Assignment 4 : MA3140

Q 1. Let $X \sim P(\lambda)$. For testing $H_0 : \lambda = 1$ vs. $H_a : \lambda = 4$, consider the test $\phi(x) = 1$, if $x > 2$; and 0, if $x \leq 2$. Find the probabilities of Type I and Type II errors.

Q 2. Let X have double exponential density $f_X(x) = \frac{1}{2\pi} \exp^{-|x|/\sigma}$, $x \in \mathbb{R}$, $\sigma > 0$. For testing $H_0 : \sigma = 1$ vs. $H_a : \sigma > 1$, consider the test $\phi(x) = 1$, if $|x| > 1$; and 0, if $|x| \leq 1$. Find the size and power of the test. Show that power is always more than the size of the test.

Q 3. Let X have Cauchy density $f_\sigma(x) = \frac{\sigma}{\pi(\sigma^2+x^2)}$, $x \in \mathbb{R}$, $\sigma > 0$. Find the most powerful test of size α for testing $H_0 : \sigma = 1$ vs. $H_a : \sigma = 2$.

Q 4. Let X have density $f_\theta(x) = \frac{2}{\theta^2}(\theta - x)$, $0 < x < \theta$. Find the most powerful test of size α for testing $H_0 : \theta = 1$ vs. $H_a : \theta = 2$.

Q 5. Let $X_1, \ldots, X_n$ be a random sample from a population with density $f_\theta(x) = \frac{1}{\Gamma\theta} x^{\theta-1} e^{-x}$, $x > 0$, $\theta > 0$. Show that the family has MLR in $\prod X_j$. Hence derive UMP test of size α for testing $H_0 : \theta \leq 3$ vs. $H_a : \theta > 3$.

Q 6. Based on a random sample of size n from $Exp(\lambda)$, derive UMP unbiased test of size α for testing $H_0 : \lambda = 1$ vs. $H_a : \lambda \neq 1$.

Q 7. For the set up in Q. 4, find LRT for testing $H_0 : \theta = 2$ vs. $H_a : \theta \neq 2$.

Q 8. Based on a random sample of size n from double exponential density $f_X(x) = \frac{1}{2\pi} \exp^{-|x|/\sigma}$, $x \in \mathbb{R}$, $\sigma > 0$, derive LRT for testing $H_0 : \sigma = 1$ vs. $H_a : \sigma \neq 1$.