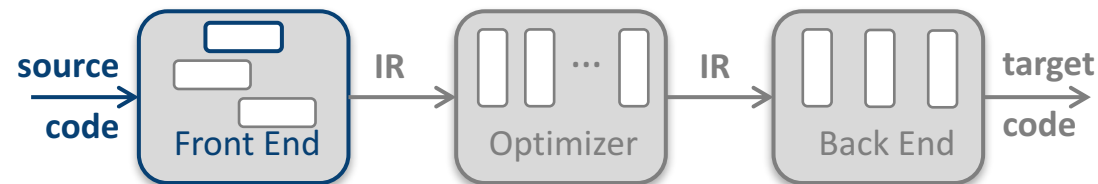




COMP 412  
FALL 2017

## Syntax Analysis, IV

### Comp 412



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# Review

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## Last Class

- Introduced **FIRST**, **FOLLOW**, and **FIRST<sup>+</sup>** sets
- Introduced the **LL(1)** condition

*A grammar  $G$  can be parsed predictively with one symbol of lookahead if for all pairs of productions  $A \rightarrow \beta$  and  $A \rightarrow \gamma$  that have the same lhs  $A$ :*

$$\text{FIRST}^+(A \rightarrow \beta) \cap \text{FIRST}^+(A \rightarrow \gamma) = \emptyset$$

- Observed that predictively parsable, or **LL(1)** grammars
- Showed how to construct a recursive-descent parser for an **LL(1)** grammar

## We did not cover

- An algorithm to construct **FIRST** sets
- An algorithm to construct **FOLLOW** sets

# FIRST and FOLLOW Sets



## FIRST( $\alpha$ )

For some  $\alpha \in (T \cup NT \cup \mathbf{EOF} \cup \varepsilon)^*$ , define **FIRST**( $\alpha$ ) as the set of tokens that appear as the first symbol in some string that derives from  $\alpha$

That is,  $\underline{x} \in \mathbf{FIRST}(\alpha)$  iff  $\alpha \Rightarrow^* \underline{x} \gamma$ , for some  $\gamma$

**FIRST** is defined over strings of grammar symbols:  $(T \cup NT \cup \mathbf{EOF} \cup \varepsilon)^*$

## FOLLOW( $A$ )

For some  $A \in NT$ , define **FOLLOW**( $A$ ) as the set of symbols that can occur immediately after  $A$  in a valid sentential form

**FOLLOW**( $S$ ) = {**EOF**}, where  $S$  is the start symbol

**FOLLOW** is defined over the set of nonterminal symbols, **NT**

To build **FOLLOW** sets, we need **FIRST** sets ...

**EOF**  $\cong$  end of file

# FIRST and FOLLOW Sets



## FIRST( $\alpha$ )

For some  $\alpha \in (T \cup NT \cup \mathbf{EOF} \cup \varepsilon)^*$ , define **FIRST**( $\alpha$ ) as the set of tokens that appear as the first symbol in some string that derives from  $\alpha$

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**FIRST** is defined over strings of grammar symbols:  $(T \cup NT \cup \mathbf{EOF} \cup \varepsilon)^*$

## FOLLOW( $A$ )

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**FOLLOW**( $S$ ) = {**EOF**}, where  $S$  is the start symbol

**FOLLOW** is defined over the set of nonterminal symbols, **NT**

To build **FOLLOW** sets, we need **FIRST** sets ...

**EOF**  $\cong$  end of file

# Conceptual Sketch: Computing **FIRST** Sets



```
for each  $x \in (T \cup \text{EOF} \cup \varepsilon)$ 
     $\text{FIRST}(x) \leftarrow \{x\}$ 
for each  $A \in \text{NT}$ ,  $\text{FIRST}(A) \leftarrow \emptyset$ 
while (FIRST sets are still changing) do
    for each  $p \in P$ , of the form  $A \rightarrow \beta$  do
         $\text{rhs} \leftarrow \text{FIRST}(B_1) - \{\varepsilon\}$ 
        Some details go here to handle  $\varepsilon$  productions
         $\text{FIRST}(A) \leftarrow \text{FIRST}(A) \cup \text{rhs}$ 
    end // for loop
end // while loop
```

