MATHEMATICS FOR ELECTRONICS ENGINEERS (UE21EC241A) PROJECT

TOPIC:WEIBULL DISTRIBUTION-AND APPLICATIONS

PROFESSOR : RAJINI M.

AKASH RAVI BHAT(PES1UG21EC025)

A.ANIRUDH SIMHA(PES1UG21EC001)

SECTION: A

The Weibull Distribution Parameters

Three parameter PDF

$$f(x) = \frac{\gamma}{\alpha}\left(\frac{(x-\mu)}{\alpha}\right)^{\gamma-1} exp\left[-\left(\frac{x-\mu}{\alpha}\right)^{\gamma}\right]$$

$$x \geq \mu;\ \gamma, \alpha > 0$$

$$\xrightarrow{\mu = 0}$$

Two parameter PDF

$$f(x) = \frac{\gamma}{\alpha}\left(\frac{x}{\alpha}\right)^{\gamma-1} exp\left[-\left(\frac{x}{\alpha}\right)^{\gamma}\right]$$

$$x \geq 0;\ \gamma, \alpha > 0$$

μ: *location parameter* γ: *shape parameter* α: *scale parameter*

Standard Weibull distribution

$$f(x) = \gamma(x)^{\gamma-1} exp[-(x)^{\gamma}]$$

$$x \geq 0;\ \gamma > 0, \alpha = 1$$

Mean $\mu = \alpha\Gamma\left(1 + \frac{1}{\gamma}\right)$

Variance $\sigma^2 = \alpha^2\left[\Gamma\left(1 + \frac{2}{\gamma}\right) - \left\{\Gamma\left(1 + \frac{1}{\gamma}\right)\right\}^2\right]$

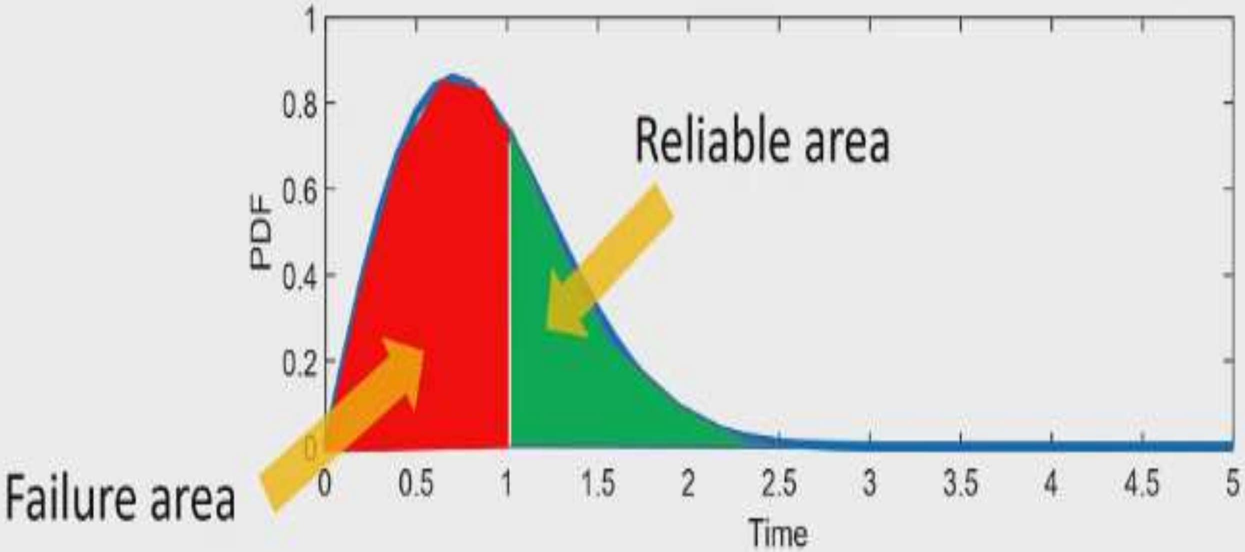
The Weibull Cumulative Distribution Function

