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# Documentation for MEMLS\_Active

## Microwave Emission Model of Layered Snowpacks

### Adapted to Include Backscattering

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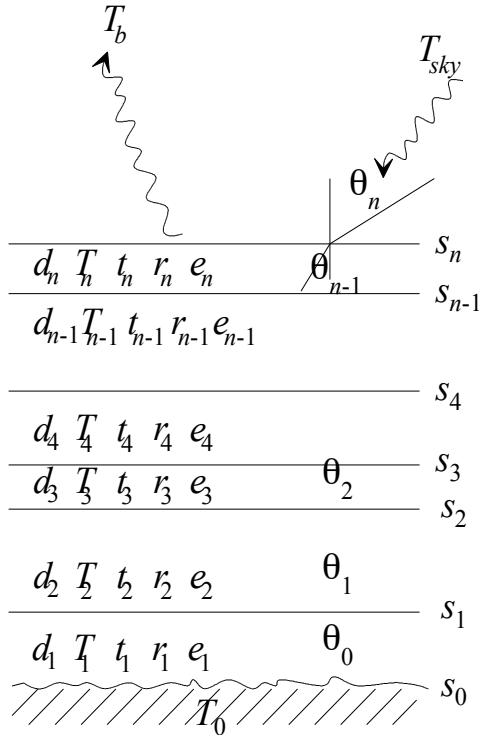
03 July 2014

### **Abstract**

A thermal Microwave Emission Model of Layered Snowpacks (MEMLS) was developed for the frequency range, 5 - 100 GHz. It is based on radiative transfer, using six-flux theory to describe multiple volume scattering and absorption, including radiation trapping due to internal reflection and a combination of coherent and incoherent superposition of reflections between the horizontal layer interfaces. In a first step, the scattering coefficient was determined empirically from measured snow samples, whereas the absorption coefficient, the effective permittivity, refraction and reflection at layer interfaces were based on physical models and on measured ice dielectric properties. In the later step, we determined the scattering coefficient by the Improved Born Approximation. Minor modifications were realised in Version 3, including weakly saline snow often present on sea ice. Here we describe a further extension of MEMLS to include a backscatter model. For this purpose the reflectivity is decomposed into diffuse and specular components, and the snowpack is allowed to be slightly undulated.

## 1. Introduction

Empirical observations revealed a wide range of microwave signatures for active and passive microwave remote sensing as shown e.g. by *Mätzler* (1987). The lack of realistic models to understand these signatures was the motivation for efforts leading to the Microwave Emission Model of Layered Snowpacks (MEMLS) in the 1990ies (*Mätzler* 1996; *Wiesmann and Mätzler* 1999). First, the microwave emission behaviour of single snow layers was investigated by *Weise* (1996) and later by *Wiesmann* (1997). Their measurements led to the empirical approach to determine the scattering coefficient of snow in the frequency range, 5 - 100 GHz, and correlation-length range, 0.05 - 0.3 mm (*Wiesmann et al.* 1998) and thus to a first version of MEMLS (*Wiesmann and Mätzler* 1999). In a further step, the model was extended to coarse-grained snow described by an exponential autocorrelation function with correlation lengths up to 0.6 mm (*Mätzler and Wiesmann*, 1999) by using the Improved Born Approximation of *Mätzler* (1998). MEMLS was validated with detailed radiometric and *in-situ* observations.

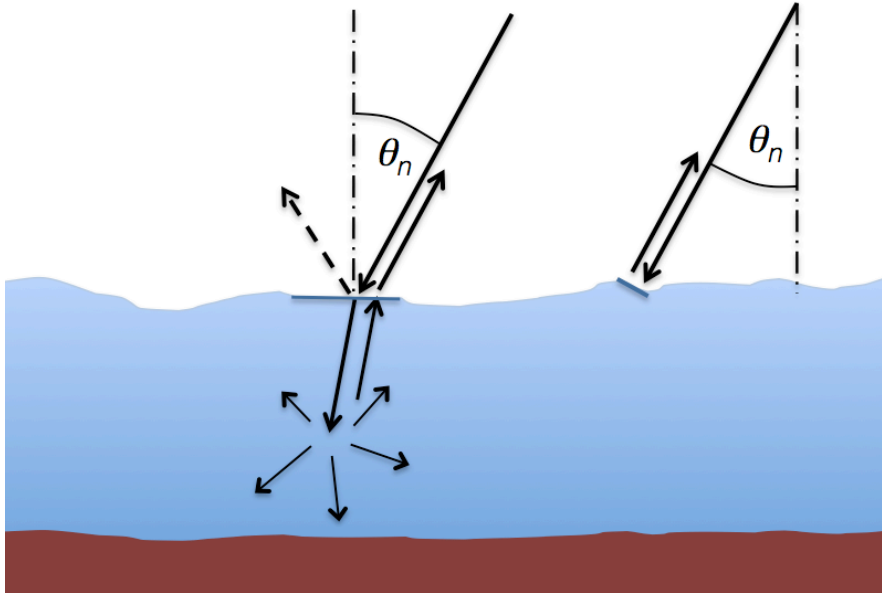


**Figure 1:** Geometry of the layered snowpack with an electromagnetic wave incident from above at an incidence angle  $\theta_n$ . Layer thickness  $d_j$ , temperature  $T_j$ , transmissivity  $t_j$ , volume reflectivity  $r_j$ , emissivity  $e_j$ , and interface reflectivity  $s_j$ , refracted angle  $\theta_{j-1}$  for layer number  $j$  ranging from 1(bottom) to  $n$  (top). Snow-ground reflectivity  $s_0$ , ground temperature  $T_0$ .

