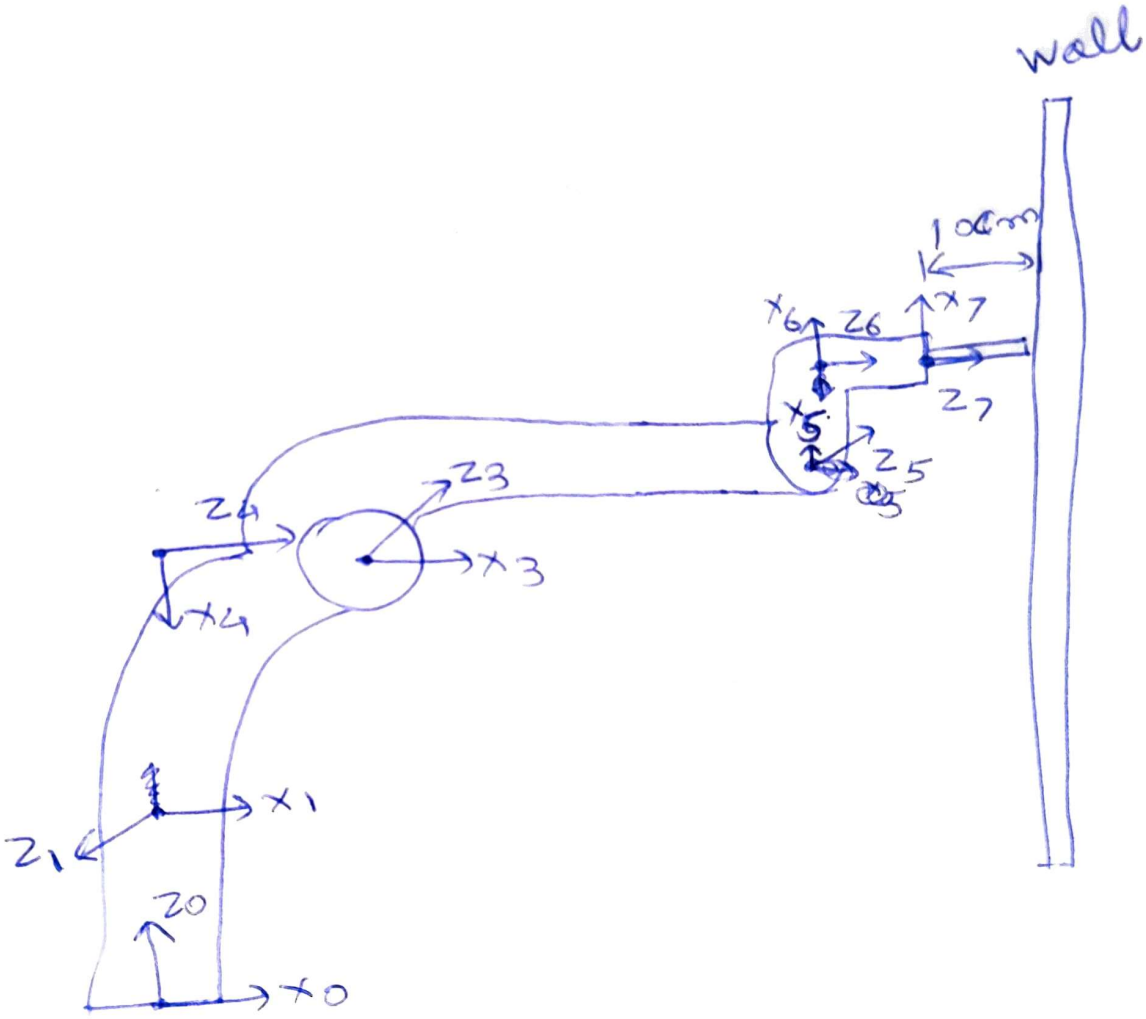