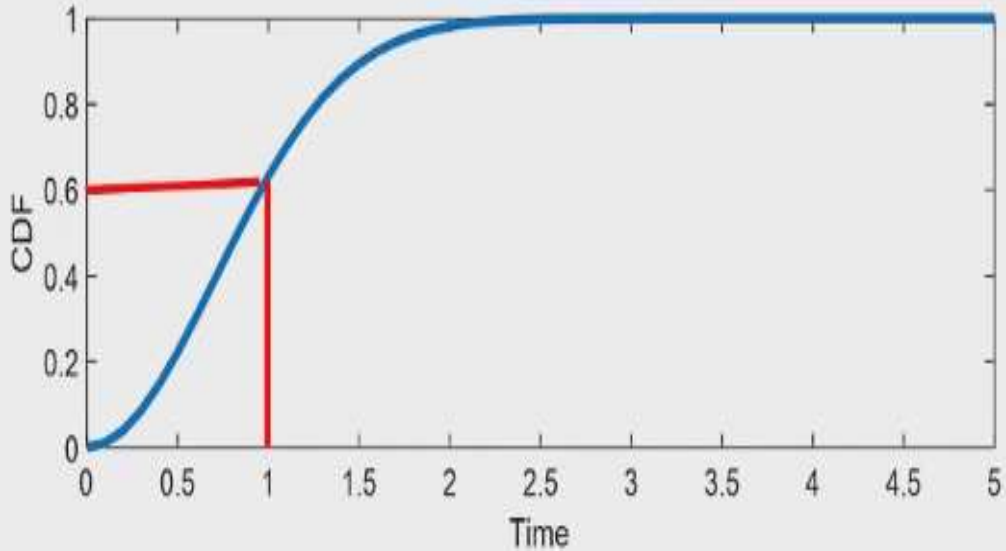
Cumulative distribution function (CDF) provides the probability of failure up to time a

$$F = P\{x \leq a\} = 1 - exp\left[-\left(\frac{a}{\alpha}\right)^{\gamma}\right]$$

The reliability at time t, will be:

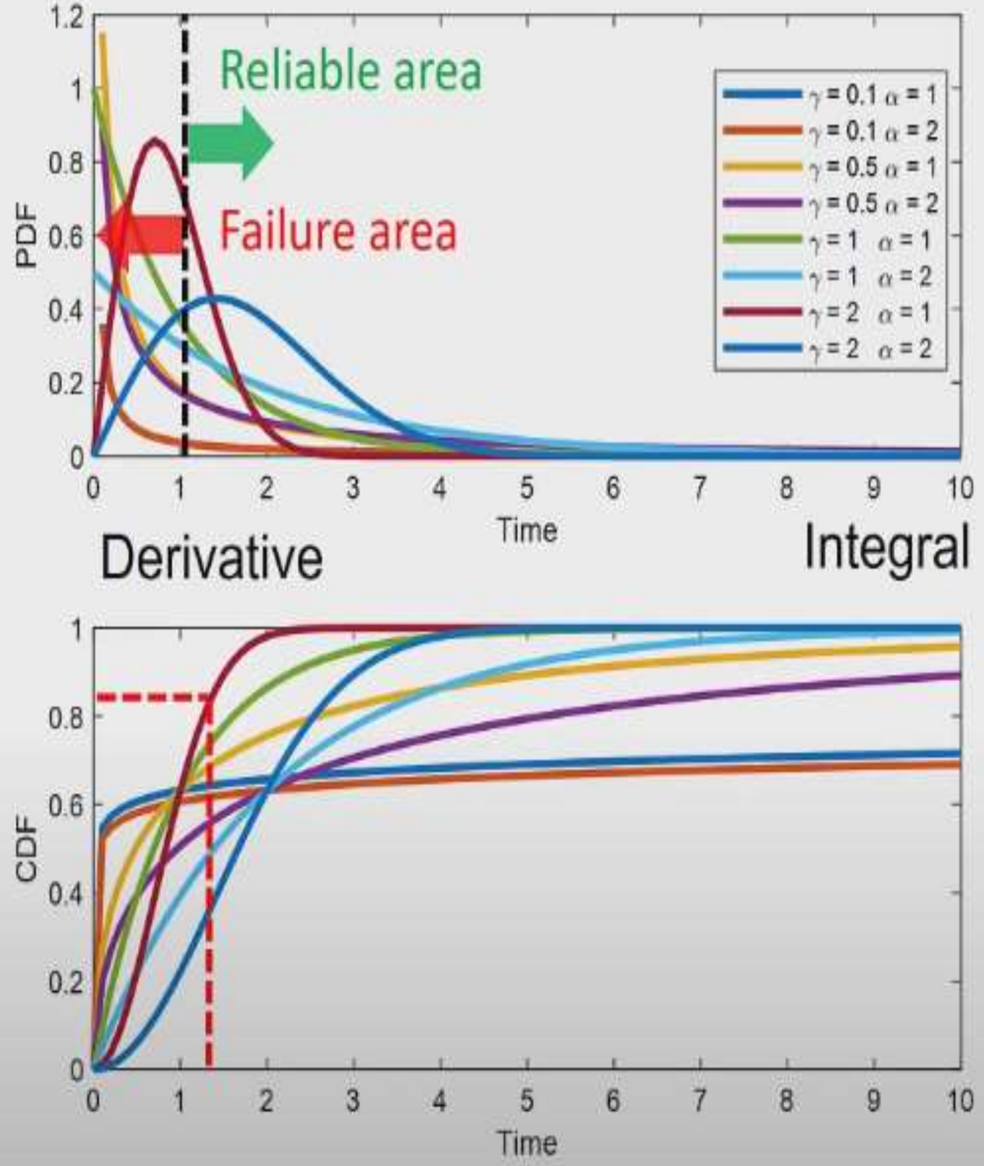
$$R = P\{x \geq a\} = 1 - F$$
$$= exp\left[-\left(\frac{a}{\alpha}\right)^{\gamma}\right]$$





The Weibull Distribution Applications

- Used extensively in **reliability engineering** as a mathematical model of **time-to-failure** for electrical and mechanical components and systems.
- Used in electrical devices such as **memory elements** or mechanical components such as **bearings**, and **structural elements in aircraft and automobiles**.
- Used in warranty analysis and utility services.
- Used in analysis of lifetime of **dental and medical implants**.
- Because of its **flexibility** can simulate distributions like normal and exponential distributions.



MATLAB IMPLEMENTATION

Example 1: Find:

P(x>20000)

P(x<10000)

P(10000<x<20000)