In MEMLS the snowcover is considered as a stack of  $n$  horizontal layers ( $j = 1, 2, \dots, n$ ) with planar boundaries at the snow surface and between snow layers (Figure 1). Each layer is characterized by snow parameters (layer thickness, correlation length, density, salinity (added in MEMLS Version 3), liquid water content and temperature) that determine the layer-radiative

properties of Figure 1. For simplicity polarisation is not indicated. The snow-ground interface is specified by an empirical reflectivity  $s_0$ . A sandwich model is used to combine internal scattering and reflections at the interfaces. Internal volume scattering is accounted by a two-flux model (up and down welling streams) derived from a six-flux approach (fluxes in all space directions). The absorption and scattering coefficients are functions of the six-flux parameters. The absorption coefficient can be obtained from density, frequency, temperature and salinity, and the scattering coefficient depends on the correlation length, density and frequency.

Here we develop a first version of MEMLS\_Active, an extension of MEMLS to a backscatter model. For this purpose we adapt the snowpack as shown in Figure 2. The undulations should yield a realistic treatment of backscattering due to roughness while the emission remains largely unaffected because specular backscatter is limited to a small fraction of the snowpack.



**Figure 2:** *Adaption of the geometry to slightly undulated snow. Waves incident at nadir angle  $\theta_n$  are refracted at the snow surface followed by volume scattering with backscatter; specular reflection without backscatter (left). Specular backscatter from tilted surface (right).*

## 2. Link between active and passive observables

At any given frequency and polarisation of electromagnetic radiation with incident direction  $(\mu_n, \phi_n)$  defined by nadir angle  $\theta_n$  (where  $\mu_n = \cos\theta_n$ ) and azimuth angle  $\phi_n$ , the reflectivity  $r$  of the surface is related to its emissivity  $e$  (in the reciprocal direction) by Kirchhoff's Law according to

$$r = 1 - e \quad (1)$$

For a more general description of Kirchhoff's Law, see Mätzler and Melsheimer (2006). Equation (1) relates the emissivity, the key quantity of passive microwaves, to the reflectivity, a

quantity related to scattering. It is this relation that allows us to link active and passive microwave remote sensing. The reflectivity represents the fraction of the incident radiation that is scattered in the hemisphere above the surface. This fraction can be represented as an integral over scattering directions in the upper hemisphere of the bistatic scattering function  $S$

$$r = \frac{1}{4\pi\mu_n} \int_{2\pi} S(\mu_n, \phi_n, \mu, \phi) d\Omega = \frac{1}{2\mu_n} \int_0^1 S(\mu_n, \mu) d\mu \quad (2)$$

Here  $d\Omega = d\mu d\phi$  is the infinitesimal solid-angle element in the scattered direction. The azimuth integration extends over 0 to  $2\pi$ , and the last expression is valid for azimuth-independent functions. The function  $S$  describes the scattering from incident direction  $(\mu_n, \phi_n)$  to  $(\mu, \phi)$  and vice versa. Chandrasekhar (1960) introduced the  $S$  function in his famous book on radiative transfer. He showed that  $S$  is reciprocal

$$S(\mu_n, \phi_n, \mu, \phi) = S(\mu, \phi, \mu_n, \phi_n) \quad (3)$$

Furthermore  $S$  is identical to the bistatic scattering cross section  $\sigma^0$  introduced by Ulaby et al. (1981), see their Equations (4.186-187), more exactly to the sum of the like- and cross-polarization terms,  $S = \sigma_{like}^0(\theta_n, \phi_n, \theta, \phi) + \sigma_{cross}^0(\theta_n, \phi_n, \theta, \phi)$ . It is also related to Peake's (1959) function  $\gamma = S/\mu_n$ , i.e. the  $1/\mu_n$  factor of Equation (2) is included inside this function. For completeness we note that  $S$  is related to, but differs from other definitions: the reflection function  $R$  used for instance by Kokhanovsky (2001) differs by a factor  $\pi$  from the Bidirectional Reflection Distribution Function (BRDF) used in optical remote sensing (Kasten and Raschke, 1974), and all quantities are related by

$$S(\mu_n, \phi_n, \mu, \phi) = \mu_n \gamma(\mu_n, \phi_n, \mu, \phi) = 4\mu_n \mu R(\mu_n, \phi_n, \mu, \phi) = 4\pi\mu_n \mu BRDF(\mu_n, \phi_n, \mu, \phi) \quad (4)$$

The  $S$  function can be highly complex. However for diffuse scattering, some simple functions are provided in the literature, see e.g. Mätzler and Rosenkranz (2007), the simplest one being for Lambert scattering

$$S_d = S_0 \mu_n \mu \quad (5)$$

where Index  $d$  indicates diffuse scattering, and  $S_0$  being a constant. By integration according to Equation (2), we find that the diffuse reflectivity  $r_d$  is independent of the incidence angle, namely  $r_d = S_0/4 = R$ , and thus equal to Kokhanovsky's  $R$ . The normalised backscattering cross section is given by  $\sigma_d^0 = S_d(\mu = \mu_n)$ , and expressing it by the  $r_d$ , we get

$$\sigma_d^0 = 4r_d \mu_n^2 \quad (6)$$

Indeed, Lambertian behaviour was found by the investigation of the HPACK Model for snow by Mätzler (2000). It is an extension of an earlier one-layer, active-passive model of Tsang et al. (1982) to include multiple-isotropic scattering in the snow as well as refraction and reflection at the snow surface. The combined effect led to Lambert scattering for the diffuse component.

