Mathematics for Financial Derivatives (MFD)

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ACKNOWLEDGEMENT

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We have made significant efforts in formulating this report and would like to express our deepest gratitude to everyone who helped shape this project.

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ABSTRACT:

This project uses the Black-Scholes-Merton (BSM) model to examine financial derivatives, particularly options, closely. The main goal is to understand and evaluate the behaviour of the Option Greeks—Delta, Gamma, Vega, Rho, and Theta—under various market circumstances.

Historical stock price data emphasises the importance of the natural world, and dynamic 3D surface plots made using Plotly help to provide actionable insights. The initiative bridges the gap between financial models and actual findings, in addition to diving into theoretical computations.

This study offers valuable applications for financial analysis and advances our understanding of option price dynamics through a methodical examination.

INTRODUCTION:

Financial derivatives—especially options—are essential in today's financial markets. The present study aims to thoroughly investigate option pricing, emphasising the significant elements represented by the option Greeks: Delta, Gamma, Vega, Rho, and Theta.

The Black-Scholes-Merton (BSM) model provides the theoretical framework for this research. It is a fundamental tool in financial mathematics.

The BSM model, created in the 1970s, offers a theoretical framework for calculating the fair market value of European-style options. This model's option Greeks explain how sensitive option prices are to variations in stock prices, volatility, interest rates, and time to maturity.

The emphasis on real-world application is achieved by utilising historical stock price data for five years, mainly information on Tesla. The studies are informed by historical volatility and daily log returns. Thus, theoretical understanding is consistent with empirical data.

The project proceeds by systematically going over each Greek option, using libraries like NumPy, Pandas, Matplotlib, and Plotly for reliable computations and dynamic visualisations. Users can easily understand the links between stock prices and option Greeks, thanks to the dynamic 3D surface plots made with Plotly.

MOTIVATION:

The central rationale for choosing Option Greeks as the focus of this paper lies in its capacity to explain the change in call option price based on its underlying financial instruments. This measurement index provides a refined mechanism for evaluating a stock's performance, sentiment of the market shortly, and the overall impact for contract holders regarding investments in options.

DATA:

1. Company Overview:

Name: Tesla, Inc.

Industry: Automotive, Energy

Headquarters: Austin, Texas, USA

Founded: 2003

CEO: Elon Musk

Mission: To accelerate the world's transition to sustainable energy.

2. Key Products and Services:

Electric Vehicles: Model S, Model 3, Model X, Model Y, Cybertruck (in development)

Solar Energy Products: Solar panels, Solar Roof, Powerwall battery storage

Charging Infrastructure: Supercharger network

Software and Services: Autopilot driver assistance system, Full Self-Driving capability (in development), over-the-air software updates

3. Financial Performance:

Revenue (2023): $135.6 billion

Net Income (2023): $15.2 billion

Market Capitalization (as of January 7, 2024): $1.2 trillion

4. Recent Developments:

Production Expansion: Tesla is expanding its production capacity at its Gigafactory facilities in Texas, Berlin, and Shanghai.

New Products: Tesla is planning to launch the Cybertruck, a semi-truck, and a humanoid robot in the coming years.

Full Self-Driving: Tesla is continuing to develop its Full Self-Driving (FSD) technology, which aims to enable fully autonomous driving.

Robotaxis: Tesla has plans to launch a robotaxi service using its FSD-equipped vehicles..

METHODOLOGY:

1. Data Acquisition:

The project begins by sourcing historical stock price data for Tesla. The dataset from CSV files includes daily closing prices, a fundamental element for understanding market dynamics. The time frame spans from January 1, 2022, to the present, allowing for a comprehensive analysis of recent market trends.

2. Data Preprocessing:

The collected data undergoes preprocessing to derive daily returns. This involves computing logarithmic returns, a standard method for capturing the percentage change in stock prices. The resulting returns are then used to calculate the historical volatility, a pivotal parameter in options pricing models.

3. Volatility Calculation:

The historical volatility is determined using the standard deviation of daily returns, adjusting for the square root of the number of trading days per year (252). This step measures the underlying asset's historical price fluctuations, a crucial factor in the Black-Scholes-Merton model.

