CONTROL SYSTEMS (UE21EC241B) PROJECT
ROLL ANGLE CONTROL

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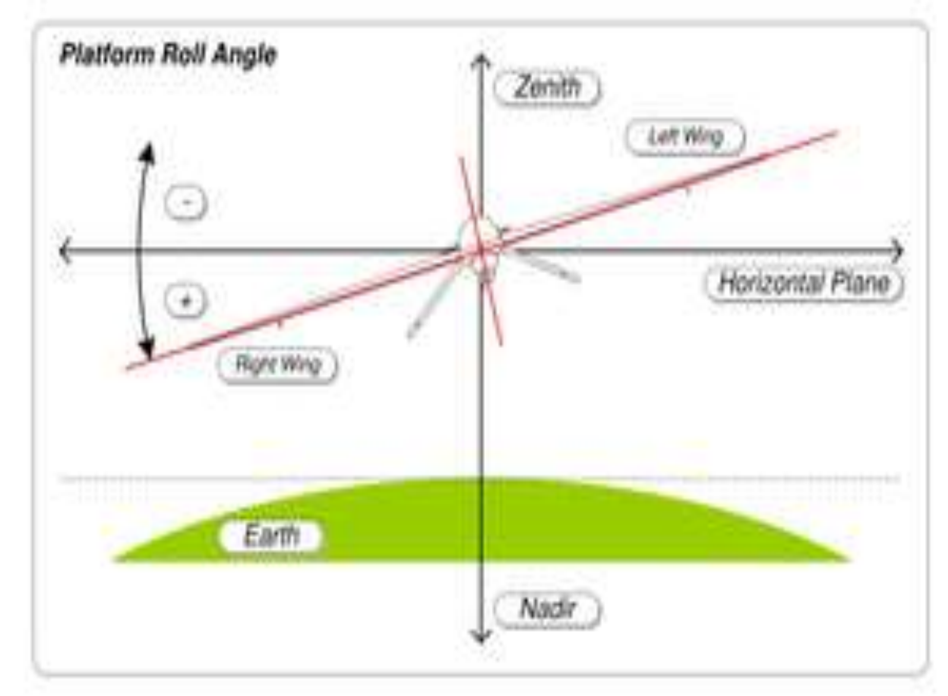
SECTION: A

INTRODUCTION

- Roll angle control (RAC) is required for the lateral stability of an aircraft. Lateral stability makes the aircraft more stable around the longitudinal axis.
- Roll angle control makes both the wings of the aircraft to be at the same level.
- If one of the wing dips below the other, then RAC tries to stabilize the system again



DATE: / /

{Roll angle control}

It can be derived using the equation of motion for a rigid body undergoing roll motion.

It relates the roll angle output to the input control signal.

Moment of Inertia:

I_{xx}, I_{yy}, I_{zz}

Equation of motion for roll motion:

$I_{xx} * \theta''(t) + (I_{zz} - I_{yy}) * \omega y(t) * \omega z(t) = \tau x(t)$

$I_{xx} * s^2 * \theta(s) + (I_{zz} - I_{yy}) * \omega y(s) * \omega z(s) = \tau x(s)$

(By taking Laplace transform)

$\theta(s) = \left(\frac{1}{I_{xx} s^2}\right)\left[\tau x(s) - (I_{zz} - I_{yy}) * [\omega y(s) * \omega z(s)]\right]$

WCPM

There the transfer function output $\theta(s)$ to the input $\tau x(s)$ can be written as -

$$\boxed{H(s) = \theta(s) / \tau x(s) = 1/(I_{xx} s^2)}$$

The open loop transfer function is -

$$\boxed{\frac{36.6}{s^3 + 9.2s^2 + 15.4s}}$$

Q1

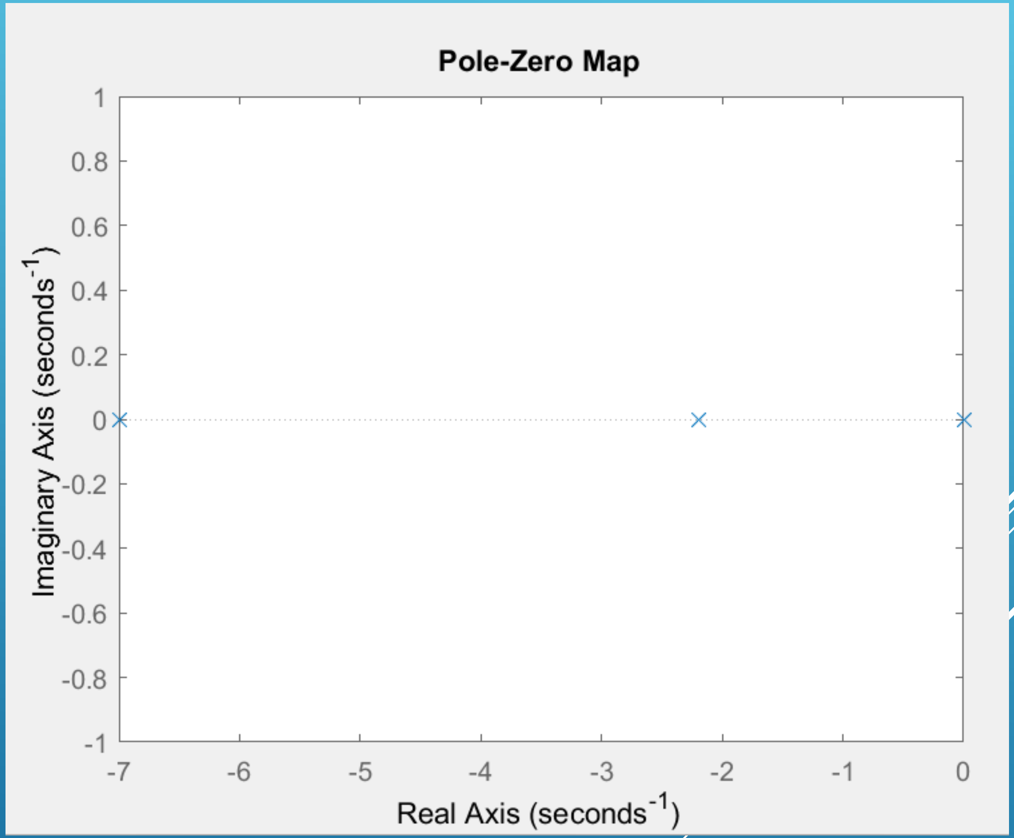
- The open loop transfer function is

$$\frac{36.6}{s^3 + 9.2 s^2 + 15.4 s}$$

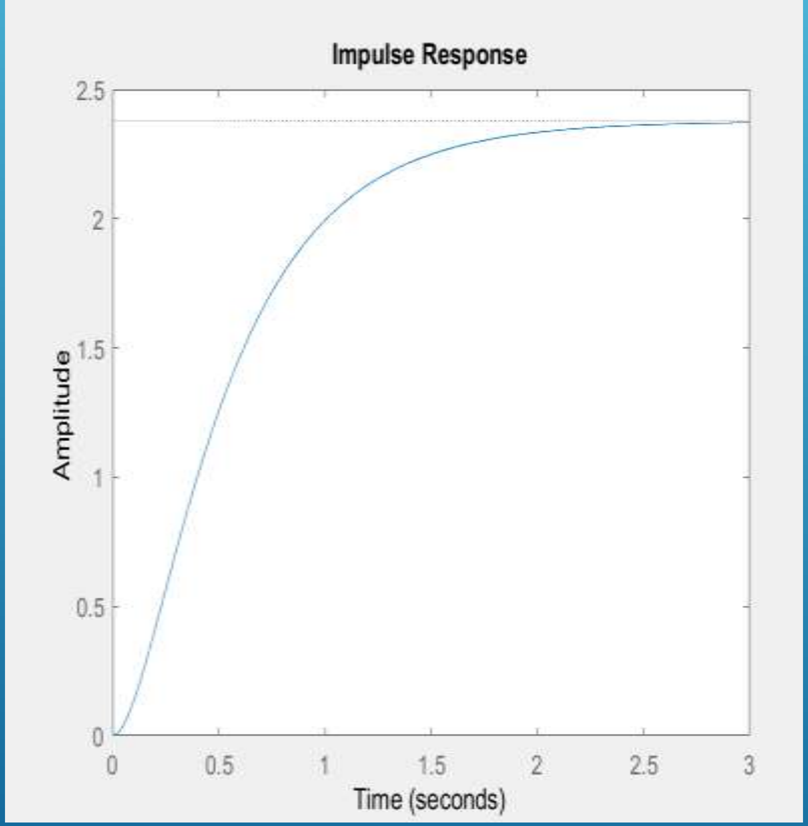
We first find the pole zero map first and it can be observed that the system is open loop stable

Poles are present at 0,-2,-7

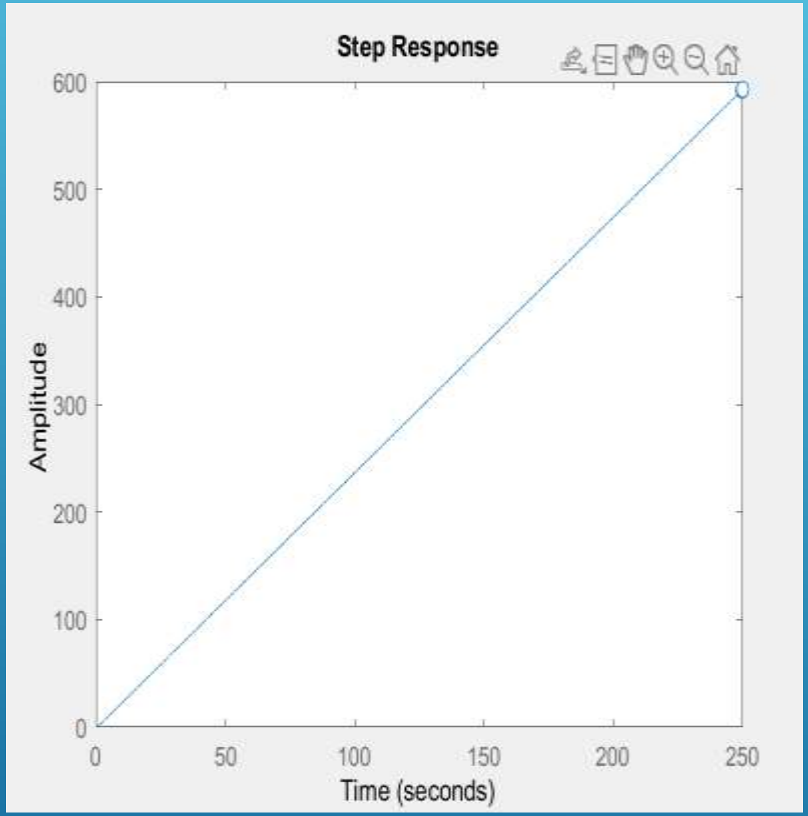
We apply different test signals and observe that the system is stable only for impulse input and unstable for step and ramp input



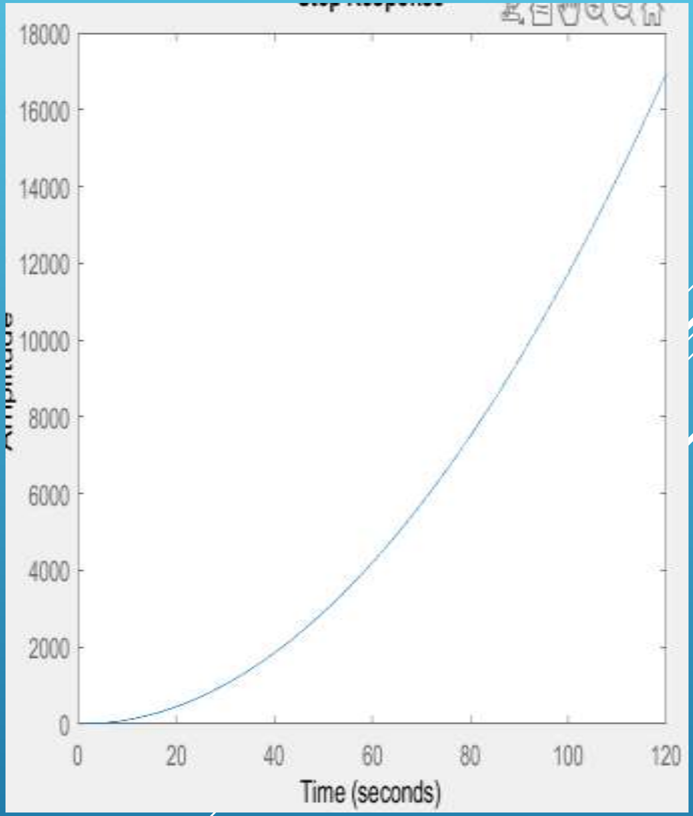
From the impulse response we can observe that the response settles to a value and hence we can call it stable



From the step response we can observe that the response is continuously increasing and unstable

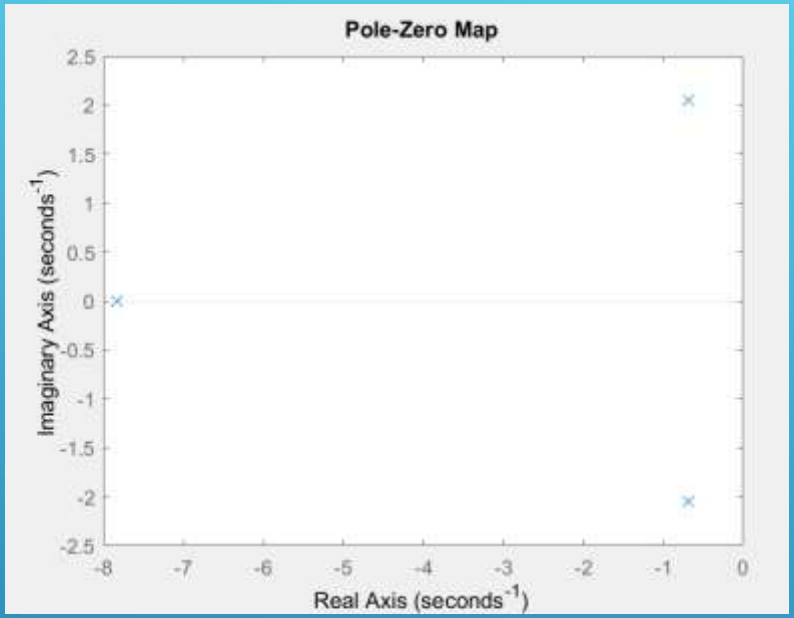


From the ramp response we can observe that the response is continuously increasing and unstable



Q2

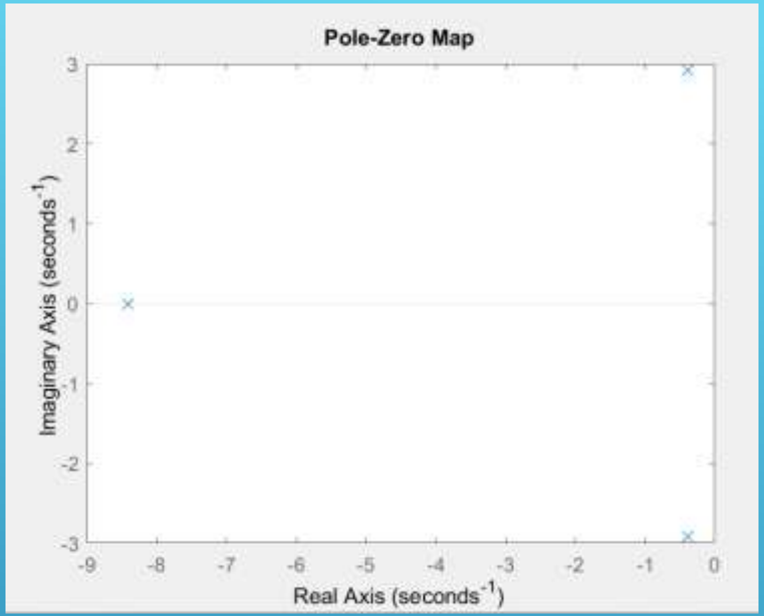
- We apply a gain K in cascade with our initial transfer function
- The theoretical method is using the RH criterion, where we use the RH table to determine the value of K.
- Using this method we find that the for $K < 3.87$ the system is stable
- To confirm this we use the pole zero plots
- We apply different integer value of K in cascade with our transfer function
- The poles lie on LHP for $K<=3$
- The system becomes unstable when we take $K = 4$



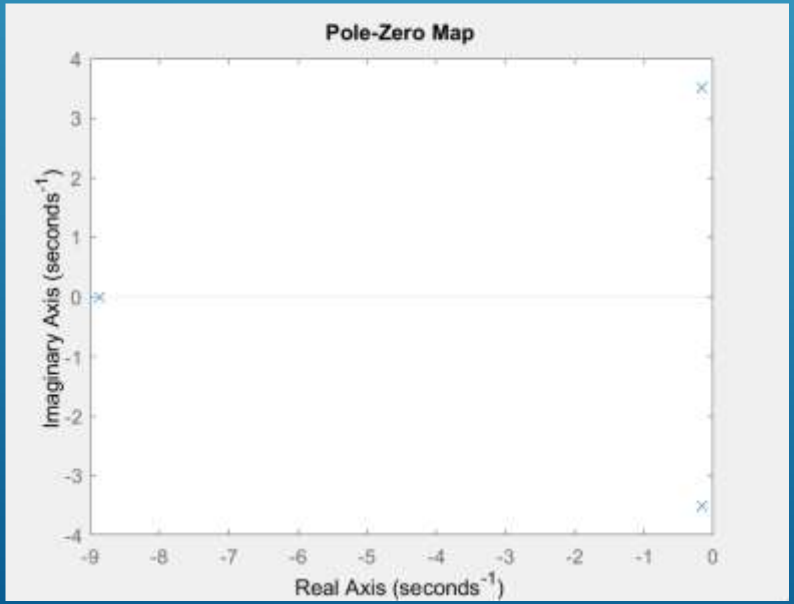
Pole-Zero Map
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})

K = 1

K = 2



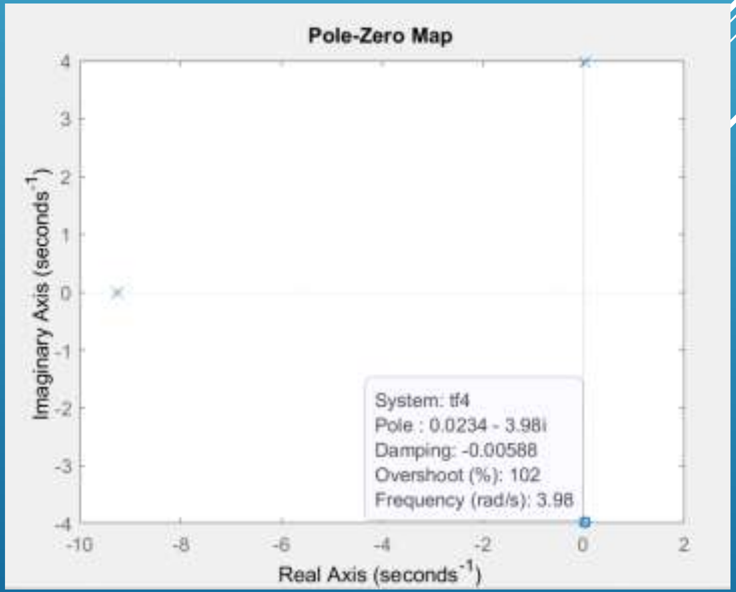
Pole-Zero Map
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})



Pole-Zero Map
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})

K = 3

K = 4

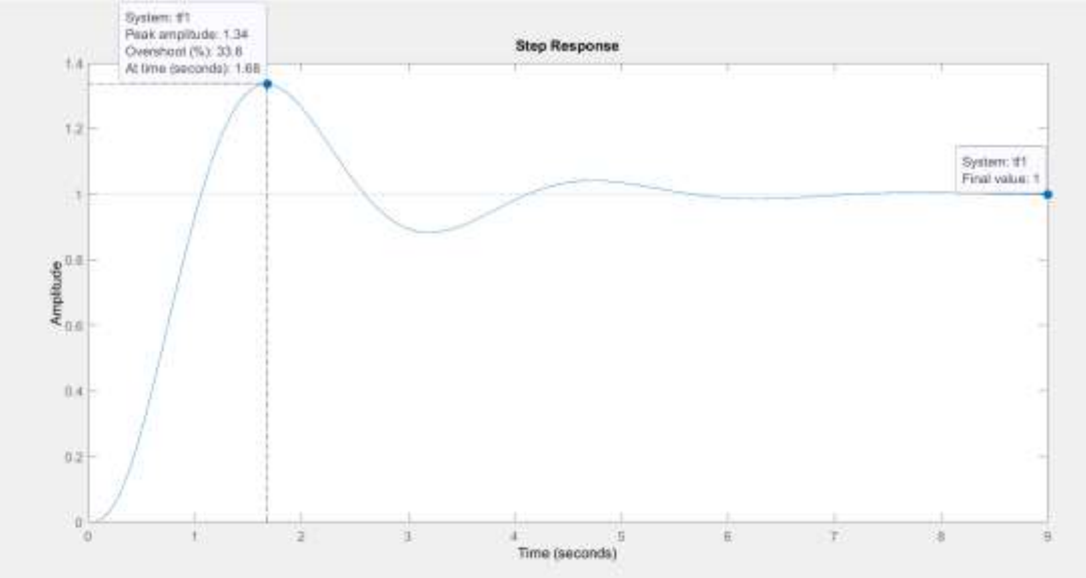


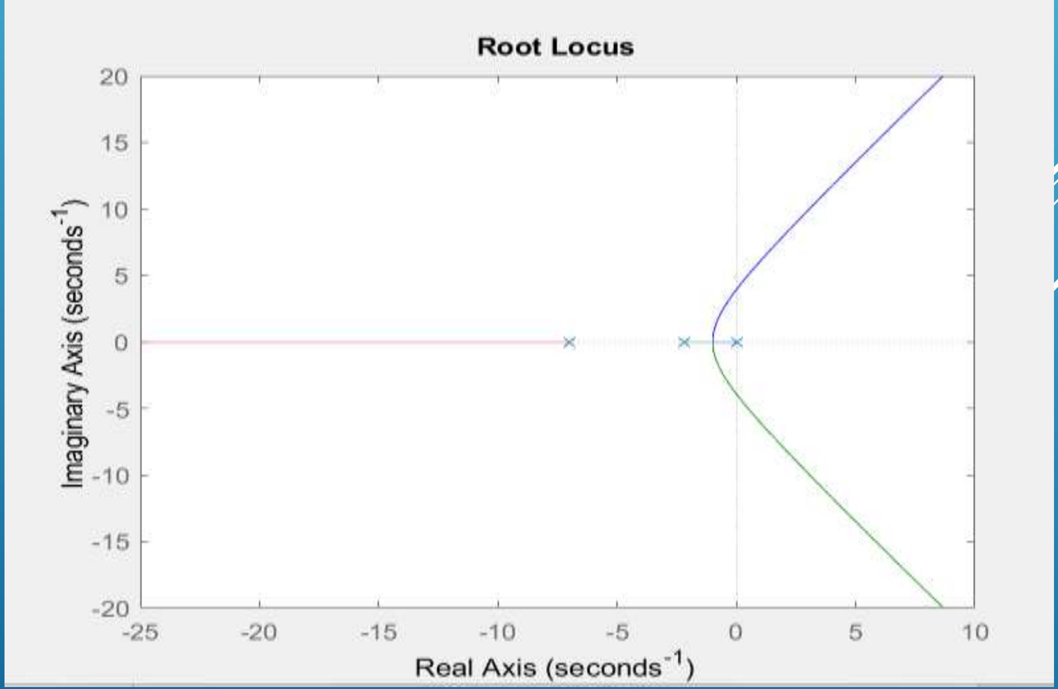
Pole-Zero Map
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
System: tf4
Pole : 0.0234 - 3.98i
Damping: -0.00588
Overshoot (%): 102
Frequency (rad/s): 3.98

