAIL}.
4. Set SUBST to NIL. (At the end of this procedure, SUBST will contain all the substitutions used to unify L1 and L2.)
5. For I ← 1 to the number of arguments in L1 :
    1. Call Unify with the i^{th} argument of L1 and the i^{th} argument of L2, putting the result in S.
    2. If S contains FAIL then return {FAIL}.
    3. If S is not equal to NIL then:
        2. Apply S to the remainder of both L1 and L2.
        3. SUBST: = APPEND(S, SUBST).
6. Return SUBST.

Resolution in Predicate Logic

We can now state the resolution algorithm for predicate logic as follows, assuming a set of given statements F and a statement to be proved P:

Algorithm: Resolution

1. Convert all the statements of F to clause form.
2. Negate P and convert the result to clause form. Add it to the set of clauses obtained in 1.
3. Repeat until a contradiction found, no progress can make, or a predetermined amount of effort has expanded.
    1. Select two clauses. Call these the parent clauses.
    2. Resolve them together. The resolvent will the disjunction of all the literals of both parent clauses with appropriate substitutions performed and with the following exception: If there is one pair of literals T1 and ¬T2 such that one of the parent clauses contains T2 and the other contains T1 and if T1 and T2 are unifiable, then neither T1 nor T2 should appear in the resolvent. We call T1 and T2 Complementary literals. Use the substitution produced by the unification to create the resolvent. If there is more than one pair of complementary literals, only one pair should omit from the resolvent.
    3. If the resolvent is an empty clause, then a contradiction has found. Moreover, If it is not, then add it to the set of classes available to the procedure.

Resolution Procedure

- Resolution is a procedure, which gains its efficiency from the fact that it operates on statements that have been converted to a very convenient standard form.
- Resolution produces proofs by refutation.
- In other words, ***to prove a statement (i.e., to show that it is valid), resolution attempts to show that the negation of the statement produces a contradiction with the known statements (i.e., that it is unsatisfiable).***
- The resolution procedure is a simple iterative process: at each step, two clauses, called the parent clauses, are compared (resolved), resulting in a new clause that has inferred from them. The new clause represents ways that the two parent clauses interact with each other. Suppose that there are two clauses in the system:

winter **V** ***summer***

¬ ***winter*** **V** ***cold***

- Now we observe that precisely one of winter and ¬ winter will be true at any point.
- If winter is true, then cold must be true to guarantee the truth of the second clause. If ¬ winter is true, then summer must be true to guarantee the truth of the first clause.
- Thus we see that from these two clauses we can deduce ***summer V cold***
- This is the deduction that the resolution procedure will make.
- Resolution operates by taking two clauses that each contains the same literal, in this example, ***winter***.
- Moreover, The literal must occur in the positive form in one clause and in negative form in the other. The resolvent obtained by combining all of the literals of the two parent clauses except the ones that cancel.
- If the clause that produced is the empty clause, then a contradiction has found.

For example, the two clauses

winter

¬ winter

will produce the empty clause.

Natural Deduction Using Rules

Testing whether a proposition is a tautology by testing every possible truth assignment is expensive—there are exponentially many. We need a **deductive system**, which will allow us to construct proofs of tautologies in a step-by-step fashion.

The system we will use is known as **natural deduction**. The system consists of a set of **rules of inference** for deriving consequences from premises. One builds a proof tree whose root is the proposition to be proved and whose leaves are the initial assumptions or axioms (for proof trees, we usually draw the root at the bottom and the leaves at the top).

For example, one rule of our system is known as **modus ponens**. Intuitively, this says that if we know P is true, and we know that P implies Q, then we can conclude Q.

$$\frac{P \qquad P \Rightarrow Q}{Q} \text{ (modus ponens)}$$

The propositions above the line are called **premises**; the proposition below the line is the **conclusion**. Both the premises and the conclusion may contain metavariables (in this case, P and Q) representing arbitrary propositions. When an inference rule is used as part of a proof, the metavariables are replaced in a consistent way with the appropriate kind of object (in this case, propositions).