## To begin, we will ignore $\varepsilon$ productions

- Initially, set **FIRST** for each nonterminal, terminal **EOF**, and  $\varepsilon$
- Then, loop through the productions and set **FIRST** for the *lhs* nonterminal to **FIRST** of the leading symbol on the *rhs*
- Need to iterate because *rhs* can start with a nonterminal

# Filling in the Details: Computing FIRST Sets



```
for each  $x \in (T \cup \text{EOF} \cup \varepsilon)$ 
     $\text{FIRST}(x) \leftarrow \{x\}$ 
for each  $A \in NT$ ,  $\text{FIRST}(A) \leftarrow \emptyset$ 
while (FIRST sets are still changing) do
    for each  $p \in P$ , of the form  $A \rightarrow B_1 B_2 \dots B_k$  do
         $\text{rhs} \leftarrow \text{FIRST}(B_1) - \{\varepsilon\}$ 
        for  $i \leftarrow 1$  to  $k-1$  by 1 while  $\varepsilon \in \text{FIRST}(B_i)$  do
             $\text{rhs} \leftarrow \text{rhs} \cup (\text{FIRST}(B_{i+1}) - \{\varepsilon\})$ 
        end // for loop
        if  $i = k$  and  $\varepsilon \in \text{FIRST}(B_k)$ 
            then  $\text{rhs} \leftarrow \text{rhs} \cup \{\varepsilon\}$ 
         $\text{FIRST}(A) \leftarrow \text{FIRST}(A) \cup \text{rhs}$ 
    end // for loop
end // while loop
```

$\varepsilon$  complicates matters

If  $\text{FIRST}(B_1)$  contains  $\varepsilon$ , then we need to add  $\text{FIRST}(B_2)$  to  $\text{rhs}$ , and ...

If the entire  $\text{rhs}$  can go to  $\varepsilon$ , then we add  $\varepsilon$  to  $\text{FIRST}(lhs)$

# Computing FIRST Sets



```
for each  $x \in (T \cup \text{EOF} \cup \varepsilon)$ 
     $\text{FIRST}(x) \leftarrow \{x\}$ 
for each  $A \in \text{NT}$ ,  $\text{FIRST}(A) \leftarrow \emptyset$ 
while (FIRST sets are still changing) do
    for each  $p \in P$ , of the form  $A \rightarrow B_1 B_2 \dots B_k$  do
         $\text{rhs} \leftarrow \text{FIRST}(B_1) - \{\varepsilon\}$ 
        for  $i \leftarrow 1$  to  $k-1$  by 1 while  $\varepsilon \in \text{FIRST}(B_i)$  do
             $\text{rhs} \leftarrow \text{rhs} \cup (\text{FIRST}(B_{i+1}) - \{\varepsilon\})$ 
        end // for loop
        if  $i = k$  and  $\varepsilon \in \text{FIRST}(B_k)$ 
            then  $\text{rhs} \leftarrow \text{rhs} \cup \{\varepsilon\}$ 
         $\text{FIRST}(A) \leftarrow \text{FIRST}(A) \cup \text{rhs}$ 
    end // for loop
end // while loop
```

Outer loop is **monotone increasing** for FIRST sets

$\Rightarrow |T \cup \text{NT} \cup \text{EOF} \cup \varepsilon|$  is bounded, so it terminates

Inner loop is bounded by the length of the productions in the grammar

# Example



Consider the *SheepNoise* grammar & its FIRST sets

```
0 Goal          → SheepNoise
1 SheepNoise    → SheepNoise baa
2               | baa
```

*Left-recursive SheepNoise Grammar*

Clearly and intuitively,  $\text{FIRST}(x) = \{\underline{\text{baa}}\}$ ,  $\forall x \in (T \cup NT)$

Symbol	FIRST Set
<i>Goal</i>	{ <u>baa</u> }
<i>SheepNoise</i>	{ <u>baa</u> }
<u>baa</u>	{ <u>baa</u> }