Q3

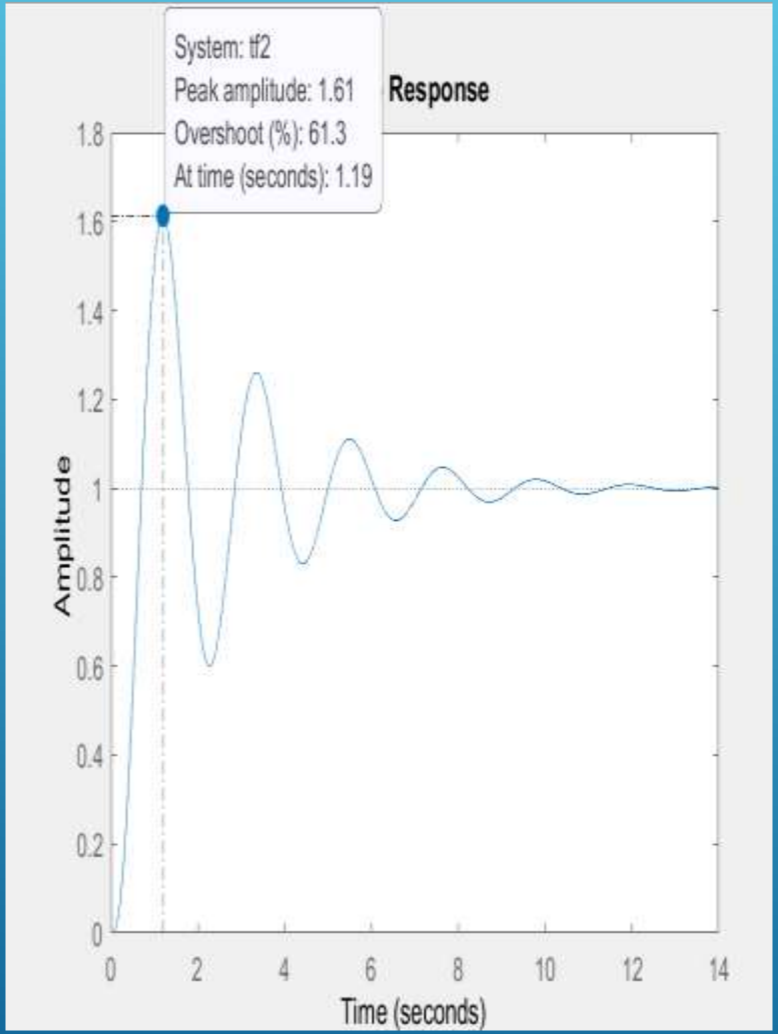
- For the proportional controller we can take different values of K
- For k = 1,we obtain the following characteristics

RiseTime:	0.6592
TransientTime:	5.2933
SettlingTime:	5.2933
SettlingMin:	0.8825
SettlingMax:	1.3359
Overshoot:	33.5871
Undershoot:	0
Peak:	1.3359
PeakTime:	1.6810

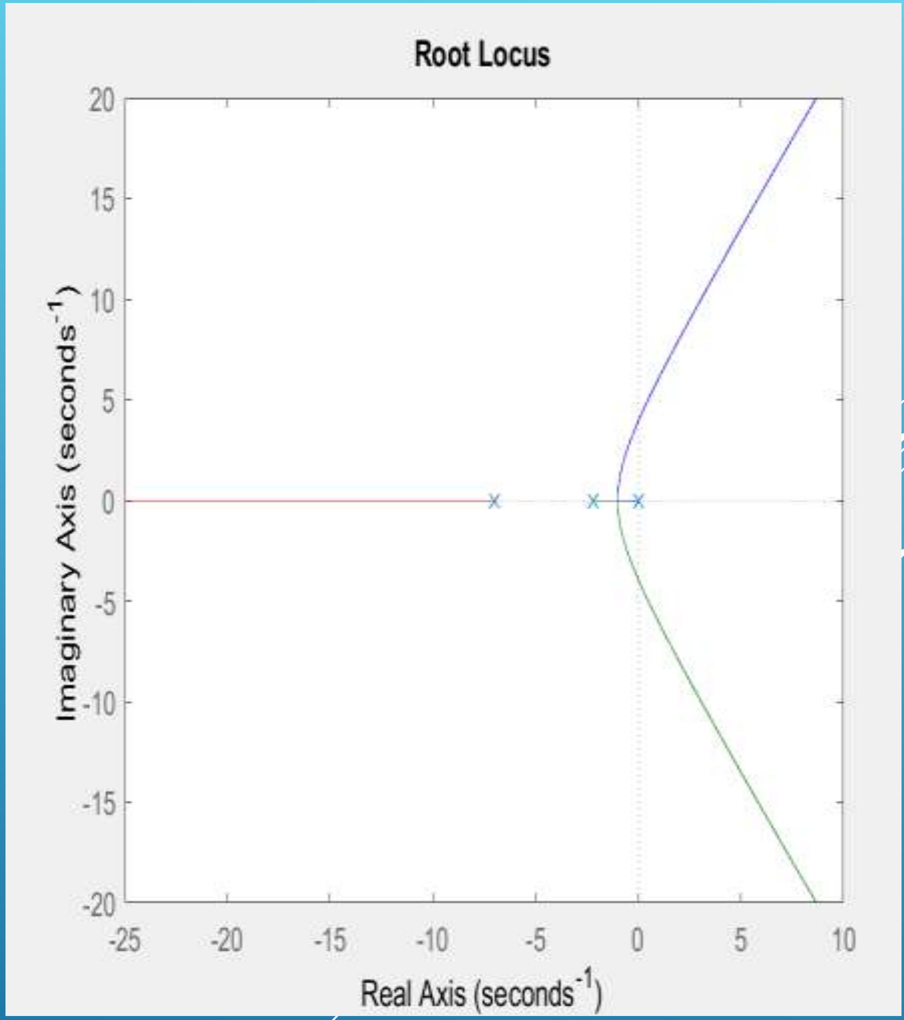




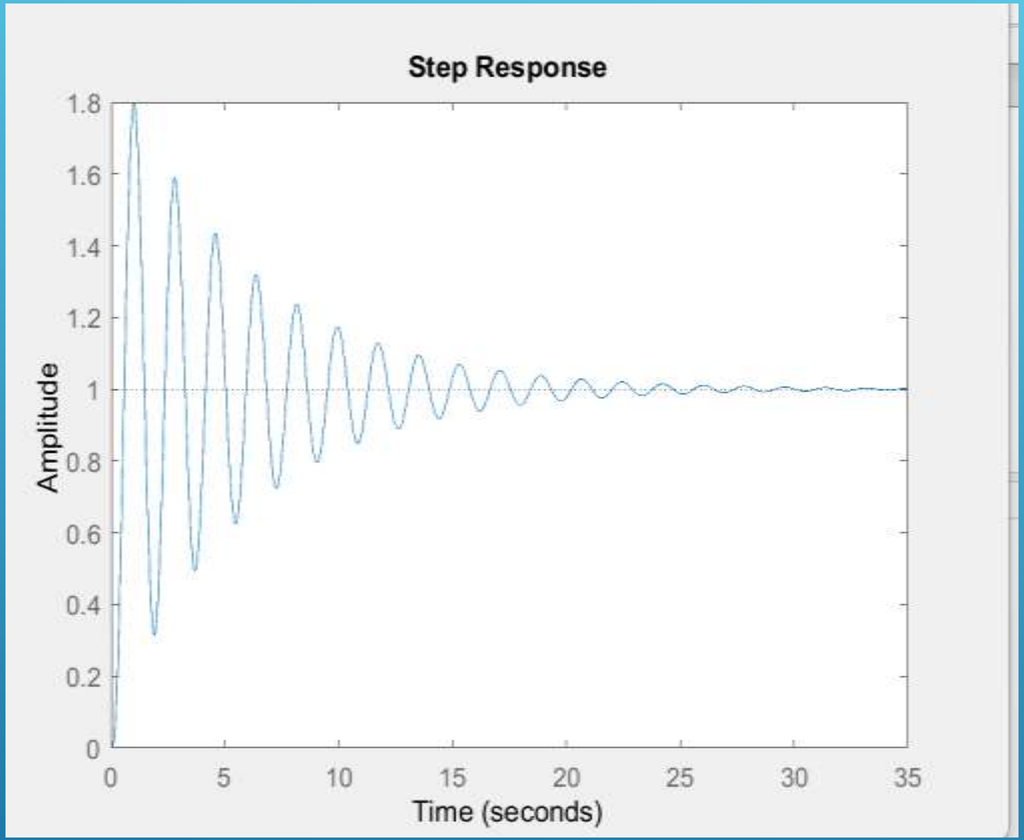
For K = 2

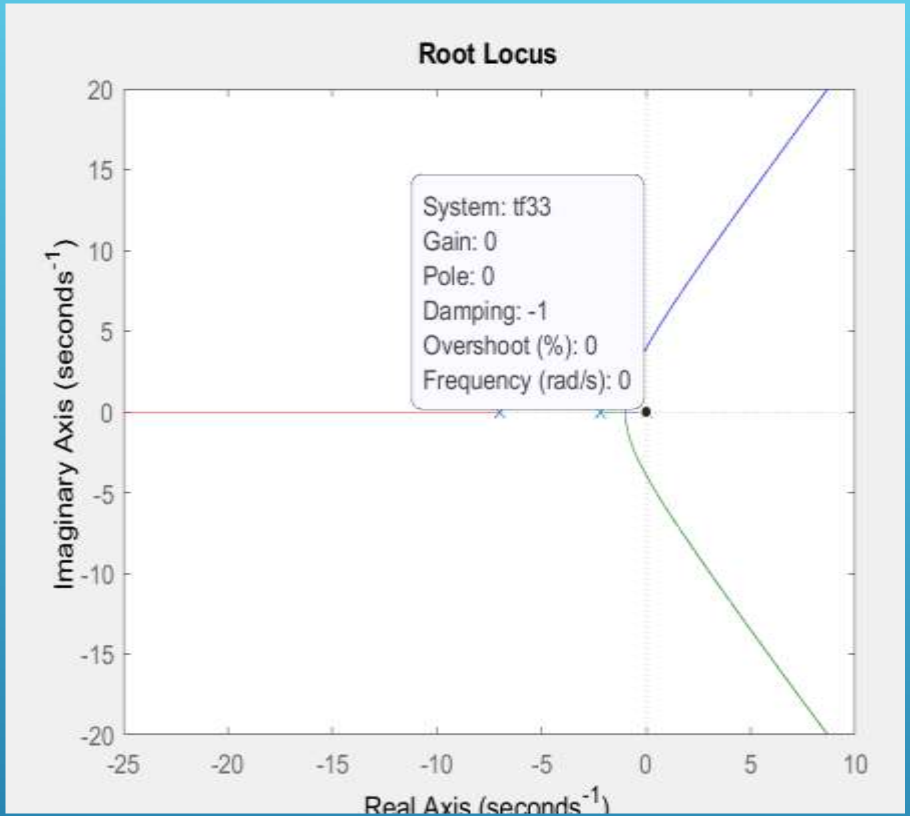



```
       RiseTime: 0.4193
  TransientTime: 9.8096
   SettlingTime: 9.8096
    SettlingMin: 0.6003
    SettlingMax: 1.6130
      Overshoot: 61.3017
     Undershoot: 0
           Peak: 1.6130
       PeakTime: 1.1946
```



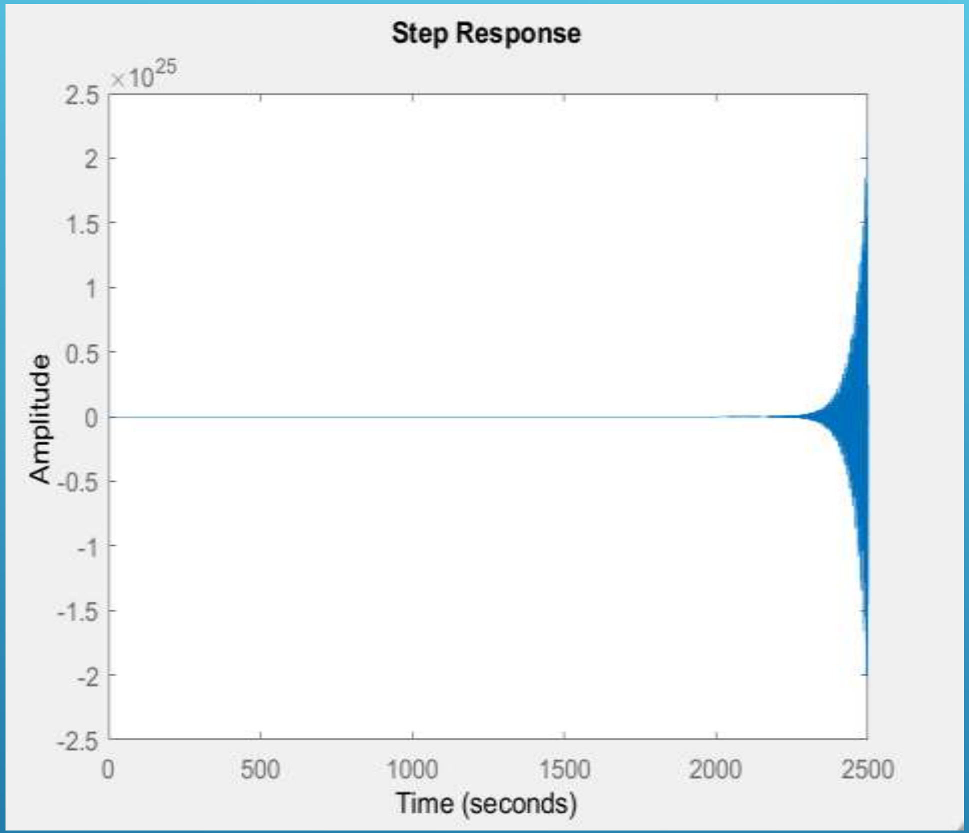
For K = 3

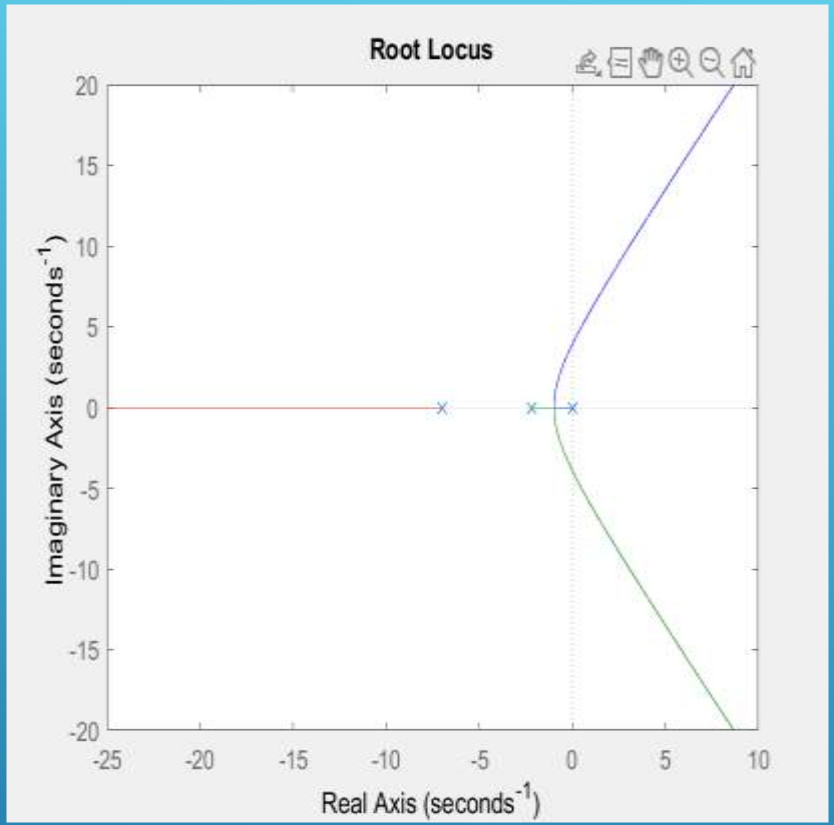





```
        RiseTime: 0.3330
   TransientTime: 22.5317
    SettlingTime: 22.5317
     SettlingMin: 0.3151
     SettlingMax: 1.7970
       Overshoot: 79.7043
      Undershoot: 0
            Peak: 1.7970
        PeakTime: 0.9979
```

K = 4





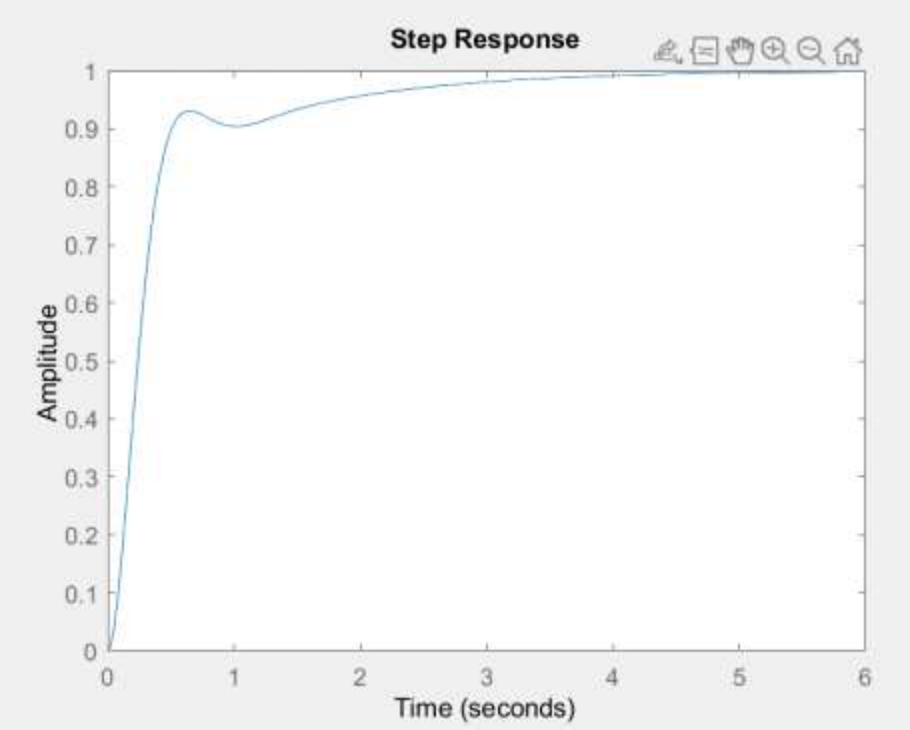
RiseTime: NaN
TransientTime: NaN
SettlingTime: NaN
SettlingMin: NaN
SettlingMax: NaN
Overshoot: NaN
Undershoot: NaN
Peak: Inf
PeakTime: Inf

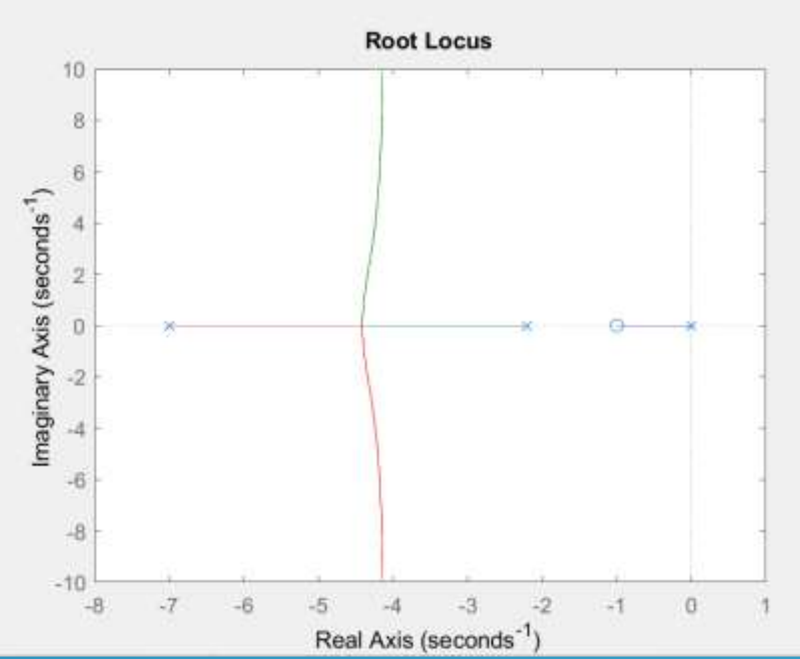
We observe that for K = 4, the system becomes unstable which is evident from observing the step response graph

We also observe that the rise time is least for K = 3 and hence it's the most suitable proportional controller for our system