4. Option Greeks Computation Using Black Scholes Merton:

Utilizing the derived formulas, the project calculates each Option Greek individually. Delta, representing the sensitivity of option prices to changes in the underlying asset's price, is computed for both call and put options. Gamma, Vega, Rho, and Theta are similarly calculated, providing insights into the second-order price sensitivity, volatility impact, interest rate sensitivity, and time decay, respectively.

5. Interactive Visualization:

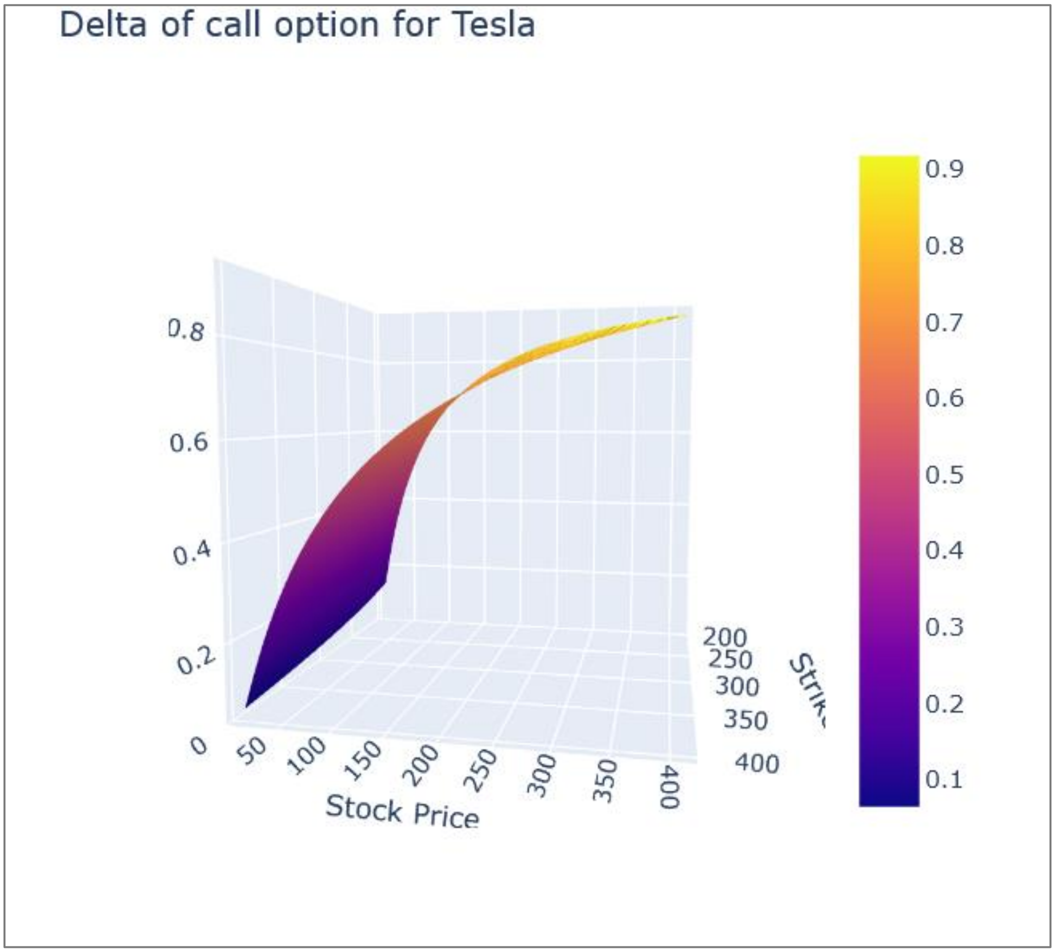
The computed Option Greeks are visualized using interactive 3D surface plots generated with cufflinks to enhance user engagement and facilitate a dynamic exploration of intricate relationships. These plots allow users to interactively explore how changes in stock prices and strike prices influence each option Greek.