# Computing FIRST Sets



```
for each  $x \in (T \cup \text{EOF} \cup \varepsilon)$ 
     $\text{FIRST}(x) \leftarrow \{x\}$ 
for each  $A \in NT$ ,  $\text{FIRST}(A) \leftarrow \emptyset$ 
while (FIRST sets are still changing) do
    for each  $p \in P$ , of the form  $A \rightarrow \beta$  do
         $\text{rhs} \leftarrow \text{FIRST}(B_1) - \{\varepsilon\}$ 
        if  $\beta$  is  $B_1 B_2 \dots B_k$  then begin;
            for  $i \leftarrow 1$  to  $k-1$  by 1 while  $\varepsilon \in \text{FIRST}(B_i)$  do
                 $\text{rhs} \leftarrow \text{rhs} \cup (\text{FIRST}(B_{i+1}) - \{\varepsilon\})$ 
            end // for loop
        end // if-then
        if  $i = k$  and  $\varepsilon \in \text{FIRST}(B_k)$ 
            then  $\text{rhs} \leftarrow \text{rhs} \cup \{\varepsilon\}$ 
             $\text{FIRST}(A) \leftarrow \text{FIRST}(A) \cup \text{rhs}$ 
        end // for loop
    end // while loop
```

See also, Fig. 3.7, EaC2e, p. 104

Initialization assigns each **FIRST** set a value

Symbol	FIRST Set
Goal	$\emptyset$
SheepNoise	$\emptyset$
<u>baa</u>	{ <u>baa</u> }

# Computing FIRST Sets



```

for each  $x \in (T \cup \text{EOF} \cup \varepsilon)$ 
     $\text{FIRST}(x) \leftarrow \{x\}$ 
for each  $A \in NT$ ,  $\text{FIRST}(A) \leftarrow \emptyset$ 
while (FIRST sets are still changing) do
    for each  $p \in P$ , of the form  $A \rightarrow \beta$  do
         $\text{rhs} \leftarrow \text{FIRST}(B_1) - \{\varepsilon\}$ 
        if  $\beta$  is  $B_1 B_2 \dots B_k$  then begin;
            for  $i \leftarrow 1$  to  $k-1$  by 1 while  $\varepsilon \in \text{FIRST}(B_i)$  do
                 $\text{rhs} \leftarrow \text{rhs} \cup (\text{FIRST}(B_{i+1}) - \{\varepsilon\})$ 
            end // for loop
        end // if-then
        if  $i = k$  and  $\varepsilon \in \text{FIRST}(B_k)$ 
            then  $\text{rhs} \leftarrow \text{rhs} \cup \{\varepsilon\}$ 
         $\text{FIRST}(A) \leftarrow \text{FIRST}(A) \cup \text{rhs}$ 
    end // for loop
end // while loop
    
```

See also, Fig. 3.7, EaC2e, p. 104

## Production 2

- (1) sets *rhs* to **FIRST**{ baa } &
- (2) copies *rhs* into **FIRST**(SheepNoise)

Symbol	FIRST Set
Goal	$\emptyset$
SheepNoise	{ <u>baa</u> }
<u>baa</u>	{ <u>baa</u> }

# Computing FIRST Sets



```

for each  $x \in (T \cup \text{EOF} \cup \varepsilon)$ 
     $\text{FIRST}(x) \leftarrow \{x\}$ 
for each  $A \in NT$ ,  $\text{FIRST}(A) \leftarrow \emptyset$ 
while (FIRST sets are still changing) do
    for each  $p \in P$ , of the form  $A \rightarrow \beta$  do
         $\text{rhs} \leftarrow \text{FIRST}(B_1) - \{\varepsilon\}$ 
        if  $\beta$  is  $B_1 B_2 \dots B_k$  then begin;
            for  $i \leftarrow 1$  to  $k-1$  by 1 while  $\varepsilon \in \text{FIRST}(B_i)$  do
                 $\text{rhs} \leftarrow \text{rhs} \cup (\text{FIRST}(B_{i+1}) - \{\varepsilon\})$ 
            end // for loop
        end // if-then
        if  $i = k$  and  $\varepsilon \in \text{FIRST}(B_k)$ 
            then  $\text{rhs} \leftarrow \text{rhs} \cup \{\varepsilon\}$ 
         $\text{FIRST}(A) \leftarrow \text{FIRST}(A) \cup \text{rhs}$ 
    end // for loop
end // while loop
    
```