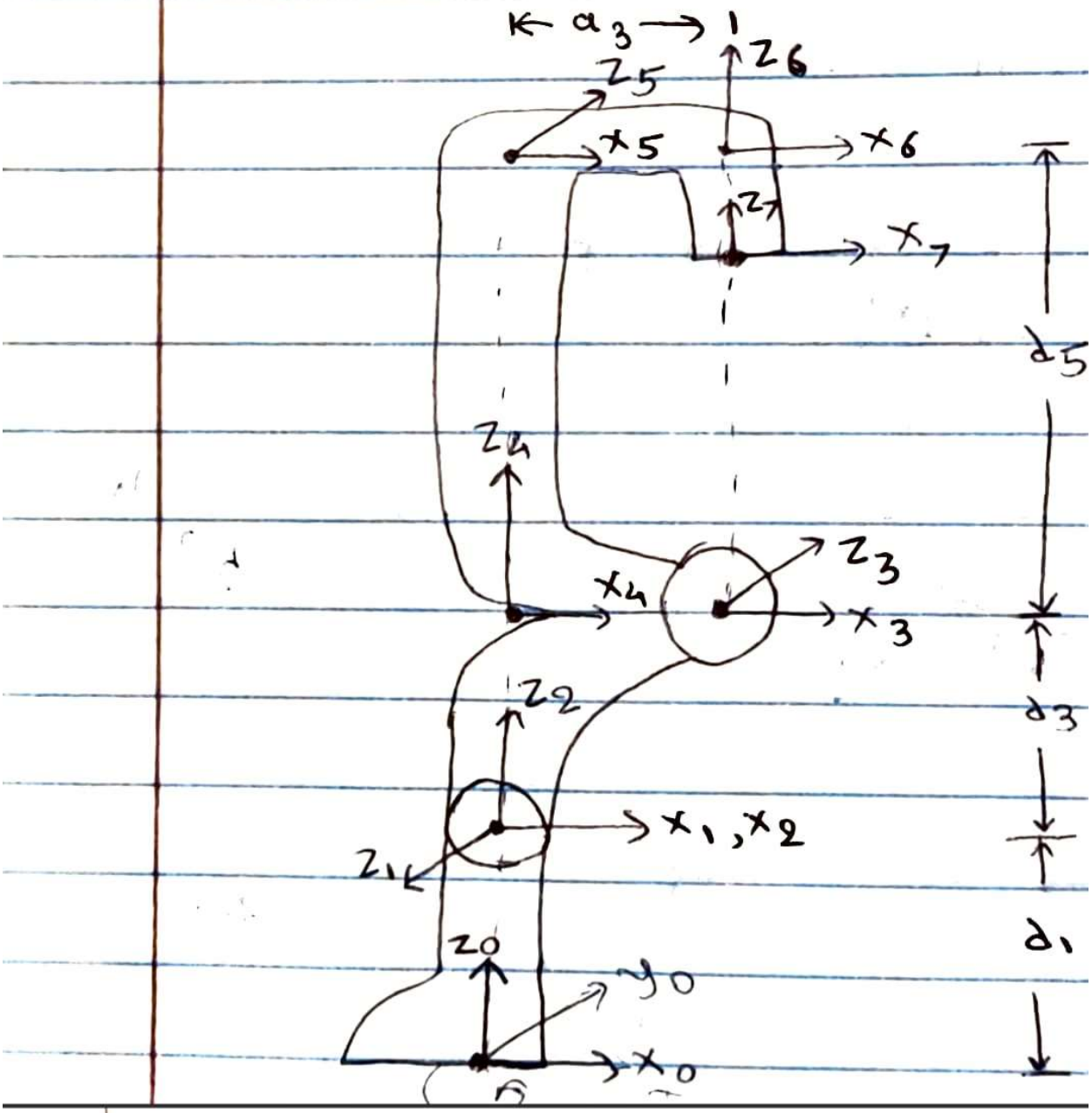