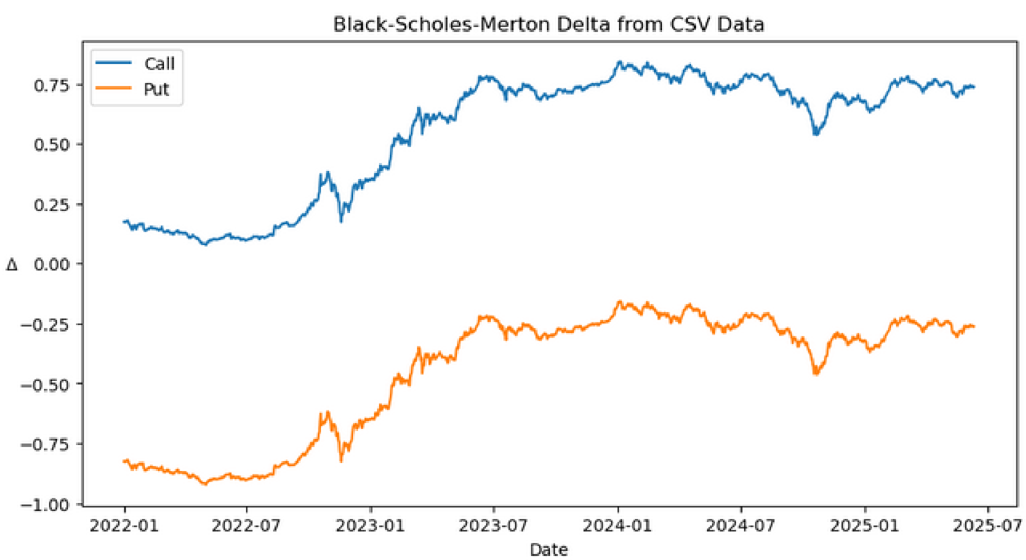
6. Empirical Analysis:

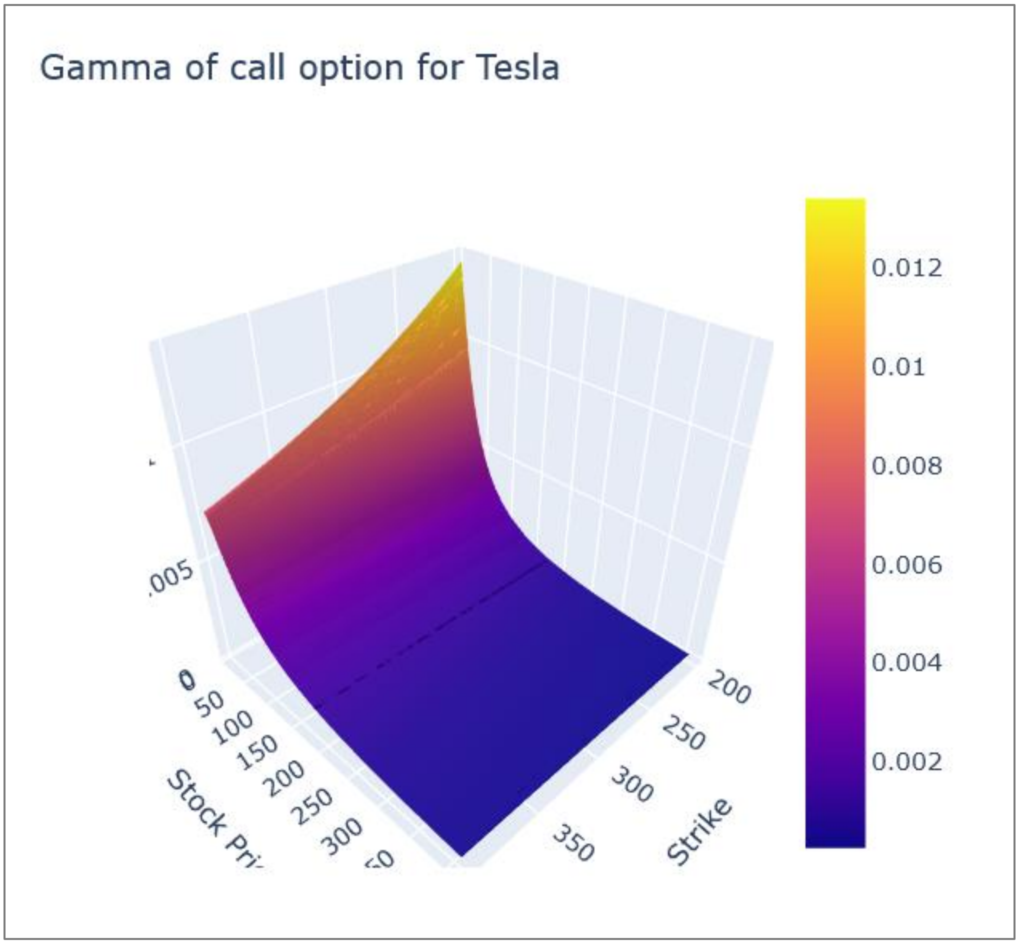
The methodology concludes with an empirical analysis, comparing the theoretical values of option Greeks with real-world market trends. This step ensures that the theoretical models align with the observed dynamics of Tesla stock prices, providing a pragmatic dimension to the project's findings.