Unspecified in Equation (6) is the separation of  $\sigma_d^0$  in its like- and cross-polarised components. For isotropic scatterers considered in HPACK, the first-order backscattering is like polarised, and cross polarisation requires higher-order scattering. However, natural snow grains have complex shapes, meaning that cross polarisation occurs for all scattering orders. Therefore we introduce an empirical relationship with a parameter,  $q$ , defining the cross-polarized part, and  $(1-q)$  representing the like-polarized part

$$\sigma_{d,ij}^0(\theta_n, \phi_n) = \begin{cases} (1-q)\sigma_{d,v}^0 & ; i = j = v \\ (1-q)\sigma_{d,h}^0 & ; i = j = h \\ q\sigma_d^0 & ; i = v, j = h; \text{ or } i = h, j = v \end{cases} \quad (7)$$

Here we took into account that the  $r_d$  and thus  $\sigma_d^0$  are slightly different for  $h$  and  $v$  polarisation. With  $\sigma_d^0$  on the bottom line of Equation (7) we understand the mean value of  $\sigma_{d,v}^0$  and  $\sigma_{d,h}^0$ . Now Equation (6) can be rewritten using the polarisation terms for incident waves at vertical and horizontal polarisation, respectively:

$$\sigma_{d,v}^0(\theta_n, \phi_n) = \sigma_{d,vv}^0 + \sigma_{d,hv}^0 = 4r_{d,v}\mu_n^2; \quad \sigma_{d,h}^0 = \sigma_{d,hh}^0 + \sigma_{d,vh}^0 = 4r_{d,h}\mu_n^2 \quad (8)$$

An additional contribution to backscattering is resulting from specular reflection as shown in Figure 2. It is limited to near-vertical incidence. For an undulating snow surface with a gaussian distribution of surface slopes, the backscattering coefficient of the specular term can be written as

$$\sigma_s^0 = r_{s,0} \frac{\exp[-\tan^2 \theta_n / (2m^2)]}{2m^2 \mu_n^4}; \text{ like polarisation, only} \quad (9)$$

where  $m^2$  is the mean-square slope, and  $r_{s,0}$  refers to  $r_s$  at normal incidence (Figure 2, right). This equation corresponds to the physical optics and to the geometrical optics solution for undulating surfaces, see Ulaby et al. (1982), Equations (12.45-46), and Kong (1986), Section 6.6. Here we generalise it from surface scattering to specular terms that fit to the observation geometry (i.e. specular reflectivity for local incidence angle  $\theta_l = 0$ ). Furthermore we note that Equation (9) describes like-polarised backscatter. For negligible anisotropy in the local surface

plane the same values are obtained for  $hh$  and  $vv$  polarisation, and the cross-polarisation terms are zero.

For both  $v$  and  $h$  polarisation the reflectivity is split in two components, the diffuse reflectivity  $r_d$  and the specular reflectivity  $r_s$ :

$$r = r_d + r_s \quad (10)$$

Whereas Equations (6) and (8) are valid for  $r_d$ , Equation (9) applies for  $r_s$ , but taken at normal incidence. With some additional effort to be described below, MEMLS provides both,  $r_d$  and  $r_s$ . And the total backscattering coefficient is the sum:

$$\sigma^0 = \sigma_d^0 + \sigma_s^0 \quad (11)$$

### 3. Determination of $r_d$ and $r_s$

#### 3.1 Determination of $r$

Apart from the physical temperatures of all snow layers including the ground temperature, the input parameters of the basic MEMLS program (`memlsmain.m`) also requires the brightness temperature  $T_{sky}$  of the incident sky radiation. The output is the brightness temperature  $T_b$  that is observed as upwelling radiation above the snowpack.

$$T_b = rT_{sky} + (1 - r)T_{eff} \quad (12)$$

Here  $T_{eff}$  is the emission-effective mean temperature of snow and ground. The reflectivity follows from computations of  $T_b$  ( $T_{b1}$ ,  $T_{b2}$ ) with two arbitrary, but different values of  $T_{sky}$  ( $T_{sky1}$ ,  $T_{sky2}$ ), such as 100 K and 0 K. The reflectivity then follows from

$$r = \frac{T_{b1} - T_{b2}}{T_{sky1} - T_{sky2}} \quad (13)$$

In MEMLS this computation is performed for both  $v$  and  $h$  polarisation by the routines, `fmain2.m` and `lmain2.m`.

#### 3.2 Determination of $r_s$

According to Figure 2 we need the specular reflectivities  $r_{s,v}$  and  $r_{s,h}$  at the observation incidence angle as well as  $r_{s,0}$  at normal incidence. For brevity we just write  $r_s$ , keeping in mind that we have to repeat the computation for all three cases. In many situations  $r_s$  can be identified by the reflectivity of the snow surface. This is especially true for wet snow and for snowpacks that consist of a single layer. However MEMLS has to consider all possible snow conditions. As an example if an old snowpack is covered by fresh snow, the dominant specular



layer may be the interface between the fresh and the old snow. Also ice lenses form dominant reflectors. Therefore MEMLS requires a program that estimates realistic specular reflectivities for all types of snow covers. The method is described in the Appendix. The result is expressed by an iterative equation

$$R_j = s_j + [(1-s_j)u_j]^2 R_{j-1} / (1 - u_j^2 s_j R_{j-1}); j=1, \dots, n \quad (14)$$

where  $s_j$  is the interface reflectivity on top of Layer  $j$ , and  $u_j = \exp(-\gamma_{ej} d_j / \mu_{j-1})$  is the coherent transmissivity of Layer  $j$  (Figure 1),  $\gamma_{ej}$  is the extinction coefficient, and  $d_j$  is the layer thickness. The specular reflectivity of the entire snowpack-ground system is given by

$$r_s = R_n \quad (15)$$

The iteration of Equation (14) starts at the lowest layer contributing to specular reflection. If this is the ground surface then  $j$  starts with  $j=1$ , and  $R_{j-1}$  on the right-hand side of Equation (14) is represented by  $R_0 = s_{s0}$ , the specular part of  $s_0$ . The first iteration gives  $R_1$ , and this value is inserted again on the right-hand side of the equation to yield  $R_2$ , and so on, until we get  $R_n$ .

