🔍 Where It Comes From (The Math)

This comes from the **Taylor Series expansion** used in numerical methods:

1. Expand T_{i+1} and T_{i-1} about T_i:

$$T_{i+1} = T_i + \Delta t \cdot \frac{dT}{dt} + \frac{(\Delta t)^2}{2} \cdot \frac{d^2T}{dt^2} + \dots$$

$$T_{i-1} = T_i - \Delta t \cdot \frac{dT}{dt} + \frac{(\Delta t)^2}{2} \cdot \frac{d^2T}{dt^2} + \dots$$

2. Add both expansions:

$$T_{i+1} + T_{i-1} = 2T_i + (\Delta t)^2 \cdot \frac{d^2T}{dt^2} + \dots$$

3. Rearranged:

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta t)^2} \approx \frac{d^2T}{dt^2}$$

✅ This is the basis for your **second derivative approximation.**

🧠 What It Tells You

Δ^2T Value	Meaning
≈ 0	Smooth change
Positive & large	**Spike or rapid rise**
Negative & large	**Drop or power-down**
Alternating +/–	**Jittery/unstable** signal or sensor fault

🎯 What Are ΔUpper and ΔLower?

From your rolling window:

- $\bar{T}(t)$ = Rolling mean
- $T_{\text{max}}, T_{\text{min}}$ = Max and min in the same window

We define:

$$\Delta_{\text{Upper}} = T_{\text{max}} - \bar{T}$$

$$\Delta_{\text{Lower}} = \bar{T} - T_{\text{min}}$$

They measure **how far temperatures deviate above** and **below** the average.

🧠 The Goal

We want the system to **automatically decide** what's "too much" fluctuation above or below average — based on **recent system behavior**.

🧮 Adaptive Thresholds for ΔUpper and ΔLower

You maintain a history of `N` windows (say 6 hours, so 6 entries), and calculate:

python Copy Edit

```python
adaptive_delta_upper_thresh = mean(delta_upper) + 2 * std(delta_upper)
adaptive_delta_lower_thresh = mean(delta_lower) + 2 * std(delta_lower)
```

✅ That means:

> If the current ΔUpper or ΔLower exceeds this threshold → system is **no longer stable**

adaptive_delta_upper_thresh = mean(delta_upper) + 2 * std(delta_upper)
adaptive_delta_lower_thresh = mean(delta_lower) + 2 * std(delta_lower)

🔁 How to Use This in Your Algorithm

✅ Step-by-step Decision:

python

```python
if T is NaN or 0:
    SystemState = "Invalid"
elif ΔUpper > adaptive_delta_upper_thresh:
    SystemState = "Upper Spike (Overheat Risk)"
elif ΔLower > adaptive_delta_lower_thresh:
    SystemState = "Lower Spike (Sensor Dip or Shutdown)"
elif ΔUpper in [1.5, 3] or ΔLower in [1.5, 3]:
    SystemState = "Warning Zone"
else:
    SystemState = "Normal"
```

This becomes your **core alarm logic** — clean, adaptive, and reactive only when needed.


```
if T is NaN or 0:
    SystemState = "Invalid"
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else:
    SystemState = "Normal"
```

✅ Here is the updated flowchart with only the **Adaptive Thresholds** box added — exactly as you requested:

📥 Download Flowchart with Adaptive Threshold Added (PNG)

🧠 Full Explanation of Each Step and Value:

🔹 1. Rolling Mean / Max / Min

- Takes last 12 readings (1 hour) and calculates:
  - Mean = average temp = base state (like Reynolds base flow)
  - Max & Min = detect fluctuation limits
- **ΔUpper = Max − Mean**
- **ΔLower = Mean − Min**

🟢 **Why?**
This tells us how much the temperature is **spiking above or dipping below** the norm.

🔹 2. Adaptive Thresholds

- Calculated as:
  - ΔUpper_Thresh = mean(ΔUpper) + 2 × std_dev(ΔUpper)
  - ΔLower_Thresh = mean(ΔLower) + 2 × std_dev(ΔLower)

🟢 **Why?**
This adapts the system to **normal variations**, so it doesn't trigger false alarms unless there's a real deviation.

🔹 3. ΔT (1st Derivative)

- Formula: `ΔT = T[i] - T[i-1]`

- Tells how fast the temperature is rising or falling.

🟢 **Why?**
Detects **gradual heating trends** — could indicate impending danger.