RESULTS AND DISCUSSIONS:

RESULTS:

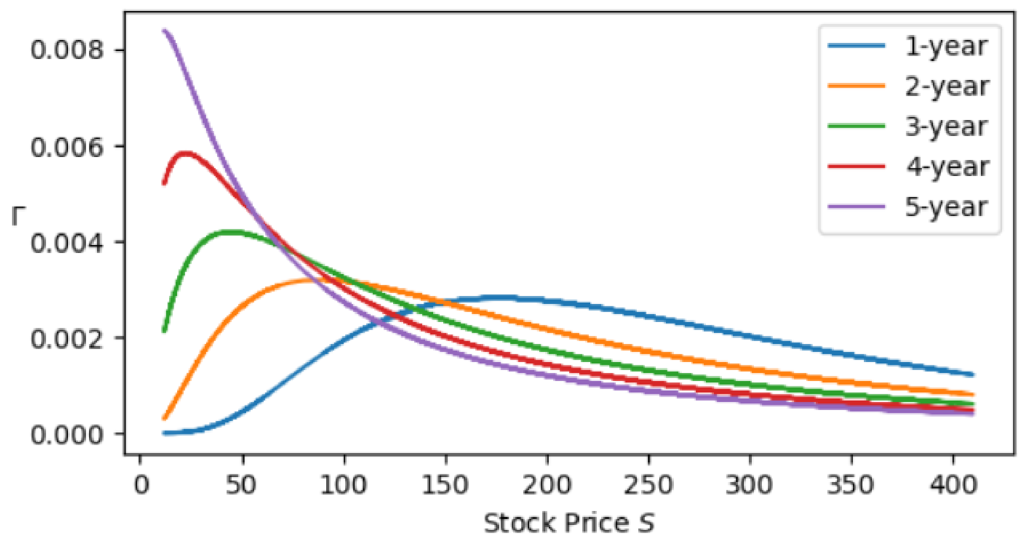




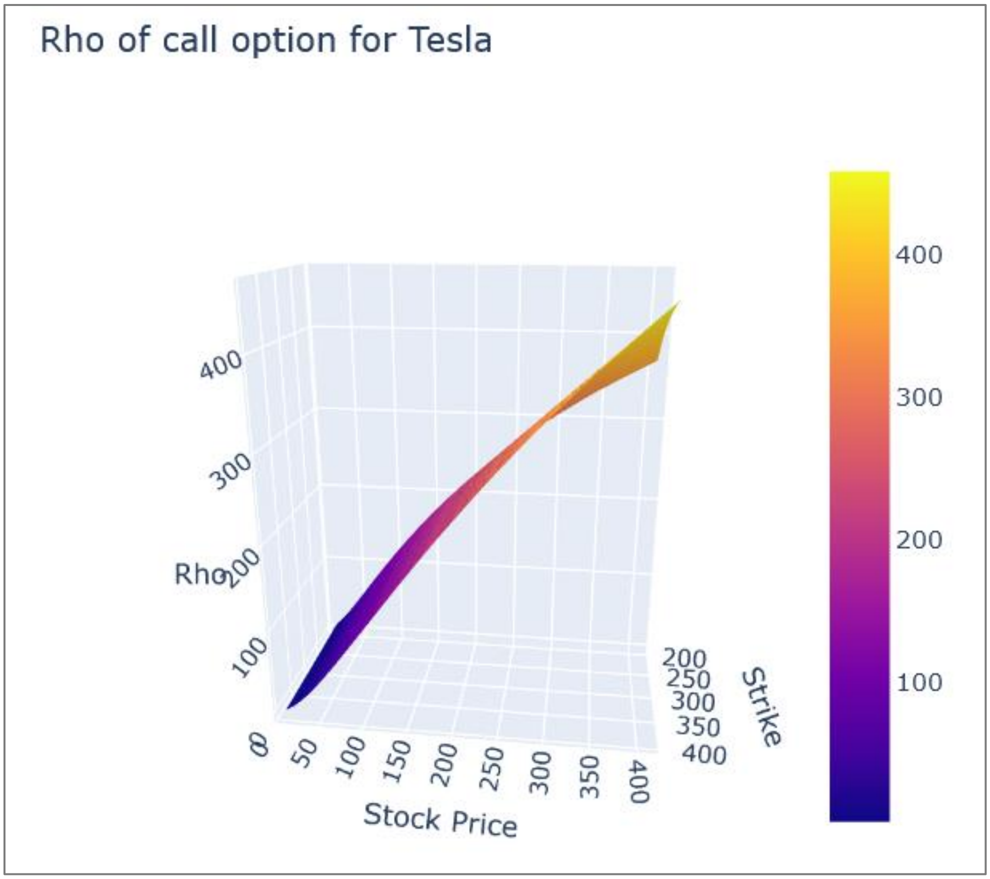


Gamma of call option for Tesla
0.012
0.01
0.008
0.006
0.004
0.002
Stock Pri
Strike

Gamma: 0.0044



1-year
