notebook

November 24, 2024

0.0.1 Data Import and Setup

Import necessary libraries for building and training a neural network, including NumPy for numerical computations, Matplotlib for visualization, and scikit-learn for dataset handling and preprocessing.

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_openml
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import OneHotEncoder
from PIL import Image
```

0.0.2 MNIST Dataset Loading

Define a function to load and preprocess the MNIST dataset. The function: - Loads the MNIST dataset using fetch_openml - Normalizes pixel values to [0,1] - One-hot encodes the labels - Returns processed features and labels

[2]:
```
# Load the MNIST dataset
def load_mnist():
    mnist = fetch_openml('mnist_784', version=1)
    X, y = mnist.data.astype(np.float32), mnist.target.astype(np.int32)

    # Normalize the data
    X /= 255.0

    # Convert y to a NumPy array and reshape it for one-hot encoding
    y = np.array(y).reshape(-1, 1)

    # One-hot encode the labels
    encoder = OneHotEncoder(sparse_output=False)
    y = encoder.fit_transform(y)

    return X, y

# Load and split the data
X, y = load_mnist()
```

```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
↪random_state=42)
```

0.0.3 Neural Network Architecture

Define the neural network class with configurable layer sizes. Initialize weights using He initialization for better training dynamics.

[3]:
```
class NeuralNetwork:
    def __init__(self, input_size, hidden_sizes, output_size):
        self.layers = len(hidden_sizes) + 1
        self.weights = []
        self.biases = []

        # Initialize weights and biases for each layer
        sizes = [input_size] + hidden_sizes + [output_size]
        for i in range(len(sizes) - 1):
            self.weights.append(np.random.randn(sizes[i], sizes[i+1]) * np.
↪sqrt(2 / sizes[i]))
            self.biases.append(np.zeros((1, sizes[i+1])))
```

0.0.4 Activation Functions

Define the necessary activation functions and their derivatives: - ReLU for hidden layers - Softmax for output layer

[4]:
```
def relu(x):
    return np.maximum(0, x)

def relu_derivative(x):
    return (x > 0).astype(float)

def softmax(x):
    exp_x = np.exp(x - np.max(x, axis=1, keepdims=True))
    return exp_x / np.sum(exp_x, axis=1, keepdims=True)
```

0.0.5 Forward Propagation with Dropout

Implement forward propagation with dropout regularization to prevent overfitting. Dropout randomly deactivates neurons during training with a specified probability.

[5]:
```
# Apply dropout to the activations
def apply_dropout(activations, dropout_rate):
    mask = np.random.binomial(1, 1 - dropout_rate, size=activations.shape)
    return activations * mask / (1 - dropout_rate)

# Modify the forward_propagation method to include dropout
def forward_propagation_with_dropout(self, X, dropout_rate=0.0):
```

```
    activations = [X]
    z_values = []

    for i, (w, b) in enumerate(zip(self.weights, self.biases)):
        z = np.dot(activations[-1], w) + b
        z_values.append(z)

        # Apply ReLU for hidden layers, softmax for the output layer
        if i < self.layers - 1:
            activation = relu(z)
            if dropout_rate > 0:  # Apply dropout only during training
                activation = apply_dropout(activation, dropout_rate)
        else:
            activation = softmax(z)

        activations.append(activation)

    return activations, z_values

NeuralNetwork.forward_propagation = forward_propagation_with_dropout
```

0.0.6 Back Propagation

Implement the backpropagation algorithm to compute gradients for network parameters. Uses the chain rule to propagate errors backward through the network.

[6]:
```
def back_propagation(self, activations, z_values, y_true):
    deltas = [activations[-1] - y_true]

    for i in reversed(range(self.layers - 1)):
        delta = np.dot(deltas[0], self.weights[i + 1].T) *␣
↪relu_derivative(z_values[i])
        deltas.insert(0, delta)