See also, Fig. 3.7, EaC2e, p. 104

## Production 0

- (1) sets *rhs* to **FIRST**(*Sheepnoise*) &
  - (2) copies *rhs* into **FIRST**(*Goal*)
- ... and one more iteration to recognize that the **FIRST** sets have stopped changing

Symbol	FIRST Set
<i>Goal</i>	{ <u>b</u> aa }
<i>SheepNoise</i>	{ <u>b</u> aa }
<u>baa</u>	{ <u>b</u> aa }

# An Example



Consider the simple parentheses grammar

- 0 *Goal*  $\rightarrow$  *List*
- 1 *List*  $\rightarrow$  *Pair List*
- 2       |  $\epsilon$
- 3 *Pair*  $\rightarrow$  LP *List* RP

where LP is ( and RP is )

Symbol	FIRST Initial
<i>Goal</i>	$\emptyset$
<i>List</i>	$\emptyset$
<i>Pair</i>	$\emptyset$
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

# An Example



Consider the simple parentheses grammar

0 *Goal*  $\rightarrow$  *List*  
 1 *List*  $\rightarrow$  *Pair List*  
 2       |  $\epsilon$   
 3 *Pair*  $\rightarrow$  LP *List* RP

where LP is ( and RP is )

- Iteration 1 adds LP to **FIRST(Pair)** and LP,  $\epsilon$  to **FIRST(List)** & **FIRST(Goal)**  
 $\rightarrow$  If we take them in order 3, 2, 1, 0
- Algorithm reaches fixed point<sup>†</sup>

Symbol	FIRST Sets		
	Initial	1 <sup>st</sup>	2 <sup>nd</sup>
<i>Goal</i>	$\emptyset$	<u>LP</u> , $\epsilon$	<u>LP</u> , $\epsilon$
<i>List</i>	$\emptyset$	<u>LP</u> , $\epsilon$	<u>LP</u> , $\epsilon$
<i>Pair</i>	$\emptyset$	<u>LP</u>	<u>LP</u>
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

<sup>†</sup> In the adversarial order (0, 1, 2, 3), propagating {LP,  $\epsilon$ } through *Pair*, *List*, and *Goal* would require one iteration for each set.

# FIRST and FOLLOW Sets



## FIRST( $\alpha$ )

For some  $\alpha \in (T \cup NT \cup \text{EOF} \cup \varepsilon)^*$ , define **FIRST**( $\alpha$ ) as the set of tokens that appear as the first symbol in some string that derives from  $\alpha$

That is,  $\underline{x} \in \text{FIRST}(\alpha)$  iff  $\alpha \Rightarrow^* \underline{x} \gamma$ , for some  $\gamma$

**FIRST** is defined over strings of grammar symbols:  $(T \cup NT \cup \text{EOF} \cup \varepsilon)^*$

## FOLLOW( $A$ )

For some  $A \in NT$ , define **FOLLOW**( $A$ ) as the set of symbols that can occur immediately after  $A$  in a valid sentential form

**FOLLOW**( $S$ ) = {**EOF**}, where  $S$  is the start symbol

**FOLLOW** is defined over the set of nonterminal symbols, **NT**

To build **FOLLOW** sets, we need **FIRST** sets ...

**EOF**  $\cong$  end of file

# Computing FOLLOW Sets



```
for each  $A \in NT$ 
     $FOLLOW(A) \leftarrow \emptyset$ 
 $FOLLOW(S) \leftarrow \{ EOF \}$ 
while ( $FOLLOW$  sets are still changing)
    for each  $p \in P$ , of the form  $A \rightarrow B_1 B_2 \dots B_k$ 
         $TRAILER \leftarrow FOLLOW(A)$ 
        for  $i \leftarrow k$  down to 1
            if  $B_i \in NT$  then                                     // domain check
                 $FOLLOW(B_i) \leftarrow FOLLOW(B_i) \cup TRAILER$ 
                if  $\epsilon \in FIRST(B_i)$                              // add right context
                    then  $TRAILER \leftarrow TRAILER \cup (FIRST(B_i) - \{\epsilon\})$ 
                    else  $TRAILER \leftarrow FIRST(B_i)$            // no  $\epsilon \Rightarrow$  truncate the right context
                else  $TRAILER \leftarrow \{B_i\}$                      //  $B_i \in T \Rightarrow$  only 1 symbol
```