Q4

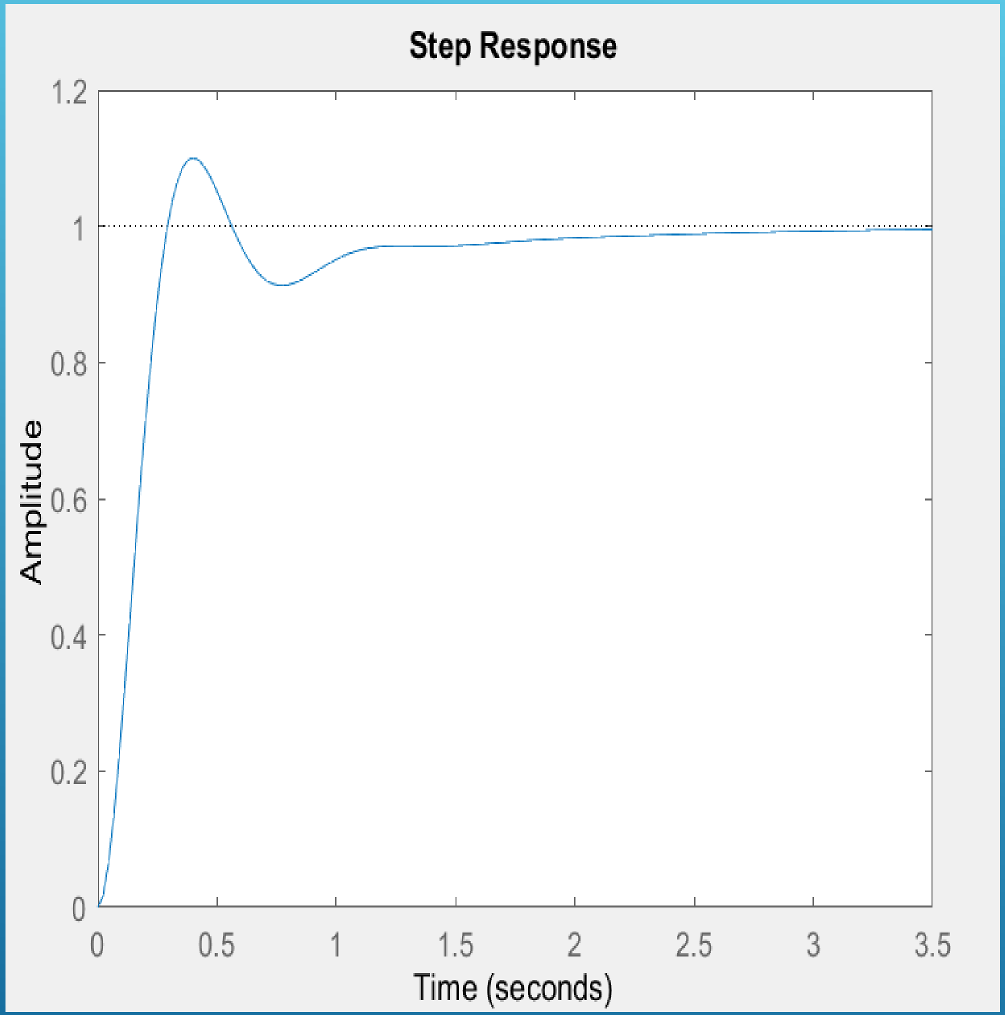
- We introduce a new PD controller into the system by cascading K(s+z) with our system. We choose different values of K and z and obtain the step response for each and determine which is more suitable for our system
- K = 1 and z = 1

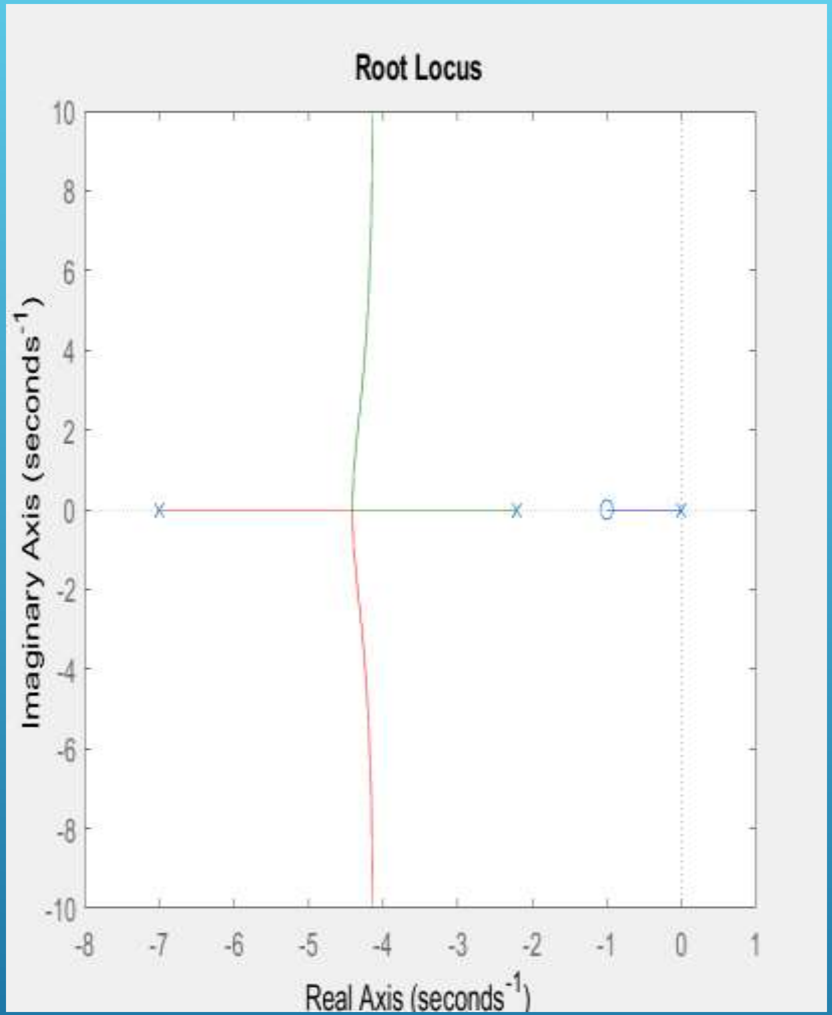





```
        RiseTime: 0.4233
   TransientTime: 2.9633
    SettlingTime: 2.9633
     SettlingMin: 0.9043
     SettlingMax: 0.9985
       Overshoot: 0
      Undershoot: 0
            Peak: 0.9985
        PeakTime: 6.1253
```

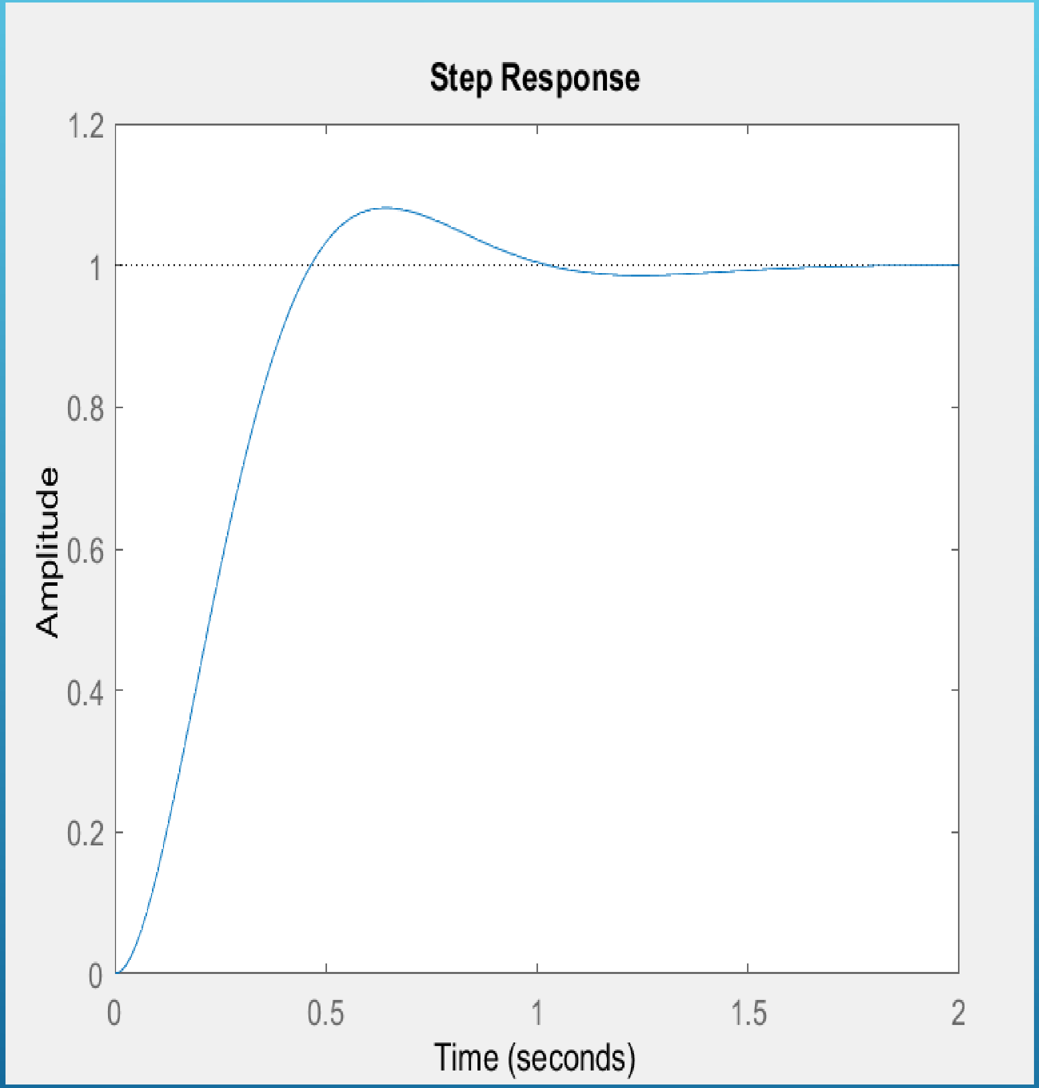
K = 2, z = 1

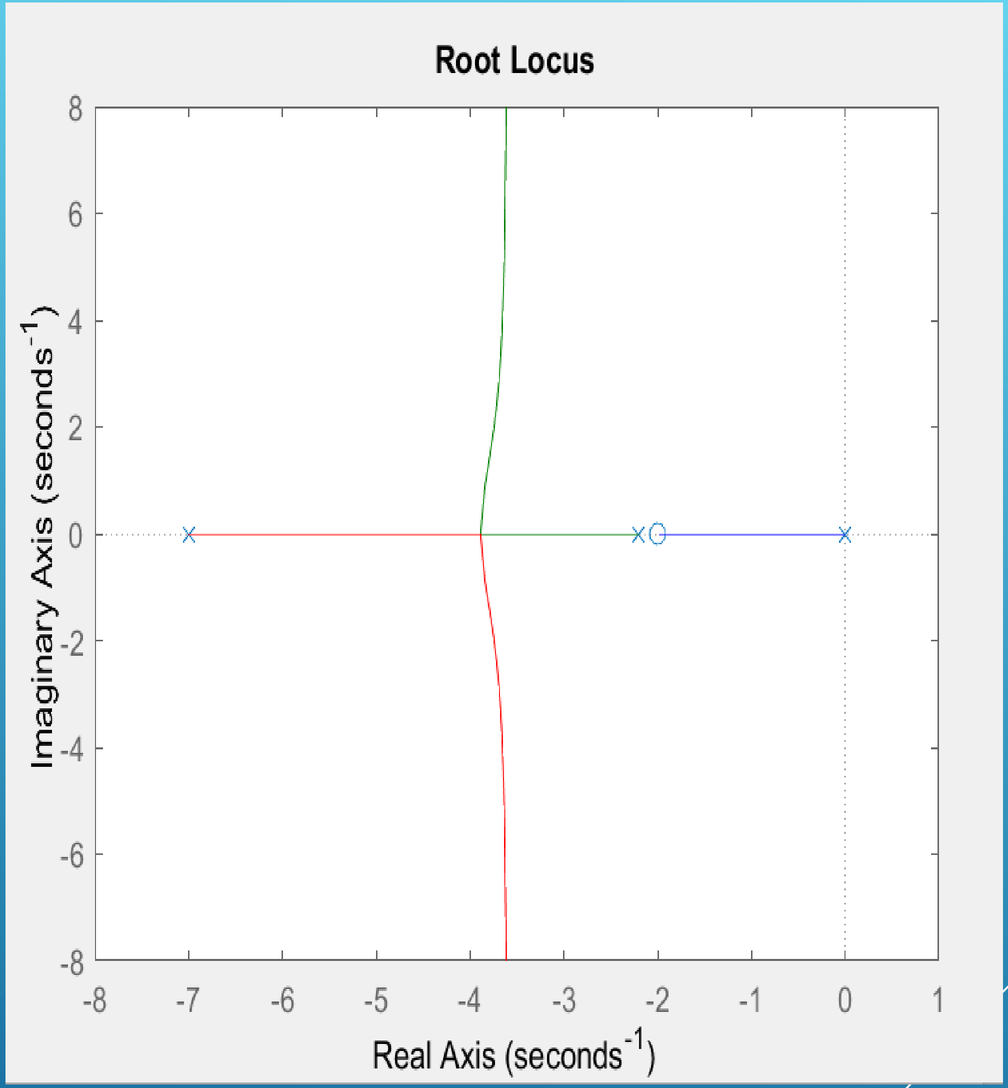





```
        RiseTime: 0.1966
   TransientTime: 1.8474
    SettlingTime: 1.8474
     SettlingMin: 0.9132
     SettlingMax: 1.1007
       Overshoot: 10.0727
      Undershoot: 0
            Peak: 1.1007
        PeakTime: 0.3996
```

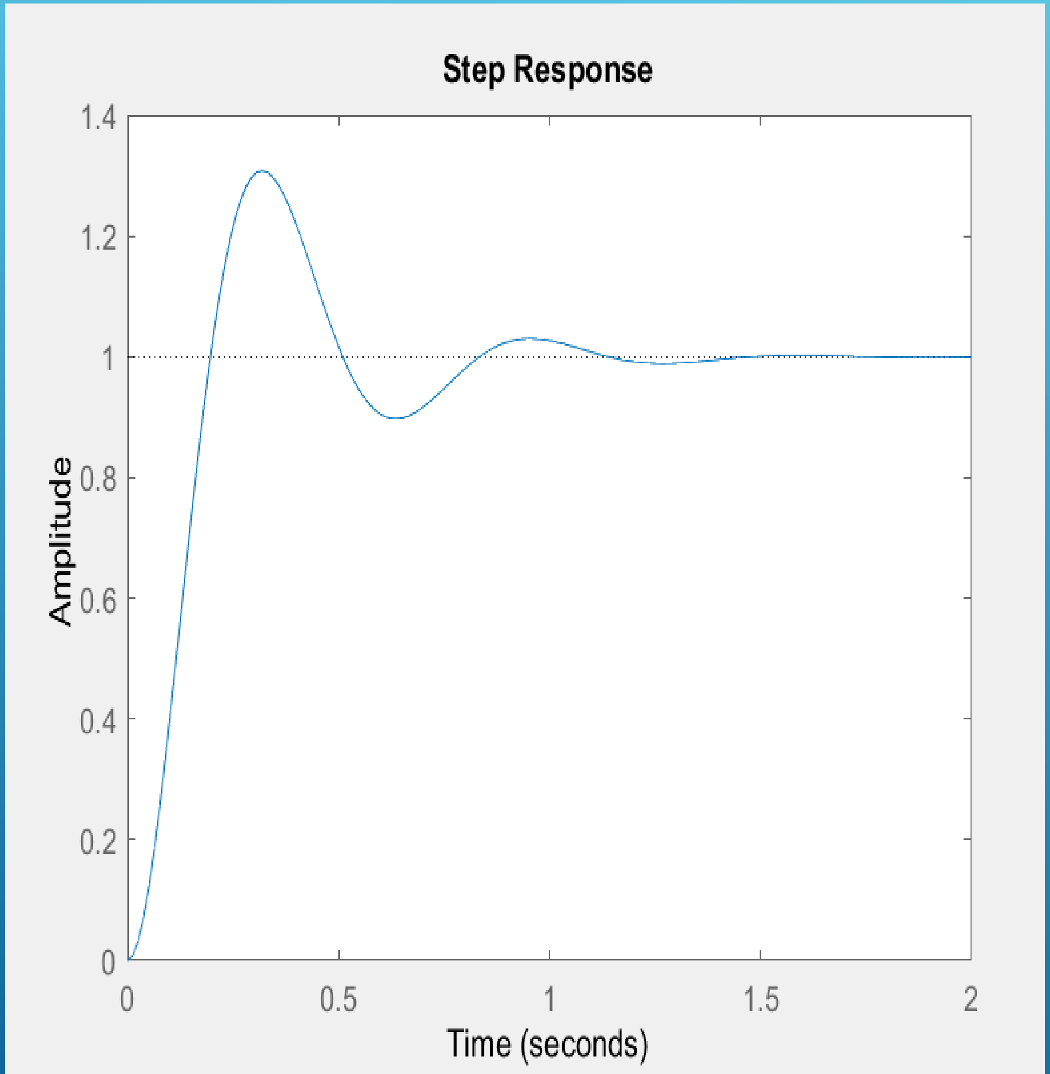
K = 1, z = 2

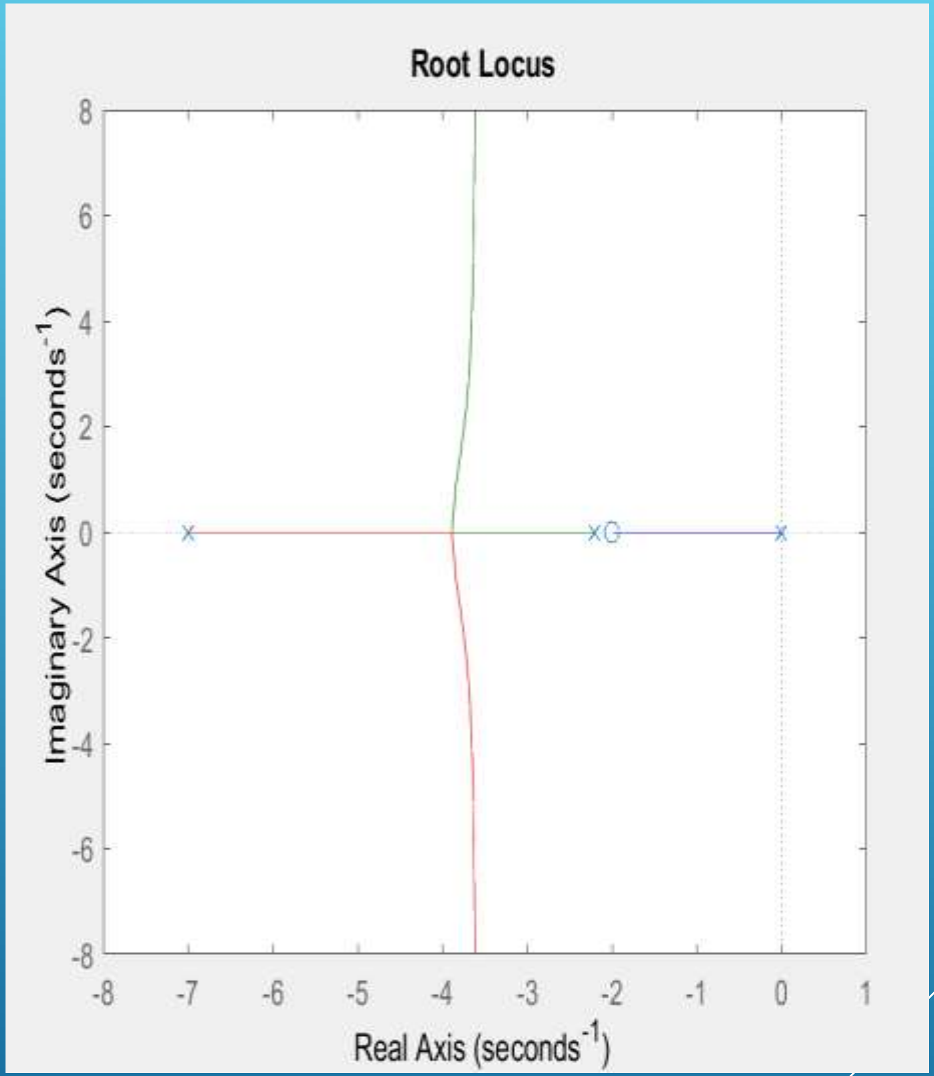




RiseTime: 0.3091
TransientTime: 0.9239
SettlingTime: 0.9239
SettlingMin: 0.9028
SettlingMax: 1.0810
Overshoot: 8.1000
Undershoot: 0
Peak: 1.0810
PeakTime: 0.6459

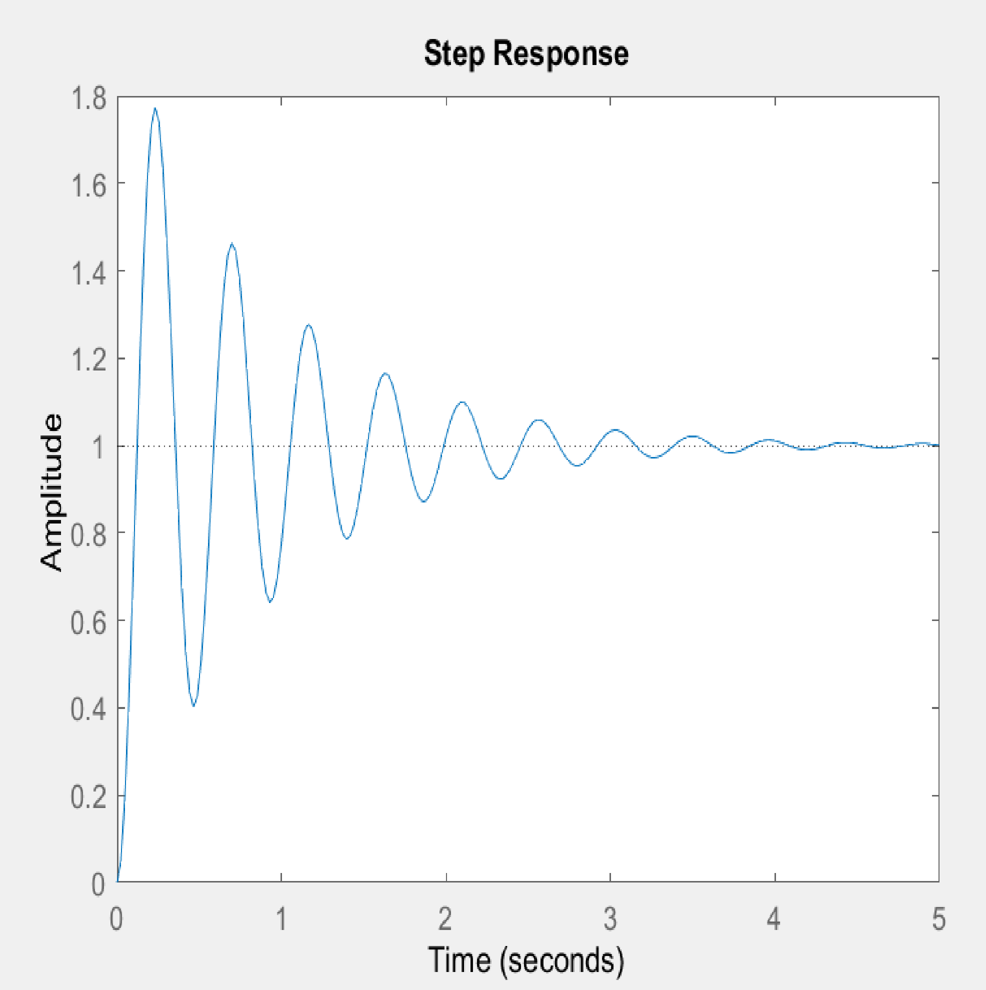
K = 3, z = 2

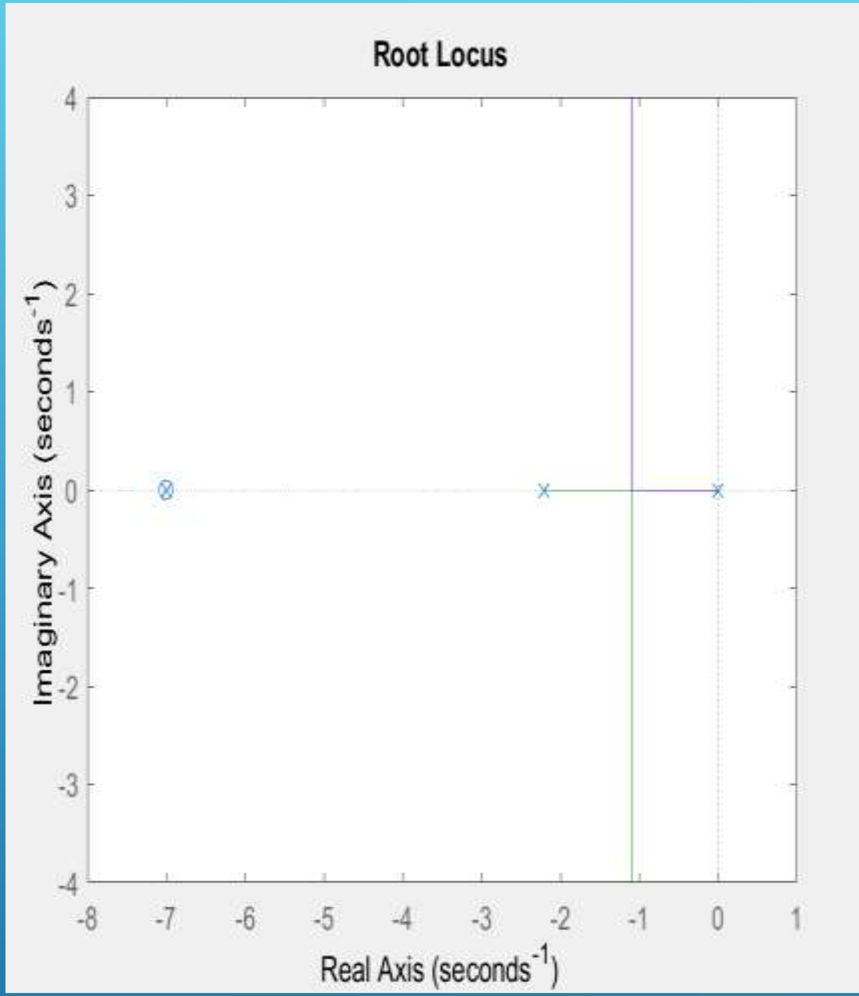




RiseTime: 0.1324
TransientTime: 1.0404
SettlingTime: 1.0404
SettlingMin: 0.8970
SettlingMax: 1.3084
Overshoot: 30.8438
Undershoot: 0
Peak: 1.3084
PeakTime: 0.3189