Expected time to failure

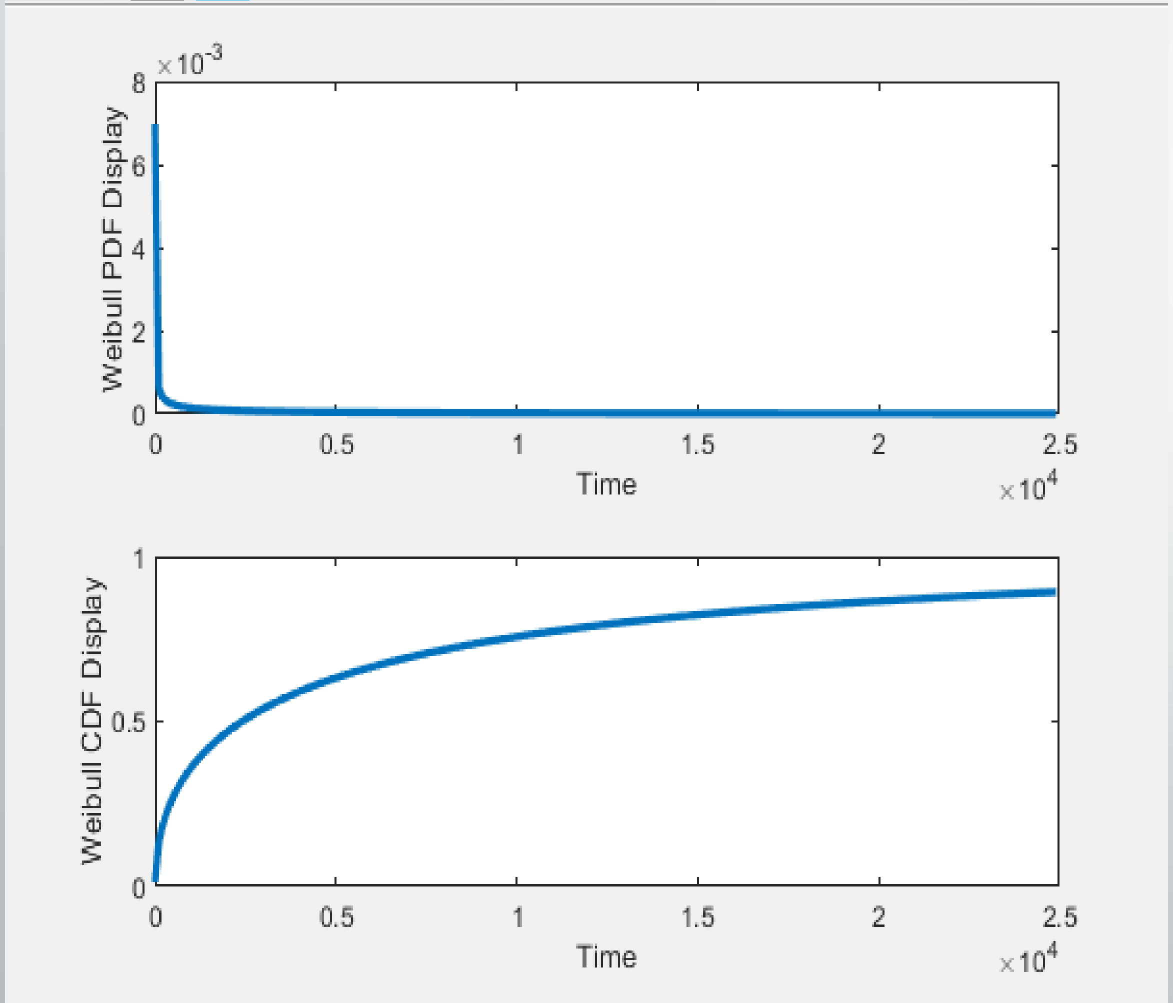
Given gamma=0.5 and alpha=5000

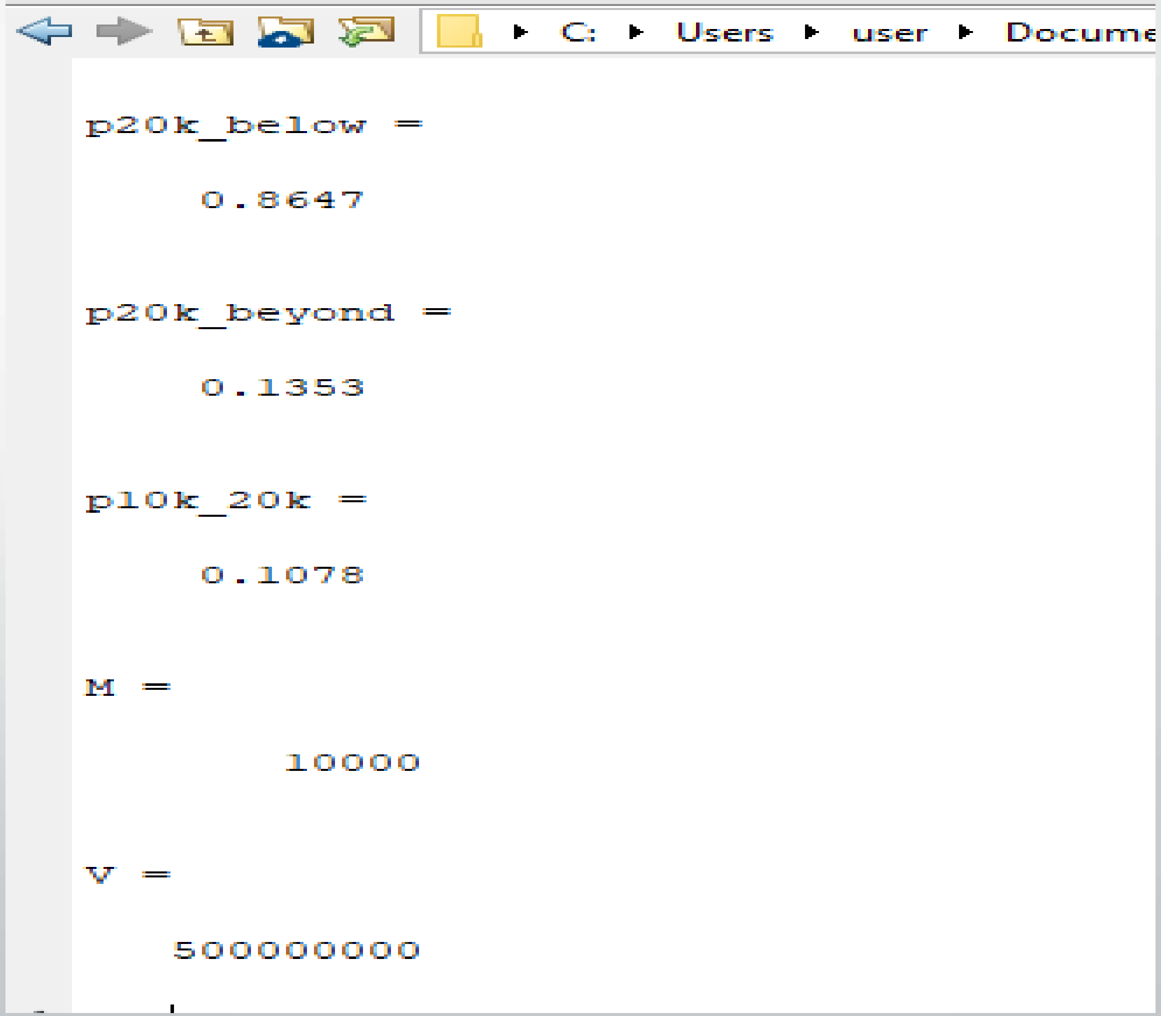
Solution:

```
%% Example Flat display
clc; clear;
alpha= 5000; gamma= 0.5;
p20k_below= wblcdf(20000, alpha, gamma)
p20k_beyond = 1- p20k_below
p10k_20k = wblcdf (20000, alpha, gamma) - wblcdf (10000, alpha, gamma)
[M, V]= wblstat (alpha, gamma)
time= 1:100:25000;
subplot (211); fx= wblpdf (time, alpha, gamma);
plot (time, fx, 'linewidth', 2.5); xlabel('Time'); ylabel('Weibull PDF Display');
subplot (212); ft= wblcdf (time, alpha, gamma);
plot (time, ft, 'linewidth', 2.5); xlabel('Time'); ylabel('Weibull CDF Display');
```

MATLAB IMPLEMENTATION

- **output:** The graphs of CDF and PDF are obtained
- we see that 10000 hour is mean failure time .





MATLAB IMPLEMENTATION

Example 2

Given 20000 tyres what percentage of tyres fail before running 6000km.