    # Compute gradients for weights and biases
    grad_w = []
    grad_b = []
    for i in range(self.layers):
        grad_w.append(np.dot(activations[i].T, deltas[i]) / y_true.shape[0])
        grad_b.append(np.sum(deltas[i], axis=0, keepdims=True) / y_true.
↪shape[0])

    return grad_w, grad_b

NeuralNetwork.back_propagation = back_propagation
```

0.0.7 Parameter Updates with L2 Regularization

Update network parameters using gradient descent with L2 regularization to prevent overfitting.

[7]:
```
def update_parameters_with_l2(self, grad_w, grad_b, learning_rate, l2_lambda):
    for i in range(self.layers):
        self.weights[i] -= learning_rate * (grad_w[i] + l2_lambda * self.
 ↪weights[i])
        self.biases[i] -= learning_rate * grad_b[i]

NeuralNetwork.update_parameters = update_parameters_with_l2
```

0.0.8 Training Loop

Implement the main training loop with: - Mini-batch gradient descent - Loss and accuracy tracking - Progress visualization

[8]:
```
# Modify train method to store loss and accuracy
def train(self, X, y, epochs, batch_size, learning_rate, l2_lambda=0.0,␣
 ↪dropout_rate=0.0):
    X = np.array(X)
    y = np.array(y)

    losses = []
    accuracies = []

    for epoch in range(epochs):
        indices = np.arange(X.shape[0])
        np.random.shuffle(indices)
        X, y = X[indices], y[indices]

        epoch_loss = 0
        for start in range(0, X.shape[0], batch_size):
            end = start + batch_size
            batch_X, batch_y = X[start:end], y[start:end]

            # Forward and backward pass
            activations, z_values = self.forward_propagation(batch_X,␣
 ↪dropout_rate=dropout_rate)
            grad_w, grad_b = self.back_propagation(activations, z_values,␣
 ↪batch_y)

            # Update parameters with L2 regularization
            self.update_parameters(grad_w, grad_b, learning_rate, l2_lambda)

            # Compute loss
            batch_loss = -np.mean(np.sum(batch_y * np.log(activations[-1]),␣
 ↪axis=1))
```

```
            epoch_loss += batch_loss

        # Calculate accuracy for monitoring
        epoch_loss /= (X.shape[0] // batch_size)
        epoch_accuracy = self.accuracy(X, y)
        losses.append(epoch_loss)
        accuracies.append(epoch_accuracy)

        print(f"Epoch {epoch + 1}/{epochs}, Loss: {epoch_loss:.4f}, Accuracy:␣
↪{epoch_accuracy:.4f}")

    # Plot the training loss and accuracy
    plt.figure()
    plt.plot(range(epochs), losses, label='Loss')
    plt.plot(range(epochs), accuracies, label='Accuracy')
    plt.xlabel('Epochs')
    plt.ylabel('Value')
    plt.legend()
    plt.title('Training Loss and Accuracy')
    plt.show()

NeuralNetwork.train = train
```

0.0.9 Prediction and Accuracy Methods

Define methods for making predictions and calculating model accuracy

[9]:
```
def predict(self, X):
    activations, _ = self.forward_propagation(X)
    return np.argmax(activations[-1], axis=1)

def accuracy(self, X, y_true):
    y_pred = self.predict(X)
    y_true = np.argmax(y_true, axis=1)
    return np.mean(y_pred == y_true)

