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Recurrent Fuzzy CMAC for Nonlinear System Modeling

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Abstract. Normal fuzzy CMAC neural network performs well because of its fast learning speed and local generalization capability for approximating nonlinear functions. However, it requires huge memory and the dimension increases exponentially with the number of inputs. In this paper, we use recurrent technique to overcome these problems and propose a new CMAC neural network, named recurrent fuzzy CMAC (RFCMAC). Since the structure of RFCMAC is more complex, normal training methods are difficult to be applied. A new simple algorithm with a time-varying learning rate is proposed to assure the learning algorithm is stable.

1 Introduction

The Cerebellar Model Articulation Controller (CMAC) presented by Albus [1] is an auto-associative memory feedforward neural network, which is a simplified mode of the cerebellar based on the neurophysiological theory. A very important property of CMAC is that it has faster convergence speed than MLP neural networks. Many practical applications have been presented in recent literatures [9].

Since the data in CMAC is quantized and the knowledge information cannot be presented, fuzzy CMAC(FCMAC) was proposed in [2] where fuzzy set (fuzzy label) is used as the input clusters instead of crisp set. Compared to normal CMAC which associates with numeric values, FCMAC can model a problem using linguistic variables based a set of If-Then fuzzy rules. Also FCMAC is more robust, highly intuitive and easily comprehended [9].

A major drawback of FCMAC is that its application domain is limited to static problems due to its feedforward networks structure. Recurrent techniques incorporate feedback, they have powerful representation capability and can overcome disadvantages of feedforward networks [4]. Recurrent CMAC network naturally involves dynamic elements in the form of feedback connections. Its architecture is a modified model of the conventional CMAC network to attain a small number of memory, it includes delay units in CMAC. There are several types of recurrent

structures, for examples, external feedback [18], internal recurrence [11], the recurrent loop is in the premise part [8], or the recurrent loop is in the consequence part [7]. In this paper, we apply recurrent method on CMAC and propose a new CMAC, named recurrent fuzzy CMAC (RFCMAC).

It is well known that normal identification algorithms (for example, gradient decent and least square) are stable for ideal conditions. In the presence of unmodeled dynamics, these adaptive procedures can go to instability easily. The lack of robustness of the parameter identification was demonstrated in [3] and became a hot issue in 1980s, when some robust modification techniques for adaptive identification was suggested [5]. Some robust modifications must be applied to assure stability with respect to uncertainties. Projection operator is an effective tool to guarantee fuzzy modeling bounded [14]. It was also used by many fuzzy-neural systems [10]. Another general approach is to use robust adaptive techniques [5] in fuzzy neural modeling. For example, applied a switch σ -modification to prevent parameters drift. By using passivity theory, we successfully proved that for continuous-time recurrent neural networks, gradient descent algorithms without robust modification were stable and robust to any bounded uncertainties [16], and for continuous-time identification they were also robustly stable [17]. Nevertheless, do recurrent fuzzy CMAC (RFCMAC) has the similar characteristics?.

In this paper backpropagation-like approach is applied to nonlinear system modeling via RFCMAC, where feedback is in the fuzzification layer of RFCMAC. The gradient decent learning is used. Time-varying learning rate is obtained by input-to-state stability (ISS) approach to update the parameters of the membership functions, this learning law can assure stability in the training process.

2 Preliminaries

The main concern of this section is to understand some concepts of ISS. Consider the following discrete-time state-space nonlinear system

$$\begin{aligned} x(k+1) &= f[x(k), u(k)] \\ y(k) &= x(k+1) \end{aligned} \quad (1)$$

where $u(k) \in \mathbb{R}^m$ is the input vector, $x(k) \in \mathbb{R}^n$ is a state vector, and $y(k) \in \mathbb{R}^l$ is the output vector. f is general nonlinear smooth function $f \in C^\infty$. Let us recall the following definitions.

Definition 1. (a) If a function $\gamma(s)$ is continuous and strictly increasing with $\gamma(0) = 0$, $\gamma(s)$ is called a \mathcal{K} -function (b) For a function $\beta(s, t)$, $\beta(s, \cdot)$ is \mathcal{K} -function, $\beta(\cdot, t)$ is decreasing and $\lim_{t \rightarrow \infty} \beta(\cdot, t) = 0$, $\beta(s, t)$ is called a \mathcal{KL} -function. (c) If a function $\alpha(s)$ is \mathcal{K} -function and $\lim_{s \rightarrow \infty} \alpha(s) = \infty$, $\alpha(s)$ is called a \mathcal{K}_∞ -function.