🔹 **4. $\Delta^2 T$ (2nd Derivative)**

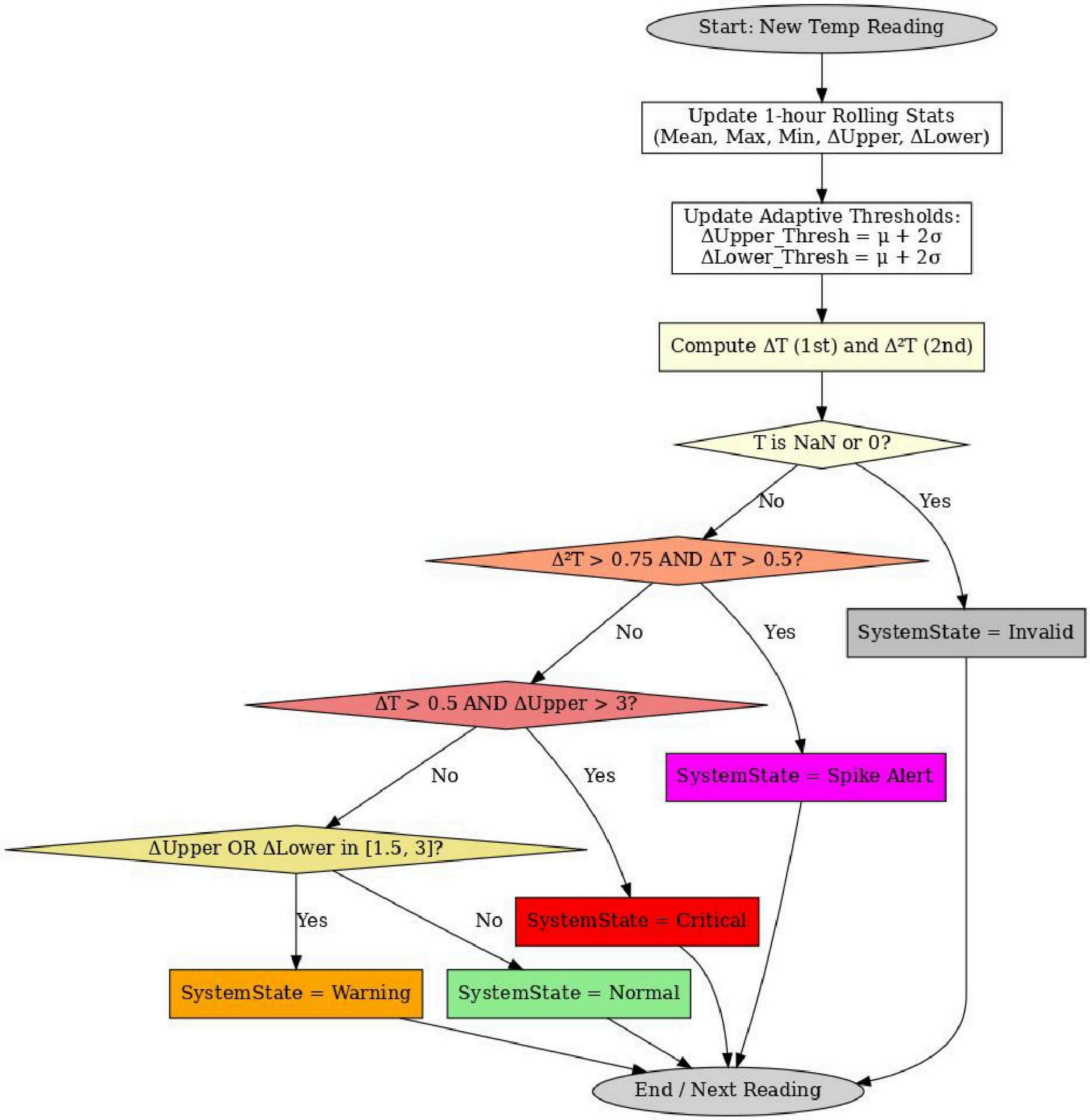
- Formula: `Δ²T = T[i+1] - 2×T[i] + T[i-1]`
- Tells how suddenly the rate of temperature is changing — i.e., spikes.

🟢 **Why?**
Perfect for catching **sudden faults, noise, or system glitches.**

🔸 **5. Conditions Checked**

Condition	Meaning
NaN or 0	Sensor error or disconnected
$\Delta^2 T > 0.75$ and $\Delta T > 0.5$	Very sudden and large spike = "Spike Alert"
$\Delta T > 0.5$ and ΔUpper > 3	Fast heating and climbing beyond normal → "Critical"
ΔUpper or ΔLower in [1.5, 3]	Slight instability → "Warning"
Else	Everything normal → "Normal"

Would you also like this explanation as a **1-page PDF summary** with the image?


Start: New Temp Reading
Update 1-hour Rolling Stats
(Mean, Max, Min, ΔUpper, ΔLower)
Update Adaptive Thresholds:
ΔUpper_Thresh = μ + 2σ
ΔLower_Thresh = μ + 2σ
Compute ΔT (1st) and Δ²T (2nd)
T is NaN or 0?
No
Yes
Δ²T > 0.75 AND ΔT > 0.5?
No
Yes
SystemState = Invalid
ΔT > 0.5 AND ΔUpper > 3?
No
Yes
SystemState = Spike Alert
ΔUpper OR ΔLower in [1.5, 3]?
Yes
No
SystemState = Critical
SystemState = Warning
SystemState = Normal
End / Next Reading