# Assignment - 4 Solutions

1.

$$\begin{aligned}\alpha &= P_{\lambda=1}(X > 2)\\ &= 1 - \frac{5}{2}e^{-1}\end{aligned}$$

$$\beta = P_{\lambda=4}(X \leq 2) = 13e^{-4}$$

2.

$$\alpha = P(|X| > 1) = e^{-1}$$

$$Power\ = P_{\sigma}(|X| > 1) = e^{-1/\sigma} > e^{-1},\ \ as\ \ \sigma > 1.$$

3. Using NP Lemma, MP test is to reject $H_o$ when

$$R(x) = \frac{f_2(x)}{f_1(x)} > k$$

$$Now\ \ R(x) = \frac{2(1+x^2)}{(4+x^2)}.$$

It can be seen that $R^{'}(x) = \dfrac{6x}{(4+x^2)^2}$. So $R(x)$ has minimum $1/2$ at $x = 0$ and supremum is 2 as $x \longrightarrow \pm\infty$. The MP test is defined as follows:

(a) If we take $k \leq 1/2$, then MP test will always reject $H_0$ at $\alpha = 1$.

(b) If we take $k \geq 2$, then MP test will always accept $H_0$ at $\alpha = 0$.

(c) If we take $1/2 < k < 2$, then MP test will reject $H_0$ when $R(x) > k$. This is equivalent to $|x| > \sqrt{\dfrac{4k-2}{(2-k)}}$. Applying the size condition, we get

$$k = \frac{4}{4 + 3cos(1-\alpha)}$$

4. Using the NP Lemma, we find the MP test will reject $H_0$ when

$$x < 1 - \sqrt{\alpha}$$

5. The solution follows from the use of MLR property.

6. The test is based on $\sum_{i=0}^{n} X_i$ which has a Gamma (n, $\lambda$).

7. The test will reject $H_0$ when $|x - 1| > k$.

8. The test is based on $\sum_{i=1}^{n} X_i$ which has Gamma(n, $1/\sigma$) distribution. The LRT is to reject $H_0$ if

$$\sum_{i=1}^{n} |X_i| > \frac{\chi^2_{2n,\alpha/2}}{2} \ \ or \ \ \sum_{i=1}^{n} |X_i| < \frac{\chi^2_{2n,1-\alpha/2}}{2}$$

**Each question carries 5 marks.**

**Total - 40**