# Chapter #5
# Internal Memory

# Error Detection & Correction

- Detection:
  - — Find one or more erroneous bits
  - — Report & resend data
- Correction:
  - —Find one or more erroneous bits
  - —Report and fix the error(s)
- Methods:
  - —Parity bit code
  - —Hamming code

# Types of Errors

- Hard failure
  - —Permanent defect
  - —Eg. broken wire, burned transistor
- Soft Error
  - —Random, non-destructive
  - —No permanent damage to memory
  - —Eg. Noise on bus, weak signal (weak 0, weak 1)

# Parity Bit Code

- Bit appended to data to determine even/odd number of 1's in data
- Parity bit sent is compared to parity bit received to determine possible error
- Types:
  - —Odd parity: parity bit is set to 1 if total number of 1's in data is even, 0 otherwise
  - —Even parity: parity bit is set to 1 if total number of 1's in data is odd, 0 otherwise
- Capability:
  - —Detect odd number of errors
  - —Correct no errors

# Hamming Code

- Length of code:

  $M$ original data bits + $K$ parity bits

- Relation between $K$ and $M$:

  $K \geq \lceil \log_2 (M + \lceil \log_2 M \rceil + 1) \rceil$

- Capability:

  —Detect multiple number of errors
  (2 errors always, >2 errors sometimes)

  —Correct one error

# Example Hamming Code Lengths

| Data bits (M) | Check bits (K) | Increase |
|---|---|---|
| 8 | 4 | 50% |
| 16 | 5 | 31.25% |
| 32 | 6 | 18.75% |
| 64 | 7 | 10.94% |
| 128 | 8 | 6.25% |
| 256 | 9 | 3.52% |

# Hamming Code Implementation
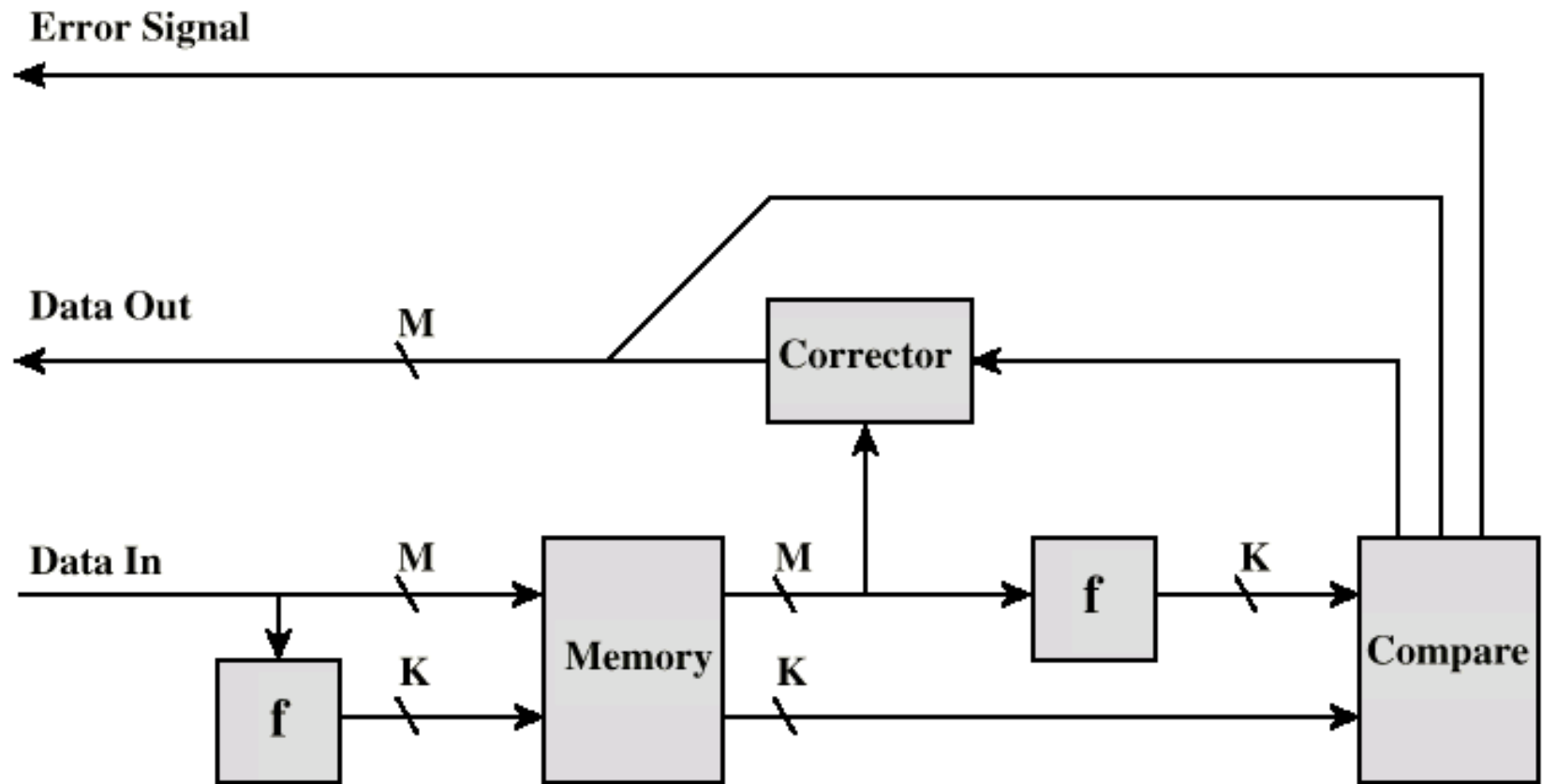
# Hamming Code w/ 5 parity bits

- Hamming code:

  16 original data bits:    $D_{16}$ to $D_1$
  5 check bits:             $P_{16}$, $P_8$, $P_4$, $P_2$, $P_1$
  21 code bits:             $C_{21}$ to $C_1$

- Recalculate parity bits:

  $P_1' = C_3 \oplus C_5 \oplus C_7 \oplus _{...} \oplus C_{21}$ (every other 1)
  $P_2' = C_3 \oplus C_6 \oplus C_7 \oplus _{...} \oplus C_{19}$ (every other 2)
  $P_4' = C_5 \oplus C_6 \oplus C_7 \oplus _{...} \oplus C_{21}$ (every other 4)
  $P_8' = C_9 \oplus C_{10} \oplus C_{11} \oplus _{...} \oplus C_{15}$ (every other 8)
  $P_{16}' = C_{17} \oplus C_{18} \oplus C_{19} \oplus C_{20} \oplus C_{21}$ (every other 16)

- Compare original & new parity bits (difference is bit in error):

  | | $P_{16}$ | $P_8$ | $P_4$ | $P_2$ | $P_1$ |
  |---|---|---|---|---|---|
  | $\oplus$ | $P_{16}'$ | $P_8'$ | $P_4'$ | $P_2'$ | $P_1'$ |