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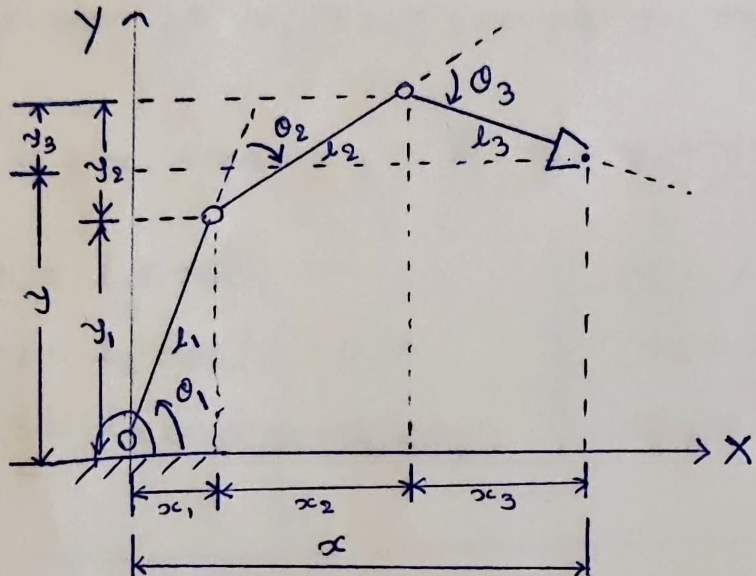
* 1.2 (1):-



From the geometry, x and y are vectorial summation of x_1, x_2, x_3 and y_1, y_2, y_3 respectively.

$\Rightarrow$ $x = x_1 + x_2 + x_3$ & $y = y_1 + y_2 + y_3$

$$x_1 = l_1 \cos\theta_1$$
$$x_2 = l_2 \cos(\theta_1 + \theta_2)$$
$$x_3 = l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$y_1 = l_1 \sin\theta_1$$
$$y_2 = l_2 \sin(\theta_1 + \theta_2)$$
$$y_3 = l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$\boxed{\begin{aligned} x &= l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ y &= l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3) \end{aligned}}$$

Also, orientation can be defined as,

$$\boxed{\phi = \theta_1 + \theta_2 + \theta_3}$$

Let's put $\sin\theta_1 = S_1$, $\sin(\theta_1 + \theta_2) = S_{12}$, $\sin(\theta_1 + \theta_2 + \theta_3) = S_{123}$

$\cos\theta_1 = C_1$, $\cos(\theta_1 + \theta_2) = C_{12}$, $\cos(\theta_1 + \theta_2 + \theta_3) = C_{123}$

for ease of calculations.

$$x = l_1 C_1 + l_2 C_{12} + l_3 C_{123}$$
$$y = l_1 S_1 + l_2 S_{12} + l_3 S_{123}$$
$$\phi = \theta_1 + \theta_2 + \theta_3$$

velocities will be,

$\dot{x}$, $\dot{y}$ and $\dot{\phi}$.

$$\boxed{\begin{aligned}
\dot{x} &= -l_1 S_1 \dot{\theta}_1 - l_2 S_{12}(\dot{\theta}_1 + \dot{\theta}_2) - l_3 S_{123}(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \\
\dot{y} &= l_1 C_1 \dot{\theta}_1 + l_2 C_{12}(\dot{\theta}_1 + \dot{\theta}_2) + l_3 C_{123}(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \\
\dot{\phi} &= \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3
\end{aligned}}$$

* 1.2(2):-

Here, the velocities are given as $\dot{x}, \dot{y}, \dot{\phi}$. As we know and derived in forward kinematics,

$$\dot{x} = -l_1 S_1 \dot{\theta}_1 - l_2 S_{12}(\dot{\theta}_1 + \dot{\theta}_2) - l_3 S_{123}(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)$$

$$\dot{y} = l_1 C_1 \dot{\theta}_1 + l_2 C_{12}(\dot{\theta}_1 + \dot{\theta}_2) + l_3 C_{123}(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)$$

$$\dot{\phi} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3$$

$$\Rightarrow \dot{x} = (-l_1 S_1 - l_2 S_{12} - l_3 S_{123})\dot{\theta}_1 + (-l_2 S_{12} - l_3 S_{123})\dot{\theta}_2 - l_3 S_{123}\dot{\theta}_3$$

$$\dot{x} = -(l_1 S_1 + l_2 S_{12} + l_3 S_{123})\dot{\theta}_1 - (l_2 S_{12} + l_3 S_{123})\dot{\theta}_2 - l_3 S_{123}\dot{\theta}_3$$

$$\dot{y} = (l_1 C_1 + l_2 C_{12} + l_3 C_{123})\dot{\theta}_1 + (l_2 C_{12} + l_3 C_{123})\dot{\theta}_2 + l_3 C_{123}\dot{\theta}_3$$

$$\dot{\phi} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3$$

In other words,

$$\dot{x} = \frac{\partial x}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial x}{\partial \theta_2}\dot{\theta}_2 + \frac{\partial x}{\partial \theta_3}\dot{\theta}_3$$

$$\dot{y} = \frac{\partial y}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial y}{\partial \theta_2}\dot{\theta}_2 + \frac{\partial y}{\partial \theta_3}\dot{\theta}_3$$

$$\dot{\phi} = \frac{\partial \phi}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial \phi}{\partial \theta_2}\dot{\theta}_2 + \frac{\partial \phi}{\partial \theta_3}\dot{\theta}_3$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial \phi}{\partial \theta_1} & \frac{\partial \phi}{\partial \theta_2} & \frac{\partial \phi}{\partial \theta_3} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$\dot{p} = J\dot{q} \quad , \text{where } \dot{p} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix}, \ \dot{q} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}, \ J = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial \phi}{\partial \theta_1} & \frac{\partial \phi}{\partial \theta_2} & \frac{\partial \phi}{\partial \theta_3} \end{bmatrix}$$

To find the joint velocities,

$$J\dot{q} = \dot{p}$$

$$J^{-1}J\dot{q} = J^{-1}\dot{p}$$

$$\boxed{\Rightarrow \dot{q} = J^{-1}\dot{p}}$$