IRIS '09

Math-o-Logica Guidelines!!!

Hey folks,

Welcome to the world of numbers. And if you have that extra bit of logic in you, Math-O-Logica would be something you would love to have a look at. Follow the guidelines and rattle your brains through the event to be the best……Or you may very well back out and rest!!!!.........

1. It is an individual event.

2. The maximum marks for every problem is written in bold within brackets.

3. The marks you score for each problem would be based on the approach followed. Keep them simple,concise, elegant and explanatory!!! Comprehensive and conclusive solution would fetch maximum marks for a problem.

4. The winner would be the one with maximum sum total.

5. In case of a tie, the bold questions would carry extra weightage.

6. Further ties would be resolved on the basis of time of submission.

7. E-mail your solutions in the given formats OR scanned copy of solutions to mathologica.iris09@gmail.com by the time 11:00 a.m. on 19th September, 2009. You may submit your handwritten solutions to room no. 225, H-8 before the above-mentioned time. Participants can use the provided contact information in case of any queries.

8. The mailed solutions should be in .doc, .jpeg or .pdf format.

9. Only one solution set will be accepted from an individual.

10. Clearly mention your Name, College name and Contact details with the solution set.

See you at IRIS'09………

Cheers,

Team Technobyte

IRIS 09

Math-o-Logica

... A Pragmatic Riddle on the Pinnacle

SECTION-1(OBJECTIVE)

Rules for the questions 1-5:

(i) Only one answer is correct.
(ii) Any wrong answer will attract penalty of 1/3 of the total credit for that question.
(iii) You have to give a logical, one paragraph discussion (not the complete solution) about the approach followed for every question of this section.

1. Assuming n to be an integer, such that the fraction m/n can be expanded as a periodic decimal fraction. For example- in 1/7 = 0.1428571428..., then the digits appearing in the period are 142857, hence number of these digits is 6 and is called the period of the decimal.

   Now suppose the number of digits appearing in the period of the decimal is odd, then the arithmetic mean of the digits in the period:

   (A) can be equal to 9/2, if number of digits appearing in period is larger than 7.
   (B) can be equal to 9/2, if number of digits appearing in period is smaller than 7.
   (C) will be equal to 9/2, if number of digits appearing in the period is 11.
   (D) will be equal to 9/2, if number of digits appearing in period is equal than 9.
   (E) can never be equal to 9/2. [4]

2. Let $p_1, p_2, \ldots\ldots, p_n$ be arbitrary permutations of integers 1, 2, ...,n .
   And f(n) be the number of these permutations such that

   (i) $p_i = 1$ for $i = 1$;
   (ii) $|p_i - p_{i+1}| <= 2$, for all i ranging from 1 to n-1.

   Which is the possible value of f(n) at n = 1999. [4]

   (A) 5 (B) 3 (C) 2 (D) 4 (E) 1

3. How many 3-digit numbers are possible which are equal to 11 times the sum of the squares of their digits.

   (A) none (B) two (C) three (D) one (E) four [3]

4. How many six digit numbers, starting with 52, are divisible by 7, 8 and 9. [3]

   (A) 19 (B) 21 (C) 20 (D) 23 (E) 22

5. Let x be the smallest possible number that leaves remainders as 1, 2, 3, 4, 5 when divided by 2, 3, 4, 5, 6 respectively.

   What will be the remainder if x is divided by 7 ?

   (A) 2 (B) 4 (C) 1 (D) 6
   (E) none of the above [3]

SECTION-2(DESCRIPTIVE)

Rules for the questions 6 - 10:

i. You have to give a complete, but concise and self-explanatory solution for each problem in this section.
ii. Comprehensive and conclusive approach would fetch maximum marks for a problem.

6. Define the sequence $s_0, s_1, s_2, \ldots\ldots$, by
   $s_0 = 0$,
   $s_1 = y$,
   $s_{n+1} = y^2 s_n - s_{n-1}$ for n = 1, 2, 3, ... where y is a positive integer.

   Prove that an ordered pair (r, s) with r <= s, gives a solution to the equation
   $Y = [(r^2 + s^2) / (rs + 1)]^{1/2}$, where r and s are non negative.

   if and only if (r, s) is of the form (s_k, s_{k+1}), where k>=0. [5]

7. If p denotes any real parameter, then determine the real solutions of the following equation:

   $(x^2 - p)^{1/2} + 2(x^2 - 1)^{1/2} = x$

   Also determine the real values of p, for which we obtain these real solutions. [3]

8. On Jonathan's marriage; each of his friends wants to give a gift to the wedding couple, packed in a separate square packet. (Note: The square packets may not be of the same sizes.)
   Let the number of friends be N and the total area of these N square packets be A.

   Now prove that these N square shaped gift packets can be packed (without overlapping) into a rectangular pot of total area 2A provided that the largest of the square packet can fit into the rectangular pot .

   (Without overlapping means the square packets should not press against each other or not be inside one another). [5]

9. Let T(p,q) be the number of different rook walks on a chessboard having p rows and q columns, beginning at the lower-left corner and ending at the upper-left corner. For example, T (p, 1) = 1 ; T (2, 2) = 2 ; T (3, 2) = 4 ; T (3, 3) = 11.

Then find a formula for T (3, q).
(A rook walk on a chess board is the path traced by a sequence of rook moves parallel to an edge of the board from one unit square to another, such that each begins where the previous move ended and such that no move ever crosses a square that has been crossed previously.)

Take a chessboard as a rectangular grid of squares and assume p and q to be natural numbers. [4]

10. King Solomon designed a prison for deadly criminals having its base in square shape. Assuming the base to be in X-Y reference frame, two assumptions can be taken:
(1)The centre of square base acts as the origin (0, 0).
(2)The X and Y axes divide the square base in four equal sized squares.

For security purpose, he set the rotating pillars with sharp swords attached to them on the line connecting lower left to upper right corner of the square base. (So that anyone tries to cross that line will get cut into pieces.)

Let two points A (p, q) and B (m, n) are both, below the line having rotating pillars, and have integer co-ordinates. A criminal wants to move from point A to B assuming that he can move only in steps of 1, either upwards or to the right, but not on the line having rotating pillars (He doesn't wants to die anyways!).

How many possible ways are there for him to travel from A to B? [4]