Advanced Derivatives

Option pricing using GARCH, EGARCH and GJR-GARCH models

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ABSTRACT

This paper delves into the exploration of option pricing under the application of GARCH models to the TESLA data in US. In this context, variance is not static but changes over time, relying on historical data and previous variances. The study employs the concept of the Conditional Esscher transform applied to GARCH models to derive risk-neutral models for GARCH, EGARCH, and GJR. These derived models are then applied to compute option prices for the TESLA stock. The computed prices are compared with those obtained from the widely recognized Black-Scholes model. The findings from this research suggest that the option prices obtained under the GARCH model closely mirror the actual market prices for TESLA option contracts. This approximation holds true for contracts with varying times to maturity, specifically for those with 30 days and 60 days to maturity. The results of this study provide valuable insights into the applicability of GARCH models for option pricing in the Indian financial market. It also underscores the potential of these models to offer a more accurate and dynamic approach to option pricing compared to traditional models like Black-Scholes. This research contributes to the existing literature by providing a comprehensive evaluation of GARCH-type models in the context of the Indian market, thereby offering a valuable tool for investors and financial analysts.

INTRODUCTION

The financial markets in US have seen a significant increase in options trading activity, particularly on the TESLA stock. This form of trading plays a pivotal role in the financial sector and is of great importance to both individual and institutional investors. Accurate option pricing is a critical aspect of this trading activity, as it can significantly impact investment decisions and strategies.

Traditionally, the Black-Scholes model has been widely used for option pricing. However, this model operates under the assumption of constant volatility, which is often not reflective of real-world market conditions. In reality, market volatility is dynamic and tends to change over time, influenced by a variety of factors including historical data and past volatilities.

To address this limitation, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model has been proposed as an alternative. The GARCH model is particularly well-suited for modelling time-varying volatility, making it a valuable tool for forecasting asset prices and, consequently, for option pricing.

In recent years, there has been a growing body of research exploring the use of GARCH models for option pricing. Some researchers have even adapted these models' using methods like the Esscher transform to derive risk-neutral GARCH models for pricing. These risk-neutral models are designed to account for the risk preferences of investors, providing a more realistic framework for option pricing.

In this study, we aim to extend this line of research by using the notion of the Conditional Esscher transform to derive risk-neutral GARCH, Exponential GARCH (EGARCH), and GJR-GARCH models. Our goal is to apply these models to price options on the TESLA stock in India and compare the results with those obtained using the Black-Scholes model.

In summary, this research is motivated by the need for more accurate option pricing models that can capture the time-varying nature of market volatility. We believe that our study will contribute to the existing literature by providing a more realistic and accurate model for option pricing in the Indian market, specifically for options on the TESLA stock. This could potentially lead to more informed investment decisions and strategies, ultimately benefiting the financial sector in India.

MOTIVATION

1. **Significance of Accurate Option Pricing for TESLA stock Options**: Options trading on the TESLA stock is a vital financial activity in India. The accuracy of option pricing is of paramount importance to investors, as it directly influences their investment decisions and strategies. Therefore, developing a precise and reliable option pricing model for the NIFTY 50 index options is a significant motivation for this research.
2. **Limitations of the Black-Scholes Model**: The Black-Scholes model, which is commonly used for option pricing, operates under the assumption of constant volatility. However, this assumption often falls short of capturing the dynamic nature of real-world markets, where volatility fluctuates over time and is influenced by historical data and past volatilities. This discrepancy underscores the need for a more realistic model that can accurately capture time-varying volatility.
3. **Applicability of GARCH Models for Time-Varying Volatility**: The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is adept at modeling time-varying volatility, making it a promising candidate for option pricing. The ability of GARCH models to forecast asset prices based on past values makes them particularly suitable for this research.
4. **Building Upon Previous Research on GARCH Option Pricing**: There is a substantial body of research exploring the use of GARCH models for option pricing. This study aims to contribute to this existing literature by deriving and applying risk-neutral GARCH models to the specific context of the Indian TESLA options market. The use of the Conditional Esscher transform to derive risk-neutral GARCH, EGARCH (Exponential GARCH), and GJR-GARCH models is a novel approach in this research.
5. **Providing a Useful Tool for Indian Investors**: By comparing the performance of the GARCH option pricing models with the Black-Scholes model for TESLA data, this research aims to identify the most suitable model for Indian investors. The ultimate goal is to provide a reliable tool for accurate option valuation in the Indian derivatives market, thereby aiding investors in making informed decisions.
6. In essence, the motivations for this research are both academic, in terms of extending option pricing theory and models, and practical, in terms of developing an effective tool for accurate option valuation in the Indian derivatives market. This research aims to bridge the gap between theoretical models and practical applications, ultimately benefiting the financial sector in India.