Don't add  $\epsilon$

# Computing **FOLLOW** Sets



**This algorithm has a completely different feel than computing FIRST sets**

For a production  $A \rightarrow B_1 B_2 \dots B_k$  :

- It works its way backward through the production:  $B_k, B_{k-1}, \dots B_1$
- It builds the **FOLLOW** sets for the *rhs* symbols,  $B_1, B_2, \dots B_k$ , not  $A$
- In the absence of  $\varepsilon$ , **FOLLOW**( $B_i$ ) is just **FIRST**( $B_{i+1}$ )
  - *As always,  $\varepsilon$  makes the algorithm more complex*

To handle  $\varepsilon$ , the algorithm keeps track of the first word in the trailing right context as it works its way back through the *rhs*:  $B_k, B_{k-1}, \dots B_1$

- It uses **FOLLOW**( $A$ ) to initialize the *Trailer* for  $B_k$ 
  - That use is the only mention of **FOLLOW**( $A$ ) in the algorithm
- *Trailer* approximates the **FIRST**<sup>+</sup> set for the trailing left context



# An Example



Consider, again, the simple parentheses grammar

```
0  Goal  → List
1  List  → Pair List
2         | ε
3  Pair  → LP List RP
```

Symbol	FOLLOW Sets	
	Initial	1 <sup>st</sup>
Goal	EOF	EOF
List	∅	EOF, RP
Pair	∅	EOF, LP

## Initial Values:

- *Goal*, *List*, and *Pair* are set to ∅
- *Goal* is then set to { **EOF** }

# An Example



Consider, again, the simple parentheses grammar

0 *Goal*  $\rightarrow$  *List*  
1 *List*  $\rightarrow$  *Pair List*  
2       |  $\epsilon$   
3 *Pair*  $\rightarrow$  *LP List RP*

Symbol	FOLLOW Sets	
	Initial	1 <sup>st</sup>
<i>Goal</i>	EOF	EOF
<i>List</i>	$\emptyset$	EOF, RP
<i>Pair</i>	$\emptyset$	EOF, LP

## Iteration 1:

- Production 0 adds **EOF** to FOLLOW(*List*)
- Production 1 adds LP to FOLLOW(*Pair*)  
→ from **FIRST**(*List*)
- Production 2 does nothing
- Production 3 adds RP to FOLLOW(*List*)  
→ from **FIRST**(*RP*)

Symbol	FIRST
<i>Goal</i>	<u>L</u> P, $\epsilon$
<i>List</i>	<u>L</u> P, $\epsilon$
<i>Pair</i>	<u>L</u> P
LP	<u>L</u> P
RP	<u>R</u> P
EOF	EOF

# An Example



Consider, again, the simple parentheses grammar

0  $Goal \rightarrow List$   
1  $List \rightarrow Pair List$   
2  $\quad \quad \quad | \quad \epsilon$   
3  $Pair \rightarrow LP List RP$

Symbol	FOLLOW Sets		
	Initial	1 <sup>st</sup>	2 <sup>nd</sup>
Goal	EOF	EOF	EOF
List	$\emptyset$	EOF, RP	EOF, RP
Pair	$\emptyset$	EOF, LP	EOF, LP, RP

## Iteration 2:

- Production 0 adds nothing new
- Production 1 adds RP to FOLLOW(*Pair*)  
→ from FOLLOW(*List*),  $\epsilon \in \text{FIRST}(\textit{List})$
- Production 2 does nothing
- Production 3 adds nothing new

Iteration 3 produces the same result  $\Rightarrow$  reached a fixed point

# Classic Expression Grammar



0	<i>Goal</i>	$\rightarrow$	<i>Expr</i>
1	<i>Expr</i>	$\rightarrow$	<i>Term Expr'</i>
2	<i>Expr'</i>	$\rightarrow$	$+ \text{Term Expr'}$
3		$ $	$- \text{Term Expr'}$
4		$ $	$\epsilon$
5	<i>Term</i>	$\rightarrow$	<i>Factor Term'</i>
6	<i>Term'</i>	$\rightarrow$	$* \text{Factor Term'}$
7		$ $	$/ \text{Factor Term'}$
8		$ $	$\epsilon$
9	<i>Factor</i>	$\rightarrow$	$( \text{Expr} )$
10		$ $	<u>number</u>
11		$ $	<u>identifier</u>