#### 4. Primary input parameters

At a given frequency  $f$ , polarization  $p$  and observation incidence angle  $\theta_n$  the following snow physical parameters are required for each layer  $j$  ( $j = 1, \dots, n$ ):

- density  $\rho_j$
- temperature  $T_j$
- salinity  $S_{ppt}$
- liquid water content  $W_j$
- correlation length  $p_{cj}$
- vertical extent  $z_j$
- physical ground temperature  $T_0$
- snow-ground reflectivity  $s_0$ , and its specular part  $s_{s0}$

From these primary parameters secondary parameters are computed as described in the previous versions of MEMLS.

- Furthermore we need the parameter  $q$

#### 5. Implementation

MEMLS was written in MATLAB 5 [MathWorks, 1996], and presently it is running on MATLAB 7. All subroutines are commented. Below are some general remarks to each subroutine and to the references. With the subroutines *fmain* and *lmain* the brightness temperature versus incidence angle and frequency, respectively, of a given snowpack can be computed. These two programs shall illustrate in which order the subroutines must be used. The main program is *memlsmain*. The main input parameters are contained in the input ascii file where each line contains the data of one snow layer, with the top line representing the bottom layer just above the ground, and the bottom line representing the top snow layer:

Each line (layer) contains the following input data:

Layer no.,  
 Layer temperature (K),  
 Volumetric liquid-water content (vol. fraction): 0-1  
 Snow density (kg/m<sup>3</sup>)  
 Layer thickness (cm)  
 Snow salinity (parts per thousand)  
 Exponential correlation length (mm)

## 5.1 The main routines and subroutines

The new main routine is called 'amemlsmain.m'. Here we list it completely.

### *amemlsmain*

```

%function result = amemlsmain(fGHz, tetad, s0h, s0v, ss0h, ss0v, ifile,
Tgnd, sccho, m, q)
function result = amemlsmain(fGHz, tetad, s0h, s0v, ss0h, ss0v, ifile, Tsky,
Tgnd, sccho, m, q)
%   Basic MEMLS program, computes the (brightness temperatures Tv and Th),
%   diffuse and specular reflectivities and backscattering coefficients
%   of a snowpack at a given frequency and incidence angle.
% Input parameters:
%   fGHz:    frequency [GHz]
%   tetad:   incidence angle [deg]
%   s0h:     snow-ground reflectivity, h-pol
%   s0v:     snow-ground reflectivity, v-pol
%   ss0h:    specular part of snow-ground reflectivity, h-pol
%   ss0v:    specular part of snow-ground reflectivity, v-pol
%   ifile:   Snowpack-Input File with
%   layer-number, temp [K], volume fraction of liquid water, density [kg/m3],
%   thickness [cm], Salinity (0 - 0.1) [ppt], expon.corr.length [mm]
%   (Tsky: sky brightness temperature [K], not used presently)
%   Tgnd:    ground temperature [K]
%   sccho:   type of scattering coefficient (11 recommended)
%   m:       mean slope of surface undulations (typical 0.05 to 0.1)
%   q:       cross pol fraction (typical 0.1 to 0.3)
% Version history:
%   3.0      ma 02.04.2007
%   3.1      ma 05.04.2007 adapted to saline snow
%   4.0      ma 2013 addition of backscatter coefficient 'MEMLS-Active'
% Uses:
%   vK2eps, epswet, epsaliceimag, pfadi, pfadc, polmix, fresnelyc,
%   slred, sccoeff, rt, layer
%
% Aux. input parameter for dielectric model of snow:
graintype=1; % 1, 2 or 3 for different assumptions:
%"1" empirical snow measurements, "2" small spheres, "3" thin shells
forwa=4; % empirical enhancement factor of extinction by diffraction
% alternative would be the use of the formula of Hallikainen & al 1996
teta = (tetad * pi) / 180; % Transformation from degrees to radians
c0=0.299793; % vac. speed of light in m/ns

y=load(ifile)
di=y(:,5);
y1=find(di>0);
num = y(y1,1);
Ti = y(:,2);
Wi = y(:,3);
roi = y(:,4);
Sppt = y(:,6);
pci = y(:,7);
  
```

```

N = length(num);
if N == 0
    return % test if there is there a layer at all
end
roi = roi./1000; % transforms density to units of g/cm3
di = di./100; % transforms thicknesses to units of m
cc=vK2eps(roi, graintype);
v =cc(:,1); % ice volume fraction
kq=cc(:,2); % K^2 ratio
epsid=cc(:,3); % epsilon of dry snow
nid=sqrt(epsid); % real refract.index of dry snow
freq=fGHz;
eii=epsaliceimag(freq,Ti,Sppt); % imag epsilon of saline ice
epsiid=v.*eii.*kq.*nid; % imag epsilon of dry snow component
epsd=epsid+i*epsiid;
eps = epswet(freq,Ti,Wi,epsd);
epsi =real(eps); % real epsilon of snow, dry or wet
epsii=imag(eps); % imag epsilon of snow, dry or wet
gai = (4*pi*freq).*(imag(sqrt(eps))./c0; %absorption coeff (1/m)
ns = sqrt(epsii); % approx. real refractive index of snow
tei = [asin(sin(teta)./ns);teta];
dei = pfadi(tei,di);
[sih,siv] = fresnelyc(tei,[epsi;1]);
[rnum,rroi,repai,repaii,rtei,rsih,rsiv,rdi,rdei,rTi,rpci,rWi,rgai,rkq]=
slred(num,roi,epsi,epsii,tei,sih,siv,di,dei,Ti,pci,freq,Wi,gai,kq);
N = length(rnum) % number of incoherent layers
[gbih,gbiv,gs6,ga2i] = sccoeff(rroi,rTi,rpci,freq,repai,rgai,sccho,rkq);
[rdei,rtei,tscat] = pfadc(teta,rdi,repai,gs6);

% Comp of specular reflectivity rsh, rsv, rs0:
rsih = [s0h;rsih];
rsiv = [s0v;rsiv];
rextc= rgai+forwa*gs6; % extinct coeff for specular/coherent reflections
Rh(1)=ss0h; Rv(1)=ss0v; % coherent refl at the ground surface
u=exp(-rextc.*rdei);
u2=u.*u;
for j=2:N+1
    Rh(j)=rsih(j)+Rh(j-1)*((1-rsih(j))*u(j-1))^2/(1-u2(j-1)*rsih(j)*Rh(j-1));
    Rv(j)=rsiv(j)+Rv(j-1)*((1-rsiv(j))*u(j-1))^2/(1-u2(j-1)*rsiv(j)*Rv(j-1));
end;
rsh=Rh(N+1);
rsv=Rv(N+1);
rs0=0.5*(rsh+rsv);

[rsih,rsiv] = polmix(tscat,rsih,rsiv);
rtei.*180./pi;

% The following 6 lines are to be used if Tbh and Tbv are required:
[ri,ti]= rt(ga2i,gbih,rdei);
Dh = layer(ri,rsih,ti,rTi,Tgnd,Tsky);
Tbh = (1-rsih(N+1))*Dh(N) + rsih(N+1)*Tsky;
[ri,ti]= rt(ga2i,gbiv,rdei);
Dv = layer(ri,rsiv,ti,rTi,Tgnd,Tsky);
Tbv = (1-rsiv(N+1))*Dv(N) + rsiv(N+1)*Tsky;

% Tb for Tsky = 0 K:
[ri,ti]= rt(ga2i,gbih,rdei);
Dh0 = layer(ri,rsih,ti,rTi,Tgnd,0);
Tbh0 = (1-rsih(N+1))*Dh0(N);
[ri,ti]= rt(ga2i,gbiv,rdei);
Dv0 = layer(ri,rsiv,ti,rTi,Tgnd,0);

