* 2.1 Trajectory Optimization

Considering the rotations of ψ_g, θ_g and ϕ_g w.r.t x, y, z (World frame) axes respectively. Rotation matrix,

$$R = R_{\phi_g} R_{\theta_g} R_{\psi_g}$$

$$= \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\phi\cos\theta & -\sin\phi & \sin\theta\cos\phi \\ \sin\phi\cos\theta & \cos\phi & \sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\phi\cos\theta & -\sin\phi\cos\psi + \sin\theta\sin\psi\cos\phi & \sin\phi\sin\psi + \sin\theta\cos\phi\cos\psi \\ \sin\phi\cos\theta & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi \\ -\sin\theta & \sin\psi\cos\theta & \cos\theta\cos\psi \end{bmatrix}$$

Converting to axis angle,

$$\theta = |k| = \cos^{-1}\left(\frac{\text{trace}(R) - 1}{2}\right)$$

For $\psi_g = 35^\circ, \theta_g = 15^\circ, \phi_g = 20^\circ$,

$$R = \begin{bmatrix} 0.9076 & -0.1406 & 0.3954 \\ 0.3303 & 0.8204 & -0.4664 \\ -0.2588 & 0.5540 & 0.7912 \end{bmatrix}$$

②

$$\theta = \cos^{-1}\left(\frac{trace(R) - 1}{2}\right)$$

$$\theta = \cos^{-1}\left(\frac{0.9076 + 0.8204 + 0.7912 - 1}{2}\right)$$

$$\boxed{\theta = 40.57^{\circ}}$$

$$\hat{k} = \frac{1}{2\sin\theta}\begin{pmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{pmatrix}$$

$$= \frac{1}{1.300}\begin{pmatrix} 0.5540 + 0.4664 \\ 0.3954 + 0.2588 \\ 0.3303 + 0.1406 \end{pmatrix}$$

$$= \frac{1}{1.3}\begin{pmatrix} 1.0204 \\ 0.6542 \\ 0.4709 \end{pmatrix}$$

$$\hat{k} = \begin{bmatrix} 0.7849 \\ 0.5032 \\ 0.3622 \end{bmatrix}$$

Therefore, it is ~~required~~ evident that in any case $\omega_x > \omega_y, \omega_z$.

Also, $\omega_x \leq 1$ deg/s, for shortest time, we need $(\omega_x)_{max} = 1$ deg/s.

$$\omega_x = 1 = 0.7849\,\omega$$

$$\boxed{\omega = 1.274 \text{ deg/s}}$$

$$\Rightarrow \boxed{\omega_y = 0.5032\,\omega = 0.641 \text{ deg/s}}$$
$$\Rightarrow \boxed{\omega_z = 0.3622\,\omega = 0.4614 \text{ deg/s}}$$

$$\omega = \theta_K / t$$

$$T_{(shortest)} = \theta_K / \omega$$
$$= 40.57 / 1.274$$
$$\boxed{T_s = 31.84\,s}$$