$\text{FIRST}^+(A \rightarrow \beta)$  is identical to  $\text{FIRST}(\beta)$   
except for productions 4 and 8

$\text{FIRST}^+(\text{Expr}' \rightarrow \epsilon)$  is  $\{\epsilon, ), \text{eof}\}$

$\text{FIRST}^+(\text{Term}' \rightarrow \epsilon)$  is  $\{\epsilon, +, -, ), \text{eof}\}$

Symbol	FIRST	FOLLOW
<u>num</u>	<u>num</u>	$\emptyset$
<u>id</u>	<u>id</u>	$\emptyset$
+	+	$\emptyset$
-	-	$\emptyset$
*	*	$\emptyset$
/	/	$\emptyset$
(	(	$\emptyset$
)	)	$\emptyset$
<u>eof</u>	<u>eof</u>	$\emptyset$
$\epsilon$	$\epsilon$	$\emptyset$
<i>Goal</i>	<u>(,id,num</u>	eof
<i>Expr</i>	<u>(,id,num</u>	), eof
<i>Expr'</i>	+, -, $\epsilon$	), eof
<i>Term</i>	<u>(,id,num</u>	+, -, ), eof
<i>Term'</i>	*, / , $\epsilon$	+, -, ), eof
<i>Factor</i>	<u>(,id,num</u>	+, -, *, / , ), eof

\*

# Classic Expression Grammar



0	<i>Goal</i>	$\rightarrow$	<i>Expr</i>
1	<i>Expr</i>	$\rightarrow$	<i>Term Expr'</i>
2	<i>Expr'</i>	$\rightarrow$	<i>+ Term Expr'</i>
3		$ $	<i>- Term Expr'</i>
4		$ $	$\epsilon$
5	<i>Term</i>	$\rightarrow$	<i>Factor Term'</i>
6	<i>Term'</i>	$\rightarrow$	<i>* Factor Term'</i>
7		$ $	<i>/ Factor Term'</i>
8		$ $	$\epsilon$
9	<i>Factor</i>	$\rightarrow$	<i>( Expr )</i>
10		$ $	<u>number</u>
11		$ $	<u>identifier</u>

Prod'n	FIRST <sup>+</sup>
0	(, <u>id</u> , <u>num</u>
1	(, <u>id</u> , <u>num</u>
2	+
3	-
4	$\epsilon$ , ), eof
5	(, <u>id</u> , <u>num</u>
6	*
7	/
8	$\epsilon$ , +, -, ), eof
9	(
10	<u>number</u>
11	<u>identifier</u>

# Recursive Descent Parsing (Procedural)



## A couple of routines from the expression parser

### **Goal( )**

```
token ← next_token( );  
if (Expr( ) = true & token = EOF)  
    then next compilation step;  
else  
    report syntax error;  
    return false;
```

### **Expr( )**

```
if (Term( ) = false)  
    then return false;  
else return Eprime( );
```

looking for number, identifier, or (,  
found token instead, or failed to find  
**Expr** or ) after (

### **Factor( )**

```
if (token = number) then  
    token ← next_token( );  
    return true;  
else if (token = identifier) then  
    token ← next_token( );  
    return true;  
else if (token = lparen)  
    token ← next_token( );  
    if (Expr( ) = true & token = rparen) then  
        token ← next_token( );  
        return true;  
    // fall out of if statement  
    report syntax error;  
    return false;
```

*EPrime, Term, & TPrime follow the  
same basic lines (Figure 3.10, EaC2e)*

# Implementing a Recursive Descent Parser

---



## A nest of if-then else statements may be slow

- A good case statement would be an improvement<sup>†</sup>
  - See EaC2e, § 7.8.3
  - Encode with computation rather than repeated branches
- Order the cases by expected frequency, to drop average cost

*Python?*

## What about encoding the decisions in a table?

- Replace if then else or case statement with an address computation
- Branches are slow and disruptive
- Interpret the table with a skeleton parser, as we did in scanning