```

```

Tbv0    = (1-rsiv(N+1))*Dv0(N);

% Tb for Tsky = 100 K:
[ri,ti]= rt(ga2i,gbih,rdei);
Dh100   = layer(ri,rsih,ti,rTi,Tgnd,100);
Tbh100  = (1-rsih(N+1))*Dh100(N) + rsih(N+1)*100;
[ri,ti]= rt(ga2i,gbiv,rdei);
Dv100   = layer(ri,rsiv,ti,rTi,Tgnd,100);
Tbv100  = (1-rsiv(N+1))*Dv100(N) + rsiv(N+1)*100;

rv       = 0.01*(Tbv100 - Tbv0); % Total reflectivity v pol
rh       = 0.01*(Tbh100 - Tbh0); % Total reflectivity h pol
rdv      = rv-rsv; % diffuse reflectivity v pol
rdh      = rh-rsh; % diffuse reflectivity h pol

mu=cos(teta);
mu2=mu.*mu;
m2=m*m;
sigma0dv =4*rdv.*mu2;
sigma0dh =4*rdh.*mu2;
sigma0dvv= (1-q)*sigma0dv;
sigma0dhh= (1-q)*sigma0dh;
sigma0hv  = q*0.5*(sigma0dv+sigma0dh);
xpon      = -(tan(teta)).^2/(2*m2);
sigma0s   = rs0*exp(xpon)/(2*m2.*mu2.*mu2);
sigma0vv  = sigma0dvv + sigma0s;
sigma0hh  = sigma0dhh + sigma0s;
result.reflec = [rv,rh,rdv,rdh,rsv,rsh,rs0];
result.sigma0 = [sigma0vv,sigma0hh,sigma0hv];
result.Tb     = [Tbv,Tbh];

```

### *memlsmain*

```

function result = memlsmain(fGHz,tetad,s0h,s0v,ifile,Tsky,Tgnd,sccho)
% Basic MEMLS program, computes the brightness temperatures Tv and Th
% of a snowpack at a given frequency and incidence angle.
%   fGHz: frequency [GHz]
%   tetad: incidence angle [deg]
%   s0h: snow-ground reflectivity, h-pol
%   s0v: snow-ground reflectivity, v-pol
%   ifile: Snowpack-Input File (ascii) with
% layer-number, temp [K], volume fraction of liquid water, density [kg/m3],
% thickness [cm], Salinity (0 - 0.1) [ppt], expon.corr.length [mm]
%   Tsky: sky brightness temperature [K]
%   Tgnd: ground temperature [K]
%   sccho: type of scattering coefficient (11 recommended)
% Version history:
%   3.0   ma 02.04.2007
%   3.1   ma 05.04.2007 adapted to saline snow
% Uses:
%   vK2eps, epswet, epsaliceimag, pfadi, pfadc, polmix, fresnelyc,
%   slred, sccoeff, rt, layer
% Copyright (c) 2007 by the Institute of Applied Physics,
% University of Bern, Switzerland
% Aux. input parameter:
graintype=1; % 1, 2 or 3 for different assumptions:
%"1" empirical snow measurements, "2" spheres, "3" thin shells
output: result=[Tbv,Tbh];

```

### *fmain2*

```

function result=fmain2(freq,tetal,teta2,s0h,s0v,ifile,Tgnd,sccho)
% MEMLS program, plots and lists the incidence angle(deg), emissivities
% eh(%), ev(%), the emitted brightness temperatures Tebh(K), Tebv(K),