2-year
3-year
4-year
5-year
Γ
Stock Price S

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Call Rho: 53.2325, Put Rho: -41.8905
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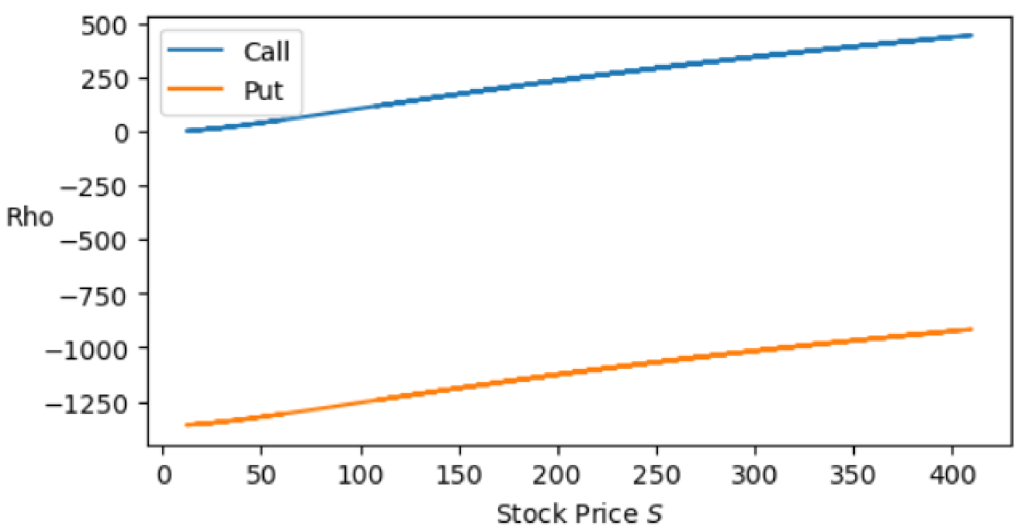