<sup>†</sup> a good case statement can be hard to find

# Building Table-Driven Top-down Parsers



## Strategy

- Encode knowledge in a table
- Use a standard “skeleton” parser to interpret the table

## Example

- The non-terminal *Factor* has 3 expansions
  - ( Expr ) or Identifier or Number
- Table might look like:

0	Goal	→	Expr
1	Expr	→	Term Expr'
2	Expr'	→	+ Term Expr'
3			- Term Expr'
4			ε
5	Term	→	Factor Term'
6	Term'	→	* Factor Term'
7			/ Factor Term'
8			ε
9	Factor	→	( Expr )
10			<u>number</u>
11			<u>identifier</u>

		Terminal Symbols								
		EOF	+	-	*	/	(	)	id.	num.
Non-terminal Symbols	<u>Factor</u>	—	—	—	—	9	—	11	10	

Cannot expand *Factor* into an operator  $\Rightarrow$  error

Expand *Factor* by rule 10 with input “number”



# Building Top-down Parsers

---



## Building the complete table

- Need a row for every  $NT$  & a column for every  $T$

# LL(1) Table for the Expression Grammar



	EOF	+	-	*	/	(	)	id.	num.
<i>Goal</i>	—	—	—	—	—	0	—	0	0
<i>Expr</i>	—	—	—	—	—	1	—	1	1
<i>Expr'</i>	4	2	3	—	—	—	4	—	—
<i>Term</i>	—	—	—	—	—	5	—	5	5
<i>Term'</i>	8	8	8	6	7	—	8	—	—
<i>Factor</i>	—	—	—	—	—	9	—	11	10

Row we built  
earlier

Figure 3.11(b), page 112, EaC2e

# Building Top-down Parsers

---



## Building the complete table

- Need a row for every  $NT$  & a column for every  $T$
- Need an interpreter for the table (*skeleton parser*)

# LL(1) Skeleton Parser



```
word ← NextWord()           // Initial conditions, including
push EOF onto Stack         // a stack to track local goals
push the start symbol, S, onto Stack
TOS ← top of Stack
loop forever
  if TOS = EOF and word = EOF then
    break & report success // exit on success
  else if TOS is a terminal then
    if TOS matches word then
      pop Stack             // recognized TOS
      word ← NextWord()
    else report error looking for TOS // error exit
  else                      // TOS is a non-terminal
    if TABLE[TOS,word] is  $A \rightarrow B_1 B_2 \dots B_k$  then
      pop Stack             // get rid of A
      push  $B_k, B_{k-1}, \dots, B_1$  // in that order
    else break & report error expanding TOS
  TOS ← top of Stack
```

# Building Top-down Parsers



## Building the complete table

- Need a row for every  $NT$  & a column for every  $T$
- Need a table-driven interpreter for the table
- Need an algorithm to build the table

Filling in  $TABLE[X,y]$ ,  $X \in NT$ ,  $y \in T$

1. entry is the rule  $X \rightarrow \beta$ , if  $y \in FIRST^+(X \rightarrow \beta)$
2. entry is *error* if rule 1 does not define

If any entry has more than one rule,  $G$  is not  $LL(1)$

Incrementally tests the  $LL(1)$  criterion on each  $NT$ .

An efficient way to determine if a grammar is  $LL(1)$

**This algorithm is the  $LL(1)$  table construction algorithm**

In Lab 2, you would have built a recursive descent parser for a modified form of **BNF** and build  $LL(1)$  tables for the grammars that are  $LL(1)$ . (A good weekend project)

# Recap of Top-down Parsing

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- Top-down parsers build syntax tree from root to leaves
- Left-recursion causes non-termination in top-down parsers
  - Transformation to eliminate left recursion
  - Transformation to eliminate common prefixes in right recursion
- **FIRST**, **FIRST<sup>+</sup>**, & **FOLLOW** sets + **LL(1)** condition
  - **LL(1)** uses left-to-right scan of the input, leftmost derivation of the sentence, and 1 word lookahead
  - **LL(1)** condition means grammar works for predictive parsing
- Given an **LL(1)** grammar, we can
  - Build a recursive descent parser
  - Build a table-driven **LL(1)** parser
- **LL(1)** parser doesn't build the parse tree
  - Keeps lower fringe of partially complete tree on the stack