Most rules come in one of two flavors: **introduction** or **elimination** rules. Introduction rules introduce the use of a logical operator, and elimination rules eliminate it. Modus ponens is an elimination rule for ⇒. On the right-hand side of a rule, we often write the name of the rule. This is helpful when reading proofs. In this case, we have written (modus ponens). We could also have written (⇒-elim) to indicate that this is the elimination rule for ⇒.

Rules for Conjunction

Conjunction (∧) has an introduction rule and two elimination rules:

$$\frac{P \qquad Q}{P \wedge Q} \text{ (∧-intro)} \qquad \frac{P \wedge Q}{P} \text{ (∧-elim-left)} \qquad \frac{P \wedge Q}{Q} \text{ (∧-elim-right)}$$

Rule for T

The simplest introduction rule is the one for T. It is called "unit". Because it has no premises, this rule is an **axiom**: something that can start a proof.

$$\frac{}{T} \text{ (unit)}$$

Rules for Implication

In natural deduction, to prove an implication of the form P ⇒ Q, we assume P, then reason under that assumption to try to derive Q. If we are successful, then we can conclude that P ⇒ Q.

In a proof, we are always allowed to introduce a new assumption P, then reason under that assumption. We must give the assumption a name; we have used the name x in the example below. Each distinct assumption must have a different name.

$$\frac{}{[x : P]} \text{ (assum)}$$

Because it has no premises, this rule can also start a proof. It can be used as if the proposition P were proved. The name of the assumption is also indicated here.
However, you do not get to make assumptions for free! To get a complete proof, all assumptions must be eventually *discharged*. This is done in the implication introduction rule. This rule introduces an implication $P \Rightarrow Q$ by discharging a prior assumption [x : P]. Intuitively, if Q can be proved under the assumption P, then the implication $P \Rightarrow Q$ holds without any assumptions. We write x in the rule name to show which assumption is discharged. This rule and modus ponens are the introduction and elimination rules for implications.

$$\frac{\begin{array}{c}[x : P] \\ \vdots \\ Q\end{array}}{P \Rightarrow Q}\ (\Rightarrow\text{-intro/x}) \qquad \frac{P \qquad P \Rightarrow Q}{Q}\ (\Rightarrow\text{-elim, modus ponens})$$

A proof is valid only if every assumption is eventually discharged. This must happen in the proof tree below the assumption. The same assumption can be used more than once.

Rules for Disjunction

$$\frac{P}{P \vee Q}\ (\vee\text{-intro-left}) \qquad \frac{Q}{P \vee Q}\ (\vee\text{-intro-right}) \qquad \frac{P \vee Q \qquad P \Rightarrow R \qquad Q \Rightarrow R}{R}\ (\vee\text{-elim})$$

Rules for Negation

A negation $\neg P$ can be considered an abbreviation for $P \Rightarrow \bot$:

$$\frac{P \Rightarrow \bot}{\neg P}\ (\neg\text{-intro}) \qquad \frac{\neg P}{P \Rightarrow \bot}\ (\neg\text{-elim})$$

Rules for Falsity

$$\frac{\begin{array}{c}[x : \neg P] \\ \vdots \\ \bot\end{array}}{P}\ (\text{reductio ad absurdum, RAA/x}) \qquad \frac{\bot}{P}\ (\text{ex falso quodlibet, EFQ})$$

Reductio ad absurdum (RAA) is an interesting rule. It embodies proofs by contradiction. It says that if by assuming that P is false we can derive a contradiction, then P must be true. The assumption x is discharged in the application of this rule. This rule is present in classical logic but not in **intuitionistic** (constructive) logic. In intuitionistic logic, a proposition is not considered true simply because its negation is false.

Excluded Middle

Another classical tautology that is not intuitionistically valid is the **the law of the excluded middle**, $P \vee \neg P$. We will take it as an axiom in our system. The Latin name for this rule is *tertium non datur*, but we will call it *magic*.

$$\frac{}{P \vee \neg P}\ (\text{magic})$$

Proofs

A proof of proposition P in natural deduction starts from axioms and assumptions and derives P with all assumptions discharged. Every step in the proof is an instance of an inference rule with metavariables substituted consistently with expressions of the appropriate syntactic class.