Definition 2. (a) A system (1) is said to be input-to-state stable if there is a \mathcal{K} -function $\gamma(\cdot)$ and \mathcal{KL} -function $\beta(\cdot)$, such that, for each $u \in L_\infty$, i.e., $\sup \{\|u(k)\|\} < \infty$, and each initial state $x^0 \in \mathbb{R}^n$, it holds that

$$\|x(k, x^0, u(k))\| \leq \beta(\|x^0\|, k) + \gamma(\|u(k)\|)$$

(b) A smooth function $V: \mathbb{R}^n \rightarrow \mathbb{R} \geq 0$ is called a ISS-Lyapunov function for system (1) if there is \mathcal{K}_∞ -functions $\alpha_1(\cdot)$, $\alpha_2(\cdot)$ and $\alpha_3(\cdot)$, and \mathcal{K} -function $\alpha_4(\cdot)$ such that for any $s \in \mathbb{R}^n$, each $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$

$$\begin{aligned} \alpha_1(s) &\leq V(s) \leq \alpha_2(s) \\ V_{k+1} - V_k &\leq -\alpha_3(\|x(k)\|) + \alpha_4(\|u(k)\|) \end{aligned}$$

These definitions imply that for the nonlinear system (1), the following are equivalent: a) it is ISS; b) it is robustly stable; c) it admits a smooth ISS-Lyapunov function.

3 Recurrent Fuzzy CMAC Neural Networks

In order to identify the nonlinear system (1), we use a recurrent fuzzy CMAC (RFCMAC). This network can be presented in Fig. 1.

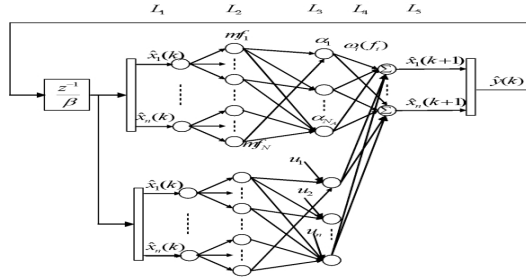


Fig. 1. Recurrent fuzzy CMAC neural networks

This network can be divided into five layers Input Layer (L_1), Fuzzified Layer (L_2), Fuzzy Association Layer (L_3), Fuzzy Post-association Layer (L_4) and Output Layer (L_5) and β is a scale constant, $\beta > 0$. The input Layer transfers input $x = (x_1, x_2, \dots, x_n)^T$ to the next layer $mf_i = x_i$, $i = 1, \dots, n$, n is the number of input variables. Each node at Fuzzified Layer corresponds to a linguistics variable which are expressed by membership functions $\mu_{A_j^i}$, there are m quantizations (membership functions) for each input. The number of the nodes in this layer is n^m . Fuzzified Layer accomplishes the fuzzification of input variables. And it corresponds to both sensor layer of CMAC and fuzzifier of fuzzy logic controller. Fuzzy Association Layer connects fuzzified layer and accomplishes the matching of precondition of fuzzy logic rule. Each node at this layer completes fuzzy implication operation (flo) to obtain firing strength $\alpha_j = \pi(x) flo\{mf_i(x_1), \dots, mf_n(x_n)\}$. If we use product rule for flo ,

$$\alpha_k = \prod_{i=1}^n \lambda_k(\mu_{A_j^i})$$

where k is association times, $k = 1 \cdots l$, l is association number, λ is the selection vector of association memory which is defined as

$$\lambda_k \left(\mu_{A_j^i} \right) = \mu_{A_{j,k}^i} = [0, 0 \cdots 1, 0 \cdots] \begin{bmatrix} \mu_{A_1^i} \\ \vdots \\ \mu_{A_m^i} \end{bmatrix}$$

where $i = 1 \cdots n$. Fuzzy post-association layer will calculate the normalization of firing strength and prepare for fuzzy inference,

$$\bar{\alpha}_k = \alpha_k / \sum_{k=1}^l \alpha_k = \left(\prod_{i=1}^{N_i} \mu_{A_j^i} \right) / \left(\sum_{j=1}^{N_A} \prod_{i=1}^{N_i} \mu_{A_j^i} \right)$$

In the output layer, Takagi fuzzy inference will be used, that is, consequence of each fuzzy rule is defined as a function of input variables (with control input)

$$\begin{aligned} R^j \text{ IF } \hat{x}_1 \text{ is } A_j^1 \cdots \text{ and } \hat{x}_n \text{ is } A_j^n \text{ THEN } \beta \hat{x}_1(k+1) \text{ is } f_1(x_1, x_2, \dots, x_n) \\ \text{ IF } \hat{x}_1 \text{ is } A_j^1 \cdots \text{ and } \hat{x}_n \text{ is } A_j^n \text{ THEN } \beta \hat{x}_1(k+1) \text{ is } f_2(x_1, x_2, \dots, x_n) u \end{aligned}$$