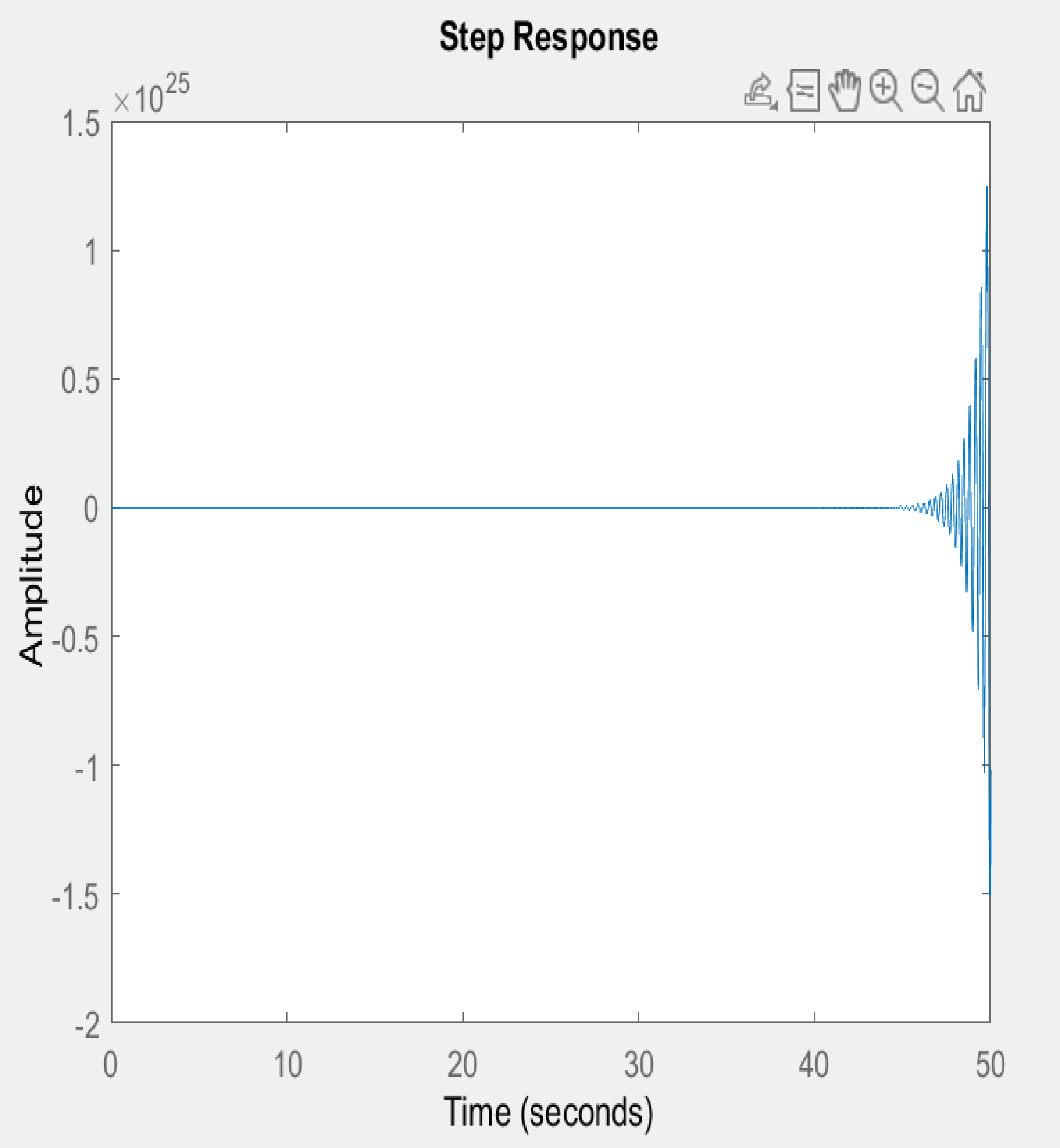
K = 5, z = 7

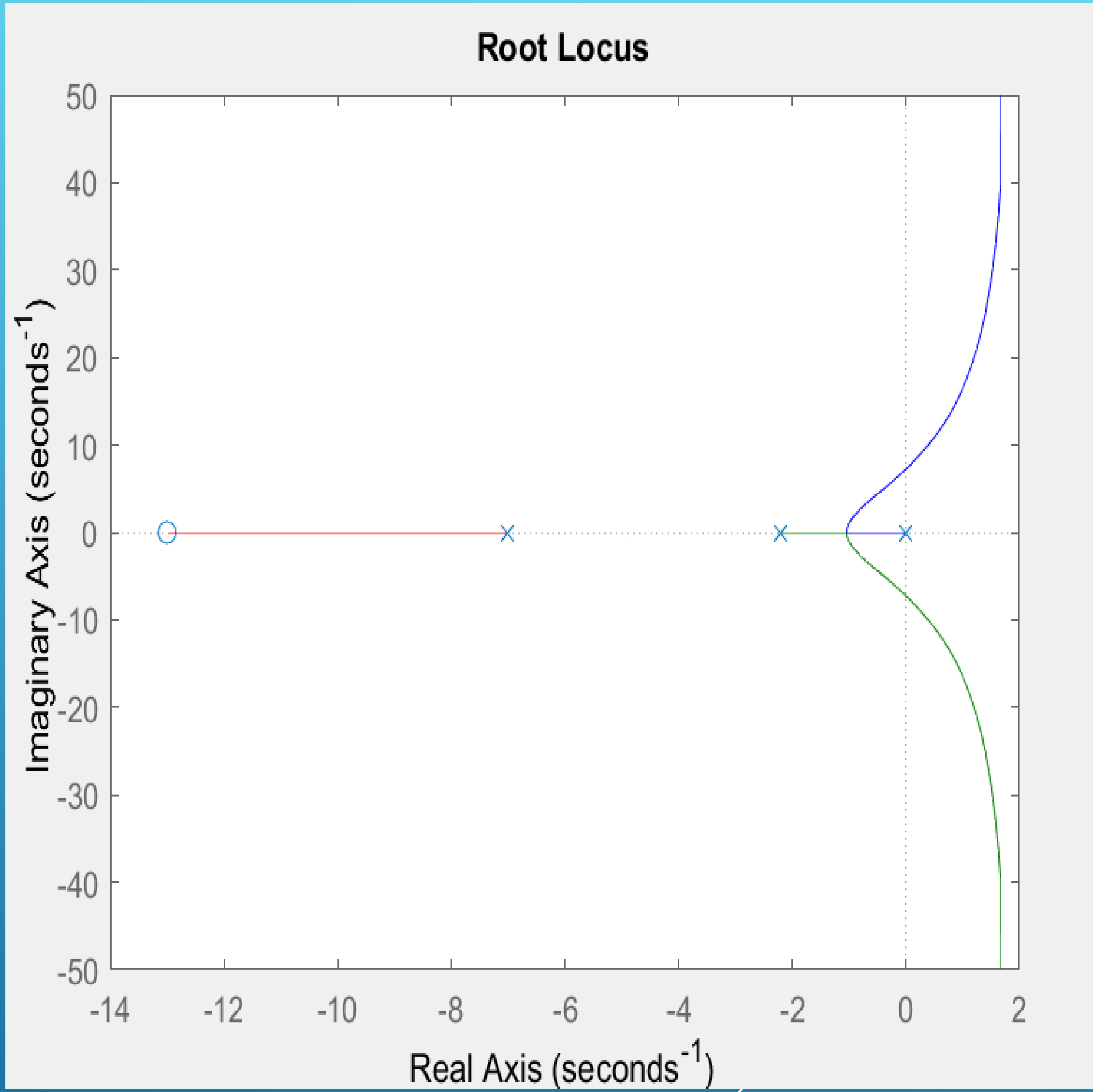




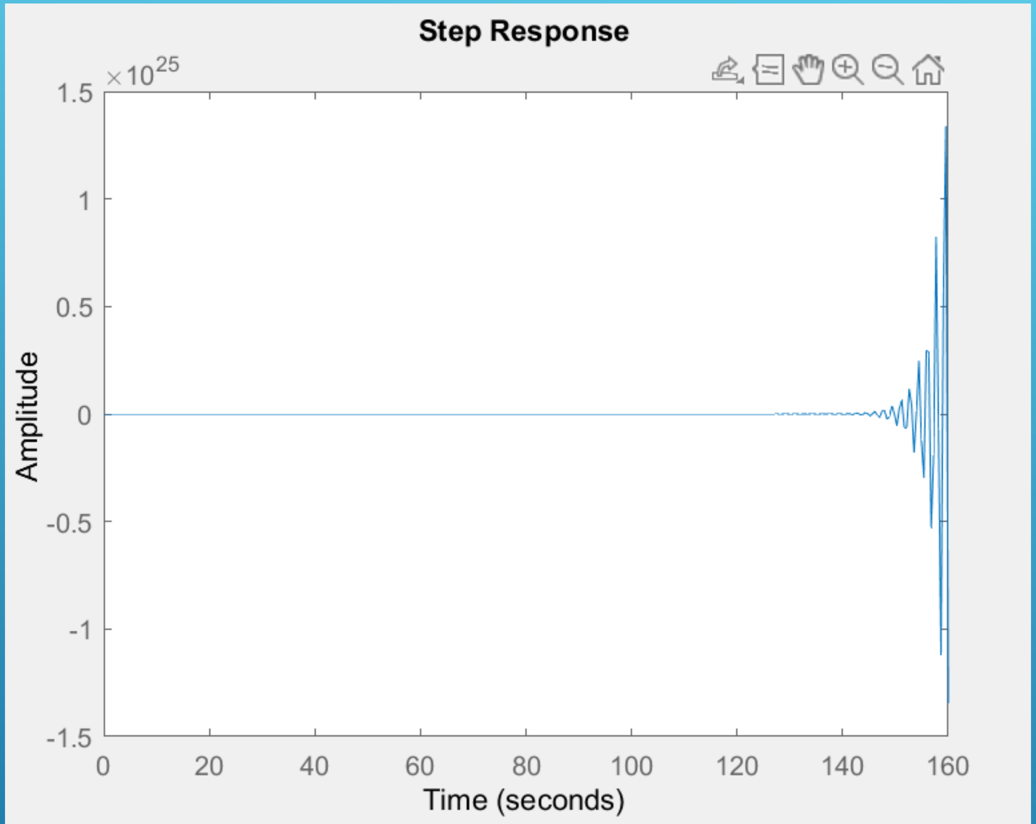
RiseTime:	0.0821
TransientTime:	3.5198
SettlingTime:	3.5198
SettlingMin:	0.4012
SettlingMax:	1.7739
Overshoot:	77.3863
Undershoot:	0
Peak:	1.7739
PeakTime:	0.2322

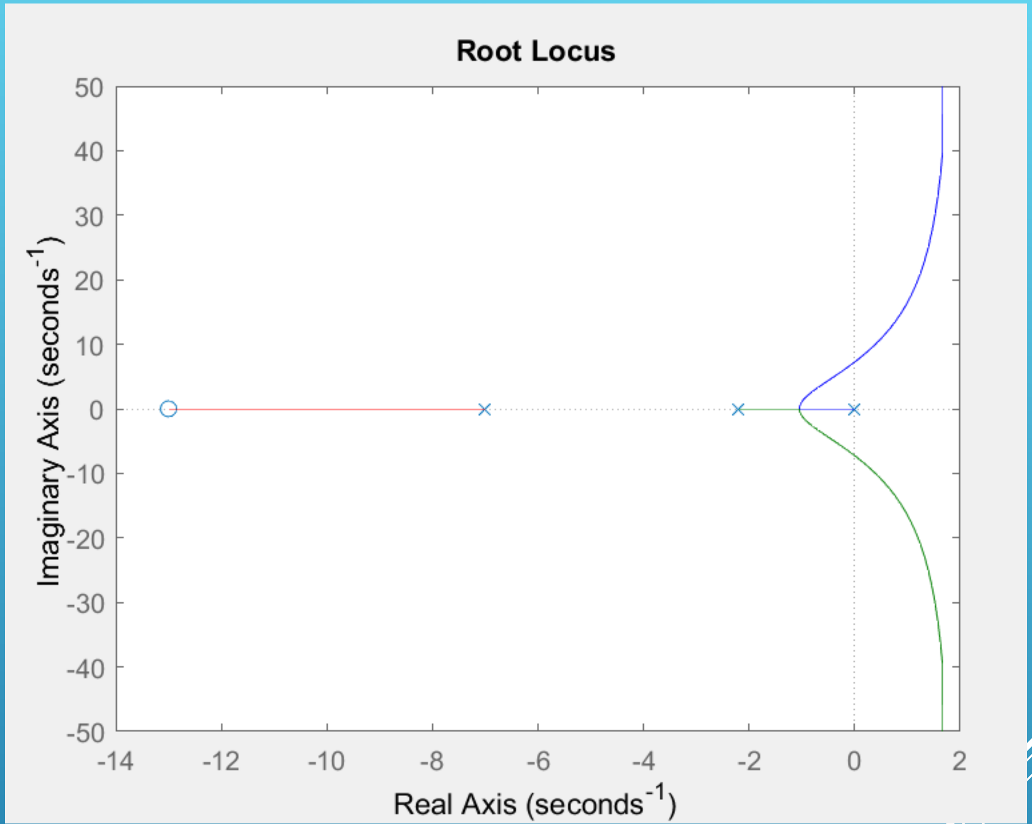
K = 9, z = 13


Step Response
×10^25
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Amplitude
0
10
20
30
40
50
Time (seconds)


Root Locus
50
40
30
20
10
0
-10
-20
-30
-40
-50
Imaginary Axis (seconds^-1)
-14
-12
-10
-8
-6
-4
-2
0
2
Real Axis (seconds^-1)

K = 2, z = 13

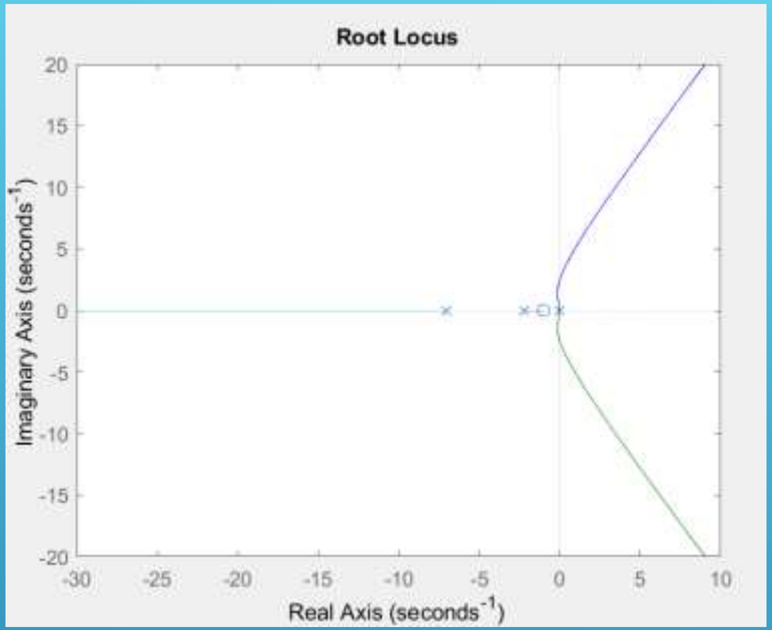


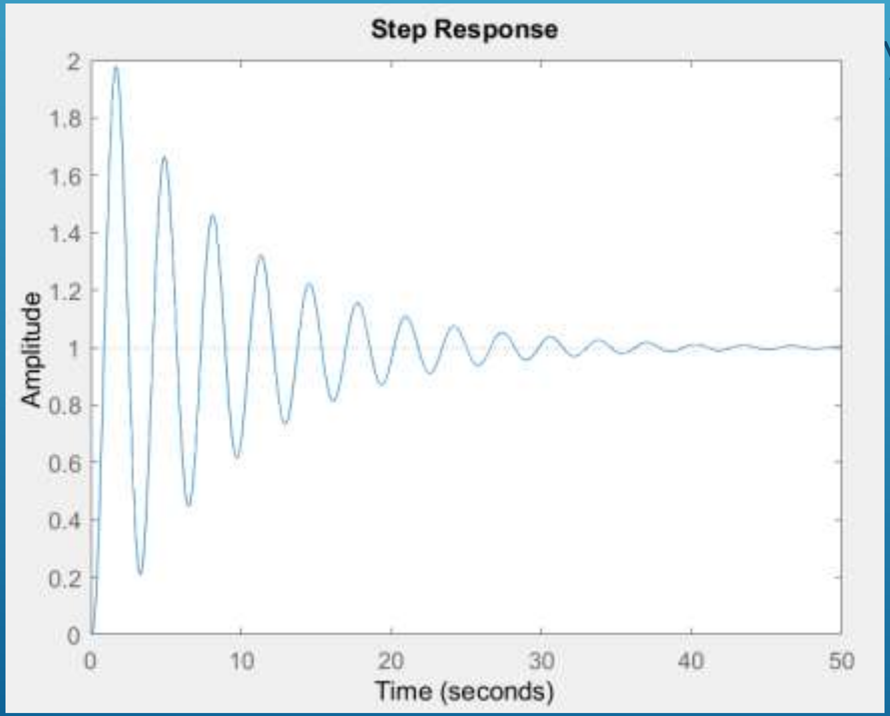


We observe that the introduction of zero allows for the system to have lower rise time for the same value of K. It makes the system more stable and allows higher values of K to be chosen, but there is a limit to this too. As we have observed from our examples, if the value of K is too high(in our example K = 9) the system tends to be unstable. Even in the case of the zero being non dominating, i.e, far away from the origin, the system ends up being unstable(as seen in our example z = 13)