NeuralNetwork.predict = predict
NeuralNetwork.accuracy = accuracy
```

0.0.10 Model Training

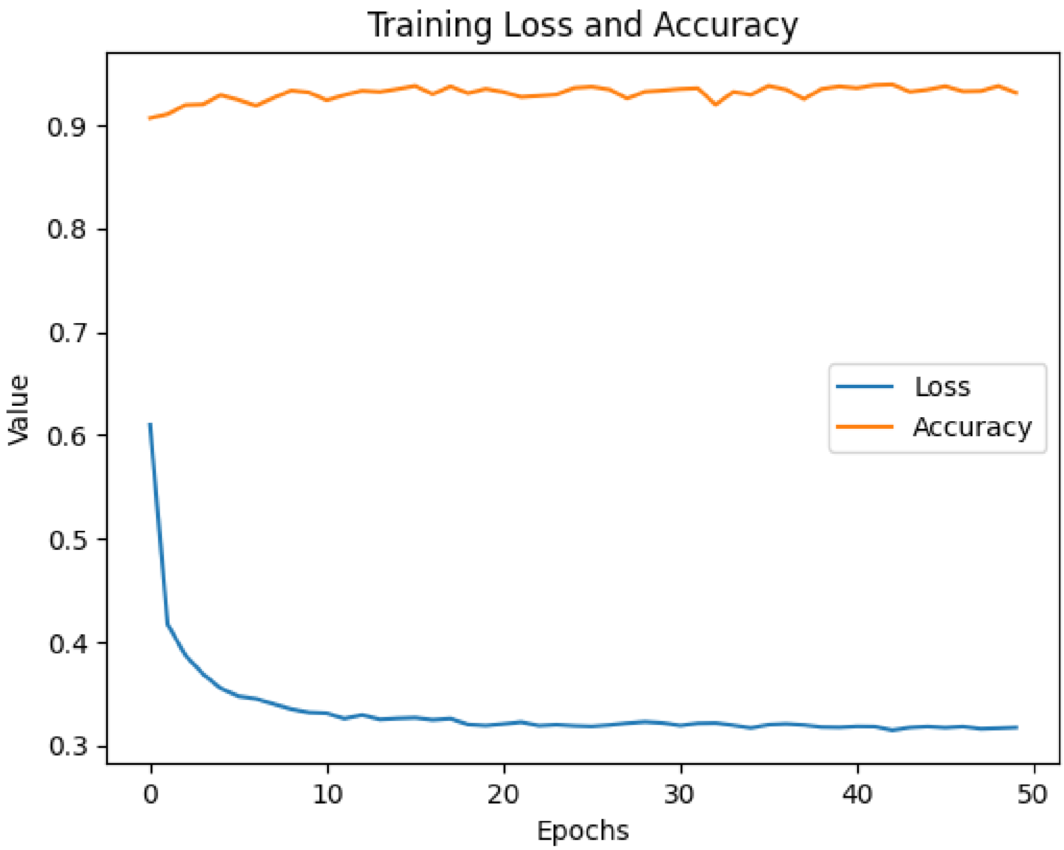
Create and train the neural network with: - 784 input features (28x28 pixels) - Two hidden layers (128 and 64 neurons) - 10 output classes (digits 0-9)

[10]:
```
# Define the network
nn = NeuralNetwork(input_size=784, hidden_sizes=[128, 64], output_size=10)

# Train the network
```

```python
nn.train(X_train, y_train, epochs=50, batch_size=64, learning_rate=0.1, dropout_rate=0.2, l2_lambda=0.01)