The output of the recurrent fuzzy CMAC can be expressed in a vector notation as

$$\begin{aligned} \beta \hat{x}(k+1) &= \sum_{i=1}^l w_{1,i} \varphi_{1,i}[x(k)] + \sum_{i=1}^l w_{2,i} \varphi_{2,i}[x(k)] u(k) \\ \text{or } \beta \hat{x}(k+1) &= W_1^T \varphi_1[x(k)] + W_2^T \varphi_2[x(k)] U(k) \\ \hat{y}(k) &= \hat{x}(k+1) \end{aligned} \quad (2)$$

where w_k plays the role of connective weight, W_j ($j = 1, 2$) is adjustable weight values, $\varphi_j(x)$ is base function defined as

$$\varphi_k = \frac{\prod_{i=1}^n \lambda_k \left(\mu_{A_j^i} \right)}{\sum_{k=1}^l \prod_{i=1}^n \lambda_k \left(\mu_{A_j^i} \right)}$$

We use l ($k = 1 \cdots l$) times to perform association from an input vector $X = [x_1, \cdots, x_n] \in \mathbb{R}^n$ to an output linguistic y . Each input variable x_i ($i = 1 \cdots n$) has m quantizations.

4 System Identification Via RFCMAC with Stable Learning

We assume the base function φ_k of CMAC is known, only the weights need to be updated for system identification. We will design a stable learning algorithm such that the output $\hat{y}(k)$ of recurrent fuzzy CMAC neural networks (2) can

follow the output $y(k)$ of nonlinear plant (1). Let us define identification error vector $e(k)$ as

$$e(k) = \hat{y}(k) - y(k)$$

According to function approximation theories of fuzzy logic and neural networks [14], the identified nonlinear process (1) can be represented as

$$\begin{aligned} \beta x(k+1) &= Ax(k) + W_1^{*T} \varphi_1[x(k)] + W_2^{*T} \varphi_2[x(k)] U(k) + \nu(k) \\ y(k) &= x(k+1) \end{aligned} \quad (3)$$

Where W_1^* and W_2^* are unknown weights which can minimize the unmodeled dynamic $\nu(k)$. The identification error can be represented by (2) and (3),

$$\beta e_p(k+1) = Ae_p(k) + \widetilde{W}_1(k) \varphi_1[x(k)] + \widetilde{W}_2^T \varphi_2[x(k)] U(k) - \nu(k) \quad (4)$$

where $\widetilde{W}_1(k) = W_1(k) - W_1^*$, $\widetilde{W}_2(k) = W_2(k) - W_2^*$. In this paper we are only interested in open-loop identification, we assume that the plant (1) is bounded-input and bounded-output (BIBO) stable, i.e., $y(k)$ and $u(k)$ in (1) are bounded. By the bound of the base function φ_k , $\nu(k)$ in (3) is bounded. The following theorem gives a stable gradient descent algorithm for fuzzy neural modeling.

Theorem 1. *If the Recurrent Fuzzy CMAC neural network (2) is used to identify nonlinear plant (1) and the eigenvalues of A is selected as $-1 < \lambda(A) < 0$, the following gradient updating law without robust modification can make the identification error $e(k)$ bounded (stable in an L_∞ sense)*

$$\begin{aligned} W_1(k+1) &= W_1(k) - \eta(k) \varphi_1[x(k)] e^T(k) \\ W_2(k+1) &= W_2(k) - \eta(k) \varphi_2[x(k)] U(k) e^T(k) \end{aligned} \quad (5)$$

where $\eta(k)$ satisfies

$$\eta(k) = \begin{cases} \frac{\eta}{1 + \|\varphi_1\|^2 + \|\varphi_2 U\|^2} & \text{if } \beta \|e(k+1)\| \geq \|e(k)\| \\ 0 & \text{if } \beta \|e(k+1)\| < \|e(k)\| \end{cases}$$

$0 < \eta \leq 1$.