Q5

- Adding a pole to our system is equivalent to have an integral controller

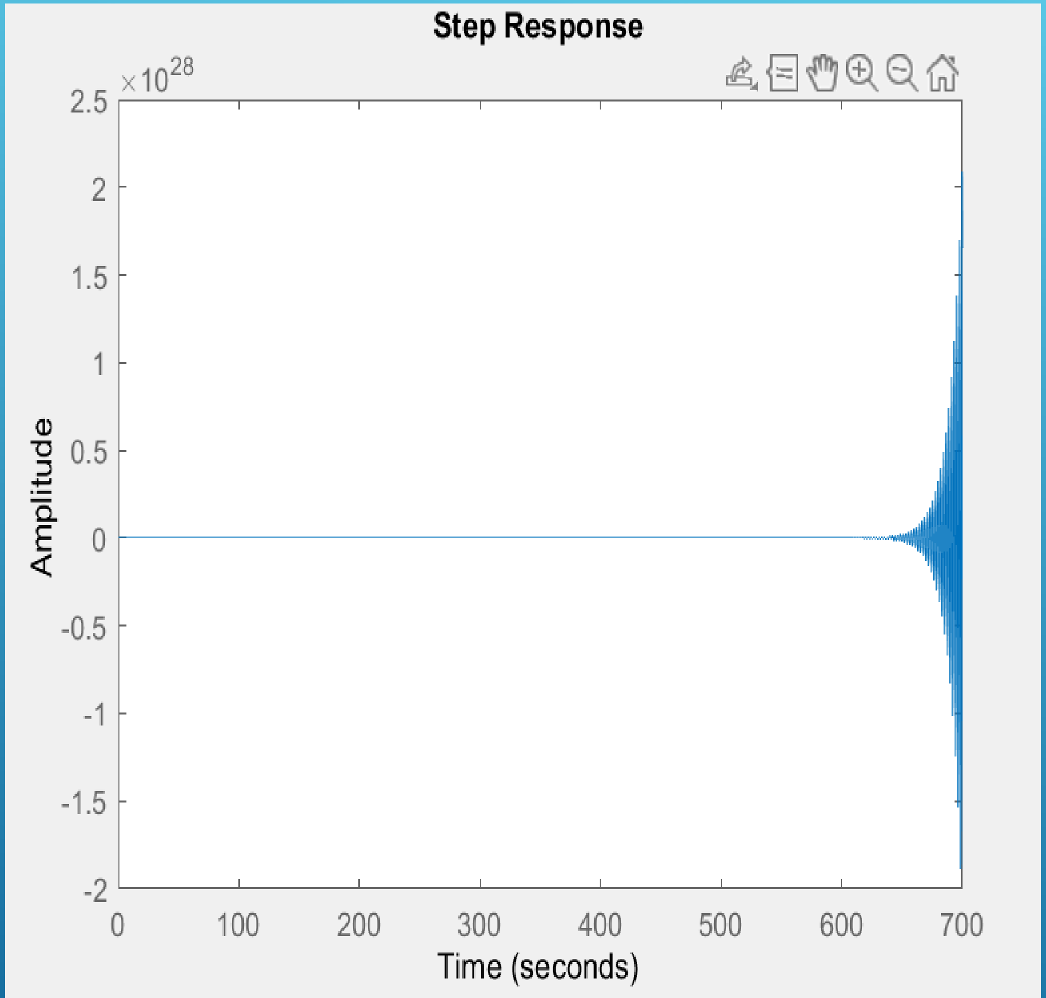




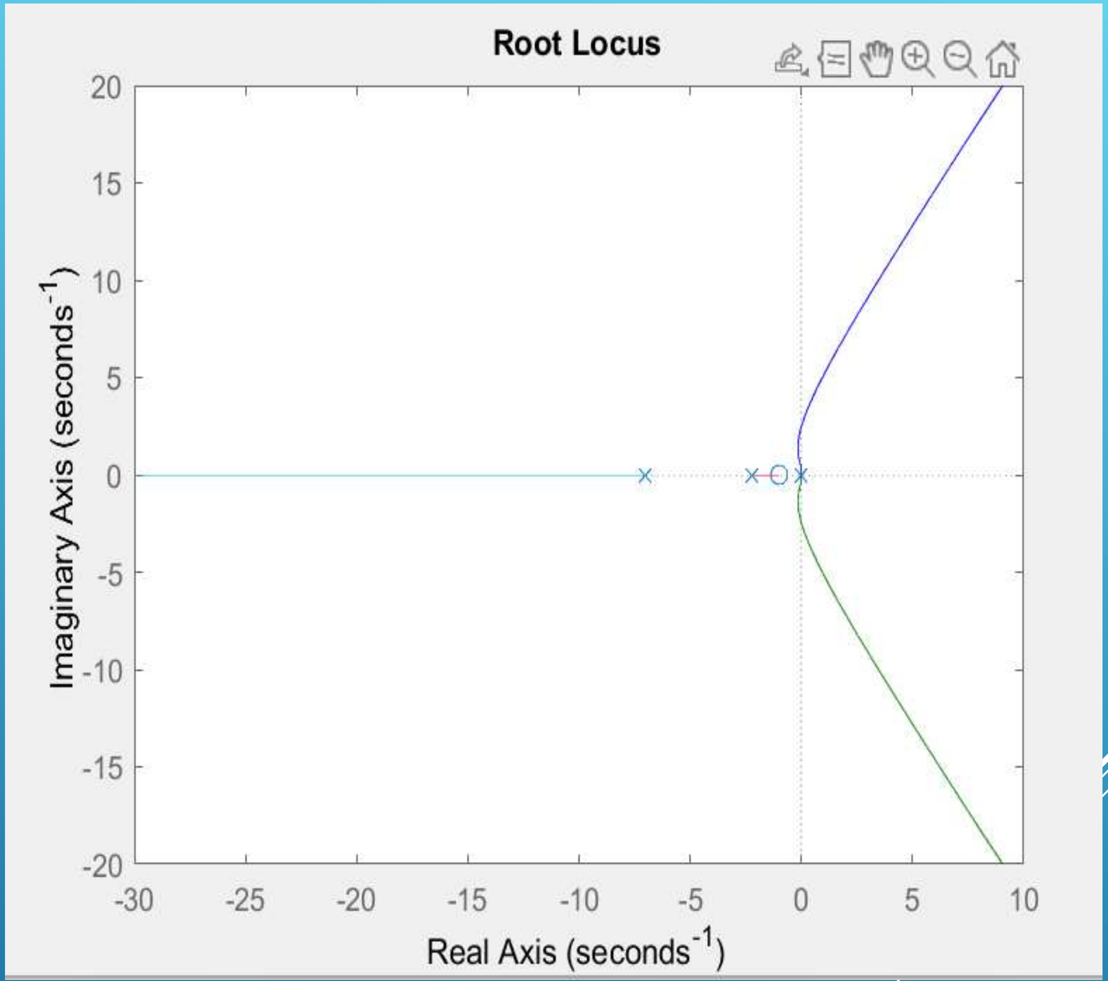
ystem

```
        RiseTime: 0.5224
   TransientTime: 35.5850
    SettlingTime: 35.5850
     SettlingMin: 0.2080
     SettlingMax: 1.9781
       Overshoot: 97.8142
      Undershoot: 0
            Peak: 1.9781
        PeakTime: 1.6454
```

K = 2, z = 1

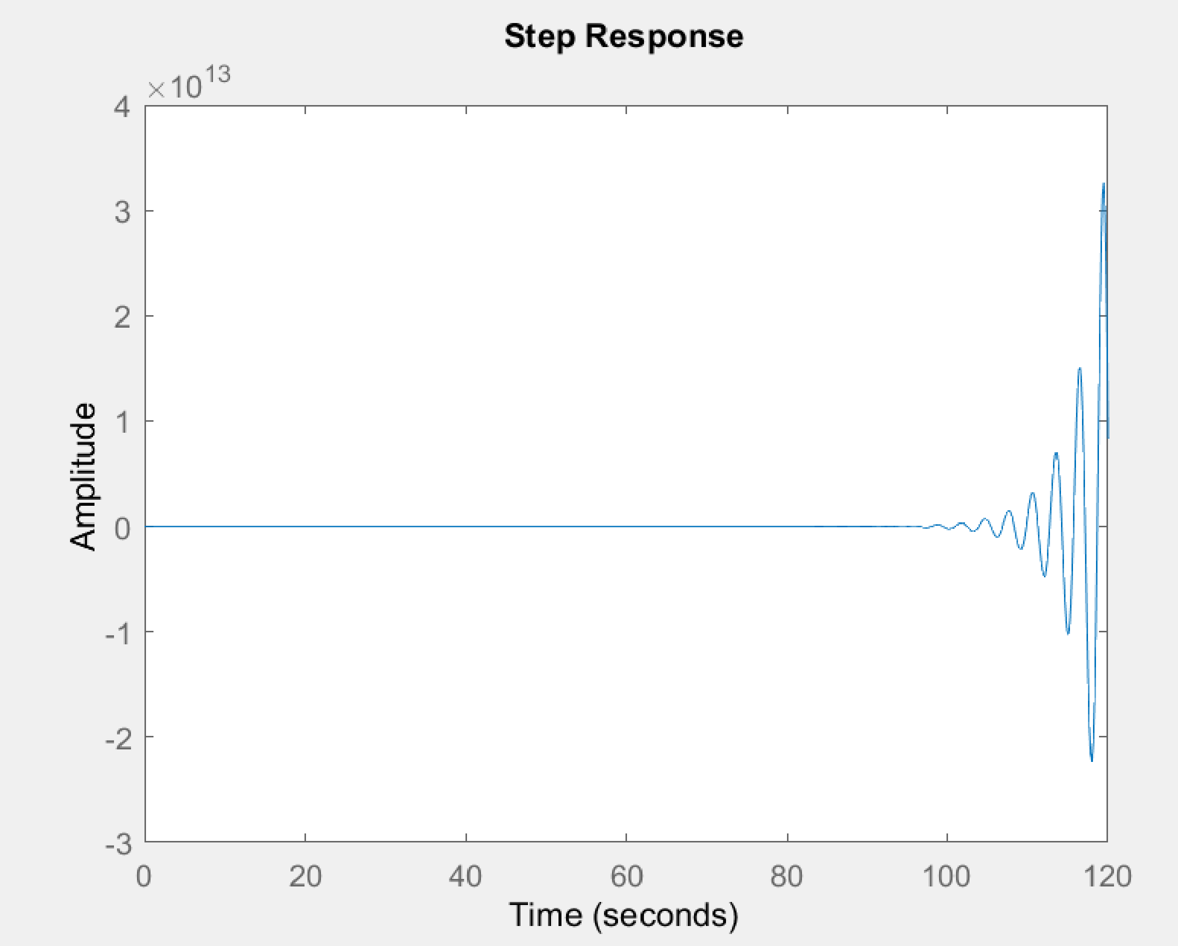


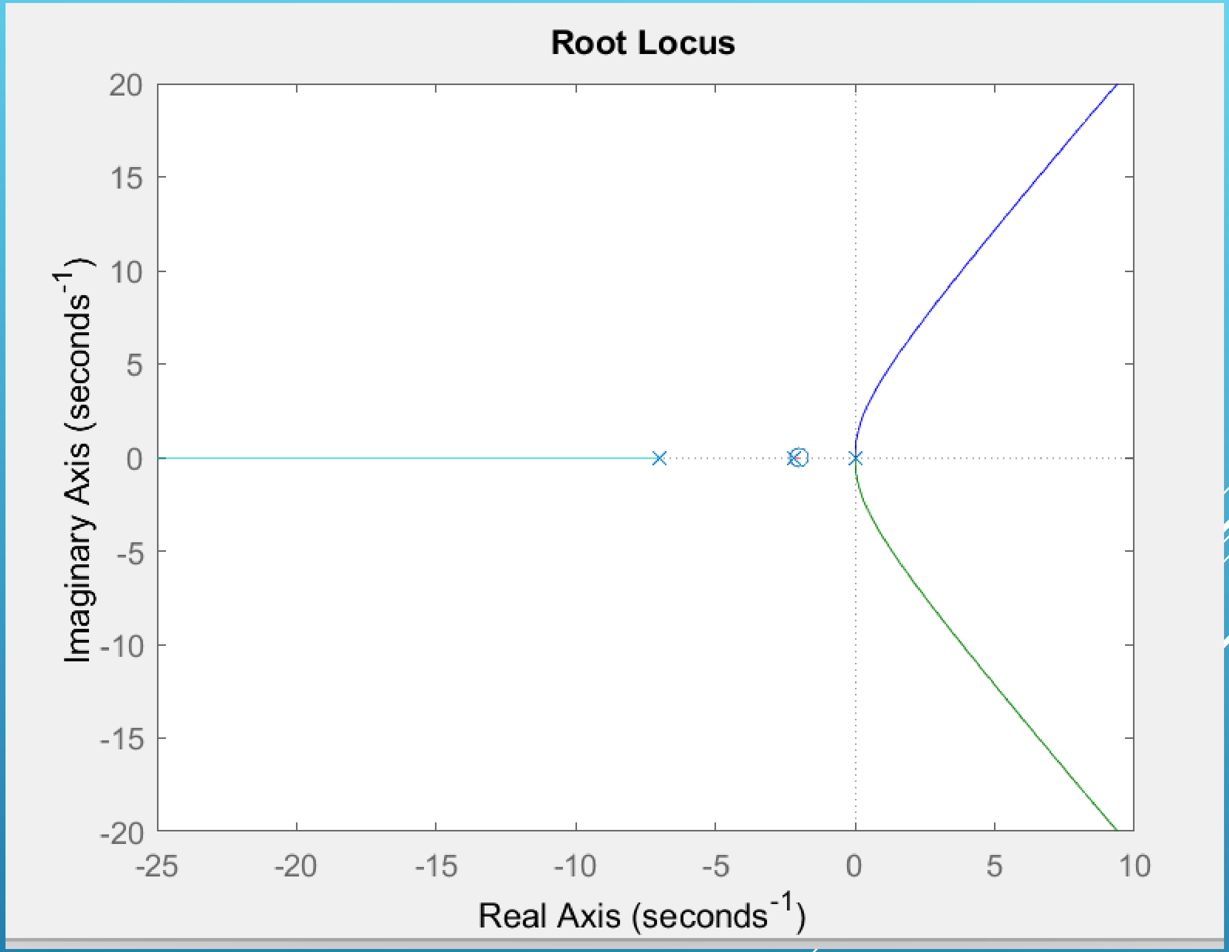
Step Response
×10^28
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Amplitude
0
100
200
300
400
500
600
700
Time (seconds)



Root Locus
20
15
10
5
0
-5
-10
-15
-20
Imaginary Axis (seconds^-1)
-30
-25
-20
-15
-10
-5
0
5
10
Real Axis (seconds^-1)

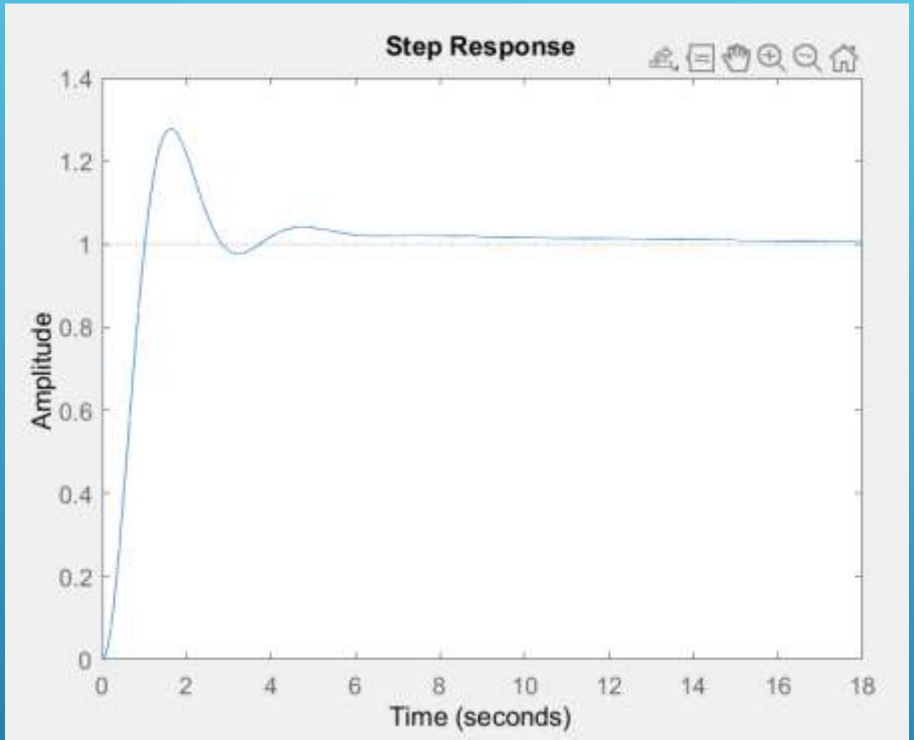
K = 1, z = 2

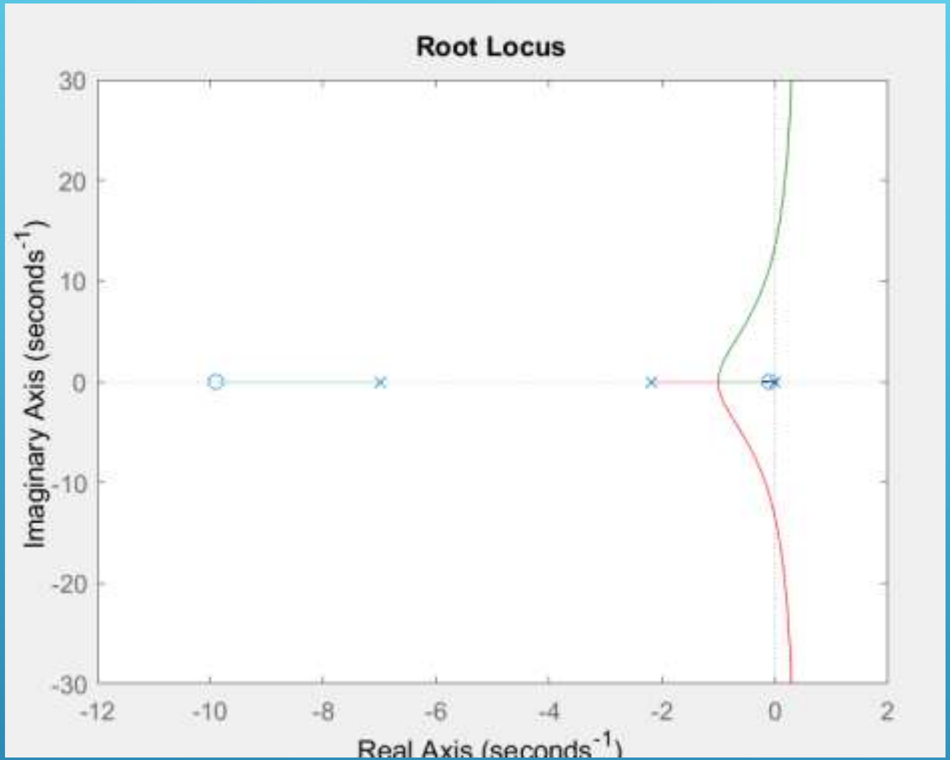




We observe from our experiments that the addition of a pole to our particular system ends up making it more unstable, rather than stable, limiting the value of K that can be chosen

K1 = 1, K2 = 0.1, K3 = 0.1

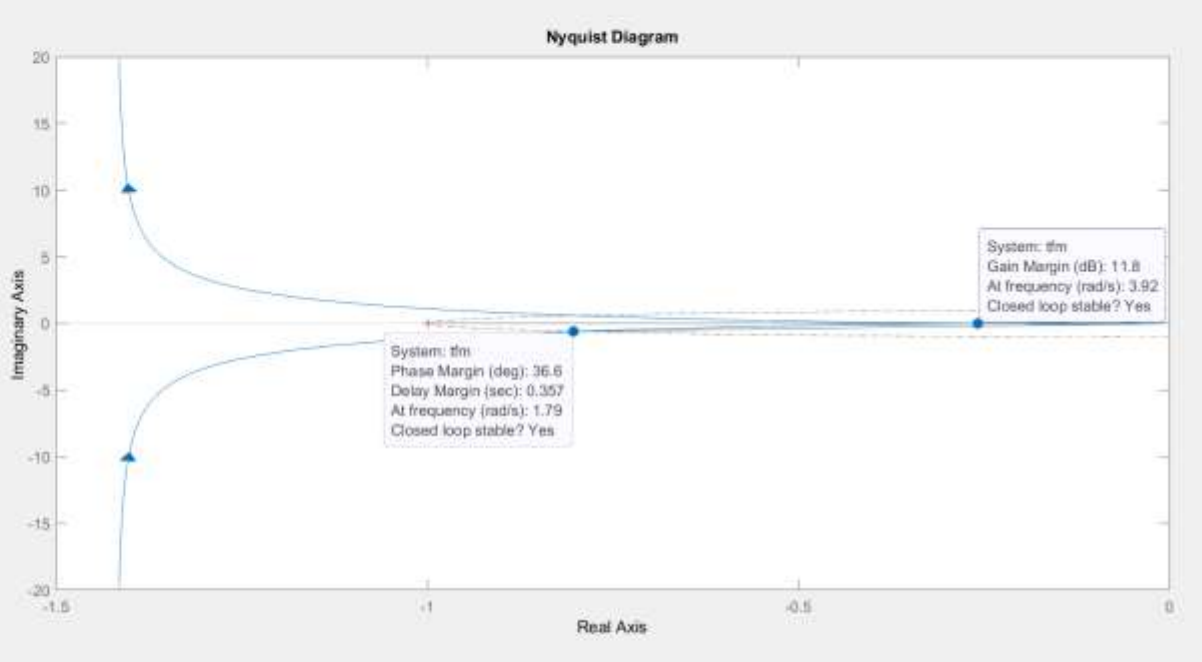


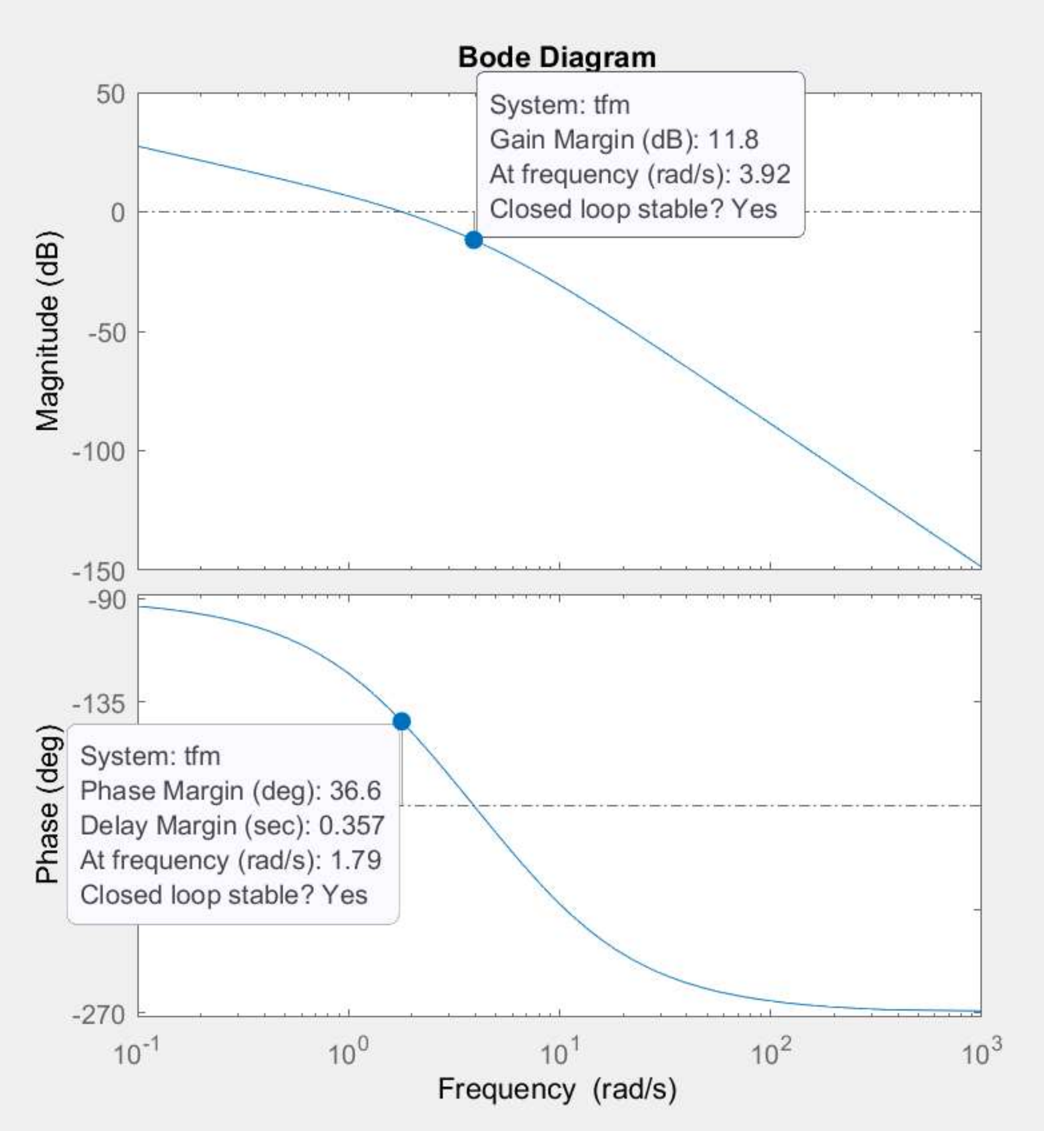


RiseTime: 0.6738
TransientTime: 8.2502
SettlingTime: 8.2502
SettlingMin: 0.9471
SettlingMax: 1.2776
Overshoot: 27.7630
Undershoot: 0
Peak: 1.2776
PeakTime: 1.6061