# Test accuracy
test_accuracy = nn.accuracy(X_test, y_test)
print(f"Test Accuracy: {test_accuracy:.4f}")
```

```
Epoch 1/50, Loss: 0.6098, Accuracy: 0.9066
Epoch 2/50, Loss: 0.4170, Accuracy: 0.9105
Epoch 3/50, Loss: 0.3871, Accuracy: 0.9192
Epoch 4/50, Loss: 0.3689, Accuracy: 0.9198
Epoch 5/50, Loss: 0.3551, Accuracy: 0.9289
Epoch 6/50, Loss: 0.3474, Accuracy: 0.9242
Epoch 7/50, Loss: 0.3449, Accuracy: 0.9185
Epoch 8/50, Loss: 0.3398, Accuracy: 0.9263
Epoch 9/50, Loss: 0.3349, Accuracy: 0.9332
Epoch 10/50, Loss: 0.3317, Accuracy: 0.9313
Epoch 11/50, Loss: 0.3310, Accuracy: 0.9239
Epoch 12/50, Loss: 0.3257, Accuracy: 0.9289
Epoch 13/50, Loss: 0.3294, Accuracy: 0.9329
Epoch 14/50, Loss: 0.3250, Accuracy: 0.9318
Epoch 15/50, Loss: 0.3260, Accuracy: 0.9346
Epoch 16/50, Loss: 0.3266, Accuracy: 0.9376
Epoch 17/50, Loss: 0.3246, Accuracy: 0.9297
Epoch 18/50, Loss: 0.3258, Accuracy: 0.9371
Epoch 19/50, Loss: 0.3200, Accuracy: 0.9305
Epoch 20/50, Loss: 0.3190, Accuracy: 0.9347
Epoch 21/50, Loss: 0.3206, Accuracy: 0.9317
Epoch 22/50, Loss: 0.3224, Accuracy: 0.9270
Epoch 23/50, Loss: 0.3189, Accuracy: 0.9281
Epoch 24/50, Loss: 0.3199, Accuracy: 0.9294
Epoch 25/50, Loss: 0.3188, Accuracy: 0.9357
Epoch 26/50, Loss: 0.3183, Accuracy: 0.9368
Epoch 27/50, Loss: 0.3196, Accuracy: 0.9341
Epoch 28/50, Loss: 0.3213, Accuracy: 0.9256
Epoch 29/50, Loss: 0.3227, Accuracy: 0.9320
Epoch 30/50, Loss: 0.3216, Accuracy: 0.9332
Epoch 31/50, Loss: 0.3190, Accuracy: 0.9346
Epoch 32/50, Loss: 0.3212, Accuracy: 0.9354
Epoch 33/50, Loss: 0.3215, Accuracy: 0.9196
Epoch 34/50, Loss: 0.3193, Accuracy: 0.9318
Epoch 35/50, Loss: 0.3167, Accuracy: 0.9291
Epoch 36/50, Loss: 0.3199, Accuracy: 0.9376
Epoch 37/50, Loss: 0.3206, Accuracy: 0.9340
Epoch 38/50, Loss: 0.3196, Accuracy: 0.9251
Epoch 39/50, Loss: 0.3176, Accuracy: 0.9347
Epoch 40/50, Loss: 0.3173, Accuracy: 0.9371
Epoch 41/50, Loss: 0.3183, Accuracy: 0.9356
```

```
Epoch 42/50, Loss: 0.3181, Accuracy: 0.9384
Epoch 43/50, Loss: 0.3144, Accuracy: 0.9390
Epoch 44/50, Loss: 0.3172, Accuracy: 0.9319
Epoch 45/50, Loss: 0.3182, Accuracy: 0.9340
Epoch 46/50, Loss: 0.3170, Accuracy: 0.9373
Epoch 47/50, Loss: 0.3181, Accuracy: 0.9324
Epoch 48/50, Loss: 0.3158, Accuracy: 0.9327
Epoch 49/50, Loss: 0.3164, Accuracy: 0.9375
Epoch 50/50, Loss: 0.3171, Accuracy: 0.9310
```



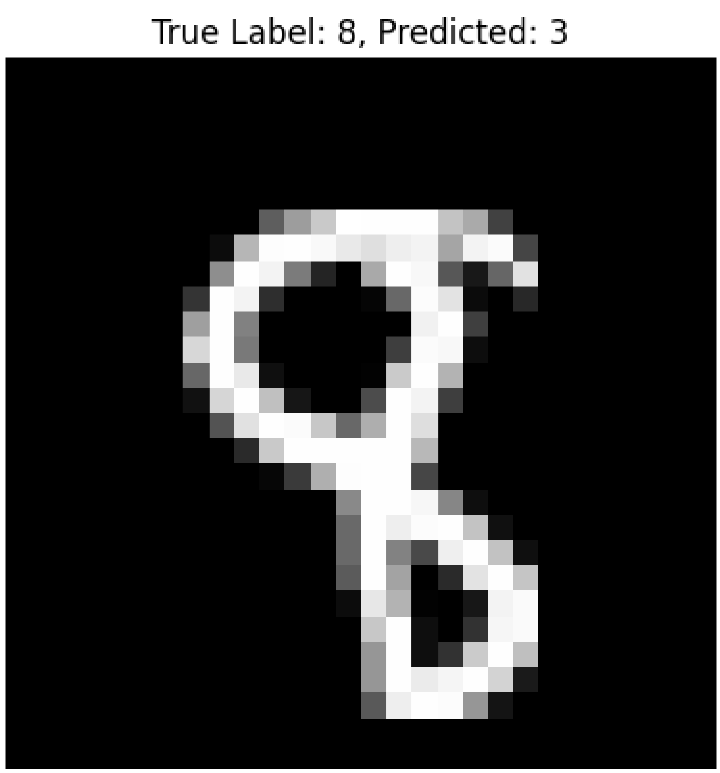