```

```

% and the effective physical temperatures Teffh(K), Teffv(K)
% of the emitting surface (ground and snowpack) of a layered snowpack.
%   fGHz: frequency [GHz]
%   teta1: start inc. angle [deg]
%   teta2: stop incidence angle [deg]
%   s0h: snow-ground reflectivity
%   s0v: snow-ground reflectivity
%   ifile: Snowpack-Input File with
% layer-number, temp [K], vol. fraction of liquid water, density [kg/m3],
% thickness [cm], Salinity (0 - 0.1) [ppt], expon.corr.length [mm]
%   Tgnd: ground temperature [K]
%   sccho: type of scattering coefficient (11 recommended)
% Version history:
%   3.0   ma 04.04.2007 adapted from fmain
%   3.1   ma 05.04.2007 adapted to saline snow
% Uses:
%   vK2eps, epswet, epsaliceimag, pfadi, pfadc, polmix, fresnelyc,
%   slred, sccoeff, rt, layer
% Copyright (c) 2007 by the Institute of Applied Physics,
% University of Bern, Switzerland
% Aux. input parameter:
graintype=1; % 1, 2 or 3 as in memlsmain:
result=[x',yh',yv',yeh',yev',yeh'./yh',yev'./yv'];
% Incidence angle(deg), eh(%), ev(%), Tebh(K), Tebv(K), Teffh(K), Teffv(K)

```

## *lmain2*

```

function result = lmain2(f1,f2,tetad,s0h,s0v,ifile,Tgnd,sccho)
% MEMLS main program, plots and lists the frequencies(GHz), emissivities
% eh(%), ev(%), the emitted brightness temperatures Tebh(K), Tebv(K),
% and the effective physical temperatures Teffh(K), Teffv(K)
% of the emitting surface (ground and snowpack) of a layered snowpack.
%   f1: start frequency [GHz]
%   f2: stop frequency [GHz]
%   tetad: incidence angle [deg]
%   s0h: snow-ground reflectivity
%   s0v: snow-ground reflectivity
%   ifile: Snowpack-Input File with
% layer-number, temp [K], vol. fraction of liquid water, density [kg/m3],
% thickness [cm], Salinity (0 - 0.1) [ppt], expon.corr.length [mm]
%   Tgnd: ground temperature [K]
%   sccho: type of scattering coefficient (11 recommended)
%
% Version history:
%   3.0   ma 04.04.2007 adapted from lmain
%   3.1   ma 05.04.2007 adapted to saline snow
% Uses:
%   vK2eps, epswet, epsaliceimag, pfadi, pfadc, polmix, fresnelyc,
%   slred, sccoeff, rt, layer
% Copyright (c) 2007 by the Institute of Applied Physics,
% University of Bern, Switzerland
% Aux. input parameter:
graintype=1; % 1, 2 or 3 as in memlsmain
result=[x',yh',yv',yeh',yev',yeh'./yh',yev'./yv'];
% Frequency(GHz), eh(%), ev(%), Tebh(K), Tebv(K), Teffh(K), Teffv(K)

```

## *borna*

```

function [gb6,gc6,gf6,gs6] = borna(k0,vfi,pci,epsi,eice,kq)
% calculates the scattering coefficient using Born Approximation
% [gb6,gc6,gf6,gs6] = borna(k0,vfi,pci,epsi,eice,kq)
%   gb6: 6-flux back scattering coefficient
%   gc6: 6-flux cross scattering coefficient
%   gf6: 6-flux forward scattering coefficient

```

```

%      gs6: 6-flux scattering coefficient
%      k0: vacuum wave number (1/m)
%      vfi: volume fraction of ice
%      pci: correlation length (mm)
%      epsi: real dielectric constant of snow
%      eice: real dielectric constant of ice
%      kq : squared E-field ratio (from bornsnk)
% Version history:
%      1.0      wi 27.05.98,
%      3.0      lb 2.3.2007, 3.1 cm 2.4.2007
% Uses: integrmui
% Copyright (c) 2007 by the Institute of Applied Physics,
% University of Bern, Switzerland

```

### *epsr*

Calculates the dielectric permittivity  $\epsilon'$  of dry snow from snow density  $\rho$ .

### *epsaliceimag*

```

function z = epsaliceimag(fGHz,TK,Sppt)
% MATLAB Function for calculating the imaginary part of the
% relative permittivity of pure ice in the microwave region,
% according to C. Matzler, IET Book (2006), chapter on ice
% Input:
% TK = temperature (K), range 20 to 273.15
% fGHz = frequency in GHz
% Sppt = salinity in parts per thousand

```

### *epsaliceimag*

```

function z = epsicereal(TK)
% Function for calculating the real relative permittivity
% of pure ice in the microwave region, according to
% Matzler, C.(ed),"Thermal Microwave Radiation - Applications for Remote
% Sensing", IET, London, UK (2006), Chapter 5.
% Input:
% TK = temperature (K), range 240 to 273.15
z = 3.1884 + 9.1e-4*(TK-273);

```

### *epswet*

```

function result = epswet(f,Ti,Wi,epsd);
% calculates complex dielectric constant of wet snow
% using Maxwell-Garnett Mixing rule of water in dry snow
% for prolate spheroidal water with experimentally determined
% depolarisation factors.
% Water temperature is at 273.15 K, with epsilon
% of water from Liebe et al. 1991.
%      epsd: complex epsilon of dry snow
%      f: frequency [GHz]
%      Ti: physical snow temperature [K]
%      Wi: wetness [volume fraction]
% Version history:
%      1.0      wi 15.7.95
%      2.0      ma 31.5.2005: Wi is volume fraction (not %)
%      3.0      ma 2.4.2007 : adjustments, new function name
% Uses: epswater (since Version 3)
% Copyright (c) 1997 by the Institute of Applied Physics,
% University of Bern, Switzerland

```

### *fresnelyc*

calculates the interface reflectivity  $s_h$ ,  $s_v$  of the snow slabs (using Fresnel) neglecting the imaginary part of the dielectric properties of snow  $\epsilon'' = 0$ .

### *fresnelrc*

calculates the interface reflection coefficients  $F_h$ ,  $F_v$  of the snow slabs (after Fresnel) neglecting the imaginary part of the dielectric properties of snow  $\epsilon'' = 0$ .

### *layer*

calculates the array D.

### *pfadc*

calculates the effective path length  $d'$  in a layer.