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Call Theta: -23.2005, Put Theta: -6.5540
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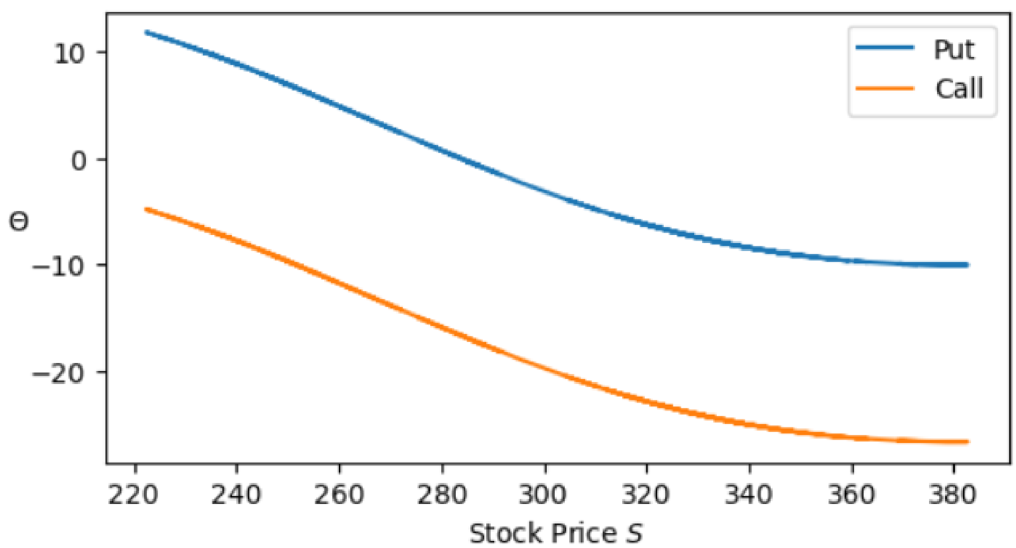
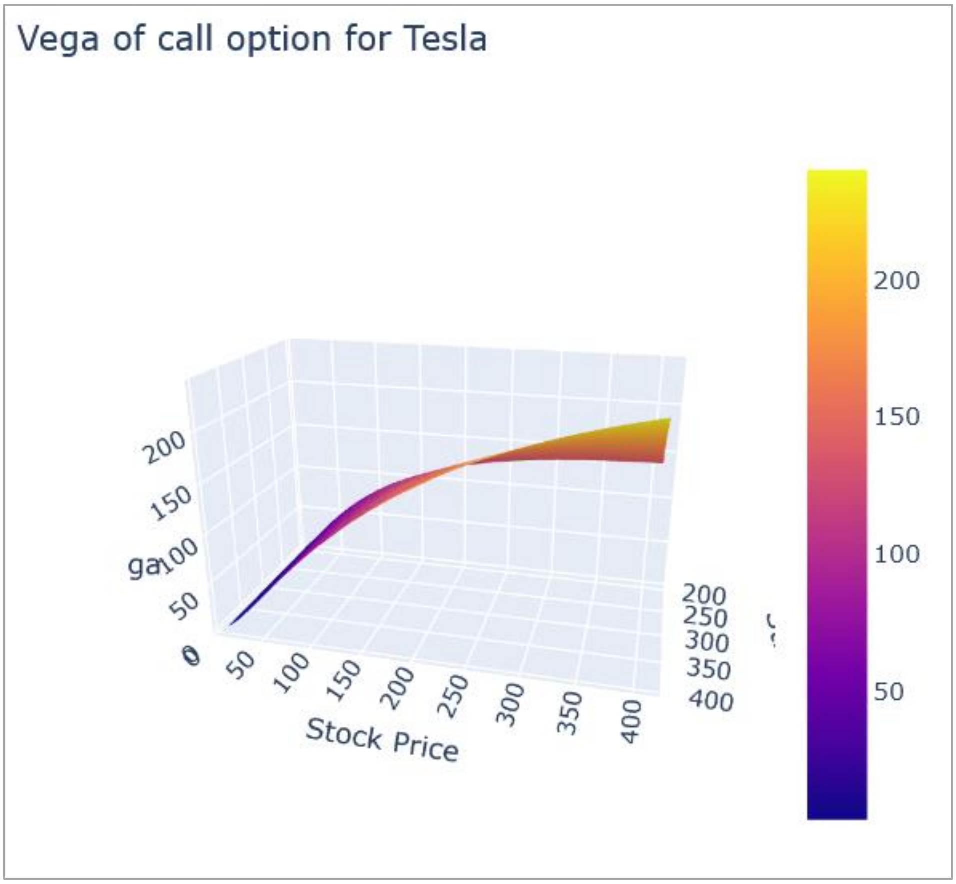
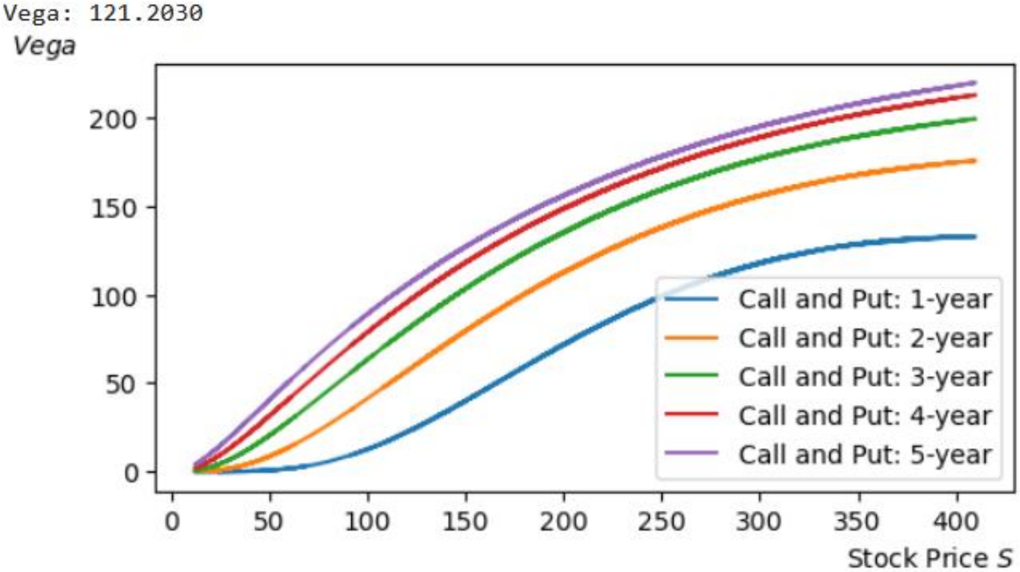