Q6

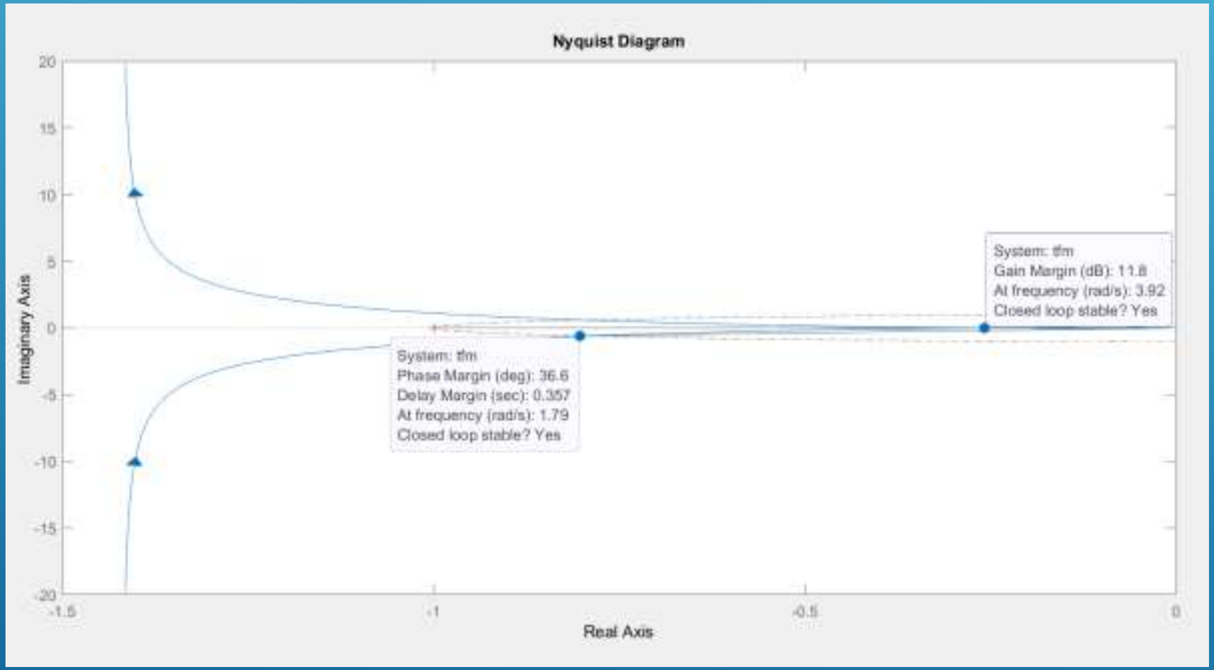
- Bode and Nyquist plots are used to obtain the stability of the system using the Open loop transfer function
- From these plots we observe that our system is open loop stable

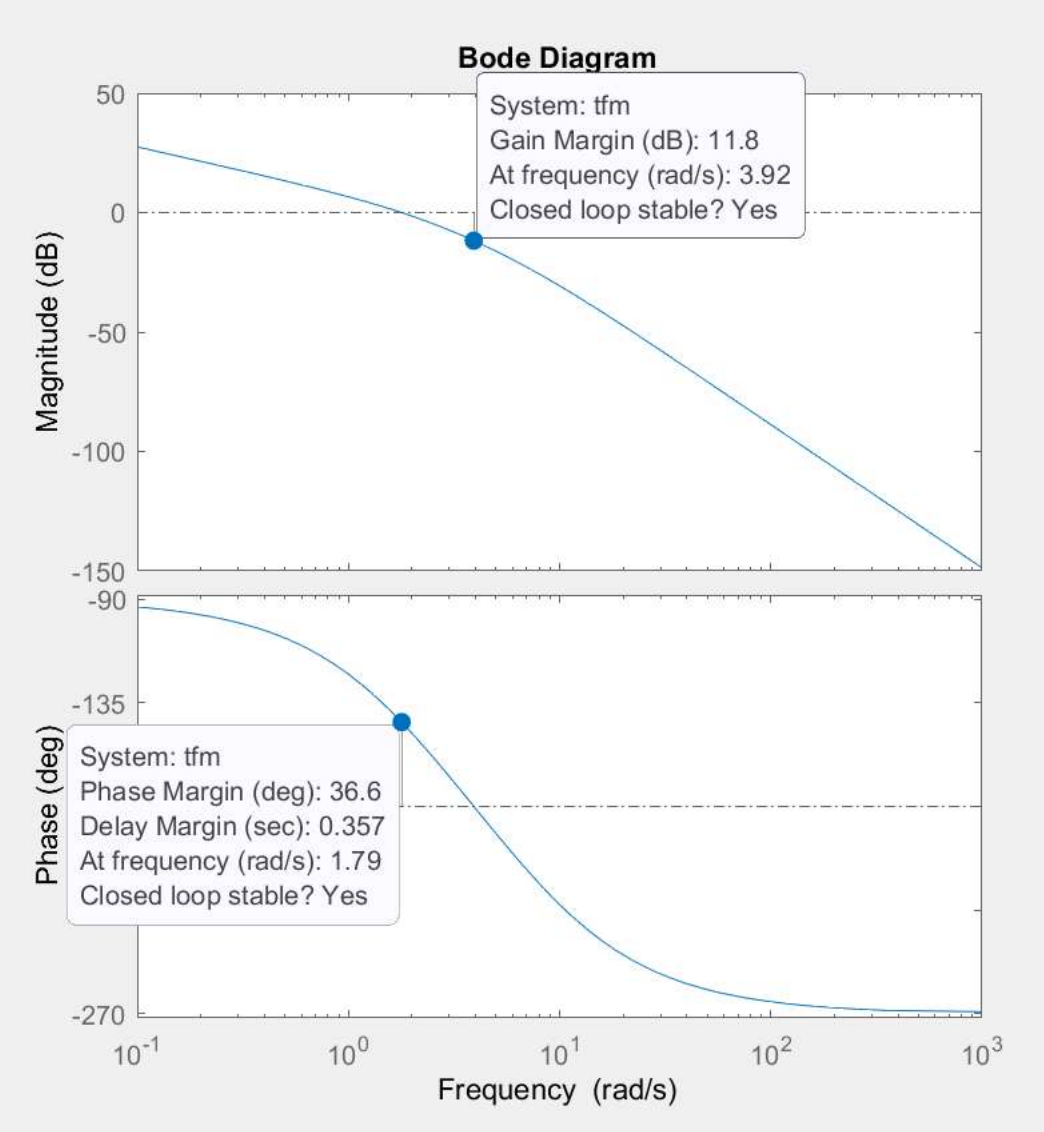




Q6

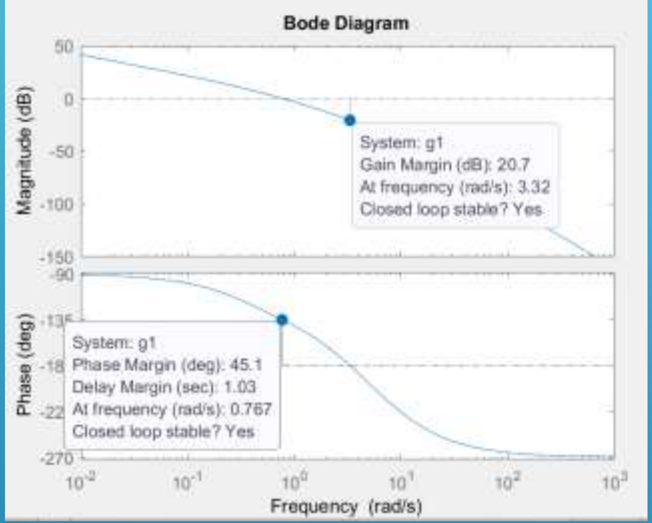
- Bode and Nyquist plots are used to obtain the stability of the system using the Open loop transfer function
- From these plots we observe that our system is open loop stable

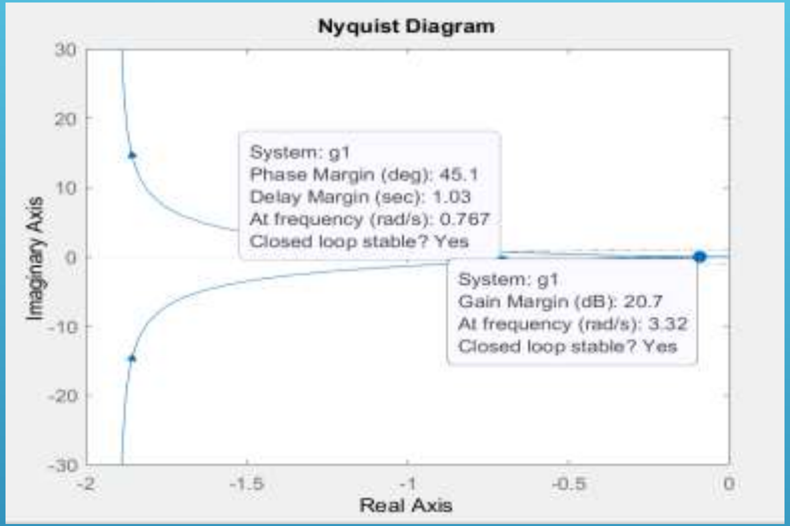




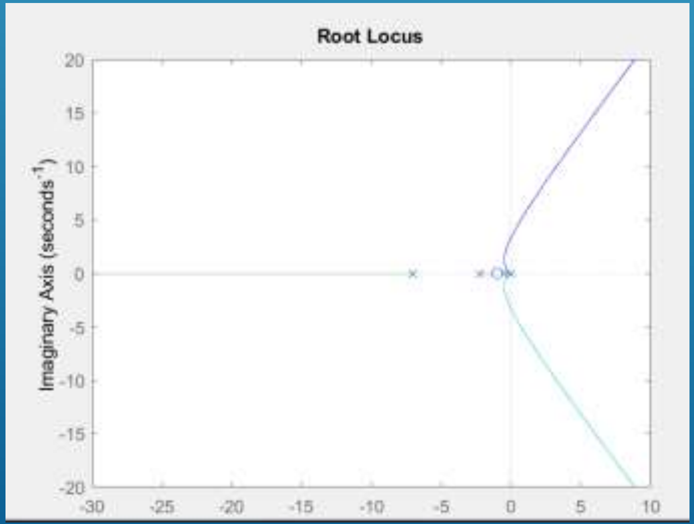
LAG COMPENSATOR

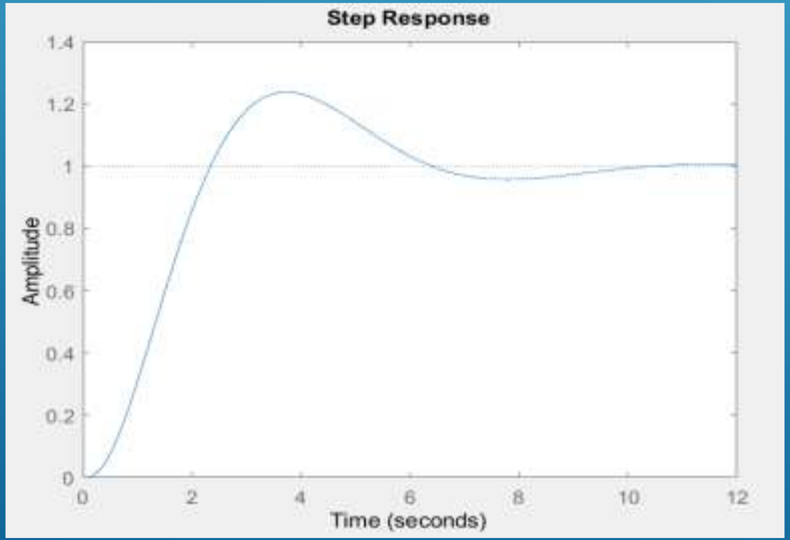
Alpha = 2, Tau = 1



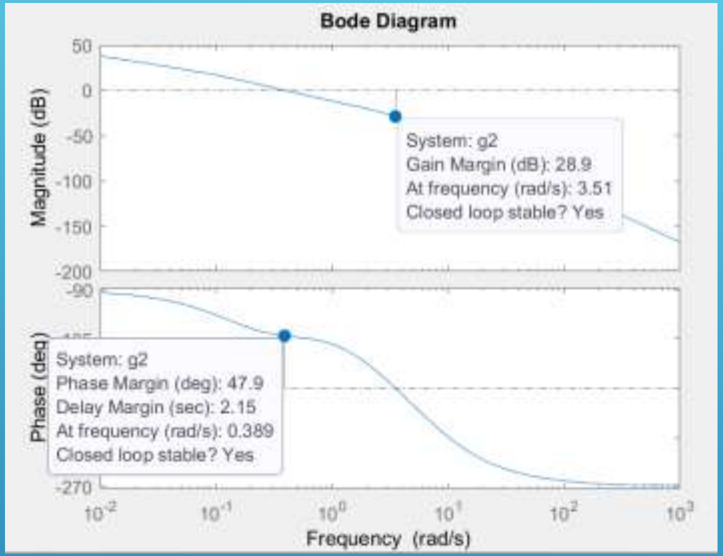


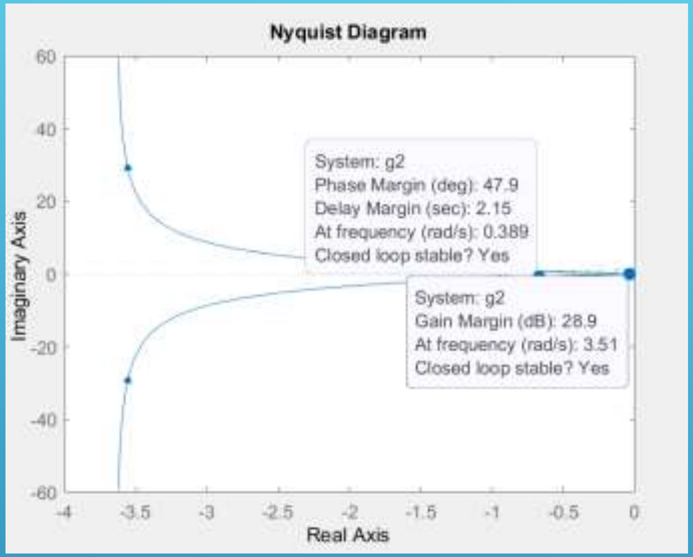

```
   RiseTime: 1.5294
TransientTime: 9.3005
SettlingTime: 9.3005
 SettlingMin: 0.9049
 SettlingMax: 1.2381
   Overshoot: 23.8112
  Undershoot: 0
        Peak: 1.2381
    PeakTime: 3.7547
```



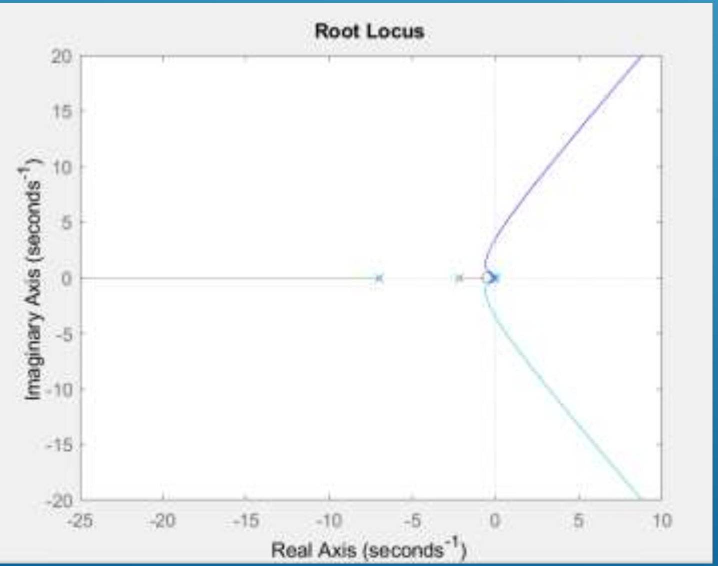


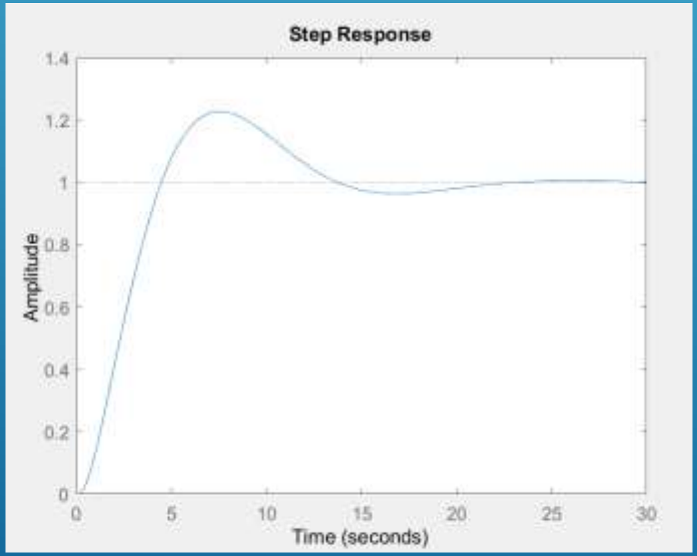
Alpha = 3, Tau = 2



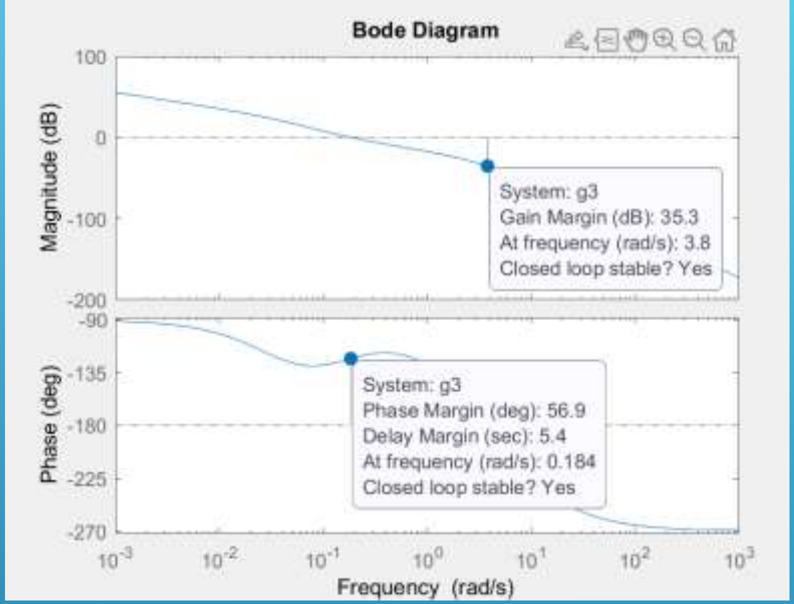


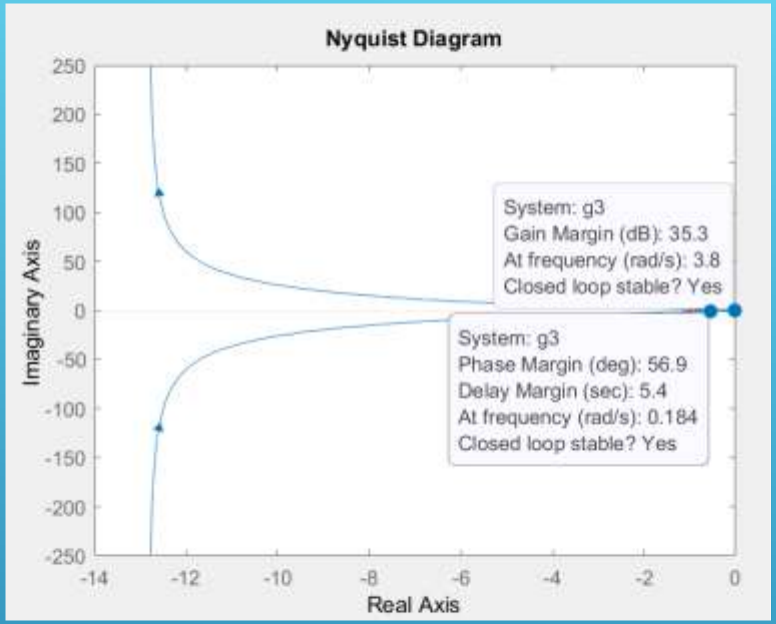
RiseTime:	3.0675
TransientTime:	19.8752
SettlingTime:	19.8752
SettlingMin:	0.9002
SettlingMax:	1.2283
Overshoot:	22.8343
Undershoot:	0
Peak:	1.2283
PeakTime:	7.5338



