

$$\begin{aligned} & \wedge \forall n \in G_NODES : nodeRcvdAcks[n] \subseteq (G_NODES \setminus \{n\}) \\ & \wedge nodeLastWriteTS \in [G_NODES \rightarrow [version : 0..G_MAX_VERSION, \\ & \quad tieBreaker : G_NODES]] \\ & \wedge nodeTS \in [G_NODES \rightarrow [version : 0..G_MAX_VERSION, \\ & \quad tieBreaker : G_NODES]] \\ & \wedge nodeState \in [G_NODES \rightarrow \{\text{"valid"}, \text{"invalid"}, \text{"invalid_write"}, \text{"write"}\}] \end{aligned}$$

The consistent invariant: all alive nodes in valid state should have the same value / TS

$$GConsistent \triangleq$$

$$\begin{aligned} \forall k, s \in G_NODES \quad : \quad & \forall nodeState[k] \neq \text{"valid"} \\ & \forall nodeState[s] \neq \text{"valid"} \\ & \forall nodeTS[k] = nodeTS[s] \end{aligned}$$

$$GInit \triangleq \text{The initial predicate}$$

$$\begin{aligned}
\wedge \text{ msgs} &= \{\} \\
\wedge \text{ nodeRcvdAcks} &= [n \in G_NODES \mapsto \{\}] \\
\wedge \text{ nodeState} &= [n \in G_NODES \mapsto \text{"valid"}] \\
\wedge \text{ nodeTS} &= [n \in G_NODES \mapsto [\text{version} \mapsto 0, \\
&\quad \text{tieBreaker} \mapsto \\
&\quad \text{CHOOSE } k \in G_NODES : \\
&\quad \quad \forall m \in G_NODES : k \leq m]] \\
\wedge \text{ nodeLastWriteTS} &= [n \in G_NODES \mapsto [\text{version} \mapsto 0, \\
&\quad \text{tieBreaker} \mapsto \\
&\quad \text{CHOOSE } k \in G_NODES : \\
&\quad \quad \forall m \in G_NODES : k \leq m]]
\end{aligned}$$

$$\begin{aligned}
& g_upd_state(n, newVersion, newTieBreaker, newState, newAcks) \triangleq \\
& \quad \wedge \textit{nodeRcvdAcks}' = [\textit{nodeRcvdAcks} \quad \text{EXCEPT } ![n] = newAcks] \\
& \quad \wedge \textit{nodeState}' = [\textit{nodeState} \quad \text{EXCEPT } ![n] = newState] \\
& \quad \wedge \textit{nodeTS}' = [\textit{nodeTS} \quad \text{EXCEPT } ![n].version = newVersion, \\
& \hspace{10cm} ![n].tieBreaker = newTieBreaker] \\
& \quad \wedge \textit{nodeLastWriteTS}' = [\textit{nodeLastWriteTS} \text{ EXCEPT } ![n].version = newVersion, \\
& \hspace{10cm} ![n].tieBreaker = newTieBreaker]
\end{aligned}$$

$$g_send_inv_or_ack(n, newVersion, newTieBreaker, msgType) \triangleq \\ \wedge \quad send([type \mapsto msgType, \\ sender \mapsto n, \\ version \mapsto newVersion, \\ tieBreaker \mapsto newTieBreaker])$$

$$\begin{aligned} g_actions_for_upd(n, newVersion, newTieBreaker, newState, newAcks) \triangleq \\ \wedge \quad g_upd_state(n, newVersion, newTieBreaker, newState, newAcks) \\ \wedge \quad g_send_inv_or_ack(n, newVersion, newTieBreaker, "INV") \end{aligned}$$