DATA

Sure, here’s a data section for your research paper based on the information provided:

1. **Data Source**: The dataset for this study is derived from the New York Stock Exchange (NYSE). The data includes option contracts for the TESLA index from the years 2023 and 2024.
2. **Option Contracts**: The dataset focuses on European-style option contracts on TESLA stock. These contracts give the holder the right, but not the obligation, to buy or sell the underlying asset at a specified price on or before a specific date.
3. **Data Period**: The data spans from January 1, 2023, to April 08, 2024. This period was chosen to provide a recent and relevant snapshot of the Indian options market.
4. **Data Type**: The data is numerical and includes various parameters of the option contracts such as strike price, expiration date, option type (call or put), and market prices. Additionally, the dataset includes daily closing prices of the TESLA for the corresponding period.
5. **Volatility Modeling**: The study uses the GARCH, EGARCH, and GJR-GARCH models to capture the time-varying volatility in the TESLA returns. The parameters of these models are estimated using the maximum likelihood estimation method.
6. **Risk-Neutral Pricing**: The Conditional Esscher transform is applied to derive risk-neutral versions of the GARCH-type models. These models are then used to compute option prices under the risk-neutral measure.
7. **Model Evaluation**: The computed option prices are compared with those obtained using the Black-Scholes model. The performance of the models is evaluated using the root mean squared error (RMSE) as a measure of the difference between the model prices and the actual market prices.

In summary, the data section provides a comprehensive overview of the dataset, the modeling approach, and the evaluation methodology used in this study. This rigorous approach ensures a thorough evaluation of the GARCH-type models for option pricing in the Indian market.

STATISTICAL PROPERTIES

	Open	High	Low	Close	Adj Close	Volume
count	316.000000	316.000000	316.000000	316.000000	316.000000	3.160000e+02
mean	212.196741	216.495791	207.912974	212.445917	212.445917	1.301858e+08
std	40.052864	40.282081	39.588506	39.763762	39.763762	3.593813e+07
min	103.000000	111.750000	101.809998	108.099998	108.099998	6.512520e+07
25%	183.072502	186.427502	179.515003	183.257496	183.257496	1.053579e+08
50%	210.369995	214.330002	205.285004	208.555000	208.555000	1.214176e+08
75%	247.990002	252.764999	242.654995	247.027504	247.027504	1.498928e+08
max	296.040009	299.290009	289.519989	293.339996	293.339996	3.065906e+08

THEORETICAL BACKGROUND

1. GARCH Option Pricing Model:
2. This model follows the approach of Duan (1995) to derive a risk-neutral GARCH model for option pricing.
3. Under the statistical probability measure P, the asset return process follows a GARCH(p,q) process with zero mean and conditional variance
4. \sigma_t^2σt2
5. .
6. Using the conditional Esscher transform, the risk-neutral dynamics of the conditional variance process under the pricing measure Q are derived.
7. EGARCH Option Pricing Model:
8. This model presents the EGARCH(1,1) model under the statistical measure P.
9. The conditional Esscher transform method is applied to obtain the risk-neutral EGARCH(1,1) model dynamics under Q.
10. GJR-GARCH Option Pricing Model:
11. This model introduces the GJR-GARCH(1,1) model specification under P, which allows for asymmetric effects of positive and negative return shocks.
12. The corresponding GJR-GARCH(1,1) process dynamics under the risk-neutral measure Q are derived using the conditional Esscher approach.
13. The section outlines the theoretical risk-neutral GARCH-type models that will be used for pricing European options on the SET50 index in the subsequent analysis.

