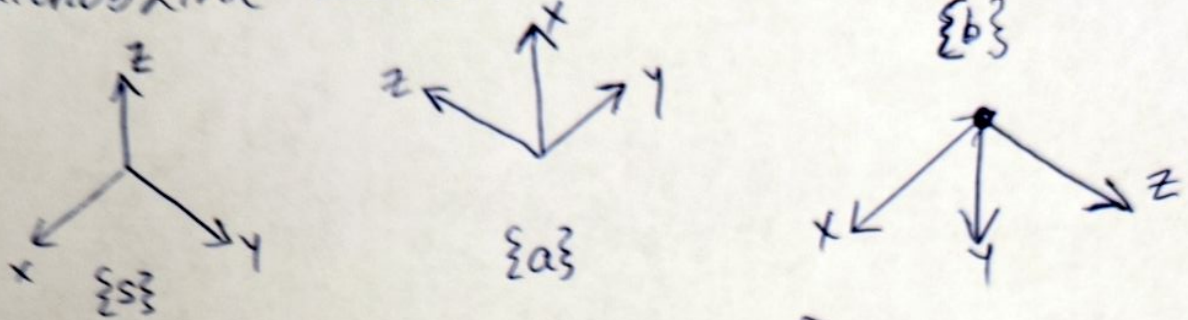
09/23/2023 **Robotics HW-3** AKASH VIDYA

Q1 (a) Drawing along with code to generate it is on GithubLink shared via Canvas!



(b) $\hat{x}_A = (0, 0, 1)$ $\quad$ $\hat{x}_B = (1, 0, 0)$

$\hat{y}_A = (-1, 0, 0)$ $\quad$ $\hat{y}_B = (0, 0, -1)$

$\hat{z}_A = \hat{x}_A \times \hat{y}_A = (0, -1, 0)$ $\quad$ $\hat{z}_B = \hat{x}_B \times \hat{y}_B = (0, 1, 0)$

$$R_{SA} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \qquad R_{SB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

(c) $R_{SB}^{-1} = R_{SB}^{T}$ (Since Rotational Matrices are orthogonal)

$$R_{SB}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

Yes, this coincides with the position of {b} w.r.t {s}!

(d) Given R_{SA} and R_{SB},

$$R_{AB} = R_{SA}^{T} \times R_{SB} = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\boxed{R_{AB} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}}$$

The matrix multiplication script used is attached in GitHub!

(e) $R = R_{SB}$ $\quad$ Rot $\hat{x} \to 90^\circ (\theta_1)$ $\quad$ $R_1 = R_{SA} \cdot R$ $\quad$ $R_2 = R \cdot R_{SA}$

For $Rot(\hat{x}, \theta_1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$ $\quad$ θ must be in rad $= \theta * \pi/180^\circ$

For $Rot(\hat{x}, \theta_1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$ $\quad$ * Calculation done on python script and available on Git!

$R_1 = R_{SA} \cdot R = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$

$$\boxed{R_1 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}}$$

Orientation R_1 corresponds to rotation of R_{SA} about $\underline{\hat{x}_A}$ axis!

$R_2 = R \cdot R_{SA} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$

$$\boxed{R_2 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

~~Orient~~ Here, R_2 corresponds to rotation of R_{SA} about $\underline{\hat{x}_S}$-axis!

Python Script used to generate coordinate frame. drawings are available on Github!

(f) ~~To~~ perform this operation,

$$p_s = R_{SB} \times p_B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\boxed{p_s = [1, 3, -2]}$$

This does not change the position of the point, it only ~~[illegible]~~ represents the point in {s} coord frame as compared to the {b} coordinates frame!

(g) $p_s = (1, 2, 3)$ $\quad p' = R_{SB} \cdot p_s$ $\quad p'' = R_{SB}^T \cdot p_s$

$$p' = R_{SB} \cdot p_s = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$$

$$\boxed{p' = (1, 3, -2)}$$

This operation moves the location of the point itself without changing the reference frame!

$$p'' = R_{SB}^T \cdot p_s = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$$

Changes coordinates without moving the point itself!

$$\boxed{p'' = (1, -3, 2)}$$

Calculations done on python and codes are on Github!

Q2 on next page!

$\boxed{Q2}$ $p = \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{2}}\right)$ $\theta_1 = 30°$ $\theta_2 = 135°$ $\theta_3 = -120°$

$$Rot(\hat{x}, \theta_1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.866 & -0.5 \\ 0 & 0.5 & 0.866 \end{pmatrix}$$
$\theta_1 = 30°$

$$Rot(\hat{y}, \theta_2) = \begin{pmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix} = \begin{pmatrix} -0.707 & 0 & 0.707 \\ 0 & 1 & 0 \\ -0.707 & 0 & -0.707 \end{pmatrix}$$
$\theta_2 = 135°$

$$Rot(\hat{z}, \theta_3) = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -0.5 & 0.866 & 0 \\ -0.866 & -0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$\theta_3 = -120°$

$p' = R_z \times R_y \times R_x \times p \,.(I)$

$$\boxed{p' = (-0.553, 0.457, -0.696)}$$

Calculations done on python and on Git!

$\boxed{Q3}$ (a) (1) $R_{02} = Rot(\hat{x}_0, \alpha) . I$

(2) $R_{03} = Rot(\hat{y}_0, \beta) \times R_{02}$

(3) $R_{04} = Rot(\hat{z}_0, \gamma) \times R_{03}$

$$\boxed{R_{04} = Rot(\hat{z}_0, \gamma) * Rot(\hat{y}_0, \beta) * Rot(\hat{x}_0, \alpha) . I}$$

(b) On step-3, if we change it to:

(3) $R_{34} = Rot(\hat{z}_3, \gamma) \Rightarrow$ Intrinsic rotation!!

$$\boxed{R_{04} = Rot(\hat{y}_0, \beta) * Rot(\hat{x}_0, \alpha) . I * Rot(\hat{z}_3, \gamma)}$$

$\boxed{Q4}$ Python Script of Q4 available on GitHub!

The two conditions for [3 X 3] matrix are:

(1) $\det|R| = 1$

(2) $R \times R^T = I$ (identity matrix)

$\rightarrow$ If both are true, then its a rotation matrix!