$$\begin{aligned}
GRead(n) &\triangleq \text{Execute a read} \\
&\wedge nodeState[n] = \text{"valid"} \\
&\wedge \text{UNCHANGED } \langle msgs, nodeTS, nodeState, nodeRcvdAcks, nodeLastWriteTS \rangle \\
\\
GWrite(n) &\triangleq \text{Execute a write} \\
&\wedge nodeState[n] = \text{"valid"} \\
&\wedge nodeTS[n].version < G_MAX_VERSION \quad \text{Only to configurably terminate the model checking} \\
&\wedge g_actions_for_upd(n, nodeTS[n].version + 1, n, \text{"write"}, \{\}) \\
\\
GRcvAck(n) &\triangleq \text{Process received Ack} \\
&\exists m \in msgs : \\
&\quad \wedge m.type = \text{"ACK"} \\
&\quad \wedge m.sender \neq n \\
&\quad \wedge m.sender \notin nodeRcvdAcks[n] \\
&\quad \wedge equalTS(m.version, m.tieBreaker, \\
&\quad \quad nodeLastWriteTS[n].version, \\
&\quad \quad nodeLastWriteTS[n].tieBreaker) \\
&\quad \wedge nodeState[n] \in \{\text{"write"}, \text{"invalid_write"}\} \\
&\quad \wedge nodeRcvdAcks' = [nodeRcvdAcks \text{ EXCEPT } ![n] = \\
&\quad \quad \quad nodeRcvdAcks[n] \cup \{m.sender\}] \\
&\quad \wedge \text{UNCHANGED } \langle msgs, nodeLastWriteTS, nodeTS, nodeState \rangle \\
\\
GSendVals(n) &\triangleq \text{Send validations once acknowledgments are received from all alive nodes} \\
&\wedge nodeState[n] \in \{\text{"write"}\} \\
&\wedge receivedAllAcks(n) \\
&\wedge nodeState' = [nodeState \text{ EXCEPT } ![n] = \text{"valid"}] \\
&\wedge send([type \mapsto \text{"VAL"}, \\
&\quad \quad version \mapsto nodeTS[n].version, \\
&\quad \quad tieBreaker \mapsto nodeTS[n].tieBreaker]) \\
&\wedge \text{UNCHANGED } \langle nodeTS, nodeLastWriteTS, nodeRcvdAcks \rangle \\
\\
GCoordinatorActions(n) &\triangleq \text{Coordinator actions for reads or writes} \\
&\vee GRead(n) \\
&\vee GWrite(n) \\
&\vee GRcvAck(n) \\
&\vee GSendVals(n)
\end{aligned}$$

$$\begin{aligned}
GRcvInv(n) &\triangleq \text{Process received invalidation} \\
&\exists m \in msgs : \\
&\quad \wedge m.type = \text{"INV"} \\
&\quad \wedge m.sender \neq n \\
&\quad \text{always acknowledge a received invalidation (irrelevant to the timestamp)}
\end{aligned}$$

$$\begin{aligned}
& \wedge \text{send}([type \mapsto \text{"ACK"}, \\
& \quad \quad \quad sender \mapsto n, \\
& \quad \quad \quad version \mapsto m.version, \\
& \quad \quad \quad tieBreaker \mapsto m.tieBreaker]) \\
& \wedge \text{IF } greaterTS(m.version, m.tieBreaker, \\
& \quad \quad \quad nodeTS[n].version, nodeTS[n].tieBreaker) \\
& \quad \text{THEN } \wedge nodeTS' = [nodeTS \text{ EXCEPT } ![n].version = m.version, \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad ![n].tieBreaker = m.tieBreaker] \\
& \quad \quad \wedge \text{IF } nodeState[n] \in \{\text{"valid"}, \text{"invalid"}\} \\
& \quad \quad \quad \text{THEN} \\
& \quad \quad \quad \quad nodeState' = [nodeState \text{ EXCEPT } ![n] = \text{"invalid"}] \\
& \quad \quad \quad \text{ELSE} \\
& \quad \quad \quad \quad nodeState' = [nodeState \text{ EXCEPT } ![n] = \text{"invalid_write"}] \\
& \quad \text{ELSE} \\
& \quad \quad \text{UNCHANGED } \langle nodeState, nodeTS \rangle \\
& \wedge \text{UNCHANGED } \langle nodeLastWriteTS, nodeRcvdAcks \rangle
\end{aligned}$$

$$\begin{aligned}
GRcvVal(n) &\triangleq \text{Process received validation} \\
&\exists m \in msgs : \\
&\quad \wedge nodeState[n] \neq \text{"valid"} \\
&\quad \wedge m.type = \text{"VAL"} \\
&\quad \wedge equalTS(m.version, m.tieBreaker, \\
&\quad \quad \quad nodeTS[n].version, \\
&\quad \quad \quad nodeTS[n].tieBreaker) \\
&\quad \wedge nodeState' = [nodeState \text{ EXCEPT } ![n] = \text{"valid"}] \\
&\quad \wedge \text{UNCHANGED } \langle msgs, nodeTS, nodeLastWriteTS, nodeRcvdAcks \rangle
\end{aligned}$$

$$\begin{aligned}
GFollowerActions(n) &\triangleq \text{Follower actions for writes} \\
&\vee GRcvInv(n) \\
&\vee GRcvVal(n)
\end{aligned}$$

$$\begin{aligned}
GNext &\triangleq \text{Coordinator and Follower actions} \\
&\exists n \in G_NODES : \\
&\quad \vee GFollowerActions(n) \\
&\quad \vee GCoordinatorActions(n)
\end{aligned}$$

$$G_Spec \triangleq GInit \wedge \Box[GNext]_{gvars}$$

$$\text{THEOREM } G_Spec \Rightarrow (\Box GTypeOK) \wedge (\Box GConsistent)$$