Proof. Select Lyapunov function as

$$V(k) = \|\widetilde{W}_1(k)\|^2 + \|\widetilde{W}_2(k)\|^2$$

where $\|\widetilde{W}_1(k)\|^2 = \sum_{i=1}^n \widetilde{w}_1(k)^2 = \text{tr} \left\{ \widetilde{W}_1^T(k) \widetilde{W}_1(k) \right\}$. From the updating law (5)

$$\widetilde{W}_1(k+1) = \widetilde{W}_1(k) - \eta(k) \varphi_1[x(k)] e^T(k)$$

So

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) = \|\widetilde{W}_1(k) - \eta(k) \varphi_1 e(k)^T\|^2 - \|\widetilde{W}_1(k)\|^2 \\ &+ \|\widetilde{W}_2(k) - \eta(k) \varphi_2 U(k) e^T(k)\|^2 - \|\widetilde{W}_2(k)\|^2 \\ &= \eta^2(k) \|e(k)\|^2 \|\varphi_1\|^2 - 2\eta(k) \left\| \varphi_1 \widetilde{W}_1(k) e^T(k) \right\| \\ &+ \eta^2(k) \|e(k)\|^2 \|\varphi_2 U(k)\|^2 - 2\eta(k) \left\| \varphi_2 U(k) \widetilde{W}_2(k) e^T(k) \right\| \end{aligned}$$

There exist a constant $\beta > 0$, such that

If $\|\beta e(k+1)\| \geq \|e(k)\|$, using (4) and $\eta(k) \geq 0$,

$$\begin{aligned}
 & -2\eta(k) \left\| \varphi_1 \widetilde{W}_1(k) e^T(k) \right\| - 2\eta(k) \left\| \varphi_2 U(k) \widetilde{W}_2(k) e^T(k) \right\| \\
 & \leq -2\eta(k) \left\| e^T(k) \right\| \left\| \beta e(k+1) - Ae(k) - \nu(k) \right\| \\
 & = -2\eta(k) \left\| \begin{matrix} e^T(k) \beta e(k+1) \\ -e^T(k) Ae(k) - e^T(k) \nu(k) \end{matrix} \right\| \\
 & \leq -2\eta(k) \left\| e^T(k) \beta e(k+1) \right\| + 2\eta(k) e^T(k) Ae(k) + 2\eta(k) \left\| e^T(k) \nu(k) \right\| \\
 & \leq -2\eta(k) \|e(k)\|^2 + 2\eta(k) \lambda_{\max}(A) \|e(k)\|^2 + \eta(k) \|e(k)\|^2 + \eta(k) \|\nu(k)\|^2
 \end{aligned}$$

Since $0 < \eta \leq 1$

$$\begin{aligned}
 \Delta V(k) & \leq \eta^2(k) \|e(k)\|^2 \|\varphi_1\|^2 + \eta^2(k) \|e(k)\|^2 \|\varphi_2 U(k)\|^2 \\
 & - \eta(k) \|e(k)\|^2 + 2\eta(k) \lambda_{\max}(A) \|e(k)\|^2 + \eta(k) \|\nu(k)\|^2 \\
 & = -\eta(k) \left[\frac{(1 - 2\lambda_{\max}(A)) \|\varphi_1\|^2 + \|\varphi_2 U(k)\|^2}{1 + \|\varphi_1\|^2 + \|\varphi_2 U(k)\|^2} \right] e^2(k) \\
 & + \eta_k^2 \nu^2(k) \leq -\pi e^2(k) + \eta \nu^2(k)
 \end{aligned} \tag{6}$$

where $\pi = \frac{\eta}{1 + \kappa} \left[1 - 2\lambda_{\max}(A) - \frac{\kappa}{1 + \kappa} \right]$, $\kappa = \max_k \left(\|\varphi_1\|^2 + \|\varphi_2 U(k)\|^2 \right)$.

Since $-1 < \lambda(A) < 0$, $\pi > 0$

$$n \min(\tilde{w}_i^2) \leq V_k \leq n \max(\tilde{w}_i^2)$$

where $n \times \min(\tilde{w}_i^2)$ and $n \times \max(\tilde{w}_i^2)$ are \mathcal{K}_∞ -functions, and $\pi e^2(k)$ is an \mathcal{K}_∞ -function, $\eta \nu^2(k)$ is a \mathcal{K} -function, so V_k admits the smooth ISS-Lyapunov function as in *Definition 2*. From *Theorem 1*, the dynamic of the identification error is input-to-state stable. The "INPUT" is corresponded to the second term of the last line in (6), i.e., the modeling error $\nu(k)$, the "STATE" is corresponded to the first term of the last line in (6), i.e., the identification error $e(k)$. Because the "INPUT" $\nu(k)$ is bounded and the dynamic is ISS, the "STATE" $e(k)$ is bounded.