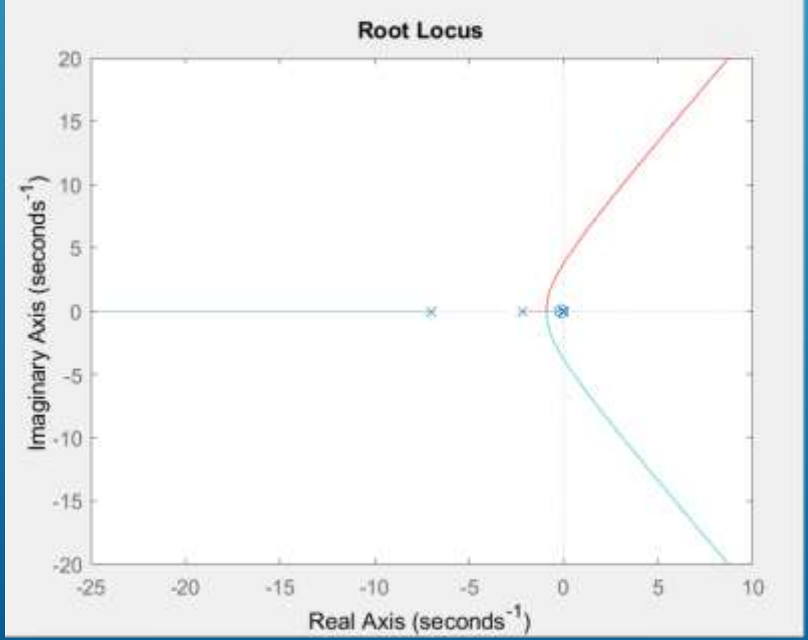


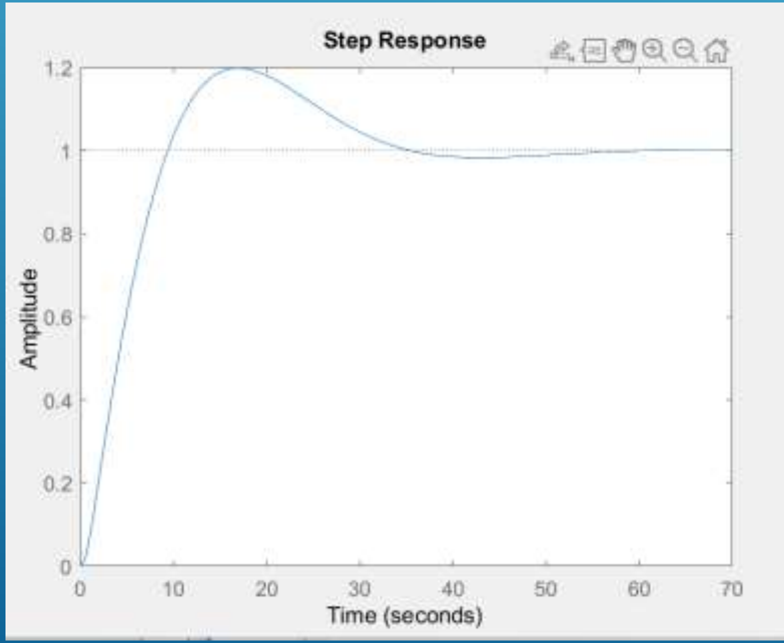
Alpha = 4, Tau = 7



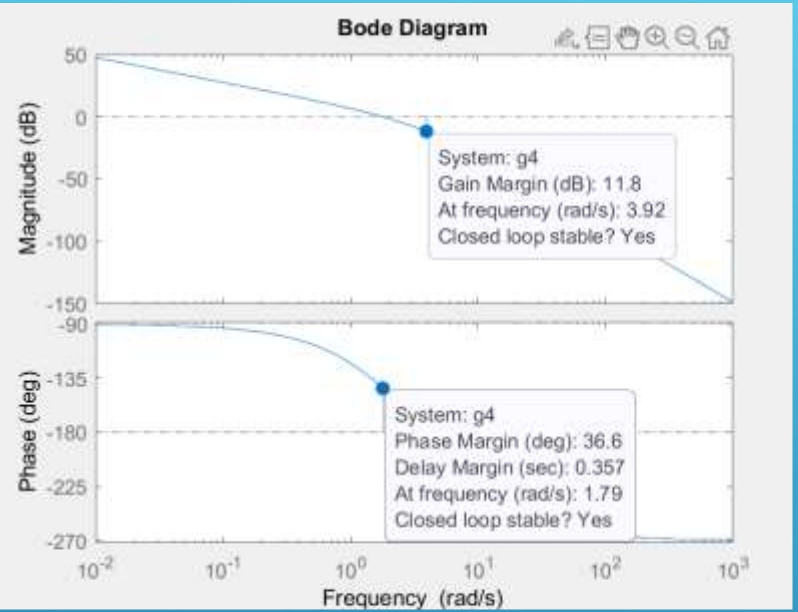


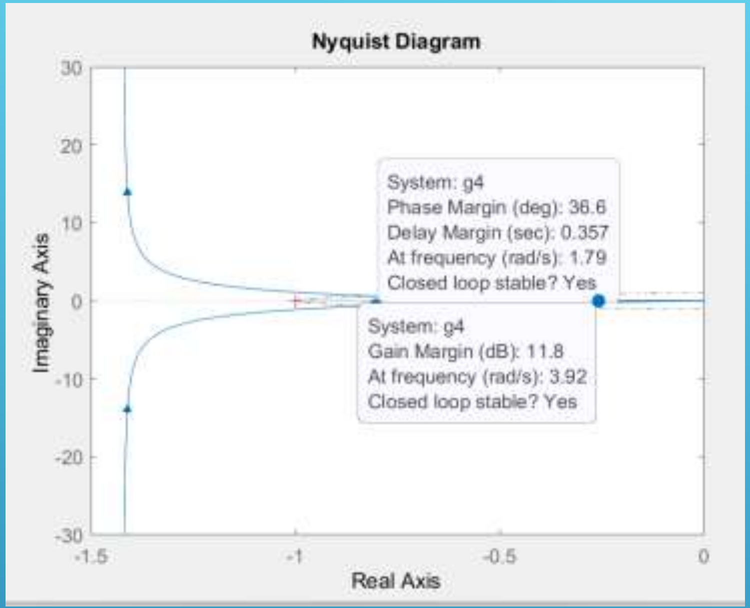

```
      RiseTime: 6.7323
 TransientTime: 32.5073
  SettlingTime: 32.5073
   SettlingMin: 0.9320
   SettlingMax: 1.1982
     Overshoot: 19.8185
    Undershoot: 0
          Peak: 1.1982
      PeakTime: 16.7029
```



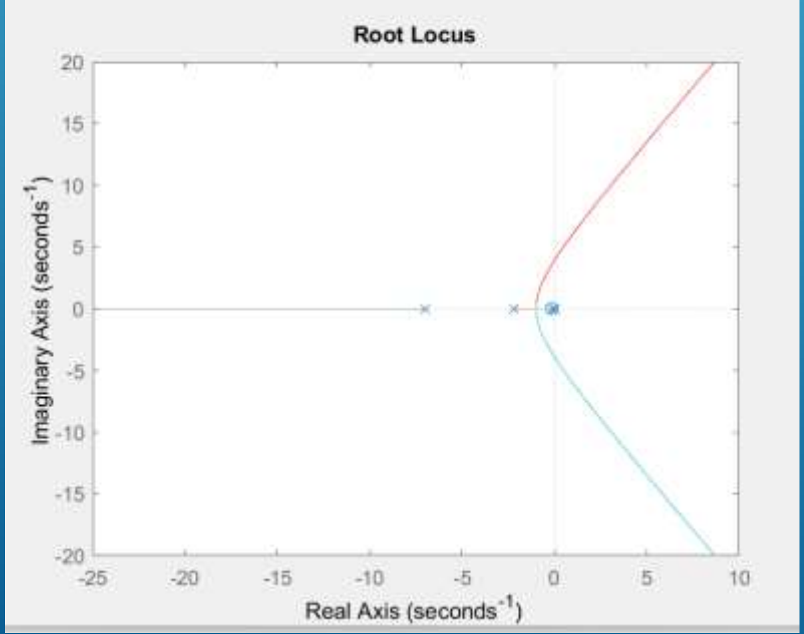


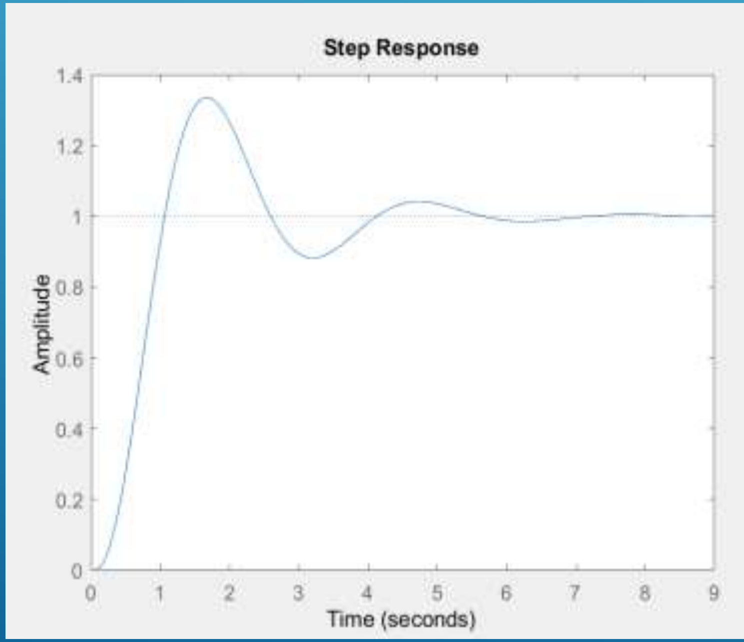
Alpha = 1, Tau = 7



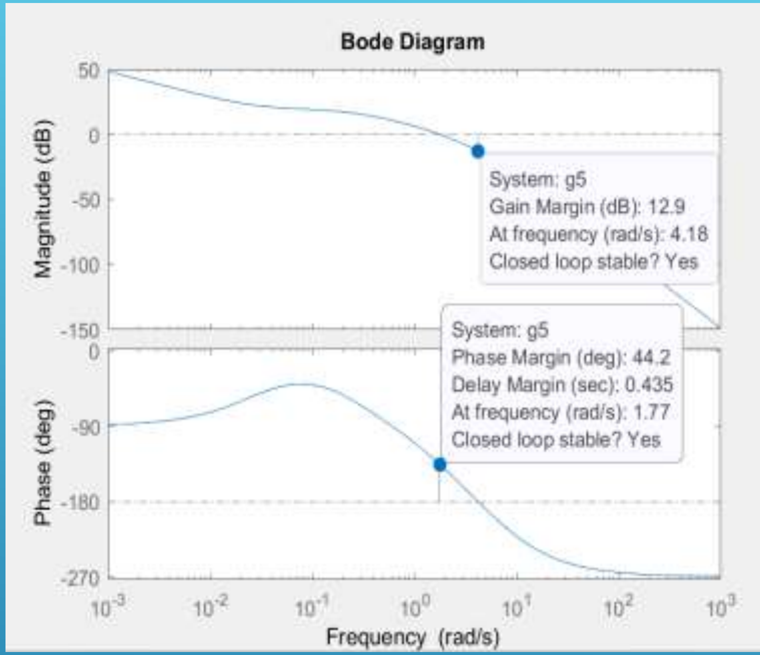


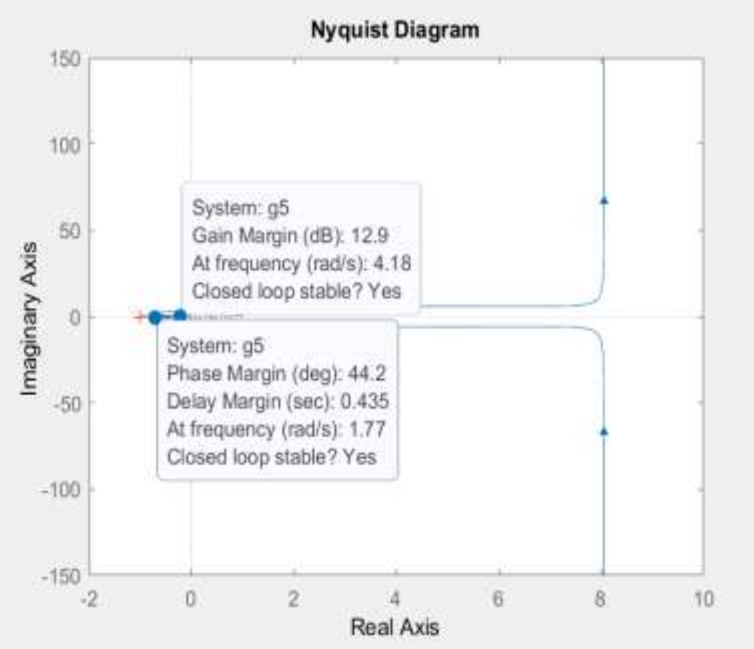
RiseTime:	0.6592
TransientTime:	5.2933
SettlingTime:	5.2933
SettlingMin:	0.8825
SettlingMax:	1.3359
Overshoot:	33.5871
Undershoot:	0
Peak:	1.3359
PeakTime:	1.6810



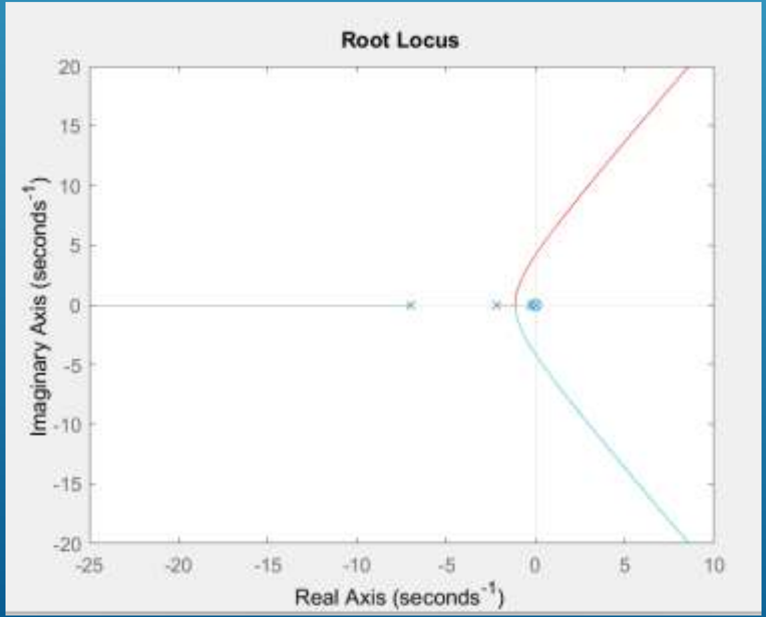


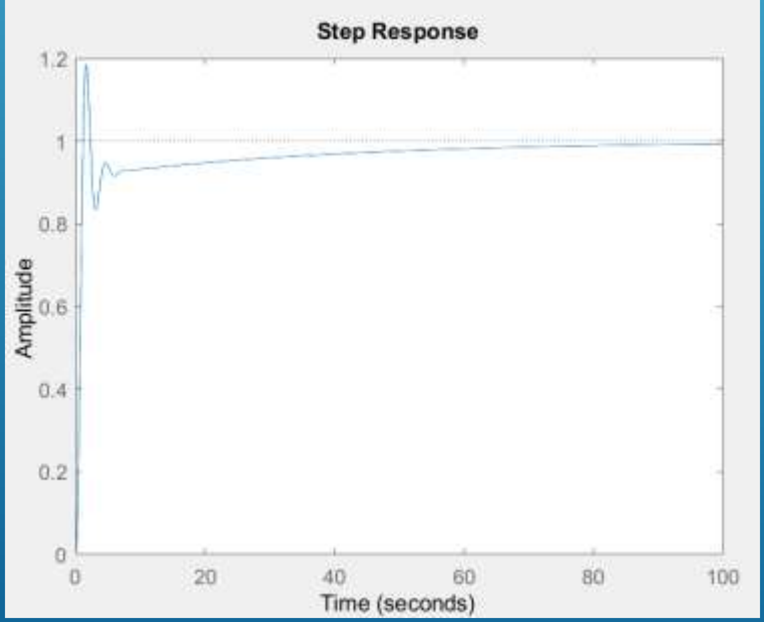
Alpha = 9, Tau = 5





RiseTime: 0.7167
TransientTime: 56.9701
SettlingTime: 56.9701
SettlingMin: 0.8351
SettlingMax: 1.1849
Overshoot: 18.4893
Undershoot: 0
Peak: 1.1849
PeakTime: 1.6312

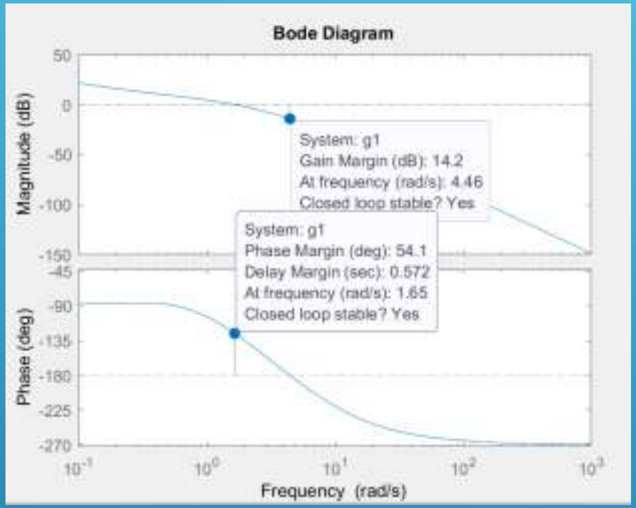


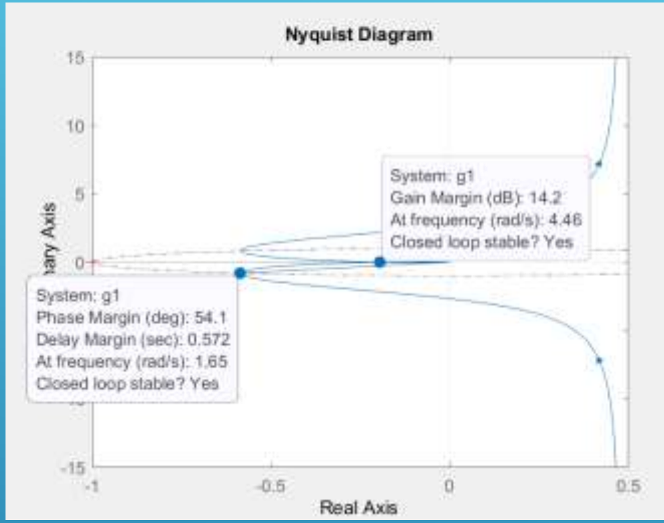


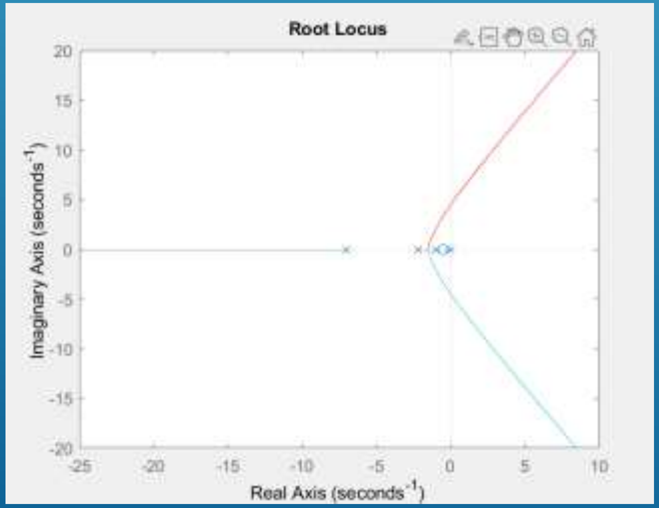
From these experiments we observe that the value of Tau must be large and Alpha be small for the best response(lowest rise time)

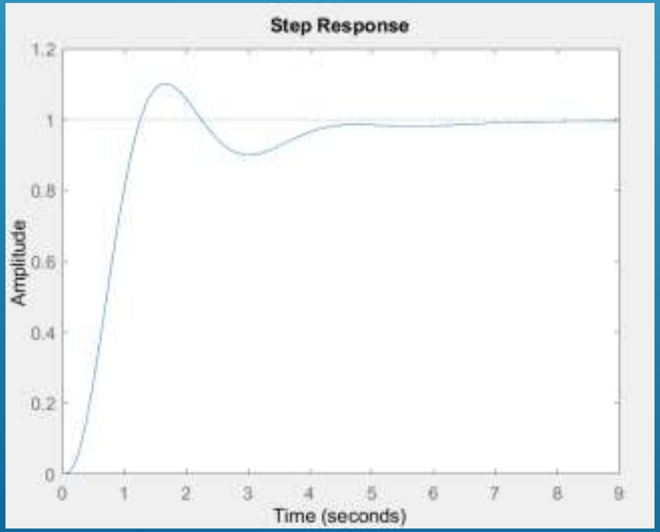
LEAD COMPENSATOR

Alpha = 2, Tau = 1



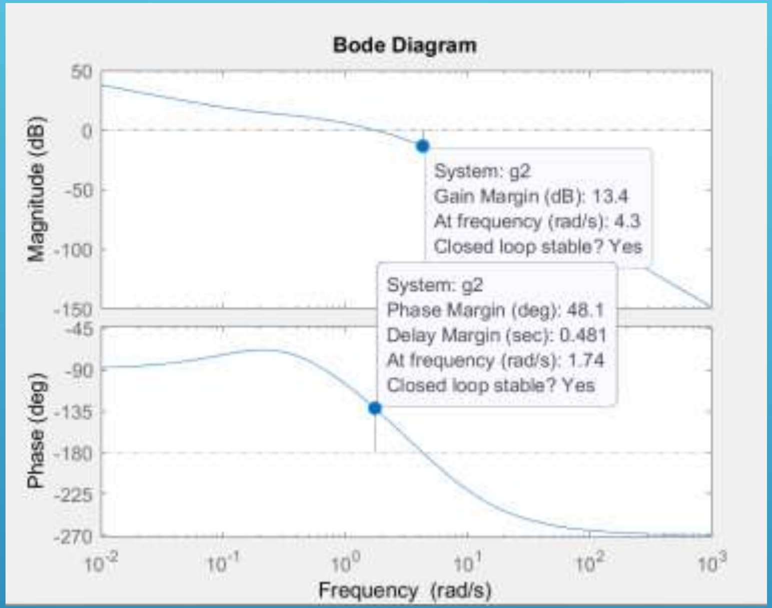


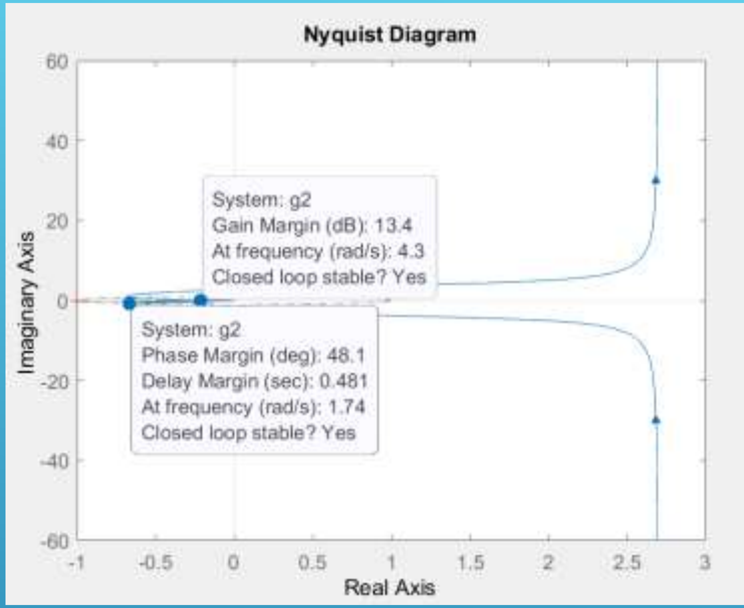




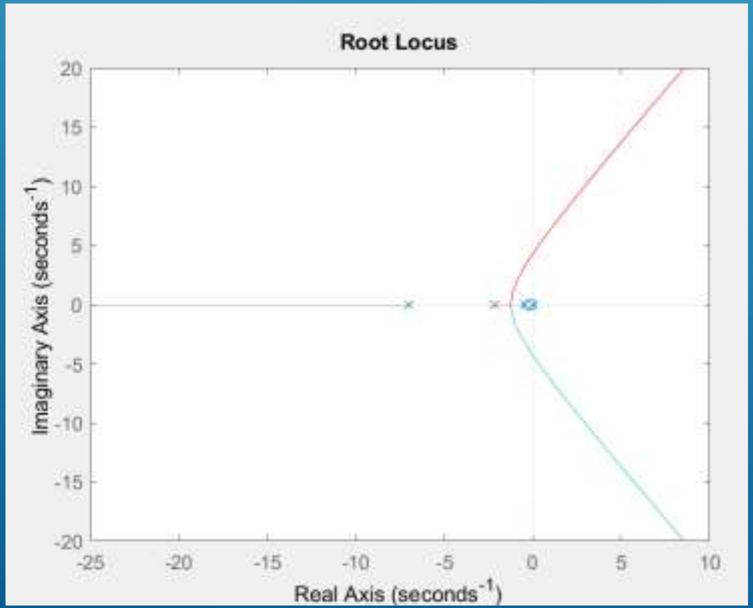

```
      RiseTime: 0.7877
 TransientTime: 4.3211
  SettlingTime: 4.3211
   SettlingMin: 0.9001
   SettlingMax: 1.1020
     Overshoot: 10.2021
    Undershoot: 0
          Peak: 1.1020
      PeakTime: 1.6480
```