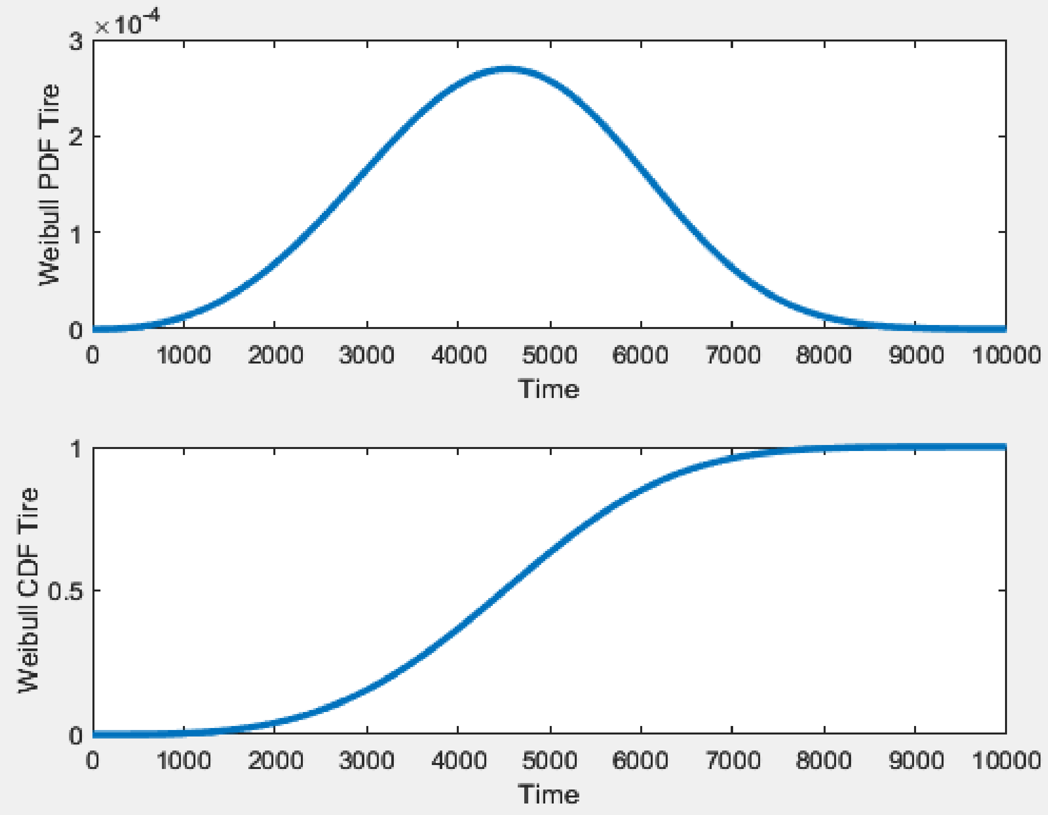
Alpha=5000 gamma=3.5

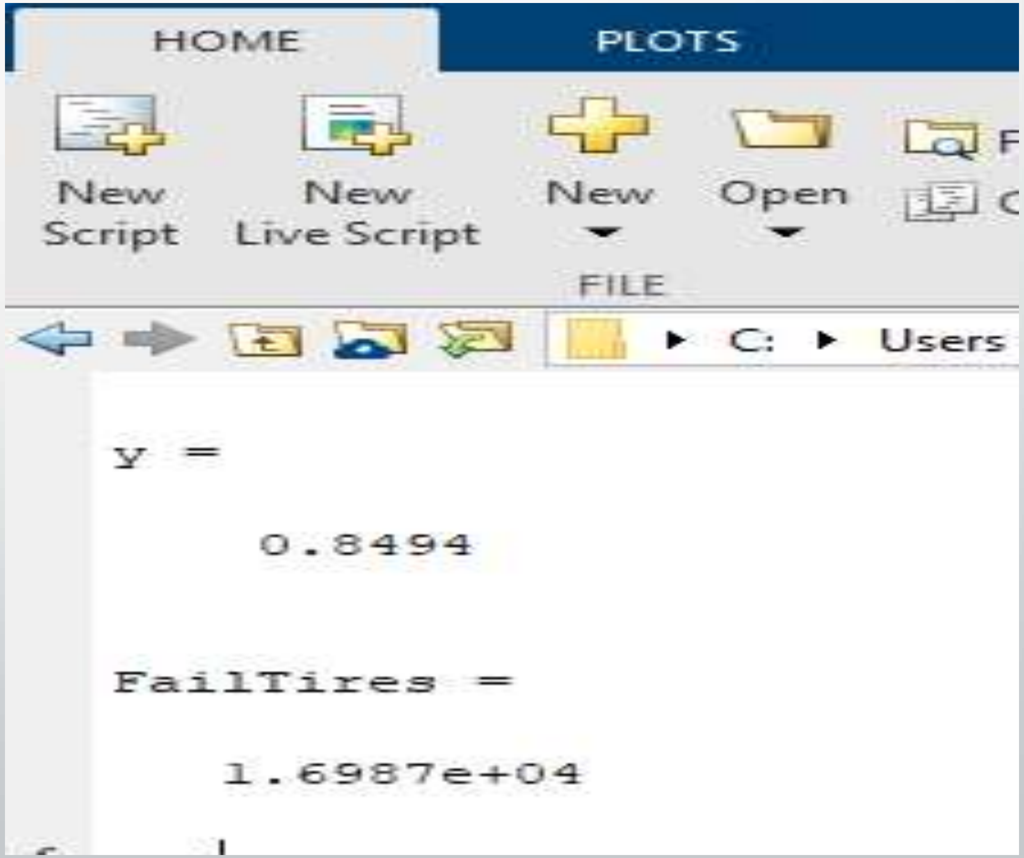
Solution:

```
%% Example Tire Weibull Distribution
clc; clear;
alpha = 5000; gamma = 3.5;
TotalTires= 20000;
y= wblcdf (6000, alpha, gamma)
FailTires= y*TotalTires
time =1:10:10000;
subplot (211); fx = wblpdf (time, alpha, gamma);
plot (time, fx, 'linewidth', 2.5); xlabel('Time'); ylabel('Weibull PDF Tire');
subplot (212); ft = wblcdf (time, alpha, gamma);
plot (time, ft, 'linewidth', 2.5); xlabel('Time'); ylabel('Weibull CDF Tire');
```

