Assignment 1 : MA3140

Q 1. Let $X_1, \ldots, X_n$ be a random sample from a distribution having pdf (pmf) $f(\cdot|\boldsymbol{\theta})$, where $\theta \in \Theta$ is unknown, and let $g(\boldsymbol{\theta})$ is the estimand. Find the **MME** and the **MLE** in each of the following cases, and see if they differ.

(i) Geometric Distribution (Geom(θ))

$$f(x|\theta) = \begin{cases} \theta(1-\theta)^{x-1}, & \text{if } x = 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases};$$

where $\Theta = (0, 1)$ and $g(\theta) = \theta$. Also, compute the MLE of $P(X_1 \geq 4)$ based on the following data: $x_1 = 2, x_2 = 7, x_3 = 6, x_4 = 5$ and $x_5 = 9$.

(ii) Uniform Distribution (U($-\theta, \theta$))

$$f(x|\theta) = \begin{cases} \frac{1}{2\theta}, & \text{if } -\theta \leq x \leq \theta \\ 0, & \text{otherwise} \end{cases};$$

where $\Theta = (0, \infty)$ and $g(\theta) = \theta$.

(iii) Two parameter Exponential Distribution (Exp(μ, σ))

$$f(x|\boldsymbol{\theta}) = \begin{cases} \frac{1}{\sigma} \exp\left(-\frac{x-\mu}{\sigma}\right), & \text{if } x > \mu \\ 0, & \text{otherwise} \end{cases};$$

where $\boldsymbol{\theta} = (\mu, \sigma)$; $\Theta = (-\infty, \infty) \times (0, \infty)$ and $g(\boldsymbol{\theta}) = (\mu, \sigma)$.

(iv) Pareto Distribution (Pa(α, β))

$$f(x|\boldsymbol{\theta}) = \begin{cases} \frac{\beta\alpha^{\beta}}{x^{\beta+1}}, & \text{if } x > \alpha, \alpha > 0, \beta > 2 \\ 0, & \text{otherwise} \end{cases};$$

where $\boldsymbol{\theta} = (\alpha, \beta)$ and $g(\boldsymbol{\theta}) = (\alpha, \beta)$. Find the estimate based on the following data: 3, 5, 2, 3, 4, 1, 4, 3, 3, 3.

(v) Normal Distribution (N(μ, σ^2))

Here, $\boldsymbol{\theta} = (\mu, \sigma^2)$; $\Theta = (-\infty, \infty) \times (0, \infty)$ and $g(\boldsymbol{\theta}) = \frac{\mu^2}{\sigma^2}$.

Q. 2 Let one observation be taken on a discrete random variable X with pmf $p(x|\theta)$, given below, where $\Theta = \{1, 2, 3\}$. Find the MLE of θ.

		θ		
		1	2	3
	1	1/2	1/4	1/4
x	2	3/5	1/5	1/5
	3	1/3	1/2	1/6
	4	1/6	1/6	2/3

Q. 3 The lifetimes of a brand of a component are assumed to be exponentially distributed with mean (in hours) θ, where $\theta \in \Theta = (0, \infty)$ is unknown. Ten of these components were independently put in test. The only data record were the number of components that had failed in less than 100 hours versus the number that had not failed. It was found that 3 had failed before 100 hours. What is the MLE of θ?

Q. 4 Let $X_1, \ldots, X_n$ be a random sample from a $Bin(1, \theta)$ distribution. where $\theta \in \Theta$ is unknown. Find MLE of θ when $\Theta = [\frac{1}{4}, \frac{3}{4}]$.

Q. 5 Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu, \sigma^2)$ distribution. where μ is known (say $\mu = \mu_0$). Find MLE of

(i) σ^2;

(ii) σ^2 when $\mu_0 = 0$;

(ii) σ^2 when $\sigma^2 \geq \sigma_0^2$;

(iv) σ^2 when $\sigma^2 \leq \sigma_0^2$.