wall
10cm
x6
z6
x7
z7
x5
z5
z4
z3
x3
x4
x1
z1
z0
x0

1. Position Kinematics -

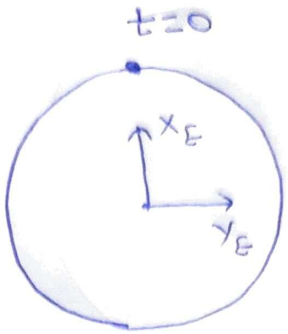


* D-H Parameters :-

Link	θ_i	d_i	a_i	α_i
1	θ_1	d_1	0	90
2	θ_2	0	0	-90
3	θ_3	d_3	a_3	-90
4	θ_4	0	$-a_3$	90
5	θ_5	d_5	0	90
6	θ_6	0	a_3	-90
7	θ_7	$-d_7$	0	0

In d_7, 10 cm added as to compensate for the length of pencil.

* End-effector frame:-



Parametric eqn of the circle w.r.t end-effector frame.

$$x_E = 0.1 \sin(\omega t + \pi/2)$$

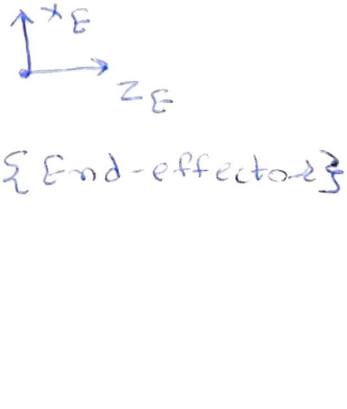
$$y_E = 0.1 \cos(\omega t + \pi/2)$$

at $t=0$, $x_E = 0.1$, $y_E = 0$ | $\omega = 2\pi/5$

$$\dot{x}_E = 0.1\,\omega \cos(\omega t + \pi/2)$$

$$\dot{y}_E = -0.1\,\omega \sin(\omega t + \pi/2)$$

* Transformation of velocity from end effector to Base frame:-



According to frame configurations,

$$x_E \cong z_0 \quad \Rightarrow \dot{x}_E = \dot{z}_0$$

$$y_E \cong -y_0 \quad \Rightarrow \dot{y}_E = -\dot{y}_0$$

$$\dot{z}_0 = 0.1\,\omega \cos(\omega t + \pi/2)$$

$$\dot{y}_0 = 0.1\,\omega \sin(\omega t + \pi/2)$$

Velocity matrix $\dot{x} = \begin{bmatrix} 0 \\ \dot{y}_0 \\ \dot{z}_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$\dot{x} = \begin{bmatrix} 0 \\ 0.1\,\omega \sin(\omega t + \pi/2) \\ 0.1\,\omega \cos(\omega t + \pi/2) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

* Jacobian :-

$$ {}^{0}_{1}T = \begin{bmatrix} | & | & | & | \\ | & | & | & | \\ 0 & 0 & z_1 & 0 \\ | & | & | & | \end{bmatrix} $$

$$ {}^{0}_{2}T = \begin{bmatrix} | & | & | & | \\ | & | & | & | \\ 0 & 0 & z_2 & 0 \\ | & | & | & | \end{bmatrix} $$

$$ {}^{0}_{4}T = \begin{bmatrix} | & | & | & | \\ | & | & | & | \\ 0 & 0 & z_4 & 0 \\ | & | & | & | \end{bmatrix} $$

$$ {}^{0}_{5}T = \begin{bmatrix} | & | & | & | \\ | & | & | & | \\ 0 & 0 & z_5 & 0 \\ | & | & | & | \end{bmatrix} $$

$$ {}^{0}_{6}T = \begin{bmatrix} | & | & | & | \\ | & | & | & | \\ 0 & 0 & z_6 & 0 \\ | & | & | & | \end{bmatrix} $$

$$
{}^{0}_{7}T = \begin{bmatrix} | & | & z_7 & x_p \\ | & | & | & | \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
J = \begin{bmatrix} \frac{\partial x_p}{\partial \theta_1} & \frac{\partial x_p}{\partial \theta_2} & \frac{\partial x_p}{\partial \theta_4} & \frac{\partial x_p}{\partial \theta_5} & \frac{\partial x_p}{\partial \theta_6} & \frac{\partial x_p}{\partial \theta_7} \\ z_1 & z_2 & z_4 & z_5 & z_6 & z_7 \end{bmatrix}
$$

$$\dot{q} = \bar{J}^{-1} \dot{x}$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \\ \dot{\theta}_7 \end{bmatrix} = \bar{J}^{-1} \begin{bmatrix} 0 \\ 0.1\,\omega \sin(\omega t + \pi/2) \\ 0.1\,\omega \cos(\omega t + \pi/2) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\theta_{new} = \theta_{old} + \dot{\theta} \times \Delta t$$