```
Test Accuracy: 0.9271
```

0.0.11 Model Testing

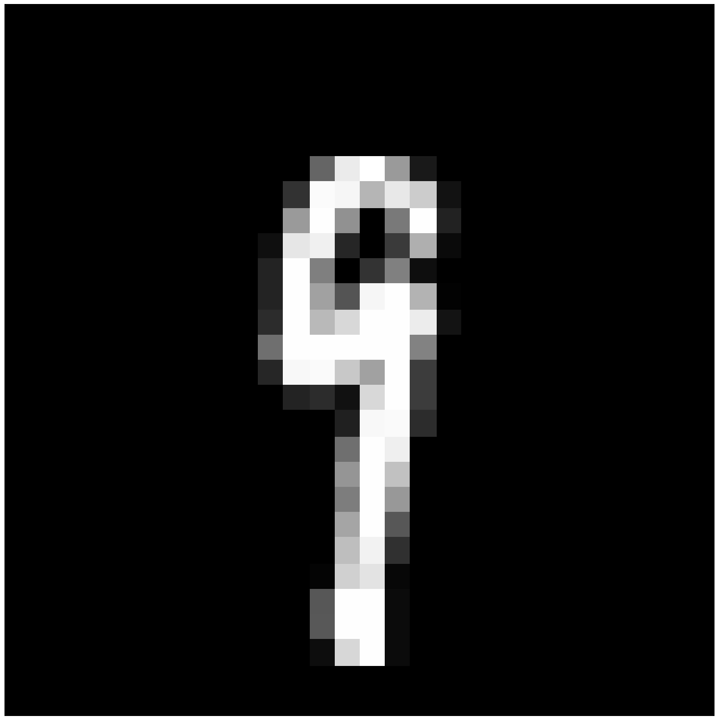
Test the trained model on random samples from the test set Visualize results with true and predicted labels

[12]:
```
# Function to test the model on random test samples
def test_random_samples(model, X_test, y_test, num_samples=5):
    # Convert X_test to numpy array if it isn't already
```

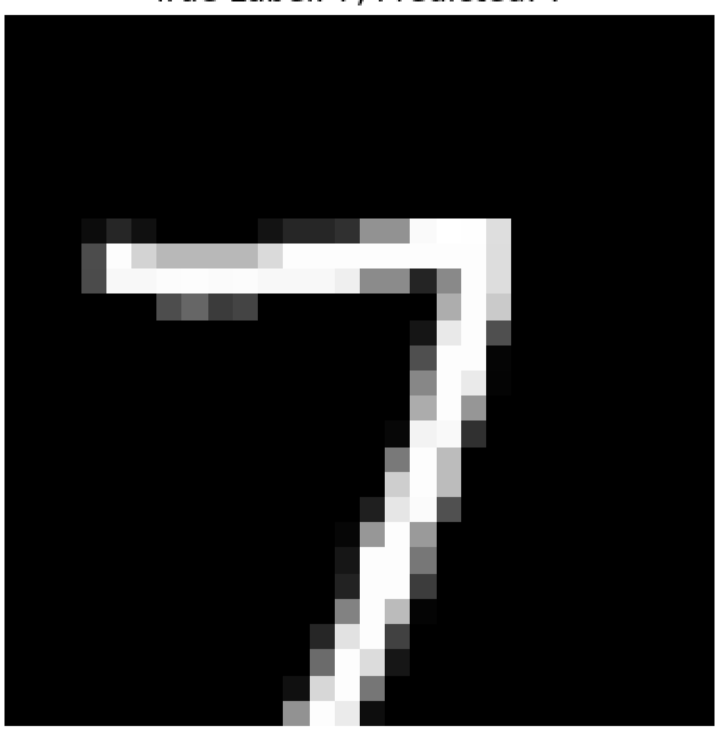
```
    X_test_array = X_test.to_numpy() if hasattr(X_test, 'to_numpy') else np.
↪array(X_test)
    y_test_array = y_test.to_numpy() if hasattr(y_test, 'to_numpy') else np.
↪array(y_test)

    indices = np.random.choice(range(X_test_array.shape[0]), num_samples,␣
↪replace=False)
    for idx in indices:
        image = X_test_array[idx].reshape(28, 28)
        true_label = np.argmax(y_test_array[idx])
        predicted_label = model.predict(X_test_array[idx:idx+1])[0]

        plt.figure()
        plt.imshow(image, cmap='gray')
        plt.title(f"True Label: {true_label}, Predicted: {predicted_label}")
        plt.axis('off')
        plt.show()