### *pfadi*

calculates the effective path length  $d'$ .

### *polder*

is a subroutine to solve the *Polder and van Santen* [1946] Equation for spheroids  $A_1 = A_2 = 0.5 (1 - A_3)$ .

### *polmix*

calculates the polarization mixing of the interface reflectivities of each layer.

### *rt*

computes the layer reflectivity  $r$  and transmissivity  $t$  from the two-flux coefficients  $\gamma_a'$ ,  $\gamma_b'$  and the path length  $d'$

### *sccoeff*

computes the six-flux scattering coefficients from snow structure information with different choices, expressed by the parameter *sccho*

```
function [gbih,gbiv,gs6,ga2i] = sccoeff(roi,Ti,pci,freq,epsi,gai,sccho,kq)
% calculates the scattering coefficient from structural parameters
% different algorithms can be chosen, by changing "sccho"
%     gbih: 2-flux scattering coefficient at h pol
%     gbiv: 2-flux scattering coefficient at v pol
%     gs6: 6-flux scattering coefficient
%     ga2i: 2-flux absorption coefficient
%     roi: density
%     Ti: physical temperature
%     pci: correlation length
%     freq: frequency
%     epsi: real part of relative dielectric constant of dry snow
%     gai: absorption coefficient
%     sccho: scattering coefficient algorithm chosen
%     kq: squared field ratio K^2 (for sccho=12)
% Version history:
% 1.0b wi 15.7.95
% 1.0 wi 23.9.97 bug fixed
% 1.1 wi 26.9.97 latest fit on experimental data was added (option 7)
% 1.2 wi 13.10.97 option 8 added, adapted scattering of a shell/sphere
% 1.3 wi 4.11.97 option 9, 10 and 11 added
% 1.4 wi 27.05.98 born approximation added (borna.m)
```

```
% 3.0      ma 03.04.2007 adapted to Version 3
% Uses:
% borna, epsicereal (both for sccho = 12 only)
% Copyright (c) 2007 by the Institute of Applied Physics,
% University of Bern, Switzerland
```

### *slred*

Given a special snow profile, then at each frequency  $f$  and incidence angle  $\theta_n$ , the following tasks have to be performed:

- Check if there is a volume scattering layer at all ( $2p_j > 4.7$ ). If there is no such layer, the snowpack emission model cannot be applied to this situation.
- Remove thin layers from the bottom until a scattering layer is found. The thickness of the scattering layer increases by the extent of the removed layers.
- Locate thin layers elsewhere from the table of the snow profile.
- Locate the thickest layer if more than one subsequent layer is found to be thin and ignore the others.
- Repeat until all thin layers have been identified.
- Compute the coherent reflectivities (at h and v polarization) of the thin layers and replace the interface reflectivities of the adjacent layers by the coherent reflectivities
- Ignore volume scattering within a thin layer, e.g. remove it from the input table of scattering layers. Only regular layers are left. The new input table is now ready to be used by MEMLS.

### *vK2eps*

```
function result = vK2eps(roi,graintype)
% Computes volume fraction, K^2 (the squared ratio between internal
% and external E-field) and real diel. constant in the
% Effective-Medium Approximation (Polder and van Santen, 1946) of
% dry snow with given depolarization factors, A=A1=A2 and A3=1-2A
% all versus snow density, roi (graintype=1), for spheres
% (graintype=2), or spherical shells (graintype=3).
%      kq:      squared E field ratio
%      roi:     dry snow density (g/cm3)
%      graintype: 1 (snow), 2 (spheres), 3 (thin sph. shells)
% Version history:
%      1.0      wi 29.5.98,
%      3.0      lb 2.3.2007, cm 2.4.2007
% Uses: epsr
% Copyright (c) 2007 by the Institute of Applied Physics,
% University of Bern, Switzerland
result=[v, kq, epseff];
```

## 6. Examples

TBD

## 7. References

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## Appendix: Specular reflectivity of the layered snowpack

Note 17, Christian Mätzler, IAP-Univ. of Bern, September 1999, adapted May 2013

The purpose of this note is to get an expression for the specular part of the reflectivity of a layered snowpack, in order to separate it from the diffuse part by subtraction from the total reflectivity using MEMLS. It is assumed here that all layer interfaces are parallel and horizontal in order to produce specular reflection. Separation between diffuse and specular reflection is required in bistatic-scattering and in backscatter models.

### 1. Model parameters

- transmissivity  $u_j$  of directed radiation in layer  $j$  ( $j=1$  to  $n$ ),
- the interface reflectivity  $s_j$  on top of each layer ( $j=1$  to  $n$ ),
- the total specular reflectivity  $R_j$  resulting from all layers below  $z_j$  ( $j=1$  to  $n$ ),
- the reflectivity  $s_0 = s_{s0} + s_{d0}$  at the bottom of the snowpack, consisting of a specular  $s_{s0}$  and of a diffuse  $s_{d0}$  component.

Assuming smooth interfaces allows us to apply the Fresnel formulae for  $s_j$ . The propagation angle  $\theta_{j-1}$  in layer  $j$  is given by Snell's law of refraction.

### 2. The layered snowpack model

The plane-parallel snowpack corresponds to the one of MEMLS and is shown in Figure A1. The parameters assigned to a specific layer are shown in more detail in Figure A2. Further explanations will be given below.