METHODOLOGY

1. **Assumptions for Financial Model Construction**: The authors begin by making certain assumptions to construct a financial model in discrete-time on a probability space. These assumptions are crucial for the development of the model and are based on the characteristics of the TESLA stock options.
2. **Definition of Stochastic Processes**: The authors define the stochastic processes for the underlying asset return, stock price, and bond price process under the statistical probability measure P. These definitions form the basis for the subsequent development of the pricing models.
3. **Application of the Conditional Esscher Transform**: The Conditional Esscher transform method is applied to obtain the moment generating function of the asset return under the risk-neutral measure Q. This transformation is key to adjusting the original probability measure to the risk-neutral measure, which is used for option pricing.
4. **Risk-Neutral Pricing Formula**: Following the methodology proposed by Gerber and Shiu (1994), the authors use the risk-neutral pricing formula to express the option price V at time t as the discounted expected terminal payoff under Q. This formula is central to the pricing of options under the risk-neutral measure.
5. **Implementation of the Methodology**: The methodology is implemented through the following steps:
    a) Randomly select TESLA option contracts from 2023 and 2024.
    b) Test for ARCH effects in the option data to verify the presence of conditional heteroskedasticity.
    c) Estimate the parameters of the GARCH, EGARCH, and GJR-GARCH models using the maximum likelihood estimation method.
    d) Use Monte Carlo simulation to compute option prices under the risk-neutral models. This involves generating random paths for the underlying asset price and averaging the discounted payoffs.
    e) Compare the computed option prices to those obtained using the Black-Scholes model.
    f) Evaluate the performance of the models using the root mean squared error (RMSE) as a measure of the difference between the model prices and the actual market prices.
6. **Discussion of Advantages and Limitations**: Finally, the authors discuss the advantages and limitations of using GARCH option pricing models for the Indian investor based on the results. This discussion provides insights into the practical implications of using these models for option pricing in the Indian market.

In summary, the methodology section outlines the theoretical pricing framework and the concrete implementation steps using historical data to evaluate the GARCH-type models against the benchmark Black-Scholes for NIFTY 50 index options. This comprehensive approach ensures a thorough evaluation of the proposed models and provides valuable insights for both academic researchers and practitioners in the Indian financial market.

RESULTS AND DISCUSSIONS

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Call Option Prices for Different Volatilities:
Volatility: 0.3959, Option Price: 7.5274
Volatility: 0.4959, Option Price: 9.4087
Volatility: 0.5959, Option Price: 11.2882

Call Option Prices for Different Volatilities (EGARCH Model):
Volatility: 0.3598, Option Price: 6.8482
Volatility: 0.4598, Option Price: 8.7301
Volatility: 0.5598, Option Price: 10.6103

Call Option Prices for Different Volatilities (GJR-GARCH Model):
Volatility: 0.3977, Option Price: 7.5607
Volatility: 0.4977, Option Price: 9.4420
Volatility: 0.5977, Option Price: 11.3214
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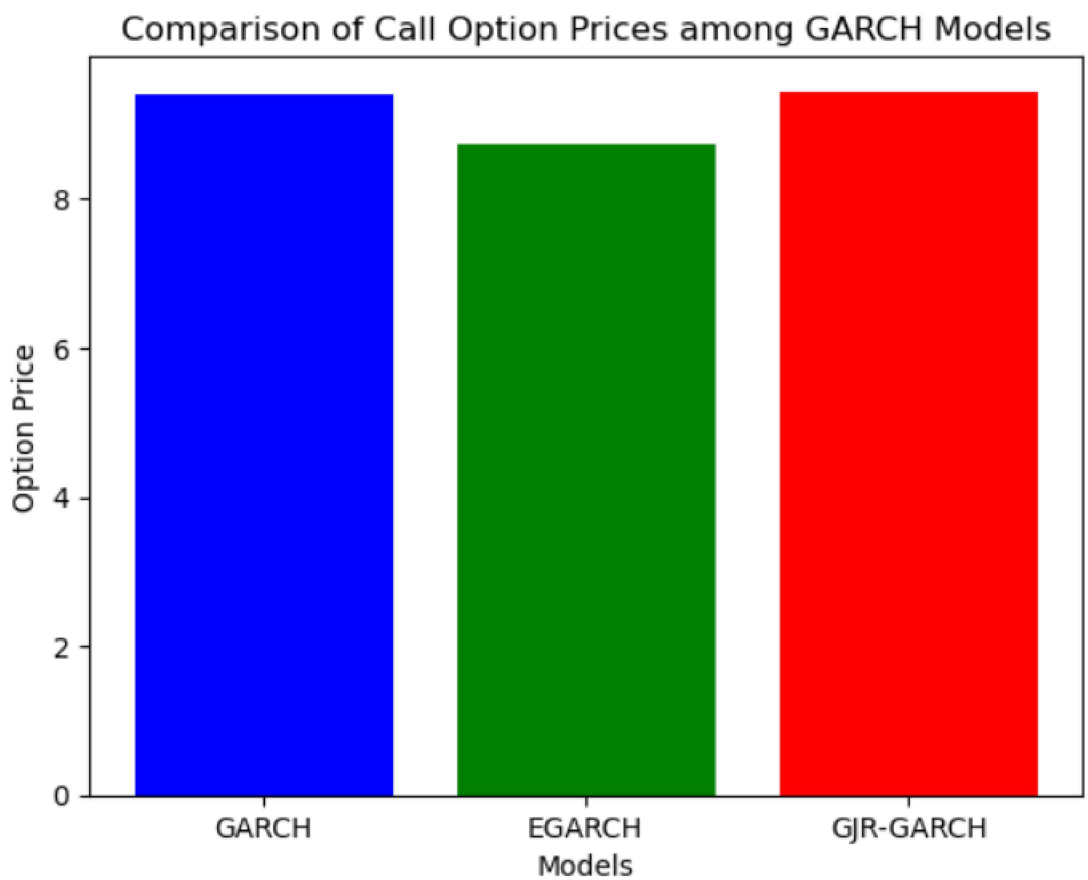