# Test the model on random samples
test_random_samples(nn, X_test, y_test, num_samples=5)
```



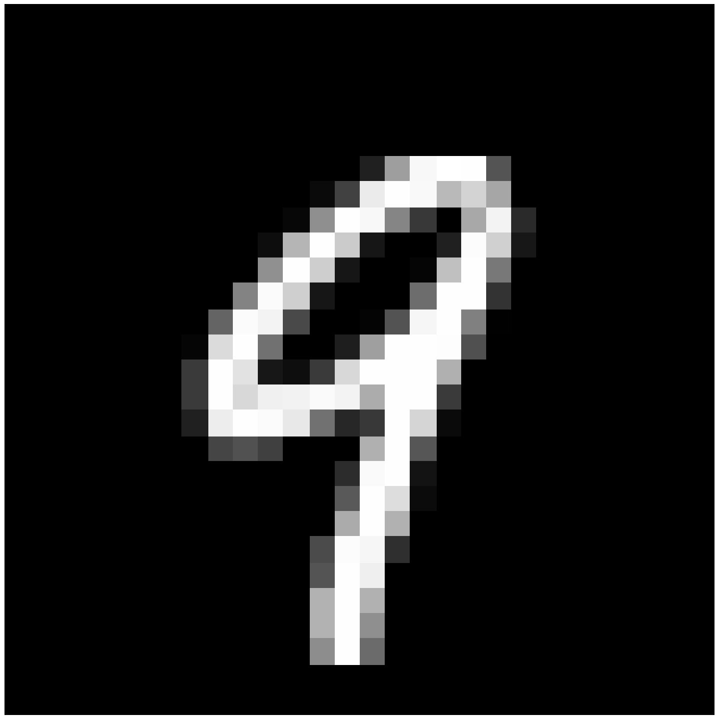
True Label: 9, Predicted: 1



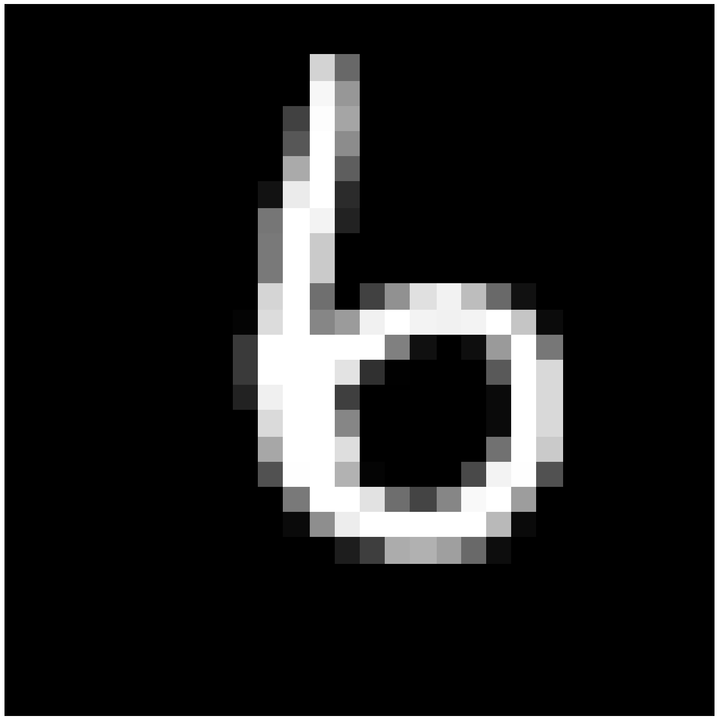
True Label: 7, Predicted: 7



True Label: 9, Predicted: 9



True Label: 6, Predicted: 6



```
[ ]:
```