Example

For example, here is a proof of the proposition (A ⇒ B ⇒ C) ⇒ (A ∧ B ⇒ C).

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{[y : A \wedge B]}\,(\text{A})}{A}\,(\wedge\text{E})\quad \dfrac{}{[x : A \Rightarrow B \Rightarrow C]}\,(\text{A})}{B \Rightarrow C}\,(\Rightarrow\text{E})\quad \dfrac{}{[y : A \wedge B]}\,(\text{A})}{B}\,(\wedge\text{E})}{C}\,(\Rightarrow\text{E})}{A \wedge B \Rightarrow C}\,(\Rightarrow\text{I,y})}{(A \Rightarrow B \Rightarrow C) \Rightarrow (A \wedge B \Rightarrow C)}\,(\Rightarrow\text{I,x})$$

The final step in the proof is to derive (A ⇒ B ⇒ C) ⇒ (A ∧ B ⇒ C) from (A ∧ B ⇒ C), which is done using the rule (⇒-intro), discharging the assumption [x : A ⇒ B ⇒ C]. To see how this rule generates the proof step, substitute for the metavariables P, Q, x in the rule as follows: P = (A ⇒ B ⇒ C), Q = (A ∧ B ⇒ C), and x = x. The immediately previous step uses the same rule, but with a different substitution: P = A ∧ B, Q = C, x = y.
The proof tree for this example has the following form, with the proved proposition at the root and axioms and assumptions at the leaves.

A proposition that has a complete proof in a deductive system is called a **theorem** of that system.
Soundness and Completeness
A measure of a deductive system's power is whether it is powerful enough to prove all true statements. A deductive system is said to be **complete** if all true statements are theorems (have proofs in the system). For propositional logic and natural deduction, this means that all tautologies must have natural deduction proofs. Conversely, a deductive system is called **sound** if all theorems are true. The proof rules we have given above are in fact sound and complete for propositional logic: every theorem is a tautology, and every tautology is a theorem. Finding a proof for a given tautology can be difficult. But once the proof is found, checking that it is indeed a proof is completely mechanical, requiring no intelligence or insight whatsoever. It is therefore a very strong argument that the thing proved is in fact true.
We can also make writing proofs less tedious by adding more rules that provide reasoning shortcuts. These rules are sound if there is a way to convert a proof using them into a proof using the original rules. Such added rules are called **admissible**.

Procedural versus Declarative Knowledge

We have discussed various search techniques in previous units. Now we would consider a set of rules that represent,

1. Knowledge about relationships in the world and
2. Knowledge about how to solve the problem using the content of the rules.

Procedural vs Declarative Knowledge
Procedural Knowledge

- A representation in which the control information that is necessary to use the knowledge is embedded in the knowledge itself for e.g. computer programs, directions, and recipes; these indicate specific use or implementation;
- The real difference between declarative and procedural views of knowledge lies in where control information reside.

For example, consider the following
Man (Marcus)
Man (Caesar)
Person (Cleopatra)
∀x: Man(x) → Person(x)
Now, try to answer the question. ?Person(y)
The knowledge base justifies any of the following answers.
Y=Marcus
Y=Caesar
Y=Cleopatra

- We get more than one value that satisfies the predicate.
- If only one value needed, then the answer to the question will depend on the order in which the assertions examined during the search for a response.
- If the assertions declarative then they do not themselves say anything about how they will be examined. In case of procedural representation, they say how they will examine.

Declarative Knowledge

- A statement in which knowledge specified, but the use to which that knowledge is to be put is not given.
- For example, laws, people's name; these are the facts which can stand alone, not dependent on other knowledge;
- So to use declarative representation, we must have a program that explains what is to do with the knowledge and how.
- For example, a set of logical assertions can combine with a resolution theorem prover to give a complete program for solving problems but in some cases, the logical assertions can view as a program rather than data to a program.
- Hence the implication statements define the legitimate reasoning paths and automatic assertions provide the starting points of those paths.
- These paths define the execution paths which is similar to the 'if then else "in traditional programming.
- So logical assertions can view as a procedural representation of knowledge.