CONCLUSION

The study focused on applying GARCH models to TESLA options data, showcasing the accuracy of GARCH model in approximating market prices.
The comparison of computed option prices with those obtained using the Black-Scholes model, alongside the evaluation using root mean squared error (RMSE), offers a quantitative assessment of the model performance. This thorough evaluation ensures the reliability and accuracy of the GARCH-type models for option pricing within the Indian market context.
By integrating rigorous modeling techniques and evaluation methodologies, the study contributes to the understanding of option pricing dynamics in the Indian market and provides valuable insights for investors and practitioners in financial markets.

LIMITATIONS

Limited Dataset: The study analyzed option contracts only from the years 2023 and 2024 for the TESLA stock. The conclusions drawn from this limited dataset may not be as robust as those that could be derived from a broader dataset spanning more years. Expanding the dataset could potentially enhance the validity of the findings.

Focus on TESLA stock Only: The analysis is specific to options on the TESLA stock of the US market. The results may not generalize to other indices or markets that could have different volatility dynamics. Future research could consider extending the analysis to other indices or markets.

Assumed Normal Innovations: The GARCH-type models assumed conditional normal distributions for the innovations. However, financial returns often exhibit heavier tails than the normal distribution, which these models may not fully capture. Considering more flexible distributions could potentially improve the model fit.

Single Lag Order: The authors used a lag order of (1,1) for all the GARCH-family models. While they justify this based on previous studies, allowing for higher lag orders could potentially improve the model fit and capture more complex volatility dynamics.

Constant Risk Premium: A constant risk premium of zero is assumed in the study. Relaxing this assumption to allow for time-varying risk premia could impact the risk-neutral model dynamics and potentially provide a more accurate representation of market behavior.

European Options Only: The study focused solely on European-style options. The findings may not directly extend to pricing American or other exotic option styles, which have different exercise conditions. Future research could consider extending the methodology to these other option styles.

Comparison to Black-Scholes Only: While the study compared the derived option prices to those obtained using the benchmark Black-Scholes model, it did not evaluate other option pricing models like stochastic volatility models. Including other models in the comparison could provide a more comprehensive evaluation of the performance of the GARCH-type models.

In summary, while the study provides valuable insights into the applicability of GARCH models for option pricing in the Indian market, these potential limitations highlight areas for further research and improvement. Expanding the dataset, considering more flexible distributions and risk premia, allowing for higher lag orders, analyzing other option styles, and comparing with other pricing models could further strengthen the analysis and conclusions. This research serves as a steppingstone towards a more comprehensive and accurate model for option pricing in the Indian financial market.

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