If $\beta \|e(k+1)\| < \|e(k)\|$, $\Delta V(k) = 0$. $V(k)$ is constant, $W_1(k)$ is constant. Since $\|e(k+1)\| < \frac{1}{\beta} \|e(k)\|$, $\frac{1}{\beta} < 1$, $e(k)$ is bounded.

Remark 1. The condition " $\eta(k) = 0$ if $\beta \|e(k+1)\| < \|e(k)\|$ " is dead-zone. If β is selected big enough, the dead-zone becomes small.

5 Simulations

We will use the nonlinear system which proposed [12] and [13] to illustrate the training algorithm for recurrent fuzzy CMAC. The identified nonlinear plant is

$$\begin{aligned}
 x_1(k+1) &= x_2(k) \\
 x_2(k+1) &= x_3(k) \\
 x_3(k+1) &= \frac{x_1(k)x_2(k)x_3(k)u(k)[x_3(k)-1]+u(k)}{1+x_2(k)^2+x_3(k)^2} \\
 y(k) &= [x_1(k), x_2(k), x_3(k)]^T
 \end{aligned}$$

The input signal is selected the same as [12][13](training)

$$u(k) = \begin{cases} 0.8 \sin(\frac{2\pi}{25}k) + 0.2 \sin(\frac{2\pi}{10}k) & k \leq 200 \\ \sin(\frac{2\pi}{25}k) & k > 200 \end{cases} \quad (7)$$

We use the following recurrent fuzzy CMAC neural networks to identify it, see Fig.1. The quantization $m = 10$. The association times $l = 10$. We use 1000 data to train the model, the training input is used as in (7), then we use another 1000 data to test the model ($u(k) = \frac{1}{2} \cos(\frac{t}{35}) + \frac{1}{2} \sin(\frac{t}{10})$). The identification results are shown in Fig. 2 and Fig. 3.

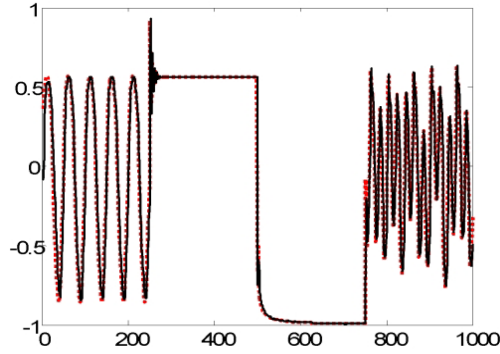


Fig. 2. RFCMAC training

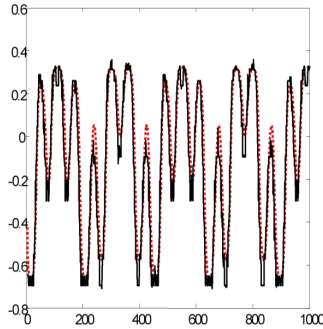


Fig. 3. RFCMAC testing

Now we compare our algorithm with normal fuzzy CMAC neural networks [2]. The training rule is (5). The identification results are shown in Fig.44 and Fig. 5. We can see that compared to normal fuzzy CMAC neural networks, recurrent fuzzy CMAC neural networks can model nonlinear system with more accuracy. By the training algorithm proposed in this paper, the convergence speed is faster than the normal one.

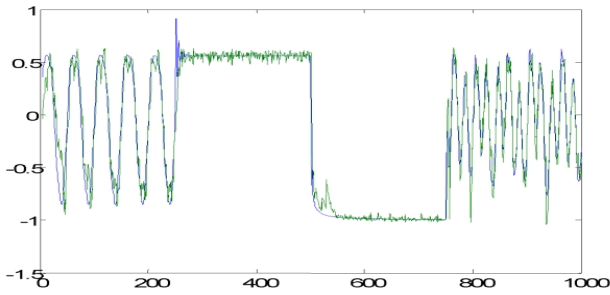


Fig. 4. Normal FCMAC training

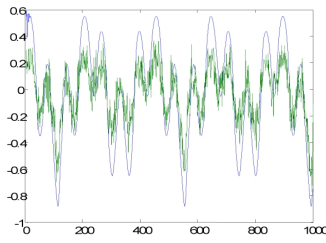


Fig. 5. Normal FCMAC testing

6 Conclusion

In this paper we propose a new CMAC structure for system identification and a simple training algorithm for recurrent fuzzy CMAC. The new stable algorithms with time-varying learning rates are stable. Further works will be done on structure training and adaptive control. Also an FPGA real-time implementation will be tested.

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