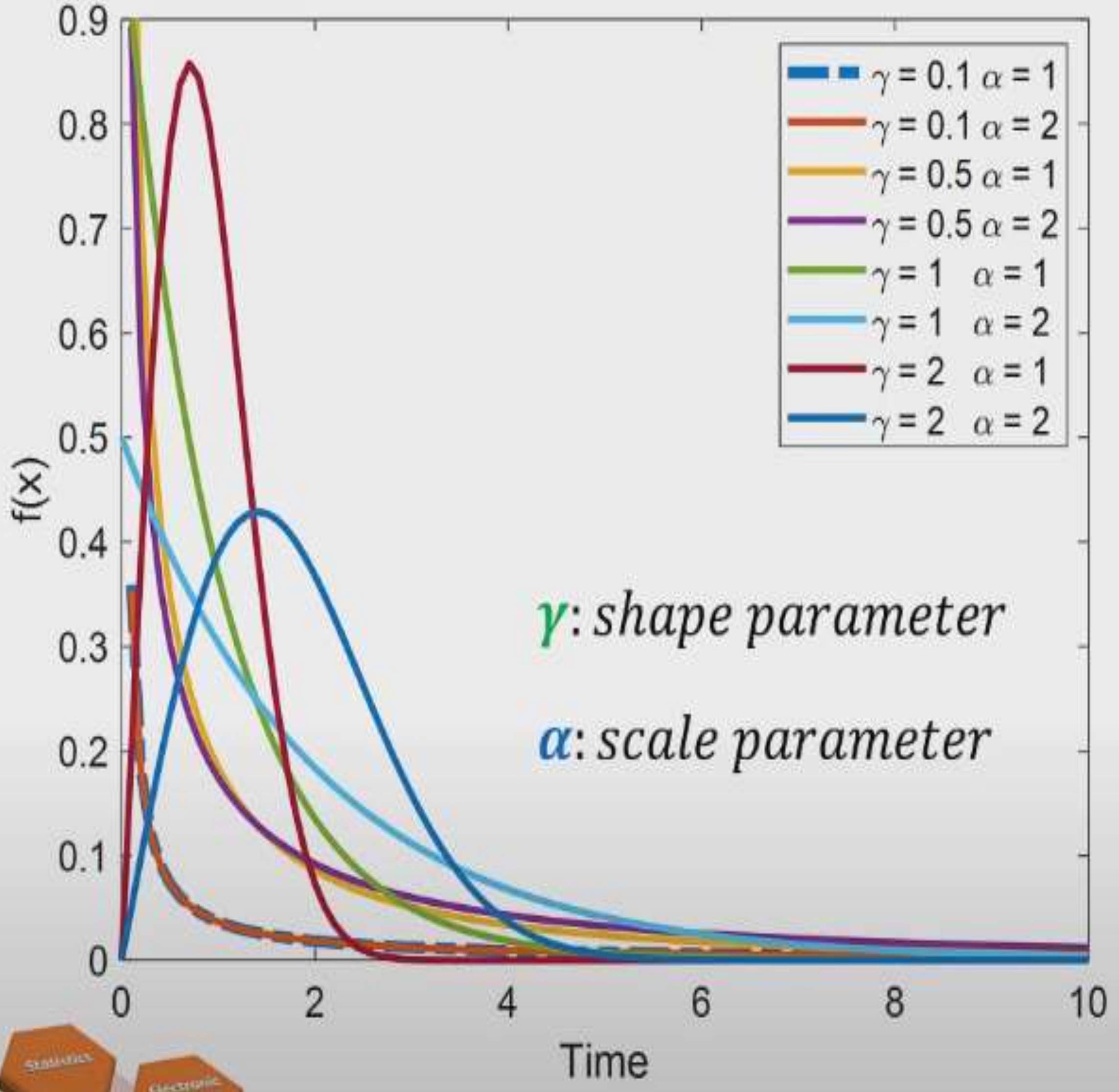
MATLAB IMPLEMENTATION

- **output:**
- we see that 85% of the tyres fail before 6000km
- which is 16987 of 20000 tyres.





The Weibull Distribution Failure Rate



$if\ \gamma = 1$ then the **failure rate is constant**

$if\ \gamma > 1$ then **failure rate increases with time**

$if\ \gamma < 1$ then the **failure rate decreases with time**

$$f(x) = \frac{\gamma}{\alpha}\left(\frac{x}{\alpha}\right)^{\gamma-1} exp\left[-\left(\frac{x}{\alpha}\right)^{\gamma}\right]$$

$$x \geq 0;\ \gamma, \alpha > 0$$

FEW MORE APPLICATIONS:

- In survival analysis
- In reliability engineering and failure analysis
- In electrical engineering to represent overvoltage occurring in an electrical system
- In industrial engineering to represent manufacturing and delivery times
- In extreme value theory
- In weather forecasting and the wind power industry to describe wind speed distributions, as the natural distribution often matches the Weibull shape
  In communications systems engineering
- In radar systems to model the dispersion of the received signals level produced by some types of clutters
- To model fading channels in wireless communications, as the Weibull fading model seems to exhibit good fit to experimental fading channel measurements

REFERENCE:

https://en.wikipedia.org/wiki/Weibull_distribution

THANK YOU