Logic Programming – Representing Knowledge Using Rules

- Logic programming is a programming paradigm in which logical assertions viewed as programs.
- These are several logic programming systems, PROLOG is one of them.
- ***A PROLOG program consists of several logical assertions where each is a horn clause i.e. a clause with at most one positive literal.***
- Ex : P, P V Q, P → Q
- The facts are represented on Horn Clause for two reasons.
    1. Because of a uniform representation, a simple and efficient interpreter can write.
    2. The logic of Horn Clause decidable.

- Also, The first two differences are the fact that PROLOG programs are actually sets of Horn clause that have been transformed as follows:-
    1. If the Horn Clause contains no negative literal then leave it as it is.
    2. Also, Otherwise rewrite the Horn clauses as an implication, combining all of the negative literals into the antecedent of the implications and the single positive literal into the consequent.
- Moreover, This procedure causes a clause which originally consisted of a disjunction of literals (one of them was positive) to be transformed into a single implication whose antecedent is a conjunction universally quantified.
- But when we apply this transformation, any variables that occurred in negative literals and so now occur in the antecedent become existentially quantified, while the variables in the consequent are still universally quantified.

For example the PROLOG clause P(x): – Q(x, y) is equal to logical expression ∀x: ∃y: Q (x, y) → P(x).

- The difference between the logic and PROLOG representation is that the PROLOG interpretation has a fixed control strategy. And so, the assertions in the PROLOG program define a particular search path to answer any question.
- But, the logical assertions define only the set of answers but not about how to choose among those answers if there is more than one.

Consider the following example:

1. Logical representation

∀x : pet(x) □ small (x) → apartmentpet(x)
∀x : cat(x) □ dog(x) → pet(x)
∀x : poodle (x) → dog (x) □ small (x)
poodle (fluffy)

2. Prolog representation

apartmentpet (x) : pet(x), small (x)
pet (x): cat (x)
pet (x): dog(x)
dog(x): poodle (x)
small (x): poodle(x)
poodle (fluffy)

Forward versus Backward Reasoning

Forward versus Backward Reasoning
A search procedure must find a path between initial and goal states.
There are two directions in which a search process could proceed.
The two types of search are:

1. Forward search which starts from the start state
2. Backward search that starts from the goal state

The production system views the forward and backward as symmetric processes.
Consider a game of playing 8 puzzles. The rules defined are
Square 1 empty and square 2 contains tile n. →

- *Also, Square 2 empty and square 1 contains the tile n.*

Square 1 empty Square 4 contains tile n. →

- *Also, Square 4 empty and Square 1 contains tile n.*

We can solve the problem in 2 ways:

1. Reason forward from the initial state
    - Step 1. Begin building a tree of move sequences by starting with the initial configuration at the root of the tree.
    - Step 2. Generate the next level of the tree by finding all rules ***whose left-hand side matches*** against the root node. The right-hand side is used to create new configurations.
    - Step 3. Generate the next level by considering the nodes in the previous level and applying it to all rules whose left-hand side match.
2. Reasoning backward from the goal states:
    - Step 1. Begin building a tree of move sequences by starting with the goal node configuration at the root of the tree.
    - Step 2. Generate the next level of the tree by finding all rules ***whose right-hand side matches*** against the root node. The left-hand side used to create new configurations.
    - Step 3. Generate the next level by considering the nodes in the previous level and applying it to all rules whose right-hand side match.
    - So, The same rules can use in both cases.
    - Also, In forwarding reasoning, the left-hand sides of the rules matched against the current state and right sides used to generate the new state.
    - Moreover, In backward reasoning, the right-hand sides of the rules matched against the current state and left sides are used to generate the new state.

There are four factors influencing the type of reasoning. They are,

1. Are there more possible start or goal state? We move from smaller set of sets to the length.
2. In what direction is the branching factor greater? We proceed in the direction with the lower branching factor.
3. Will the program be asked to justify its reasoning process to a user? If, so then it is selected since it is very close to the way in which the user thinks.
4. What kind of event is going to trigger a problem-solving episode? If it is the arrival of a new factor, the forward reasoning makes sense. If it is a query to which a response is desired, backward reasoning is more natural.

