Mid-sem Exam
MA 3140 - Statistical Inference

Time Allowed: 45 Minutes **Maximum Marks: 20**

Q 1. Let $X_1, \ldots, X_n$ $(n \geq 2)$ be a r.s. from $U(0, \theta)$, $\theta \in \Theta = (0, \infty)$ is an unknown parameter. Among the estimators of θ, which are based on the MLE and belong to the class $\mathcal{D} = \{\delta_\alpha(\boldsymbol{X}) : \delta_c(\boldsymbol{X}) = cX_{(n)}, c > 0\}$, find the estimator having the smallest MSE, at each parametric point. $\boxed{\text{5 MARKS}}$

Q 2. Let $X_1, \ldots, X_n$ $(n \geq 2)$ be a r.s. from a population with mean μ and variance σ^2.

(i) Show that the estimator $\sum_{i=1}^n a_i X_i$ is an unbiased estimator of μ if $\sum_{i=1}^n a_i = 1$.

(ii) Among all the unbiased estimators of the form (called linear unbiased estimators), find the one with minimum variance, and calculate the variance. $\boxed{\text{2+3=5 MARKS}}$

Q 3. Let $X_1, \ldots, X_n$ $(n \geq 2)$ be a r.s. from $N(0, \sigma^2)$.

(i) Find the CRLB for the variance of an unbiased estimator of σ.

(ii) Define $T_1 = \alpha \sum_{i=1}^n |X_i|$ and $T_2 = \beta(\sum X_i^2)^{1/2}$. Find α and β such that T_1 and T_2 are unbiased estimators of σ. Also, check whether any of these estimators attain CRLB.

(iii) Of the two estimators T_1 and T_2, defined in (ii) above, which one is a better estimator? $\boxed{\text{2+5+3=10 MARKS}}$