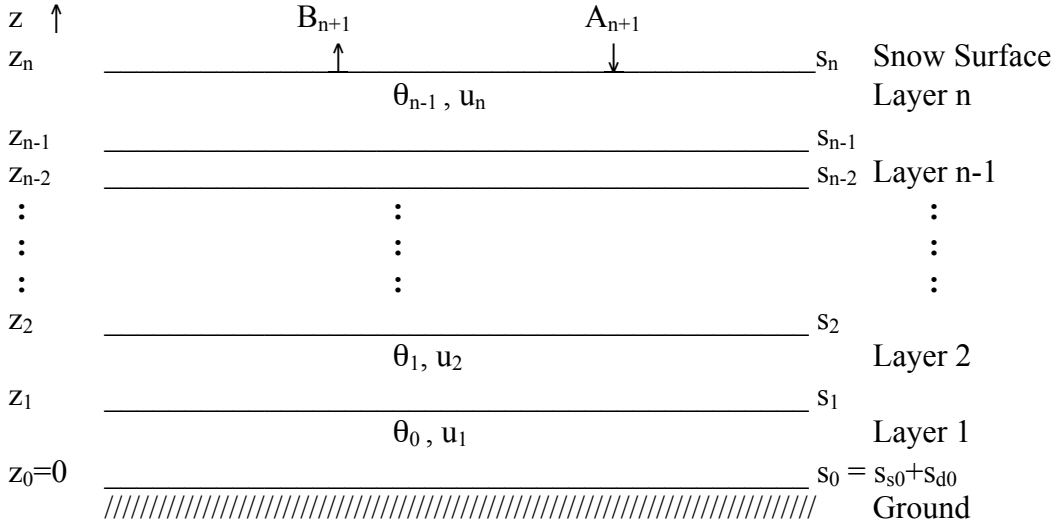


Figure A1: The  $n$ -layer snowpack.

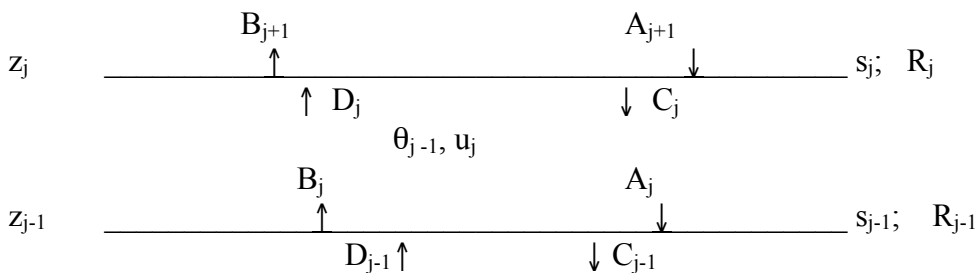


Figure A2: The parameters of a selected layer,  $j$ .

Note that  $A_j$ ,  $B_j$ ,  $C_j$  and  $D_j$  are downwelling and upwelling intensities just above and below the boundaries of the snow layer. In order to compute the specular reflectivity we select sufficiently large directional intensities to allow thermal radiation to be neglected. This means that we illuminate the scene from above with an intensive noise source.

The transmissivity  $u_j$  of the directed intensity passing through layer  $j$  is given by

$$u_j = \exp(-\gamma_e \cdot d_j / \cos \theta_{j-1}) \quad (1)$$

where  $d_j = z_j - z_{j-1}$  is the thickness of layer  $j$  and where  $\gamma_e = (\gamma_a + \gamma_b + \gamma_r + 4\gamma_c)$  is the extinction coefficient (here determined by the six-flux model).

Considering Figure A2 we can derive the following equations relating the directional intensities at the boundaries:

$$A_j = u_j C_j \quad ; j=1 \text{ to } n \quad (2)$$

$$B_j = R_{j-1} A_j \quad (3)$$

$$C_j = (1-s_j) A_{j+1} + s_j D_j \quad (4)$$

$$D_j = u_j B_j \quad (5)$$

$$B_{j+1} = R_j A_{j+1} = (1-s_j) D_j + s_j A_{j+1} \quad (6)$$

Furthermore at the bottom we have

$$R_0 = s_{s0} \quad (7)$$

where  $s_{s0}$  is the specular part of the ground-snow interface reflectivity. In order to solve these equations for  $R_j$ , we first eliminate the  $D_j$  and  $C_j$  in (4) and (6) by using (2) and (5). In this way we get

$$u_j(1-s_j) A_{j+1} = A_j - u_j^2 s_j B_j \quad (8)$$

$$B_{j+1} = (1-s_j) u_j B_j + s_j A_{j+1} \quad (9)$$

Dividing (8) by  $A_j$  and (9) by  $A_{j+1}$  we get, together with (6) and (3)

$$u_j(1-s_j) A_{j+1}/A_j = 1 - u_j^2 s_j R_{j-1} \quad (8)$$

$$R_j = (1-s_j) u_j R_{j-1} A_j/A_{j+1} + s_j \quad (9)$$

Eliminating the ratio  $A_{j+1}/A_j = (1 - u_j^2 s_j R_{j-1})/[u_j(1-s_j)]$  in (9) leads to

$$R_j = s_j + [(1-s_j) u_j]^2 R_{j-1} / (1 - u_j^2 s_j R_{j-1}) \quad (10)$$

Equation (10) is an iterative equation to solve for the total specular reflectivity  $r_s = R_n$ , starting from (7), then inserting  $R_0$  on the right-hand side of (10) for  $j=1$  to find  $R_1$ . This value is then used on the right-hand side of (10) for  $j=2$  to find  $R_2$ . This procedure is continued until we get to  $j=n$ .

The described procedure is to be applied to horizontal and vertical polarization separately. For  $v$  polarization we call  $R_n = r_{sv}$ , and for  $h$  polarization we call  $R_n = r_{sh}$ . These are the specular parts of the total reflectivities  $r_v$  and  $r_h$  of MEMLS. The diffuse components  $r_{dv}$  and  $r_{dh}$  are thus

$$r_{dv} = r_v - r_{sv} ; r_{dh} = r_h - r_{sh} \quad (11)$$

The diffuse components should be nearly the same at both polarizations. This property can be tested by computing  $r_{dv}$  and  $r_{dh}$  from (11), by taking the total reflectivities from MEMLS and the specular reflectivities from the method described here.