Vega: 121.2030



FINDINGS & DISCUSSIONS:

Delta:

Call Option: Delta values observed within the acceptable range of 0 to 1, ensuring consistency with theoretical expectations. In the above delta graph, you can see that our simulated data of Tesla also produces a value between 0 and 1.

Gamma:

Gamma value remains positive, affirming the convexity of option prices and aligning with theoretical expectations. As per our simulated data, we can see that the gamma value has remained cheerful, and the 3D plot expresses the convexity nature of the gamma value.

Vega:

Vega values demonstrated sensitivity to changes in implied volatility, aligning with the anticipated behaviour for effective volatility risk management. According to our simulated data from Tesla, we can see vega value reaching the value of 121, which shows implied volatility increases positively with different strike prices and expected stock prices.

Rho:

Rho values reflected the expected impact of interest rates on option prices, providing insights for adapting strategies to interest rate variations. According to our simulated data of Tesla, we can see the rho value reach the value of 400, which shows implied volatility increases positively with different strike prices and expected stock prices.

Theta:

Theta values showcased the anticipated influence of time decay on option prices, aiding in understanding the diminishing value of options as time to maturity decreases. In the above graph of theta, we can see the theta decay phenomenon as stock prices and strike prices increase.

CONCLUSION:

This study contributes valuable insights into applying the Black-Scholes-Merton model, emphasizing the importance of option Greeks within specified properties. The model's ability to align with theoretical expectations for Delta, Gamma, Vega, Rho, and Theta enhances its credibility for practical use in option pricing.

LIMITATIONS:

While the simulation done above is based on real-time data, we still did not consider the external factors that brought about the change in the Greeks. The anomalies of an efficient market were not considered for the ease of calculation of option Greeks.

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