Example 1 of Forward versus Backward Reasoning

- It is easier to drive from an unfamiliar place from home, rather than from home to an unfamiliar place. Also, If you consider a home as starting place an unfamiliar place as a goal then we have to backtrack from unfamiliar place to home.

Example 2 of Forward versus Backward Reasoning

- Consider a problem of symbolic integration. Moreover, The problem space is a set of formulas, which contains integral expressions. Here START is equal to the given formula with some integrals. GOAL is equivalent to the expression of the formula without any integral. Here we start from the formula with some integrals and proceed to an integral free expression rather than starting from an integral free expression.

Example 3 of Forward versus Backward Reasoning

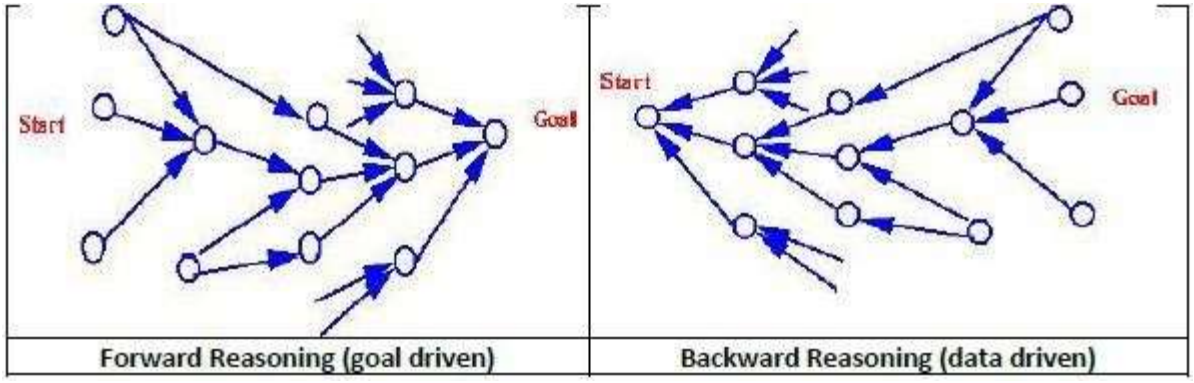
- The third factor is nothing but deciding whether the reasoning process can justify its reasoning. If it justifies then it can apply. For example, doctors are usually unwilling to accept any advice from diagnostics process because it cannot explain its reasoning.

Example 4 of Forward versus Backward Reasoning

- Prolog is an example of backward chaining rule system. In Prolog rules restricted to Horn clauses. This allows for rapid indexing because all the rules for deducing a given fact share the same rule head. Rules matched with unification procedure. Unification tries to find a set of bindings for variables to equate a sub-goal with the head of some rule. Rules in the Prolog program matched in the order in which they appear.

Combining Forward and Backward Reasoning

- Instead of searching either forward or backward, you can search both simultaneously.
- Also, That is, start forward from a starting state and backward from a goal state simultaneously until the paths meet.
- This strategy called Bi-directional search. The following figure shows the reason for a Bidirectional search to be ineffective.



Forward versus Backward Reasoning

- Also, The two searches may pass each other resulting in more work.
- Based on the form of the rules one can decide whether the same rules can apply to both forward and backward reasoning.
- Moreover, If left-hand side and right of the rule contain pure assertions then the rule can reverse.
- And so the same rule can apply to both types of reasoning.
- If the right side of the rule contains an arbitrary procedure then the rule cannot reverse.
- So, In this case, while writing the rule the commitment to a direction of reasoning must make.

Symbolic Reasoning Under Uncertainty

Symbolic Reasoning

- The reasoning is the act of deriving a conclusion from certain properties using a given methodology.
- The reasoning is a process of thinking; reasoning is logically arguing; reasoning is drawing the inference.
- *When a system is required to do something, that it has not been explicitly told how to do, it must reason. It must figure out what it needs to know from what it already knows.*