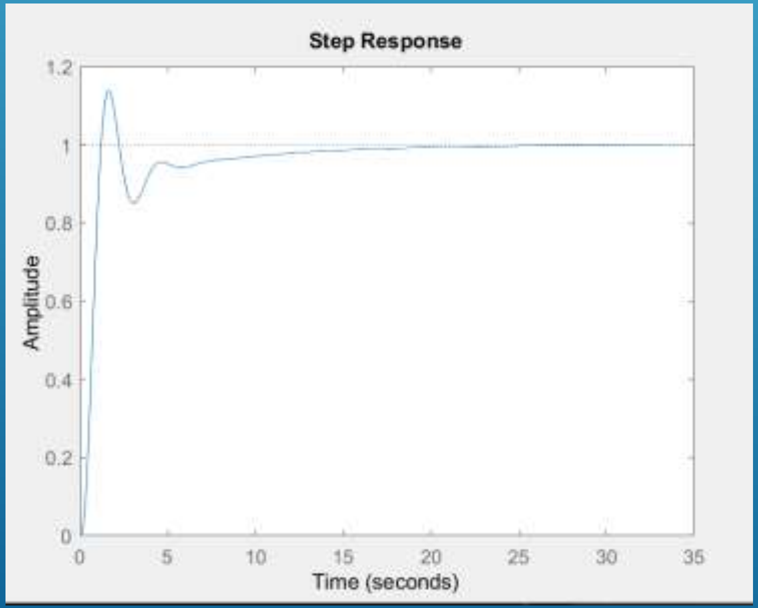
Alpha = 3, Tau = 2



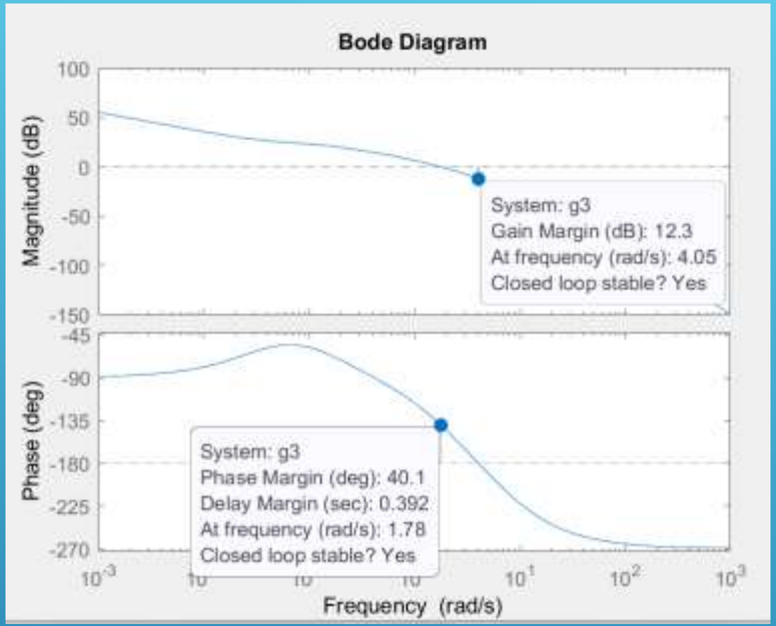


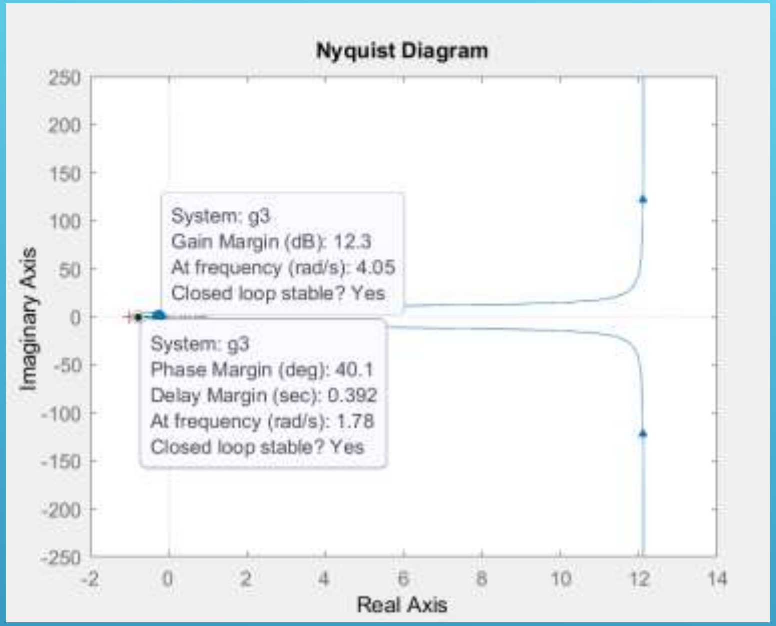

```
      RiseTime: 6.7323
 TransientTime: 32.5073
  SettlingTime: 32.5073
   SettlingMin: 0.9320
   SettlingMax: 1.1982
     Overshoot: 19.8185
    Undershoot: 0
          Peak: 1.1982
      PeakTime: 16.7029
```



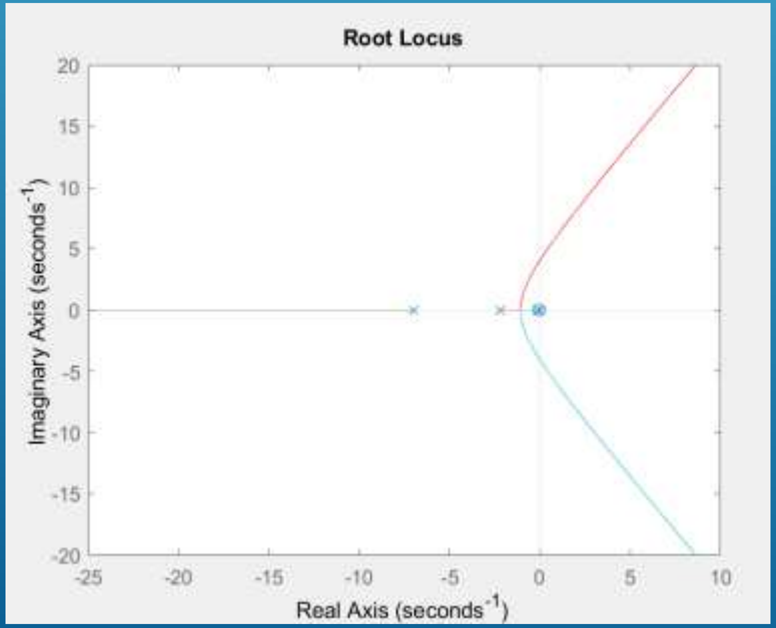


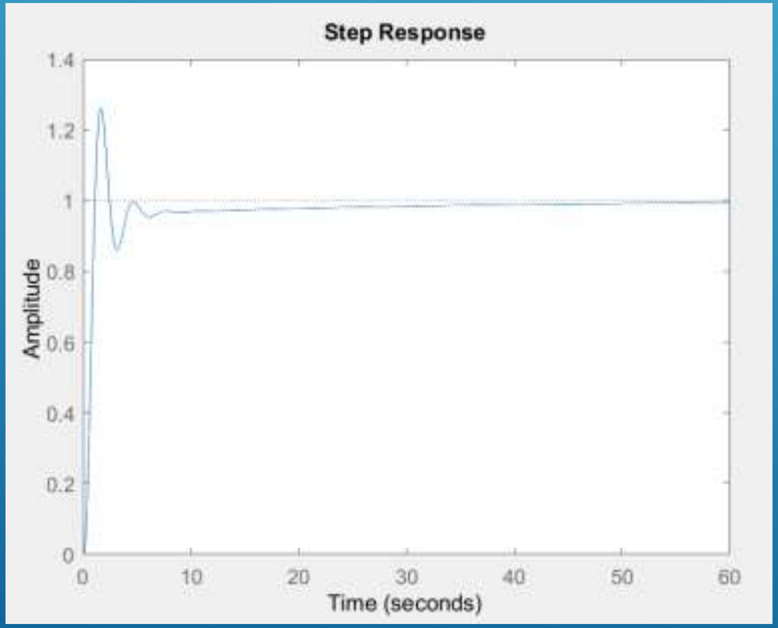
Alpha = 4, Tau = 7



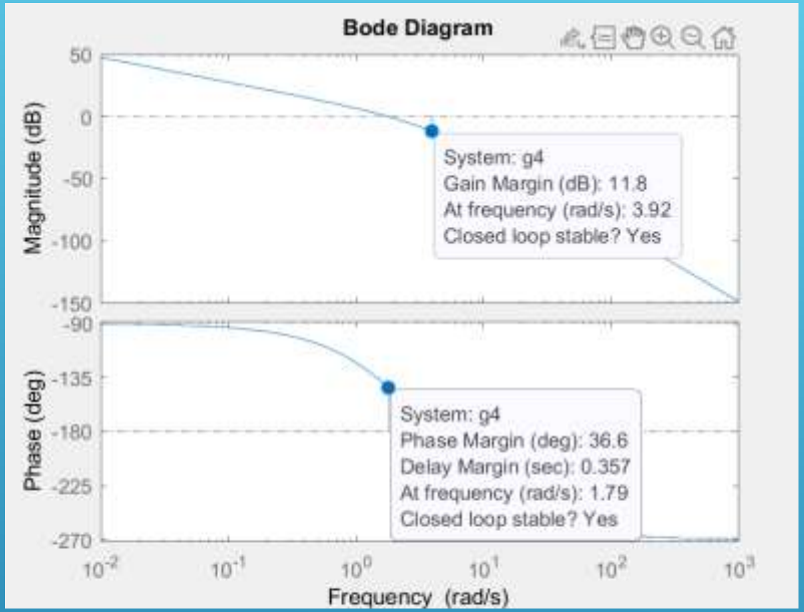


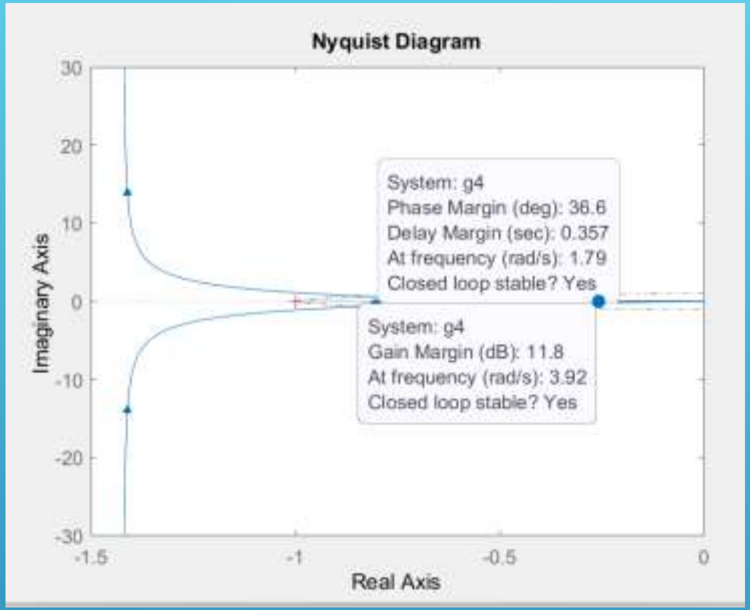
RiseTime:	0.6826
TransientTime:	22.7317
SettlingTime:	22.7317
SettlingMin:	0.8611
SettlingMax:	1.2611
Overshoot:	26.1147
Undershoot:	0
Peak:	1.2611
PeakTime:	1.6298

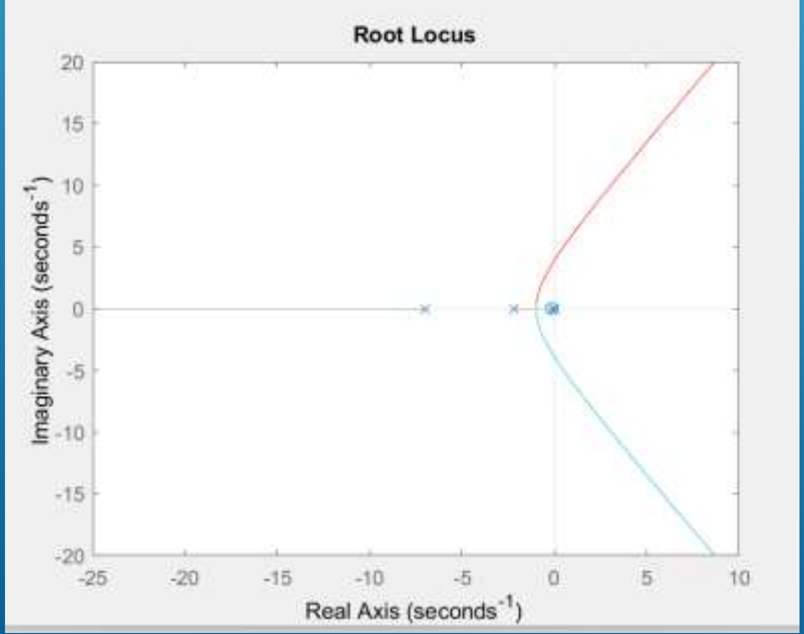


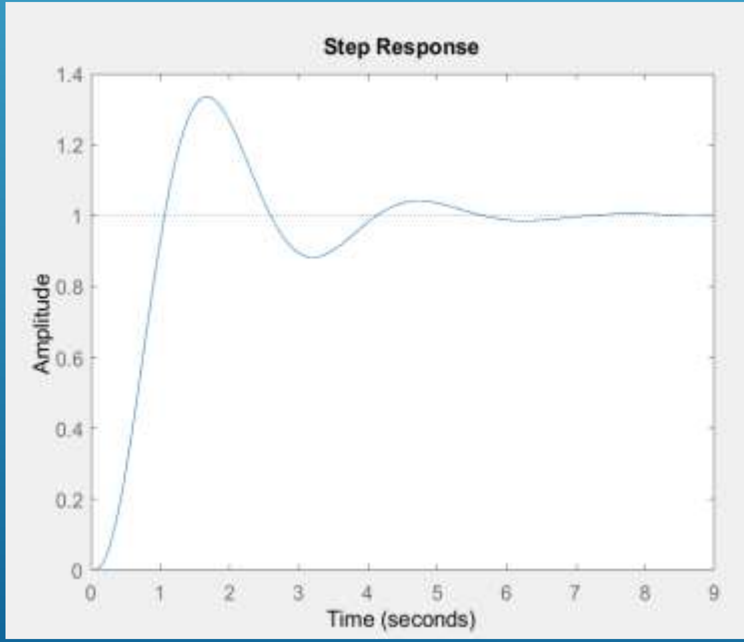


Alpha = 1, Tau = 7



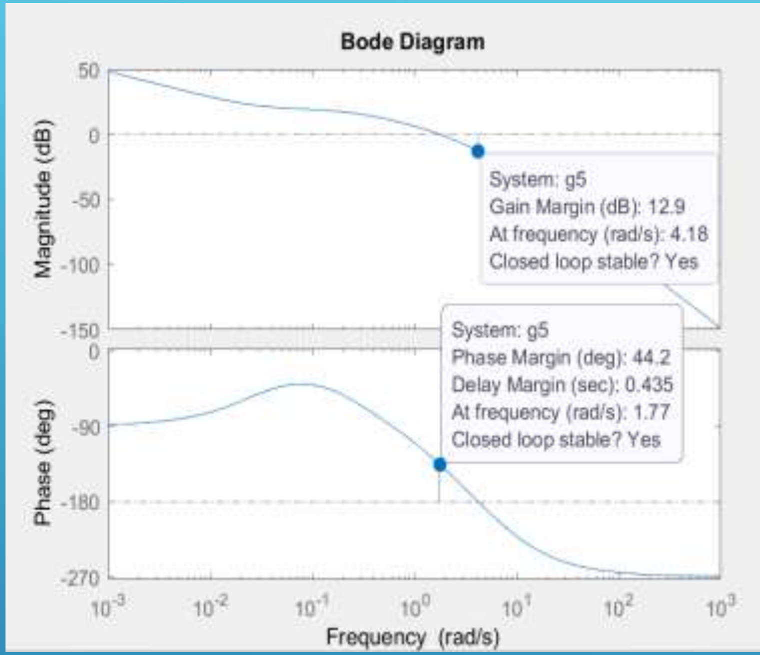


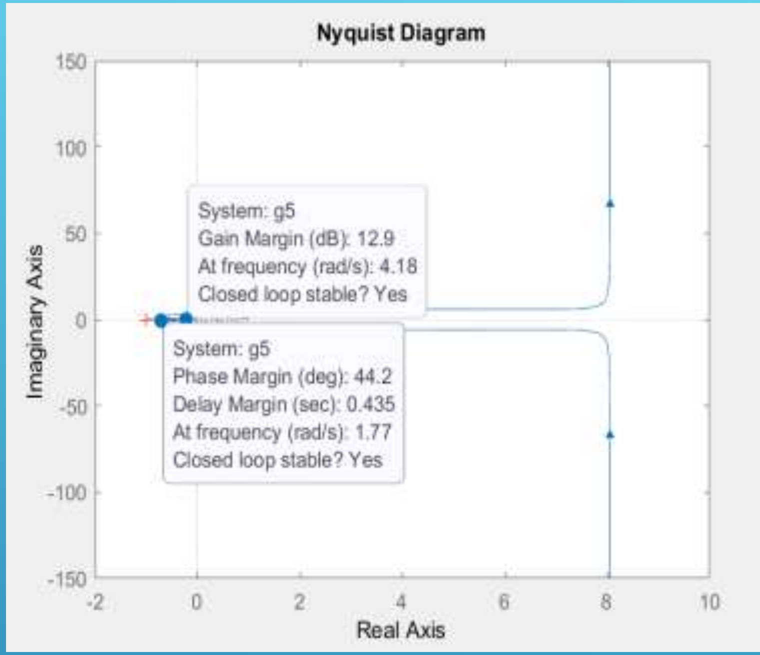




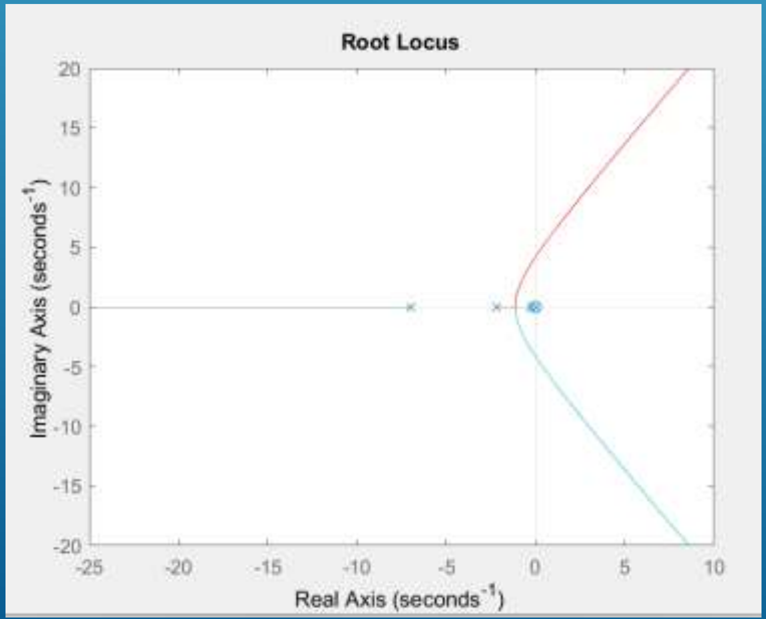
RiseTime:	0.6592
TransientTime:	5.2933
SettlingTime:	5.2933
SettlingMin:	0.8825
SettlingMax:	1.3359
Overshoot:	33.5871
Undershoot:	0
Peak:	1.3359
PeakTime:	1.6810

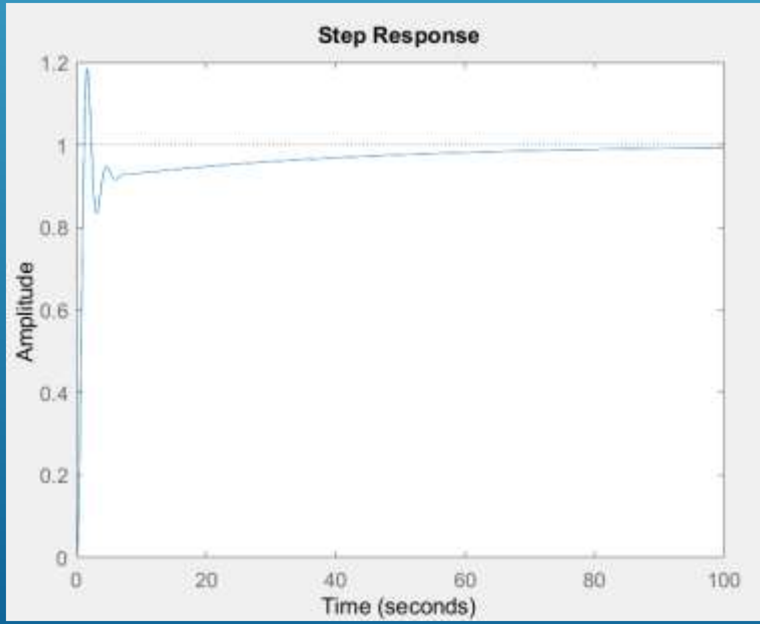
Alpha = 9, Tau = 5






```
       RiseTime: 0.7167
  TransientTime: 56.9701
   SettlingTime: 56.9701
    SettlingMin: 0.8351
    SettlingMax: 1.1849
      Overshoot: 18.4893
     Undershoot: 0
           Peak: 1.1849
       PeakTime: 1.6312
```





From these experiments we observe that the value of Tau must be large and Alpha be small for the best response(lowest rise time)

THANK YOU ☺